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LEAK RATE PREDICTION FOR LEAK-BEFORE-BREAK ASSESSMENTS ON NUCLEAR PIPEWORK USING A NEW FINITE FINITE ELEMENT METHOD

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ABSTRACT

A finite element model which utilizes the extended finite element method (XFEM) to model leaks through cracks is presented in this paper. Preliminary calculations using Ecrevisse in Code Aster indicates there is a case to include thermo-mechanical effects in Leak-before-Break analysis. This is because the leak rate can reduce due to crack closure effects. The preliminary work motivated the development of a specific finite element which incorporated a leak rate model and was coupled to the structure through a convection law and pressure boundary condition. Convergence studies were performed on the XFEM thermal model to validate its suitability and optimum convergence rates were seen. The mechanical model was validated by comparing the finite element approximated crack opening displacement with analytical solutions and the errors were less than 1% for a relative element size of more than 0.01. Thermo-mechanical simulations were then carried out using a simple 2D plate with a central crack. The leak rate was shown to decrease by about 30% when the fluid was 10^{o}C hotter than the structure.

INTRODUCTION

The leak rate calculation in a Leak-before-Break assessment is subject to considerable error due to the uncertainty in the crack shape input parameters and the thermal hydraulics of the flow. Typically a crack will have a path through the thickness of the pipe or pressure vessel which deviates considerably from the idealised diverging or converging channel. In order to model the flow through this geometry in detail an expensive CFD analysis would be required. Therefore simplified 1D thermal hydraulic codes are preferred for engineering assessments, and these use additional pressure loss terms which account for changes in direction and corners (Taggart and Budden, 2008), (Beck et. al., 2005), (Rudland et. al., 2002). For two phase flow there is the added complexity of phase change through the crack. SQUIRT (Paul et. al., 1990) is a computer code based on the Henry-Fauske Homogeneous non-equlilibrium model (Henry and Fauske, 1971) which can model steam evolution through a crack. The work presented here is a first step in coupling the leaking thermofluid to the structure so that thermo-mechanical effects can be investigated. It is hypothesized that a leaking fluid through a crack will heat up the crack walls causing crack closure and hence reducing leak rate. This is shown to be another factor that may cause inaccurate over-prediction of leak rate. The thermal hydraulics of the flow is also considered in some detail by using the leak rate calculational tool Ecrevisse in Code Aster (EDF, 2013).

MODELING IN CODE ASTER

Preliminary calculations were performed to assess the thermo-mechanical behaviour of a structure containing a through wall crack using Ecrevisse in Code Aster. The mesh used was a simple 2D representation of a pipe wall, a plate of width 50mm and depth 10mm (Figure 1).

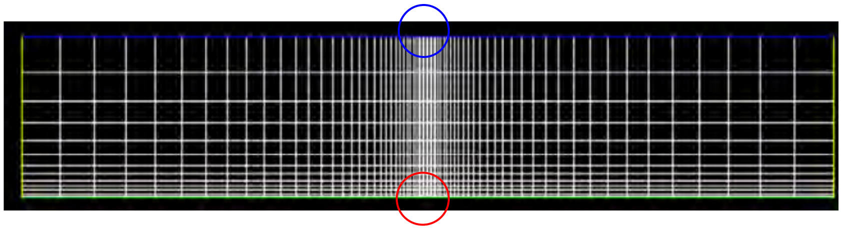

Figure 1. Mesh used for simulations. The crack is located at the centre of the plate in the vertical direction. The crack in inlet is circled in red and the crack outlet is circled in blue.

The plate was fixed at both ends with Encastre boundary conditions so that any expansion due to heat transfer from the fluid could cause crack closure. Two different situations were considered to investgate both the structural response when a leaking single phase fluid is present and the thermal hydraulic behaviour of a water-steam mixture. For the air model, inlet conditions of the fluid were a pressure of 1MPa and temperature of 140^{o}C. For the water model, inlet conditions were a pressure of 16MPa and a temperature of 300^{o}C. The crack was considered to have parallel walls with no change in crack length through in the out of plane direction. The crack opening displacements (COD) for the air model ranged from 90 to 140 microns, and the length was 100mm, where a rectangular opening was considered. For the water model, the crack was 60mm in length and the COD was 50 microns. Material properties are approximate and correspond to those of Austenitic Steel with a Young's modulus of E=200GPa, Poisson's Ratio ν=0.3, Thermal Conductivity k=20W/mK, and Thermal Expansion Coefficient α=1.282x10^{-5}.

Heat transfer occurs at the inner and outer surfaces and along the crack walls. The heat transfer coefficients (HTC) are fixed as 8 W/m^{2}K on the inner surface and 4 W/m^{2}K on the outer surface. Along the crack walls, correlations are used to calculate the HTC. Ambient temperatures on the inside are inlet fluid temperatures and for the outside they are 20 ^{o}C for the air model and 100 ^{o}C for the water model.

Results

Figure 2 shows the temperature distribution in the plate for the air model after the structure reaches a steady state solution. The highest and lowest temperatures are seen at the inlet and the outlet respectively.

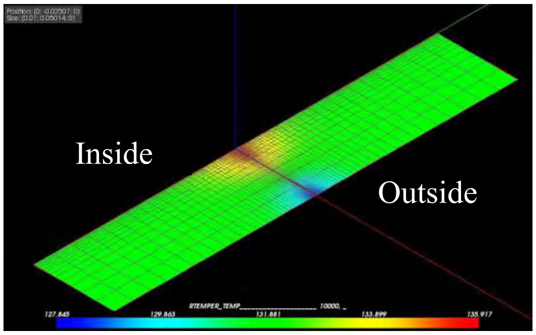

Figure 2. Temperature distribution for the air model

Figure 3 shows the effect of including heat transfer on the crack walls. There is a large reduction in leak rate which varies non-linearly with crack opening displacement.

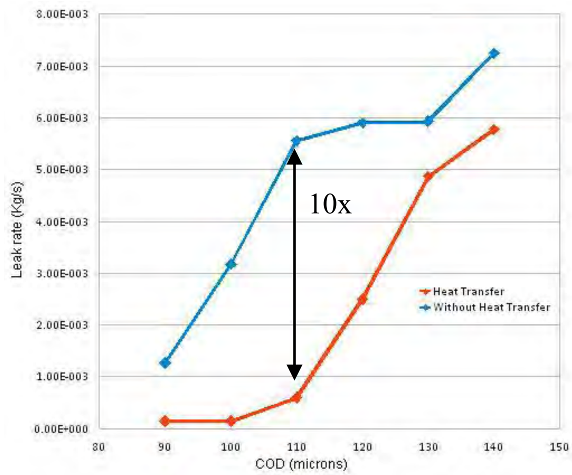

Figure 3. Leak rates for air model, with and without heat transfer with crack walls. Arrow indicates a factor of ten difference.

Temperature and pressure plots of the water model are given in Figs. 4 and 5 respectively. The plots show the effect of including heat transfer, as well as sensitivity to different friction correlations. Figure 4 indicates that when heat transfer is turned off in the model, i.e. adiabatic, the temperature remains constant until a point through the thickness where it suddenly drops off. This is due to the pressure falling below saturation pressure and the water flashes to steam thereby reducing its temperature. When heat transfer is included the temperature gradually decreases with a smooth profile. The pressure is shown to only be slightly affected when heat transfer is included.

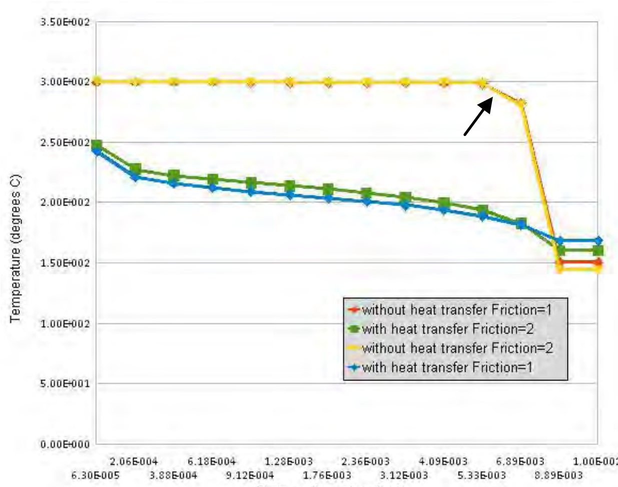

Figure 4. Temperature of water/steam through thickness with and without heat transfer. Two different friction correlations are considered. The arrow indicates where flashing occurs.

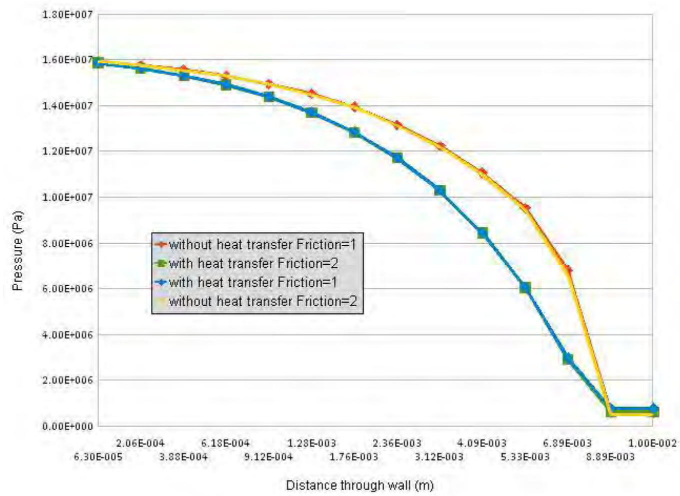

Figure 5. Pressure of water through thickness

Summary of preliminary work

In summary the preliminary calculations in Code Aster show that leak rates can be reduced considerably when thermo-mechanical effects such as crack closure are considered. It is worth noting that this effect is exaggerated here due to the boundary conditions and in reality the thermal closure may not be so pronounced. However, it emphasizes the point that thermal dilation, which causes the crack to close, is having a large influence here. The plots of fluid properties show that heat transfer to the crack walls can also change these. The sudden drop in temperature in Fig 4 is due to flashing and there may be a situation where heat could be drawn from the structure when flashing occurs in the crack. More work was thus required to investigate this phenomena.

PROPOSED LBB MODEL

A new tool is being developed, initially with a prototype code in MATLAB, and there is an on-going implementation in Code-Aster. The purpose of this tool is to investigate the thermo-mechanical closure effect and thermal hydraulics of flow using a special element based on the extended finite element method (XFEM). Therefore a brief descriptive summary of XFEM is given. The fluid models are based on DAFTCAT (Chivers et. al., 1996) and SQUIRT (Paul et. al., 1990) which describe single and two phase flow respectively. The coupling of the fluid to the structure is in the form of a pressure boundary condition, as well as a convection law.

XFEM

Standard finite elements suffer poor convergence rates when the approximation space contains discontinuous or singular behaviour. This is particularly relevant to crack problems as they exhibit a strain singularity at the crack tip and a jump in displacement along the crack plane. Therefore, using the partition of unity method (PUM), one can improve the convergence behaviour by introducing enrichment functions which describe the complex physics (Belytshko, 1999). Typically for an elastic crack problem these functions are the Heaviside and Westergaard asymptotic functions for the crack plane and crack tip respectively.

$$u(\underline{x}) = \sum_{i \in \Omega} N_i(\underline{x}) u_i + \sum_{i \in \Omega_c} N_i(\underline{x}) H(\chi(\underline{x})) a_i + \sum_{i \in \Omega_m} N_i(\underline{x}) \Phi_i(\underline{x}) b_i + \sum_{i \in \Omega_{ct}} \sum_{j=1}^{4} N_i(\underline{x}) \Psi_{ij}(\underline{x}) c_{ij} \qquad (1)$$

Equation (1) shows that there are the standard elements, Ω, with standard shape functions, then there are elements along with the crack face, material, and crack tip denoted by Ω_c, Ω_m, and Ω_{ct} respectively. These are the enriched elements, and the additional degrees of freedom a, b and c are multiplied by the enrichment functions, H(χ(x)), Φ(x) and Ψ(x). These are the Heaviside step function for jumps in displacement at the crack, the Moes Bimaterial function (Möes et. al., 2003) for jumps in strain or heat flux, and the branch functions for crack tip singularities in strain or heat flux respectively.

Fluid models

Single phase flow can be calculated in the computer code DAFTCAT (Chivers et. al., 1996), which uses a discharge coefficient C_D and the average crack opening area A.

$$\dot{m} = C_D (\rho_0 p_0)^{1/2} A \qquad (2)$$

C_D is calculated based on the dimensions of the crack and the friction and accommodates for lamina, turbulent or critical flow. The crack opening area is denoted by A, the inlet pressure by p_0 and the inlet density by ρ_0. The type of flow regime is very sensitive to crack opening area and if this changes due to closure from wall heating, the flow may enter a new regime and change the flow rate dramatically. This motivates the development of a thermo-mechanical model with heat transfer on the crack faces.

Two phase flow is a complicated phenomenon and requires many simplifying assumptions in order for it to be modeled in a practical way. A key assumption for this application is that the flow is assumed to be homogeneous, so there is no slip between the liquid and vapour phase. One such model which utilizes this simplification is the Henry-Fauske model (Henry and Fauske 1971), which is a homogeneous non-equilibrium model. The Henry-Fauske model is implemented in the computer code SQUIRT (Paul et. al., 1990) and can be summarised by the following system of non-linear simultaneous equations:

$$\Psi(G_c, p_c) = G_c^{\,2} - \frac{1}{\left[\dfrac{X_c \upsilon_{gc}}{\gamma_0 p_c} - \left(\upsilon_{gc} - \upsilon_{Lc}\right) N \dfrac{dX_E}{dp}\right]} = 0 \tag{3}$$

$$\Omega(G_c, p_c) = p_c + p_e + p_a + p_f + p_k + p_{aa} - p_0 = 0 \tag{4}$$

where

G_c is the mass flow rate per unit area,
X_c is the non-equilibrium mixture quality,
v_{gc}, v_{lc} are the specific volumes (1/density) of the vapour and liquid respectively,
p_c is the critical pressure,
X_E is the equilibrium mixture quality (volume fracture of vapour),
p is the pressure,
N is a ramp function, equal to 0 at $X_E = 0$ and 1.0 at $X_E \geq 0.05$, and
γ_0 is the isentropic exponent

This model accounts for thermal non-equilibrium between the liquid and vapour phase through a relaxation correlation. With this modification, the quality can change from the equilibrium value, giving a more accurate representation of the flow physics. The subscripts in (4) denote the different types of pressure changes, which are: critical, entrance, phase change acceleration, friction, head losses at crack path turns, area change loss and inlet pressure.

Thermo-mechanical Coupling

The coupling of the fluid to the structure is acheived using a convection law to model the heat transfer from the fluid to the crack wall. As well as this the fluid pressure is applied to the crack face.

$$\bar{\underline{q}}_{1,2} = h(T_{1,2} - T_{bulk})\underline{n}_{1,2} \tag{5}$$

where the subscript 1,2 denotes the two crack faces with their respective temperatures, T, and normals, n. The heat transfer coefficient, h, can be calculated from correlations. Here the Titus Boelter correlation was chosen as a good approximation.

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n} \quad \text{and} \quad h = \frac{k}{D_h} Nu \tag{6}$$

where n = 0.4 for fluid heating and 0.33 for cooling, with the conduction, *k*, and hydraulic diameter, D_h. Pr is the Prandtl number, Nu is the Nusselt number and Re is the Reynolds number.

The thermo-mechanical finite element model follows a similar formulation to that of Duflot (2008). The thermal model contains a discontinuity in heat flux along the crack due to the presence of a hot fluid, which acts like a line heat source. This means the temperature is taken to be continuous along the crack with a discontinuous first derivative. Therefore the second term from the standard Westergaard expansion for linear elastic fracture mechanics $\Psi_2(r,\theta)$ is used:

$$\Psi_j(r,\theta) = \left\{ r^{1/2}\sin(\theta/2), r^{1/2}\cos(\theta/2), r^{1/2}\sin(\theta/2)\sin\theta, r^{1/2}\cos(\theta/2)\sin\theta \right\} \tag{7}$$

where *r* and *θ* are polar coordinates centred at the crack tip.

IMPLEMENTATION

The finite element approximation was substituted into the variational weak form and the finite element equations were implemented in a 2-D MATLAB code. The code incorporates a single and two phase flow model based on the above formulation. For the two-phase case a Newton-Raphson iteration scheme was used to solve the simultaneous non-linear equations.

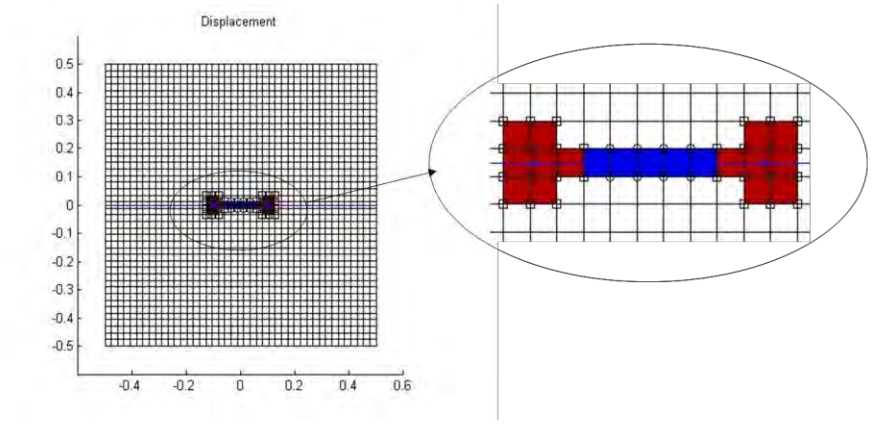

Figure 6. Mesh used for thermomechanical simulations, 3994 DoF for mechanical model and 1955 DoF for thermal model. The close up region shows the enriched elements, blue are crack face, red are crack tip. The elements containing the crack include heat flux and pressure on the crack faces.

RESULTS

Initially a convergence study was performed to assess the accuracy of the mechanical and thermal models. This was undertaken for the mechanical model by comparing an exact crack opening displacement solution against the finite element approximation for varying mesh sizes. For the thermal case an analytical temperature model was derived via a Green's function method. This was used with L2 and energy error norm to quantify the error for varying mesh sizes. The results of this are shown in Figures 7 and 8. After the model was validated, thermo mechanical simulations were carried out with standard operating conditions of a typical Pressurised Water Reactor (PWR), P=155Bar, T=300^{o}C. The mesh used for the simulations is shown in Fig. 6. Various crack lengths were simulated at three different fluid temperatures of 290^{o}C, 295^{o}C and 300^{o}C. Properties of the fluid are those of water and obtained from steam tables. Approximate material properties of Stainless Steel are taken with E=200GPa, ν=0.3, α=1.282x10^{-5} and k=20W/mK. The effect of bulk fluid temperature on crack opening and leak rate is shown in Figures 10. Displacements of 0.1mm and -0.1mm were imposed on the upper and lower sides of the plate and the temperatures were fixed at 290^{o}C, which was also taken as the reference temperature in the thermal strain calculation.

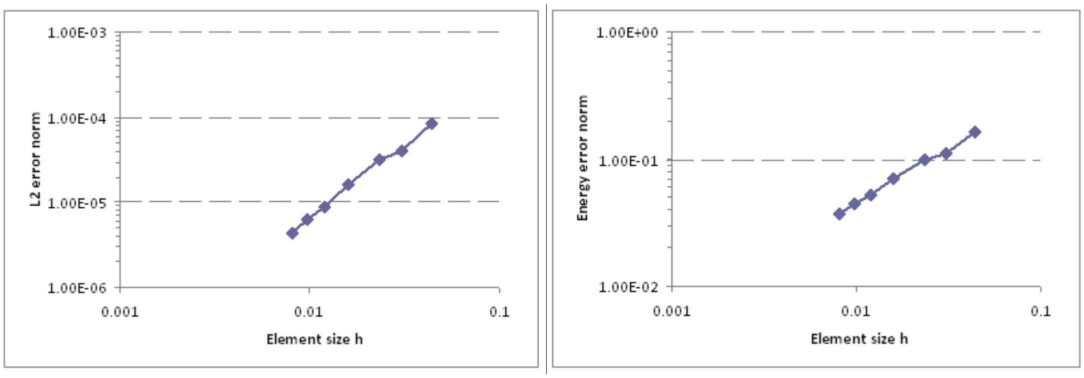

Figure 7. L2 and energy error norm against element size

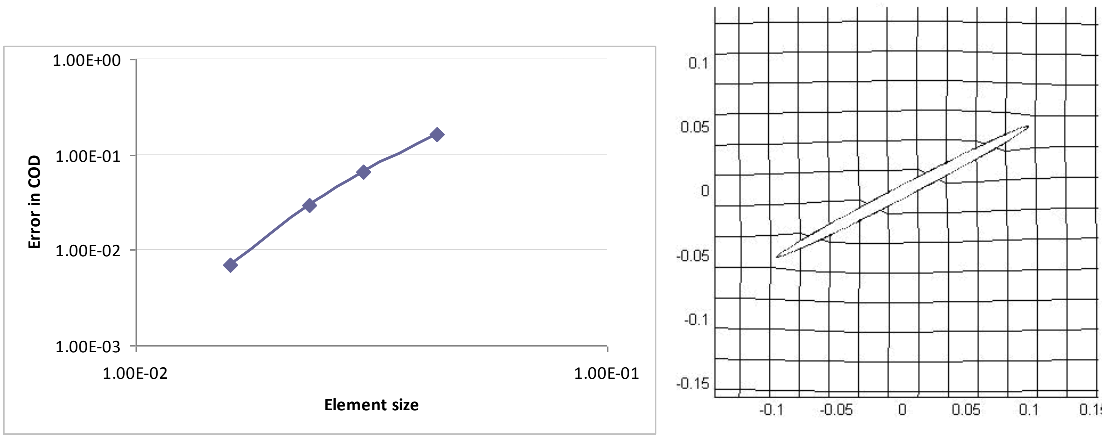

Figure 8. COD error against element size and visualization of crack opening (enhanced with scale factor) for an angled crack which has tips at (-0.1, -0.05) and (0.1, 0.05)

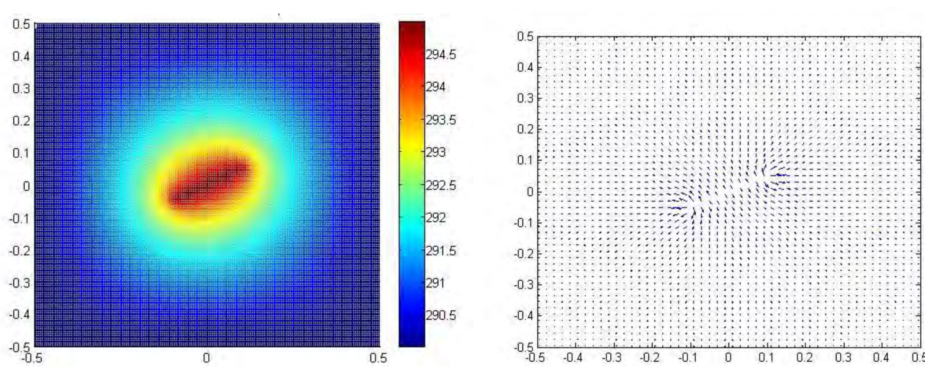

Figure 9. Temperature and heat flux vector plots for plate with hot angled crack centred at the origin

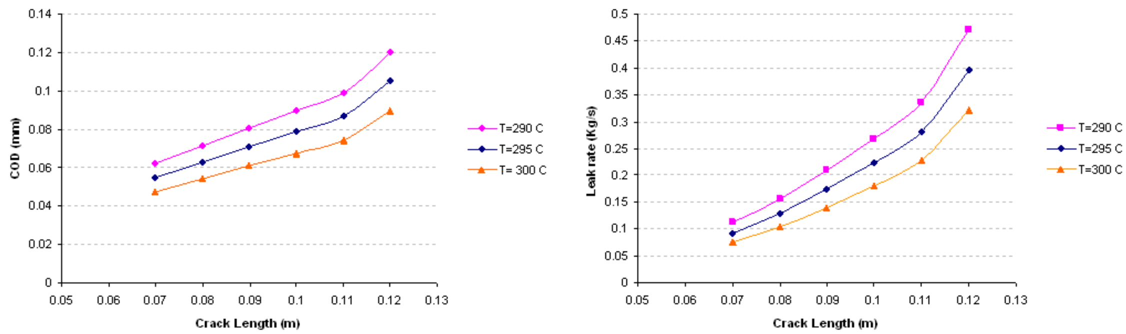

Figure 10. Leak rate and crack opening displacement against crack length for different bulk fluid temeperatures

DISCUSSION

The results indicate that the method employed here is an effective means of analyzing Leak-before-Break. The crack opening area shown in figure 8 indicates that with even a fairly course mesh the characteristic ellipse for crack opening can be obtained from the model. The mechanical and thermal XFEM models give accurate results with optimum rates of convergence shown in Figs.7 and 8. This justified the use of this model in analyzing thermo mechanical crack closure when a leaking fluid heats the walls of a crack. Thermo-mechanical simulations for standard operating conditions of a PWR show that there is an effect of closure when there is a temperature differential between fluid and structure. Figure 9 shows the temperature distribution and heat flux vectors for a plate containing an angled crack which is subject to heat transfer. There are peaks in heat flux at the crack tips due to the asymptotic behaviour of heat flow around singularites, as well as a discontinuity along the crack. The effect seen in Fig. 10 is a reduction of up to 30% in leak rate for the temperatures considered. This is attributed to the crack closure as the reduction in crack opening follows the trend of the reduction in leak rate. It is also worth noting that the crack closes more as the crack length increases.

As mentioned in the fluid mechanics section of this paper, a change in crack opening could potentially cause a change in the flow regime. This would change the way the leak rate is calculated and could give vastly different answers. For example if the crack opening was reduced by a sufficient amount, the flow may become lamina. The flow could also start to follow a path around the corners of the peaks and troughs of the crack surface roughness. Ignoring the above scenario could cause the leak rate to be over predicted. In this work the heat transfer is only modeled in the structure, there is no dependency of the fluid on heat flux. This is known to affect the onset of flashing, because if heat is being lost from the fluid through the crack then the saturation pressure is lowered, see Fig. 4 for example. When heat transfer is included, flashing may not occur because the fluid temperature is lowered to below saturation temperature. This means single phase flow would exist through the crack which would give a higher mass flow rate. Also, when flashing does occur, there is a net flow of heat into the fluid, thereby reversing the flow of heat and actually cooling the walls. This cooling could cause the crack opening to increase, resulting in a higher leak rate.

CONCLUSIONS AND FURTHER WORK

In this work a new method for evaluating the crack opening and leak rate elements of Leak-before-Break has been proposed. The model is designed to calculate crack opening areas and leak rates using the extended finite element method. The thermo mechanical nature of the model meant it was possible to investigate the effect of crack closure which occurs when the fluid heats the crack faces. A reduction in leak rate up to 30% is seen when the bulk fluid temperature is sufficiently hotter than the structure. However, it is suggested that this effect may have larger ramifications due to the flow regime shift as crack opening changes. Further work could involve an analysis of crack paths for cracks found in reactor pipes. Once a more complete understanding of how the crack paths exist in real cracks it will be possible to carry out more realistic CFD calculations to analyse how the flow develops through the crack. For example, identifying the location of the choking plane, or where water starts to vaporize to steam for a complex crack path would be a useful study. It would also be useful to model a crack through the thickness of a component to understand how the crack opening changes due to heat transfer and pressure effects. This is a development of the work carried out here and is being realized, utilizing XFEM and Ecrevisse in Code Aster. The fluid model Ecrevisse includes a heat term so it is capable of modeling the heat transfer phenomena in flashing as discussed in the report.

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