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REQUIREMENT OF ROCKING SPECTRUM IN CANADIAN NUCLEAR STANDARDS

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ABSTRACT

The Canadian Nuclear Standards CSA N289.1 (2008) and CSA N289.3 (2010) define the Design Basis Earthquake (DBE) for a Nuclear Power Plant (NPP) and also provide a generic response spectrum in this regard which can be scaled to suit a particular site. In NPPs on several occasions the analysis of a rocking object is required. In such situations, a designer has to go through the literature containing rocking theories by many authors. Rocking has not been discussed in the Canadian standards. In the absence of any guideline from the Canadian standards and in order to simplify the analysis a designer tends to rely on the reserve energy method equating the rocking object to a single degree of freedom (SDOF) oscillator with an equivalent frequency. The seismic acceleration pertaining to that frequency is obtained from an applicable response spectrum and is compared to the overturning acceleration. However, a rocking object is not a SDOF oscillator and its behavior cannot be predicted by a typical response spectrum representing the peak response of a family of SDOF oscillators to an earthquake. This paper compares the results obtained from the reserve energy method and from the rocking equation of motion and concludes that the simplification of rocking analysis can be achieved by establishing a rocking spectrum. Inclusion of guidelines in the Canadian standards with regard to rocking is warranted.

INTRODUCTION

According to CSA N289.3 (2010), the DBE of NPPs in Canada is represented by a response spectrum, similar to the NBK spectrum developed by Newmark, Blume and Kapoor (1973) and recommended by the USNRC (1973) regulatory guide 1.60. In addition to this, CSA N289.3 (2010) recommends that the sites located on the eastern coast (closer to the Atlantic ocean), may be rich in high frequency content and hence its effect on the seismic design should be evaluated. However, irrespective of the frequency content, the recommended response spectra are based on the response of a family of SDOF oscillators which do not serve the design or seismic evaluation requirements of typical free standing unanchored objects prone to rocking due to a seismic event in a NPP. Some of the typical free standing unanchored objects are scaffolds, tool cabinets, radiation shield of masonry walls, nuclear waste storage containers etc. Due to the complexity of operations, some of the free standing objects are sometimes left inside the concrete containment structures such as the reactor building between outages. In the past, such objects have been analyzed and evaluated by converting a rocking object into an equivalent SDOF oscillator as demonstrated by Wesley, Kennedy, and Ritcher (1980) on the basis of the reserve energy technique recommended by Blume (1960). As established by Makris and Konstantinidis (2003), it is erroneous to represent a rocking object as an equivalent SDOF oscillator in order to obtain its response from a response spectrum. They recommend establishing a rocking spectrum in order to truly predict the behavior of a rocking object subject to ground motion. This requirement is very essential in determining the beyond design basis capacity of an existing NPP where anchoring each and every object or reinforcing the existing masonry shielding walls is very cumbersome and sometimes economically prohibitive. In the light of the above facts, this paper examines the behavior of a rectangular rocking block and explores the differences between the two approaches. It is concluded that establishing a rocking spectrum is essential in order to facilitate the prediction of the behavior of a rocking object in a NPP.

ROCKING ANALYSIS BASED ON RESPONSE SPECTRUM

This method initially recommended by Wesley et al (1980) in the context of masonry structures is explained in Figure 1. As evident from Figure 1, this method is simple and hence popular because it relies on representing a rocking object by an equivalent SDOF oscillator in order to obtain the seismic acceleration from a given response spectrum. There are other sophisticated methods such as the one explained in ASCE 43-05 which too obtains the seismic acceleration from the response spectrum given in USNRC (1973) regulatory guide 1.60.

Figure 1 shows a rocking object with weight W, mass m, width 2b, height 2h, subject to force F at the center of gravity with the displacement d. The force displacement diagram (Figure 1(b)) shows the minimum force required to cause rocking as F_r, which would be more than F since the force required reduces as the displacement increases, opposite to the known behavior of a spring-mass oscillator. According to Wesley et al (1980), the area of the force displacement diagram of a rocking object is equal to the area of the similar diagram of a SDOF spring-mass oscillator for a given allowable displacement. Slope of this diagram leads to the equivalent stiffness k_{eq}. The mass of the object is known and hence the frequency of the SDOF becomes known which can be plugged in a given response spectrum (Figure 2) in order to obtain the seismic acceleration. This acceleration can be compared with the overturning acceleration leading to the pass/fail criterion. The method appears to be simple and practical. However there are many factors which cannot be simplified in order to obtain an equivalent SDOF. Some of them are listed below:

- The force required to cause displacement decreases as the displacement increases. This behavior is quite the opposite of a SDOF oscillator.
- Damping is unknown. As seen later, the rocking analysis leads to a damping parameter which is angle α.
- The inertial force and the restoring force due to the self-weight both act on the object against the motion whereas in a SDOF it is only the inertial force.

An example is solved below with this method taking the input from the response spectrum given in Figure 2. A rectangular block is considered with the following parameters –

1. The block is rigid.
2. No sliding is assumed. Only rocking without sliding is considered.
3. Aspect ratio h/b = 2
4. Vertical seismic acceleration is zero. Only lateral acceleration is considered.

Equating the overturning moment with the restoring moment leads to

$$F_r = \frac{b}{h}W \quad (1)$$

At displacement d, the area under the curve in Figure 1(b)

$$A = \frac{d}{2}\left[F_r + \frac{(b-d)F_r}{b}\right] = dF_r\left(1 - \frac{d}{2b}\right)$$

Equating it to the area in Figure 1(c)

$$\frac{F_{eq}d}{2} = dF_r\left(1 - \frac{d}{2b}\right)$$

Thus the equivalent force (with equivalent acceleration a_{eq}) is

$$F_{eq} = ma_{eq} = F_r\left(2 - \frac{d}{b}\right)$$

The equivalent stiffness can be calculated as

$$k_{eq} = \frac{F_{eq}}{d} = \frac{F_r}{d}\left(2 - \frac{d}{b}\right)$$

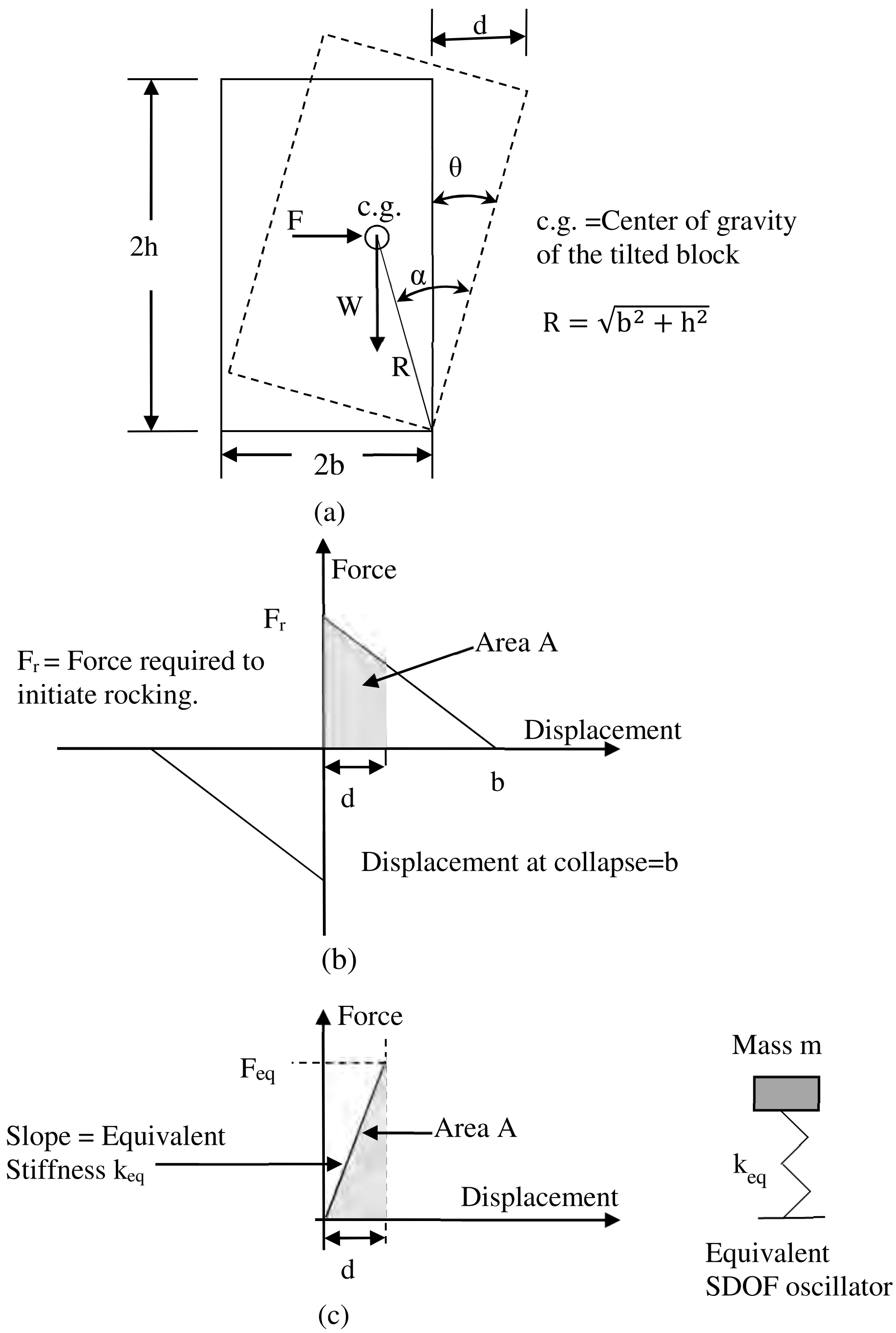


Figure 1. Rocking analysis by reserve energy method.

According to the above equation if b approaches infinity, equivalent stiffness would be maximum

$$k_{eqmax} = \frac{2F_r}{d}$$

Equivalent maximum frequency,

$$f_{eqmax} = \frac{1}{2\pi}\sqrt{\frac{k_{eqmax}}{\frac{W}{g}}} = \frac{1}{2\pi}\sqrt{\frac{2F_r g}{Wd}}$$

where g is the gravitational acceleration. Substituting for F_r from Equation (1) gives

$$f_{eqmax} = \frac{1}{\pi\sqrt{2}}\sqrt{\frac{bg}{hd}} \quad (2)$$

The above frequency depends on the geometric properties and the displacement. The allowable displacement for an object leads to the maximum frequency which can be input to the response spectrum in order to find acceleration leading to the maximum force. This maximum force can be compared with F_r in order to obtain pass-fail criteria. The maximum allowable displacement cannot be more than b which leads to the minimum frequency,

$$f_{eqmin} = \frac{1}{\pi\sqrt{2}}\sqrt{\frac{g}{h}} \quad (3)$$

The above equations establish that the equivalent frequency and acceleration are functions of displacement. Considering the block given in Table 1, a frequency-acceleration combination curve, for various values of displacement is drawn in Figure 2 which is superimposed on the response spectrum obtained from the seismic margin study of a NPP in Canada (Dar and Hanna (2012)) which has been scaled up to an arbitrary value to demonstrate its intersection with the rocking block curve. This intersection leads to the frequency-acceleration combination applicable to the block.

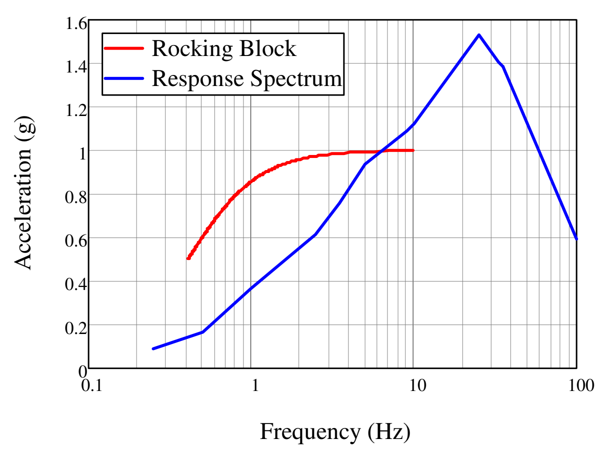


Figure 2. Rocking curve (by reserve energy method) and response spectrum

EQUATION OF MOTION OF A ROCKING OBJECT

Makris and Konstantinidis (2003) have considered a different approach which has been recognized by the standard ASCE 41-06. They recommend establishing rocking spectrum in order to obtain the rotation from the frequency parameter (as defined later) of a rocking object. The following equation of motion of a rocking object is considered in this approach, which is well known in the literature (Yim et al (1980)):

$$I_o\ddot{\theta} + mgR\sin(-\alpha - \theta) = -m\ddot{u}_g R\cos(-\alpha - \theta)\,, \qquad \theta < 0 \tag{4}$$

$$I_o\ddot{\theta} + mgR\sin(\alpha - \theta) = -m\ddot{u}_g R\cos(\alpha - \theta)\,, \qquad \theta > 0 \tag{5}$$

where I_o is the moment of inertia of the rigid block, $\ddot{u}_g$ is the ground acceleration and α $\left(= \tan^{-1}\left(\frac{b}{h}\right)\right)$ and R are defined in Figure 1(a). Rotation θ and ground displacement u are functions of time. The double dots represent the second derivative of a variable with respect to time. For a rectangular block($I_o = \frac{4}{3}mR^2$), the above equations (4 and 5) can be rewritten in the following format:

$$\ddot{\theta} = -p^2\left\{\sin[\alpha\,\mathrm{sgn}(\theta) - (\theta)] + \frac{\ddot{u}_g}{g}\cos[\alpha\,\mathrm{sgn}(\theta) - (\theta)]\right\}$$

where sgn is the signum function and $p = \sqrt{\frac{3g}{4R}}$ is the frequency parameter of the rectangular block. The solution to the above equation is found by state-space formulation (Makris and Konstantinidis (2001)) as given below:

$$\{y\} = \begin{Bmatrix}\theta \\ \dot{\theta}\end{Bmatrix} \tag{6}$$

with the time derivative vector as

$$\{f\} = \{\dot{y}\} = \begin{Bmatrix}\dot{\theta} \\ -p^2\left\{\sin[\alpha\,\mathrm{sgn}(\theta) - (\theta)] + \frac{\ddot{u}_g}{g}\cos[\alpha\,\mathrm{sgn}(\theta) - (\theta)]\right\}\end{Bmatrix} \tag{7}$$

Mathcad 15.0® was used to find the numerical solution to the above equation with the ODE solver based on Runge Kutta method. This solution includes series of solutions accounting for the energy loss due to the impact in every cycle of rocking. First, a solution is obtained with zero initial conditions and then impact is detected in this solution by locating the reversal in the direction of rotation. The solution beyond this point is discarded. The energy loss at this point is introduced by reducing the velocity as explained below. The reduced velocity and the rotation (= zero at impact) obtained from the first cycle of solution constitute the new initial conditions and the second cycle of solution is obtained. The impact is detected again, the second solution beyond this point is discarded, new initial conditions are noted and the third solution cycle is obtained. The process is continued till the target response time is reached or the object overturns or it dampens to zero amplitude. Adding all the cycles together leads to the total solution. Reduction in the angular velocity is obtained by incorporating the co-efficient of restitution given by Housner (1963) by equating the angular momentum before and after the impact.

$$r = \frac{\dot{\theta}_2^{\,2}}{\dot{\theta}_1^{\,2}} \tag{8}$$

where $\dot{\theta}_1$is the angular velocity before the impact and $\dot{\theta}_2$is the angular velocity after the impact and the coefficient of restitution r is given by:

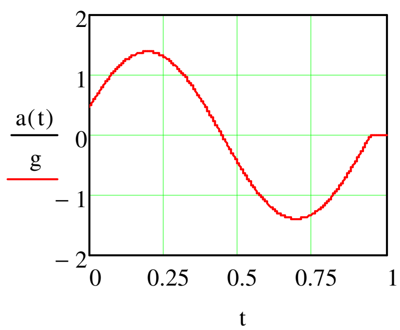
$$r = \left[1 - \frac{3}{2}\sin^2\alpha\right]^2 \tag{9}$$

For comparison of the two methods, sine pulse acceleration (Figure 3) is considered. The aspect ratio $\left(\frac{h}{b}\right)$ is considered as 2. The details of the block and the sine pulse acceleration loading are given in Table 1 and Figure 3 respectively. The reason for choosing sine pulse is the fact that it continues to increase in magnitude past the onset of rocking. The starting acceleration for the sine pulse is taken as the acceleration required to begin rocking.

Table 1: Details of rocking block

h (m)	b (m)	$\left(\frac{h}{b}\right)$	$R = \sqrt{b^2 + h^2}$ (m)	$p = \sqrt{\frac{3g}{4R}}$ (radians/s)	$\frac{2\pi}{p}$ (s)	$r = \left[1 - \frac{3}{2}(\sin\alpha)^2\right]^2$
1.5	0.75	2	1.677	2.094	3	0.49

Figure 4 shows the results of the rectangular rocking object (Table 1) with the different levels of acceleration. The rotation (θ) and angular velocity ($\dot{\theta}$) are shown as dimensionless parameters by dividing them by α and the frequency parameter p. It is observed that with the increase in acceleration (Figure 4(a) and (b)), the amplitude increases and the object overturns (Figure 4(c)). However, it starts rocking again at nearly double the overturning acceleration (Figure 4(d)). This phenomenon is very typical of a rocking block and cannot be envisaged if a rocking object is analyzed as a SDOF system.

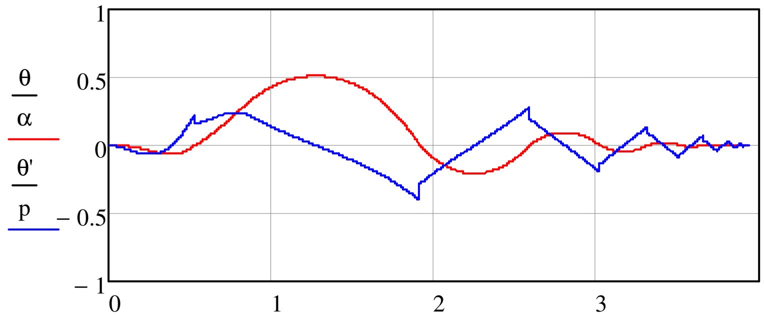


$$a(t) = a_p \sin(\omega_p t + \psi)$$
$$\omega_p = 3p$$
$$a_p = 3\alpha g$$
$$\psi = \sin^{-1}\frac{\alpha g}{a_p}$$
$$a(0) = \alpha g$$

Figure 3. Sine pulse loading.

ROCKING SPECTRUM AND COMPARISON OF METHODS

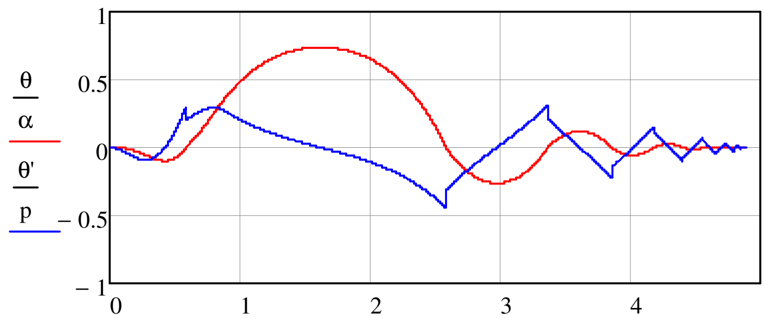
In order to compare the two methods of analysis, by the reserve energy method and by solving the rocking equations of motion, it is necessary to create a rocking spectrum for an earthquake represented by a SDOF response spectrum. CSA N289.3 (2010) recommends a generic response spectrum for the nuclear power plants but also recommends formulation of a site specific response spectrum for the east coast sites in Canada having high frequency content. However, no generic response spectrum is given by the standard in this regard. Figure 1 shows a scaled up east coast response spectrum. According to CSA N289.3 (2010), time history compatible to the response spectrum should be generated for the purpose of seismic analysis. An artificial time history was created for the response spectrum in Figure 1 by utilizing the software STAADPro v8i®. As explained later in Figure 6, utilizing the method given by Nigam and Jennings (1969) led back to the original response spectrum. A rocking spectrum was created (Figure 5) for this time history. As shown in Figure 5(c), no overturning takes place for any value of the frequency parameter for the given α=0.46 in Table 1. This is also visible in Figure 5(a) for the high frequency sine pulse in Figure 5(b). This is in contrast to the intersection of the rocking curve with the response spectrum in Figure 1 which predicts rocking and low displacement at approximately 6 Hz frequency. In reality the block would attract the zero period acceleration (ZPA) since it will not at all respond to the response spectrum in Figure 1.



(a) Rocking at $a_p = 1.4\alpha g$.

t

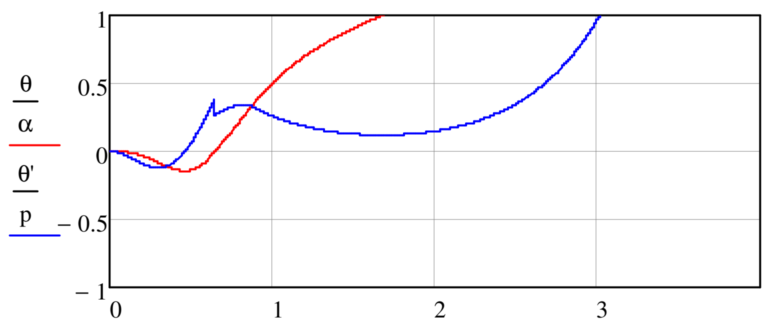
Time (Sec)



(b) Rocking at $a_p = 1.5\alpha g$.

t

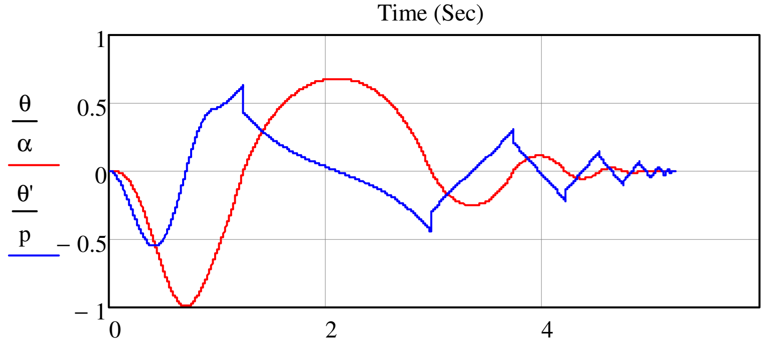
Time (Sec)



(c) Overturning at $a_p = 1.62\alpha g$.

t

Time (Sec)

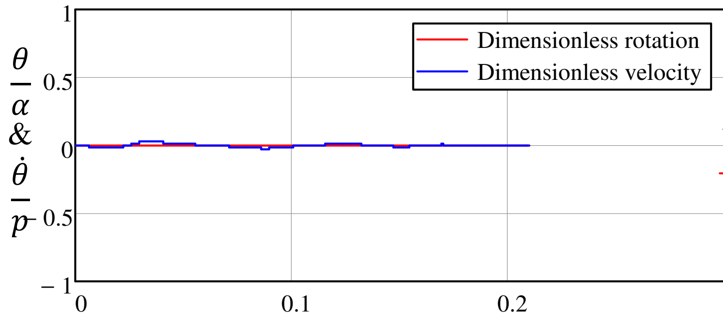


(d) Rocking resumes at $a_p = 3\alpha g$.

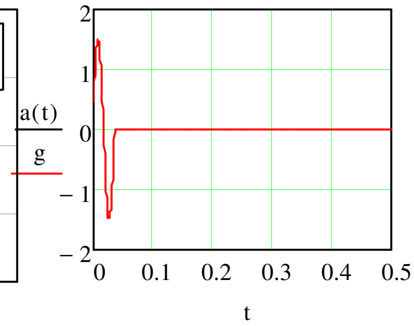
t

Time (Sec)

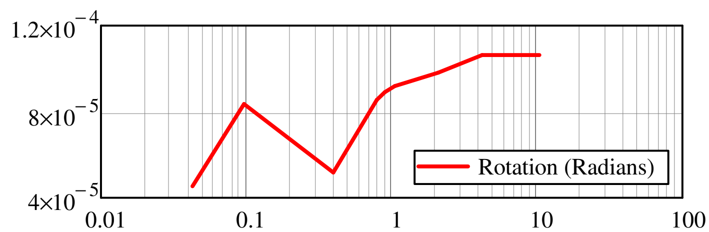
Figure 4. Rotation and angular velocities time histories due to sine pulse loading.



(a)



(b)



(c)

$$a(t) = a_p \sin(\omega_p t + \psi)$$
$$\omega_p = 80p$$
$$a_p = 3.2\alpha g$$
$$\psi = \sin^{-1}\frac{\alpha g}{a_p}$$
$$a(0) = \alpha g$$

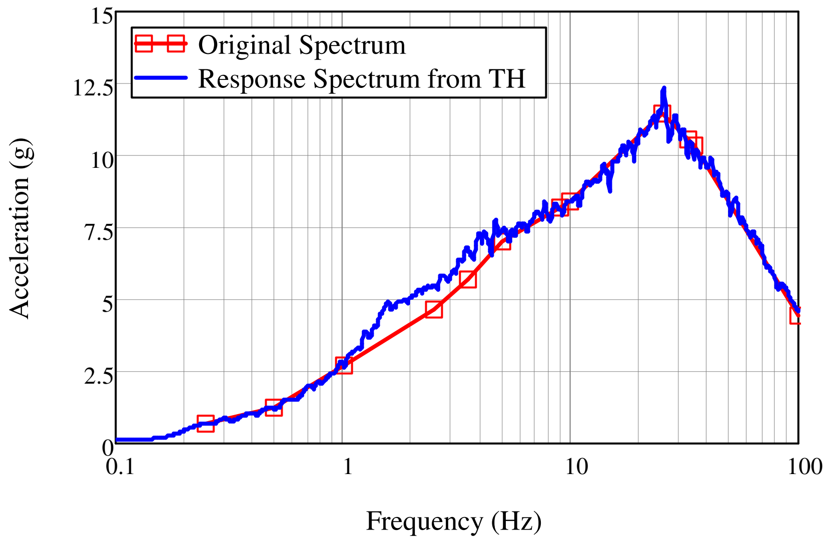
Figure 5. (a) Rotation and angular velocities time histories. (b) High frequency sine pulse.
(c) Rocking spectrum with negligible rotation

In order to incorporate high levels of acceleration for the ease of obtaining rocking spectrum, the response spectrum in Figure 1 was further scaled up and a time history was obtained (Figure 6 (b)) by the software STAADPro v8i®. This time history was used to obtain a response spectrum by the method given by Nigam and Jennings (1969). As shown in Figure 6(a), both the response spectra are in agreement. This time history was used to create the rocking spectra given in Figure 7 by varying the damping parameter α and the frequency parameter p. The rocking spectra in Figure 7 are applicable to the ground motion depicted by the response spectrum in Figure 6(a). The vertical lines in the rocking spectra depict the corresponding overturning ranges to their left.

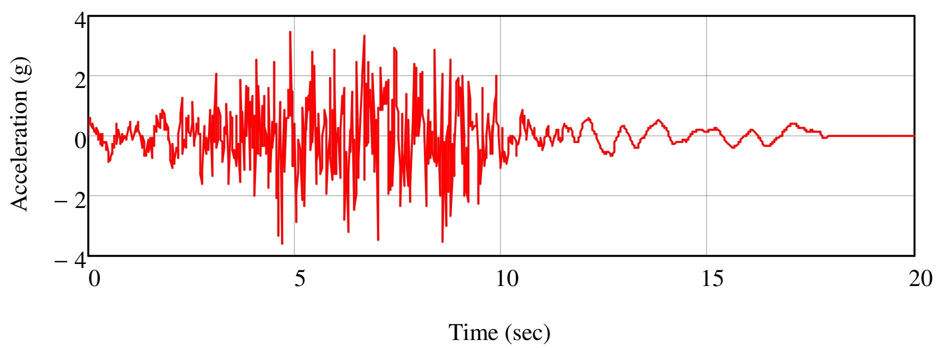
Comparison of Two Methods

The following points are observed in comparison of the two methods -

1. Looking at the equations of motion and their solution, it is clear that a rocking object cannot be represented by an equivalent SDOF oscillator.
2. Equation (1) considers the maximum applicable acceleration to be dependent on the width to height ratio, whereas it is observed that rocking can sustain more acceleration than this value.
3. The rocking spectrum gives immediate information about overturning. In Figure 7, the straight vertical lines depict the limit on overturning. This information is not directly given by the equivalent SDOF oscillator response spectrum.
4. In a rocking spectrum, the angle α and the frequency parameter p lead to the knowledge of the entire rocking behavior.
5. Reserve energy method does not account for a rocking object's geometry. Rocking spectrum is dependent on the frequency parameter which is governed by the geometry of the object.



(a) Original Response Spectrum and Response Spectrum from Time History



(b) Time History obtained by STAADPro v8i®

Figure 6. Original response spectrum (5% damped) utilized for the artificial time history compared with the one obtained by the method from Nigam and Jennings (1969).

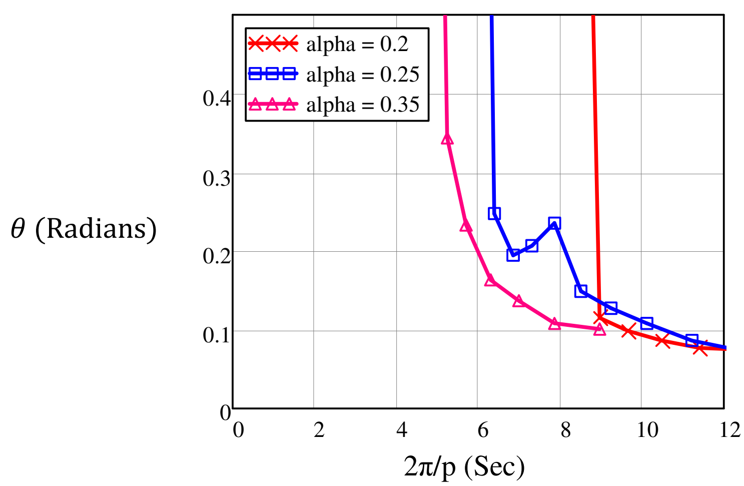


Figure 7. Rocking spectra for the time history in Figure 6.

CONCLUSION

It is concluded that a rocking object's response cannot be predicted by considering an equivalent SDOF oscillator. Canadian nuclear standards do not have any provisions in this regard and are required to address the issue of rocking directing the user of the standard to the correct methodology. Rocking has been considered in the standards ASCE 43-05 and ASCE 41-06. ASCE 43-05, in its examples, utilizes the response spectrum given in the USNRC (1973) regulatory guide 1.6. This requires further investigation. The rocking spectrum is a very useful tool that minimizes the time required in the analysis of rocking objects. Once determined for all the floors of a NPP, it can be very helpful in providing quick solutions to many routine rocking problems related to storage drums, tool cabinets, free standing scaffolds etc.

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