ABSTRACT

LEE, KIBOK. Universal Flying Restart Strategy for Asynchronous and Synchronous Motors. (Under the direction of Dr. Srdjan Lukic.)

This research focuses on the development of universal flying restart strategy for asynchronous and synchronous motors controlled by scalar control. In many industrial settings, momentary power disruptions commonly occur, resulting in tripping of large electric machines, which then have to be brought to zero speed before the machine can be restarted. This approach can result in frequent interruptions in an industrial process, which can have negative effects on productivity. A more practical control implementation would restart the machine back to the original speed as soon as power is restored, not having to wait for the machine to be at a standstill. This concept is known as flying restart. In addition, this method can be applied in areas such as pumps or fans which are initially rotating without being fed by an inverter. This dissertation presents the need of a method to achieve flying restart and proposes novel restart strategies for induction motor (IM), permanent magnet synchronous motor (PMSM) and synchronous reluctance motor (SynRM).

As the first topic, the basic principle of scalar control method is introduced for IM and PMSM. In case of PMSM, the scalar control may have an instability issue by external disturbance. To deal with this problem, a stabilizing loop is adopted from the literature that is capable of emulating the damping effect, thus stabilizing the machine. The stability of system is compared in two cases which are with the stabilizing loop and without the stabilizing loop. Simulation and experimental tests validate the results.

Secondly, the novel restart method for induction motor is presented. The biggest challenge is to develop a restart algorithm suitable for any induction machine, regardless of
motor parameter and ratings. To implement that, this proposed method uses only motor
parameters on the machine nameplate, which must be available for motor control
implementation. Our approach is to excite the machine at descending frequencies, starting
with the rated value, and determine the synchronous speed by calculating the input power
and the input power perturbation. Experimental results validate the performance of the
proposed algorithm.

The third contribution of this work is a novel restart method for synchronous motors
which includes the PMSM and SynRM. Unlike the induction motor which needs only the
synchronous speed, PMSM and SynRM require the rotor speed and position for
implementing a flying restart. The pulse method using zero voltage vector is suggested for
PMSM, where both the speed and the position can be extracted from the resulting current. In
case of SynRM, the one active voltage vector instead of the zero voltage vector is used for
rotor speed and position estimation. Both proposed methods use only motor parameters given
by the nameplate so that the universal application can be implemented. The performance is
verified through simulations and experimental tests.

In summary, the flying restart methods for the induction motor, the permanent magnet
synchronous motor and the synchronous reluctance motor are developed with the aim of
universal application and show the good performance. The detailed procedure for
implementing the proposed algorithm have been described.
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Universal Flying Restart Strategy for Asynchronous and Synchronous Motors

by

Kibok Lee

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DEDICATION

To my wife, Eunjee Lee.
BIOGRAPHY

Kibok Lee received the B.S. and M.S. degrees in electrical engineering from Korea University, Seoul, Korea, in 2005 and 2007, respectively. From 2007 to 2011, he worked at LG electronics Home appliance R&D Center, Seoul, Korea. Since August 2011, he has been pursuing his Ph.D. degree at North Carolina State University in Raleigh, North Carolina, where he is currently working as a Research Assistant at the Future Renewable Electric Energy Delivery and Management (FREEDM) Systems Center. His primary areas of interest are motor drives, power electronics, electric vehicles and systems for wireless inductive power transfer.
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CHAPTER 1: INTRODUCTION

1.1. Electric Motors and Drives Systems

Electric motors are widely used in many applications for precise position control and variable speed control, and in very wide power range from several watts to several of megawatts. In general, electric motors are used to convert electric energy into mechanical energy. Electric motors have been widely used in industrial applications. Industry is consuming more than 40% of total world electricity and about 70% of the electricity consumed by industry is being used by motors [1, 2].

The physical operating principle of electric motors is to produce the mechanical force by the interactions between a magnetic field and winding currents. Energy conversion by electromagnetic induction was demonstrated by the British scientist Michael Faraday in 1821. The first commutator DC electric motor was invented by the British scientist William Sturgeon in 1832. Several inventors followed William Sturgeon in the development of DC motors which can generate useful torque using DC electric power. In 1888, Nikola Tesla invented the first practicable AC motor such as synchronous motor, reluctance motor and wound type induction motor [3]. Soon after, the development of three phase power, distributed windings and squirrel cage type induction motor followed. The efficiency of electric AC motor is increased during the 19th century and these kind of motors have been widely used in both industry and residential applications [4].

In recent times, DC motors are generally used for speed or position control purposes due to the precise torque control capability. AC induction motors are the most widely used in industry because of robust and simple structure and low price. Electric motors are classified mainly as DC motors and AC motors corresponding on the input power type as shown in Table 1.

About 80% of electric motors used in industry is induction motors which can be directly connected to the grid. However, the energy saving has become an increased concern in industrial applications because of the increase of oil price and the issue of environment conservation. As a result, interest is rising in high efficiency electric motor drives, and efficient machine designs.
Table 1. Classification of the electric motors.

<table>
<thead>
<tr>
<th>Electric Motors</th>
<th>DC Motors</th>
<th>AC Motors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wound Field</td>
<td>Permanent Magnet</td>
<td>BLDC</td>
</tr>
<tr>
<td>• Compound</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>• Series</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>• Shunt</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
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<tr>
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</tr>
</tbody>
</table>

Abbreviations: BLDC (Brushless DC), IPMSM (Interior permanent magnet synchronous motor), SPMSM (Surface permanent magnet synchronous motor), SynRM (Synchronous reluctance motor), SynRM-PM (Synchronous reluctance motor-permanent magnet), SRM (Switched reluctance motor).

Nowadays, electric motor drives enable the motors to operate at higher efficiency and performance. The power switches of motor drives are controlled usually by a microprocessor or DSP (digital signal processor). These motor drives enable to use complex control methods for electric motors so that tight motion control and energy efficiency can be achieved.

1.2. Control Methods for Electric Motors

Fig. 1 shows the motor drive system which consists of AC/DC rectifier and DC/AC inverter. The electric motors can be controlled in different ways depending on the applications. This section introduces the scalar control and the vector control. The scalar control is divided in the open loop control and closed loop control, which is determined by the use of feedback such as the phase current. Vector control method which is suited for the higher performance applications is divided in the field oriented control (FOC) and the direct torque control (DTC).
1.2.1. Scalar Control

One control method of AC motors for variable speed application is the scalar control which is simple and needs a relatively low cost drive. The scalar control is a good control method in many applications such as fans, pumps, blowers and so on where high dynamic performance is not required. The performance of the scalar control method depends on motor parameters and the applied load condition. The feedback of the position and speed of the rotor are not required in the scalar control. In addition, scalar control does not need the high performance DSP as in the case of the vector control.

The principle of the scalar control is to maintain the constant ratio of the magnitude and the frequency of supply voltage in whole operating speed range. By controlling the $v/f$ ratio as constant, the stator flux of the motors can be maintained as constant in the steady state condition. Then, the motor is neither over-excited nor under-excited, and the motor can produce the same torque in whole operating speed range. This is the basic principle of the scalar control. The $v/f$ ratio is determined by the information of the motor nameplate such as the rated voltage and the rated speed. In general, the stator resistance voltage drop is compensated at low speed range.

This scalar control is used mainly for induction motors. However, PMSM drives that do not require fast dynamic performance can be also controlled in $v/f$ mode to ensure easier commissioning by the end-user. Only a few parameters about the machine are needed, such as the $v/f$ ratio and machine ratings. Any speed or position feedback is not needed for the operation of these schemes.
1.2.2. Vector Control

Another control method of AC motors for variable speed application is the vector control, which has the fast response and the precise position and speed regulation. The vector control is used in many applications such as a robot, a crane, in automotive industry and so on when high dynamic performance and high efficiency are required. Unlike the scalar control, vector control requires the feedback of the position and speed of the rotor. It can be obtained by using the mechanical sensors which increases the system cost and has the reliability issues. Therefore, many speed estimation algorithms (so called, sensorless algorithm) are developed to remove the use of the mechanical sensors. For these reason, the higher performance DSP for the complex calculation is required compared to those which are used for scalar control.

The vector control method is to control the magnitude, the frequency and the angle of supply voltage. Vector control enables torque and flux of the motor to be controlled independently. The torque current component and the flux component always have 90° degree spatially which is similar to the DC motor. Therefore, the instantaneous torque control is possible by using the several equations and the information of rotor position. To implement vector control, the d-q transformation is required. The speed and position estimation of the vector control is based on the back-emf and motor parameters including the inductance of the motor.

Although the vector control method has many advantages such as the high performance and the fast response, the vector control algorithm is more complicated and the system cost is more expensive than the scalar control.

1.3. Needs of the Flying Restart Algorithm

In many industrial settings, momentary power disruptions commonly occur, resulting in tripping of large electric machines, which then have to be brought to zero speed before the machine can be restarted. This approach can result in frequent interruptions in an industrial process, which can have negative effects on productivity. Flying restart is to start the machine back to the original speed as soon as power is restored, not having to wait for the machine to be at a standstill. In addition, this method can be applied in areas such as pumps or fans which are initially rotating at low speed without being fed by an inverter.
Momentary power disruptions can happen by many causes in an electricity network. It will cause a problem on electric equipment. It will depend on the magnitude and duration of the voltage drop and on the sensitivity of equipment. In general, many types of electric equipment which include the variable frequency drive for operating machines are sensitive to voltage drop. In face of such momentary power disruptions, the variable frequency drive for operating machines can generate the error, and then will stop feeding machines. Once the variable frequency drive is stopped, it has to restart machines from zero speed. This issue is especially problematic in applications with a large shaft inertia where it may take minutes for the machine shaft to reach standstill. It can result in frequent interruptions in an industrial process, which can have negative effects on productivity. In such applications, it would be beneficial to resume the machine operation as soon as power is restored. The flying restart that is capable of restarting in rotating condition can be one of solutions for these issues. The next section introduces the classification of momentary power disruption. In general, it is related to the duration and magnitude of voltage drop.

1.3.1. Momentary Power Interruptions [5]

<table>
<thead>
<tr>
<th>Voltage disruptions</th>
<th>Blackout</th>
<th>Momentary interrupt</th>
<th>Voltage sag or Brownout</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
<td>Few minutes ~ few weeks</td>
<td>Few seconds</td>
<td>3~10 cycles (voltage sag)</td>
</tr>
<tr>
<td>Duration</td>
<td>100%</td>
<td>100%</td>
<td>Few minutes (Brown out)</td>
</tr>
<tr>
<td>Voltage deviation</td>
<td></td>
<td></td>
<td>10 ~ 90%</td>
</tr>
<tr>
<td>Waveform</td>
<td><img src="image" alt="Waveform" /></td>
<td><img src="image" alt="Waveform" /></td>
<td><img src="image" alt="Waveform" /></td>
</tr>
</tbody>
</table>

Table 2. Classification of voltage disruptions.
Momentary power interruptions are brief disruptions in electric network. Momentary power interruptions are a complete loss of voltage and usually lasts no longer than a few seconds. It is caused by a temporary fault on a power line. The faults can be cleared by relay reclosing and the power can be restored quickly. It is different with a blackout that is the total loss of power and requires the repairs to restore power.

1.3.2. Voltage Sag

The voltage ‘sag’ is a decrease in voltage to between 10% and 90% of nominal voltage for one-half cycle to one minute [6]. Most voltage sags last from 50 to 170 milliseconds or from 3 to 10 cycles, and those sags does not drop below 50% of nominal voltage [7]. If it lasts for minutes or hours, it is called as brownout. Brownouts can cause issues with electric equipment that require certain voltage levels to function. Table 3 shows the number of voltage incidents per year [8].

<table>
<thead>
<tr>
<th>Voltage Deviation</th>
<th>The duration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt; 0.02 Seconds</td>
</tr>
<tr>
<td><strong>Voltage Swell</strong></td>
<td></td>
</tr>
<tr>
<td>110 – 120%</td>
<td>&gt; 700</td>
</tr>
<tr>
<td>106 – 110%</td>
<td>&gt; 700</td>
</tr>
<tr>
<td><strong>Normal Voltage</strong></td>
<td></td>
</tr>
<tr>
<td>87 – 106%</td>
<td>-</td>
</tr>
<tr>
<td><strong>Voltage Sag</strong></td>
<td></td>
</tr>
<tr>
<td>70 – 87%</td>
<td>&gt; 240</td>
</tr>
<tr>
<td>25 – 87 %</td>
<td>&gt; 240</td>
</tr>
</tbody>
</table>

There are several factors to cause a voltage sag. The external causes of a sag are lightning, animal and human activity, and normal and abnormal utility equipment operation. The internal causes of a sag are starting large electric loads such as motors which generate the large inrush current to start [7].
1.4. Research Objectives

The main objective of this thesis is to develop the universal flying restart algorithm which has the good performance. The universal flying restart algorithm means that it can be used in any motor drive systems such as a scalar control or a vector control. In addition, although a motor is replaced which means that motor parameters is changed, the universal algorithm will not require additional tuning procedures.

The conventional restart algorithms need the additional hardware or motor parameters such as the stator inductance. The use of additional hardware will limit the universal application. And several conventional methods use the motor parameters such as the stator inductance. These algorithms can be implemented only in motor drive systems using a vector control which generally requires the motor parameter information for the controller design. In case of the vector control, the parameter information is obtained by a self-commissioning or user measurement. On the other hand, motor drive systems using a scalar control method do not have motor parameters such as the stator inductance. Therefore, those conventional restart algorithms cannot be used in motor drive systems using a scalar control method.

This thesis will focus on the development of restart algorithm for the motor drive system using scalar control method which uses just the nameplate motor information such as the rated power, current, voltage, speed and so on. Then, the developed algorithm can be applied to a motor drive system using the scalar control as well as the vector control method. In addition, the developed algorithm does not require additional hardware besides the equipment installed in commercial inverters. And this algorithm includes the method to eliminate the need of additional tuning procedure. Therefore, the developed restart algorithm can be used in universal applications.

1.5. Outline of Dissertation

This thesis presents the development of restart algorithms for synchronous and asynchronous motors. The developed restart algorithms are to use only the nameplate information so that results in a universal application. To verify the performance of the proposed algorithm, the modeling and the simulations are performed with MATLAB and Simulink. The experimental tests are implemented with Opal-RT.
The dissertation is organized in the following manner:

Chapter 1 provides background and motivation of the development of universal restart algorithm for motors using variable-frequency drive.

Chapter 2 introduces the scalar control method for induction motor (IM) and permanent magnet synchronous motor (PMSM) which are used to implement the proposed restart method. Next, equations for the stability analysis of PMSM are derived and the stability analysis is implemented with simulation. In addition, the need of stabilizing loop for PMSM is explained and the damping effect of the stabilizing loop is investigated. To verify the result of stability analysis and the damping effect of stabilizing loop, the simulation and experimental tests are implemented.

Chapter 3 presents the novel restart algorithm for IM which uses the input power and the input power perturbation for searching the rotor speed. The relation between the rotor speed and the input power and input power perturbation is investigated in detail. And the important features of proposed restart algorithm are explained such like using only the motor nameplate information and no requirement of any tuning work. The performance of proposed restart method is verified with simulation and experimental results.

Chapter 4 describes the universal restart algorithm for PMSM. The conventional restart methods are explained and the disadvantage of the conventional methods is discussed. Then, this chapter suggests two effective restart algorithms minimizing the estimation error and eliminating the need of stator inductance information that result in a universal application for PMSM. The simulation and experimental results verifies the performance of the proposed restart method.

Chapter 5 presents the restart algorithm for synchronous reluctance motor (SynRM). This chapter proposes an effective restart algorithm that results in a universal application for SynRM. This method can eliminate the need of stator inductance information without installing the additional hardware equipment. And this method uses the simple method to estimate the rotor speed and position. In addition, the algorithm to minimize the speed estimation error is also suggested. The performance of proposed restart method is verified with simulation and experimental results.

Chapter 6 summarizes thesis and concludes the major contributions of this research and discussion for future work.
CHAPTER 2: IMPLEMENTATION OF SCALAR CONTROL

This chapter introduces the scalar control method for the IM and PMSM which will be used to implement the proposed restart method. It also includes stability analysis of the PMSM and presents the stabilizing loop to implement the frequency modulation method. The simulation and experimental results verify the stability analysis results and the damping effect of the stabilizing loop for PMSM. In this chapter, the scalar control method for SynRM is not included because it is almost similar to one of PMSM. And the stabilizing loop of PMSM can be applied to the scalar control of SynRM.

2.1. Scalar Control for the Induction motors

The operation of induction motor using the scalar control (v/f mode) is a popular method in variable speed drives. By controlling the v/f ratio as constant, the stator flux of the motors can be maintained as constant in the steady state [9-12]. Then, the motor is neither over-excited nor under-excited, and the motor can produce the same torque in whole operating speed range. This is the basic principle of the scalar control. Fig. 2 shows the equivalent circuit of induction motor at steady state. The stator flux linkage is expressed as:

\[ \lambda_s \propto \frac{E_s}{f} \]  

(1)

\( \lambda_s \) is the stator flux linkage and \( f \) is the stator frequency. As the speed is changed by the stator frequency control, the voltage should be adjusted in proportional to the stator frequency in order to maintain the constant flux. Then, the generated electric torque and the rotor current can be maintained as constant in whole operating speed range.
The electric torque can be simply calculated as ignoring the voltage drop by the stator leakage inductance.

$$ T_e = \frac{E_s^2}{(R_s/s_i^2 + (X_{s_r})^2)} R_s \omega_s = \left( \frac{E_s}{\omega_s} \right)^2 \frac{R_s \omega_s}{R_i^2 + (\omega_s L_s)^2} $$

The rotor current is also determined as:

$$ I_r = \frac{E_s}{\sqrt{(R_s/s_i^2 + (X_{s_r})^2)}} \left( \frac{E_s}{\omega_s} \right) \frac{\omega_s}{R_i^2 + (\omega_s L_s)^2} $$

From the above equations, the torque and the rotor current will be independent to the stator frequency by controlling the $v/f$ ratio as constant. To maintain the magnitude of stator flux linkage ($\lambda_s$) be constant, a simple
The stator voltage drop compensation method is required due to the voltage drop by the stator resistance. The applying voltage $V_s$ to the stator terminal is determined as [9-13]:

$$
V_s = R_s I_s \cos \phi + \sqrt{\frac{E_{\text{rated}}}{f_{\text{rated}}} f^*} + (R_s I_s \cos \phi)^2 - (R_s I_s)^2
$$

(4)

where $R_s$ is the stator resistance, $I_s$ is stator current, $\phi$ is the power factor angle, $E_{\text{rated}}$ is the magnitude of $E_s$ at the rated frequency $f_{\text{rated}}$ and $f^*$ is the reference frequency. It requires only the motor rating information, the stator resistance value and the measured phase currents. Although the current magnitude $I_s$ and the power factor $\cos \phi$ of (4) are steady state quantities, the instantaneous measured values can be used during the calculation of the voltage reference magnitude ($V_s$) [10]. The stator current magnitude $i_s$ can be obtained by measuring two phase currents as assuming the balanced system.

$$
i_s = \sqrt{\frac{1}{3} (i_{a*} + i_{b*})^2 + i_{c*}^2}
$$

(5)

The $i_s \cos \phi$ can be calculated by transforming the measure two phase currents to the stator voltage reference frame as:

$$
i_s \cos \phi = \frac{2}{3} \left[ i_{a*} \cos \theta_e + i_{b*} \cos \left( \theta_e - \frac{2\pi}{3} \right) + i_{c*} \cos \left( \theta_e + \frac{2\pi}{3} \right) \right]
$$

(6)

The angle ($\theta_e$) of the stator voltage vector in the stationary reference frame is the known value. Therefore, the stator voltage can be calculated by using (4)-(6).

In addition, the scalar control method for induction motor can also use a slip compensation method. It is used to compensate the difference between the actual speed and the reference speed caused by a load dependency [10, 14, 15]. However, the slip compensation method is not explained in this dissertation because it is not used in the implementation of restart method for induction motor in next chapter.
2.2. Scalar Control of Permanent Magnet Synchronous Motor

Permanent magnet synchronous motor (PMSM) in many industrial applications has replaced an induction motor due to their higher efficiency and power density [16, 17]. In many applications that high dynamic performance is not required [18-20], the PMSM is controlled in \( v/f \) mode to ensure easier commissioning by the end-user. Only a few parameters about the machine are required such like the \( v/f \) ratio and machine ratings. Any speed or position feedback is not needed for the operation of these schemes.

2.2.1. Electrical and Mechanical Equations of the PMSM

In general, the electrical equation of the interior-type PMSM (IPMSM) is expressed in the \( d-q \) rotor reference frame. Fig. 3 shows the equivalent circuit of the IPMSM in the rotor reference frame.

![Fig. 3. The equivalent circuit of the interior-type PMSM in the rotor reference frame; (a) \( d \)-axis circuit (b) \( q \)-axis circuit.](image)
The $d$-axis and $q$-axis stator voltages are obtained respectively as [16, 17, 21-25]:
\[
v_d = r_d i_d + \frac{d\lambda_d}{dt} - \omega_r \lambda_q \\
v_q = r_q i_q + \frac{d\lambda_q}{dt} + \omega_r \lambda_d
\] (7)

Where $v_d$ and $v_q$ are $d$-axis and $q$-axis stator voltages, respectively. $r_s$ is the stator resistance. $i_d$ and $i_q$ are $d$-axis and $q$-axis stator current, respectively. $\lambda_d$ and $\lambda_q$ are $d$-axis and $q$-axis stator linkage flux, respectively and $\omega_r$ is the electrical speed of the motor. The stator linkage flux ($\lambda_d$ and $\lambda_q$) are expressed as the following equations.
\[
\lambda_d = L_d i_d + \lambda_m \\
\lambda_q = L_q i_q
\] (8)

Where $L_d$ and $L_q$ are $d$-axis and $q$-axis stator inductance, respectively. $\lambda_m$ is the flux linkage generated by the permanent magnet. The electromagnetic torque can be derived with the input power of the motor. The input power of the motor in the synchronous reference frame can be expressed as:
\[
P_m = \frac{3}{2} (v_d i_d + v_q i_q)
\] (9)

Substituting (7) and (8) into (9) results in
\[
P_m = \frac{3}{2} \left[ r_d \left( i_d^2 + i_d^2 \right) \right] + \left[ i_d \frac{d\lambda_d}{dt} + i_q \frac{d\lambda_q}{dt} \right] + \omega_r \left[ \lambda_m i_q + (L_d - L_q) i_q i_q \right]
\] (10)

The first term is the stator copper loss, the second term is the loss of the change in electromagnetic energy storage and the final term is the mechanical output. As dividing the input power of (10) with the mechanical rotor speed, the electromagnetic torque equation can be obtained as:
\[ T_e = \frac{n}{2}\left[ \lambda_d i_d + (L_d - L_q)i_d i_q \right] \tag{11} \]

where \( n \) is the number of the pole. The surface mounted permanent motor (SMPM) does not have the saliency so that \( d \)-axis and \( q \)-axis stator inductance are equal. Therefore, the torque equation of SMPM can be obtained as eliminating second term of (11).

Sometimes, PMSMs having the damper winding in the rotor are used to assure the synchronization of the rotor with the stator voltage frequency. As considering the damper winding, the stator flux equations of (8) are replaced as:

\[ \lambda_d = L_d i_d + L_{md} i_D + \lambda_m \]
\[ \lambda_q = L_q i_q + L_{mq} i_Q \]

Where \( L_{md} \) and \( L_{mq} \) are \( d \)-axis and \( q \)-axis magnetizing inductance, respectively. \( i_D \) and \( i_Q \) are \( d \)-axis and \( q \)-axis damper winding current, respectively. The electromagnetic torque can be derived with the above same procedure.

\[ T_e = \frac{n}{2}\left[ \lambda_d i_d + (L_d - L_q)i_d i_q + L_{md} i_D i_q + L_{mq} i_Q \right] \tag{13} \]

Third and fourth terms in bracket are generated by the existence of the damper winding. As known in the principle of the induction motor, the damper winding current is determined by the slip of the stator voltage frequency and the rotor speed. In addition, the mechanical equation for motors can be expressed as:

\[ T_e - T_i = \frac{2}{n} J \frac{d\omega}{dt} + \frac{2}{n} B\omega \tag{14} \]

2.2.2. Closed Loop Scalar Control for PMSM

There are two different scalar control methods of an open loop [24, 26, 27] and a closed loop [28-32]. The open loop control method is simpler because any feedbacks such like phase current and a rotor speed are not
required to the controller. However, it needs a low startup frequency and a voltage-boosting for low-speed operation. In general, the open loop control method can be used just for induction motor. On the other hand, the closed loop control method can implement a higher control performance than an open loop control method and it can be used for driving PMSM as well as IM. In addition, it can provide a higher accuracy for determining the voltage reference with the phase current feedback to the controller. In this chapter, the closed loop scalar control method is only considered because it is more conventional method and one used in the proposed restart method.

2.2.2.1. Voltage Reference Calculation

The constant \( v/f \) control method used in the proposed restart algorithm maintains the constant flux linkage in order to avoid stator flux linkage saturation. In addition, the motor can produce the constant torque regardless of the applied stator frequency as maintaining the constant flux linkage [28, 33]. Fig. 4 shows the voltage vector diagram at steady state condition.

The magnitude of voltage vector can be calculated with the resistance voltage drop compensation as [28]:
\[ V_s = R_s I_s \cos \phi + \sqrt{E_s^2 + (R_s I_s \cos \phi)^2 - (R_s I_s)^2} \]  

(15)

Where \( \phi \) is the phase angle between the stator voltage and current vector. \( E_s \) is the voltage induced by the stator flux and set equal to \( E_{emf} \) in order to make the low stator voltage requirement and to minimize the no load current. Then, \( E_s \) can be determined as:

\[ E_s = E_{emf} = 2\pi f_e \lambda_m \]  

(16)

where \( f_e \) is the applied stator voltage frequency. The current magnitude \( I_s \) and the power factor \( \cos \phi \) can be obtained through the same equations used to determine the stator voltage of the induction motor. The instantaneous stator current magnitude \( i_s \) can be obtained by measuring two phase currents as assuming the balanced system.

\[ i_s = \sqrt{\frac{1}{3}(i_{a1} + i_{a2})^2 + i_{aw}^2} \]  

(17)

By transforming the measure two phase currents to the stator voltage reference frame, the \( i_s \cos \phi \) can be calculated as:

\[ i_s \cos \phi = \frac{2}{3} \left[ i_{aw} \cos \theta_e + i_{aw} \cos \left( \theta_e - \frac{2\pi}{3} \right) - (i_{aw} + i_{aw}) \cos \left( \theta_e + \frac{2\pi}{3} \right) \right] \]  

(18)

The angle (\( \theta_e \)) of the stator voltage vector in the stationary reference frame is the known value. By substituting (16)-(18) into (15), the magnitude of the stator voltage reference can be calculated.
2.3. Stability Analysis of \(v/f\) Control

This section presents the stability analysis of interior permanent synchronous motor (IPMSM) used in the next chapter 4 [23, 27, 28, 33-35]. The linearized model is used to analyze the stability of the IPMSM. The stability of the IPMSM can be evaluated by checking the eigenvalues of the linearized model. To simplify the stability analysis, this section considers only the IPMSM not including the damper winding. However, the analysis of motor having the damper winding can be done using the same procedure and the derivation is explained in detail in appendix B.

The electrical and mechanical equations were obtained in (7), (11) and (14). The load angle shown in Fig. 4, which is the angle between the applied voltage and the back-emf voltage, can be expressed as:

\[
p(\delta) = \omega_e - \omega_r
\]

where \(p\) is a differential symbol, \(\omega_e\) is the angular speed of the applied voltage and \(\omega_r\) is the rotor speed. With those equations, the following differential equations of IPMSM are written as:

\[
\begin{align*}
\frac{di_q}{dt} &= \frac{1}{L_q} V_e \cos \delta - \frac{r}{L_q} i_q - \frac{1}{L_q} \omega_e (\lambda_w + L_d i_d) \\
\frac{di_d}{dt} &= -\frac{1}{L_d} V_e \sin \delta - \frac{r}{L_d} i_d + \frac{1}{L_d} \omega_e L_d i_d \\
\frac{d\omega_e}{dt} &= \frac{3}{2J} \left( \frac{n}{2} \right)^2 \left[ \lambda_w i_q + (L_d - L_q) i_d i_q \right] - \frac{1}{J} B_m \omega_e - \frac{n}{2J} T_l \\
\frac{d\delta}{dt} &= \omega_e - \omega_r
\end{align*}
\]

These equations are nonlinear and can be solved only with the aid of a computer. Therefore, the linearized equations of the nonlinear system are required to analyze the stability of the machine. To obtain a linear model, a small-signal model which is linearized about a quiescent operating point is used. Then, the linearized differential equations can be expressed in this state-space form.
\[ p(x) = Ax + Bu \]  

(21)

In here, \( A \) is the state matrix, \( B \) is the input matrix, \( x \) is the state vector and \( u \) is the input vector. The state vector and the input vector include both the steady state value and the small signal value as shown in the following equation:

\[
x = X + \Delta x = \begin{bmatrix} I_q \\ I_d \\ \delta_0 \end{bmatrix} + \begin{bmatrix} \Delta I_q \\ \Delta I_d \\ \Delta \delta \end{bmatrix}; \quad u = U + \Delta u = \begin{bmatrix} 0 \\ 0 \\ T_l \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \Delta T_l \end{bmatrix}
\]  

(22)

where \( \Delta x \) is the perturbations for state variables \( x \) and \( \Delta u \) is the input perturbation. \( I_q, I_d, \omega_o \), and \( \delta_o \) are the steady state values at the operating point. \( T_l \) and \( \Delta T_l \) is the load torque and the torque perturbation, respectively. In here, \( \Delta V_s \) and \( \Delta \omega_e \) are considered as zero. If the small signal components are much small compared to the steady state values, the machine can be treated as linear systems with regard to small signal model. Then, the behavior for small variations around the quiescent operating point can be analyzed. The steady state values can be calculated by using DC equations with the given operating speed and torque.

\[
0 = \frac{1}{L_q} V_s \cos \delta_o - \frac{r_s}{L_q} I_q - \frac{1}{L_q} \omega_o (\lambda_m + L_d I_d)
\]

\[
0 = -\frac{1}{L_d} V_s \sin \delta_o - \frac{r_s}{L_d} I_d + \frac{1}{L_d} \omega_o I_q I_q
\]

\[
0 = \frac{3}{2} J \left( \frac{n}{2} \right)^2 \left[ \lambda_m I_q + (L_d - L_q) I_d I_q \right] - \frac{1}{J} B_m \omega_e - \frac{n}{2J} T_{l,0}
\]

\[
0 = \omega_e - \omega_o
\]  

(23)

As mentioned before, the magnitude of the stator flux vector is selected to be equal to that of the rotor permanent-magnet flux.

\[
\lambda_s = \sqrt{\left(\lambda_{qs}^*\right)^2 + \left(\lambda_{ds}^*\right)^2} = \sqrt{\left(L_q I_{qs}^*\right)^2 + \left(L_d I_{ds} + \lambda_m\right)^2} = \lambda_m
\]  

(24)
This equation can be expressed again with the function of the $I_q$ and $I_d$ as:

$$
(I'_q)^2 = \frac{-\left(I_d I'_d\right)^2 - 2\lambda_m I_d I'_d}{I_q^2}
$$

(25)

To find the $I_q$ and $I_d$ value, the torque equation of (23) is rewritten as:

$$
\left[\frac{4}{3n^2} \left(2B_m \omega_s + nT_i\right)\right]^2 = \left(I'_q\right)^2 \left(\lambda_m + (I_d - L_q I'_d)\right)^2
$$

(26)

By substituting (25) into (26), the following equation is obtained as:

$$
L_d (L_d - L_q) \left(I'_d\right)^4 + \left[2\lambda_m (L_d - L_q)\right]^2 + 2L_d \lambda_m (L_d - L_q) \left(I'_d\right)^3 \\
+ \left[4\lambda_m (L_d - L_q) + L_d \lambda_m^2\right] \left(I'_d\right)^2 + 2\lambda_m^3 I'_d + \frac{L_q^2}{L_d} \left(4 \frac{2B_m \omega_s + nT_i}{3n^2}\right) = 0
$$

(27)

By solving the fourth order equation (27), the steady state value of $I_d$ at the given speed and load torque condition is obtained. Then, the steady state value of $I_q$ corresponding to the $I_d$ value can be calculated by using (25). The $d$-axis and $q$-axis voltages can be calculated using (23) as:

$$
V_q' = V_s \cos \delta_0 = r_i I_q' + \omega_s \left(\lambda_m + L_d I'_d\right) \\
V_d' = V_s \sin \delta_0 = r_i I_d' + \omega_s L_q I'_q
$$

(28)

With the known steady state values, the small signal model can be expressed in this state-space form. When deriving a small signal model as substituting (22) into (20), second order terms which is the products of small signals can be neglected because it is assumed that the small signals are much smaller than the DC steady state values.
The final term $\Delta T_l$ of (29) is the $u$ vector which represents the forcing function. If $u$ is set equal to zero, the general solution of the linear differential equation become as:

$$x = e^{At}K$$  \hspace{1cm} (30)

The exponential $e^{At}$ represents the response of the system. If all roots of the characteristic equation of $A$ have negative real parts, the stability of the small signal model is assured. The characteristic equation of $A$ is defined as:

$$\det (A - \lambda I) = 0$$ \hspace{1cm} (31)

The stability analysis for the motor used in the restart test is done with the above explained procedure. The machine parameters for the simulation are given in Table 7. Fig. 5 shows the stability test results under different load conditions (no load, half load and full load).
Fig. 5. The loci of the poles corresponding to the rotor speed under different load conditions (a) no load condition (b) half load condition [12Nm] (c) full load condition [24Nm].

In these plots, the x-axis is the real part and the y-axis is the imaginary part. The stator poles of the PMSM are located on the left side of the plots. The rotor poles are located on the right side of the plots. It is because the electrical dynamics of machine are much faster than the mechanical dynamics of it. In the simulation, the applied stator voltage frequency is changed from 0Hz to 150Hz [elec.]. As increasing the applied voltage frequency, the rotor pole migrates to the positive plane under all load conditions. As a result, the simulation results show that the closed loop $v/f$ control method for the PMSM, which is used in the restart test of the next chapter 4, will be unstable and lose synchronism within specific speed ranges. For these reasons, the damping is added to improve the stability of the system. One solution is the damper winding in the rotor as mentioned.
before. However, it requires the physical modification in the rotor. Therefore, a more general solution is to add the damping to the system by using a stabilizing algorithm in the closed loop v/f control [18, 28-33, 36].

2.3.1. Improvement of System Stability by Using Frequency Modulation

In this section, the stabilizing loop which generates the frequency modulation signal is derived for PMSM not including the damper winding. The stability problem can be solved by introducing a frequency modulation method to increase the system stability [18, 28-33, 36]. The derivation of the stabilizing loop for PMSM including the damper winding is included in appendix C. As considering the small signal model of the PMSM, it is noticeable that the rotor speed perturbation generates the input power perturbation at the operation point [28, 31, 33]. If the copper loss and the electromagnetic energy storage are ignored, the input power perturbation can be written as:

\[
\Delta P_{in} = \left( \frac{2}{n} \right)^2 J_0 \omega_0 \frac{d\Delta \omega_r}{dt} + 2 \left( \frac{2}{n} \right)^2 B_0 \Delta \omega_r + \frac{2}{n} T_i \Delta \omega_r
\]

Therefore, the frequency modulation \( \Delta \omega_r \) of the stabilizing loop can be determined proportional to the input perturbation. It can be expressed as:

\[
\Delta \omega_r = -K_p \Delta P_{in}
\]

where \( K_p \) is the proportional gain. \( \Delta P_{in} \) can be extracted by using a high pass filter from the calculated input power through (15) and (18). It can be written as:

\[
\Delta P_{in} = \frac{s}{s + \frac{L_i}{R_i}} P_{in}
\]
where $\tau_h$ is the time constant of a high pass filter. To analyze the stability of the machine with the stabilizing loop, the differential form about the frequency modulation variable ($\Delta \omega_e$) should be derived. The differential form of the following equation can be generated as substituting (34) into (33).

$$\frac{d\Delta \omega_e}{dt} = -\frac{1}{\tau} \Delta \omega_e - K_p \frac{dp_m}{dt}$$  \hspace{1cm} (35)

The input power of the machine can be written as:

$$p_m = \frac{3}{2} \left[ v_q i_q + v_d i_d \right] = \frac{3}{2} V \left[ i_q \cos \delta - i_d \sin \delta \right]$$  \hspace{1cm} (36)

The differentiation of (36) results in

$$\frac{dp_m}{dt} = \frac{3}{2} V \left[ \frac{di_q}{dt} \cos \delta - i_q \sin \delta \frac{d\delta}{dt} - \frac{di_d}{dt} \sin \delta - i_d \cos \delta \frac{d\delta}{dt} \right]$$  \hspace{1cm} (37)

By substituting (20) into (37), the differential terms can be eliminated as:

$$\frac{dp_m}{dt} = \frac{3}{2} V \left[ \frac{1}{L_q} V \cos \delta - \frac{r}{L_q} i_q - \frac{1}{L_q} \omega_e (\lambda_m + L_d i_d) \right] \cos \delta - i_q (\omega_e - \omega_r) \sin \delta$$

$$\left[ -\left( -\frac{1}{L_d} V \sin \delta - \frac{r}{L_d} i_d + \frac{1}{L_d} \omega_d L_q i_q \right) \sin \delta - i_d (\omega_d - \omega_r) \cos \delta \right]$$  \hspace{1cm} (38)

By substituting (38) into (35), the final differential form of the frequency modulation can be obtained as:

$$\frac{d\Delta \omega_e}{dt} = -\frac{\Delta \omega_e}{\tau} - k_pr \frac{3}{2} V \left[ \frac{1}{L_q} V \cos \delta - \frac{r}{L_q} i_q - \frac{1}{L_q} \omega_e (\lambda_m + L_d i_d) \right] \cos \delta - i_q (\omega_e - \omega_r) \sin \delta$$

$$\left[ -\left( -\frac{1}{L_d} V \sin \delta - \frac{r}{L_d} i_d + \frac{1}{L_d} \omega_d L_q i_q \right) \sin \delta - i_d (\omega_d - \omega_r) \cos \delta \right]$$  \hspace{1cm} (39)
However, it still includes both the steady state and the small signal values. By using \( (22) \), the small signal of the frequency modulation can be derived as:

\[
\frac{d\Delta \omega}{dt} = k \cdot \frac{3}{2} \frac{V_s}{\tau k_p}
\]

As a result, the stability of closed loop \( v/f \) control including the stabilizing loop can be analyzed using all the differential equations obtained in \((29)\) and \((40)\). Fig. 6 shows the results of the stability analysis. The rotor poles under all different load conditions are located in the negative plane at all frequency range. It means that the operation of the PMSM becomes stable by adding the frequency modulation control.
However, the poles of the system can be moved to the negative plane when the proportional gain $K_p$ of the stabilizing loop is selected properly. To investigate the effect of the $K_p$ for the system stability, the closed loop transfer function of the small signal model is derived. The characteristic equation of the system including the stabilizing loop can be expressed as [28]:

$$s^2 + \left( \frac{B}{J} + \frac{2k_e \omega_0 K_p}{n} \right) s + \frac{k_e}{2J} \left[ n + \left( \frac{8}{n} \right) BK_e \omega_0 + 2T_n K_p \right] = 0 \quad (41)$$

Where $k_e$ is the slope of the load angle–torque curve at the operating point. The roots of (41) are obtained as:
From (42), if the $K_p$ is inversely proportional to the rotor speed, the poles of the system will be almost independent on the rotor speed. Therefore, $K_p$ gain can be selected as the following equation.

$$K_p = \frac{C}{\omega_b}$$  \hspace{1cm} (43)

Where $C$ is the constant value. To find the proper gain value, the stability of the system is again investigated as changing the constant $C$ of the stabilizing loop gain. Fig. 7 shows the analysis result under the rated speed and load torque.

Fig. 7. The loci of the poles corresponding to the stabilizing gain under the fixed speed and load conditions.
The simulation result shows that the system becomes unstable from the above 2.5 gain value. As considering additional stability tests under the different conditions, the stabilizing loop gain is selected as 0.8 value. To verify the stability analysis result of the small signal model, the step load test at the rated speed is done with the Simulink of MATLAB. Fig. 8 shows the result of the step load test, which the full rated load is applied at the rated speed. Although the step load is applied, the system maintains the stable condition like the result obtained in Fig. 6.

The additional test is done to verify the result of Fig. 7 as changing the stabilizing loop gain under the rated speed and load condition. When the stabilizing gain is selected as zero and the load is applied, the system starts having the mechanical and electrical oscillation. After the gain is changed to 0.8, the system become stable. When the stabilizing gain is increased to 3, the system loses synchronism.
Fig. 9. The stability result corresponding to the stabilizing gain under the rated speed and load condition.

2.4. Experimental Results

To verify the simulation results of Fig. 5 and Fig. 6, the experimental test are done under the no load condition. As shown in the simulation result of Fig. 5 (a), the experimental test of Fig. 10 (a) shows the same result that the rotor loses synchronism at high speed. In contrast, Fig. 10 (b) shows the stable operation in case of using the stabilizing loop, which is matched with the result of Fig. 6 (a).
Fig. 10. The stability result corresponding to the rotor speed (a) without the stabilizing loop (b) with the stabilizing loop; CH1: the phase current (10A/div), CH2: the rotor speed (400 rpm/div).

The test result of Fig. 11 verifies the simulation results of Fig. 7 and Fig. 9. The test is implemented as changing the stabilizing loop gain during operating at the fixed speed. The rotor is rotating at 1200rpm with the stabilizing loop having 0.8 gain. Suddenly, the stabilizing loop gain is changed from 0.8 to 0. The result shows that the rotor loses synchronism when the stabilizing loop gain becomes zero.
Fig. 11. The stability result corresponding to the stabilizing gain; CH1: the phase current (20A/div), CH2: the stabilizing gain, CH3: the rotor speed (400 rpm/div).

The additional tests are implemented to show the system stability corresponding to the load variation at different fixed speeds. The step load is applied to the motor at each speed and the motor maintains synchronism at every condition. These results also verifies with the simulation results of Fig. 6.

Fig. 12. The stability result with the step load test; CH1: the load torque (5Nm/div), CH2: the rotor speed (600 rpm/div.), CH3: the phase current (20 A/div).
2.5. Conclusion

This chapter explained the basic principle of scalar control methods for IM and PMSM which will be used to drive the machine when testing the flying restart algorithms. It also presented the stability analysis method for PMSM. In addition, the stabilizing loop to generate the frequency modulation signal was derived to add the damping effect to the system. The stability of the PMSM was analyzed in two cases which are with the stabilizing loop and without the stabilizing loop. Simulation and experimental tests have conducted to verify the stability analysis results and the damping effect of the stabilizing loop.
CHAPTER 3: A UNIVERSAL RESTART STRATEGY FOR INDUCTION MACHINES

3.1. Introduction

In this chapter, we propose an approach to implement the flying restart for Induction motor through a search algorithm which determines the rotating speed so that the correct voltage vector can be applied and thus minimize the inrush current during the restart. Beyond the development of the algorithm, implementation issues will be considered to provide general guideline for the application of the developed algorithm.

The goal of this work is to develop a flying restart controller that is capable of restarting an induction machine driving a high inertia load such as a fan or turbine. The proposed approach borrows concepts from similar work done on induction machines [37-40]. However, [37, 38] methods make use of the current controller to maintain the constant phase current during the rotor speed search, which requires a controller gain tuning depending on machine parameters such like the inductance and the resistance. In many applications where high dynamic performance is not required, the induction machine is controlled in \(v/f\) mode [9-13, 15, 40-46] instead of the complex vector [17, 21, 22, 25, 47-55] and DTC method [56-59] to ensure easier commissioning by the end-user. This \(v/f\) control method uses only a few parameters about the machine such as the \(v/f\) ratio and machine ratings. As a result, restart methods suggested in [37, 38] cannot be used in the \(v/f\) control mode. The approach described in [39] uses the DC current sensor which is not essential in driving an induction motor. In addition, the flow of DC current will be very small during the rotor speed estimation because it is proportional to the total power consumption, and will therefore cause the estimation error. The approach described in [40] is based on finding the phase angle between the stator current vector and the stator linkage voltage vector. This method will be very sensitive to the current sensing error thus requiring a large current to ensure precise estimation. In addition, the rotor speed searching time for these methods will depend on the controller gain and motor parameters. Therefore, these approaches may require algorithm tuning that is specific to the machine operating conditions.

In this chapter we propose an effective restart algorithm minimizing a stator current during the rotor speed estimation and eliminating the need to know the machine parameters and additional measurements such as a DC
current, which results in a universal restart algorithm for induction motors. In addition, the proposed restart method does not require any tuning work and the rotor speed searching time will be independent of machine parameters. The challenges in implementing the approach for IM stems from the fact that (1) the residual magnetizing flux is not present after a power interruption lasts several hundreds of milliseconds and (2) error in identifying the rotor speed can cause large inrush currents at the beginning of the restart process, and when the negative slip is applied to the motor, the braking torque is generated.

3.2. Proposed Restart Method for Induction Machine

As described earlier, the goal of this work is to develop an algorithm suitable for scalar control of IM that allows the machine to continue operating after a short power outage. This proposed restart method uses only the parameters known with the nameplate. It means that additional parameters information and the tuning procedure are not required for applying the restart algorithm. The proposed method to search the rotor speed requires just the applied stator voltage and the measured stator currents.

In developing the restart algorithm we have made the following assumptions: (1) during the outage, the drive loses power, but the controller has knowledge of the \( v/f \) ratio; (2) the controller recognizes the speed command prior to and after the fault and (3) the controller monitors the input power (i.e. recognizes when power was lost and when power was restored).

In this section the complete scheme for searching the rotor speed is described. To implement the proposed method, the input power and the input power perturbation are used and the calculation of these quantities is derived using the stator voltage and currents. To verify the validity of the proposed algorithm, the relation between the rotor speed and the input power and the input power perturbation is analyzed.

3.2.1. Rotor Speed Search Using the Input Power

The basic concept proposed herein is to excite the machine with a small constant voltage and variable frequency, starting from the rated and reducing at a constant rate towards zero, and to measure the resulting
machine currents. By using the applied voltage and the measured currents, the input power of machine can be calculated. When the applied voltage and the rotor speed are at the same frequency which means the zero slip, it will correspond to a near-zero input power. It means that the rotor speed can be simply detected by monitoring the input power.

In this section, the relation of the motor variable parameters (ex. the impedance, the current and the torque) and the slip is investigated and the derivation of the input power is explained in detail. From the machine equivalent circuit shown in Fig. 2, the input impedance can be represented as:

\[
Z_s = R_s + jX_{ls} + \left[ jX_m / \left( \frac{R_s}{s} + jX_{lr} \right) \right] = |Z_s| \angle \phi
\]

(44)

where \( s \) denotes the slip, \( X_{ls} \& X_{lr} \) is the leakage reactance of the stator and the rotor, \( X_m \) is the mutual reactance and \( \phi \) is the input power factor. The stator current is determined as:

\[
I_s = \frac{V}{Z_s} = I_m + I_r
\]

(45)

The stator current is the sum of the magnetizing inductance current and the rotor current. When the slip becomes zero \((s=0)\), the rotor current will be zero due to the infinite rotor impedance. Then, the input impedance \((Z_s)\) will be only expressed as the function of stator parameters. Fig. 13 shows the stator current corresponding to the stator frequency and the rotor speed. The stator current will be minimized when the slip is zero.
The rotor current can be obtained using the simplified equivalent circuit of induction motor shown in Fig. 14. The rotor current is calculated as:

\[
I_r = \frac{V_i}{(R_r + R_\phi / s) + j(X_p + X_m)}
\]  

(46)

It is noticed that the leakage reactance of (46) is proportional to the frequency and becomes very small in low frequency area so that the larger rotor current can flow in that area. In a negative slip area which the stator voltage frequency is slower than the rotor frequency, the rotor current becomes much larger than that of the positive slip area. This is because the real part of the impedance is cancelled out due to the negative rotor resistance in the negative slip. Therefore, as is clear from Fig. 13, in case the rotor is rotating at the low speed, the negative slip area should be avoided to prevent the overcurrent problem during the rotor speed searching.
Fig. 14. The simplified equivalent circuit of induction motor.

In addition, the airgap power and the mechanical power can be calculated by using the rotor current and resistance. The mechanical power can be obtained by subtracting the rotor power loss from the airgap power:

\[
\begin{align*}
p_{\text{airgap}} &= I_r^2 \frac{R_r}{1-s}, \\
p_{\text{mech}} &= I_r^2 \frac{(1-s)R_r}{s}.
\end{align*}
\]  (47)

The airgap power and the mechanical power will be zero in the zero slip region because the rotor current became zero. The input power consists of the motor losses, the change of the electromagnetic stored energy and the mechanical output power as expressed in the following equation.

\[
P_{\text{input}} = P_{\text{loss}} + P_{\text{stored}} + T_e \cdot \omega_r.
\]  (48)

The power loss \(P_{\text{loss}}\) and the change of the stored energy \(P_{\text{stored}}\) can be negligible around the zero slip because the rotor current is almost zero and the stator current is minimized as shown in Fig. 13. Therefore, the input power will be proportional to the mechanical power. The mechanical power becomes zero as defined in (47) when the slip is zero. As a result, the input power will be almost zero at the zero slip. Therefore, the rotor speed can be searched by detecting the zero input power.

The torque can be obtained by dividing the mechanical power with the rotor speed \(\omega_r\) as:
By substituting (46) into (49), the torque can be defined again as:

\[ T_e = \frac{1}{\omega_s} \frac{V_s^2}{\left( R_s + R_t / s \right)^2 + \left( X_{ds} + X_{qs} \right)^2} \frac{R_t}{s} (1 - s) \]  

(50)

If the rotor speed is assumed as the constant value, the torque is only the function of the slip. Fig. 15 shows the stator current and the torque corresponding to the stator frequency change in case a rotor speed is constant. In the area where the slip is low, the torque is proportional to the slip. On the contrary, the torque is inversely proportional to the slip in the high slip area. The input power curve corresponding to the slip will be almost similar to the torque curve of Fig. 15 if the variation of the rotor speed is not very large.

---

**Fig. 15. Torque curve corresponding to the slip.**
The basic concept of the proposed method for the rotor speed search is derived from this investigation. The input power required for finding the rotor speed is calculated as:

\[ P_{\text{input}} = \frac{3}{2} V_s I_s \cos \phi \]  

(51)

In here, \( I_s \) can be obtained by measuring two phase currents of stator and \( I_s \cos \phi \) can be calculated by transforming currents to the stator voltage reference frame.

\[ I_s \cos \phi = \frac{2}{3} \left[ i_{a_s} \cos \theta_e + i_{a_s} \cos \left( \theta_e - \frac{2\pi}{3} \right) - (i_{a_s} + i_{a_s}) \cos \left( \theta_e + \frac{2\pi}{3} \right) \right] \]  

(52)

where \( \theta_e \) is the angle of the stator voltage vector in the stationary reference frame. The instantaneous input power can be calculated using (51) and (52), and the \( V_s \) is a stator voltage used during searching the rotor speed. It is chosen as a small value considering the rated current for the machine to avoid the overcurrent.

Fig. 16 shows the rotor speed searching scheme using the input power. The stator frequency is initially set at the rated frequency, and the output of controller is added to the rated frequency. The input feedback of the integral controller is the input power calculated by (51). The stator frequency can be automatically adjusted by using a simple integral controller so that the input power can reach to zero point. At that time, the stator frequency will settle in the rotor speed. As a result, the rotor speed can be detected using this method.

Fig. 16. The rotor speed searching scheme using the input power.
However, the goal of the proposed method is to develop the universal restart algorithm for the induction motors. If the controller of the above scheme uses the fixed gain value, the frequency searching time will depend on the amount of input power which is the function of the slip, the applied voltage, motor parameters, the controller gain and the rotor speed. It means that the controller gain should be tuned every time depending on the conditions. It will restrict the universal application of the restart algorithm. In addition, the controller can be unstable on right side of the torque curve of Fig. 15 because it has the positive closed-loop feedback. Therefore, the additional monitoring parameter excepting the input power is required to resolve these issues. Then, the proposed method can be used universally to the restart of the induction motors. The next section suggests to use the input power perturbation and explains the derivation of that.

3.2.2. Need of Input Power Perturbation for Rotor Speed Search

As mentioned earlier, it is assumed that the power loss and the change of the stored energy are negligible and the rotor speed is constant due to a high inertia of the machine. Therefore, the input power curve will be proportional to the torque curve corresponding to the slip as shown in Fig. 15.

\[ p_{\text{input}} = T_e \cdot \omega_r \]  

(53)

Further, the perturbation of the input power is expressed as:

\[ \Delta p_{\text{input}} = \Delta T_e \cdot \omega_r + T_e \cdot \Delta \omega_r \]  

(54)

In here, \( \Delta \omega_r \) is almost zero because the rotor speed is assumed as constant during the rotor speed searching. Therefore, the second term of (54) can be neglected and the perturbation of the torque (\( \Delta T_e \)) will be proportional to the perturbation of the input power (\( \Delta p_e \)).

\[ \Delta T_e = k \cdot \Delta p_{\text{input}} \]  

(55)
When the input power perturbation is zero, the torque will be the maximum. As a result, the input power will be also the maximum when the input power perturbation is zero. $\Delta P_{\text{input}}$ can be extracted by passing the input power through a first order high pass filter.

$$\Delta P_{\text{input}} = \frac{s}{s + \frac{1}{\tau_h}} P_{\text{input}}$$

(56)

Where $\tau_h$ is the time constant of the high pass filter. The max torque can be detected by monitoring the input power perturbation as shown in Fig. 17. The simulation shows the input power, the input power perturbation and the torque corresponding to the slip. The parameters of induction motor in simulations is listed in appendix E.

Fig. 17. The torque and the input power at the zero input power perturbation.
The simulation result verifies that the torque and the input power are almost the maximum at zero input power perturbation point. Therefore, the stable area for the integral controller, which has the negative closed-loop feedback, can be also found as monitoring the input power perturbation. To resolve the above mentioned stability issue, the stator frequency is reduced with the constant slope in the unstable area and it is reduced with the integrator controller in the stable area.

The maximum torque of the conventional induction motor for fan or pump drive applications is generally generated near zero slip [22]. The classification of the induction motors was listed in Table 4 and the torque-speed curve of different type of induction motors is plotted in Fig. 18. The slip value generating the maximum torque can be obtained as differentiating (50) with slip.

\[
s_{\text{max}} = \frac{R_s}{\sqrt{R_s^2 + (X_{ds} + X_{ls})^2}}
\]

(57)

From this investigation, it is known that the rotor speed searching spends most time with the constant slope and spends just the short time with the integral controller. It means that the searching time can be less dependent on the motor parameters and it will depend on mostly the constant slope. The proposed algorithm used the 60Hz/sec constant slope considering the electric time constant and the first-order filter time delay used for the calculation of the input power and the input power perturbation. A very large constant slope makes the detection of the zero input power perturbation difficult due to calculation delays.

In summary, the proposed method uses the input power and the input power perturbation. At the start of the procedure, the stator frequency is reduced with constant slope from the rated frequency. After the input power perturbation crosses the zero point, the integral controller starts to adjust the stator frequency. The rotor speed searching time will be almost similar in all different conditions as using the constant slope. In addition, the integral controller will be used only for the short time in the stable area so that the controller gain will not affect greatly the speed searching time and the stability of the system. The proposed method used the fixed small gain (0.1) of integral controller in all test conditions. Therefore, the proposed method will not require gain tuning for
a wide range of induction machines. As a result, the proposed restart method can be used universally for induction motor restart.

3.2.2.3. NEMA Classification of Induction Motors

This section introduces the NEMA classification of induction motors. The induction motors are classified into four different categories as listed in Table 4. The four NEMA (National Electrical Manufacturers Association) designs have unique speed-torque relationships suitable to different type of applications. The most important factors to classify the type are the rotor resistance and leakage inductance. Fig. 18 shows the characteristic by torque-speed curve in each type.

As mentioned before, a flying restart algorithm is the most necessary for restarting an induction motors driving a high inertia load such as a fan or turbine. Conventional induction motors designed for fan or pump applications belongs to class A or B, and have the peak torque near zero slip [22]. As a result, the proposed restart algorithm spends most time in the constant slope area to detect the peak torque and short time in the area using the integral controller to determine the rotor speed.

<table>
<thead>
<tr>
<th>Types</th>
<th>Characteristics</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class A</td>
<td>Low starting torque, High starting current, low operation slip (s=0.005~0.015), High efficiency</td>
<td>Low starting torque load, Fans or pumps, Suitable to operate with inverter</td>
</tr>
<tr>
<td>Class B</td>
<td>Low starting torque, Low starting current (75% of design A), low operation slip</td>
<td>Suitable for constant speed Fans, blowers and pumps</td>
</tr>
<tr>
<td>Class C</td>
<td>Higher starting torque, Low starting current, low efficiency</td>
<td>Compressors, Conveyors</td>
</tr>
<tr>
<td>Class D</td>
<td>Highest starting torque, low starting current, high operation slip, low efficiency</td>
<td>Cranes, hoists, punch press High-impact load</td>
</tr>
</tbody>
</table>
3.3. Implementation of the Proposed Restart Method

The key performance criteria of the proposed restart method for an induction motor is to successfully estimate the rotor speed with only motor parameters given by the nameplate and restart the motors without causing the overcurrent and the braking torque. Fig. 19 shows the complete scheme for searching the rotor speed using the input power and the input power perturbation. The procedure for searching the rotor speed is explained as following steps.

- Step 1 – The frequency of stator voltage is set as the rated frequency. The applied stator voltage is increased gradually from zero. The voltage increase is stopped when the stator current reaches about 10% of the rated current.
- Step 2 – The stator frequency is reduced with the constant slope while applying the fixed small voltage.
• Step 3 – Monitoring the input power perturbation (it can be obtained by passing the input power through the high pass filter). Find the zero crossing point of the input power perturbation.

• Step 4 – When the input power perturbation is zero, the stator frequency is adjusted by using the integral controller instead of using the constant slope. Start to monitor the input power (it can be calculated by using the reference stator voltage & phase currents).

• Step 5 – After the input power reaches the zero point corresponding to almost zero slip, the integral controller will make the stator frequency settle at the rotor speed.

• Step 6 – After fixing the stator frequency, increase the stator voltage up to the rated v/f ratio.

• Step 7 – If the stator voltage is increased up to the rated v/f ratio, increase both the stator frequency and the stator voltage together keeping the v/f ratio, to bring the machine back to the reference speed.

In step 3, the zero crossing point of the input power perturbation will indicate the max torque and input power as shown in Fig. 17. In step 4, the input reference value of the integral controller is zero. It will make the input power become zero as the controller generates a proper output value. In step 5, the short settling time is required because an integral controller generally has the oscillation around the searched rotor speed. It will depend on the integral controller gain. In step 6, the rated v/f ratio is already the known value with the nameplate of the machine. In step 7, the stator frequency is increased up to the previous speed command value. It is assumed that the controller is recognizing the speed command prior to and after the fault.

Fig. 19 shows the complete scheme for determining the rotor speed. It is monitoring both the input power and the input power perturbation. When the input power perturbation is the positive, the switch of Fig. 19 turns on the upper side so that the stator frequency is decreased with constant slope. When the input power perturbation crosses the zero point, the switch moves to the lower side. The integral controller will start to adjust the stator frequency. While searching the rotor speed, the stator voltage command for the inverter uses the value calculated in step 1. After the input power approaches zero, the stator voltage command will increase as considering the rated v/f ratio.
3.4. Issue of the Residual Stator Voltage

Sometimes, a short power supply interruption from the distribution network can occur. In this case, the inverter power will be restored in shortly after the loss of power. When the rotor speed searching is started, the induction motor can be still energized because the residual stator voltage is not disappeared. If the inverter switching is started in this condition, it can cause the inrush current. The inrush current will depend on the motor parameters and the power interruption time. In this section we investigate this inrush current due to the residual stator voltage and the time the residual stator voltage remains.

Fig. 20. The equivalent circuit of the induction motor with the zero input voltage.
Fig. 20 shows the equivalent circuit when the zero stator voltage is applied. As mentioned before, the restart method is started the rotor speed searching with the rated frequency and the zero magnitude of the applied voltage. \( E_m \) of Fig. 20 is the voltage across the mutual inductance and it is determined as:

\[
E_m = \frac{d\lambda_m}{dt} = L_m \frac{d(i_s + i_r)}{dt}
\]  \( (58) \)

In general, the stator current become zero very fast because the stator current is freewheeled through the anti-parallel diodes of inverter and the DC-link capacitor while the inverter loses the power. Thus, the stator current is assumed as zero. Then, the mutual inductance voltage is mostly generated by the rotor current. The rotor current seen from the stator side will be defined as:

\[
i_r = I_r \cos(\omega_r t + \theta_0)
\]  \( (59) \)

When the rotor current is seen from the stator side, the frequency of rotor current will be approximately similar to the electric rotor speed \( \omega_r \). \( \theta_0 \) is the arbitrary initial rotor current angle when applying the stator voltage. By substituting (59) into (58), \( E_m \) can be expressed again as:

\[
E_m = L_m \frac{di_r}{dt} = -\omega_r L_m I_r \sin(\omega_r t + \theta_0)
\]  \( (60) \)

When the zero voltage is applied to the stator, the stator inrush current is calculated as:

\[
0 = R_s i_s + L_m \frac{di_s}{dt} + E_m
\]  \( (61) \)

The equation of (61) can be solved using the Laplace transform as:

\[
i_s(t) = -\frac{E_m}{R_s} \left(1 - e^{-\frac{R_s t}{L_m}}\right)
\]  \( (62) \)
In here, it is assumed that the rotor speed and the magnitude of rotor current are constant because it is considering the short instants which the zero stator voltage is applied. Because the peak inrush current is generated in the very short time, it is mostly determined by the stator leakage inductance as:

$$\frac{\Delta i(t)}{\Delta t} = -\frac{E_m}{L_s} e^{\frac{-t}{R_s L_m}} = -\frac{E_m}{L_s}$$

(63)

In general, the leakage inductance value of the induction motor is very small. Therefore, if the residual voltage is remained, the inrush current will be huge.

In addition, this section investigates the relation between the remained time of the residual stator voltage ($E_m$) and the motor parameters. Fig. 21 shows the vector diagram of the rotor flux in the rotor flux reference frame.

The rotating speed of the rotor flux is same with the stator voltage frequency.

The electrical equation of the induction motor can be expressed in the $d$-$q$ rotor flux reference frame rotating with the $\omega_r$ speed [17, 21, 22, 25, 47-55].
The \(d\)-\(q\) axis rotor flux equations are written as:

\[
v_{dr} = 0 = R_i i_{dr} + \frac{d\lambda_{dr}}{dt} - (\omega_r - \omega_e)\lambda_{qr}
\]

\[
v_{qr} = 0 = R_i i_{qr} + \frac{d\lambda_{qr}}{dt} + (\omega_r - \omega_e)\lambda_{dr}
\]

(64)

As shown in Fig. 21, the rotor flux is aligned in \(d\)-axis so that the \(q\)-axis rotor flux \(\lambda_{qr}\) is zero. The \(d\)-axis rotor voltage equation can be simplified and the relation between the rotor \(d\)-axis current \(i_{dr}\) and rotor flux \(\lambda_{dr}\) can be obtained as:

\[
\lambda_{dr} = L_i i_{dr} + L_m i_{qs}
\]

(65)

\[
i_{dr} = -\frac{p\lambda_{dr}}{R_r}
\]

(66)

By substituting (66) into the \(d\)-axis rotor flux equation of (65), the relation between the stator \(d\)-axis current \(i_{ds}\) and rotor flux \(\lambda_{dr}\) can be obtained as:

\[
\lambda_{dr} = \lambda_m + \frac{L_m}{L_i} i_{ds} = \frac{L_m}{1 + \frac{L_i}{R_r} p} i_{ds}
\]

(67)

From (67), it can be known that the rotor flux has the relation of \(L_m\) gain and \(L_i/R_r\) time constant \((T_r)\) delay with the \(d\)-axis stator current. Therefore, after the stator current become zero, it will take about several \(T_r\) time for the rotor flux to be almost zero. Typically, 10kW and 100kW induction motors take several hundreds of milliseconds and few seconds for the rotor flux to disappear, respectively [37, 39].

As a result, the inrush current can be easily occurred due to the small leakage inductance if the residual stator voltage is remained. And the remained time of the mutual flux will be dependent on the rotor parameters. While implementing the restart method for induction motors, if the inrush current is generated by the zero magnitude
voltage in step 1, the rotor speed searching should be stopped and the waiting time will be required for the rotor flux to disappear. That waiting time will be determined by the motor power rating which is known from the nameplate.

3.5. Simulation Results

In order to verify the restart algorithms explained above, a Simulink model was built, and some additional features that cannot be easily measured on the experimental tests were investigated. The needs for the correct speed estimation are investigated and the results for the Simulink simulation tests are given below.

Fig. 22 shows the restart simulation results that the rated stator voltage and frequency are applied to the motor at the unknown rotor speed condition. This result shows that the huge overcurrent was occurred at the beginning due to the higher magnitude voltage than what is expected and the high slip between the reference frequency and the actual rotor speed.

![Graphs showing simulation results](image)

Fig. 22. The restart method from the rated frequency.
And Fig. 23 shows the restart simulation result that the reference frequency was started with the zero frequency and was increased gradually with constant slope up to the rated frequency. As shown in the Fig. 23, the huge braking torque is generated at the beginning and it will have negative effects on the induction motor and its load. As a result, the rotor frequency searching algorithm is necessary for the induction motor to restart without the inrush current or the overcurrent issues.

![Graph showing speed, current, and torque over time](image)

**Fig. 23.** The slow restart method from the zero frequency.

Fig. 24 shows a simulation result including the rotor speed searching, the applied stator voltage, the stator phase currents, the input power, the input power perturbation and the electric torque. It is assumed that a fault occurred and it resulted in a temporary power loss. The motor is rotating due to a high inertia and the speed is reduced depending on the inertia, the friction and the load condition. At t=0s, the power is restored, and the frequency searching algorithm is started. After approaching the actual rotor speed, the applied voltage is adjusted to the rated constant \(v/f\) ratio, and the speed is increased gradually to the reference speed. It was taken about 1.5 seconds to settle back at the reference speed. In here, the rotor speed searching time is just taken about
200ms. If the increasing slope of frequency is selected as larger value, the time to settle back will be shortened. In the procedures to settle back to the reference speed, if the algorithms are optimized, the total restart time can be minimized.

Fig. 24. The proposed restart results with induction motor. Plots top to bottom: machine speed (green) and speed estimate (blue); machine phase current; input power; input power perturbation; electric torque.
3.6. Experimental Results

A set of experiments was conducted to validate performance of the proposed restart method. As shown in Fig. 25, the dynamo test bed consists of a commercial induction motor manufactured by ABB for the test purpose and an induction motor for supplying the load torque. The parameters for each motor are listed in Table 5 and Table 6, respectively. The \( v/f \) control and the proposed restart method are implemented using the OPAL-RT and the voltage source inverter (VSI). The inverter used is APS-100T120 having the rating of 1200V & 100A IGBT switches. The PWM switching and the current & voltage sensing frequency are set as 5 kHz. And the DC-link voltage of the inverter is fed by the DC power supply. The input DC voltage is 500V and is always maintained as constant. For the proposed restart control, only two phase currents sensors and a DC-link voltage sensor of the inverter are used. The load motor is fed by a commercial ABB voltage source inverter (VSI) and the torque & the speed were monitored using the analog output of the ABB inverter.
Table 5. The parameters of the induction motor for the test purpose.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>[kW]</td>
<td>$P_{out}$</td>
<td>7.5</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>[rpm]</td>
<td>$N_r$</td>
<td>1745</td>
</tr>
<tr>
<td>Rated Torque</td>
<td>[Nm]</td>
<td>$T_e$</td>
<td>47</td>
</tr>
<tr>
<td>Rated Voltage (line-line)</td>
<td>[V]</td>
<td>$V_s$</td>
<td>220 / 440</td>
</tr>
<tr>
<td>Phase Current (RMS)</td>
<td>[A]</td>
<td>$i_s$</td>
<td>30.8 / 15.4</td>
</tr>
<tr>
<td>Pole</td>
<td>-</td>
<td>$n$</td>
<td>4</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>[Ω]</td>
<td>$R_s$</td>
<td>0.608</td>
</tr>
<tr>
<td>Rotor Resistance</td>
<td>[Ω]</td>
<td>$R_r$</td>
<td>0.535</td>
</tr>
<tr>
<td>Magnetizing Inductance</td>
<td>[mH]</td>
<td>$L_m$</td>
<td>151.897</td>
</tr>
<tr>
<td>Stator Leakage Inductance</td>
<td>[mH]</td>
<td>$L_d$</td>
<td>3.869</td>
</tr>
<tr>
<td>Rotor Leakage Inductance</td>
<td>[mH]</td>
<td>$L_r$</td>
<td>5.824</td>
</tr>
<tr>
<td>Inertia</td>
<td>[Nm/rad·s$^2$]</td>
<td>$J$</td>
<td>0.054</td>
</tr>
<tr>
<td>Inverter DC Link Voltage</td>
<td>[V]</td>
<td>$V_{dc}$</td>
<td>500</td>
</tr>
<tr>
<td>PWM Switching Freq.</td>
<td>[kHz]</td>
<td>$f_{sw}$</td>
<td>5</td>
</tr>
<tr>
<td>Current Sampling Time</td>
<td>[μs]</td>
<td>$t_s$</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 6. The parameters of the induction motor for supplying the load.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>[kW]</td>
<td>$P_{out}$</td>
<td>7.5</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>[rpm]</td>
<td>$N_r$</td>
<td>1760</td>
</tr>
<tr>
<td>Rated Torque</td>
<td>[Nm]</td>
<td>$T_e$</td>
<td>40</td>
</tr>
<tr>
<td>Rated Voltage (line-line)</td>
<td>[V]</td>
<td>$V_s$</td>
<td>230 / 460</td>
</tr>
<tr>
<td>Phase Current (rms)</td>
<td>[A]</td>
<td>$i_s$</td>
<td>28.4 / 14.2</td>
</tr>
<tr>
<td>Pole</td>
<td>-</td>
<td>$n$</td>
<td>4</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>[Ω]</td>
<td>$R_s$</td>
<td></td>
</tr>
<tr>
<td>Stator Inductance</td>
<td>[mH]</td>
<td>$L_d$</td>
<td></td>
</tr>
<tr>
<td>Inertia</td>
<td>[Nm/rad·s$^2$]</td>
<td>$J$</td>
<td></td>
</tr>
</tbody>
</table>

To verify the need of both the constant slope and integral controller of Fig. 19, the experimental tests were implemented. Fig. 26 shows the results of the rotor frequency searching tests in case of only using the integral controller. First, the test motor was rotated at the fixed speed not fed by the inverter but used the speed control of the load motor coupled with the test motor. In this condition, the rotor speed searching algorithm was implemented. The purple line of plots is the actual speed of the motor. The integral controller gain was selected as 0.5. In first case (600 rpm) of three tests, the speed searching algorithm was failed due to the overcurrent. It was caused by the use of too large gain in the integral controller. In general, too large gain of controller can cause the overshoot from the set point value. As mentioned in section 3.2, the large negative slip by the
overshoot caused the large stator current. From these results, if the only integral controller is used in the rotor speed searching method, the integral controller gain selection will be very critical issue.

Fig. 26. The experimental results with using the integral controller for rotor speed searching (a) 600 rpm (b) 900 rpm (c) 1200 rpm; CH1: the phase current (1A/1V), CH2: the input power (10W/1V), CH3: the actual rotor speed (300rpm/1V), CH4: the reference speed (300 rpm/1V).
Fig. 27 shows the experimental results using a different integral controller gain value of 0.5 and 0.1. In case of using 0.1 gain, the rotor speed searching is succeed at 600 rpm condition unlike the 0.5 gain case. The small gain (0.1) reduced the overshoot of the stator frequency below the rotor speed so that the overcurrent is not generated. However, the problem is that the searching time of case using 0.1 gain is much longer than that of 0.5 gain case. These results know that the controller gain affects both the stability of the searching algorithm and the searching time. Therefore, the speed searching algorithm using only the integral controller requires the gain tuning as depending to the input power, the rotor speed and motor parameters. This restart method will not be appropriate for the universal application for induction motors. To resolve that, the proposed restart method suggested to combine the constant slope with the integral controller to search the rotor speed. It was mentioned in detail in section 3.2.2.

![Fig. 27](image)

(a) (b)

Fig. 27. The experimental results using a different integral controller gain (a) 0.5 gain (b) 0.1 gain; CH1: the phase current (1A/1V), CH2: the input power (10W/1V), CH3: the actual rotor speed (300rpm/1V), CH4: the reference speed (300 rpm/1V).

Fig. 28 shows the result of the input power ($p_e$) and input power perturbation ($\Delta p_e$) corresponding to the slip. The purple line is the stator voltage frequency. It was decreased with a constant slope from the rated frequency (60 Hz) to the actual rotor speed. The test motor was rotating at the constant speed (900 rpm) by using the speed control of the load motor coupled with the test motor. The red line is the input power and the green line is the input power perturbation. As explained in section 3.2.2, the input power ($p_e$) has almost the peak value, when the input power perturbation ($\Delta p_e$) was crossed the zero point. The slight difference between the peak input power and the zero input power perturbation is present due to the delay of the first-order low pass filter used to
eliminate the high frequency noise component of input power perturbation. Nevertheless, this test result is matched well with the analytical analysis result of section 3.2.2.

Fig. 28. The experimental results of the input power and the input power perturbation corresponding to the slip (900 rpm); CH1: the phase current (1A/1V), CH2: the input power (10W/1V), CH3: input power perturbation (10W/1V), CH4: the reference speed (300 rpm/1V).

Fig. 29 shows the results of the rotor frequency searching tests in case of using both the constant slope and the integral controller. The constant slope is 60Hz/seconds and the integral controller gain is 0.1. The gain used in Fig. 26 tests was 0.5. The small gain made the rotor speed searching algorithm become less sensitive to the integral controller. The test motor was rotating at the constant speed not fed by the inverter but used the speed control of the load motor coupled with the test motor. The purple line is the actual speed of the motor. By using the combined method of the constant slope and the integral controller, the rotor speed search was also succeed in 600 rpm condition and the speed searching time was almost constant (about 1 second) at every speed conditions unlike cases using only the integral controller.

As a result, the restart method using both the constant slope and the integral controller made it more stable than case of using only integral controller. It is because the integral controller is used only in stable area and the small gain of it does not generate large overshoot around the zero slip. The other important advantage is that the searching time is almost constant in all different conditions because it is mostly dependent on the constant slope. Therefore, this restart method can be used for the universal application for induction motors.
Fig. 29. The experimental results with using both the constant slope and the integral controller for rotor speed searching (a) 600 rpm (b) 900 rpm (c) 1200 rpm; CH1: the phase current (1A/1V), CH2: the input power (10W/1V), CH3: the actual rotor speed (300rpm/1V), CH4: the reference speed (300 rpm/1V).
Fig. 30 shows the test results of the complete restart algorithm. First, the motor is rotating at the reference speed. The inverter feeding induction motor is stopped by intentionally for 1.5 seconds. During that time, the motor speed is reduced depending on the inertia, the friction and the load condition. After 1.5 seconds, the speed searching algorithm is started. The stator voltage frequency is set the rated frequency (60Hz) and the magnitude of that is increased gradually from zero while monitoring the stator current. Next, the stator voltage frequency is reduced by using both the constant slope and integral controller mentioned before. Once the stator voltage frequency is approached to the rotor speed, which means that the input power becomes almost zero, the magnitude of the stator voltage is increased gradually to meet the rated \( v/f \) ratio. And the stator voltage frequency will go back to the former reference speed maintaining the rated \( v/f \) ratio. The test results verify that the proposed restart method can implement without causing any inrush current. In addition, the estimated speed (green line) is matched well with the actual speed (purple line).
Fig. 30. The experimental results with using both the constant slope and the integral controller for rotor speed searching (a) 900 rpm (b) 1200 rpm (c) 1500 rpm; CH1: the phase current (1A/1V), CH2: the magnitude of the stator voltage (10V/1V), CH3: the actual rotor speed (300rpm/1V), CH4: the reference speed (300 rpm/1V).
Fig. 31 shows the effect of the residual stator voltage. The inverter feeding induction motor is stopped by intentionally for 0.5 seconds and 1.5 seconds. In the first case (0.5s), there is the inrush current due to the residual stator voltage. Sometimes, the huge inrush current can be occurred depending on the motor parameters. Therefore, the restart method will need the inrush current protection logic. When the overcurrent is measured at the beginning of the speed searching, the inverter will automatically stop and spend the additional waiting time. As mentioned in section 3.4, the additional waiting time will be determined by the inverter rating or the motor rating power. In general, the nameplate gives the rating power of the machines. The delay of several hundred of milliseconds is used for 10kW and the delay of few seconds is used for 100 kW induction machines [37, 39].

![Graph](image1)

(a)

![Graph](image2)

(b)

Fig. 31. The experimental test results for the residual stator voltage (a) 0.5 seconds (b) 1.0 seconds; CH1: the phase current (1A/1V), CH2: the stator voltage (10V/1V), CH3: the actual rotor speed (300rpm/1V), CH4: the reference speed (300 rpm/1V).
3.7. Conclusion

This chapter described the proposed flying restart algorithm for induction motors in detail. The experimental results have conducted to validate the performance of the proposed method. The goal of proposed restart method was to develop the universal algorithm for induction motors. First of all, this proposed method uses only motor parameters known by the nameplate. In general, the nameplate includes the machine rating information such as the power, the current, the speed and the voltage. Secondly, this restart method does not require any tuning work and the rotor speed searching time will be almost constant although the motor is replaced. In addition, the proposed restart method is designed to be less sensitive to the motor parameters and other conditions (such as rotor speed, input power and so on) as monitoring the input power perturbation as well as the input power. Finally, this restart method does not require the additional hardware such as the speed sensors, phase voltage sensors, DC-link current sensor which are not installed in general commercial inverters.
CHAPTER 4: A UNIVERSAL RESTART STRATEGY FOR PERMANENT MAGNET SYNCHRONOUS MACHINES

4.1. Introduction

Another emerging trend in industrial applications is the replacement of induction machines with permanent magnet synchronous machines (PMSM), due to their higher efficiency and power density [47, 60]. In many applications that high dynamic performance is not required, the machine is controlled in V/Hz mode to ensure easier commissioning by the end-user [17-20, 26-33, 35, 61-66]. As a result only a few parameters about the machine are known, such as the V-Hz ratio and machine ratings.

The goal of this work is to develop a universal restart method that is capable of restarting a PMSM driving a high inertia load. The proposed approach borrows concepts from similar work done on induction machines [37-39, 67, 68] and more recently PMSM [69-72]. However, the method suitable for PMSM and presented in [69-72] makes use of the d-q transform and requires the knowledge of machine parameters, which can cause an estimation error due to a variation by temperature and current, to estimate speed and position information. In addition to those methods, high frequency injection methods can be used for a restart strategy [73-77]. In general, high frequency injection methods have been proposed for sensorless vector control and estimate the rotor speed and position even at zero speed. However, those methods require a demodulation process and an observer or a state filter so that it increases the complexity of restart algorithm. The challenges in implementing the approach for PMSM stems from the fact that (1) the magnetizing flux is always present in PM machines, (2) absence of damper windings can cause machine to lose synchronism, (3) slight error in identifying the speed/position of the back-emf can cause large inrush currents at the beginning of the restart process and (4) when a scalar control is used, machine parameter information is typically not known. Although machine parameters can be identified through self-commissioning [78], this is not done because scalar control does not require motor parameters.
In this chapter we propose an effective restart algorithm minimizing the estimation error and eliminating the need of stator inductance information so that result in a universal application for PMSM. The remainder of this chapter explains the basic concept existing restart method and describes the proposed restart algorithm in detail.

4.2. Basic Principle of Restart Method for PMSM

This section describes the basic concept of conventional restart methods to obtain the speed and position information used in restart algorithm. The concept of restart methods for PMSM is to excite the machine with a zero voltage vector and measure the resulting machine current [69-72]. The speed and the position of rotor are estimated by using the resulted current and motor parameter.

![Diagram of applying zero voltage vector to motor](image)

Fig. 32. Applying zero voltage vector to motor.

4.2.1. Conventional Rotor Speed Estimation Method

To estimate the rotor speed, the zero voltage vector \([1, 0, 1, 0, 1, 0]\) is applied to the motor as shown in Fig. 32. The other zero voltage vector \([0, 1, 0, 1, 0, 1]\) can be also used for that. Two zero voltage vectors are applied to
the motor for \( t_{\text{pulse}} \) time with the short delay time (\( \tau \)) between two pulses as shown in Fig. 33 (a). Fig. 33 (b) shows the current vectors generated by two zero voltage vectors.

![Diagram](a) Two zero voltage pulses and (b) the current vectors generated by two voltage vectors.

While applying the zero voltage vector to the motor, two phase currents are measured with current sensors. The resulting stator phase currents are transformed to the stationary reference frame as:

\[
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix} \begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

(68)

The angle (\( \theta_i \)) of each current vector in the stationary reference frame is calculated by using the following equation.

\[
\theta_i = \tan^{-1} \left( \frac{I_c}{I_a} \right)
\]

(69)

The first and second current vector angles are \( \theta_i \) and \( \theta_{i2} \), respectively. If the delay time between two pulses is short enough, the rotor speed can be assumed as the constant. Therefore, the speed of the rotor can be estimated by using two zero voltage vector pulses. The rotor speed can be defined as:
where $t_{\text{pulse}}$ is the time which the zero vector is applied to the stator winding, $\tau$ is the delay time between two pulses and $\omega_r$ is the electrical angular frequency of the rotor. When choosing the delay time ($\tau$), the rated speed of the rotor should be considered because the interval time should be shorter than the time which is spent for the rotor to rotate one time [71]. In other word, if the second pulse is applied after the rotor rotates the electric angle $2\pi$ [rad], the rotating speed of the rotor can be estimated incorrectly.

\begin{align}
\omega_{\text{real}} &= \frac{(\theta_{t2} + 2\pi \cdot N) - \theta_{t1}}{t_{\text{pulse}} + \tau} \quad \text{(Actual)} \\
\omega_{\text{est}} &= \frac{\theta_{t2} - \theta_{t1}}{t_{\text{pulse}} + \tau} \quad \text{(Estimation)}
\end{align}

where $N$ is the rotor rotation number between two pulses, $\omega_{\text{real}}$ is an actual speed and $\omega_{\text{est}}$ is the estimated speed using two pulses. The estimation speed will be just same with the actual speed when $N$ is zero. Therefore, the following equation should be satisfied to prevent the wrong speed estimation.

$$\theta_{t2} - \theta_{t1} = \omega_{\text{real}} \cdot \tau < 2\pi$$

(72)

In case a rotor direction information is also required, $2\pi$ of (72) should be replaced with $\pi$. Then, although the rotor is rotating in an opposite direction, the rotor angle and the rotation direction can be estimated correctly.

### 4.2.2. Conventional Rotor Position Estimation Method

Analyzing the machine equivalent circuit, the resulting equation describing the PM machine operation in rotor reference frame can be represented as [16, 17, 21-25]:

\[ \begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_i + pL_d & -\omega L_q \\ \omega L_d & R_i + pL_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega L_r \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]

(73)
where \( p \) denotes the derivative operator, \( R_s \) is the winding resistance, \( v_d \) & \( v_q \) are the \( d-q \) axis stator input voltage, \( i_d \) & \( i_q \) are the \( d-q \) axis stator current, \( L_d \) & \( L_q \) are the \( d-q \) axis stator inductance, \( \lambda_f \) is the magnet flux linkage, and \( \omega_r \) is the electrical angular frequency of the rotor. When applying a zero vector (\( v_d = 0, v_q = 0 \)), it is assumed that the applied time (\( t_{\text{pulse}} \)) is much smaller than the stator time constants (\( \tau_d = L_d/R_s, \tau_q = L_q/R_s \)). Then, the stator resistance can be neglected and the equation (73) can be simplified as the following:

\[
\begin{bmatrix}
0 \\
0 
\end{bmatrix} = \begin{bmatrix}
p L_d & -\omega_r L_q \\
\omega_r L_d & p L_q
\end{bmatrix} \begin{bmatrix}
i_d(t_{\text{pulse}}) \\
i_q(t_{\text{pulse}})
\end{bmatrix} + \omega_r \lambda_f \begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

(74)

As solving the equation (74) using Laplace transform, the resulting currents can be calculated as [70-72]:

\[
\begin{bmatrix}
i_d(t_{\text{pulse}}) \\
i_q(t_{\text{pulse}})
\end{bmatrix} = \begin{bmatrix}
-\frac{\lambda_f}{L_d} \left(1 - \cos \omega_r t_{\text{pulse}} \right) \\
-\frac{\lambda_f}{L_q} \sin \omega_r t_{\text{pulse}}
\end{bmatrix}
\]

(75)

The derivation is explained in detail in appendix A. In (75), the rotor speed was already estimated by two pulses using the equation (70) and \( t_{\text{pulse}} \) is the time which applies the zero voltage vector. And \( d-q \) axis stator inductance and \( \lambda_f \) are already the known motor parameters. Thus, \( d-q \) axis rotor reference stator current (\( i_d \) & \( i_q \)) can be calculated through (75). With those information, an angle difference (\( \theta_0 \)) between the stator current vector angle (\( \theta_i \)) and the rotor \( d \) axis angle can be derived as:

\[
\theta_0 = \tan^{-1}\left( \frac{i_q}{i_d} \right)
\]

(76)

By using the angle information obtained from (69) and (76), the rotor angle in the stationary reference frame can be estimated as:

\[
\theta_r = \theta_{r2} + \theta_0
\]

(77)
As using this two pulses method, the rotor speed and position can be estimated through the above explained procedure. However, these conventional restart methods have some issues to use as the universal application method for PMSM using scalar control method. First of all, those methods need the knowledge of $d$-$q$ axis stator inductance. In general, the inductance values are not required in a scalar control method and the motor nameplate also does not support those information. Secondly, the conventional methods are using the PWM switching time as the duty of zero voltage vector. Sometimes, the full duty of zero voltage vector can cause the overcurrent problem in case of having a very high B-emf due to a high rotor speed or a big magnet flux constant and having a small stator impedance. Finally, the $\theta_0$ angle estimation of conventional methods is sensitive to the motor parameter variation and is coupled with the estimated rotor speed. The $d$-$q$ axis inductance can be easily varied by the temperature and the stator current. And the estimated rotor speed can have some error due to the current sensing error. From these reasons, the rotor angle ($\theta_0$) can have the estimation error which can cause the failure at the restart instant. Therefore, conventional restart methods are not available for the universal application of PMSM and the other restart method should be considered.

4.3. Proposed Restart Method

As described earlier, the goal of this work is to develop an algorithm suitable for scalar control of PMSM that allows the machine to continue operating after a short power outage. In developing the algorithm we have made the following assumptions: (1) during the outage, the drive loses power, but the controller does not and (2) the controller has knowledge of the $v/f$ ratio, and any stabilization loop gains that were in effect during the machine operation prior to the fault; (3) the controller recognizes the speed command prior to and after the fault and (4) the controller monitors the input power (i.e. recognizes when power was lost and when power was restored).

In this section, we present an implementation of restart method for universal application, which does not require the additional information excepting motor parameters given by the nameplate. In addition, this method can automatically determine the duty of zero vector pulse considering the rated current to prevent an over-current.
and the delay time between two pulses considering the rated rotor speed and the acceptable estimation error. And the proposed method is developed to be less sensitive to the motor parameter variation and the rotor speed estimation error. We will also discuss the speed and the rotor angle estimation error by the current sensing error and the motor parameter variation.

4.3.1. Rotor Speed Estimation of the Proposed Method

Similar to conventional methods, the rotor speed can be easily estimated by using (69) and (70). And the time delay ($\tau$) between two pulses can be selected to satisfy (72). In addition, this proposed method suggests the minimum delay time between two pulses to limit the speed estimation error. First, the speed estimation uses the angle of two current vectors generated by two zero voltage vectors. Therefore, the cause of the speed estimation error is come from the estimation error of current vector angle ($\theta_l$) by the current sensing error. To calculate the speed estimation error, the maximum estimation error of current vector angle should be calculated in advance.

The maximum current vector angle error is calculated as considering the current measurement error. It is assumed the worst case that the current sensor has the 1% sensing error and 1% offset of rated current.

\[
\begin{align*}
    i_{\alpha m} &= i_\alpha + \Delta i_{\alpha \_error} + i_{\alpha \_offset} = I_m (1 + k_\alpha) \cos \theta_l + I_{\text{rated}} k_{\alpha \_offset} \\
    i_{\beta m} &= i_\beta + \Delta i_{\beta \_error} + i_{\beta \_offset} = I_m (1 + k_\beta) \sin \theta_l + I_{\text{rated}} k_{\beta \_offset}
\end{align*}
\]

(78)

where $i_{\alpha m}$ & $i_{\beta m}$ is the measured $\alpha-\beta$ axis current, $\Delta i_{\alpha \_error}$ & $\Delta i_{\beta \_error}$ is $\alpha-\beta$ axis current error, $I_m$ is the magnitude of current vector, $K_\alpha$ & $K_\beta$ is the error % of current sensor and $K_{\alpha \_offset}$ & $K_{\beta \_offset}$ is the DC offset % of current sensor. In general, the DC offset of current sensor is compensated before the motor is fed by the inverter. Therefore, the DC offset errors can be ignored in (78). The angle difference between the real current vector and the measured current vector is expressed as [72]:

\[
\begin{align*}
    \theta_l &= \tan^{-1} \left( \frac{i_\beta}{i_\alpha} \right) \\
    \theta_{lm} &= \tan^{-1} \left( \frac{i_{\beta m}}{i_{\alpha m}} \right) = \tan^{-1} \left( \frac{I_m (1 + k_\beta) \sin \theta_l}{I_m (1 + k_\alpha) \cos \theta_l} \right)
\end{align*}
\]

(79)
Fig. 34 shows the estimation error of current vector angle in case the current sensing has the 1% error of rated current. The maximum angle error is $0.6^\circ$ [deg] in the worst current sensing error condition.

![Diagram showing the estimation error of current vector angle]

Fig. 34. The estimation error of current vector angle due to the current sensing error.

With this result, the maximum speed estimation error can be expressed as:

$$\omega_{\text{est}} = \frac{\theta_{m2} - \theta_{m1}}{N_{\text{delay}} \cdot T_{\text{sw}}} = \frac{\theta_{m2} - \theta_{m1} + 2\Delta\theta_{\text{max}}}{N_{\text{delay}} \cdot T_{\text{sw}}} = \omega_{\text{r}} + \frac{2\Delta\theta_{\text{max}}}{N_{\text{delay}} \cdot T_{\text{sw}}}$$  \hspace{1cm} (80)

Where $N_{\text{delay}}$ is the number which divides the delay time ($\tau$) by the switching period time ($T_{\text{sw}}$), $\Delta\theta_{\text{max}}$ is maximum angle error and $T_{\text{sw}}$ is the switching period time of the voltage source inverter. The maximum speed of the motor is known in the nameplate and if the maximum speed estimation error ($\text{error}_{\text{speed}}$) is set, the range of the number ($N_{\text{delay}}$) can be determined as:

$$\frac{2\Delta\theta_{\text{max}} [\text{rad}]}{(\omega_{\text{max}} \cdot \text{error}_{\text{speed}})T_{\text{sw}}} < N_{\text{delay}} < \frac{2\pi}{\omega_{\text{max}}T_{\text{sw}}}$$  \hspace{1cm} (81)
In this dissertation, the 5% error ($error_{speed}$) of the rotor speed estimation is considered as a reasonable value. It was validated with the experimental test. The speed estimation error can be reduced as increasing $N_{delay}$ in the allowable range. The maximum number of (81) that is determined by using (72).

4.3.2. Rotor Position Estimation of the Proposed Method

Similar to existing restart methods, the angle estimation between the current vector and the rotor position is started from (75) and (76). As mentioned before, the stator resistance in the process deriving (75) was neglected and it will be validated with the simulation result. As substituting (75) into (76), the following equation can be obtained as:

$$\theta_0 = \tan^{-1}\left(\frac{i_q}{i_d}\right) = \tan^{-1}\left(\frac{L_d \sin \omega r t_{pulse}}{L_q \left(1 - \cos \omega r t_{pulse}\right)}\right)$$

(82)

It is noticeable that (82) is expressed as a function of two variables. One thing is the inductance ratio ($L_d/L_q$) and the other one is the rotor speed multiply the zero vector time ($\omega r \cdot t_{pulse}$). The angle between the current vector and the rotor position can be plotted as shown in Fig. 35.
In general, the ratio of $d$-axis and $q$-axis inductance of PMSM is under 5. If $\omega_r \cdot t_{\text{pulse}}$ is limited as a smaller value than 0.035, the angle between the current vector and the rotor position will be always above 85° [deg] for any PMSM. Therefore, although the angle between a current vector and a rotor position is considered as 90° [deg], the rotor position estimation error will be always below 5° [deg]. The basic concept of the rotor position estimation in the proposed method is come from this assumption. Therefore, (77) of the conventional restart methods can be replaced as:

$$\theta_{es} = \theta_{iz} + \pi/2$$  \hspace{1cm} (83)

In here, the current vector angle ($\theta_{iz}$) is calculated by sensing two phase currents when the second zero voltage vector is applied. The reason why 0.035 is selected is because the position estimation error of less than 5° [deg] is considered as a reasonable value. This assumption is validated with the experimental test.
4.3.3. Comparison of Rotor Position Estimation Error with Conventional Methods

As mentioned before, the conventional method [69-72] suitable for PMSM makes use of the $d$-$q$ transform and requires the knowledge of machine parameters, which can be varied by a temperature and a stator current. The theta estimation error by the $L_q$ inductance variation is investigated with the simulation. In general, the $L_q$ variation is more critical than the $L_d$ variation in the interior permanent motor (IPMSM). And the conventional method and the proposed method are compared with respect to the theta estimation error. The worst variation of $L_q$ is assumed as 100% of the initial value [72]. First, the error of conventional method can be calculated as:

\[
\begin{bmatrix}
    i_d(t_{\text{pulse}}) \\
    i_{q,\text{real}}(t_{\text{pulse}})
\end{bmatrix} = \begin{bmatrix}
    -\frac{\Delta L}{L_q} \left(1 - \cos \omega t_{\text{pulse}} \right) \\
    \frac{2 \omega}{L_q + \Delta L_q} \sin \omega t_{\text{pulse}}
\end{bmatrix}
\]

(84)

where $i_{q,\text{real}}$ is the $q$-axis current changed by the $L_q$ inductance variation, $\Delta L_q$ is the inductance variation. This one will be a real $d$-$q$ axis current value considering the parameter variation. With (84), the angle between the rotor and the current vector can be calculated as:

\[
\theta_{0,\text{real}} = \tan^{-1} \left( \frac{i_{q,\text{real}}}{i_d} \right)
\]

(85)

The obtained real theta of (85) will be compared with that of (76). The theta of (76) was obtained in ideal condition not considering the $L_q$ variation. Then, the estimation theta error by the $L_q$ inductance variation can be expressed as:

\[
\Delta \theta_{0,\text{error}} = \theta_{0,\text{real}} - \theta_0
\]

(86)

Fig. 36 shows the rotor position estimation error by the $L_q$ variation in conventional methods. The error is increased as the $L_q$ variation increases. The maximum error of the conventional method is about 20°[deg] when
the nominal value is 0.5 or 2. This maximum error can cause the failure by the overcurrent at the restarting instants. It will be verified with the experimental test in the next section.

Fig. 36. Rotor position estimation error by the $L_q$ variation in conventional methods.

In addition, the conventional method will also have the additional error source. As mentioned before, the estimated rotor speed ($\omega_r$) can get the error by the current sensing error. Therefore, it will also affect to the rotor position angle estimation.

$\Delta \omega_r$ is the speed estimation error by the current sensing error. The rotor angle estimation error will be dependent on the speed estimation error as:

$$\theta_{0,\text{real}} = \tan^{-1} \left( \frac{L_d}{L_q} \frac{\sin\left((\omega_r + \Delta \omega_r) t_{\text{pulse}}\right)}{1 - \cos\left((\omega_r + \Delta \omega_r) t_{\text{pulse}}\right)} \right)$$  (88)
By the inductance variation and the speed estimation error, the conventional rotor position estimation method can cause the error above $20^\circ$ [deg] in worst case which can cause the failure at the restart instant.

On the contrary, the proposed method assumed that the angle ($\theta_0$) between the rotor position and the zero current vector is always $90^\circ$ [deg]. Fig. 35 depicts the angle between the rotor position and the zero current vector as changing the ratio of $L_q$ and $L_d$. The ratio of $L_q$ and $L_d$ in PMSM is generally under 5. Although considering the worst case which the variation of $L_q$ is 100%, the ratio of $L_q$ and $L_d$ will be under 10. Then, the angle is always above $80^\circ$ [deg] because the $\omega r \cdot t_{pulce}$ is limited as a smaller value than 0.035. Therefore, the maximum error of the proposed method will be under $10^\circ$ [deg]. In addition, the proposed method does not use the estimated speed to calculate the angle ($\theta_0$). Thus, the error of speed estimation will not affect to the angle calculation. As a result, the proposed method is more robust to the $L_q$ inductance variation and the speed estimation error than the conventional methods.

4.4. Implementation of the Proposed Restart Method

The proposed restart method uses total four zero voltage pulses to estimate the speed and the position of rotor. Fig. 37 shows the overall estimation procedure using four zero voltage vector pulses. The procedure is explained as followings. In addition, Fig. 38 shows the flowchart of the proposed restart method.

![Fig. 37. The overall logic of the proposed restart method.](image-url)
Step 1 – Applying the first zero vector pulse having a small duty value $D_{init}$ (ex. 10% duty). Adjusting the duty by checking the resulted current ($I_{pulse}$).

Step 2 – $\alpha$ time is spent to wait for the pulse current to be attenuated to zero. The second zero vector pulse is applied with an adjusted duty ($D_1$). Then, the stator phase currents are measured at $t_1$ time. The angle ($\theta_{11}$) of the current vector is calculated by using (69).

Step 3 – After $\tau/2$ time is spent, the third zero pulse is applied with a half duty ($D_1/2$) to check a rotation direction. In here, the $\tau$ time is already determined using (81) with the rated speed and the switching frequency.

Step 4 – $\tau/2$ time is again spent from the third zero vector for applying a fourth zero vector pulse. The duty is the $D_1$ which is used in the second vector. In this step, the speed and the position of the rotor are estimated with (70) and (83).

Step 5 – The validation of the estimation is checked in this step. If $\omega_r \cdot t_{pulse}$ is a larger value than 0.035, the estimation is repeated from the step 2 with an adjusted duty ($D_{new} \leq 0.035/((\omega_r \cdot t_{sw}))$).

Step 6 – In the step 5, if $\omega_r \cdot t_{pulse}$ is a smaller value than 0.035, the restart is started after the stator current is attenuated to zero. The command voltage is calculated by using B-emf constant. After the voltage is applied to the motor, the stabilizing loop [28] of the v/f control is started from the next switching step with the current feedback.
In the step 1, the purpose of small duty pulse is only to determine the next pulse duty $D_1$. The reason using a small duty is to prevent the overcurrent during applying a zero voltage vector. The duty for next pulse can be adjusted by detecting the current increasing slope. The current slope is derived as:

$$K_{\text{slope}} = \frac{E_{\text{emf}}}{I_q} = \frac{\Delta I_{\text{pulse}}}{T_{\text{pulse}}} \left(= \frac{\Delta I_{\text{pulse}}}{D_{\text{pulse}} T_{\text{pulse}}} \right)$$  \hspace{1cm} (89)

In here, $K_{\text{slope}}$ is the slope of current variation, $E_{\text{emf}}$ is back-emf induced by the magnet flux linkage and $\Delta I_{\text{pulse}}$ is the variation of current magnitude during applying a zero voltage vector. The slope of current variation is assumed as a constant value because the zero vector pulse time is short enough to neglect the stator resistance.
Therefore, it is assumed that the current variation is linear and the pulse current is proportional to the duty. The new duty is derived to make the magnitude of current vector be about one-fifth of the rated current. It can prevent from the big braking torque and the overcurrent by too large current and the current sensing error by too small current compared to the current sensor rating. In the step 3, when checking the rotation direction, the very precise position information is not required. Therefore, a half duty of $D_1$ is used. In the step 4, the switching period delay and the current sampling delay are compensated to the rotor position estimation. The compensated position ($\theta_{\text{comp}}$) can be obtained as:

$$\theta_{\text{comp}} = \theta_{\text{est}} + \frac{\omega_{\text{est}} T_{\text{sw}}}{2} + \omega_{\text{est}} T_{\text{sw}}$$

(90)

The first term on the right side is the estimated rotor position calculated in (83). The second term is for a switching period delay compensation. Fig. 39 shows the compensated and uncompensated cases of the switching period delay. The blue line is the estimated theta of the rotor and the green line is the actual theta of the rotor. The estimated position of the rotor is updated at every switching period. Therefore, the estimated rotor position is delayed than the actual position of the rotor. To compensate this delay, the half of the switching period should be added to the rotor theta like the second term of (90).
Fig. 39. Switching period delay compensation (a) without compensation (b) with compensation.

The third term is for a current sampling delay compensation. Fig. 40 includes the zero voltage vector pulse, the actual current and the sampled current by the Opal-RT. As shown in the Fig. 40, the sampled current of the controller (Opal-RT) has the delay of one switching period than the actual current. After the controller generates the PWM, the current occurred by PWM is measured by current sensors and the values of that are reflected to the controller on next switching period. Therefore, the current sampling delay time should be compensated to the rotor theta like the third term of (90).
This proposed method can be available for universal application for PMSM because that does not require the additional information excepting motor parameters given by the nameplate and the duty of zero voltage vector pulses is automatically determined as considering the rated current to prevent an overcurrent. In addition, the position estimation error of the proposed method is less than that of the existing method used in [71] although the variation of inductance and the rotor speed estimation error by the current sensing error exist. The maximum estimation error will be under 10°[deg] as shown in Fig. 35 because the product of the rotor speed and the zero vector time ($\omega_r \cdot t_{pulse}$) is always smaller than 0.035. In addition, this method can automatically determine the delay time between two pulses considering the rated rotor speed and the acceptable estimation error.

4.5. Simulation Results

In order to verify the restart algorithms explained above, a Simulink model was derived, and some additional features that cannot be easily measured on the experimental setup were investigated. The needs for the correct position and speed estimation are investigated. And the results for these two important Simulink simulation tests are given below, the verification test for the assumption which neglected the resistance in (74) and the auto duty adjustment test.
4.5.1. Needs of the Correct Position and Speed Estimation.

The first simulation is done to investigate the effect of the wrong estimation. Fig. 41 shows the worst case which the applied voltage is in opposite direction with the B-emf vector. It will cause a huge inrush current in a stator winding. The current will determined by the following equation.

\[ V_s - E_{emf} = R_s i_s + L_s \frac{di_s}{dt} \]  \hspace{1cm} (91)

In (91), the applied stator voltage is added with the B-emf voltage generated by the rotor rotating flux. The huge overcurrent can be easily generated at the high speed condition because the applied voltage and B-emf is proportional to the rotor speed.

Fig. 41. Worst case inrush current due to imprecise determination of the position of the back-emf vector.

Fig. 42 shows the restart simulation results in two cases. Fig. 42 (a) is the simulation result having the wrong position estimation. The simulation includes the actual rotor speed and the applied voltage reference speed, the phase current, the actual theta of the rotor and the estimated theta. In this simulation, the huge inrush current is generated although the speed estimation was correct. Fig. 42 (b) shows the case which the speed and the position of rotor are estimated correctly. Unlike a result of Fig. 42 (a), the overcurrent and the speed oscillation are not generated at the restart instant.
Fig. 42. Restart simulation waveform (a) with wrong rotor position estimation & with correct speed estimation (b) with the correct position and speed estimation; CH1: the actual speed and the applied voltage speed, CH2: the phase current, CH3: the actual theta of the rotor, CH4: the estimated theta of the rotor.
4.5.2. Verification of the Assumption ($R_s$ is ignored in the derivation for the position estimation)

When the rotor position is estimated in the proposed restart method, there is one assumption that an applied zero vector pulse time ($t_{pulse}$) is much smaller than the stator time constants. With that assumption, the stator resistance is neglected in (74). It is verified by using simulation results. The simulation is implemented at rated speed condition with real parameters of PM motor used for the experimental test. Table 7 shows the PM motor parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>[kW]</td>
<td>$P_{out}$</td>
<td>12</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>[rpm]</td>
<td>$N_r$</td>
<td>3000</td>
</tr>
<tr>
<td>Rated Torque</td>
<td>[Nm]</td>
<td>$T_e$</td>
<td>24</td>
</tr>
<tr>
<td>Phase Current (RMS)</td>
<td>[A]</td>
<td>$i_s$</td>
<td>23.4</td>
</tr>
<tr>
<td>Pole</td>
<td>-</td>
<td>$n$</td>
<td>6</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>[Ω]</td>
<td>$R_s$</td>
<td>0.12</td>
</tr>
<tr>
<td>Stator d-axis Inductance</td>
<td>[mH]</td>
<td>$L_d$</td>
<td>1.04</td>
</tr>
<tr>
<td>Stator q-axis Inductance</td>
<td>[mH]</td>
<td>$L_q$</td>
<td>1.50</td>
</tr>
<tr>
<td>Magnet Flux</td>
<td>[vs/rad]</td>
<td>$\lambda_m$</td>
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</tr>
<tr>
<td>Inertia</td>
<td>[Nm/rad·s$^{-2}$]</td>
<td>$J$</td>
<td>0.059</td>
</tr>
<tr>
<td>Bemf (line-line)</td>
<td>[V]</td>
<td>$E_{emf}$</td>
<td>336</td>
</tr>
<tr>
<td>Inverter DC Link Voltage</td>
<td>[V]</td>
<td>$V_{dc}$</td>
<td>500</td>
</tr>
<tr>
<td>PWM Switching Freq.</td>
<td>[kHz]</td>
<td>$f_{sw}$</td>
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</tr>
<tr>
<td>Current Sampling Time</td>
<td>[μs]</td>
<td>$t_s$</td>
<td>200</td>
</tr>
</tbody>
</table>

In the below simulation, the blue line is the result of (73) which the stator resistance is not neglected. The green line is the result which the resistance is neglected like the assumption of the proposed method. If the applied time of zero voltage vector is set under 5% relative to the stator time constant, the green line are almost consistent with the blue line. In general, the stator time constant is several tens of millisecond and the inverter PWM switching time is shorter than a millisecond. The stator time constant of test PM motor and the inverter switching time have 12.5ms and 0.2ms, respectively. Therefore, the assumption of (74) is available and the stator winding resistance can be neglected when calculating the pulse current and rotor position.
4.5.3. Duty Adjustment of the Zero Voltage Vector Pulses.

The proposed restart method is including the duty adjustment function of zero voltage vector pulse. The purpose of the first pulse is only used to calculate the next pulse duty. The duty for next pulse can be adjusted by detecting the slope of the increasing current as discussed in section 4.4. The reason using a small duty in the first duty is to prevent the overcurrent during applying a zero voltage vector. Fig. 44 shows the simulation result of the duty adjustment of zero voltage vector. The rotor speed is set as 3000 rpm which is the rated speed of test motor. The first zero voltage vector is applied with the 10% duty and the magnitude of current vector corresponding the zero vector was 3.6A. This magnitude of current vector was matched well with the calculation by (89). With the measured magnitude value, the duty of next zero voltage vector is automatically
changed as 18% in order to make the current vector magnitude be one-fifth of the rated current. As mentioned in section 4.4, the duty of third zero vector is the half of second zero vector. And the duty of fourth zero vector is again 18%. With these four zero vectors, the speed and the position of rotor are calculated and the motor can be restarted.

The main advantage of suggested duty adjustment algorithm is to prevent the overcurrent due to the large duty during applying the zero voltage vectors. And the magnitude of current vector can be controlled as a reasonable constant value with the adjusted duty at all different conditions. Too small magnitude of current vector compared to the current sensor rating can cause the angle estimation error of current vector due to the current sensing error. It will be again discussed with experimental test results.

Fig. 44. The duty adjustment of the zero voltage vector pulse.
4.6. Experimental Results

A set of experiments was conducted to validate performance of the proposed restart method. Fig. 45 shows that the dynamo test bed consists of a permanent magnet synchronous motor for the test purpose, an induction motor for supplying the load torque, a voltage source inverter (VSI) and torque & speed transducer which is just for monitoring a real speed and load torque. The test PM motor parameters are already given in Table 7 and the load IM motor parameters are given in Table 8. The inverter used is APS-100T120 having the rating of 1200V & 100A IGBT switches. The \(v/f\) control and the proposed restart method are implemented using the OPAL-RT. For these control, only two phase currents sensors and DC link voltage sensor of inverter are used.

![Fig. 45. The dynamo set configuration for the experimental test.](image-url)
### Table 8. The load motor & system parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>[kW]</td>
<td>$P_{out}$</td>
<td>11.2</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>[rpm]</td>
<td>$N_r$</td>
<td>1730</td>
</tr>
<tr>
<td>Rated Voltage (line-line)</td>
<td>[V]</td>
<td>$V_s$</td>
<td>95</td>
</tr>
<tr>
<td>Rated Torque</td>
<td>[Nm]</td>
<td>$T_e$</td>
<td>62</td>
</tr>
<tr>
<td>Phase Current (RMS)</td>
<td>[A]</td>
<td>$i_s$</td>
<td>80</td>
</tr>
<tr>
<td>Pole</td>
<td>-</td>
<td>$n$</td>
<td>4</td>
</tr>
<tr>
<td>System Inertia (coupled condition)</td>
<td>[Nm/rad·s²]</td>
<td>$J$</td>
<td>0.059</td>
</tr>
</tbody>
</table>

#### 4.6.1. DC Offset Compensation of Current Sensors

If the current sensor is not ideal, several types of deviations can be occurred. First of all, when the current of the stator winding is zero, the output of the current sensors should be zero. If it is not compensated properly, it can cause the estimation error of the current vector angle ($\theta_i$). Therefore, the DC offset of sensors should be compensated before applying the zero voltage vector for the rotor speed and position estimation. It can be easily implemented by adding the DC offset compensation loop to the restart algorithm. Fig. 46 shows the experimental result applying the zero voltage vector without the DC offset compensation. The test is done by applying the zero voltage vector to the test motor at every switching frequency while remaining a constant speed using the speed control of the load motor. The blue waveform on Fig. 46 is the magnitude of the current vector. The reason why the DC offset and the oscillation are generated can be explained with the following equation.

$$U = I_{mag} \cos(\omega t) + I_{offset};$$

$$V = I_{mag} \cos\left(\omega t - \frac{2\pi}{3}\right) + I_{offset};$$

$$W = I_{mag} \cos\left(\omega t + \frac{2\pi}{3}\right) + I_{offset};$$

In (92), the 3-phase DC offsets are assumed as the same value for the simplification of calculation. The magnitude of the current vector ($I_s$) can be derived as:
\[ I_e^2 = I_\alpha^2 + I_\beta^2 = I_{\text{max}}^2 + 3I_{\text{offset}}^2 + 4I_{\text{offset}} \cos \left( \omega t - \frac{2\pi}{3} \right) \]  

(93)

Second and third terms on the right side of (93) are generated due to the DC offset of current sensors. Specifically, third term generates the oscillation corresponding to the rotor speed. This analysis result is consistent with the experimental result as shown in the Fig. 46. The blue waveform has the DC offset and the oscillation corresponding to the rotor speed.

![Fig. 46. The current sensing & the estimated rotor theta without the current offset compensation; CH1: the zero vector current magnitude (2A/div), CH2: the stator phase current after A/D conversion (5A/div), CH3: the estimated theta of the rotor, CH4: the real stator phase current (5A/div).](image)

In addition, the estimation error of current vector angle due to the DC offset is investigated with the simulation. It is assumed that the DC offset has the 1% of the current sensor rating (50A). \( I_{\text{max}} \) is set as 5A which is about one-fifth of the rated current. Fig. 47 shows the simulation result of the worst DC offset case. The first row of simulation shows both the ideal case and the actual case having the DC offset of current sensors. And the theta difference between the actual and the ideal one are plotted in the second row. The maximum theta error is about 8° [deg].
Fig. 47. The rotor position estimation error due to the DC offset of current sensors.

Fig. 48 is the experimental result of the case that DC offsets are properly compensated for the precise rotor theta estimation. Test results of Fig. 46 - Fig. 48 show that DC offset can affect to the estimation of current vector angle. As a result, the proper DC offset compensation is required for the precise position and speed estimation.

Fig. 48. The current sensing & the estimated rotor theta with the current offset compensation; CH1: the zero vector current magnitude (2A/div), CH2: the stator phase current after A/D conversion (5A/div), CH3: the estimated theta of the rotor, CH4: the real stator phase current (5A/div).
4.6.2. Current Sensor Resolution Issue

In order to investigate the issue of sensing error by the current sensor resolution explained above, experimental tests were implemented with two current sensors having a different rating. The speed of the test motor is maintained at 1200 rpm using the speed controller of the load motor coupled with the test motor. The test results are given below, the test with a 100A LEM sensor and the test with a 50A LEM sensor.

![Fig. 49. The effect by the current sensor resolution (a) 100A current sensor (b) 50A current sensor; CH1: the zero vector current magnitude (1A/V), CH2: the stator phase current after A/D conversion (5A/div), CH3: the estimated theta of the rotor, CH4: the real stator phase current (5A/div).](image)

The test using a 50A sensor has better accuracy in the theta estimation than the test using a 100A sensor. However, the case using a 50A sensor has still a theta estimation problem because the increasing slope of theta in Fig. 49(b) is not constant. It means that the current sensing value is including a significant error because the stator current magnitude is too small compared to the current sensor rating. In general, a current sensor has a maximum 1% error of the rating according to the specification. That is the reason why the test using a 50A sensor also has the significant estimation error. Therefore, a larger duty to increase the stator current is required for the precise theta estimation. Fig. 50 shows the additional test results to verify the above analysis. The used duties are 10, 15, 30 and 50% of a PWM switching frequency. As the stator current is increased by adjusting a zero voltage vector duty, the theta estimation become more precisely and the increasing slope of theta become constant. However, the duty selection requires a tradeoff between current measurement accuracy, theta estimation accuracy, and overcurrent. The small current causes a current measurement error. In contrast, the large current causes the overcurrent (or a big braking torque) and the estimation error of rotor position as shown in Fig. 35. Consequently, the proposed restart method uses an automatic duty adjustment algorithm to make the
pulse current be about one-fifth of the rated stator current. The automatic duty adjustment algorithm was explained at section 4.4.

Fig. 50. The effect by the current sensing error (50A current sensor) (a) 10% zero vector duty (b) 15% zero vector duty (c) 30% zero vector duty (d) 50% zero vector duty at 1200rpm; CH1: the zero vector current magnitude (1A/V), CH2: the stator phase current after A/D conversion (1A/V), CH3: the estimated theta of the rotor, CH4: the real stator phase current (1A/V).

4.6.3. Speed Estimation Accuracy Check Corresponding to the Delay Time between the Applied Zero Voltage Vectors

In order to investigate the speed estimation accuracy corresponding to delay time between the applied zero voltage vectors explained above, experimental tests were implemented with two different delay time. The speed of the test motor is maintained at 1200 rpm. The test results are given below, the test with a 4 $T_{sw}$ (PWM switching time) and the test with 20 $T_{sw}$. In here, 20 $T_{sw}$ is selected to a value within the range which obtained by substituting the parameters given in Table 7 into (81).
Fig. 51. The speed estimation corresponding to the zero pulse delay time (1200 rpm) (a) 4 T_{sw} (b) 20 T_{sw}. CH1: the estimated speed (400 rpm/div), CH2: the stator phase current after A/D conversion (5A/div), CH3: the estimated theta of the rotor, CH4: the real stator phase current (5A/div).

As known in (80), the speed estimation error is inverse proportional to the delay time (β) of Fig. 37. The blue line and the purple line of Fig. 51 are the rotor speed estimated using (70) and the estimated current vector angle in the stationary frame, respectively. The theta and speed estimation of Fig. 51 (a) has a short updating time than ones of Fig. 51 (b). However, the speed estimation of (a) has much bigger error than one of (b) as predicted by (80). On the other hand, the speed estimation of (b) is almost constant. As a result, the proposed restart method used the 20 T_{sw} delay time between the applied zero voltage vector to estimate the rotor speed. The test method to implement Fig. 51 (b) is described well in Fig. 52. As mentioned before, the second small duty vector is just for detecting the rotating direction and is not used for the speed estimation.

Fig. 52. The description of 20 T_{sw} delay time between the applied zero vectors.
4.6.4. Implementation of the Proposed Restart Algorithm

Fig. 53 shows experimental results including the estimated speed and position of the rotor, the stator current and the actual speed of the transducer when the mechanical rotor speed is 600, 1200, 1800 and 2400rpm. The restart tests are implemented as the following steps. First, the motor is operating at the reference speed. The inverter feeding PM motor is stopped by intentionally for 2 seconds. During that time, the motor speed is reduced depending on the inertia, the friction and the load condition. After 2 seconds, the speed and position estimation of the rotor is implemented and the motor is again fed by the inverter. Then, the \( \frac{v}{f} \) control is started with the stabilizing loop [28] and the rotor speed is increased to the previous reference speed used before the temporary power failure occurs. The test results of Fig. 53 verify that the restart using the proposed method can implement without causing any inrush current. In addition, the estimated speed (cyan line) is matched well with the real speed (blue line) at the restart instants.

Fig. 54 shows restart experimental results done with three different load conditions (0, 5 and 10Nm). The restart tests are implemented as the following steps. First, the motor is operating at the 1200 rpm with a constant load torque. The inverter feeding PM motor is stopped by intentionally. The motor speed is reduced depending on the inertia, the friction and the load condition. Test results of Fig. 54 verify that the proposed method can be implemented in high load conditions.
Fig. 53. Restart waveforms (a) 600rpm (b) 1200rpm (c) 1800rpm (d) 2400rpm; CH1: the rotor speed from the transducer (400rpm/div), CH2: the estimated rotor speed (400rpm/div), CH3: the estimated position of the rotor, CH4: stator phase current (20A/div).
Fig. 54. Restart test at different load torque conditions; (a) 0 Nm (b) 5 Nm (c) 10 Nm; CH1: the phase current (20A/div), CH2: the actual rotor speed (400rpm/div), CH3: the applied load torque (5Nm/div), CH4: the estimated rotor speed (400rpm/div).
4.6.5. Need of Stabilizing Loop in Restart Algorithm

A set of experiments were conducted to validate the need of stabilizing loop in the PMSM restart algorithm. The need of stabilizing loop is explained in section 2.3. The experimental results are summarized in Fig. 55 and Fig. 56. The restart algorithm is again tested in two cases which are without the stabilizing loop and with the stabilizing loop. Fig. 55 shows the test results done without the stabilizing loop at three different speed conditions (600, 1200 and 1800 rpm). Although the motor does not lose synchronism, there are the oscillation and the overcurrent problems at the restart instant.
Fig. 55. The test results without the stabilizing loop (a) 600 rpm (b) 1200 rpm (c) 1800 rpm; CH1: the phase current (20A/div), CH2: the rotor speed (400 rpm/div), CH3: the estimated theta of the rotor, CH4: the estimated rotor speed (400 rpm/div).
Fig. 56. The test results with the stabilizing loop (a) 600 rpm (b) 1200 rpm (c) 1800 rpm; CH1: the phase current (20A/div), CH2: the rotor speed (400 rpm/div), CH3: the estimated theta of the rotor, CH4: the estimated rotor speed (400 rpm/div).
On the other hand, the results of Fig. 56, which is the case including the stabilizing loop, show the good performance at restart instant. These results verified that the restart algorithm should include the stabilizing loop from the restart instant.

4.6.6. Investigation about the Acceptable Estimation Error

Experimental tests to evaluate how much estimation error is acceptable were implemented at 1200rpm because the analytical approach is not easy. The overcurrent is set as 35A considering the rated current. The error is intentionally added to the estimated position and speed value. Although these tests cannot be generalized for all motor, the restart tests are succeeded up to 15° position error and 5% speed error. In worse conditions, the restart is failed due to the overcurrent caused by the estimation error. For a more precise evaluation, analytical derivations will be required. However, it will be out of range of this dissertation. Fig. 57 depicts the current vector depending on the estimated theta and speed error. The magnitude of current vector will increase as the theta or the speed error increases. Fig. 58 shows the test results done with three different theta error conditions (10, 15 and 20° [deg]). Fig. 59 shows the test results implemented with three different speed error conditions (0, 5 and 10% error of the rotor speed). As a result, the estimation error should be minimized to prevent the overcurrent.

![Diagram](attachment:image.png)

Fig. 57. The vector diagram showing the stator current vector caused by the magnitude and phase angle difference between the B-emf (induced voltage from the stator flux) and the applied voltage vector.
Fig. 58. The theta estimation error tests (a) 10° [deg] (b) 15° [deg] (c) 20° [deg]; CH1: the phase current (20A/div), CH2: the actual rotor speed (400rpm/div), CH3: the rotor position, CH4: the estimated rotor speed (400rpm/div).
Fig. 59. The speed estimation error tests (a) 0\% (b) 5\% (c) 10\%; CH1: the phase current (20A/div), CH2: the actual rotor speed (400rpm/div), CH3: the rotor position, CH4: the estimated rotor speed (400rpm/div).
4.7. Second Proposed Method by Estimating L_d & L_q Inductance

In previous section, the restart method for PM motor is developed without using the motor inductance information. Therefore, this proposed method always has theta estimation error because the angle between the rotor and the current vector is assumed as 90°[deg.]. To minimize the estimation error, this section suggests the method using the \( L_d \) & \( L_q \) inductance information which is estimated during the restart process. This proposed method uses three zero voltage pulses and one active voltage pulse in the restart process to estimate \( L_d \) & \( L_q \). This method is different with the conventional methods that uses the inductance value measured by additional procedure before running motor. This section describes the method to estimate the \( L_d \) & \( L_q \) inductance and explains the implementation method. And the inductance estimation error is analyzed by simulation.

4.7.1. Overview of Second Proposed Method

Fig. 60 shows the overall estimation procedure using four voltage vector pulses. The first pulse is used to adjust the next pulse duty. By applying second pulse, the rotor direction is detected and the current vector angle is estimated in stationary reference frame. And as applying third pulse, the rotor speed and position can be estimated. It is almost same with the previous proposed method in shown Fig. 37. However, this method includes the forth non-zero voltage pulse to estimate the \( L_d \) inductance. And \( L_q \) inductance is estimated by using the resulted current by second or third pulse after knowing the rotor speed.

Fig. 60. The overall logic of the second proposed restart method.
Fig. 61 shows the flowchart of the proposed restart method. The highlighted part is for the $L_d$ & $L_q$ inductance estimation. And this part also includes the rotor position estimation. The rotor position is re-estimated with the estimated inductance.

### 4.7.2. $L_q$ Inductance Estimation Method

In section 4.2.2, the resulted current by zero voltage vector was analyzed. The $d$-$q$ axis current was expressed as:

$$
\begin{bmatrix}
    i_d(t_{pulse}) \\
    i_q(t_{pulse})
\end{bmatrix} =
\begin{bmatrix}
    \frac{\dot{\lambda}_d}{L_d}(1 - \cos \omega t_{pulse}) \\
    -\frac{\dot{\lambda}_q}{L_q} \sin \omega t_{pulse}
\end{bmatrix}
$$

(94)
If it is assumed that the pulse time is short enough, \(\cos(\omega t_{\text{pulse}})\) and \(\sin(\omega t_{\text{pulse}})\) of (94) can be considered as 1 and \(\omega t_{\text{pulse}}\). Then, the \(d-q\) axis current can be simplified as:

\[
\begin{bmatrix}
i_d(t_{\text{pulse}}) \\
i_q(t_{\text{pulse}})
\end{bmatrix}
= \begin{bmatrix}
0 \\
-\frac{\lambda_f}{L_q}\omega t_{\text{pulse}}
\end{bmatrix}
\] (95)

The error percentage induced by this assumption is plotted in Fig. 62. In the proposed method, as \(\omega t_{\text{pulse}}\) is limited by 0.035, the error is very small as shown in Fig. 62. Therefore, this assumption is reasonable.

Fig. 62. The error caused by assuming (a) \(\cos(\omega t_{\text{pulse}}) = 1\), (b) \(\sin(\omega t_{\text{pulse}}) = \omega t_{\text{pulse}}\).
By measuring the resulted current by the zero voltage vector, \( q \)-axis current can be obtained because most of resulted current is aligned with \( q \)-axis as shown in (95). Then, \( L_q \) inductance value can be calculated using the estimated speed by three zero voltage pulses. Fig. 63 shows the test result to verify the \( L_q \) inductance estimation. In both tests (30\%, 50\% duty at 1,200 rpm), the magnitude of resulted current is measured and the inductance is estimated using (95). It results in the estimation value of 1.56mH and 1.46mH. It is almost matched with real \( L_q \) inductance value of 1.50mH.

![Test result of \( L_q \) inductance estimation at 1,200 rpm](image)

(a) (b)

Fig. 63. Verification test result of \( L_q \) inductance estimation at 1,200 rpm (a) 30\% duty (b) 50\% duty; CH1: the resulted current magnitude (1A/V), CH2: the stator phase current after A/D conversion (5A/div), CH3: the estimated theta of the rotor, CH4: the real stator phase current (5A/div).

4.7.3. \( L_d \) Inductance Estimation Method

To estimate \( L_d \) inductance, forth non-zero voltage pulse is applied as shown in Fig. 60. Before applying forth non-zero voltage vector (active voltage vector), the rotor position and speed is already estimated at \( t_3 \) through applying three zero voltage vector. Therefore, forth active voltage vector (\( V \)) can be applied to the \( d \)-axis in rotor reference frame as shown in Fig. 64.
The active voltage vector can be expressed as:

\[ V = V_d + jV_q = |V| \cos(\theta_{st}) + j|V| \sin(\theta_{st}) \]  \hspace{1cm} (96)

If active voltage vector \((v_d = V, v_q = 0)\) is applied in \(d\)-axis, the equation for PM motor can be obtained as:

\[ \begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + pL_d & -\omega_s L_q \\ \omega_s L_d & R_s + pL_q \end{bmatrix} \begin{bmatrix} i_d(t_{pulse}) \\ i_q(t_{pulse}) \end{bmatrix} + \omega_s \lambda_f \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]  \hspace{1cm} (97)

where \(V\) is the magnitude of the applied voltage vector. As assuming that the applied pulse time \(t_{pulse}\) is much shorter than the stator time constants \((\tau_d = L_d/R_s, \tau_q = L_q/R_s)\), the stator resistance can be neglected and the equation (97) can be simplified as the following:

\[ \begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{bmatrix} pL_d & -\omega_s L_q \\ \omega_s L_d & pL_q \end{bmatrix} \begin{bmatrix} i_d(t_{pulse}) \\ i_q(t_{pulse}) \end{bmatrix} + \omega_s \lambda_f \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]  \hspace{1cm} (98)

As solving the equation (98) using Laplace transform, the resulting currents can be calculated as:

\[ \begin{bmatrix} i_d(t_{pulse}) \\ i_q(t_{pulse}) \end{bmatrix} = \frac{\lambda_f \left( \cos(\omega_s t_{pulse}) - 1 \right) + V \frac{\sin(\omega_s t_{pulse})}{\omega_s}}{L_d} \begin{bmatrix} -\omega_s L_q \\ pL_q \end{bmatrix} \]  \hspace{1cm} (99)

When deriving \(d-q\) axis current, the initial condition of those current was assumed as zero. As mentioned before, \(\cos(\omega_s t_{pulse})\) and \(\sin(\omega_s t_{pulse})\) of (99) can be considered as 1 and \(\omega_s t_{pulse}\) so that the equation (99) can be simplified as:
When applying the active voltage vector in $d$-axis, the resulted $d$-$q$ axis current is derived in (100). The $d$-axis current includes the $L_d$ inductance value. By measuring the resulted $d$-axis current, $L_d$ inductance value can be extracted. To obtain $d$-axis current, two methods are suggested. First method is using Park’s transformation. After measuring two phase current, phase current is transformed to $d$-$q$ axis as:

$$
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix} = \frac{2}{3}
\begin{bmatrix}
\cos(\theta_{est}) & \cos\left(\theta_{est} - \frac{2\pi}{3}\right) & \cos\left(\theta_{est} + \frac{2\pi}{3}\right) \\
-\sin(\theta_{est}) & -\sin\left(\theta_{est} - \frac{2\pi}{3}\right) & -\sin\left(\theta_{est} + \frac{2\pi}{3}\right)
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
-(I_a + I_b)
\end{bmatrix}
$$

(101)

And then, the $L_d$ inductance can be calculated by using $I_d$ current as:

$$
L_d = \frac{V_{t_pulse}}{I_d}
$$

(102)

The second method is more precise method for $L_d$ estimation than the first method. The first method uses the estimated theta ($\theta_{est}$) that may have the estimation error. On the other hand, the second method uses just the magnitude information of the measured current. However, this method requires more complex calculation. The magnitude of the measured current can be calculated as:

$$
I_s^2(t_{pulse}) = I_a^2(t_{pulse}) + \frac{1}{3}\left(I_a(t_{pulse}) + 2I_b(t_{pulse})\right)^2
$$

(103)

In here, $I_s$ is the magnitude of the resulted current vector. It will be same with one derived from (100) and it is expressed as:
\[
I_i^2(t_{\text{pulse}}) = I_d^2(t_{\text{pulse}}) + I_q^2(t_{\text{pulse}}) = I_{\text{pulse}}^2 \left( \frac{V}{L_d} \right)^2 + \left( \frac{\alpha v_\lambda_i}{L_q} \right)^2 \]  \hspace{1cm} (104)

In (104), b-emf and \( L_d \) inductance is already estimated value. Then, \( I_q \) current can be calculated and \( L_d \) inductance can be obtained as:

\[
L_d = \frac{V}{\sqrt{\left( \frac{I_i(t_{\text{pulse}})}{I_{\text{pulse}}} \right)^2 - \left( \frac{\alpha v_\lambda_i}{L_q} \right)^2}} \]  \hspace{1cm} (105)

4.7.3.1. Realistic Issue for Implementation

In previous section, we discussed about the method for \( L_d \) estimation. The proposed method is applying an active voltage in \( d \)-axis in rotor reference frame. However, in order to apply the voltage exactly to \( d \)-axis, it requires at least two active vector at one switching period because VSI only can make eight voltage vector (\( V_0 \sim V_7 \)) with combinations of six switches status. To make all active vector (phase: 0 ~ 360°[deg.]) in one switching period, SVPWM or another general PWM methods should be used. Fig. 65 shows switches status during applying the zero voltage pulses and SVPWM.

![Fig. 65. Switching condition at estimation and normal SVPWM mode.](image-url)
In the figure, $S_u$, $S_v$, $S_w$ are the switching condition. If $S_{uvw}$ is high, upper switches of the inverter each legs are turned on. On the other hand, the low statue indicates that switches are turned off. In SVPWM mode, it is applying four voltage vector including two zero voltage vector ($V_0$ & $V_7$) and two active vector. As shown in (94), the current is always generated by the zero voltage vector if PM motor is rotating. The current will not be zero before two active voltage vector is applied. Therefore, the equations from (97) to (100) derived for $L_d$ estimation are not correct any more because it is assuming that an initial stator current is zero. As a result, this switching method cannot be used for $L_d$ estimation. The alternative method is suggested in next section.

4.7.3.2. Alternative Switching Method by Realistic Issue

As mentioned in previous section, the active voltage vector should be applied in the condition that all switched are opened. Thus, this method suggests using one of six active vectors ($V_1 ~ V_6$) generated by VSI for $L_d$ estimation. Fig. 66 shows switches status at each mode. First of all, to estimate the speed and position of rotor, the zero voltage vector is applied. And one active voltage vector is applied for $L_d$ estimation. After estimating the rotor speed and position again, the motor is restarted with a normal SVPWM mode.

![Fig. 66. Switching condition of active voltage vector for $L_d$ estimation.](image-url)
By using this switching scheme, the equations derived from (97) to (100) can be used for $L_d$ estimation. However, the applied active voltage cannot be aligned with $d$-axis as shown in Fig. 67.

![Diagram](image)

**Fig. 67.** Angle between the applied active voltage vector and the rotor position.

Thus, the resulted current is derived again by the following procedure. First, because the applied voltage is not aligned with $d$-axis, it has $d$-$q$ components and (98) is replaced with the following equation as:

\[
\begin{align*}
\begin{bmatrix} V \cos(\theta_{V-d}) \\ V \sin(\theta_{V-d}) \end{bmatrix} &= \begin{bmatrix} pL_d & -\omega_r L_q \\ \omega_r L_q & pL_q \end{bmatrix} \begin{bmatrix} i_d(t_{\text{pulse}}) \\ i_q(t_{\text{pulse}}) \end{bmatrix} + \omega_r \lambda_f \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\end{align*}
\]

(106)

where $\theta_{V-d}$ is the angle between the applied voltage and the rotor position. As solving the equation (106) using Laplace transform, the resulting currents can be calculated as:

\[
\begin{align*}
\begin{bmatrix} i_d(t_{\text{pulse}}) \\ i_q(t_{\text{pulse}}) \end{bmatrix} &= \frac{\begin{bmatrix} (V \sin(\theta_{V-d}) - \omega_r \lambda_f)(1 - \cos \omega_r t_{\text{pulse}}) + V \sin \omega_r t_{\text{pulse}} \cos(\theta_{V-d}) \\ \frac{\omega_r L_d}{\omega_r L_q} \frac{V \cos(\theta_{V-d}) \cos \omega_r t_{\text{pulse}} - 1 + \sin \omega_r t_{\text{pulse}} (V \sin(\theta_{V-d}) - \omega_r \lambda_f)}{\omega_r L_q} \end{bmatrix}}{\omega_r L_d}
\end{align*}
\]

(107)

When deriving $d$-$q$ axis current, the initial condition of those current was assumed as zero. As mentioned before, $\cos(\omega_r t_{\text{pulse}})$ and $\sin(\omega_r t_{\text{pulse}})$ of (107) can be considered as 1 and $\omega_r t_{\text{pulse}}$ so that the equation (107) can be simplified as:
In (108), only $L_d$ is unknown value and it can be calculated by the same procedure used in (101)-(105).

4.7.3.3. Selection of Applied Active Voltage Vector

When selecting the active voltage vector ($V_0$-$V_6$), the closest active voltage vector with $d$-axis is chosen. In Fig. 60, the speed and rotor position is estimated by three zero voltage vector before applying forth active voltage vector. Thus, the active voltage vector can be applied to the closest position with $d$-axis and $\theta_{V,d}$ is always smaller than 30° [deg.] as shown in Fig. 68.

The reason selecting the closest voltage vector is explained with the following equation and figure.
\[ I_s^2(t_{\text{pulse}}) = I_d^2(t_{\text{pulse}}) + I_q^2(t_{\text{pulse}}) = I_{\text{pulse}}^2 \left( \frac{V \cos \theta_{v,d}}{L_d} \right)^2 + \left( -\omega_c \lambda_f + \frac{V \sin \theta_{v,d}}{L_q} \right)^2 \]  \hspace{1cm} (109)

The current magnitude can be obtained by using \( d-q \) axis current of (108) and it is the function of \( \theta_{v,d} \). Fig. 69 shows the current magnitude corresponding to \( \theta_{v,d} \).

![Graph showing the current magnitude corresponding to \( \theta_{v,d} \).](image)

Fig. 69. Is Current magnitude corresponding to \( \theta_{v,d} \).

The simulation result shows that the closer \( \theta_{v,d} \) is zero, the smaller the resulted current magnitude is. Therefore, the active voltage vector is selected as the closest one with \( d \)-axis of rotor reference frame to prevent the overcurrent and the large braking torque. Fig. 70 shows the simulation result for \( L_d \) inductance estimation while motor is running at 1,200 rpm.
Fig. 70. Simulation result for $L_d$ inductance estimation.

In the simulation, the active voltage vector is applied to the closest position with $d$-axis at every two switching period. The sector number indicates the number of active voltage vector ($V_1$~$V_6$) and the range of $\theta_{V_d}$ is from $-30^\circ$ to $30^\circ$ [deg.]. The simulation also includes the phase and $d$-$q$ axis current. The $L_d$ inductance is estimated with $d$-$q$ axis current and the estimated value is $1.06mH$. It is matched well with the real $L_d$ value ($1.05mH$) listed in Table 7. It verifies that the analytical analysis derived in this section is matched well with the simulation result.
4.7.3.4. Duty Selection of Applied Active Voltage Vector

This section explains the method how to select a duty of forth applied voltage pulse. The pulse duty should be selected properly to prevent the overcurrent and the big braking torque. The duty selection of forth voltage is related with the duty of second and third voltage and it is derived with the following procedure. As explained before, the duty of second and third pulses is adjusted for the current magnitude to be one-fifth of the motor rating current. The following equation is the resulted current by second and third zero voltage pulse and it is simplified from (75).

\[
\begin{bmatrix}
    i_d(t_{\text{pulse}}) \\
    i_q(t_{\text{pulse}})
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    -\frac{\alpha \dot{\lambda}_f t_{\text{pulse}}}{L_q}
\end{bmatrix}
\quad (110)
\]

The adjusted pulse time \( t_{\text{pulse}} \) makes the current magnitude to be about one-fifth of the motor rating current.

\[
\begin{align*}
    i_q(t_{\text{pulse}}) &= \frac{\alpha \dot{\lambda}_f t_{\text{pulse}}}{L_q} = \frac{1}{5} I_{\text{rating}} \\
\end{align*}
\quad (111)
\]

Similarly to this concept, the duty of forth pulse is selected for the current magnitude to be about one-fifth of the motor rating current. From (108), \( I_d \) current generated by the forth active voltage pulse is expressed as:

\[
i_d(t_{\text{pulse},\text{new}}) = \frac{V_{t_{\text{pulse},\text{new}}} \cos \theta_{V,d}}{L_q}
\quad (112)
\]

If we want to make \( I_d \) current to be one-fifth of the rating current, it can be expressed as:

\[
\frac{V_{t_{\text{pulse},\text{new}}} \cos \theta_{V,d}}{L_q} = \frac{1}{5} I_{\text{rating}} = \frac{\alpha \dot{\lambda}_f t_{\text{pulse}}}{L_q}
\quad (113)
\]

In (113), \( t_{\text{pulse},\text{new}} \) can be calculated as:
\[ t_{\text{pulse \_ new}} = \frac{L_d}{L_q} \frac{\omega_r \lambda_f t_{\text{pulse}}}{V \cos \theta_{v \cdot d}} \quad (114) \]

When calculating \( t_{\text{pulse \_ new}} \), the ratio of \( L_d \) and \( L_q \) was assumed as 1 because it is not known in this step. If the actual ratio of \( L_d \) and \( L_q \) is 5, the new pulse duty will cause 5 times larger current than the expected one (1/5 of rating current). However, it is still smaller than the rating current. In addition to \( I_d \) current, \( I_q \) current is also generated by forth voltage vector and the magnitude of it is investigated. \( I_q \) current is derived in (108) and it is given by:

\[ I_q(t_{\text{pulse \_ new}}) = \frac{-\omega_r \lambda_f}{L_q} t_{\text{pulse \_ new}} + \frac{V t_{\text{pulse \_ new}} \sin \theta_{v \cdot d}}{L_q} \quad (115) \]

The absolute value of \( I_q \) current is maximum when \( \theta_{v \cdot d} \) is 0° or 30°[deg.] because the sign of the first and second term is different. The following equation shows the boundary of the \( I_q \) current.

\[ \frac{-\omega_r \lambda_f}{L_q} t_{\text{pulse \_ new}} < I_q(t_{\text{pulse \_ new}}) < \frac{-\omega_r \lambda_f}{L_q} t_{\text{pulse \_ new}} + \frac{V t_{\text{pulse \_ new}}}{2L_q} \quad (116) \]

The left boundary of (116) includes the b-emf term (\( \omega_r \lambda_i \)) and it is always smaller than \( V \) of \( I_d \) current in (112). In addition, \( L_q \) is larger than \( L_d \). Therefore, the absolute value of left boundary is smaller than one of \( I_d \) current. Although the right boundary of (116) has the positive value, it cannot be larger than \( I_d \) current. As a result, the resulted current \( (I_s) \) will not exceed the rating current in most condition if \( t_{\text{pulse \_ new}} \) of forth active voltage pulse is selected with the above explained method.

\[ I_s = \sqrt{I_d(t_{\text{pulse \_ new}})^2 + I_q(t_{\text{pulse \_ new}})^2} < \sqrt{2} \cdot I_d(t_{\text{pulse \_ new}}) \quad (117) \]
4.7.4. $L_d$ Estimation Error Caused by Rotor Position Estimation Error

In section 4.7.3, $L_d$ inductance is estimated by applying one of active voltage vectors. During calculating $L_d$ inductance, the angle ($\theta_{V-d}$) between the active voltage vector and the rotor position is used. It is noticeable that the angle ($\theta_{V-d}$) can have the error because the rotor position is the estimated one. Therefore, the $L_d$ inductance estimation can also have the error. In this section, the $L_d$ estimation error is calculated and the effect is investigated.

\[ \theta_{error} = \theta_{est} - \theta_{act} \]  \hspace{1cm} (118)

Fig. 71. The applied active voltage vector, the actual rotor $d$-$q$ axis and the estimated rotor $\gamma$-$\delta$ axis.

In Fig. 71, $d$-$q$ axis is the actual rotor reference frame and $\gamma$-$\delta$ axis is the estimated rotor reference frame. $\gamma$-axis is aligned with the rotor position ($\theta_{est}$) estimated by three zero voltage pulses of proposed method and it always has small error with the actual rotor position ($\theta_{act}$) because the proposed method assumes the angle ($\theta_0$) between the rotor and current vector angle ($\theta_i$) as $90^\circ$[deg.]. The error angle between the real and the estimated rotor position is given by:
While calculating $L_d$ inductance, the measured phase current is transformed to $d$-$q$ axis. Actually, it is not $d$-$q$ axis current but the $\gamma$-$\delta$ axis current because the transformation used the estimated angle ($\theta_{est}$). Therefore, the equation shown in (108) used for $L_d$ inductance estimation should be replaced as:

$$L_{d_{est}} = \frac{V_{t_{pulse}} \cos \theta_{\gamma-d}}{i_d(t_{pulse})} \quad (119)$$

If the estimated theta has some error, $I_\gamma$ current has the difference with $I_d$ current and it will cause $L_d$ estimation error. To find the difference between two values, the $d$-$q$ current of (108) is transformed to $\gamma$-$\delta$ axis and it is obtained as:

$$
\begin{bmatrix}
    i_d(t_{pulse}) \\
    i_q(t_{pulse})
\end{bmatrix}
= \begin{bmatrix}
    \cos(\theta_{error}) & -\sin(\theta_{error}) \\
    \sin(\theta_{error}) & \cos(\theta_{error})
\end{bmatrix}
\begin{bmatrix}
    i_d(t_{pulse}) \\
    i_q(t_{pulse})
\end{bmatrix}
\quad (120)
$$

The difference between $d$-$q$ axis current and $\gamma$-$\delta$ axis current is expressed as the function of $d$-$q$ axis current as:

$$
\begin{bmatrix}
    i_d(t_{pulse}) \\
    i_q(t_{pulse})
\end{bmatrix}
- \begin{bmatrix}
    i_d(t_{pulse}) \\
    i_q(t_{pulse})
\end{bmatrix}
= \begin{bmatrix}
    \cos(\theta_{error}) - 1 & -\sin(\theta_{error}) \\
    \sin(\theta_{error}) & \cos(\theta_{error}) - 1
\end{bmatrix}
\begin{bmatrix}
    i_d(t_{pulse}) \\
    i_q(t_{pulse})
\end{bmatrix}
= \begin{bmatrix}
    0 & -\theta_{error} \\
    \theta_{error} & 0
\end{bmatrix}
\begin{bmatrix}
    i_d(t_{pulse}) \\
    i_q(t_{pulse})
\end{bmatrix}
\quad (121)
$$

From (121), the difference between $d$-axis current and $\gamma$-axis current can be calculated as:

$$i_{\gamma}(t_{pulse}) - i_{d}(t_{pulse}) = -\theta_{error} i_{q}(t_{pulse}) = -\theta_{error} \left[ -\omega \lambda_{i} t_{pulse} + \frac{V_{t_{pulse}} \sin \theta_{\gamma-d}}{L_q} \right] \quad (122)$$

By using (119) and (122), the $L_d$ estimation error is expressed as:

$$L_{d_{est}} - L_d = V_{t_{pulse}} \cos \theta_{\gamma-d} \left( \frac{1}{i_{d}(t_{pulse}) - \theta_{error} i_{q}(t_{pulse})} - \frac{1}{i_{d}(t_{pulse})} \right) \quad (123)$$
With the above equation, $L_d$ estimation error is plotted in Fig. 72. In the simulation, $\theta_{error}$ is calculated at every rotor speed as assuming the worst theta estimation condition. The simulation result shows that the estimation error of $L_d$ inductance estimated by forth active voltage vector is small although the forth active voltage vector is applied to the rotor position estimated by three zero voltage vector of the proposed method. As a result, although the proposed method always causes the rotor position estimation error, $L_d$ inductance estimation error is small enough to ignore. This estimated $L_d$ inductance can be used for final rotor position estimation in the next step.

4.7.5. Rotor Position Estimation Using Estimated $L_d$ & $L_q$ Information

In section 4.7.2 and 4.7.3, $L_d$ & $L_q$ inductance was estimated. Then, it is not necessary to assume that the angle ($\theta_h$) between the rotor and the current vector is 90° [deg.]. The accurate angle ($\theta_h$) can be calculated by using the estimated $L_d$ & $L_q$ inductance. To find the angle, the $d$-$q$ axis current is calculated by using the following equation.
\[
\begin{bmatrix}
  i_d(t_{\text{pulse}}) \\
  i_q(t_{\text{pulse}})
\end{bmatrix} = \begin{bmatrix}
  \frac{\lambda_r}{L_d} (1-\cos \omega t_{\text{pulse}}) \\
  -\frac{\lambda_r}{L_q} \sin \omega t_{\text{pulse}}
\end{bmatrix}
\]

(124)

With those information, the angle \( (\theta_0) \) can be derived as:

\[
\theta_0 = \tan^{-1}\left(\frac{i_q}{i_d}\right)
\]

(125)

With the angle \( (\theta_0) \) between the rotor and current vector obtained by (125) and the current vector angle \( (\theta_I) \) obtained by (69), the rotor angle in the stationary reference frame can be estimated as:

\[
\theta_r = \theta_{Iz} + \theta_0
\]

(126)

where \( \theta_r \) is the rotor angle. The angle \( (\theta_0) \) and angle \( (\theta_I) \) was plotted in Fig. 33(b). The method to calculate the rotor angle \( \theta_I \) is exactly same with conventional method. However, this proposed method includes the procedure for \( L_d \) and \( L_q \) inductance estimation while motor is running. And this estimated inductance value is used for the rotor position estimation. On the other hand, the conventional methods uses the inductance value measured by additional procedure before running motor.

4.8. Conclusion

This chapter explained the basic concept of existing restart methods and described the proposed restart methods in detail. The goal of proposed restart method was to develop the universal algorithm for PM motors. First of all, this proposed method uses only motor parameters known by the nameplate. In general, the nameplate does not include the information such as \( d-q \) axis stator inductance which the conventional restart methods used. In addition, this proposed methods can automatically determine the duty of zero voltage vector
pulses as considering the rated current to prevent an overcurrent and the delay time between two pulses considering the rated rotor speed and the acceptable estimation error. As a result, this proposed restart method can be used for universal application of PMSM. The maximum rotor position estimation error of the proposed method is less than that of the existing method used in [71] although the variation of inductance and the rotor speed estimation error by the current sensing error exist. The maximum estimation error will be under $10^\circ$ [deg] because the $\omega_r \cdot t_{\text{pulse}}$ is always limited as a smaller value than 0.035. The experimental results have conducted to validate the performance of the proposed restart method. Moreover, experimental result includes the restart test in both cases which are with the stabilizing loop and without that, and it verified the need of stabilizing loop at the restart instant. In addition, this chapter suggests the method for $L_d$ & $L_q$ estimation while implementing the flying restart. This estimated inductance value is used for rotor position estimation. This proposed method is verified with the simulation and the result was matched well with the analytical analysis result.
CHAPTER 5: A UNIVERSAL RESTART STRATEGY FOR SYNCHRONOUS RELUCTANCE MACHINES

5.1. Introduction

The goal of this work is to develop a universal restart method that is capable of restarting SynRM driving a high inertia load. The proposed approach borrows concepts from similar work done on induction machines [37-39, 67, 68] and more recently PMSM [69-72]. However, these methods are not suitable for SynRM due to the following facts.

1) The absence of b-emf voltage makes it difficult to use the zero voltage pulse method used in PMSM. Unlike PMSM, SynRM does not have the permanent magnet in the rotor and it uses only the reluctance torque. The $d-q$ equation of SynRM is given by:

$$
\begin{align*}
\dot{i}_d(n+1) &= i_d(n) + \frac{dT}{L_d} [v_d(n) - r_i d(n) + \omega_L L_d i_d(n)] \\
\dot{i}_q(n+1) &= i_q(n) + \frac{dT}{L_q} [v_q(n) + r_i q(n) - \omega_L L_q i_q(n)]
\end{align*}
$$

During the free running mode, $i_d(n)$ and $i_q(n)$ current of (127) can be assumed as zero. Therefore, although the zero voltage ($v_d(n) = v_q(n) = 0$) is applied, the current ($i_d(n+1)$ & $i_q(n+1)$) will not be generated in the stator.

2) No current conduction in the rotor makes it impossible to use the searching method used in induction motor.

Fig. 73 shows the simulation result as changing the applied stator voltage frequency at the fixed rotor speed. Although the stator and rotor speed are matched, there is no special features in the phase current and the power.
3) Motor parameter information such like an inductance value is limited. Especially, when a scalar control method is used, those information is not required to operate the motor. Thus, the proposed method should not use any motor parameter information to estimate the rotor speed and position excepting the motor rating information listed in motor nameplate.

In this chapter we propose an effective restart algorithm that result in a universal application for SynRM. This method can eliminate the need of stator inductance information without installing the additional hardware equipment. And this method used the simple method to estimate the rotor speed and position. In addition, the algorithm to minimize the speed estimation error is also suggested. The remainder of this chapter describes the proposed restart method in detail.
5.2. Principle of Synchronous Reluctance Motor

The electrical equation of the synchronous reluctance motor (SynRM) is expressed in the \( d-q \) rotor reference frame. Fig. 74 shows the equivalent circuit of the SynRM in the rotor reference frame.

The \( d \)-axis and \( q \)-axis stator voltages are obtained respectively as:

\[
\begin{align*}
v_d &= r_s i_d + \frac{d}{dt} \lambda_d - \omega_r \lambda_q \\
v_q &= r_s i_q + \frac{d}{dt} \lambda_q + \omega_r \lambda_d
\end{align*}
\]  

(128)
where \(v_d\) and \(v_q\) are \(d\)-axis and \(q\)-axis stator voltages, respectively. \(r_s\) is the stator resistance. \(i_d\) and \(i_q\) are \(d\)-axis and \(q\)-axis stator current, respectively. \(\lambda_d\) and \(\lambda_q\) are \(d\)-axis and \(q\)-axis stator linkage flux, respectively and \(\omega_r\) is the electrical rotor speed. The stator linkage flux (\(\lambda_d\) and \(\lambda_q\)) are expressed as the following equations.

\[
\lambda_d = L_d i_d \\
\lambda_q = L_q i_q \tag{129}
\]

Where \(L_d\) and \(L_q\) are \(d\)-axis and \(q\)-axis stator inductance, respectively. The electromagnetic torque equation can be derived with the input power of the motor. The input power of the motor in the synchronous reference frame can be expressed as:

\[
P_{\text{in}} = \frac{3}{2} \left( v_d i_d + v_q i_q \right) \tag{130}
\]

Substituting (128) and (129) into (130) results in.

\[
P_{\text{in}} = \frac{3}{2} \left( r_s \left( i_d^2 + i_q^2 \right) \right) + \frac{1}{2} \left[ i_d \frac{d\lambda_d}{dt} + i_q \frac{d\lambda_q}{dt} \right] + \omega_r \left( L_d - L_q \right) i_d i_q \tag{131}
\]

The first term is the stator copper loss, the second loss is the change in electromagnetic energy storage and the final term is the mechanical output. As dividing the input power with the mechanical rotor speed, the electromagnetic torque equation can be obtained as:

\[
T_e = \frac{n}{2} \left( L_d - L_q \right) i_d i_q \tag{132}
\]

Where \(n\) is the number of the pole. The maximum torque per ampere (MTPA) in SynRM is generated when the magnitude of \(i_d\) and \(i_q\) is same, which means that the current vector is located in 45° [deg.] from the \(d\)-axis frame.
5.3. Proposed Restart Method for SynRM

In developing the restart algorithm we have made the following assumptions: (1) during the outage, the drive loses power, but the controller has knowledge of the \( v/f \) ratio; (2) the controller recognizes the speed command prior to and after the fault and (3) the controller monitors the input power (i.e. recognizes when power was lost and when power was restored). In this section the complete scheme for searching the rotor speed and position is described.

Analyzing the machine equivalent circuit in Fig. 74, the equation describing the SynRM machine operation in rotor reference frame can be represented as:

\[
\begin{bmatrix}
    v_d \\
    v_q
\end{bmatrix} =
\begin{bmatrix}
    R_s + pL_d & -\omega_r L_q \\
    \omega_r L_d & R_s + pL_q
\end{bmatrix}
\begin{bmatrix}
    i_d \\
    i_q
\end{bmatrix}
\]

where \( p \) denotes the derivative operator, \( R_s \) is the winding resistance, \( v_d \) & \( v_q \) are the \( d-q \) axis stator input voltage, \( i_d \) & \( i_q \) are the \( d-q \) axis stator current, \( L_d \) & \( L_q \) are the \( d-q \) axis stator inductances and \( \omega_r \) is the electrical angular frequency of the rotor. The concept proposed herein is to excite the machine with one of six active voltage (\( V_1 \sim V_6 \)) every two switching frequency. And the speed and position of rotor is extracted with the resulting machine current. The following sections describes the proposed method in detail.

5.3.1. Applying Active Voltage Vector

3-phase voltage source inverter (VSI) can generate eight voltage vectors (\( V_0 \sim V_7 \)) by the switching state. Fig. 75 shows the output voltage vector of VSI. In Fig. 75. (\( S_a, S_b, S_c \)) represents the switching state of each legs of VSI. When \( S_{abc} = 1 \), the upper side switch of leg turns on. On the other hand, 0 indicates that the lower side switch turns on. Upper and lower side switches of each legs are complementary.
To simplify the implementation, the proposed method uses $V_1$ voltage vector. When $V_1$ voltage vector is applied to the motor, the each phase voltage of motor is determined as:

$$v_{as} = \frac{2}{3}V_{dc}, \quad v_{bs} = -\frac{1}{3}V_{dc}, \quad v_{cs} = -\frac{1}{3}V_{dc}$$  \hspace{1cm} (134)$$

In here, $V_{dc}$ is the DC-link voltage of VSI. $V_1$ voltage vector is aligned with $\alpha$-axis in stationary reference frame. The proposed method uses this $V_1$ vector to estimate the speed and position of rotor. Fig. 76 shows $V_1$ voltage vector and pulses applied to motor. While the rotor is rotating with $\omega_r$ speed, $V_1$ vector is applied for $t_{\text{pulse}}$ time and it is repeated every two switching period as shown in Fig. 76 (b).

Fig. 76. (a) Applying $V_1$ voltage vector and (b) $V_1$ voltage vector pulse and resulting current.
When applying $V_1$ voltage vector, it is assumed that the applied pulse time ($t_{pulse}$) is much shorter than the stator time constants ($\tau_d = L_d/R_s$, $\tau_q = L_q/R_s$). Then, the stator resistance can be neglected. Therefore, (133) is expressed again as:

$$
\begin{bmatrix}
v_d \\
v_q
\end{bmatrix} = 
\begin{bmatrix}
pL_d & -\omega_r L_q \\
\omega_r L_d & pL_q
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix}
$$

(135)

By using this simplified equation, the resulted current by $V_1$ voltage vector can be calculated.

### 5.3.2. Resulted Current by $V_1$ Voltage Vector

To calculate the resulted $d$-$q$ axis current, $V_1$ voltage vector of Fig. 76 (a) can be expressed in the rotor reference frame as:

$$
\begin{aligned}
v_d &= 2V_{dc} / 3 \cos \theta_r \\
v_q &= -2V_{dc} / 3 \sin \theta_r
\end{aligned}
$$

(136)

where $\theta_r$ is the actual rotor angle in stationary reference frame. By substituting (136) into (135), the resulted $d$-$q$ axis current by $V_1$ voltage vector can be obtained by solving (135) using Laplace transform as:

$$
\begin{bmatrix}
i_d(t_{pulse}) \\
i_q(t_{pulse})
\end{bmatrix} = 
\frac{2V_{dc}}{3}
\begin{bmatrix}
\cos (\omega_r t_{pulse}) \sin (\theta_r) + \sin (\omega_r t_{pulse}) \cos (\theta_r) - \sin (\theta_r) \\
\cos (\omega_r t_{pulse}) \cos (\theta_r) - \sin (\omega_r t_{pulse}) \sin (\theta_r) - \cos (\theta_r)
\end{bmatrix}
\begin{bmatrix}
\omega_r L_d \\
\omega_r L_q
\end{bmatrix}
$$

(137)

When deriving $d$-$q$ axis current, the initial condition of those current was assumed as zero. Because $t_{pulse}$ is the short time, $\cos(\omega_r t_{pulse})$ and $\sin(\omega_r t_{pulse})$ can be considered as 1 and $\omega_r t_{pulse}$, respectively. Then, (137) can be simplified as:
\[
\begin{bmatrix}
    i_d(t_{\text{pulse}}) \\
    i_q(t_{\text{pulse}})
\end{bmatrix} \approx \frac{2V_{dc}t_{\text{pulse}}}{3} \begin{bmatrix}
    \cos(\theta_r) \\
    \frac{L_d}{L_q} \\
    \sin(\theta_r)
\end{bmatrix}
\] (138)

From (138), the resulted three phase current can be calculated using inverse Park’s transformation as:

\[
i_d(t_{\text{pulse}}) = \frac{2V_{dc}t_{\text{pulse}}}{3} \left( \frac{1}{2} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + \frac{1}{2} \cos \left( 2\theta_r \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \right) \right);
\]

\[
i_b(t_{\text{pulse}}) = \frac{2V_{dc}t_{\text{pulse}}}{3} \left( \frac{1}{4} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + \frac{1}{2} \cos \left( 2\theta_r - \frac{2\pi}{3} \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \right) \right);
\]

\[
i_c(t_{\text{pulse}}) = \frac{2V_{dc}t_{\text{pulse}}}{3} \left( \frac{1}{4} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + \frac{1}{2} \cos \left( 2\theta_r + \frac{2\pi}{3} \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \right) \right)
\] (139)

These three phase currents obtained consist of two terms. The first term is DC-offset current and the second term is the portion oscillated by the rotor position. The rotor position can be estimated by extracting the AC terms from three phases current. It is noticeable that the AC current frequency is twice faster than the rotor electric speed. In case of applying other active vectors (ex. \(V_3\) & \(V_5\)), the AC terms of resulted currents is changed and it is derived in Appendix F.

In Fig. 77, the simulation result shows the resulted phase current and the generated torque by applying \(V_1\) voltage vector. The machine parameters are listed in Table 9. While the rotor is rotating at 1,200rpm, the \(V_1\) voltage pulse is applied and its duty is 50%. The top one is the actual electric rotor angle, the middle one is the phase current. The current magnitude is matched well with the equation derived in (139). The bottom one is the generated torque by voltage pulse.
The generated instantaneous torque by $V_1$ voltage vector is derived by substituting (138) into (132) as:

$$T_r = \frac{3n}{8} \left( \frac{2V_{dc} t_{pulse}}{3} \right)^2 \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \sin(2\theta_r)$$

(140)

where $n$ is the pole number of machine. The magnitude of instantaneous torque is determined by DC-link voltage, the applied $V_1$ voltage pulse time and the $d$-$q$ inductance. Most inverter systems use the PWM switching frequency above 1 kHz so that the applied voltage will be not too big and the magnitude of instantaneous torque will be not too big to worry about the speed fluctuation. In addition, the generated average torque per one revolution is zero. As a result, although the proposed method can generate the acoustic noisy, it will rarely make a mechanical issue in terms of the rotor speed.
Table 9. The test motor parameter.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>[kW]</td>
<td>$P_{out}$</td>
<td>18.5</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>[rpm]</td>
<td>$N_r$</td>
<td>1800</td>
</tr>
<tr>
<td>Rated Torque</td>
<td>[Nm]</td>
<td>$T_e$</td>
<td>98</td>
</tr>
<tr>
<td>Rated Voltage (line-line)</td>
<td>[V]</td>
<td>$V_s$</td>
<td>380</td>
</tr>
<tr>
<td>Phase Current (rms)</td>
<td>[A]</td>
<td>$i_s$</td>
<td>43</td>
</tr>
<tr>
<td>Pole</td>
<td>-</td>
<td>$n$</td>
<td>4</td>
</tr>
<tr>
<td>Stator Resistor</td>
<td>[Ω]</td>
<td>$R_s$</td>
<td>0.19</td>
</tr>
<tr>
<td>Stator $d$-axis Inductance</td>
<td>[mH]</td>
<td>$L_d$</td>
<td>35</td>
</tr>
<tr>
<td>Stator $q$-axis Inductance</td>
<td>[mH]</td>
<td>$L_q$</td>
<td>17</td>
</tr>
<tr>
<td>Inertia</td>
<td>[Nm/rad·s²]</td>
<td>$J$</td>
<td>0.059</td>
</tr>
<tr>
<td>Inverter DC Link Voltage</td>
<td>[V]</td>
<td>$V_{dc}$</td>
<td>540</td>
</tr>
<tr>
<td>PWM Switching Freq.</td>
<td>[kHz]</td>
<td>$f_{sw}$</td>
<td>5</td>
</tr>
</tbody>
</table>

5.3.3. Proposed Position Estimation Method

As mentioned in previous section, the resulted three phase current by $V_1$ voltage vector includes both the DC-offset term and the AC oscillation term. In order to use three phase currents for the rotor position estimation, DC-offset terms in (139) should be eliminated properly. The proposed method uses the averaging method and the DC-offset is obtained by averaging the $a$-phase current for several revolutions of the rotor. When selecting the method to obtain the DC-offset, the averaging method was compared with the LPF method. In case of the averaging method, the time to reach at the steady state value depends on the current sampling time (several kHz) and the current frequency (several tens of Hz). On the other hands, LPF always causes the time delay determined by the filter time constant. The cutoff frequency of LPF cannot be set as a value which exceeds the rotor minimum electric frequency (ex. 5Hz). The time delay of LPF will be shown in the simulation result of Fig. 78. In addition, if the current frequency is close with the cutoff frequency of LPF, the AC terms is not eliminated properly. It will cause the DC-offset calculation error. Fig. 78 shows the DC-offset value of the a-phase current obtained in (a) the averaging method and (b) the LPF method. In the simulation, the rotor electric frequency is 40Hz and the current sampling frequency is 5kHz. Unlikely the averaging method, the LPF method cause much longer time delay and calculation error. Therefore, the simulation result verifies the
superiority of averaging method so that the proposed method uses the averaging method to obtain the DC-offset of phase current.

Fig. 78. DC-offset of a-phase current obtained by (a) Averaging method (b) LPF method.

In the average method, the DC-offset calculation error corresponding to the averaging time is discussed. The calculation error can be reduced as increasing the averaging time. Fig. 79 shows the DC-offset term and AC term of the phase current. At every half period of the current, the DC-offset calculation error is expressed in Fig. 79. $f$ is the frequency of phase current. The error is calculated with the following equation as:

$$\text{Error} = \frac{AC_{\text{sum}}}{AC_{\text{sum}} + DC_{\text{sum}}} = \frac{1}{1 + \frac{B \pi}{AT_f} f}$$  \hspace{1cm} (141)
In here, $T_f$ is the period of current. In (141), $B$ is larger than $A$ because a-phase current is always positive value while applying $V_1$ voltage vector. Thus, the minimum ratio of $B$ and $A$ can be assumed as 1. For example, if the electric rotor speed is assumed as 5Hz, the current frequency will be 10Hz as explained in (139). Then, the minimum averaging time is about 0.3[sec.] to make the DC-offset calculation error be below 10%.

Fig. 79. The DC-offset calculation error corresponding to the averaging time.

The proposed method uses the a-phase current to calculate DC-offset current in case of applying $V_1$ voltage vector. When $V_1$ voltage is applied, the magnitude of a-phase current is biggest among three phase current. It was already explained in previous section 5.3.2 and it will be verified with the experimental test. Therefore, as calculating DC-offset with a-phase current, the DC-offset calculation error caused by the current sensing error can be minimized. DC-offsets of $b$ & $c$ phase are a half of a-phase DC-offset and have the negative value. By subtracting each DC-offset from the measured three phase current, only AC terms including the rotor position information can be obtained as:
\[ i_{a_{ac}}(t) = K \cos(2\theta_t); \quad i_{b_{ac}}(t) = K \cos\left(2\theta_t - \frac{2\pi}{3}\right); \quad i_{c_{ac}}(t) = K \cos\left(2\theta_t + \frac{2\pi}{3}\right) \] (142)

In here, \( i_{a_{ac}}, \ i_{b_{ac}} \) and \( i_{c_{ac}} \) are AC terms of measured three phase current which DC-offset is eliminated and \( K \) is \( V_{dc} \cdot T_{pulse}3 \cdot (1/L_d - 1/L_q) \). Actually, as measuring two phase current through two current sensors, three phase current can be obtained. To extract the rotor position, Clarke transform is used. Then, \( \alpha-\beta \) axis current in stationary reference frame is obtained as:

\[
\begin{bmatrix}
    i_{\alpha} \\
    i_{\beta}
\end{bmatrix} = T(0) \begin{bmatrix}
    i_{a_{ac}} \\
    i_{b_{ac}} \\
    i_{c_{ac}}
\end{bmatrix}
\] (143)

With \( \alpha-\beta \) axis current, the rotor position can be estimated by using the following equation.

\[
\theta_{est} = \frac{1}{2} \tan^{-1}\left(\frac{i_{\beta}}{i_{\alpha}}\right)
\] (144)

In here, \( \theta_{est} \) is the estimated rotor angle. In proposed method, the rotor position is estimated by measuring two phase current resulted by \( V_1 \) voltage vector. In ideal case, the estimated theta and the actual theta will be same. However, the measured phase current may have the error due to the current sensor resolution or external noise. The estimation error is generated and the error angle is expressed as:

\[
\Delta\theta_{error} = \theta_{est} - \theta_r
\] (145)

\( \theta_r \) is the actual rotor angle and \( \Delta\theta_{error} \) is the error between the estimated angle and the actual angle. \( \Delta\theta_{error} \) will depend on the accuracy of current sensing.
5.3.4. Proposed Speed Estimation Method

While estimating the rotor speed and position, the voltage vector is applied every two switching period as shown in Fig. 76 (b). The rotor speed can be easily estimated by using two estimated theta obtained in (144). If the interval time between two estimated theta is short enough, the rotor speed can be assumed as the constant. Therefore, the rotor speed can be calculated by using the following equation as:

\[
\omega_{est} = \frac{\theta_{est2} - \theta_{est1}}{t_{pulse} + \tau}
\]  
(146)

As shown in Fig. 76(b), \( \theta_{est1} \) and \( \theta_{est2} \) are the theta estimated at \( t_1 \) and \( t_3 \), respectively. \( t_{pulse} \) is the time which the \( V_1 \) vector is applied to the stator winding, \( \tau \) is the time between two voltage pulses and \( \omega_{est} \) is the estimated electrical angular frequency of the rotor. As mentioned before, the estimated theta can have the error caused by the current sensing error. If \( \tau \) is selected as long enough value, the speed estimation can be more precise as shown in the following equation.

\[
\omega_{est} = \frac{\theta_{est2} - \theta_{est1}}{t_{pulse} + \tau} = \frac{\theta_{est2} - \theta_{est1} + 2\Delta \theta_{error}}{t_{pulse} + \tau} = \omega_r + \frac{2\Delta \theta_{error}}{t_{pulse} + \tau}
\]  
(147)

However, when choosing the interval time (\( \tau \)), the rated speed of the rotor should be considered so that the interval \( \tau \) is shorter than the time taken for one electrical revolution [71]. In other words, if the \( \theta_{est2} \) is calculated after the rotor rotates the electric angle \( 2\pi \) [rad] from the \( \theta_{est1} \) calculated at \( t_1 \), the rotor rotating speed can be estimated incorrectly.

\[
\begin{align*}
\omega_r &= \frac{(\theta_{est2} + 2\pi \cdot N) - \theta_{est1}}{t_{pulse} + \tau} \quad \text{(Actual)} \\
\omega_{est} &= \frac{\theta_{est2} - \theta_{est1}}{t_{pulse} + \tau} \quad \text{(Estimated)}
\end{align*}
\]  
(148)
where \( N \) is the rotor rotation number between two theta, \( \omega_r \) is an actual speed and \( \omega_{est} \) is the estimated speed. The estimated speed will be the same with the actual speed only when \( N \) is zero. Therefore, the following equation should be satisfied to prevent the wrong speed estimation.

\[
\theta_{z_2} - \theta_{z_1} = \omega_r \cdot (\tau + t_{\text{pulse}}) < \pi
\]  

(149)

During calculating \( \tau \) in (149), \( \omega_r \) and \( t_{\text{pulse}} \) are selected as \( \omega_{\text{rated}} \) and 100% duty, respectively. Then, the difference between two estimated theta will not exceed \( \pi \) at any speed condition. \( \omega_{\text{rated}} \) is the maximum speed known by the motor nameplate. Unlike the restart method of PMSM, the restart method of SynRM uses \( \pi \) instead of using \( 2\pi \). It is because the current frequency of SynRM is twice faster than the rotor electric speed as shown in (142).

When estimating the speed with (146), the additional issue is happened in low speed condition. Because the difference between two estimated theta is too small in low speed condition, it causes the speed estimation error. Fig. 80 shows the speed estimation in low speed condition. The time interval used for the speed estimation was 50\( T_{sw} \). It was calculated by substituting \( \omega_{\text{rated}} \) into \( \omega_r \) of (149). During the estimation mode, the estimated speed is not constant. And the restart is failed due to the overcurrent. Therefore, additional compensation logic is required.

![Fig. 80. The speed estimation in low speed (electric 5Hz speed); CH1: the estimated theta, CH2: the estimated speed [10Hz/div.], CH3: the interval time (\( \gamma \)) [250T_{sw}/div.], CH4: the actual phase current [20A/div.].](image-url)
Fig. 81 shows the test result of the speed estimation and the restart in case the interval time is adjusted at low speed condition. At the beginning, the interval time is set as 50Tsw and the speed is estimated. With the estimated speed, the new interval time is calculated. When calculating the new interval time using (149), \( \omega_r \) is replaced with \( \omega_{est} \) instead of \( \omega_{rated} \). In here, as considering that \( \omega_{est} \) has some error, the new interval time is calculated as:

\[
\tau_{new} = 0.9 \cdot \frac{\pi}{\omega_{est}}
\]  

(150)

And the maximum new interval time is limited below 500Tsw because too low speed condition will require very long interval time. Fig. 81 shows that the interval time is automatically changed from 50Tsw to 500Tsw at 5Hz speed. In case of using 500Tsw interval time, the speed estimation is much better than one of 50Tsw so that the restart is succeed in low speed condition.

![Diagram](image-url)

Fig. 81. The speed estimation in low speed with the interval time adjustment (electric 5Hz speed); CH1: the estimated theta, CH2: the estimated speed [10Hz/div.], CH3: the interval time (\( \tau \)) [250T_{sw}/div.], CH4: the actual phase current [20A/div.].
5.4. Implementation of Proposed Restart Method

The key performance criteria of the proposed restart method for SynRM motors is to successfully estimate the rotor speed and position with only motor parameters given by the nameplate and restart the motors without causing the overcurrent and the big braking torque. The proposed complete scheme for estimating the rotor speed and position is shown in Fig. 82. And the estimation procedure is explained as following steps.

- Step 1 – Applying $V_1$ voltage pulses with the fixed duty.
- Step 2 – Measuring the resulted two phase current by $V_1$ voltage pulses.
- Step 3 – Averaging the phase current ($i_a$) and eliminating DC-offset from three phase current.
- Step 4 – Transformation to stationary reference frame ($i_{abc} \rightarrow i_{dq}$).
- Step 5 – Estimate the rotor speed & position.
- Step 6 – Calculation of new interval time and the estimation of the rotor speed and position.
- Step 7 – Theta compensation.
- Step 8 – The command voltage is calculated by using the rated $v/f$ ratio.
- Step 9 – Stator voltage is increased gradually from zero until reaching to the command voltage.
- Step 10 – Starting the stabilizing loop [28] from the next switching step with the current feedback.
In the step 1, the initial duty of the applied voltage is set as 50%. In general, the inductance of SynRM is relatively larger than other types of motors. If the minimum of the $L_d$ & $L_q$ inductance and the PWM switching frequency are assumed as several tens of milli-henry [mH] and several kHz and the maximum of DC-link voltage is assumed as smaller value than 1000 [V], the generated current will not exceed the rating current excepting the extreme cases. The magnitude of generated current is determined by the following equation as:

$$I_{mag} = \sqrt{i_d(t_{pulse})^2 + i_q(t_{pulse})^2} = \frac{2V_{dc} t_{pulse}}{3} \sqrt{\frac{\cos(\theta)^2}{L_d^2} + \frac{\sin(\theta)^2}{L_q^2}} < \frac{2V_{dc} t_{pulse}}{3L_q}$$  (151)

If the current exceeds the rating current during estimating the speed and position, the duty is reduced to the half of initial duty and the estimation is restarted again. Fig. 83 shows the switching condition during applying $V_1$ voltage pulses for the estimation.

![Fig. 83. The switching condition during applying the active voltage vector.](image)

In the figure, $S_u$, $S_v$, $S_w$ are the switching condition. If $S_{uvw}$ is high, upper switches of the inverter each legs are turned on. On the other hand, the low statue indicates that switches are turned off. Before the fault is occurred,
the inverter is using the normal SVPWM mode to run the motor. After the fault, all switches are opened. The estimation mode is started after the power is recovered. In this mode, \( V_1 \) voltage vector is applied with 50% duty. And all switches are opened in the next switching period. And then \( V_1 \) voltage vector is applied again with the same duty. The reason requiring all switches open condition is explained in the following. First, all switches in proposed method are opened after applying \( V_1 \) voltage vector. While all switches are opened, the current is flowed by the anti-parallel diodes. It is shown in Fig. 84. During that time, the DC-link voltage is applied to reduce the flowing phase current. Therefore, the phase current will reach to zero before starting the next switching period.

![Fig. 84. The current flow in the proposed method in case of (a) applying \( V_1 \) and (b) opening all switches.](image)

On the other hands, if \( V_0 \) zero voltage pulse is applied instead of opening all switches after \( V_1 \) voltage applied, the phase current does not go back to zero before applying \( V_1 \) voltage. During applying \( V_0 \) voltage pulse, the phase current decreases depending on the stator resistor dissipation. When \( V_1 \) voltage pulse is applied again, the phase current is increased. Thus, the phase current is continuously increased and the inductor will be saturated. Fig. 85 shows the \( a \)-phase current generated by \( V_1 \) and \( V_0 \) voltage vector. Therefore, this switching method cannot be used for the restart method for SynRM.
Fig. 85. $a$-phase current generated by $V_1$ and $V_0$ voltage vector.

Fig. 86 shows the current flow in the inverter when $V_1$ and $V_0$ voltage is applied. While applying $V_1$ voltage pulse, $2/3V_{dc}[V]$ voltage is applied to $a$-phase stator. In case of applying $V_0$ voltage pulse, $0[V]$ is applied. Therefore, the average of applied voltage is not zero so that the stator inductor will be saturated.

In the step 7, the estimated theta ($\theta_{est}$) is aligned with $d$-axis of motor in the rotor reference frame. The estimated theta is compensated and the compensated theta ($\theta_{comp}$) is given by:

$$\theta_{comp} = \theta_{est} + \frac{\pi}{2} + \omega_{est} T_{sw}$$  \hspace{1cm} (152)
By adding $\pi/2$ [rad.] to $\theta_{est}$, the compensated theta ($\theta_{comp}$) can be aligned with $q$-axis. Moreover, the third term ($\omega_{est} T_{sw}$) in (152) is to compensate the current sampling delay. It is also mentioned in PM restart method of section 4.4. 

![Diagram](image)

Fig. 87. Vector diagram of theta compensation.

Fig. 87 shows the applied voltage and the resulted current at the restart moment. The stator voltage is applied to $q$ axis to ensure a positive torque at the restart instant. $\theta_z$ is the phase of motor impedance and has a value between 0 and 90°[deg.] because a motor is an inductive load. Then, $d$-$q$ axis current and the generated torque will be always positive value. The torque of SynRM is determined by the following equation as:

$$T_r = \frac{n}{2} \left( L_d - L_q \right) i_d i_q$$  \hspace{1cm} (153)

In the equation, the maximum torque will be generated when the $d$-$q$ axis current have the same positive value. In other word, the torque will be maximum when the phase of $I_s$ is 45°[deg.].

In the step 9, the stator voltage frequency is set as the estimated frequency. The applied stator voltage is increased gradually from zero. The voltage increase is stopped when reaching the rated $v/f$ ratio. The purpose increasing the voltage gradually is to prevent the inrush current which can be generated as applying the step.
voltage. In the proposed method, the slope of the voltage increase is selected as 1000 $[V/sec.]$ that is about twice faster slope than the rated voltage slope of motor. The rated speed and voltage slope of motor are 60 $[Hz/sec.]$ and 360 $[V/sec.]$, respectively. Thus, 1000 $[V/sec.]$ is considered as the reasonable value and it was verified with experimental tests.

5.5. Simulation Results

Simulation results are given in Fig. 88. A fault occurs resulting in a temporary power loss. At $t = 0$, power is restored and the speed and position estimation is started. And then, the machine settles back at its reference speed about 600ms later. The simulation shows the estimated speed and theta, the applied stator voltage and the resulted phase current. With the proposed method, the speed and position estimation are good and the restart for SynRM is succeed without any inrush current.

![Simulation Results](image)

Fig. 88. Flying restart simulation results. A fault occurs resulting in a temporary power loss. At $t = 0$, power is restored, and the machine settles back at its reference speed about 600ms later. Plots top to bottom: machine speed (blue) and speed estimate (red); machine angle (blue) and estimated angle (red); applied stator voltage; machine phase current.
5.6. Experimental Results

A set of experiments will be conducted to validate the performance of the proposed restart method. As shown in Fig. 89, the dynamo test bed consists of a synchronous reluctance motor for the test purpose and an induction motor for supplying the load torque. The scalar control and the proposed restart method are implemented using OPAL-RT.

Fig. 89. The test motor parameters & dynamo set configuration.

First, the test motor was rotated at the fixed speed not fed by the inverter but used the speed control of the load motor coupled with the test motor. Fig. 90 shows experimental results including the $a$-phase current, AC oscillation term and DC-offset term of the stator current when the mechanical rotor speed is 600rpm.

Fig. 90. The resulted current by applied voltage vector; CH1: the phase current sensed by A/D converter of DSP [1A/div], CH2: the DC-offset of sensed phase current [1A/div], CH3: the phase current which DC-offset is eliminated ($i_{a, ac}$) [1A/div], CH4: the actual phase current ($i_a$) [1A/div].
Fig. 91 shows the test results of the speed and position estimation by applying $V_1$ voltage vector. First, the test motor was rotated at the fixed speed by the load motor coupled with the test motor. In this condition, the rotor speed and position estimation algorithm was implemented by applying $V_1$ voltage pulses. The speed condition was 600rpm and 1200rpm. The pulse duty was 50%, the DC-link voltage was 300V and PWM switching frequency was 5kHz. The blue line is the measured phase current (D/A converter output) and the cyan line is the AC term of $a$-phase current which DC-offset is eliminated. The estimated speed (green line) is matched well with the actual speed during the speed searching time. In addition, these result verifies that the resulted phase current does not depend on the rotor speed. The phase current magnitude just depends on the applied voltage magnitude and the stator inductance as shown in (139).

![Fig. 91. The estimated position and speed waveforms (a) 600rpm [20ms/div] (b) 1200rpm [10ms/div]; CH1: the sensed phase current ($i_a$) [1A/div], CH2: the AC term of phase current which DC-offset is eliminated ($i_{a1}$) [1A/div], CH3: the estimated position of the rotor, CH4: the estimated speed of the rotor [600rpm/div].](image-url)
Fig. 92 shows the speed estimation error in the beginning due to using the averaging method. The inverter feeding SynRM motor is stopped by intentionally. During that time, the motor speed is reduced depending on the inertia, the friction and the load condition. The purple line is the estimated speed obtained through the measured phase current. As shown in Fig. 79, the DC-offset has the error in the beginning and it also causes the speed estimation error. Therefore, the proposed method requires a little time for the error of DC-offset to be small enough. In Fig. 92, it took about 100 [msec.] to get a correct value.

Fig. 92. Initial speed estimation error caused by using the averaging method CH1: the sensed phase current (i_a) [1A/div], CH2: the estimated theta, CH3: the estimated rotor speed [600rpm/div.], CH4: the stator phase current (20A/div).

Fig. 93 shows experimental results of the proposed restart method including the estimated speed and position of the rotor and the stator current when the mechanical rotor speed is 600, 900, 1200 and 1500rpm. The restart tests are implemented as the following steps. First, the motor is operating at the reference speed. The inverter feeding SynRM motor is stopped by intentionally for 1.5 seconds. During that time, the motor speed is reduced depending on the inertia, the friction and the load condition. After 1.5 seconds, the speed and position estimation of the rotor is implemented and the motor is again fed by the inverter. Then, the v/f control is started with the stabilizing loop [28]. The test results of Fig. 93 verify that the performance of the proposed method is good to estimate the rotor speed and position and the restart is implemented without causing any inrush current.
Fig. 93. Restart waveforms of the complete proposed method (a) 600rpm (b) 900rpm (c) 1200rpm (d) 1500rpm; CH1: the estimated rotor position, CH2: the estimated speed (600rpm/div.), CH3: the magnitude of stator voltage (100V/div.), CH4: the stator phase current (20A/div.).
Fig. 94 shows the test results with different voltage increase slope. As mentioned before, the voltage increase slope in the proposed method is selected as 1000 [V/sec.]. Fig. 94 shows the results when 3000 [V/sec.] slope is selected. At slow speed condition, the restart is failed due to the overcurrent. It means that too fast voltage increase can cause the overcurrent issue.

Fig. 94. Test results with 3000 [V/sec.] voltage change slope (a) 600rpm (b) 900rpm (c) 1200rpm (d) 1500rpm; CH1: the estimated rotor position, CH2: the Angle of applied stator voltage, CH3: the magnitude of stator voltage (20V/1V), CH4: the stator phase current (20A/div).

The experimental test results of Fig. 95 verify that the stator voltage should be applied in $q$-axis at the restart moment. The tests are done with different stator voltage angle. The rotor speed condition was 900rpm. The restart tests are failed at 0° and 45°[deg.]. It is because these condition generates the negative torque at the restart instant. On the other hand, the restart tests are succeed at 90° and 135°[deg.]. As mentioned before, the best performance is shown when applying the stator voltage to 90°[deg.].
5.7. Conclusion

This chapter described the proposed restart algorithm for SynRM in detail. The goal of proposed restart method was to develop the universal algorithm for SynRM motors. Therefore, this proposed method uses only the measured phase current and motor rating information known by the nameplate. In general, the nameplate includes the machine rating information such as the power, the current, the speed and the voltage. In addition, this method does not require the additional hardware such as the speed sensors, phase voltage sensors, DC-link current sensor which are not installed in general commercial inverters. In addition, the algorithm to minimize the speed estimation error is also suggested. The interval time for the speed estimation is automatically adjusted for the restart in low speed condition. It was to minimize the speed estimation error at low speed. Experimental results have conducted to validate the performance of the proposed method. As a result, this proposed method is available for the universal application of SynRM.
CHAPTER 6: SUMMARY AND FUTURE WORK

6.1. Major Contributions

The need of flying restart method in the industry field is explained and the novel universal flying restart strategies for asynchronous and synchronous motors are developed. Major contributions of the research work are as follows.

CHAPTER 1: INTRODUCTION

This chapter introduced the basic background and features of electric motor and drive system. And the need of flying restart method in an industry field is explained.

CHAPTER 2: IMPLEMENTATION OF SCALAR CONTROL

This chapter explained the basic principle of scalar control methods for IM and PMSM which were used to drive the machine when testing the flying restart algorithms. It also presented the stability analysis method for PMSM. In addition, the stabilizing loop to generate the frequency modulation signal was derived to add the damping effect to the system. The stability of the PMSM was analyzed in two cases which are with the stabilizing loop and without the stabilizing loop. The simulation and experimental tests have conducted to verify the stability analysis results and the damping effect of the stabilizing loop.

CHAPTER 3: A UNIVERSAL RESTART STRATEGY FOR INDUCTION MACHINES

This chapter described the proposed flying restart algorithm for induction motors in detail. The most important aspect of the developed restart strategy is to realize the universal application for induction motors. First of all, this proposed method used only motor parameters known by the nameplate. In general, the nameplate includes the machine rating information such as the power, the current, the speed and the voltage. In addition, this restart method does not require any tuning work although the motor is replaced. And the rotor speed searching time is almost constant at different conditions. The proposed restart method is designed to be
less sensitive to the motor parameters and other conditions (such as rotor speed, input power and so on). It is implemented by monitoring the input power perturbation as well as the input power. Finally, this restart method does not require the additional hardware such as the speed sensors, phase voltage sensors, DC-link current sensor which are not installed in general commercial inverters. The performance of the proposed method was verified by the experimental tests.

**CHAPTER 4: A UNIVERSAL RESTART STRATEGY FOR PERMANENT MAGNET SYNCHRONOUS MACHINES**

This chapter explained the basic concept of existing restart methods and described the proposed restart algorithm in detail. The goal of proposed restart method was to develop the universal algorithm for PM motors. First of all, this proposed method uses only motor parameters known by the nameplate. In general, the nameplate does not include the information such as $d$-$q$ axis stator inductance which the conventional restart methods used. In addition, this proposed method can automatically determine the duty of zero voltage vector pulses as considering the rated current to prevent an overcurrent and the delay time between two pulses considering the rated rotor speed and the acceptable estimation error. As a result, this proposed restart method can be used for universal application of PMSM. The maximum rotor position estimation error of the proposed method is less than that of the existing method used in [71] although the variation of inductance and the rotor speed estimation error by the current sensing error exist. The maximum estimation error will be under 10° [deg] because the $\omega_r \cdot t_{pulse}$ is always limited as a smaller value than 0.035. The experimental results have conducted to validate the performance of the proposed restart method. Moreover, experimental result includes the restart test in both cases which are with the stabilizing loop and without that, and it verified the need of stabilizing loop at the restart instant. In addition, this chapter suggests the method for $L_d$ & $L_q$ estimation while implementing the flying restart. This estimated inductance value is used for rotor position estimation. This proposed method is verified with the simulation and the result was matched well with the analytical analysis result.

**CHAPTER 5: A UNIVERSAL RESTART STRATEGY FOR SYNCHRONOUS RELUCTANCE MACHINES**

This chapter described the proposed restart algorithm for SynRM in detail. The goal of proposed restart method was to develop the universal algorithm for SynRM motors. Therefore, this proposed method uses only
the measured phase current and motor rating information known by the nameplate. In general, the nameplate includes the machine rating information such as the power, the current, the speed and the voltage. In addition, this method does not require the additional hardware such as the speed sensors, phase voltage sensors, DC-link current sensor which are not installed in general commercial inverters. In addition, the algorithm to minimize the speed estimation error is also suggested. The interval time for the speed estimation is automatically adjusted for the restart in low speed condition. It was to minimize the speed estimation error at low speed. Experimental results have conducted to validate the performance of the proposed method. As a result, this proposed method is available for the universal application of SynRM.

6.2. Future Work

All proposed methods are tested with the motor drive system using a scalar control. For the verification of universal application, proposed methods need to be tested with the motor drive system using a vector control. In addition, second proposed method of PMSM is verified with only simulation. The performance needs to be verified with experimental test. In addition, the restart method for SynRM using only DC-link current sensor was already developed and It will be published after filing patent.
REFERENCES


APPENDICES
Appendix A: Derivation of the $d$-$q$ Axis Currents in the Rotor Reference Frame Induced by Zero Voltage Vector

In this section, the derivation omitted in section 4.2.2 is explained in detail. As mentioned in section 4.2, the stator resistance can be neglected because a short time ($t_{\text{short}}$) is much smaller than the stator time constants.

\[
\begin{bmatrix} 0 \\
0 \end{bmatrix} = \begin{bmatrix} pL_d & -\omega_r L_q \\
pL_q & pL_q \end{bmatrix} \begin{bmatrix} i_d(t_{\text{short}}) \\
i_q(t_{\text{short}}) \end{bmatrix} + \omega_r \lambda_f \begin{bmatrix} 0 \\
1 \end{bmatrix}
\]

(154)

And the zero voltage vector is applied to the stator when the stator current is zero so that initial value of Laplace transform is zero. Once these equations are Laplace transformed, the following equation can be obtained as:

\[
0 = sL_d I_d(s) - \omega_r L_q i_q(s); \\
0 = \omega_r L_d I_d(s) + sL_q i_q(s) + \omega_r \lambda_f
\]

(155)

By using the first equation of (155), the relation between the stator $d$-axis current ($i_d(s)$) and the stator $q$-axis current ($i_q(s)$) in $s$-domain can be obtained as:

\[
I_d(s) = \frac{\omega_r L_q}{sL_d} i_q(s)
\]

(156)

By substituting (156) into the second equation of (155), the stator $q$-axis current ($i_q(s)$) can be obtained as:

\[
I_q(s) = -\frac{\lambda_f}{L_q} \frac{\omega_r}{s^2 + \omega_r^2}
\]

(157)

The inverse Laplace transform of (157) results in the stator $q$-axis current in time domain.

\[
i_q(t_{\text{short}}) = -\frac{\lambda_f}{L_q} \sin(\omega_t t_{\text{short}})
\]

(158)
And substituting (157) into (156) results in the stator $d$-axis current in s-domain.

$$I_d(s) = -\frac{\omega_f \lambda_f}{s L_d} \left( \omega_f \right) = -\frac{\omega_f \lambda_f}{L_d} \left[ \frac{1}{s^2 + \omega_f^2} \right]$$

(159)

The inverse Laplace transform of (159) results in the stator $d$-axis current in time domain.

$$i_d(t_{short}) = \frac{\lambda_f}{L_d} \left[ 1 - \cos(\omega_f t_{short}) \right]$$

(160)

From (158) and (160), the $d$-$q$ axis currents are rewritten as:

$$\begin{bmatrix} i_d(t_{short}) \\ i_q(t_{short}) \end{bmatrix} = \begin{bmatrix} \frac{-\lambda_f}{L_d} \left[ 1 - \cos(\omega_f t_{short}) \right] \\ \frac{-\lambda_f}{L_q} \sin(\omega_f t_{short}) \end{bmatrix}$$

(161)
Appendix B: Derivation of the Small Signal Model of PMSM Having the Damper Winding

In this section, the equations for the stability analysis of PM motor having the damper winding are derived through the same procedures of section 2.3. First, the electrical and mechanical equations of PM motor are derived and the small signal equations through the linearization is extracted.

The electrical equation of the interior-type PMSM is expressed in the $d$-$q$ rotor reference frame [31, 63]. Fig. 96 shows the equivalent circuit of the IPMSM having the damper winding in the rotor reference frame.

![Equivalent Circuit Diagram](image)

Fig. 96. The equivalent circuit of IPMSM having the damper winding in rotor reference frame; (a) $d$-axis circuit (b) $q$-axis circuit.

The $d$-axis and $q$-axis stator voltages are obtained respectively as:
\[
\begin{align*}
v_d &= r_s i_d + \frac{d\lambda_d}{dt} - \omega \lambda_q; \\
v_q &= r_s i_q + \frac{d\lambda_q}{dt} + \omega \lambda_d
\end{align*}
\] (162)

The stator linkage flux ($\lambda_d$ and $\lambda_q$) are expressed as the following equations [16, 17, 21-25].

\[
\begin{align*}
\lambda_d &= L_d i_d + L_{md} i_D + \lambda_m; \\
\lambda_q &= L_q i_q + L_{mq} i_Q
\end{align*}
\] (163)

In here, $L_{mq}$ and $L_{md}$ are the $d$-axis and $q$-axis mutual inductance between the stator and rotor damper winding, respectively. $i_D$ and $i_Q$ are $d$-axis and $q$-axis damper winding current, respectively.

The $d$-axis and $q$-axis rotor voltages are obtained respectively as:

\[
\begin{align*}
v_D &= 0 = R_D i_D + \frac{d\lambda_D}{dt}; \\
v_Q &= 0 = R_Q i_Q + \frac{d\lambda_Q}{dt}
\end{align*}
\] (164)

where $R_D$ and $R_Q$ are the $d$-axis and $q$-axis rotor damper winding resistance, respectively. $\lambda_D$ and $\lambda_Q$ are the $d$-axis and $q$-axis rotor linkage flux, respectively. The rotor linkage flux are expressed as the following equations.

\[
\begin{align*}
\lambda_D &= L_{md} i_d + L_{qD} i_D + \lambda_m; \\
\lambda_Q &= L_{mq} i_q + L_{qQ} i_Q
\end{align*}
\] (165)

With the above equations, the differential equations can be derived. Then, the small signal model can be obtained with the linearization. The state vector $x$ of the small signal model will be $[i_d i_q i_D i_Q \omega \delta]^T$. By substituting (165) into (164), the differential equations for $i_D$ and $i_Q$ can be derived as:
\[
\begin{align*}
\frac{di_D}{dt} &= -\frac{R_2}{L_D}i_D - \frac{L_{ma}}{L_D} \frac{di_d}{dt}; \\
\frac{di_q}{dt} &= -\frac{R_2}{L_q}i_q - \frac{L_{ma}}{L_q} \frac{di_q}{dt};
\end{align*}
\] (166)

(166) has two differential terms and it cannot be expressed in the state-space form. Therefore, the differential terms of \(i_d\) and \(i_q\) should be replaced with other state variables. By using (162), (163) and (166), the differential equations for \(i_d\) and \(i_q\) can be derived as:

\[
\begin{align*}
\frac{di_d}{dt} &= -\frac{1}{L_d}V_s \sin\delta - \frac{r_s}{L_d}i_d + \frac{L_{ma}R_D}{L_d(1-k_D^2)}i_d - \frac{1}{L_d} \omega_L (L_q i_q + L_{ma} i_q); \\
\frac{di_q}{dt} &= \frac{1}{L_q}V_s \cos\delta - \frac{r_s}{L_q}i_q + \frac{L_{ma}R_Q}{L_q(1-k_Q^2)}i_q - \frac{1}{L_q} \omega_L (\lambda_m + L_d i_d + L_{ma} i_d)
\end{align*}
\] (167)

In here, \(v_d\) and \(v_q\) are replaced with \(V_s \sin\delta\) and \(V_s \cos\delta\), respectively. When deriving this model, it is considered that the input voltage \(v_s\) is constant steady-state values. \(k_D\) and \(k_Q\) are used for the simplification of the equation and expressed as:

\[
\begin{align*}
k_Q^2 &= \frac{L_{ma}^2}{L_d L_q}; \\
k_D^2 &= \frac{L_{ma}^2}{L_d L_q}
\end{align*}
\] (168)

By substituting (167) into (166), the final differential equations for \(i_D\) and \(i_Q\) can be derived as:

\[
\begin{align*}
\frac{di_D}{dt} &= \frac{L_{ma}}{L_d} \frac{1}{L_d(1-k_D^2)} V_s \sin\delta + \frac{L_{ma}}{L_q} \frac{r_s}{L_q} i_d - \frac{L_{ma}}{L_q} \frac{1}{L_q(1-k_Q^2)} \omega_L (L_q i_q + L_{ma} i_q); \\
\frac{di_Q}{dt} &= -\frac{L_{ma}}{L_q} \frac{1}{L_q(1-k_Q^2)} V_s \cos\delta + \frac{L_{ma}}{L_q} \frac{r_s}{L_q} i_q - \frac{L_{ma}}{L_q} \frac{1}{L_q(1-k_Q^2)} \omega_L (\lambda_m + L_d i_d + L_{ma} i_d)
\end{align*}
\] (169)
The electric torque equation can be expressed as:

\[
T_e = \frac{3}{2J} \left( \frac{n}{2} \right)^2 \left[ \lambda_d i_q - \lambda_q i_d \right]
\]  
(170)

By substituting (163) into (170), the differential equation for rotor speed \( \omega_r \) can be obtained as:

\[
\frac{d\omega_r}{dt} = \frac{3}{2J} \left( \frac{n}{2} \right)^2 \left[ \lambda_m i_q + \left( L_d - L_q \right) i_d i_q + L_{md} i_q^2 - L_{mq} i_q^2 \right] - \frac{1}{J} B_m \omega_r - \frac{n}{2J} T_i
\]

(171)

The differential equation for the load angle (\( \delta \)) is expressed as:

\[
\frac{d\delta}{dt} = \omega_e - \omega_r
\]

(172)

From (167), (169), (171) and (172), the machine state equations can be obtained as:

\[
\begin{align*}
\frac{di_d}{dt} &= \frac{1}{L_q \left( 1 - k_{\delta}^2 \right)} V_s \cos \delta - \frac{r_s}{L_q \left( 1 - k_{\delta}^2 \right)} i_q + \frac{L_{ma} R_q}{L_q \left( 1 - k_{\delta}^2 \right) L_{m}} i_q - \frac{1}{L_q \left( 1 - k_{\delta}^2 \right)} \omega_r \left( \lambda_m + L_d i_d + L_{md} i_d \right) \\
\frac{di_q}{dt} &= -\frac{1}{L_d \left( 1 - k_{\delta}^2 \right)} V_s \sin \delta - \frac{r_s}{L_d \left( 1 - k_{\delta}^2 \right)} i_d + \frac{L_{md} R_d}{L_d \left( 1 - k_{\delta}^2 \right) L_{m}} i_d + \frac{1}{L_d \left( 1 - k_{\delta}^2 \right)} \omega_r \left( \lambda_m + L_d i_d + L_{md} i_d \right) \\
\frac{di_{m}}{dt} &= \frac{L_{md}}{L_{m}} \frac{1}{L_q \left( 1 - k_{\delta}^2 \right)} V_s \sin \delta + \frac{L_{md}}{L_d \left( 1 - k_{\delta}^2 \right) L_{m}} r_s \frac{R_q}{L_q \left( 1 - k_{\delta}^2 \right)} i_q - \frac{1}{L_{m}} \omega_r \left( \lambda_m + L_d i_d + L_{md} i_d \right) \\
\frac{di_{q}}{dt} &= \frac{L_{md}}{L_d \left( 1 - k_{\delta}^2 \right)} V_s \cos \delta - \frac{r_s}{L_d \left( 1 - k_{\delta}^2 \right)} i_d + \frac{L_{ma} R_q}{L_d \left( 1 - k_{\delta}^2 \right) L_{m}} i_d - \frac{1}{L_d \left( 1 - k_{\delta}^2 \right)} \omega_r \left( \lambda_m + L_d i_d + L_{md} i_d \right) \\
\frac{d\omega_r}{dt} &= \frac{3}{2J} \left( \frac{n}{2} \right)^2 \left[ \lambda_m i_q + \left( L_d - L_q \right) i_d i_q + L_{md} i_q^2 - L_{mq} i_q^2 \right] - \frac{1}{J} B_m \omega_r - \frac{n}{2J} T_i \\
\frac{d\delta}{dt} &= \omega_e - \omega_r
\end{align*}
\]  
(173)
The state variables consist of the steady state value and the small signal value. By separating two values, the linearized small signal model can be expressed as the following form.

\[
\frac{d\Delta x}{dt} = A\Delta x + B\Delta u
\]  

(174)

Where \(\Delta x = [\Delta i_d, \Delta i_q, \Delta i_D, \Delta \omega, \Delta \delta]^T\), \(B = [0, 0, 0, -n/(2J), 0]^T\) and \(\Delta u = \Delta T_L\). And A matrix is expressed as:
\[ A = \begin{bmatrix}
\frac{r_z}{L_q (1-k_\phi^2)} & \frac{\alpha_0 L_d}{L_q (1-k_\phi^2)} & \frac{L_{mq} R_q}{L_q (1-k_\phi^2)} & \frac{\lambda_m + L_d I_d'}{L_q (1-k_\phi^2)} & \frac{V_z \sin (\delta_0)}{L_q (1-k_\phi^2)} \\
\frac{\alpha_b L_q}{L_d (1-k_\phi^2)} & -\frac{r_z}{L_d (1-k_\phi^2)} & \frac{\alpha_0 L_{mq}}{L_d (1-k_\phi^2)} & \frac{L_{mq} R_D}{L_d (1-k_\phi^2)} & \frac{L_d I_q'}{L_d (1-k_\phi^2)} & \frac{V_z \cos (\delta_0)}{L_d (1-k_\phi^2)} \\
\frac{L_{mq}}{L_q} & \frac{r_z}{L_q (1-k_\phi^2)} & \frac{\alpha_b L_d}{L_q (1-k_\phi^2)} & -\frac{1}{L_q (1-k_\phi^2)} & \frac{l_{mq} \lambda_m + L_d I_d'}{L_q (1-k_\phi^2)} & \frac{L_q I_q'}{L_q (1-k_\phi^2)} & \frac{V_z \sin (\delta_0)}{L_q (1-k_\phi^2)} \\
\frac{L_{mq}}{L_d} & \frac{r_z}{L_d (1-k_\phi^2)} & \frac{\alpha_b L_{mq}}{L_d (1-k_\phi^2)} & -\frac{1}{L_d (1-k_\phi^2)} & \frac{l_{mq} \lambda_m + L_d I_d'}{L_d (1-k_\phi^2)} & \frac{L_d I_q'}{L_d (1-k_\phi^2)} & \frac{V_z \cos (\delta_0)}{L_d (1-k_\phi^2)} \\
\frac{3}{2J} \left( \frac{n}{2} \right)^2 \left[ \lambda_m + (L_d - L_q) I_d' \right] & \frac{3}{2J} \left( \frac{n}{2} \right)^2 \left( L_d - L_q \right) I_d' & \frac{3}{2J} \left( \frac{n}{2} \right)^2 L_{mq} I_d' & \frac{3}{2J} \left( \frac{n}{2} \right)^2 L_{mq} I_q & \frac{1}{J} B_m & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
\end{bmatrix} \]

(175)
Appendix C: Derivation of Stabilizing Loop for PMSM Having the Damper Winding

In this section, the equation of the frequency modulation for the PM motor having the damper winding are derived. As explained in section 2.3.1, the frequency modulation ($\Delta \omega_e$) of the stabilizing loop can be determined in proportional to the input perturbation. And $\Delta p_m$ can be extracted by using a high pass filter from the calculated input power. The frequency modulation signal can be expressed as:

$$ \frac{d\Delta \omega_e}{dt} = -\frac{1}{\tau} \Delta \omega_e - K_p \frac{dp_m}{dt} \tag{176} $$

The input power of the machine can be written as:

$$ p_m = \frac{3}{2} [v_q i_q + v_d i_d] = \frac{3}{2} V_s \left[ i_q \cos \delta - i_d \sin \delta \right] \tag{177} $$

By differentiating (177), the following equation can be obtained.

$$ \frac{dp_m}{dt} = \frac{3}{2} V_s \left[ \frac{di_q}{dt} \cos \delta - i_q \sin \delta \frac{d\delta}{dt} - \frac{di_d}{dt} \sin \delta - i_d \cos \delta \frac{d\delta}{dt} \right] \tag{178} $$

Substituting (167) and (172) into (178) results in

$$ \frac{dp_m}{dt} = \frac{3}{2} V_s \left[ \begin{array}{c} \left( \frac{1}{L_q (1-k_q^2)} \right) V_s \cos \delta - \frac{r_s}{L_q (1-k_q^2)} i_q \cdot \sin \delta + \frac{L_m R_q}{L_q (1-k_q^2)} i_q - \frac{1}{L_q (1-k_q^2)} \omega_s \left( \lambda_m + L_{ad} i_d + L_{md} i_d \right) \cos \delta \\
- \frac{1}{L_d (1-k_d^2)} V_s \sin \delta - \frac{r_s}{L_d (1-k_d^2)} i_d \cdot \cos \delta + \frac{L_m R_q}{L_d (1-k_d^2)} i_d - \frac{1}{L_d (1-k_d^2)} \omega_s \left( \lambda_d + L_{md} i_q + L_{md} i_q \right) \sin \delta \\
- i_d \left( \omega_s - \omega_r \right) \sin \delta \\
- i_q \left( \omega_s - \omega_r \right) \cos \delta \end{array} \right] \tag{179} $$
By substituting (179) into (176), the final differential equation of the frequency modulation signal can be obtained as:

\[
\frac{d\Delta \omega_s}{dt} = -\frac{\Delta \omega_s}{\tau_h} - k_p \left( \frac{3}{2} \right) V_s \cos \delta - \frac{r_i}{L_q (1-k_i^2)} i_q + \frac{L_m R_i}{L_q (1-k_i^2) L_D} i_D - \frac{1}{L_q (1-k_i^2)} \omega_s \left( \lambda_m + L_i i_d + L_m i_d \right) \cos \delta
\]

\[
- \frac{r_s}{L_q (1-k_i^2)} V_s \sin \delta - \frac{L_m R_D}{L_q (1-k_i^2) L_D} i_D + \frac{1}{L_q (1-k_i^2)} \omega_s \left( L_i i_q + L_m i_q \right) \sin \delta
\]

\[
= \frac{L_m R_i}{L_q (1-k_i^2) L_D} i_D - \frac{1}{L_q (1-k_i^2)} \omega_s \left( L_i i_q + L_m i_q \right) \sin \delta
\]

The state variables of (180) are including the steady state and small signals and it is nonlinear equation. Thus, it is required the linearization using the following equations.

\[
\cos \delta = \cos (\delta_0 + \Delta \delta) = \cos \delta_0 - \Delta \delta \sin \delta_0
\]

\[
\sin \delta = \sin (\delta_0 + \Delta \delta) = \sin \delta_0 + \Delta \delta \cos \delta_0
\]

\[
i_q = I_q + \Delta i_q
\]

\[
i_d = I_d + \Delta i_d
\]

\[
\omega_s - \omega_s = (\omega_0 + \Delta \omega_s) - (\omega_0 + \Delta \omega_s) = \Delta \omega_s
\]

By substituting (181) into (180), the linearized equation of the frequency modulation signal can be obtained. The second order terms which is the products of small signals can be neglected because it is assumed that the small signals are much smaller than the steady state values.
\[
\frac{d \Delta \omega_j}{dt} = \\
\begin{cases}
\Delta i_d \left( \frac{r_s}{L_q(1-k^2_\omega)} \cos \delta_0 + \frac{L_d \omega_h}{L_d(1-k^2_\omega)} \sin \delta_0 \right) + \Delta i_d \left( -\frac{r_s}{L_q(1-k^2_\omega)} \sin \delta_0 + \frac{L_d \omega_h}{L_d(1-k^2_\omega)} \cos \delta_0 \right) \\
+ \Delta i_d \left( -\frac{L_m R_q}{L_q(1-k^2_\omega)L_q} \cos \delta_0 + \frac{L_m \omega_h}{L_d(1-k^2_\omega)} \sin \delta_0 \right) + \Delta i_d \left( \frac{L_m R_q}{L_d(1-k^2_\omega)L_q} \sin \delta_0 + \frac{L_m \omega_h}{L_d(1-k^2_\omega)} \cos \delta_0 \right) \\
+ k_p \frac{2}{\tau} V_s \frac{3}{2} \Delta \omega + \Delta \omega \left( \frac{\lambda_m + L_d I_q}{L_q(1-k^2_\omega)} \cos \delta_0 + \frac{L_q I_q}{L_q(1-k^2_\omega)} \sin \delta_0 - I_q \sin \delta_0 - I_d \cos \delta_0 \right) \\
+ \Delta \delta \left( -I_q \frac{r_s}{L_q(1-k^2_\omega)} - \frac{\omega_h(\lambda_m + L_d I_d)}{L_q(1-k^2_\omega)} \right) + V_s \sin 2\delta_0 \left( \frac{1}{L_q(1-k^2_\omega)} - \frac{1}{L_d(1-k^2_\omega)} \right) \\
+ \cos \delta_0 \left( -I_q \frac{r_s}{L_q(1-k^2_\omega)} I_d + \frac{\omega_h L_q I_q}{L_d(1-k^2_\omega)} \right) \\
+ \Delta \omega \left( I_q \sin \delta_0 + I_d \cos \delta_0 - \frac{2}{3 V_s \tau \omega_h} \right)
\end{cases}
\]

(182)

From (174), (175) and (182), the linearized state-space model of PM motor with damping winding which includes the stability control can be written as:
\[ \begin{bmatrix} \frac{r}{L_4(1-k_1^2)} & -\frac{\alpha L_4}{L_4(1-k_1^2)} & \frac{L_4 R_2}{L_4(1-k_1^2)} & -\frac{\alpha L_4 R_2}{L_4(1-k_1^2)} & \frac{L_4}{L_4(1-k_1^2)} & \frac{V_2 \sin(\delta_1)}{L_4(1-k_1^2)} & 0 \\ \frac{r_2}{L_2(1-k_2^2)} & \frac{\alpha L_2}{L_2(1-k_2^2)} & \frac{L_2 R_2}{L_2(1-k_2^2)} & -\frac{\alpha L_2 R_2}{L_2(1-k_2^2)} & \frac{L_2}{L_2(1-k_2^2)} & \frac{V_2 \cos(\delta_2)}{L_2(1-k_2^2)} & 0 \\ \frac{r_3}{L_3(1-k_3^2)} & \frac{\alpha L_3}{L_3(1-k_3^2)} & \frac{L_3 R_2}{L_3(1-k_3^2)} & -\frac{\alpha L_3 R_2}{L_3(1-k_3^2)} & \frac{L_3}{L_3(1-k_3^2)} & \frac{V_3 \sin(\delta_3)}{L_3(1-k_3^2)} & 0 \\ \frac{r_4}{L_4(1-k_4^2)} & \frac{\alpha L_4}{L_4(1-k_4^2)} & \frac{L_4 R_2}{L_4(1-k_4^2)} & -\frac{\alpha L_4 R_2}{L_4(1-k_4^2)} & \frac{L_4}{L_4(1-k_4^2)} & \frac{V_4 \cos(\delta_4)}{L_4(1-k_4^2)} & 0 \\ \frac{r_5}{L_5(1-k_5^2)} & \frac{\alpha L_5}{L_5(1-k_5^2)} & \frac{L_5 R_2}{L_5(1-k_5^2)} & -\frac{\alpha L_5 R_2}{L_5(1-k_5^2)} & \frac{L_5}{L_5(1-k_5^2)} & \frac{V_5 \sin(\delta_5)}{L_5(1-k_5^2)} & 0 \\ \frac{r_6}{L_6(1-k_6^2)} & \frac{\alpha L_6}{L_6(1-k_6^2)} & \frac{L_6 R_2}{L_6(1-k_6^2)} & -\frac{\alpha L_6 R_2}{L_6(1-k_6^2)} & \frac{L_6}{L_6(1-k_6^2)} & \frac{V_6 \cos(\delta_6)}{L_6(1-k_6^2)} & 0 \\ \end{bmatrix} \]
Appendix D: Stability Test Results of PMSM Having the Damper Winding (w/ & w/o stabilizing loop)

This section includes the control stability analysis of interior permanent synchronous motor (IPMSM) having the damper winding. The linearized model and the stabilizing loop are derived to analyze the stability of the IPMSM in appendix B and C. The parameters of the machined used for the simulation and experiment tests are given in Table 10.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Values</th>
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<tbody>
<tr>
<td>Rated Power</td>
<td>[kW]</td>
<td>$P_{out}$</td>
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</tr>
<tr>
<td>Rated Speed</td>
<td>[rpm]</td>
<td>$N_r$</td>
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<td>Rated Torque</td>
<td>[Nm]</td>
<td>$T_e$</td>
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<td>$n$</td>
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<td>Stator Resistance</td>
<td>[$\Omega$]</td>
<td>$R_s$</td>
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<td>Stator d-axis Inductance</td>
<td>[mH]</td>
<td>$L_d$</td>
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<td>Stator q-axis Inductance</td>
<td>[mH]</td>
<td>$L_q$</td>
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<td>Magnet Flux</td>
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</tr>
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<td>[Nm/rad·s$^{-2}$]</td>
<td>$J$</td>
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<td>Bemf (line-line)</td>
<td>[V]</td>
<td>$E_{emf}$</td>
<td>460</td>
</tr>
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</table>

Fig. 97 shows the stability analysis result of $v/f$ control not using the stabilizing loop. Although this motor has the damper winding, the motor can be unstable in the conditions which the above half load is applied. When the load is applied to the motor, some rotor poles is located on the positive plane. The unstable region that poles are located on the positive plane is from $3Hz$ to $10Hz$.

Fig. 98 shows the stability analysis result of $v/f$ control using the stabilizing loop. In here, the stabilizing gain ($k_p$) is selected as 2 through the simulation test. With the stabilizing loop, the motor is always stable on the all operating speed range although the over-load is applied to the motor.
Fig. 97. The loci of the rotor poles corresponding to the rotor speed under different load conditions in \( v/f \) control without the stabilizing loop.

Fig. 98. The loci of the rotor poles corresponding to the rotor speed under different load conditions in \( v/f \) control with the stabilizing loop.
To verify this simulation result, the experimental test was implemented in both cases. Fig. 99, Fig. 100 and Fig. 101 shows the results of the step load test under rotor speed 5, 7 and 11Hz, respectively. The tests not using the stabilizing loop were unstable in 5 and 7Hz. However, the motor was stable in 11Hz as shown in Fig. 97. It was exactly matched with the simulation result. In contrast, the tests using the stabilizing loop were always stable regardless of the rotor speed.

![Fig. 99](image_url)  
(a)  
(b)  
Fig. 99. The experimental test results with 4Nm step load under 5Hz speed (a) without the stabilizing loop (b) with the stabilizing loop; CH1: the speed reference (1Hz/div), CH3: the rotor speed (1Hz/div), CH4: the applied load (1.33Nm/div).

![Fig. 100](image_url)  
(a)  
(b)  
Fig. 100. The experimental test results with 4Nm step load under 7Hz speed (a) without the stabilizing loop (b) with the stabilizing loop; CH1: the speed reference (1Hz/div), CH3: the rotor speed (1Hz/div), CH4: the applied load (1.33Nm/div).
Fig. 101. The experimental test results with 4Nm step load under 11Hz speed (a) without the stabilizing loop (b) with the stabilizing loop; CH1: the speed reference (1Hz/div), CH3: the rotor speed (1Hz/div), CH4: the applied load (1.33Nm/div).
Appendix E: Induction Motor Parameters Used in the Simulation

The below table lists the parameters used in the simulations of chapter 3. This parameters is given by the induction motor (50hp) of MATLAB Simulink.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>[kW]</td>
<td>( P_{\text{out}} )</td>
<td>37.3</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>[rpm]</td>
<td>( N_r )</td>
<td>1780</td>
</tr>
<tr>
<td>Rated Voltage (line-line)</td>
<td>[V]</td>
<td>( V_s )</td>
<td>460</td>
</tr>
<tr>
<td>Pole</td>
<td></td>
<td>( n )</td>
<td>4</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>[Ω]</td>
<td>( R_s )</td>
<td>0.099</td>
</tr>
<tr>
<td>Rotor Resistance</td>
<td>[Ω]</td>
<td>( R_r )</td>
<td>0.058</td>
</tr>
<tr>
<td>Magnetizing Inductance</td>
<td>[mH]</td>
<td>( L_m )</td>
<td>30.390</td>
</tr>
<tr>
<td>Stator Leakage Inductance</td>
<td>[mH]</td>
<td>( L_{is} )</td>
<td>0.867</td>
</tr>
<tr>
<td>Rotor Leakage Inductance</td>
<td>[mH]</td>
<td>( L_{lr} )</td>
<td>0.867</td>
</tr>
<tr>
<td>Inertia</td>
<td>[kg.m^2]</td>
<td>( J )</td>
<td>0.4</td>
</tr>
<tr>
<td>Friction</td>
<td>[N.m.s]</td>
<td>( F )</td>
<td>0.022</td>
</tr>
</tbody>
</table>
Appendix F: Resulted Phase Current by Active Vector

This case shows that $V_3$ voltage vector is applied to estimate the rotor speed and position instead of using $V_1$ voltage vector. The $d$-$q$ axis voltage can be expressed in the rotor reference frame as:

$$
\begin{align*}
 v_d &= 2V_{dc}/3 \cos(2\pi/3 - \theta_r) \\
 v_q &= -2V_{dc}/3 \sin(2\pi/3 - \theta_r)
\end{align*}
$$

By using the same procedure used in (136) - (139), the resulted three phase current can be calculated as:

$$
\begin{align*}
 i_a(t_{pulse}) &= \frac{2V_{dc}t_{pulse}}{3} \left( -\frac{1}{4} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + \frac{1}{2} \cos \left( 2\theta_r - \frac{2\pi}{3} \right) \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \right); \\
 i_b(t_{pulse}) &= \frac{2V_{dc}t_{pulse}}{3} \left( \frac{1}{2} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + \frac{1}{2} \cos \left( 2\theta_r + \frac{2\pi}{3} \right) \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \right); \\
 i_c(t_{pulse}) &= \frac{2V_{dc}t_{pulse}}{3} \left( -\frac{1}{4} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + \frac{1}{2} \cos(2\theta_r) \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \right)
\end{align*}
$$

Fig. 102. Applying $V_3$ voltage vector for the estimation of the rotor speed and position of SynRM.
Fig. 103. Applying $V_5$ voltage vector for the estimation of the rotor speed and position of SynRM.

This case shows that $V_5$ voltage vector is applied to estimate the rotor speed and position instead of using $V_1$ voltage vector. The $d$-$q$ axis voltage can be expressed in the rotor reference frame as:

$$\begin{align*}
v_d &= 2V_{dc}/3\cos(4\pi/3 - \theta) \\
v_q &= -2V_{dc}/3\sin(4\pi/3 - \theta)
\end{align*}$$

(186)

By using the same procedure used in (136) - (139), the resulted three phase current can be calculated as:

$$\begin{align*}
i_a(t_{pulse}) &= \frac{2V_{dc}t_{pulse}}{3} \left( -\frac{1}{4} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + \frac{1}{2} \cos \left( 2\theta + \frac{2\pi}{3} \right) \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \right) \\
i_b(t_{pulse}) &= \frac{2V_{dc}t_{pulse}}{3} \left( -\frac{1}{4} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + \frac{1}{2} \cos \left( 2\theta \right) \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \right) \\
i_c(t_{pulse}) &= \frac{2V_{dc}t_{pulse}}{3} \left( \frac{1}{2} \left( \frac{1}{L_d} + \frac{1}{L_q} \right) + \frac{1}{2} \cos \left( 2\theta - \frac{2\pi}{3} \right) \left( \frac{1}{L_d} - \frac{1}{L_q} \right) \right)
\end{align*}$$

(187)

As a result, any active vector ($V_1$-$V_6$) can be used for the estimation. When selecting an active vector used, the correct theta compensation should be done.