Development of New Correlations for Improved Integrity Assessment of Nuclear Reactor Piping Components

J.Chattopadhyay*, B.K.Dutta, K.K.Vaze
Reactor Safety Division, Hall-7, Bhabha Atomic Research Centre, Mumbai, INDIA-400085
*Email: jchatt@barc.gov.in

ABSTRACT

To investigate several unresolved issues and to improve upon the existing equations for integrity assessment of piping components, a comprehensive Component Integrity Test Program (CITP) was initiated at BARC, India. As a part of this program, several fracture tests have been conducted on straight pipes and pipe bends, which forms a valuable data base. Simultaneously, analytical work have been undertaken to propose the improvements in the existing equations for optimized and more accurate integrity assessment of piping components. As an outcome of this analytical investigations, generalized equation of $\eta_{pl}$ and $\gamma$ have been proposed to evaluate J-R curve, study of transferability of fracture properties from specimen to component has been done to highlight the role of constraint parameter, new limit moment equations of elbows have been proposed and new J and COD estimation schemes of throughwall cracked elbows have been proposed. All these newly proposed equations have been experimentally validated with the test data generated under CITP.

INTRODUCTION

Integrity assessment of piping components is very essential for safe and reliable operation of all types of process power plants. It is especially important for nuclear power plants because of the application of leak-before-break (LBB) concept which involves detailed integrity assessment of primary heat transport piping systems taking into account the postulated cracks. The mechanical evaluation of pipe failures has evolved over time. The main effort in evaluating the mechanical and structural behavior or pressurized components started about 1950. Since that time, numerous investigations have been performed to assess the loading capacity and failure behavior of piping components. Investigations have also focused on determining failure loads and quantifying the margins of safety. While a considerable work has already been done in the development of integrity assessment procedure of cracked/un-cracked piping components, some issues are still unresolved or not fully understood, especially regarding elbows.

Against this backdrop, a comprehensive Component Integrity Test Program was initiated at Reactor Safety Division, BARC, India. In this program, large number of full scale tests on straight pipe and elbows of various sizes with various crack configurations subjected to different loading conditions were carried out. Subsequently, these test data base was utilized scientifically to develop a number of new equations that may be used for improved integrity assessment of piping components. The present report first describes briefly the tests carried out and then its scientific use to develop new equations.

EXPERIMENTAL WORK

In the experimental investigations, fracture mechanics tests are carried out on cracked pipes and elbows under quasi-static monotonic loading. Total 45 tests consisting of 27 pipes of various sizes (200 - 400 mm diameter) with circumferential cracks of various angles (30°-150°), configurations (throughwall/surface), materials (base/weld) and 18 elbows of various sizes (200 - 400 mm diameter) with throughwall cracks of various angles (60°-120°), locations (extrados/intrados/crown), configurations (circumferential/axial) and in-plane bending modes (opening/closing) have been tested. However, out of these, test results of 7 pipes and 10 elbows are reported here, which are utilized for validation of the newly developed equations. The geometric details of the pipe and elbow test specimens are given in Table 1 and 2 respectively. More details of the tests are described in [1-2]. Tests on small tensile and TPB specimens, machined from the pipe of same material and heat, have also been done to evaluate the actual stress-strain and fracture resistance properties of pipe/elbow material. Table 3 shows the mechanical properties obtained from these tests.
Table 1 Details of Pipe Test Specimens

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Outer Dia. (mm)</th>
<th>Thickness (mm)</th>
<th>Outer Span (mm)</th>
<th>Inner Span (mm)</th>
<th>Crack angle, 20°</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP BM TWC8-1*</td>
<td>219</td>
<td>15.15</td>
<td>4000</td>
<td>1480</td>
<td>60.0</td>
</tr>
<tr>
<td>SP BM TWC8-2</td>
<td>219</td>
<td>15.10</td>
<td>4000</td>
<td>1480</td>
<td>90.0</td>
</tr>
<tr>
<td>SP BM TWC8-3</td>
<td>219</td>
<td>15.29</td>
<td>4000</td>
<td>1480</td>
<td>120.0</td>
</tr>
<tr>
<td>SP BM TWC8-4</td>
<td>219</td>
<td>15.11</td>
<td>4000</td>
<td>1480</td>
<td>150.0</td>
</tr>
<tr>
<td>SP BM TWC16-1</td>
<td>406</td>
<td>32.38</td>
<td>5820</td>
<td>1480</td>
<td>90.9</td>
</tr>
<tr>
<td>SP BM TWC16-2</td>
<td>406</td>
<td>32.15</td>
<td>5820</td>
<td>1480</td>
<td>121.4</td>
</tr>
<tr>
<td>SP BM TWC16-3</td>
<td>406</td>
<td>32.36</td>
<td>5820</td>
<td>1480</td>
<td>153.0</td>
</tr>
</tbody>
</table>

*SP = Straight Pipe, BM = Base Metal, TWC = Through Wall Crack, First number represent the nominal pipe diameter in inch and second number represents the test no.

Table 2 Details of Elbow Test Specimens

<table>
<thead>
<tr>
<th>Test reference number</th>
<th>Rb (mm)</th>
<th>D (mm)</th>
<th>tavg (mm)</th>
<th>Moment arm length* (mm)</th>
<th>Crack orientation</th>
<th>Crack location</th>
<th>Bending mode</th>
<th>Crack angles after fatigue pre-crack (20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELTWIN8-1</td>
<td>207</td>
<td>219</td>
<td>19.1</td>
<td>825.72</td>
<td>Circumferential</td>
<td>Intrados</td>
<td>Opening</td>
<td>94.96°</td>
</tr>
<tr>
<td>ELTWIN8-2</td>
<td>207</td>
<td>219</td>
<td>18.8</td>
<td>825.72</td>
<td>Circumferential</td>
<td>Intrados</td>
<td>Opening</td>
<td>125.16°</td>
</tr>
<tr>
<td>ELTWEX8-4</td>
<td>207</td>
<td>219</td>
<td>19.3</td>
<td>825.72</td>
<td>Circumferential</td>
<td>Extrados</td>
<td>Closing</td>
<td>98.24°</td>
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<tr>
<td>ELTWEX8-6</td>
<td>207</td>
<td>219</td>
<td>19.0</td>
<td>825.72</td>
<td>Axial</td>
<td>Crown</td>
<td>Closing</td>
<td>2a=109.2 mm</td>
</tr>
<tr>
<td>ELTWIN16-1</td>
<td>609</td>
<td>406</td>
<td>36.4</td>
<td>840.22</td>
<td>Circumferential</td>
<td>Intrados</td>
<td>Opening</td>
<td>95.89°</td>
</tr>
<tr>
<td>ELTWIN16-2</td>
<td>609</td>
<td>406</td>
<td>36.8</td>
<td>840.22</td>
<td>Circumferential</td>
<td>Intrados</td>
<td>Opening</td>
<td>122.79°</td>
</tr>
<tr>
<td>ELTWEX16-3</td>
<td>609</td>
<td>406</td>
<td>35.1</td>
<td>840.22</td>
<td>Circumferential</td>
<td>Extrados</td>
<td>Closing</td>
<td>64.85°</td>
</tr>
<tr>
<td>ELTWEX16-4</td>
<td>609</td>
<td>406</td>
<td>35.7</td>
<td>840.22</td>
<td>Circumferential</td>
<td>Extrados</td>
<td>Closing</td>
<td>94.11°</td>
</tr>
<tr>
<td>ELTWEX16-5</td>
<td>609</td>
<td>406</td>
<td>37.6</td>
<td>840.22</td>
<td>Circumferential</td>
<td>Extrados</td>
<td>Closing</td>
<td>124.0°</td>
</tr>
<tr>
<td>ELTWCR16-6</td>
<td>609</td>
<td>406</td>
<td>36.2</td>
<td>840.22</td>
<td>Axial</td>
<td>Crown</td>
<td>Opening</td>
<td>2a=210 mm</td>
</tr>
</tbody>
</table>

* It is the perpendicular distance of between the loadline and middle of elbow cross section to convert load to moment

Table 3 Mechanical Properties of SA 333 Gr 6 steel at Room Temperature

<table>
<thead>
<tr>
<th></th>
<th>200 mm NB pipe material</th>
<th>400 mm NB pipe material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress, σy</td>
<td>288 MPa</td>
<td>312 MPa</td>
</tr>
<tr>
<td>Ultimate tensile stress, σu</td>
<td>420 MPa</td>
<td>459 MPa</td>
</tr>
<tr>
<td>Young’s modulus of elasticity, E</td>
<td>203 GPa</td>
<td>203 GPa</td>
</tr>
<tr>
<td>Percentage elongation</td>
<td>36.2</td>
<td>39.1</td>
</tr>
<tr>
<td>Percentage reduction in area</td>
<td>76.64</td>
<td>76.15</td>
</tr>
<tr>
<td>Poisson’s ratio, ν</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Ramberg-Osgood coefficient (α) with reference stress equal to yield stress</td>
<td>10.759</td>
<td>10.249</td>
</tr>
<tr>
<td>Ramberg-Osgood hardening exponent (n)</td>
<td>4.301</td>
<td>220 N/mm</td>
</tr>
<tr>
<td>Initiation toughness, (Ji)SZW</td>
<td>4.23</td>
<td>236 N/mm</td>
</tr>
</tbody>
</table>

ANALYTICAL WORK

Various analytical work have been undertaken in RSD, BARC to propose the improvements in the existing equations for optimized and more accurate integrity assessment of piping components, specially for Break-Preclusion or Leak-Before-Break analysis of nuclear power plant piping. The test data developed above have been extensively used for experimental validation of the proposed improvements. The major analytical works are as follows:

- Generalized equation of ηpl and γ to evaluate J-R curve
• Study of transferability of fracture properties from specimen to component
• New limit moment equations of elbows
• New J and COD estimation schemes of throughwall cracked elbows

These analytical works are described in brief in the subsequent sections.

GENERALIZED EQUATION OF ηPL AND γ TO EVALUATE J-R CURVE

The evaluation of J-integral from test data generally requires the experimental load vs. load-line-displacement and load vs. crack growth data. Rice et al [3] proposed splitting the total J-integral into elastic (Je) and plastic (Jp) components:

\[ J = J_e + J_p \]  

\[ J_e = \frac{K^2}{E} \]  

where, \( E' = E \) for plane stress case and \( E' = \frac{E}{1-\nu^2} \) for plane strain case, \( K \) is the elastic stress intensity factor, \( E \) is the Young’s modulus and \( \nu \) is the Poisson’s ratio. The general expression to evaluate \( J_p \) from experimental data is as follows [4-5]:

\[ J_p = \int \eta_{pl} P d\Delta_{pl} + \int \gamma J_p da \]  

where, \( P \) is the total applied load, \( \Delta_{pl} \) is the plastic load-line-displacement due to crack only, \( a_0 \) is the initial crack length per crack tip, \( a \) is the current crack length per crack tip, \( \eta_{pl} \) and \( \gamma \) are two geometry and loading dependent functions.

The earlier \( \eta_{pl} \) and \( \gamma \) functions had been derived from dimensional analyses that were specific to the geometry and loading conditions. No general formula was available. Chattopadhyay et al [6] derived the limit load-based general expressions of \( \eta_{pl} \) and \( \gamma \) functions as follows:

\[ \eta_{pl} = \frac{\partial F_L}{\partial A} \frac{1}{F_L} \quad \text{and} \quad \gamma = \frac{\partial^2 F_L}{\partial a^2} \frac{\partial a}{\partial F_L} \]  

Utilizing these general expressions, new \( \eta_{pl} \) and \( \gamma \) functions for following pipe/elbow geometry under various loading conditions, for which no solutions are available in the open literature, have been derived.

• Throughwall circumferentially cracked thick pipe under combined bending and tension
• Pipe with constant depth part-through circumferential crack under combined bending moment and axial tension
• Pipe with semi-elliptical part-throughwall circumferential crack under axial tension
• Pipe with semi-elliptical part-throughwall circumferential crack under combined bending moment and axial tension
• Pipe with full circumferential part-throughwall crack under axial tension
• Elbow with throughwall circumferential crack under in-plane bending moment
  • Crack at extrados under closing moment
  • Crack at intrados under opening moment

• Elbow with throughwall axial crack at crown under in-plane bending moment
  • Closing moment
  • Opening moment

Details of these derivations and experimental/numerical validations of some of these new \( \eta_{pl} \) and \( \gamma \) functions are available in [7-8].

STUDY OF TRANSFERABILITY OF FRACTURE PROPERTIES FROM SPECIMEN TO COMPONENT

Ductile tearing resistance of a material is conventionally characterized by a J-resistance (J-R) curve, which is obtained from laboratory fracture specimens. It is now well-known that J-R curve shows geometry dependence due to influence of stress triaxiality. Two-parameter fracture mechanics approach have been tried to describe the effect of constraint on ductile tearing resistance and thereby resolving the issue of transferability of specimen J-R curve to components. In the two-parameter fracture mechanics approach, the first parameter reflects the scale of crack tip
deformation (e.g. J-integral) and the second parameter is used to quantify the level of stress triaxiality. If the triaxial conditions are found to be similar then it is believed that the J-R curves are transferable.

The multi-axiality quotient, ‘q’ as proposed by Clausmeyer et al [9] and later modified by Pavankumar et al [10] is used as constraint parameter. The parameter is as defined below:

$$A_{nq} = \frac{\int_{x_{nq}}^{x_{m}} q \, dx}{\int_{x_{nq}}^{x_{m}} q \, dx}$$

Where, $q = \frac{1}{\sqrt{3}}(\sigma_e/\sigma_m)$ with $\sigma_e = \text{von-Mises effective stress}$, $\sigma_m = \text{hydrostatic stress}$, $q_c$ is the critical value of multi-axiality quotient (0.27), $dx$ is the distance across the ligament, $J$ is the J-integral, $\sigma_0$ is the yield stress.

These parameters (i.e. $q$ and $A_{nq}$) have been evaluated for side grooved TPB specimen (a/w = 0.5) which are machined from 200 mm NB pipes, 200 mm NB pipes having various sizes of throughwall circumferential crack subjected to four point bending load (see Table 1 for details) and also for 200 mm NB elbows having throughwall circumferential cracks at extrados/intrados subjected to closing/opening bending moment (see Table 2 for details). Figure 1 shows the variation of ‘$A_{nq}$’ with J-integral for TPB specimen and pipes and elbows. It may be seen that stress triaxiality ahead of crack tip, quantified by the parameter ‘$A_{nq}$’ is almost identical for all these pipes and elbows and TPB specimen. This implies that J-R curves generated from all these components and specimens should be same. Figure 2 shows the J-R curve generated from TPB specimen, 3 pipes and 3 elbows mentioned in Tables 1 and 2. The J-R curves from pipes are taken from [1-2]. The J-R curves of elbows have been evaluated using the newly proposed ‘$\eta_p$’ and ‘$\gamma$’ functions and details are available in [6-8]. Figure 2 shows that J-R curve from all these components and specimens are indeed identical, because of identical stress triaxialities in a region ahead of the crack tip. This shows the role of stress triaxialities in the transferability of J-R curve from specimen to component.

**DEVELOPMENT OF NEW LIMIT LOAD EQUATIONS FOR PIPE BENDS**

Pipe bends or elbows are commonly used components in a piping system. It is important to know its limit moment for the safe operation of the plant. The term ‘limit load’ is used in this paper in a generic sense to indicate plastic collapse load. In this paper, plastic collapse load has always been evaluated by twice elastic slope (TES) criterion. Elbows may potentially contain cracks due to manufacturing defects or service related degradation mechanisms. It is very important to know the effect of cracks on the plastic collapse moment (PCM) of elbows for integrity assessment of the piping system. The PCM of any cracked component is generally expressed as product of PCM of defect-free component and a weakening factor due to the presence of crack. Therefore, before studying the PCM of any cracked component, one should know the PCM of a defect-free component. In comparison to the straight pipe, the deformation characteristics of pipe bend has additional complexities due to ovalisation of elbow cross section, which makes the deformation behaviour completely different for opening and closing mode of
bending moment. Additionally, bending moment induces both axial and circumferential stresses at a significant level in pipe bends, which makes it imperative to postulate both circumferential and axial crack configurations. Further, an elbow is often subjected to combined internal pressure and bending moment in actual service condition. Internal pressure affects the load carrying capacity of elbows (specially thin ones) quite significantly. Finally wall thickness and bend radius of elbows also determine its deformation characteristics. These large numbers of variables make the analysis of pipe bends quite elaborate and complex. As a part of the comprehensive program initiated by RSD, BARC, new closed-form equations have been proposed to evaluate plastic collapse moments of pipe bends considering most the variables mentioned above. The following cases have been studied.

Three crack configurations: (i) Defect free elbow, (ii) Throughwall circumferentially cracked elbow (iii) Throughwall axially cracked elbow

For each crack configurations two in-plane bending modes: (i) Closing mode, (ii) Opening mode
For each bending mode two loadings: (i) Pure in-plane bending moment, (ii) Combined loading of internal pressure and in-plane bending moment

For each of the above geometry and load configuration, several radii to thickness ratio (R/t) and crack sizes (for cracked elbows) have been considered. The following sections describe briefly the basic methodology followed to develop these limit moment equations, results and discussion. More details may be found in [11].

Methodology

The finite element method is used to conduct the parametric study to develop the equation for plastic collapse moment (PCM) of pipe bends. Geometrically, a 90° cracked elbow is characterized by three parameters, namely, $R_b/R$, $R/t$ and a parameter to denote crack size, which is $2\theta$ for circumferential crack and $a/D_m$ for axial crack, where $a$ is the semi-axial crack length, $\theta$ is the semi-circumferential crack angle, $R_b$ is the mean bend radius of elbow and $R$, $D_m$ and $t$ are the mean radius, mean diameter and wall thickness of the elbow cross section respectively. Table 5 shows different combinations of these parameters taken in the study. In the present analyses, the elbow is connected with straight pipes of length equal to the six times the mean cross sectional radius.

<table>
<thead>
<tr>
<th>Crack configuration: Circumferential</th>
<th>Crack configuration: Axial</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bending Mode : Closing</strong></td>
<td><strong>Bending Mode : Opening</strong></td>
</tr>
<tr>
<td>$R : 250$ mm</td>
<td>$R : 250$ mm</td>
</tr>
<tr>
<td>$R/t : 5,7,5,10,15,20$</td>
<td>$R/t : 5,10,15,20$</td>
</tr>
<tr>
<td>$R_b/R : 3$</td>
<td>$R_b/R : 3$</td>
</tr>
<tr>
<td>$20 : 0^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$</td>
<td>$20 : 0^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$</td>
</tr>
<tr>
<td>$p : 0, 0.2, 0.4, 0.6, 0.8, 0.9$</td>
<td>$p : 0, 0.2, 0.4, 0.6, 0.8, 0.9$</td>
</tr>
<tr>
<td>No. of cases : 180</td>
<td>No. of cases : 144</td>
</tr>
</tbody>
</table>

| **Bending Mode : Closing** | **Bending Mode : Opening** |
| $R : 250$ mm | $R : 250$ mm |
| $R/t : 5, 10$ | $R/t : 5, 10$ |
| $R_b/R : 2$ | $R_b/R : 2$ |
| $20 : 0^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$ | $20 : 0^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$ |
| $p : 0, 0.2, 0.4, 0.6$ | $p : 0, 0.2, 0.4, 0.6$ |
| No. of cases : 48 | No. of cases : 24 |

| **Bending Mode : Closing** | **Bending Mode : Opening** |
| $R : 250$ mm | $R : 250$ mm |
| $R/t : 5, 20$ | $R/t : 5, 20$ |
| $R_b/R : 2$ | $R_b/R : 2$ |
| $20 : 0^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$ | $20 : 0^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$ |
| $p : 0, 0.2, 0.4, 0.6$ | $p : 0, 0.2, 0.4, 0.6$ |
| No. of cases : 48 | No. of cases : 24 |

Total no. of cases: 607

Elastic-perfectly plastic material model is used in this study. The other material properties chosen are as follows: Young’s modulus = 200 GPa, Yield stress = 300 MPa, Poisson’s ratio = 0.3. Choice of these fixed values of the above-mentioned parameters does not affect the results for other cases as all the results are expressed in normalized form. Pure in-plane bending moment has been applied at the end plane of connecting straight pipe. In the present study, closing and opening bending moments are considered separately. In case of combined loading, the load is split in two components: a constant internal pressure applied first and then in-plane bending moment monotonically increasing in definite steps. This is the usual loading sequence in actual service condition. Internal pressure is normalized as, $p = PR/(t\sigma_y)$, where $P$ is the applied internal pressure and $\sigma_y$ is the material yield stress.
Results and Discussion

For each case, PCM is evaluated from the moment rotation curve by TES method. Before analyzing cracked elbows, defect-free elbows are analyzed. Subsequently, the weakening factor because of the presence of crack has been quantified by evaluating the ratio of PCM of cracked and defect-free elbows ($X = M_f/M_o$). Finally, closed-form equations of PCM are proposed by curve fitting the FE data. The results of defect-free elbows are first presented and then the effect of cracks on the PCM is shown.

**Defect-free elbow**

Figure 3 shows the variation of normalized PCM ($m_o = M_o/(4R^2t\sigma_y)$) with normalized internal pressure ($p = PR/(t\sigma_y)$) for various elbow thicknesses ($R/t$).

![Figure 3 Variation of normalized closing PCM ($m_o$) with normalized internal pressures ($p$) of defect-free elbows](image)

The ovalisation of the elbow cross section plays an important role in its collapse. The application of uniform internal pressure opposes the ovalisation of the elbow cross-section, thus delaying the collapse phenomenon. Ovalisation is more prominent in case of thin walled elbow. That is why internal pressure enhances the limit moments significantly in thin walled elbow. However, if the internal pressure is increased beyond a limit, the hoop stress due to internal pressure nullifies the beneficial effect on the limit moments and finally the limit moment starts reducing with further increase in internal pressure. The FE data of these normalized parameters ($m_o$, $p$, $h$) have been best fitted and two separate equations are proposed to evaluate the PCM of defect-free elbow under combined loading of internal pressure and in-plane closing and opening bending moment respectively:

$$m_o = \frac{M_o}{4R^2t\sigma_y} = \left[ 1.075h^{2/3} + \frac{2.071p^{1.418}}{h^{0.223}} + 8.41p^{12.129} \right] [1 - p]$$  \hspace{1cm} (6)

$$m_o = \frac{M_o}{4R^2t\sigma_y} = \left[ -0.0617 + 1.0485h^{1/3} + \frac{1.2182p}{h^{1/3}} + 7.8509p^{0.6431} \right] [1 - p]$$  \hspace{1cm} (7)

Applicability: $2 \leq (R_o/R) \leq 3$, $5 \leq (R/t) \leq 20$, $0 \leq p \leq 1$

**Throughwall Circumferentially Cracked Elbow**

**Closing mode**

Figure 4a shows the variation of PCM of TCC elbows with crack angles for $R/t$ values. Figures 4b shows the effect of internal pressure on the PCM of TCC elbows subjected to combined internal pressure and closing bending moment for one typical case of $R/t = 5$. All the data points are generated for long radius elbows ($R_o/R = 3$).
It is seen from Fig.4a that there is a threshold crack angle below which a through wall circumferential crack at elbow extrados does not weaken an elbow. This threshold crack angle increases with increasing $R/t$.

![Graph showing variation of normalized plastic collapse moment with crack angle](image)

Fig. 4  Variation of normalized plastic collapse moment with crack angle: (a) for pure closing moment and (b) combined internal pressure and closing moment

It may be seen from Fig.4b that application of internal pressure reduces the threshold crack angle and increases the weakening due to the crack (i.e. reduces the weakening factor $X$).

The FE data of these normalized parameters ($M_o$, $p$, $2\theta$) have been best fitted and equations are proposed to evaluate the PCM of TCC elbows under pure in-plane bending moment and combined internal pressure and in-plane bending moment. The proposed equations are as follows:

$$M_L = M_o X$$

$M_o$ defined by Eq.(6) and $X$ defined as a function of $(\theta/\pi)$ and normalized pressure ($p$) for various $R/t$. In case of pure in-plane closing bending moment without any internal pressure, the proposed equations of $X$ are as shown in Table 6.

<table>
<thead>
<tr>
<th>R/t</th>
<th>$A_o$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$\theta$ limits</th>
<th>$X$ values</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.1194</td>
<td>-0.7236</td>
<td>-2.0806</td>
<td>for $45^\circ \leq \theta \leq 150^\circ$ and $X = 1$ for $20 &lt; 45^\circ$</td>
<td>0.9, 0.8</td>
</tr>
<tr>
<td>7.5</td>
<td>1.1185</td>
<td>-0.3420</td>
<td>-2.5200</td>
<td>for $60^\circ \leq \theta \leq 150^\circ$ and $X = 1$ for $20 &lt; 60^\circ$</td>
<td>0.2, 0.4</td>
</tr>
<tr>
<td>10</td>
<td>0.9655</td>
<td>1.0152</td>
<td>-4.6800</td>
<td>for $60^\circ \leq \theta \leq 150^\circ$ and $X = 1$ for $20 &lt; 60^\circ$</td>
<td>0.9, 0.6</td>
</tr>
<tr>
<td>15</td>
<td>1.1400</td>
<td>0.3000</td>
<td>-3.6000</td>
<td>for $90^\circ \leq \theta \leq 150^\circ$ and $X = 1$ for $20 &lt; 90^\circ$</td>
<td>0.2, 0.4</td>
</tr>
<tr>
<td>20</td>
<td>0.6400</td>
<td>3.4200</td>
<td>-7.920</td>
<td>for $90^\circ \leq \theta \leq 150^\circ$ and $X = 1$ for $20 &lt; 90^\circ$</td>
<td>0.2, 0.4</td>
</tr>
</tbody>
</table>

In case of combined loading of internal pressure and in-plane closing bending moment, the proposed equations are as follows:

For $R/t = 5$

$$X = 1.1194 - 0.7236 \left( \frac{\theta}{\pi} \right) - 2.0806 \left( \frac{\theta}{\pi} \right)^2 - 3.4164 p^{0.8408} \left( \frac{\theta}{\pi} \right)^{2.1758}$$

(9)

For $R/t = 7.5$

$$X = 1.4423 - 1.495 \left( \frac{\theta}{\pi} \right)^{0.6687} - 2.9803 p^{0.7136} \left( \frac{\theta}{\pi} \right)^{1.7345}$$

(10)
For $R/t = 10$

\[
X = 1.6039 - 1.0847 \left( \frac{\theta}{\pi} \right)^{0.4082} - 3.1773 \rho^{0.4807} \left( \frac{\theta}{\pi} \right)^{1.4381} \tag{11}
\]

For $R/t = 15$

\[
X = 1.4298 - 0.0789 \left( \frac{\theta}{\pi} \right)^{0.5435} - 3.3789 \rho^{0.2336} \left( \frac{\theta}{\pi} \right)^{0.9644} \tag{12}
\]

For $R/t = 20$

\[
X = 7.7803 - 6.8959 \left( \frac{\theta}{\pi} \right)^{0.0231} - 4.1061 \rho^{0.5464} \left( \frac{\theta}{\pi} \right)^{1.2776} \tag{13}
\]

Applicability: $0 \leq X \leq 1, 2 \leq (R_b/R) \leq 3, 2 \theta \leq 150^\circ, 1 > p \geq 0.1$

For intermediate $R/t$, $X$ can be linearly interpolated between the adjacent $R/t$ values. However, for conservative results, the equation applicable for next lower $R/t$ may be chosen. Please note that the above Eqs.(9-13) should not be used for $p = 1$. For $0.1 > p > 0$, $p = 0.1$ values may be used for conservative result.

**Opening mode**

For pure in-plane bending moment, the proposed equation is in the form of Eq.(8) with $M_o$ defined by Eq.(7) and $X$ defined as a function of $(\theta/\pi)$ as follows:

\[
X = 1.0 - 0.8 \left( \frac{\theta}{\pi} \right) \quad \text{for} \quad 0^\circ \leq 2\theta \leq 45^\circ \tag{14}
\]

\[
= 1.127 - 1.8108 \left( \frac{\theta}{\pi} \right) \quad \text{for} \quad 45^\circ \leq 2\theta \leq 150^\circ
\]

Figure 5 shows the effect of internal pressure on the PCM of TCC elbows subjected to combined internal pressure and opening bending moment. All the data points have been generated for long radius elbows $(R_b/R = 3)$. It has been observed that for opening mode, the weakening factor $(X)$ is not a strong function of internal pressure as in the case of closing mode.

Accordingly, on a conservative estimate, a lower bound curve of weakening factor $(X)$ versus crack angle $(2\theta)$ is plotted for each $R/t$. A two-part curve-fitting is done on this lowest bound curve and the proposed equations are as follows (see Fig.5):

\[
X = 1 - 1.2776 \left( \frac{\theta}{\pi} \right) \quad \text{for} \quad 0^\circ \leq 2\theta \leq 45^\circ \tag{15}
\]

\[
= 1.3053 - 4.2133 \left( \frac{\theta}{\pi} \right) + 3.8386 \left( \frac{\theta}{\pi} \right)^2 \quad \text{for} \quad 45^\circ \leq 2\theta \leq 150^\circ
\]

Figure 5b compares the predictions of Eq.(15) for combined loading with that for pure in-plane opening moment loading. It may be noted from Fig.5b that internal pressure moderately increases the weakening effect (i.e. decreases the weakening factor $X$) of a throughwall circumferential crack at elbow intrados, compared to the pure in-plane opening moment loading ($p = 0$).

**Throughwall Axially Cracked Elbow**

As in the case of TCC elbows, PCM is evaluated from the moment rotation curve by TES method for TAC elbows as well. Penetration of crack faces signifying crack closure effect has been observed during this analysis. Consequently, contact analysis has been adopted for all the axially cracked elbows under both closing and opening bending moment to correctly capture the deformation behaviour. It is seen that consideration of contact analysis strengthens the elbow and hence enhances the PCM.
Fig. 5 Variation of weakening factor ($X$) with int. pressure and crack angle ($2\theta$) for TCC elbows: (a) Lowest bound curve for all $R/t$ and $p$ values, (b) comparison with un-pressurized case

Figure 6 shows the effect of crack on the PCM of TAC elbows subjected to closing bending moment. All the data points are generated for long radius elbows ($R_\theta/R = 3$). Axial crack size ($2a$) is normalized as crack angle ($2\alpha$) using the following relation:

$$a = R_\theta \alpha$$

(16)

where, $R_\theta$ is the mean bend radius. Accordingly, the normalized crack size parameter ($a/D_m$) is expressed as:

$$\frac{a}{D_m} = (R_\theta/D_m) \alpha$$

(17)

Normalization of crack size using crack angle ($\alpha$) is preferred here to the parameter ($a/D_m$), because $\alpha$ is independent of bend radius ($R_\theta$) which is not the case for '$a/D_m$'. Figure 6 also compares the present results with the predictions by Zahoor [12]. It may also be observed from Fig.6 that the weakening factor ($X$) of short radius elbows as predicted by Zahoor [12] is closer to the present results.

Fig. 6 Variation of normalized plastic collapse moment with axial crack angle for closing moment for: (a) pure closing moment (b) combined int. pressure and closing moment

The FE data of these normalized parameters ($X$, $\alpha$, $R/t$) have been best fitted and equations are proposed to evaluate the PCM of TAC elbows under in-plane closing bending moment. The basic form of the equation is same as Eq.(8) with $M_o$ defined by Eq.(6) and $X$ defined as a function of $\alpha/\pi$ and $R/t$. The proposed equation of $X$ is as follows:
To investigate the effect of internal pressure on the weakening factor ($X$) for TAC elbow under closing moment, a systematic study is carried out where, the above set of TAC elbows have been analyzed under combined internal pressure and bending moment loading. It has been observed that the effect of internal pressure on the $X$ vs. $\alpha$ variation is negligible. Therefore, Eqs.(6,8,18) may be used even for combined loading of internal pressure ($p \leq 0.4$) and in-plane closing moment without losing much accuracy in a conservative manner.

Following the same procedure for opening bending moment, the proposed equation of weakening factor is as follows:

$$X = 1.0 - 2.11 \left( \frac{\alpha}{\pi} \right) + \frac{1.958}{R^{0.5885}} \left( \frac{\alpha}{\pi} \right)^{0.8855} \left( \frac{R}{t} \right)^{0.2346}$$  \hspace{1cm} \text{Applicability: } 5 \leq R/t \leq 20, \hspace{0.5cm} 0 \leq 2\alpha < 90^\circ, \hspace{0.5cm} 2 \leq (R_b/R) \leq 3 \hspace{1cm} (18)$$

**DEVELOPMENT OF ELASTIC-PLASTIC J AND COD ESTIMATION SCHEME FOR THROUGHWALL CIRCUMFERENTIALLY CRACKED ELBOW UNDER IN-PLANE BENDING MOMENT**

LBB qualification of nuclear power plants requires detailed fracture analysis of piping components with postulated throughwall cracks. For this purpose, the estimation of elastic-plastic $J$-integral and COD is very essential. In LBB analyses, $J$-integral is used to calculate crack initiation and unstable ductile tearing load while COD is used to calculate crack opening area in the evaluation of leakage size crack. Elastic-plastic finite element analysis (FEA) is the most general technique to evaluate these parameters. However, FEA often requires large computational time, expertise and resources, which make the computation quite expensive. Moreover, it has to be carried out on a case-by-case basis for each piping component. To circumvent these problems, simple $J$ and COD estimation schemes emerged. However, $J$ and COD estimation schemes for cracked elbows, one of the very important geometry for LBB analyses is quite limited. In the present paper, simple $J$ and COD estimation schemes for elbow with throughwall circumferential crack at extrados/intrados subjected to in-plane closing/opening bending moment have been proposed. The following paragraphs describe very briefly the methodology followed to develop the estimation schemes. More details may be found in [13-15].

**Methodology**

Non-linear finite element analysis is carried out to determine the $J$-integral and COD of throughwall circumferentially cracked (TCC) elbows for various geometric and material combinations. Incremental flow theory of plasticity considering both geometric and material non-linearities is adopted in the present analysis. The crack is located at extrados and intrados for closing and opening bending moment respectively. Material stress-strain properties are assumed to follow Hook’s law up to yield stress and beyond yield stress, to follow Ramberg-Osgood relation. The equations are as follows:

$$\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} \quad \text{for } \sigma \leq \sigma_y \quad \text{and} \quad \frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + \alpha \left( \frac{\sigma}{\sigma_y} \right)^n \quad \text{for } \sigma > \sigma_y$$  \hspace{1cm} (20)

where, $\sigma$ is the true stress, $\varepsilon$ is the true strain, $\sigma_y$, $\sigma$, $\alpha$ and $n$ are the yield strain, yield stress, Ramberg-Osgood coefficient and hardening exponent respectively with $E\varepsilon_y = \sigma_y$, where, $E$ is the Young’s modulus. The values for some of these parameters have been fixed as: $E = 200$ GPa, $\alpha = 1$, $\sigma_y = 300$ MPa and $\nu = 0.3$, where, $\nu$ is the Poisson’s ratio. However, specific values of these parameters do not affect the estimation schemes. The Ramberg-Osgood hardening exponent ($n$) has been varied as: $n = 3,5,7$, which cover a wide range of steel material.

$J$-integral values are evaluated by finite element method. The COD at inside and outside surface are directly obtained from the FEA output. Once the elastic-plastic $J$-integral and COD are obtained from FEA, the elastic parts of these parameters are subtracted to get the fully plastic $J$-integral and COD.
There is no solution available for elastic COD of TCC elbow subjected to in-plane bending moment. Accordingly, it has been developed here. The equation of elastic COD is as follows:

\[ \delta_e = \frac{4MV_2}{\pi D_o^2 E} \]  

(21)

where, \( D_o \) is the outer diameter of elbow cross section, \( V_2 \) is a parameter which depends on \( R/t \) and crack angle (\( 2\theta \)). The \( V_2 \) parameter is calculated from the initial few elastic load steps for each geometry. It is calculated separately for COD at inside and outside surface and for all combinations of \( R/t \) and \( 2\theta \). The plastic component of \( J \)-integral and COD are obtained from FEA as follows:

\[ J_p = J^E - J_e \quad \text{and} \quad \delta_p = \delta^E - \delta_e \]  

(22)

where, \( J^E \) and \( \delta^E \) are the total \( J \)-integral and COD respectively obtained from FEA and \( J_e \) and \( \delta_e \) are their elastic components. The plastic components of \( J \)-integral and COD for throughwall circumferentially cracked pipe subjected to bending moment are, in general, expressed as follows [12]:

\[ J_p = \alpha \sigma_y \varepsilon_y \pi R \left( 1 - \frac{\theta}{\pi} \right)^2 h_1 \left( \frac{M}{M_L} \right) \left( \frac{M}{M_L} \right)^n \] \quad \text{and} \quad \delta_p = \alpha \varepsilon_y \pi R h_2 \left( \frac{M}{M_L} \right) \]  

(23)

where, \( M \) is the applied moment, \( M_L \) is the limit moment which is proportional to \( \sigma_y \) and \( \alpha \) and \( n \) are the constants in Ramberg-Osgood Eq.(20), \( h_1 \) and \( h_2 \) are the non-dimensional plastic influence functions which ideally depend only on \( R/t \), \( n \) and \( 2\theta \).

The same form is retained in this paper for plastic components of \( J \)-integral and COD for TCC elbow subjected to in-plane closing/opening moment.

**Proposed \( J \) and COD estimation schemes**

The geometry constants and also the plastic influence functions i.e. \( h_1 \) for \( J \)-integral in Eq.(23) and \( h_2 \) for COD at inside and outside surface in Eq.(23) are evaluated for various geometric cases and Ramberg-Osgood hardening index, \( n = 3, 5, 7 \). Detailed values of these constants are available in [12-15]. The proposed \( J \)-integral and COD equations are as follows:

\[ J = J_e + J_p; \quad \delta = \delta_e + \delta_p; \quad J_e = \frac{K^2}{E} \]  

\[ J_p = \frac{1}{\alpha^{n+1}} \sigma_y \varepsilon_y \pi R \left( 1 - \frac{\theta}{\pi} \right)^2 h_1 \left( \frac{\theta}{\pi}, n, \frac{R}{t}, \frac{M}{M_L} \right) \left( \frac{M}{M_L} \right)^{n+1} \] \quad \text{for} \quad \frac{M}{M_L} \leq 1.0 \]  

(24)

\[ \alpha^{1/n} \] in the above eqn. is replaced with \( \alpha \) for \( M/M_L \geq 1.2 \) and it is linearly interpolated for \( 1.0 < M/M_L < 1.2 \)

\[ \delta_p = \frac{1}{\alpha^{1/n}} \varepsilon_y \pi R h_2 \left( \frac{\theta}{\pi}, n, \frac{R}{t}, \frac{M}{M_L} \right) \left( \frac{M}{M_L} \right)^n \] \quad \text{for} \quad \frac{M}{M_L} \leq 1.0 \]  

(25)

\[ \alpha^{1/n} \] in the above eqn. is replaced with \( \alpha \) for \( M/M_L \geq 1.2 \) and it is linearly interpolated for \( 1.0 < M/M_L < 1.2 \)

where, the subscripts ‘e’ and ‘p’ indicate the elastic and fully plastic values respectively, \( M_L \) is the limit moment.

**CONCLUSIONS**

Several new equations have been proposed for improved and more accurate integrity assessment of piping components. These equations have also been experimentally validated using the test data generated as a part of the comprehensive Component Integrity Test Program by RSD, BARC.
REFERENCES


