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# "CHESHIRE CAT" EFFECT IN SSI ANALYSIS PERFORMED BY SASSI

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## ABSTRACT

SASSI code is a kind of industrial standard in soil-structure interaction (SSI) analysis. However, it also has some shortcomings. The author came across one of them: the procedure of outcropping is modeled approximately, and certain error arises from different approaches to the dynamic stiffness matrix of one and the same nodal system a) in the initial infinite soil media and b) later on in the finite "outcropped" volume of the same initial soil. As a result, the "internal" nodes in the outcropped volume after the procedure of outcropping are still linked by some "artificial" stiffness matrix, though physically they should not be linked at all. So, in some sense the presence of the initial soil stays in this volume even after this soil goes – that is why the author called it "Cheshire Cat Effect". Naturally, this "artificial" stiffness is proportional to the actual stiffness of the initial soil.

After a new soil fills the outcropped volume, "true" stiffness matrix of this soil is added to the "artificial" stiffness matrix mentioned above. If a new soil is more or less comparable to the old one in terms of stiffness, "artificial" matrix does not spoil "true" stiffness matrix, and the results in terms of integral impedances are reasonable. However, if hard soil is substituted with soft soil, the "Cheshire Cat Effect" leads to the unacceptable errors in terms of integral impedances.

The author discusses a sample from practice, demonstrating this effect. The combined asymptotic method (CAM) requiring the integral impedances made it possible to discover it. Then the author proposes the way to address the problem: soft part must be included into the initial soil model, and hard lateral walls must be a part of "structure" in terms of SASSI.

## INTRODUCTION

SASSI code presented in Lysmer et al (1980) with several further reincarnations (see Ostadan (2006); Tabatabaie (2007); Ghiocel, (2009)) is an industry standard in SSI analysis. It is a powerful tool, and the results have been verified by experiments (e.g., in Lotung). However, one should always keep in mind certain basic principles of the SASSI methodology and each time try to condense and verify SASSI results against physical logic and simplified models, because sometimes SASSI can provide strange and unrealistic results. The author came across such a case and presents the results of special investigation.

The whole case started when the author tried to demonstrate the impact of a "soft soil pillow" in a comparatively hard rock around the basement on structural seismic response of NPP reactor building. Some colleagues wondered if that could be a way to change seismic response due to the shift of natural frequencies and due to soil damping (like some kind of seismic insulation).

The first step of combined asymptotic method (CAM) developed for SSI analysis in Tyapin (2010) requires integral impedances of rigid basement resting on soil foundation. This problem in our case seems obvious: one a) takes infinite rocky half-space as initial soil, b) outcrops some finite volume of this rock, then c) refills this volume with a new soft soil and finally d) embed rigid basement into this soft soil. All these steps are easily performed in SASSI. After integral impedances of the rigid basement were obtained, the author compared their static values to those got by simple formulae for the same basement on rectangular pillow made of the same soft soil resting on rigid rock. Surprisingly, the results for vertical translational integral impedance after SASSI proved to be considerably lower than that prescribed by simple formula. This fact forced the author to organize special investigation. The process and the results of this investigation are presented in the paper.

## SAMPLE SSI MODEL AND THE COMPARISON OF STATIC INTEGRAL IMPEDANCES

Let us consider rigid basement 80 x 80 m with embedment depth 8 m. “Soil pillow” containing this basement is 144 x 144 m in plan (thus making 32 m from the basement edge to the pillow edge) with embedment depth 24 m (thus making 16 m from the bottom of the basement to the bottom of the pillow). Soil in the pillow consists of two layers: the first one 8 m up from the bottom of the basement and the second one 16 m down from the bottom of the basement. Soil pillow is embedded into the rocky half-space (hard enough, but still flexible). The whole scheme is shown in Fig.1.

Figure 1. Scheme of the sample model: rigid basement embedded into two-layered soil pillow in rocky half-space

Table 1 shows the properties of the rock and two layers of soil. Poisson’s coefficient ν is linked to the ratio of shear wave velocity $V_s$ and primary wave velocity $V_p$:

$$V_p^2 / V_s^2 = 2*(1-\upsilon)/(1-2\upsilon); \quad \upsilon = 0,5*\{1-\frac{V_s^2/V_p^2}{1-V_s^2/V_p^2}\} \tag{1}$$

Table 1: Properties of rock and two layers of soil

| Material | $V_s$, m/s | $V_p$, m/s | Poisson’s coefficient | Mass density ρ, t/m$^3$ | Material damping |
|---|---|---|---|---|---|
| Rock | 1500 | 3000 | 0,33 | 2,2 | 0,01 |
| Lower soil layer | 400 | 1132 | 0,429 | 2,1 | 0,02 |
| Upper soil layer | 400 | 980 | 0,40 | 2,0 | 0,02 |

Let us fix a very low frequency (minimum allowed by SASSI for time step 0.01 s and Fourier number 4096) – practically static 0.0244 Hz. Let us after that gradually increase stiffness of the rock in Table 1 via increasing $V_s$ with constant Poisson’s coefficient ν. Both soil layers stay the same as in Table 1. Physically speaking we are to arrive to some asymptotic ultimate integral impedance values determined by rigid rock and flexible soil. These levels may be estimated from below as stiffness of rectangular pillow 80 x 80 x 16 m made of the soil from the lower layer:

$$\mathrm{Re}\, C_x = \rho\, V_s^2 L^2 / H = 2.1 \times 400^2 \times 80^2 / 16 = 1.344E8 \tag{2}$$

$$\mathrm{Re}\, C_z = 2(1+\nu)\rho\, V_s^2 L^2 / H = 2 \times (1+0.429) \times 2.1 \times 400^2 \times 80^2 / 16 = 3.841E8 \tag{3}$$

SASSI model of the outcropped volume consists of 18 x 18 x 3 elements (each element is a cube with 8 m side). Fig.2 shows real parts of the integral translational impedances (horizontal and vertical) in kN/m.

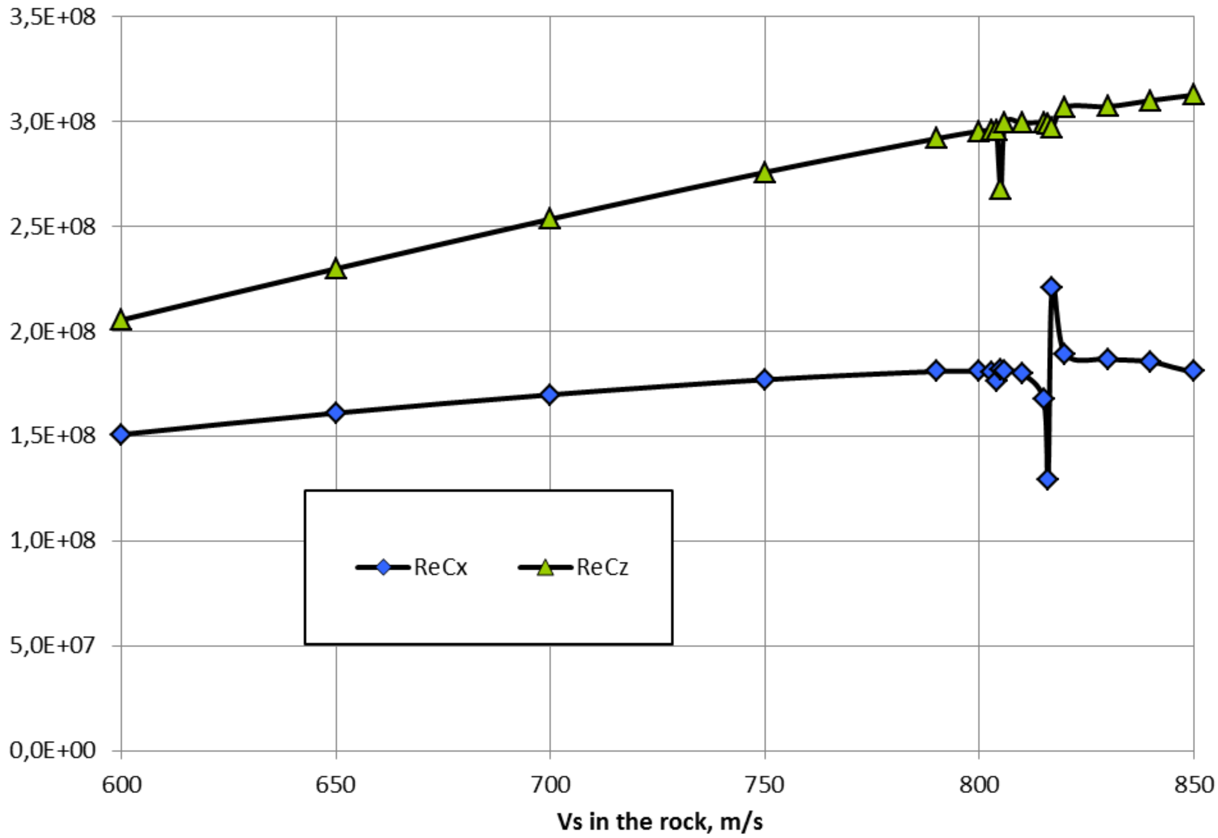


Figure 2. Real parts of the integral translational impedance (horizontal and vertical)

We see that for the shear wave velocities $V_s$ in the rock less than 800 m/s the impedances follow physical logic: they increase along with increase of the rock stiffness. However, for greater values of $V_s$ there appear oscillations. The conclusion is that the results got for $V_s$ =1500 m/s (see Table 1) by this approach will not be trustable.

## INVESTIGATION

Looking at these surprising results, the author decided to carry out the investigation in order to a) find the mechanism of this effect, b) find the way to get appropriate results for such cases. The tool for investigation is CAM again, but this time the procedure for obtaining the integral impedance matrix *C* for rigid basements by means of condensation of nodal impedance matrix *D* with certain displacement fields, proposed in Tyapin (2010), is applied to the system with two rigid basements. The system of two rigid basements on the common soil foundation has the integral impedance matrix 12 x 12 consisting of four blocks 6 x 6. The block (1,1) of this integral impedance matrix contains the dynamic stiffness of the first basement when the second basement is fixed. Initially the algorithm was developed for several structures interacting through the soil (the so-called “structure-soil–structure interaction”), but here it enables the analysis of the ultimate case with rigid rock. Let us set the second “basement” as a bottom and walls of “soil pillow”, as shown in Fig.3 by bold black lines.

Now we should get in block (1,1) the integral impedance of the first basement (i.e. actual one) with rigid half-space surrounding the soil pillow. This result should not depend on the parameters of the rocky half-space, as rock stays beyond the fixed "second basement", and only soft soil remains between this fixed "second basement" and the actual first basement.

Figure 3. Scheme of "two rigid basements" for the soil pillow

However, the results of SASSI for very low frequency mentioned above (practically in the static case) show considerable influence of the rocky stiffness, especially on the vertical impedance, as shown in Fig.4.

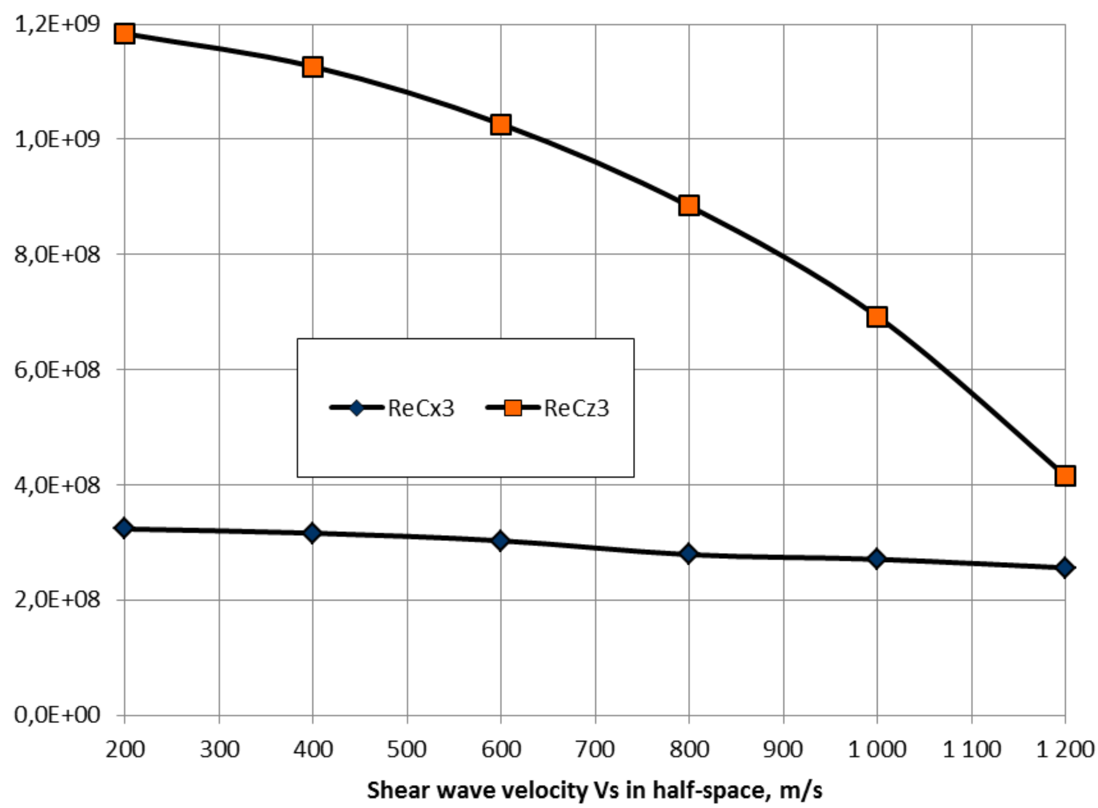


Figure 4. Real parts of integral translational impedances of the actual basement in "two-base" scheme (fixed bottom and walls of the soil pillow, equivalent to rigid rock)

Fig.4 shows our effect very clearly, because for "two-base" system any dependence of the impedances on the rock properties does not have physical meaning. When we calculated conventional "one-base" impedances, we got inappropriate results in a sample case, when shear wave velocity in a rock

(800 m/s) was only about two times greater than in the pillow. In fact, these inappropriate results moved the author to investigate the whole problem.

## RESULTS OF THE INVESTIGATION

SASSI ideology of SSI analysis is based on the "outcropped volume" concept, developed by Lysmer et al (1980). For a certain set of nodes three dynamic stiffness matrices are developed one after another. The first one $D_{soil}$ links the nodes in the infinite horizontally-layered half-space. The second matrix $D_{excav}$ links the same nodes in the outcropped volume of the same soil. The third matrix $D_{struc}$ links the same nodes in the same volume, but filled in by the new content – basement of a structure with surrounding backfill (see Fig.1). "Structure" in SASSI terms means the whole content of this volume. The resulting dynamic stiffness matrix is $[D_{soil}\text{-}D_{excav}+D_{struc}]$.

If the above mentioned volume is embedded (i.e. not a plain surface area), then all the nodes can be split into the "internal" and the "external" subsets. The "external" subset consists of the nodes which belong not only to the considered volume, but at the same time to an "external" part of the soil foundation. "External" nodes are not similar to the "boundary" nodes, because the upper boundary most often is not included into the "external" subset.

Physically speaking, after subtracting $D_{excav}$ from $D_{soil}$ we should get zero dynamic stiffness for the "internal" nodes. In practice the matrix $D_{soil}\text{-}D_{excav}$ does not provide zero stiffness for the "internal" nodes due to the specifics of getting $D_{soil}$ (Lysmer et al, 1980). Thus, though the initial soil has gone from the outcropped volume after subtraction, some impact of it still stays. It is somewhat like the famous smile of the Cheshire Cat from "Alice in Wonderland": it stays after the Cat himself has gone. That is why the author calls $[D_{soil}\text{-}D_{excav}]$ the "Cheshire stiffness" matrix.

The elements of the "Cheshire stiffness" matrix $D_{soil}\text{-}D_{excav}$ are comparatively small as compared to the elements of $D_{soil}$ or $D_{excav}$. That is why they do not matter a lot, if the third stiffness $D_{struc}$ is the same order as $D_{excav}$ or greater. Very often this is really the case: e.g., hard basement in soft soil; or "competent" soil substituting weak soil in "soil pillow" under the basement. The SASSI results are reasonable in such cases, in spite of the "Cheshire Cat" effect. However, when a soft "soil pillow" is put under the basement in a hard rock site in attempt to decrease seismic response, then the "Cheshire stiffness" may become comparable to $D_{struc}$ and thus spoil the results of SASSI analysis.

## POSSIBLE SOLUTION

The author suggests the way to address this problem – to model soft soil of a pillow in SASSI as a part of the initial horizontally-layered half-space, as shown in Fig.5.

Figure 5. System with an infinite soil pillow

In this case we have to model the walls of pillow separately as a part of a "structure" (e.g., special one-row "wall" of elements with rocky properties surrounding a pillow volume), as shown in Fig.6.

Figure 6. System with a one-row "wall" around soil pillow

The same "two basements" check was performed for such a scheme. Only the rock stiffness was variable. In principle, properties of the "walls" should be varied along with properties of a rocky half-space, but in the "two base model" they are out of second basement; so, they do not influence the block (1,1) of the impedance matrix.

Fig.7 shows the previous results from Fig.4 (marked "C") in comparison to the results of a new calculation using layered initial soil (marked "CC").

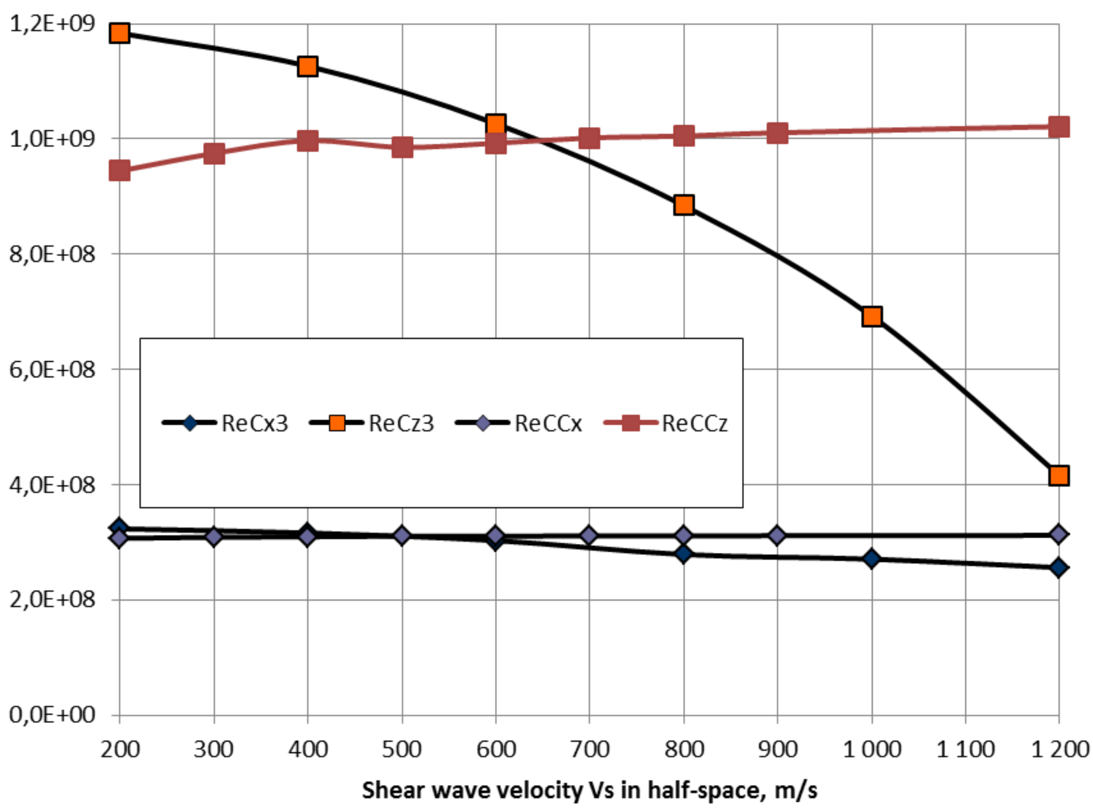


Figure 7. Impedances for "two-base" scheme from Fig.4 (marked "C") in comparison to the results of a new calculation using layered initial soil (marked "CC").

Fig.7 shows that the "Cheshire Cat" effect has not gone completely for a new system, as the underlying half-space still impacts the upper part in $D_{soil}$ (see vertical impedance CCz in Fig.7). However, the scale of this effect is far less. Vertical impedance is rather far from the level given by Equation 3 and is close to the "constraint" version of this equation:

$$\mathrm{Re}\,C_z = \rho\, V_p^2 L^2 / H = 2.1 \times 1132^2 \times 80^2 / 16 = 10.764E8 \qquad (4)$$

After this check the conventional "one-base" impedance calculations enable obtaining reasonable results even for comparatively hard rock. Here we should set up size of the "wall". Let us take "wall" thickness twice as "soil" element size – i.e. 2 x 8 m =16.0 m. Final model will have 20 x 20 x 3 elements. Fig.8 shows old results from Fig.2 (marked "Cx2" and "Cz2") in comparison to new results (marked "CCC"). "Two-base" results from Fig.7 (marked "CC") are also shown – they should form ultimate limits for CCC when flexible rock becomes harder.

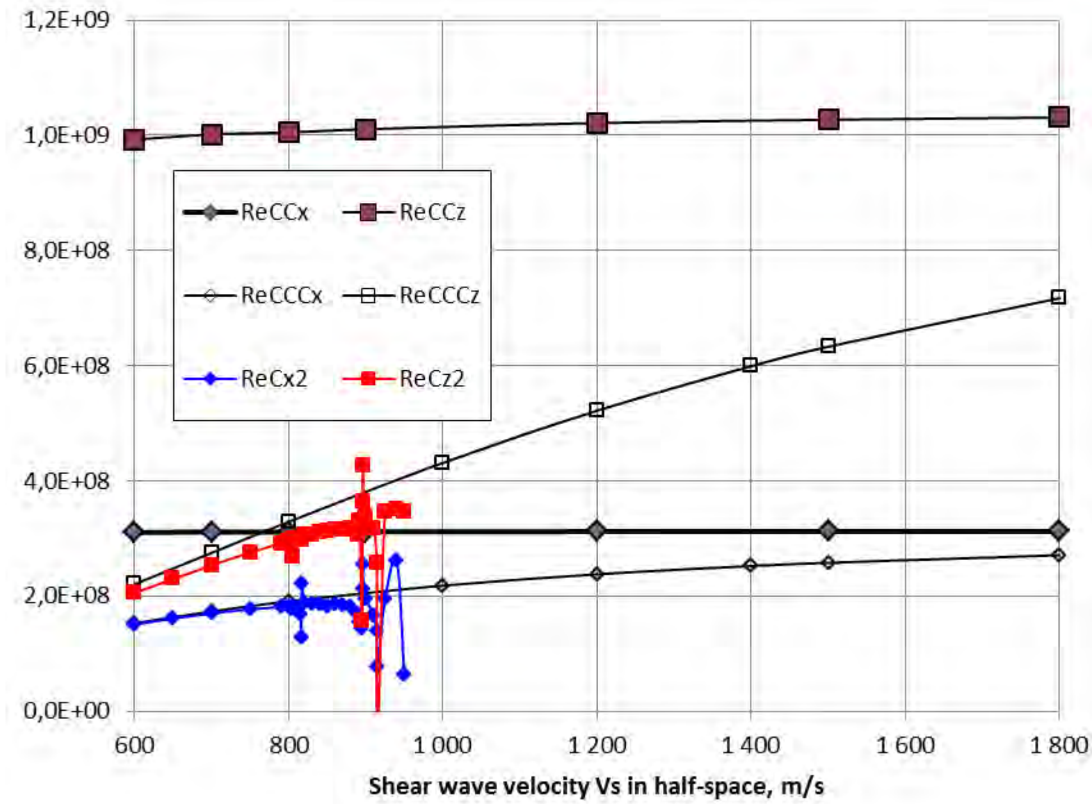


Figure 8. Impedances for "one-base" scheme from Fig.2 (marked "Cx2" and "Cz2") in comparison to the results of a new calculation using layered initial soil (marked "CCC") and previous results from Fig.7 (marked "CC")

For low shear wave velocities in the rock ($V_s$ below 800 m/s) horizontal impedance Cx2 in Fig.8 is almost similar to CCCx. However, vertical impedance Cz2 even for comparatively small shear wave velocities is different from CCCz. This is not a surprise, because Fig.4 showed "Cheshire Cat effect" for vertical impedance even with $V_s$ about 400 m/s.

Comparing CC to CCC in Fig.8, we are able to give some insight on the common question: how hard should be rock to be considered rigid? This is an important question in practice, because rigid rock enables implementation of completely different analytical approach using fixed walls and bottom of soil pillow – and without conventional SSI problems originated by infinite geometry of soil foundation (see Tyapin (2012)). ASCE4-98 proposed threshold velocity $V_s$ = 1100 m/s. However, in our case we see, that even for $V_s$ = 1800 m/s horizontal impedance in Fig.8 is 10% lower than corresponding' "rigid" analogue, and vertical impedance is 30% lower than corresponding "rigid" analogue. This is not enough to make final conclusion, because even poor approximation of impedance can lead to the reasonable transfer functions if dynamic inertia is comparatively small (i.e., $\omega^2 M$ is much smaller than $C_x$ in the "seismic" frequency range). However, the considerable difference between “flexible rock” and “rigid rock” impedances must be taken into account.

Why is the impact of the rock stiffness so surprisingly important in our case? In fact, effective stiffness of the soil pillow and effective stiffness of the rocky half-space work sequent. Together they form impedances for the rigid basement. As soil pillow is comparatively thin (thickness to horizontal size makes 1:5), its’ stiffness is controlled by its’ thickness and its’ module. For the rocky half-space there is no “thickness”. However, formulae for impedances from ASCE4-98 show that for homogeneous half-space the horizontal basement’s size plays a role of thickness in the effective stiffness control. For example, formula for vertical impedance from ASCE4-98 may be re-written as

$$k_z = \frac{4GR}{(1-\nu)} = \frac{\pi R^2 \ \rho V_p^2}{\pi(1-\upsilon)^2 R / 2(1-2\nu)} \tag{5}$$

Thus, effective stiffness of comparatively thin soil pillow can be of the same order as effective stiffness of the rocky half-space: the difference in modules is compensated by the difference in controlling “thickness”. As a result, change in the rock module (changing effective rock stiffness proportionally) may have a great impact on the resulting impedance even for the hard rock.

## CONCLUSIONS

The main conclusion is that formal implementation of SASSI code to the case when soft soil pillow is placed in the rocky site under the basement may lead to inappropriate results. The origin and mechanism of this effect are explained in the present paper. To avoid the difficulties the author suggested an artificial analytical scheme and demonstrated its’ effectiveness.

One more conclusion is that the “two-base” scheme used in combined asymptotic method (CAM) enables “ultimate case” solutions with rigid rock. This is important for the check of the results in terms of impedances and also for the check of the assumption about rigidity of the rock.

Assumption about rigid rock at the bottom of the site should be used very carefully, because not only modules or wave velocities in the rock control the applicability of such an assumption.

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