ESTIMATION OF RELIABILITY LEVELS ASSOCIATED WITH FITNESS FOR SERVICE CODES

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ABSTRACT

This paper presents the estimation of the reliability levels assessed for cracked components found acceptable as per the fitness for service evaluation code. The code considered in this study is the Failure Assessment Diagram based acceptance criteria of ASME Section XI. This acceptance criterion is built on the concepts of fracture mechanics. The parameters which enter the acceptance criteria are applied loading, crack size and the material properties (tensile and fracture) of the component being inspected. These parameters are known to exhibit uncertainty in their values. The fracture mechanics model used also has an associated modeling uncertainty. The codes address these uncertainties by providing factor of safety on the loading parameters. However, this does not give an indication of the level of safety associated with the component being evaluated. Estimation of the reliability based on the concepts of structural reliability gives an estimate of the safety level. This paper presents a study to estimate the probability of failure of the cracked components considered safe as per the code procedures. The study is done for representative carbon (SA 333 Gr.6) and stainless steels (X6CrNiNb1810) used for straight pipes in the nuclear power plants. The uncertainty in the material properties is characterized by probability distribution functions. The data for this is obtained from the experiments conducted on these materials. The fitting was checked for Normal, lognormal and Weibull distributions. The modeling uncertainty in the fracture mechanics model is evaluated using the component integrity tests performed on the pipes. The probability of failure is obtained using the structural reliability methods like Monte Carlo simulation and First Order Reliability Method (FORM).

INTRODUCTION

Fitness for service codes ensures safe operation of the piping in plants. These codes address the acceptance criteria for the service induced flaws which are detected during the in-service inspection of piping components. The treatment of flaws as cracks gives a conservative estimation for continued operation. The acceptance of crack like flaws is based on the principles of fracture mechanics. Based on the ductility of the piping material the failure mode in the presence of crack is modeled as linear elastic fracture mechanics (brittle materials), net section collapse (very ductile materials) and elasto-plastic fracture mechanics for materials in between. Stress intensity factors are used as the failure criteria for brittle materials. This factor is measured against the material fracture toughness for safety assessment. Handbook solutions for a variety of cracked geometries are available to estimate the stress intensity factor. The limit load is used as the acceptance criteria for very ductile materials. Handbook solutions are again available for a variety of crack configurations in piping components. The elasto-plastic fracture mechanics is applicable to a variety of steels used for piping components in plants. The assessment of safety of such components requires the evaluation of the J-integral and its comparison with material's fracture toughness. The availability of J-integral solutions for the common piping geometries, crack configurations and load combinations is limited. The estimation of J-integral is an involved process and requires considerable expert knowledge. Thus for such materials a fitness for service assessment using the inspection data is a tedious process for the plant personnel. To address this difficulty and to provide a quick assessment of cracked components approximate methods have been given. The most popular amongst these is the Failure Assessment Diagram based method [1, 2]. The fitness for purpose codes across industries use this estimation procedure [2, 3, 4, 5]. Section XI of the ASME Boiler and Pressure Vessel Code [6] gives the flaw acceptance criteria for the cracked components in nuclear power plants. One of the assessment methods given in this code for analyzing the acceptance of cracked piping is based on Failure Assessment Diagram. This appears in the non-mandatory appendix-H of the code. The flaw acceptance procedure is dependent on the piping geometry, crack size, tensile and fracture properties and the applied load. Many of these parameters exhibit considerable scatter in their values. The code specifies a suitable factor of safety on stress to address this uncertainty. The factor of safety though ensures safe operation but do not quantify the safety levels. The safety levels can be quantified using the principles of "Structural Reliability". The safety levels are quantified in terms of reliability level or its complimentary, the "probability of failure". The probabilistic methodology involves quantifying the uncertainty in the design variables using probability density functions. This fitting is achieved using the established statistical procedures if sufficient data is available. The design equations given by the code are treated as the "limit state". The probability that the limit state is violated is estimated using the structural reliability methods. The code should ensure that the probability of failure remains the same across components and materials. An alternate way of design called "Partial Safety factor based approach" has evolved which ensures this consistency. This involves the use of a separate factor of safety on each of the uncertain design parameters in the code equation rather than a single factor of safety. This method of designing is used in civil engineering since long. Few mechanical design codes have appeared which use this methodology for design [3, 7]. The Partial Safety Factor based design ensures that all designs correspond to the same probability of failure. Thus, to determine this probability of failure, the reliability levels associated with the existing code needs to be evaluated. There is modeling uncertainty associated with the fracture mechanics models which gives the acceptance criteria. This is estimated using the test data.

This paper assesses the reliability levels of the acceptance criteria given in the ASME Section XI, Non-mandatory Appendix H [6] for nuclear piping. The estimation is performed for two kinds of steel, SA 333 Gr6 (carbon steel) and X6CrNiNb1810 (austenitic steel). Thereby, straight pipes with circumferential part through wall cracks are considered. The uncertainty in the tensile and fracture properties is quantified using the test data on these materials. The modeling uncertainty is also estimated using the component tests on the piping. Structural reliability methods like Monte Carlo simulations and the First Order Reliability Method are employed to estimate the probability of failure.

FAILURE ASSESSMENT DIAGRAM (FAD)

The FAD based method was developed to simplify the elastic plastic fracture analysis. The J-integral calculation can be broken up into two parts, elastic and plastic. The elastic part can be estimated using the stress intensity factor solutions. The Calculation of the plastic part of the J integral is difficult. The R6 methodology helps in performing a detailed EPFM analysis using the stress intensity factor only. The method is a double criteria approach. It simultaneously checks for failure against plastic collapse too. Considering fig. 1, the x-axis represents a ratio between the applied load M and the limit load M_l. The failure based on net section collapse criteria is when this ratio exceeds 1. The y-axis represents the ratio between the applied J and the fracture toughness J_c. The failure based on fracture mechanics *(crack growth initiation)* will happen when this ratio is greater than 1. Thus the safe and the unsafe regions can be defined. It can be seen that the applied J has an elastic (J_e) and a plastic (J_p) part. The elastic part contribution to the J decreases as the load increases. FAD based methods envisages a failure assessment line (FAL), which joins all the points that demarcate the elastic and the plastic part of the J integral. If such a line is obtained, only the elastic part of the J may be evaluated and then a check for safe condition is made such that the ratio J_e/J_c is within the FAL at the given applied load.

The basic equation for FAL can be given as (eq. (1))

$$K_r = \sqrt{\frac{J_e}{J}} \qquad \begin{array}{l} J = J_e + J_p \\ J_e = \text{elastic part of J; } J_p = \text{platic part of J} \end{array} \tag{1}$$

For assessment of safety FAD based methods use two ratios K_r' and L_r'. K_r' is a measure of nearness to brittle fracture. K_r' is defined by eq.2.

$$K_r^{'} = \sqrt{\frac{J_e}{J_c}} = \frac{K_I}{K_c} \qquad \begin{array}{l} \text{K}_\text{I} = \text{stress intensity factor} \\ K_c = \sqrt{J_c E} \text{ and } K = \sqrt{J_e E} \end{array} \tag{2}$$

$$J_c, K_c = \text{ fracture toughness} \quad E = \text{ Young's Modulus}$$

S_r' is a measure of nearness to plastic collapse. S_r' is defined by eq.3.

$$S_r^{'} = \frac{M}{M_l} \qquad \begin{array}{l} M = \text{ applied load} \\ M_l = \text{ limit load} \end{array} \tag{3}$$

The eq.(1) involves estimation of the plastic part of the J-integral. Approximate but conservative FAL have been derived which eliminate the need for estimation of the plastic part of the J-integral [1, 2]. The FAD given in ASME Boiler and Pressure Vessel code, Section 11, Appendix H uses one approximation which is given by Bloom [1]. The Failure Assessment Diagram given in ASME is shown in fig. 2. Separate curves are given for austenitic and ferritic steels. The failure assessment procedure in ASME for pipes with circumferential part-through wall flaws is as follows.

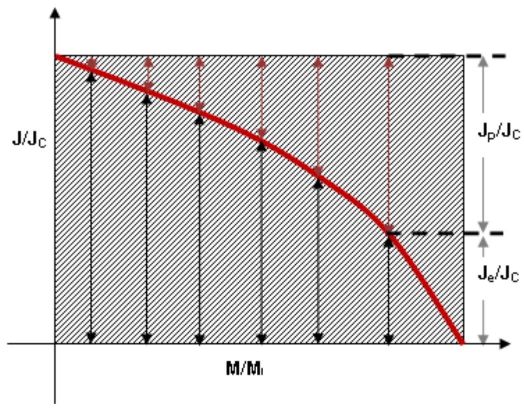

Figure 1: Failure Assessment Diagram (FAD)

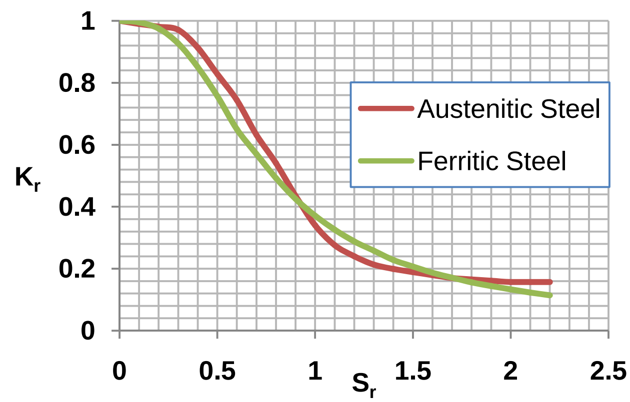

Figure 2: FAD from ASME Section XI, Appendix H

The nearness to plastic collapse is measured by the parameter S_r' given by eq. (4).

$$S_r' = (SF_m)\sigma_m/\sigma_m'$$
$$\sigma_m' = \sigma_y \psi \Gamma_m$$
$$\psi = \frac{-\pi\sigma_b}{8\sigma_m} + \left[\left(\frac{\pi\sigma_b}{8\sigma_m}\right)^2 + 1\right]^{0.5}$$
$$\Gamma_m = \left[R_2^2 - R_c^2 + (1-\theta/\pi)(R_2^2 - R_c^2)\right] \Big/ (R_2^2 - R_1^2)$$
$$R_c = R_1 + a \qquad (4)$$

$\sigma_m =$ primary membrane stress at flaw
$SF_m =$ structural factor on primary membrane stress
$\sigma_y =$ yield strength
$\sigma_b =$ primary bending stress at flaw
$R_1 =$ inside radius
$R_2 =$ outside radius
$a =$ flaw depth

The nearness to brittle fracture in absence of secondary stresses is measured by the parameter Kr' given by eq. 5.

$$K_r' = (J_e/J_R)^{0.5}$$
$$J_e = K_I^2/E'$$
$$K_I = (SF_m)\sigma_m F_m(\pi a)^{0.5} + (SF_b)\sigma_b F_b(\pi a)^{0.5} \qquad (5)$$

$J_R =$ fracture toughness for prescribed crack extension
$E' = E/(1-\nu^2)$
$SF_b =$ structural factor on primary bending stress
$F_m, F_b =$ geometrical factors, defined in the code

The factors of safety SF_m and SF_b are given for different service levels of loading. The values of SF_m for Service level A, B, C, D are 2.7, 2.4, 1.8 and 1.3 respectively. The values of SF_b for Service level A, B, C, D are 2.3, 2.0, 1.6 and 1.4 respectively. The ASME Section XI permits the use of J-resistance curves for assessing the safety. In this paper we have considered the crack growth initiation as the failure mode.

STRUCTURAL RELIABILITY

A structural component fails when the applied loads 'S' exceed its resistance 'R' i.e. R – S < 0. Generally, this condition is expressed in terms of a failure equation or a limit state equation 'g'. Let g(**x**) be a function of variables x_1, x_2, x_3,…. x_n. The failure is defined by the condition g(**x**) < 0. The variables **x** are modeled as random variables in the probabilistic analysis. The probability density functions for these random variables are obtained by using statistical techniques on data. The failure probability P_f is defined by eq. (6).

$$P_f = \text{Probability}(g < 0) \qquad (6)$$

The probability of failure can be written in terms of the marginal density functions f_i of the random variables **x**, equation 7.

$$P_f = \int_{g\le 0} f_X(\mathbf{x})d\mathbf{x} = \int_{g\le 0} f_1(x_1)\ldots f_n(x_n)dx_1\ldots dx_n \qquad (7)$$

Thus the calculation of probability of failure essentially reduces to the calculation of the integral given by equation 7. The techniques broadly fall in two categories. The first category includes approximation methods like First Order Reliability Method (FORM) which are known as fast probability integration methods. The second category comprises of simulation-based methods based on Monte Carlo Simulation. The FORM is very powerful and requires very less computational resources but may not give correct results for highly nonlinear failure functions. A classical Monte Carlo method gives the most accurate results but is computationally very expensive. It may not work for low probability numbers ($<10^{-6}$).

First Order Reliability Method (FORM)

The FORM method is based on the premise that, if a limit state function is linear and the random variables are un-correlated and have normal distribution, then the reliability index 'β' of failure is given by the shortest distance between the failure state equation and the origin in the standard normal space. The probability of failure is estimated from β using eq. 8.

$$P_f = \Phi(-\beta) \qquad \Phi(\bullet) = \text{standard normal distribution function} \tag{8}$$

Engineering problems usually have non-linear limit states. The stochastic variables are generally non-normal and correlated. To solve such problems, first the correlations of non-normal variables are transformed into equivalent correlation of normal variables by applying Nataf's transformations [8]. The correlated variables are then decoupled using Choleski's decomposition method. The resulting problem is then solved using the Rackwitz-Fiesser algorithm [9,10]. It involves the transformation of non-normal variables into an equivalent normal space. The failure function is linearized at the point of interest by using linear terms of Taylor's expansion. The point of interest is the point of minimum distance from the origin in the reduced space. It is popularly called as design point or most probable point (MPP). This point is arrived by iterative solutions. This method is very fast. The method is not suitable if the limit state function is highly non-linear or there are multiple design points.

Classical Monte Carlo method (CMC)

In this method the random variables are simulated randomly based on their distribution. These simulated variables are then fed into the limit state equation to check the failure. This exercise is repeated a number of times. The probability of failure is given by eq. 9.

$$P_f = \frac{N_f}{N} \qquad \begin{aligned} N_f &= \text{number of failures} \\ N &= \text{number of simulations} \end{aligned} \tag{9}$$

The error estimate in the simulation methods is given by the coefficient of variation (cov) of the result. It is given by eq. 10.

$$\text{cov} = \sqrt{\frac{(1-P_f)}{NP_f}} \tag{10}$$

In this paper the limit state is given by eq. 11 and shown in fig. 3. The secondary stresses are not considered in this study.

$$g = R_L - R_{ASS} \qquad \begin{aligned} R_{ASS} &= \text{distance of assessment point from origin} \\ R_L &= \text{distance of crack growth intitiation point from origin} \end{aligned} \tag{11}$$

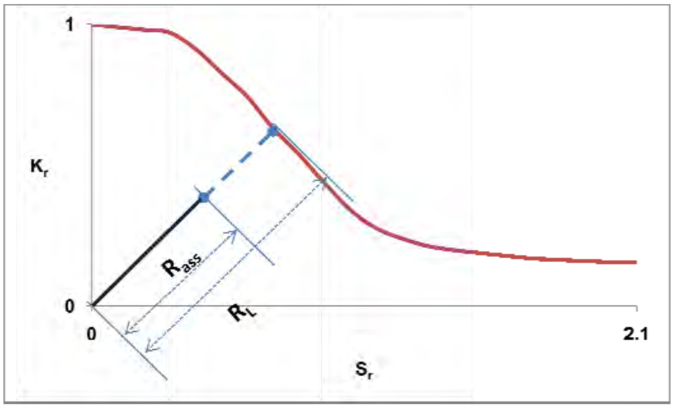

Figure 3: Limit State

STATISTICAL ESTIMATION

The estimation of the probability of failure requires knowledge of the distribution function of the uncertain variables involved in the design equation. The design variables involved in the flaw acceptance criteria are the geometry of the pipe (radius, thickness, crack size), tensile properties, fracture properties and the applied loading. The uncertainty in the radius and thickness of the pipe typically is not appreciable [11]. The uncertainty in crack size is due to the non-destructive testing technology used for sizing the crack. Bloom [12] has used the lognormal distribution for the crack depth with a coefficient of variation (cov) of 30% for cracks less than 5mm and 20% for cracks greater than 5mm. This paper is the basis for the Code Case N-654 [7] which gives the partial safety factor based guidelines for flaw acceptance. In this paper the same distribution is used for crack depth. In the same paper the distribution for load is given as a Type I extreme value distribution with a cov of 10-30%. The distribution of the tensile properties and fracture properties is estimated from the test data. A Similar exercise for similar materials has been reported earlier by Roos et. al. [13].

As a part of the comprehensive Component Integrity Test Program in RSD, BARC, a large number of fracture tests was conducted on full scale pipes and pipe bends of carbon steel material SA333Gr6 of diameters ranging from 200 mm to 600 mm with various sizes and configurations (e.g. circumferential, axial etc.) of crack under various loading conditions. Additionally, small specimens (e.g. tensile, CT and TPB) machined from these piping components were tested to generate the tensile and fracture properties of these materials. Statistical analyses of these test data are presented in this paper. The straight pipe tests conducted at the MPA Stuttgart refer to a series of experiments performed within several research programs in close cooperation with the German industry. The research was focused on the austenitic steel "X 6 CrNiNb 18 10" (DIN/EN steel name), DIN/EN steel number 1.4550. Besides the large component tests, small specimen tests were also performed in order to determine the fracture toughness (acc. to ASTM E813-89) and the tensile properties (acc. to DIN 50125) of the material.

The commonly used distributions for tensile and fracture properties of steels are the Normal, Lognormal and Weibull distribution. Characterizing the uncertainty in the variables of the limit state function in the form of a probability distribution function is the backbone of a good structural reliability analysis. This study uses the data obtained from specimen tests done on SA333Gr6 and X6CrNiNb1810. There are two main methods for estimating the distribution associated with a random variable. The first one is based on a graphical approach where the histogram of the data is plotted and an estimate of the distribution is made. The data is later plotted on the appropriate probability paper to make the assessment. This method is intuitive and gives a graphical picture of the chosen distribution. The other method is more formal and is based on the principles of statistics. This method is termed as “Goodness of Fit test”. The estimation of a probability distribution is more quantifiable and reliable using this method. In this method the practice is to assume a probability distribution function and then the statistical check is made on this assumption. The statistical check can be made for the density function or the distribution function. The tests which use the density function are termed as “area tests”. When the distribution function is analyzed for the test purpose, the test is termed as “distance test”. The Chi-square test is the most popular method within the category of “area tests”. It however works well mainly if the sample size is large. In the category of distance tests, the popular ones are the Anderson-Darling (AD) and Kolmogorov-Smirnov tests. These can work with small sample sizes as well as with large sample sizes. The AD test has been used here for ascertaining the distribution function.

AD Test for Normality [14]

As the data set is small, the Anderson-Darling Goodness of Fit technique is utilized to ascertain a suitable distribution for the given material property. The test requires the knowledge of the distribution parameters. The mean and standard deviation is estimated from the sample. The test statistic is given by eq. 12.

$$AD = \left[\sum_{i=1}^{n} \frac{1-2i}{n} \left\{ \ln\left(F_0\left[z_i\right]\right) + \ln\left(1-F_0\left[z_{n+1-i}\right]\right) \right\} \right] - n \qquad (12)$$

$F_0 =$ Normal Distribution Function
$n =$ sample size
$z_i =$ sorted sample

This statistic is used for estimating the Observed Significance Level (OSL) or the p-value. The OSL is generally compared with 0.05 as the acceptability criteria. The OSL is calculated using the following formulation [15]

$$
\begin{aligned}
& ast = AD(1 + 0.75/n + 2.25/n^2) \\
& 0.600 < ast < 13 \quad OSL = \exp(1.2937 - 5.709ast + 0.0186ast^2) \\
& 0.340 < ast < 0.600 \quad OSL = \exp(0.9177 - 4.279ast - 1.38ast^2) \\
& 0.200 < ast < 0.340 \quad OSL = 1 - \exp(-8.318 + 42.796ast - 59.938ast^2) \\
& ast < 0.200 \quad OSL = 1 - \exp(-13.436 + 101.14ast - 223.73ast^2)
\end{aligned} \tag{13}
$$

AD Test for Lognormal Distribution [14]

The procedure for testing for the Lognormal distribution is similar to the Normal distribution. The natural logarithmic values of the sample are taken for checking as per the AD test used for checking Normality.

AD Test for Weibull Disribution [14]

The distribution function for a random variable following a Weibull distribution is given in eq. 13.

$$F(x) = 1 - \exp\left[-\left(\frac{x-\delta}{\theta}\right)^{\zeta}\right] \tag{13}$$

x = random variable

$F(\bullet)$ = distribution function

ζ = shape parameter

θ = scale parameter

δ = location parameter

Before the AD test can be applied for the Weibull distribution, the parameters of the distribution need to be estimated. There are four methods to estimate these parameters [16]. These are Probability plotting, Hazard plotting, Maximum likelihood and Linear estimators. For limited samples the method of hazard plotting gives the best results [16]. This study uses the Hazard plotting method to estimate the distribution parameters. The Hazard function for the Weibull distribution is given by eq. 14.

$$H(x-\delta) = -\ln[1 - F(x-\delta)] \tag{14}$$

H = Hazard function

F = Weibull distribution

δ = location parameter

Using eq. 13 and eq. 14.

$$\ln H(x-\delta) = \zeta \ln(x-\delta) - \zeta \ln\theta \tag{15}$$

Plotting $\ln(x-\delta)$ on the x-axis and $\ln H(x-\delta)$ on the y-axis and fitting a straight line through these points helps in determining ζ from slope and θ from y-intercept. The value of δ is assumed initially and varied parametrically such that the best straight line fit is achieved.

The test statistic for AD test in case of Weibull distribution is given by [14, 16]

$$AD = \left[\sum_{i=1}^{n} \frac{1-2i}{n}\left\{\ln\left(F_0[z_i]\right) + \ln\left(1 - F_0[z_{n+1-i}]\right)\right\}\right] - n \tag{16}$$

F_0 = Weibull Distribution Function

n = sample size

z_i = sorted sample

The OSL is given by eq. 17.

$$
\begin{aligned}
& ast = AD(1 + 0.2/\sqrt{n}) \\
& OSL = 1/\{1 + \exp(-0.1 + 1.24\ln(ast) + 4.48ast)
\end{aligned} \tag{17}
$$

The OSL is generally compared with 0.05 as the acceptability criteria.

The methodology followed is demonstrated for the Yield strength (σ_y) of SA333Gr6. The results for all other materials were obtained in similar manner and are presented in tabular form.

σ_y (MPa)	255.8	319	304.8	303.5	326.3	328.7	332.2	321.2	310.2	341.9	304.1
Sample Size = 11	Mean = 313.43			Standard Deviation = 22.90			Coefficient of variation = 0.07				

Testing for Normal Distribution using Anderson-Darling Goodness of Fit Test (Typically the test is done for a significance level of 0.05)

The calculations are shown in tabular form.

i	Sorted Data z_i	$F_0(z_i)$	$\ln[F_0(z_i)]$	n+1-i	$F_0(z_{n+1-i})$	$1-F_0(z_{n+1-i})$	$\ln(1-F_0(z_{n+1-i}))$	$\ln(F_0(z_i))$ + $\ln(1-F_0(z_{n+1-i}))$	multiply with (1-2i)/n
1	255.8	0.006	-5.128	11	0.893	0.107	-2.236	-7.364	0.669

2	303.5	0.332	-1.102	10	0.794	0.206	-1.579	-2.681	0.731
3	304.1	0.342	-1.073	9	0.748	0.252	-1.377	-2.450	1.114
4	304.8	0.353	-1.041	8	0.713	0.287	-1.248	-2.289	1.457
5	310.2	0.444	-0.812	7	0.633	0.367	-1.002	-1.814	1.484
6	319	0.596	-0.517	6	0.596	0.404	-0.907	-1.424	1.424
7	321.2	0.633	-0.458	5	0.444	0.556	-0.587	-1.044	1.234
8	326.3	0.713	-0.338	4	0.353	0.647	-0.436	-0.774	1.055
9	328.7	0.748	-0.291	3	0.342	0.658	-0.418	-0.709	1.096
10	332.2	0.794	-0.231	2	0.332	0.668	-0.404	-0.635	1.097
11	341.9	0.893	-0.113	1	0.006	0.994	-0.006	-0.119	0.227
								Total	11.589

AD = 11.589 – 11 = 0.589

$ast = AD(1 + 0.75/n + 2.25/n^2) = 0.640$

$OSL = \exp(0.9177 - 4.279ast - 1.38ast^2) = 0.09$

Typically the decision is made over a critical level = 0.05. If the OSL value is greater than 0.05, this means that the Normal distribution is a possible candidate (not the only one) to represent the data. Had OSL been less than 0.05 then the Normal distribution surely would have been disqualified as distribution to represent this data.

Testing for Lognormal Distribution using Anderson-Darling Goodness of Fit Test

Here the fitting was done after taking the natural logarithm of the data points. The OSL obtained was 0.04, hence the lognormal distribution was ruled out as a possible distribution.

Testing for Weibull Distribution using Anderson-Darling Goodness of Fit Test

Using the data, a straight line fit to the following equation is done.

$$\ln H(x-\delta) = \zeta \ln(x-\delta) - \zeta \ln\theta$$

The value of δ is varied from 0 to the minimum point of the data set (255.8 in this case). For each value the straight line is fitted. The value of δ is chosen such that the best fit line is obtained. In this case, the best fit was obtained for δ = 0.

The calculations to estimate the distribution parameters are shown in the table.

i	Sorted Data z_i	$z_i - \delta$	n+1-i	$h(z_i)=1/(n+1-i)$	$H(z_i) = \sum h(z_i)$	$\ln(z_i - \delta)$	$\ln(H\{z_i\})$
1	255.8	255.8	11	0.091	0.091	5.544	-2.398
2	303.5	303.5	10	0.100	0.191	5.715	-1.656
3	304.1	304.1	9	0.111	0.302	5.717	-1.197
4	304.8	304.8	8	0.125	0.427	5.720	-0.851
5	310.2	310.2	7	0.143	0.570	5.737	-0.562
6	319	319	6	0.167	0.737	5.765	-0.306
7	321.2	321.2	5	0.200	0.937	5.772	-0.066
8	326.3	326.3	4	0.250	1.187	5.788	0.171

9	328.7	328.7	3	0.333	1.520	5.795	0.419
10	332.2	332.2	2	0.500	2.020	5.806	0.703
11	341.9	341.9	1	1.000	3.020	5.835	1.105

Plotting $\ln(x-\delta)$ on the x-axis and $\ln H(x-\delta)$ on the y-axis and fitting a straight line through these points helps in determining ζ from slope and θ from y-intercept. For the given set the graph is shown in fig. 4.

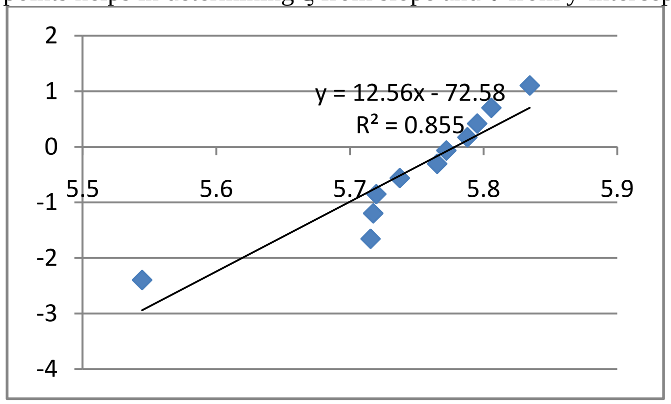

Figure 4: Estimating parameters for the Weibull distribution

From the straight line fit, the parameter ζ is 12.56. The parameter θ is calculated from the y-intercept and is 323.33.

$$F(x) = 1 - \exp\left[-\left(\frac{x-\delta}{\theta}\right)^{\zeta}\right]$$

$$\zeta = 12.56 \quad \theta = 323.33 \quad \delta = 0$$

AD test calculations show that the OSL value is 0.05. Comparing the OSL of the three distributions, the Normal distribution is the best choice among these distributions. Similarly, the distribution functions were obtained for the tensile and fracture properties of SA333Gr6 and X6CrNiNb1810 steel. The results are shown in Table 1.

Table 1: Statistical Data

	σ_y			σ_u			J_c		
Material	Distribution	Mean	Standard Deviation	Distribution	Mean	Standard Deviation	Distribution	Mean	Standard Deviation
SA 333 Gr6	Normal	313.42	22.9	Normal	508.03	28.7	Lognormal	295.72	69.94
X6CrNiNb1810	Lognormal	256.92	1.074	Fixed	615.38	-	Lognormal	280.8	41.38

MODELING UNCERTAINTY

There is always uncertainty in the modeling of failure. In case of the FAD based procedure the modeling uncertainty will be due to the assessment of K_I, limit load and the FAL. The uncertainty would be different for different crack configurations and for different materials. In order to approximate this uncertainty, we have compared the limit state given by the FAD defined in the code with the experimental data. The modeling uncertainty is estimated as (eq.16) [17]. This is shown in fig. 5.

$$M_U = R_{EXP} - R_{FAD} \tag{16}$$

Following this, the limit state for the estimation of the probability of failure is given by eq.17.

$$g = R_L + M_U - R_{ASS} \qquad \begin{array}{l} R_{ASS} = \text{ distance of assessment point from origin} \\ R_L = \text{ distance of crack growth intitiation point from origin} \end{array} \tag{17}$$

The experimental points on the ASME FAD for SA 333 Gr6 steel are shown in Fig. 6. The modeling uncertainty is represented by a Normal distribution. The mean of M_U is 0.3 and the standard deviation is 0.14.

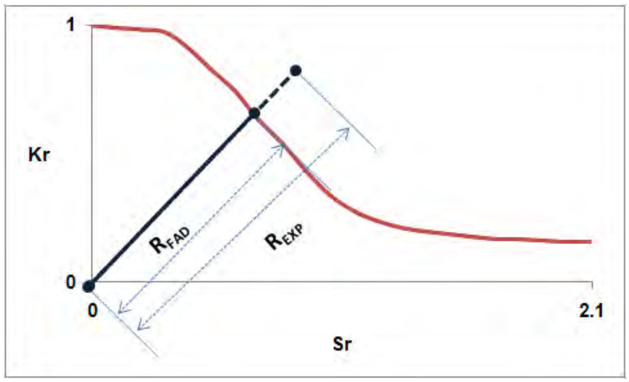

Figure 5: Modeling Uncertainty

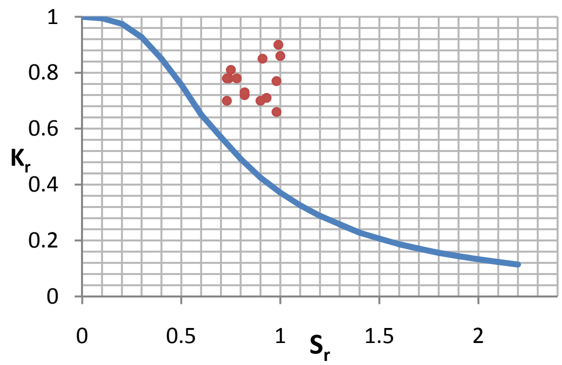

Figure 6: Experimental data for SA 333 Gr6

PROBABILITY OF FAILURE

The probability of failure is estimated for a representative pipe for both the materials. For SA333 Gr. 6, the pipe taken has an outer diameter of 508 mm, thickness of 40 mm, and is subjected to an internal pressure of 9.8MPa. For X6CrNiNb1810 the pipe considered has an outer diameter of 331 mm, thickness of 32 mm, and is subjected to an internal pressure of 16 MPa. The bending moment considered for the study is the maximum permitted by the code. The reliability levels are estimated by the code equations without the factors of safety. The probability of failure is estimated for the allowable flaw sizes given by ASME Section XI, Appendix H. Fig. 7 shows the reliability index plotted for the service Level D condition for SA333 Gr6 pipe. These curves are drawn considering the uncertainty in the material properties only. The reliability index is related to the probability of failure by eq.8 (β = 3.1 $P_f = 9.68x10^{-4}$, β = 3.6 $P_f = 1.59x10^{-4}$). The reliability index is plotted for different a/t ratios and different crack lengths). The reliability index is calculated using Monte Carlo as well as FORM. The difference in results is not significant. The reliability index across the crack size is in a narrow range. Fig. 8 shows the results for the SA333 Gr6 pipe for Service Level A, B, and C. It is seen that for each Service Level the variation in the reliability levels for different crack sizes is not significant. The reliability index was subsequently obtained considering the uncertainty in the load and crack size also. Fig. 9 gives the reliability index for Level A loading. The minimum reliability index obtained is 3.5 which would correspond to a probability of failure of $2x10^{-4}$. The reliability index for Level D loading for all crack configurations was in the narrow range of 1.21 to 1.23. The corresponding probability of failure is around 0.1 which is quite high. It can be argued that the load value used in the design calculation is a conservative estimate and is typically 95 percentile value. The results were obtained considering the design load as the 95 percentile value. The corresponding mean value is obtained considering the Gumbel distribution with 30% cov. The results for this case are shown in fig. 10. The minimum reliability index in this case is 2.3, which corresponds to a probability of failure of 0.01. The reliability levels were further investigated using the modeling error term. The results are shown in fig. 11.

Similar studies were undertaken for the pipe made of X6CrNiNb1810 steel. The reliability index for Level A loading for all crack configurations considering the uncertainty in the material properties, crack size and loading is shown in fig.12. The reliability index for Level D loading for all crack configurations considering the uncertainty in the material properties, crack size and loading varied in a narrow range with the minimum of 1.27 (P_f = 0.1). The design loading is now taken as the 95 percentile as with the previous material and the reliability index obtained is shown in fig. 13.

CONCLUSION

This paper has tried to quantify the reliability levels associated with the ASME Section XI, FAD based flaw assessment criteria. The uncertainty in the crack geometry, material properties, load and the modeling is considered while evaluating the reliability. Of these, the uncertainty in the material properties and the modeling is evaluated from the actual test data on the two steels (SA333 Gr.6 and X6CrNiNb1810). For the SA333 Gr.6 steel the minimum probability of failure for Level A loading is of the order of $2x10^{-4}$ for the maximum loading allowed for the crack configurations. The consideration of 95 percentile load and the modeling uncertainty gives a minimum probability of failure of 4.3x10^{-6}. This study does not take the credit of the use of the J-Resistance curve permitted by the code. The code target of the probability of failure of the order of 10^{-6} for Level A loading [12] is met. For Level D loading the probability of failure considering the design load as the mean load came out to be quite high (0.1 per occurrence of Level D loading). If the design load is taken as the 95 percentile load the minimum probability of failure was estimated to 10^{-2} per occurrence of Level D loading. The consideration of modeling uncertainty gave an estimate for Level D as 1.6x10^{-3} per occurrence of Level D loading. This number is again as per the code target of 10^{-3} [12]. Similar studies for the X6CrNiNb1810 steel show that for Level A loading the minimum probability of failure is 5x10^{-5}. This estimate does not consider the J-resistance curve and the modeling uncertainty. For Level D the minimum probability of failure is 8x10^{-3} per occurrence of Level D loading. This again is the code desired level. The sensitivity analysis has shown that the reliability estimates are primarily dependent on uncertainty in the crack size and the modeling uncertainty. Thus, to increase the reliability levels the crack sizing is most important.

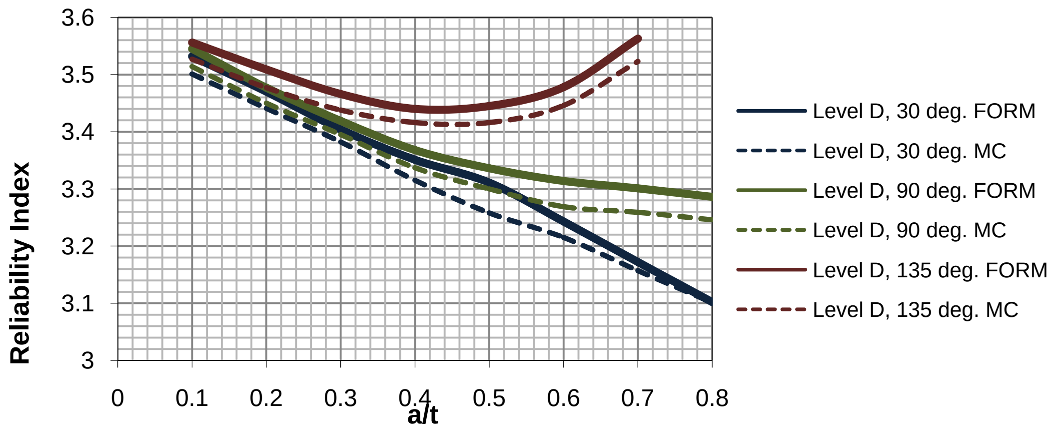

Figure 7: SA333 Gr. 6, Level D, FORM, Monte Carlo Results

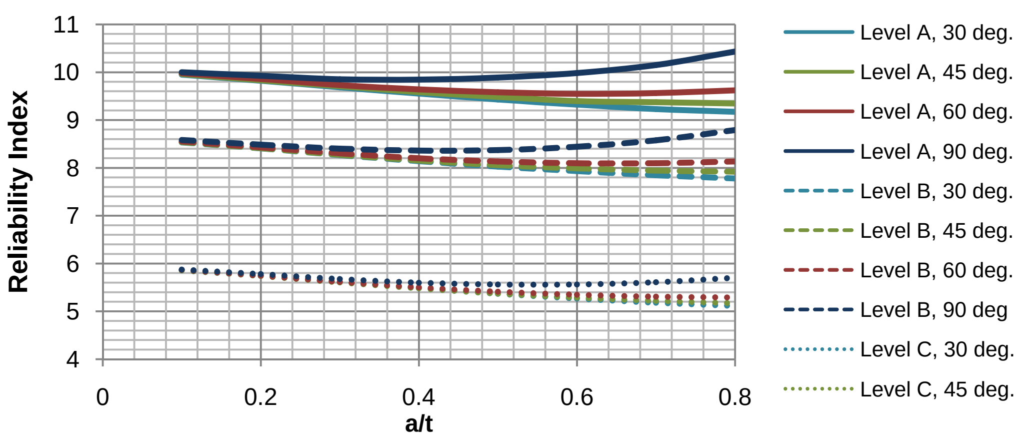

Figure 8: SA 333 Gr.6, Level A, B, C, FORM Results

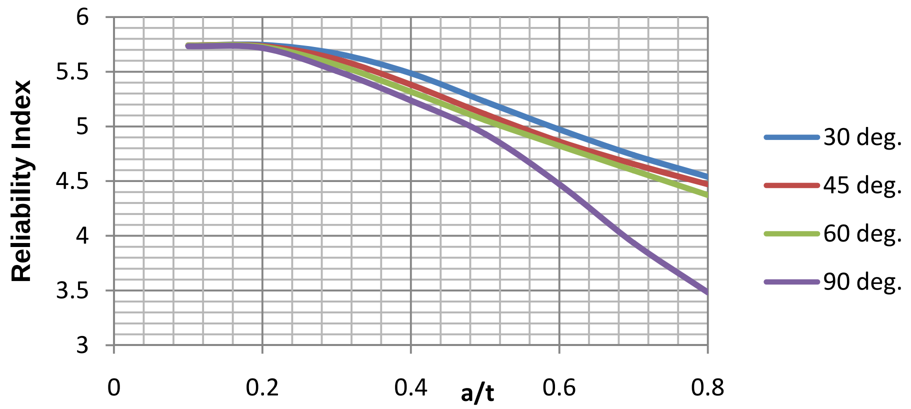

Figure 9: SA 333 Gr.6 with uncertainty in load, crack size, Level A loading

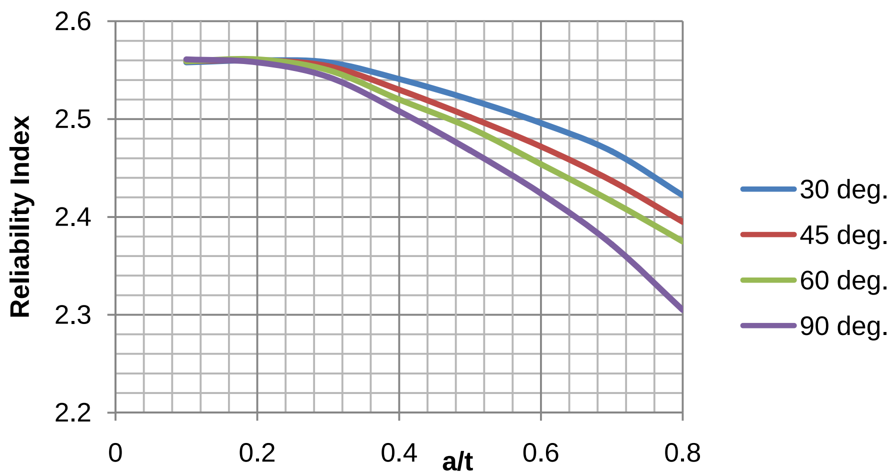

Figure 10: SA 333 Gr.6 with uncertainty in load, crack size, Level D loading (95 percentile load)

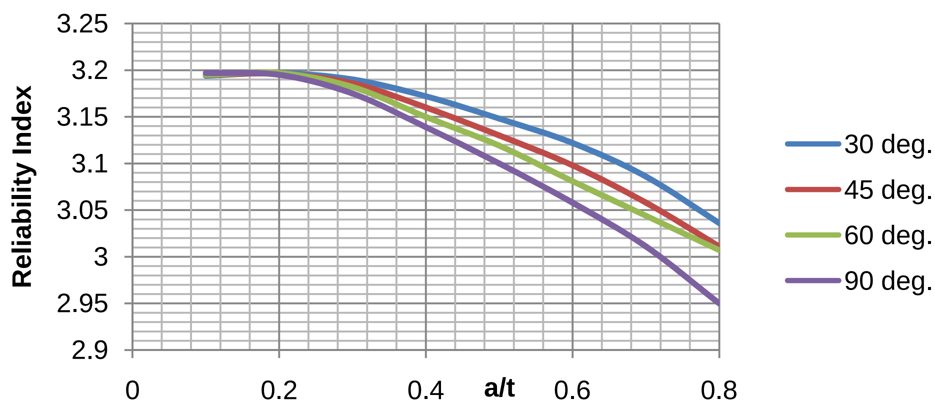

Figure 11: SA333 Gr6, with modeling uncertainty

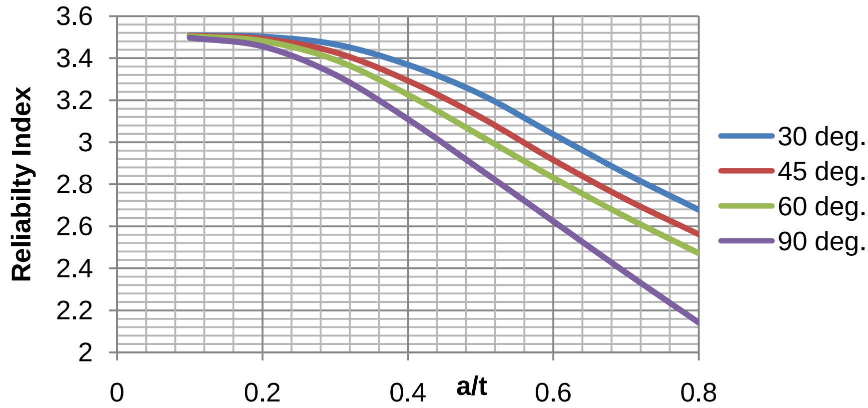

Figure 12: X6CrNiNb1810, with uncertainty in load, crack size, Level D loading (95 percentile load)

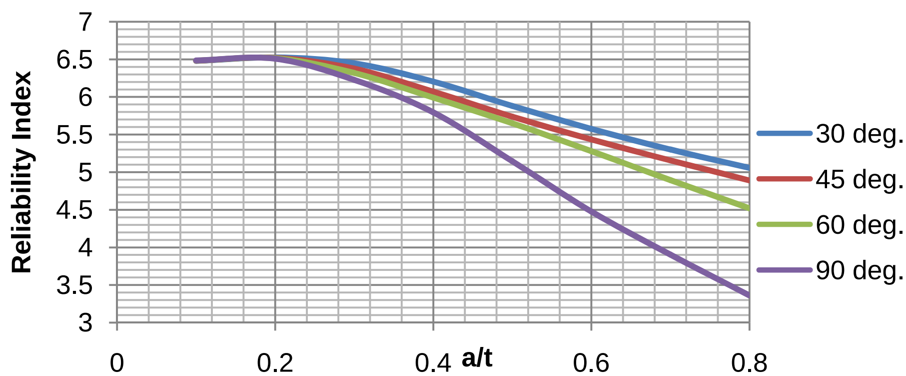

Figure 13: X6CrNiNb1810, with uncertainty in load, crack size, Level A loading

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