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TEST RIG DEVELOPMENT FOR SEISMIC QUALIFICATION OF BOTTOM-MOUNTED REACTIVITY CONTROL MECHANISM OF RESEARCH REACTORS

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ABSTRACT

A new research reactor employing bottom-mounted reactivity control mechanism (RCM) is under development and its safety function which is a shutdown of the reactor under earthquake events should be verified through a test. However, the real system is too heavy to be excited artificially, and hence the size of a test rig should be reduced somehow. As a preliminary study for development of a seismic test rig, this paper presents how to reduce the length of an extension shaft which is a main component of RCM and its reasonability will be assessed.

In this paper, a simplified beam model for the shaft was used. Instead of reduction in length of the shaft, the inner/outer radius and water gap size between the shaft and tube were modified in order to match its natural frequency and displacement due to seismic excitation to those of the real system. Furthermore, a proper mass which does not increase the stiffness was inserted into the hollow shaft to match its natural frequency. Then, dynamic equation was derived for the beam model and an optimization problem was defined and solved. The result shows that the design modification is reasonable for description of dynamic characteristics of the real system under earthquake events.

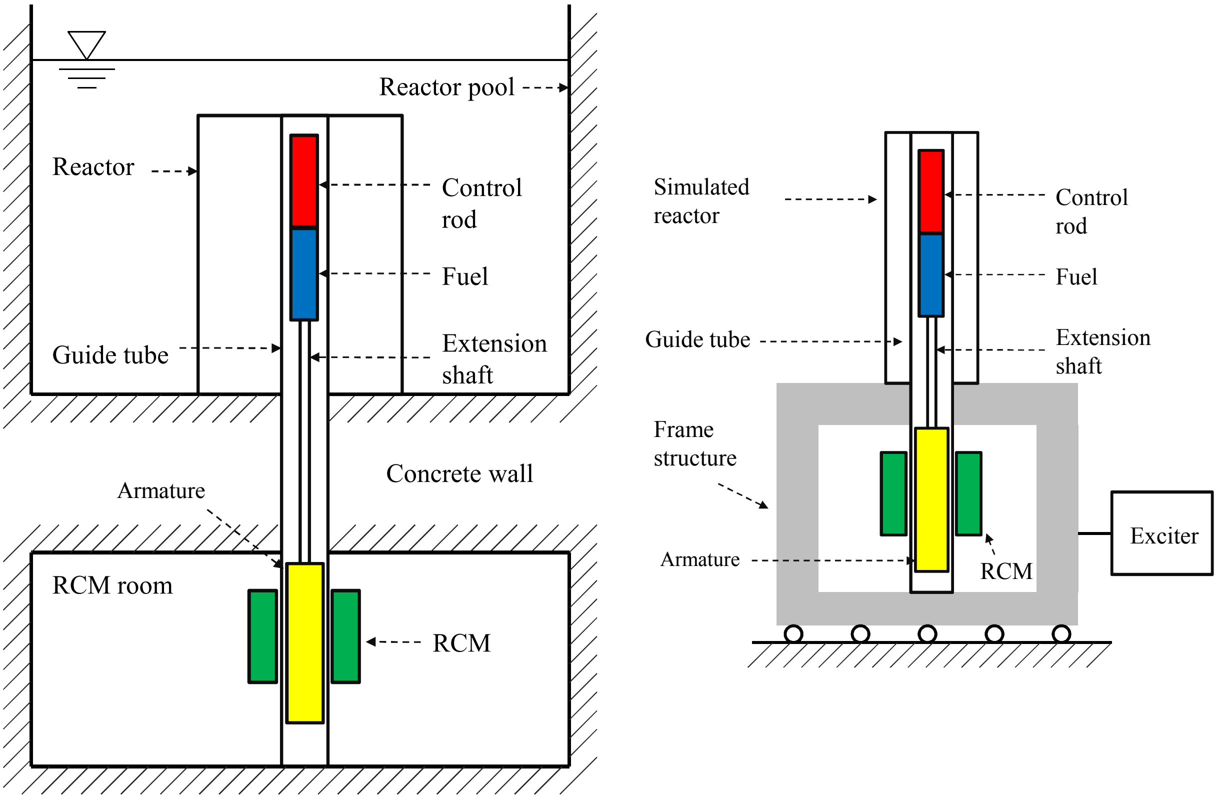
INTRODUCTION

A new research reactor employing bottom-mounted reactivity control mechanism (RCM) is under development. As schematically shown in figure 1(a), the reactor is located at the bottom of the water-filled reactor pool, and its reactivity control rods are driven by the RCM located in the RCM room below the reactor pool. Although the safety function of the RCM during the earthquake events should be verified by test, the whole seismic system encompassing the reactor, RCM and reactor pool is too big and heavy to be excited. Therefore, a seismic test rig with a reduced size needs to be developed.

The functions of RCM are to control the reactivity of reactor and to shut down the reactor safely by gravity drop of control rods. The operability and shutdown function shall be maintained under operating basis earthquake (OBE) and safe shutdown earthquake (SSE). The factors which can affect the drop time are dynamic characteristics of components in the reactor and a collision between the shaft and the guide tube due to seismic excitations. Hence, the test rig should substantially reflect these characteristics.

The proposed seismic test rig is described in Figure 1(b). The main differences between the real system and the test rig are the reduction in size of the concrete wall, length of the extension shaft and size of the reactor. The concrete wall is expected to be very stiff so that it can be treated as a rigid body below 33Hz, and hence, it will be substituted by a frame structure which can be stiff enough to be a rigid body. On the other hand, natural frequencies of the extension shaft are expected to be much higher than the original system, so that its structural modification is essential. The modification of the shaft can be accomplished by various methods, such as insertion of a proper mass inside the hollow shaft or installation of a lumped mass at the middle of the shaft, etc. In this paper, it will be shown how to modify the extension shaft, maintaining dynamic characteristics of the real system, as a preliminary research for development of a seismic test rig. Here, the dynamic characteristics of the shaft can be represented by its natural frequency and a seismic input magnitude which makes a collision between the shaft and the guide tube.

In the beginning, a simplified model for the extension shaft will be presented, followed by a derivation of its dynamic equations. Then, dynamic characteristics of the real shaft will be presented, and an optimization problem for reduction in size of the shaft will be defined and solved. Finally, reasonability of the solution will be assessed.



(a) Real reactivity control mechanism (b) Seismic test rig

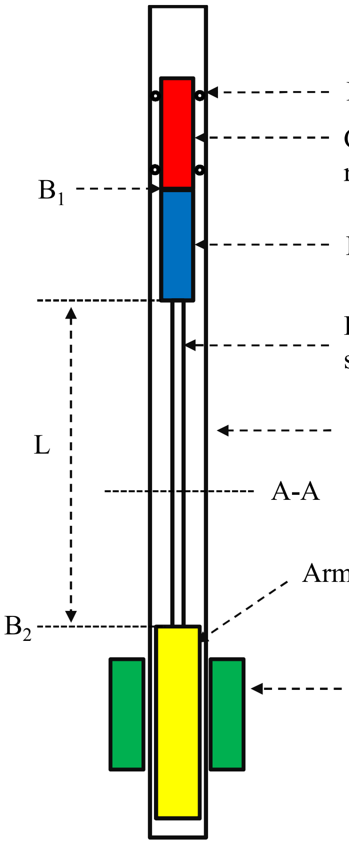
Figure 1. Schematic diagram of reactivity control mechanism and its seismic test rig.

SIMPLIFIED BEAM MODEL FOR REACTIVITY CONTROL MECHANISM

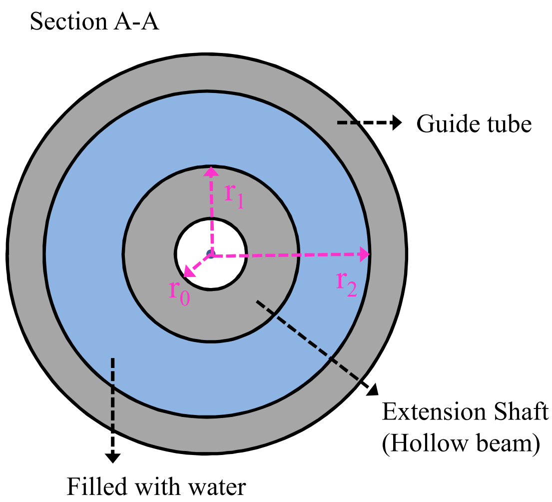
The real RCM is a quite complicated system so that it is quite hard to use it directly for design purpose. Therefore, a simplified model in order to describe dynamic characteristics of the real system will be presented in this section.

Schematic diagram of the original RCM is shown in figure 2(a) and a cross sectional view of extension shaft is also shown in figure 2(b). The RCM consists of a control rod, a fuel, an extension shaft, an armature, a guide tube and an electro-magnet assembly. It should be noted that the scale of figure 2(a) is not same as the real size and the length of the extension shaft (L) is much longer than others. The upper end of the guide tube is connected to reactor, the middle is fixed in the concrete wall, and the lower end is fixed on the bottom of RCM room, therefore, dynamic characteristics of the RCM and reactor should be considered together for a precise analysis. However, as a preliminary research, the reactor is excluded in the model and the guide tube is considered as a rigid wall for simplification.

There are rollers between the control rod and the guide tube, so that the control rod can slide against the guide tube. Furthermore, the gap size between the armature and the guide tube is very small and it can be considered as a sliding condition. Therefore, a beam model with a fixed-fixed boundary condition at point B_1 and B_2 in figure 2(a) would be an appropriate boundary condition for the RCM. However, because the moment of inertia and length of fuel is much larger and shorter than those of the extension shaft, it would be reasonable to assume it as a fixed-fixed beam with length L.



(a) Front view of RCM



(b) Cross sectional view of extension shaft

Figure 2. Schematic diagram of RCM

DYNAMIC EQUATIONS FOR SIMPLIFIED MODEL

As mentioned before, the important feature which should be considered during drop of RCM is a collision between the extension shaft and guide tube. Therefore, a reduced model should have the same first natural frequency and mode shape as the real one because the first mode has the largest displacement and the natural frequency is related to the input magnitude of seismic excitations. In addition, a collision must occur in the reduced model by the input magnitude which causes a collision of the real system. In this section, dynamic equations in order to calculate natural frequencies and displacement by a harmonic excitation will be presented.

A general dynamic equation for a Euler-Bernoulli beam is expressed as equation (1) and its natural frequency (ω_k) is shown as equation (2)[1].

$$\rho \frac{\partial^2}{\partial t^2} w(x,t) + d \frac{\partial}{\partial t} w(x,t) + EI \frac{\partial^4}{\partial x^4} w(x,t) = F(x,t) \tag{1}$$

$$\omega_k = \left(\frac{\mu_k}{L}\right)^2 \sqrt{\frac{EI}{\rho}} \tag{2}$$

where, w is displacement, ρ mass per unit length, d viscous damping coefficient, E Young's modulus, I moment of inertia, F external force, L length of the beam, and μ_1 is 4.73 for the first mode of a beam with a fixed-fixed boundary condition. Equation (1) can be rewritten using modal combinations ($w(x,t) = \sum_{k=1}^{\infty} q_k(t)u_k(x)$) as below:

$$\sum_{k=1}^{\infty} \rho u_k(x)\left\{\ddot{q}_k(t) + 2\zeta_k\omega_k\dot{q}_k(t) + \omega_k^{\ 2}q_k(t)\right\} = D(x)f(t) \tag{3}$$

where, q_k denotes modal coordinates, ω_k natural frequency, ζ_k damping ratio, and $D(x)$ spatial distribution of the external force. u_k is the eigenfunction of k^{th} mode and expressed for a fixed-fixed beam by:

$$u_k(x) = \cosh\beta_k x - \cos\beta_k x - \alpha_k(\sinh\beta_k x - \sin\beta_k x) \tag{4}$$

where, $\beta_k = \mu_k / L$ and $\alpha_k = \left(\cosh\mu_k - \cos\mu_k\right)/\left(\sinh\mu_k - \sin\mu_k\right)$.

As already mentioned before, the displacement of the beam is a main interest and the first mode has the largest displacement, so that the response of the first mode shape should be calculated. Multiplying $u_1(x)$ to equation (3), integrating from 0 to L and using orthogonality of eigenfunctions (u_k) yield:

$$\rho\gamma_1\left\{\ddot{q}_1(t) + 2\zeta\omega_1\dot{q}_1(t) + \omega_1^{\ 2}q_1(t)\right\} = \int_0^L u_1(x)D(x)dx \cdot f(t) \tag{5}$$

where, γ_1 is equal to $\int_0^L u_1 \cdot u_1 dx$. By dividing equation (5) by $\rho\gamma_1$, a dynamic equation in modal coordinate can be obtained as below:

$$\ddot{q}_1(t) + 2\zeta\omega_1\dot{q}_1(t) + \omega_1^{\ 2}q_1(t) = b_1 \cdot f(t) \tag{6}$$

where, b_1 is equal to $\int_0^L u_1 \cdot D(x)dx/(\rho\gamma_1)$. When $f(t)$ is a unit sinusoidal excitation with the first natural frequency (ω_1), the response magnitude in modal coordinate is expressed by:

$$|Q| = \frac{b_1}{2\zeta\omega_1^{\ 2}} \tag{7}$$

In spatial coordinate, the maximum displacement of the extension shaft can be derived as:

$$w_{\max} = w\left(\frac{L}{2}\right) = u\left(\frac{L}{2}\right)\cdot|Q| = u\left(\frac{L}{2}\right)\cdot\frac{b_1}{2\zeta\omega_1^{\ 2}} \tag{8}$$

DYNAMIC CHARACTERISTICS OF REAL EXTENSION SHAFT

The real extension shaft is a hollow beam submerged in water as shown in figure 2(b). In this case, mass per unit length becomes[2]:

$$\rho = \pi\left(r_1^2 - r_0^2\right)\rho_s + \pi r_1^2 \rho_w \left(\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}\right) \tag{9}$$

where, r_0 and r_1 are inner and outer radius of the extension shaft, r_2 inner radius of the guide tube, ρ_s density of the shaft, ρ_w density of water. The second term in left-hand side is an added mass term caused by water. Also, the external force ($D(x)$) due to seismic excitation appears as

$$D(x) = \left\{\pi\left(r_1^2 - r_0^2\right)\rho_s - \pi r_1^2 \rho_w\right\} \cdot \ddot{u}_g \tag{10}$$

where, $\ddot{u}_g$ is a magnitude of ground acceleration. The second term in the brace is a buoyant mass term due to water. Finally, the first natural frequency and the maximum displacement of the shaft can be calculated by substituting equation (9) and (10) into (2) and (8) with parameters shown in table 1. The result shows that the natural frequency is 8.86Hz and maximum displacement is 3.6cm. Here, the gap size between the shaft and the guide tube is 0.85cm as can be seen in table 1 and the maximum displacement is 3.6cm with unit gravitational acceleration (=1*g*) input. Therefore, it can be seen that a collision will occur by seismic excitations with 0.236*g* magnitude at 8.86Hz.

Table 1: Parameters of real extension shaft

Parameter	Value
Inner radius of shaft (r_0) [mm]	10.5
Outer radius of shaft (r_1) [mm]	16.5
Inner radius of guide tube (r_2) [mm]	25.0
Length of shaft (L) [m]	4
Damping ratio (ζ)	0.03
Density of shaft (ρ_s) [kg/m^3]	8100
Density of water (ρ_w) [kg/m^3]	1000
Young's modulus of shaft (E) [GPa]	205
Magnitude of ground acceleration ($\ddot{u}_g$) [m/s^2]	9.806 (=1*g*)

DESIGN OF REDUCED MODEL

As mentioned before, the natural frequency of the modified model should be same as the real one and the modified one also should collide with the guide tube when the real one has a collision. However, the natural frequency of a beam is inversely proportional to L^2 and the maximum response is proportional to L^4, which means that a shorter shaft leads to much higher natural frequency and smaller displacement than the real one. Therefore, inner, outer radius of shaft (r_0, r_1) and inner radius of guide tube (r_2) will be modified, and some material which has a large density without stiffness will be filled into the hollow shaft.

Because of material inside the shaft, mass per unit length and external force (equation (9) and (10)) become as follow:

$$\rho = \pi\left(r_1^2 - r_0^2\right)\rho_s + \pi r_0^2 \rho_0 + \pi r_1^2 \rho_w \left(\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}\right) \tag{11}$$

$$D(x) = \left\{\pi\left(r_1^2 - r_0^2\right)\rho_s + \pi r_0^2 \rho_0 - \pi r_1^2 \rho_w\right\} \cdot \ddot{u}_g \tag{12}$$

where, ρ_0 denotes density of filling. In order to satisfy the same natural frequency and a collision with the same input magnitude, an optimization problem which is a function of r_0, r_1 and r_2 has been formulated as below:

$$\min_{r_0, r_1, r_2} \left[\left\{\omega_{real} - \omega_{reduced}\right\}^2 + S\left\{ \left(\frac{w_{\max}}{r_2 - r_1}\right)_{real} - \left(\frac{w_{\max}}{r_2 - r_1}\right)_{reduced} \right\}^2 \right] \tag{13}$$

where, ω_{real} and $\omega_{reduced}$ denote the first natural frequency of the real and reduced model and S means scaling factor. $w_{\max}$ is the maximum displacement due to seismic input of 1g magnitude at the first natural frequency. The ratio of $w_{\max}$ to (r_2-r_1) means the seismic input magnitude which causes a collision in unit g. And a constraint equation was given by:

$$0 < r_0 < r_1 < r_2 < 10\text{cm} \tag{14}$$

Table 2: Parameters of reduced extension shaft

Parameter	Value
Length of shaft [m]	2.2
Damping ratio	0.03
Density of shaft [kg/m^3]	8100
Density of water [kg/m^3]	1000
Density of filling (mercury) [kg/m^3]	13600
Young's modulus of shaft [GPa]	205
Magnitude of ground acceleration [m/s^2]	9.806 (=1g)

Table 3: Possible design variables for reduced shaft model - solutions for equation (13)

	Real model	Design #1	Design #2
r_0 [mm]	10.5	2.9	8.8
r_1 [mm]	16.5	5.7	10.0
r_2 [mm]	25.0	18.7	23.5
Thickness of shaft (r_1-r_0) [mm]	6.0	2.8	1.2
Gap of water (r_2-r_1) [mm]	8.5	13.0	13.5
Maximum displacement [mm]	36.0	55.3	57.3
First natural frequency [Hz]	8.86	8.86	8.86
Input magnitude causing collision [g=9.806m/s^2]	0.2360	0.2355	0.2360

The optimization problem has been solved using 'fmincon' function in Matlab with parameters in table 2, and two possible solutions were obtained as shown in table 3. Both of two designs show the same natural frequency as one of the real model (8.86Hz) and a collision takes place by the very similar input magnitude compared to the real one (0.236g). However, the thickness of the shaft for solution #2 is 1.2mm which is much thinner than the real one (6mm), so that the shaft can be easily damaged during drop tests with seismic excitations. Therefore, the solution #1 seems to be the best choice.

CONCLUSION

In this paper, a reduced extension shaft model for a seismic test rig has been developed as a preliminary research with many simple assumptions. It shows that the length of the shaft can be reduced from 4m to 2.2m while dynamic characteristics of the shaft are still maintained. However, very simple assumptions were used in this study, such as a fixed-fixed boundary condition for the extension shaft, a rigid wall assumption for the guide tube, and so on. In the real system, the guide tube is not a rigid body and it is connected to the reactor, so that the boundary condition for the shaft is actually not a fixed-fixed condition. Moreover, the coupling effects of the shaft and the guide tube should also be considered in the model. Therefore, a further study needs to be carried out in order to obtain more precise results in the future.

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