# GENERAL PLASTIC COLLAPSE LOAD EQUATIONS OF PIPE BEND WITH OR WITHOUT CRACK UNDER IN-PLANE BENDING

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## ABSTRACT

Elbows exhibit highly strained regions and are vulnerable to plastic collapse. It has been observed that available equations in literature for evaluation of plastic collapse load of a pipe bend have limited applicability and do not cover wide range of pipe bend radius ratios and bend angles which are used in power plant piping. Moreover, the elbow collapse load equation should approach to straight pipe collapse load as elbow bend radius increases or bend angle decreases. Generally, the available equations do not satisfy this asymptotic behaviour of an elbow. About 600 number of elastic plastic and geometric nonlinear finite element analyses of elbows having different geometric parameters bend radius, pipe radius, thickness and crack size (in case of cracked elbow), have been performed. For each of the elbow the in-plane plastic collapse moments have been evaluated from M-rotation curve by twice elastic slope (TES) method. Further two weakening factors were defined to quantify the degrees in plastic collapse moment, one due to elbow doubly curved geometry and second due to presence of crack. The geometric weakening factor was evaluated by comparing elbow plastic collapse moment with corresponding straight pipe collapse load while the crack weakening factor was evaluated by comparing cracked elbow plastic collapse moment with corresponding un-cracked elbow plastic collapse moment. The simple equation were developed for these weakening functions which can be used with pipe plastic collapse load equation for prediction of plastic collapse load of elbows.

## INTRODUCTION

Failure resistant design of any structure or component has always been the objective of designer. Pipe bends or elbows being more flexible among other piping components generally are more vulnerable for plastic collapse during the design basis accident events. Hence to prevent pipe bends failing due to plastic instability or by excessive plastic deformations, the integrity assurance needs accurate plastic collapse load information. In recent past, considerable research has been done to develop accurate plastic collapse equations of straight pipe with or without cracks, and to limited extent for pipe bend or elbow with or without cracks. However, presently available plastic collapse load equations for pipe bends have limited range of applicability and generally do not cover wide range of bend geometry, such as different bend angles and bend radius. Ideally, the elbow plastic collapse load should approach to straight pipe plastic collapse load on increasing of pipe bend radius or/and on reducing the pipe bend angle. Similar asymptotic trend is expected for the elbows with circumferential through wall cracks. The plastic collapse load solutions for cracked elbows are required during their integrity demonstration in leak before break design. The presently available plastic collapse equation of pipe bends with or without cracks do not satisfy the above asymptotic tends and have limited applicability in term of bend radius and bend angles of pipe bend.

Keeping above in mind, effort was made to develop equation applicable to any pipe bend, with or without cracks. Finite element analyses of around 600 pipe bends, having different geometric and crack parameters were performed. This included cases on healthy (no crack) elbows and elbow cases with circumferential through wall cracks at intrados (for in-plane opening bending) and at extrados (under in-plane closing bending load) locations. The parameters considered were elbow bend radius ($r_b$) to mean radius (R) ratio ($r_b$/R: 2, 3, 6, 9, 12 and 18) pipe radius (R) to thickness (t) ratio (R/t: 5, 7.5, 10, 15, 20 and 30), pipe bend angle ($\psi$: 30, 45, 60, 90, 135, 150 degrees) and circumferential through wall crack (total angle $2\theta$: 36, 72, 108, 144, 180 degrees). One quarter of elbow was modelled using three dimensional 20 nodded brick elements and elastic perfectly plastic analysis was performed which included the geometric nonlinearity. The plastic collapse load for each case has been evaluated using twice elastic slope method. A relational study was performed to evaluate the elbow geometry weakening factor (W) which is defined as the ratio of plastic collapse load of healthy elbow to plastic collapse load of corresponding pipe (no crack). A crack weakening factor (H), evaluated from the analyses results of the cracked elbow and defined as the ratio of plastic collapse load of cracked elbow to plastic collapse load of corresponding healthy elbow was evaluated. The elbow plastic collapse load can be obtained from corresponding pipe collapse load by multiplying it with applicable weakening factors W and H. The weakening factors W and H dependence on the R/t, $r_b$/R, bend angle, crack angle and elbow characteristic (λ) was studied. Finally simple expressions for above weakening factors, W and H, were developed for closing and opening bending which covered the entire domain of the parameters. These developed equation satisfies the asymptotic trends like when $r_b$/R tends to infinity or/and bend angle tends to zero, the elbow geometry weakening factor (W) approaches to unity. Similarly for cracked elbows, the crack weakening factor (H) tends to corresponding value for straight pipe (h) when $r_b$/R tends to infinity or/and bend angle ψ tends to zero. The developed closed form equations for weakening factors were validated with recent literature equations in their validity range and the goodness of the fitting have also been checked with the FE evaluated values for these factors. They were found in good agreement and mostly within ±5% error band.

## OVERVIEW OF ELBOW PLASTIC COLLAPSE LOAD EQUATIONS

Most of the elbow plastic collapse moment ($M_{CL}^{E}$) equations available in literature, relates it to the corresponding of straight pipe ($M_{CL}^{P}$) through a weakening function ($W_f$). The geometry weakening factor $W_f$, basically quantifies the loss in load carrying capacity of an elbow due to its doubly curved geometry, with respect to corresponding pipe and is defined as below.

$$W_f = \frac{\text{Collpase Load of Elbow}}{\text{Collpase Load of Pipe}} = \frac{M_{CL}^{E}}{M_{CL}^{P}} \tag{1}$$

The $W_f$ depends on the elbow characteristic parameter, λ defined as $r_b t/R^2$. Here $r_b$ is pipe bend radius, R is pipe mean radius and t is pipe wall thickness. The plastic collapse load of elbow may further decrease due to presence of cracks. A crack weakening factor H(θ) may be defined as ratio of the cracked elbow plastic collapse load ($M_{CL}^{CE}$) to the corresponding healthy elbow plastic collapse load ($M_{CL}^{E}$). Hence the equation for elbow plastic collapse load (with or without crack) may be given in following general form.

$$M_{CL}^{CE} = W_f * H(\theta) * M_{CL}^{P} \tag{2}$$

Here, $M_{CL}^{P}$ is plastic collapse load of corresponding pipe. For defect free elbows, H(θ) is equal to 1. The plastic collapse load ($M_{CL}^{P}$) of straight pipe under bending load is given as

$$M_{CL}^{P} = 4 * R^2\, t\, \sigma_y \tag{3}$$

It may be noted that the Eq.3 of pipe PCL is based on thin shell theory. For thick pipe PCL calculation, an additional correction factor is generally used and is given in Zahoor [9]. However, the Eq. 3 is popular and mostly used for pipe PCL calculation and in present study also, this was adopted for normalising the elbow PCL. Hence the $W_f$ and H(θ) in Eq.2 must be used with PCL of pipe evaluated using Eq.3. Here, the material constitutive behaviour is assumed as elastic perfectly plastic and $\sigma_y$ is the yield stress. The crack weakening factor for a through-wall circumferential cracked straight pipe is given as

$$h(\theta) = \mathrm{Cos}(\theta/2) - \frac{1}{2}\mathrm{Sin}(\theta) \tag{4}$$

In past many investigators [1-9] have proposed equations to evaluate plastic collapse load of an elbow under in-plane bending. Most of these equations have been derived from finite element analysis of standard elbows (where bend radius is 1.5 times of pipe diameter) or short radius elbow (where bend radius is equal to pipe diameter). These expression are not general since they does not satisfy following asymptotic behaviour

- For a given pipe section, when the bend radius is increased the collapse load of the elbow must approach to the collapse load of corresponding straight pipe
- For a given pipe section, when the bend angle of the elbow is reduced the collapse load of the elbow must approach to the collapse load of corresponding straight pipe

In other word under above two asymptotic conditions the elbow geometry weakening factor, $W_f$ should approach to 1.0 and the crack weakening factor H(θ) should approach to corresponding pipe h(θ). Below is the summary of plastic collapse equations, geometry weakening factor $W_f$ and crack weakening factor H(θ), developed over year, available in literature. Spence and Findlay [1] obtained expression for in-plane lower bound limit moment of an elbow as

$$W_f = 0.8\lambda^{0.6} \text{ for } \lambda \le 1.45$$
$$= 1.0 \quad \text{ for } \lambda > 1.45 \tag{5}$$

The Section III of ASME Boiler and Pressure Vessel Code, (ASME) [2] allows designing of piping and piping components against plastic collapse by simplified analysis using stress indices and following equation.

$$B_1 \frac{PD_o}{2t} + B_2 \frac{D_o}{2I} M \le 1.5 S_m \quad \text{where } B_2 = 1.3/\lambda^{2/3} \tag{6}$$

Here $B_1$ and $B_2$ are stress indices for internal pressure and bending. For elbows, in absence of internal pressure, the elbow collapse load, based on the ASME $B_2$ stress index (which is equivalent to inverse of weakling factor), are in good agreement with the lower bound limit load given by Spence and Findlay [1]. Calladine [3] has developed in-plane lower bound limit moment as

$$W_f = 0.935\lambda^{2/3} \text{ for } \lambda \le 5 \tag{7}$$

Both the above given theoretical expressions (Eq.6 and Eq.7) were based on small displacement analyses and assume ideal plastic material behaviour. Based on large displacement analysis Goodall [4] proposed the maximum load carrying capacity of the defect free elbow subjected to closing bending moment as

$$W_f = \frac{1.04\lambda^{2/3}}{1+\beta} \text{ for } \lambda \le 5 \text{ Where } \beta = \beta(\lambda, \sigma_o, E, \nu, R/t) \tag{8}$$

Based on experimental investigations, Touboul [5] has proposed different equations for collapse load under in-plane closing bending and in-plane opening bending.

$$W_f = 0.715\lambda^{2/3} \text{ (Closing) and } = 0.722\lambda^{1/3} \text{ (Opening)} \tag{9}$$

Based on large number of finite element analyses, Chattopadhayay [6] has also proposed an equation for collapse moment of elbow considering pipe constraining effect and material as an elastic perfectly plastic and geometry nonlinearity in the analysis.

$$W_f = 1.075\lambda^{2/3} \text{ (Closing) and } W_f = 1.048\lambda^{1/3} - 0.0617 \text{ (Opening)} \tag{10}$$

Recently, Y.J.Kim [7] has given a equation for collapse moment of elbow under closing bending moment based on large displacement analysis and wide range of combinations of $r_b/R$ and R/t but the resulting values of $\lambda$ are limited from $\lambda$=0.1 to $\lambda$=0.5. It was shown that collapse moment not only depends on $\lambda$ but also on R/t ratio.

$$W_f = A_c(\lambda + K_c)^{n_c} \quad \text{Where, } A_c = 0.800(R/t)^{-0.017};\ K_c = 1.460(R/t)^{-0.311};\ n_c = 0.423(R/t)^{0.127} \tag{11}$$

For elbows with circumferential cracks, Miller [8] was the first to propose, the plastic collapse moment equation for a elbow with through-wall circumferential cracks based at extrados subjected to in-plane bending moment, the $W_f$ is used as given by Calladine, eq.5, and H(θ) is given as below

$$H(\theta) = 1 - \frac{3\theta}{2\pi} \tag{12}$$

Zahoor [9] proposed a plastic collapse moments of through wall circumferential cracked elbows under in-plane bending moment where the $W_f$ is used as given by Calladine, and H(θ) as given below

$$H(\theta) = 1 - 0.2137\left(\frac{a}{D}\right) - 0.0485\left(\frac{a}{D}\right)^2 - 1.0559\left(\frac{a}{D}\right)^3 \tag{13}$$

Where a = 0.5*D*θ and D is mean diameter. This has applicability, where $a/D<0.8$, $\lambda<0.5$, and $D/t>15$. Recently Chattopadhayay [6] carried out large number of FE calculations on through wall circumferential cracked elbows and proposed a series of equations for crack weakening function for opening and closing bending loadings for various applicability ranges. For opening bending moment, the H(θ) is given by eq.-14 while for closing bending moment a set of equations for several R/t and crack size applicability ranges was given. The geometry weakening factor $W_f$ is used as given by eq.10

$$H(\theta) = 1 - 0.8\left(\frac{\theta}{\pi}\right) \quad \text{for } 2\theta \le 45^o \quad \text{and} \quad H(\theta) = 1.127 - 1.8108\left(\frac{\theta}{\pi}\right) \quad \text{for } 45^o \le 2\theta \le 150^o \tag{14}$$

It is observed that all above equations have limited applicability and do not cover wide range of bend radius to pipe radius ($r_b/R$) ratios and the elbow bend angles which are used in power plant piping. Moreover, the elbow collapse load equation should approach to corresponding straight pipe collapse load, or in other words, the $W_f$ should approach to 1.0, as elbow bend radius increases or bend angle decreases. None of above equations satisfies this asymptotic behaviour. All equations, except Eq.6 and Eq.9 assume that any two elbows with same elbow characteristic ($\lambda$) will have same $W_f$. Keeping this fact in mind, a series of finite element analyses of elbows having different geometric parameters bend radius, pipe radius and thickness have been performed.

## FINITE ELEMENT ANALYSES

Rigorous elastic plastic finite element analyses of around 600 elbows were carried out covering wide range of elbow geometry parameters such as elbow bend radius ($r_b$) to mean pipe radius (R) ratio and pipe radius (R) to thickness (t) ratio, pipe bend angle ($\psi$), and crack sizes ($\theta/\pi$) for cases of cracked elbows. The geometric details are given in Table-1. It shows that there are several elbows with equal elbow characteristic, $\lambda$, while their R/t and $r_b$/R are different.

Table 1: Geometry details of the elbows analyzed (Mean Pipe Radius, R=250mm)

| | | |
|---|---|---|
| Bend radius, $r_b/R$ | : | 2, 3, 6, 9, 12 and 18<br>Note: R is taken as 250 mm |
| Pipe radius, R/t | : | 5, 7.5, 10, 15, 20 and 30 |
| Bend angle, $\psi$ | : | 30, 45, 60, 90, 135, 150 degrees |
| Crack size, $\theta/\pi$ | : | 0 (healthy or defect free) & 0.1, 0.2, 0.3, 0.4, 0.5 (through wall circumferential) |

| $r_b/r$ → | 2 | 3 | 6 | 9 | 12 | 18 |
|---|---|---|---|---|---|---|
| R/t ↓ | Elbow Characteristics, $\lambda = (r_b/r) / (R/t)$ | | | | | |
| 5 | 0.4 | 0.6 | 1.2 | 1.8 | 2.4 | 3.6 |
| 7.5 | | 0.4 | 0.8 | 1.2 | 1.6 | 2.4 |
| 10 | 0.2 | 0.3 | 0.6 | 0.9 | 1.2 | 1.8 |
| 15 | | 0.2 | 0.4 | 0.6 | 0.8 | 1.2 |
| 20 | 0.1 | 0.15 | 0.3 | 0.45 | 0.6 | 0.9 |
| 30 | | 0.1 | 0.2 | 0.3 | 0.4 | 0.6 |

Due to double symmetry, one quarter of the elbow was modelled using three dimensional 20 noded brick elements with connected pipes with length equal to 3 times pipe diameter on both elbow end in order to account for constraint effect posed by connected piping. The material constitutive behaviour is assumed as elastic perfectly plastic and finite element analysis was performed including the geometric nonlinearity effects. The material's Young's Modulus, yield stress and Poisson's ratio is taken equal to 203GPa, 250 MPa and 0.3 respectively. Figure 1 shows meshes for two typical cases, small bend radius thin elbow ($r_b$/R=2 and R/t=30) and other extra large bend radius thick elbow ($r_b$/R=9 and R/t=5). Pure in plane bending moment has been applied to the end plane of connected straight pipe. Here, it is observed that under closing bending the M-$\phi$ curve becomes almost flat or some time starts decreasing after certain load while under opening bending it continues to rise as the applied moment increases. This is due to the difference in the deformation (ovalization) behaviour of elbow cross-section (see Fig.2) when subjected to closing bending, the elbow geometrically softens and for opening bending, it geometrically hardens. The collapse moments have been evaluated from M-$\phi$ curve by twice elastic slope (TES) method. If the TES line intersect the M-$\phi$ curve after maximum moment (generally happens for very thin elbows under closing bending), the plastic collapse moment is taken equal to the maximum moment on the M-$\phi$ curve. For the cracked elbow plastic collapse load evaluation, the circumferential through wall crack is modelled at extrados or intrados depending on the mode of bending moment applied. The extrados crack is assumed for closing mode of bending moment and intrados crack is assumed for opening mode of bending moment. The analyses for healthy elbow, under closing bending, were performed at all stations of pipe geometry R/t, bend radius $r_b$/R, and bend angle $\psi$. In past, Gupta [10] has reported some of the results that is for all R/t, $r_b$/R but for bend angle of 90° alone, for healthy elbow, under closing bending. Here the data were presented in context of ASME $B_2$ stress index which intend to ensure integrity of elbows against plastic collapse mode of failure. In case of opening bending load, only selected station of R/t, $r_b$/R and $\psi$ were analysed. In case of cracked elbow, for both opening & closing bending load, the analyses were carried out only at selected station of R/t, $r_b$/R and for $\psi$=90°.

## EVALUATION OF WEAKENING FACTORS

The plastic collapse load ($M_{CL}^E$) using TES method were evaluated for all the above described elbow geometries under closing and opening bending loading. The geometric weakening factors, $W_f$, were evaluated as the ratio of the healthy elbow plastic collapse load ($M_{CL}^E$) and corresponding pipe plastic collapse load (using Eq.3), and the crack weakening factors H($\theta$) were evaluated as the ratio of the cracked elbow plastic collapse load equations with corresponding healthy elbow plastic collapse load. The evaluated geometric weakening factor $W_f$ , is given in table-2 while the crack weakening factor H($\theta$), is given in table-3.

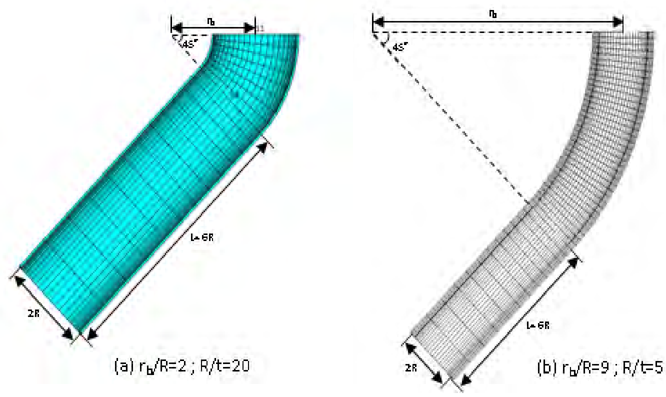



Fig.1: Typical FE meshes (a) small $r_b$ thin elbow, $r_b/R=2$ & $R/t=30$ (b) extra large $r_b$ thick elbow, $r_b/R=9$ & $R/t=5$

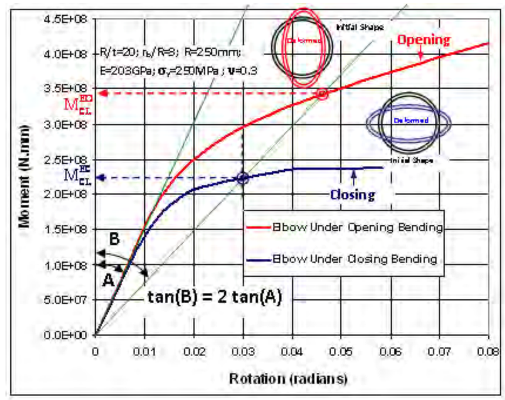



Fig.2: Typical M-$\phi$ curve for closing & opening moment, $M_{CL}^{E}$ evaluation using TES method

Table-2: Geometric weakening factor, $W_f$, for Elbow subjected to in-plane bending

| $W_f$ for Closing Bending | | | | | | |
|---|---|---|---|---|---|---|
| $r_b/R\rightarrow$ | 2 | 3 | 6 | 9 | 12 | 18 |
| R/t ↓ | Bend Angle ψ=30° | | | | | |
| 5.0 | 0.88 | 0.91 | 0.94 | 0.96 | 0.97 | 0.97 |
| 7.5 | 0.80 | 0.83 | 0.89 | 0.92 | 0.94 | 0.96 |
| 10.0 | 0.72 | 0.76 | 0.83 | 0.87 | 0.90 | 0.94 |
| 15.0 | 0.61 | 0.64 | 0.72 | 0.77 | 0.81 | 0.87 |
| 20.0 | 0.53 | 0.56 | 0.63 | 0.68 | 0.72 | 0.79 |
| 30.0 | 0.43 | 0.45 | 0.51 | 0.55 | 0.59 | 0.65 |
| R/t ↓ | Bend Angle ψ=60° | | | | | |
| 5.0 | 0.74 | 0.80 | 0.91 | 0.95 | 0.96 | 0.97 |
| 7.5 | 0.62 | 0.68 | 0.81 | 0.89 | 0.93 | 0.96 |
| 10.0 | 0.53 | 0.59 | 0.72 | 0.81 | 0.87 | 0.94 |
| 15.0 | 0.41 | 0.46 | 0.57 | 0.66 | 0.74 | 0.85 |
| 20.0 | 0.34 | 0.38 | 0.48 | 0.56 | 0.62 | 0.74 |
| 30.0 | 0.25 | 0.29 | 0.37 | 0.42 | 0.47 | 0.56 |
| R/t ↓ | Bend Angle ψ=150° | | | | | |
| 5.0 | 0.57 | 0.68 | 0.89 | 0.95 | 0.96 | 0.97 |
| 7.5 | 0.44 | 0.53 | 0.75 | 0.88 | 0.93 | 0.95 |
| 10.0 | 0.36 | 0.44 | 0.63 | 0.78 | 0.87 | 0.93 |
| 15.0 | 0.27 | 0.33 | 0.48 | 0.60 | 0.71 | 0.84 |
| 20.0 | 0.22 | 0.27 | 0.39 | 0.49 | 0.58 | 0.74 |
| 30.0 | 0.16 | 0.20 | 0.29 | 0.36 | 0.43 | 0.55 |

| $W_f$ for Closing Bending | | | | | | |
|---|---|---|---|---|---|---|
| $r_b/R\rightarrow$ | 2 | 3 | 6 | 9 | 12 | 18 |
| R/t ↓ | Bend Angle ψ=45° | | | | | |
| 5.0 | 0.80 | 0.85 | 0.92 | 0.95 | 0.96 | 0.97 |
| 7.5 | 0.70 | 0.75 | 0.84 | 0.90 | 0.93 | 0.96 |
| 10.0 | 0.61 | 0.66 | 0.76 | 0.83 | 0.88 | 0.94 |
| 15.0 | 0.49 | 0.53 | 0.63 | 0.70 | 0.76 | 0.86 |
| 20.0 | 0.41 | 0.45 | 0.53 | 0.60 | 0.66 | 0.76 |
| 30.0 | 0.32 | 0.35 | 0.41 | 0.47 | 0.51 | 0.60 |
| R/t ↓ | Bend Angle ψ=90° | | | | | |
| 5.0 | 0.65 | 0.74 | 0.90 | 0.95 | 0.96 | 0.97 |
| 7.5 | 0.53 | 0.60 | 0.78 | 0.88 | 0.93 | 0.96 |
| 10.0 | 0.44 | 0.51 | 0.67 | 0.79 | 0.87 | 0.93 |
| 15.0 | 0.33 | 0.39 | 0.52 | 0.62 | 0.72 | 0.85 |
| 20.0 | 0.27 | 0.31 | 0.42 | 0.51 | 0.59 | 0.73 |
| 30.0 | 0.20 | 0.24 | 0.32 | 0.39 | 0.45 | 0.56 |
| R/t ↓ | Bend Angle ψ=135° | | | | | |
| 5.0 | 0.58 | 0.69 | 0.89 | 0.95 | 0.96 | 0.97 |
| 7.5 | 0.45 | 0.54 | 0.76 | 0.88 | 0.93 | 0.95 |
| 10.0 | 0.37 | 0.45 | 0.64 | 0.78 | 0.87 | 0.93 |
| 15.0 | 0.28 | 0.34 | 0.48 | 0.61 | 0.71 | 0.84 |
| 20.0 | 0.23 | 0.27 | 0.39 | 0.50 | 0.58 | 0.71 |
| 30.0 | 0.17 | 0.20 | 0.29 | 0.36 | 0.43 | 0.54 |

| $W_f$ for Opening Bending | | | | | | |
|---|---|---|---|---|---|---|
| $r_b/R\rightarrow$ | 2 | 3 | 6 | 9 | 12 | 18 |
| R/t ↓ | Bend Angle ψ=45° | | | | | |
| 5.0 | 0.86 | 0.90 | 0.94 | 0.96 | 0.97 | |
| 7.5 | | 0.84 | | 0.94 | | 0.97 |
| 10.0 | 0.75 | 0.79 | 0.86 | 0.91 | 0.94 | 0.96 |
| 15.0 | | 0.72 | 0.80 | 0.86 | | 0.94 |
| 20.0 | 0.64 | | 0.75 | | 0.86 | 0.92 |
| 30.0 | | 0.63 | 0.71 | 0.77 | 0.82 | 0.89 |
| R/t ↓ | Bend Angle ψ=90° | | | | | |
| 5.0 | 0.72 | 0.80 | 0.93 | 0.96 | 0.97 | |
| 7.5 | | 0.70 | | 0.93 | | 0.97 |
| 10.0 | 0.55 | 0.62 | 0.79 | 0.89 | 0.93 | 0.96 |
| 15.0 | | 0.48 | 0.69 | 0.81 | | 0.94 |
| 20.0 | 0.42 | | 0.63 | | 0.83 | 0.91 |
| 30.0 | | 0.44 | 0.57 | 0.68 | 0.78 | 0.87 |
| R/t ↓ | Bend Angle ψ=135° | | | | | |
| 5.0 | 0.64 | 0.76 | 0.92 | 0.96 | 0.97 | |
| 7.5 | | 0.63 | | 0.93 | | 0.96 |
| 10.0 | 0.46 | 0.55 | 0.77 | 0.88 | 0.93 | 0.95 |
| 15.0 | | 0.46 | 0.66 | 0.80 | | 0.93 |
| 20.0 | 0.34 | | 0.57 | | 0.82 | 0.90 |
| 30.0 | | 0.37 | 0.52 | 0.65 | 0.75 | 0.84 |

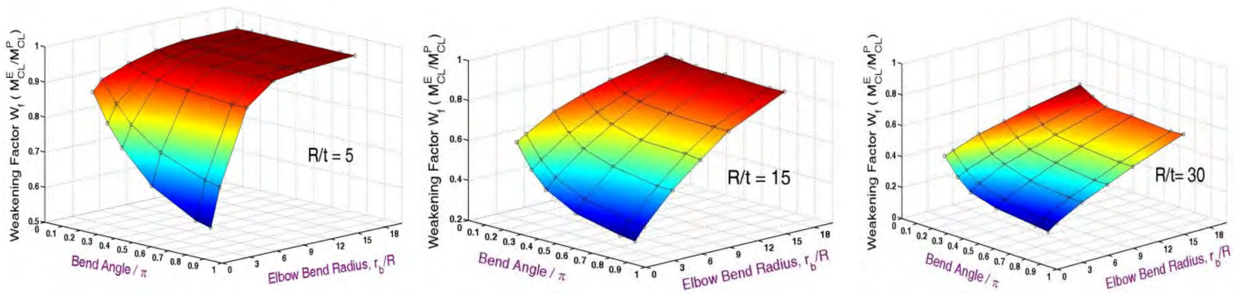



Fig. 3: Surface plots of FE evaluated $W_f$ for R/t 5, 15 and 30 elbows under closing bending

Figure-3 plots the results for three cases of R/t as 5, 10 and 30 and for closing bending moment. This clearly show the asymptotic trend discussed earlier. When the bend radius ($r_b$) increases $W_f$ approaches to 1.0 or in other words the Plastic Collapse load of the elbow approach to that of straight pipe. Similarly, when the elbow bend angle ψ is decreased, the elbow Plastic Collapse load approaches to that of pipe. However, it is observed, that for a given R/t, the plastic collapse load has strong dependence on bend radius than on bend angle.

Table-3: Crack weakening factor H(θ), for elbows with ψ=90° and subjected to in-plane bending

| H(θ) for Closing Bending (Extrados Crack) | | | | | | | H(θ) for Opening Bending (Intrados Crack) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r_b/R\rightarrow$ | 2 | 3 | 6 | 9 | 12 | 18 | $r_b/R\rightarrow$ | 2 | 3 | 6 | 9 | 12 | 18 |
| R/t ↓ | Crack Angle θ/π=0.1 | | | | | | R/t ↓ | Crack Angle θ/π=0.1 | | | | | |
| 5.0 | 1.00 | 1.00 | 0.97 | 0.95 | | | 5.0 | 0.96 | 0.96 | 0.93 | 0.94 | | |
| 7.5 | | 1.00 | | 0.97 | | | 7.5 | | 0.98 | | 0.94 | | |
| 10.0 | 1.01 | 1.03 | 1.00 | | 0.98 | 0.98 | 10.0 | 0.98 | 0.98 | 0.97 | | 0.94 | 0.92 |
| 15.0 | | 1.02 | 1.00 | 1.02 | | 1.02 | 15.0 | | 0.98 | 0.98 | 0.98 | | |
| 20.0 | | | 1.00 | | 0.99 | | 20.0 | | | 0.98 | | 0.99 | |
| 30.0 | | | 0.99 | 1.03 | 1.03 | | 30.0 | | | 1.01 | 1.03 | 1.00 | 0.97 |
| R/t ↓ | Crack Angle θ/π=0.2 | | | | | | R/t ↓ | Crack Angle θ/π=0.2 | | | | | |
| 5.0 | 0.94 | 0.91 | 0.83 | 0.79 | | | 5.0 | 0.82 | 0.80 | 0.76 | 0.76 | | |
| 7.5 | | 0.97 | | 0.83 | | | 7.5 | | 0.85 | | 0.76 | | |
| 10.0 | 1.02 | 1.02 | 0.95 | | 0.83 | 0.79 | 10.0 | 0.89 | 0.87 | 0.82 | | 0.76 | 0.75 |
| 15.0 | | 1.02 | 1.00 | 0.98 | | 0.85 | 15.0 | | 0.91 | 0.87 | 0.84 | | 0.79 |
| 20.0 | | | 1.00 | | 0.97 | | 20.0 | | | 0.91 | | 0.86 | |
| 30.0 | | | 1.00 | 1.02 | 1.02 | 1.00 | 30.0 | | | 0.97 | 0.96 | 0.91 | 0.83 |
| R/t ↓ | Crack Angle θ/π=0.3 | | | | | | R/t ↓ | Crack Angle θ/π=0.3 | | | | | |
| 5.0 | 0.77 | 0.73 | 0.63 | 0.61 | | | 5.0 | 0.61 | 0.60 | 0.58 | 0.60 | | |
| 7.5 | | 0.81 | | 0.63 | | | 7.5 | | 0.64 | | 0.57 | | |
| 10.0 | 0.90 | 0.88 | 0.79 | | 0.64 | 0.60 | 10.0 | 0.67 | 0.65 | 0.62 | | 0.58 | 0.57 |
| 15.0 | | 0.96 | 0.89 | 0.84 | | 0.64 | 15.0 | | 0.70 | 0.67 | 0.64 | | 0.60 |
| 20.0 | | | 0.95 | | 0.86 | | 20.0 | | | 0.71 | | 0.66 | |
| 30.0 | | | 1.00 | 0.98 | 0.96 | 0.92 | 30.0 | | | 0.78 | 0.77 | 0.72 | 0.64 |
| R/t ↓ | Crack Angle θ/π=0.4 | | | | | | R/t ↓ | Crack Angle θ/π=0.4 | | | | | |
| 5.0 | 0.55 | 0.53 | 0.45 | 0.43 | | | 5.0 | 0.43 | 0.42 | 0.40 | 0.40 | | |
| 7.5 | | 0.60 | | 0.45 | | | 7.5 | | 0.44 | | 0.40 | | |
| 10.0 | 0.67 | 0.66 | 0.58 | | 0.45 | 0.43 | 10.0 | 0.45 | 0.45 | 0.43 | | 0.40 | 0.40 |
| 15.0 | | 0.75 | 0.70 | 0.62 | | 0.46 | 15.0 | | 0.47 | 0.46 | 0.44 | | 0.42 |
| 20.0 | | | 0.77 | | 0.66 | | 20.0 | | | 0.49 | | 0.46 | |
| 30.0 | | | 0.85 | 0.82 | 0.79 | 0.71 | 30.0 | | | 0.55 | 0.55 | 0.51 | 0.45 |
| R/t ↓ | Crack Angle θ/π=0.5 | | | | | | R/t ↓ | Crack Angle θ/π=0.5 | | | | | |
| 5.0 | 0.36 | 0.34 | 0.29 | 0.27 | | | 5.0 | 0.29 | 0.27 | 0.26 | 0.25 | | |
| 7.5 | | 0.40 | | 0.29 | | | 7.5 | | 0.29 | | 0.25 | | |
| 10.0 | 0.42 | 0.45 | 0.38 | | 0.29 | 0.27 | 10.0 | 0.29 | 0.29 | 0.27 | | 0.25 | 0.26 |
| 15.0 | | 0.51 | 0.48 | 0.41 | | 0.29 | 15.0 | | 0.30 | 0.29 | 0.28 | | 0.27 |
| 20.0 | | | 0.57 | | 0.43 | | 20.0 | | | 0.32 | | 0.29 | |
| 30.0 | | | 0.66 | 0.62 | 0.57 | 0.46 | 30.0 | | | 0.36 | 0.36 | 0.34 | 0.29 |

Table-4: Threshold crack angle ($\theta_{th}$) under in-plane bending moment

| $r_b/R\rightarrow$ | 2 | 3 | 6 | 9 | 12 | 18 | $r_b/R\rightarrow$ | 2 | 3 | 6 | 9 | 12 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R/t ↓ | Threshold crack angle ($\theta_{th}$) under closing bending moment | | | | | | R/t ↓ | Threshold crack angle ($\theta_{th}$) under Opening Bending Moment | | | | | |
| 5.0 | 31.7 | 27.8 | 15.8 | | | | 5.0 | 18.9 | 14.4 | 11.3 | | | |
| 7.5 | | 34.4 | | 16.3 | | | 7.5 | | 22.4 | | 11.7 | | |
| 10.0 | 46.6 | 44.2 | 32.8 | | 17.2 | 16.6 | 10.0 | 27.5 | 24.5 | 19.9 | | 12.1 | 9.6 |
| 15.0 | | 50.8 | 44.9 | 36.0 | | 20.6 | 15.0 | | 28.7 | 24.8 | 21.9 | | 15.2 |
| 20.0 | | | 49.6 | | 36.5 | | 20.0 | | | 28.1 | | 23.0 | |
| 30.0 | | | 54.3 | 53.1 | 51.2 | 48.0 | 30.0 | | | 34.0 | 32.9 | 28.3 | 20.2 |

## DEVELOPMENT OF GENERAL PLASTIC COLLAPSE MOMENT EQUATIONS

It is well known that the collapse load for in plane closing bending is lowest when compared with in plane opening or out of plane bending collapse load. Hence the in plane closing collapse load forms the basis for ensuring the integrity of an elbow against plastic collapse. In view of this, efforts have been made for fitting a closed form equation for the elbow geometric weakening factor $W_f$ for closing bending. Based on the results as shown in table-2, for healthy elbow subjected to closing bending moment a relational study were performed and it was noted that the plastic collapse load, or the $W_f$ depend on all three parameter that are R/t, $r_b/R$ and $\psi$ and elbow characteristic $\lambda$ alone is in-adequate to fit the $W_f$. Analyses of several elbows with equal $\lambda$, resulted in different $W_f$. In view of the fact that geometric weakening factor always be less than 1.0 and it approaches to 1.0 as $r_b/R$ or $\lambda$ approaches infinity (i.e. elbow approaches to pipe) a general equation of the form given bellow is consider for the analysis.

$$\frac{1}{W_f} = 1 + \frac{C_1}{\lambda^{C_2}} \tag{15}$$

Here the $C_2$ was evaluated from regression analysis of log(1/$W_f$-1) and log$\lambda$ data for each of the $r_b$/R and $\psi$ station. Afterward, evaluated $C_2$ dependence on $r_b$/R and $\psi$ were investigated and it was found that, $C_2$ weakly depends on the elbow bend angle. Finally the $C_2$ was approximated as a linear function of $r_b$/R and given as

$$C_2 = 0.085\left(\frac{r_b}{r}\right) + 0.59 \tag{16}$$

The $C_2$ as given by eq. 16 will ensure the asymptotic trend with respect to $r_b$/R (or $\lambda$). Using eq.15 & 16 the $C_1$ was evaluated for all R/t, $r_b$/R and $\psi$ stations. The relational study of $C_1$ with R/t, $r_b$/R and $\psi$ performed and a closed-form of equation was fitted as given bellow:

$$C_1 = 8\left(\frac{\psi}{\pi}\right)^{1.414}\left(\frac{R}{t}\right)^{0.43}\lambda^{-0..008} - 7.67\left(\frac{\psi}{\pi}\right)^{1.443}\left(\frac{R}{t}\right)^{0.443} \tag{17}$$

This have resulted in a general plastic collapse load equation, Eq.18, for elbow closing bending, applicable to wide range of R/t, $r_b$/R and $\psi$ as well as meets the asymptotic behaviour discussed earlier. The Eq.19 is used to calculate the % error associated with prediction of $W_f$ using the proposed Eq.18.

$$\frac{1}{W_f} = 1 + \frac{8\left(\frac{\psi}{\pi}\right)^{1.414}\left(\frac{R}{t}\right)^{0.43}\lambda^{-0..008} - 7.67\left(\frac{\psi}{\pi}\right)^{1.443}\left(\frac{R}{t}\right)^{0.443}}{\lambda^{\left[0.085\left(\frac{r_b}{r}\right)+0.59\right]}} \tag{18}$$

$$\%\text{error} = [\ W_f(\text{Eq. 18})/\ W_f(\text{table. 2})-1]*100 \tag{19}$$

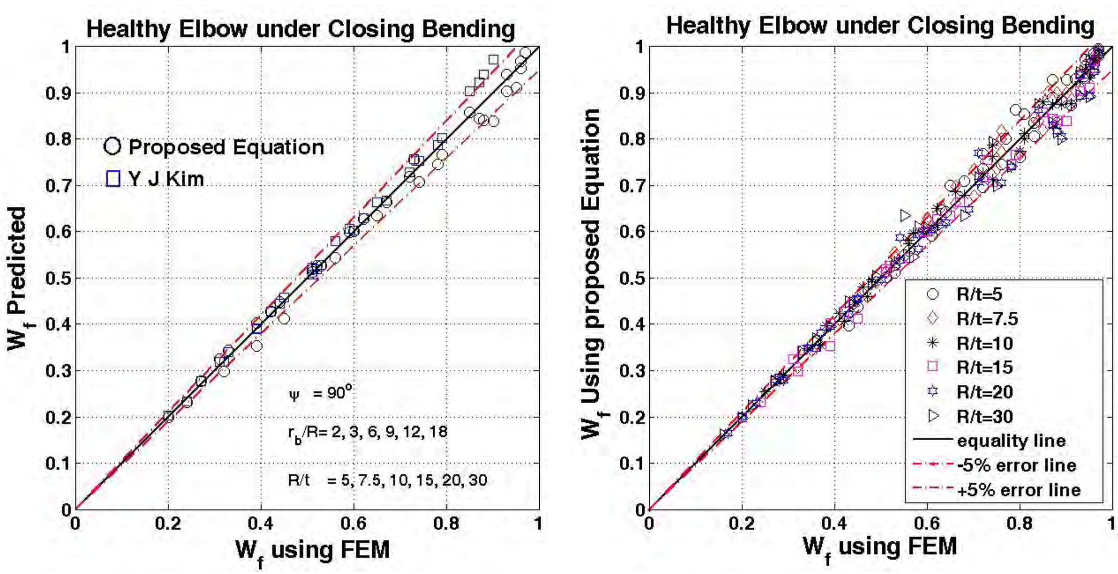



Fig. 4: $W_f$ plot of (a) Predicted Vs FE evaluated for 90° elbow using the Kim, Eq.11 and new proposed Eq.18.
(b) Predicted new proposed Eq.18 Vs FE for all $\psi$, $r_b$/R and R/t

Figure 4(a) plot compares the predictions from the new equation with the Eq. 11 given by Kim [7] for elbows having $\psi$ =90° and $\lambda \leq 0.5$. The two equations were found in good agreements except for cases with higher $\lambda$ value. Comparison of prediction with many other literature equation was reported earlier Gupta [10]. Figure 4(b) plots the predictions using new Eq. vs $W_f$ evaluated using FEM. This show that most of the data points lie within the ±5% error band.

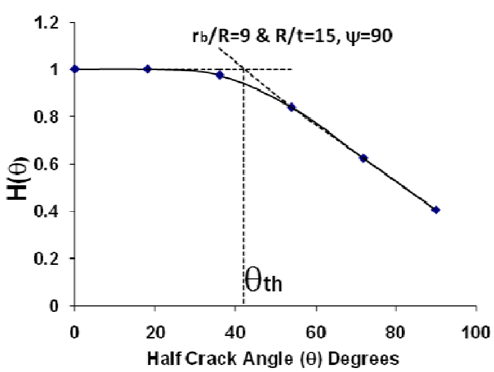
**Collapse Load for Cracked Elbow:**

Results of the crack weakening factor H(θ) have been given in table 3 for through-wall circumferentially cracked elbow subjected to closing/opening bending moment, while the corresponding elbow geometry weakening factor is reported in table-2 and also flitted and given by Eq.18. The H(θ) results show that there is a threshold crack angle ($\theta_{th}$) below which there is no effect of crack on the plastic collapse load for given geometry as also shown in fig 5a for a typical case. Hence for each case of geometry this threshold value of crack angle was evaluated and tabulated in table 4. To evaluate threshold crack angle for given geometry Factor H (θ) is plotted against semi-circumferential crack angle and data points are fitted in single line as shown in fig 5a and angle corresponding to point where this straight line cut the line H(θ)=1 is designated as threshold value. Ideally for pipe threshold crack angle should be zero as maximum stress point is on the extreme fiber which is not happening in case of elbow. Hence equation for the threshold value is assumed such that it should approach to zero as $r_b/R$ approaches to infinity. For threshold crack ($\theta_{th}$) equation is assumed to have functional form as given below.

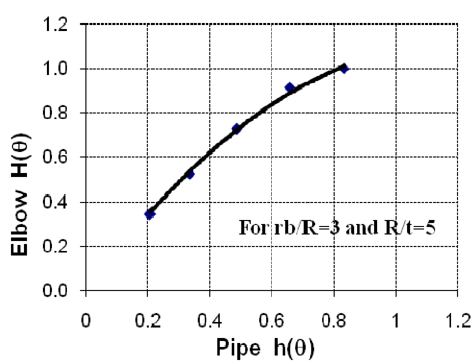
$$\theta_{th} = d_1 e^{[-((r_b/R)d2]} \tag{20}$$

The $d_1$ and $d_2$ were obtained as function of R/t using eq.20 and table-4 data. The $d_1$ and $d_2$ variation were fitted as linear function of R/t. The H(θ) variations, for crack sizes larger than the threshold crack ($\theta_{th}$), were investigated with respect to the corresponding pipe's crack weakling factor that is h(θ), as given by Eq. 4. It was observed that the H(θ) can be a fitted as a second order function of h(θ) with zero intercept, as shown in fig 5b for a typical R/t, and $r_b/R$ case. Hence a general equation of the form given bellow is considered for the analysis as given bellow.

$$H(\theta) = 1 \quad \text{for } \theta \le \theta_{th} \quad \text{and} \quad H(\theta) = (1 - D_1)h - D_2 h^2 \quad \text{for } \theta > \theta_{th} \tag{21}$$




(a) Typical H(θ) Vs. θ variation for an elbow




(b) Typical Elbow H(θ) Vs. Pipe h(θ) variation

Fig. 5: Determination of crack weakening factor and threshold circumferential crack angle for elbows

Based on the fact that H (θ) approaches to h (θ) as $r_b/R$ approaches infinity (i.e. elbow approaches to pipe) constant $D_1$ and $D_2$ are assumed to have functional forms such that they approaches to 0 with increase in λ or $r_b/R$.

$$D_1 = a_1 e^{-[(r_b/R).b1]} \qquad D_2 = a_2 e^{-[(r_b/R).b2]} \tag{22}$$

For each case, table-3, the constant $D_1$ and $D_2$ were evaluated. Further data analysis was carried out for above equations for each of R/t and the constant $a_1$, $a_2$, $b_1$ and $b_2$ were obtained as function of R/t. Then $a_1$ and $a_2$ are fitted linearly as function of R/t and $b_1$, $b_2$ were fitted using polynomial or exponential fit depending on the data trends. Finally close form equation for the crack weakening factor H(θ), for through wall circumferential cracked elbow under closing and opening bending, have been developed and is given in table-5.

Figure 6(a) plot compares the predictions from the new equation (table-5) with the equations given by Chattopadhayay [6] and Miller [8] for elbows having R/t=10 and ψ =90°. The plot show that results of $r_b/R$=2 and 3 are very close and in good agreement with both new and Eq. given by Chattopadhayay. However, Chattopadhayay's Eq. fails to predict H(θ) where bending radius is large and also does not meets the asymptotic trend. The new equation reasonability predicts for all cases and meets the asymptotic trend. Figure 6(b) plots the predictions using new Eq. vs H(θ) evaluated using FEM. This show that most of the data points lie within the ±5% error band.

Table-5: Proposed Close form Equation for Elbow with Circumferential through wall crack

| $H(\theta)=1$ for $\theta \le \theta_{th}$ and $H(\theta)=(1-D_1)h - D_2 h^2$ for $\theta > \theta_{th}$<br>Where, $h=\mathrm{Cos}(\theta/2) - 0.5\mathrm{Sin}(\theta)$ and $\theta$ = Half Crack Angle | |
|---|---|
| For Closing Bending (Extrados through-wall crack) | For Opening Bending (Intrados through-wall crack) |
| $\theta_{th}=\left[0.025\frac{R}{t}+0.72\right]\mathrm{Exp}\left[-0.8\lambda\left(\frac{R}{t}\right)^{0.08}\right]$ | $\theta_{th}=\left[0.012\frac{R}{t}+0.36\right]\mathrm{Exp}\left[-0.4\lambda\left(\frac{R}{t}\right)^{0.2}\right]$ |
| $D_1=\left(0.006\left(\frac{R}{t}\right)^2-0.32\left(\frac{R}{t}\right)+0.32\right)\mathrm{Exp}\left[\frac{r_b}{R}\left(0.004\frac{R}{t}-0.17\right)\right]$ | $D_1=\left(-0.01\left(\frac{R}{t}\right)-0.35\right)\mathrm{Exp}\left[-0.2\frac{r_b}{R}\mathrm{Exp}\left(-0.1\frac{R}{t}\right)\right]$ |
| $D_2=\left(0.32\frac{R}{t}-0.8\right)\mathrm{Exp}\left[-1.5\frac{r_b}{R}\right]$ | $D_2=0$ |

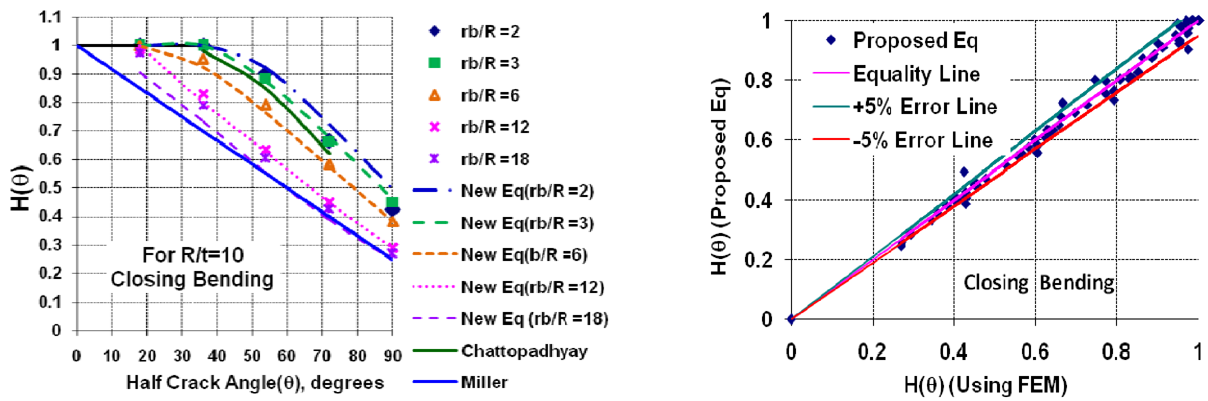



Fig. 6: Crack weakening factor, H(θ) plots for Closing Bending (a) H(θ) vs θ plots for data evaluated using FEM, proposed equation, Chattopadhayay and Miller (b) H(θ) predicted from new equation Vs. H(θ) evaluated using FEM

Figure 7(a) plot compares the predictions from the new equation (table-5) with the equations given by Chattopadhayay [6] and Miller [8] for elbows having R/t=10 and $\psi$ =90°. The plot show that the Eq. given by Chattopadhayay, Miller provide lower bound values of H(θ) and does not meets the asymptotic trend. The new equation reasonability predicts for all cases and meets the asymptotic trend. Figure 7(b) plots the predictions using new Eq. vs H(θ) evaluated using FEM. This show that most of the data points lie within the ±5% error band.

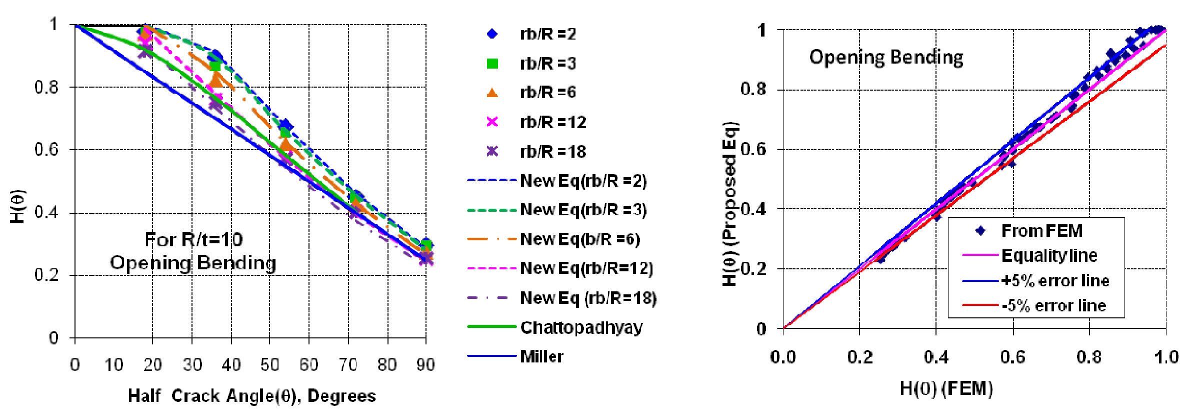



Fig. 7: Crack weakening factor H(θ) for Opening Bending (a) H(θ) vs θ plots for data evaluated using FEM, new equation, Chattopadhayay and Miller (b) H(θ) predicted from new equation vs. H(θ) evaluated using FEM

## CONCLUSION

The investigation has highlighted that equations available in literature for evaluation of plastic collapse load of an elbow have limited applicability and do not cover wide range of pipe bend radius ratios and bend angles which are used in power plant piping. These equations fail to represent asymptotic trend where the plastic collapse moment of an elbow approaches to that of pipe when the bend radius is increased keeping R/t constant or when the bend angle is decreased. Most of literature equation for $W_f$, are function of $\lambda$ alone, however, the study involving large number of nonlinear finite element calculations, revealed that $W_f$ also depends on $r_b/R$, R/t and $\psi$. Based on the FE results, a new close form equation (Eq.18) has been developed, which is function of R/t, $r_b/R$ and bend angle $\psi$ and also satisfies the asymptotic trend discussed earlier.

A crack weakening factor $H(\theta)$, defined as the ratio of plastic collapse load of cracked elbow to plastic collapse load of corresponding elbow with no crack, was also evaluated using FE analysis for large number of elbow geometries and crack sizes. The FE evaluated elbow $H(\theta)$ for both closing and opening bending, were studies in relation with corresponding pipe $h(\theta)$, along with its variation with R/t and $r_b/R$ parameters. The $H(\theta)$ results revealed a threshold crack angle ($\theta_{th}$) below which there is no effect of presence of crack on the plastic collapse load of elbow, i.e. $H(\theta)=1$. The $\theta_{th}$ was evaluated for each of the geometry case and an equation (approach to zero as $r_b/R$ approaches to infinity) was fitted for $\theta_{th}$. Further an close form equation was developed for $H(\theta)$ which reduces to $h(\theta)$ when $r_b/R$ increase and so satisfies asymptotic trend.

The plastic collapse load of an elbow without or with crack (circumferential) and under closing or opening bending can be obtained from corresponding pipe collapse load by multiplying it with a geometric weakening factor $W_f$ and a crack weakening factors $H(\theta)$. The FE evaluated database as well as close form equations for the $W_f$ and $H(\theta)$ have been presented.

## REFERENCES

[1] Spence, J. and Findley, G. E. (1976). "Limit load for pipe bend s under in-plane bending" Proc.,2nd International Conference on Pressure Vessel Technology, San Antonio, Texas, 393-399

[2] ASME, Boiler and Pressure Vessel Code, Section III, Subsection NB, 2004

[3] Caladine, C. R. (1974) "Limit analysis of curved tubes", J. Mech. Eng. Sci. vol 16, 85-87

[4] Goodal I. W., "Large Deformation in Plastically Deforming Curved Tubes Subjected to In-Plane Bending" Reserch division report, RD/B/N4312. Central Electricity Generating Board, UK, 1978

[5] Touboul, F. et. al. (1989) "Design criteria for piping component against plastic collapse: Application to pipe bend experiments", Proc.,6th International Conference on Pressure Vessel Technology, Beijing, China,73-84

[6] Chattopadahya, J. et. al. (2004) "Closed form collapse moment equation of through wall circumferentially cracked elbows subjected to in-plane bending moment" ASME J. of Pressure Vessel Technology, vol.126, 307-317.

[7] Kim, Y. J. and Chang, S. O. (2006). "Closed form plastic collapse loads of pipe bends under combined pressure and in-plane bending", J. Engineering fracture Mechanics, vol.-73, issue-11, pg.1437-1454

[8] Miller, A. G., 1988, ''Review of Limit Loads of Structures Containing Defects,'' Int. J. Pressure Vessels Piping, 32, pp. 197–327.

[9] Zahoor, A., 1991, Ductile Fracture Handbook, Vol. 3, EPRI-NP-6301-D,N14-1, Research Project 1757–69, Electric Power Research Institute, Palo Alto, CA.

[10] Suneel K. Gupta, Vivek Bhasin, K.K. Vaze, A.K. Ghosh and H.S. Kushwaha, "General Limit Load and B2 Stress Index Equation for Pipe Bends under In Plane Bending", 19th Conference on Structural Mechanics in Reactor Technology (SMiRT-19), Toronto, Canada, August, 12-17, 2007.