SPATIO-FREQUENTIAL CHARACTERIZATION OF NON-UNIFORM TURBULENCE EXCITATIONS USING INVERSE TECHNIQUES: PART 1 - EXTRACTION OF THE MODAL RESPONSES FROM THE VIBRATORY MEASUREMENTS

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ABSTRACT

Flow-induced vibrations are particularly important as a problem field in reactor technology. Beyond the need to prevent fluidelastic instability phenomena which can lead to catastrophic failures, it is necessary to also account for the long-time effects of the flow turbulence excitations. In two companion papers we address the problem of identifying, from vibratory experiments, the spectral and spatial parameters of turbulent flow excitations with non-uniform velocity profiles. These features are crucial for the predictive dynamical analysis of flow-excited nuclear components, such as steam generators tubes or fuel rods. Our recent work on this topic will be extended here to deal with flows which display significant changes in their spatial properties. Under these conditions, the frequency content of the excitation spectra in different regions of the tube cannot be assumed similar. We start by developing in Part 1 an effective modeling technique for predicting the vibrations of multi-supported tubes subjected to non-uniform turbulent flows. Then we propose an original method for the experimental identification of the spectral and spatial features of flow excitations, pursued in Part 2. The proposed approaches are illustrated with identification results based on realistic numerical simulations of a multi-supported tube subjected to a transverse flow with a triangular velocity profile.

INTRODUCTION

Flow-induced vibrations (FIV) are particularly important as a problem field in reactor technology. Beyond the need to prevent fluidelastic instability phenomena, which can lead to catastrophic failures, it is necessary to also account for the long-time effects of the flow turbulence excitations, see for instance Chen (1987). These are typically obtained from experiments, either directly measured or inferred from the vibratory responses. For multi-supported tubular components the need remains for a correct identification of the flow excitation spectral content and spatial distribution, which are crucial for the predictive analysis of nuclear components such as steam generator tubes and fuel rods, as extensively discussed in papers by Axisa et al. (1988), Antunes et al. (2008, 2009, 2011, 2012) and Piteau et al. (2012).

Indeed, even if CFD computations of the flow turbulence already produce exploitable results, see Moussou et al. (2011), these are an extremely demanding task. Therefore, the excitation is typically obtained from experiments, where the random forces are either directly measured or inferred from the system responses. In order to simplify such task we recently extended to axial flows the concept of "Equivalent Dimensionless Excitation Spectrum" $\Phi_{E}(fD/V)$ which conveniently encapsulates the effects of the axial correlation length $\lambda_\alpha(f)$ and of the convection velocity $V_C(f)$, see Antunes et al.
However, the need remains for a correct identification of the flow excitation spectral content and spatial distribution, which strongly depends on the velocity profile $V(x)$.

Therefore, the topic addressed in this paper is the identification of the spectral and spatial features of random flow excitations. Previous efforts in this area which are relevant to the proposed work include the pioneer work by Bolotin (1984) and the closely related theoretical and experimental study by Granger and Perrotin (1999), as well as recent work by Park et al. (2009) and Hwang et al. (2009, 2011). All these authors addressed the problem of source identification under the assumption of near-linear behavior and using identification strategies based on a modal formulation of the system dynamics. Other interesting publications, not specific to flow-induced vibrations but also focusing on the identification of distributed excitations from dynamical structural responses, include Lin et al. (2001), Liu & Shepard Jr (2006) and Jiang & Hu (2008).

In Part 1 of the present work we start by developing an effective modeling technique for predicting the vibrations of multi-supported tubes subjected to strongly non-uniform turbulent flows. Then we propose an original method for the experimental identification of the spectral and spatial features of flow excitations, which will be pursued in Part 2, see Antunes et al. (2013b). The present research follows the general identification framework recently developed by Antunes et al. (2013a). The source identification will be performed from a set of measured vibratory responses, as follows: (1) The modal response spectra and modeshape amplitudes at the measurement locations are first extracted through a blind decomposition of the physical response matrix; (2) The continuous modeshapes are interpolated from the identified values at the measurement locations; (3) The system modal parameters are identified from the modal responses; (4) Inversion from the modal response spectra is performed for identification of the modal excitation spectra; (5) Finally, equivalent physical excitation spectra at different regions of the tube, as well as the flow velocity profile, are estimated.

The proposed approach is illustrated with results based on realistic numerical simulations of a multi-supported tube, for linear supports with no clearances nor friction, which were performed using techniques described in detail by the authors in several publications, see Axisa et al. (1988), Antunes et al. (2008, 2009, 2011, 2013a) and Piteau et al. (2012). In order to highlight how the space distribution of the flow velocity affects the turbulence excitation features, illustrative computations and identifications will be based on a triangular velocity profile.

**TURBULENCE EXCITED VIBRATIONS**

Figure 1 shows a generic system consisting on a beam-like structure with length $L$ and diameter $D$ subjected to a flow excitation with density $\rho$ and velocity profile $V(x) = \bar{V}(x)$ (along a "wet" length $L_f = x_b - x_a$), the normalized flow profile $u(x)$ being referred to the average velocity:

$$\bar{V} = \frac{1}{L_f} \int_{x_a}^{x_b} |V(x)| dx$$

![Figure 1. Multi-supported tube subjected to a flow with generic velocity profile $V(x)$.](image)

It can then be shown, see Axisa et al. (1990) and Antunes et al. (2012), that under linear conditions the cross-spectra of the physical responses along the system at locations $x_i$ are given as:
\[ S_{ij}(f) = S_{yy}(x_i, x_j, f) = \sum_{n=1}^{N} \sum_{m=1}^{N} \phi_n(x_i) \phi_m(x_j) S_{q_{nq_m}}(f) \quad ; \quad i, j = 1, 2, \ldots, R \]  

or, in matrix form:

\[
\begin{bmatrix}
S_{11}(f) & S_{12}(f) & \cdots & S_{1R}(f) \\
S_{21}(f) & S_{22}(f) & \cdots & S_{2R}(f) \\
\vdots & \vdots & \ddots & \vdots \\
S_{R1}(f) & S_{R2}(f) & \cdots & S_{RR}(f)
\end{bmatrix}
= \begin{bmatrix}
S_{q_{1q_1}}(f) & S_{q_{1q_2}}(f) & \cdots & S_{q_{1q_R}}(f) \\
S_{q_{2q_1}}(f) & S_{q_{2q_2}}(f) & \cdots & S_{q_{2q_R}}(f) \\
\vdots & \vdots & \ddots & \vdots \\
S_{q_{Rq_1}}(f) & S_{q_{Rq_2}}(f) & \cdots & S_{q_{Rq_R}}(f)
\end{bmatrix} \begin{bmatrix}
\Phi_1 \\
\Phi_2 \\
\vdots \\
\Phi_R
\end{bmatrix}^T
\]

where the modal matrix is \( \Phi = \{ \{ \phi_1 \} \{ \phi_2 \} \cdots \{ \phi_N \} \} \), with modal vectors \( \{ \phi_n \} = \{ \phi_n(x_1), \ldots, \phi_n(x_N) \}^T \), \( n = 1, 2, \ldots, N \). Formulations similar to (2) and (3) can be established to relate the physical and modal velocities, as well as the physical and modal accelerations. The modal responses are formulated by the following cross-spectra, with \( m, n = 1, 2, \ldots, N \) modes:

\[ S_{q_{nm}}(f) = H_n(f)H_m^*(f) S_{FF}(x_1, x_2, f) \int_0^L \int_0^L \phi_n(x_1) \phi_m(x_2) S_{ff}(x_1, x_2, f) \, dx_1 \, dx_2 \]  

where \( H_n(f) = \left[ 4\pi^2 m_n \left( f_n^2 - f^2 + 2if f_n \zeta_n \right) \right]^{-1} \), with the modal parameters \( m_n \), \( f_n \), \( \zeta_n \) and \( \phi_n(x) \) of the flow-subjected tube, while \( S_{ff}(x_1, x_2, f) \) is the cross-spectral function of the turbulence field, given as:

\[ S_{ff}(x_1, x_2, f) = \left[ \Phi_{FF}(x_1, f) \Phi_{FF}(x_2, f) \right]^{1/2} \gamma(x_1, x_2, f) \]

where \( \gamma(x_1, x_2, f) \) is the space-coherence function of the turbulence eddies and the local excitation spectrum may, in dimensionless form, be approximated as:

\[ \Phi_{FF}(x_1, f) \approx \left( \frac{1}{2} \rho V^2 \left[ u(x) \right]^2 D \right)^{1/2} \frac{D}{V} \Phi_{FF} \left( \bar{f}_R \right) \quad ; \quad \bar{f}_R = \frac{fD}{V} \]

Then, we may write (4) as follows:

\[ S_{q_{nm}}(f) = H_n(f)H_m^*(f) S_{FF_m}(f) \]

where the modal forces:

\[ S_{FF_m}(f) = \Phi_{FF}(f)L_{nm}^2(f) \]

depend on the correlation integrals (joint and cross-acceptances):

\[ L_{nm}^2(f) = \int_0^L \int_0^L \phi_n(x_1) \phi_m(x_2) \left[ u(x_1)u(x_2) \right]^{1/2} \gamma(x_1, x_2, f) \, dx_1 \, dx_2 \]

Now, for near-homogeneous turbulence the space-coherence may be approximated by exponential functions, using Corcos (1964) model. For transverse (T) and axial (A) excitations, this leads to the well-known results:

\[ \gamma_T(x_1, x_2, f) = \exp \left( -\frac{|x_2 - x_1|}{\lambda_T(f)} \right) \quad ; \quad \gamma_A(x_1, x_2, f) = \exp \left( -\frac{|x_2 - x_1|}{\lambda_A(f)} \right) \exp \left( -i \frac{x_2 - x_1}{V(f)/2\pi f} \right) \]

where \( \lambda_{a}(f) \) is a correlation length and \( V(f) \) the convection velocity for the turbulence fluctuations. Then, under the physically plausible hypothesis of small correlation length, \( \lambda_{a}(f) / L \ll 1 \), it can be shown from (6)-(10) that both cases can be approximated by a common formulation, see Axisa et al. (1990) and Antunes et al. (2012):

\[ S_{FF_m}(f) = \left( \frac{1}{2} \rho V^2 D \right)^{1/2} \frac{D}{V} \Phi_{EQ}^{TA} \left( \frac{fD}{V_{T,A}} \right) \frac{C_{nm}^2}{L_{nm}} \]

with the "Equivalent Reference" turbulence spectra defined as (where \( L_R \) is a reference length, such that \( \lambda_{f}(f) / L_R \ll 1 \):
THE CASE OF STRONGLY NON-UNIFORM FLOWS

It may be noticed that the assumption of a common average velocity \( \overline{V} \) for the full system, when enforced into (6) through the average reduced frequency \( \overline{f}_r = f D / \overline{V} \), leads to a common spectral "shaping" of the excitation irrespective of the local velocity \( V(x) \). In other words, when converting from the dimensionless spectrum to the local physical excitation using formulation (6), the local velocity affects the spectral amplitude but not the spectral content. This approximation may be tolerated for moderate changes in \( V(x) \), but may potentially introduce significant errors if the velocity profile displays abrupt changes. A manner to circumvent this problem is to decompose the space-domain \([0, L]\) in several regions \( p = 1, 2, \ldots, P_{cal} \) with lengths \( L_p = x_p^f - x_p^i \), where "local" average velocities \( \overline{V}_p \) are defined. Then we can replace the global velocity profile defined using (1) by:

\[
V(x) = \sum_{p=1}^{P_{cal}} \overline{V}_p u_p(x) \quad \text{with} \quad \overline{V}_p = \frac{1}{L_p} \int_{x_p^i}^{x_p^f} V(x) \, dx \quad \text{and} \quad u_p(x) = \begin{cases} \frac{V(x)}{\overline{V}_p} & \left( x_p^i \leq x \leq x_p^f \right) \\ 0 & \left( x \leq x_p^i ; x \geq x_p^f \right) \end{cases}
\]  

so that the modal cross-spectra (11) are now given as:

\[
S_{F_n F_m}(f) \approx \sum_{p=1}^{P_{cal}} \left( \frac{1}{2} \rho \overline{V}_p^2 D \right) \frac{D}{V_p} \Phi_{EQR}^{\text{r,A}} \left( \frac{f D}{V_p} \right) \left( C_{nm}^p \right)^2 ; \quad n,m = 1, 2, \ldots, N
\]

with the correlation integrals (13) now computed for each region as:

\[
\left( C_{nm}^p \right)^2 = 2 \int_{x_p^i}^{x_p^f} \phi_n(x) \phi_m(x) \left[ \frac{V(x)}{V_p} \right]^4 \, dx \quad ; \quad n,m = 1, 2, \ldots, N \quad ; \quad p = 1, 2, \ldots, P_{cal}
\]

This generalized formulation will be used in the following for computing the system vibratory responses when subjected to a strongly space-varying flow velocity profile \( V(x) \). However one must notice that, in order to use formulation (14)-(16) in a correct manner, the region lengths \( L_p \) must be chosen such that they always be significantly larger than the correlation length, hence \( \lambda_c(f) \ll L_p \) \( \forall p \). This requirement can be easily observed in practice.

ILLUSTRATIVE SYSTEM

In the present paper all the identification work will be based on a realistic test case generated through time-domain numerical simulations. We will not detail here the computational techniques used, which have already been discussed by Axisa et al. (1988), Antunes et al. (2008, 2009, 2011, 2013) and Piteau et al. (2012). The system used for the numerical simulations is shown in Figure 2, consisting on a tube with length \( L = 5 \text{ m} \), diameter \( D = 20 \text{ mm} \) and mass per unit length (including added mass)
The tube is subjected to water flow with \( \rho = 1000 \text{ kg/m}^3 \) and average transverse velocity \( \bar{V}_r = 5 \text{ m/s} \), assuming a triangular velocity profile as shown in Figure 2. For the excitation we used the "Equivalent Reference Spectrum" (12) proposed by Axixa et al. (1990), here truncated below \( f_R = 0.01 \):

\[
\Phi_{EQR}^T (f_R) = \begin{cases} 
0 & \text{if } f_R < 0.01 \\
4 \times 10^{-4} (f_R)^{-0.5} & \text{if } 0.01 \leq f_R \leq 0.2 \\
3 \times 10^{-6} (f_R)^{-3.5} & \text{if } f_R > 0.2
\end{cases}
\]  

(17)
Computation of the vibratory responses has been performed by decomposing the tube length in \( P_{col} = 12 \) regions of equal length. The resulting computed velocity spectra (auto-spectra and absolute values of the cross-spectra) at the response locations \( x_r \) are shown superposed in Figure 3. The RMS amplitude of the response along the tube is shown in Figure 4.

These plots show that, for the support configuration of this system, vibratory energy is well distributed, which is often the case. Figure 5 shows the cross-covariance coefficient matrices \([\langle F_n(t)F_m(t) \rangle]\) and \([\langle q_n(t)q_m(t) \rangle]\), computed respectively from the time-domain modal excitations and velocities. These results clearly show that, even when the modal excitations are coupled (this is typical for non-uniform velocity profiles), the modal responses are almost uncoupled. In other words, the spectral matrix \([S_{qq}(f)]\) of modal responses is near diagonal. We have found that, with respect to the modal basis (constrained at every support) used for these computations, the modal responses are nearly decoupled whatever the excitation flow profile. Then, formulation (2) reduces to the often used simplification:

\[
S_q(f) = S_{YY}(x_i, x_j, f) = \sum_{n=1}^{N} \phi_n(x_i)\phi_n(x_j)S_{qq_n}(f) \quad ; \quad i, j = 1, 2, \ldots, R
\]  (18)

or, in matrix form:

\[
\begin{bmatrix}
S_{11}(f) & S_{12}(f) & \cdots & S_{1R}(f) \\
S_{21}(f) & S_{22}(f) & \cdots & S_{2R}(f) \\
\vdots & \vdots & \ddots & \vdots \\
S_{R1}(f) & S_{R2}(f) & \cdots & S_{RR}(f)
\end{bmatrix}
\approx
\begin{bmatrix}
S_{qq_1}(f) & 0 & \cdots & 0 \\
0 & S_{qq_2}(f) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & S_{qq_N}(f)
\end{bmatrix}
\begin{bmatrix}
\Phi^T
\end{bmatrix}
\]  (19)

a feature that we will put at use in our identification strategy.

**IDENTIFICATION OF THE MODAL RESPONSES**

Given a number of modal auto-spectral responses \( \hat{S}_{qq_n}(f), \quad n = 1, 2, \ldots, N \) (where \( N \) is the number of system modes significantly excited), formulation (19) shows how these can be directly related to a set of vibratory measurements \( S_{YY}(x_i, x_j, f), \quad i, j = 1, 2, \ldots, R \). In condensed notation:

\[
\begin{bmatrix}
S_{YY}(f)
\end{bmatrix} = \begin{bmatrix}
\Phi
\end{bmatrix} \begin{bmatrix}
S_{qq}(f)
\end{bmatrix} \begin{bmatrix}
\Phi^T
\end{bmatrix}
\]  (20)

In order to use (20) for extracting the modal responses from the set of physical measurements, a very precise knowledge of the modal matrix \([\Phi]\) is needed. Furthermore, well-posedness of the inverse problem implies that \( R \geq N \) as well as the adequate numerical conditioning of \([\Phi]\). As discussed by Antunes et al. (2013), an effective technique for achieving a satisfying isolation of the modal responses,
with no a priori information on the system modal properties, belongs to the class of Blind Source Separation (BSS) methods, a research area which is fast spreading into many fields, such as signal or image processing and system identification, for medical, engineering and science applications, see Cardoso (1998). In short, given a system such that the components of a source input vector \( \{S\} \) are operated through a mixing matrix \( [A] \), blind identification techniques attempt to extract both unknown \( \{S\} \) and \( [A] \) from the output vector:

\[
\{Y\} = [A]\{S\} \tag{21}
\]

To overcome the indeterminacy of the inverse problem stemming from (21), blind identification techniques typically use a priori information about the components of \( \{S\} \). In the present case, equation (21) may be seen as an analogue to the instantaneous mixture of the "source" vector of modal responses \( \{Q(t)\} \) through the modal matrix \( [\Phi] \), in order to obtain the vector of physical measurements:

\[
\{Y(t)\} = [\Phi]\{Q(t)\} \tag{22}
\]

Here, due to the nature of the excitation and the assumed linearity of the system, both \( \{Q(t)\} \) and \( \{Y(t)\} \) will be Gaussian, therefore higher-order statistics do not apply to our problem. On the other hand, as we have shown before, matrix \( [S_{qq}(f)] \) is in practice near-diagonal, which implies second order independence of the modal components of \( \{Q(t)\} \). For such problem, the closely related AMUSE (Algorithm for Multiple Unknown Signals Extraction) and SOBI (Second Order Blind Identification) methods appear well suited, see Tong et al. (1991) and Belouchrani et al. (1997). These techniques exploit the correlation information contained in the measured response signals and have been recently used for modal identification and machine monitoring by several authors. These algorithms are known to be effective in separating "colored" sources with distinct power spectra, which is the case of modal responses for systems with non-degenerate modes.

As usual in blind source separation, even successful identifications suffer from two inescapable ambiguities: source permutation and scaling. These ambiguities, which must be resolved through additional knowledge on the problem, are easily understood. Firstly, system (21) is invariant if the columns of \( [A] \) and the corresponding lines of \( \{S\} \) are interchanged. Secondly, results of (21) are similar if any column of \( [A] \) and the corresponding line of \( \{S\} \) are multiplied and divided, respectively, by any arbitrary constant. These ambiguities may pose difficulties in some fields, but not in the present application. Actually, we order the identified sources such that the corresponding auto-spectra \( S_{qq}(f) \) pertain to increasing modal frequencies. On the other hand, because the columns of \( [A] \) are physically related to the system modeshapes, these will be normalized as usual, which resolves scaling issues.

Both the SOBI and AMUSE methods work in the time domain, using the correlation matrix of the physical responses, which is related to the spectral matrix through the Fourier transform \( [S_{qq}(f)] = \mathcal{F}([R_{qq}(\tau)]) \), hence \( [R_{qq}(\tau)] = \mathcal{F}^{-1}([S_{qq}(f)]) \). From equation (20), we can write:

\[
[R_{yy}(\tau)] = [\Phi][R_{qq}(\tau)][\Phi]^T \tag{23}
\]

Because this equation is true for any value of the time delay \( \tau \), it may be shown that the mixing matrix \( [\Phi] \) in (22) can be estimated by joint-diagonalization of (23) for any two different time delays \( \tau_1 \) and \( \tau_2 \). This technique, at the heart of the AMUSE method developed by Tong et al. (1991), has been extended in the SOBI algorithm by Belouchrani et al. (1997), who used a large number of time delays thus increasing robustness to noise. For details on finding the orthogonal matrix \( [\Phi] \) that diagonalizes a set of correlation matrices \( [R_{yy}(\tau_k)] \), \( k = 1, 2, ..., K \), the reader is referred to the aforementioned references.
Figure 6. Real and identified modeshapes from the simulated response measurements.

Figure 7. Real and identified auto-spectra of the modal responses.

Figure 8. Real and identified auto-correlations of the modal responses.
After identification of the unscaled mixing matrix $[\hat{\Phi}]$ at the response locations, we estimated the corresponding continuous modeshapes $\hat{\phi}_n(x)$ along the tube by performing a spline interpolation through the values $\{\hat{\phi}_n\}$ of each modal column of the mixing matrix $[\hat{\Phi}]$. Then, normalization of the various modeshapes enabled to obtain the modal scaling constants $C_n$ such that each $\hat{\phi}_n(x) = C_n \hat{\phi}_n(x)$ displays maximum unit amplitude (the modeshape normalization rule used in this paper). Finally, the scaled spectra and correlation functions are computed from the unscaled SOBI results as $\hat{S}_{qq_{nn}}(\tau) = \hat{S}_{qq_{nn}}(\tau) / C_n^2$ and $\hat{R}_{qq_{nn}}(\tau) = \hat{R}_{qq_{nn}}(\tau) / C_n^2$. For the spline modeshape interpolation, we used the identified amplitudes at the response locations, while also postulating zero amplitudes at the pinned ends of the tube and at the intermediate support locations, a legitimate option because, for the modes of interest, the support stiffnesses are large compared to the tube generalized stiffness. For the first 10 modes of the system, Figure 6 illustrates the identified modeshapes, while Figures 7 and 8 show the identified modal response auto-spectra and auto-correlation functions, all these results being quite satisfying.

CONCLUSION

In this paper we start an extensive study on the dynamics of tubes excited by highly non-uniform turbulent flows. An effective method is proposed for simulating the random excitation by such flows, based on a decomposition of the excitation domain in several regions, each with a representative average velocity. This enables an adequate spectral shaping of the physical excitation, which depends on the local flow velocity. Then we develop an approach for extracting the relevant structural and excitation dynamical features of the flow-excited structure. The proposed technique enables the identification of the turbulence excitation field, from a set of vibratory measurements, using a minimum of a priori information on both the structure and the flow. The first stage of the identification procedure, presented in Part 1 of this work, is the extraction of modal responses from a set of vibratory measurements. A blind identification technique for decomposing the response correlation matrix, the SOBI method, proved convincing for the difficult problem addressed. The results obtained, based on realistic numerical simulations of a multi-supported tube subjected to a non-uniform flow velocity field, are quite satisfying. From these results, an effective strategy for identifying the flow excitation features is built in Part 2 of this work, see Antunes et al. (2013).

REFERENCES


