Comparison of Stress Intensity Factor Calculation for Surface Flaws Using Polynomial Stress Fit Versus Using Universal Weight Functions for Highly Nonlinear Through-wall Stress Distributions

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ABSTRACT

In order to evaluate flaws detected during in-service inspection of nuclear power plant components, linear elastic fracture mechanics flaw evaluations based on the ASME Section XI flaw evaluation procedure and acceptance criteria are performed. The magnitude of stresses in the flawed component is needed to determine the crack tip stress intensity factors, along with the geometry of the component, flaw dimensions and orientation. The stress intensity factors can then be used in further analyses of the detected flaw such as crack growth calculations as well as determining the maximum acceptable flaw sizes for continued operation.

For some of the pressurized water reactor (PWR) power plant weldments, the through-wall residual stress distributions can be highly nonlinear. Appendix A of ASME Section XI and API-579 provide the methodology for calculating crack tip stress intensity factor based on 3rd and 4th order polynomial fits of the through-wall stress distribution, respectively. For highly nonlinear through-wall stress distributions where 3rd or 4th order polynomial fits may not adequately represent these stress distributions, closed-form equations for calculating crack tip stress intensity factors based on the Universal Weight Function Method may be more accurate. In this method, the through-wall stress distribution is divided into multiple piece-wise segments and the stress distribution in each segment is curve fitted by either a linear equation or a polynomial. This method has shown to provide higher accuracy in the stress intensity factors calculated for complicated stress distributions.

In this paper, stress intensity factors calculated based a 4th order polynomial stress fit are compared with that based on the Universal Weight Function Method for several highly nonlinear through-wall stress distribution profiles. The resulting crack tip stress intensity factors are then used in stress corrosion crack growth calculations. The goal of the study is to assess the impact on the crack growth results for highly nonlinear through-wall stress distributions using the crack tip stress intensity factors calculated based on a 4th order polynomial stress fit and the Universal Weight Function Method.

INTRODUCTION

Flaws detected during in-service inspections of nuclear power plant components need to be shown to be acceptable for continued operation or otherwise need to be repaired. For ASME governed plants, the acceptability calculations consist of first determining the maximum allowable end-of-evaluation period flaw sizes per Section XI IWB-3600 based on linear elastic fracture mechanics, elastic plastic fracture mechanics, or limit load (fully plastic) flaw stability analyses. The maximum allowable end-of-evaluation period flaw size is the largest flaw size allowed at the end of a specified operating period. Subsequently, crack growth calculations are performed to determine the time required for the detected indication to reach the end-of-evaluation period flaw sizes based on fatigue crack growth, Primary Water Stress Corrosion Crack (PWSCC) Growth, or other credible crack growth mechanisms. The crack growth analysis result is highly dependent on the stress intensity factor (K_I), which is based on the magnitude of the stresses, flaw geometry, and flaw configuration.

The stress intensity factor expressions for various flaw shapes, sizes and configurations are provided in Appendix A of the current ASME Section XI [1] for a 3rd order polynomial fit on the stresses, while K_I expressions for a 4th order stress polynomial fit are provided in API-579 [2]. However, in some cases even a fourth-order polynomial stress fit may not adequately represent certain highly non-linear welding residual stress profiles such as those in dissimilar metal welds with prior weld repairs. Therefore for these highly
non-linear stress profiles, the stress intensity factor can also be calculated using the universal weight function method described in Xu et al. [3]. With this method, the stress profile is divided into several segments such that a piece-wise linear stress fit can be performed for each division. Based on the fitted stress distribution for each segment, the stress intensity factor can then be calculated using the weight function method [3].

This paper investigates the $K_I$ calculated based on the weight function method versus the polynomial stress fit method for multiple residual stress profiles at nozzle to safe end dissimilar metal (DM) weld locations in typical PWR reactor coolant loop components. The calculated $K_I$ based on these two methods will then be used to determine the crack growth under the PWSCC mechanism, which is prevalent in PWR components made of nickel-base alloys and weldments (e.g. Alloy 600 and Alloy 82/182). The DM welds are made of PWSCC susceptible material, Alloy 82/182; therefore, crack growth through the wall thickness under high temperature operating conditions is relatively fast, on the order of one to two years. Therefore, it is important to calculate accurate stress intensity factors for residual stress distributions that are highly non-linear in order to provide a better assessment on the flaw acceptability for continued plant operation.

The DM weld configurations that are investigated in this paper are reactor vessel nozzle DM welds with 50% pre-service inside surface repairs, a reactor vessel nozzle DM weld with an inlay repair, and a steam generator nozzle double V-groove DM weld. An example of a typical reactor vessel DM weld geometry is shown in Figure 1. A typical reactor vessel nozzle DM weld with an inlay repair is shown in Figure 2. Weld inlay is defined as a corrosion resistant barrier applied to the inside surface of the component between the PWSCC susceptible material and the reactor coolant, requiring excavation of some portion of the susceptible material. The inlay seals the PWSCC susceptible Alloy 82/182 material from the corrosive environment using more corrosion resistant Alloy 52/152 material. A full circumferential 50% inside surface weld repair is also assumed before the inlay repair. The third DM weld configuration is for a typical steam generator nozzle as shown in Figure 3. The steam generator double V-groove weld is created by welding from the middle out to the surface which results in the thinnest portion of the weld being at the center of the weld thickness. Similar to the reactor vessel nozzle DM welds, a full circumferential 50% through-wall inside surface weld repair is assumed for the steam generator double V-groove weld. It should be noted that the effect of normal operating piping loads is also included in the residual stresses for all cases.

Stress intensity factors are calculated for both axial and circumferential inside surface flaws and are used to calculate primary water stress corrosion crack growth through the DM welds. PWSCC crack growth results are compared for the stress intensity factors calculated based on the weight function method and the 4th order polynomial stress fit for the residual stress profiles in each of the DM weld configurations in order to determine whether there is any difference in using the weight function method in place of the traditional polynomial fit method for calculating stress intensity factors.

![Figure 1. Typical Reactor Vessel Nozzle DM Weld](image1)

![Figure 2. Typical Reactor Vessel Nozzle DM Weld With Inlay Repair](image2)
STRESS INTENSITY FACTOR CALCULATION

This paper compares the PWSCC crack growth of postulated flaws in the DM weld based on the 4th order polynomial stress fit $K_I$ expression versus the universal weight function. Semi-elliptical part through-wall axial and circumferential planar inside surface flaws in a cylinder are postulated at each DM weld configuration. The stress intensity factors are calculated at the deepest point along the crack front for axial flaws with a constant aspect ratio (flaw length/flaw depth ratio) of 2, and for circumferential flaws with a constant aspect ratio of 6. An axial aspect ratio of 2 is appropriate for axial flaws since the crack growth is limited to the susceptible DM weld area and the flaw does not propagate into the nozzle or the safe end material.

**Polynomial Stress Fit**

Stress intensity factors were first calculated using the polynomial fit method documented in API-579 [2]. API-579 represents the stress distribution profile for all flaws with a fourth order polynomial as shown in eqn (1).

$$\sigma = \sigma_0 + \sigma_1 \frac{a}{t} + \sigma_2 \left(\frac{a}{t}\right)^2 + \sigma_3 \left(\frac{a}{t}\right)^3 + \sigma_4 \left(\frac{a}{t}\right)^4$$

Where $\sigma_i$ are the stress profile curve fitting coefficients, $a$ is the distance from the surface where the crack initiates to the crack tip, $t$ is the wall thickness, and $\sigma$ is the stress perpendicular to the plane of the crack. The stress intensity factor can then be expressed as shown in eqn (2).

$$K_I = \left(\pi a/Q\right)^{0.5} \sum_{j=0}^{4} G_j \left(\frac{a}{c}, \frac{a}{t}, R, \Phi \right) \sigma_i \left(\frac{a}{t}\right)^j$$

Where $c$ is the half crack length along the surface, $R$ is the inside radius of the weld, $\Phi$ is the angular position on the crack front, $G_j$ are the boundary correction factors provided in API-579 [2], and $Q$ is the shape factor of a semi-elliptical crack.

**Universal Weight Function**

Stress intensity factors were also calculated using the Universal Weight Function documented in Xu et al. [3]. According to [3], the stress intensity factor can be calculated as shown in eqn (3).

$$K_I = \int_0^a m(x,a) \sigma(x) \, dx$$

Where $\sigma(x)$ is the stress distribution, $a$ is the crack depth, $m(x,a)$ is the Mode I weight function, and $x$ is the distance from the surface and moving positive towards the tip of the crack. The weight function is dependent on the geometry of both the structure and the crack. Since stress distributions are typically obtained at discrete locations, the stress distributions between the discrete points need to be approximated.
According to Reference 3, there are limitations with assuming a quadratic approximation between the discrete points, and thus the calculations done in Reference 3 utilized a piece-wise linear variation between the discrete points. The calculations performed in this paper also utilize a piece-wise linear variation between the stress points.

The purpose of using the Universal Weight Function is to capture the peaks and variations in the stress profile that are missed when fitting the stresses to a polynomial function. There are times when a polynomial fit will over- or under-estimate the stresses through a DM weld. With the Universal Weight Function method, the non-linear stress profile is broken into several segments where the stress distribution is fairly linear within those segments such that a piece-wise linear stress fit is considered to be adequate for each division.

**PWSCC CRACK GROWTH**

The PWSCC crack growth rates for the nickel-base material from EPRI MRP-115 [4] are calculated based on applied stress intensity factors and temperature. For the evaluation contained in this paper, all DM welds are assumed to be made of Alloy 182 material, while the weld inlay is of Alloy 52/52M material. Alloy 52/52M weld metal is known to be more PWSCC resistant than the Alloy 182 material; therefore, a PWSCC crack growth rate which is $1/10^{th}$ the PWSCC crack growth rate of Alloy 182 [4] was conservatively used to approximate that for Alloy 52/52M. An operating temperature of 620°F is used for the calculation of PWSCC crack growth. In general, the PWSCC crack growth rate can be calculated as shown in eqn (4).

$$\Delta a = \alpha (K_f)^n \Delta \text{time}$$

Where $\Delta a$ is the crack growth per unit time, $\alpha$ is the crack growth amplitude, $n$ is the power-law exponent and $\Delta \text{time}$ is the time step size being used in the evaluation.

**REACTOR VESSEL NOZZLE DM WELD WITH 50% PRE-SERVICE REPAIR**

Residual stresses (Figure 4) were calculated for a typical reactor vessel nozzle DM weld (Figure 1) with a 50% pre-service repair using finite element modelling techniques. As shown in Figure 4, the 4th order polynomial fit applied to the stresses follows the basic shape of the axial and hoop stress profiles, however the fit does not accurately represent certain peaks and valleys. There are points where the difference between the actual stress profiles and the fitted stress profiles is greater than 30%. The peaks and valleys for both the axial and hoop stresses between 50% and 90% of the through wall thickness are missed by the polynomial fit.

![Figure 4. Reactor Vessel Nozzle DM Weld with 50% Repair Normal Operating Stresses](image)

Based on the stress profiles shown in Figure 4, the through-wall stress intensity factors were then calculated for an axial flaw (AR = 2) using both the polynomial stress fit method and the Universal Weight Function as shown in Figure 5. The difference between the stress intensity factors calculated using the polynomial fit method and the Universal Weight Function method is not significant. The two curves in Figure 5 are in good agreement up until about 60% through the wall thickness and the maximum difference
is approximately 6 ksi√in which occurs at 70% through the wall thickness. The explanation of the divergence can be seen in the hoop stress profiles in Figure 4 where between 55% and 88% through-wall the fitted stress profile varies considerably from the actual stress profile. The stress intensity factors are then used to calculate the stress corrosion crack growth through the weld (Figure 6).

As shown in Figure 6 the crack growth using both stress intensity factor methodologies is approximately the same. The axial crack growth in this particular scenario is not sensitive to the variations in the stress intensity factor; therefore, there is little to no impact from using the Universal Weight Function for axial flaws in a typical reactor vessel nozzle with a 50% pre-service DM weld repair even though there are some differences in the calculated stress intensity factors.

As shown in Figure 6 the crack growth using both stress intensity factor methodologies is approximately the same. The axial crack growth in this particular scenario is not sensitive to the variations in the stress intensity factor; therefore, there is little to no impact from using the Universal Weight Function for axial flaws in a typical reactor vessel nozzle with a 50% pre-service DM weld repair even though there are some differences in the calculated stress intensity factors.

Based on the axial stress profile in Figure 4, the through-wall stress intensity factors were calculated for a circumferential flaw with an AR of 6 using both the polynomial stress fit method and the Universal Weight Function (see Figure 7). The universal weight function follows the stress profile through the wall since it accounts for the peaks and valleys in the stresses; this explains the fluctuation in its stress intensity factor as compared to the polynomial fit stress intensity factor profile. The stress intensity factors are then used to calculate the stress corrosion crack growth through the weld as shown in Figure 8 for the circumferential flaw.

As shown in Figure 8 the crack growth using the polynomial fit method is slower than that calculated using the Universal Weight Function for a flaw starting at 10% of the through-wall thickness. The 10% flaw using the universal weight function reaches a depth of 75% through-wall 18 months faster than the same flaw.
using the polynomial fit method. Once the flaw reaches 25% of the wall thickness, the crack growth is essentially the same, and the sensitivity to the differences in the stress intensity factor diminishes and both flaws grow at an equivalent rate. Based on the crack growth results shown in Figure 8 the polynomial fit method provides non-conservative results which underestimate the crack growth values provided by the weight function method.

**REACTOR VESSEL NOZZLE DM WELD WITH INLAY**

Residual stresses were calculated for a reactor vessel nozzle DM weld with a 50% pre-service repair and an inlay repair (see Figure 2). As shown in Figure 9, the 4th order polynomial fit applied to the stresses follows the basic shape of the axial and hoop stress profiles, however the fit misses many of the small peaks/valleys in the stresses. There are points where the difference between the actual stress profiles and the fitted stress profiles are greater than 20 ksi. Based on the stress profiles shown in Figure 9 the through-wall stress intensity factors were then calculated for an axial flaw with an aspect ratio of 2 using both the polynomial stress fit method and the Universal Weight Function (see Figure 10).

![Figure 9. Reactor Vessel Nozzle DM Weld with Inlay Normal Operating Stresses](image)

As shown in Figure 10 there is a fair amount of variation between the stress intensity factors calculated using the polynomial fit method and the Universal Weight Function methods. The largest difference in stress intensity values is approximately 8 ksi/\(\sqrt{\text{in}}\) which occurs at approximately 54% through the wall thickness. The explanation of the divergence can be seen in the hoop stress profiles in Figure 9 where the polynomial fit does not capture the peaks of the actual stress profile. The stress intensity factors are then used to calculate the stress corrosion crack growth through the weld as shown in Figure 11.

As discussed previously there were some differences that existed in the stress intensity factors calculated using the polynomial fit method and the Universal Weight Function, which can be seen in Figure 10; however the difference in the stress intensity factors resulted in very little difference in crack growth. As shown in Figure 11 the crack growth for the polynomial fit method is slightly faster through the weld inlay material (up to 5% of the wall thickness), however both methodologies are approximately the same once the flaw has grown into the DM weld material. The crack growth was not particularly sensitive to the variations in the stress intensity factor and there is little to no gain achieved by using the Universal Weight Function for axial flaw evaluations for a typical reactor vessel nozzle with a DM weld inlay repair.
Based on the stress profiles shown in Figure 9 the through-wall stress intensity factors were calculated for a circumferential flaw with an aspect ratio of 6 using both the polynomial stress fit method and the Universal Weight Function (see Figure 12). As shown in Figure 12 there is some variation through the wall between the stress intensity factors calculated using the polynomial fit and the Universal Weight Function methods. The explanation of the divergence can be seen in the axial stress profiles in Figure 9. The stress intensity factor variation mirrors the stress profile variation in that both are similar in magnitude and they actually fluctuate between which methodology produces higher stress intensity factors through the wall thickness. The greatest difference between the two is approximately 8 ksi√in. The stress intensity factors are then used to calculate the stress corrosion crack growth through the weld as shown in Figure 14. Figure 13 demonstrates the difference in stress intensity factors between the two methods for the inside 10% of the wall thickness. Since the crack growth rate through the weld inlay is slower than through the DM weld material, the slight difference in stress intensity factors will have a larger impact as shown in Figure 14.
The crack growth due to PWSCC for a circumferential flaw in a reactor vessel nozzle with an inlay repair is shown in Figure 14. As shown in Figure 14 the crack growth using the Universal Weight function is slower than that calculated using the polynomial fit method. The slight difference in stress intensity factors as the flaw growth through the inlay repair layer (up to 5% of wall thickness) is the main cause of the difference in crack growth. Once the flaw grows through the inlay the crack growth for both methods is similar. Overall the difference between the two stress intensity factor methods once the flaw grows through the inlay is insignificant.

**STEAM GENERATOR NOZZLE DOUBLE V-GROOVE WELD**

Residual stresses were calculated for a typical steam generator double V-groove weld (Figure 3) using finite element modelling techniques. As shown in Figure 15, the 4th order polynomial fit applied to the stresses follows the basic shape of the axial and hoop stress profiles, however, as with the other stress profiles, the fit misses many peaks/valleys. There are points where the difference between the actual stress profiles and the fitted stress profiles are greater than 10 ksi.

![Figure 15. Steam Generator Nozzle Double-V DM Weld Normal Operating Stress](image)

The through-wall stress intensity factors were then calculated for an axial flaw with an aspect ratio of 2 using both the two stress intensity factor methods (see Figure 16). There are some variations between the stress intensity factors calculated using the polynomial fit method and the Universal Weight Function methods. The shapes of the two are fairly similar up to 30% of the through-wall thickness, when the stress intensity factors calculated using the universal weight function become limiting. This changes at approximately 55% of the through wall thickness when the polynomial fit method becomes limiting. The divergence is due to the hoop stress profiles in Figure 15. The stress intensity factors are then used to calculate the stress corrosion crack growth through the weld as shown in Figure 17.
Per Figure 16, there were some differences in the stress intensity factors calculated using the polynomial fit method and the Universal Weight Function; however these differences in the stress intensity factors resulted in negligible difference in crack growth. The crack growth using both methodologies is approximately the same. The axial crack growth was not sensitive to the variations in the stress intensity factor and there is little impact from using the Universal Weight Function for a typical steam generator nozzle double V-groove weld.

Based on the stress profiles shown in Figure 15, the through-wall stress intensity factors were calculated for a circumferential flaw with an aspect ratio of 6 using both the polynomial stress fit method and the Universal Weight Function (Figure 18). As shown in Figure 18 there is some variation through the wall between the stress intensity factors calculated using both methods. The explanation of the divergence can be seen in the axial stress profiles in Figure 15. The greatest difference between the two stress intensity factor profiles at any location is approximately 11 ksi√in. The stress intensity factors are then used to calculate the stress corrosion crack growth through the weld (Figure 19).

As discussed previously there were some differences that existed in the stress intensity factors calculated using the polynomial fit method and the Universal Weight Function, which can be seen in Figure 18; however, the difference in the stress intensity factors resulted in very little difference in the crack growth. As shown in Figure 19 the crack growth calculated by using both methods is approximately the same throughout the wall thickness. Based on the crack growth results shown in Figure 19 there would be little to no gain achieved by using the more precise universal weight function for the calculation of stress intensity factors used in PWSCC growth evaluations.
CONCLUSION & DISCUSSION

Based on the investigation of various welding residual stress profiles for multiple DM weld nozzle configurations, it was determined that for most cases the difference between the crack growth calculated using the weight function and polynomial fit methods was small or insignificant even though there were some differences in the stress intensity factor profiles. The results did show one case where there was substantial difference in the crack growth between both methodologies, as demonstrated in Figure 8.

The crack growth for a circumferential flaw in a 50% inner diameter repaired reactor vessel nozzle was the only case that demonstrated meaningful difference in the crack growth between the two methodologies. Based on a comparison of the stress intensity factors shown in Figure 7 it does not seem that there is any more difference between the two methodologies than what was displayed in many of the other cases. The key difference for this case is that the stress intensity factors were lower than all other cases. The lower stress intensity factors cause slower growth which allows the differences in the stress intensity factors to compound. This accounts for approximately 25% faster crack growth using the weight function method which is far beyond the difference of any other case (the highest of which is approximately a 6% difference in growth rate). The 25% faster crack growth is constant regardless of the temperature used in the analysis. Based on the crack growth results, there are some cases for nonlinear through-wall stress profiles when using the more realistic weight function method to calculate stress intensity factors may result in a different crack assessment than if the polynomial fit method were used.

Also, based on the crack growth results for most cases, the exponent on the stress intensity factor (see Eqn. 4) used in calculating the PWSCC crack growth rate \( n=1.6 \) for Alloy 82/182) is not large enough in most cases to produce meaningful differences in the crack growth rates using stress intensity factors calculated from either method. The results did in fact show that there were differences in the stress intensity factors calculated so there may be a greater impact on other crack growth laws which utilize a larger exponent on the stress intensity factor. Therefore it is expected that using a crack growth law with a larger exponent on the stress intensity factor, such as for stress corrosion cracking in BWR components, would result in a larger difference between the results of the weight function and polynomial fit methods.

While using the weight function method did demonstrate some differences in crack growth, far greater impact may be found when applying the weight function to evaluations where the stress intensity factor is compared to the fracture toughness of a material. The data demonstrates that the polynomial fit method misses some of the stress intensity factor peaks and valleys. These peaks and valleys could be very important in determining non-ductile failure where the crack tip stress intensity factor is compared to the lower bound of crack arrest \( (K_{Ia}) \) or the lower bound of static initiation \( (K_{Ic}) \) fracture toughness values.

It would be necessary to determine on a case-by-case basis which method for crack tip stress intensity factor calculation is the most appropriate.

NOMENCLATURE

\( a \)  flaw depth  in.
\( c \)  half crack length along surface  in.
\( G_i \)  boundary correction factors
\( K_I \)  stress intensity factor  ksi√in
\( m(x,a) \)  mode 1 weight function
\( n \)  Power-law exponent for PWSCC crack growth
\( Q \)  shape factor of an elliptical crack
\( R \)  inside radius of weld  in.
\( t \)  weld thickness  in.
\( x \)  distance from the surface moving forwards toward the crack tip  in.
\( \sigma \)  stress perpendicular to the plan of the crack  ksi
σ, stress profile curve fitting coefficients
α, crack growth rate amplitude
Φ, angular position along the crack front

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