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THE LIMIT LOAD ANALYTICAL MODEL FOR THICK-WALLED PIPE WITH AXIAL SURFACE DEFECT

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1 ABSTRACT

The limit load analytical model of stress distribution near the axial crack originated from the external as well as internal surface of the thick-walled pipe is proposed. The starting point in modelling of the ultimate plastic state was previously developed the thin-walled pipe model (see Orynyak 2006). Three additional peculiarities of stress distribution are included in the given model:

i. First, the radial stresses distribution and their accounting for in the Tresca's limit condition.
ii. Second, the crack location and related to it the interaction (in the limit state sense) of the hoop stress (retained the inner pressure) with the axial one (due to the local bending moment).
iii. Third, the hoop stress redistribution in the circumferential direction with the possibility of plastic hinge forming in the plane, which is opposite to the crack plane.
iv. The easy to use analytical formulas are derived. An analysis of them and a comparison with the experiment results are carried out.

2 INTRODUCTION

The pipelines and piping defects repair, as a rule, is connected with their stoppage and incurs big economic losses. On the other hand, the pipes failure due to defects is potentially threatening in view of the economic and social consequences. Therefore, the assessment of the residual strength of pipes with defects should be as accurate as possible and based on the scientifically and experimentally justified methods. Over the last 40 years, such an analysis was based on so-called empirical formulas of the Battelle Memorial Institute (BMI), which are used in the calculation procedures almost in all normative documents in various industries. In particular, for a surface rectangular defect, the expression to determine the limiting pressure p_{LL} has the form (see Kiefner 1973):

$$p_{LL}R/\sigma_f t = \alpha_{LL}(\lambda,\tau) = \tau/(1-(1-\tau)/M_1(\lambda)), \tag{1}$$

where σ_f is the stress, which characterize the material property to resist the plastic failure; $\tau = (1 - a/t)$ is the dimensionless residual thickness of the net section; $\alpha_{LL}(\lambda,\tau)$ is the dimensionless limiting pressure or the coefficient of strength reduction; $\lambda = c/\sqrt{Rt}$ is the relative defect length; a is its depth; c is the half-length of the defect; and $M_1(\lambda)$ is some function which for a defect of a zero length is $M_1(0)=1$ and increases with an increase in λ, i.e., $M_1(\lambda>0)>1$. Now in most standards such as API RP 579 (2000) and SINTAP (1999) formula (1) for the description of the ***local*** limit state attained in the pipe wall ligament is intended.

For a through defect ($a = t$), at the BMI, a similar formula was proposed (see Kiefner 1973):

$$\alpha_{LL}(\lambda) = 1/M_2(\lambda) \tag{2}$$

where M_2 is another monotonic function with similar properties as compared with M_1 (in fact, sometimes it can be the same in various application).

Let us perform a comparative analysis of the limiting pressure for the surface (1) and through (2) cracks. It is evident that, for a very deep defect (at $\tau \to 0$), the residual strength $\alpha_{LL}(\tau,\lambda) \to 0$. On the other hand, the strength of a through defect is a nonzero value; therefore, $\alpha_{LL}(\tau \to 0,\lambda) < \alpha_{LL}(\lambda)$. When observing a defect of a very small depth, $\alpha_{LL}(\tau \to 1,\lambda) \to 1$ at any λ and $\alpha_{LL}(\tau \to 1,\lambda) > \alpha_{LL}(\lambda)$. Since functions $\alpha_{LL}(\tau,\lambda)$ and $\alpha_{LL}(\lambda)$ are monotonic, for surface defects, there exist such $\tau_C(\lambda)$ that the strengths of surface and through defects of equal length are equal, i.e., $\alpha_{LL}(\tau_C,\lambda) = \alpha_{LL}(\lambda)$. Thus, if the pressure for a pipe with a surface defect achieves a critical value and $\tau < \tau_C(\lambda)$, then after the growth of a surface defect to a through one, its further development is stopped since the through defect strength is greater than the surface one. Such a phenomenon has received the name of "leak before break," and it is widely used in the safety substantiation of critical components.

At the same time, it was experimentally observed that surface defects often demonstrate a strength, which is significantly higher than that predicted by formula (1), especially for deep defects. This is connected with the inherent property of formula (1) which gives zero residual strength for crack of any length at $\tau \to 0$. To make more accurate predictions, Ewing (1983) based on formula (2) for through defects has artificially produced a formula for computing the limiting pressure for surface defects. It was assumed that a pipe with a surface defect can be considered as two infinite pipes: the first is defect free with wall thickness $(t-a)$, and the second has wall thickness a and a through defect of length c. The limiting pressure is determined as the sum of two limiting pressures for each of the pipes:

$$p_{LL}^G R/\sigma_f t = \alpha_{LL}^G(\tau,\lambda_1) = \tau + (1-\tau)/M_2(\lambda_1)\,;\ \lambda_1 = c/\sqrt{Ra} \tag{3}$$

The limit pressure introduced in such a way has acquired in the literature the name "global". It is supposed that in this case limit state of pipe material is attained well beyond the crack ligament in contrast to the "local" limit state. Both formulas (1) as well as (3) to application in international standards such as API RP 579 (2000) and SINTAP (1999) are recommended. However, the cases of application of each of them are not clearly fixed and can not be understandable without physical justification. This causes many problems connected with the analysis of real defects and, in particular, in application of the "leak before break" concept. If the failure of surface defects occurs according to the more popular and recommended by standards global formula, it is impossible to understand why the leak phenomenon takes place in practice, because in most cases the formula (3) gives larger values of residual strength than formula (2).

An alternative to the use of empirical formulas (1)-(3) is the numerical analysis. The finite element method (FEM) to determine the limiting load of cylinders with axial defects (see Batte 1997, Andrade 2004) and to construct alternative formulas is more extensively used. As an example, we mention the formula by Leis (1997) for an axial crack of constant depth:

$$p_{LL} R/\sigma_u t = \alpha_{LL}^1(a,c) = \left[1 - a\left(1 - \exp\left(-0.157c/\sqrt{R(t-a)}\right)\right)/t\right] \tag{4}$$

In formula (4), as the material characteristic, the ultimate tensile strength σ_u, is selected. This choice was made due to results of tests of defect free pipes under inner pressure until to destruction. Since the work of Leis (1997) many standards become to use σ_u as the materials property, for example the well known defect assessment procedure DNV-RP-F101 (1999).

It should be noted that the existing commercial FEMs do not have ready universal procedures of computations for the case of a perfect plastic behaviour of a body or even for very large plastic deformations. Use of the FEM requires the development of special elements, special iteration procedures, and the introduction of additional subjective criteria of discerning the limit state on the calculation graphs.

These methodological problems for thin-walled pipes becomes more difficult for thick-walled pipes, which are used in piping systems of nuclear and fossil power engineering, the chemical industry, and where the problem of substantiation of the integrity is no less important.

The current state of the problem of the evaluation of thick-walled pipes with defects is described in publications by Staat (2004) and Staat et al. (2007). In these works based on the existing concept of a local and global solution and with allowance for the known solution of the theory of plasticity for the ultimate pressure of a defect-free thick-walled pipe, the following empirical formulas are proposed:

$$P_{\text{global}}/\sigma_u = D\left[\ln\left(r_2/(r_2-a)\right)/M_2 + \ln\left((r_2-a)/r_1\right)\right] \tag{5}$$

$$P_{\text{local}}/\sigma_u = D\left(r_1/\left(r_1+(a/2)\right)\right)\ln\left(1+t/r_1\right)\left(1-(a/t)\right)/\left(1-\left(a/M_{FL}t\right)\right) \tag{6}$$

Based on more than 300 experiments with pipes having surface defects, the global formula better describes their results is concluded. Here, as a material characteristic, the ultimate tensile stress σ_u is chosen; $D=1$ under conditions of a two-dimensional stress state; $D=2/\sqrt{3}$ for a two-dimensional strain state; $M_2=\sqrt{1+1.61\left(c^2/(r_2-a)a\right)}$ for a thick-walled pipe with an axial external defect of depth a; $M_{FL}=\sqrt{1+1.61\left(c^2/r_1t\right)}$; and r_1, r_2 are the inner and outer radii, respectively.

A useful approach to defect analysis is the analytical modelling. The limit plastic behaviour model of thin-walled pipes with surface axial defects was developed at the Institute for Problems of Strength (Ukraine) by Krasovskii and Orynyak (1990) and given also in Orynyak (1994), Orynyak (2006), et. It not only gives an understandable mechanism and the nature of the failure but the computation results correlate well with real full-scale tests data. Besides the model physically explain the *local* nature of fracture in the sense that the availability of the ligament prevents the redistribution of the shear stress and some stress components according to the shell theory can be omitted in the analysis. Based on the static theorem of theory of plasticity the model is described in Orynyak (2006).

The goal of the present investigation consists in the elaboration of the above model to the thick-walled pipe. In this case, the following is taken into account: (i) the surface defect location (at the internal or external surface of the pipe wall), which affects the limit value of the local bending moment. (ii) The possibility of plastic hinge formation in the opposite to the crack plane, which is allows resisting some additional part of pressure. The idea of this is borrowed from the work of Staat (2004). (iii) The modelling of radial stresses distribution and accounting for them in the Tresca limit condition is the main problem in our investigation. The choice of the Tresca criterion as the limit condition for the material at the ultimate state, for example, can be justified by the results of investigation of material behaviour under multiaxial loading by Lebedev et al. (1983). Besides the older destructive experiments on the ultimate capacity of thick-walled defect free pipes under inner pressure and bending moment by Zverkov (1958) also justify the application of the Tresca criterion as well as the choice of ultimate strength as the ultimate property.

3 LIMIT LOAD MODEL FOR A CRACKED PIPE

3.1. Basic assumptions for the thin-walled model

The model is fully described in work by Orynyak (2006). For thin-walled pipes, its basis is the equilibrium equations for a cylindrical shell. Due to the “local” limit state character, the shear force and axial force are taken to be zero and the rest equation can be written in the simplified form:

$$p = N_\varphi / R + dQ_x / dx \,,\; dM_x / dx = Q_x \tag{7}$$

where the ultimate value of the circumferential force N_φ is determined through the residual net thickness of the section and is:

$$N_\varphi = \sigma_u (t - a(x)) \tag{8}$$

where x is the axial coordinate.

Since $\sigma_u (t - a_{\max}) < Rp_{LL} < \sigma_u t$ (where $a_{\max}$ is the maximum defect depth), the transverse force increment dQ_x / dx is, at first, positive and, then, negative, which provides the presence of the interval $\Delta x = x_1 - x_0$, where Q_x is positive. This interval determines the limit state area which boundaries are the points x_0 and x_1. At this points the values of transverse force $Q_x(x_0) = Q_x(x_1) = 0$. Note that for a symmetrical defect, if the coordinates origin is connected with the symmetry centre, the coordinate $x_0 = 0$. According to the second equation of (7), the increment of the applied moments ΔM_{appl} takes place over this interval. The limit state condition is equality of the applied moment to the difference between the limit bending moments in the pipe wall at the beginning and the end of the limit area:

$$\Delta M_{appl}\Big|_{x_0}^{x_1} = [M_x(x_1)] - [M_x(x_0)] \tag{9}$$

For the simplest case of the absence of other force components (see Orynyak et al. 1994, Orynyak et al. 1996), the limit moments are:

$$[M_x(x_1)] = -[M_x(x_0)] = \sigma_u t^2 / 4 \tag{10}$$

3.2 Defect location

This problem was treated little in the literature mainly because of the difficulty of applying a rectangular defect on the inner surface. The defect location (at the internal or external surface) affects, in the first place, the limit values of moments since the value of the increment of applied moments is caused by the defect geometry (8). Consider initially the end of limit area where $x = x_1$, i.e. section *II* (Fig. 1). Here for both variants of the defect location the negative axial stresses from the bending moment interact with positive circumferential stresses from internal pressure. Taking into account the Tresca theory, for simplicity, we take $[M(x = x_1)] \approx 0.5 \sigma_u t^2 / 4$ (see Orynyak 2006).

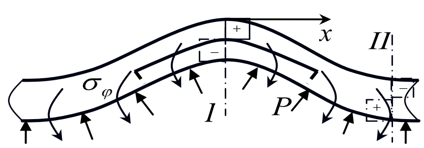

Figure 1. Interaction of circular and axial stresses at points $x_0 = 0$ and $x = x_1$ in a cracked pipe.

Now consider section *I* ($x = x_0$). The internal defect makes that circumferential stresses at the internal surface are equal to zero ($\sigma_\varphi = 0$); therefore the negative axial stress due to bending moment can exist here and the interaction of stresses is absent. We can assume that $[M(x = 0)]$ attain the maximal value, i.e. $[M(x=0)]=\sigma_u t^2/4$. In the case with the external defect, at the inner surface we have positive circumferential stress and negative axial ones, which should interact in the Tresca criterion sense. To be consistent with the thin-walled model (10) we assume that $[M(x=0)]=0.5\sigma_u t^2/4$.

Thus, the internal defect occurrence is taken into account through a correction of the limit bending moment and leads to increase of it in 1.5 times with respect to the case of the external defect.

3.3 Radial stresses effect

First, consider the external defect (Fig. 2). Instead of first equation of (7), we write the differential equilibrium equation in the radial direction for a thick-walled axial-symmetric cylinder:

$$d\sigma_r/dr+(\sigma_r-\sigma_\varphi)/r+d\tau_x/dx=0 \tag{11}$$

In this equation, radial stresses σ_r are negative (compressive); therefore, in the limit state $(\sigma_\varphi-\sigma_r)=\sigma_u$. Determine the changes in transverse forces over the first part of the limit area interval, i.e., in the defect section ($0<x<c$). To do this integrate Eq. (11) within the range $R_1\le r\le R_a=R_2-a$. Assuming approximately that $\int\tau_x dr=Q_x$, we obtain:

$$dQ/dx\big|_I=P-\sigma_u\ln(R_a/R_1) \tag{12}$$

where P is the internal pressure, R_1 is the inner pipe radius, R_2 is the outer pipe radius, and a is the rectangular defect depth.

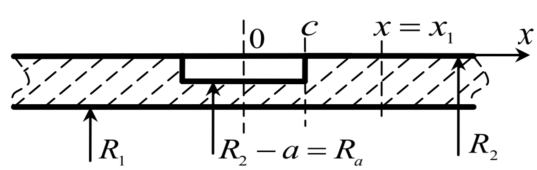

Figure 2. Axial section of a cracked thick-walled pipe.

In expression (12), the influence of the finite wall thickness is taken into account through the latter term (compare with first equation of (7) for thin-walled pipes). The drawback of Eq. (12) is that, for an infinite defect of depth $a\to t$ (second term in right side of (12) equal to zero), it gives an increment of transverse forces at any nonzero pressure. However, for thick-walled pipes this is incorrect due to possible formation of stressed zone in the lower part of pipe.

Let us consider the cross section of the thick-walled pipe part with the axial surface defect (Fig. 3: UPS is the cracked upper part of the section, and LPS is the lower part of the section). In this section, the upper part with the defect is in the limit state due to the circumferential and radial stresses. However, its lower part strength capacity has not yet exhausted and can additionally resist to some part of pressure. Let us designate it as ΔP. Since, in the upper part of the section, the load-carrying capacity is exhausted, the mentioned part of the pressure produces a force and moment in its lower part. In work by Staat (2004), equating the applied $M=2PR(R+a/2)$ and limit $[M_{Staat}]=\sigma_u a^2/4$ moments, is obtained that $\Delta P/\sigma_u=a^2/4R_1(2R_1+a)$. This is result of "global" model where a pipe with surface defects is presented as a defect free pipe and a pipe with a through defect. In the present work, to determine the additional part of the pressure, we used the known Girkman (1931) equation, which gives the admissible combination of axial forces and moments in any section $abs(M/[M]) + (N/[N])^2 = 1$, where $[N] = \sigma_u t$ is the maximum possible value of the force if no other force factors act. Taking into account $N/[N] \approx 1 - \sigma_u/(P\ln(R_2/R_1))$ and letting $M = 2\Delta P R_1(R_1 + t/2)$ and $[M] = \sigma_u t^2/4$, we find the additional part ΔP of the pressure that can be held by each defective cross-section:

$$\Delta P/\sigma_u = t^2 \cdot \left(1 - \left(P/\sigma_B \ln(R_2/R_1)\right)^2\right) / \left(8R_1(R_1 + t/2)\right) \quad (13)$$

Thus the model of a thick-walled pipe, in the limiting case $a/t \to 1$ and $c \to \infty$, gives a nonzero value $\Delta P \neq 0$. For example, at $t/R_1 \to 1$, we have $\Delta P \approx 0.2\sigma_u$.

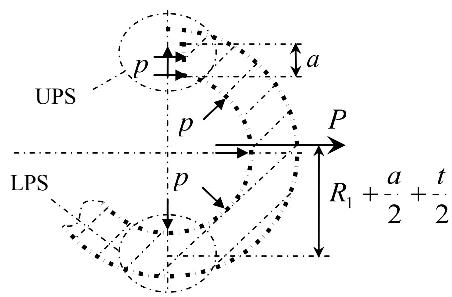

Figure 3. Cross section of a cracked thick-walled pipe.

Taking into account Eq. (13), expression (12) takes the following form:

$$dQ/dx\big|_I = P - \sigma_u \ln(R_a/R_1) - \Delta P \quad (14)$$

Further, the procedure of determining the limit pressure is in complete agreement with that for a thin-walled pipe (see Orynyak 2006). Firstly, we determine the coordinate x_1, where values of Q is zero. Further, we determine the value of the applied bending moment at the first part (from 0 to c), by integrating expression (14) two times: $M_I = c^2[P - \sigma_u \ln(R_a/R_1) - \Delta P]/2$. The bending moment M_{II} at the second part of the limit area (from c to x_1) is determined similarly: $M_{II} = 0.5 \cdot c^2(P - \sigma_u \ln(R_a/R_1) - \Delta P)^2 / \{P - \sigma_u \ln(R_2/R_1)\}$. The sum of values M_I and M_{II} gives the value of the applied moment $\Delta M_{appl}\big|_{x_0}^{x_1}$.

Using the condition of the limiting state (9) with allowance for (13), we determine the expression for the dimensionless strength reduction factor $\overline{\alpha}_{\text{ext}}$ for the external defect:

$$\overline{\alpha}_{\text{ext}} = \left(1 + 2\lambda^2 a_{\text{ext}} b_{\text{ext}}\right) / \left(1 + 2\lambda^2 b_{\text{ext}}\right) \quad (15)$$

where for the convenience of the presentation of results, we introduce the notion $\overline{\alpha}_{\text{ext}} = P/(\sigma_u \ln(R_2/R_1))$, where the resulting constants are: $a_{\text{ext}} = (\ln(R_a/R_1) + \Delta P/\sigma_u)/\ln(R_2/R_1)$, $b_{\text{ext}} = R_1(\ln(R_2/R_1) - \ln(R_a/R_1) - \Delta P/\sigma_u)/t$. It is evident that, at $\lambda \to 0$ $\overline{\alpha}_{\text{ext}} \to 1$. This corresponds to the essence of the "reduction factor" concept.

The procedure of determining the expression for the strength reduction factor $\overline{\alpha}_{\text{in}}$ for the internal defect is similar. The difference is that when determining the ΔP, it is necessary to take into account the pressure effect on the defect side surfaces, which leads to an increase of the moment in the lower part of the section (Fig. 3). The additional pressure ΔP is determined from the following equation $\Delta P/\sigma_u = t^2\left(1-\left(P/\sigma_u \ln(R_2/R_1)\right)^2\right)/4(2R_1+a)(R_1+a/2+t/2)$. The aggregate result for the dimensionless strength reduction factor $\overline{\alpha}_{\text{in}}$ for the internal defect is described by the formula:

$$\overline{\alpha}_{\text{in}} = \left(1+4\lambda^2 a_{\text{in}} b_{\text{in}}/3\right)/\left(1+4\lambda^2 b_{\text{in}}/3\right) \quad (16)$$

where $a_{\text{in}} = \left(\ln(R_2/R_a)+\Delta P/\sigma_u\right)/\ln(R_2/R_1)$, $b_{\text{in}} = R_1\left(\ln(R_2/R_1)-\ln(R_2/R_a)-\Delta P/\sigma_u\right)/t$.

The fact that, for an internal defect, the factor of 4/3 (instead of 2) is used in expression (16) is related to the greater ability of pipe walls at the section $x=0$ to resist the applied moment.

The main problem in performing calculation according to (15) and (16) is that ΔP depends on the looking for value of P. Thus, an iteration procedure should be applied. The idea is in prescribing some values of ΔP, subsequent calculation of P and successive adjustment of ΔP by the method of division in half. The starting point in choosing of initial ΔP is important. Analyzing both models (expressions (15) and (16)), we observe that the value $\Delta P/\sigma_u$ should be within the limits of $0<\Delta P/\sigma_u<A$. Here, for the external defect, the upper limit has the value $A=\min\left(\ln(R_2/R_1)-\ln(R_a/R_1);\, t^2/8R_1(R_1+t/2)\right)$, and for the internal defect, $A=\min\left(\ln(R_2/R_1)-\ln(R_2/R_a);\, t^2/4(2R_1+a)(R_1+a/2+t/2)\right)$.

3.4 Validation and discussion

From the analysis of basic assumptions of the model, it is evident that, on the one hand, the increment of applied moments is greater for the internal defects; on the other hand, limit moment is greater for this defect, too. To see which defect (external or internal) has the greater residual strength, we construct graphs of the dependence of the coefficient α (expressions (15) and (16)) on the relative defect length λ at various ratios t/R_1. The comparison is performed for the same defect depth $\tau=0.5$. It is evident (Fig. 4) that residual strength of a thin-walled pipe with an internal defect is bigger for intermediate crack lengths and smaller for long defects as compared with external one. With increase in the wall thickness, the external defect has bigger pressure resisting capacity.

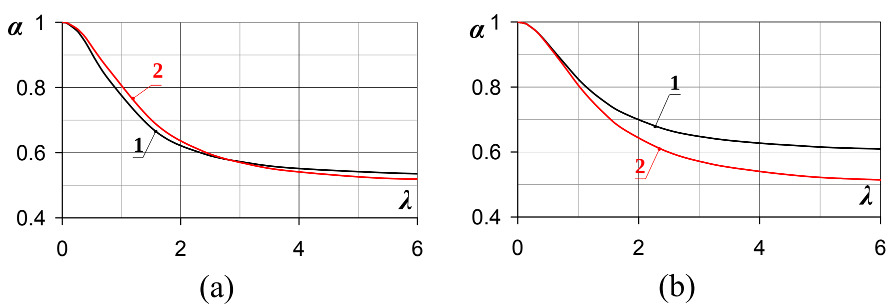

Figure 4. Strength reduction factor as a function of the external and internal defects relative length: (a) $t/R_1=1/10$, $\tau=0.5$ and (b) $t/R_1=1/2$, $\tau=0.5$: (*1*) external defect, (*2*) internal defect.

Compare the results of analytical models with experimental data. The following question arises: What is to be selected as the measure of determining the accuracy of the model? According to Staat (2004) and Staat et al. (2007), the prediction accuracy by models can be characterized by the relative error defined as $(\alpha_{\text{exp}}-\alpha_{\text{theor}})/\alpha_{\text{theor}}$, where α_{exp} is the experimental value and α_{theor} is the theoretical value. Such a choice of the accuracy parameter can be vague from the practical point of view. It is evident that, at $\alpha_{\text{theor}}\to 0$, even a small divergence between α_{exp} and α_{theor} can give a large value of eps. If it is assumed that $\alpha_{\text{formula}} = 0.03$ and $\alpha_{\exp} = 0.01$, then taking into account the possible errors in crack dimensioning, engineering attitude to this strength as compared to the initial defectless strength $\alpha_{\text{init}} = 1$, it can be concluded that a good agreement is observed. At the same time, according to this evaluation, the error is 200%.

In our opinion the model accuracy may be determined in the following way: $eps = (\alpha_{\exp} - \alpha_{\text{theor}})/\alpha_{\text{init}}$, where $\alpha_{\text{init}} = 1$. Based on such definition of the error of prediction we will try to demonstrate that the corresponding model for internal crack and not the reverse better describes thick-walled pipes with internal defects. To do this, we use an experimental base (about 300 full-scale tests) by Staat (2004). Let us present some of them (material St35, $\sigma_u = 486\text{MPa}$, $\sigma_y = 336\text{MPa}$, $t = 4$ mm, $R_1/t = 10$). In Table 1, which gives the results for internal defects, the following notation is given: $\overline{\alpha}_{\text{in}}$ - according to formula (16); $\overline{\alpha}_{\text{ext}}$ - according to formula (15); $\alpha_{\exp}$ is the experimental value; eps_{in} and eps_{ext} are the corresponding divergences from experimental data as proposed above. With the purpose of evaluating the proposed models, we determine the mean divergences of theoretical data from the test: $eps^{\text{m}}_{\text{ext}} = -0.022$ and $eps^{\text{m}}_{\text{in}} = -0.025$. The absolute mean divergences are $\left|eps^{\text{m}}_{\text{ext}}\right| = 0.0316$ and $\left|eps^{\text{m}}_{\text{in}}\right| = 0.027$. According to the second measure of accuracy, it is appropriate to analyze the residual strength of thick-walled pipes with an internal defect by the analytical model of a pipe that takes into account the defect location exactly on the internal surface. At the same time, using the first measure of accuracy, we can draw the different conclusion. This testifies about the need for performing the more precise tests for external and internal cracks comparison.

Table 1: Comparison test results for the internal defects by Staat (2004) with formulas (15) and (16).

№	λ	τ	$\overline{\alpha}_{\text{in}}$	$\overline{\alpha}_{\text{ext}}$	$\alpha_{\exp}$	eps_{in}	eps_{ext}
1	3.105	0.8	0.814	0.813	0.812	0.002	0.001
2	1.769	0.5	0.632	0.614	0.71	-0.078	-0.096
3	3.656	0.5	0.524	0.531	0.556	-0.032	-0.025
4	9.63	0.5	0.482	0.502	0.479	0.003	0.023
5	4.009	0.35	0.377	0.384	0.397	-0.02	-0.013

Analyze data for pipes with similar characteristics that have an external defect (Table 2). To draw a conclusion with respect to the adequacy of the presented models with experimental data, let us compare their mean divergences $eps^{\text{m}}_{\text{in}} = 0.00377$ and $eps^{\text{m}}_{\text{ext}} = -0.00631$, and the absolute ones $\left|eps^{\text{m}}_{\text{in}}\right| = 0.0439$ and $\left|eps^{\text{m}}_{\text{ext}}\right| = 0.0552$. As in previous case, it is impossible to draw the univocal conclusion about the better accuracy of the corresponding model intended for the given defect location.

Now we make the similar comparison of the results presented in Tables 1 and 2 with the results predicted by the models (5) and (6) of Staat (2004). According to them for the data of Table 1, it can be obtained the following values of absolute mean divergences: $\left|eps^{\text{m}}_{\text{global}}\right| = 0.0394$, $\left|eps^{\text{m}}_{\text{local}}\right| = 0.0532$. It is evident that accuracy of our model is better in this case. For the data of Table 2, we obtain that according to formulas (5) and (6) the absolute mean divergences are $\left|eps^{\text{m}}_{\text{global}}\right| = 0.056$ and $\left|eps^{\text{m}}_{\text{local}}\right| = 0.0878$. Evidently, our models results are more accurate in this case, too.

Table 2: Comparison test results for the external defects by Staat (2004) with formulas (15) and (16).

№	λ	τ	$\overline{\alpha}_{\text{in}}$	$\overline{\alpha}_{\text{ext}}$	α_{exp}	eps_{in}	eps_{ext}
1	2.594	0.725	0.816	0.795	0.729	0.087	0.066
2	4.56	0.75	0.792	0.782	0.748	0.044	0.034
3	0.786	0.5	0.869	0.826	0.786	0.083	0.04
4	1.061	0.425	0.754	0.697	0.733	0.021	-0.036
5	2.83	0.5	0.595	0.571	0.6	-0.005	-0.029
6	4.796	0.5	0.545	0.534	0.6	-0.055	-0.066
7	8.726	0.5	0.522	0.519	0.489	0.033	0.03
8	8.648	0.475	0.497	0.494	0.463	0.034	0.031
9	2.948	0.3	0.393	0.367	0.391	0.002	-0.024
10	4.913	0.25	0.291	0.281	0.321	-0.03	-0.04
11	3.341	0.225	0.3	0.279	0.182	0.118	0.097
12	1.572	0.1	0.349	0.283	0.349	0	-0.066
13	1.179	0.1	0.464	0.38	0.578	-0.114	-0.198

In the proposed analysis, a significant scatter of results is sometimes observed (Table 2), particularly for short defects ($\lambda \to 0$). One of the possible causes of such a scatter can be the technology of the defect fabrication, because of which the determination error of $\overline{\alpha}_{\text{in}}$ and $\overline{\alpha}_{\text{ext}}$ arises. With a decrease in the milling cutter diameter, through which the defect is fabricated, its shape approximates a rectangular one. To evaluate the possible error of residual strength prediction related to the procedure of the defect fabrication approximately assume that the milling cutter diameter D_{c} is equal to the pipe wall thickness, i.e., $D_{\text{c}} \approx t$. A surface defect made in such a way can be considered as rectangular in two ways. If it is assumed that the length of the defect includes curvilinear parts, the expression for the defect relative length is: $\lambda_1 = c/\sqrt{Rt}$ (Fig. 5). On the other hand, in the length calculation of a rectangular defect we can ignore the mentioned parts and the relative length can be as: $\lambda_2 = (c-t)/\sqrt{Rt}$. Evident that difference in lengths is $\Delta\lambda = \lambda_1 - \lambda_2 = \sqrt{t/R}$, which increase with increase in the pipe wall thickness. As example this influence is illustrated for very short and deep ($\tau = 0.5$, $\tau = 0.3$) defects, the error is $\Delta\alpha_{0.5}$ and $\Delta\alpha_{0.3}$, respectively (Fig. 5).

4 CONCLUSION

Local analytical model of the stress distribution in a pressurized thick-walled pipe with axial surface defects at attaining the limit plastic state have been developed. This model, along with allowance for the effect of radial stresses, takes into account the defect location (on the internal or external surface). This allows obtaining the easy to use analytical formulas for predicting the ultimate inner pressure. The analytical results are compared with full-scale tests of thick-walled defective pipes. The engineering measure of error of prediction is suggested according to which the proposed analytical models have a good predictive ability to describe the experimental results. At the same time, the available experimental results do not confirm neither disproves the influence of the crack location on the pipe residual strength.

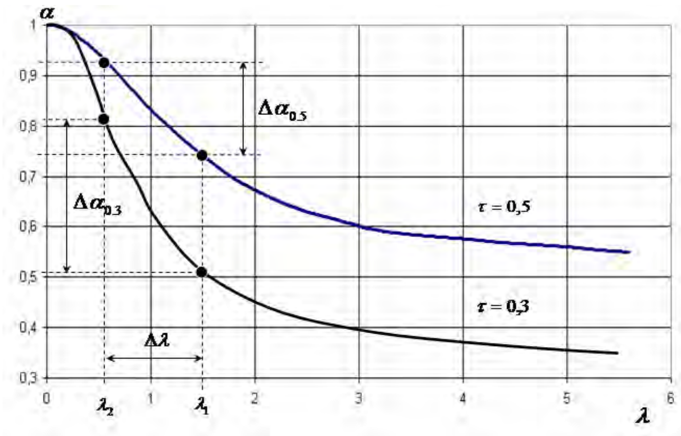

Figure 5. The defect shape influence on the calculation accuracy.

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