THE LIMIT LOAD ANALYTICAL MODEL FOR PIPE BRANCH WITH AXIAL SURFACE DEFECT

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1 ABSTRACT

The analytical limit load model for a defected branch pipe under inner pressure is suggested. Only axial surface crack at the crotch of the main pipe is considered. The main principles of the modeling are described in Orynyak (2006) and used here. The procedure of solution consists of the following steps:

i. The zone of the hole (opening) is substituted by the imaginary axial surface crack of the same length as the hole diameter. The depth of this crack is determined by equating the residual strength of pipe with a through crack (global limit load solution) with such value of the pipe with surface crack (local limit load).
ii. The limit load model of the branch pipe is constructed which gives the additional reinforcement to the main pipe (additional ligament depth).
iii. The real surface defect is considered as the continuation of the above imaginary one. The residual strength of the pipe with a crack of the complex form is calculated according to the local limit load model.

The results of three full scale experiments of the branch pipe with the axial crack of the same depth but different lengths are presented and compared with an analytical predictions. The good agreement between them is obtained. The numerical calculations by FEM are performed and results of experimental and numerical dependences of crack surface mutual displacement with an inner pressure are compared.

2 INTRODUCTION

In spite that a limit load is becoming one of the key parameters in the fracture mechanics analysis there are relatively few results for a branch pipe junction. This can be explained by two reasons. First, the real geometry of the branch is relatively complex one and it can not be described by a simple combination of the few geometrical parameters. Second, the combination of loading can include the inner pressure, bending and torsion moments with different redistributions between two ends of the main pipe and branch pipe.

The most common case of loading is an inner pressure, which can be considered here as the sole factor of loading. The strength of unreinforced branch junction is smaller as compared with the straight pipe, which requires the local thickening of material near the opening. For example, in the Russian Code PNAE G-7-002-86 (1989) for design of NPP equipment and piping the residual strength of branch junction, φ_c, with main pipe of diameter D_m and wall thickness T and the hole diameter d (inner diameter of the run pipe) is calculated:

$$\varphi_c = \varphi_d \left(1 + \Sigma A \Big/ \left(2(T-c)\sqrt{D_m (T-c)}\right)\right), \qquad (1a)$$

where ΣA is the sum of areas of all reinforcement elements, c is the increment to the wall thickness due to possible thinning at production or operation (for example, corrosion), φ_d is the strength reduction of main pipe due to opening:

$$\varphi_d = 2\left(d/\sqrt{D_m(T-c)}+1.75\right)^{-1}. \tag{1b}$$

The most loaded zone of the branch pipe junction under inner pressure is located in the plane of symmetry, which contains the axis of the main pipe. This zone is usually called as a «crotch» (Fig 1). Namely in this zone the crack is usually postulated, for example, for brittle strength assessment of reactor pressure vessel (see Akhurst 1983 and Satya 1980). Thus this zone will be considered in this paper and the crack will be postulated as directed parallel to the axis of the main pipe.

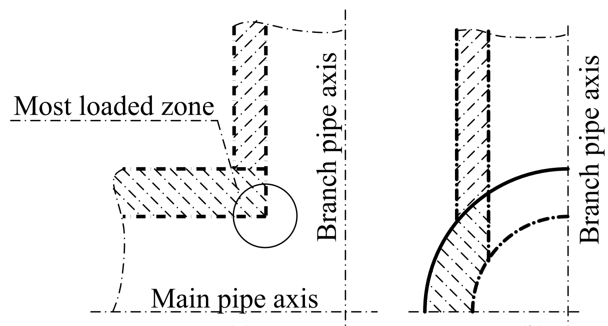

Figure 1. Scheme of pipe branch junction: (a) crotch and (b) flank.

There are three main approaches for the limit load assessment in literature, namely empirical, numerical and analytical.

1. Empirical methods originate from the late 60-ies and were constructed in the Battelles Institute (see Kiefner 1973) based on large number of full-scale experiments with defected straight pipes under inner pressure. In spite of a little progress in their development, they are very convenient in the practical applications and still prevail in the normative documents around the world. Their main drawback is that they can hardly be modified for different loading and geometry configuration as compared with experimental conditions.
2. At recent time FEM becomes a very popular tool for limit loading assessment of cracked component including the pipe branch connections (see Andrade 2004, Kim 2008, Myeong 2012). Nevertheless the application of FEA requires the special elements, a lot of time recourses and a special "numerical" definition of a limit loading based on the dependence between loading and displacement parameters. Besides, FEA cannot gives hints about which combination of dimensionless geometrical and physical parameters have the crucial influence on the limit state in order to present the results of parametrical computation in the most convenient form.
3. Analytical methods are widely popular for the simple geometries when the system of loading can be reduced to the equivalent force and bending moment in the considered defected section (see Miller 1988). Such formulas have been obtained for the standards specimens for the determination of the fracture mechanics characteristics. A number of analytical models based on the lower bound theorem of the theory of plasticity were suggested by Kitching and Zarrabi (1981), Kitching et. al. (1970), Orynyak and Borodii (1994). As they are based on the construction of the admissible stress distribution, they give the understandable mechanism of deformation and allow constructing the useful formulas for limit state assessment.

With respect to the pipe branch junction, it was noted (see Xuan et. al. 2000) that existing approaches did not give satisfactory solution even for defectless one. Thus, such a model was suggested by Xuan et. al. (2003) which is based on the modelling of the statically admissible stress distribution.

Based on our previous experience in limit load modeling (for example, Orynyak and Ageev (2009), Orynyak and Borodii (1994), Orynyak (2006)), here we use the similar premises to obtain the analytical solution for pipe branch junction loaded by inner pressure. In contrast to our previous completely "local limit load" solution (see Orynyak 1999) which gave too conservative results here we take into account the possible shear force redistribution in the zone of opening. This redistribution was modeled in construction of global limit load solution (see Orynyak and Torop 1996) and can lead to increase of limit pressure. In order to stay within the bounds of accessibility here we propose more easy way of accounting for stress redistribution within the zone of opening. It consists of two steps. First, strength reduction coefficient for defect free opening is formally determined. It can be obtained either from Battelle-like limit load solution for pipe with through crack with length $2c = d$ or from conventionally adopted formulas like (1b). Second step consists in application of simpler "local" approach for crack with surface crack of length d and some unknown fictitious depth. Its value is determined by equating the residual strength of the global solution with the local one.

Thus, the following objections of the work can be formulated:

i. To develop the analytical model of limit pressure calculation for a pipe branch junction with an axial surface crack situated near the crotch of branch.
ii. To perform the number of full scale experiments on the defected branch junction by inner pressure until the fracture.
iii. To construct the FE model of the junction and obtain the dependence of the junction points displacement with respect the inner pressure.
iv. To compare the analytical, experimental and numerical results.

3 LIMIT LOAD MODEL FOR A CRACKED PIPE BRANCH

3.1 Determination of the fictitious crack depth in the zone of opening

The limit load global model for the pipe with an through crack was suggested in Orynyak and Torop (1996). In this model all force and moments components in the equilibrium equation are considered which lead to the very complicated stress redistribution modeling. In local model it was assumed that availability of the ligament for a surface crack prevents the relative displacement of the pipe points in the circumferential direction and we can set that shear force $L = 0$. Eventually this allows considering only two of five equilibrium equations:

$$P = N_\varphi / R + dQ_x / dx; \tag{2a}$$

$$dM_x / dx = Q_x; \tag{2b}$$

Introduce the notion of the fictitious dimensionless surface crack depth $0 \le z = \left(T_{\text{n.f.}} / T\right) \le 1$, where T is the wall thickness of the main pipe and connect the origin of x axis with the center of symmetry. Then according to the limit load modeling take that $N_\varphi = \sigma_u \cdot zT$, at $x < c$ and $N_\varphi = \sigma_u \cdot T$ at $c < x$, where σ_u is the ultimate strength of material. Write the increments of the transverse force Q_x on the intervals I and II of the limit area with their lengths being equal to c and x_1 respectively (Fig 2):

$$\left(dQ_x / dx\right)\Big|_I = \sigma_u T R^{-1}\left(\alpha_1 - z\right); \; \left(dQ_x / dx\right)\Big|_{II} = \sigma_u T R^{-1}\left(1 - \alpha_1\right), \tag{3}$$

where α_1 is residual strength of the zone of opening determined according to global limit load solution.

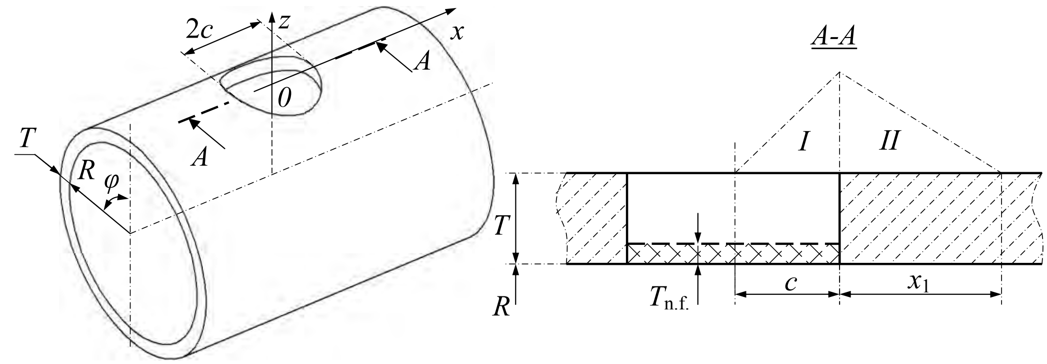

Figure 2. Straight pipe with a hole modelling

The procedure of subsequent solution of (3) and (2) with determination of the increment of applied bending moment are given in details in Orynyak and Ageev (2009), Orynyak (2006). Note only that the boundary of limit area $x = x_1$ is determined from the condition of the maximum of the applied bending moment: $Q_x(x = x_1 + c) = 0$ (see Fig 2). Then applied bending moment is equated to the limit bending moment (capacity of pipe wall to resist to applied bending moment), i.e. $\Delta M_{appl}\Big|_0^{x_1} = [M_x(x_1 + c)] - [M_x(0)]$ where the bending capacity is approximately taken to be equal to $[M_x(x_1 + c)] = -[M_x] = \sigma_u t^2/4$. Thus the solution for the dimensionless strength reduction coefficient for the pipe with a surface crack has the form:

$$\alpha_1 = P_{LL} R/(\sigma_u T) = \left(1 + 2\lambda_1^2 z(1 - z)\right) / \left(1 + 2\lambda_1^2 (1 - z)\right), \tag{4}$$

where $z = T_{n.f.}/T$ is the looking for fictitious net-section thickness and $\lambda_1 = \sqrt{c^2/(RT)}$ is the dimensionless length of the crack (opening). It can be found if we equate the value of α_1 to the residual strength of the opening or the through crack of the length equal to c. Accordingly to API RP 579 (2000) it is calculated by formula:

$$\alpha_1 = \left(1 + 0.3797\lambda_B^2 - 0.001236\lambda_B^4\right)^{-0.5}, \text{ для } \lambda_B \le 9.1, \tag{5}$$

where $\lambda_B = 1.818c/\sqrt{RT}$.

3.2 Accounting for the reinforcement from the branch pipe

The branch pipe is able to resist some additional part of the circumferential force ΔN_φ in the main pipe, which leads to additional reinforcement of the zone of opening. This can be possible only in case when the strength of the branch pipe is higher then the strength of the main pipe with an opening, i.e. α_1.

Denote the additional dimensionless force in the main pipe as $\Delta\alpha = \Delta N_\varphi/(\sigma_u T)$. Letting that the same axial force $\Delta N_x = \Delta N_\varphi$ is acts from the main pipe on the branch one we obtain that the branch pipe is loaded by additional axisymmetrical transverse force $Q_{x,0} = \Delta N_\varphi$. Then according to eq (2) the distribution of the transverse forces along the axis of the branch pipe of the radius r and wall thickness t will have the form:

$$Q_x = \Delta\alpha\sigma_u T + \left(P_{LL} - \sigma_u t/r\right)x\,. \tag{6}$$

Here x is the axial coordinate along the branch pipe. The length of the limit area, $x = x_2$, is determined from the condition $Q_x(x = x_2) = 0$. Thus:

$$x_2 = \Delta\alpha Tr/\left(t\left(1 - \alpha_2 rT/(tR)\right)\right), \tag{7}$$

where α_2 is the looking for strength reduction coefficient for the branch junction with an axial defect.

Taking into account the usual limit state condition (this time for the branch pipe) $\Delta M_{appl}\big|_0^{x_2} = \left[M_x(x_2)\right] - \left[M_x(0)\right]$, and accounting for Eq. (7), obtain the relationship between the $\Delta\alpha$ and α_2, i.e. the value of possible enlargement of pressure due to the branch pipe with respect to attained level of pressure.

$$\Delta\alpha = (t/T)\sqrt{t\left(1 - \alpha_2 rT/(tR)\right)/2r}\,. \tag{8a}$$

Now according to our local limit load model we calculate the strength of the defectless branch junction. The dimensionless pipe wall thickness in the zone of opening is $z + \Delta\alpha$ and repeating the calculation procedure we obtain for the strength reduction coefficient α_2:

$$\alpha_2 = P_{LL}R/(\sigma_u T) = \left(1 + 2\lambda_1^2(z + \Delta\alpha)(1 - z - \Delta\alpha)\right)/\left(1 + 2\lambda_1^2(1 - z - \Delta\alpha)\right). \tag{8b}$$

It can be calculated by a few iterations. Initially we take that $\Delta\alpha = 0$ and calculate α_2. Then accordingly to (8a) we find a new value of $\Delta\alpha$ and refine it with accounting of the previous value of it (as a mean value). Then the value of α_2 is recalculated.

3.3 Limit state of the defected branch junction (symmetrical solution)

In this part we consider the symmetrical model of the defected junction with crack of depth a and length b (Fig 3). The defects are situated on the outer surface of main pipe and they are symmetrical with respect to the axis of the branch pipe.

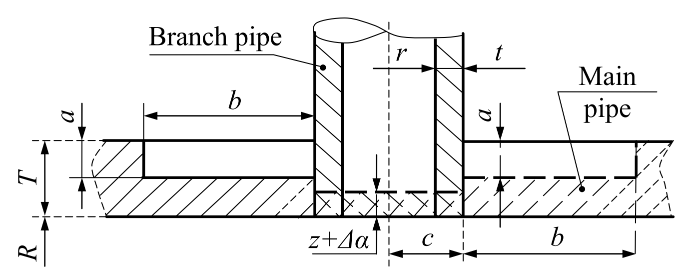

Figure 3. Scheme of pipe branch junction with symmetrical surface defects in axial direction.

Now we will consider that the limit area consists of three intervals. The first interval goes to the end of the zone of opening. The second interval is connected with the crack and the third one last until the transverse force becomes equal to zero. The increments of forces Q_x on each of them (Fig 4) is:

$$\left(dQ_x/dx\right)\big|_I = \sigma_u TR^{-1}\left(\alpha_3 - z - \Delta\alpha\right),\ \left(dQ_x/dx\right)\big|_{II} = \sigma_u TR^{-1}\left(\alpha_3 - \tau\right),\ \left(dQ_x/dx\right)\big|_{III} = \sigma_u TR^{-1}\left(\alpha_3 - 1\right),\quad (9)$$

where $\tau = 1 - \left(a/T\right)$ is dimensionless net-section thickness (interval *II*). Integrating (8) and equating $Q_x(x = x_3)\big|_{III} = 0$ we find the coordinate of boundary of the limit area. The maximal value of the applied moment is determined by twice integration of (9) from $x = 0$ to $x = x_3$:

$$\Delta M_{appl} = 0.5\sigma_u TR^{-1}\left(\left(\alpha_3 - z - \Delta\alpha\right)\left(c + 2b + 2x_1\right)c + \left(\alpha_3 - \tau\right)\left(b + 2x_1\right)b + x_1^2\left(1 - \alpha_3\right)\right). \quad (10)$$

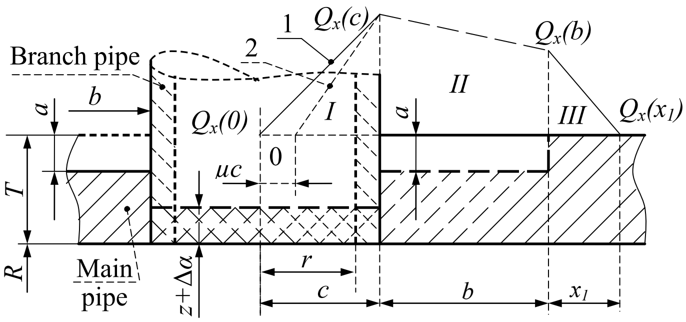

Figure 4. The Q_x distribution: 1 - symmetrical solution, 2 - unsymmetrical solution.

The dimensionless strength reduction coefficient, α_3, is found from the condition of equality of applied and limit moments:

$$\alpha_3 = \frac{P_{LL}R}{\sigma_u T} = \frac{1 + 4\lambda_1\lambda_2\left(z + \Delta\alpha\right)\left(1 - \tau\right) + 2\lambda_1^2\left(z + \Delta\alpha\right)\left(1 - z - \Delta\alpha\right) + 2\lambda_2^2\tau\left(1 - \tau\right)}{1 + 4\lambda_1\lambda_2\left(1 - \tau\right) + 2\lambda_1^2\left(1 - z - \Delta\alpha\right) + 2\lambda_2^2\left(1 - \tau\right)}, \quad (11)$$

where $\lambda_2 = b/\sqrt{RT}$ is the dimensionless length of the surface defect. The additional strength due to the branch pipe $\Delta\alpha$ is calculated from (8a), where α_3 according to (11) should be used instead of α_2. The iteration procedure for $\Delta\alpha$ calculation can be the same as in the above part **3.2**.

3.4 Unsymmetrical solution for defected branch junction

The case of nonsymmetrical defect with depth a and length b can be very easily considered within the proposed approach. We put that crack is situated on the right part of the branch. In this case the shift of the diagram of distribution of the transverse forces Q_x to the right (to the defect) on some distance μc, $\mu > 0$ (see Fig 4) will take place.

Thus to the left side of pipe branch junction we have the calculation case ***3.2*** and to the right side we have case ***3.3.*** According to the limit load conception the branch junction will attain the limit state when the left side residual strength α_2 (defectless part) will be equal to the right side, α_3. This condition serve us for the determination of the value of the shift distance μ. This is done with additional iteration procedure where upper and lower border of tentative μ are established on each iteration. In each calculation the mean value of μ is used and lower or upper border is replaced by mean value in dependence of which value α_2 or α_3 is bigger. The left side strength is calculated for dimensionless length of the opening $\lambda_1 = c(1+\mu)/\sqrt{RT}$ и $\lambda_B = 1.818c(1+\mu)/\sqrt{RT}$, and the right side strength for the length $\lambda_1 = c(1-\mu)/\sqrt{RT}$ и $\lambda_B = 1.818c(1-\mu)/\sqrt{RT}$.

3.5 The experimental results

To check the results of calculation the full scale experiments of pipe branch junction with one side (unsymmetrical) defect were performed on the pneumohydraulic test stand of the G.S.Pisarenko Institute for Problems of Strength. Three full scale tests were loaded by inner pressure until the fracture. The pipe specimen drawings are shown on Fig 5.

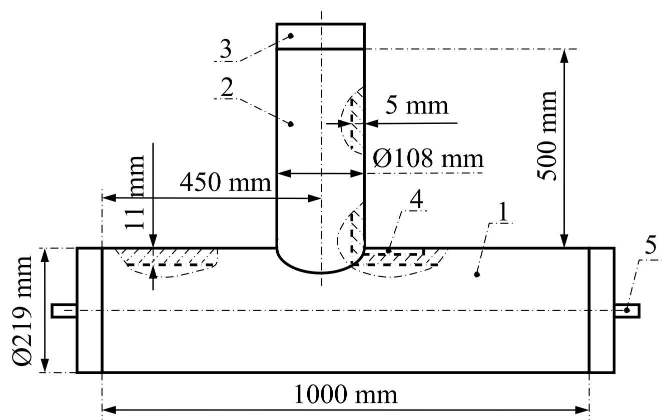

Figure 5. Notched pipe branch drawing: 1 - main pipe, 2 - branch pipe, 3 - cover plate, 4 - notch, 5 – nozzle

The length of the main pipe (Ø219×11 mm) and branch pipe (Ø108×5 mm) are equal to 1000 mm and 500 mm correspondently. The material of the junction is the austenitic steel 08Kh18N10T. To exclude the influence of the ends on the results the lengths of both pipes were big enough. To the ends of pipes were welded the thick walled plates. The surface axial notches have the width equal to 3 mm and have the plain bottom.

The following factors had influence on the choice of notch and pipes dimensions:

- Technical capacity of the test stand and availability of material;
- The strength reduction factor due to opening and branch pipe should have practical significance (be within 0.5-0.8 range)
- Additional strength reduction due to notch should be noticeable. From other hand the notch should be not very deep, when a small error in its dimension would lead to big errors in the predicted capacity.
- When the number of specimens is small, one geometrical parameter should be varied only to not obscure its influence on the residual strength.

In our case the initial dimension and results of testing (the value of experimental pressure of fracture, $P_{LL}^{\exp}$) are given in Table 1. Note that initially it was planned that depth of each defect would be approximately 6.5 mm, but the actual depth have been made different. As example, on the Fig 6 the photo of specimen №1 before and after the test are presented.

Table 1: Test results.

№ specimen	a , mm	b , mm	$P_{LL}^{\exp}$, MPa
1	6.5	40	28.60
2	6.5	60	28.63
3	5.5	80	32.33

(a) (b)

Figure 6. Pipe branch junction photo: (a) - before failure, (b) - after failure.

3.6 The comparison of the theoretical and experimental results

The results of analytical prediction by the proposed method as well as experimental results are presented graphically on Fig 7. X-coordinate characterizes the generalized length of the defect which include the length of the opening and the length of the artificial defect, the ordinate axis shows the values of strength reduction coefficient α . The black and red curves depict the analytical symmetrical solution according to eq. (11) for crack depth $a = 5.5$ mm and $a = 6.5$ mm, correspondently. The discrete round points depict unsymmetrical analytical solution for $a = 6.5$ mm, while triangular black points depict unsymmetrical solution for $a = 5.5$ mm. Experimental results are characterized by squared points and dimensionless residual strength is obtained as $\alpha = P_{LL}^{\exp} R / (\sigma_u T)$.

The ultimate strength of the material used in normalization of the experimental results was obtained according to Russian standards (see GOST 1497-84, 1975) by tensile tests of three cylindrical specimens of five-diameter length and was taken to be $\sigma_u = 615$ MPa. The specimens were cut out in the circumferential direction.

The Fig 8 shows the dependence of the inner pressure P versus defect surfaces opening displacement, Δ, which was determined during the experiments (dashed line). The analytically calculated results for this displacement by FEM software «ANSYS» are shown by solid line. These results relate to the specimen №2 (see Table 1). For reference here is shown the conventionally adopted value of P_{LL} determined by twice elastic angle method. Evidently it gives a lower result as compared with experimental data. The similar results (not shown here) were obtained for specimens №1 и №3. With respect to the whole curve description, it can be noted that numerical curve lies slightly higher (about 10%) than experimental one. In any case it can not predict the ultimate value of pressure.

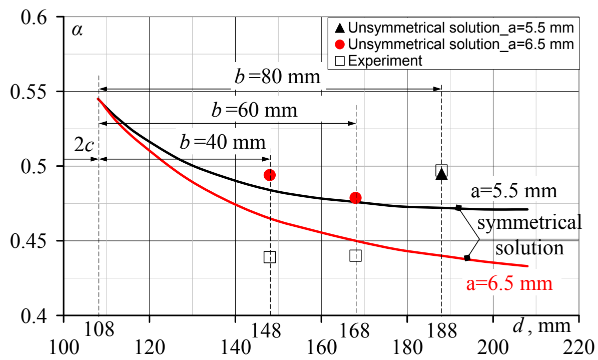

Figure 7. Theoretical and experimental data comparison.

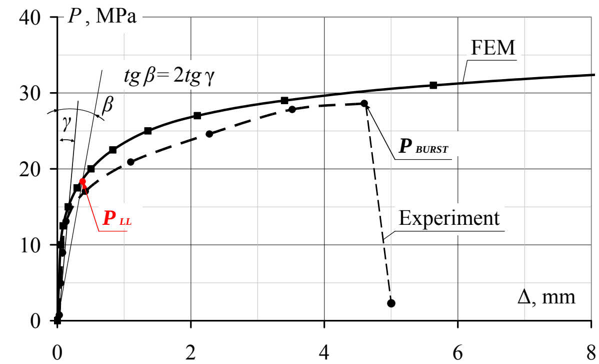

Figure 8. P_{LL} calculation by ANSYS.

4 CONCLUSION

i. Based on lower bound theorem of plasticity the local theoretical model of limit state of branch pipe junction with a surface crack is constructed and analytical formulas for the residual strength are derived. They take into account the reinforcement from the branch pipe. The modeling of the zone of opening is performed formally based on the known formulas for the global limit state of a pipe with a through defect and equating it with "local" residual strength of a surface defect, which gives the depth of an imaginary surface defect.

ii. The choice of the specimens dimensions are justified and three full scale experiments of pipe branch junction with an artificial defects by inner pressure until to fracture. The comparison has shown the good correspondence between the experimental and analytical results.

iii. With respect to FEM modeling of pressure – notch surface displacement relationship, the experimental results are slightly below the numerical results (up to 10 %). Besides, it is very problematic to single out from the numerical graph the point of supposed fracture.

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