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MODAL APPROACH TO THE PLATFORM SSI SEISMIC ANALYSIS

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ABSTRACT

The main idea of modal approach is that after coming to the "generalized coordinates" one gets a set of decoupled 1D equations of motion. Such a decoupling is easily achieved in inertial and stiffness terms of the equation of motion due to the special properties of natural modes; problems arise with damping term. Homogeneous systems (made of one and the same material) usually enable getting damping matrix in the generalized coordinates with comparatively small off-diagonal elements. These elements are simply neglected in modal approach providing desirable decoupled equations of motion. However, conventional soil-structure platform model (i.e. the model of the soil-structure system used in impedance approach to SSI) has heterogeneous damping: one in the "soil support", another one in the structure. This heterogeneous damping usually ends up in the considerable values of the off-diagonal elements of the "generalized damping matrix". The attempts to neglect these elements lead to inappropriate results of modal analysis.

The author illustrates this effect using a very simple 1D model and proposes to address the difficulties using combined asymptotic method (CAM). In CAM a new platform model with homogeneous damping is analyzed (thus enabling modal approach), and the difference between "true" damping and "platform" damping is accounted for in the modified platform excitation.

INTRODUCTION

Modal approach is widely used in seismic analysis (see ASCE4-98). It often helps to save computational resources. Moreover, it is a base for spectral seismic analysis. The main idea of this approach is that after coming to the so-called "generalized coordinates" one gets a set of decoupled 1D equations of motion. Such a decoupling is easily achieved in inertial and stiffness terms of the equation of motion due to the special properties of natural modes; problems arise with damping term. Homogeneous systems (made of one and the same material) usually enable getting "generalized damping matrix" (i.e. damping matrix in the generalized coordinates) with comparatively small off-diagonal elements. These elements are simply neglected in the modal approach providing desirable decoupled equations of motion. However, conventional soil-structure platform model (i.e. the model of the soil-structure system used in impedance approach to SSI according to ASCE4-98) has heterogeneous damping: one in the "soil support", another one in the structure. This heterogeneous damping usually ends up in the considerable values of the off-diagonal elements of the "generalized damping matrix". The attempts to neglect these elements lead to inappropriate results of modal analysis.

Forty years ago some specialists (see German "KTA" codes) believed that one can cut down the diagonal elements of the "generalized damping matrix" (say, to 20% of dimensionless damping) and thus compensate the missing off-diagonal elements in modal approach. In ASCE4-98 they have changed the mind: if diagonal elements of the "generalized damping matrix" exceed some "limit value" (corresponding to the dimensionless modal damping coefficient 0.2) modal approach is just inappropriate.

The recently developed combined asymptotic method (CAM) (see Tyapin (2009); Tyapin (2010)) enables a new insight into these problems. They are discussed using the simplest SSI model allowed by ASCE4-98 – structural model with rigid base mat supported by "soil" springs and viscous dashpots (hereinafter called "initial" ones) resting on the rigid platform excited by accelerogram. The "exact" solution for material damping in the structure and viscous damping in the soil support is obtained in the frequency domain without modal superposition. The proposed modal solutions are compared to the “exact” solution in terms of the transfer functions.

SAMPLE SSI MODEL AND EXACT SOLUTIONS IN THE FREQUENCY DOMAIN

Let us use for the illustrative purposes a very simple sample 1D model with 3 DOFs shown (without damping) in Fig.1.

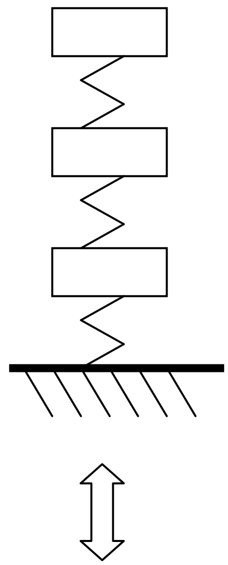

Figure 1. Sample 1D model shown without damping

Parameters of the model are given in the Table 1 together with natural frequencies and natural modes of the fixed-base system.

Table 1: Parameters of the sample model and natural frequencies/modes of the fixed-base SSI system

Number from below	Mass, tones	Stiffness of springs, kN/m	Natural frequencies, Hz	Modal displacement of the lower mass, $t^{-1/2}$	Modal displacement of the middle mass, $t^{-1/2}$	Modal displacement of the upper mass, $t^{-1/2}$
1	1	1.3E3	3.0025	0.4703	0.6069	0.6407
2	1	3.25E3	11.297	0.8535	-0.1281	-0.5051
3	1	6.75E3	20.073	-0.2245	0.7844	-0.5782

Natural frequencies in Table 1 are typical for SSI models of the NPP reactor buildings in vertical direction. Let the lower spring in Fig.1 represent “soil stiffness” and let other two springs together with three masses represent structure. Then we can use formula from ASCE4-98 linking coefficient of viscosity c_{soil} for soil dashpot to the stiffness coefficient k_{soil} of the soil spring

$$c_{soil} / k_{soil} = 0{,}85 r / V_s \tag{1}$$

Here r is equivalent radius of the base mat, V_s is shear wave velocity in the soil. For further calculations r=40 m, V_s=400 m/s, so soil viscosity coefficient in kN/(m/s) is

$$c_{soil} = k_{soil} \times (0{,}85r / V_s) = 1{,}3 \times 10^3 \times 0{,}85 \times 40 / 400 = 110{,}5 \qquad (2)$$

Viscosity coefficient c_z should be directly added to the term (1,1) of damping matrix [C] in the initial coordinates.

Let us consider the equation of motion in the frequency domain:

$$-\omega^2 [M]\{U\} + \{[K_{str}] + [K_{soil}]\}\{U\} = [K_{soil}]\{U_g\} \qquad (3)$$

Here ω is a circular frequency; [M] is a mass matrix (real matrix n x n); [K_{str}] is a stiffness matrix for structure (complex matrix n x n); [K_{soil}] is a stiffness matrix for soil spring and dashpot (complex matrix n x n with a single non-zero element (1,1) in our sample case); {U_g} is a platform displacement in the frequency domain (complex column matrix n x 1 with a single non-zero element (1,1) in our sample case); {U} is a structural response displacement (complex column matrix n x 1). Imaginary part of the only non-zero element of complex matrix [K_{soil}] is "viscous", i.e. proportional to ω with coefficient c_{soil}.

The best damping model for structure (though not appropriate in the time domain) is material damping. Let us consider it in the form

$$[\mathrm{Im}\,K_{str}] = 2\lambda\,[\mathrm{Re}\,K_{str}] \qquad (4)$$

Let us use λ=0.05 in our sample. Equation 3 leads to

$$\{U\} = [TF]\{U_g\};\,[TF] = \{-\omega^2 [M] + [K_{str}] + [K_{soil}]\}^{-1} [K_{soil}] \qquad (5)$$

Now we have all the information necessary to obtain [TF] – in the first column we have the transfer functions from platform to all three masses (from displacements to displacements; the same from accelerations to accelerations). The results will be shown below.

The alternative form of the Equation 3 may be written down using relative displacements {X}, linked to the absolute displacements {U} by

$$\{U\} = \{X\} + \{U_b\}\,u_g \qquad (6)$$

Here {U_b} is a real column matrix of "rigid" nodal displacements together with unit displacement of platform in the selected direction of excitation; u_g is scalar complex platform displacement in this direction (the only non-zero element in {U_g}). The important note is that structural stiffness does not "work" for rigid displacements; i.e. [K_{str}]{U_b}=0. With this regard we can substitute Equation 6 into Equation 3 and get

$$-\omega^2 [M]\{X\} + \{[K_{str}] + [K_{soil}]\}\{X\} = \omega^2 [M]\{U_b\}u_g + [K_{soil}]\{U_g\} - [K_{soil}]\{U_b\}\,u_g \qquad (7)$$

The last two terms in the right-hand part of Equation 7 give zero, and we get the Fourier image of the well-known equation of motion (e.g., see ASCE4-98):

$$-\omega^2 [M]\{X\} + \{[K_{str}] + [K_{soil}]\}\{X\} = \omega^2 [M]\{U_b\}u_g \qquad (8)$$

Now let us apply conventional modal approach. Modal superposition is described by

$$\{X\} = [\Phi]\{Y\} \tag{9}$$

Here [Φ] is a mode matrix of size (n x m); {*Y*} is a vector of relative displacement in generalized coordinates (m x 1); m is number of modes considered.

Natural modes [Φ] have two valuable special properties. First of all, different modes (i.e. columns of [Φ]) are "orthogonal by masses", enabling convenient scaling to unit matrix [*E*] (the so-called "normalization by masses")

$$[\Phi]^T [M]\,[\Phi] = [E] \tag{10}$$

Besides, different modes are also "orthogonal by stiffness"; as normalization described by Equation 10 has been already performed, the resulting matrix in the right-hand part of the analogue of Equation 10 will be non-unit, but carrying natural frequencies ω_j^2 (j=1,…,m) instead:

$$[\Phi]^T [\mathrm{Re}\, K]\,[\Phi] = diag[\omega_j^2] \tag{11}$$

If damping in the system is homogeneous, i.e. Equation 4 is valid not only for $[K_{str}]$, but for the total $[K]=[K_{str}]+[K_{soil}]$, then we come to the complex equation

$$[\Phi]^T [K]\,[\Phi] = diag[\omega_j^2 (1+2i\lambda)] \tag{12}$$

Combining Equations 7…12, and multiplying Equation 8 by $[\Phi]^T$ from the left, one comes to the 1D equations of motion in generalized coordinates and in the frequency domain (j=1,…,m):

$$-\omega^2\, Y_j + 2i\lambda\,\omega_j^2\, Y_j + \omega_j^2\, Y_j = \omega^2\, \Gamma_j\, u_g \tag{13}$$

Here Γ_j is modal participation factor j, equal to

$$\Gamma_j = \frac{\{\Phi_j\}^T [M]\{U_b\}}{\{\Phi_j\}^T [M]\{\Phi_j\}} = \{\Phi_j\}^T [M]\,\{U_b\} \tag{14}$$

This is a "material" variant of modal approach applicable to the systems with homogeneous damping. The main inconvenience of this variant is that 1D equations of motion must be solved in the frequency domain. However, "material" modal approach may be a base for "material" spectral method. The only difference with conventional spectral method is that spectral values are defined via maximal absolute response of 1D "material" SDOF oscillators, and not of SDOF "viscous" ones.

RAYLEIGH DAMPING: DIFFERENT OPTIONS

In the time domain "material" damping is not applicable, and instead of Equation 8 we have

$$[M]\{\ddot{X}\} + \{[C_{str}] + [C_{soil}]\}\{\dot{X}\} + \{[K_{str}] + [K_{soil}]\}\{X\} = -[M]\{U_b\}\ddot{u}_g \tag{15}$$

Here all matrices are real; in our sample the only non-zero elements in $[K_{soil}]$ and $[C_{soil}]$ are k_{soil} and c_{soil} respectively. Rayleigh damping is based on the special form of the matrix $[C_{str}]$:

$$[C_{str}] = \alpha\,[M] + \beta\,[K_{str}] \tag{16}$$

Let us set up two boundary frequencies for our sample structure f_b=3 Hz and f_e =20 Hz. Let us take "target" value of damping λ=0.05 mentioned above. Then Rayleigh coefficients are calculated as

$$\alpha = 4\pi\,\lambda\,\frac{f_b\ f_e}{f_b + f_e} = 1{,}639\ s^{-1}$$
$$\beta = \lambda\,\frac{1}{\pi\,(f_b + f_e)} = 6{,}91978 \times 10^{-4}\,s \tag{17}$$

Structural part $[C_{str}]$ of damping matrix $[C]$ in (t s^{-1})=kN/(m/s) is given by Equation 16:

$$[C_{str}] = 1{,}639 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 6{,}91978 \times 10^{-4} \times \begin{bmatrix} 3{,}25\times10^3 & -3{,}25\times10^3 & 0 \\ -3{,}25\times10^3 & 10{,}0\times10^3 & -6{,}75\times10^3 \\ 0 & -6{,}75\times10^3 & 6{,}75\times10^3 \end{bmatrix} =$$
$$= \begin{bmatrix} 3{,}8879 & -2{,}2489 & 0 \\ -2{,}2489 & 8{,}5588 & -4{,}6709 \\ 0 & -4{,}6709 & 6{,}3099 \end{bmatrix} \tag{18}$$

After the addition of the soil dashpot viscosity coefficient c_{soil} given by Equation 2 we get full damping matrix

$$[C] = \begin{bmatrix} 114{,}3879 & -2{,}2489 & 0 \\ -2{,}2489 & 8{,}5588 & -4{,}6709 \\ 0 & -4{,}6709 & 6{,}3099 \end{bmatrix} \tag{19}$$

The main difference with previous "material" case is that Equation 8 in relative displacements was equivalent to Equation 3 in absolute displacements; however, Equation 15 with Rayleigh damping is not equal to the time-domain analogue of Equation 3, because of the first term in the right-hand part of Equation 16. This part of structural damping (unlike the second part) "works" on rigid structural displacements, so it makes difference, what is the basis for the relative motion. Here we have three different options. "Option A" is to make structural damping work on absolute velocities, not relative ones. Then analogue of Equation 3 in the frequency domain will be

$$\{-\omega^2[M] + [K_{str}] + i\omega[C_{str}] + [K_{soil}] + i\omega[C_{soil}]\}\{U\} = \{[K_{soil}] + i\omega[C_{soil}]\}\{U_g\} \tag{20}$$

"Option B" is in line with ASCE4-98 and Equation 15: Rayleigh damping works on relative velocities calculated as compared to the platform motion. Then one has instead of Equation 20 another equation:

$$\{-\omega^2[M] + [K_{str}] + i\omega[C_{str}] + [K_{soil}] + i\omega[C_{soil}]\}\{U\} = \{[K_{soil}] + i\omega[C_{soil}]\}\{U_g\} + i\omega[C_{str}]\{U_b\}u_g \tag{21}$$

"Option C" means that Rayleigh damping for structure works on relative velocities calculated as compared to the rigid basement's motion. In this case we have

$$\{-\omega^2[M]+[K_{str}]+i\omega[C_{str}]+[K_{soil}]+i\omega[C_{soil}]\}\{U\}-i\omega[C_{str}]\{U_b\}u_b=\{[K_{soil}]+i\omega[C_{soil}]\}\{U_g\} \quad (22)$$

Here u_b is basement's displacement in the frequency domain participating also in $\{U\}$, so Equation 22 means that effective damping matrix becomes non-symmetric.

The comparison of the transfer functions in all three Rayleigh options to the "material" ones is given further.

CONVENTIONAL MODAL APPROACH FOR SSI MODEL

Let us transfer our sample damping matrix given by Equation 19 to the generalized coordinates using matrix of modes taken from Table 1:

$$[\Phi]=\begin{bmatrix}0{,}4703 & 0{,}8535 & -0{,}2245\\ 0{,}6069 & -0{,}1281 & 0{,}7844\\ 0{,}6407 & -0{,}5051 & -0{,}5782\end{bmatrix} \quad (23)$$

In generalized coordinates we get

$$[\Phi]^T\,[C]\,[\Phi]=\begin{bmatrix}26{,}127 & 43{,}994 & -11{,}572\\ 43{,}994 & 84{,}965 & -21{,}001\\ -11{,}572 & -21{,}001 & 18{,}170\end{bmatrix} \quad (24)$$

Conventional modal approach requires neglecting the off-diagonal terms of this matrix.

Using conventional "modal full" damping one comes to the following formulae for the transfer functions from platform to masses:

$$\{TF\}=[1 \quad 1 \quad 1]^T+[\Phi]\,\{Y\}, \quad Y_j=\Gamma_j\omega^2[-\omega^2+i\omega\,c_j+\omega_j^2]^{-1} \quad (25)$$

Here modal damping parameters c_j for our sample system are taken from the diagonal of matrix given by Equation 24. They correspond to the following dimensionless parameters $\lambda_j=c_j/(2\omega_j)$: λ_1=0,6925; λ_2=0,5985; λ_3=0,0720. We see that the first two coefficients are far greater than "material" values of λ, usually making 4%...7%. This is typical for soil-structure interaction, where "wave" damping dominates over "material" one.

One more option is called "cut-down modal damping". It is similar to the previous option, but in Equation 25 values of c_j (j=1,2) are artificially cut down to $0.4\omega_j$ (corresponding to $\lambda_1=\lambda_2=0{,}2$); c_3 and λ_3 stay the same as before. Some specialists still believe that such cutting down can provide necessary conservatism.

Now let us look at the results in terms of absolute values of the transfer functions from platform to structural masses. "Material structure plus viscous soil" damping results are compared to three options of "Rayleigh structure plus viscous soil" results, to the "full modal damping" results and to the "cut-down modal damping" results. The results for the lower mass are shown in Fig.2, the results for the middle mass – in Fig.3, the results for the upper mass – in Fig.4. There is one more curve in each Figure, which will be explained below.

The "Rayleigh damping" results in all three options are close to the "material damping" results. Some difference can be seen, but it is small. The reason is that soil damping is represented in full in all four variants ("material" and three options of "Rayleigh" ones), and it is more important in our case than structural damping. In "Rayleigh damping C" the results around the first peak are more conservative, though for the upper frequencies it is not always true.

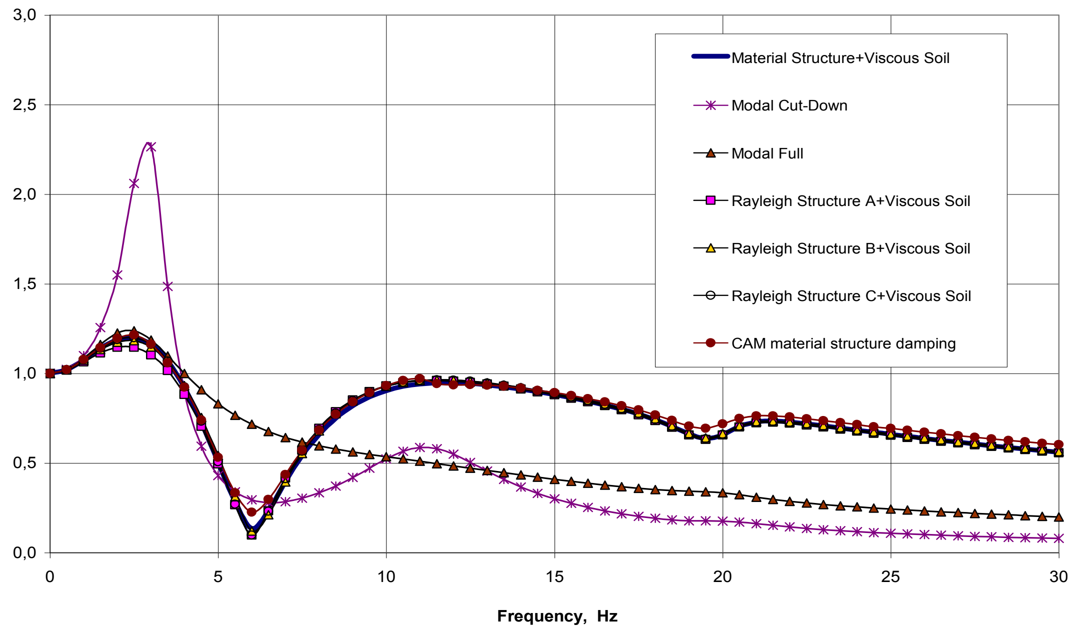

Figure 2. Absolute values of the transfer functions from the platform to the lower mass

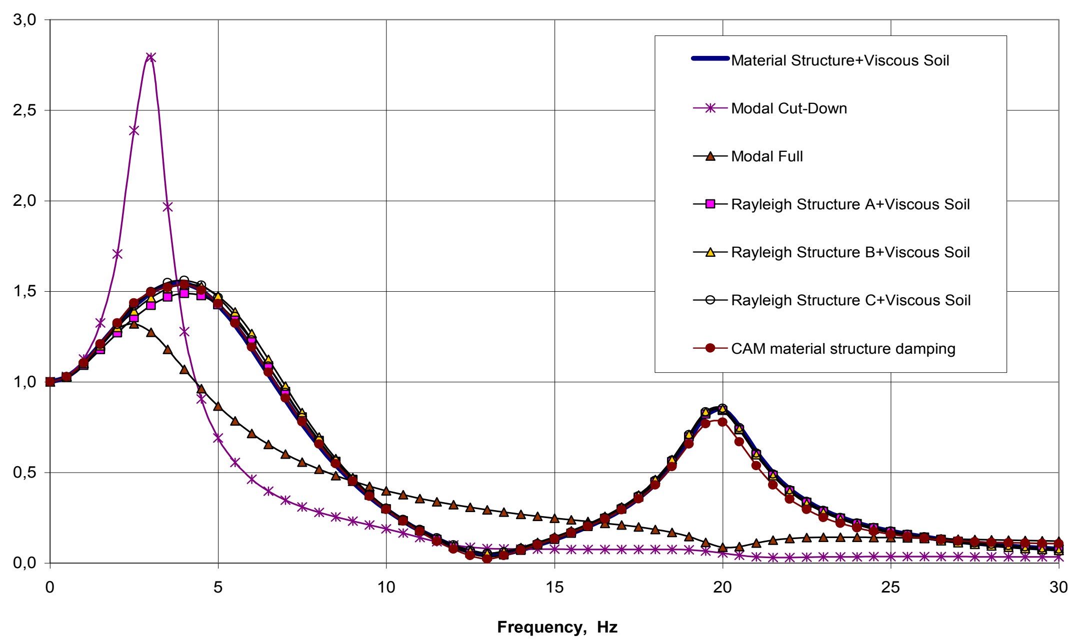

Figure 3. Absolute values of the transfer functions from the platform to the middle mass

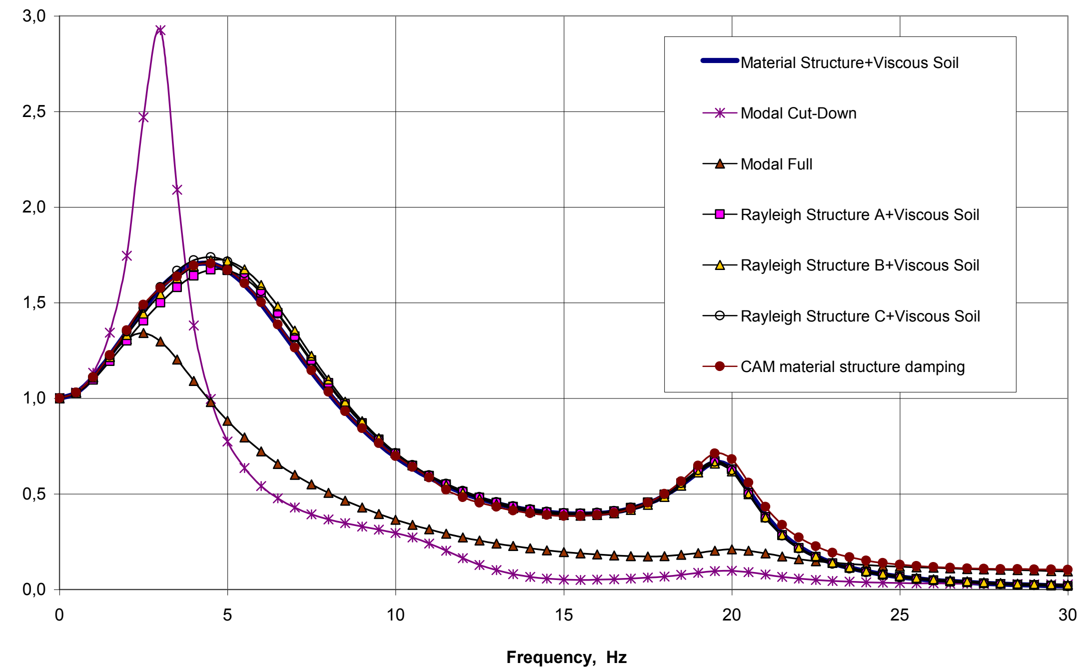

Figure 4. Absolute values of the transfer functions from the platform to the upper mass

The "full modal damping" results are satisfactory only for low frequencies. For upper frequencies they are considerably non-conservative. The "cut-down modal damping" results are excessively conservative around the first peak, but non-conservative for the upper frequencies. Note that the "cut-down" results in some frequency intervals are less than the "full" results. This is due to the phase combination.

COMBINED ASYMPTOTIC METHOD

Combined asymptotic method (CAM) proposed in Tyapin (2009); Tyapin (2010) provides powerful tool to address the described problems. The first option of CAM is to obtain the motion of the rigid basement in the frequency domain. Upper structure is "condensed" to the complex frequency-dependent matrix 6 x 6 (called "dynamic inertia matrix") linking response forces acting on the weightless rigid basement from the upper structure to the accelerations of this basement. In case of material damping in the upper structure "dynamic inertia" is as follows:

$$[M(\omega)] = [M_0] + \sum_{j=1}^{n} \frac{\omega^2}{\Omega_j^2 - \omega^2 + 2i\lambda_j\Omega_j^2} \{S_j\}^T \{S_j\} \tag{26}$$

Here $[M_0]$ is the conventional real inertia matrix 6 x 6; i is an imaginary unit; Ω_j is natural frequency of j-th mode for the fixed-base upper structure (different from ω_j in Equation 4!); $\{S_j\}$ is a line 1 x 6 of the participation factors for the j–th mode (the inertial normalization of the natural modes is assumed); λ_j is the j-th modal damping coefficient calculated in the FEM codes along with natural frequencies/modes of the fixed-base structural model. We see that in static case (i.e. for zero frequency) "dynamic inertia" is similar to the conventional "static" inertia even for flexible structure.

In our sample case the fixed-base upper structure has modal parameters given in the Table 2.

Table 2: Natural frequencies/modes of the fixed-base upper structure

Number	Natural frequencies, Hz	Modal displacement of the lower mass, $t^{-1/2}$	Modal displacement of the middle mass, $t^{-1/2}$	Modal displacement of the upper mass, $t^{-1/2}$	Participation factor, $t^{1/2}$
1	6.023	0.0	-0.6189	-0.7855	-1.4044
2	19.700	0.0	-0.7855	0.6189	-0.1666

Transfer function from the platform to the basement (the lower mass in Fig.1) will be given by scalar expression

$$TF = [k_{soil} + i\omega\, c_{soil} - \omega^2 M(\omega)]^{-1}[k_{soil} + i\omega\, c_{soil}] \quad (27)$$

It is the same as the transfer function with “material structures plus viscous soil” in Fig.2.

Changing circular frequencies ω in Equation 26 for ordinary frequencies f in Hz, we get for our sample structure

$$M(f) = 3.0 + \frac{f^2 * 1.4044^2}{6.023^2 - f^2 + 2*0.05*6.023^2 i} + \frac{f^2 * 0.1666^2}{19.7^2 - f^2 + 2*0.05*19.7^2 i} \quad (28)$$

Real and imaginary parts of complex dynamic inertia are shown in Fig.5.

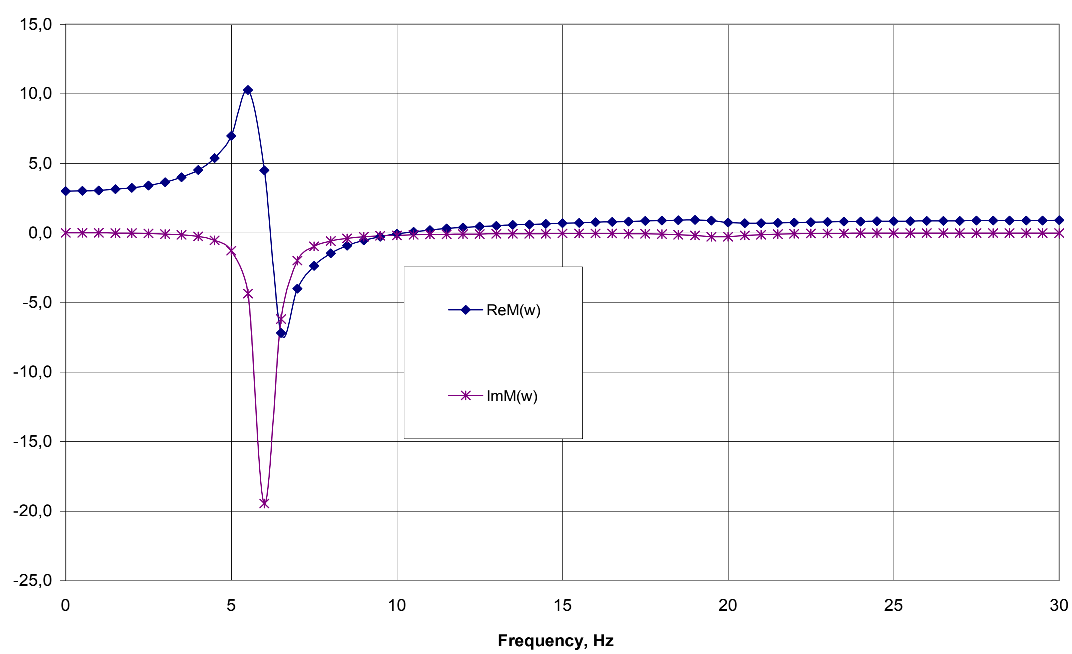

Figure 5. Real and imaginary parts of “dynamic mass” for a sample structure.

Then one can obtain the motion of the basement using the platform accelerations and the transfer functions (given by Equation 27) with Fast Fourier Transform (FFT). As a result, one has a fixed-base upper structure with prescribed base motion and more or less homogeneous damping. Modal approach and spectral approach are appropriate for this model.

The second option of CAM is more sophisticated. The idea is to substitute the initial SSI model by a new SSI model, having artificial homogeneous damping. Then the difference between the initial soil dashpot given by Equation 1 and the new soil dashpot (not viscous, but "material" one instead) should be compensated by the modification of the platform motion. We have six components of the platform motion to be modified, so we can place six conditions to be met. In Tyapin (2009) and Tyapin (2010) these conditions were as follows: the response of the rigid base mat in the new SSI model should be the same as for the initial model. Formula for the modified platform excitation is given in Tyapin (2009) as

$$\{V_0(\omega)\} = [T(\omega)]\{U_0(\omega)\};$$
$$[T(\omega)] = [D_{soil}(\omega)]^{-1}\{[D_{soil}(\omega)] - \omega^2[M(\omega)]\}\{[K_{soil}(\omega)] - \omega^2[M(\omega)]\}^{-1}[K_{soil}(\omega)] =$$
$$= [E] + [D_{soil}(\omega)]^{-1}\{[D_{soil}(\omega)] - [K_{soil}(\omega)]\}\{[K_{soil}(\omega)] - \omega^2[M(\omega)]\}^{-1}\omega^2[M(\omega)] \quad (29)$$

Here $[E]$ is a unit matrix 6 x 6; $[D_{soil}]$ is "platform" complex impedance matrix 6 x 6, $[K_{soil}]$ is "initial" complex impedance matrix. As structural damping is a "material" one, "platform" damping should also be a "material" one with the same coefficient λ:

$$[D_{soil}(\omega)] = (1 + 2i\lambda)\,[\mathrm{Re}\,K_{soil}(\omega)] \quad (30)$$

The last stage of CAM will be dynamic analysis of homogeneous platform SSI model, following Equations 12…14. If "material" 1D solutions of Equation 13 are unavailable, one can use time-domain solutions for viscous SDOF systems instead (both in modal and in spectral methods). The error will not be greater than for conventional analysis of the fixed-base structure.

The seventh curves in Figures 2…4 (marked "CAM material structure damping") are the results of conventional modal approach with constant prescribed modal damping coefficients λ in viscous 1D oscillators and with modification of the platform excitation according to the Equation 29. We see that the difference with "material" results is small. If viscous 1D oscillators are replaced by "material" 1D oscillators with the same coefficients λ and the same platform excitation, this difference completely goes.

CONCLUSION

The main conclusion is that even quite simple platform SSI models with soil springs and dashpots are not so easy to analyze by modal or spectral methods because of heterogeneous damping in the systems. Modal responses in the generalized coordinates prove to be coupled through the damping matrix. The combined asymptotic method described in Tyapin (2010) can help a lot to minimize the error arising in the modal approach. Initial SSI model is replaced by another SSI model with homogeneous damping, and the difference in damping is compensated via special modification of the platform excitation.

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