COMPLIANCE CORRELATION FOR THROUGHWALL CRACKED PIPE TO MEASURE CRACK GROWTH

J.Chattopadhyay¹, W. Venkatramana, B.K.Dutta, K.K.Vaze

Reactor Safety Division, Bhabha Atomic Research Centre, Mumbai – 400085, India (jchatt@barc.gov.in⁴)

ABSTRACT

Crack growth is generally measured during fracture experiment of specimen or component. The unloading compliance technique is commonly used for this purpose due to its simplicity. It infers the crack length from unloading compliance of cracked component. The pre-requisite of this technique is the availability of an equation that correlates crack length with unloading compliance. While such correlations are available for compact tension (CT) and Three Point Bend (TPB) specimens, it is not available for big components like pipe or pipe bend. Development of such a correlation for throughwall circumferentially cracked (TCC) straight pipe under bending, therefore, forms the objective of the present study. However, the challenge to develop such correlation for TCC pipe is that the equation should not only contain crack length as a function, it should also contain the current deformation or load level as a parameter. This is attributed to the fact that the circular cross section of the pipe ovalizes during deformation leading to change of bending stiffness of the cracked body. New compliance correlations have been proposed for TCC pipe under bending load considering these complexities. Elastic-perfectly plastic material behavior has been assumed to characterize the material stress-strain response. However, it has been shown that error due to this approximation with respect to the actual stress-strain behavior is negligible if one chooses flow stress equal to average of yield and ultimate strength. The proposed correlations are expressed in terms of normalized parameters to make them independent of specific values of geometric dimensions like, radius, thickness and span length of four-point bending loading system. Effectiveness of this normalization has also been verified by carrying out a sensitivity study.

INTRODUCTION

Fracture experiments are conducted on small specimens and full scale components for various fracture toughness parameters like \( K_{IC} \), \( J_{IC} \) and J-R curve (e.g. Wilkowski et al (1989) and Chattopadhyay et al (2000)). These parameters are required in the detailed fracture mechanics analysis of various real-life components (Chattopadhyay et al (1999)). They are derived from load versus load line displacement and load versus crack growth data, measured during fracture experiments. While load and load-line-displacement are measured directly from the in-built devices in the modern servo-hydraulic machines, crack growth measurement is a challenge in any fracture experiment. Various methods, e.g. unloading compliance, potential drop (Anderson (1994)) and image processing techniques (Chattopadhyay et al (2000)) are commonly employed to measure crack growth. The unloading compliance technique is the most commonly used and also the simplest method to measure crack growth during fracture test of small specimens, e.g. Compact Tension (CT) and Three Point Bend (TPB) etc. (ASTM (1999)). This technique allows crack growth to be inferred from the load and displacement transducers that are part of any fracture mechanics test. A specimen can be partially unloaded at various points during the test in order to measure the elastic compliance, which can be related to the current crack length. As the crack grows, the specimen becomes more compliant (less stiff). Figure 1 shows the pictorial representation of the above technique. The pre-requisite of this technique is the availability of an equation that correlates crack length with unloading compliance. In the absence of such equations for full scale components e.g. cracked pipes and elbows, compliance technique is rarely used to measure crack growth during fracture experiments of such components. It may be mentioned that fracture experiments of piping components with various crack configurations are carried out worldwide to study various fracture mechanics issues (e.g. Wilkowski et al...
(1989) and Chattopadhyay et al (1999, 2000)) in the context of application of Leak-Before-Break (LBB) concept to primary heat transport (PHT) system piping in PHWR and PWR. Throughwall circumferentially cracked (TCC) straight pipe under four point bending load is a common component for fracture studies. Straight pipes are critical components of PHT system piping of nuclear reactors.

Conventionally, compliance correlations are derived by generating compliance (reciprocal of stiffness of load-deflection curve) versus crack length by performing small displacement linear elastic finite element analysis. However, it does not account for the large geometric deformation that may occur during the loading of the specimen. The unloading compliance may be influenced by the increasing/decreasing stiffness of the specimen because of the change in basic geometry during deformation. Therefore, it is of interest to study the effect of deformation on the unloading compliance. An earlier study by Chattopadhyay et al (2006) had shown that the deformation has practically no effect on the compliance of the TPB specimens. In other words, unloading compliance of TPB specimen depends exclusively on current crack length (a/w). This behaviour was attributed to the fact that deformation of TPB specimens does not affect the area moment of inertia (i.e. bending stiffness). However, the same is not true for pipe. The same study showed that deformation of TCC pipe significantly affects the unloading compliance. In other words, the unloading compliance of TCC pipe depends not only on the current crack length but also on the deformation level.

![Figure 1 The unloading compliance technique to measure crack growth during fracture experiment](image)

**PRESENT WORK**

As mentioned earlier, an equation correlating crack length and unloading compliance is the prerequisite for using the compliance technique to measure crack growth during fracture experiments. While these types of correlations are available for small specimens e.g. CT, TPB etc. (ASTM (1999)), no such correlation is available for TCC pipe. Development of such a correlation, therefore, forms the objective of the present study. However, the challenge to develop such correlation for TCC pipe is that the equation should not only contain crack length as a function, it should also contain the current deformation or load level as a parameter. Consequently, nonlinear finite element analysis considering both geometric and material non-linearity has been carried out for straight pipes of different R/t ratio with throughwall circumferential cracks of various sizes. The analyses have been performed up to load with large scale plastic deformation. First, an equation correlating compliance and crack angle of TCC pipe has been developed from initial elastic solution of load vs. crack mouth opening displacement (CMOD). Subsequently, the equations have been modified to include the effect of load / deformation.
FINITE ELEMENT ANALYSIS

The finite element method is used to develop the compliance correlations. Straight pipe with throughwall circumferential crack under four point bending load (Fig.2) is considered for analysis. Each pipe is loaded a little beyond the theoretical plastic collapse load with periodic unloading. Theoretical plastic collapse load is calculated using the following equation:

\[ P_L = \frac{16R^2t\sigma_f}{Z - L} \left[ \cos\left(\frac{\theta}{2}\right) - 0.5\sin(\theta) \right] \]  

(1)

where, \( R \) is the mean radius of the pipe cross section, \( t \) is the pipe wall thickness, \( \theta \) is the semi-circumferential crack angle, \( \sigma_f \) is the material flow stress, \( Z \) and \( L \) are the outer and inner span of the four point bending load respectively.

\[ \lambda_0 = \frac{C_oEI}{\pi R^2} \]  

(3)

where, \( \lambda_0 \) is the normalized initial elastic compliance (\( C_o \)), \( E \) is the Young’s modulus of the pipe material, \( I \) is the area moment of inertia of the pipe cross section and \( R \) is the mean pipe radius.

Compliance values at higher load levels (\( C \)) are normalized with respect to the initial elastic value (\( C/C_o \)). Applied moment is also normalized with respect to the theoretical collapse moment as follows:

\[ m = \frac{M}{M_L} ; \quad M_L = \frac{P_L (Z - L)}{4} \]  

(4)

where, \( m \) is the normalized applied moment and \( M_L \) is the theoretical plastic collapse moment of pipe containing a throughwall circumferential crack. The rationale of normalizing the compliance and load level is to make the equations independent of specific geometric dimensions of pipe. Thus, normalized
compliances ($\lambda$) have been generated for various values of normalized load levels ($m$) and crack angles ($\theta/\pi$).

**Geometric details**

Mean radius ($R$) = 50 mm, $R/t = 5, 10, 15$ and $20$, $2\theta = 30^0, 60^0, 90^0, 120^0, 150^0, 180^0$, Inner Span ($L$) = 1480 mm, Outer Span ($Z$) = 4000 mm

**Material Properties**

Elastic-perfectly plastic material model is used in this study. The other material properties chosen are as follows: Young’s modulus: 203 GPa, Yield stress: 288 MPa, Poisson’s ratio: 0.3. Although, it has been shown later that choice of these specific values does not affect the compliance correlations significantly.

**Finite Element Model**

Twenty-noded solid elements are used to model the straight pipes. Because of symmetry, only one fourth of the pipe is modelled. The total number of elements varied between 832-468 with 1-2 element(s) across the thickness of the pipe and number of nodes varied between 4917 - 3499. Fig.3 shows a typical FE mesh of straight pipe.

Figure 3 Typical finite element mesh

**RESULTS AND DISCUSSION**

Figure 4 shows the typical moment versus CMOD data for $R/t = 20$. From these data, initial elastic compliance ($C_o$) has been calculated by linear regression of the initial 5% data. It is then normalized using Eq.(3). Zahoor (1989-91) proposed the elastic CMOD ($\delta_e$) of throughwall circumferentially cracked pipe subjected to pure bending moment as follows:

\[
\delta_e = f_2.\pi R^2 \frac{M}{EI}
\]

\[
f_2 = 4\left(\frac{\theta}{\pi}\right) \left[1 + A \left(6.071\left(\frac{\theta}{\pi}\right)^{1.5} + 24.15\left(\frac{\theta}{\pi}\right)^{2.94}\right)\right]
\]

\[
(5)
\]

\[
(6)
\]
$$A = [0.125(R/t)-0.25]^{0.25} \quad \text{for} \quad 5 \leq R/t \leq 10$$
$$= [0.4(R/t)-3]^{0.25} \quad \text{for} \quad 10 \leq R/t \leq 20$$

$$I = \pi (R_o^4 - R_i^4)/4$$

By rearranging the above equations, one can write:

$$\lambda_o = C_o EI / \pi R^2 = f_2$$

Figure 5 shows the comparison of normalized initial elastic compliance, $\lambda_o = C_o EI / \pi R^2$ between the present values calculated from the FE results and the Zahoor’s prediction as per Eqs.(6-9). A good matching may be observed. This validates the general accuracy of the present data. Subsequently, unloading compliance values have been calculated and normalized with respect to initial elastic compliance ($\lambda / \lambda_o = C/C_o$) at various load levels expressed in normalized form as per Eq.(4). Figure 6 shows the variation of normalized compliance ($\lambda / \lambda_o = C/C_o$) with normalized load level ($m = M/M_L$) for various crack angles for $R/t = 20$. It is observed that except for relatively thicker pipe and smaller crack angles, there is mostly geometric hardening leading to decrease of compliance with increased load or deformation. Geometric hardening increases with decrease in relative thickness ($t/R$), increase in crack angles and load / deformation. Geometric softening is observed for $R/t = 5$ for crack angles of $2\theta = 30^\circ$–$90^\circ$ and for $R/t = 10$ for crack angles of $2\theta = 30^\circ$–$60^\circ$. However, the degree of geometric softening is almost insignificant compared to geometric hardening. The degree of geometric hardening is most prominent for thinner pipe ($R/t = 20$) in the order of $10\%$ for $2\theta = 180^\circ$ for normalized load level of $m = M/M_L = 1$. This geometric hardening is attributed to ovalization of the circular cross section of the pipe during deformation. Figure 7 shows the cracked cross section of a pipe with $R/t = 20$ and $2\theta = 180^\circ$ after deformation at close to plastic collapse moment. The deformation is shown 10 times magnified for clarity. It is seen that circular cross section of the pipe ovalizes during deformation in such a way that area moment of inertia of the cross section is increased leading to increased bending stiffness and hence lower bending compliance.
Figure 5 Comparison of initial elastic compliance with Zahoor’s (1989-91) equations

Figure 6 Variation of normalized compliance ($\lambda/\lambda_o = C/C_o$) with normalized load level ($m = M/M_L$) for various crack angles for $R/t = 20$

Figure 7 Ovalization of circular cross section of a pipe at the crack plane
PROPOSED COMPLIANCE EQUATIONS

The present FE results and Zahoor’s equations (Eqs.5-9) of \( C_o \), match quite closely (Fig.5), which validates the present numerical model. However, Zahoor (1989-1991) expressed CMOD as a function of moment and crack length, where as for use of unloading compliance method to measure crack growth during fracture tests, crack length needs to be expressed as a function of compliance values. Therefore, a new correlation expressing crack length \((\theta/\pi)\) as a function of normalized initial elastic compliance \(\lambda_0 = \frac{C_o E I}{\pi R^2}\) is first proposed. Equation (10) shows the correlation:

\[
\frac{\theta}{\pi} = 1.68471 \left( 2.52469 \lambda_o^{0.760581} - 2.39055 \lambda_o^{0.772705} \right) \left( \frac{R}{t} \right)^{0.136538}
\]  

Equation (10) shows the correlation:

Figure 8 shows the comparison of FE data points and predictions of the proposed equation (10). The average error is around 1.42%. Subsequently, equations correlating the ratio of compliance at large deformation level and the initial elastic compliance \( (\lambda/\lambda_o = C/C_o) \) with normalized crack angle \((\theta/\pi)\) and load level \((m = M/M_L)\) are proposed for various \( R/t \) values. It is done through regression analyses on the finite element data. Four different equations (Eqs.11-14) of normalized compliance \( (C/C_o) \) have been proposed for \( R/t = 5, 10, 15 \) and 20. They are as follows:

For \( R/t = 5 \)

\[
\frac{C}{C_o} = 1 - \left[ 0.301024 \left( \frac{\theta}{\pi} \right)^{1.2335} - 0.3012 \left( \frac{\theta}{\pi} \right)^{1.234} \right] \left[ -0.0339 m^{0.736439} - 0.249899 m^{1.5864} \right]
\]  

For \( R/t = 10 \)

\[
\frac{C}{C_o} = 1 + \left[ 0.000696284 \left( \frac{\theta}{\pi} \right)^{1.0719} - 0.0007 \left( \frac{\theta}{\pi} \right)^{1.07516} \right] \left[ -0.0248 m^{0.727444} + 19.9127 m^{1.6218} \right]
\]  

For \( R/t = 15 \)

\[
\frac{C}{C_o} = 1 - \left[ 0.8238 \left( \frac{\theta}{\pi} \right)^{3.97111} - 0.849025 \left( \frac{\theta}{\pi} \right)^{4.0437} \right] \left[ -0.1758 m^{-0.99484} + 0.0452771 m^{1.5565} \right]
\]  

For \( R/t = 20 \)

\[
\frac{C}{C_o} = 1 + \left[ 1.28195 \left( \frac{\theta}{\pi} \right)^{0.8804} - 1.3015 \left( \frac{\theta}{\pi} \right)^{0.886615} \right] \left[ -0.876637 m^{1.0408} + 0.8358 m^{1.07393} \right]
\]  

EFFECT OF STRAIN HARDENING ON THE COMPLIANCE

The FE analyses to generate the compliance correlations are carried out assuming elastic-perfectly plastic material behavior to simplify the material characterization. However, actual materials show strain hardening behavior. Therefore, a study has been done to check the effect of material strain hardening on compliance values in comparison to postulated elastic-perfectly plastic material behavior. Two cases have been taken for studies with \( R/t = 5 \) and 20, and \( 20 = 90^\circ \) to represent two extreme cases of \( R/t \) within the range considered for the present investigation. To represent a realistic material property, true stress – true strain data of one nuclear grade piping carbon steel SA333Gr6 material is taken. Figure 9 shows the stress-strain diagram of the material. For each geometric case, four sets of results are shown
with different material parameters, namely, (i) actual true stress – true strain data, (ii) elastic-perfectly plastic at $YP = 288 \text{ MPa}$, (iii) elastic-perfectly plastic at $UTS = 420 \text{ MPa}$ and (iv) elastic-perfectly plastic at flow stress $= 354 \text{ MPa}$ (average of $YP$ and $UTS$). Figure 10 shows the comparison for one case, namely, $R/t = 20$. It may be seen from the figure that if actual true stress – true strain material characterization is idealized as elastic-perfectly plastic material behaviour at flow stress equal to average of yield and ultimate strength, compliance values are very close to each other. It proves that elastic-perfectly plastic material idealization does not affect significantly the compliance values provided one chooses perfect plasticity at appropriate flow stress.

Figure 8 Fitting of FE data to propose initial elastic compliance for various crack angles and $R/t$: Comparison of FE data with predictions of proposed equations

Figure 9 Stress-strain diagram of SA333Gr6 material
INDEPENDENCE OF NORMALIZED COMPLIANCE WITH RESPECT TO CHANGES OF GEOMETRIC PARAMETERS

To make the compliance value independent of geometric parameters e.g. radius, thickness etc., it has been normalized as: \( \lambda = CEI/\pi R^2 \). A study has been undertaken to investigate the effectiveness of the normalization of compliance. Two cases of mean radius \((R)\) of 50 and 100 mm are taken for studies with \(R/t = 5\) and 20, and \(2\theta = 90^\circ\) to represent two extreme cases of \(R/t\) within the range considered for the present investigation. Figure 11 shows that the difference is within acceptable limit.

Also a study has been undertaken to investigate the effect of change of inner and outer span lengths of four point bending loading on the normalized compliance \((C/C_0)\). One typical case of \(R/t = 20\) and \(2\theta = 90^\circ\) has been taken for study. In addition to the standard outer and inner span lengths of 4000 and 1480 mm, analysis for another set of outer and inner span lengths of 2000 and 740 mm respectively has been carried out and compared with the standard set results. Figure 12 shows the comparison, which shows that the difference is within acceptable limit.

These exercises prove that the normalization of the compliance parameters makes the correlations independent of specific geometric dimensions.
CONCLUSION

New compliance correlations have been proposed for throughwall circumferentially cracked straight pipe under bending load for measurement of crack growth during fracture experiments. Unlike the conventional compliance correlations, the presently proposed one accounts for the ovalisation of original circular cross section of pipe during deformation. Elastic-perfectly plastic material behaviour has been assumed to characterize the material stress-strain response. However, it has been shown that error due to this approximation with respect to the actual stress-strain behavior is negligible if one chooses flow stress equal to average of yield and ultimate strength. The proposed correlations are expressed in terms of normalized parameters to make them independent of specific values of geometric dimensions like, radius, thickness and span length of four-point bending loading system.

REFERENCES