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VERIFICATION OF SEISMIC STABILITY MODELING

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ABSTRACT

Seismic stability evaluation of a building structure is necessary to ensure that sliding and/or rocking will not negatively impact the function and serviceability of the building. Proper finite element modeling is essential in obtaining accurate results. Verification of the modeling techniques is done by comparing the results to either: (a) A numerical method using prediction-corrector algorithm to solve the elastic-plastic problem utilizing spreadsheet program or (b) The solution obtained using an alternate finite element program. This paper presents the verification method and test problems.

INTRODUCTION

The prediction of rigid body sliding, rocking or sliding-rocking under input motions is important in the design of nuclear power plants to ensure that no interactions with adjacent structures occur. Different types of contact elements are frequently used to model sliding interface in order to predict sliding and rocking responses. This paper verifies the nonlinear sliding/rocking predictions of one type of contact elements. The same methodology can be used to verify other types of contact elements.

The basic approach of the verification is by comparing the results of the non-linear finite element time-history analysis with the results obtained from solving the equations of motion. Since the equations of motion are non-linear due to the effect of friction and sliding, a numerical approach is used to solve these equations. Closed form solution may be obtained but are in general lengthy and difficult to solve. Comparison of the results of two finite element programs is used in problems that are difficult to solve using hand or simple numerical calculations.

METHODOLOGY

Two basic problems are used to verify the results of the finite element analysis. The first problem consists of a single degree of freedom (SDOF) system, Figure 1(a), which is composed of a mass M connected to the ground by an element that has a compression stiffness Kz and no tension stiffness as well as having a lateral stiffness (also known as sticking stiffness) of Kx that is limited by the sliding at the base. Sliding occurs when the lateral reaction load at the base equals to μFz, where Fz is the normal compression force on the ground and μ is the coefficient of friction. Figure 1(c) shows the element stiffness behavior. Figures 1(a) and 1(b) also represent two ANSYS (2007) finite element modeling techniques (a) using a single CONTA178 and (b) using two CONTA178 elements in a series. Two loading input were applied on each model. The first loading is half a sinusoidal acceleration-time history which is applied on the mass; and the second loading is half sinusoidal displacement-time history which is applied on the base of the structure, see Figure 2. The results are compared with the solution of a single degree of freedom (SDOF) system using numerical integration to solve the non-linear dynamic problem. The basic approach is discussed in AL-Shawaf (2007) and will be summarized here.

The second problem investigates the rocking effect and is shown in the idealization on Figure 7. Again two finite element modeling techniques were utilized (a) using a single CONTA178 and (b) using two CONTA178 elements in a series. The result is compared with the results of SAP2000 (2009).

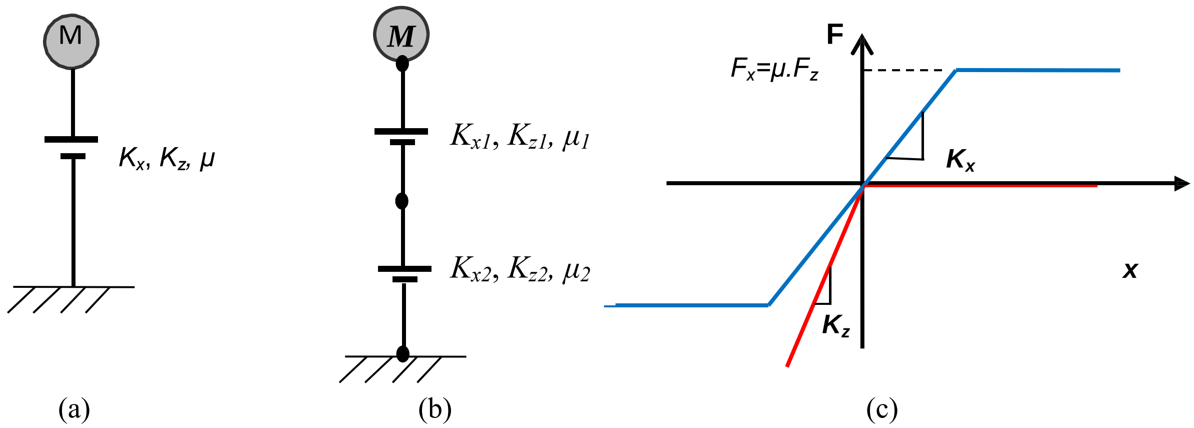


Figure 1. Perfectly Elastic-Plastic Oscillator Non-linear Spring.

(a) Single CONTA178 Element, (b) Double CONTA178 Element, (c) Element Stiffness Behavior

ELASTIC-PLASTIC ANALYSIS OF A SDOF SYSTEM

The equation of motion for an elastic single degree of freedom system can be written as shown below. See, for example, Biggs (1964).

$$F - K_x\, x - Cv = Ma \qquad (1)$$

where:

F = applied force		v = velocity
K_x = lateral stiffness of the system		M = mass
x = lateral displacement		a = acceleration
C = damping		

This equation can be re-written for an elastic-plastic single degree of freedom system by including the restoring force R instead of the term $K_x x$. The term plastic in this contest refers to the sliding behavior once the lateral load reaches μFz, where Fz is the normal compression force on the ground and μ is the coefficient of friction

$$F - R - Cv = Ma \qquad (2)$$

Using the average acceleration method, the velocity and displacement is expressed as:

$$v_t = v_{t-1} + 1/2(a_t + a_{t-1})\Delta t \qquad (3)$$
$$x_t = x_{t-1} + 1/2(v_t + v_{t-1})\Delta t \qquad (4)$$

Substituting Eq. (3) into Eq. (2) and rearranging,

$$a_t = \frac{1}{M + \frac{C\Delta t}{2}}\left[F - R - C\left(v_{t-1} + \frac{a_{t-1}\Delta t}{2}\right)\right] \qquad (5)$$

The term R is non-linear and depends on the displacement x at time t. To solve this problem numerically, a predictor-corrector method is utilized, Dede et al. (1984). In this method, the displacement x at time t is estimated (predicted) and then corrected. A convergence of this procedure can be obtained in a single iteration if the value of Δt is small enough. A recommended value for Δt is < T/10, where T is the period of the structure. The following step-by-step procedure is used:

1. At t = 0, compute a_t ($t = t_0$) from Eq. (2) and the initial conditions
2. Increment time: $t = t + \Delta t$
3. At $t = t + \Delta t$, set $a_t = a_{t-1}$ (t-1 = 0, initially)
4. Compute v_t and x_t from Eqs. (3) and (4)
5. Compute R
6. Compute a_t from Eq. (5)

If this is the predicting pass, return to Step 4. If this is the correcting pass, set $x_{t-1} = x_t$, $v_{t-1} = v_t$, $a_t = a_{t-1}$, and return to Step 2.

APPLICATION

This method was implemented using a spread sheet program. The following notes and rules were used:

1. The initial conditions for the predictor-corrector numerical method are zero velocity and displacement.
2. The damping value corresponds to the structural component under consideration. The critical damping is

   $$Cc = 2\, M\, \omega_n, \text{ where } \omega_n \text{ is the natural frequency} \tag{6}$$

   For damping value corresponding to 5% of critical damping, C = 0.05Cc. In the problems solved in this paper the damping is ignored and entered as a very small value.
3. The stress state (i.e., elastic or plastic) is monitored and updated at each time step. The plastic displacement is accumulated accordingly.
4. The plastic displacement (i.e., the sliding displacement) is equal to the shift in the origin of the stress-strain curve in the subsequent unloading from the plastic state and reloading of the system. Hence, the equation for the resistance, R, is defined as follows:

   If $(x_i - xp_{i-1}) > 0.0$ (loading)

   $$R = \text{Minimum of } [K(x_i - xp_{i-1}) \text{ or } Ru] \tag{7}$$

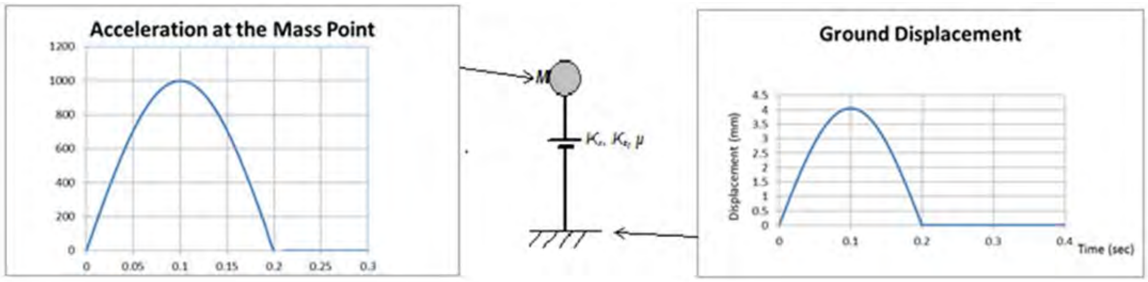
   Else (unloading)

   $$R = \text{Maximum of } [K(x_i - xp_{i-1}) \text{ or } -Ru] \tag{8}$$

   where:

   x_i = the displacement at time step i
   xp_{i-1} = the accumulated plastic displacement (deformation) at time step i-1
   = the shift of the origin of the resistance curve
5. The convergence of the numerical method is checked by monitoring the percent change of the displacement of the corrected pass and the predicted pass as follows:

$$\text{Change} = \frac{x_{correct} - x_{predict}}{x_{correct}} \tag{9}$$



(a) (b)

Figure 2. Load Cases (a) Mass Acceleration-time history, (b) Support Displacement-time history

Sliding of a Perfectly Elastic-Plastic System subjected to Mass Acceleration

The above approach was used to analyze a sliding perfectly elastic-plastic oscillator (PEPO) that is subjected to mass acceleration-time history Fig. 2(a). The applied force-time history F is equal to the mass M multiplied by the acceleration-time history. The input parameters are as follows:

M = 1000 tonne, K_x = 986960 kN/m, $\omega = (K_x/M)^{1/2}$= 31.416 rad/s, f_x= 5Hz, T_x=0.2 s

$$a_x(t) = \begin{cases} a \cdot \sin(\theta \cdot t) & for \quad 0 \le t \le t_d = \pi/\theta; \quad where: a = 1.0 m/s^2 = 1000 mm/s^2 \\ 0 & for \quad t > t_d = \pi/\theta; \quad where: \ \theta = \omega_x/2; \ t_d/T_x = 1.0 \end{cases}$$

μ = 0.5, g = 1.0, F_z = 1000kN, Max Friction Resistance = μF_z = 500kN

Using a Δt = 0.01 sec, the results obtained from the spread sheet are shown on Figure 3.

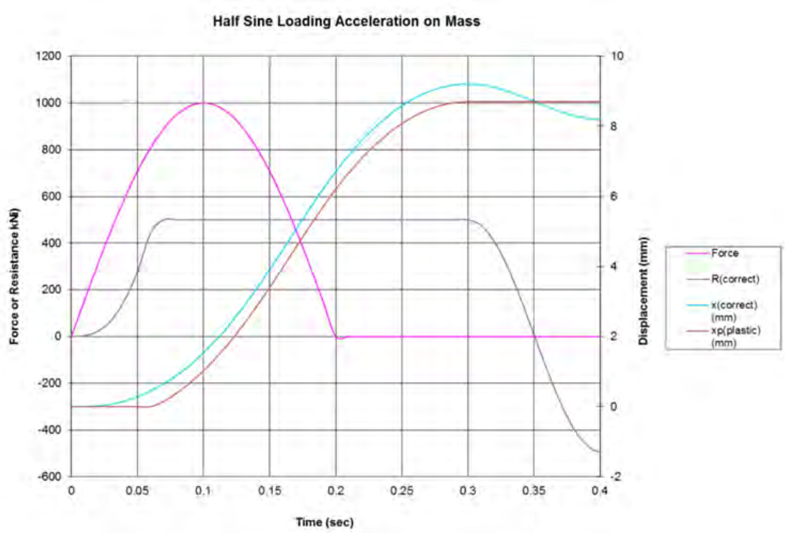


Figure 3. Applied Force, Resistance Force, Displacement and sliding from a SDOF system subjected to half sine acceleration time-history on the Mass.

The results show that the resistance (R) reaches maximum frictional resistance of 500kN at approximately 0.07 sec when sliding (xp) initiates and continues until 0.3 sec. The displacement (x) builds up even after the force goes to zero. As the structure stops from sliding, the system returned to a linear response (i.e., no slipping) with the resisting force decreasing and eventually reversing in sign.

Sliding of a Perfectly Elastic-Plastic System subjected to Ground Displacement

For a linear system subjected to ground acceleration, the equation of motion can be written as shown in Eq. 10, Biggs (1964).

$$-Ma_g - K_x u - Cv = Ma \tag{10}$$

where:

a_g = Ground Acceleration, u = Displacement of the mass with respect to the ground

Similarly for a non-linear system the equation of motion can be written as

$$-Ma_g - R - Cv = Ma \tag{11}$$

The restoring force R is a function of the relative displacement. The same numerical approach that is described earlier is use to solve above equation. The ground displacement time history must be adjusted to remove any sliding that occurs in order to obtain the relative displacement that the structure experience, i.e.

$$xg\text{-}eff_i = xg_i - xp_{i-1} \tag{12}$$

where: xg_i = the ground displacement at the step i
$xg\text{-}eff_i$ = the effective ground displacement experienced by the structure

The effective ground displacement, xg-eff, is differentiated twice to obtain an acceleration time history which is multiplied by the mass and entered in a spread sheet program as the forcing function -ma_g. It should be noted that the calculated input acceleration may show a high numerical value or sudden change at a time points where there is an abrupt change in displacement such as at time t=0 or when sliding occurs. Such anomaly will not affect the overall solution results especially when the time step is very small.

The input parameters are as follows:

M = 1000 tonne, K_x = 986960 kN/m, $\omega = (K_x/M)^{1/2}$= 31.416 rad/s, f_x= 5Hz, T_x=0.2 s

$$x_0(t) = \begin{cases} A \cdot \sin(\theta \cdot t) & for \quad 0 \le t \le t_d = \pi/\theta; \;\; where: A = 1000/\theta^2 = 4.053mm \\ 0 & for \quad t > t_d = \pi/\theta; \;\; where: \;\theta = \omega_x/2; \; t_d/T_x = 1.0 \end{cases}$$

μ = 0.5, g = 1.0, F_z = 1000kN, Max Friction Resistance = μF_z = 500kN

Using a Δt = 0.001 sec, the results obtained from the spread sheet are shown on Figure 4.

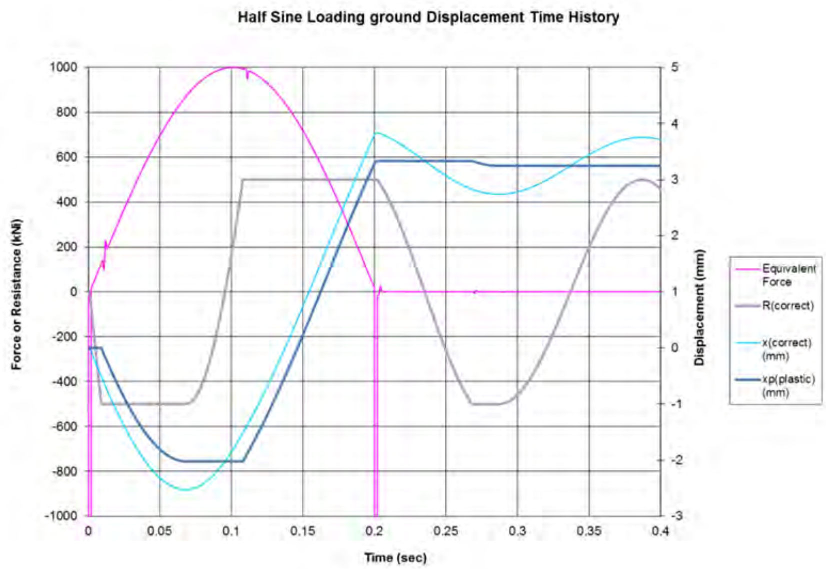


Figure 4. Equivalent applied Force, Resistance Force, Displacement and sliding for a SDOF system subjected to half sine ground displacement time-history.

The results show that the shape of the equivalent applied force on the mass. Note the sudden changes in the force which is a numerical anomaly caused by differentiation of the displacement twice. This occurs at time points such as initiation of movement, sliding or stopping. The resistance curve shows the limit when the maximum friction force is reached and sliding occurs. Sliding (xp) changes from negative value to positive value. The curve represents the relative displacement of the mass with respect to the ground.

The above two problems were analyzed using ANSYS finite element program and employing two modeling techniques namely, (a) Single CONTA178 element Figure 1(a) and (b) Double CONTA178 elements, Figure 1(b).

The Single CONTA178 Element Model input parameters are as follows:

M = 1000 tonne, K_x = 986,960 kN/m, K_z = 3,947,840 kN/m and μ = 0.5, where K_x and K_z are the horizontal and vertical stiffnesses

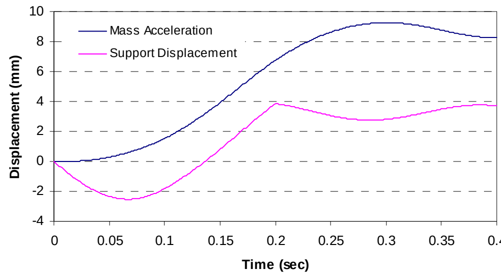
The Double CONTA178 Element Model input parameters are as follows:

M =1000 tonne, K_{x1} = 1.01E8 kN/m, K_{z1} = 4.01E8 kN/m, μ_1 = 0.5 and
K_{x2} = 996700 kN/m, K_{z2} = 3987000 kN/m, $\mu_2 = \infty$

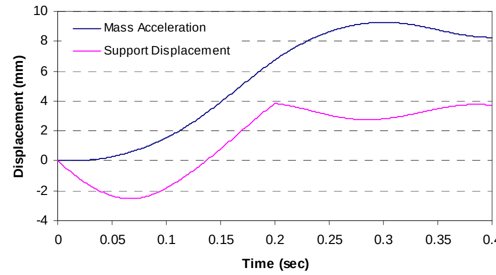
The springs in series results in combined horizontal and vertical stiffnesses of
K_x = 986,960 kN/m, K_z = 3,947,840 kN/m, respectively.

The two loading cases, i.e. the mass acceleration and the ground displacement, were analyzed using Δt = 0.001sec and the default ANSYS solution control parameters including convergence tolerance of 0.5% on forces and 5% on displacements. No weak stiffness is assumed across an open gap. The ANSYS pure penalty method is used for contact solution algorithm

The results are shown on Figures 5 and 6. Comparison of these two figures with Figures 3 and 4 demonstrate that double CONTA178 element model duplicate the results of sliding displacements shown on Figures 3 and 4 whereas the single CONTA178 element model cannot distinguish the sliding displacements from the relative displacements.

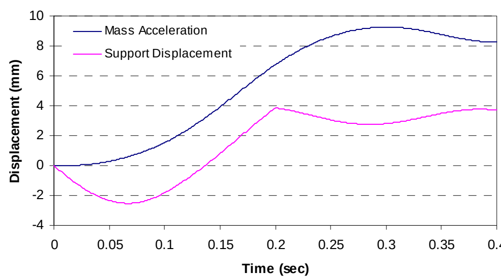


(a)

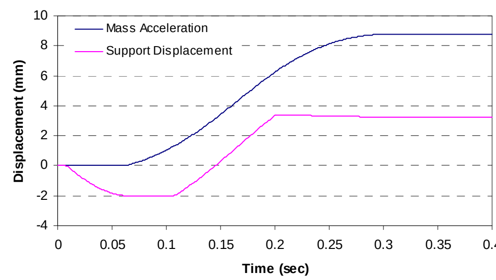


(b)

Figure 5: Relative Displacement Response with (a) Single CONTA178 Element (b) Double CONTA178 Elements



(a)



(b)

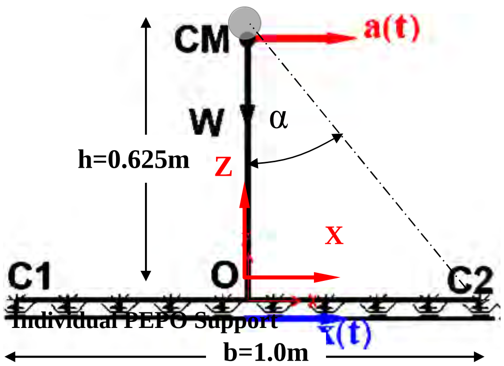
Figure 6: Sliding Displacement Response with (a) Single CONTA178 Elements (b) Double CONTA178 Elements

Sliding with Rocking of a Rocking Perfectly Elastic-Plastic System

A simplified two-degree of freedom system is studied for the combined effect of the sliding and rocking (uplift). The test-verification model for this system as a Rocking Perfectly Elastic-Plastic Oscillator (RPEPO) is shown in Figure 7.

The RPEPO model is equivalent to a rigid body model with the translational mass concentration in single joint. The body is supported by 10 equally distributed single PEPO elements. The PEPO stiffness's are determined to match the model fundamental frequencies of $\omega_X = 10\pi$ and $\omega_Z = 2 \cdot \omega_X = 20\pi$. The fundamental frequencies and frequency of 5-cycle sinusoidal input function are setup to simulate multi-cycle harmonic resonance condition in sliding direction and parametric resonance with the potential energy "pumping effect" in direction of gravity. Here, the RPEPO test model is used to compare the system nonlinear response by combined sliding and rocking behavior due to mass acceleration or support displacement inputs. For verification, the results of ANSYS analysis are compared to SAP-2000 results.

Two ANSYS modeling techniques are used for each individual PPO support namely (a) single CONTA178 and (b) double CONTA178 elements, Figure 1(a) & 1(b) respectively.



RPEPO Test-Verification Model Parameters:

Instability Angle: $\tan(\alpha) = 0.5/0.625 = 0.8 > \mu = 0.5$

Masses: $M_X = M_Z = 1000$ tonne

Single PEPO Stiffnesses: $K_X \approx 2{,}400{,}000$ kN/m; $K_Z = 394{,}784$ kN/m

Fundamental Frequencies: $\omega_X = 10\pi \rightarrow T_X = 0.2$s; $\omega_Z = 20\pi \rightarrow T_Z = 0.1$s

Vertical Load: $W = M \cdot g = 9.81$ MN ($g = 9.81\text{m/s}^2$)

Horizontal Time-History Loads:

- Mass Acceleration Input: $a(t) = a \cdot \sin(\theta \cdot t)$
- Support Displacement Input: $x(t) = A \cdot \sin(\theta \cdot t)$

where: $\theta = \omega_X$; $a = 0.1g = 0.98\text{m/s}^2$ and $A = a/\theta^2 = 0.99$mm

Figure 7: Rocking Perfectly Elastic-Plastic Oscillator (RPEPO)

RPEPO with Single CONTA178 Element input parameters are as follows:

$M = 1000$ tonne, $K_x = 2{,}400{,}000$ kN/m, $K_z = 394{,}784$ kN/m, $\mu = 0.5$

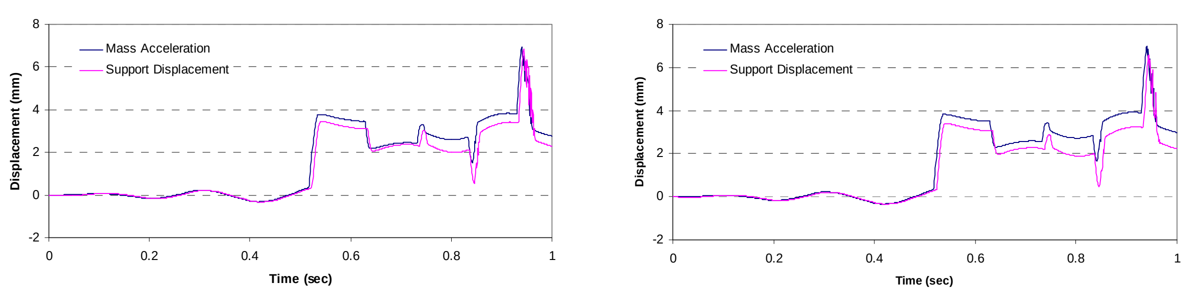
RPEPO with Double CONTA178 Elements input parameters are as follows:

$M = 1000$ tonne, $K_{x1} = 2.45E8$ kN/m, $K_{z1} = 4.00E8$ kN/m, $\mu_1 = 0.5$
$K_{x2} = 2.45E6$ kN/m, $K_{z2} = 0.3986E6$ kN/m, $\mu_2 = \infty$

Two CONTA178 springs in series results in combined horizontal and vertical stiffness for each nonlinear support spring of $K_x = 2400000$ kN/m, and $K_z = 394784$ kN/m respectively.

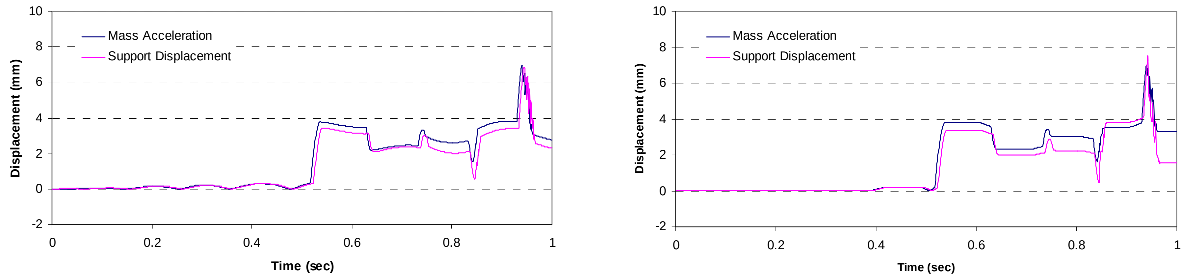
The two loading cases, i.e. the mass acceleration input and the support displacement input were analyzed using $\Delta t = 0.001$ sec, and the default ANSYS solution control parameters including convergence tolerance of .5% on forces and 5% on displacements. No weak stiffness is assumed across an open gap. The ANSYS pure penalty method is used for contact solution algorithm

From ANSYS solutions of single and double CONTA178 element models, Figure 8 and Figure 9 plots are generated showing the relative and sliding displacements of the mass, respectively. Figures 10 and 11, were also generated showing the center mass horizontal and vertical motions, respectively.



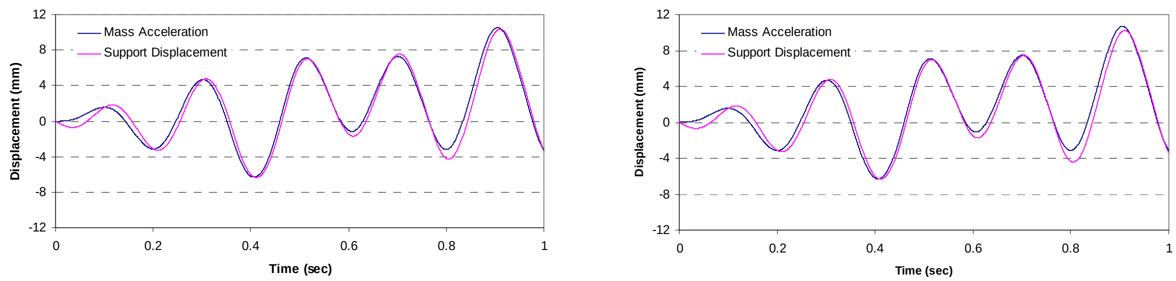
(a) (b)

Figure 8: Relative Displacements with RPEPO for point O
(a) Single CONTA178 Elements (b) Double CONTA178 Elements



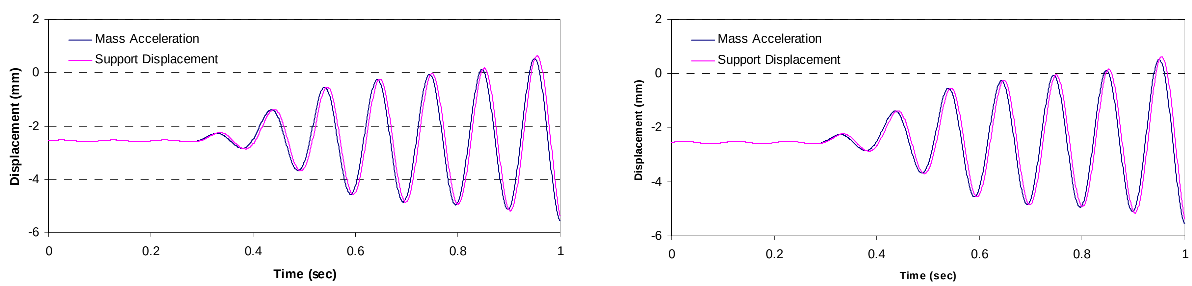
(a) (b)

Figure 9: Sliding Displacements with RPEPO for point O
(a) Single CONTA178 Elements (b) Double CONTA178 Elements



(a) (b)

Figure 10: Center Mass Horizontal Motion
(a) Single CONTA178 Elements (b) Double CONTA178 Elements



(a) (b)

Figure 11: Center Mass Vertical Motion
(a) Single CONTA178 Elements (b) Double CONTA178 Elements

The results shown on Figures 8 through 11 for both single and double element model duplicate the results obtained using SAP-2000 (2009) program.

CONCLUSION

Test problems were solved using two modeling techniques. Results were verified against solutions obtained by numerical methods or by an alternate finite element program.

The results of ANSYS analysis using CONTA178 finite element show that in some cases the program does not distinguish between the relative and sliding displacement if a single element modeling technique is used between the ground and the structure. Nevertheless, the results show higher displacement and thus sliding results are conservative.

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