CRACK RESISTANCE CURVES FOR THIN WALLED TUBES: A FRACTURE MECHANICS APPROACH

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ABSTRACT

Tubes with wall thickness (t) less than 1mm and with a large radius to wall thickness ratio (r/t) may be categorized as ‘thin walled tubes’. Such tubes are extensively used in nuclear industry as components. Because of various mechanisms, these components are susceptible to axial cracking. For safety analysis of such tubular components, knowledge about fracture mechanics parameters such as shape function \([f(a/W)]\) for finding stress intensity factor (SIF) and other functions such as \(\eta\) and \(\gamma\) for calculation of elastic-plastic fracture parameters such as J integral for axial cracking is essential. Due to limitations stemming from the geometry of the thin walled tubes, it is impossible to directly estimate the parameters for such tubes by standard tests. Grigoriev et. al. developed a new technique known as pin loading tension test (PLT) in which a machined section of such tube itself is used for fracture toughness testing. Because of the similarity of loading technique between the PLT method and the standard compact tension (CT) method, it has a potential for characterizing fracture behavior for such tubes. For fracture behavior assessment of any component with a non-standard method, experiment vis-à-vis computational work is essential. Although in recent past, a few attempts are noticed to use the PLT method for characterizing fracture behavior of both virgin tubes and delayed hydride cracking, attempts to assess the \(f(a/W)\), \(\eta\) and \(\gamma\) functions are limited. These functions may not be unique unlike CT specimens as there is one more new variant, i.e. tube diameter (d). The current work involves employment of the PLT test method for deriving the crack resistance curve for a few cladding tubes with different geometries upon finding the necessary parameters such as \(f(a/W)\) \(\eta\) and \(\gamma\) functions by experiment and finite element method (FEM) analysis. Attempts are also made to assess effect of tube geometry on the derived functions. Experiments are done for \(f(a/W)\) and J-R curve determination using PLT method upon fabricating specimens and loading jigs for all tubes. \(f(a/W)\) is determined for all tubes using two independent analysis methods and the results are compared. FEM analysis is performed for all tubes to find the SIF expressions and \(f(a/W)\) using displacement extrapolation method. A limit load analysis is also made to find \(\eta\) and \(\gamma\) functions for all the tubes. The results are compared. Load normalization method is adapted for finding J-R curves for the tubes using the derived \(f(a/W)\), \(\eta\) and \(\gamma\) functions. The results obtained are compared with the results published for standard specimens of similar materials. \(f(a/W)\) for different thin walled tubes are derived both experimentally and \(\eta\) and \(\gamma\) functions are also derived using FEM and compared for all tubes. The effect of tube geometry on the derived functions is assessed. Using the derived functions, crack resistance curve for the tubes are determined and compared with published results.

INTRODUCTION

The axial failure of thin-walled tubes (wall thickness (t) ≤ 1mm << tube diameter (D)) like fuel claddings made of Zircaloy has been a long standing concern of the nuclear industry and has received much attention recently in the research literature [1-5]. The classical fracture mechanics procedure (e.g., ASTM standard test techniques [6]) cannot be applied directly to the thin-walled cladding geometry. This is because of the presence of large plastic zone sizes (compared to the thickness) at the axial crack tips of the cladding and hence it violates the assumptions of linear elastic fracture mechanics (LEFM). Curve compact tension (CCT) specimens, as fabricated for thick walled pressure tubes [2] cannot be fabricated from the fuel cladding as the cladding wall, much thinner than the pressure tube, does not meet the requirements of plane strain condition. During testing, the bending effects resulting from the cladding specimen curvature could also complicate the stress field around the crack tip. Hence, data generated for pressure tube material cannot be scaled down for cladding tubes. Again, reshaping of the tubes by flattening in order to make standard flat compact tension (CT) specimen may lead to uncertainty due to change in microstructure and residual stress distribution [7]. Because of the above limitations, investigation related to fracture toughness assessment of cladding tubes for nuclear fuel is challenging.
Pin-loading tension (PLT) test, proposed by Grigoriev et al. [7] to evaluate the fracture toughness of Zircaloy-2 fuel claddings has the advantage of strain localization at a side of the specimen thereby providing better accuracy in displacement measurement while compared with standard uniaxial tension test [Fig. 1(a,b&c)]. Also, the analogy between PLT and standard CT specimen makes it suitable for testing thin walled tubular components in a standard fashion [Fig. 1(c)]. Samal et al. [8] have already derived the shape functions for the SIF expression regarding the axial splitting of cladding tube used in Indian Boiling Water Reactors (BWR) by detailed PLT experiment and finite element method (FEM) and compared the result with reported data [9].

In the current work, a comprehensive experimental program has been undertaken to study the response of PLT specimens of varying diameter, thickness and material properties with different initial crack lengths. The shape functions necessary for evaluation of the stress intensity factors have been derived using experimental data and FE analysis. The effect of tube geometry on shape functions is also critically discussed to some extent. A limit load analysis has also been done for finding $\eta$ and $\gamma$ functions necessary for calculation of $J_p$ through experiment and using those functions J-R curves for select tubes have been derived using both single specimen load normalization approach and multiple specimen approach.

MATERIALS, THE SPECIMENS, LOADING JIGS AND FIXTURES

In total, seven thin walled tubes with different geometry and history are considered in the current study. Table 1 elaborates the tubes’ geometry, the material, application, and other details. The detailed design of the specimen has been discussed in the earlier works [7-9]. The schematic diagram of the specimen (along with the dimensions) obtained through electro-discharge machining (EDM) from the cladding tube is shown in Fig.1 (d). A set of seven specimens [Fig. 1 (e)] with notch lengths ranging 1 to 7 mm is fabricated for each of the tubes 1-6 (Table 1). Additionally, for tubes 3 and 4, specimens with constant notch lengths of 1.5 mm are fabricated for fracture toughness testing. The detailed design of the fixture (Fig. 2) has been discussed in the earlier works [7-9].

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Material</th>
<th>Used as</th>
<th>Inner diameter (mm)</th>
<th>Wall thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube 1</td>
<td>SRA</td>
<td>Zircaloy-4</td>
<td>Cladding tube</td>
<td>12.3</td>
</tr>
<tr>
<td>Tube 2</td>
<td>SRA</td>
<td>Zircaloy-4</td>
<td>Cladding tube</td>
<td>10.8</td>
</tr>
<tr>
<td>Tube 3</td>
<td>SRA</td>
<td>Zircaloy-4</td>
<td>Cladding tube in Indian PHWR</td>
<td>14.2</td>
</tr>
<tr>
<td>Tube 4</td>
<td>RXA</td>
<td>Zircaloy-2</td>
<td>Cladding tube in Indian BWR</td>
<td>12.4</td>
</tr>
</tbody>
</table>
Depending on the inner diameter of the thin walled tubes, seven sets of fixtures are fabricated. The material used for fabrication of the fixture is a martensitic stainless steel of the grade SS403 for tubes 1-4 and 7 (Table 1) whereas it is Nimonic-90 for tube 5 and stainless steel of the grade 17-4PH for tube 6. The assembly of the specimen along with the fixture is shown in Fig. 2(c). Besides these, pairs of clevis grips and studs suitable for coupling the specimen fixture assembly and pins for transmission of load are also fabricated.

**EXPERIMENTAL AND FEM WORK**

**Experiment for determining shape function**

For tubes 1-6 (Table 1), all the seven specimens from each set of the tubes are attached to the respective fixture one by one. While making the specimen-fixture assembly, care is taken to ensure that all the four notches lie on the contact plane of the fixture halves. The specimens are loaded in displacement control mode with a pull rate of 0.2 mm/min with a servo-hydraulic actuator controlled by a 24-bit controller having load frame of 25 kN capacity up to a point where considerable load drop occurs after the peak load. The crack opening displacement (COD) is measured using a clip-on gage and the timed load-COD data are digitally recorded.

From the load-COD (crack opening displacement) response, the best linear part is fitted by linear regression analysis to find the compliance for each specimen for each of the six sets. Also, the notch lengths of the specimens are normalized in the form of $a/W$. The resultant data are analyzed by methods proposed in Ref. [9-11]. The details of the two independent analytical approaches for calculation of the shape function for evaluation of stress intensity factor in the opening mode as described in our earlier work [8].

**FEM for determining SIF, $\eta$ and $\gamma$ functions**

The whole PLT experimental setup (i.e., the tubular specimen as well as the mandrel) has been discretized in 3D domain for finite element analysis. Taking into account of the symmetry in specimen geometry, crack and loading configuration, one-fourth of the test setup has been modeled[Fig. 3(a)]. A prescribed displacement is applied at the end of the mandrel in y-direction and the movement of the loading point is restricted in the x-direction (i.e., direction of length of the tube) as this point is attached to the loading axis of the machine. The un-cracked ligament of the tubular specimen is fixed in y-direction (however, this is free to move in x and z-directions). The line of the loading pin (which acts like a fulcrum) is also fixed in y-direction only. The symmetric plane of the tubular specimen as well as of the mandrel is fixed in z-direction only. We have modeled seven different tubular specimens with different initial crack lengths which vary from 1 mm to 7 mm for SIF calculation for all the seven cases. The tubular specimens are loaded beyond the limit loads of the specimens and hence elasto-plastic finite element
Analysis has been carried out in this work. In the FE discretization, very fine finite elements are used in the crack front in order to simulate the large stress gradient. The sizes of the 3D elements are 0.2×0.2×0.2 mm³ respectively. The FE discretization consists of 20-noded iso-parametric brick elements with 3x3x3 Gauss point integration.

A complete elasto-plastic analysis has been carried for each specimen for applied displacements of the order of 7 mm which are well beyond the limit loads of the specimens. For evaluating the stress intensity factor (SIF), the analysis is carried out for the applied displacement of 0.01 mm in each specimen and the corresponding compliance values are calculated for each value of a/W. We have followed the displacement extrapolation method to calculate the stress intensity factor (SIF), where the SIF can be written as (neglecting higher order terms)

\[
K_i = \frac{E'}{4} \frac{2\pi}{r}' \nu \left(1 - \nu^2\right) \quad \text{(1)}
\]

Where \(K_i\) is stress intensity factor in opening mode of the crack, \(v\) is the opening displacement ahead of crack tip, \(r\) is the distance from the crack tip, \(E' = E/\left(1 - \nu^2\right)\) for plane strain loading condition and \(E' = E\) for plane stress loading condition, \(E\) and \(\nu\) being the Young’s modulus and Poisson’s ratio for the material respectively.

For deriving the geometric functions (\(\eta\) and \(\gamma\)) for evaluation of plastic part of J-integral (\(J_{pl}\)), we require the expression of limit load of the PLT specimens as a function of crack length to width ratio \((a/W)\). The maximum load in the load-displacement diagram corresponds to the limit load of the specimen. This also corresponds to the plastic collapse of the remaining ligament in the specimen. The limit load values are plotted against the crack length to width ratio \((a/W)\) of the specimens and this curve is used for further analysis in the \(\eta\) and \(\gamma\) factor evaluation. For derivation of an analytical expression for the \(\eta\) and \(\gamma\) functions from the expressions of limit load as function of \(a/W\), the material can be assumed to follow elastic-perfectly plastic behavior with a limit load \(F_L\) as the ultimate response of the specimen and further deformation is limited to increasing of the applied displacement at constant load. Hence, the energy utilized for such perfectly plastic deformation \(U_p\) can be obtained as the area under the load-displacement diagram with a constant load \(F_L\). For a given applied displacement, the plastic J-integral, \(J_{pl}\) can be defined as

\[
J_{pl} = \frac{U_p}{A_{uc}} \eta = -\frac{\partial U_p}{\partial P} \quad \text{(2)}
\]

Where \(A\) and \(A_{uc}\) are cracked area and area of un-cracked remaining ligament respectively. From Eq. (23), the expression for the \(\eta\) factor can be derived as [12-13]

\[
\eta = \frac{1 - \frac{a}{W}}{F_L} \frac{1}{\partial (a/W)} \left(\frac{\partial F_p}{\partial a/W}\right) \quad \text{(3)}
\]

The above expression for \(J_{pl}\) is valid for a stationary crack. For a growing crack, the solution for \(J_{pl}\) is corrected with a \(\gamma\) factor, which can be derived as [12-13]
\[ \gamma = \eta - 1 - \left(1 - \frac{a}{W}\right) - \frac{1}{\eta} \frac{\partial \eta}{\partial (a/W)} \]  

(4)

**Experiment for fracture toughness testing**

Prior to the fracture toughness testing, all the specimens having constant notch length of 1.5 mm fabricated from tubes 3 and 4, after coupling with the respective fixture were subjected to fatigue loading to generate a sharp crack of length \(-1.5\) mm ahead of the notch tip [14] in the table-top servo hydraulic actuator. While loading cyclically, the crack size is visually monitored with a traveling microscope and also indirectly, by compliance measurement by insertion of a crack opening displacement (COD) gauge at the knife edge of the fixture halves. The best six pre-cracked specimens having almost equal crack at both sides from each tube are thereafter loaded under displacement control with six different total applied displacement values so as to generate six different extents of crack extensions. Then all the six specimens are heated in air at 300°C for one hour to mark the crack extension and then subjected to fatigue reloading for the final cleave-opening. The crack lengths before and after fracture toughness testing are measured under a stereomicroscope by nine point averaging method. The method of analyzing the data for calculating J-R curve has been described in our earlier work [15].

**RESULTS AND DISCUSSION**

Results for the two analytical approaches for finding shape function from experiment are shown in Fig. 4 (a&b) for all the six tubes. For comparison purpose, the available result for tube7 [9] is also included here. Both the methods give similar values for the above function. The difference in the two approaches lies in the method of obtaining the function of the derivative of compliance with respect to \(a/W\). In approach-1, a function is fitted for the variation of the derivative of compliance with respect to \(a/W\) and used in the calculation of \(f(a/W)\), whereas in approach-2, a function is fitted for the derivative of compliance with respect to \(a/W\) and used in the calculation of \(f(a/W)\). The shape functions obtained for tubes 1-6 are shown in Table 2 for both analytical approaches.

![Fig. 4: (a&b) showing the comparison between various shape functions derived through analysis of experimental data by approach 1 and approach 2 and (c) Comparison of shape functions obtained through experiment and FEM.](image)

**Table-2: The derived expressions for shape functions for all tubes.**

<table>
<thead>
<tr>
<th>Tube</th>
<th>Shape factor expressions for approach 1</th>
<th>Shape factor expressions for approach 2</th>
</tr>
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<tbody>
<tr>
<td>Tube 1</td>
<td>(f(a/W) = 249.61214 - 1189.61338(a/W)) + 1856.91076(a/W)^2 - 878.6484(a/W)^3</td>
<td>(f(a/W) = \frac{0.51\exp(1.38(-\ln(1-a/W))^{0.77}}{\exp(1.38(-\ln(1-a/W))^{0.77}}) + (\frac{\ln(-1-a/W)}{\ln(1-a/W)}) (^{1/2})</td>
</tr>
<tr>
<td>Tube 2</td>
<td>(f(a/W) = 742.49701 - 3331.64268(a/W)) + 4869.3208(a/W)^2 - 2260.6756(a/W)^3</td>
<td>(f(a/W) = \frac{0.48\exp(1.34(-\ln(1-a/W))^{0.77}}{\exp(1.34(-\ln(1-a/W))^{0.77}}) + (\frac{\ln(-1-a/W)}{\ln(1-a/W)}) (^{1/2})</td>
</tr>
<tr>
<td>Tube 3</td>
<td>(f(a/W) = 416.9 - 1903.9(a/W)) + 2847.7(a/W)^2 - 1330.3(a/W)^3</td>
<td>(f(a/W) = \frac{0.49\exp(1.35(-\ln(1-a/W))^{0.77}}{\exp(1.35(-\ln(1-a/W))^{0.77}}) + (\frac{\ln(-1-a/W)}{\ln(1-a/W)}) (^{1/2})</td>
</tr>
<tr>
<td>Tube 4</td>
<td>(f(a/W) = 570.3294 - 2571.32467(a/W)) + 3782.18256(a/W)^2 - 1758.41953(a/W)^3</td>
<td>(f(a/W) = \frac{0.52\exp(1.29(-\ln(1-a/W))^{0.77}}{\exp(1.29(-\ln(1-a/W))^{0.77}}) + (\frac{\ln(-1-a/W)}{\ln(1-a/W)}) (^{1/2})</td>
</tr>
</tbody>
</table>
In order to compare the experimental load-crack opening displacement (COD) data with that obtained from FE analysis, we carry out the elasto-plastic analysis beyond the maximum load carrying capacity of the specimen and a representative result is shown in Fig. 3 (b). It can be seen that the present FE model could very well reproduce the experimental results, not only in the elastic regime, but also in the whole range of experimental load-COD curve. In order to obtain the stress intensity factor, the data of crack opening displacement vs. square root of distance from the crack tip is plotted for all the seven specimens and the resulting data is fitted through straight lines to obtain the constants according to Eq. (1). The SIF obtained at an applied load line displacement of 0.01 mm for all the specimens for tube 3 as a function of a/W is show in Fig. 3(c). The SIF values as function of a/W for the inside, middle and outside surfaces for an applied displacement of 0.01 mm can be fitted with suitable polynomials and are represented as

\begin{align}
K_{\text{inside surface}} &= -256.81 + 1303.78(a/W) - 1887.84(a/W)^2 + 927.12(a/W)^3 \\
K_{\text{middle surface}} &= -226.35 + 992.79(a/W) - 1180.82(a/W)^2 + 469.88(a/W)^3 \\
K_{\text{outside surface}} &= -376.83 + 1634.19(a/W) - 2108.87(a/W)^2 + 926.14(a/W)^3
\end{align}

(5)

The SIF solution will be different at different locations along the crack front (i.e., at the inside, middle and outside surface of the fuel tube). As the inside surface of the crack front is closer to the centre of loading compared to the outside surface, the SIF solution is higher for smaller a/W values and the differences among the results at the three locations along the crack front vanishes as the a/W is increased. However, the SIF solutions obtained from the experimental data provide a single value which can be deemed as the average value across the thickness of the specimen. In order to obtain the shape function, \( f(a/W) \), we use Eq. (2) as the SIF solutions are known as functions of a/W. The results from FE analysis compare very well with those of the analytical solutions [Fig. 4(c)]. The results of the shape functions \( f(a/W) \) for tube 3, for the inside, middle and outside surfaces as a function of a/W can be fitted with suitable polynomials and are represented as

\begin{align}
\begin{aligned}
f(a/W)_{\text{inside surface}} &= -416.48 + 2220.26(a/W) - 3899.27(a/W)^2 + 2330.56(a/W)^3 \\
f(a/W)_{\text{middle surface}} &= -346.58 + 1835.49(a/W) - 3228.79(a/W)^2 + 1957.98(a/W)^3 \\
f(a/W)_{\text{outside surface}} &= -424.62 + 2213.44(a/W) - 3846.07(a/W)^2 + 2296.22(a/W)^3
\end{aligned}
\end{align}

(6)

When the FEM results for tubes 3, 4 and 7 are compared, the following common analytic expressions could be derived to take into account effect of tube diameter on the shape function:

\begin{align}
\begin{aligned}
f\left(\frac{a}{D}, \frac{D}{W}\right) &= -30.2637 + 158.1416 \left(\frac{a}{W}\right) - 273.385 \left(\frac{a}{W}\right)^2 + 162.8713 \left(\frac{a}{W}\right)^3 \\
&+ 23.5347 - 14.4964 \left(\frac{D}{W}\right) - 10.0554 \left(\frac{D}{W}\right)^2 + 5.7543 \left(\frac{D}{W}\right)^3
\end{aligned}
\end{align}

(7)

where \( D \) is nominal or mean diameter, i.e. \( D=(D_i+D_o)/2 \). Using these shape functions, the SIF values in the opening mode of the tubular specimens with any value of \( a/W \) and loaded in the similar condition as prevails in this setup can be determined. These solutions can be further used in structural integrity assessment methods of these types of thin tubular specimens with axial through-wall cracks.
The derived $\eta$ and $\gamma$ functions for tubes 1-4 have been shown in Fig. 5 with the limit load functions. It is found that the functions are not that much sensitive as much shape functions are. In Fig. 6 the J-R curves derived using single specimen and multiple specimen approaches are shown for tubes 3 and 4. Single specimen J-R curves are showing a higher trend than the multiple specimen result in both case as single specimen results are crack growth corrected with the derived $\gamma$ functions. Ample reproducibility in the result is observed.

![Fig. 5: showing (a) limit load, (b) $\eta$ function, and (c) $\gamma$ function for tubes 1-4 obtained through FEM.](image)

![Fig. 6: J-R curves for (a) tube 1 and (b) tube 2.](image)

**CONCLUSIONS**

A new test technique is presented to assess the fracture behavior of thin-walled tubes used as components in nuclear industry. Two analytical approaches are used to obtain the shape function needed for evaluation of stress intensity factor from experimental data. A full elasto-plastic FE analysis has been performed for the whole experimental setup. The results of stress intensity factor solutions obtained from FE analysis has been compared with those obtained from the experiment and the results are found to match well. Also, a common expression can be derived for all tubes to take into account the effect of tube diameter with change in shape function. Using these shape functions, the SIF values in the opening mode of the tubular specimens with any geometry can be derived. The necessary functions for calculation of $J$ from are derived through a thorough limit load analysis with FEM and using the result and load normalization approach, J-R curves for select tubes have been derived.

**REFERENCES**