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CONCRETE TRANSIENT THERMAL CREEP IMPLEMENTATION IN FINITE ELEMENT SOFTWARE

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ABSTRACT

Transient Thermal Creep is an extra strain developing in concrete above 100°C when it is heated under compression. It is an important phenomenon as for moderately high temperatures and stress to strength ratio it can be large enough to completely suppress thermal expansion. Its implementation in finite element software is not straightforward because this phenomenon only happens during the first heating under compression and is not recovered during cooling. As the phenomenon depends on both the evolution of stress and temperature it leads to further difficulties when stress is not constant.

A way to implement Anderberg & Thelandersson transient thermal creep model in finite element software is presented, and numerical results are derived. They are compared to experimental results in order to calibrate the model. Multi-axial loadings are then considered, and the model is modified for these cases using the stress triaxiality as a correction coefficient. The subsequent numerical results are in good agreement with the experimental data for both uniaxial and biaxial tests.

The behavior of this numerical model when stress is not constant is then investigated. The example of a beam restrained at both ends in the vertical direction and heated to 250°C highlights the fact that too large time steps lead to prediction errors. A method to avoid this issue is presented.

INTRODUCTION

Transient Thermal Creep (TTC) is a phenomenon which has been studied for over forty years (Nishizawa, Okamura (1972); Thelandersson (1987)). When a concrete specimen is loaded in compression before being heated to a high temperature (above 100 °C), an extra strain develops compared to when it is loaded after having been heated. This extra strain is known as Load Induced Thermal Strain (LITS). TTC is by far the main component of LITS. It has been investigated by several authors who created a number of different phenomenological models based on experimental results (a review of these models was done by Youssef and Moftah (2007)). The mechanisms involved in TTC are not yet fully understood though. The experiments carried out were mostly specimens loaded with a constant load in one direction and heated from room temperature to high temperature – sometimes as high as 800°C (Anderberg, Thelandersson (1976)).

Consequently most of the characteristics of this phenomenon are well known as long as a uniaxial load is considered.

Under certain low frequency fault conditions, the Pre-Stressed Concrete Pressure Vessels (PCPVs) of Advanced Gas Cooled Reactors (AGRs) in the UK can be heated to moderately high temperatures (about 250°C) for several weeks before cooling down. It is necessary to assess its safe performance during this period of high temperature and after the vessel has cooled down.

TTC is a very important phenomenon in such fault conditions, the pressure vessel being loaded in compression. Tests were carried out by Petkovski, Crouch, (2008) in the University of Sheffield to investigate more precisely on TTC at moderately high temperatures with multiaxial loads. These tests underline the fact that TTC does not depend linearly on the stress when multiaxial loads are applied, so the existing models should be modified when such loads are considered. It was chosen in the present study to modify Anderberg and Thelandersson's (1976) model to make it more accurate for the temperature range considered and for multiaxial loads.

Implementation of TTC in a finite element code should take into account its main characteristics. It is necessary to accurately model the fact that it only happens during the first heating under compression, and that it is an irrecoverable strain. Assuming the stress varies linearly over a time step, it is possible to solve the equations in case the stress is not constant, which is the case in the PCPVs under certain fault conditions.

The open-source finite element software *Code_Aster* was used. It is developed by EDF Research & Development, and did not yet include TTC.

MODIFICATION OF ANDERBERG AND THELANDERSSON'S MODEL

Anderberg and Thelandersson's model is one of the first mathematical models proposed to model TTC. The main equation of this model is equation (1).

$$\dot{\varepsilon}_{ttc} = k_{tr}\dot{\varepsilon}_{th}\frac{\sigma}{\sigma_c} \quad (1)$$

The TTC strain in the loaded direction ε_{ttc} is supposed to be proportional to the free thermal expansion ε_{th} and the stress to compressive strength ratio $\frac{\sigma}{\sigma_c}$. k_{tr} is a parameter supposed to be between 1.8 and 2.3. Assuming $\varepsilon^{th} = \alpha\,\Delta T$ where α is the thermal expansion coefficient, and assuming temperature varies linearly over time lead to equation (2).

$$\frac{d\varepsilon_{ttc}}{dT} = k_{tr}\alpha\frac{\sigma}{\sigma_c} \quad (2)$$

In order to derive a 3 dimensional version of this equation, a new material parameter ν_{ttc} - similar to the Poisson ration - is needed. Assuming TTC depends linearly on the stress leads to equation (3), where the double underlines indicate a second order tensor.

$$\frac{d\underline{\underline{\varepsilon}}_{ttc}}{dT} = \frac{k_{tr}\alpha}{\sigma_c}\left[(1+\nu_{ttc})\underline{\underline{\sigma}} - \nu_{ttc}\,Tr\left(\underline{\underline{\sigma}}\right)\underline{\underline{1}}\right] \quad (3)$$

Using this equation in the numerical model gives good results when a uniaxial load is applied. But experimental results show that TTC does not depend linearly on the stress when multiaxial loads are applied. That is why a correction factor $C_m$was introduced in order to improve the accuracy of results for such cases. This correction factor is proportional to a stress triaxiality ratio used by several authors in fracture mechanics (for example Brunig, Chyra, Albrecht, Driemeier, Alves (2008)).

$$C_m = \frac{\mathrm{Tr}\left(\underline{\underline{\sigma}}\right)}{\sqrt{\underline{\underline{\sigma}}:\underline{\underline{\sigma}}}} \quad (4)$$

Equation (3) can then be modified to :

$$\frac{d\underline{\underline{\varepsilon}}_{ttc}}{dT} = \frac{k_{tr}\alpha}{\sigma_c}\,C_m\left[(1+\nu_{ttc})\underline{\underline{\sigma}} - \nu_{ttc}\,Tr\left(\underline{\underline{\sigma}}\right)\underline{\underline{1}}\right] \quad (5)$$

This way, there is no difference with equation (3) when uniaxial loadings are applied as $C_m = 1$. On the other hand when a multiaxial compressive load is applied $C_m > 1$ which means the amplitude of TTC increases along with the triaxiality of the load.

IMPLEMENTATION IN FINITE ELEMENT SOFTWARE

In the implementation considered, the strain is divided explicitly into its three main components:

$$\underline{\underline{\boldsymbol{\varepsilon}}} = \underline{\underline{\boldsymbol{\varepsilon}}}_{\boldsymbol{\sigma}} + \underline{\underline{\boldsymbol{\varepsilon}}}_{\boldsymbol{th}} + \underline{\underline{\boldsymbol{\varepsilon}}}_{\boldsymbol{ttc}} \quad (6)$$

ε_σ is the mechanical strain, ε_{th} the thermal strain and ε_{ttc} the TTC strain. This approach is recommended in Gernay, Franssen (2011) to avoid the shortcomings described by Law, Gillie, Pankaj (2008) when TTC strain is implicitly integrated in the mechanical strain: in this explicit approach TTC strain is separated from other strains to make it possible to take into account the different characteristics of TTC, in particular the fact that it is an irrecoverable strain which only occurs during the first heating under compression.

Let us derive this equation with respect to temperature, using equation (5). The double underlines are removed for simplification. E and υ are the material's Young modulus and Poisson ratio. In the scope of this study the material is assumed to be elastic and isotropic, and its parameters are assumed not to depend on temperature.

$$\frac{d\varepsilon}{dT} - \alpha = \frac{1+\upsilon}{E} * \frac{d\sigma}{dT} - \frac{\upsilon}{E} * Tr\left(\frac{d\sigma}{dT}\right) 1 + \frac{k_{tr}\alpha}{\sigma_c} C_m \left[(1+\nu_{ttc})\sigma - \nu_{ttc}\, Tr(\sigma) 1\right] \quad (7)$$

C_m is considered to be constant over the time step, which means the stress direction is assumed not to change significantly over a time step. Still it is not possible to solve this first order differential equation without making any further hypothesis. That is why the assumption that evolution of stress is linear during a time step is made – see equation (8). A similar assumption is made in De Borst and Peeters (1989).

$$\sigma = \sigma_{i-1} + \frac{\Delta\sigma}{\Delta \mathrm{T}} * (T - T_{i-1}) \quad (8)$$

Let us integrate equation (7) over time step i. The Newton algorithm in the code gives a prediction of the total strain. The goal of the material law is to deduce from this strain the internal stress. The thermal strain (and drying/ hydration strain if relevant) is given by a previous computation. The TTC strain at step $i-1$ is also known.

$$\underline{\underline{\varepsilon_i}} - \underline{\underline{\varepsilon_i^{th}}} - \underline{\underline{\varepsilon_{i-1}^{ttc}}} = \underline{\underline{\varepsilon_i^{\sigma}}} + \Delta\underline{\underline{\varepsilon}}^{ttc} \quad (9)$$

All the terms on the left-hand side of equation (9) are known at the beginning of the time step.

Integrating equation (7) using equations (8) and (9) leads to equation (10):

$$\begin{aligned} \varepsilon_i - {\varepsilon_i}^{th} - \varepsilon_{i-1}^{ttc} &= \frac{1+\nu}{E}\sigma_i - \frac{\nu}{E} * Tr(\sigma_i) * 1 \\ &+ \frac{k_{tr}\alpha}{\sigma_c} C_m \Bigg[(1+\upsilon_{ttc}) * \left(\sigma_{i-1}\frac{\Delta T}{2} + \sigma_i \frac{\Delta T}{2}\right) - \upsilon_{ttc} \\ &* Tr\left(\sigma_{i-1}\frac{\Delta T}{2} + \sigma_i\frac{\Delta T}{2}\right) * 1\Bigg] \end{aligned} \quad (10)$$

which can be written as equation (11):

$$\varepsilon_i^{th} - \varepsilon_{i-1}^{ttc} - \frac{k_{tr}\alpha}{\sigma_c} C_m * \frac{\Delta T}{2} * \left((1 + \upsilon_{ttc}) * \sigma_{i-1} - \upsilon_{ttc} * Tr(\sigma_{i-1})\right) = \frac{1 + \nu_{app}}{E_{app}} \sigma_i - \frac{\nu_{app}}{E_{app}} * Tr(\sigma_i) * 1 \quad (11)$$

where:

$$E_{app} = \frac{1}{\frac{1}{E} + \frac{1}{2} * \frac{k_{tr}\alpha\Delta T}{\sigma_c} C_m} \quad (12)$$

$$\nu_{app} = \left(\frac{\upsilon}{E} + \frac{1}{2} * \frac{k_{tr}\alpha\Delta T\, \nu_{ttc}}{\sigma_c} \right) * E_{app}$$

are apparent Young modulus and Poisson ratio. All terms on the left hand side of the equation are known at the beginning of the time step, so it is possible to deduce the stress at the end of the time step σ_i from equation (11), from which the elastic strain and TTC strain can be deduced:

$$\varepsilon_i^{\sigma} = \frac{1 + \nu}{E} \sigma_i - \frac{\nu}{E} * Tr(\sigma_i) * 1 \quad (13)$$

$$\Delta\varepsilon^{ttc} = \frac{1 + \nu_{app}}{E_{app}} \sigma_i - \frac{\nu_{app}}{E_{app}} * Tr(\sigma_i) * 1 - \varepsilon_i^{\sigma}$$

With this implementation, the different characteristics of TTC can be taken into account in the code. No TTC strain develops if the temperature is below a user-defined threshold T_{boil} - default is 100°C. No TTC strain develops if the temperature is below the maximum temperature reached by the integration point. TTC strain only develops if the load is in compression – which means no TTC strain develops if $\sigma_{11} + \sigma_{22} + \sigma_{33} \geq 0$. And TTC strain is not recoverable.

IDENTIFICATION AND VALIDATION OF THE MODEL

Tests were carried out in the University of Sheffield (Petkovski, Crouch (2008)) to investigate on the behavior of Transient Thermal Creep with multiaxial loads at moderately high temperatures. During these tests, a concrete cube (0.1m*0.1m*0.1m) is loaded uniaxially, biaxially or triaxially and heated from 20 to 250°C. The concrete material parameters are given by previous experiments: $E = 47\ GPa$, $\nu = 0.25$, $\alpha = 1.62 * 10^{-5}\ K^{-1}$ and $\sigma_c = 57MPa$. These tests are reproduced numerically in order to deduce the material parameters k_{tr} and ν_{ttc} and then to validate the model. A thermo-mechanical computation is run on a quadratic mesh.

Identification With a Uniaxial Test

The uniaxial experimental results are used to calibrate the parameters, so that the same values of TTC at 250°C are obtained with the computation. The TTC parameters are found to be $k_{tr} = 1.46$ and $\nu_{ttc} = 0.37$.

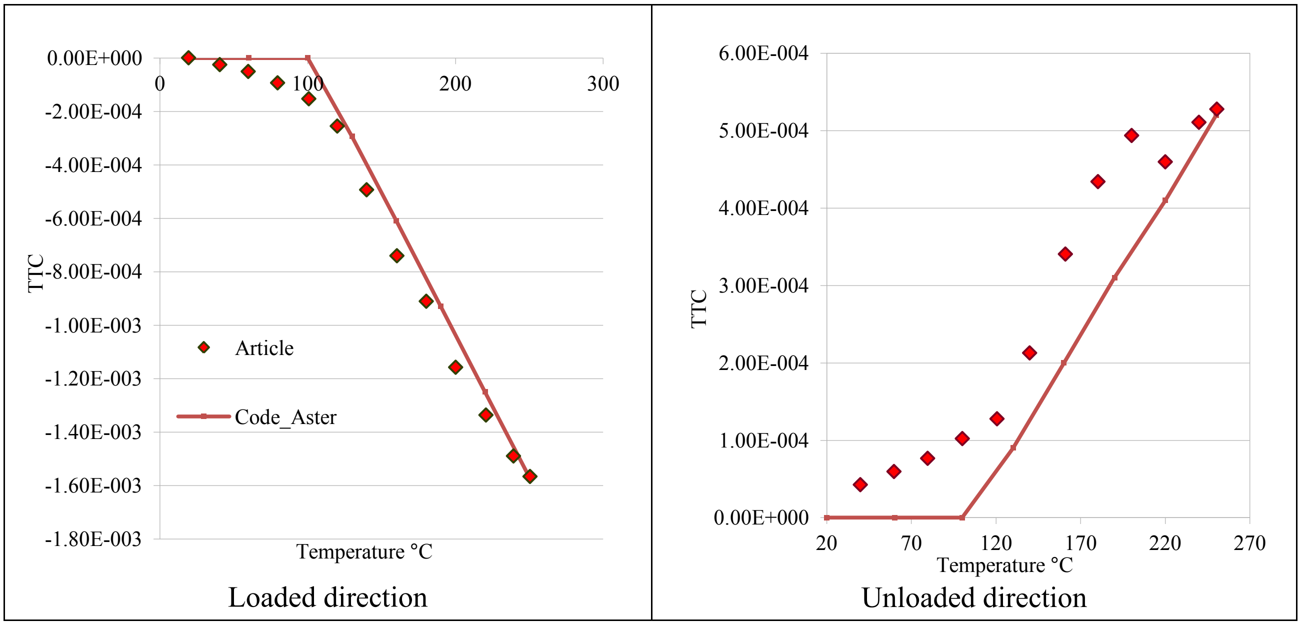


Figure 1: Results of uniaxial tests

Validation With a Biaxial Test.

The identified parameters are used to simulate a biaxial test. Results of numerical analysis are in accordance with the experimental data, in the loaded and unloaded directions. Without the multiaxial correction factor, the results are much worse with more than 30% error at 250°C.

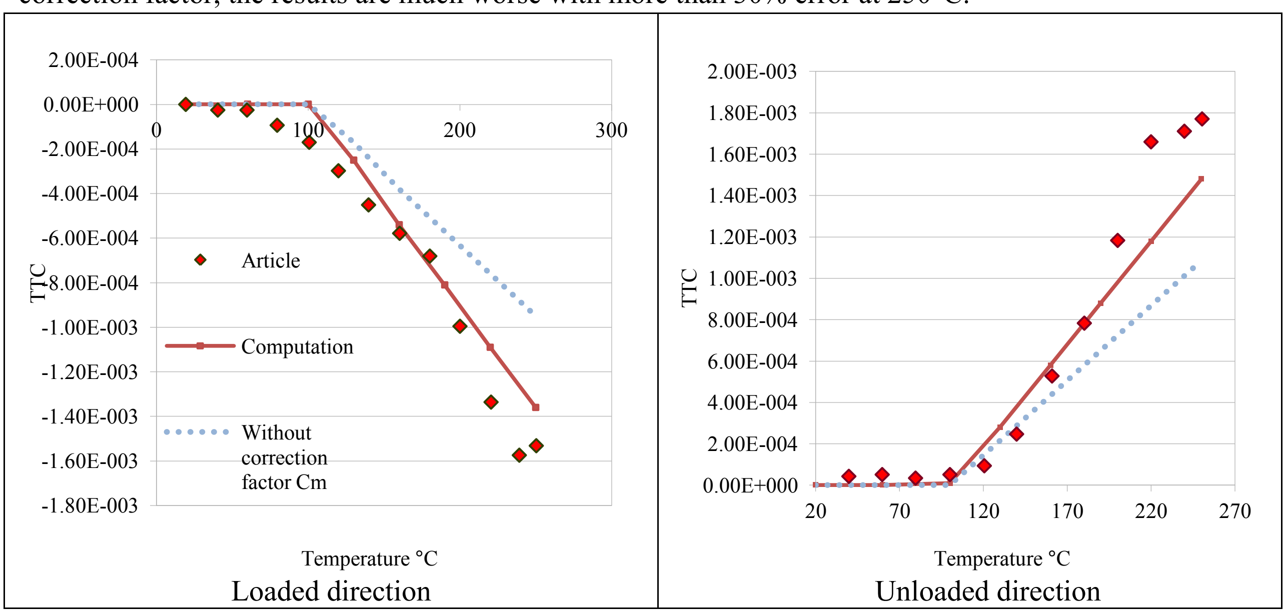
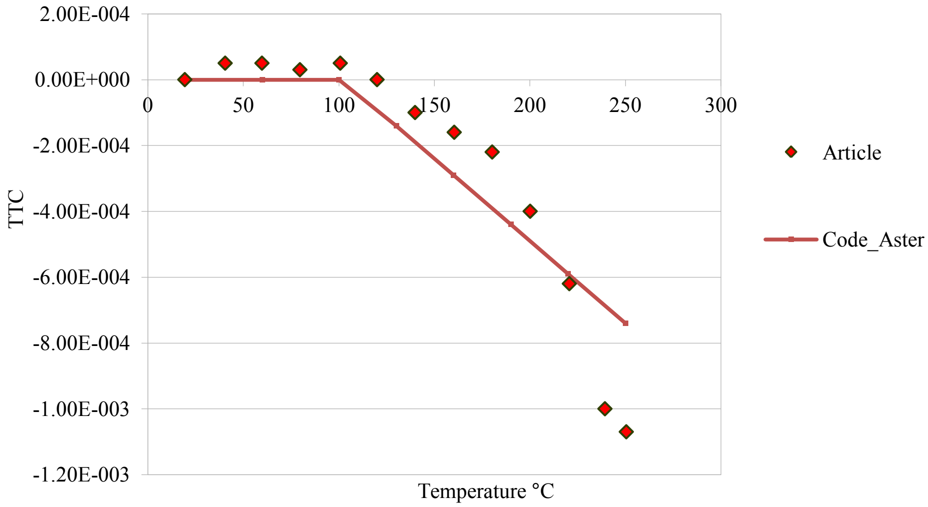


Figure 2: Results of biaxial tests.

Triaxial Test.

The same identified parameters are used to simulate a triaxial test. The results are not so good. Despite the multiaxial correction factor, there is still a 30% error at 250°C. It was nonetheless chosen not to change it in order to keep a simple formulation of the model.



INVESTIGATION OF THE CASE OF A NON CONSTANT STRESS

The assumption was made that the stress evolution was linear over a time step. This assumption is of course verified when stress is constant as for the tests presented above. But as soon as stress is not constant, it may be wrong. The example studied here is the example of an element restrained in the vertical direction at its top and bottom, heated from 20 to 250°C. The analytical solution will be compared to the numerical results obtained with the assumption of a linear stress. For simplification the temperature threshold above which TTC starts was set to 20°C. It makes sense as this test is not meant to be realistic - it is just an investigation on the error induced by the implementation compared to the analytical solution.

Analytical Solution

The total strain in the vertical direction is zero. Consequently,

$$\varepsilon_\sigma + \varepsilon_{ttc} + \varepsilon_{th} = 0 \tag{14}$$

Deriving (14) with respect to temperature gives:

$$\frac{1}{E} * \frac{d\sigma}{dT} + k_{tr}\alpha \frac{\sigma}{\sigma_c} C_m + \alpha = 0 \tag{15}$$

Assuming the stress is zero at 20°C leads to the following solution:

$$\sigma = \frac{\sigma_c}{k_{tr}}\left(1 - e^{-\frac{k_{tr}E\alpha}{\sigma_c}C_m(T-20)}\right) \tag{16}$$

Comparison With Numerical Results

This same test was performed with *Code_Aster,* with 1, 2 and 5 time steps in order to highlight the influence of the number of time steps on the quality of the result. The results are presented on Figure **3**: Evolution of stress on a restrained element.

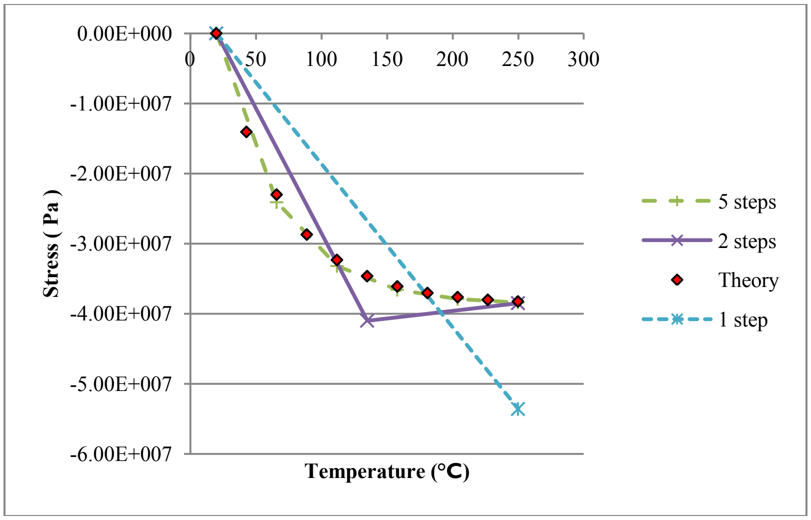


Figure 3: Evolution of stress on a restrained element

With 5 time steps, the results are in good accordance with the analytical solution. The error is always inferior to 5% and is only 0.4% at 250°C.

On the other hand with 2 time steps the stress is overestimated by 19% after the first time step (T= 135°C). After the second time step this error goes down to 1% though (T=250°C).

With only 1 time step the stress is overestimated by 40%.

It is easy to explain these differences. The longer the time step is, the worse the assumption of linearity of stress evolution gets, as stress is exponential in this example. Let us focus on the 2 time steps case. After the first step, the error made is quite big, but after the second one it is almost zero. This underlines not only that this formulation converges efficiently towards the analytical solution, but also that what is important is not the length of the time step but the difference of stress between the beginning and the end of the step. Between 135 and 250°C, stress does not change much, so the hypothesis of linearity of stress evolution is appropriate and the result is precise.

A Criterion For The Automatic Subdivision Of Time Steps

In order to avoid too big prediction errors a solution is to subdivide the time steps over which stress varies a lot into smaller time steps over which the assumption of linearity of the stress evolution is reasonable. On the other hand time steps are subdivided even if the stress evolution is linear, which means even if no prediction error would be made, potentially leading to unnecessarily longer computation time.

The criterion chosen for this purpose is presented in equation (17).

$$I = \frac{1}{|\varepsilon_\sigma + \varepsilon_{ttc}|} * |\,\sigma_i^- - \sigma_{i-1}^-| * \frac{Tr\left(\underline{\underline{\sigma}}_{\,i-1}\right)}{\sqrt{\underline{\underline{\sigma}}_{\,i-1} : \underline{\underline{\sigma}}_{\,i-1}}} * \frac{k_{tr}\alpha\Delta T}{\sigma_c} \approx \frac{\Delta\varepsilon_{ttc}}{\varepsilon_\sigma + \varepsilon_{ttc}} \qquad (17)$$

Where σ_i^- stands for the compressive part of the stress tensor at time step index i.

The user can then define a threshold for this indicator I over which subdivision of the time step is required. For a threshold of 0.1, and initially one time step the following result is obtained:

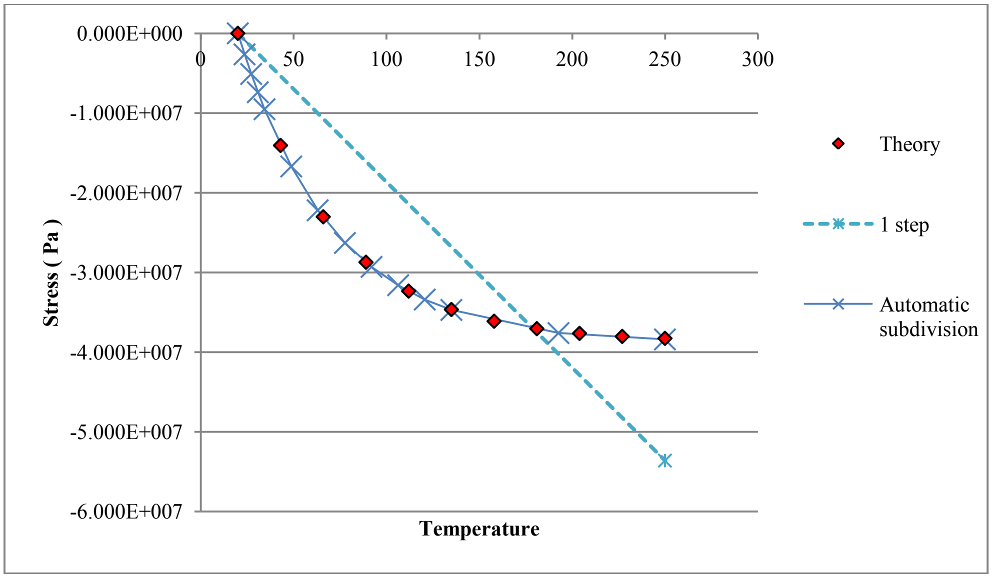


Figure 4: Evolution of stress with automatic subdivision of time steps

The results are in good accordance with the analytical ones. As expected time steps are subdivided finely at the beginning when stress changes a lot and coarsely at the end when $T > 150°C$, for a total of 13 time step.

CONCLUSIONS

A new model for TTC was presented, focusing on moderately high temperature and multiaxial loadings. In this model the amplitude of TTC depends, in addition to temperatures, not only on the stress value but also on the stress triaxiality. It was implemented successfully in a finite element code. The numerical results are in good accordance with the experimental results – excepted for triaxial loads where TTC strain is underestimated by 30%. The implementation proposed gives good results when stress is not constant provided the time steps are chosen carefully. A method to automatically subdivide time steps over which a prediction error could be made was proved to be reasonably efficient, making it easier for the user to handle TTC in its model.

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