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DETERMINATION OF AN INTERACTION RULE FOR TWO DEFECTS SUBMITTED TO A MODE I OF FRACTURE IN AN INFINITE MEDIA

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ABSTRACT

This article deals with the question of crack interaction when two cracks are close to each other and then, a conventional approach based on isolated crack consideration cannot be applied.

Within that frame, a bibliography and an exhaustive 2D modeling was performed in order to define precisely interaction limits and defect consideration in the case of two isolated and infinite length cracks submitted to a mode I loading. This large 2D modeling was then completed by a 3D prospective modeling dealing with the question of finite length defect consideration in the interaction evaluation.

The work, completed by perspectives for the continuation of this development is presented. It shows that, for a uniform mode I loading and elastic behavior, interaction limits can be defined through simple geometric considerations.

INTRODUCTION

This work is performed in the frame of integrity assessment, and more precisely in the frame of crack-like defect evaluation in the case where two defects, close to each other, may interact and thus may increase their respective noxiousness.

In such a case, the conventional approach adopted by the codes devoted to integrity assessment consists in defining interaction rules, in terms of relative distance between the cracks, prescribing the cases when the cracks have to be evaluated separately (as isolated cracks) and the cases when they have to be merged together and then treated as a unique and envelop crack.

Due to the difficulty of the problem and the large number of parameters governing the potential interaction, those interaction rules are generally conservative, both in terms of interaction distance and equivalent envelop defect. This may lead to large overestimations of the defect noxiousness evaluation.

In order to relax the conservatism of those interaction criteria, a specific work was launched within AREVA. The global objectives of that work were first to improve the understanding on defect interaction, then define interaction rules more appropriate for the multiple defects problem encountered.

For that purpose, a bibliography review was achieved to define clearly the needs of the problem, the lacks and limits of existing formula and rules. Then, for a complete definition of the interaction domains, a F.E. analysis based on a parametric model was performed. First and exhaustively on a 2D parametric model, this interaction rule was completed by prospective 3D calculations which cannot, of course, be as exhaustive as 2D modeling.

This article gives a synthesis of that work. It describes the model specifically developed for the study, then the proposition of the interaction domains for two 2D cracks in a mode I solicitation.

Then, based on available results and the prospective 3D F.E. calculations, a correction taking into account finite length of cracks is made. Finally, perspectives for a continuation of this work are proposed.

BIBLIOGRAPHY ANALYSIS

Existing results – scopes and impact on our problem definition

There are not so many papers dealing with defect interaction which is a complex problem, both in terms of analytical developments and F.E. modeling. Nevertheless, in the *Murakami (1987)* compendium, different analytical solutions interesting for our purpose are proposed. Those solutions are covering essentially mode I (θ_L=90° – see fig. 1 for notations). We are using here:

- Two equal or unequal aligned cracks (H=0) solutions: those data are based on analytical solutions and thus constitute useful results for the definition of the limits but also for F.E. model validation;
- Two equal elliptical cracks and two unequal penny-shape cracks in the same plane (H=0) solutions which are also very useful for the 3D model validation;
- Two parallel penny-shape cracks in two different planes (H≠0) solutions which, at this step, constitute important results for the problem definition.

On the other hand, *Hasegawa (2009)* proposes a consistent and exhaustive review of the interaction rules adopted in the Japanese, European and American codes, and associated to that review, a F.E. modeling work for two 2D parallel cracks in a mode I of loading (the scope we are looking at this step of our work). For our purpose, many results can be deduced from that work:

- In the codes, interaction rules are defining the limit distance for grouping or not the defects under consideration. Even if that point is not explicitly evoked by *Hasegawa*, one can easily understand that those grouping rules are directly linked to the potential damage under consideration (i.e. brittle fracture, ductile fracture, fatigue crack propagation…) and thus this has to be precised.
- In his analysis, *Hasegawa* considers two cracks of different sizes and the interaction at the maximum loading crack tip. He then logically shows that the dominating crack is the largest one and the interaction to quantify is the effect of the smallest crack on the largest one.
- In this work devoted to brittle fracture, the interaction is quantified in an elastic analysis as the amplification of the stress intensity factor K of the dominated crack tip. By doing this, he clearly shows that the interaction coefficient is proportional to the smallest crack size. This point is illustrated on fig. 2 where the intersection of the 10% interaction curves with the bisecting line (H=S) are plotted in terms of a_1/a_2 ratio (see fig. 1 for definitions). Those two values being quasi-proportional, we can easily conclude that a unique interaction domain based on a_2 can be defined.

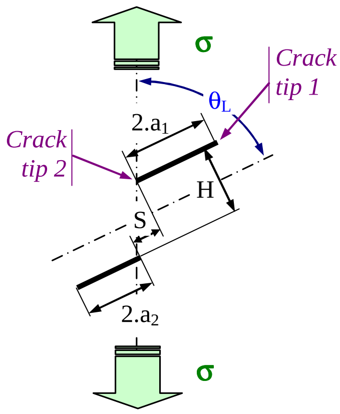


Figure 1: Definition of the geometrical problem

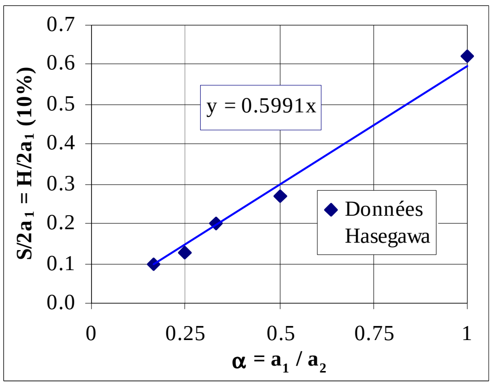


Figure 2: Illustration of the proportionality between interaction and smallest crack size

- *Hasegawa* work was focused on the coalescence of cracks in brittle fracture. Based on tests, he has concluded that a criterion at 6% of interaction is adequate. In a more general case, for example for potential crack coalescence in fatigue or in ductile tearing, this criterion is not appropriate *a priori*. A criterion linked to the potential damage and the associated fracture mechanics approach has to be defined.

- *Hasegawa (2009)* has also considered the difference between 2D and 3D. He shows that, in terms of interaction, the 2D approach presents a consequent conservatism since it considers the crack as infinitely long. The fig. 3 deduced from *Murakami (1987)* gives an evaluation of that conservatism for equal elliptical cracks in the same plane.

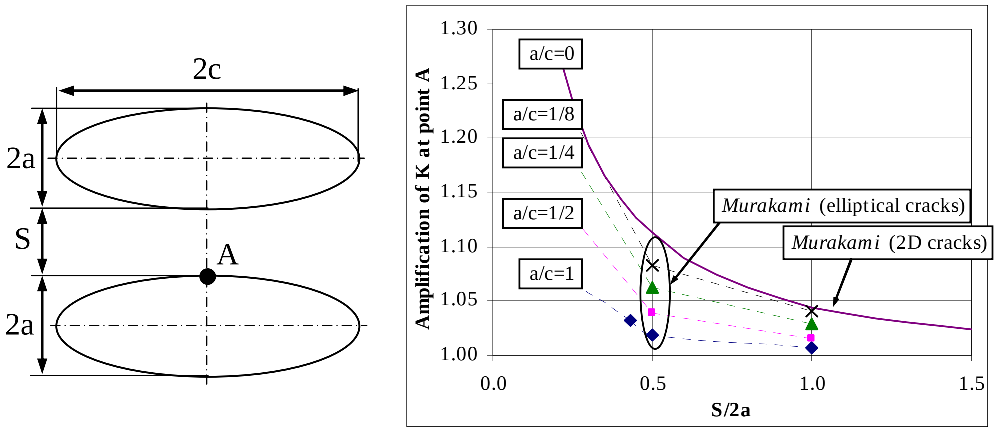


Figure 3: Influence of the crack elongation on the amplification of K at point A

Meyer et al. (2000) propose a numerical elastic-plastic analysis of the interaction between two cracks, in any orientation. The question discussed here is linked to short crack interaction in fatigue. Thus, again, the results presented in this paper could not be transposed easily from that configuration to another. Nevertheless, this work provides very interesting results on the very complex crack interaction in an elastic-plastic strain field. In the major cases of the presented results, the interaction is amplified by the plastic flaw (i.e. the interaction limits could be significantly enlarged, up to a factor of 2). But inverse cases (cases where the interaction is reduced) can be observed. This work thus shows that the plasticity may have an important impact. But it must be precised that, in this work, plasticity is large with a ratio between plastic and elastic strain around 5.

Synthesis – Interaction definition

In our work, we are looking at parallel cracks, thus all the results presented in the bibliography are useful for the definition of interaction limits. However, for our targeted final objectives, the stress distribution may be non-uniform, leading to a mode I+II of loading, and may be significant (in comparison to the material yield stress). Those three aspects will have to be taken in consideration in a second step.

In the first step presented here, we focus on the uniform mode I loading in the elastic domain. We are not specifically focused on brittle fracture. Thus, despite the *Hasagawa* choice, we adopted an interaction limit linked to a K amplification of 10%. This choice is motivated by the fact that this value is in accordance with the assumption originally adopted in support of *RSE-M (2010)* interaction rule definition. In addition, we consider that for a 10% amplification of K, the process zones corresponding to the interacting crack tips remain independent.

We finally define three domains: a high interaction domain corresponding to a K amplification larger than 10%, a weak interaction domain corresponding to a K amplification between 10% and 2.5% and a negligible interaction domain corresponding to a K amplification lower than 2.5% (which appears to be a reasonable limit regarding a conventional K calculation precision).

INTERACTION DOMAIN BASED ON A 2D PARAMETRIC F.E. MODEL

Objectives of the model

In this first step of analysis, a specific parametric model was developed in *Cast3M (2012)* F.E. code in order to determine precisely the different domains of interaction. It is used for mode I type of loading in this article, but this model is also developed to analyze mode II and combination of mode I+II.

As discussed previously, we are considering the effect of the smallest crack on the largest one. In that case, *Hasegawa (2009)* has shown that the interaction can be normalized by the size of the smallest defect. This assumption can be verified with the *Murakami (1987)* compendium which provides an analytical solution for aligned unequal cracks (fig. 3). As it can be seen on that figure, the interaction is quite constant for a constant S/a_2 ratio with varying a_2/a_1 ratios. The amplification slightly increases with an increasing a_2/a_1 ratio, which means that the numerical study and interaction domain definition can be conservatively determined by modeling equal length crack.

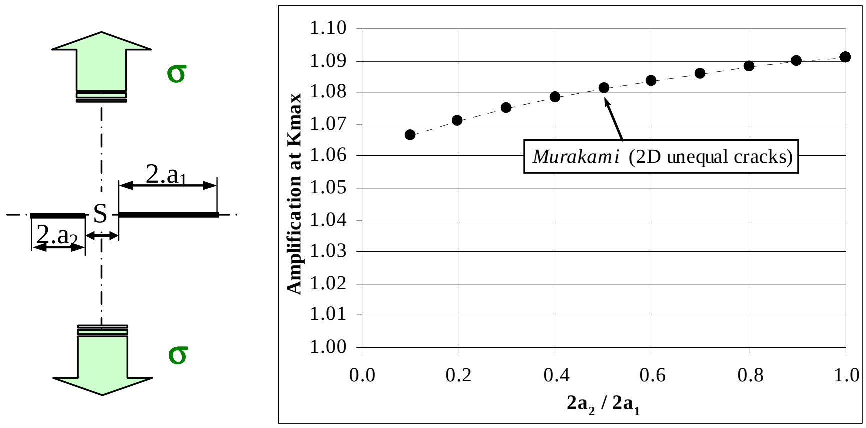


Figure 3: Amplification of max K for a constant $S/2a_2 = 0.6$

Element of validation

Before the development of the interaction limits, a validation of the model is required. For that purpose, we have used the existing analytical and numerical results which provide useful values to evaluate the precision of our numerical F.E. results. For the 2D model, we rely on:

- The analytical solutions for mode I and II for an isolated crack (see comparison in table I);
- The analytical solutions for aligned cracks in mode I from the *Murakami* compendium (see fig. 4);
- The *Hasegawa* mode I numerical solutions (example here for $a_2/a_1=1$, $H/2a_2=0.2$ and variable $S/2a_2$ – see fig. 5).

All those comparisons are made in terms of influence function defined by:

$$F = \frac{K_{eq}}{\sigma.\sqrt{\pi.a}}, \quad (1)$$

where K_{eq} is the equivalent stress intensity factor directly deduced from the G parameter determined by the F.E. model through the $G(\theta)$ approach (*Cast3m, 2012*):

$$K_{eq} = \sqrt{\frac{E.G}{1-\nu^2}}, \tag{2}$$

As it can be seen in those comparisons, the comparison of our F.E. model to analytical solution is very good providing us a high confidence in the precision of our model. The comparison to *Hasegawa (2009)* results is also good but shows slightly higher values than our results in particular for larger $S/2a_2$.

Table 1: Comparison to analytical solutions for isolated cracks ([1]see fig. 1 for θ_L definition)

Loading	F at tip 1	F at tip 2	Ref. solution	difference
Mode I+II (θ_L=20°)[1]	0.3419	0.3419	Sin(20°)	< 0.2%
Mode I (θ_L=90°)	0.9920	0.9920	1.	< 0.1%

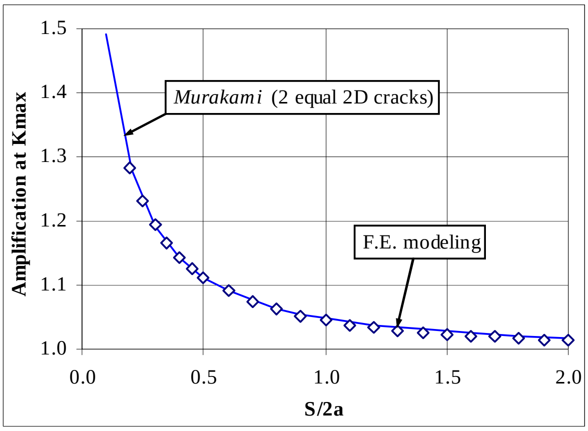


Figure 4: Comparison to *Murakami* compendium

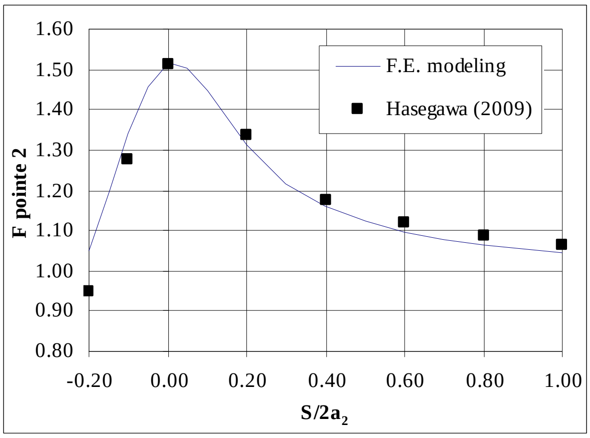


Figure 5: Comparison to *Hasegawa* solution

Interaction limit definition

While the model is validated, the numerical work consists of the definition of developing K_{eq} amplification curves for fixed $H/2a_2$ values and varying $S/2a_2$ values , for the two crack tips (because dominating crack tip changes when S becomes negative – see fig. 6, 7 and 8). Then, the limits of the domains (10% and 2.5% of interaction) can be defined (for uniform mode I loading at that step).

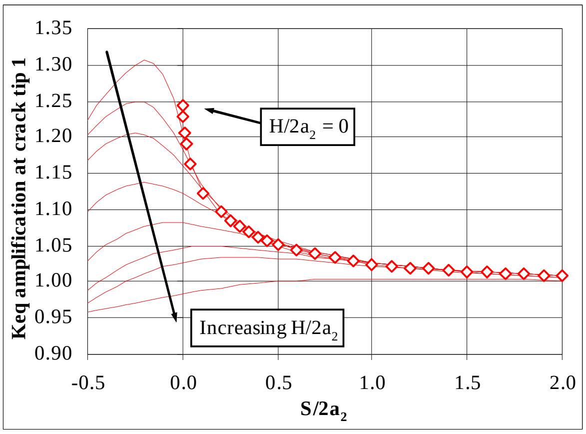


Figure 6: K_{eq} amplification at crack tip 1

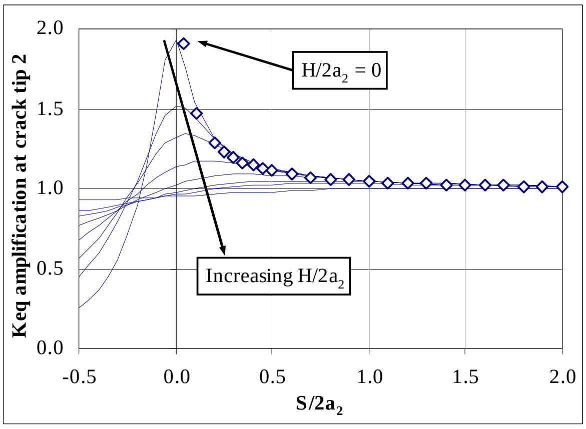
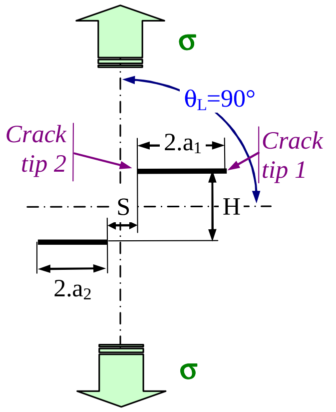


Figure 7: K_{eq} amplification at crack tip 2



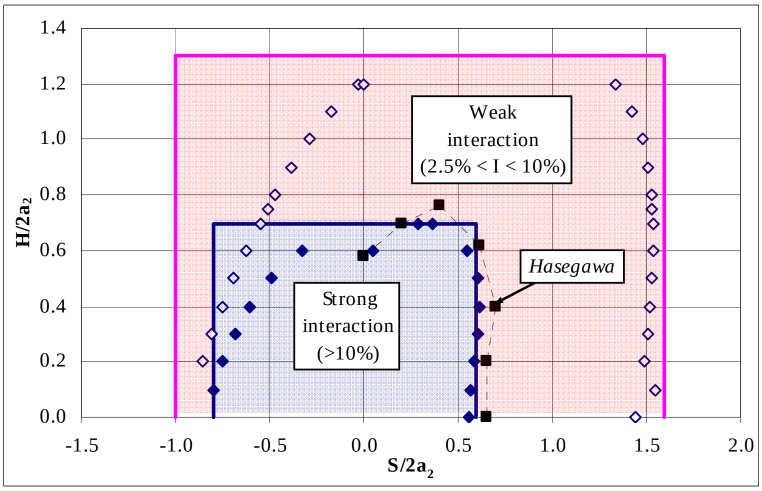


Figure 8: interaction limit definition for loading mode I (θ_L=90°)

The figure 8 gives a synthesis of the different domains determined for this study. We can see that:

- The interaction domain is slightly smaller than the one determined by Hasegawa (2009), which is consistent with the previous results shown in fig. 5.
- The interaction domain is different within positive and negative domains, but nevertheless and in particular for positive values of S, simple limits could be defined: respectively $S/2a_2 x H/2a_2$ = 0.6x0.7 and 1.6x1.3 for 10% and 2.5 amplification.

PROSPECTIVE RESULTS BASED ON A 3D PARAMETRIC F.E. MODEL

Equivalent defect definition

As discussed previously, it appears that the interaction estimated from a 2D analysis could be over-estimated for an elliptical defect since it considers an infinitely long defect. On the other hand, it also appears that the interaction distance is quasi proportional to the crack size.

This means that, in a first approximation, the interaction distance can be assumed as proportional to K_{eq}^2 and thus a correction based on the ratios of K_{eq}^2 for elliptical and infinitely long crack of the same width (i.e. same '*a*') can be assumed. For uniform mode I loading and at the maximum loaded point of the crack front, the correction becomes:

$$\left[\frac{K_I(a/c)}{K_I(a/c=0)}\right]^2 = \frac{1}{[E(a/c)]^2}, \tag{3}$$

where E(a/c) is the complete elliptic integral of second kind and c is the finite half length of the elliptical defect (see fig. 3). For our purpose, we then propose the definition of an equivalent defect size a_{eq} which has to be considered in the interaction distance definition:

$$a_{eq} = \frac{a}{[E(a/c)]^2}, \tag{4}$$

This correction was applied on the available data for *Murakami (1987)* compendium on fig. 9: elliptical cracks of same size with varying a/c ratios and penny-shape cracks (a/c=1) of different sizes. On these graphs, the K_{max} amplification is expressed in terms of a_{eq} and the results are compared to the curve corresponding to the infinitely long defect (analytical results for aligned 2D cracks). As it can be seen, the results are very good and validate the proposed correction which appears to be reasonably conservative.

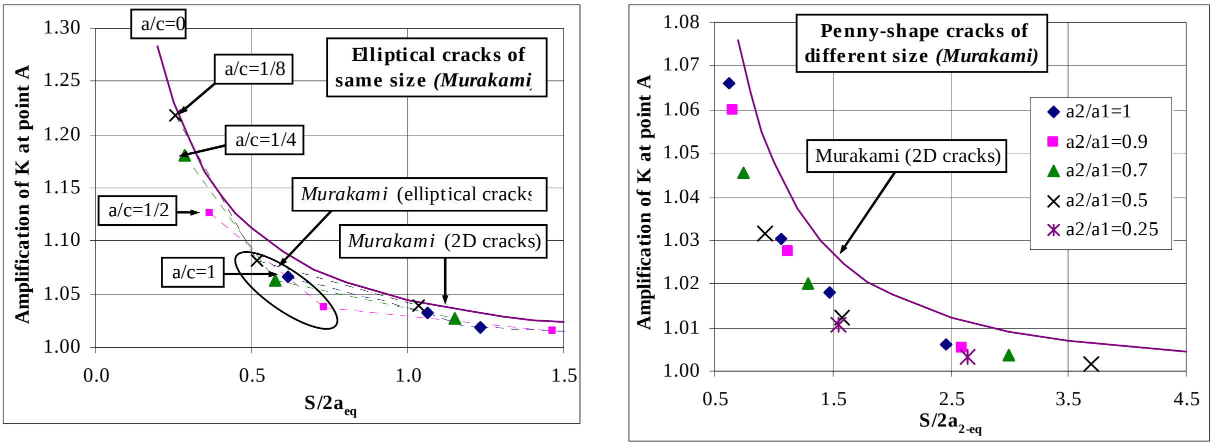


Figure 9: Equivalent defect size proposition based on *Murakami* compendium

Objectives and description of the model

A 3D model was initiated in order to validate the previous length correction for the evaluation of interaction distance. For that purpose, a parametric model which allows to model different crack sizes in different positions (in terms of longer axis versus the smaller one) was developed in *Cast3M (2012)*.

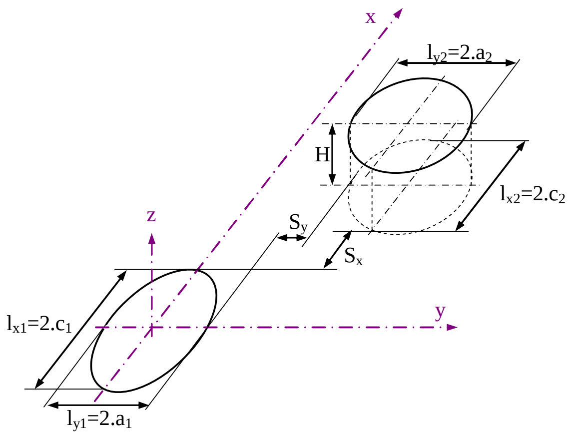


Figure 10: Definition of the 3D problem

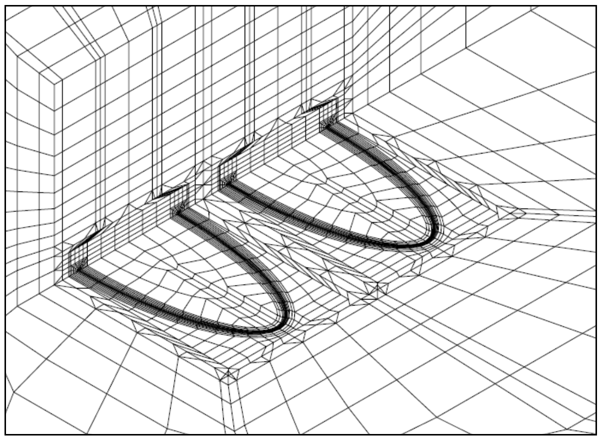


Figure 11: Example of 3D mesh

Table 2: Synthesis of the 3D reference F.E. models and results

#	l_{y1}/l_{x1}	l_{y2}/l_{x2}	$S_y/2.a_1$	$S_x/2.a_1$	H/a_1	a_2/c_2	$S/2.a_{eq}$	Ampl.
#1	1/3	1/3	0.5	0	0	0.333	0.620	1.064
#2	1/3	1	0.5	0	0	1	1.234	1.020
#3	1/3	3	0.5	0	0	0.333	0.620	1.022
#4	1/3	1/3	0	0.5	0	0.333	0.620	1.004
#5	1/3	1	0	0.5	0	1	1.234	1.002
#6	1	1	0.5	0	0	1	1.234	1.022

At this step, this model allows to analyze co-planar cracks (H=0 – see fig. 10 for geometrical definition). Within that frame, a set of reference configurations was defined (see table 2) in order to investigate the effect on dissymmetry on the interaction.

Element of validation

Again, before the determination of reference interaction values, a validation phase was performed to validate the meshing procedure which is quite complex in that case. For that purpose, a set of validation cases was treated and compared to:

- The *Irwin* solution for isolated cracks (see an example on fig. 12). In that case, a very good correlation with *Irwin* solution has been observed with an error lower that 0.5%.
- The *Murakami* solution (equal penny-shape and elliptical cracks). Again, in that case, a good correlation is obtained with the available results with, for elliptical cracks, a value determined by the F.E. model slightly lower.

However, the good correlation between our model and the Irwin solution gives us a good confidence in the precision of the mesh.

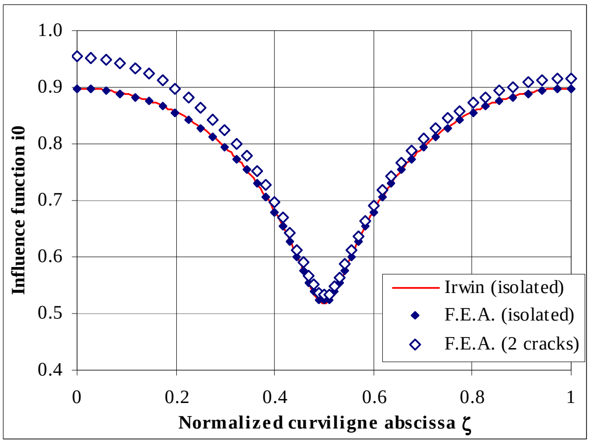


Figure 12: Comparison of the F.E. model to Irwin solution (a/c = 1/3)

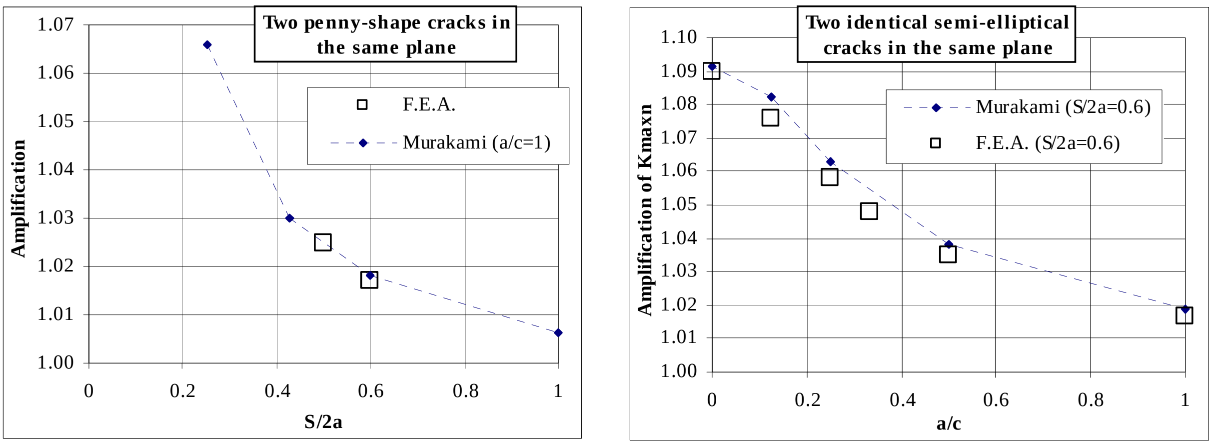


Figure 13: Comparison of F.E. results to Murakami data

First results – validation of a 'length correction'

The interaction values (amplification of max K for the largest defect) are plotted against the equivalent distance on figure 14. We represent on that figure the data from the reference configurations (given in table 2) and the data from the validations cases (penny-shape and elliptical data are the one from *Murakami* compendium).

We can see on that figure that, in each case, 2D formulation remains envelop, that is to say the proposed correction is valid. All the data correspond to the cases where the maximum loaded points of the two cracks are in front of each other (configuration represented on fig. 3) and are in the same trend similar to the 2D one. At the opposite, in the case where this is the minimum loaded points which are in front of each other (case #4), the interaction is much lower. This means that with the current definition of the a_{eq}, an intrinsic margin still exists but is not used here for simplicity.

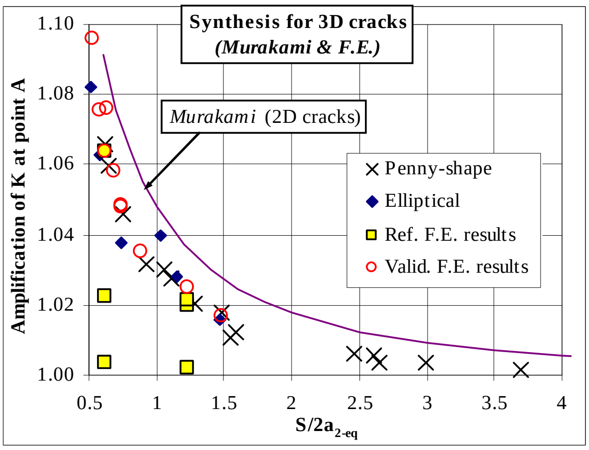


Figure 14: Validation of the length correction

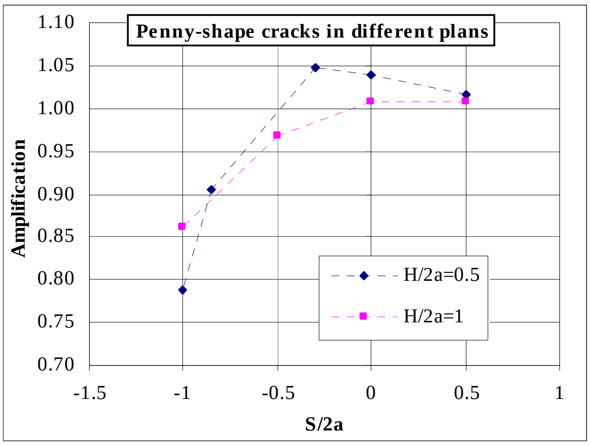


Figure 15: Available results for H≠0

Final expression of the interaction domain

From the results obtained in mode I, a simple strong interaction limit (i.e. amplification larger than 10% in our case) can be defined by the following formula:

$$S_{10\%} = 0.6 \min(2.a_{1\text{-}eq} \,;\, 2a_{2\text{-}eq}) \;\&\; H_{10\%} = 0.7 \min(2.a_{1\text{-}eq} \,;\, 2a_{2\text{-}eq}), \qquad (5)$$

with: $a_{i\text{-}eq} = a_i / [E(a_i/c_i)]^2$, *2.*$a_i$ *and 2.*c_i *being respectively smaller and larger axes of the crack.*

Finally, if the in-plane distance (minimum distance between the 2 cracks in the plane) is larger than $S_{10\%}$ **or** the out of plane distance (distance H in figure 10) is larger than $H_{10\%}$, then the two cracks remain separated cracks. If no, they have to be grouped in the same envelop crack. The same limit can be defined for the weak interaction limit of 2.5%.

PERSPECTVES

As it has been shown in the previously presented work, a simple geometrical limit can be defined to evaluate the possible interaction between two cracks close to each other. However, the present validation is limited to cracks in the same plane and complementary validations have to be performed for non-coplanar cracks. That case appears to be more difficult because, as it is shown on the fig 15, for a given H/2a value, the S/2a value at maximum of interaction is varying, eventually in negative domain for S. This problem will thus require a specific model and a parametric analysis.

In addition, limits presented here were established for a uniform mode I loading. The case of varying loading in space or cumulated mode I+II has to be investigated.

Finally, the effect of plasticity has also to be considered: as it has been shown by *Meyer (2000)*, the plasticity may increase significantly the interaction distances and thus, for the investigation on potential damage as crack coalescence in ductile regime, the strong interaction domain may be increased.

CONCLUSIONS

This paper presents an analysis dedicated to the interaction between two cracks submitted to a mode I of loading. Based on a bibliography review and 2D then 3D F.E. modeling, the following main results have been obtained:

- A significant amount of solutions exist for mode I loading, aligned or co-planar cracks. For other configurations, the number of available results is much more reduced and specific F.E. models have to be developed;
- Regarding the integrity assessment question, the interaction is quantified here as the amplification of the SIF on the dominating crack tip (compared to an isolated crack submitted to the maximum equivalent stress intensity factor K_{eq}). In that case, the interaction becomes the effect of the smallest crack on the larger one;
- For mode I, it has been shown that interaction limits can be defined by simple geometrical formulae normalized by the size of the smallest crack. In that frame, two domains are defined here: a strong interaction domain corresponding to an interaction larger than 10% (i.e. an amplification of max K_{eq}) and a weak domain corresponding to an interaction between 10 and 2.5%.
- For elliptical cracks, a length correction is proposed based on the Irwin analytical solution. This correction consists in the calculation of an equivalent length which has been validated through the results from the bibliography and 3D modeling.

For strong interaction of 10%, the final limits for two elliptical cracks become finally:

$$S_{10\%} = 0.6 \min(2.a_{1\text{-}eq} \; ; 2a_{2\text{-}eq}) \;\&\; H_{10\%} = 0.7 \min(2.a_{1\text{-}eq} \; ; 2a_{2\text{-}eq}),$$

with: $a_{i\text{-}eq} = a_i / [E(a_i/c_i)]^2$, $2.a_i$ *and* $2.c_i$ *being respectively smaller and larger axes of the crack.*

The work presented here corresponds to a first phase for mode uniform mode I and for an elastic material behavior. It will be followed by a second phase dedicated to more complex cases with non-uniform loading and elastic-plastic behavior.

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