



EXPERIMENTAL AND THEORETICAL SEISMIC MODELING OF A SURFACE FOUNDATION IN FINITE-DOMAIN SIMULATIONS

Mahdi Soudkhah¹, Ronald Y.S. Pak²

¹ SSI Engineer, Bechtel Corporation, Frederick, MD (msoudkha@bechtel.com); formerly research associate, University of Colorado, Boulder, CO

² Professor, Dept. of Civil, Environmental, and Architectural Engineering, University of Colorado, Boulder, CO

ABSTRACT

A fundamental experimental study to explore the vertical, horizontal, and rocking responses of a plate footing resting on a horizontal sand stratum under seismic excitations is reported in this paper. By means of centrifuge modeling, multiple series of tests were performed using an absorbing boundary container and a shake table which delivered simultaneous vertical-horizontal-rocking motions to the soil. To evaluate the applicability of the elastodynamic theory to a stratum configuration, relevant sets of complex-valued foundation impedance functions and foundation input motion functions were computed for a square rigid footing bonded to the surface of an infinite horizontal stratum as well as the employed finite-size centrifuge model, both with a square-root depth-dependent shear modulus profile. Through a comparison of the experimental data with the classical 1-D stratum and 3-D finite-domain solutions, it is shown that boundary effects in geotechnical centrifuge models need to be accounted for generally to ensure a proper interpretation and synthesis of the data.

INTRODUCTION

Verification of the computational seismic response of surface and embedded foundations under multi-directional plane-wave vibrations remains one of the challenging problems in geotechnical and structural earthquake engineering (Richart et al 1970, Kramer 1996, Chopra 2007, Chin 2008). Credibility and soundness of industrial foundation design, system characteristics identification, and health monitoring of buildings, require thorough experimental field studies which are in most cases difficult to achieve due to the large size of the foundation or structure and higher cost of the field operations. A practical and economical solution for this complication would be utilizing the scaled modeling techniques in elevated gravity (e.g. Schofield 1980, Ko 1988a, 1988b). While a laminar box placed on a shake-table is appropriate for one-dimensional and large-amplitude soil motion problems (Hushmand et al. 1988, Law et al. 1991, Zeng and Steedman 1993), a large soil container with physical absorptive boundaries has been found to be effective for three-dimensional soil-structure interaction (SSI) simulations of foundations under forced vibrations or base excitations (e.g., Pak and Guzina 1996, Ashlock 2000 and 2006, Pak et al. 2012, Soudkhah and Pak 2012a, 2012b).

In this paper, an experimental-analytical study of the seismic soil-structure interaction with a surface foundation on a soil stratum using centrifuge and elastodynamic modeling is presented. Conducted on the 400 g-ton centrifuge at the University of Colorado, Boulder, the experimental research program employed a large steel rectangular container with ductseal lining and a thick aluminum shake table on elastomeric support at the bottom. Measureable ground motions were generated in the soil via the base table using an electromagnetic exciter. By means of random vibration techniques, time histories and frequency response functions (FRF) of the foundation under seismic base excitations were processed and evaluated. To synthesize the experimental centrifuge data from seismic excitations, the shake-table, absorptive boundary, the soil medium, and the surface foundation were included in the computer

modeling with the aid of an advanced boundary element code BEASSI (Pak and Guzina 1999). Relevant sets of complex-valued foundation impedance functions, foundation input motion functions and free-field motions were computed for a rigorous assessment of both 1-D and 3-D syntheses of the experimental transfer function data.

EXPERIMENTAL SETUP

Soil Model, Container and Centrifuge

A fundamental experimental program on the seismic response of a surface foundation on a soil stratum was carried out using the 400 g-ton centrifuge at the University of Colorado, Boulder. A dry uniform and fine silica F-75 Ottawa sand with a specific gravity of 2.65, coefficient of uniformity $C_u = 1.71$, and coefficient of curvature $C_c = 1.01$ was used. To obtain a soil model with a uniform density, the sand was pluviated from a calibrated height through a slotted plate attached to the bottom of a suspended hopper, yielding a uniform density of 1730 kg/m^3 , a void ratio of $e = 0.53$, a relative density of 86%, and a friction angle of $\phi' = 40^\circ$. To study the seismic response, a large rectangular container of a plan dimension of 1.2 m by 1.0 m was used with a layer of 0.035m thick ductseal lining for the inner wall. A soil depth of 0.18m was used, yielding a width-to-depth ratio and length-to-depth ratio of 5.2 and 6.3, respectively. Serving as the shake-table in this study, an in-box high-strength aluminum plate of a $1.12\text{m} \times 0.98\text{m} \times 0.038\text{m}$ resting on elastomeric pads (see Soudkhah 2010 and Pak et al. 2012) was placed on the base of the container (see Figure 1 and Figure 2a). Under the overburden weight of the soil as well as the significant friction between it and the container, the thin elastomeric support is stiff in the vertical direction and soft in the horizontal direction.

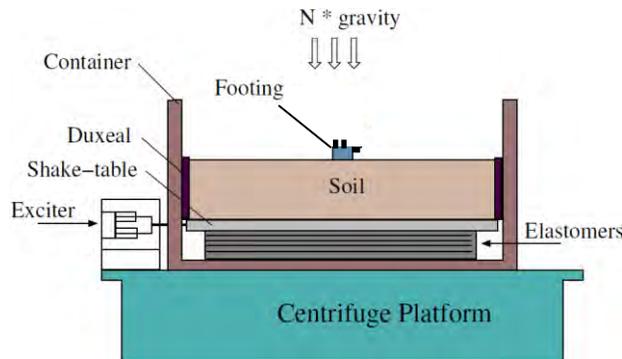


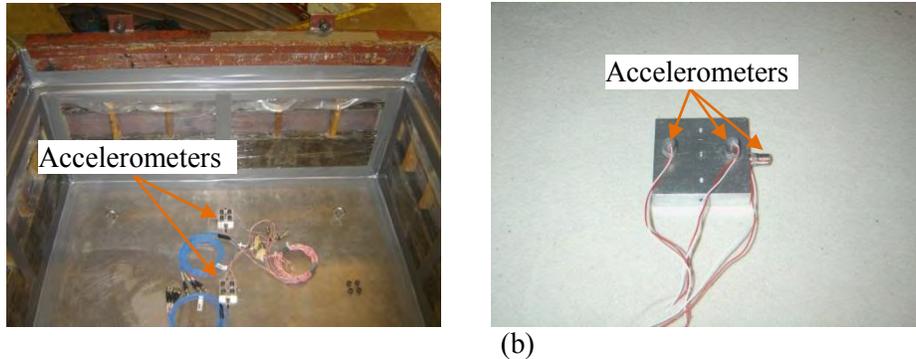
Figure 1. Assembly of seismic tests on a surface foundation in shake-table

Data Acquisition Systems and Measurement Approach

The dynamic data acquisition and real-time control systems (2 sets of 8-channel National Instruments (NI) PXI-4472 module) resides on the centrifuge arm and can be controlled by a wireless remote desktop connection. The PXI-4472 modules provide the necessary accelerometer excitation current and signal conditioning as well. The miniaturized PCB-352C67 accelerometers from PCB Piezotronics used in this study all have exceptionally low cross-sensitivity of less than 1.5%. The power and signal conditioning for the accelerometer were provided by a PCB Model 483A signal conditioner mounted on the centrifuge arm. To simulate seismic motion over a wide frequency band, a Labwork-ET 139 electromagnetic shaker was used as the source of excitation for the aluminum shake-table.

Scaled-Model Surface Foundation, Instrumentation, and Measurements

To explore the seismic soil-foundation interaction problem experimentally, an aluminum square plate footing with the base width of 0.055m was used. The instrumentation configurations used in this study are shown in Figure 2(b) and the resulting foundation inertial properties are summarized in Table 1. The plate footing B02 was placed on a 0.18m-deep horizontal sand stratum which was tested at centrifugal acceleration levels of 22, 33, 44, 55 and 66 g levels. For the plate footing B02, prototype bearing pressures p_{pr} are relatively low (see Table 2) and the prototype foundation dimensions are up to 3.6m at the g-levels considered.



(a) (b)
 Figure 2. Physical arrangement of instrumented (a) shake-table and (b) plate footing on the soil

Table 1: Specifications of footing in experiments

Footing	Width (m)	Height (m)	Mass (Kg)	Moment of Inertia (kg. cm ²)	Centroid (cm)
Plate	0.055	0.0123	0.12261	0.273	0.613

Table 2: Foundation-soil average contact pressures including instrumentations in kPa

Footing	Nominal g-level				
	22g	33g	44g	55g	66g
B02	8.70	13.0	17.3	21.7	26.0

The basic configuration of the seismic test and the instrumentation plan for the tests are shown schematically in Figure 3. To monitor the motion of the shake table and the soil, accelerations at the center and quarter-point at both the base and the top of the soil were measured. For ease of referencing, the following acronyms of the measurements on top of the shake-plate are used: (a) Bottom vertical-centric (BVC), (b) Bottom horizontal-centric (BHC), (C) Bottom transverse-centric (BTC), (d) Bottom vertical-quarter point (BVQ), (e) Bottom horizontal-quarter point (BHQ), (f) Bottom transverse-quarter point (BTQ). Similarly, the designations of the acceleration measurements on the footing are as follows: (a) vertical-centric (VC), (b) vertical-eccentric (VE), (c) horizontal-centric (HC).

In this study, both time and frequency domain measurements of the accelerations were recorded. For direct physical insights as well as theoretical considerations, a frequency response function (FRF) between the input and the output was defined as

$$H(f) = \frac{\tilde{Y}(f)}{\tilde{X}(f)} \quad (1)$$

where f is the frequency in Hz, $\tilde{X}(f)$ and $\tilde{Y}(f)$ are the Fourier transforms of the input $X(t)$ and output $Y(t)$ that characterize the system's dynamic behavior fully in the frequency domain. Referred to as the 'seismic acceleration', the experimental transfer function of particular interest in this problem is

$$A^*(f) = \frac{\tilde{a}_p(f)}{\tilde{a}_b(f)} = \frac{\tilde{U}_p(f)}{\tilde{U}_b(f)} \quad (2)$$

which relates the transformed measured acceleration $\tilde{a}_p(f)$ and its corresponding displacement $\tilde{U}_p(f)$ at a control point P on the footing to the imposed basal acceleration $\tilde{a}_b(f)$ and its displacement $\tilde{U}_b(f)$ at the bottom of the model induced by seismic vibration. In what follows, the seismic acceleration function $A_{HC/BHC}^*(f)$ which relates the horizontal acceleration of the footing to the horizontal base acceleration and $A_{VC/BVC}^*(f)$ and $A_{VE/BVC}^*(f)$ which relate the corresponding vertical-centric and -eccentric components are of particular interest.

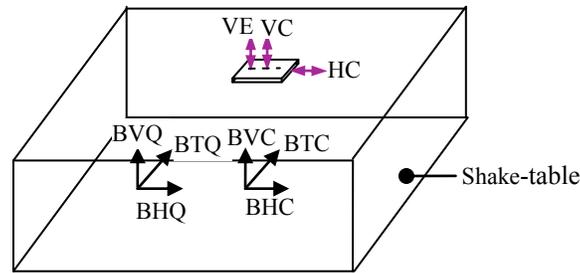


Figure 3. Instrumentation and notation for acceleration measurement locations of seismic tests

EXPERIMENTAL RESULTS

Time history of Seismic Vibrations

Making use of the side-mounted exciter as shown in Figure 1, forced seismic tests were performed on the sandy soil by random vibration of the shake-table. Shown in Figure 4(a) are the typical time histories of the acceleration of the shake-table under exciter-induced shaking of the table/seismic vibration at different centrifuge g -levels. As shown from the figure, the magnitude and frequency of BHC and BVC vibrations are consistent for all examined g -levels but the frequency of the horizontal vibrations seems to be greater than the vertical vibrations. The vibration of the shake-table is expected to be due to the generated motions of the side-mounted shaker, however, it is possible to observe vibrations in both horizontal and vertical directions due to the ambient vibrations transferred by the centrifuge arm's rotation. More detailed discussion on the ambient condition of the centrifuge test environment can be found in Soudkhah and Pak (2012).

Shown in Figure 4(b) are the typical time histories of the acceleration of the Footing B02 at HC, VE, and VC locations under seismic vibration at different centrifuge g -levels. Regarding the horizontal and vertical motions of the footing, the time histories of the accelerometers indicate that both horizontal and vertical amplifications occurred in the seismic tests. By comparing the times histories of the vibrations in shake-table and Footing B02 as shown in Figures 4(a) and 4(b), it is evident that amplification of the motions occur at the footing due to resonance. As observed, the magnitude of the acceleration of the footing on the soil surface especially in the horizontal direction varies from 1 to 4 m/s^2 which is generally higher than the base acceleration measured on the shake-table.

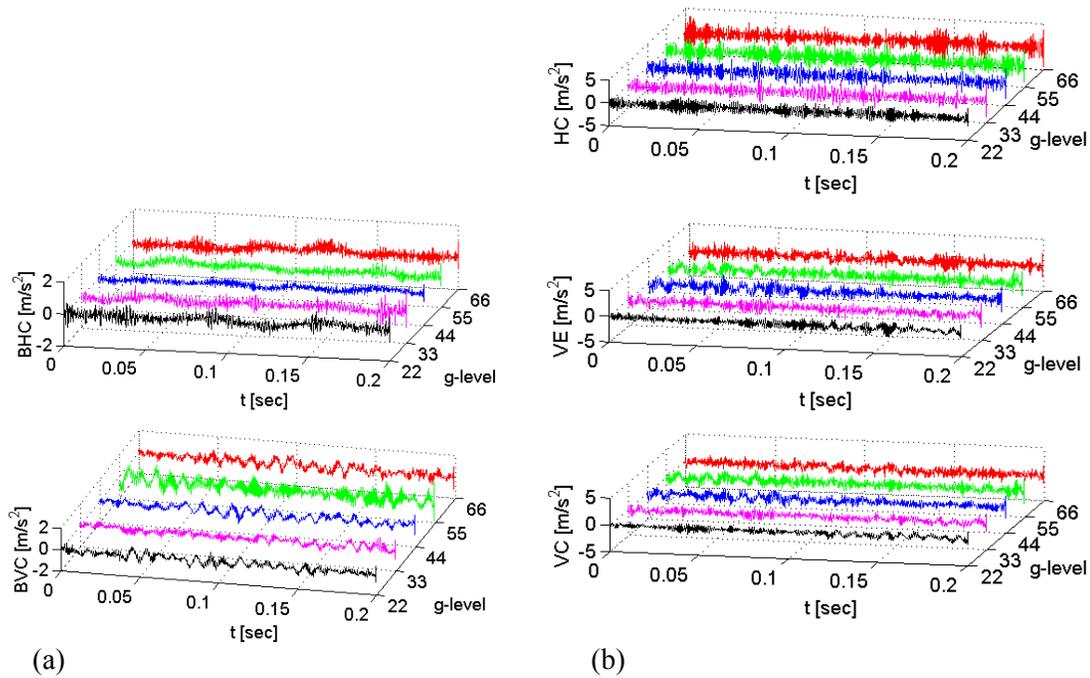


Figure 4. Typical time histories of accelerometer measurements on (a) shake-table and (b) foundation at 22, 33, 44, 55, and 66g seismic tests

Seismic acceleration functions for Seismic response

Representative model-scaled horizontal and vertical seismic accelerances at different g levels for Footing B02 are shown in Figure 5. In contrast to the heavier footings, the horizontal accelerances for the light footing only show two relatively sharp peaks. They correspond to the in-situ free-field soil resonances. One can also observe that there is the fundamental inertia peak of seismic accelerances is expected to exhibit a gradual shift towards higher frequencies and lower magnitudes with decreasing footing size/mass. Compared to the forced vibration test results for Footing B02 in Pak et al. (2012) the inertial resonance peaks of the horizontal and vertical modes are minor in the seismic tests as shown in Figures 6 and 7. The inertia resonance peaks measurements for this footing in the horizontal F_{1H}^{Fdn} and vertical F_{1V}^{Fdn} directions along with the first and second free-field/seismic resonance frequencies in the horizontal and vertical directions, i.e. f_{1H}^{FF} , f_{2H}^{FF} , f_{1V}^{FF} , f_{2V}^{FF} , are summarized in Table 3. The prototype scale frequencies can be obtained by dividing the model scale frequencies presented in Table 3 by the centrifugal acceleration level of the tests (see Ko 1988a and 1988b). As shown in Figures 6(b) and 7(b) and Figures 6(c) and 7(c), the seismic accelerance of the footing for HC and VC modes are very close to the free-field tests. Indeed, owing to the low mass of Footing B02, the seismic and free-field acceleration responses are similar in terms of resonance frequency and magnitude.

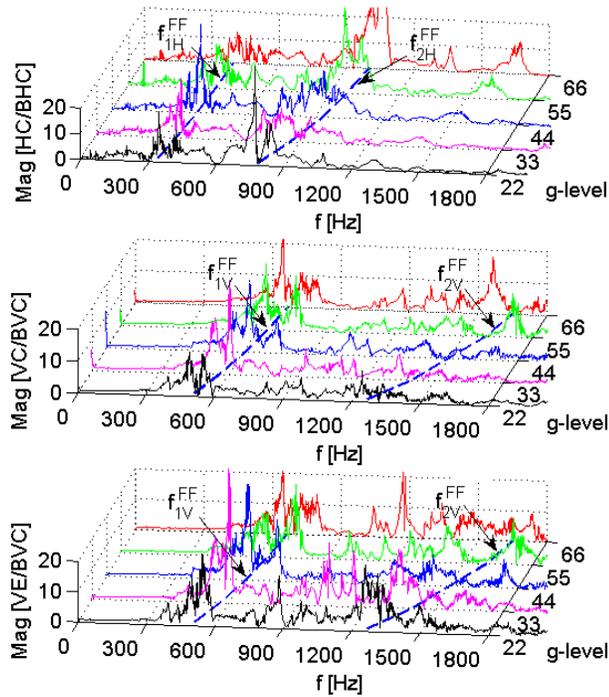


Figure 5. Accelerance measurements [$m/s^2/m/s^2$] in model scale for Footing B02 at 22, 33, 44, 55, and 66g seismic tests.

Table 3: Fundamental model scale inertia and free-field equivalent frequencies in seismic tests of Footing B02

g-level	f_{1H}^{Fdn} Hz	f_{1H}^{FF} Hz	f_{2H}^{FF} Hz	f_{1V}^{Fdn} Hz	f_{1V}^{FF} Hz	f_{2V}^{FF} Hz
22	850	330	770	1200	495	1250
33	910	350	860	1340	560	1400
44	970	365	910	1430	600	1480
55	1020	378	950	1520	650	1620
66	1090	400	1000	1600	670	1700

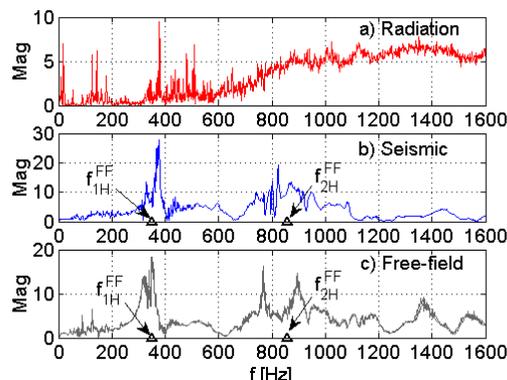


Figure 6. Comparison of horizontal (HC) acceleration measurements for Footing B02 at 33g in radiation, seismic and free-field tests: Red: Radiation (HC/VE [$m/s^2/N$]), Blue: Seismic (HC/BHC [$m/s^2/m/s^2$]), Black: Free-field (HC/BHC [$m/s^2/m/s^2$]).

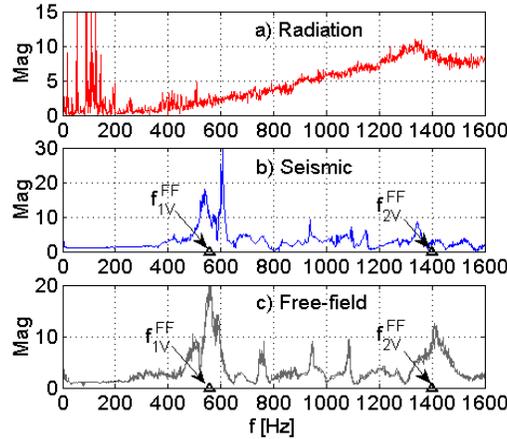


Figure 7. Vertical centric (VC) acceleration measurements for Footing B02 at 33g in radiation, seismic and free-field tests: Red: Radiation (VC/VE [m/s²/N]), Blue: Seismic (VC/BVC [m/s²/m/s²]), Black: Free-field (VC/BVC [m/s²/m/s²]).

ANALYTICAL ASSESSMENTS

Computational Model

To synthesize meaningfully the experimental centrifuge data from seismic tests, the components of the soil-shake table system, which includes the ductseal panels, the aluminum shake-table plate, and the elastomeric shake-table supports, were modeled as continua (see Figure 8) using a 3-D boundary element model (Pak and Guzina 1999). The material properties of the shake-table components can be found in Soudkhah and Pak (2012b). Using this continuum model, the theoretical foundation input motion function per base motion of the shake-table in the horizontal U_{x0}/U_{BHC} , and vertical U_{z0}/U_{BVC} directions are obtained and plotted in Figure 9 as a function of the dimensionless frequency of excitation which is defined as

$$\bar{\omega} = \frac{2\pi fb}{\sqrt{G/\rho}}, \quad (4)$$

where ρ is the density of the soil, f is the frequency of excitation, b is the half width of the footing, and G is the free-field shear modulus of the soil at depth b .

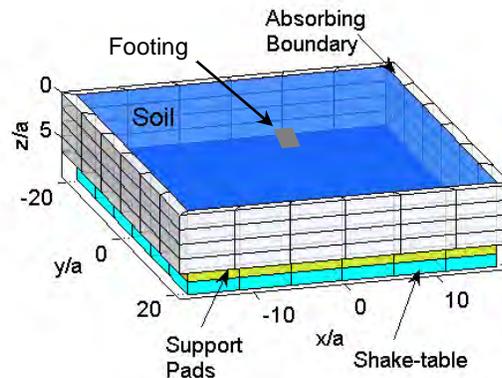


Figure 8. Boundary element meshes used for 3-D finite-domain computational modeling

To highlight the difference between using an infinite- versus a finite-domain continuum model, sets of analytical foundation input motion functions per unit base acceleration of a pure-square root-modulus for both stratum models are plotted in Figure 9. The horizontal and vertical foundation input motions of the infinite-stratum model are generally smooth, except at the frequencies (e.g., $\bar{\omega}_{1s}^{1D} = 0.34$, $\bar{\omega}_{2s}^{1D} = 0.92$, $\bar{\omega}_{1p}^{1D} = 0.59$) which are consistent with the resonance frequencies of 1D analytical solutions for the pure-square root modulus profile. For the finite-domain centrifuge soil block model, the natural frequencies of the horizontal modes can be seen to be higher generally (e.g., $\bar{\omega}_{1s}^{blk} = 0.39$, $\bar{\omega}_{2s}^{blk} = 0.95$) than those for the infinite stratum, with the ratio of $\bar{\omega}_{2s}^{blk}/\bar{\omega}_{1s}^{blk} = 2.43$ which is different from the ratio of 2.67 for the pure-square root soil-modulus infinite-stratum model (see also the 1D analytical solution in Idriss and Seed 1968). The response of U_{zo}/U_{BVC} mode for the block model as shown in Figure 9(b) is close to the infinite-stratum model in terms of resonance frequency ($\bar{\omega}_{1p}^{1D} = 0.59$) but has some additional oscillations beyond the fundamental frequency. In what follows, the two analytical models will be examined in terms of their usefulness in synthesizing the experimental data.

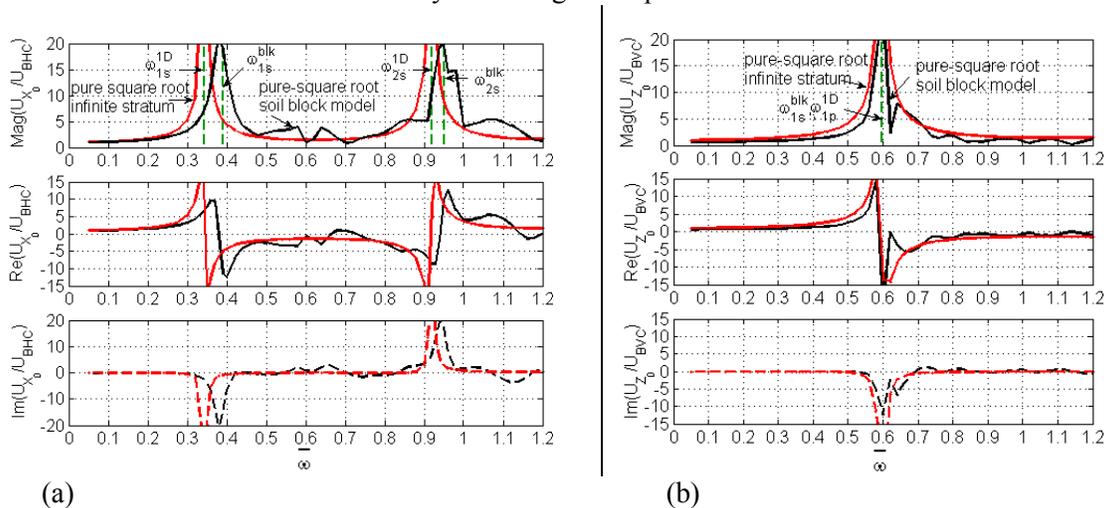


Figure 9. Foundation input motion in (a) horizontal U_{xo}/U_{BHC} and (b) vertical U_{zo}/U_{BVC} directions of soil block model versus pure-square root infinite stratum model.

Results

To evaluate the performance of the analytical models, the seismic acceleration or acceleration-to-acceleration transfer functions of the finite-domain and infinite stratum models are compared to the corresponding experimental measurements of Footing B02. For the case of Footing B02, a typical comparison of the experimental and theoretical horizontal and vertical seismic acceleration functions $A^*(f)$ is shown in Figure 10 for the 33g tests. In this comparison, the theoretical pure-square root soil block model and infinite stratum model were chosen to best match the fundamental resonant frequency observed in the experimental HC/BHC seismic acceleration measurement. Upon matching the fundamental frequencies in the horizontal direction, the soil block model with a shear modulus of 43.8 MPa at depth b can be seen to match well the VC/BVC and VE/BVC acceleration measurements as well. In contrast, the matching of the fundamental HC/BHC resonance frequency by the infinite stratum model requires a shear modulus of 56.5 MPa at depth b which is approximately 30% greater than the shear modulus of the matched soil block model. Most importantly, one can see from the figure that the agreement between the infinite stratum model and the HC/BHC, VC/BVC, and VE/BVC acceleration measurements is all generally poor in regard to the resonance frequencies and other peaks of the accelerances. For the cases examined, the use of the pure-square root infinite stratum model to synthesize the experimental measurements have lead to about a 30% higher dynamic shear modulus of the soil or an

under-prediction of the fundamental resonance frequency of the system by 14%. Accordingly, it is evident that a theoretical continuum model of the shake-table finite domain model can be strongly recommended for a proper interpretation of three-dimensional SSI experiments in a centrifuge setting.

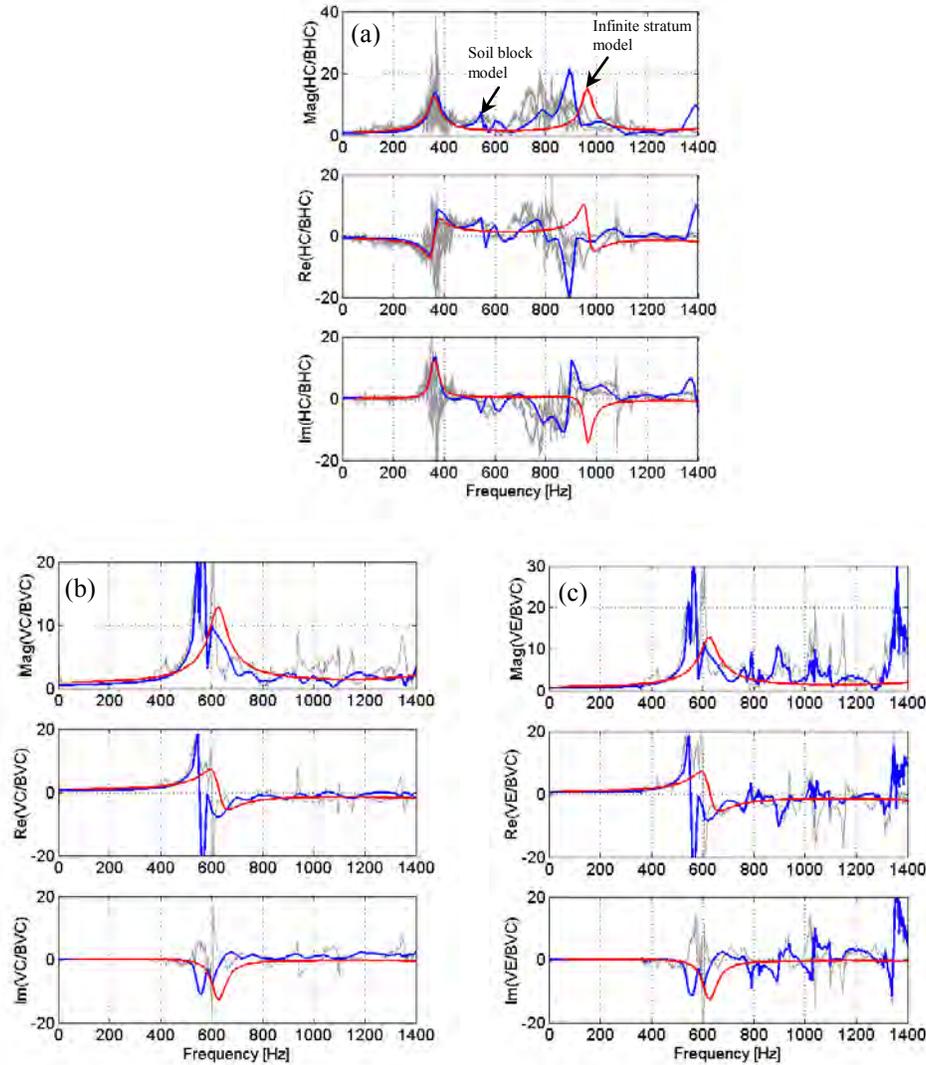


Figure 10. HC 1st resonance-fitted pure-square root soil block model solution to measured (a) HC/BHC, (b) VC/BVC, and (c) VE/BVC seismic acceleration for 33g tests of Footing B02.

CONCLUSION

In this paper, a parallel experimental-analytical investigation of the seismic behavior of a plate surface foundation on sandy soil stratum by means of centrifuge modeling is highlighted. The key findings and conclusions can be summarized as follows:

- To explore fully the underlying characteristics of the seismic soil-foundation interaction problem on a soil stratum, parallel basic studies of the free-field, forced radiation, and seismic excitations can produce significantly more insights.
- Horizontal and vertical resonance frequencies of the soil model generally increase with the centrifugal g-level and the fundamental resonance frequencies of both horizontal and vertical vibrations tend to be sharper and clearer than those of the higher modes.

- Through a comparison of the generated seismic data for a lightweight footing and the free-field measurements, the similarity of the resonance frequencies of the free-field and seismic tests is confirmed.
- Boundary effects in geotechnical centrifuge modeling may prevent direct extrapolation of results to full-scale infinite-domain SSI problems.
- Fundamental three-dimensional mechanics modeling should be used to achieve a proper interpretation of centrifuge simulations of three-dimensional seismic foundation SSI problems even with the physical absorbing boundary method.

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