Ratcheting in cylindrical pipes due to an axially oscillating sharp temperature front

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ABSTRACT

When hot fluid within a pipe moves cyclically along its axis, the pipe is differentially heated, and, so, experience varying amounts of thermal strain. If the temperature is high enough, plastic strains may also be produced. This plastic strain then accumulates over several cycles leading to the phenomenon known as thermal ratcheting. Considering the elevated temperatures found within nuclear reactors where the working fluid is often hot sodium, thermal ratcheting leads to appreciable shape change that adversely affects the pipe’s performance. Here, we consider two different types of heat transfer scenarios, and conclude that this has an important impact on the ultimate outcome of a shakedown analysis. Further, we consider computationally the possibility of the pipe shaking down and its relation to different heat transfer models.

INTRODUCTION

Ratcheting is the accumulation of plastic strain in a component subjected to cyclic loading. There are, broadly, two types: (a) material ratcheting and (b) structural ratcheting. The former is caused due to material constants during loading and unloading not being the same. This may thus occur under uniform stress distribution, e.g., uniaxial-stress controlled ratcheting with mean stress, or biaxial ratcheting such as shear-strain controlled ratcheting with constant tensile stress. The latter type of ratcheting occurs due to a stress gradient. Thermal ratcheting due to cyclically moving high-temperature fluid falls under this category.

In the past, thermal ratcheting was associated quintessentially with the system first identified and analyzed by Bree [1]. He investigated the elasto-plastic behavior of a thin tube when a radially varying temperature field is imposed on top of a uniform internal pressure. The pressure’s presence made the loading and unloading unsymmetric leading to an accumulation of plastic strain, i.e., ratcheting. He primarily employed a uniaxial model and for the most ignored work hardening and creep. Later, Bree [2] extended his previous work by employing a biaxial model that takes into account the effect of the stress in the tube’s axial direction.

We consider the system shown in Fig. 1. Hot fluid, shaded in gray, oscillates along the axial direction. After a cycle the pipe deforms as shown. This deformation accumulates over several cycles. More complicated deformed shapes are also possible. This phenomenon of accumulated plastic strains due to cyclic thermal loads is termed thermal ratcheting. Furthermore, plastic strains can continue to increase with number of cycles.

The system of our interest (Fig. 1) was first investigated by Goodman [3]. He assumed an elastic perfectly-plastic pipe that is subjected to an axially oscillating infinitely sharp temperature front, i.e., the temperature field has a jump. Asymmetry during loading and unloading was introduced by assuming different heat transfer coefficients in the hot and cold regions. He showed that hot front leads to an inward ratchet, while the cold front causes the outward ratchet. He estimated the extent of ratcheting strain per cycle and also showed that under his assumed conditions, the system shakes down. This latter information is perhaps of most use from the point of view of design.

Lee et al. [4] evaluated the ratcheting behavior of a thin-walled 304 SS pressurized cylinder under an axially moving temperature distribution using elastic-plastic constitutive theory of Chaboche [5]. They implemented this constitutive model in ABAQUS as a user subroutine UMAT and showed that residual thermal deformation was continuously accumulated over number of cycles.

Kobayashi and Ohno [6] discussed the effect of kinematic hardening by applying the finite element analysis. They investigated four different hardening rules: (a) perfectly-plastic model (PP), (b) the linear kinematic hardening rule (LKH), (c) the classical non-linear kinematic hardening rule of Armstrong and Fredrick [7], and (d) the Ohno and Wang (OW) rule [8,9]. They neglected isotropic hardening, cyclic hardening and the temperature dependence of all...
material parameters. They concluded that thermal ratcheting in the current system depends strongly on the kinematic hardening rule employed.

Lee et al. [10] studied progressive inelastic deformation in 316L stainless steel cylinder under a moving temperature distribution, and compared with ABAQUS driven finite element simulations. The thermal load was applied by heating the test cylinder up to 550°C and cooled nine times. The temperature distribution in axial direction was measured. The Chaboche-Rousselier non-linear combined hardening model of isotropic and kinematic hardening was implemented in ABAQUS as a user-defined UMAT subroutine. After nine thermal load cycles, experimental and computational results were found to be comparable.

![Fig. 1: Thermal ratcheting due to cyclic moving high temperature fluid](image)

Here, we investigate the effect of heat transfer models on the predicted outcome of a ratcheting analysis of the structure shown in Fig. 1. We will also be interested in probing how sensitive is shakedown to the manner in which heat transfer is modeled. To this end, we will model the material as an isotropic elastic perfectly-plastic material that obeys a von Mises yield criterion, and consider two very different heat transfer models.

**SETUP**

We simulate a Steel SS 304 pipe subjected to an oscillating temperature front as shown in Fig. 1 and 2(b). The geometric dimensions of the pipe are radius \( R = 170 \text{ mm} \), height \( H = 150 \text{ mm} \) and thickness \( t = 1 \text{ mm} \). The material and thermal properties for use in an appropriate perfect-plasticity model are provided in Tables 1 and 2, respectively.

<table>
<thead>
<tr>
<th>E (GPa)</th>
<th>( \nu )</th>
<th>( \sigma_Y ) (MPa)</th>
<th>( \alpha (^\circ\text{C}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>169</td>
<td>0.30</td>
<td>154</td>
<td>18.54 ( \times 10^6 )</td>
</tr>
</tbody>
</table>

| Heat transfer coefficient between steel and air | 28 W/ m\(^2\)/°C |
| Heat capacity | 500 J/kg \(^\circ\text{C}\) |
| Thermal conductivity | 16.3 W/m\(^2\)/°C (0-100 °C) and 21.4 W/m\(^2\)/°C (100-500 °C) |
| Density | 8030 kg/m\(^3\) |

Because the pipe has two axes of symmetry, only one-fourth of the pipe is considered as the domain for analysis; see Fig. 2(a). The mechanical boundary conditions are as follows:
**Surface GCDH:** Due to X-Z plane of symmetry, the normal component of the displacement vector and both the shear components of the traction vector vanish on this boundary. Thus, the natural boundary conditions are $T_{YZ}=0$, $T_{YX}=0$, while the essential boundary condition is that the circumferential displacement $U_Y = 0$.

**Surface FEAB:** Similarly, $T_{XZ}=0$, $T_{XY}=0$ and $U_X=0$.

**Surface FGCB:** We consider that material points on this surface are prevented from any axial motion, though they may move transversely without restriction. Thus, the normal component of the displacement vector and both the shear components of the traction vector vanish on this boundary, and we have the natural boundary condition $T_{ZX}=0$, $T_{ZY}=0$, while the essential boundary condition is $U_Z=0$.

**Surfaces FGHE, CDAB and EHDA:** These are free surfaces, so that on these surfaces the tractions vanish.

The thermal cycle consists of four stages; see Fig. 2(b):

**Stage 1 (thermal loading):** At the beginning of each cycle, the cylinder is at the ambient temperature of $T_0$. Then temperature increases by $\Delta T$ in the region $z \leq z_l$ to form a temperature front at $z = z_l$. This is taken to correspond to the introduction of the hot fluid into the pipe. The temperature is assumed to decrease linearly from $T_0 + \Delta T$ to $T_0$ over a length $\Delta z$. This is done to avoid numerical issues.

**Stage 2:** The temperature front moves up by a distance of $z_u - z_l$. This corresponds to the hot fluid’s progressive motion along the pipe.

**Stage 3:** The temperature front comes back by the same distance $z_u - z_l$. This corresponds to the lowering of the hot fluid’s level.

**Stage 4 (thermal unloading):** The pipe’s temperature decreases to the initial uniform temperature distribution of $T_0$. This corresponds to the hot fluid being removed from the pipe.

Here, we take $T_0 = 25 \, ^\circ C$, $\Delta T = 250 \, ^\circ C$, $\Delta z = 5 \, mm$, $z_l = 50 \, mm$ and $z_u - z_l = 50 \, mm$.

Finally, we describe the two different heat transfer models that we consider:

**Instant conduction:** This mode assumes that the heat transfer to the pipe from the fluid and the ambient air is instantaneous. Furthermore, while the pipe does not conduct along its axis, it is thin enough to allow sudden radial heat transfer, i.e., radial gradients in temperature are ignored. This will introduce some ambiguity in those regions where the pipe’s inner surface contacts the hot fluid, but the outer surface is open to air. This is resolved by taking the heat transfer coefficient with the fluid to be far greater than the corresponding one with the ambient. This approximates the case when the pipe’s thermal conductivity is far less than all other heat transfer coefficients. Thus, when the hot fluid moves up, the temperature of the pipe that comes in direct contact increases instantly by $\Delta T$, and when hot fluid recedes, the pipe’s temperature decreases immediately by $\Delta T$, i.e. there is instant heating and cooling.

![Fig. 2: (a) Domain considered for analysis. (b) Cylindrical pipe and temperature distribution in one cycle.](image)
**Finite conduction:** In this case, heat transfer between the fluid and the pipe’s inner surface is taken to be infinite. However, pipe’s thermal conductivity, as well as, its ambient heat transfer coefficient is comparable. Furthermore, radial conduction is not instantaneous. Thus, the full heat transfer problem needs to be solved. This is done computationally in ABAQUS assuming the original undeformed geometry, so that the temperature field’s time history may be resolved prior to computing the stresses and strains. The hot fluid is taken to traverse along the pipe’s axis at the rate of 1 mm/s.

**COMPUTATION**

A user subroutine UTEMP was written in the finite element package ABAQUS to simulate potentially large deformation thermo-elasto-plastic problem. As mentioned previously, the material was assumed perfectly-plastic and isotropic. In ABAQUS, we employed C3D8R type element, i.e. an 8-noded linear brick element with reduced integration. Full Newton solution technique is invoked to solve the non-linear equations. Only 1/4th of the specimen is considered for analysis because of symmetry. The mesh contained four elements in the radial direction, 50 elements in the circumferential direction and 300 elements in along the axis. Thus, we employed 60000 (4 x 50 x 300) elements with 76755 nodes.

![Graph (a)](image1)

![Graph (b)](image2)

![Graph (c)](image3)

![Graph (d)](image4)
Fig. 3: Temperature distribution along axial direction of the pipe at 1 mm/sec for both the cases (a) just after thermal loading when the hot fluid’s front is at $z = 50$ mm, (b) when the hot fluid’s front is at $z = 75$ mm, (c) when the front proceeds to $z = 100$ mm, (d) when the front recedes to $z = 75$ mm, (e) when the front returns to $z = 50$ mm, (f) and (g) at various times during thermal unloading for the two heat transfer models.
Fig. 4: Change in circumferential stress distributions at the middle surface, i.e., at $r = R + t/2$, with the thermal load cycle (a) just after thermal loading when the hot fluid’s front is at $z = 50$ mm, (b) when the hot fluid’s front is at $z = 75$ mm, (c) when the front proceeds to $z = 100$ mm, (d) when the front recedes to $z = 75$ mm, (e) when the front returns to $z = 50$ mm, and (f) just after thermal unloading.
Fig. 5: Change in radial displacement with the thermal load cycle (a) just after thermal loading when the hot fluid’s front is at z = 50 mm, (b) when the hot fluid’s front is at z = 75 mm, (c) when the front proceeds to z = 100 mm, (d) when the front recedes to z = 75 mm, (e) when the front returns to z = 50 mm, and (f) just after thermal unloading.

Fig. 6: Change in von-Mises stress with equivalent plastic strain (a) over 30 thermal load cycles, employing the instant conduction model for heat transfer, and (b) over 15 thermal load cycles with the finite conduction heat transfer model.

DISCUSSION AND CONCLUSION
Figures 3 plots the temperature distribution along the axial direction of the pipe’s middle surface \( r = R + t / 2 \) for the cases of finite heat transfer and instant heat conduction. We note the markedly different temperature responses during the thermal unloading of the two different heat transfer models. This, as we will see below, impacts greatly the stress displacement response.

Figures 4 and 5 plot, respectively, the circumferential (hoop) stress \( \sigma_{\theta\theta} \) and the radial displacement at various stages of the hot fluid’s motion over one cycle, i.e., one thermal load cycle. Results derived from both heat transfer models are shown.

From Fig. 4(a), we observe that there is, initially, little difference in the hoop stress computed based on temperature fields obtained from either of the two heat transfer models, except that the section between \( z = 0 \text{mm} \) to \( z = 30 \text{mm} \) experiences more tensile stress when heat transfer takes place at a finite rate because of a radial temperature gradient. We observe the effect of finite heat transfer rate in the axial direction in Figs. 4(b) and 4(c) wherein the cylinder is under less tensile stress ahead of the front; recall that in instant conduction there is, in fact, no axial heat transfer, so that the pipe experiences greater hoop stress ahead of the front; cf. temperature fronts Figs. 3(b) and 3(c).

Finally, Figs. 4(d)-(f) show the very different distributions of circumferential stress as the hot fluid’s level lowers. This exemplifies the significant effect different heat transfer models have on the stress distribution.
Similarly, in Figs. 5(a)-(c) there is very little difference between the outcomes due to the two heat transfer models. However, in Figs. 5(d) and 5(e), there is an appreciable mismatch in the predicted radial displacements, and, in fact, post-unloading, in Fig 5(f), we see a significant change in ultimate radial displacement profile; far more thermal ratcheting is observed when the more accurate finite heat transfer model is employed.

Figure 6 displays the von Mises stress as a function of the equivalent plastic strain over several thermal load cycles. We observe from Fig. 6(a) that the pipe shakes down within 15 thermal load cycles if the instant conduction model for heat transfer is invoked. However, Fig. 6(b) shows that if the potentially more accurate finite heat transfer model is followed, shakedown is not observed. In this context we mention the results of Goodman [3] who confirmed shakedown of this system employing the same material model, provided that the heat transfer involved no heat conduction in the pipe’s axial direction, no heat transfer with the ambient air and instantaneous heat transfer with the hot fluid. This heat transfer model has some elements of both models considered above, but is obviously not as accurate the finite heat conduction model. This confirms the importance of accurately modeling heat transfer: systems that are predicted to shake down with an approximate heat transfer model may, in fact, not shake down.

REFERENCES


