Transactions, **SMiRT-22**
San Francisco, California, USA - August 18-23, 2013
Division VI Design and Construction Issues

IMPLEMENTATION AND APPLICATION OF NON-ITERATIVE DESIGN PROCEDURES FOR REINFORCED CONCRETE CONTAINMENTS SUBJECTED TO COMBINED AXIAL FORCE AND BENDING MOMENT

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ABSTRACT

A non-iterative design methodology for reinforced concrete containments subjected to combined axial force and bending moment is presented by Jung et al. (2012). A prerequisite to the proposed methodology is a known four-sided polygon region where a demand pair is located on the non-dimensionalized interaction diagram. The purpose of this paper is to identify and present an efficient numerical scheme that can be used to locate a capacity polygon where a particular demand point is located. The identified scheme is the key element to the implementation of the non-iterative reinforced concrete design methodology. A portion of a Reactor Containment Building (RCB) is used as a representative design section to demonstrate the efficiency of the developed design methodology to handle a large number of design cases.

INTRODUCTION

A non-iterative design methodology for reinforced concrete containments subjected to combined axial and bending is presented by Jung et al. (2012). The methodology eliminates a cumbersome need to perform trial and error solutions to obtain a reinforicng area requried for combined axial force and bending moment. The purpose of this paper is to identify and present an efficient numerical scheme that can be used to locate a capacity polygon where a particular demand point is located, which is a prerequisite to implementation of the proposed non-iterative reinforced concrete design methodology. A portion of a Reactor Containment Building (RCB) is used as a representative design section to demonstrate the efficiency of the developed design methodology to handle a large number of design cases.

REVIEW OF NON-ITERATIVE DESIGN METHODOLOGY

Jung et al. (2012) demonstrate that the required reinforcing area for a section subjected to combined axial force and bending moment can be efficiently computed in the following manner:

- Step 1: First construct two non-dimensionalized capacity curves approximated by a combination of capacity polygon segments that are expected to bound all possible design cases including the demand point,
- Step 2: Divide the area enclosed by the lower- and upper-bound capacity segments into several four-sided capacity polygons,
- Step 3: Locate the capacity polygon where the demand point is located and identify associated lower- and upper-bound capacity segments,
- Step 4: Finally determine the required reinforcing area for the demand point by linear interpolation between the minimum and maximum reinforcing ratios associated with the pre-defined lower- and upper-bound capacity segments, respectively.

For illustration purposes, the aforementioned steps are applied to the construction of a non-dimensionlized interaction diagram for a rectangular section (e.g., 12 by 24 or 24 by 12) with $f_c^{'} = 7$ ksi and $f_y = 60$ ksi. The following simplifications and assumptions are made:

- An exact smooth interaction diagram is approximated by a total of 11 line segments with 12 control data points labeled “CP1” through “CP12”. Each of the twelve data points corresponds to a certain value of Z, which is an arbitrarily chosen value and multiplied to rebar yield strain, ε_y, to obtain ε_s ($=Z\varepsilon_y$). Positive values of Z correspond to positive (compressive) strain. For example, Z = -1 correspond to $\varepsilon_s = -1\varepsilon_y$, which is the yield strain in tension. For a given Z value, the corresponding internal axial force and bending moment to maintain internal equilibrium are calculated.
- A symmetric layout of rebars (i.e., the same reinforcing area for both tension and compression faces of the section) is assumed. This is reasonable since it is often the case that reinforced concrete structures have symmetric reinforcing patterns for practical reasons such as convenience and ease of rebar placement on site.
- The spacing between the centroids of the tension and compression reinforcing layers is termed the “effective depth” and assumed to be fixed relative to the full depth of the section. The ratio of the effective depth to the full depth is termed the “effective depth ratio,” and annotated with γ. For the sake of this paper, this parameter is set to 0.85.
- Applicable limiting stresses and strains are those for factored load combinations including primary and secondary loading effects per ACI 359 (2004).

Figure 1 shows resulting interaction diagrams for reinforcing values 0.01 and 0.05, respectively. Each interaction diagram is approximated by a total of 11 capacity line segments as mentioned earlier. The area enclosed by the lower- (0.01) and upper-reinforcing (0.05) capacity segments are split into 11 regions labeled as Region 1 through Region 11 from top to bottom as shown in Figure 1.

For an example application, consider a demand point, (M, P) = (0.92, 3.21984) located in Region 4 as shown in Figure 1. It is important to note that this point is intentionally positioned right on the capacity segment for the reinforcing ration of 0.02. For a close-up view, Region 4 with this demand point is isolated and shown in Figure 2. In additon to the capacity segments, there are three lines drawn in Figure 2. The first line ($P_1 = a_1 \times M_1 + b_1$) is formed by connecting CP4 points for the reinforcing ratios, 0.01 and 0.05 while the the second line ($P_2 = a_2 \times M_2 + b_2$) is defined by CP5 points. These two lines meet at the Center of Rotation (COR) at (M,P) = (-0.05343, 2.064977). The COR and demand point passes through the third line ($P3 = a_3 \times M_3 + b_3$). This line meets with the capacity segment associated with the reinforcing ratio of 0.01 at (M, P) = (0.789159, 3.064609) which is termed the lower intersection point. The same line meets with the capacity segment associated with the the reinforcing ratio of 0.05 at (M, P) = (1.311899, 3.684776), which is termed the upper intersection point. The distance between these lower and upper intersection points is calculated by

$$d_1 = \sqrt{((0.789159 - 1.311899)^2 + ((3.064609 - 3.684776)^2)} = 0.811088$$

The distance between the demand point and the lower capacity point is obtained by

$$d_2 = \sqrt{((0.789159 - 0.92)^2 + ((3.064609 - 3.21984)^2)} = 0.203014$$

The required reinforcing ratio is computed by the following equation

$$\rho_{req'd} = \rho_{minimum} + (\rho_{maximum} - \rho_{minimum}) \times d_2 / d_1 \tag{1}$$

$$= 0.01 + (0.05 - 0.01) \times 0.203014 / 0.811088 = 0.02$$

As expected, the required reinforcing ratio is 0.02 for the given demand point. Thus, any demand point inside this region can be designed by a simple linear interpolation between 0.01 and 0.05. Following the same procedures, demand points located in other capacity regions can be handled similarly.

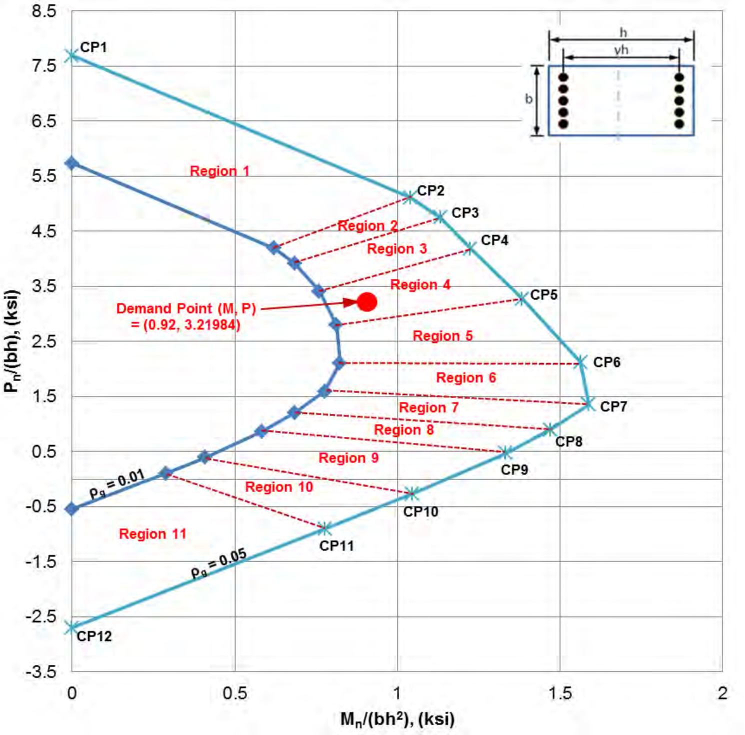

Figure 1. ACI 359 Factored (Primary + Secondary) Load Combination - Nondimensional Interaction Diagrams for Reinforcing Ratios, 0.01 and 0.05 (f_c'= 7 ksi, f_y = 60 ksi, γ = 0.85)

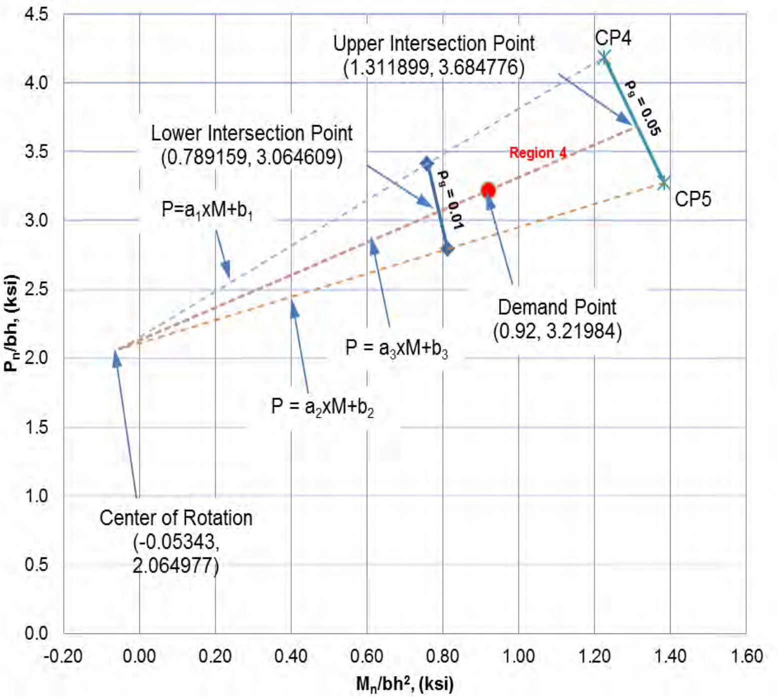

Figure 2. Portion of Nondimensionalized Interaction Diagrams of Reinforcing Ratios, 0.01 and 0.05 for a Rectangular Section per ACI 359 with Focus on Control Points (CPs) 4 and 5

NUMERICAL SCHEME TO IDENTIFY AN APPLICABLE CAPACITY POLYGON

From the review of the non-iterative design methodology, it can be seen that a prerequisite to the proposed methodology is a known four-sided polygon region where a demand pair is located. Then, a legitimate question arises as to how one can determine in which region a particular demand data point is located. Mathematically speaking, this is identical to a problem whether a point is located inside a polygon or not. Consider Figure 3 showing an arbitrarily-shaped closed polygon made up of N vertices (x_i, y_i) where i ranges from 0 to N-1. The last vertex (x_N, y_N) is assumed to be the same as the first vertex (x_0,y_0), that is, the polygon is closed.

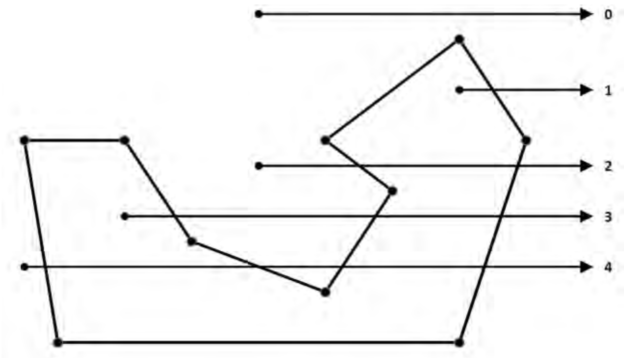

Figure 3. Number of Intersection Points between Parallel Lines Emanating to Right from Points Located Inside and Outside of a Closed Polygon

To determine the status of a point (x_p, y_p), consider a horizontal ray emanating from (x_p,y_p) and to the right as shown in Figure 3. If the number of times this ray intersects the line segments making up the polygon is even, then the point is outside the polygon. Whereas if the number of intersections is odd, then the point (x_p,y_p) lies inside the polygon. Alternatively, consider Figure 4 showing a line segment between two points, P0(x_0,y_0) and P1(x_1,y_1), where P0 and P1 are the starting and ending nodes, respectively, to form a line vector between P0 and P1.

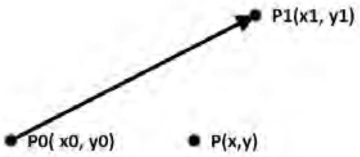

Figure 4. Relationship between a Line Segment and a Point

Then, another point, P(x,y), has the following relationship to the line segment:

$$(y-y_0)(x_1-x_0) - (x-x_0)(y_1-y_0) \quad (2)$$

If this is less than 0, then P is to the right of the line segment. If greater than 0, it is to the left. If it is equal to 0, then it lies on the line segment. This property can be used to determine whether a point is inside a polygon. For illustration purposes, consider a four-sided polygon in Figure 5.

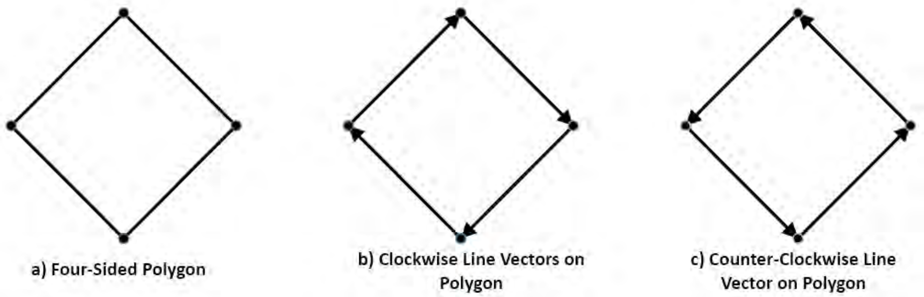
a) Four-Sided Polygon
b) Clockwise Line Vectors on Polygon
c) Counter-Clockwise Line Vector on Polygon

Figure 5. Four-sided Polygon with Clockwise and Counter-Clockwise Line Vectors

If line vectors are defined clockwise as shown in b) of Figure 5, all points inside the polygon are to the right of the line segments. Whereas all these points are to the left of the line segment if line vectors are defined counter-clockwise as shown in c) of Figure 5.

Either of the two methods can be used to identify a region where a particular demand point is located in Figure 1. For the sake of discussion herein, the first method is utilized as follows to identify the applicable capacity region:

- Step 1: Starting with Region 1, identify the maximum and minimum P and M values of control points . If the demand point falls inside the region bounded by these extrema, proceed to step 2. If not, move on to the next region until an applicable capacity region or polygon is identified.
- Step 2: Relative to the identifed capacity region or polygon, draw a horizontal line to the right of the demand point long enough to pass through the capacity polygon associated with the upper bound reinforcing ratio.
- Step 3: Count the number of intersections the horizontal line drawn in Step 2 meets with the capacity polygon. If the number of intersections is 0 or 2, the demand point is outside the capacity region. For this case, go to Step 1 to move on to the next capacity region. If the number of intersections is 1, the demand point is inside the capacity region. Then, move on to Step 4.
- Step 4: Use Equation (1) to determine the required reinforcing for the demand point.

For the example case considered previously, it can be easily seen in Figure 6 that the demand point (0.92. 3.21984) is located in a region bounded by (M_{min}, M_{max}) = (0.812599,1.38358) and (P_{min}, P_{max}) = (2.793681,4.18408) (Step 1). A horizontal line (P = 0.92) emanating from the demand point is drawn to the right of the demand point until it passes through the capacity polyon (Step 2). Figure 6 shows that the horizontal line meets with the only one line labeled Line 3 of the four lines in Region 4. Thus, the demand point is inside Region 4 (Step 3). The required reinforcing for the demand point can be obtained by a linear interpolation of reinforcing ratios between 0.01 and 0.05 using Equation (1) (Step 4).

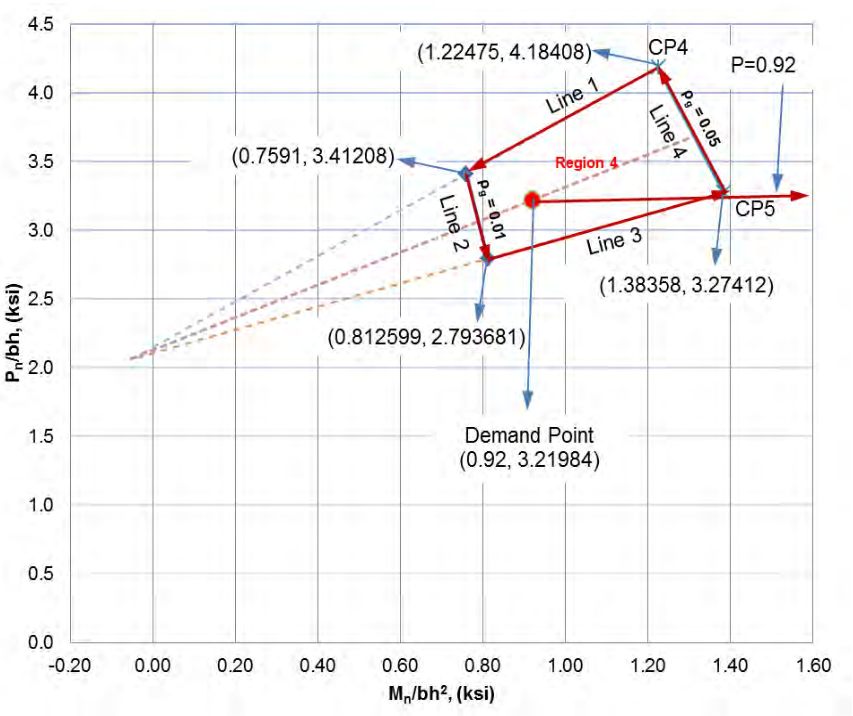

Figure 6. Intersection of Horizontal Line Emanating from Demand Point with Capacity Region 4.

DESIGN TOOL DEVELOPMENT

The non-iterative design methodology along with the numerical scheme identified in this paper is implemented as part of a reinforced concrete design tool as shown in Figure 7. This tool has been developed using Visual Basic Application (VBA) augmented by Dynamic Link Libraries (DLL). For computational efficiency, three reinforced concrete design modules for combined axial and bending, in-plane shear, and out-of-plane shear design were developed using Fortran90 and compiled into a DLL. Then, this design DLL is included as part of the VBA scripts that are embedded inside an Excel file. The Excel file serves as the interface between the design DLL and design input/output files (e.g., text files including design demands and results). Alternatively, worksheets inside the Excel file can be used to provide the design DLL with input design data and present resulting design data obtained from the design DLL.

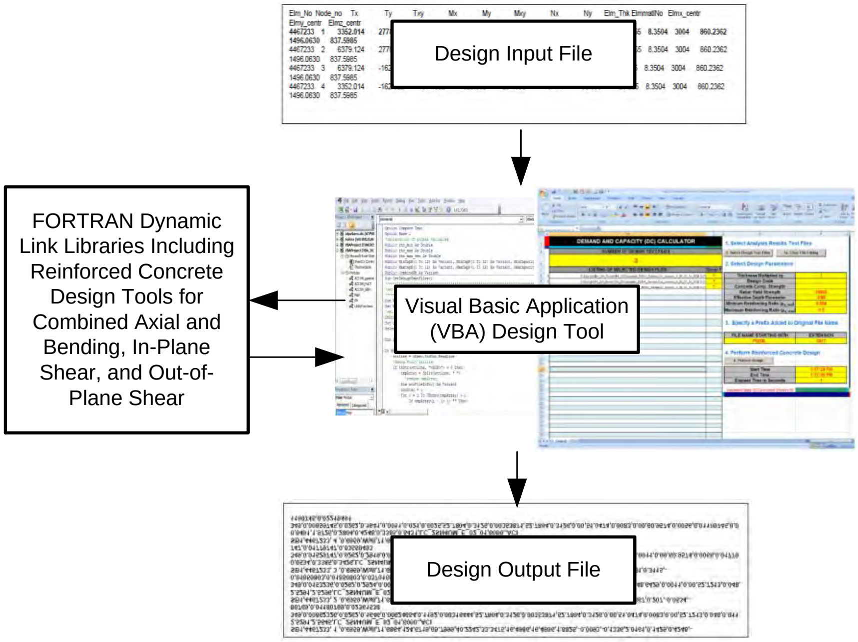

Figure 7. Visual Basic Application (VBA) Design Tool Augmented by Reinforced Concrete Design Dynamic Link Libraries (DLL)

ILLUSTRATIVE DESIGN EXAMPLE

The design tool is applied to perform the design of a portion of a RCB as shown in Figure 8. The design section is 51 inches thick with a concrete compressive strength ($f_c^{'}$) of 7 ksi and rebar yield stress (F_y) of 60 ksi. As shown in Figure 8, the design section is discretized into 16 shell elements with element local X axis aligned with the meridional (vertical) direction of the RCB cylinder while element local Y axis with the hoop (horizontal) direction. For the sake of discussion herein, focus is placed on computation of orthogonal reinforcing along the meridional direction for combined axial and bending design.

There are four applicable load combinations considered for the design section: factored load combinations involving primary and secondary loading termed FACT (P+S), factored load combinations involving primary loading termed FACT (P), service load combinations involving primary and secondary loading termed SERV (P+S), and service load combinations involving primary loading termed SERV (P). Taken as a whole, they add up to nearly five thousand load combinations. When design forces and moments are extracted from element nodes in lieu of element centroid, the total number of design cases are approximately twenty thousands with each element having four nodes as shown in Figure 8. Despite some variations in computing time among computers with different system configurations, it has been found that all these design cases can be processed by the design DLL in around ten seconds for a computer system with Intel Core i5-2520 M CPU and 8 GB RAM.

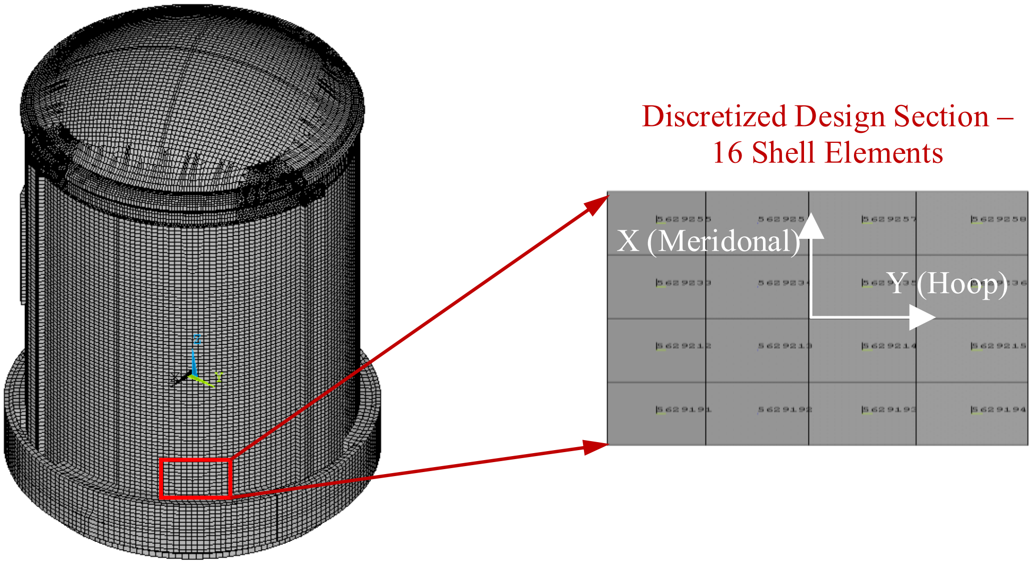

Figure 8. Design Section - Portion of Reactor Containment Building (RCB)

The design results are then sorted with respect to the required reinforcing area for each element. Of the sorted results, top 1000 meridional design pairs (axial force and moment) are identified. The same process is repeated for all 16 elements included in the design section. This results in a total of 16000 meridional design pairs and they are all plotted in Figure 9, together with corresponding capacity curves each of which is constructed based on the maximum reinforcing requirement of a particular design load category (e.g., FACT (P+S) or SERV (P+S)). It is important to note that there are only three capaciy curves shown in Figure 8: FACT (P+S), FACT (P) and SERV (P+S). Of the four load combination categories initially considered, design demands from SERV (P) never rank within top 1000 entries in terms of reinforcing area for all 16 elements. From Figure 9, it is interesting to note that the required maximum reinforcing ratios are 0.00711 for FACT (P+S), 0.004 for FACT (P), and 0.009449 for SERV (P+S). In case of FACT (P), it is noteworthy that the maximum required reinforcing ratio is 0.004 and the capacity curve associated with this reinforcing ratio is far away from a cluster of the design demand points. This is because the reinforcing ratio of 0.004 is the code-specified minimum required reinforcing ratio. If the design demand requires the reinforcing ratio less than this minimum requirement, the use of the minimum reinforcing ratio is enforced in the design tool DLL. In summary, the controlling load combiantion for the design section under consideration is SERV (P+S) and the associated reinforcing ratio is 0.009449.

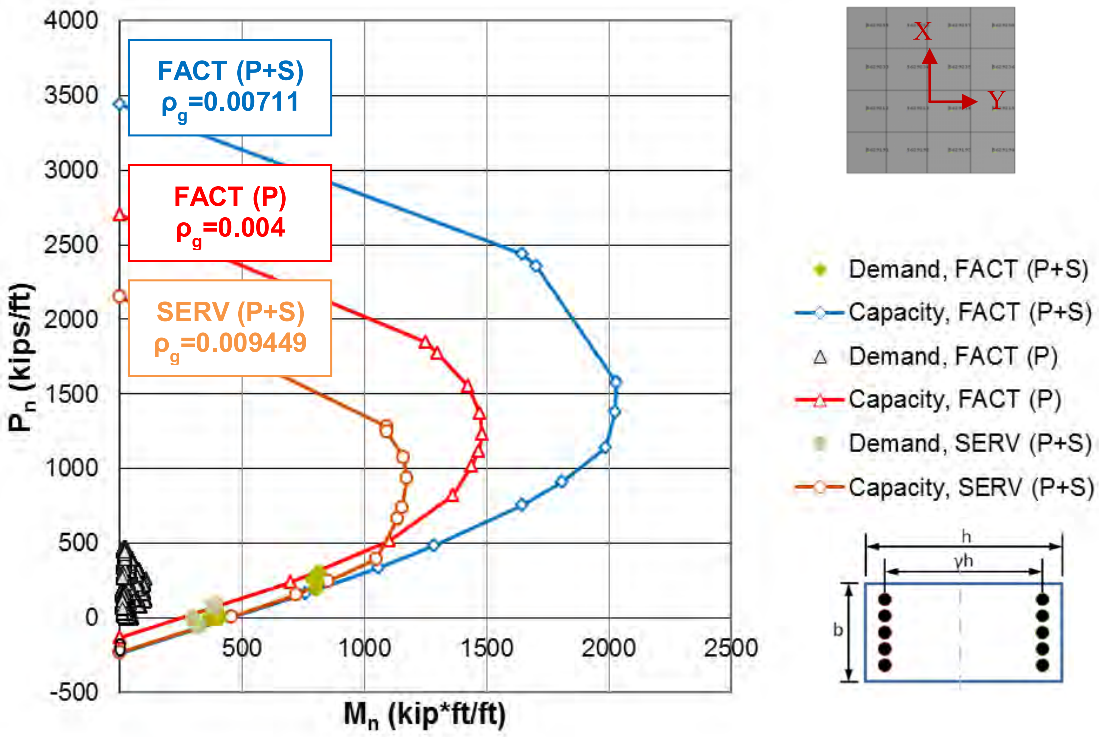

Figure 9. XY Scatter Plot of Meridional (Vertical) Axial and Bending Demand Pairs (FACT (P+S), FACT (P), and SERV (P+S)) and Corresponding Capacity Curves Approximated by Capacity Segments, $\gamma = 0.85$

CONCLUSION

To complement the non-iterative reinforced concrete design methodology developed for the efficient combined axial and bending design, this paper has identified and presented the efficient numerical scheme that can be used to locate a particular polygon where a particular section demand is located. The identified scheme is the key element to the implementation of the non-iterative reinforced concrete design methodology as part of the design tool. It has been demonstrated through the illustrative design example that the non-iterative design methodology along with the identified numerical scheme can be used to efficiently handle a large number of combined axial and bending design cases without iteration.

REFERENCES

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