Transactions, SMiRT-22
San Francisco, California, USA - August 18-23, 2013
Division III Applied Computation, Simulation and Animation

GENERATION OF SPECTROGRAMS AND ACCELEROGRAMS BY WAVELETS ON SPREADSHEET

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ABSTRACT

This paper is aimed to propose an efficient way of programing on a spreadsheet (EXCEL® for instance) the generation of i) a representation of a spectrogram of any accelerogram, in order to ease the description of the distribution of the frequencies all along the time history of the seismic signal and/or ii) a synthetic accelerogram that matches any given response spectrum, both uses being independent. The algorithm implemented in the spreadsheet is the one proposed by Suarez & Montejo (2002) which is based on the wavelet transform properties.

RECALL OF SIGNAL ANALYSIS AND RECONSTRUCTION BY WAVELET TRANSFORMS

Any signal f(t) may be analyzed by the “mother wavelet” expressed by:

$$\Psi(u)=e^{-\zeta.\Omega.|u|}.\sin(\Omega.u) \qquad \text{with } \zeta = 0.05 \text{ and } \Omega = \pi.$$

The following figure represents the shape of the wavelet expressed with u = (t-p)/s where p is a variable of position and s a variable related to a frequency (i.e. proportional to the inverse of a frequency).

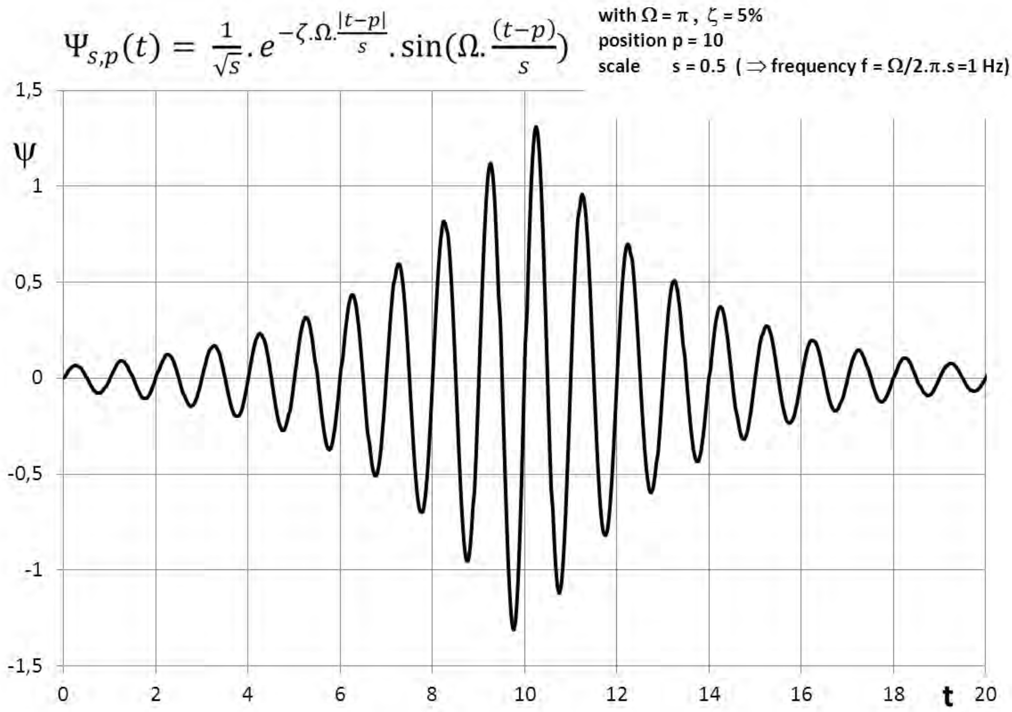

Fig. 1 - Typical shape of a wavelet signal.

The variable of position p varies as the time variable t, according to the time step dt used in the accelerogram definition (generally about 1 ms), whereas the frequency term s is limited to a total amount of 63, which is a good compromise between accuracy and CPU time. The s values are not regularly spaced: they are given by the expression $s=2^{n/8}$, n varying from 12 down to -50, allowing the frequencies to respectively vary from 0.18 Hz up to 38.05 Hz.

The transform of the signal f(t) by wavelet is made according to the convolution:

$$C(s,p)=\frac{1}{\sqrt{s}}.\int_{-\infty}^{+\infty} f(t).\Psi(\frac{t-p}{s}).dt$$

The computation of this integral is simply done by trapezoidal rule, the integral being actually definite, t varying from 0 to T_f , T_f being the time at the end of the signal.

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The absolute value of C(s,p) function gives series of kinds of Fourier spectrum for any given position p. The mapping of the absolute value of the function C(s,p), after a quadratic smoothening by a Hann window for instance, may qualitatively illustrate the location of the dominant frequencies along the signal, as shown in Figure 2.

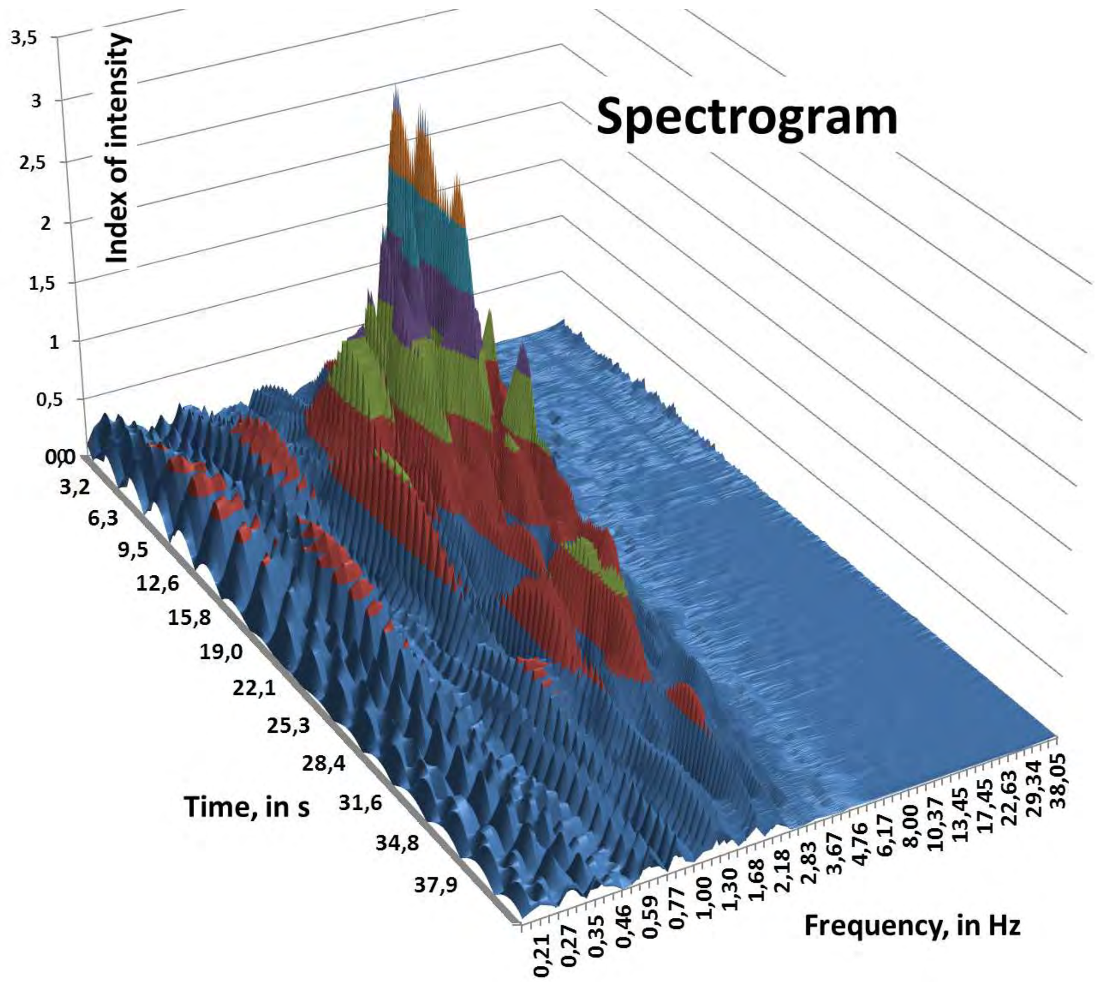


Fig. 2 – Example of spectrogram of an accelerogram showing the frequency content and the location of their maxima, by using the convolution C(s,p) of the accelerogram by wavelet.

The spectrograms obtained by this method are satisfactorily compared to those ones given by the method developed by Conte and Peng (1997), that can also be easily programed on spreadsheet.

A "detail function" D(s,t) (according to the terminology of the authors of the Ref. [1]) is generated by using the following expression:

$$D(s,t)=\frac{1}{s^{5/2}}.\frac{1}{K_\Psi}.\int_{-\infty}^{+\infty}C(s,p).\Psi(\frac{t-p}{s}).dp$$

where $K\psi$ is expressed by:

$$K_\Psi = \int_0^{+\infty}\frac{|\Psi(\omega)|^2}{\omega}.d\omega$$

K_ψ□ is a constant only depending from Ω and ζ:

$$K_\Psi = \frac{-4.\zeta.(\zeta^2-1)+\pi.(\zeta^2+1)^2+2.(\zeta^2+1)^2.\tan^{-1}(\frac{1}{2.\zeta}-\frac{\zeta}{2})}{4.\zeta.(\zeta^2+1)^2.\Omega^2}$$

For ζ=5% and $\Omega = \pi$, $K_\Psi \approx 3.18243$

As for C(s,p), the computation of this integral is simply done by trapezoidal rule, p varying from 0 to T_f, T_f being the time at the end of the signal.

The reconstruction of the signal is obtained by:

$$f(t) = \int_0^{+\infty} D(s,t).dt$$

The integral computation has to be done by using the Simpson's rule, due to the little number of terms (63 frequencies), with a special attention to the fact that the spacing ds between two successive values of s is not constant.

The D(s,t) functions give series of kinds of elementary accelerograms $D_s(t)$ (= D(s,t) for a given frequency term s) whose summation gives the original signal f(t). Some examples of D(s,t) are hereafter reproduced:

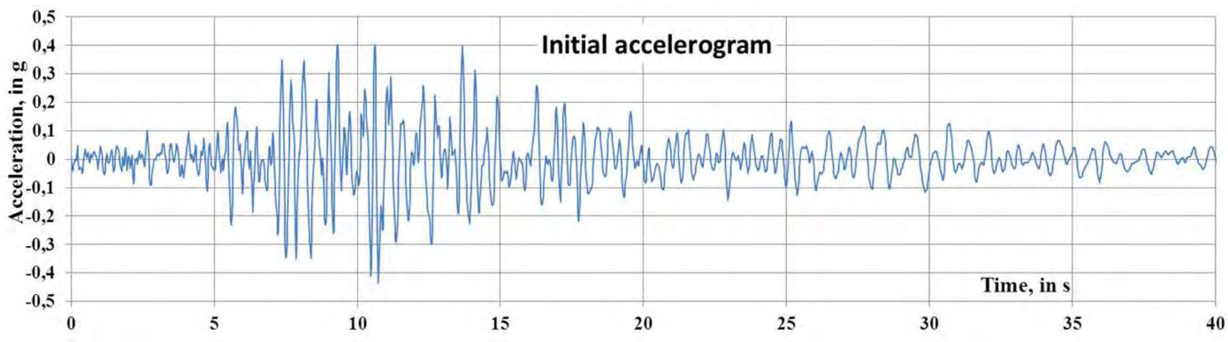


Fig. 3 – Initial accelerogram.

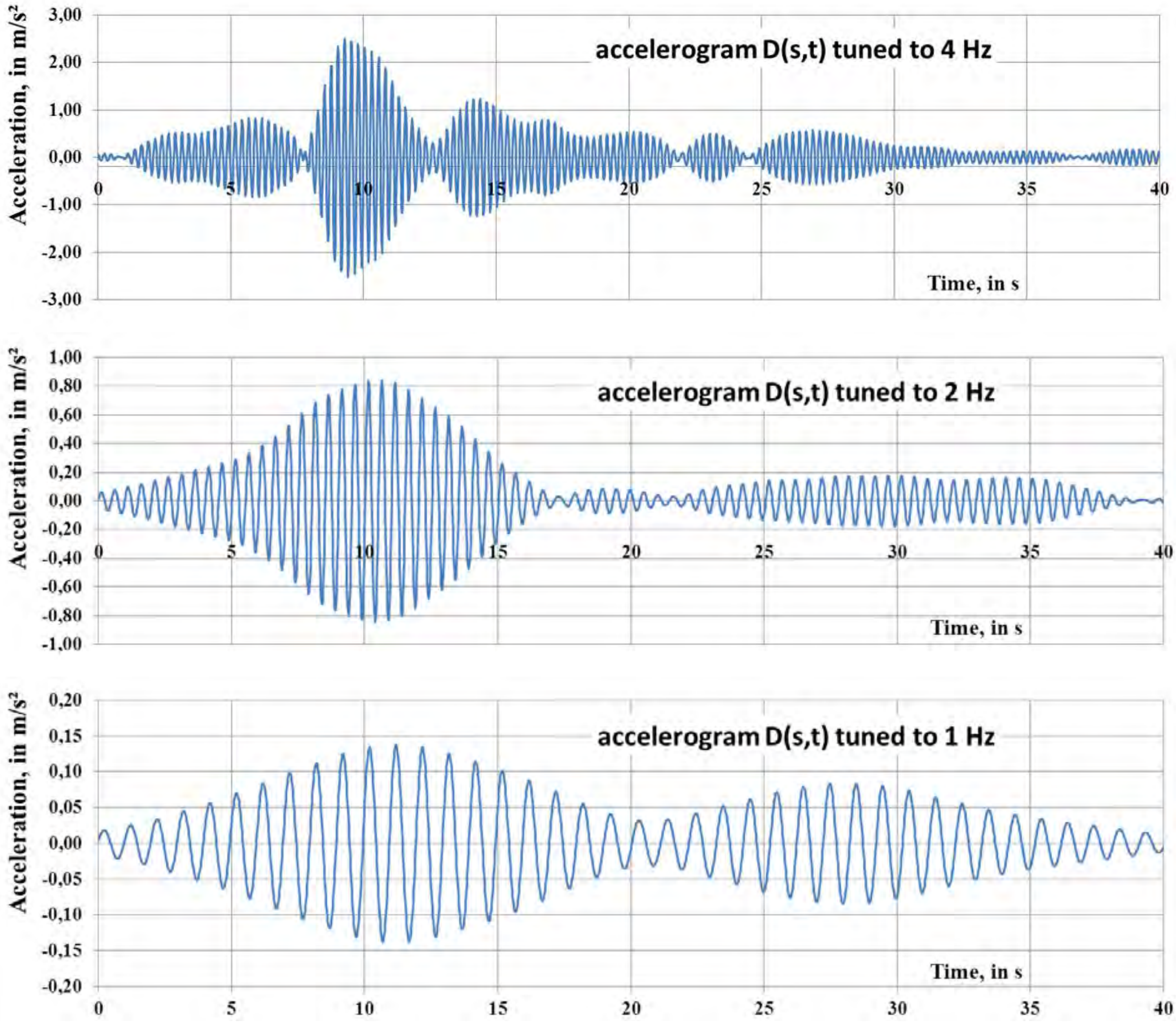


Fig. 4 – Examples of elementary accelerograms D(s,t) for different s values corresponding to the analyzing frequencies 4, 2 and 1 Hz.

ACCELEROGRAM GENERATION

The spectrum response of the accelerogram to be generated shall fit to a given spectrum response, defined by the pseudo-accelerations curve $Sa_{Target}(s)$. The generation is based on the iterative use of C(s,p), D(s,t) functions and signal reconstruction by summation of D(s,t): the response spectrum of the accelerogram at the iteration N gives the pseudo-accelerations Sa(s), the iteration N+1 of the accelerogram is done by the summation of each preceding $D_s(t)$ functions factored by the ratio $Sa_{Target}(s)/Sa(s)$. The process is convergent after only a few iterations.

The computation of the response spectrum may be easily programmed on spreadsheet, as indicated in Rambach J.-M. (2012): the pseudo-acceleration is simply obtained by direct integration of the canonical ODE for a linear oscillator submitted to the signal f(t), at several frequencies, followed by the research of the maximum value of displacement that gives the pseudo-acceleration after its multiplication by the square of the corresponding pulsation.

The initial accelerogram and its response spectrum may be of any type (as far as its frequency content is rich enough), the only requirement being the duration which is unchanged. The use of the recording of a natural accelerogram is recommended.

The following flowchart indicates the algorithm completed by its inputs and outputs:

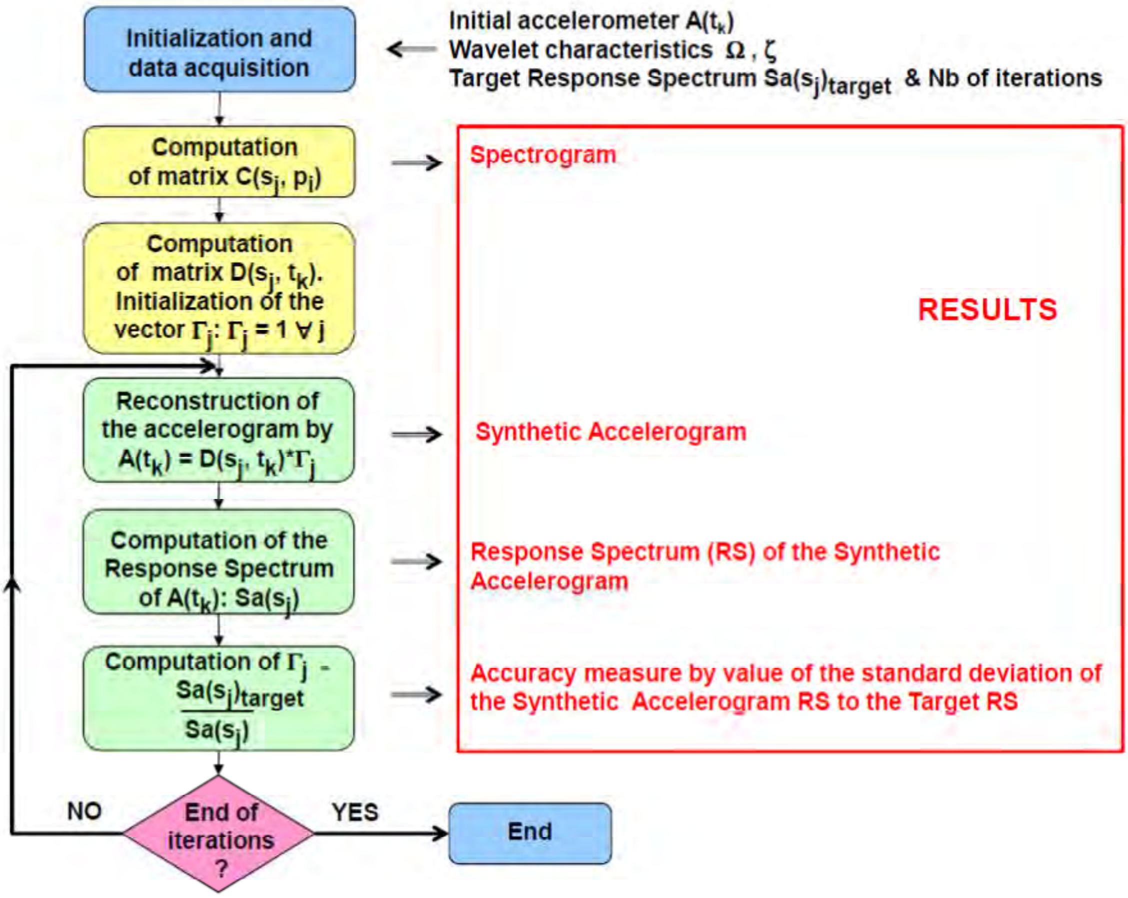


Fig. 5 – Flow chart illustrating the algorithm of the computation

The data necessary for running the computation are:

- an accelerometer f(t) described by some thousands of values and a time step dt, see Fig.3 for instance,
- a target response spectrum described by few couples of values (frequency, pseudo-acceleration),
- the number of iterations allowed for the matching of the response spectrum of the synthetic accelerogram with respect to the target response spectrum.

The computation of the matrices $C(s_j,p_i)$ and $D(s_j,t_k)$ and of the response spectrum $Sa(s_j)$ of the accelerograms $A(t_k)$ is to be done by using VBA subroutines called from the spreadsheet by macro commands.

The computational time is reasonable, around 3 to 4 minutes.

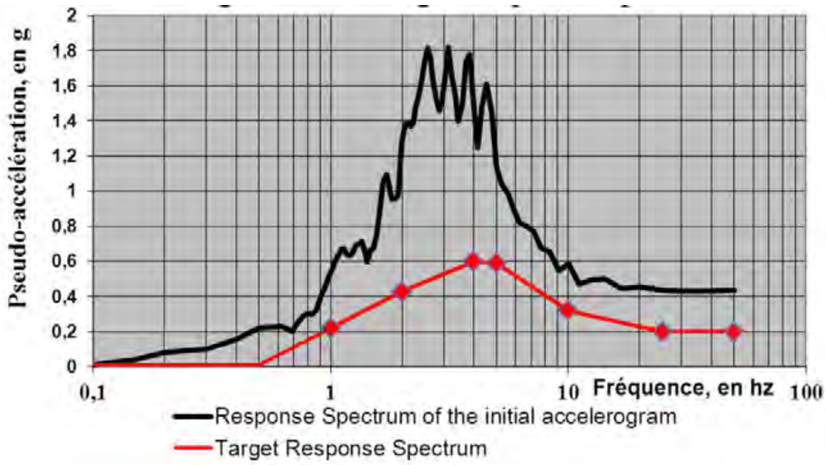
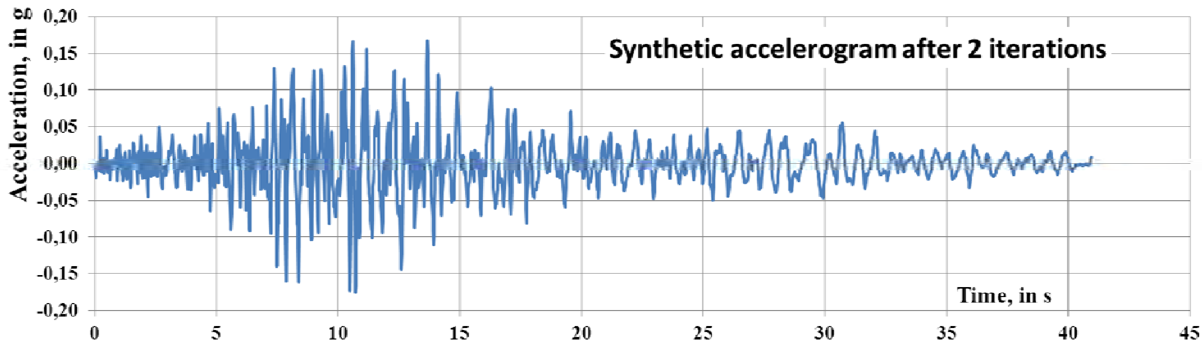


Fig. 6 – Response spectrum of the initial accelerogram and the targeted response spectrum before computation

The following Figs. represent some intermediate results showing the evolution of the accelerogram and of corresponding response spectrum with the iterations progress, after 2 iterations and 5 iterations.



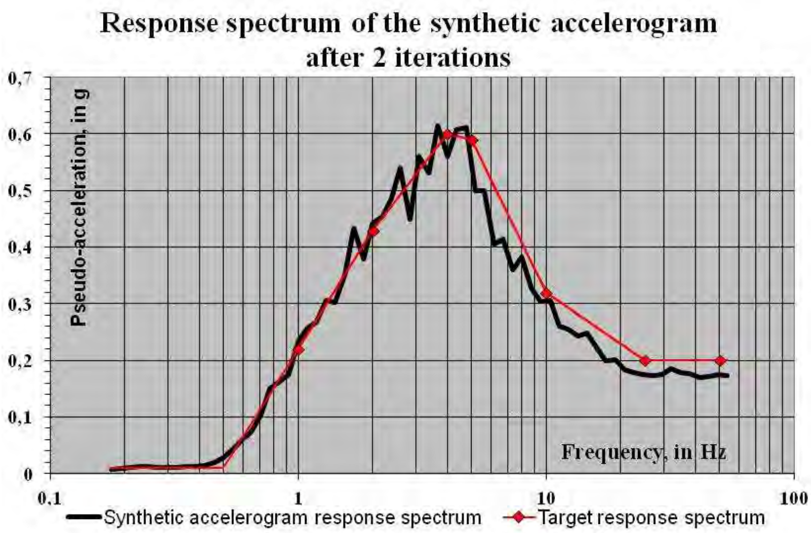
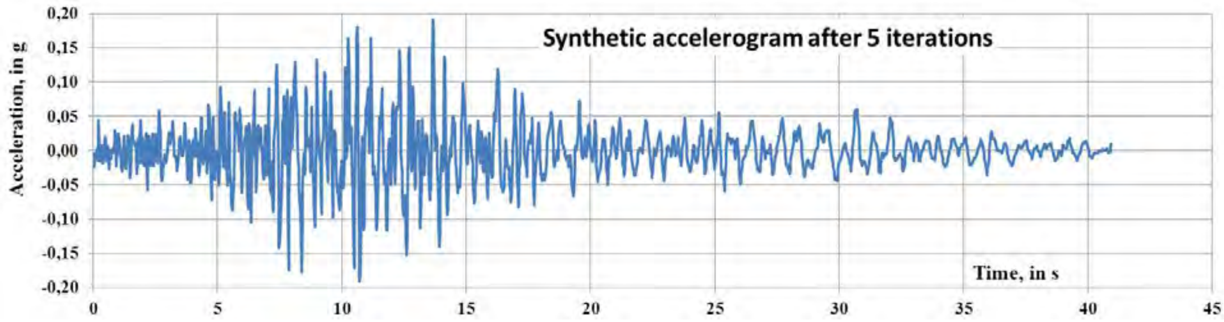


Fig. 7 – Synthetic accelerogram and corresponding response spectrum after 2 iterations

The matching of the response spectrum to the target response spectrum is rather fair after only 2 iterations.

After 5 iterations:



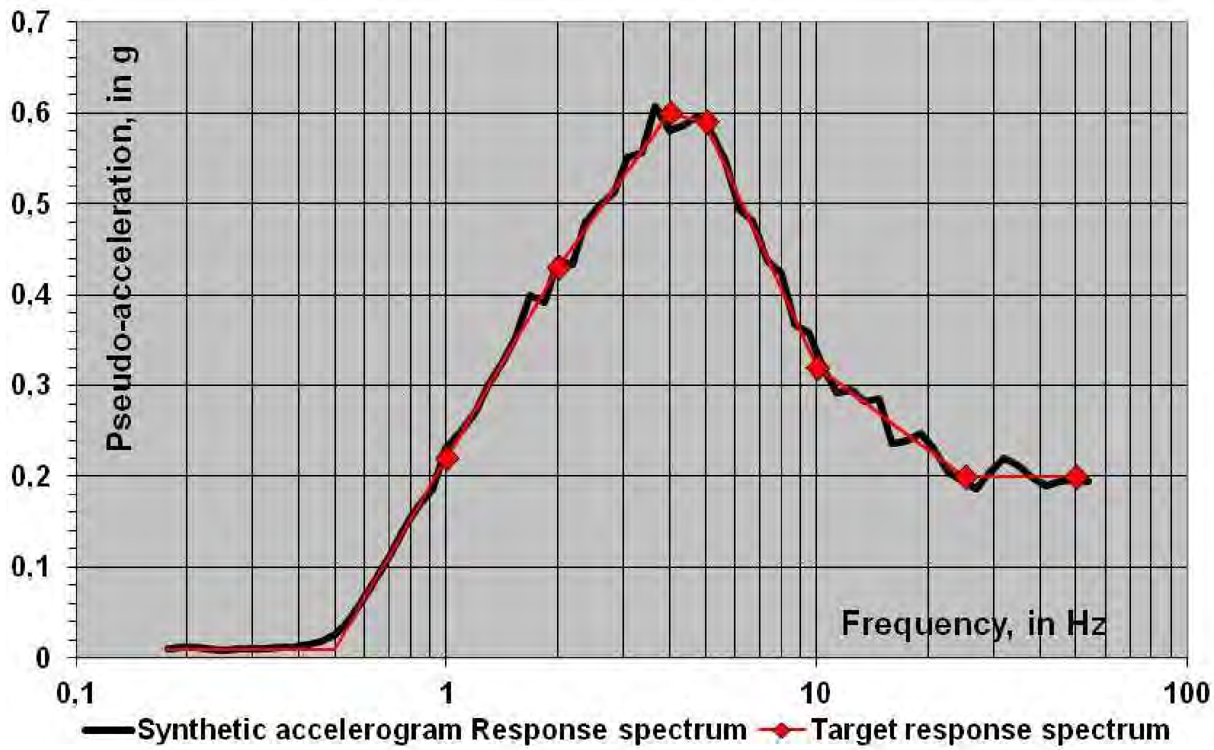


Fig. 8 – Synthetic accelerogram and corresponding response spectrum after 5 iterations

The matching of the response spectrum to the target response spectrum is satisfactory after only 5 iterations. Further iterations do not induce better matching, because of the insufficient number of frequencies selected in the signal analyzing process. Moreover, the energy content of the signal increases with the number of iterations: for non-linear analyses this effect may be detrimental to the representativeness of the synthetic accelerogram.

The comparison of the spectrograms of the initial accelerogram (see hereafter Fig. 9) vs. the synthetic accelerogram (see hereafter Fig. 10) after 5 iterations shows clearly the global enrichment in energy of the signal. This is a classical consequence of the synthesis of accelerogram whose is targeted to fit a given response spectrum.

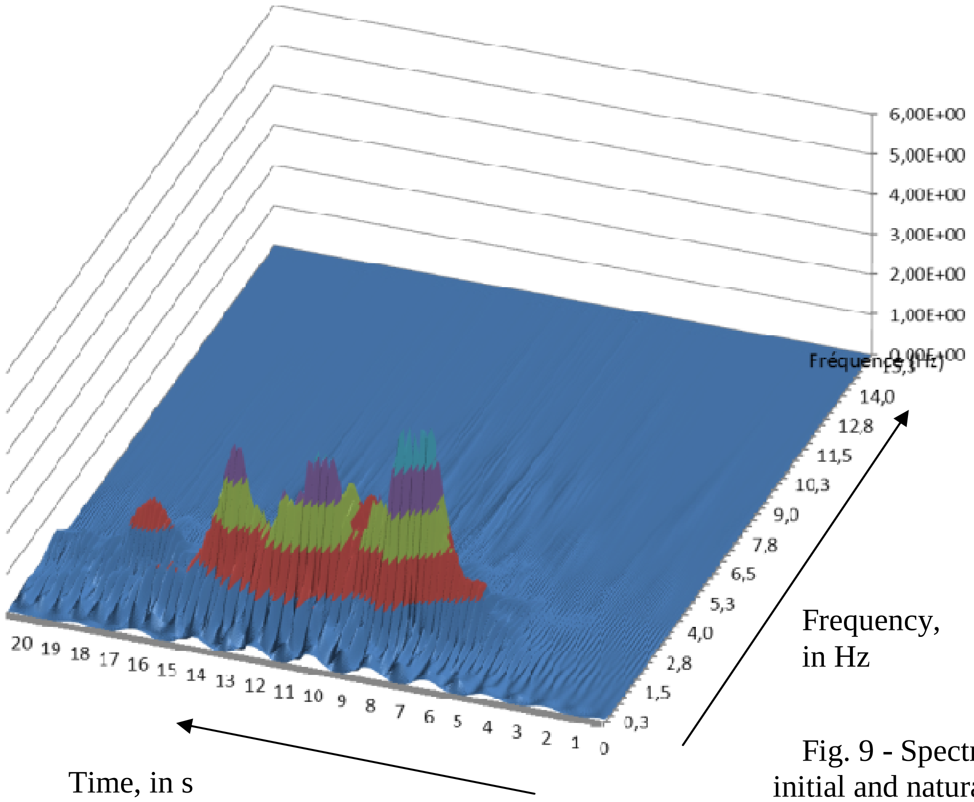


Fig. 9 - Spectrogram of the initial and natural accelerogram

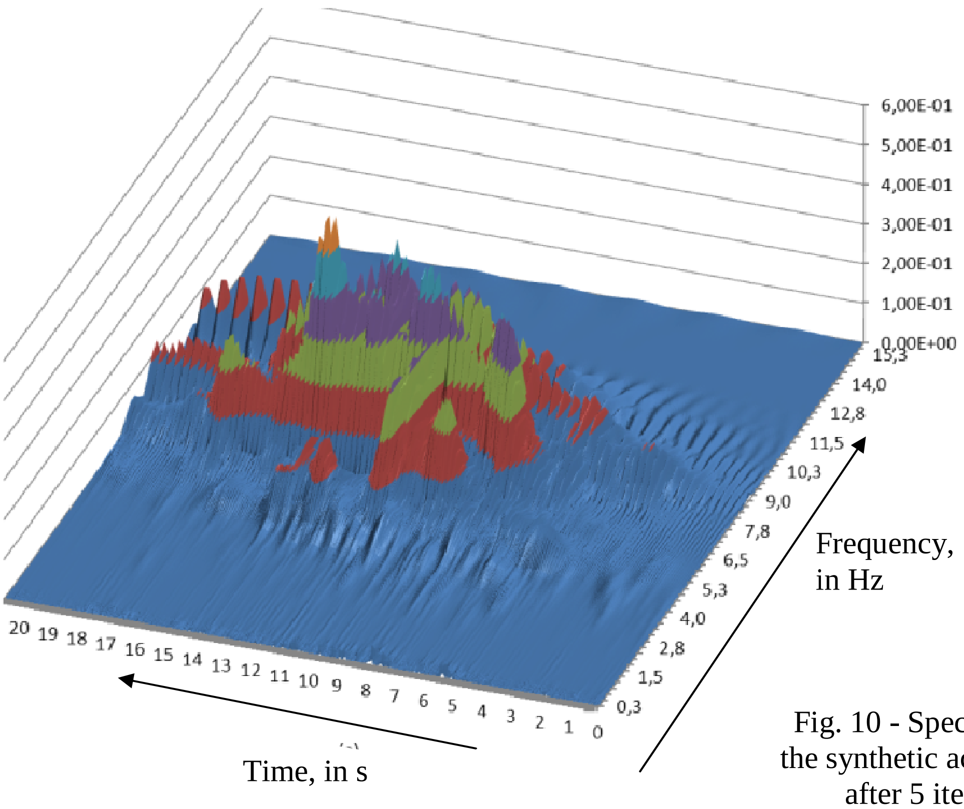


Fig. 10 - Spectrogram of the synthetic accelerogram after 5 iterations

CONCLUSION

The interest of this efficient method of generation of synthetic accelerogram relies on the fact that it can be easily implemented on a spreadsheet and on the fact that the spectrogram, being a natural by-product of the wavelet transform, may be used for illustrating the content of the accelerogram during its successive stages.

REFERENCES

Suarez, L. E. and Montejo, L. A. (2002). “Generation of artificial earthquakes via the wavelet transform”, *International Journal of Solids and Structures* 42 (2005) 5905–5919.

Conte, J.P. and Peng B.F. (1997). “Fully Nonstationary Analytical Earthquake Ground-Motion Model”, *Journal of Engineering Mechanics,* January 1997, 15-24

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