Time dependent risk assessment of structures by efficient sampling techniques

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ABSTRACT. The problem of failure (damage) risk assessment of structures is considered for the case when the relevant structural parameters are random and the loading parameters are modeled as stochastic processes. The component and subsystem failures are represented in terms of flow networks, and a technique for orientated sampling is adapted so as to fit with the former models.

1 INTRODUCTION

Several ways to represent the component and subsystem failures in a structural or NPP system are available, ranging from Boolean decompositions (including the minimal cutsets) to certain types of networks. Many of them become hardly manageable for large and complex systems. Some models for failure events are discussed in Section 2. A procedure for converting a fault / success tree into a (flow) network is presented. It offers and appropriate "logical" (or functional) support for defining a probabilistic / stochastic model for time invariant and time dependent reliability problems. Formal connections with the failure domains in the space of random parameters that characterize the uncertain behaviour of structures under stochastic load effects are presented in Section 3, which also includes an algorithm for building time-dependent failure domains corresponding to the cutsets in a flow network that models a system. Finally, techniques for the time dependent risk assessment are presented in Section 4, making use of the models developed in Sections 2 & 3. An adapted algorithm of orientated sampling is presented. The main references giving the basis for our approach were [1], [2], [3], [4].

2 FAULT TREES AND FLOW NETWORKS FOR FAILURE EVENT DESCRIPTION

Assume that a system $S$ to be analyzed consists of $m$ components. The failure of a component $i$ is assume to be an "elementary" failure event and we denote it by $F_i \ (1 \leq i \leq m)$. Such an event is assumed to be not further decomposable into "smaller" events. Depending on the logical structure of the system (or subsystem in an NPP), the failure of the system can be written as a union of intersections (or an intersection of unions) of component events:

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\[ F_S = \bigcup_{i,j} F_{ij} \]  
\[ \text{or} \quad F_S = \bigcap_{k} \bigcup_{i} F_{ki} \]  
(2.1)  
(2.2)

where \( F_{ij} \) is the failure of component \( j \) in the "failure mode" \( i \), while \( F_{ki} \) is the failure of component \( k \) within the chain \( i \) of components (as a series subsystem). In the PSA of NPPs (under the "common cause failure" models) it is assumed that the failure of the system occurs when at least \( k \) out of the \( m \) components fail. This event is denoted by \( F_{k/m} \). For a large/complex system, the number of events of this type may become very large and a selection is therefore necessary. If the logical structure of the system is represented by a fault tree \( T \) with the associated network \( N \), only the events corresponding to minimal cutsets in \( N \) have to be taken into account. Efficient techniques exist for identifying the minimal cutsets in a tree \( T \) / network \( N \).

Let \( C \) be the set of such cutsets: \( C = \{ c_j : j = 1, 2, \ldots, J \} \), with \( 1 \leq J \leq \sum_k \binom{m}{k} \). Then the failure of system \( S \) can be represented in two ways:

\[ F_S = F_{k/m} = \bigcup_{j=1}^{J} \bigcap_{i=1}^{m} F_{ij} = \bigcup_{j=1}^{J} \bigcap_{i \in c_j} F_i \]  
(2.3)

where \( F_{ij} \) has been previously explained, while \( F_i \) stands for the failure of component \( i \). The second representation in (2.3) is obviously consistent with the cutset approach. It may be turned into a Boolean expression with \( \bigcup \) replaced by a Boolean sum \( \Sigma \) and \( \bigcap \) replaced by a Boolean product \( \Pi \). Thus, the second expression of \( F_s \) in (2.3) becomes

\[ F_s = \sum_{j=1}^{J} \prod_{i \in c_j} F_i \]  
(2.4)

A widely used representation tool, especially for the PSA of NPP systems, makes use of the so-called fault trees. We have not yet found a formal definition for a general F-tree, but it is known that the structure of such a tree can be described as follows. An F-tree has a finite number of levels \( L_0, L_1, \ldots, L_N \). Each level consists of a set of events \( E_{ij} \), immediately followed by a gate that can be either an AND gate \( \bigcap \) or an OR gate \( \bigcup \). Several arcs exit down from each gate or — alternatively — a simple arborescence (branching) exits with its branches connected to the events at the lower level \( L_{i+1} \). The events can be either composite (marked by a box) or elementary, that is component failure events (encircled). All the leaves (terminal nodes) of an F-tree \( T \) are component failure events. They may occur at various levels in \( T \). Sometimes, a specification occurs inside an AND gate, e.g., \( \geq 3/4 \) (see, as a typical example, Figure 4-2 of [5]). Let us try to formalize the structure of a general F-tree \( T \).

\[ T = (E \cup T E, U) \]  
(2.5)

where \( E \cup T E = \bigcup_{l=0}^{N-1} \bigcap_{j=1}^{n_l} E_{ij} \), \( E \cup T E = \bigcup_{l=0}^{N-1} \bigcap_{j=1}^{m_l} F_{ij} \), and \( U \) is the set of arcs (or edges) of \( T \), with \( n_l = \) the number of \( E \)-nodes on level \( l \), while \( m_l = \) the number of "terminal events" (i.e., component failures) on level \( l \). Every arc in \( U \) joins a gate on level \( l \) to an
level \( \ell \) to an event on level \( \ell + 1 \). In other words, every arc in \( T \) is of the form
\[
\mathbf{u} = (G_{\ell,j}, E_{\ell+1,k}), \quad \text{or} \quad \mathbf{u} = (G_{\ell,j}, F_{\ell+1,k}),
\]
where \( F_{\ell+1,k} \) stands for the failure of component \( k \) on level \( l \) in \( T \). The "top node" \( E_{G_0} \) in \( T \) corresponds to the top event consisting in the failure of the (sub)system under analysis.

A cutset in an F-tree \( T \) can be defined as a subset of "terminal events" whose lowest dominant node in \( T \) is just the top event represented by \( E_{G_0} \). Certainly, the events under an AND gate should fail all together, while only one of the events under an OR gate has to fail for going up in \( T \).

We give an example of an F-tree in Fig.1, associated to a complex system (similar to one given in [6]) that consists of eight interfacing components \( A, \ldots, G \). The system is defined as available (it operates) if any series of components (from \( A \) to \( G \)) is operable.

The letters \( (A, B, \ldots) \) in circles represent the failure of the respective components. The events denoted by subscripted \( E \)'s are complex events, each of them coupled with an AND / OR gate. According to the assumption of the system availability it will fail when no series of components from \( A \) to \( G \) will remain operable. A "success sequence" for the system is modeled by a partial arborescence in the "success tree" of the system, which is nothing else than the fault tree with the OR and AND gates interchanged. It is easy to identify the three success sequences corresponding to the F-tree in Fig.1. We give them by a Boolean expression:
\[
S = A B D G H + A C E G H + A C F G H
\]
where \( A \) means that this component does not fail (an so on). The failure of a component, say \( A \), is denoted by \( \bar{A} \). and we can write down the nine minimal cutsets for this system:
\[
\begin{align*}
c_1 &= \bar{A}, \\
c_2 &= \bar{B} \bar{C}, \\
c_3 &= \bar{C} \bar{E}, \\
c_4 &= \bar{D} \bar{E} \bar{F}, \\
c_5 &= \bar{C} \bar{G}, \\
c_6 &= \bar{D} \bar{E} \bar{F},
\end{align*}
\]
\[ c_3 = \overline{E} \overline{F}, \quad c_7 = \overline{E} \overline{F} \overline{G}, \]
\[ c_4 = \overline{C} \overline{D}, \quad c_8 = \overline{G}, \quad c_9 = \overline{H}. \]

A (flow) network \( N \) with labeled arcs can be rather easily associated with a fault / success tree \( T \). The network corresponding to the system in our example is

![Diagram](image)

Fig.2 The labeled network corresponding to the F-tree \( T \) in Fig.1

Each arc is labeled by the symbol denoting the non-failure event of a component. This simple example shows that the existence of multiple arcs between two nodes is possible, but these arcs should wear distinct labels; in our example, \( E \) and \( F \) on the two arcs linking nodes 4 and 5.

The general algorithm for getting a labeled network \( N_f = (X, U^*, \lambda) \) from an F-tree \( T = (EG \cup TE, U) \), consists in defining how the nodes and arcs of \( N \) are built.

**Step 1.** A function \( \nu : EG \rightarrow X = \{1, 2, \ldots, \text{card} X\} \) is defined by \( \nu (EG_0) = 1 \); and the other nodes together with the arcs in \( U^* \) are built in Steps 2, 3 & 4.

**Step 2.** A CF (context free) grammar \( G(T) \) is built, with a rule corresponding to each \( E \)G-node in \( T \); the \( G_{t,j} \) nodes are mapped on the nonterminal symbols of \( G(T) \), while the component nodes in \( TE \) correspond to the terminal symbols. The symbols in the r.h.s. of each rule are connected by + if \( G_{t,j} \) is an OR gate, and they are simply juxtaposed when it is an AND gate. The symbols under an AND gate are separated by a line |, meaning that the alternatives around it are both acceptable.

**Step 3.** Each + in the r.h. sides of \( G(T) \) is replaced by an integer \( 2, 3, \ldots, \text{card} X \), such that the earlier numbered \( E \)-nodes keep the same node number at left.

**Step 4.** The grammar \( G(T) \) generates a language over \( TE \cup X \), with the node symbols 1 and \( \text{card} X + 1 \) respectively preceding and succeeding each string in \( L[G(T)] \). The network \( N \) is assembled from this set of strings that represent \( TE \)-labeled paths. The function \( \nu \) is built in Step 3, and the labeling function \( \lambda : U^* \rightarrow TE \) (that assigns a component symbol to each arc in \( N \)) is uniquely determined by the strings in \( L[G(T)] \).

Let us illustrate this algorithm by means of the earlier example, given by the F-tree \( T \) in Fig.1. We do not give the rules of \( G(T) \), but only the three possible derivations:

\[ EG_0 \rightarrow A \ 2 \ 2 \ A \ 2 \ E \ G_1 \rightarrow A \ 2 \ 2 \ E \ G_2 \rightarrow A \ 2 \ 2 \ B \ 3 \ D \ 4 \ 2 \ G_3 \rightarrow A \ 2 \ 2 \ B \ 3 \ D \ 4 \ G \ 6 \ H; \]

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\[ EG_0 \rightarrow A \ 2 \ EG_1 \rightarrow A \ 2 \ EG_3 \rightarrow A \ 2 \ C \ 5 \ EG_4 \ 4 \ EG_5 \rightarrow A \ 2 \ C \ 5 \ E \ | \ F \ 4 \ EG_5 \rightarrow \]

\[ \begin{cases} A \ 2 \ C \ 5 \ E \ 4 \ G \ 6 \ H \\ A \ 2 \ C \ 5 \ F \ 4 \ G \ 6 \ H \end{cases} \]

The resulting "language" with the three TE-labeled paths consists of

\[ 1 \ A \ 2 \ B \ 3 \ D \ 4 \ G \ 6 \ H \ 7, \ 1 \ A \ 2 \ C \ 5 \ E \ 4 \ G \ 6 \ H \ 7, \ 1 \ A \ 2 \ C \ 5 \ F \ 4 \ G \ 6 \ H \ 7. \] (2.8)

Hence \( X = \{1, 2, \ldots, 7\} \) and the network \( N \) of Fig.2 can be easily retrieved from these paths.

3 STOCHASTIC FLOW NETWORKS AND CUTSET RISK EVALUATION

The evaluation of the probability of failure / unavailability for a complex system (either structural or in an NPP) which was previously modeled by a fault tree \( T \) or a corresponding network \( N \) obviously needs a way to associate probabilities to the basic (component) events, together with an assumed structure of dependence among them or among subsets of F-events (for instance cutsets). Such a probabilistic / stochastic structure may be available from statistical evidence and/or from engineering judgement, based upon the study of behaviour of certain classes of systems during extended periods of time.

If a network \( N_s = (X, U^*, \lambda) \) models the failure logic of the system, with the set of its arc \( U^* \) in 1-to-1 correspondence with the set of components (or basic events) \( TE = \{F_1, F_2, \ldots, F_m\} \), a failure-probability function \( \pi : U^* \rightarrow [0, 1] \) or a reliability function \( R \) have to be (respectively) defined as

\[
(\forall u_t \in U^*) \pi(u_t) = q_t \quad (\forall u_t \in U^*) \ R(u_t) = p_t = 1 - q_t.
\] (3.1)

Alternatively (as in [6]), a 0-1 valued random variable

\[ X_t = \begin{cases} 0 & 1 \\ q_t & p_t \end{cases} \ (1 \leq i \leq m), \] (3.2)

can be assigned to each component / arc in \( N \). It takes value 0 when component \( i \) fails and 1 otherwise. With these "indicator" variables, it is very easy to characterize the survival / failure along a certain path in \( N \):

\[
\prod_{t=1}^{l} X_t = 1 / \prod_{t=1}^{l} X_t = 0
\] (3.3)

for a complete path (from source to sink) in \( N \) of the form \((u_{t_1}, u_{t_2}, \ldots, u_{t_l})\). Clearly, the random variables \( X_t \) are assigned to each component \( i \) or to each arc \( u_t \in U^* \). We extend the random function \( X \) from arcs to cutsets:

\[ X : C \rightarrow X(C), \ X(c_j) = \sum_{t \in c_j} X_t. \] (3.4)

It is clear that the system will fail due to cutset \( c_j \) if and only if

\[
\sum_{t \in c_j} X_t = 0, \quad \text{with} \quad q(c_j) = \prod_{t \in c_j} q_t = \text{not} \ Q_j.
\] (3.5)
The system availability / failure can be respectively characterized by

\[
S : \prod_{j=1}^{J} X(c_j) > 0 / F_S : \prod_{j=1}^{J} X(c_j) = 0,
\]  

(3.6)

with the corresponding probabilities

\[
P(S \text{ available}) = \sum_{j=1}^{J} (1 - Q_j) - \sum_{j,k} [1 - q(c_j \cap c_k)] + \ldots
\]  

(3.7)

\[
P(F_S) = \sum_{j=1}^{J} Q_j - \sum_{j,k} q(c_j \cap c_k) + \ldots
\]  

(3.8)

The \(q\)-probabilities of the cutsets and intersections thereof are given by (3.5) provided the component failures are independent. Otherwise the correlation between \(X_i\)'s has to be taken into account.

Eq.(3.8) is numerically illustrated with the data Table 1:

\[\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{Comp.} & A & B & C & D & E & F & G & H \\
\hline
q_i & .02 & .04 & .03 & .05 & .07 & .04 & .03 & .05 \\
\hline
\text{Cutset} & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \\
\hline
Q_j & 2-2 & 12-4 & 112-6 & 15-4 & 9-4 & 140-6 & 84-6 & 3-2 & 5-2 \\
\hline
\end{array}\]

From these data, Eq.(3.8) with intersections of up to three events in the second sum gave a system failure probability \(P(F_S) \approx 103,936 - 6 - 1,948 - 6 = 101,988 - 6 = 0.101988\).

According to the classical reliability theory, the failure probability of a system or subsystem is evaluated as a probabilistic measure of a failure domain \(D_j\) in the space of design parameters, the U-space. The boundary of \(D_j\) consists of several failure surfaces, each of them characterized by a limit state function \(g_k(U) = 0\). Thus, the F-domain can be written as

\[
D_j = \bigcap_k \{U : g_k(U) \leq 0\}.
\]  

(3.9)

Once the set \(C\) of the cutsets for system \(S\) identified, a risk measure should be evaluated for each minimal cutset \(c_j \in C\). This depends on the F-domain associated with every \(c_j \in C\). For a time-dependent problem, the failure domain will depend on \(t : D_j(t)\). This dependence can be analytically expressed either by adding the time variable to the vector \(U\) under \(g_k\), or by letting the limit state functions themselves to depend on \(t\). Under both approaches, the boundary of \(D_j(t)\) will depend on time. Such a situation (for a 2-dimensional vector \(U\)) is described in Fig.2. Since all (or part of) the parameters in \(U\) are random / stochastic, the resulting load effect on component \(i\) within the cutset \(c_j\) will be
defined as a stochastic process \( L_j(t) \) for \( t \in [0, T] \), with \( L \) depending on the limit state functions, which are now time dependent: \( g_j(U(t)) \) with \( U \) - the random vector of the (resistance and load) parameters that are relevant to the failure mode identified by the cutset \( c_j \). In fact, this vector is partitioned as \( U(t) = [R^T \ S^T(t)] \) where \( R \) stands for the random vector of resistance parameters while \( S(t) \) is the stochastic loading vector process. Consequently, the failure domain is characterized by

\[
D_j(t) = \bigcap_{l \in c_j} \{ g_{U}(U(t)) < 0 \} \tag{3.10}
\]

where \( g_{U} \) is the limit-state function of component \( i \) within the cutset \( j \).

4 RISK ASSESSMENT BY ORIENTATED SAMPLING

The evaluation of the failure risk of a system with random strength parameters subjected to stochastic (time-variant) loads is not a simple problem. For one-dimensional stochastic processes, the number of outcrossings is analyzed by G.I.Schueler in [1]. The \( n \)-dimensional stochastic processes are considered, e.g., in [6]. If the cutset \( c_j \) consists of (exactly) \( k \) failures among \( m \) potentially failing components, we adopt a formula from the latter reference for evaluating its probability:

\[
P[c_j, t] = \left[ \frac{\int_{D_j} \int_{0}^{t} w(x, \tau) d\tau \ dx}{(k-1)!} \right]^{k-1} \exp \left[ -\int_{0}^{t} f(\tau) d\tau \right] f(t) \tag{4.1}
\]

where \( w(x, \tau) \) is the joint pdf of \( X(t) \) and \( f(t) \) is the joint pdf of \( S(t) \). The integral in Eq.(4.1) gives the probability that all the failure events in cutset \( c_j \) occur until time \( t \).

These probabilities can be evaluated under specific assumptions on the pdfs involved and using selected sampling techniques. Among the latter ones, the orientated simulation seems to be rather efficient (as argued by Puppo & Bertero in [7]. It selects the randomly generated point inside / outside of \( D_j(t) \cap H_\star \), where \( H_\star \) is a hypercone in the \( n \)-space of polar coordinates. The proposed technique to assess the time dependent failure risk consists in replacing the time independent
limit state (see Eq. (7) of [7]), by time-dependent ones, \( g_x(U,t) \). The failure probability will be estimated (as in FORM method) by an integral of the form

\[
P(F_x) = \int_{0}^{T} \int_{D_f} f_U(U,t) du dt
\]  

(4.2)

where \( f_U(U,t) \) is the time-dependent probability density function of the random vector \( U \). The interior integral in (4.2) may depend on \( t \) not only due to the \( t \)-dependent pdf but also through the (possibly) \( t \)-dependent \( D_f \). Next, the \( U \)-parameters are transformed into the \( Z \)-space of the polar coordinates, and integral (4.2) is correspondingly reformulated and evaluated. Certainly, distributional assumptions should be selected for the vector \( Z \). The time dependence can be taken into account in terms of the mean point \( m = E[Z] = m(t) \) and covariance / correlation matrix \( \text{cov}_Z(t) / \text{corr}_Z(t) \). The adapted algorithm of orientated sampling starts with generating a \( t \)-dependent random point from (e.g.) a normal pdf \( \zeta(t) \), and then the points \( z_i(t) \) are checked for belonging to \( D_f(t) \cap \delta_z \), where \( \delta_z \) is an angular domain around the direction of \( z \). The algorithm outputs a distance \( \beta \) to \( \partial D_f \), that is a reliability index involved in computing \( R(t) = 1 - p_f(t) \).

REFERENCES


