Abstract

WHITEHEAD, ASHLEY NICOLE. Preservice Teachers’ Visions, Implementation, and Post-Lesson Analyses of Mathematical Tasks: A Longitudinal, Case-Study Design. (Under the direction of Dr. Temple Walkowiak).

Tasks are central to mathematics instruction and influence the discourse and mathematical representations that are used within a classroom (Munter, 2014). Using a qualitative case-study design, three preservice elementary teachers were examined in how they develop in their visions, implementation, and post-lesson analyses of mathematical tasks throughout their teacher preparation program and into their first year of teaching. The three participants were selected from a larger sample based on their scores on the Mathematical Knowledge for Teaching assessment in Number and Operations (Hill & Ball, 2004; Hill et al., 2005). The case study analyses included interviews and video-recorded lesson observations. Each case’s results are presented chronologically over the three-year study followed by the results of a cross-case comparative analysis. Overall, the participants’ memories and experiences in the K-12 classroom as well as their methods course work and field placements influenced their visions and implementation of mathematical tasks. The participants with stronger MKT provided more sophistication in the way they analyzed their lessons, and they were able to provide ways to informally assess students rather than relying on correct answers as a means of the lesson’s effectiveness. Furthermore, the preservice teacher participants described a lack of horizontal and vertical curricular knowledge throughout the three-year study. These findings suggest implications for mathematics teacher educators: increase MKT in preservice and novice teachers; refine field placements in order to align cooperating teachers’ visions with that of the teacher preparation as well as provide more consecutive days in the classroom for preservice teachers; and provide opportunities for
reflection during methods courses in order to refine visions of mathematical tasks, as well as shift the focus during analysis from the teacher to the students.
Preservice Teachers’ Visions, Implementation, and Post-Lesson Analyses of Mathematical Tasks: A Longitudinal, Case-Study Design

by
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Biography

Ashley Nicole (Stonecipher) Whitehead was born in Aurora, Illinois on July 14, 1986. She is the daughter of Sondra and Bill Westerfield and Rex Stonecipher. She graduated from Princeton Community High School in 2004. She attended Oakland City University and graduated with a Bachelor of Science in Mathematics Education in 2008. She then attended Indiana State University and graduated with a Master’s of Science in Applied Mathematics in 2010. Upon graduation, Ashley took a position at Austin Peay State University as an Instructor of Mathematics. In 2011, she married Jared Whitehead, and in 2012 returned to Indiana to take a position as a Mathematics Instructor at the University of Southern Indiana. In 2013, Ashley began her pursuit of a Ph.D. in mathematics education at North Carolina State University. During her time at NC State, Ashley worked as a research assistant on two NSF-funded grants. These included: Preparing to Teach Mathematics with Technology (PTMT) under the direction of Dr. Hollylynne Lee, Dr. Karen Hollebrands, and Dr. Allison McCulloch, and Accomplished Teachers of Mathematics and Science (Project-ATOMS) under the direction of Dr. Temple Walkowiak. Additionally, during her time at NC State, Ashley served as an instructor teaching Calculus for Elementary Education. Upon graduation, Ashley plans to continue her research in developing preservice elementary teachers’ mathematical knowledge for teaching as well as furthering her understanding of how their visions relate to their instruction in the classroom.
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Chapter 1: Introduction

Over the last several decades, the achievement of K-12 students has been at the forefront of educational reform. Beginning with the National Council of Teachers of Mathematics’ (NCTM) *Curriculum and Evaluation Standards for School Mathematics* in 1989, to the *Principles and Standards for School Mathematics* in 2000, and most recently with the *Common Core State Standards* in 2010 (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010) educators have focused on the mathematics knowledge and skills that students need in order to be successful members of society once they enter the workforce or college upon graduation from high school. The current *Common Core State Standards* includes eight Standards for Mathematical Practice (SMPs) that are important for students in all grades (NGA & CCSSO, 2010). These Mathematical Practices suggest that students should be able to: problem solve, model with mathematics, reason abstractly, engage in productive discourse, use appropriate tools, attend to precision, and look for both structure and repeated reasoning within mathematics problems. It is through these practices that students gain flexible understandings of mathematics and can apply mathematics learned in the classroom to practical situations in real life.

With the current aim to develop the aforementioned practices and mathematical understanding (NGA & CCSSO, 2010), teachers need to be able to engage students in conceptual learning consistently and effectively in their own classrooms. Similar to the Standards for Mathematical Practice, NCTM released the *Principles to Actions* (2014) document that includes a list of research-based Mathematics Teaching Practices. These eight
practices, for teachers rather than students, focus specifically on: engaging students in tasks that promote problem solving, allowing students to use mathematical representations, supporting productive struggle while students learn mathematics, facilitating productive discourse and eliciting student thinking, establishing goals, and posing meaningful questions that will help develop procedural fluency through conceptual understanding. It is through these practices that teachers help students gain the flexibility needed to understand math and help them apply it to real-life situations.

**Statement of the Problem**

Although standards-based teaching practices are important (NCTM, 1989; 2000; NGA & CCSSO, 2010), many preservice teachers struggle with implementing the Mathematics Teaching Practices due to their past experiences with mathematics (i.e., focused on memorization and procedures) (Gurbuzturk, Duruhan, & Sad, 2009). Before entering their teacher preparation program, future preservice teachers develop preconceived notions of teaching based upon their time as learners of mathematics (Feiman-Nemser, 1983; Masingila & Doerr, 2002). It is during this “apprenticeship of observation” (Feiman-Nemser, 1983; Kennedy, 1999) that preservice teachers create ideals of what good teaching looks like. However, once they become a preservice teacher of mathematics, these ideals, or visions to which they strive (Hammerness, 2001), often do not align with the standards-based practices of their teacher preparation program. Nevertheless, teacher preparation programs can help to refine preservice teachers’ visions of instruction (Gurbuzturk, Duruhan, & Sad, 2009; Swars, Smith, Smith, & Hart, 2009). Through this refinement, mathematics teacher educators are also likely impacting how preservice teachers enact lessons in their field placement.
classrooms (Hammerness, 2001) as well as how they analyze lessons after implementation. To understand how teacher preparation programs can help preservice teachers refine their visions, implementation, and post-lesson analyses, an examination of preservice teachers’ development, is warranted.

While teacher development itself is broad, one particular area in which mathematics teacher educators can focus is on the mathematical tasks preservice teachers choose to implement with their students. Tasks are a central component of mathematics instruction, thereby central to the work of teachers, and influence the discourse and mathematical representations that are used in the classroom (Munter, 2014). The authors of the Principles to Actions document suggest that:

To ensure that students have the opportunity to engage in high-level thinking, teachers must regularly select and implement tasks that promote reasoning and problem solving. These tasks encourage reasoning and access to the mathematics through multiple entry points, including the use of different representations and tools, and they foster the solving of problems through varied solution strategies (NCTM, 2014, p. 17).

Therefore, in order to understand one piece of teacher development, it is essential to understand how preservice teachers refine and develop their visions, implementation, and analyses of mathematical tasks over time.

**Purpose of Study**

The current research study is part of a larger NSF-funded grant, Project ATOMS, that followed four cohorts of elementary teachers throughout their teacher preparation program
and into their beginning year(s) of teaching. Three teachers, from one of the four cohorts, were selected as case studies in order to better understand their journey of learning to teach. By understanding how they develop in learning to teach, as well as understanding how the situations they were a part of influenced their learning to teach process, it is my hope that mathematics teacher educators can provide the support and resources novice teachers need to deliver quality mathematics instruction to current and future elementary students.

Therefore, the purpose of this research project is three-fold. The first purpose is to understand how preservice teachers develop in their visions and implementation of mathematical tasks throughout their teacher preparation program and into their first year of teaching. This is tightly coupled with the second purpose, which is to examine the alignment between visions and implementation of mathematical tasks throughout the study. Through these two key purposes, mathematics teacher educators can help preservice teachers further their visions of what high-quality mathematical tasks look like in the classroom and support them in implementing instruction that is aligned with their visions. A third, and final, purpose of this study is to understand how preservice teachers develop in their post-lesson analyses of their implemented tasks throughout a three-year time span. By examining how preservice teachers analyze their implemented lessons, mathematics teacher educators can help early teachers reflect in a meaningful and critical way in order to improve their quality of instruction.

**Significance**

The proposed study is significant to the field of mathematics education in two main ways. First, most studies focused on tasks have studied practicing teachers (Boston, 2012;
By examining teachers in their early years of teaching we, as researchers and mathematics teacher educators, can begin to better understand the implications preservice and novice teachers’ visions have on their enacted lessons, as well as the many other factors that play into their quality of instruction (i.e. MKT, field placements, etc.).

Second, longitudinal studies in the field of mathematics education are few and far between (Ensor, 2001; Gellert, Hernandez, & Chapman, 2012). Although there have been longitudinal studies published around mathematical tasks (i.e. Desimone, Hochberg, & McMaken, 2016; Fennema et al., 1996; Stein et al., 1996), these studies have not focused on how tasks and visions relate to one another (Ensor, 2001). Furthermore, there is a gap in the literature on how preservice and novice teachers develop over time in their implementation and analyses of tasks (Ball & Bass, 2002; Osana, LaCroix, Tucker, Bradley, & Desrosiers, 2006). This study aims to fill these gaps as well as add to the existing literature base by providing rich, descriptive evidence of three preservice teachers’ journeys as they develop in learning to teach mathematics.

**Definition of Terms**

It is beneficial to the reader to define several terms that will be used consistently throughout this study.

1. **Cooperating Teacher**: A K-12 teacher who partners with the university by mentoring, hosting, and supervising a preservice teacher during a field experience.
2. **Mathematical Knowledge for Teaching (MKT):** A specialized type of knowledge that teachers need in order to teach mathematics effectively. It is comprised of both subject matter knowledge and pedagogical content knowledge (Ball et al., 2008).
   a. **Subject Matter Knowledge:** The knowledge a teacher has related to the content needed to teach mathematics effectively.
   b. **Pedagogical Content Knowledge:** The knowledge a teacher has related to the content as well as how it fits with their students and the curriculum.

3. **Mathematical Task:** An activity whose purpose is to “focus students’ attention on a particular mathematical idea” (Stein et al., 1996, p. 460).

4. **Novice Teacher:** An individual in their first year of teaching.

5. **Preservice Teacher:** An individual in their teacher preparation program at a college/university.

6. **Professional Visions for Teaching:** Defined by Hammerness (2001) as “a set of images of ideal classroom practice for which teachers strive” (p. 143).

7. **Standards-Based Mathematics Teaching Practices:** These instructional practices, as described in standards and recommendation documents (NCTM, 1989; 2000; NGA & CCSSO, 2010) require that students engage in processes (e.g. communicating mathematical ideas, using multiple representations) with the goal to build conceptual understanding of mathematical ideas.

8. **Teacher Preparation Program:** Methods courses, field experiences, and student teaching during the preservice teacher’s junior and senior year.
Organization of Study

In this chapter, the study has been introduced with its purpose and significance explained. Chapter 2 provides a thorough literature review on teacher development, which also encompasses components such as: the learning to teach process, as well as teachers’ visions, implementation, and post-lesson analyses of mathematical tasks. The chapter concludes with the conceptual framework guiding the study based upon theoretical foundations. Chapter 3 presents the sampling process, data collection procedures, and methods for data analysis. Chapters 4-6 report on the findings of each participants’ visions, implementation, and post-lesson analyses throughout the three-year study. Within these chapters, the overall research questions of the study are addressed. In order to best answer these questions, a cross-case comparison is conducted in Chapter 7. Finally, a summary and discussion of the overall findings as well as implications for mathematics teacher educators are presented in Chapter 8.
Chapter 2: Literature Review

For the last 25 years, there has been a push to focus on standards-based instruction in school mathematics (NCTM, 1989, 2000; NGA & CCSSO, 2010). Beginning with the National Council of Teachers of Mathematics’ (NCTM) *Curriculum and Evaluation Standards for School Mathematics* in 1989, the goal of conceptual understanding for all students has been at the forefront of mathematics education (Ma, 1999). To meet this goal, teachers need to not only have the common content knowledge necessary to *know* mathematics, but also a specialized content knowledge that allows them to *teach* mathematics (Ball et al., 2008; Newton, Evans, Leonard, & Eastburn, 2012). Furthermore, content knowledge alone is not enough; teachers also need the pedagogical knowledge of students to be effective leaders in the classroom (Ball et al., 2008; Cardetti & Truxaw, 2014). However, the acquisition of these skills does not happen all at once; rather, “learning to teach takes place over time” (Cochran-Smith, 2011, p. 22) through participation in authentic experiences as a learner of mathematics and as a teacher of mathematics (Feiman-Nemser, 1983; Lave & Wenger, 1991; Masingila & Doerr, 2002).

**Teacher Development**

Over the years, researchers have described how teachers acquire skills as they develop in learning to teach (Berliner, 1991; Dreyfus & Dreyfus, 1980). In the first few years of teaching, teachers are learning the rules and norms for teaching, as well as are establishing routines (Berliner, 1991). Once they gain experience, teachers begin to become more flexible in their teaching, begin to recognize patterns that are helpful while teaching, and are intuitive about the situations they face (Berliner, 1991; 1992; 2004). However, this development does
not happen only once the teacher has their own classroom; rather, the experiences that preservice teachers have before and during their teacher preparation programs also influence how they develop as teachers.

In 1983, Feiman-Nemser discussed a continuum that focused on a teacher’s path as they developed in learning to teach. The continuum, broken up into four main parts, begins even before the preservice teacher enters their teacher preparation program (see Figure 1). In the *pre-training* stage, which occurs before any formal training takes place, preservice teachers bring ideals about teaching from their own backgrounds as students (Hollingsworth, 1989). It is during this “‘apprenticeship of observation’ that students internalize models of teaching that are activated [once] they become teachers” (Feiman-Nemser, 1983, p. 6). Once the preservice teacher enters into formal preparation, they enter what is known as the *preservice* stage. During this time, the preservice teacher takes methods coursework, has field experiences in classrooms, and even becomes a student teacher. Next, as the novice teacher moves into their first year(s) of teaching, they become part of the *induction* stage. These first couple of years of teaching are critical in the learning-to-teach process (Feiman-Nemser, 1983). Some believe that “what happens during the first year of teaching determines not only whether someone remains in teaching, but also what kind of teacher they become” (Feiman-Nemser, 1983, p.15). Although typically discussed as the first year of teaching, the induction stage can last a couple of years into the teacher’s career. Finally, as the teacher transitions out of the induction stage, they move into the *inservice* stage, which “covers the rest of the teacher’s career” (Feiman-Nemser, 1983, p. 4).
Since learning is a social endeavor (Cobb & Yackel, 1996; Lave & Wenger, 1991), Putnam and Borko (2000) suggest we consider both the individual teacher as well as the situations in which they are participants. The next section describes how the learning to teach process is situated within teacher development. This means that although each phase is a time point within a teachers’ development, the experiences within each phase also influence how a teacher develops in learning to teach.

**Experiences in learning to teach.** As preservice teachers transitioned from grade to grade during their K-12 schooling, they participated in classroom environments and observed their own teachers teach. Through these interactions, as learners of mathematics, preservice teachers come into teaching with a notion of what they believe to be effective or non-effective practices (Feiman-Nemser, 1983; Masingila & Doerr, 2002). It is during this time that the preservice teacher is in their pre-training phase. In the case of the current study, the pre-training phase is described in terms of the preservice teachers’ memories of K-12 mathematics (i.e., their affect of K-12 mathematics, experiences in K-12 mathematics, etc.).
Once in their teacher preparation program, during their *preservice* phase, preservice teachers engage in authentic teaching experiences such as classroom field experiences and student teaching practicums that are part of their legitimate peripheral participation (Lave & Wenger, 1991). Legitimate peripheral participation is a process that moves the preservice teacher from the periphery of the community of teachers towards the center, all while gaining access to the group’s (i.e., community of teachers) behavior and skills. As the preservice teacher gains experience, they begin to incorporate more of the customs that are relevant to their community of practice (Lave & Wenger, 1991). Feiman-Nemser (1983) describes how some studies have even shown that after spending time in their classroom, student teachers “become like their cooperating teachers” (p.15). In the case of the current study, the schools in which the preservice teachers participate (i.e. during field experiences, student teaching), the interactions they have with their cooperating teachers, and the teacher preparation program itself, all influence their development.

Finally, once novice teachers are in their own classrooms they enter into the *induction* and *inservice* phases. It is during these phases that teachers continue to enter into communities of practice (e.g. the school in which they teach, the team of teachers they teach with, the community in which they teach) and develop more skills through these social practices that help them to develop their teaching abilities. For the purposes of this study, only the *pre-training* through the *induction* phase will be examined.

**Teacher preparation programs.** “Although learning to teach must be situated within the real materials of practice, learning to teach does not necessarily need to occur solely in real time in the ‘crucible of the classroom’” (Darling-Hammond, Hammerness, Grossman,
Rust, & Shulman, 2005, p. 427). Darling-Hammond and colleagues (2005) continued by suggesting that activities such as watching videos of good teaching and examining work samples from the classroom are all useful as representations of practices that preservice teachers may not have experienced as learners of mathematics themselves. Feiman-Nemser (2001) similarly suggests that mathematics teacher educators should model the teaching they are promoting in order to expose preservice teachers to practices they may not have seen during their own K-12 schooling. That is to say, teacher preparation programs should encourage preservice teachers to “not adapt to existing conditions, but to challenge current practices and to work for change” (Grossman, 1992, p. 171).

In addition to the methods courses in which preservice teachers gain experience, teacher preparation programs include field experiences and a student teaching component, which is oftentimes described as the most important part of preservice teacher preparation (Feiman-Nemser, 1983; 2001). These field experiences help preservice teachers connect what they are learning in their methods coursework to what they experience in an actual elementary school classroom (Feiman-Nemser, 1983). Although there is not a standard number of hours or way field experience components are held (this is up to the individual university), we do know that learning to teach does not stop once teachers graduate (Feiman-Nemser, 1983). Learning to teach continues over the course of a teacher’s career, as they move from ‘survival mode’ during their first few years, to mastery later in their career (Feiman-Nemser, 1983; 2001).

In order to understand this development of teachers over time, specifically related to tasks, it is important to understand how they develop in: their professional visions of
mathematical tasks; their implementation of mathematical tasks; and what they attend to when analyzing their implemented instruction. The next three sections focus on these areas in which teachers develop. Furthermore, a section on how their mathematical knowledge for teaching (MKT), relates to their overall development is presented.

**Professional Visions for Teaching**

Teacher beliefs are complex and are comprised of several components. Philipp (2007) suggests, “beliefs are lenses through which we humans view the world” (p. 309). One area in which teachers develop beliefs is in their professional visions of how to teach mathematics (Duffy, 2002; Gurbuzturk et al., 2009; Hammerness, 2001, 2008). Hammerness (2001) defines *visions* as “a set of images of ideal classroom practice for which teachers strive” (p. 143). Duffy (2002) extends Hammerness’ definition by saying “when teachers have a vision, they assume control over instructional decision making in order to achieve the mission [student learning]” (p. 334).

Oftentimes, preservice teachers bring in their visions of what good teaching looks like from their previous experiences in K-12 schooling (Feiman-Nemser, 1983; Masingila & Doerr, 2002); however, these visions might be flawed from what is considered best practices for teaching mathematics. Feiman-Nemser (2001) suggests that “before [preservice teachers] can embrace new visions, [they] need opportunities to examine critically their taken-for-granted, often deeply entrenched beliefs so that these beliefs can be developed or amended” (p. 1017). Otherwise, the visions preservice teachers enter their teacher preparation program with will continue to influence their ideals and teaching practice (Feiman-Nemser, 2001). Therefore, although prior experiences make change difficult (Thompson, 1992), with the
right intervention, preservice teachers’ visions can be refined through experiences in their teacher preparation programs (Gurbuzturk et al., 2009).

One way in which teacher preparation programs can help preservice teachers to refine their visions is through writing prompts. Duffy (2002) had preservice teachers write about their visions using sample prompts such as: “what is the most important thing you want your students to learn from you?” and “if you were to meet your students fifteen years from now, what do you hope they will tell you was the most important thing they learned from you” (p. 336). These prompts, encouraged reflection and allowed preservice teachers to articulate their visions. The hope is that through reflection, preservice teachers are able “to develop a clearer sense of their purposes for teaching and of their commitment to the profession” (Hammerness, 2003, p. 55).

In addition to solely defining a vision, Hammerness (2001) claims that a teacher’s professional vision can be used as a “measuring stick, indicating how far current practice [sits] from where one [wants] to be” (p. 146). This idea of comparing teachers’ visions to their instructional practice is one component of the current study and helps to fill a void in the literature. By making their visions explicit, teachers can understand how far or how closely aligned they are with their current instructional practices and hopefully improve their efforts related to teaching and learning (Hammerness, 2001). Despite this process being beneficial for some, others may feel their visions are unattainable in comparison to their current instructional practice (Hammerness, 2003). Hammerness (2003) describes that for these teachers, realizing that “episodic visions” can occur, these are visions in which:
teachers recognize that their classes will not be ideal every day, but rather that those instances may occur once or twice a semester, after several weeks, or even months of careful scaffolding. If the end result of their work is good… then this can help them deal with the notion that practice is not always perfect (p. 54).

Although past experiences in K-12 mathematics as well as teacher preparation programs influence teachers’ visions, these are not the only factors to consider. Gurbuzturk and colleagues’ (2009) findings suggest that female teachers have higher aspirations than male preservice teachers (although both want to perform well). The authors suggest that this can be related to the findings that female preservice teachers report better previous experiences in elementary school than males. Additionally, Hammerness’ (2008) found that school context factors, such as the type of school of employment (i.e. traditional, charter, private), affect their visions of instruction.

**VHQMI framework.** In order to better understand teachers’ visions of effective mathematics teaching, Munter (2009) conducted interviews with teachers, principals and mathematics coaches focusing on the way they described and characterized high-quality instruction. The results from these interviews, paired with previous research, formed the basis of a coding scheme that measured teachers’ visions of high-quality instruction. Initially, Munter (2009) used Hiebert et al.’s (1997) coding scheme, which focused on essential components of mathematics instruction; however, additional codes emerged and evolved as he spoke with participants. The result of this research led to the four main dimensions and rubrics he used to classify “visions of high-quality mathematics instruction” (VHQMI; Munter, 2014). These four areas include: *role of the teacher* (i.e., a teacher who supports
students to learn mathematics conceptually), *classroom discourse* (i.e., a classroom in which student-to-student talk occurs and does not solely depend on the teacher), *mathematical tasks* (i.e., high in cognitive demand – *doing mathematics*), and *student engagement* (i.e., students are engaged in problem solving activities). For the purposes of this study, and to limit the scope of the research to one area, the focus is on *mathematical tasks* (see Chapter 3; Table 3).

**Implementation of Mathematical Tasks**

Understanding how to select, adapt, and implement quality mathematical tasks is important to the work of teachers. Stein, Grover, & Henningsen (1996) define a mathematical task as a “classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea” (p. 460). Overall, mathematical tasks are “indicators of instructional quality” (Boston, 2012, p. 81); however, the implementation of a task has the greatest impact on student learning outcomes when students are exposed to tasks high in cognitive demand (Smith & Stein, 1998; Tarr et al., 2008). Oftentimes the cognitive demand of a task declines during implementation (Henningsen & Stein, 1997), but through the use of “questioning, encouraging conceptual connections, and holding students accountable for explanations and meanings,” (Boston, 2012, p. 82) teachers can keep the level of cognitive demand high.

In order to better prepare preservice teachers for their future classrooms, mathematics teacher educators should stress the importance of the tasks in which their preservice teachers engage, and be explicit about the tasks that they help preservice teachers choose for their elementary students (Ben-Chaim, Keret, & Ilany, 2007; Chapman, 2013). Peressini and colleagues (2004) describe task selection as:
situated in particular classrooms filled with students who bring with them different experiences and backgrounds. Teachers must take into account their own particular students’ knowledge and interests, what is known about the ways in which students learn particular mathematical ideas, and common student confusions and misconceptions about those ideas (p. 78).

These tasks should also be mostly high in cognitive demand, meaning they require students to conceptualize the mathematics they are doing as well as sometimes connect it to real-life examples (Smith, Hughes, Engle, & Stein, 2009; Smith & Stein, 1998). By doing so, elementary students will be more engaged throughout mathematics lessons (Osana et al., 2006).

Types of mathematical tasks. In order to better understand cognitive demand of tasks, I defer to Smith and Stein’s (Smith & Stein, 1998; Stein & Smith, 1998; Stein, Smith, Henningsen, & Silver, 2009) work around mathematical tasks. Table 1 shows examples of each type of mathematical task as presented by Smith and Stein (1998). Lower-level demand tasks are those involving memorization or procedures without connections. Although there is a time and place for these tasks, if students are consistently asked to perform only lower-level demand tasks, they will not be as engaged, they will not gain the conceptual understanding of mathematical ideas, and they will not gain the problem-solving skills necessary for today’s society (Smith & Stein, 1998). Higher-level demand tasks require students to engage in procedures with connections or doing mathematics. Smith and Lane (1996) suggest that by starting with higher-level demand tasks, students will have more opportunities to think critically, problem solve, and reason about mathematics.
Table 1.
Description of Types of Tasks (Adapted from Smith & Stein, 1998, p. 348)

<table>
<thead>
<tr>
<th>Type of Task</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memorization</td>
<td>Solved through without a procedure, by reproducing a previously learned fact.</td>
<td>What is the rule for dividing fractions?</td>
</tr>
<tr>
<td>(lower level)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedures without Connections</td>
<td>Focus is on the use of procedure with no connection to the concept.</td>
<td>Divide: $\frac{5}{6} \div \frac{1}{2}$</td>
</tr>
<tr>
<td>(lower level)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedures with Connections</td>
<td>Focus is on the procedure, but only in order to develop a deeper understanding of the concept. Can be represented in multiple ways and require some cognitive effort.</td>
<td>Find how many ( \frac{1}{2} ) pieces are in ( \frac{5}{6} ). Use pattern blocks to help you draw and explain your solution.</td>
</tr>
<tr>
<td>(higher level)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doing Mathematics</td>
<td>Requires the student to use considerable cognitive effort, allows the student to explore and understand the nature of the concept, and does not have an explicit pathway for solving.</td>
<td>Create a real-world situation for the problem $\frac{5}{6} \div \frac{1}{2}$. Solve the problem without using a rule and explain your solution.</td>
</tr>
<tr>
<td>(higher level)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although it is important to consider the cognitive demand of the task, we must also consider the task as it moves from the teacher to the students. Before implementation, the task is chosen from a set of curricular or instructional materials. The cognitive demand of the task can then be influenced by the teacher’s: goals; knowledge of content, pedagogy, and own students; and modifications to the task as it was originally written (Stein et al., 1996). This all occurs between the “tasks as they appear in curricular/instructional materials” and “tasks set up by teachers” (i.e. planned, adapted, etc.). Next, as the task moves from set up to implementation, the cognitive demand can be changed by the classroom norms, as well as the
students and teacher themselves (Stein et al., 1996). As the task is implemented, cognitive demand can shift due to the uncomfortable nature of high-level tasks as well as the struggle that accompanies them. Students may offload the task by asking the teacher for help which can lead to a step-by-step breakdown of the problem at hand. In order to keep the task engaging and high-level, teachers need to make instructional choices that allow for conceptual connections as well as questioning students and holding them accountable for their own explanations (Boston, 2012). Once the task has been implemented, teachers need to reflect on their instruction through post-lesson analyses. These are important because they inform next instructional steps and help the teacher make adaptations to the task for future use (NCTM, 2007).

**Analyses of Mathematical Tasks**

One final part of understanding how teachers teach, is to understand how they analyze their lessons post-implementation. This portion of the teaching process is where teachers examine “students’ learning, the mathematical tasks, the environment, and classroom discourse to make ongoing instructional decisions” (NCTM, 2007, p. 10). Many researchers believe that by developing how preservice teachers analyze their implemented tasks, they will continuously improve their teaching and analysis skills (Cavanagh & McMaster, 2015; Haciomeroglu, 2009; Parks, 2008; Santagata & Angelici, 2010).

In order to understand how preservice and novice teachers develop in their analysis skills, one must understand what teachers attend to when reflecting on their own teaching. Through their reflection, teachers identify important aspects within a classroom situation and reason about the interactions that occurred (van Es & Sherin, 2002). Although all teachers
attend to events in the classroom, novice teachers’ analyses are less refined than expert teachers (Santagata, 2010; Sherin & van Es, 2005). Expert teachers focus on student learning as well as understanding the events in the classroom with “more detail and more insight than novices” (Santagata, 2010, p. 75).

In a study conducted by Santagata, Zannoni, and Stigler (2007), they used video-based lessons with preservice teachers to understand how they analyzed the lessons and to what they specifically attended. At the beginning of their course, preservice teachers were asked to analyze a video-taped lesson and reflect on the teacher’s practices. Santagata and colleagues (2007) noticed that the preservice teachers mostly: described what was observed in the video (elaboration), made comments about the lesson overall in regard to what was effective (links to evidence), used their developing mathematical knowledge to make comments about general instructional choices of the teacher (mathematics content), did not comment on students’ behavior but rather focused on the teacher’s actions (student learning), and viewed the lesson in a mostly positive light (critical approach). By the end of the course, the preservice teachers’ analyses focused on: providing alternative suggestions and effects the lesson had on student learning (elaboration), gave specific instances of how the lesson was effective (links to evidence), used their increased mathematical knowledge to discuss, more in depth, the teacher’s choices in the lesson (mathematics content), focused on student behavior and learning as a result of the tasks (student learning), and discussed issues they saw overall (critical approach).

The findings of this study were similar to that of Cavanagh and McMaster’s (2015) study of nine secondary preservice teachers who engaged in a year-long program focused on
observing, co-teaching, and reflecting. Both studies found that preservice teachers typically do not describe more than what is observed in the classroom during their reflections. Furthermore, preservice teachers tend to focus on the teacher’s actions when reflecting, rather than describe the student learning that took place. These findings reiterate Cavanagh and McMaster’s (2015) belief that “developing teachers’ reflective practice is not easy” (p. 475). However, Santagata and colleague’s (2007), present hope that through continual refinement of their analysis skills, preservice teachers can become more reflective in their post-lesson analyses.

In addition to using video as a reflective practice, other researchers highlight the importance of writing or discussing with colleagues as a means for reflection. Davis (2006), examined journal entries from 25 of her preservice teachers, focused on their reflection of teaching. Her results found that through the journal writing process, preservice teachers were able to make sense of their thinking; however, she notes that it should be used in conjunction with other collaborative reflective practices. Despite journaling being a practical reflective tool, Day (1999), suggested that teachers reflect on their own teaching practices rather than on observed practices of others, when reflecting through writing. Cavanagh and McMaster’s (2015) also extended Davis’ (2006) research related to collaboration, by finding that preservice teachers who collaborated via discussions with their peers acting as “critical friends” (p. 475) were able to “attain higher levels of critical reflection” (p. 475). The implications of these studies suggest that, although preservice and novice teachers should reflect on their own practices, they should not do so alone because the reflections tend to occur more infrequently and in a disorganized way.
The research provided, up to this point, indicates the need to examine teachers’ visions, implementation, and post-lesson analyses in order to understand teacher development. Additionally, we have also seen the importance of considering the role of preservice teachers’ past experiences when examining how they develop (Feiman-Nemser, 1983; Lave & Wenger, 1991; Masingila & Doerr, 2002). One last piece to examine when considering how preservice teachers develop when learning to teach, is their mathematical knowledge for teaching and how it relates to instruction (Charalambous, 2015; Hill et al., 2008; Hill & Charalambous, 2012).

**Mathematical Knowledge for Teaching**

In the late 19th and early 20th century, beginning teachers’ state board examinations were comprised of 95% subject matter knowledge (Shulman, 1986). It was during this time that Dewey (1904) recognized a tension between theory and practice in teacher education. This tension remains prominent still today. Although the progression of mathematics education has cycled from teaching for skills to teaching for understanding, the 1970s saw an increased emphasis on teaching basic mathematics, such as arithmetic (Schoenfeld, 2007). This weakened focus on subject matter shifted the focus of teacher preparation to the ability to teach (Shulman, 1986). However, Shulman, in his 1986 presidential address to the American Educational Research Association, referred to this absence of subject matter knowledge as the “missing paradigm.” Shulman (1986) urged for teacher preparation programs to focus teachers’ subject matter knowledge on three main categories: content knowledge, pedagogical content knowledge, and curricular knowledge. Content knowledge (CK) referred to what teachers must know in relation to “the accepted truths in a domain” as
well as “why it is worth knowing” (Shulman, 1986, p. 9). Pedagogical content knowledge (PCK), a new term for the field, related to knowing the topic but also the pedagogical side of teaching the topic. For instance, a teacher must know what misconceptions students may have of the topic or what instructional methods may be appropriate or effective for a given topic or goal. Finally, curricular knowledge referred to knowing both the lateral curriculum (knowing the topics in a particular grade level) and the vertical curriculum (knowing the topics in preceding and later grades).

Although Shulman (1986) emphasized content knowledge for teaching, he also understood the importance and need of pedagogical understanding as well. He claimed, “mere content knowledge is likely to be as useless pedagogically as content-free skill” (Shulman, 1986, p. 8). This new focus led many researchers in the 1990s and early 2000s to concentrate their research not only on pedagogical knowledge for teaching but also on content knowledge again (Ball & Bass, 2000; Ball, 1990, 2000; Borko et al., 1992; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Hill et al., 2005; Ma, 1999; Thompson & Thompson, 1996). Furthermore, researchers (Ball, 2000; Ball & Bass, 2000) laid out three challenges for the mathematics education community related to the relationship between knowledge and pedagogy: 1) re-examine the type(s) of content knowledge necessary for good teaching; 2) develop an awareness among educators and the general public of the myth that knowing content meant a teacher could teach the content well; and 3) create opportunities for teachers to learn subject matter and apply it in various teaching contexts.

With most of the foundation laid through qualitative research, Ball, Hill and colleagues (Ball et al., 2008; Hill, Ball, & Schilling, 2008; Hill, Blunk, et al., 2008)
developed a framework for mathematical knowledge for teaching (Figure 2). This framework consists of two major components that extended Shulman’s (1986) work regarding subject matter knowledge and pedagogical content knowledge. Within each of the major components were three sub-components that helped to define MKT as “the mathematical knowledge needed to carry out the work of teaching mathematics” (Ball et al., 2008, p. 395).

![Figure 2. Mathematical Knowledge for Teaching Framework (Ball et al., 2008).](image)

Each of the sub-components are defined separately; however, they are not completely distinct from one another (Ball et al., 2008). On the left half of the framework, subject matter knowledge (SMK) consists of: common content knowledge (CCK; or the knowledge that most people know regarding mathematics), specialized content knowledge (SCK; or the knowledge that is specific to teaching such as understanding uncommon approaches to solving a problem), and horizon content knowledge (or the knowledge of how topics are...
related to each other in mathematics) (Ball et al., 2008). On the right side of the framework, pedagogical content knowledge (PCK) consists of: knowledge of content and students (KCS; or the knowledge to anticipate students’ thinking and knowing common misconceptions that may arise), knowledge of content and teaching (KCT; or the knowledge that allows teachers to select and sequence their instruction in order to reach mathematical goals), and knowledge of content and curriculum (or the knowledge that Shulman deemed lateral and vertical knowledge of curriculum) (Ball et al., 2008).

In the 1990s and early 2000s, most of the research on MKT was qualitative and sought to measure student achievement (Hill, Ball, Blunk, Goffney, & Rowan, 2007; Hill et al., 2005; Hill, Blunk, et al., 2008; Schilling & Hill, 2007). However, in 2000, researchers with the Study of Instructional Improvement (SII) and the Learning Mathematics for Teaching (LMT) projects (Hill & Ball, 2004; Hill et al., 2005; Hill, Blunk, et al., 2008; Schilling & Hill, 2007) at the University of Michigan developed quantitative measures for measuring MKT. Once created and validated, the LMT-MKT assessments became the most viable effort for measuring MKT (Izsak, Jacobson, Araujo, & Orrill, 2012). Goos (2013) went as far as describing it as “landmark” (p. 975).

For the current study, the LMT-MKT assessment, focused on Number and Operations in grades K-5, was utilized. Similar to other researchers (Charalambous, 2015; Hill et al., 2005; Hill, Blunk, et al., 2008; Izsak et al., 2012), the LMT-MKT assessment is paired with other data sources to examine other factors in the study (e.g. professional visions, implementation of tasks, analysis of lessons). Little is known about how visions and analysis of implementation are related to MKT; however, prior studies have indicated a link between

**MKT and mathematics instructional quality.** Hill and colleagues (2008) defined mathematical quality of instruction (MQI) as “the rigor and richness of the mathematics lesson”. In 2015, Copur-Gencturk conducted a longitudinal study of 21 teachers that focused on subject matter knowledge. Upon completion of their study, it was shown that teachers with higher content knowledge for teaching were “more successful in presenting mathematical concepts clearly and accurately throughout the lesson” (p. 305) and focused more on students’ explanations and justifications. Similarly, despite the fact that Hill et al. (2008) could not draw general conclusions from their research of four case studies, they noted that low-MKT teachers did not provide well thought out explanations and did not attend to language in the same manner that high-MKT teachers did. Additionally, in another study, researchers (Hill & Charalambous, 2012) found that as MKT declined so did the MQI. For example, they found that teachers with low MKT only had stronger instruction when they were closely following the curriculum materials. On the other hand, high-MKT teachers quickly understood students’ ideas, launched tasks successfully, and stayed faithful but improved upon the curriculum materials.

In the current study, MKT was utilized for participant selection in order to provide insights about preservice teachers’ developing MKT in relation to their visions, implementation, and post-lesson analyses. By selecting participants based upon MKT, this study hopes to better understand how preservice teachers develop in the learning to teach process. The research questions guiding this study are presented below.
Research Questions

The review of the literature shows a need for understanding how preservice and novice teachers develop in their visions and implementation of mathematical tasks. There have been few longitudinal studies in mathematics education that examined beginning teaching (Ensor, 2001), and this study begins to fill that void. Furthermore, this study uses previous research around MKT being positively related to mathematical quality of instruction to help select cases for in-depth case study analyses. The use of a qualitative case study design provides descriptions of the changes taking place within the participants from preservice years into their first year of teaching. By following these teachers over time, we see how their pre-training, preservice, and induction phases influence their development as a teacher of mathematics. Therefore, the following research questions guide this study:

1. How do preservice and novice elementary teachers, with varying MKT, develop in their visions and implementation of mathematical tasks?
   
a. To what extent are their visions and implementation of mathematical tasks aligned?

2. How do preservice and novice elementary teachers, with varying MKT, develop in their analysis of their implemented mathematical tasks?

Conceptual Framework

The conceptual framework (Figure 3) guiding this study draws on the literature provided previously in this chapter. Overall, this study is situated in teachers’ experiences as they develop in learning to teach. These experiences, during their pre-training, preservice, and induction phase of learning to teach are both a part of teacher development as well as
influence teacher development (Feiman-Nemser, 1983; Lave & Wenger, 1991; Masingila & Doerr, 2002). Additionally, although there are several ideas in which to focus in teacher development, for the purposes of this study tasks were highlighted because they are central to instruction (Munter, 2014) and influence many of the other components of a classroom (i.e. discourse, representations). Furthermore, I acknowledge the many components of tasks (i.e. planning, beliefs about tasks, implementation of tasks), but for this study I focused on how preservice teachers develop in their visions, implementation, and post-lesson analyses of mathematical tasks over three years (Research Question 1 & 2). Finally, the dotted line between visions and implementation indicates how this study will examine the alignment between visions and implementation (Research Question 1a), with the arrow from analysis to implementation as a reminder to the reader that analysis refers to the teacher’s post-lesson analyses of their implemented tasks.
Figure 3. Conceptual framework.
Chapter 3: Methodology

A longitudinal, case-study design was utilized by purposefully selecting three participants and investigating their development of visions, implementation, and post-lesson analyses of tasks over a three-year period. In this chapter, I describe the study’s design, context, participant selection, as well as data collection and analysis methods. Additionally, limitations and ethical considerations are discussed.

Research Design

In order to answer the research questions, a longitudinal qualitative case-study approach (Creswell, 2013) was used to better understand preservice teachers’ authentic experiences (Patton, 1990, 2002; Yin, 2009, as cited in Creswell, 2013) in elementary classrooms. Creswell (2013) defined case-study research as “a qualitative approach in which the investigator explores a real-life, contemporary bounded system or multiple bounded systems over time, through detailed, in-depth data collection involving multiples sources of information and reports a case description and case themes” (Case Study Research, Paragraph 1). More specifically, a multiple case study (Creswell, 2013) was used to examine three purposefully selected cases related to the same issue: visions, implementation, and post-lesson analyses of mathematical tasks. Finally, a within-case analysis was used, followed by a cross-case analysis to discuss the findings from the collective group of cases (Creswell, 2013).

Context

The study took place within a larger research project, Project ATOMS, an evaluation study of a STEM-focused elementary teacher preparation program. The larger research
project consisted of 245 participants across four cohorts of graduates of the program. The STEM-focused teacher preparation program represented the common demographic of novice elementary teachers as noted in the National Survey of Science and Mathematics Education (NSSME; Banilower et al., 2013): white and female (96% are female, 87% are white, and 83% of ATOMS participants are both white and female.).

General education courses. During their freshman and sophomore years, preservice teachers were required to take 9 courses consisting of a mix of mathematics, science, and engineering in order to meet their general education requirements, as well as complete 15 hours of field experience during the spring of their sophomore year. These courses are helpful to the students and are above and beyond what most elementary teacher preparation programs offer (Banilower et al., 2013). The required 9 courses included those such as: *Math for Elementary Teachers*, which focused on a conceptual understanding of mathematics topics as well as physics and engineering design courses for elementary teachers. The *Math for Elementary Teachers* course was taught in the Fall of 2010, with 16% of the larger research study participants taking the course; in Fall 2011 and later, the course was redesigned and became *Calculus for Elementary Education*, which focuses on a conceptual understanding of calculus with elementary content woven throughout. The redesigned course is 2 semesters, and 5% of the larger research study participants took only one semester of the course, while 9% of the participants took both semesters of the course (those who did not take the course were enrolled in a traditional calculus course offered by the university).

Teacher candidacy. Upon completion of the general education classes and the *Introduction to Elementary Education* course, preservice teachers are admitted to teacher
candidacy. As part of their teacher preparation, preservice teachers complete three methods courses during the fall of their junior year focused on grades K-2 mathematics, science, and engineering. In the spring of their junior year, preservice teachers complete their second mathematics and science methods courses focused on grades 3-5. During these two semesters, preservice teachers complete a total of 172 hours of field placement work (86 hours in each semester). The methods coursework is designed with activities that emphasize: teaching for conceptual understanding, high-cognitive demand tasks, fixed and growth mindsets, representing mathematics in multiple ways, modeling and using a variety of discourse strategies, attending to mathematical language, understanding learning trajectories, and using evidence to support what they see happening in an elementary classroom (Smith, 2015).

During the senior year of their teacher preparation program, preservice teachers are placed in an elementary classroom throughout the entire duration of the year. This allows the preservice teacher an in-depth experience in an elementary school environment. Throughout the entire year, preservice teachers completed a total of 646 hours of student teaching work. Although STEM-focused elementary teacher preparation programs are rare, this particular program allows us to examine the effects of the program on preservice teachers as well as how their visions and implementation of mathematical tasks develop throughout their time in the program.

**Participant Selection Process**

The focus of the current study was a subset of one of the four cohorts (denoted as the P-Cohort). At the beginning of their junior year or beginning of their full-time elementary
education coursework, 36 of the 55 cohort members volunteered to participate in an additional in-depth case study component of Project ATOMS. In order to ensure the volunteers were representative of the larger population of students enrolled in the teacher preparation program, GPAs were utilized to obtain a stratified sample. Twenty of 36 volunteers were selected to participate. Of these 20 participants, 16 remained in the study throughout their junior, senior, and first year of teaching. These sixteen participants were videoed and interviewed extensively regarding lessons they taught for both mathematics and science throughout their junior and senior years in the teacher preparation program as well as throughout their first year of teaching.

The original cohort of 55 participants consisted of 54 members who completed the teacher preparation program in the spring of 2014. Of these 54 participants, 91% were females and 9% were males. Additionally, 92% were white, 2% were African American, 2% were Asian, 2% were American Indian, and 2% were not specified. In order to maintain confidentiality of the 16 participants that agreed to be followed in depth throughout the three-year study, as well as because gender and race are not salient to their stories in this study, gender-neutral pseudonyms as well as female pronouns were used for reporting throughout the study. Furthermore, of the participants in the original cohort (n=54), 9 people took the Math for Elementary Teachers course in the Fall of 2010, 5 people took two semesters of Calculus for Elementary Education in Fall 2011 or later, while only 1 person took only one semester of Calculus for Elementary Education. The remaining participants (n=38) either took no calculus or the standard calculus course (1 participant had no data reported).
In order to select participants for in-depth case study analyses, the LMT-MKT assessment (Hill & Ball, 2004; Hill et al., 2005; Hill et al., 2008; Schilling & Hill, 2007) in Number and Operations (K-5) was used. The assessment is multiple-choice and scored by using logits from the Rasch model. Logits were used because they are equal in size and helped to describe MKT better than using a linear measure (e.g., a difference in an individual’s score from 10% to 20% was much more substantial than a difference of 70% to 80%; therefore, logits were used).

The assessment was given to participants at four time points throughout the study (see Figure 11 for a breakdown of data collected throughout the study). The participants completed the assessments online via the Teacher Knowledge Assessment System (TKAS) at the University of Michigan. A member of the Project ATOMS research team administered the assessments face-to-face with participants with one exception; the participants completed the assessment at the end of their first year of teaching at their convenience at a location of their choice. The assessments were scored by the online TKAS, and results, reported as both raw scores and IRT scores, were downloaded by the lead PI of the Project ATOMS team.

The participants in the study were, on average, higher than the national average in terms of their MKT (Figure 4).
However, in order to examine the participants’ MKT in comparison to one another, z-scores in relation to the P-cohort were calculated. Furthermore, change from the beginning to the end of the teacher preparation program (from time point 2 to time point 4) was used to select participants, since experiences in the program were fairly consistent among participants. Of the 16 participants who were followed over the three-year time span, 15 had complete MKT data throughout the program (Figure 5).

*Figure 4. P-Cohort average MKT in comparison to the national average.*
Figure 5. MKT scores of 15 participants during teacher preparation program.

Tertiles were created based upon the change in their MKT from the beginning to the end of the teacher preparation program. These tertiles were then examined for those who: increased more than 0.5 standard deviations from the group mean (Figure 6), those who decreased more than 0.5 standard deviations from the group mean (Figure 7), or stayed within 0.5 standard deviations from the group mean (Figure 8).
Three groups. After carefully examining the 15 participants with complete MKT data throughout the program, as well as examining the participants within each of the three tertiles, participants were selected for further in depth analysis. These three participants represent those who increased more than 0.5 standard deviation from the mean, decreased more than 0.5 standard deviation from the mean, or stayed within 0.5 standard deviation from the mean. However, each participant also exhibited characteristics that allowed for further study beyond their change in MKT from the beginning to the end of the program. These characteristics include: remaining above average in terms of MKT throughout the program, remaining below average in terms of MKT throughout the program, and remaining fairly average in terms of MKT throughout the program. These groups and their participants are described in more detail below.

Increase of MKT throughout program. Jamie was chosen from a group of participants (five of the eleven) who increased in MKT more than 0.5 standard deviations from the group mean from the beginning to the end of the teacher preparation program (figure 6).
She was chosen because she had the largest change throughout the program in terms of her MKT (change of 1.67 standard deviations from the P-cohort mean), and she also was consistently below average in terms of her MKT.

**Decrease of MKT throughout program.** Jordan was chosen from a group of participants (three of the eleven) who decreased in MKT more than 0.5 standard deviations from the group mean from the beginning to the end of the teacher preparation program (figure 7).
Although Jordan was chosen because she decreased in MKT from the beginning to end of her teacher preparation program, she also was consistently above-average in terms of her MKT, as compared to her cohort.

**Stable MKT throughout program.** Charlie was chosen from a group of participants (three of the eleven) who remained stable (did not change more than 0.5 standard deviations from the group mean) in terms of their MKT from the beginning to the end of the teacher preparation program (figure 8).
Figure 8. Participants who remained stable in MKT from beginning of teacher preparation program to the end of the program.

Although Charlie was chosen because she changed less than 0.5 standard deviations from beginning to the end of the program (change of 0.03 standard deviations from the P-cohort mean), she also was the only “stable” participant who had complete video and interview data throughout the three-year study.

Data Collection

Once the cases were selected, data that had been collected during the participants’ time in the study (2012-2015) were used for analysis (Figure 9). The data included: a total of six video-recorded mathematics lessons (twice during both their junior and senior years in the teacher preparation program as well as twice during their first year of teaching) post-lesson, cognitive interviews for each lesson taught (for a total of 6 each); and benchmark interviews at the beginning and end of year (as well as middle of the year interviews during their junior and senior years) for each year in the study (for a total of 8 each). Although a
middle of the year lesson was captured during the participants’ first year of teaching, these were not used in this study in order to stay consistent with the beginning and end of each of the three years. A further description of each type of data collected throughout the study is presented in the sections below.
Figure 9. Timeline of data collected relevant to current study.
**Video-recorded mathematics lessons.** Throughout the teacher preparation program, participants were asked to video record themselves teaching multiple lessons. For the purposes of this study, participants were asked to videotape themselves teaching a lesson to a group or class of elementary students four times throughout the teacher preparation program. Table 2 provides an overview of the assignment that lead to teaching the lesson as well as when it occurred during the program. All videos recorded during the teacher preparation program were recorded by the cooperating teacher or the participant’s field placement partner.

In the first year of teaching, teachers were again asked to teach three lessons, with the intent to be videotaped by a member of the Project ATOMS team. The researcher acted as a non-participant observer (Creswell, 2013), such that they did not interfere with the lesson or classroom environment. They would simply take field notes and comply with the open-ended observational protocol during the lesson being observed. All videos throughout the study were stored on two password-protected external hard drives and stored with a number-pseudonym instead of the participants’ actual names.
Table 2.
*Description of Videotaped Lessons and Cognitive Mathematics Interviews*

<table>
<thead>
<tr>
<th>Time Point During Study</th>
<th>Field Placement</th>
<th>Assignment for Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall of Junior Year</td>
<td>Students were enrolled in a mathematics methods course that focused on grades K-2. The students were asked to videotape themselves teaching a lesson to their field placement classroom.</td>
<td>Students were asked to plan a mathematics lesson with their field placement partner suitable for their K-2 classroom and implement it while being videotaped.</td>
</tr>
<tr>
<td>Spring of Junior Year</td>
<td>Students were enrolled in a mathematics methods course that focused on grades 3-5.</td>
<td>Students were asked to plan a mathematics lesson with their field placement partner suitable for their 3-5 classroom and implement it while being videotaped.</td>
</tr>
<tr>
<td>Fall of Senior Year</td>
<td>Students were placed in an elementary classroom (grade varied for each participant) part time.</td>
<td>Students were asked to plan a mathematics lesson and videotape the implementation of the lesson in their student teaching classroom.</td>
</tr>
<tr>
<td>Spring of Senior Year</td>
<td>Students were placed in an elementary classroom (grade varied for each participant) full time.</td>
<td>Students were asked to plan a mathematics lesson and videotape the implementation of the lesson in their student teaching classroom.</td>
</tr>
<tr>
<td>1st Year of Teaching</td>
<td>Teachers taught at a variety of elementary schools in the southeastern region of the US.</td>
<td>Teachers were asked to plan and implement a mathematics lesson at the beginning, middle, and end of the school year.</td>
</tr>
</tbody>
</table>
**Interviews.** Teachers were interviewed throughout the study using a standardized open-ended interview (Johnson & Turner, 2003). This type of interview was used in order to maintain question consistency across participants. All interviews conducted with the participants were audio-recorded using two recording devices. This resulted in a total of 16 interviews throughout the three-year study, with only 14 being used in this study since the middle of the senior year lesson was not used in order to stay consistent with the previous two years. Furthermore, all interviews were transcribed for data analysis using a transcription company, unrelated to the project, and were saved with pseudonyms rather than the participants’ actual names.

**Post-lesson, cognitive interviews.** Following a videotaped lesson, a cognitive interview that was related to the previously taught lesson was held with the participant (See Table 2 for a description of the assignment). These lessons helped describe the planning, implementation, and post-lesson analysis process that the participants went through for their lesson. During the junior and senior years, interviews were held in-person between a member of the Project ATOMS team and the participant within a short period of time after the preservice teacher taught the corresponding lesson. However, during the first year of teaching, all post-lesson, cognitive interviews were held immediately after or within 48 hours of the lesson.

**Benchmark interviews.** At three time points throughout each year of the study, an interview was conducted with the participant to gauge items such as: confidence, preparedness to teach, their visions and beliefs on teaching, etc. Benchmark interviews were
conducted at the beginning, middle, and end of each year in the study. A member of the Project ATOMS team interviewed the participant either in-person or via Skype for each benchmark interview.

**Data Analysis**

Data from benchmark interviews, video-recorded mathematics lessons, and cognitive interviews were coded chronologically for each case throughout all three years before moving on to the next case. In order to maintain consistency with coding, a subsample of all data was coded by additional coders to achieve reliability. A further discussion of how this was achieved is presented later in this chapter.

Before coding, all interview questions from the overall larger study were read in order to decide upon questions that pertained to the preservice teacher’s visions and their analysis of implemented mathematical tasks. These questions were read by an additional member of the Project ATOMS team and agreed upon before being selected to use in the analysis of each participant (see Appendix A). Once solidified, the questions were imported into NVivo 10, a qualitative software package, for each participant.

Each case was coded chronologically before moving on to the next case. For example, Charlie was coded from the beginning of her junior year (J_BOY) in terms of her visions, implementation, and post-lesson analysis, before moving on to the end of the junior year. This occurred for each of her six time points. Once finished, Jamie and Jordan were coded similarly. Before moving on in the coding process, coding reliability checks were done at the J_BOY and S_EOY time points for each case (See reliability and trustworthiness for more details).
**Benchmark interviews: Visions.** In order to understand the preservice teacher’s visions as they relate to mathematical tasks, predetermined questions were used to narrow down the data (see Appendix A). The tasks rubric of the VHQMI framework was used to code for visions of high-quality mathematical tasks (see Table 3). After reading a response to a given question, the participant’s answer was coded into one of the corresponding levels of the VHQMI tasks rubric. Once all questions were coded for a particular interview, an overall, holistic level was given to describe the participant at a given time point. This process was used for all benchmark interviews coded throughout the three-year study. In order to obtain a holistic level of the participant’s visions, values were assigned based upon the levels that appeared most often. For example, if a participant had mostly level 2’s, and one or two level 1’s, then a holistic level 2 was given for the time point. However, if the participant had mostly 1’s and only one instance of a level 2, then they received a holistic level 1 for the time point. Halves were given as well due to some time points needing to reflect the participant as in between levels. Once each time point was coded, a summary of their visions was written in order to understand the codes in more detail as well as reduce the data from the coding process. In the results section, a description of the teacher’s development of visions throughout each year of the study, as well as their development overall will be described. Furthermore, a comparison of their visions of mathematical tasks (level on the tasks rubric of the VHQMI) versus their implemented tasks (level on the IHQMI tasks rubric) will be compared and described in detail.
Table 3.  
*Visions of high-quality mathematics instruction tasks rubric (Munter, 2014).*

<table>
<thead>
<tr>
<th>Level</th>
<th>VHQMI Description</th>
<th>Example(s)</th>
</tr>
</thead>
</table>
| 4     | Emphasizes tasks that have the potential to engage students in “doing mathematics” (Stein et al., 1996; Smith & Stein, 1998), allowing for “insights into the structure of mathematics” and “strategies or methods for solving problems” (Hiebert et al., 1997). | • Students should be engaged in challenging questions that have ambiguous or multiple routes to a solution in order to generate multiple solution paths and strategies for discussing/comparing, thus promoting students’ flexibility in applying problem-solving strategies (Russell, 2000)  
• “Questions that pertain to their lives around them or connected to things they’ve done in previous days… and require the kids to learn a concept not just by being told what it is and how to do it but to actually think about what it is they were doing and then coming up with the why or ‘Oh, look, this worked for all these problems? ’… do some critical thinking” |
| 3     | Emphasizes tasks that have the potential to engage students in complex thinking, including tasks that allow multiple solution paths or provide opportunities for students to create meaning for mathematical concepts, procedures, and/or relationships. “Application” is characterized in terms of problem solving. However, tasks described lack complexity, do not press for generalizations, do not emphasize making connections between strategies or representations, or require little explanation (Boston, 2012). Instead, they emphasize connections to “the real world” or “prior knowledge”. Reasons for multiple strategies are not tied to rich discussion or making connections between ideas. | • Tasks should have “more than one solution or maybe different ways to approach it so that different ideas are accepted and could be possible.”  
• “A problem can be solved different ways, because there are different ways of thinking and kids need to know that there’s not just one set way to do things.”  
• “Have multiple entry points for students, multiple solution tasks that require children to really think and put a lot of information together in order to answer the question.”  
• “Open-ended so it doesn’t have a right answer, and it talks about how things fit together instead of what the answer is.”  
• “I would look for tasks that accessed some sort of prior knowledge yet took the kids a little bit further to build on that knowledge.”  
• “I want to see a lot of different ways of doing the same thing… some kids can get past the visual and they’re into the abstract mode much quicker and then they don’t want to waste their time and be bored.” |
Table 3 (continued).

<table>
<thead>
<tr>
<th>Level</th>
<th>Task Orientation</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Promotes “reform”-oriented aspects of tasks without specifying the nature of tasks beyond broad characterizations (e.g., “hands-on”, “real-world connections”, “higher-order”), and without elaborating on their function in terms of providing opportunities for “doing mathematics” (Stein, Grover &amp; Henningsen, 1996; Smith &amp; Stein, 1998). “Application” is characterized in terms of “real-world” context and/or students being active.</td>
<td>“Hands-on activities, instead of doing worksheets…maybe build something or work it out with some kind of a model… the application of what they’ve learned is really important.”&lt;br&gt;• “Higher order thinking problems with application.”&lt;br&gt;• “Bring in the outside world to try to get the kids engaged.”&lt;br&gt;• “Not doing straight book work” (instead, “cutting out puzzle pieces and making two puzzles to prove Pythagorean’s Theorem”).&lt;br&gt;• Tasks should include “time to move and use those manipulatives and things.”</td>
</tr>
<tr>
<td>1</td>
<td>Emphasizes tasks that provide students with opportunity to practice a procedure before then applying it conceptually to a problem (Hiebert et al., 1997).</td>
<td>“First is to understand what the concept is, and what the formula is, and how to do it in terms of the numerical way. Second is applying it… if it’s put into a word problem.”</td>
</tr>
<tr>
<td>0</td>
<td>Either (a) does not view tasks as inherently higher or lower quality or (b) does not view tasks as a manipulable feature of classroom instruction.</td>
<td>(a) Depends on the teacher, “whatever works for them”; “Depends on the class”; “The thing that actually gets them to start asking questions.”&lt;br&gt;(b) “We’re supposed to be using the CMP book which is pretty much, this is what you do and here’s what the teacher should say and it even tells you how it should run.”</td>
</tr>
</tbody>
</table>

Although the participant was coded using a numerical level from the VHQMI codebook, the holistic levels allowed for them to be understood as the defining characteristics of the level in which they were coded. The qualitative descriptors used in the VHQMI codebook, coupled with the responses given by the participant, are used to describe their trajectory of development of professional visions for mathematics teaching over the three-year time period. These predetermined descriptors, based upon research used to create the VHQMI, helped to provide rich-descriptive evidence for the participant’s vision.
**Video-recorded mathematics lessons: Implementation.** In order to code the implementation of mathematical tasks, the videos obtained throughout the study were first watched in their entirety (Erickson, 2012) and notes were taken to understand the tasks that occurred. During this first viewing, time-stamps were placed to denote the beginning of each new task in the overall lesson. A task was defined as an activity whose purpose is to “focus students’ attention on a particular mathematical idea” (Stein et al., 1996, p. 460). For example, a lesson may consist of one task that takes up the entire time, or several smaller tasks throughout.

Next, each video was re-watched and coded using the Implementation of High-Quality Mathematics Instruction (IHQMI) rubric for mathematical tasks. The IHQMI (see Table 4) was created for the current study by adapting Munter’s (2014) VHQMI framework. Rather than focusing on how teachers describe their vision of tasks (VHQMI), the IHQMI focuses on how a teacher implements tasks in the classroom. Although levels 1-4 on VHQMI and IHQMI correspond to how the teacher envisions a task and how they implement the task, respectively, there is some discrepancy at a level 0. For example, on the VHQMI, a level 0 is described as envisioning the lesson to be “fun” or not describing the cognitive demand of the task in general. However, on the IHQMI, a task coded as a level 0 is either focused on memorization or practicing a procedure with no connection to why students are using it.
Table 4.  
Implementation of high-quality mathematics instruction (adapted from VHQMI; Munter, 2014).

<table>
<thead>
<tr>
<th>Level</th>
<th>IHQMI Description</th>
<th>Example Features of Tasks: (Tasks in this level may include the following)</th>
</tr>
</thead>
</table>
| 4     | The task chosen has students “doing mathematics” (Stein et al., 1996; Smith & Stein, 1998). | • The task has students engaged in solving a problem that is relevant to their lives or the mathematics they’ve done recently that can be addressed using a variety of ways.  
  • Students are using appropriate manipulatives and tools, as well as discussing their solution strategies.  
  • The teacher might also take the task one step further in having the students generalize or develop an algorithm based upon discussed solution strategies. |
| 3     | The task chosen has the potential to engage students in multiple solution paths or higher-order thinking. However, the task does not have students making generalizations or making connections between other mathematical topics or the world around them. Additionally, the reason for the multiple solution strategies and/or open-endedness of the task is not tied to rich discussions between ideas. | • The task has students solving a problem that allows them to provide several different ways to approach the problem.  
  • Students use their prior knowledge as well as manipulatives or tools to help them solve the problem in a way that is meaningful to them.  
  • However, the teacher does not wrap up the task with a discussion of how students solved the problem differently. |
| 2     | The task is “reform”-oriented in the sense that it is a hands-on task, or relates somehow to the “real world”. However, it does not provide students with the opportunity for “doing mathematics”. In other words, students are not engaged in higher-order thinking, but only performing a task with a real-context in order to help them engage and participate more fully. Multiple solution strategies are able to be used, but the task/teacher does not encourage the use of additional strategies, | • The task has the students using manipulatives or tools for the sake of using them.  
  o For example, the teacher sets out base-ten blocks while students are adding two digit numbers. The students do not necessarily need the manipulatives because they are focused more on the procedure to solve the problem.  
  • The task has students solving problems related to the real world or brings in activities that require problem solving. The real-world nature of the problems is used to help engage the students in the task. |
Table 4 (continued).

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The task provides students with the opportunity to practice a procedure before applying it conceptually to a problem (Hiebert et al., 1997).</td>
<td>- The task has the teacher solving a problem first to show students how to do the problem procedurally, and then students are given time to work on similar word problems using the algorithm to solve.</td>
</tr>
<tr>
<td>0</td>
<td>The task is focused on memorization or procedures without connections.</td>
<td>- The task focuses on having students do rote memorization (i.e. timed multiplication tests) or has the students practicing a procedure without a connection to why they are practicing it.</td>
</tr>
</tbody>
</table>

Similar to the coding used with the VHQMI, each task was given a level on the IHQMI rubric throughout the lesson (see Table 4). Once completed, a holistic level was created to understand the teacher’s implementation of mathematical tasks at a particular time point. This process was used for all video-recorded lessons coded throughout the three-year study. In order to obtain a holistic level of the participant’s implementation, the time spent on each task, coupled with the task’s level on the IHQMI were considered. For example, if a participant had three tasks: Task A, coded at a level 2 lasting thirty minutes; Task B, coded at a level 1 lasting five minutes; and Task C, coded at a level 1 lasting another 5 minutes, then a holistic level 2 was given because much of time was spent at a level 2. Half scores between whole numbers (e.g., 1.5, 2.5) were given as well due to some time points needing to reflect the participant as in-between levels (i.e. twenty minutes at a level 1 and twenty minutes at a level 2 would be a holistic level 1.5). Once each time point was coded, a summary of their implementation was written in order to understand the codes in more detail as well as reduce
the data from the coding process. In chapters 4-6, a description of their development of implementation throughout each year of the study, as well as their development overall all will be described.

**Post-lesson, cognitive interviews: Analysis.** In addition to understanding the implementation of the mathematical tasks, a mixture of apriori codes and thematic-content analysis (Vaismoradi, Turunen, & Bondas, 2013) was used to understand how the participants reflected upon and analyzed their own teaching through the use of post-lesson cognitive interviews. These interviews provided insight into how the teacher planned and modified the lesson as well as what they attended to post-implementation. Understanding what adaptations were made to the lesson was crucial for this study since most preservice teachers do not have control over the tasks they use in a cooperating teacher’s classroom. However, the way they adapt the given curriculum as well as the adaptations made in the moment helped to understand their own implementation rather than the intended instruction of the mentor teacher.

In order to code how the preservice teacher analyzed their implemented lesson, a predetermined set of interview questions, see Appendix A, was used along with apriori codes, see Table 5 (not mutually exclusive) that focused on how the participant: critiqued the lesson, provided evidence of the lesson’s effectiveness, analyzed their MKT, and analyzed the student learning that occurred (Santagata, Zannoni, & Stigler, 2007). Although Santagata and colleagues’ (2007) coded for “elaboration”, it was not coded for in this study. However, a description is given in the results if the teacher did not provide in-depth rationales during their analysis.
Table 5.  
**Coding of participants’ post-lesson analyses. (Adapted from Santagata et al., 2007)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Sub Code</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Approach</td>
<td>General</td>
<td>The teacher’s comments were mostly affective about the lesson.</td>
<td>“I enjoyed teaching this lesson and the students had fun”.</td>
</tr>
<tr>
<td>What is the teacher’s focus when they are critiquing the lesson?</td>
<td>Specific</td>
<td>The teacher describes what they saw happen and/or provides alternative solutions.</td>
<td>“I did not have enough materials to make the task rich for all students. Next time, I would be sure to be better prepared”.</td>
</tr>
</tbody>
</table>
Table 5 (continued).

<table>
<thead>
<tr>
<th>Student Learning</th>
<th>Focus on Teacher</th>
<th>The teacher mostly focuses on the teacher’s actions or errors.</th>
<th>“I was very familiar and comfortable teaching this topic. Because of this, I was able to have a discussion about the solution strategies used to solve the problem in the given task”.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus on Students</td>
<td>The teacher focused on student behavior and student learning as a result of the tasks given.</td>
<td>“A group of students were off task and I had to continually stop the lesson to redirect their behavior. Because of this, student learning was affected”.</td>
<td></td>
</tr>
<tr>
<td>Focus on Resources</td>
<td>The teacher focused on student learning occurring (or not occurring) due to the materials, an outside influence (i.e. mentor teacher, etc.).</td>
<td>“I wish the materials I had in the lesson were better”.</td>
<td></td>
</tr>
<tr>
<td>Uncertainty</td>
<td>The teacher is unsure or cannot point to specific evidence as to whether student learning occurred because of the tasks in the lesson.</td>
<td>“I hope that they learned…”</td>
<td></td>
</tr>
</tbody>
</table>

In terms of how they critiqued the lesson, if applicable, a response was coded into a sub-category as to whether they were specific in their critique (i.e. described what they saw happen) or were general (i.e. mostly affective about the lesson). The evidence of the lesson’s effectiveness was also sorted into sub-categories as to whether their evidence was specific (i.e. provided instances of effectiveness) or was mostly general (i.e. referred to the lesson overall). To understand how they analyzed their MKT, sub-categories based upon Ball and colleagues’ (2008) work were used. These sub-categories included: common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), knowledge of content and teaching (KCT), horizontal curriculum knowledge (HCK),
and vertical curriculum knowledge (HCK). Each of the six sub-categories helped to further understand how the teacher was making sense of their implemented lesson through the MKT they were using in the moment. Finally, student learning was sorted into four sub-categories that described what the teacher believed was a source of students’ learning throughout the lesson. These included: a focus on the teacher (i.e. their own actions or errors), a focus on the students (i.e. behavior), a focus on resources (i.e. not having the necessary materials), and uncertainty (i.e. being unsure of whether student learning occurred).

As each preselected question was read, the response was coded into one or more of the categories mentioned above as well as placed into a sub-category, if possible. Once coded, the responses for each category were summarized for each time point. Throughout the coding process, new codes or sub-codes were created based upon responses given by the participants in the study. For example, “MKT” replaced Santagata and colleagues’ (2007) “mathematical content” category because the participants also referred to pedagogical content knowledge throughout their post-lesson cognitive interviews. Similarly, “uncertainty” and “resources” were added as sub-codes for the category “Student Learning” because these were relevant to the participants in the current study. Overall, the coding process was iterative and the coding scheme was further fine-tuned as each new case was coded throughout the data analysis process.

**Additional coding.** Once all three major components were coded for each participant (visions, implementation, and post-lesson analysis), each post-lesson cognitive interview was read to understand how the participant planned their lesson prior to implementation. Descriptions were summarized as to how they planned the lesson, how it was modified, and
where the lesson plan was obtained (i.e. from a curriculum, online, from a cooperating teacher, etc.). Furthermore, each participant’s J_MOY “Getting to Know You” interview was read and summarized in order to understand their memories and experiences in K-12 mathematics. Finally, once all cases were coded, their results were written chronologically (from the B_JOY to the 1_EOY) with notes being kept in the margins and within a spreadsheet to note any themes emerging within each case. These notes allowed for themes to become apparent around how each case developed over time as well as similarities and differences amongst each case.

Table 6. **Overview of Data Analysis Process**

<table>
<thead>
<tr>
<th>Data Analysis Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Read through all interview questions from larger study to narrow down questions pertaining only to visions and analysis of tasks. Had a second team member verify the questions being used.</td>
<td>This was helpful in order to reduce the data and to focus only on questions pertaining to visions and analysis of tasks.</td>
</tr>
<tr>
<td>2. Imported all questions related to visions and analysis for each participant into NVivo 10. See Appendix A for list of questions considered.</td>
<td>This was helpful in order to easily code and look for themes in the cross-case comparison.</td>
</tr>
<tr>
<td>3. Began coding with Charlie. Read J_BOY visions questions and coded any statements using the VHQMI rubric. Gave a holistic level to capture the time point.</td>
<td>This was used to understand their visions at a particular time point.</td>
</tr>
<tr>
<td>4. Wrote a summary of Charlie’s visions at the J_BOY.</td>
<td>This helped me to gain an understanding of the codes that I had created during this time point. It was also a data reduction step.</td>
</tr>
</tbody>
</table>
Table 6 (continued).

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>Watch Charlie’s J_BOY videotaped lesson entirely. Take notes and create time-stamps of when tasks occurred.</td>
<td>This allowed me to gain an understanding of the lesson and to note the changing of tasks throughout.</td>
</tr>
<tr>
<td>6.</td>
<td>Watch Charlie’s J_BOY lesson again, coding using the IHQMI rubric for each task. Once complete, I gave the overall lesson a holistic level to capture the time point.</td>
<td>This was used to understand their implementation of tasks at a particular time point.</td>
</tr>
<tr>
<td>7.</td>
<td>Read questions from cognitive-interviews pertaining to Charlie’s post-lesson analysis during her J_BOY lesson. Coded using apriori codes.</td>
<td>This allowed me to understand their analysis of their lesson at a particular time point.</td>
</tr>
<tr>
<td>8.</td>
<td>Wrote a summary of Charlie’s analysis of her tasks at the J_BOY.</td>
<td>This helped me to gain an understanding of the codes that I had created during this time point. It was also a data reduction step.</td>
</tr>
<tr>
<td>9.</td>
<td>Met with outside coder to discuss our coding of Charlie’s J_BOY visions, implementation, and post-lesson analysis.</td>
<td>This allowed me to maintain consistent coding throughout.</td>
</tr>
<tr>
<td>10.</td>
<td>Repeated steps 3-8 for Charlie’s J_EOY, S_BOY, and S_EOY.</td>
<td>This helped me to understand Charlie as a case.</td>
</tr>
<tr>
<td>11.</td>
<td>Met with an outside coder to discuss our coding of Charlie’s S_EOY visions, implementation, and post-lesson analysis.</td>
<td>This allowed me to maintain consistent coding throughout.</td>
</tr>
<tr>
<td>12.</td>
<td>Repeated steps 3-8 for Charlie’s 1_BOY and 1_EOY lessons.</td>
<td>This helped me to understand Charlie as a case.</td>
</tr>
<tr>
<td>13.</td>
<td>Repeated steps 3-12 for Jamie and Jordan.</td>
<td>This helped me to understand Jamie and Jordan as a case.</td>
</tr>
</tbody>
</table>
Table 6 (continued).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>14. Refined coding of the post-lesson analyses and recoded Charlie and Jamie.</td>
<td>As I met with outside coders, I developed additional codes “MKT”, “Uncertainty”, and “Resources” as part of the coding scheme for the post-lesson analyses. I recoded the first two cases to make sure I did not miss these codes when I did their initial coding.</td>
</tr>
<tr>
<td>15. Read and described how each case planned each of their 6 lessons.</td>
<td>This allowed me to understand the modifications they made to a lesson.</td>
</tr>
<tr>
<td>16. Read and described a lesson the cases felt was effective from their own K-12 experiences.</td>
<td>This allowed me to understand their background memories of elementary mathematics.</td>
</tr>
<tr>
<td>17. Wrote each case chronologically. Created notes in the margins and in a spreadsheet to notice themes that were emerging.</td>
<td>These allowed me to notice how each case developed over time as well as similarities and differences amongst them.</td>
</tr>
</tbody>
</table>

Reliability and Trustworthiness

In order to ensure the study’s findings are reliable and trustworthy, I provided triangulation of the data through the use of a longitudinal study, by collecting responses to the same questions throughout the study, and by using multiple data sources throughout the study (Creswell, 2013). Additionally, for the coding conducted throughout this study, six of the eighteen time points (3 participants with 6 time points each) were double coded by outside coders (see Table 7).
Table 7.
*Description of coders used for reliability.*

<table>
<thead>
<tr>
<th>Participant and Timepoint</th>
<th>Outside Coder</th>
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<tbody>
<tr>
<td>Charlie J_BOY</td>
<td>Coder 1 for visions, implementation and analysis.</td>
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<tr>
<td>Charlie S_EOY</td>
<td>Coder 1 for visions; Coder 2 for implementation and analysis.</td>
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<td>Jamie J_BOY</td>
<td>Coder 1 for visions; Coder 3 for implementation and analysis.</td>
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<tr>
<td>Jamie S_EOY</td>
<td>Coder 1 for visions, implementation and analysis.</td>
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<tr>
<td>Jordan J_BOY</td>
<td>Coder 1 for visions; Coder 2 for implementation and analysis.</td>
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<tr>
<td>Jordan S_EOY</td>
<td>Coder 1 for visions; Coder 3 for implementation and analysis.</td>
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Three outside coders were brought in to check for reliability in the coding process. Coders 1 and 2 of the three outside coders were a part of the Project ATOMS team. Outside coder 1, checked for reliability of visions coding throughout all six time points. However, coders 1, 2, and 3 rotated for the implementation and analysis coding. Each outside coder was given instructions on how to code, and then asked to read the interview questions and/or watch the implemented lesson video attending to time-stamps given to them. Upon completion of the coding, meetings were arranged with each outside coder to discuss how the interviews or lessons were coded. Any discrepancies that arose were discussed and agreed upon before continuing any additional coding. Furthermore, until a coding meeting occurred, no further coding of cases took place to ensure reliability with the future time points.
Chapter 4: Jamie

At the beginning of her junior year, Jamie reflected on her past experiences with mathematics. Overall, Jamie felt positive about her school experiences; however, she did not remember much about how she was taught. She recalled loving her second-grade teacher because she made her “feel really comfortable [because she] was just very nurturing.” Although she went to tutoring outside of school, she remembered feeling “pretty confident in math [because she] always caught on quick.” She continued by saying, “I wouldn’t say I enjoyed doing math problems, but I did feel a sense of accomplishment when I did well, so I liked that.”

In middle school, Jamie had a teacher in eighth grade that she “didn’t really like as a person or a teacher.” Although the teacher retired and was replaced by another, she continued to do poorly in the class. Jamie’s struggles with math continued through high school, and she lamented that “it wasn’t like I did bad, but I never really was the best in my class…I was just kind of okay with math.” She reflected on an effective lesson she had in high school, by saying it was one where “you do this, this is how you do it, and that’s it.” She felt it was effective because “you just memorize it, but then [it was not really effective] because you don’t know why you are doing it.” Once in college, Jamie took several math classes, which included calculus and statistics. She felt she did okay in calculus, but she did not go into detail about other experiences beyond listing the courses.

Jamie is representative of a case that increased in her mathematical knowledge for teaching from the beginning of the junior year in the teacher preparation program to the end of the program in her senior year (Figure 10). Although she increased more than one and a
half standard deviations, she remained below the cohort average by more than a half standard deviation. The case findings are described by presenting Jamie’s trajectory from the teacher preparation program through the first year of teaching on her visions and implementation of mathematical tasks as well as her post-lesson analyses.

*Figure 10.* Jamie’s change in MKT from beginning to end of the teacher preparation program.

**Beginning of Junior Year**

**Visions.** Jamie provided her vision of tasks through a description of a lesson she felt was effective. She described that students should have “the teacher show them how to do it, then they could do it themselves with the teacher’s help, and then they could work on it on their own and [the teacher] would give them feedback based on what they did on their own”. Jamie felt this was effective because “they can just be introduced to it” by the teacher and have some general idea of how to do it before they go work independently. Additionally, the
independent work allowed students time to “show you they can do it on their own” and by doing it themselves the teacher “can kind of see what they’ve accomplished or if they still need to work with you.” Jamie felt that as the students were working on “hands on stuff… or something that can keep their attention,” the teacher should be there to help fix mistakes.

Jamie’s visions are that of a level 0.5 on the VHQMI rubric at this time point. It is unclear if she is describing a procedural lesson or one with an application-based aspect when she used the pronoun “it” in her statement regarding the teacher not wanting “it to be too hard for them… and it should build their confidence.” This statement led her to be coded as a level 0. However, despite her lack of clarity, she describes a lesson that is of the “practice and apply” mindset during this time point (Level 1). Therefore, Jamie’s vision of high-quality mathematical tasks was coded as 0.5 on the VHQMI at the beginning of her junior year.

**Implementation.** Jamie’s lesson was taught to a kindergarten class, in which she was placed during the fall of her junior year. The lesson she was asked to teach focused on items that were alike or different. She planned the lesson by asking her cooperating teacher if she had any plans she would like for her to teach. From a book of lessons given to her by her cooperating teacher, Jamie chose a worksheet for her students to complete. However, her classroom did not have all of the resources the worksheet suggested to compare (i.e. cone, sphere, triangle). Instead, Jamie showed a circle and square and asked students to compare and contrast the shapes.

To begin her lesson, Jamie’s first task lasted ten minutes and had students seated on the carpet as a whole group. Jamie showed the students a square and a circle and asked them whether the shapes were same or if they were different. When students began responding
they were different, she asked them to describe how. Although Jamie tried to draw the students’ attention to the defining attributes of shapes (e.g., a square has vertices), the students chose to focus on the color and size of the shapes. To extend their discussion, Jamie selected two students, both girls, to stand at the front of the room and asked the other students what was “alike and different” about them. The students focused on the length of the girls’ names, their height, and their clothing being different colors. In order to draw their attention to gender, Jamie brought a male student to the front of the room for the students to compare as well.

For the next eight minutes, Jamie gave the students a worksheet of items that were alike and different. She gave an example of one problem showing a lemon, a boot, and another lemon, and asked the students “which is different?” A student came to the front of the room and circled the boot, and Jamie asked why it was different. The students then moved to their tables and completed the worksheet on their own. Similar to the lemon example, the worksheet showed several problems with three shapes, two the same and one different, and students were asked to circle the object that was different.

*Overall, Jamie’s implementation for her beginning-of-the-junior year lesson was coded as a level 2 on the IHQMI tasks rubric.* Her first task (level 2.5) had the potential to engage the kindergarteners in higher order thinking and multiple solution paths, but she did not ask questions to get the students to think about the task differently (i.e. other than just color and size). However, she did use real-world contexts and did not have a prescribed way to solve the task. Her second task (level 1.5) was procedural in nature (i.e. only asked the
students to circle their answer and not explain any reasoning); however, Jamie went around and asked students to verbally explain why they circled the object they did.

**Post-lesson analysis.** Jamie began her post-lesson analysis by positively critiquing her students’ behavior by saying that they completed the worksheet when given time to do so individually, and that they were not “out in another land” like she perceives them to be sometimes. She reflected upon the lesson by saying she wished she would have been “a bit more prepared for [the lesson] and could have thought about it a little bit more so that it would have been more effective for [the students]”. The evidence Jamie provided as to whether the lesson was effective or not, focused on how the students had the opportunity to think about similarities and differences between objects when they might not have had the opportunity to do so before. She mentioned that they “haven’t really ever noticed that, even though they kind of know in their minds, but they just didn’t ever really think about it, so I feel like it went well because of that”. She continued by saying that the lesson was effective because “I was trying to get across to them that some things are different and that you can compare them and when you’re picking them out of a group you can explain why it’s different or the same.” She indicated she saw them accomplishing this goal when they completed their worksheets individually.

Jamie critiqued the examples she chose to provide at the beginning of class, by saying that she could have done a better job and that she did not know if she confused the students or not. Her *mathematical content knowledge* was shown through her critiques of her knowledge of content and teaching (KCT), because she felt she could have done better, if she would have selected better examples to use with her students. Furthermore, she used her
common content knowledge (CCK) to make an instructional decision to include a third student, Carlos, when she originally chose two girls to come to the front of the classroom because she wanted to show that the genders were different.

Jamie’s analysis of the students’ learning was shown through her reflection on herself as a teacher. She described that without the whole-group discussion before the independent work, that the students would have just “circled every single one of them [on the worksheet] because they wouldn’t know or they just wouldn’t be sure of how to do it, so I feel like I kind of explained that you could notice or watch out for those things in simple terms…” When Jamie focused on the students in her post-lesson analysis, she gave credit to their behavior by saying that “for the most part they paid attention… and there wasn’t anybody that just completely had no idea.” Additionally, she mentioned that if she would have had better materials and explained the material better she could have increased student learning.

End of Junior Year

Visions. At the end of her junior year, Jamie did not comment on a specific lesson that she felt was effective. Instead, she commented on what she felt should be going on inside a classroom: the teacher should show students how to do a problem and then the students should go and work on it on their own. While students were working (in small groups, but doing their work individually), Jamie felt it was the teacher’s job to check on students and make sure they were understanding the material. Jamie failed to comment on whether tasks should be higher or lower level, but stressed the importance of using a lot of discourse to have a good lesson. Similar to the beginning of her junior year, she was of the mindset that students should “practice and apply” (level 1), but she did not comment on the nature of the
Therefore, these traits placed Jamie at a level 0.5 on the VHQMI rubric at the end of her junior year.

**Implementation.** Near the end of Jamie’s junior year, she completed a field placement in a fourth-grade classroom where she taught a lesson on comparing fractions and writing equations. She planned the lesson by using the lesson plan given to her by her cooperating teacher. The fourth-grade teachers on the team were responsible for planning each subject, and math was no exception. Therefore, Jamie used pre-existing slides for her math lesson that were created by a teacher on the fourth-grade team.

Jamie’s lesson began by her working with a small group of students focused on word problems. This first task lasted five minutes, and involved Jamie putting an example on the smart board that involved rides at an amusement park. Before beginning the problem, she asked someone to remind everyone what a ‘multiple’ is. One student replied by saying “it’s like 3, 6, 9, 12, 15...”. Jamie then used the student’s definition to help the class write equations involving the rides at the amusement park and the number of people who had ridden on the rides. For example, the “Loop-da-loop (L) had 80 people ride while the Carousel (C) only had 40 people ride”. Therefore, the equations the class wrote were \( C \times 2 = L \) and \( L \times \frac{1}{2} = C \). Once finished with the example, Jamie released them to work on the remaining problems independently.

Although focused on the same types of problems, the second task lasted for twelve minutes, and had students working independently. While the students were working alone, Jamie walked around and helped them as they asked questions. Some students were confused and constantly raised their hands for Jamie to come help them. After many students raised
their hands, Jamie exclaimed, “I don’t think I showed you enough.” She then proceeded to quickly work through another example without any explanation of why she wrote the equations the way she did. Once finished she asked the class “does everyone see how I did that?”

**Overall, Jamie’s implementation for her end-of-the-junior year lesson was coded as a level 1 on the IHQMI tasks-rubric.** Her first task (level 1) had a real-world context, but the problems were solved procedurally. Throughout her second task (level 1) Jamie used examples of ways to solve the problems together; however, she rushed through the task and did not capitalize on moments where higher-order thinking could have taken place. Instead, Jamie created a procedure for writing equations, and the students used her examples to solve similar problems on their own.

**Post-lesson analysis.** Jamie’s critique of her lesson focused mostly on whether she understood where the students were at in terms of the curriculum and whether or not they understood what she was doing. She commented that “I don’t think it went bad, but I think that it could’ve gone a lot better if I was more familiar with what they had been doing [in class].” Jamie’s evidence of the lesson’s effectiveness was rooted in whether the students could figure out the correct answers to the problems. She mentioned that “their examples on their papers were correct… so I feel like they learned a little something from it”.

Furthermore, Jamie felt the real-world applications were effective because it “got the students thinking in a different way and in more detail.”

In Jamie’s post-lesson analysis, her mathematical knowledge for teaching was described during her interviews through her lack of understanding the students’ instructional
experiences before she taught the lesson (i.e. horizontal curriculum knowledge; HCK). She elaborated on this by saying “I didn’t know what she [the cooperating teacher] had been doing before”, and she wished she would have sat down with the teacher to get a better understanding beforehand. Despite the students’ struggles as shown on the video, Jamie described her instruction by saying “I just wanted to show them a quick example, because they knew all the vocabulary. They knew what multiples were.” She continued by saying, “I just wanted them to think of it on their own and come up with examples, and I think they were doing that.” This is indicative of her knowledge of content and students (KCS) because she believed she was able to incorporate additional examples throughout the lesson in order to help the students in their understanding of the material.

Finally, Jamie analyzed the students’ own learning, by focusing on students getting correct answers and on her own teaching as a source for learning. She wished that she had been more familiar with the topic to help them better understand since some were confused. However, “after a little bit of help they could get it and move on and do more examples on their own.”

**Beginning of Senior Year**

**Visions.** At the beginning of her senior year, Jamie continued to envision good tasks as those where the teacher first shows the students multiple ways to do the problem and then allows students “time to practice with the support of the teacher and then by themselves”. She felt the tasks might have the students “memorizing some things, but they won’t truly understand the process if they don’t have the base knowledge that they need.” Therefore, she felt that in order to plan a task, the teacher must decide on “where the students are at” and
“plan for your students instead of just looking at the standards and planning based on those.” Jamie also felt that the way she was “taught isn’t the best way to teach your students,” meaning that you “can’t just sit them down and just do problems after problems after problems. You have to start at the beginning and create a deeper understanding for math so they can solve a lot of different types of problems.” Therefore, Jamie’s vision of mathematical tasks at the beginning of her senior year was coded as a level 1 on the VHQMI rubric. Although she mentioned needing to have a deeper understanding (level 2), she also mentioned needing to “practice and apply” (level 1) as well as tasks that focus on memorization (Level 0).

Implementation. During Jamie’s senior year, she was placed in a fifth-grade classroom. Near the beginning of the year, Jamie was required to administer an assessment and subsequently teach a mini-lesson to a selected group of students based on their assessment performance. Jamie’s assessment focused on rounding. Jamie chose three students who responded incorrectly on all questions focused on rounding to the hundredths place. Her planned remediation lesson only included the three students because the others in the class did well, and she felt they did not need re-engagement. She chose to use a chart of place values she found online, and then reinforced the rules for rounding, since she felt “it just seemed that they didn’t have the understanding of the rules for rounding.”

Jamie began her lesson on rounding decimals by reviewing how to round decimals to the hundredths place. She asked her small group “what does rounding up mean? When do you round up?” Jamie’s students provided her with the rule for rounding a number up if the
place value to the right is 5 or higher, and to round down if the place value to the right is 4 or lower. Since this was a discussion (not a task) it was not included for coding.

Next, Jamie gave the students several problems and asked them whether they were going to round up or down. The students used the rounding rules to write their answers on a white board and showed her their answer when they were finished. She additionally asked them to round a number to the tenths place, while asking them to point to the tenths place. She asked the students “what number do we need to look at to round”, and once the students solved the problem she gave another example.

Although the video cut off before the end of the lesson, the portion that was viewable was coded as a level 0. Therefore, overall, Jamie’s beginning-of-the-senior year lesson was coded as a level 0 on the IHQMI tasks rubric. Jamie and her students were focused on practicing a procedure for rounding, rather than understanding why they round up or down.

Post-lesson analysis. Jamie’s critique of her lesson focused on whether the students could obtain a correct answer to the rounding problems or not. She mentioned that “if we didn’t do that lesson, I think they would still have been okay” since it was a review lesson. However, Jamie reflected in her interviews by saying that “at first they [the students] were kind of giving me answers that weren’t exactly right and I would clarify, and after that it was a breeze.” Her evidence that the lesson was effective, was based upon the students’ ability to “get the correct answer” when working in her small group.

Jamie briefly referred to her mathematical knowledge for teaching when she mentioned that she only did the reinforcement lesson with her students because their test results showed they were struggling. This is representative of her knowledge of content and
students (KCS). Additionally, Jamie saw herself as the source for student learning, by stating that once she reinforced the rounding rules and reminded them how to round, the students were able to complete the problems without any difficulty.

**End of Senior Year**

**Visions.** Although at the end of her senior year Jamie did not specifically discuss a task that she considered to be effective, she mentioned that it is the teacher’s job to show students how to work through a problem, and then allow them time to try similar problems on their own (level 1). However, Jamie failed to describe the types of tasks the teacher should be working on with students (i.e., procedural or conceptual). Jamie tended to focus mostly on allowing the students to work in groups to engage and motivate them to learn (level 0).

*Therefore, Jamie’s vision of mathematical tasks was consistent with a level 0.5 on the VHQMI at the end of her senior year.*

**Implementation.** During her student teaching, Jamie was placed in a fifth-grade classroom where she taught a lesson on fraction word problems. She wanted the students to not focus on using key words; instead, she wanted to “make sure they [could] take the word problem and apply it to solving their problem.” Jamie did not use an existing lesson plan; instead she chose problems from their math workbooks and the previous end-of-grade test to complete as a review with the entire class.

To begin her first task, which lasted fifteen minutes, Jamie called on a student to work through a problem, and she and the student engaged in a discussion of the key word “of.” Jamie told the students to not focus on the key word because “most of the time when we see ‘of’ it means to multiply… but not every single time.” This confused students, and
they pushed her as to which situations it applies and which ones it does not. Jamie told them they will work through a few more problems, and it will hopefully clear up the confusion.

Throughout the word problems, Jamie focused on “how do we solve this?” or “is this our answer?” She encouraged the students to draw a picture to represent the problem, and asked the students “what would this picture look like?” A student in the class asked her again if “there is another word that tells you to multiply” and Jamie replied with “‘of’ is the only thing you will probably be looking for.”

In between word problems, Jamie switched to a naked-number problem that involved multiplying 2/3 and 9/5. This second task lasted for three minutes of the overall lesson. During this task, a student in the class asked if the answer to the problem “will still be the same if it is set up differently?” (i.e., commutative property). Jamie replied with “yes, but in a word problem it can look different.”

The third task lasted eleven minutes, and took up the remainder of the class. Students worked as a whole group to solve word problems together with Jamie. Many students called out answers, worked on their own, or laid their heads on their desks. However, Jamie told her students “don’t get stressed out. This is why we practice these together.” She tried to clear up confusion occurring in the word problems by drawing out a picture to represent the multiplication of fractions and answered the students’ questions regarding key words a final time by saying “you’re not going to see a multiplication word problem without seeing the word ‘of’.”

*Overall, Jamie’s end-of-the-senior year lesson was coded as a level 1 on the IHQMI tasks rubric.* Her first task (level 1) had students engaged in practicing a procedure, despite
using word problems. The second task involved a naked-number problem (level 0) and was procedural without a conceptual application. Finally, the third task (level 1) had the students, once again, practicing a procedure despite using word problems. Although Jamie told students to not use key words to solve the problems, she still maintained an emphasis on how “of” means to multiply when pushed on it by her students.

**Post-lesson analysis.** Jamie’s critique of her lesson focused on how the lesson was confusing for the students. However, she anticipated that would be the case and planned to talk through the problems together as a whole group. Overall, she felt “it went okay,” but she highlighted the students needing help selecting the parts needed to solve the problems. If she were to teach the lesson again she would “teach it with word problems first because that’s probably the best way to introduce the language, and it introduces how you would set up the problem and why you set it up this way.”

Jamie felt that the lesson “kind of helped some people… but you can tell they still needed practice on it.” Although she did not provide reasons as to why she knew this, she elaborated by saying some students were okay and some were not. In addition to this evidence of the lesson’s effectiveness, she also mentioned that the lesson “was a learning experience” for the students. She felt that it provided the practice she felt students needed to “pick things out of the word problems to solve”.

Jamie’s own mathematical knowledge for teaching was used to make decisions about the lesson, such as letting it be a whole-group “discussion and letting them [the students] work on the problems together.” This is representative of her knowledge of content and students (KCS) because she anticipated the students needing to work cooperatively to
complete the problems. She mentioned that “they have done some word problems before, but not that many” so she chose to talk about it as a whole group. Despite teaching a lesson on word problems after the students already knew the procedure for multiplying fraction, Jamie felt that if she were to teach the lesson again she would teach it using word problems first, rather than teaching an algorithm first. She felt that “they’re never going to be given just two fractions that they have to multiply; that’s just not realistic, it’s not real world.” This is indicative of her knowledge of content and teaching (KCT) because Jamie realized that she would need to re-sequence the way the material was taught for her future students.

Furthermore, Jamie’s analysis of herself as the teacher was a main source for student learning. Despite the students’ struggles with using key words, as shown on the video, Jamie felt that by letting students practice as a whole group it “kind of helped some people,” and that some students began to see that “maybe those key words were not as big of a deal as maybe they have learned in the past.” Additionally, she felt that she did a good job reaching those that were having trouble because she kept an open discussion that allowed students to be comfortable asking questions.

**Beginning of First Year of Teaching**

Jamie’s first year of teaching placement was in a second-grade classroom, in a rural school district in the southeastern United States. The elementary school consisted of 84% free and reduced price lunch, and was a Title 1 school during the time of data collection. The population of students included: 41.0% Hispanic, 27.6% Black, 27.2% White, 2.4% Two or more races, 1.2% Asian, 0.4% American Indian, and 0.2% Pacific Islander.
**Visions.** At the beginning of her first year of teaching, Jamie’s vision of effective tasks involved “keeping [students’] involvement level and their attention and all that stuff. You have to do various things, don’t just sit there and do one thing because their attention level will not stay for very long.” Jamie did not focus on tasks being inherently higher or lower level; however, she said that she would start with some type of modeling and then allow students to put into action what they did together. Similar to before, she did not specify what her modeling looked like (e.g., application-based, conceptual), but she gave attention to the teacher making sure students answered questions and stayed on task. Although Jamie was stable in her mindset of “practice and apply” (level 1), she did not give a description of the types of tasks that students were practicing (level 0). Furthermore, she focused on engaging students and keeping their behavior on track more than she had during previous time points (level 0); therefore, overall, her vision of mathematical tasks was consistent with that of a level 0 on the VHQMI at the beginning of her first year of teaching.

**Implementation.** At the beginning of her first year of teaching, Jamie taught a lesson on addition of two-digit numbers. She planned the lesson alone and did not use an existing lesson plan. Instead she “just kind of thought about what they needed to know and [wanted] to keep it simple for the first day of talking about it.” She chose to use base-ten blocks for the lesson, because the students had already used one strategy for solving similar problems before, and she wanted to give them another way to solve.

Jamie’s lesson began as a whole group on the carpet at the front of the room. She asked her students “what is the only math strategy that we have talked about to ‘prove’ our answers?” The students responded with “a number line”. This led into their discussion of
recalling previous strategies they used in first grade (e.g., drawing pictures, using ten frames), and her telling them they would be using base ten blocks to ‘prove’ their work today. This piece of the lesson was not coded because the students were engaged in a discussion rather than a task.

Next, Jamie wrote the problem $16 + 23$ on the board, and asked the students to walk her through the procedure to solve using a number line, starting with 16. Once they did, she pulled out the base ten blocks and modeled how to add the two numbers using the blocks. She told the students “we’re doing the same thing as before, but just using a different strategy to prove our answer.” Jamie continued the lesson by showing the students two more examples ($22 + 16$ and $43 + 39$) and allowing the students to walk her through the process of using the base ten blocks.

Once she finished modeling the problems, the students worked in partners to solve similar problems using base-ten blocks. She gave each pair a white board to use, and they were to write the problem on their board and represent it with their blocks. She encouraged them to trade 10 ones for a rod if possible. As the students worked, Jamie walked around the room and checked their answers to see if they were correct. Most of the students stayed on task and used the blocks to model the addition problems given to them.

Towards the end of the lesson, Jamie had the students clean up and come back to the carpet as a whole group. She noticed that they were having trouble with the problem $84 + 16$ and the problem $67 + 38$. She worked these problems on the document camera with her modeling how to use the blocks to solve.
Overall, Jamie’s lesson at the beginning of her first year of teaching was coded as a level 1 on the IHQMI tasks-rubric. Despite allowing students to use manipulatives throughout, she selected problems that did not have any context. Additionally, she continued to use the “practice and apply” approach within her lesson.

Post-lesson analysis. Jamie critiqued her lesson by saying her students “responded well to it. I think they liked it, and I think they caught on to everything.” She continued by saying that her group “is kind of the higher group of students, so they generally already know how to solve this stuff, but it’s just kind of looking at it in different ways.” She further provided evidence of the lesson’s effectiveness by saying that the students were able to get the correct answers quickly and were working together.

Jamie did not know how much her students would know about addition strategies coming into her class, which is reflective of her mathematical knowledge for teaching. She said, “I kind of went in it thinking that they wouldn’t know anything about it because I didn’t know they had really done that much of it in first grade.” Jamie’s statements indicate her lack of knowledge around the vertical curriculum (VCK). However, she mentioned that if she were to teach the lesson again she would “have them present their answers to me… just some type of thing that will slow them down and have a way for me to really see the answer and get a chance for the whole group to see how the different students solve it.” She felt this would help with the students’ learning of the material, as well their own prior knowledge that they brought to the class from first grade.
End of First Year of Teaching

Visions. Jamie’s vision of mathematical tasks at the end of her first year of teaching are focused on the students being engaged and ways to keep their attention. She mentioned that,

The most effective lessons are the lessons where the students are really involved; their interest is not lost. They’re paying attention. They’re actively participating. Just anything that really keeps their interest, I feel can go the longest (sic) way. I mean, it could be a great lesson, but if they don’t find it interesting then you’re not going to get anything accomplished.

She continued by saying,

It is inevitable. I mean, it has to start out with you sitting there almost talking to them. I feel like I haven’t found a way around that. . . you can’t just magically give it to them and have it work out. [It has to be something] that they’re touching or they’re doing. . . I just try to keep things moving quick. That’s the only way I’ve found to keep their interest.

Jamie’s overall visions of mathematical tasks are consistent with that of a level 0 on the VHQMI at the end of her first year of teaching. She is focused on students’ ability to maintain interest in a topic, rather than the level of the task itself. She mentioned briefly that it is beneficial for students to “have their hands on something,” but she did not elaborate on how that played into the cognitive demand of the task.

Implementation. Jamie’s end-of-year-lesson on repeated addition and the use of arrays was planned without the use of an existing lesson plan. She came up with the idea
based upon other tasks they had done in class that kept them moving and switching things up. She wanted all students to have the chance to practice writing equations as well as draw arrays so she chose to create two stations through which they would rotate.

Jamie began her lesson by having the students work quietly at their desks on a worksheet. This first task lasted almost ten minutes and had students solving problems that were a mix of naked-number and word problems focused on addition and subtraction of two- and three-digit numbers. Once they were finished, Jamie called on students to read the problem, describe the method they used to solve, and give their final answer. She had the other students in the classroom show with a thumb up or down if they got the right answer.

After checking their worksheet, Jamie asked the students to move as a whole group to the carpet at the front of the room. This second task, which lasted ten minutes, focused on wrapping up their work on arrays so they could move into equal partitioning the next day. This meant that her lesson would be a review of things the students had already learned. She began by showing the students a picture of two rows with three stars in each. The students were then asked to write two different number sentences to represent the picture. One student came up and wrote \(2 + 2 + 2 = 6\), and another student wrote \(3 + 3 = 6\). Jamie gave two more examples to the class, a picture with four rows with two circles in each row, and another picture of an array with three rows by five columns. She asked students to come up and write two number sentences for each picture. Before sending them to work independently, she wrote the expression \(4 + 4 + 4\), and asked for a student to draw two different arrays that could match the expression.
Back at their seats, students worked on similar problems individually. This third task lasted twenty-four minutes and was structured into two centers. Jamie asked half of the class to create number sentences to represent 12 different arrays drawn on paper plates around the room. The students passed each plate along while at their desks in order to write number sentences for all 12 arrays. The other half of the class was given a list of repeated addition sentences on the board, and they were asked to draw their own array to match the equation. The students were asked to work independently, but they were allowed to whisper to a neighbor if they needed help.

Jamie wrapped up the lesson with a quick quiz that students were to complete by themselves. Although the camera did not pan into the worksheet, Jamie mentioned to the class that “you shouldn’t have any problem on this from what I saw as I was walking around the room.” This task was not coded due to not being able to see the problems that students were solving.

*Overall, Jamie’s end-of-the-first-year-of-teaching lesson was coded as a level 1 on the IHQMI task rubric.* Her first task (level 0.5) had students engaged in some application of addition and subtraction, but they mostly focused on a procedure for solving. Her second task (level 1) had students procedurally writing equations for arrays, while her third task (level 1) asked students to practice similar problems as they had done as a whole class. Although the students could write two different equations for each array or draw their own picture, they were modeling the same behavior from the beginning of class (i.e. an array with 2 rows and 3 columns could either be written as $2 + 2 + 2$ or $3 + 3$). Therefore, this was indicative of the “practice and apply” mentality on the IHQMI framework.
Post-lesson analysis. Jamie’s overall critique of her lesson was that “it went fine [because] it was more like a review-type situation [since] we’ve been working on arrays for three days now.” She mentioned that if she were to teach the lesson again, she would try to come up with more than two things for her students to practice. This is indicative of her mathematical knowledge for teaching, specifically her knowledge of content and students (KCS); however, she did not elaborate on what the other stations might be.

The evidence Jamie provided that the lesson went well and student learning took place, focused on her “walking around [the classroom] and [seeing that] they were able to get things completed quickly.” Jamie also commented on her students’ learning by saying that, “they’re just learning at this point to be a little bit more independent and to use the models and really come up with their own way of thinking.” Additionally, she felt that after she had a chance to look at the formal assessment they completed at the end of class, she would be able to see if they understood the lesson or not.

Summary of Jamie

Development of Visions. Jamie’s visions of effective tasks maintained relative consistency throughout her teacher preparation program and into her first year of teaching (Figure 11). On the VHQM1 rubric, her articulation of her vision aligned with indicators ranging from 0 to 1. Throughout the duration of her junior year, Jamie felt it was the teacher’s job to model a task for students before allowing them to work independently on it for practice. Although she was of the “practice and apply” mentality, she did not clarify what types of tasks she was working on with students (i.e. high or low level of cognitive demand).
Therefore, Jamie was placed at a level 0.5 during her junior year in her teacher preparation program.

Once finished with methods coursework and into her student teaching placement, Jamie felt that good tasks were those where the teacher modeled how to solve and then allowed the students time to practice. However, she also included in her visions that it is the teacher’s job to “create a deeper understanding for math so [students] can solve a lot of different types of problems.” This level 1 way of thinking was fleeting, as Jamie reverted back to her previous beliefs (level 0.5) by the end of her senior year.

At the beginning of her first year of teaching, Jamie briefly commented on her “practice and apply” vision; however, her primary focus was on making tasks engaging for students and keeping their attention. Therefore, she was coded at a level 0 during this time point. By the end of her first year of teaching, Jamie maintained her visions of a level 0 by focusing on students’ attention during a task and not the type of task being implemented with the students.
**Development of Implementation.** Jamie’s overall implementation of mathematical tasks varied widely throughout her teacher preparation program (from a level 0 to a level 2), with more consistency occurring during her first year of teaching (level 1). At the beginning of her junior year, Jamie’s lesson to her kindergarten class was coded as a blend of a level 1 and a level 3 on the IHQMI; therefore, overall, she was coded as a level 2 (Figure 12). She showed the ability in the lesson to engage students in a real-life context that had the potential for multiple solution paths and higher-order thinking (level 3); however, she finished the lesson with the students completing a worksheet that was procedural in nature (level 1). By the end of her junior year, her lesson to fourth graders on solving word problems and writing equations was coded as a level 1 because of the procedural nature of the lesson, despite its real-world context.
While in her fifth-grade student teaching classroom during her senior year, Jamie wavered between a level 0 and 1. At the beginning of the year, her lesson on rounding was procedural in nature and focused the students’ attention on the rules used for rounding, rather than a conceptual understanding of rounding (level 0). By the end of her senior year, she incorporated real-life contexts into her lesson on fraction multiplication. Throughout the lesson, Jamie tried to help students understand how to solve the problems using a picture; however, due to the students learning the algorithm first, they relied on their procedure and the use of key words, rather than drawing the pictures for understanding (level 1).

During Jamie’s first year of teaching, her two observed lessons were coded as a level 1. In her beginning-of-the-year lesson on addition with base-ten blocks, Jamie allowed students to use manipulatives, but she gave naked-number problems rather than those with a context. Furthermore, she modeled a procedure for using the blocks to solve the problems and then allowed students to continue solving similar problems on their own (level 1). This lesson was similar to her end-of-year lesson on addition with arrays. Although she had students connecting the equation and array representations, she chose naked-number problems and modeled a procedure for writing the equations before allowing the students to solve similar problems on their own (level 1).
Development of post-lesson analyses. Jamie’s analyses of her lessons focused on critiquing the lesson, providing evidence for her critique as to why the lesson was effective or not, examining her own mathematical knowledge for teaching, and examining students’ learning based upon the tasks provided. Below is a breakdown of each category, with her trajectory described in detail.

Critical approach. Throughout her time in the study, Jamie’s critiques of her implemented lessons tended to mostly focus on the students liking the lessons she implemented. During her junior year, Jamie critiqued her lessons by focusing on whether students were on task and if she felt prepared for the lessons. Jamie mentioned that both lessons would have gone better if she had been more familiar with what the students had been doing in class prior to her lesson. Once in her senior year, Jamie’s critiques focused on the students being confused and not getting correct answers. In her end-of-senior-year lesson,
Jamie provided a way to improve her lesson in the future by suggesting that she start with word problems first and then move into working on problems without a context. During her first year of teaching, Jamie’s critiques were focused on the students liking the lesson and how the students responded to the overall lesson. She critiqued her lesson by suggesting that she come up with other activities for the students to do, but she did not provide specifics as to what those activities might be.

**Links to evidence.** Throughout her time in the study, Jamie’s evidence of the lessons’ effectiveness focused on whether students were able to work quickly through the tasks she provided as well as obtain correct answers. During her junior year, Jamie’s evidence that her lessons were effective focused on the students providing correct answers to the problems they were given. Her focus on correct answers continued throughout her senior year and even through her first year of teaching. Jamie’s attentiveness to correct answers as a means for effectiveness were coupled with her beliefs that if students could complete the task quickly, that meant they understood it.

**Mathematical knowledge for teaching.** Throughout her post-lesson, cognitive interviews, Jamie referred to her own MKT, specifically her CCK, KCS, KCT, as well as her HCK and VCK. In her junior year, she specifically referenced instances that were coded as her knowledge of content and teaching (KCT), her horizontal curriculum knowledge (HCK), and her common content knowledge (CCK). She felt she was ill-prepared for her lessons which might have confused her students (KCT). She also felt that she did not know what her students had been doing previously in their class, which was a struggle for her due to the lack of time in their classroom and grade level (HCK). Despite her concerns, she felt she made a
good decision when she chose an additional student to come to the front of the room during her lesson on similarities and differences because it allowed her to show how the genders were different (CCK).

Throughout her senior year, Jamie continued to describe instances of her knowledge of students and teaching coupled with the mathematical content (KCS and KCT). In her decision to provide a reinforcement lesson based on test results of struggling students in her first lesson, she used her knowledge of her students as it relates to mathematical content. Additionally, Jamie’s realization of needing to restructure the way she taught multiplication of fractions (i.e. word problems first and then teaching of the algorithm) was representative of her KCT.

Finally, in her first year of teaching, Jamie focused on her lack of vertical curriculum knowledge (VCK) and her knowledge of content and students (KCS). Due to her novice status, she was unsure what her second-grade students had learned during their time in first grade (VCK). Additionally, she wanted to change the way she taught her end-of-year lesson because she felt she needed other stations for the students to complete (KCS).

**Student learning.** Throughout her time in the study, Jamie focused on student learning occurring in her lessons by attending to herself as the teacher as well as the materials she available to use. Jamie’s analysis of her students’ learning during her junior year mainly focused on herself as a teacher. She felt she was their main source of learning, by describing how she could have explained the material better, but she still helped them solve the problems correctly. Additionally, she mentioned that she observed student learning occurring
throughout the year because she noticed students getting correct answers to the problems she gave them.

In her senior year, Jamie continued to focus on herself as the source for student learning, by mentioning how she allowed students to work in groups, how she fostered discussions, and how she gave reminders of rules to help the students complete problems. Once into her first year of teaching, Jamie still focused on herself as a source of student learning by mentioning that “I was telling them a more specific way to solve…” but she also described evidence of student learning because she could visually see it on their assessments.
Chapter 5: Jordan

At the beginning of her junior year, Jordan reflected on her past experiences with mathematics. Growing up, Jordan went to a private school for grades K-8. In elementary school, she “loved math… it was my strong point.” Jordan recalled doing timed tests and drills which were “purely memorization, and I was so good at that.” In middle school, her fondness of math continued because of the use of “the plug and chug method” for formulas.

Once in high school, Jordan switched to a public school. She had a hard time making friends since most of the students already knew each other, and she did not make the volleyball team. These factors, coupled with her feeling that her teachers did not care because they did not check up on her to make sure she was doing her work, left Jordan feeling miserable. Shortly after, she switched to a smaller new-to-the-area high school, and it made her experiences much more enjoyable. She liked math until she enrolled in calculus because she found it “really difficult.”

In college, Jordan had “fairly positive experiences”, but felt that some of the classes were really large and were hard for her to focus in them. She enjoyed her methods courses and began to have a “stronger liking towards [math] again” because she was excited to teach students about math. She expressed looking forward to being the type of teacher that takes an interest in her students’ lives and wanted to teach abroad someday.

Jordan is representative of a case who decreased in her mathematical knowledge for teaching from the beginning of her junior year in the teacher preparation program to the end of the program in her senior year (Figure 13). Although she decreased one standard deviation, she remained above the cohort average by more than a half standard deviation. As
Jordan moved through teacher preparation and into her first year of teaching, her vision and implementation of mathematical tasks, as well as her post-lesson analyses, were documented. Her developmental trajectories are presented below.

*Figure 13.* Jordan’s change in MKT from beginning to end of the teacher preparation program.

**Beginning of Junior Year**

**Visions.** At the beginning of her junior year, Jordan discussed an effective lesson focused on subitizing because she was learning about it in her methods courses. She stated the lesson would be effective because it allowed students to understand numbers by seeing a “picture representation [and a] symbolic number.” Jordan indicated this as important because it shows students how to break apart numbers differently and make connections “to the underlying principle of what [a number] is” (level 2). Jordan continued by saying that students’ engagement with tasks should have a “conceptual understanding” of a topic (level
2), but she also placed emphasis on different activities throughout a lesson in order to keep students engaged (level 0).

*These beliefs placed Jordan’s vision of mathematical tasks as a level 1.5 on the VHQMI framework at the beginning of her first year of teaching.* Her vision of the “engaging” feature of having multiple tasks was coded as a level 0 because it did not describe the cognitive demand of the task. However, her visions of tasks being conceptual and connected to underlying principles were coded as a level 2.

**Implementation.** Jordan’s beginning-of-junior-year lesson was taught to a second-grade class involving partners of ten. She mentioned that the students had experienced “a lot of practice with it, so [the lesson] ended up being more of a review.” Following the recommendation of her cooperating teacher, Jordan used a lesson from the district’s curriculum guide, but she made modifications to the lesson. For one modification, she chose to not use the context of the problem given because she felt “it was not very engaging.” Instead, Jordan chose to make the scenario about the students in the group and their siblings. Additionally, she chose to modify the lesson by asking the students to give the partner of a number, for example “what is the partner of seven that makes ten?”

Jordan’s lesson was taught to a small group of students around a table and with each student having an individual whiteboard. Her first task, lasted roughly eight minutes, and began by asking students “how many of you have 1 sibling?” Several students raised their hands, and she called on one student to ask his name and his sibling’s name. The student, Matthew, had a sister named Caroline, who they were going to use for the class example. Jordan asked the students to write “Matthew” on one side of their boards and “Caroline” on
the other with a line down the middle. Next, she asked the students to “imagine that Matthew’s mom bought a bag of his favorite cookies… and they split the cookies up”. On the board, Jordan wrote that Matthew received nine cookies, and Caroline only received one cookie. She then asked, “are there any other ways to break apart the ten cookies?” The students gave several partners that made ten; however, they did not explain their thinking about how they knew the pair made ten.

Next, Jordan changed the focus of the lesson to her second task, which lasted five minutes, by asking the students what patterns they noticed about the list of “break-aparts” that they created as a group. Despite the high-level nature of the task, Jordan followed up her question by asking the students “which break-aparts have the same partners, but just in different orders?” This lowered the cognitive demand of the task because the students were off-task, had to be redirected for roughly three minutes, and were doing something procedural in nature by reading to her what they saw on the board.

For the next eleven minutes, Jordan’s third task required students to provide several quick facts such as “what is a partner of 7?” or “what is a partner of 2?”. One student described to her “math mountains” that they did in first grade, and she asked the students to draw a math mountain with 10 at the top and its partners at the bottom of the mountain. She gave them several facts to practice on their own white boards, and students completed the task easily and quickly.

The fourth and final task of the lesson lasted seven minutes, and asked that the students work with a partner to create other “break-aparts” on their board. During this time,
many of the students were off-task and caused disruptions. Jordan tried to redirect their attention back to the lesson until the end of the video.

*Overall, Jordan’s lesson at the beginning of her junior year was coded as a level 0.5 on the IHQMI tasks rubric.* Despite her attempts to use a real-world context for the first eight minutes of her lesson, the remaining 26 minutes were spent at a level 0. Although this was a review lesson for the students, they were consistently engaged in memorization of the pairs that make 10 and did not engage in higher-order thinking except during the short duration of identifying patterns in the cookie problem scenario.

**Post-lesson analysis.** Although she felt the lesson “went well”, Jordan *critiqued* her lesson by focusing on her own use of language with her students. She mentioned that “it was just awkward, and I [was] not practicing good math language when I was asking them questions.” She continued by saying that “I wasn’t comfortable with the language so I felt like I needed to be more comfortable saying break apart to make ten and partners so that they would get the most repetition of hearing the two interchanged.” This, coupled with wishing that she would have “elaborated a little bit more… 9 + 1 and 1 + 9 are not the same equation” were indicative of her specialized content knowledge (SCK) when she described her *mathematical knowledge for teaching* throughout her post-cognitive interview. Additionally, she mentioned her knowledge of content and students (KCS) by saying that if she were to redo the lesson, she would have the students hold up their white boards with their math mountains because she felt that not all students had a chance to think through the problems before someone would shout out an answer.
In terms of the *evidence* that she provided for the lessons’ effectiveness, Jordan mentioned that it went well because of the way the students responded. When describing how she wrote the students’ partners of ten in order on the board (i.e. $9 + 1$, $8 + 2$, $7 + 3$, etc.) she said:

When I asked them about partners they were able to tell me that while the first side [first addend or “partner”] was going down, counting down from ten, the second one [partner] was counting up. So that kind of meant they understood the whole internal process of adding to one side and subtracting from the other.

She continued by saying that she noticed their correct answers on their whiteboards; therefore, she knew they were understanding the material. Jordan’s evidence for the lesson’s effectiveness also provided evidence of *student learning*; however, Jordan further elaborated on student learning by saying that she knew learning took place because “they [the students] learned different partners of ten and I know that because when I was able to ask them orally or give them one of the partners and ask for the other ones.”

**End of Junior Year**

**Visions.** Jordan’s visions at the end of her junior year were focused on having a task that was “high in cognitive demand for the students because that means that the students have to be engaged in the task that they are completing… and there’s the appropriate level of challenge there.” She continued by saying that students should be “actively engaged with the material and asking questions” throughout a lesson. Although she wanted students to be engaged, her visions were consistent with a level 0 because she was not focused on the task itself, but rather keeping the students’ interest in the lesson. However, Jordan continued in
her interview by saying that she believes students “should be actively exploring the mathematical concepts through representations or any manipulatives, and the teacher should be walking around and fostering the discourse and asking questions” (level 2). This, paired with her beliefs of having “high in cognitive demand tasks” and tasks that are hands-on and involve “higher-order thinking” (level 2), are representative of a level 2, which focuses on reform-oriented lessons. Therefore, her overall level on the task rubric of the VHQM at the end of her junior year was a 1.5.

**Implementation.** Jordan’s lesson at the end of her junior year was taught to a group of fifth-grade students who were exploring volume. She used an existing lesson plan given to her by her cooperating teacher to plan a lesson with her field placement partner, Charlie (also a participant in this study). In describing the goal of her lesson, she mentioned, “the main goal of the lesson wasn’t to actually teach. It was for the students to have time to explore with the multi-link cubes and construct [rectangular prisms].” She commented that her focus of the lesson was to allow students to build an understanding of volume, since they had not worked with it before, and relate it to the area of the base of the prism.

Jordan began her first task, which lasted fifteen minutes, by asking one student to come to the board to record responses, while she walked around the room and asked students about their response to “what is volume?” This discussion led the class into their task for the day, which was to explore volume by creating rectangular prisms out of multi-link cubes. Jordan asked the students to work in groups around the classroom to create prisms with either 12 or 36 cubes. Some students knew that the volume could be determined by multiplying the length times the width times the height, and this allowed some groups to create their shapes
quickly. However, other students focused on the factors of 12 or 36 to help them find all of
the prisms.

Once back at their seats, Jordan’s second task asked students to select a prism for
their group to share with the class. Jordan and her class spent the next fifteen minutes
discussing how length, width, and height could be interchanged, and how multiplying the
dimensions resulted in the number of cubes used to create the prism. Additionally, Jordan
discussed how to write volume with proper notation and how knowing the area of the prism’s
base and the prism’s height allowed them to find the volume of the prism. She then passed
out their daily math worksheet (not coded) and asked the students to work individually.

*Overall, Jordan’s task with the multi-link cubes coupled with her discussion placed
her at a level 3 for implementation on the IHQMI task rubric at the end of her junior year.*
The tasks required the students to use multiple solution paths to create the prisms out of
cubes, engaged students in higher-level thinking, and the cubes were used for more than just
keeping the students’ interest in the lesson. Although the lesson was chunked into two
different tasks, they worked together to give the lesson its overall level.

**Post-lesson analysis.** Jordan’s major *critique* of her end-of-junior-year lesson was
that she introduced the algorithm for volume too soon. She mentioned that “I just think I
introduced it too soon and didn’t allow them enough time to really just work with the
manipulatives before trying to connect the meaning.” However, she believed that giving her
students the opportunity to work with multi-link cubes went well and “they learned that just
because two figures can have the same volume they don’t look the same.” She continued to
provide *evidence* that it went well by saying “I know this [that it went well] because their
homework (which she saw after teaching the lesson) was to write what they have learned with regard to volume and how their understanding of volume has been furthered through the [multi-link] cubes.”

Jordan also felt that student learning took place because “the students had time to play around with the cubes and build without any direction.” She mentioned that:

I don’t think you could hear with the video, but one girl was talking and said that they were missing one of their combinations for 36 and she’s like ‘okay, well let’s think about our facts, so we have the 1, we have the 2, we have 3…’ so she was really thinking about dividing the 36 and what could she divide it by to make that the base.

Her understanding was definitely much further than a lot of the students.

Additionally, Jordan elaborated on student learning occurring as a result of her own mathematical knowledge for teaching as shown through her specialized content knowledge (SCK). She mentioned that “once I talked to them about the base and how the base connected to the area, the length times width, then they started thinking about that as they built.” This provides evidence of her SCK because in order to understand where the volume formula comes from, one must have an understanding of volume as “area of the base” times the height of a prism.

**Beginning of Senior Year**

**Visions.** At the beginning of her senior year, Jordan described a lesson from her junior year in which students used a number line and felt it was effective because “it was getting them [the students] to see a more visual representation.” Although she did not describe the number line lesson in detail, Jordan stated that it allowed students to see and
verbalize their mistakes. Jordan also commented on the effectiveness of hands-on manipulatives for lessons, but she did not describe why they were effective.

Before her methods courses, Jordan felt that she just focused on “remembering all the cute things that your teachers did like art projects, and now my vision is doing everything with a purpose.” Jordan described her changing vision by saying that “I guess my vision has changed more into wanting to teach through inquiry and actually having students be engaged in the material.” Furthermore, she wanted students to be “using manipulatives and other visual models with a purpose, and showing students conceptual reasons or creating that conceptual understanding of why they’re doing something, not just a procedure.” Additionally, Jordan wanted to come up with tasks that will help “construct student’s knowledge of concepts and get them that conceptual understanding, not just a page of problems where you’re performing algorithms.” Overall, Jordan’s visions at the beginning of her senior year align with a level 2 on the VHQMI tasks rubric. Jordan is focused on reform-oriented teaching practices (i.e. inquiry based, hands on, for conceptual understanding), but she does not focus on the types of tasks and how those will help her teach conceptually (i.e., she does not focus on the open-endedness of a task, allowing for multiple solution methods, etc.).

Implementation. Jordan began her senior year in a fourth-grade classroom teaching a lesson on story problems involving addition and subtraction. As part of her teacher preparation program’s requirements, she created an assessment for her students that had two problems that used the standard algorithm and two story problems. Her analysis of the
assessment was that her students did the algorithms correctly; however, some struggled with the wording of the story problems.

Jordan designed the lesson based on her assessment, originally for six students who were ELL, but she ended up teaching it to the whole class. Due to this last-minute change, it caused some engagement issues for those who had mastered the topic. Despite these changes, Jordan taught the lesson focusing on story problems and the language used within.

The first task, which lasted four minutes, began with the students working in groups with individual white boards. Jordan put the problem “John had ___ Skittles and some M&Ms. He had ___ more M&Ms than Skittles. How many M&Ms does he have?” She specifically did not put numbers in the problem so that students would have to use arbitrary red and blue strips (i.e. bar models) to represent the quantities in the problem to show which had more. She also chose to not use numbers so students would focus on the words and less on quantities.

For the remainder of the lesson, approximately 17 minutes, Jordan’s second task included a variety of story problems. She placed a problem on the white board at the front of the classroom, and similar to before, the students solved using their strips and white boards. For example, the students solved “a zoo cares for 27 lions and 16 tigers. How many fewer tigers than lions are at the zoo?” The students were asked to use their strips to decide which represented tigers and which represented the lions. Once they knew which one should have more, they were asked to solve the problem. Between each word problem, Jordan gave her students time to solve and work together, and then come up to the board and present their
solutions. Most of the solutions were presented procedurally, and they quickly discussed the procedure before moving on to the next problem.

Overall, Jordan’s beginning-of-the-senior-year lesson was coded at a level 2 on the IHQMI tasks rubric. Her first task (level 4) focused on understanding the word-problem and sense-making rather than on procedural mathematics. Her second task (level 2) focused her students’ thinking on real-world problems in order for them to understand the problems themselves, and the students were able to use other strategies to solve; however, she only encouraged the use of the standard algorithm along with the strips.

Post-lesson analysis. Although a critique, Jordan’s analysis of her beginning-of-the-senior-year lesson also provided insight into her mathematical knowledge for teaching by focusing on her knowledge of content and teaching (KCT). Jordan commented that, “I think the [bar models] would have been appropriate if it were for a smaller group, but I don’t think I made the connection between the bar models… I just don’t think I went as far as I could have.” She further provided evidence of the lesson’s effectiveness and her knowledge of content and students (KCS) by saying “some students, when I walked around, I had to really push them to talk about their bar model… and I knew that could have been the students who said they had used bar models before…” However, she continued by saying that “some students who at the beginning were holding up the wrong bars were grasping it a lot better towards the end when they were able to talk with their group and with the closing question in their journal.”

Additionally, Jordan analyzed the student learning that occurred by saying “I think they learned more about thinking logically through a problem because whenever I walked
around in the group I could hear them talking about using the bar models and talking about ‘well, we said that this value has to be the bigger one’.” This showed Jordan that her students were focused on creating their own learning through exploring the bar models.

**End of Senior Year**

**Visions.** At the end of her senior year, Jordan’s emphasized several times throughout her interview the need for lessons to be hands-on and engaging for students. She commented on the same lesson as she did at the beginning of her senior year regarding the number line, and her feelings of its effectiveness. This time, she described the lesson by saying that during the lesson a number line was placed on the floor for students to understand that the space in between numbers was what they were counting, not the actual numbers themselves. She felt this was a good lesson because the teacher did not have the students just “working through a ton of procedural problems.” Instead, she liked that the students would have “two or three [problems] where they are really able to have all those conversations and make sure they are really working as mathematicians and explaining their thinking, talking it out, showing multiple representations…”

Jordan continued by saying that for some students with shorter attention spans, having “something that is completely hands-on for the entire class where they are essentially learning the lesson and having practice opportunities” is effective because they are unable to focus for long stretches of time. She stated:

having activities that are engaging for the students are going to be the ones that allow students to either have some kind of hands-on practice or the higher order thinking
questions that they are grappling with like investigation stations. Not just standing in
front of the room and teaching a lesson.

*Overall, Jordan’s reform-oriented beliefs are what placed her at a level 2 in terms of her
vision of high-quality mathematical tasks at the end of her senior year.*

**Implementation.** Jordan’s lesson at the end of her senior year was taught to the same
fourth-grade class, but this time focused on comparing fractions with different-size wholes.
She planned for mostly group work throughout the lesson “because if I just stand up there
and instruct, they’re just not having it. They talk.” Jordan used a pre-existing lesson plan
from her county’s adopted curriculum with minimal adaptations. She felt the intended lesson
was good because it was “modeling first and then the students go off and work on their own
and then come back together;” therefore, little change was needed to the lesson. She only
decided to add in a bit of review of the previous topics in the beginning to refresh the
students’ memories before they began the lesson.

The first task began with a quick four-minute review on how to compare fractions
using the symbols for greater than, less than, and equal. Jordan put two questions on the
overhead and asked the students to discuss within their groups: “How do you compare
fractions with the same numerator but different denominators?” and “How do you compare
fractions with different numerators but the same denominators?” After allowing the students
a minute to talk in their groups, they discussed as a whole group and provided examples for
their explanations.

For the next ten minutes, Jordan’s second task involved two story problems that
involved objects that were different sizes. She asked the students to compare the fractions of
different-size wholes for the two problems on the board. For example, one problem read: “Amelia and her five friends are making sandwiches with two different loaves of bread. One with a short loaf and another with a longer loaf. Amelia cuts each sandwich into six pieces, but her friends claim the pieces are not the same size from each loaf. Are her friends correct?” Jordan asked the students if they agreed and encouraged them to explain their thinking when giving an answer to the class. Jordan also challenged their thinking with one question by asking the students to think about how the friends in the problem could get the same size amount given that the bread loaves were different sizes to begin with. The students came up with a variety of answers and solutions to her challenge question.

Jordan’s third task began with a brief discussion as she was sending students off to work on an activity sheet. Jordan drew her students’ attention to one of the problems and quickly explained how they should not have to calculate anything in the problem if they were to just look at the wholes instead. The remainder of the class, ten minutes, was spent working on problems similar to what they completed as a whole group. After showing the students working on the problems with their group, the camera shut off and came back on when the students were presenting their solutions on the board. The students used a variety of methods to solve (i.e. pictures, symbols), but Jordan described their solutions instead of allowing the students to do so themselves.

Finally, for the last three minutes of class, Jordan wrapped up the lesson with a discussion of what they would do the next day in class (not coded). She drew a number line on the board and asked students to place the “1/2s” on the board. She then asked where the “1/4s” are, and why \( \frac{1}{4} \) is between 0 and \( \frac{1}{2} \). One student responded with how the line is now
broken into four equal pieces and that $2/4$ has the same value as $\frac{1}{2}$. Jordan agreed with the student, and described how they were going to play a game the next day to see if they could find where certain numbers lie on the number line.

*Overall, Jordan was coded as a level 2 on the IHQMI tasks rubric at the end of her senior year.* She had potential throughout the lesson for a level 3 because the tasks required the students to use higher-order thinking. However, she repetitively reduced the cognitive demand of the task, placing her implementation at a level 2 for most tasks.

**Post-lesson analysis.** Jordan’s *critique* of her lesson focused on how she wished she would have implemented her visual representations more effectively. She commented that “now that I’m further in the lessons I feel like drawing it out wasn’t a bad thing then… you should always be reinforcing visual modeling… [because] pictures are so concrete.” This critique of her own lack of experience was more so with the unit itself and how she did not know what to expect later on in her classroom, which is representative of her lack of horizontal curriculum knowledge (HCK) as part of her *mathematical knowledge for teaching*.

Despite her critiques, Jordan believed the lesson was effective and provided *evidence* by saying that the two students who shared their models were unusual because they are never ones to volunteer or participate. She also commented on her group getting correct answers by saying that they were “able to make comparisons, think about the size of parts, and compare either denominators or the numerators being the same.” Furthermore, she also indicated that the real-life, multi-step problems “were appropriate because they first did a guided practice problem with me and then went into their partner work… the questions were realistic like they were going to be working with later… and they had them doing more than one step.”
Although Jordan focused on student learning through the evidence she provided of the lesson’s effectiveness, she also analyzed the resources used throughout the lesson. She felt the resources she had available were beneficial to student learning because they were appropriate, and forced the students to look at visual representations “instead of trying to get numbers in there too quickly.” She focused on some students who used numbers and calculated which fractions were more or less, and saw them observing their peers looking at the size of the parts and wholes to obtain the same answer. She also stated that through her own instruction students were able to focus on the size of the wholes rather than the numbers, and this enforced their own learning.

**Beginning of First Year of Teaching**

Jordan began her first year of teaching in a fifth-grade classroom in a public charter school in a city in the southeastern United States. Her non-Title 1 school consisted of almost 650 students, 10% of which had free or reduced-priced lunches. The population of students was comprised of: 51.8% White, 14.7% Asian, 11.5% Black, 9.7% who were two or more races, 6.7% Pacific Islander, 5.5% Hispanic, and 0.2% American Indian.

**Visions.** At the beginning of her first year of teaching, Jordan described an effective lesson as one where students are “collaborating to solve problems, using different methods to solve them, and showing each other their methods.” These visions are aligned with level 3 descriptors on the VHQMI rubric. However, when Jordan gave a specific example of a task involving addition, she mentioned that students would have manipulatives to represent the problem that they have and the “word problems and language would match up.” She also felt that “students need to be able to do more than just on a sheet of paper like a fact test… they
need to be able to read a real-life scenario or word problem and create the equation themselves.” She also wanted her students to be engaged, so she envisioned the teacher as someone who finds a lesson where “the students are engaged, and where the students are going to grow and get some sort of personal connection to it.” These visions are mostly a level 2 (her vision of engagement a level 0), because they focus on using hands-on tools and real-life scenarios which are indicative of reform-oriented lessons; however, she did not describe why these types of lessons are important or how she could use them to push students to higher levels of thinking. Therefore, overall at the beginning of her first year of teaching, Jordan was coded as a level 2.5 on the VHQMI tasks rubric.

Implementation. Jordan’s beginning-of-the-year lesson was a review lesson on division. Although her school administration wanted her grade level to stay at the same pace, she indicated that she was beginning to move ahead of the other teachers. For this lesson, she wanted her students to review whole-number division and what it meant to divide before they moved on to division of decimals the following week. Jordan did not use an existing lesson to plan for the review lesson, but rather she found a variety of resources online to use to supplement the lesson (e.g. videos on regrouping, virtual manipulatives). She recalled the focus in her teacher preparation program to make sure students conceptualize the mathematics they were learning, the reason why she wanted to use blocks with her students to help them understand the meaning of division.

She began the lesson by asking her students to discuss with their small groups “how does multiplication help us when dividing?”. She called on students after a few minutes to discuss what their own group had talked about. One student compared multiplication and
division to addition and subtraction and how they are “opposites”. She then asked the students to discuss “what are we doing when we divide? For example, if we know 8 groups of 7 is equal to 56, and we’re making groups, then what are we doing when we divide?”

After more discussion, one student explained by saying “56 divided by 8 groups means there are 7 in each group.” This eight-minute discussion led the class into their task for the day (not coded).

For the next 14 minutes of class, Jordan’s first task showed her students how to use online base-ten blocks to model division with 2-digit divisors. She also had physical base-ten blocks for the students to use as well. Together, they walked through a problem of dividing 299 by 23 using the base-ten blocks to solve.

The remainder of class, approximately 1 hour, was spent in various centers throughout the classroom. The first center, Jordan gave the students four division problems, without a context, for the students to solve using the base-ten blocks (either physically or virtually). She asked them to work with a partner in their group and use the blocks to solve the problems. The overall task at this center required the use of hands-on tools for solving naked-number problems.

The second center involved an ongoing class project in which the students had to design a playground and come up with a budget to use. She asked the students to come up with an estimated cost, actual cost, and a cost after tax. This center was higher in cognitive demand because it asked the students to design their own playground as well as their own budget.
The third and final center, which was not coded, involved a computer station in which the students were asked to complete games and problems using a math software. Jordan did not describe this center, as most students seemed to understand their task, and the camera did not focus on the computer screens.

*Overall, Jordan’s ninety-minute lesson at the beginning of her first year of teaching was coded as a 2.5 on the IHQMI tasks rubric.* Jordan’s first task (level 2) used manipulatives to solve her division problem posed to the class, while her center rotations included a variety of tasks of different levels of cognitive demand. One center had students focused on a task that did not have a prescriptive method for solving (playground problem – level 4), while one center had students engaged in hands-on manipulatives with naked-number problems (level 1.5).

**Post-lesson analysis.** Jordan was very *critical* of this lesson at the beginning of her first year of teaching. She felt she was holding her students back because she was using “filler projects while [she waited] for the other teachers to get where we are.” She also felt that it was hard to show division using two-digit divisors because of all the groups you had to represent. She went on to state that she assumed virtual manipulatives would be better, but once I got there and started doing it I didn’t think it was effective because they didn’t see the 23 groups. I don’t know how I would have done it better, maybe getting 23 kids to stand up and hand them actual manipulatives would have worked better so then they could have visualized the 23 groups.
Jordan felt her *evidence* of the effectiveness of the lesson was lacking, despite feeling that “it went well”. She stated that:

I don’t have that much evidence after today since I didn’t walk around as much as I thought, but I did when we were in whole group, I was walking around and talked to their table groups… so I guess I have that as evidence.

She mentioned more specifically that when she walked around she heard students “using words such as regrouping or creating equal groups.” However, she felt she would have had “more evidence if [she] were able to give an exit ticket, but [they] got started on centers so late.”

In terms of her own *mathematical knowledge for teaching*, Jordan referred to her choice of activities in the centers which was representative of her knowledge of content and teaching (because she chose tasks that allowed students to dig deeper into the material) as well as her knowledge of content and students (because she chose certain tasks due to the students who needed challenges or were struggling). She mentioned that she chose to use the computer activity because she has “students who are at different levels in the class,” and the computer lesson “makes sure that they are getting those background skills mastered.” Her Playground Project was chosen because it, “incorporated three or four previously worked-on skills to see how all of those would fit together in a real-life situation.” Finally, the independent work was chosen because she felt most students would want to use the standard algorithm to divide the numbers, but she “chose the manipulatives to show regrouping.”

Finally, Jordan analyzed her students’ learning by focusing on the resources she used as a means for learning. She mentioned that the computer activity gave her students a pretest
and placed them on a pathway that allowed for “learning of different things.” Jordan also mentioned that the students helped further their own learning because they:

learned that they can regroup to make equal groupings when they divide and I think some of them learned that when you divide you’re making equal groups, because it seemed like at first when we began the lesson that not all of them knew exactly what division is.

Furthermore, in the Playground Project, Jordan felt that students “learned about budgeting money… they practiced skills that I’ve already taught: rounding decimals, rounding whole numbers, multiplying to get tax, adding and subtracting decimals…” Despite not all of the topics being covered during the day’s lesson, the students had been working through each of the topics over the course of several days.

**End of First Year of Teaching**

After approximately one quarter of the school year, Jordan resigned from her fifth-grade position at the public charter school. A few weeks later, she accepted a position as a fourth-grade teacher at a public elementary school in the same geographical area and spent the remainder of the school year at that school. The school consisted of almost 600 students, 88% of which received free or reduced-priced lunch, making the school eligible for Title I funding. The population of students was comprised of: 54.0% Black, 34.5% Hispanic, 8.0% White, 2.3% that were of two or more races, 0.8% Asian, and 0.3% American Indian.

**Visions.** At the end of her first year of teaching, *Jordan’s vision of high-quality mathematical tasks was coded as a level 2.5 on the VHQMI tasks rubric.* Her vision of tasks was coded as a level 2 because she thought students should be using various strategies to
solve problems so “they can do it in several ways, then you know you’ve been effective in teaching it because they understand what they’re doing not just doing a procedure.” This is a level 2 because it shows the need to not just learn a procedure. She also shared descriptions of her vision of tasks that are consistent with a level 3; she stated that students should be “collaborating to work out problems [and] using good math talk to guide their conversation.” She also said the teacher should find tasks that are “higher thinking questions where students are going to be stretching their limits.”

Implementation. Jordan’s end-of-year lesson focused on a review of fractions using math centers. She had been trying to incorporate centers into her math lessons more at her new school; however, this was the third time they were able to try them. She planned the lesson by trying to “incorporate things we’ve done in the past, and not isolate the math units so much.” The lesson was compiled from resources she found on the internet, as well as story problems she made up for them to solve. She chose to focus on equivalent fractions and story problems “because they were two of the big units,” but she also included addition, subtraction, multiplication, mixed numbers, and improper fractions because some students still struggled with those concepts when they used the standard algorithm.

The lesson began with the students in one of five centers throughout the room. Every fifteen minutes they switched into a new group. The first group worked on cards with QR codes that asked them to “write the missing number to make an equivalent fraction.” Jordan asked the students to fill in the blanks using their knowledge of multiplication facts or to cross multiply to find the answer. They could then check their work using the QR code provided on the card. Some students struggled because they did not know their multiplication
facts as quick as other students; however, some students did not feel challenged as one student exclaimed “I need something more challenging.”

The second center had students working at the board to complete several story problems that involved fractions, including mixed numbers. For example, one problem asked students to solve “Quaneisha needs 8 ¼ yards of ribbon for a craft project. She has 3 ¾ yards. How many more yards does she need?” The students worked together to solve the problem on the board and Jordan suggested they use a picture to help them.

The third center involved students completing a worksheet on decimals. The directions asked the students to “write the decimal and the fraction shown by each square” and included decimals with tenths and hundredths. A picture of a flat with 100 boxes was shaded to represent a certain fractional amount, and the students were to write the decimal and fraction it represented.

The fourth center asked students to complete a worksheet involving improper fractions and mixed numbers. The worksheet’s directions were to cut out tiles and glue them into the box with the equal improper fraction or mixed number. For example, if the number 2 ¾ was shown, the student had to find the number 11/4 and glue it next to the corresponding mixed number.

The fifth, and final center, worked on cards similar to the ones with the QR codes, except they required the students to solve story problems. For example, one card asked the students to solve “A ticket booth at the carnival sold 1/10 of the tickets at 9am, another 3/10 of the tickets at 10am, and then 2/10 of the tickets at 11am. How many of the tickets are there left after the 11am ticket sales?”.
Before ending the lesson, Jordan asked the students to complete an exit ticket in which they were to: 1) make an equivalent fraction for \( \frac{4}{6} = \frac{12}{?} \); 2) add using a method of your choice: \( 1\frac{5}{10} + 4\frac{9}{10} \); 3) subtract using a method of your choice: \( 5\frac{1}{5} - 2\frac{4}{5} \); 4) \( \frac{13}{4} \) = make into a mixed number; and 5) \( 5 \times 2\frac{1}{2} = ? \). Although students could use a variety of strategies for the problems, the problems were lacking a context.

**Overall, Jordan was coded as a level 1 at the end of her first year of teaching on the IHQMI tasks rubric.** She had five stations and an exit ticket that lasted similar lengths of time during the lesson and each was coded as a level 0, 3, 1, 0, 2, and 1, respectively. Her first, third, and fourth centers as well as her exit ticket were procedural in nature, while her second task had students solving real-world scenarios as well as discussing and critiquing each other’s work. Her fifth center was similar in that it had students using a real-world scenario to solve a problem but it did not require them to engage in discussions of the problem.

**Post-lesson analysis.** Jordan’s main critique for her end-of-year lesson focused on the time it took to complete the centers. She commented that “it takes so much time going over every center when I felt like it’s all review stuff… but I guess reviewing a few things especially with drawing helped at the beginning.” She also critiqued the format of her groups and mentioned that if she were to do the centers again, she would give her students a way to organize their work better. Despite feeling good about the lesson, Jordan commented that “it’s still not where I would want them to be… there were a lot of off-task conversations, but it went better than the other two times so I’m not like it’s a total fail.”
Jordan’s evidence for her lesson’s effectiveness was analyzed through her mention of her students’ “making some gains where there were gaps before”. She noticed the students helped each other see other ways to solve the problems by making connections between their drawings and other students’ algorithms, and vice versa. Although she saw individual students making gains in their learning, she commented that “I’m not sure as a whole group they really learned anything… because it was review.” She mainly wanted the students to “review and take all of their knowledge that they’ve learned about fractions and apply it in different situations.” She felt that her encouragement to draw pictures helped students to “put themselves into the situation” and make a connection between the picture and the algorithm. However, Jordan recognized the students lost steam, and behavioral issues increased as the lesson went along.

In terms of her mathematical knowledge for teaching, Jordan focused on her reason for choosing each of the centers the students completed. Similar to her beginning-of-the-year lesson, Jordan used her knowledge of content and students as well as her knowledge of content and teaching to select appropriate tasks for her students. She chose equivalent fractions and story problems because “they were two of the big units, but then I selected and just mixed up the multiplication, subtraction, and addition altogether.” She chose to include the mixed numbers and improper fraction center because “if they were doing the standard algorithm way and weren’t drawing it out, they still struggled with converting an improper fraction to a mixed number. So, I was hoping that it would help a little bit.” Finally, she chose to put the story problems on the white board because she felt they would struggle the most with those, and she wanted to be able to see what they were doing as they worked
through the problems. She originally did not plan for an exit ticket, but due to the behavioral issues and students losing focus in their stations, she decided to include it to assess their understanding.

**Summary of Jordan**

**Development of Visions.** Throughout her three years in the study, Jordan’s visions of high-quality mathematical tasks increased from a level 1.5 to a level 2.5. During her junior year, Jordan had visions that wavered between a level 0 and a level 2 on the VHQMI (Figure 14) for mathematical tasks. She felt that the tasks should be engaging for students, which is indicative of a level 0 because she does not describe the task itself as being “inherently higher or lower quality.” However, she did describe important features of tasks as making sure students have a “conceptual understanding and not just take everything at face value”, as well as making sure students are “actively exploring the mathematical concepts… and making sure you have appropriate representations.” These qualities are representative of a level 2 because she is focused on higher-order thinking and hands-on activities.

Once in her senior year, Jordan consistently aligned with a level 2 on the VHQMI because she described effective lessons as “having some kind of hands-on practice or higher-order thinking questions that they’re grappling with, or like investigations stations. Not just standing in front of the room and teaching a lesson.” She did not want the students “working through a ton of procedural problems,” but rather felt it was more effective to “have two or three where they are really able to have all those conversations and make sure they are really working as mathematicians and explaining their thinking, talking it out, showing multiple representations…” In her description of an effective lesson she had recently seen in practice,
she described a lesson involving a number line that allowed students to have a visual representation which she felt was effective. She also felt the lesson was effective because it allowed them to see and verbalize their mistakes.

By the end of her first year of teaching, Jordan’s vision of mathematical tasks increased slightly to a level 2.5 on the VHQMI rubric, despite changing schools mid-year. Jordan felt that an effective lesson had the students “collaborating to solve problems and use different methods to solve them and show each other their methods.” She also felt that real-life scenarios were beneficial to students to help them make a personal connection to the problem, which helped engage students in the mathematics they were learning. Additionally, Jordan felt that students should have the opportunity to engage in higher-order thinking problems that required them to “stretch their limits.”

*Figure 14.* Jordan’s visions of mathematical tasks.
**Development of Implementation.** Throughout her time in the study, Jordan’s implementation of mathematical tasks wavered between a level 0.5 and a level 3, with most of her tasks being coded as a level 2. During her junior year, Jordan enacted two lessons, one with a 2nd grade class involving making partners of 10 and one with a 5th grade class involving volume of rectangular prisms. During her beginning-of-the-year lesson, Jordan began with real-world context that engaged her students in thinking about all the ways to make 10 and the patterns they noticed in the break-aparts they made (level 2-3). However, once this short segment of the lesson was complete, the cognitive demand of the lesson dropped, and the students spent the remainder of the lesson following a procedure or using previously memorized knowledge (level 0). Despite her few minutes at a level 2 and 3, her lesson was mostly coded as a level 0; therefore, this placed her at a level 0.5 overall. At the end of the year, Jordan allowed her 5th grade class to explore volume using multi-link cubes. This lesson allowed for multiple solution paths to find the number of rectangular prisms for the given number of cubes, and students were engaged in discussions throughout. Therefore, it was coded as a level 3 on the IHQMI.

Despite the inconsistency in her junior year, once in her 4th grade student-teaching placement, Jordan consistently enacted two level-2 lessons (Figure 15). At the beginning of the year, Jordan taught a lesson involving story problems and comparison of numbers. Despite moments of allowing for higher-order thinking and sense-making (level 3), her lesson was coded as a level 2 because she focused on real-world problems but encouraged one approach to solve. At the end of her senior year, her lesson on comparing fractions had potential to be a level 3; however, she reduced the cognitive demand of the tasks by not
engaging students in meaningful discussions of the ideas being presented in class. Jordan asked the students to solve real-world problems and tried to focus on making sense of the fractions they were comparing (level 2).

During her first year of teaching, Jordan switched elementary schools and moved from teaching 5th grade to 4th grade. During her observed beginning-of-the-year lesson, Jordan engaged her students in a review lesson on division. She broke the class into three centers in which they rotated throughout the lesson. The centers ranged from a level 2 to a level 3 on the IHQMI, therefore her overall level was a 2.5 at the beginning of the year. At the end of the year, Jordan implemented centers with her students once again; however, this time she chose to do five centers for a review lesson with her students. The centers ranged from a level 0 to a level 3, although most were a level 0. This placed Jordan at a level 1 on the IHQMI at the end of her first year of teaching.

![Jordan's Implementation of Mathematical Tasks](image)

*Figure 15. Jordan’s implementation of mathematical tasks.*
Development of post-lesson analyses. During her time in the program and her first year of teaching, Jordan analyzed each lesson she video recorded. Her analysis focused on critiquing the lesson, providing evidence for her critique or as to why the lesson was effective or not, examining her own mathematical knowledge for teaching, and examining students’ learning based upon the tasks provided. Below is a breakdown of each category, with her trajectory described in detail.

Critical approach. Throughout her time in the study, Jordan’s critiques of her implemented tasks focused mostly on herself as a teacher as well as ways to improve her future lessons. During her junior year, Jordan’s main critiques focused on her language and her instructional techniques. For example, she felt that she was not precise in her mathematical language and wished she would have made clear that “9 + 1 and 1 + 9 are not the same equation.” In terms of her instructional techniques, she felt that she did not give her students enough time to think through the problems she gave them and introduced algorithms too soon without “allowing them enough time to really just work with the manipulatives.”

Throughout student teaching and her first year of teaching, Jordan continued to focus on her own instruction as a part of her critical approach. In her senior year, she felt she did not take the lesson as far as she could have. She wished she would have made more connections with the bar models in her first lesson, but felt that she did better with her visual representations in her second lesson. Once in her own classroom during her first year of teaching, Jordan believed she held her students back because of the pacing issues with the other teachers at her school. She also felt that even though she originally thought virtual
manipulatives would be helpful to her lesson, she found that they did not work as she intended. By the end of her first year, Jordan continued to feel her instruction could improve. For instance, she felt her lesson format could have been better (i.e. giving her students a better place to organize their work), and the design of the centers she created could have been organized.

*Links to evidence.* Throughout her six implemented lessons in the study, Jordan’s evidence of the lessons’ effectiveness focused mostly on student conversations as a means of effectiveness as well as whether the students were able to obtain correct answers. During her junior year, Jordan’s evidence that her lessons were effective focused on students’ understanding of the material taught. For example, during her beginning of the year lesson, she was able to know the lesson was effective based on the students’ explanations about the partner sums of 10 as well as the correct answers they provided on their white boards. During her end-of-year lesson, she used their homework as a way to understand what they learned throughout the lesson. Jordan also overheard conversations as they occurred throughout the room, one specifically focusing on how to find the missing combination for their rectangular prisms.

Similar to her junior year, during Jordan’s senior year, she continued to focus her analysis on the way students discussed and understood the topics being taught. In her beginning-of-the-year lesson, she noticed that students could use the bar models and talk about them within their groups. She also noticed that many who struggled in the beginning were grasping the ideas and could write about the concept in their journals. In her end of the year lesson, she noticed that some students volunteered to participate that normally do not
and that students were able to get correct answers to the problems when comparing denominators or numerators that were the same.

Unlike the previous two years, at the beginning of her first year of teaching, Jordan seemed unsure about the evidence that she had as to whether the lesson was effective or not. She mentioned several times that she “thought [the lesson] went well,” but did not provide reasons as to why. She further commented that she was unable to walk around as much as she would have liked, so she did not have the evidence she would have if she were to give an exit ticket. By the end of the year, however, Jordan noticed that her students “made some gains where there were gaps before”, because she could see students helping each other throughout the classroom and when she checked in on their groups she saw correct answers to the worksheets she gave them. Therefore, she knew the lesson was effective due to the students’ ability to complete the worksheet correctly, despite the students not being “where [she] would want them to be” mathematically.

**Mathematical knowledge for teaching.** Throughout her time in the study, Jordan referred to her own mathematical knowledge for teaching, specifically her SCK, KCS, KCT, as well as her HCK. At the beginning of her teacher preparation program during her junior year, Jordan mentioned a struggle she had with using the correct language to use throughout the lesson. This description of her specialized content knowledge (SCK) was coupled with her description of her knowledge of content and students (KCS). Jordan wished she would have allowed for students more time to think through the math-mountain problems on their own instead of allowing other students to call out the answers. She felt this was an issue because it did not give some students enough time to think through the problems. During her
end-of-the-junior-year lesson, Jordan focused positively on her SCK by mentioning how she was able to discuss with her students how area connected to volume by looking at the rectangular prisms they built.

Throughout her senior year, Jordan continued to refer to her KCS; however, she also discussed moments that were coded as her knowledge of content and teaching (KCT) and her horizontal curriculum knowledge (HCK). For example, she felt the bar models were not as effective as she could have made them out to be to help promote student learning (an analysis of her KCT); however, she tried to use them to push students in their thinking about what they represented in terms of the problem (KCS). During her end of the year lesson, Jordan examined her own experience as a teacher, and not knowing what to expect in future lessons with her students (representative of her HCK) when she discussed how she is now further along in the lessons and it was “not a bad thing” to draw out what is occurring in the problem.

Once in her first year of teaching, Jordan’s lessons focused on the use of mathematical centers in which students rotated through. Jordan chose the centers because she wanted students to dig deeper into some of the topics they were reviewing (KCT). However, she chose other centers because she felt some students needed challenged or were struggling with the material (KCS).

**Student learning.** Throughout her time in the study, Jordan’s analysis of student learning occurring focused on herself as their teacher, the evidence she had from students’ written artifacts, as well as students’ ability to work together and discuss problems. During her junior year, Jordan’s analysis of her students’ learning and whether it occurred focused
on the students’ ability to provide answers to her questions orally and in their homework. She felt the students had time in her volume lesson to explore and build without direction from her, which helped them create their own learning. Furthermore, she felt that her connection of area to volume helped students think about how they are related while they built their prisms.

Although Jordan felt that students’ correct answers were a sign of their learning during her junior year, she noticed a variety of aspects led to student learning during her senior year. For example, in her beginning-of-the-year lesson, she noticed that students were furthering their own lesson through discussions among their groups and exploring the bar models they were given. In her end of the year lesson, students were also working with their peers and comparing methods to see that they were indeed getting the same answer even though they solved the problem differently. She also noticed that the problems themselves aided in student learning because they forced students to look at the visual representations rather than “trying to get numbers in [the problem] too quickly.”

Once in her first year of teaching, Jordan again focused on a variety of aspects that attributed to student learning despite not being sure if learning occurred during her end-of-year lesson. For instance, she noted that the resources themselves helped students learn throughout the lesson (i.e. the computer station placed students on a pathway that was appropriate for them). She also commented that students helped each other make gains in their learning by showing each other how to solve problems differently (i.e. using a picture versus using an algorithm). Furthermore, Jordan felt that she helped students further their learning by giving them more practice in the skills she already taught (i.e. the Playground
Project), and by encouraging them to “put themselves in the situation”, by drawing out a story problem instead of jumping straight to an algorithm to solve.
Chapter 6: Charlie

At the beginning of her junior year in college, Charlie reflected on her past experiences with mathematics. Overall, she had very positive experiences with school; however, Charlie felt that math was more challenging than other subjects. In elementary school, Charlie was anxious about timed multiplication quizzes and became frustrated at the lack of methods taught for solving problems. This frustration led her to “go home and cry [while] doing homework.” Even though she took higher-level math in middle school (algebra), she was still frustrated with the way math was taught.

Once in high school, she felt she did not struggle as much. This was largely due in part to the pre-calculus teacher she had that would tell her “You are not bad at math. You just have to work harder.” Before this teacher, Charlie “hated math and thought it was just the worst, most pointless thing ever.” However, despite her previous feelings she used her newfound confidence and took calculus her senior year.

In college, Charlie felt better about math, which she attributed to her high-school teacher and the boost of confidence she was given; however, she still “didn’t really enjoy it very much.” The one thing she did like about math was that “it has a definite answer… you just have to find it.” Despite her negative past with mathematics, Charlie began to change her view once enrolled in her elementary mathematics methods courses. She spoke of her methods courses by saying: “they gave me an explanation of why math is the way it is, and I have really felt like I’m repairing all the damage that I did in elementary school.”

Charlie is representative of a case that remained stable in her mathematical knowledge for teaching, in comparison to her cohort members, from the beginning of her
junior year in the teacher preparation program to the end of the program in her senior year (Figure 16). Although she remained stable in terms of her MKT scores, she performed more than a half standard deviation above the cohort average while in the program. Below I present Jamie’s trajectory of visions, implementation, and post-lesson analyses from her teacher preparation program into her first year of teaching.

**Figure 16.** Charlie’s change in MKT from beginning to end of the teacher preparation program.

**Beginning of Junior Year**

**Visions.** At the beginning of her teacher preparation program in her junior year, Charlie felt that students should first “understand the underlying concepts” and then have “lots of practice.” She mentioned that, “I think it is important to show a few problems and then have [students] do it.” Due in part to her own frustrations with how she was taught mathematics, Charlie also valued the importance of students having multiple ways of solving
a problem, instead of the teacher only showing one way. Her vision of “practice and then apply” in regard to mathematical tasks aligned with a Level 1 on the VHQMI task rubric at the beginning of her junior year.

**Implementation.** Charlie’s beginning-of-the-year lesson was taught as a mini-lesson to the students in her cooperating teacher’s kindergarten class. She developed the lesson based on data collected from a previous lesson when she worked with a student individually. Charlie was tasked with linking the lesson back to a target concept in which the student struggled; however, she shared that the lesson provided by her cooperating teacher already addressed the target concept so she did not alter the lesson in any way. The lesson she taught focused on counting as well as comparisons of objects and numbers.

Charlie’s first task to begin her lesson, lasted just over two minutes and began with the students in a whole group on the carpet. She held up cards containing numerals as well as dots (i.e. the numeral 4 and four dots) and asked her students to use their fingers to represent the number. However, many of the students were counting the dots and were not actually able to subitize the number of dots on the cards.

For the next six minutes, Charlie’s second task asked that two students come to the front of the class and hold a number line while another student pointed to the numbers as the class counted. The students started at several different numbers on the line; for example, they began at 4 and counted up to 10 and then counted backwards to 0. Charlie varied the speed in which students counted, by asking them to count very quickly up to 5, and then count very slowly the rest of the way to ten.
The third task of the lesson, which lasted two minutes, had the students remain on the carpet, but instead face a hundreds-chart while they worked on the number of the day. For their particular lesson, the number was 19. This allowed the students to break the number apart into the number sentence $10 + 9$ and count from 1-19. However, unconnected, they then counted by 10s to 100.

After they discussed the number of the day, they turned their attention to “alike and different” using shoes from students in the class. This fourth task lasted for three and a half minutes, and had students discuss what was alike (i.e. color) and what was different (i.e. size) about the shoes. This discussion of alike and different led the students into their fifth, and final task, that lasted six minutes, in which they were asked to “create 10” using tiles stuck to a board in the room. Charlie asked two students to create a group of ten on the board and then asked the class to compare the two groups of tiles. She had the students focus on how they were the same and how they might look different depending on how the students arranged their tiles. Charlie also provided different groupings of the tiles in order for the students to view the tiles arranged differently.

*Overall, Charlie’s beginning-of-the-junior-year lesson was coded as a level 1.5 on the IHQMI tasks rubric.* Her first three tasks were all procedural in nature (level 1); however, her fourth and fifth tasks were coded as a level 2. These tasks either included a real-world application or they had students thinking in non-prescribed ways (despite only two students being called upon to create tile groupings).

**Post-lesson analysis.** Charlie’s analysis of her first math lesson was that it “went well, but there’s a lot that I already know that I did not necessarily do wrong, but that I
could’ve done much better.” Although she provides a critique of the lesson, she is unsure of how to improve the lesson for future students other than picking different shoes for her ‘alike and different’ lesson. She mentioned how students had seen some of the material before, and that discussion portion of the lesson went well because the students “had a lot of right answers, so they understood it [the lesson].” Despite this evidence, she provided of the lesson’s effectiveness, Charlie did not provide other examples of how the lesson “went well”.

Charlie’s referenced her mathematical knowledge for teaching specifically her common content knowledge (CCK) by saying she “didn’t even understand the concept [making groups of five] really, so that part I could improve on a lot.” She also referenced her specialized content knowledge (SCK) by saying she confused her students when she said two things “are still equal, and are the same”, because the previous day her cooperating teacher discussed with the students how “‘equal’ does not mean ‘same as’” (Faulkner, 2013); however, Charlie used the language interchangeably. Additionally, Charlie commented on her knowledge of content and teaching (KCT), by saying how it was beneficial that she provided additional arrangements of the tiles on the board, in order for students to view the numbers broken apart differently.

When discussing what the students learned throughout the lesson, Charlie focused on her own actions and errors, rather than focusing on the students’ actions. For example, her uncertainty that student learning occurred was shown through her language of using words such as “I think…”; “I hope they learned that [the task] was effective”; “I’m not sure… if that was the first time that they had learned it. I would hope that they got it because I think that part went well.” She did not provide evidence as to why she thought or hoped the lesson
was effective and was also clouded by her lack of time in the classroom as to whether students had seen or experienced the material before.

**End of Junior Year**

**Visions.** Charlie’s vision at the end of her junior year was reflective of what she had seen in her field-placement teachers’ classroom the semester before. She described an effective lesson as being one in which the teacher in the classroom had the students counting and using their fingers to show $10 + 1 = 11$, and so on. She felt it is important to “understand what numbers are, where they come from, and what they’re composed of”. She also mentioned the activity her cooperating teacher, from the previous semester, facilitated where the students had to match cards that said “$10 + 4$” to the card that said “14”, and felt it was effective because “it was really giving them a foundation of base ten and the way numbers are composed”. Therefore, the vision of tasks Charlie held at the end of her junior year was consistent with a level 2 on the VHQMI at this time point. She valued reform-oriented aspects of tasks (e.g., the hands-on nature of the counting lesson or using the matching cards to give meaning to counting). Additionally, she valued connecting different representations throughout the lesson instead of focusing only on showing students one way to represent an answer.

**Implementation.** Charlie’s end-of-junior-year lesson was taught to a fifth-grade class around the topic of volume. She planned the lesson with her field-placement partner, Jordan, using a county-wide curriculum. Charlie mentioned that they “didn’t really have to plan much, we just had to edit the lesson and think about how we were going to go about it.”
Although she described editing the lesson, she only provided details of the edits by saying they incorporated more discussion than the lesson originally intended.

To begin the lesson, Charlie asked the students to provide examples of prisms. One student suggested a shoebox; to which Charlie probed their thinking to further think about the differences between prisms and rectangles. She used this discussion of 2D and 3D shapes to then discuss the dimensions and attributes of a rectangular prism.

Once they finished the whole class introduction to prisms, she asked students to break off into two groups (she was only working with half the class) to build as many rectangular prisms as they could using 12, 20, and 36 cubes. One group built prisms with 12 and 36 cubes, and the second group built prisms with 20 and 36 cubes. As the students created a new prism, she asked them if they had created a “similar shape” by rotating it. While working in their small groups one student hypothesized that the volume of the shape is just the dimensions multiplied together.

After students had the opportunity to build all prisms with their given number of cubes, they came back together as a whole group to discuss how many different rectangular prisms they could build with 36 cubes. Although the class agreed that only 8 prisms could be built, some students seemed confused, due to their expressions and questions amongst the table members. However, Charlie did not hear their discussions and moved on to another class discussion about the dimensions that composed the prisms. Some students explained their process for finding all of the prisms, and suggested that they used factors of 36, and would make all figures with that factor before moving onto the next.
Overall, Charlie's task and its discussion lasted seventeen minutes and was coded as a 3 on the IHQMI task rubric. She used manipulatives to allow students to explore volume of rectangular prisms, and students created the prisms however they wished. Charlie wrapped up the task by discussing the multiple strategies used, and students used this task to build their understanding that volume is the product of the length, width, and height of a rectangular prism. Although there was a wrap-up discussion for the task, Charlie did not go into detail about how all eight prisms were found.

Post-lesson analysis. Charlie’s critique of her end-of-junior-year lesson was that “it went well and they [the students] achieved the objective based upon what I watched in the lesson and completed at the end of the lesson”. She mentioned that she liked how the students had time to explore and work things out for themselves; although she never provided a reason for why this is important. The other critiques of her lesson focused on her own mathematical language, which she found challenging at times. Charlie mentioned that if she were to do things differently she would watch the way she phrased things so they would not become a misconception for her students; however, she did not provide examples of which phrases she would change if given the opportunity.

In terms of the evidence she provided as to whether the lesson was effective, Charlie focused on how students were able to build the prisms correctly. To Charlie, this meant they understood the topic and met the lesson objective (understand the connection between area and volume). However, her evidence did not point specifically to what made the lesson effective other than describing that the lesson was facilitated just “okay” because of the fuzziness of the directions and not being explicit from the beginning. Once students had a
chance to play with the cubes, Charlie felt the fuzziness cleared up because students were able to create the correct prisms, her evidence of student understanding and lesson effectiveness.

Charlie described her *mathematical knowledge for teaching* throughout her interview when she focused on the language that she used within the lesson, specifically using words interchangeably (coded as SCK). Despite her lack of elaboration on the words she chose to interchange, Charlie believed that doing so could impact *student learning*, and “it can very easily become a misconception if I’m saying something [wrong] and they hear me saying it.” Additionally, Charlie mentioned the students’ performance on their assessment, their discussion at the end of the lesson, and how many were able to “explore on their own without me having to tell them,” as her evidence of student learning occurring throughout the lesson. Finally, Charlie felt she missed an opportunity for student learning because she ran out of time and could not help a group find the final prism they were missing.

**Beginning of Senior Year**

**Visions.** As she began her senior year, post- math methods courses, *Charlie’s visions centered mostly around a level 2 on the VHQMI.* She believed that “a lot of effective lessons are ones that have a real-world context”; for example, she mentions problems that deal with money or the economy. Although she did not provide a reason for needing to have these real-life contexts, she knew it is important to do so. She also mentioned that “you have to teach the foundations first, and if you’re doing things procedurally then the underlying knowledge is never really there”. This was also indicative of reform-oriented mathematics practices (consistent with that of a level 2); however, she again did not mention why it is important.
Finally, Charlie emphasized her reform-oriented beliefs by saying that “a lot of group work and a lot of manipulative work” was important, but depending on where students are you might have them “working on facts”.

**Implementation.** Once her mathematics methods courses were over, and Charlie was in her student teaching placement part-time, she taught a lesson to her cooperating teacher’s third-grade class involving rounding. The lesson was designed to provide reinforcement of the concept with three students who received the lowest averages on an assessment. Charlie felt that the students “needed some visual representation” to help them understand rounding and used this as motivation to plan the lesson the way she did.

Charlie began her small group lesson with a task that lasted just over five minutes. In this first task, she gave students cubes and asked them to quickly assess whether they had close to 10 or closer to 20 cubes. They answered with quick responses as to their estimates. Next, Charlie asked the students to find 27 on their hundreds chart and circle the nearest ten. They quickly saw it was closer to 30 and circled it; they continued this for 52, as well as 75. Many students were unsure which group of ten to circle since 75 fell in the middle of 70 and 80; however, one student circled 80, and she asked “why did you circle 80? What’s the rule?”, to which they responded if the ones digit is 5 or more, round up.

Once they practiced rounding two-digit numbers, Charlie had the students move on to a second task, which lasted fourteen minutes, using three-digit numbers. In this task, she asked the students to round with respect to a variety of place values. For example, she asked them “how can you use your hundreds chart to round your number, 427, to the nearest 10?” She also had the students use dry-erase boards to write a three-digit number and show to the
nearest hundred and ten. Although she asked for answers to the problems, she did not ask students to explain their reasoning.

To wrap up the lesson, Charlie asked the students to solve a problem in their heads, $28 + 73$. She wrote it horizontally first and then rewrote it vertically “the way they like to see it”. Unlike typical “number talks” in classrooms (Parrish, 2010) where students share their mental math orally, Charlie asked students to return to their seats, for the final three minutes of the lesson, to work on the problem on their own and to show their work.

*Overall, Charlie’s beginning-of-the-senior-year lesson was coded as a 0.5 on the IHQMI task rubric.* She used manipulatives for brief moments throughout her lesson (less than $\frac{1}{4}$ of the overall lesson, and only during task 1); however, she mostly used the manipulatives to enforce the procedure for rounding (level 1). In her second task (level 0), she strictly had the students focusing on using the “five or higher, round up” rule for rounding throughout the task. Finally, in her exit ticket (level 1), Charlie asked her students to solve a naked-number problem, but they were allowed to solve using any method they wished.

**Post-lesson analysis.** Charlie’s critique of her lesson was that she should have done something differently than what she did throughout the lesson; however, she did not provide specifics of what that might be. Charlie felt limited to assessing the students by only being able to “go off of what they told me during the lesson”, and gauging their non-verbal body language, such as facial expressions. Additionally, she seemed unsure in the evidence she provided of the lesson’s *effectiveness* by saying “I hope they have a better idea of how to round…” and “I think they kind of started to understand it.”
Charlie’s mathematical knowledge for teaching was described throughout her interviews in the choices she made during her lesson. For example, she felt she “could have done better with saying your number of cubes is between 200 and 300, which of the two hundreds, is it closer to?” This is representative of her specialized content knowledge (SCK) as well as her knowledge of content and students (KCS) because she focused on her language used in teaching, as well as what her students might find confusing. Although doubtful if student learning occurred (i.e. “I hope…”, “I think…”), she did make note of learning occurring because of students’ struggle with making sense of how to round a number to the nearest hundred versus to the nearest ten.

End of Senior Year

Visions. Charlie’s vision of good tasks at end-of-senior year focused on a lesson that she considered effective. She spoke of a lesson in which students were split into groups with several tools and manipulatives, and they had to represent a given fraction using the objects. She mentioned how throughout the lesson students were talking with one another, explaining their thinking, and the teacher was not providing a lot of direct instruction. Although Charlie felt the teacher should be “teaching why things are the way they are and encouraging a lot of discourse”, she also feels the teacher should be “teaching foundational skills”. The tasks that the teacher selects should have students “thinking and making them actually do some work and develop skills and not just memorize random stuff”.

Charlie’s visions by the end of her senior year were coded as a level 2 on the VHQMI task rubric. She valued the use of manipulatives, and wanted students doing problems that required higher-order thinking. However, Charlie’s vision is not higher on the VHQMI rubric
because she made no mention of the use of open-ended tasks, multiple solution methods, or engaging in discussions that compared solution methods of the students.

**Implementation.** Charlie’s end-of-senior-year lesson in her third-grade classroom focused on real-life word problems involving area and perimeter. She chose the lesson by combining two different curriculum lesson plans provided by her cooperating teacher. Charlie edited the lesson by removing some of the examples that the intended curriculum provided because she felt they were too repetitive. She commented that “I would only do one or two of [the problems] instead of six or seven because it just took too much time and [the students] would get out of control.” Instead, she chose to have the students work in small groups or independently to complete the problems. However, Charlie tried to “find tasks that would ask them harder questions” or used “real world application instead of just ‘find the area’.”

To begin her whole-group lesson, Charlie’s first task gathered the students on the carpet, and focused on the word problem: “Amir is getting carpet in his bedroom. His room measures 7 feet by 15 feet. How many square feet of carpet will he need for his bedroom?” Each student was given a dry-erase board, and she suggested they begin by drawing an area model. Charlie asked that they draw and label the figure appropriately (making sure the longer side is labeled as 15 feet, etc.), and then asked them to use any strategy to find the answer. This task lasted fourteen minutes once students finished sharing their solutions and strategies with the class.

After their whole-group discussion, Charlie sent students back to their seats to work on more problems using their dry-erase boards. This second task, which lasted almost twenty
minutes, had students draw pictures individually, but then discuss the problem with their small-group and decide on a best strategy to solve the problem. Although some students were working on the task at hand, many were off-task and shouting across the room. When it came time to share strategies, students were still off-task and not paying attention to what was being presented to the whole group.

Approximately halfway through the lesson, students were given a third task, which consisted of a worksheet with problems they had started the previous day. They were asked to finish solving any problems they had not already solved. Students worked individually to complete the problems (type unknown, due to camera placement in the room), and were given a five-minute exit ticket similar to the problems they solved at the beginning of class once they were finished.

Overall, Charlie’s end-of-the-senior-year lesson was coded as a level 2 on the IHQMI task rubric. Her first two tasks were coded as a level 2 because they utilized a real-world context and asked students to share strategies. However, by guiding the students to draw an area model to solve and describing how to label their picture, Charlie reduced the cognitive demand of the first task. Similarly, the cognitive demand of the second task was reduced, this time by the disruptions caused throughout the lesson, the “practice and apply” nature of the task itself, and because of the lack of connections made between students’ solution methods. Finally, the worksheet task was not coded due to not seeing the problems being solved on the worksheet; however, the exit ticket was coded as a level 2 because it was similar to the previous problems solved during the lesson.
Post-lesson analysis. Charlie began her critique of the lesson by saying she felt that only about 50% of the students understood the lesson. She revised the intended lesson plan by having the students work individually due to her students getting out of control, and she also cut back on the number of problems the students solved for the same behavior reasons. Despite these changes, Charlie felt the lesson went well and that she did a good job scaffolding the lesson. Her evidence that the lesson went well focused on students being engaged and getting correct answers. She elaborated by saying the students “were pretty engaged”, “weren’t playing”, and were “doing problems correctly”.

In terms of her mathematical knowledge for teaching, Charlie did not focus her discussion on any specific instructional choices that were made except for her use of scaffolding in the lesson. She commented that she gave problems to students who “needed to be challenged”, but also worked separately with “the kids who were struggling”. This is indicative of her knowledge of content and students (KCS) because it shows she anticipated what her students might find confusing and adapted the lesson accordingly. Charlie described the students’ learning as being driven by their own struggle with the exit ticket, but also by being engaged in the lesson and showing their learning through correct answers. In terms of her own contribution to their learning, Charlie felt that she “did a good job” because she would “read the problem and then let [the students] think about it.” She continued by saying she “gave [the students] enough time to do the problems” in order to feel successful with the material.
Beginning of First Year of Teaching

Charlie’s first-year-of-teaching placement was in a second-grade classroom in a suburban school district in the southeastern United States. The Title 1 elementary school consisted of 725 students, 80% of which received free and reduced price lunch during the time of data collection. The population of students included: 43.9% Black, 37.6% Hispanic, 12.3% White, 3.9% two or more races, 2.1% Asian, 0.14% American Indian, and 0.14% Pacific Islander.

Visions. Charlie began her first year of teaching by describing a lesson that she taught recently as “kind of effective”. In the lesson, she described that she was teaching addition, and the students did place value drawings to understand regrouping, as well as created representations with place value blocks. This activity led the students to finally represent the numbers using the standard algorithm for addition. She viewed manipulatives as important, and stressed the importance of “actually teaching them something and helping them understand what they’re doing… not just completing a bunch of problems, but actually understand why”.

Although Charlie aligned with many descriptors of a level 2 on the VHQMI rubric, her vision also included some level 1 descriptors. She had a “practice and apply” mentality in which she believed that the “teacher [should] first models whatever the topic is, then have [the students] work with the teacher to kind of solve a similar problem, and then students should be working independently.” This was reiterated in her description of the task she mentioned as effective when she said, “it was a lot of modeling and then a lot of the students doing it with me, and then they worked independently on it.”
mathematical tasks during the beginning of her first year of teaching was coded as a level 1.5 on the VHQMI task rubric.

Implementation. At the beginning of her first year of teaching, Charlie taught a lesson to her fourth-grade class around one-digit by three-digit multiplication. She used the county’s existing curriculum to plan the lesson; however, she chose to pull problems from previous days’ lessons and have the students complete them. Additionally, she chose to only present one method to solve the problems to her students, instead of the three methods the intended curriculum provided because she thought “they were so similar and very confusing for the students”.

Charlie’s lesson began with the entire class working individually on a timed test, although the video did not show the types of questions her students were answering. The students had four minutes to complete the problems, and once completed they quickly went over the answers as a class.

Next, they moved on to another task, which lasted eight minutes, and involved multiplication with hundreds. Charlie began with a word problem from their homework and asked students to circle key words and numbers from the problem. Many of the students were not engaged and spent the time talking to each other or working on something else at their desks.

Once they were finished checking homework, Charlie’s main task of the lesson focused on showing the class several arrays that went from 7 x 3 to 70 x 3, to 700 x 3, and finally to 7000 x 3. She asked the class “why don’t we have to break the numbers apart?” to which a student replied, “you can just add the zeros on because you know 7 x 3”. This
activity led into the next problem, which had students using area models to break 384 into 300 + 80 + 4 and multiply each piece by 3. Many students were again off task and talking with those around them. Charlie asked them to share with a partner how they solved the problem and whether they came up with the same answer.

After practicing as a group, Charlie had students solve similar multiplication problems individually or with a partner while she worked with a small group at the back table on the same problems. For students who finished early, they practiced their multiplication flash cards with a partner. The students’ task of applying what they saw in the whole-group instruction to their problems they solved with a partner, lasted almost forty minutes of the fifty-two-minute lesson. Finally, the lesson ended with a four-minute exit ticket in which Charlie told the students “this is exactly like what you’ve been doing”. She explained that it was only for her to see how they were doing and how she can teach them better.

Overall, Charlie’s beginning-of-the-first-year-of-teaching lesson was coded as a level 1 on the IHQMI task rubric. Her first task of checking homework was coded as a level 0 because the students were checking their homework and were only focused on memorized procedures. The majority of the lesson was coded as a level 1 due to the procedural nature of the problems the students were solving as well as Charlie’s “practice and apply” mentality to implementing the tasks. Finally, her exit ticket was coded as a level 0 because students were using an already learned procedure to mimic the problems they had previously solved.

Post-lesson analysis. Charlie’s critique of her lesson was that overall it “went pretty well [because students] got the concept very quickly”. She originally thought the students would be “totally lost” and the concept would be hard for them, but for many she felt the task
was too easy and she wished she had planned for more to do. The evidence she provided for the lesson’s effectiveness focused on how the students had already worked on multiplication of two-digit numbers so she felt it helped them in the current lesson. She also believed that the lesson was effective and student learning occurred because of the students’ written work and exit tickets. These written artifacts allowed her to “see how well they were able to complete [the problems] and showed [her] if they were too hard for them or not”.

Her own mathematical knowledge for teaching was challenged through her work with a small group of students. She realized when she called a group of students to the back table to work with her she “didn’t know how to explain it [the material] very well and didn’t know how to answer their questions very well”. In order to improve, Charlie felt she needed to be more careful with her language in the future, but did not elaborate as to how she would do so. This is representative of her specialized content knowledge (SCK) as well as her knowledge of content and students (KCS) because she is focusing on her language used in teaching, as well as what her students might find confusing. Furthermore, she mentioned that she felt she picked appropriate problems for her students to solve (KCT); however, she did not give justification as to why she picked the problems she did.

End of First Year of Teaching

**Visions.** Charlie’s vision of an effective task, as she finished her first year of teaching, came from a lesson she taught during a field experience in her teacher preparation program. She brought up a lesson she did with students that involved using little hand-held clocks and changing time depending on whether it was a quarter, or half, etc. She mentioned the lesson was effective because “they were engaged at the beginning because they got to use
manipulatives”. Charlie felt that students should constantly be working and not just watching the teacher teach. However, she commented that the teacher should be demonstrating and “checking to see if the kids are doing it the right way”. Overall, Charlie’s visions at the end of her first year of teaching aligns with a level 1.5 on the VHQMI. She valued the use of manipulatives, but she did not provide details about why they were important (indicative of a level 2). Additionally, she continued to value “practice and apply” as a method for teaching, which aligns with a level 1.

Implementation. At the end of her first year of teaching, Charlie taught a lesson to her fourth-grade class around converting between units of measurement. Charlie mentioned that she is focused on preparing for state standardized assessments at the end of the academic year; therefore, she was “kind of speeding through the last topics.” She mentioned that she was being selective with what she chose to use from the county curriculum. “I’m just kind of pulling resources from everywhere just trying to get it done,” she commented. This led Charlie to create her lesson by compiling several resources together divide her students into groups at stations to review the previous days’ material.

Charlie began the lesson as a whole group, with the students seated on the carpet in front of the white board. She discussed the rotations they would be using throughout their math workshop and then allowed the students to move into groups. The first group worked independently on a weight and capacity worksheet. She allowed them to use anything in their folders to help (e.g., gallon man, vocabulary). Although this was a worksheet, the camera never zoomed into the types of problems the students were completing. Therefore, this activity was not included in the scoring of her tasks. The second group worked individually,
with online computer games that involved measurement. They had the option of four different games focusing on converting units or deciding on what units they should use to measure an object. The third group worked with a partner to solve as many task cards as possible. Each task card had a story about an animal in which the students had to identify and defend their reasoning for selecting the appropriate unit for measuring their animal. If they could change the measurement to a larger unit, they were challenged to do so. The fourth, and final group, met with Charlie to solve word problems. One of the problems the students were tasked with solving was “if you have 2 gallons of juice, how many pint servings could you serve to friends”. Students used their “gallon man” drawing for help and converted the gallons into pints in order to solve.

As students rotated into Charlie’s group, the students at the partner and independent work stations began to get off task. By the last rotation, most students had lost interest and had to be constantly redirected by her to stay on task. This required her to leave her word-problem station and go correct behavior issues around the room.

*Overall, Charlie’s end-of-the-first-year-of-teaching lesson was coded as a level 2 on the IHQMI task rubric.* Each group spent roughly fifteen minutes at each of the four stations throughout the room. The group working with Charlie was focused on solving real-life problems, and students used their prior knowledge to help them solve. However, Charlie spent most of the time guiding them through how to solve the problem, and students were not given much time to grapple with the problem at hand. This group was coded as a level 2. The computer games were coded as a level 2 because they were open-ended and allowed students multiple solution paths to determine the appropriate units. Finally, the partner work was
coded as a level 2 due to the real-life context and allowing students to solve in meaningful ways that made sense. However, in all groups, students were not challenged to compare solution strategies, and Charlie reduced the cognitive demand of her group by guiding them through the lesson.

**Post-lesson analysis.** Charlie’s analysis of her lesson, began with a *critique* that she felt uncertain about the quality of the lesson and that it was “just an okay lesson”. She struggled with the notion of wanting to use groups, but she also knew that students in her class struggled socially with it. She mentioned that if she were to do the lesson again she would keep the tasks the same but structure the groups differently. Her *evidence* of the lesson’s effectiveness and whether *student learning* occurred, focused on how she saw some of the students struggling with independent work which told her that “it might have been a little bit too hard for some of them to do independently.” Charlie also felt uncertain about the lesson in general by saying “I don’t know” several times when asked about how components of the lesson went.

Charlie relied on her *mathematical knowledge for teaching*, specifically her knowledge of content and teaching (KCT) to select tasks that she thought would provide differentiation for all students. For example, she mentioned that she selected: one task with a picture that she felt would be easier to do; another task with reading involved, which she chose to do with a partner for students who struggled with reading; and the task with word problems, which she chose to be with her so she could help those who needed help. Unlike her rationale for these tasks, she chose the computer rotation because she knew her students
enjoyed computer time. Also, she did not comment on the effectiveness of the computer station on student learning.

Summary of Charlie

Development of Visions. Charlie’s development in her vision of high-quality mathematical tasks throughout her teacher preparation program and into her first year of teaching were characterized as mostly a level 1-2 on the VHQM framework (see Figure 17). At the beginning of her junior year, Charlie viewed good tasks as being ones in which students first practiced with the teacher, and then solved on their own (Level 1). Additionally, she valued the importance of having multiple ways in which to solve a problem. At the end of her methods courses, Charlie valued the lessons she observed by her classroom cooperating teacher. She still viewed multiple ways to solve a problem as important and also encouraged the use of different representations (Level 2). At the conclusion of her student teaching and the end of the program, Charlie focused on more reform-oriented aspects of tasks such as real-world contexts, using manipulatives, and having students participate in group work (Level 2). However, once into her first year of teaching, Charlie began to slide back in her visions, by valuing “practice and apply” methods again. She still valued the use of manipulatives, but more so focused on the teacher modeling a problem and allowing the students time to practice on their own (Level 1.5).
Charlie's visions of high-quality mathematical tasks throughout the study.

**Development of Implementation.** Charlie’s implementation of mathematical tasks varied considerably throughout her teacher preparation program and into her first year of teaching, ranging from 0.5 to 3 on the IHQMI framework (see Figure 18). However, most of her tasks settled around a level 1.5. Charlie used many of the lesson plans given to her by her cooperating teachers while in the teacher preparation program. At the beginning of her junior year, Charlie taught a lesson in which her implemented tasks focused mostly on procedures without connections. However, she made sure to include manipulatives and engage all students in the lesson (Level 1.5). At the end of her methods courses, Charlie taught a lesson in which students explored the volume formula for rectangular prisms. This lesson allowed for group work, exploration, and multiple solution paths (Level 3).

Once into her senior year, Charlie taught a lesson in which she mostly focused on procedures and did not connect to the concept of rounding (Level 0.5). By the end of her
senior year, she focused her lesson on real-life word problems and encouraged multiple ways to solve (Level 2). However, she still used a “practice and apply” method in which she modeled the problems first before allowing students to try on their own.

Finally, once in her first year of teaching, Charlie continued with her “practice and apply” approach to teaching (Level 1), but by the end of the year she tried using a workshop approach that allowed students to be more in charge of their learning (Level 2). However, she struggled with this concept because students were off-task and causing disruptions in the classroom. These behavior issues caused Charlie to lose time with her station focused on helping students make sense of word problems.

![Charlie's Implementation of Mathematical Tasks](image)

*Figure 18. Charlie’s implementation of mathematical tasks throughout the study.*
Development of post-lesson analyses. Each of Charlie’s post-lesson analyses focused on critiquing the lesson, providing evidence for her critique as to why the lesson was effective or not, examining her own mathematical knowledge for teaching, and examining students’ learning based upon the tasks provided. Below are descriptions of each category, with her trajectory described in detail.

Critical approach. Throughout her time in the study, Charlie’s critiques of her implemented tasks focused on trying to provide alternative suggestions to improve her future lessons, but not being sure what those changes would look like. During her junior year, Charlie felt her lessons “went well,” but she still felt there were things she could have done differently. In her first lesson of the year she was unsure how to improve the lesson mathematically; however, in her second lesson she provided detail by saying she would have been more careful with her mathematical language. Despite her feelings of needing to improve, she felt it was beneficial that she gave students time to explore and think through the problems for themselves. Overall, however, Charlie did not provide other examples of why she felt the lessons “went well” or how she would change them for future students.

Once in her senior year, Charlie’s first lesson analysis remained similar to that of her junior year lessons due to her feeling that she should have done something differently, but she did not point to what it might be. However, the second lesson in her senior year provided more reasoning as to why she felt that only 50% of her students understood the lesson. This time, Charlie pointed to her students’ behavior issues as to why she had to enact the lesson the way she did. She also critiqued her lesson by saying she did a good job scaffolding the lesson to help students who needed challenged or were struggling.
Throughout her first year of teaching, Charlie’s critiques of her lesson ranged from “it went pretty well” in the beginning of the year, to it was “just an okay lesson” by the end of the year. In the beginning of the year, she felt her students grasped the concept quickly, which was why she critiqued the lesson in a positive manner. However, by the end of the year, her behavior issues in the class led her to believe the lesson did not go as well as she would have wished. Charlie commented that if she were to repeat the lesson again she would structure the groups differently while keeping the tasks the same.

**Links to evidence.** Throughout her six implemented tasks, Charlie’s evidence of the lessons’ effectiveness centered around whether students were able to obtain correct answers to the problems she posed. She additionally struggled with being able to provide any evidence at all during her senior year and first year of teaching. During her junior year, Charlie focused mostly on whether the students could provide the correct answers to the problems. For example, in her beginning-of-the-year lesson, she mentioned that the lesson went well because the students “had a lot of right answers”. Similarly, in her end-of-the-year lesson she felt any confusion in the lesson cleared up when the students were able to create the correct rectangular prisms. Charlie’s focus on correct answers as a means for effectiveness continued throughout her senior year. For example, during her end-of-the-year lesson she mentioned that students “were pretty engaged” and were “doing problems correctly”.

Despite her focus on correct answers, Charlie also felt uncertain about whether the lesson was effective or not during her beginning-of-the-senior-year lesson and her first-year-of-teaching lesson. She used language such as “I hope…”, “I think…” or “I don’t know”
when asked about how the lesson went. Furthermore, Charlie focused on students’ previous knowledge as a means for effectiveness during her beginning-of-the-first-year-of-teaching lesson since she believed it helped her students learn during their current lesson.

**Mathematical knowledge for teaching.** Throughout her three years in the study, Charlie referred to her own MKT, specifically her SCK, KCS, KCT, as well as her CCK. Specifically, during her junior year, Charlie described moments that were coded as common content knowledge (CCK), her specialized content knowledge (SCK), and her knowledge of content and teaching (KCT). She felt that she struggled with the concept itself while teaching (CCK), but she also felt that she struggled with the proper mathematical language while teaching her students (SCK). Despite her struggles, she commented on her ability to provide additional arrangements of the tiles during her first lesson in order for her students to view the numbers differently (KCT).

Once in her senior year, Charlie continued to focus on her SCK during her post-lesson analysis, but also focused on her knowledge of content and students (KCS) as well. For example, she described the language she used during her rounding lesson and felt she could have done a better job in providing questions and responses to her students (SCK). Furthermore, during Charlie’s second lesson during her senior year, she specifically chose problems to use in her lesson that would help students who needed to be challenged as well as help those who were struggling (KCS).

During her first year of teaching, Charlie provided descriptions of her MKT which were coded as SCK, KCS, and KCT. For example, during her first lesson, she felt she should be more careful with language moving forward in order to not confuse her students (SCK and
KCS). However, during her second lesson of the year, she focused on the types of tasks she specifically chose for the students to complete in centers, which was indicative of her KCT.

**Student learning.** Throughout her lessons, Charlie’s analysis of whether student learning occurred, focused on herself as a teacher, whether the students provided written artifacts for her to see their progress, as well as the struggles she observed throughout her lessons. During her junior year, Charlie’s analysis of student learning fixated on herself rather than the students. During both lessons, Charlie focused on the language errors she committed as a hindrance for student learning. She mentioned that “it can very easily become a misconception if I’m saying something [wrong] and they hear me saying it.” The focus on herself continued during the end of her senior year lesson when she mentioned that she “did a good job” reading problems to the students and letting them think about it.

Despite her feelings of being the source of student learning, Charlie also focused on the students themselves as a means for learning. Although she was unsure if learning occurred during her beginning-of-the-senior-year lesson, she felt the students were able to struggle with rounding and that was productive for them. Furthermore, in her beginning-of-the-first-year-of-teaching lesson, she felt that students’ written artifacts were evidence that learning occurred. However, during the second lesson during that same year, Charlie did not comment about students’ learning during her analysis.
Chapter 7: Cross Case Comparison

In order to explain my research questions related to the preservice teachers’ visions, implementation, and post-implementation analyses of mathematical tasks, I present a cross-case comparison of the three participants in the study. The three sections compare and contrast the three cases in relation to: their visions of mathematical tasks; their implementation of mathematical tasks; and their post-lesson analyses, respectively. Additionally, I use the phases of the Learning to Teach continuum (e.g. pre-training, preservice, and induction) that were within my conceptual framework to help frame the emerging themes. These phases are important to consider because a teacher’s development is situated in the experiences they have while in each phase.

Cross-Case Comparison: Visions

Throughout the three-year study, the participants described their visions of high-quality mathematical tasks in relation to their experiences while in their pre-training and preservice phases (Table 8). Although experiences in the induction phase also influence novice teacher’s development while learning to teach (Feiman-Nemser, 1983), the teachers in this study did not draw upon those situations when describing their visions in their benchmark interviews. Rather, they mostly drew upon experiences they had as preservice teachers when describing their visions during their first year of teaching.
### Table 8. Cross Case Comparison of Visions.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Jamie</th>
<th>Jordan</th>
<th>Charlie</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-training</strong></td>
<td>MKT*</td>
<td>MKT*</td>
<td>MKT*</td>
</tr>
<tr>
<td>Struggled with mathematics.</td>
<td>Increased MKT; Below average MKT</td>
<td>Decreased MKT; Above average MKT</td>
<td>Stable MKT; Average MKT</td>
</tr>
<tr>
<td>Recalled learning mathematics through memorization.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Influences</strong></td>
<td>Mostly did not describe visions of an effective lesson, but rather gave traits of an effective lesson.</td>
<td>Described observations she saw during her junior year in her cooperating teacher’s classroom as a means for describing her visions of an effective lesson.</td>
<td>Described observations she saw during her junior and senior years while in her cooperating teacher’s classroom as a means for describing her visions of an effective lesson.</td>
</tr>
<tr>
<td><strong>Preservice</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Overall Vision</strong></td>
<td>Envisioned good tasks as those where the teacher models a procedure and students practice it on their own.</td>
<td>Envisioned tasks as those where students are engaged and learning conceptually.</td>
<td>Envisioned tasks as reform-oriented and wanted to show multiple ways to solve a problem.</td>
</tr>
<tr>
<td><strong>Overall Vision Development</strong></td>
<td>Increased during teacher-preparation program, but decreased below where she began as a junior by the end of her first year of teaching.</td>
<td>Increased continually throughout all three years.</td>
<td>Increased during teacher-preparation program, but decreased to almost where she began as a junior by the end of her first year of teaching.</td>
</tr>
</tbody>
</table>

*in relation to P-cohort average*
**Pre-training.** During their junior year, each of the preservice teachers answered questions regarding their experiences in K-12 mathematics. All three participants recalled positive feelings about elementary school mathematics; however, they also recounted memories of learning through memorization and procedures. For Jamie and Charlie, this way of learning created struggles and frustration with mathematics. Jamie recalled mathematics becoming increasingly more difficult as she progressed throughout her middle and high-school career, while Charlie mentioned that she cried over timed multiplication quizzes and became frustrated with only knowing one way to solve a problem as she moved into higher-level math courses. On the other hand, Jordan excelled at timed tests and drills because they were “purely memorization and [she] was so good at that”.

Throughout their teacher preparation program and first year of teaching, all three teachers envisioned tasks in ways that related to their experiences as learners of mathematics. Despite knowing that memorization was not effective for her as a learner, Jamie envisioned the teacher as someone who should modeled a procedure for students and then allow them to practice the procedure on their own. However, Charlie and Jordan both envisioned effective tasks quite differently than how they were taught. As a novice teacher, Charlie envisioned good teaching as showing students multiple ways to solve a problem. She also valued real-world problems and reform-oriented practices. Similarly, Jordan envisioned good tasks as those that were not memorization, but instead engaging, conceptual, and connected “to the underlying principle.”

**Preservice.** Throughout the duration of the study, each teacher described an effective lesson as a means for describing their visions of effective tasks. When asked this question,
Jamie did not describe a particular lesson, but she instead described what she believed to be effective teaching practices. However, Jordan and Charlie both drew on their experiences during their field placements. For example, during her senior year, Jordan described a lesson that she observed during her junior year where students used a number line placed on the classroom floor to understand they were counting spaces between numbers, not the numbers themselves. She felt this was effective because students were “not just working through a ton of procedural problems.” Similarly, Charlie drew on a field-experience observation that occurred during her junior year, in which students were counting and using their fingers as a way to “understand what numbers are”. Additionally, she described another lesson she observed during her senior year in which students were using manipulatives to represent a given fraction and were talking with one another and the teacher was not providing a lot of direct instruction. She felt this was effective because the students were developing skills and not just memorizing random stuff”. Furthermore, during her first year of teaching, Charlie continued to pull from her lessons taught during her field placements and described them as effective because she had students using manipulatives and working together.

**Overall vision and development.** Overall, Jordan, Charlie, and Jamie made gains in their visions of mathematical tasks respective to the VHQMI framework during the teacher preparation program; however, once into their first year of teaching, their trajectories were different. Jordan increased further, Charlie remained similar to where she started, and Jamie decreased lower than where she started. As shown in figure 19, Jordan increased in her visions between her junior and senior year (from a level 1.5 to a 2), and then again from her completion in the teacher preparation program and into her 1st year of teaching (from a level
This is indicative of her overall visions of tasks which include tasks where students are engaged and learning conceptually. Charlie, increased in her visions during her methods coursework throughout her junior year (level 1 to a 2); despite staying stable throughout the rest of her teacher-preparation program, she decreased during her 1st year of teaching almost back to where she began in her junior year (level 2 to a 1.5). This is indicative of her overall visions of tasks which include tasks that are reform-oriented with multiple solution strategies presented to students. Finally, Jamie increased in her visions from the end of her junior year and into her senior year (level 0.5 to a 1); however, she decreased back to a level 0.5 by the end of her senior year. Once into her first year of teaching, she continued to decrease further to a level 0. This is indicative of her overall visions of tasks which include tasks where the teacher models the procedure, and the students practice it on their own.

Furthermore, in relation to one another, Jamie, Charlie, and Jordan all entered the program with visions similar to one another (level 0.5, level 1, and level 1.5 respectively). However, throughout the program, their experiences, and possibly their MKT, influenced their development of mathematical visions. This led the three participants to end the program differently, with wider spread visions (Jamie – level 0, Charlie- level 1.5, Jordan – level 2.5).

Finally, in terms of their MKT, Jamie was below the P-cohort average throughout the teacher preparation program (despite increasing throughout), and also remained below Jordan and Charlie in her visions of high-quality mathematical tasks. Despite being classified as “average MKT”, Charlie was slightly above the cohort average in terms of MKT, while Jordan was more than 0.5 standard deviations above. As shown in their vision trajectories
(Figure 19), both teachers performed similarly on the VHQMI in terms of their visions of high-quality mathematical tasks (level 1-2). However, once in their first year of teaching Jordan surpassed Charlie by envisioning tasks at a level 2.5 versus Charlie at a level 1.5.

![Case-Studies' Visions of High Quality Mathematical Tasks](image)

*Figure 19. Case Studies’ Visions of High Quality Mathematical Tasks.*

**Cross-Case Comparison: Implementation**

As part of their participation in the study, each participant was videoed at the beginning and end of each year. Through these videos, participants were coded according to the IHQMI framework regarding their implementation of tasks. Similar to before, by engaging in a variety of experiences over time (*pre-training, preservice, induction*), we can see how they develop in the tasks they chose to implement with their students (Table 9).
Table 9.  
*Cross Case Comparison of Implementation.*

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Jamie</th>
<th>Jordan</th>
<th>Charlie</th>
</tr>
</thead>
<tbody>
<tr>
<td>MKT*</td>
<td>Increased MKT; Below average MKT</td>
<td>Decreased MKT; Above average MKT</td>
<td>Stable MKT; Average MKT</td>
</tr>
<tr>
<td>Pre-training</td>
<td>Struggled with mathematics. Recalled learning mathematics through memorization.</td>
<td>Loved mathematics because she was good at memorizing procedures.</td>
<td>Struggled with mathematics. Recalled being frustrated with timed tests and only knowing one way to solve a problem.</td>
</tr>
<tr>
<td>Influences</td>
<td>Used tasks given to her by her CT or tasks she found online. She modified only if she lacked resources to implement the task faithfully.</td>
<td>Used tasks given to her by her CT. She modified the context to make it more engaging for her students.</td>
<td>Used tasks given to her by her CT. She modified the tasks to include more discourse or to exclude repetitive examples.</td>
</tr>
<tr>
<td>Preservice</td>
<td>Implemented engaging tasks based upon what she felt that students “needed to know”.</td>
<td>Implemented centers in order to review material and stay on pace with other grade-level teachers.</td>
<td>Implemented tasks based upon a variety of resources from her county’s curriculum.</td>
</tr>
<tr>
<td>Induction</td>
<td>Overall lessons average = 1.00 on IHQMI.</td>
<td>Overall lessons average = 1.83 on IHQMI.</td>
<td>Overall lessons average = 1.67 on IHQMI.</td>
</tr>
<tr>
<td>Overall Development</td>
<td>Mostly implemented tasks focused on “practice and apply”.</td>
<td>Mostly implemented tasks focused on conceptual understanding.</td>
<td>Mostly implemented tasks that used real-world scenarios, but through a “practice and apply” approach.</td>
</tr>
</tbody>
</table>

*in relation to P-cohort average*
**Pre-training.** Despite Jamie’s positive experiences in K-12 school, she struggled with mathematics as a student. She recalled being taught many concepts through memorization, and this transferred into her practices once she became a preservice and novice teacher herself. Although her beginning-of-the-junior-year lesson was reform-oriented, her remaining five lessons were focused on showing students how to do a procedure and then asking them to practice it on their own. Similarly, Jordan, was also taught with memorization, which she enjoyed. However, once in the classroom, Jordan implemented tasks that were focused on students understanding “the underlying principle.” Finally, despite Charlie’s frustrations with K-12 school, due to only being taught one method and having to complete timed tests, she implemented these same practices with her students once she became a first-year teacher. In her classroom, she was observed giving timed tests as well as showing students how to solve a problem and asking them to solve other problems in the same way.

**Preservice.** Throughout their time in field-placement classrooms, Jamie, Jordan, and Charlie all were at the mercy of their cooperating teachers in terms of the tasks they could implement with their elementary students. Oftentimes, the cooperating teachers would give the preservice teachers a task to be taught to their students, and it was up to the PSTs to modify and implement the task with their elementary students. Jamie, for instance, used worksheets provided by her cooperating teacher, and only modified the task if they were lacking the resources required for the intended task. If she was not given a task, she used online resources to select tasks for the students to complete based upon what she felt they
needed at that moment. Jordan, also used lesson plans given to her by her cooperating teachers; however, she only modified them if she felt the context was not engaging enough for her students. Similar to the other two preservice teachers, Charlie also used the tasks given to her by her cooperating teacher. Unlike the other two, Charlie modified the tasks more often to include more discourse if she felt it was necessary, or to get rid of examples that were too repetitive.

**Induction.** Once in their first-year-of-teaching schools, all three preservice modified lessons from the intended curriculum to use with their students. For example, Jamie created tasks based upon “what they needed to know” and tasks that would keep her students engaged and on-task throughout. It was also during this year that she settled into a “practice and apply” routine with her students. Jordan, consistently tried to incorporate centers into her math lessons; however, she struggled with behavioral issues once she changed schools midway through her first year of teaching (also dropping from a level 2.5 to a level 1 on the IHQMI from beginning to end of the year). Additionally, Jordan planned the centers as a review of the material they had previously covered in class and also in order to stay on pace with the other teachers in her grade level. Finally, Charlie used a “pick and choose” method for selecting resources from her county’s curriculum in order to implement with her students. Due to end-of-grade tests, she felt the need to quickly get through the material she was required to before the end of the year, all while reviewing previous material.

**Overall implementation and development.** Overall, all three participants struggled with behavioral issues throughout their preservice and induction phases, which brought down
the level of their tasks throughout their time in the three-year study. Due to students being off-task and causing disruptions, teachers were unable to implement the tasks as intended.

Despite their implementation varying widely (i.e. more than 1.5 standard deviations) during their first three time points, all three participants ended both the teacher preparation program and their first year of teaching with similar levels of implementation (no more than 1 standard deviation apart). Additionally, while each participant did not have a clear trajectory throughout their teacher preparation program, once into their student teaching and first year of teaching their implementation seemed to become more stable. Overall, Jamie trended towards a level 1 on the IHQMI by the end of her first year of teaching (overall average of lessons = 1.00); Jordan trended towards a level 2 (overall average of lessons = 1.83); and Charlie trended towards a level 1.5 (overall average of lessons = 1.67; see Figure 20).

Furthermore, Jamie, who had below average MKT throughout the program, implemented tasks at lower levels than both Charlie and Jamie. Meanwhile, Jordan, who had above average MKT throughout the teacher preparation program implemented tasks that were mostly higher than her peers throughout the three-year study. Similarly, Charlie implemented tasks that were coded in between her above and below average peers for most time points throughout the study.
Comparison of Visions and Implementation. At the beginning of the study, all three cases were selected because of the change in their MKT scores from the beginning of the teacher preparation program through the end of the program. MKT was chosen because of the positive relationship between it and quality of instruction (Hill et al., 2008; Hill & Charalambous, 2012). Although Jamie increased in her MKT from the beginning of the teacher preparation program until the end, she only slightly increased in her visions during the beginning of her senior year, and then returned to the same visions she entered the program with (see figure 21). Additionally, she steadily decreased in her implementation, as coded on the IHQMI, with a slight increase in implementation during the end of her senior year, but not as high as her first junior year lesson. Furthermore, she was below average in terms of her MKT in relation to her cohort’s average and consistently performed lower on the VHQMI and IHQMI than her other two peers.
Jordan was selected because she decreased in her MKT throughout the teacher preparation program; however, she increased in both her visions and her implementation throughout the teacher preparation program (see figure 22). Despite her decrease in MKT from the beginning of the program to the end of the program, Jordan had above-average MKT in relation to her cohort. Similarly, she most often performed higher on the VHQMI and the IHQMI than her other two peers.

Figure 21. Jamie’s visions and implementation of high-quality mathematical tasks.

Jamie's Visions and Implementation of High-Quality Mathematical Tasks

Jordan was selected because she decreased in her MKT throughout the teacher preparation program; however, she increased in both her visions and her implementation throughout the teacher preparation program (see figure 22). Despite her decrease in MKT from the beginning of the program to the end of the program, Jordan had above-average MKT in relation to her cohort. Similarly, she most often performed higher on the VHQMI and the IHQMI than her other two peers.
Charlie was selected as a case because she stayed stable in her MKT throughout the teacher preparation program and she stayed close to the cohort’s average MKT. Overall, Charlie increased from the beginning of the teacher preparation program (a level 1) to the end of the program (a level 2) in her visions, as coded on the VHQMI (see figure 23). Additionally, although she increased in her implementation (a level 1.5 to a level 2) from the beginning of the program to the end of the program, she had a variety of scores in between (level 3 and level 0.5).
Charlie’s Visions and Implementation of High-Quality Mathematical Tasks

Overall, in terms of how their visions of high-quality mathematics tasks compared to that of their implementation, little alignment occurred during their time in the teacher preparation program. This may have occurred due to the preservice teachers having little choice over what they teach since they were in their cooperating teachers’ classrooms. However, once into their student teaching and first year of teaching, more alignment was seen between all three participants. Jordan had a direct alignment between her visions and implementation during her senior year and the beginning of her first year of teaching. Despite still using centers during the end of the year, she changed schools to a Title 1 school, where she reduced the cognitive demand of the tasks required of the students in each center. Charlie also showed alignment between her visions and implementation during her student teaching and first year of teaching, as she was only within 0.5 levels on the VHQM and IHQM.

Finally, although Jamie ended at a level 0 on the VHQM, and a level 1 on the IHQM, the

Figure 23. Charlie’s visions and implementation of high-quality mathematical tasks.
disconnect comes from the coding rubrics themselves and the translation of the levels from how they envisioned a task to how they implemented the task. For example, a level 0 in terms of visions on the VHQMI, means Jamie wanted tasks that were engaging and fun for her students, whereas a level 0 on the IHQMI meant the task was memorization based (i.e. typically not a fun lesson for students). Despite more of an alignment between the higher levels on the two rubrics, this may cause a first glance of Jamie’s comparison on the VHQMI and IHQMI graphs to not see any alignment.

Cross-Case Comparison: Post-Lesson Analyses

Throughout the study, participants were interviewed after each video-recorded lesson in order to conduct a self-analysis of the implemented lesson. Four overarching codes adapted from Santagata, Zannoni, and Stigler’s (2007) work were used to understand the teacher’s post-lesson self-analysis. These included: critique of the lesson, evidence of the lesson’s effectiveness, MKT, and student learning. Although each code had sub-codes, for the purposes of this cross-case comparison only the major codes will be examined and themes discussed (Table 10).
Table 10. 
**Cross Case Comparison of Post-Lesson Analyses.**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Jamie</th>
<th>Jordan</th>
<th>Charlie</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>MKT</strong></td>
<td><strong>MKT</strong></td>
<td><strong>MKT</strong></td>
</tr>
<tr>
<td><strong>Critique of the Lesson</strong></td>
<td>Mostly focused on students liking the lesson she implemented.</td>
<td>Focused on providing alternative suggestions to improve her lessons for future students.</td>
<td>Focused on, but struggled with, providing alternative suggestions to improve her lessons for future students.</td>
</tr>
<tr>
<td><strong>Evidence of the Lesson’s Effectiveness</strong></td>
<td>Also, focused on students working quickly as a means for effectiveness.</td>
<td>Also, focused on student conversations as a means of effectiveness.</td>
<td>Struggled during her senior year and first year of teaching to provide evidence of the lesson’s effectiveness.</td>
</tr>
<tr>
<td><strong>Components of Post-Lesson Analysis</strong></td>
<td>All three focused on correct answers.</td>
<td>All three discussed their lesson by providing evidence of using KCS, and KCT during the lesson.</td>
<td>All three discussed their lesson by providing evidence of using KCS, and KCT during the lesson.</td>
</tr>
<tr>
<td><strong>MKT</strong></td>
<td>Also, discussed a struggle with HCK and VCK throughout.</td>
<td>Also, discussed a struggle with HCK during senior year.</td>
<td>Also, discussed a struggle with CCK during junior year.</td>
</tr>
<tr>
<td><strong>Student Learning</strong></td>
<td>All three focused on themselves when analyzing whether student learning occurred.</td>
<td>All three focused on the students to understand if student learning occurred. Specifically, their written artifacts and their ability to discuss and work together.</td>
<td>All three focused on the students to understand if student learning occurred. Specifically, their written artifacts and struggles with mathematics.</td>
</tr>
</tbody>
</table>

*in relation to P-cohort average*
**Critique of the lesson.** After each of the six observed lessons, the novice teachers provided critiques as to how they felt their lessons went. When analyzing her lesson during her senior-year lesson, Jamie felt the lesson would have went better if she were to teach the lesson introducing word problems first and then the algorithm. Similarly, Jordan provided examples throughout many of her lessons that suggested alternative ways she would teach the lesson in the future. For example, during her junior and senior years she recognized that she introduced procedures too quickly and wanted to do a better job connecting representations to one another. Additionally, by the end of her first year of teaching, Jordan focused on the time it took to complete centers and wanted to restructure them in order to combat off-task students, but mentioned that her lesson went better than it had during her previous two tries with centers. Despite knowing she wanted to improve, Charlie struggled with providing alternatives suggestions for her lessons. She simultaneously felt that her lessons “went well” but also, she “could’ve done much better”. However, she did not elaborate how she could improve besides briefly mentioning during her first year of teaching that she should structure the groups differently but keep the tasks the same.

**Evidence of the lesson’s effectiveness.** Throughout their post-lesson analyses, all three preservice teachers focused on students’ abilities to obtain correct answers as evidence of the lesson’s effectiveness. Statements from each such as “their examples on their papers were correct… so I feel like they learned a little something” (Jamie), or that students “had a lot of right answers” (Charlie), or even noticing “correct answers on their whiteboards” (Jordan) provided evidence that each teacher valued students’ ability to obtain a correct answer as a means for the lesson being effective.
Despite focusing on correct answers, each participant also focused on other aspects they noticed while analyzing their implemented tasks. For example, Jamie felt that if students could complete the task quickly then the students understood what was being taught. Jordan focused on the conversations she overheard happening within the classroom as a means for whether the students were understanding the material. Charlie, however, felt the most uncertain whether the lesson was effective during her senior and first year of teaching by using phrases during her interviews such as: “I hope…”, “I think…” or “I don’t know”.

**MKT.** In order to understand how the preservice teachers referred to their mathematical knowledge for teaching, the MKT framework was used in conjunction with their post-lesson analyses. Overall, all three teachers described instances, throughout their interviews, that were coded as KCS, and KCT, although some provided more evidence of these dimensions than others. Additionally, each provided instances of other MKT dimensions such as CCK, SCK, HCK, and VCK.

When describing their lessons, Jamie, Jordan, and Charlie all described moments where they used their KCS or KCT. For example, their descriptions of how they provided support to struggling students, or students who needed challenged throughout their lessons, referenced their KCS. Similarly, when describing how they structured a particular lesson or how they would change a lesson in the future based upon what they know about the content and teaching all three teachers were describing their KCT.

Although they had similarities when discussing their MKT, each had differences as well. Charlie described her weak knowledge of content during her junior year (CCK), while Jamie used her CCK to further an example for students. When focusing on the language they
used with their students, Charlie and Jordan were describing their SCK. They mentioned that by not using proper mathematical language, something they learned during their methods coursework, they might communicate misconceptions to their students which could impact them in future lessons. Jordan and Jamie lacked horizontal curriculum knowledge (HCK) during their teacher preparation program because they both mentioned not knowing what their students had learned previously or where they were going with future lessons. Additionally, during Jamie’s first year of teaching she discussed struggling with vertical curriculum knowledge (VCK) as well by not having a grasp of what her students knew from first grade now that they were second graders.

**Student learning.** Although connected to the other dimensions, each preservice teacher provided insights as to whether they felt that student learning had occurred. At some point during their six lessons, each teacher felt that they were the source for student learning. Jamie, for example, continually felt she was the reason student learning occurred because she helped them get right answers, let them work together, helped created discussions in the classroom, and even told “them a more specific way to solve…” Jordan felt she helped student learning occur because she allowed students to explore, make connections, and she encouraged them to draw pictures instead of jumping straight to an algorithm. Finally, Charlie focused on how she allowed students time to think through problems as a means for student learning occurring.

Although similar in terms of focusing on themselves as a source for learning, each preservice teacher also focused on additional aspects of student learning. Charlie and Jordan both focused on students’ written artifacts as a means for whether learning occurred.
Additionally, Charlie focused on students’ struggle with the mathematics while Jamie focused on the materials she had available as a means for students’ learning. Unlike the other two participants, Jordan provided explanations for student learning occurring by describing how her students were able to discuss in groups, work together, and compare solution strategies whereas her peers did not.

**Overall development of post-lesson analyses.** Throughout their three years in the program, similarities emerged between all three cases’ post-lesson analyses. These similarities include: a focus on correct answers as a means of a lesson’s effectiveness; focusing on themselves as a source of student learning occurring; and also by describing instances throughout their lessons that was indicative of their mathematical knowledge for teaching, specifically their KCS, and KCT.

Despite these similarities, differences emerged as well. In regard to their MKT, Jamie, who had below average MKT in relation to her cohort average, analyzed her lesson by focusing on whether students liked her lesson and whether they could quickly complete the tasks she gave them. On the other hand, Jordan, who had above average MKT in relation to her cohort average, analyzed her implementation of mathematical tasks by focusing on whether students were discussing and working together as evidence of student learning occurring. Additionally, she provided several alternatives for ways in which she could improve her lessons for future students. Finally, Charlie, who had average MKT in relation to her cohort average, had traits of both of her peers. Like Jordan, she wanted to provide alternative suggestions to improve her lessons; however, like Jamie she did not actually provide any within her interviews. Furthermore, Charlie struggled to provide much evidence
of her lesson’s effectiveness during her senior year and first year of teaching other than whether the students were able to obtain correct answers. However, she did describe her students’ struggle with mathematics as evidence for their learning.

In addition to these differences, Jaime and Jordan struggled with curricular knowledge as identified through their lack of HCK and/or VCK. Jamie, lacked HCK and VCK when she described instances of not knowing where her students were knowledge wise within their grade level, or not knowing what knowledge they possessed when coming from a previous grade level. Similarly, Jordan struggled with HCK during her senior year by describing how she did not know what her students had been doing prior to her teaching her lessons. Furthermore, Charlie and Jordan both focused on using proper mathematical language (SCK) and how they worried they might communicate misconceptions to their students. Finally, Charlie and Jamie both described instances of their CCK. Charlie mentioned her own struggle with the lesson’s content during her junior year, while Jamie used her CCK to select students to use in her examples (based upon gender).
Chapter 8: Summary and Discussion

The following chapter has four main purposes. These purposes include: 1) summarizing the major findings from the study and discussing how those findings relate to previous research; 2) understanding the implications for mathematics teacher educators; 3) describing the limitations of the study; and 4) providing recommendations for future research.

The goal of this research was to understand how preservice elementary teachers develop in their visions, implementation, and post-lesson analyses of mathematical tasks throughout a teacher preparation program and into their first year of teaching. This study is important to the field of mathematics education because most studies, focused on tasks, have examined practicing teachers (Boston, 2012; Fennema et al., 1996; Galant, 2013; Silver & Stein, 1996; Stein, Grover, & Henningsen, 1996). Additionally, it fills a void in the literature related to how teachers select, plan, and evaluate tasks over time (Ball & Bass, 2002; Osana, LaCroix, Tucker, Bradley, & Desrosiers, 2006). Although generalizations cannot be made from a case-study analysis, it is my hope that through this research, mathematics teacher educators can better understand preservice elementary teachers’ visions, implementation, and analyses of their mathematical tasks. Furthermore, by providing these examples, we can better understand the role MKT plays in teachers’ visions, implementation, and post-lesson analyses, as well as help teachers align their visions with their instruction.

Summary of Major Findings

In order to understand how preservice teachers develop throughout their teacher preparation program and into their first year of teaching, a longitudinal, case-study approach
was used. The case-study approach was appropriate for this study because it allowed preservice teachers to be followed in-depth over time, and examined a variety of artifacts that led to the understanding of their visions, implementation, and post-lesson analyses of mathematical tasks. Through the use of interviews and video-recorded lessons, each case was described over six time points throughout the three-year study in order to address the research questions of the study. As a reminder to the reader, these research questions are as follows:

1. How do preservice and novice elementary teachers, with varying MKT, develop in their visions and implementation of mathematical tasks?
   a. To what extent are their visions and implementation of mathematical tasks aligned?

2. How do preservice and novice elementary teachers, with varying MKT, develop in their analysis of their implemented mathematical tasks?

   Using the benchmark interviews at the beginning and end of each year, the preservice teachers in the study described their visions of high-quality mathematical tasks as they developed into novice elementary teachers. Additionally, video-taped lessons at similar time points were examined in order to understand each participant’s development in their implementation of mathematical tasks. These two key pieces helped address the first research question. The findings are centered around the theoretical frameworks driving this study, specifically focusing on the experiences teachers have when developing in learning to teach (pre-training, preservice, and induction). Within each of these time points, themes emerged in the cross-case comparison such as: how their K-12 memories influence their vision and
implementation of mathematical tasks, how field placements and cooperating teachers impact preservice teachers’ visions and implementation of mathematical tasks, the role of methods coursework in influencing visions and implementation, the relationship MKT plays when understanding a teacher’s visions and implementation of mathematical tasks, and the impact resources plays when creating and implementing a lesson. I elaborate on each theme in more detail in the discussion section that follows.

Next, the post-lesson cognitive interviews were used in order to answer the second research question. Drawing from the literature (Santagata, Zannoni, & Stigler, 2007), four main apriori codes were used to interpret how the participants developed in the way they analyzed their implementation of mathematical tasks. These four codes were: critique of the lesson, evidence of the lesson’s effectiveness, MKT, and student learning. A description of how each preservice teacher developed throughout the study in terms of these four codes, as well as how they compared with one another was conducted in order to inform this research question. Within each of these codes, themes emerged in the cross-case comparison such as: the role MKT plays in teachers’ post-lesson analyses of mathematical tasks, preservice teachers’ focus on themselves as a means for student learning, their focus on correct answers as a means of the lesson’s effectiveness, and preservice teachers’ lack of HCK during field placements. I elaborate on each theme in more detail in the discussion section that follows.

Discussion

An examination of the findings for the two guiding research questions is presented below. In the first section, I describe each of the themes that emerged while examining the
Visions and Implementation. Throughout the study, several themes emerged while examining the preservice teachers’ visions and implementation of mathematical tasks. These themes are not provided as generalizations of all preservice elementary teachers; however, they provide insight into three teachers in a STEM-focused teacher preparation program with varying MKT. The themes include: how their K-12 memories influence their vision and implementation of mathematical tasks, how field placements and cooperating teachers impact preservice teachers’ visions and implementation of mathematical tasks, the role MKT plays when understanding a teacher’s visions and implementation of mathematical tasks, and the impact resources plays when creating and implementing mathematical tasks.

K-12 memories’ influence on visions and implementation of mathematical tasks. During their junior year in the teacher preparation program, all three participants were asked to recall their memories of K-12 mathematics. These memories contained preconceived notions of what teachers believed to be effective or non-effective practices based upon their own experiences as a student of mathematics (Feiman-Nemser, 1983; Masingila & Doerr, 2002). Two participants, Jamie and Charlie, both recalled struggling with mathematics and being taught procedurally. Despite these memories and feelings of frustration with mathematics, Jamie envisioned similar practices and continued these practices in her own classroom. Although Charlie was frustrated with the methods she was taught as a student of mathematics, she envisioned a classroom where students were shown multiple ways to solve a problem as well as envisioned reform-oriented practices. However, once in her first year of
teaching classroom, she reverted to how she taught as a K-12 student. Similar to the other two participants, Jordan, who loved mathematics, also recalled being taught procedures. Rather than mimicking what she had been exposed to during her own time as a student, she embraced reform oriented practices in her visions as well as in her own classroom. This finding validates Gurbuzturk and colleagues’ (2009) claim that with the right intervention, preservice teachers’ visions can be changed through experiences in their teacher preparation programs. Although this is only representative of three cases, these findings suggest that students’ memories and experiences in the K-12 classroom transfer to their visions and teaching practices.

*Field placements and cooperating teachers’ impact on visions and implementation of mathematical tasks.* Throughout their time as a preservice teacher, each participant was asked to describe their vision of an effective lesson in the classroom. Jordan and Charlie both drew on observations of lessons that occurred while in a cooperating teacher’s classroom. Furthermore, when asked to implement tasks during their field placements, all three preservice teachers used tasks given to them by their cooperating teachers with only slight modifications. This is consistent with findings by Moore (2003) and Valencia and colleagues (2009), such that preservice teachers try to please their cooperating teachers in order to fit into their classroom settings rather than try what they have learned during methods coursework. These findings suggest the significance of field placements in attributing to a preservice teacher’s visions and implementation. Furthermore, these findings suggest the importance of placing preservice teachers in a classroom in which the cooperating teacher’s visions align with that of the university.
Role of methods coursework in influencing visions and implementation of mathematical tasks. During their junior year, Jamie, Jordan, and Charlie were enrolled in two methods courses, specifically focused on K-2 and 3-5 mathematics. Upon completion, or during these courses, an increase in their visions of high-quality mathematical tasks was seen, as coded on the VHQMI. For Jamie and Jordan, this increase occurred post-methods coursework, while for Charlie it occurred during methods coursework. Furthermore, in terms of their implementation, Charlie and Jordan both implemented higher-quality mathematical tasks after completing one methods course. The methods coursework, that the participants in this study received, emphasized the importance of cognitively demanding tasks (Stein, Smith, Henningsen, & Silver, 2009). However, it is noted that more time could have been spent on reflecting and connecting their visions of high-quality mathematical tasks with their implementation of tasks while in their field placements. Therefore, these findings suggest the importance of methods coursework and allowing time to reflect on how their visions and implementation increase over time, as well as how well their visions and implementation are aligned with one another (Feiman-Nemser, 2001).

MKT’s role in understanding preservice teachers’ visions and implementation of mathematical tasks. All three participants were selected to be included in this study based upon the change in their MKT from the beginning to the end of their teacher preparation program. Jamie, Jordan, and Charlie were selected from tertiles that focused on those who: increased, decreased, or stayed stable in their MKT from the beginning of the program to the end of the program, respectively. Additionally, the three chosen participants represented
preservice teachers from their cohort who had above average MKT (Jordan), below average MKT (Jamie), or average MKT (Charlie), all in comparison to their cohort average.

Overall, the change in MKT from the beginning to the end of the teacher preparation program did not seem to influence the visions or implementation seen throughout the study; however, whether they were above, below, or average in relation to their cohort was seen as the most notable influence. Again, although generalizations cannot be made, MKT seemed to indicate the quality of instruction given to students throughout the study as well as the teacher’s visions of quality tasks. These findings hold true from previous research conducted by Hill and colleagues (2008) as well as Hill & Charalambous (2012) such that teachers with higher MKT also had higher quality of instruction in their classrooms. These findings show the importance in developing strong MKT in preservice and novice teachers in order to increase the quality of instruction given to elementary students.

**Impact of resources when creating and implementing mathematical tasks.** During their Junior year, Jamie, Jordan, and Charlie implemented the lessons given to them by the cooperating teachers, faithfully with little modifications. Once in their senior year, Jordan continued implementing the lesson with only some adaptations, while Charlie and Jamie began developing their lessons from multiple resources. However, once in their first year of teaching placements, all three novice teachers implemented tasks that were created from a variety of resources. Holstein and Keene (2013) suggest that teachers’ beliefs about teaching influence the fidelity in which they implement a curriculum. However, “if a teacher does not implement the materials as the authors intended, she or he could undermine the effectiveness of the new research-based curriculum” (Holstein & Keene, 2013, p. 618). The teachers in the
current study used a variety of reasons for implementing the tasks they used in their classroom such as focusing on what students “need to know” or creating a review to stay on pace with the other teachers in the grade level. These findings suggest the importance of providing novice teachers with resources that are standards-based and developed from research practices, as well as stressing the importance of implementing curriculum faithfully. Otherwise, teachers turn to less-reputable sources found online to fill the void in their classrooms. Despite feeling that these lessons are “more engaging” for their students, they lack coherence within the lesson and across multiple lessons as well.

**Post-lesson analyses.** In order to understand how the preservice teachers developed in the way they analyzed their lessons, post-lesson cognitive interviews were held with each participant. Sherin and van Es (2005) suggest that “the ability to notice classroom interactions is a key feature of teaching expertise” (p. 477). In order to understand what preservice teachers attend to in their analysis of implemented classroom lessons, four key features from research by Santagata, Zannoni, and Stigler (2007) were used. This analysis led to themes emerging from the cross-case comparison between participants. These themes include: the role MKT plays in teachers’ post-lesson critiques of mathematical tasks, preservice teachers’ focus on themselves as a means for student learning, preservice teachers’ focus on correct answers as a means of the lesson’s effectiveness, and preservice teachers’ lack of HCK during field placements.

**Role of MKT when critiquing task implementation.** Throughout each of their six lessons, the participants were asked how they felt the lesson went overall. Jordan, the participant with above average MKT, provided several alternatives to improve her lesson for
future students; while Jamie, the participant with below average MKT, provided little improvements, but rather focused on whether her students liked the lesson she implemented. Charlie, the participant with average MKT, knew she needed to make improvements, but struggled with what those improvements would look like.

These findings are similar to that of Santagata and colleagues (2007), in which they found teachers who had less sophistication in their analysis skills tended to focus positively on the lesson, whereas those with more sophistication discussed issues they saw overall and provided alternative suggestions to improve upon. Additionally, the findings of the current study are important for teacher preparation programs because, although overall generalizations cannot be made, in these instances teachers with more mathematical knowledge for teaching were able to provide stronger justifications for improvement to their lessons.

Focus on the teacher as a source for student learning. Throughout their time in the study, all three teachers pointed to themselves as a means for students’ learning. This is also similar to the findings of Santagata and colleagues’ (2007) study of preservice teachers, which emphasized the lack of development in preservice teachers’ analyzing abilities around student learning. This is important for mathematics teacher educators as they help to understand what novice teachers attend to while looking for student learning. Additionally, these findings provide evidence for the need of preservice teachers to gain experience focusing on what students are doing throughout the lesson and less on what they are doing as teachers.
Focus on correct answer as a means of the lesson’s effectiveness. When asked to provide evidence of why they believed the lesson was effective, all three participants focused on correct answers at some point during their time in the study. Although Jordan additionally focused on students’ conversations and Jamie focused on students working quickly to complete assignments, they felt that if students could provide the correct answers either orally or on a written assignment then they were understanding the material. The teachers in this study were able to provide specific evidence of the lesson’s effectiveness, similar to the teachers during the post-intervention phase of Santagata and colleagues’ (2007) study, rather than making general comments about the lesson overall. However, these findings show it is important to help preservice teachers focus on the importance of informal assessment (i.e. class discussions, students working collaboratively, critiquing the work of others, etc.) as well as use informal assessment as an indicator of student learning. Furthermore, these findings continue to point to the importance of developing MKT, as Jordan (who had above average MKT) was able to provide evidence of informal assessment whereas the other two were not.

Lack of horizontal content knowledge during field placements. Although Santagata, Zannoni, and Stigler (2007) noticed their teachers attending to the choices a teacher made as a means for describing the teachers’ mathematical content knowledge, I chose to further extend their subcodes by utilizing the MKT framework developed by Ball and colleagues (2008). In doing so, I was able to understand how Jordan and Jamie both cited struggles with their horizontal knowledge for teaching while in their teacher preparation program. Oftentimes they felt that their field placements only allowed them to see what was happening
in the classroom on the days they were there, and when asked to teach a lesson they did not
know what their students had learned before when they were not in the classroom. These
findings are important for teacher preparation programs because they suggest the need for
longer field placements for preservice teachers. Rather than being in a classroom a few days
a week, preservice teachers need to see the day-in and day-out workings of a classroom in
order to understand what their students know prior to teaching a lesson.

Implications for Mathematics Teacher Educators

This study shows how three teachers, with varying MKT developed in their visions
and implementation of mathematical tasks as well as how they developed in their post-lesson
analyses. As evident in existing research (Hill and Charalambous, 2012; Hill et al., 2008) as
well as this study, an increase in mathematical knowledge for teaching is needed to provide
quality tasks and instruction to students. Additionally, this study provided evidence of the
need for refining field placements for preservice teachers, as well as modifications to teacher
preparation programs in order to allow novice teachers time to reflect on their visions,
implementation, and analyses of lessons. Therefore, I present three implications for
mathematics teacher educators from this study. These include: 1) increasing MKT in
preservice and novice teachers; 2) refining field placements for PSTs; and 3) providing time
for reflection throughout their teacher preparation program.

Importance of increasing MKT in preservice and novice teachers. Although the
findings of this study are limited to three participants, they still illustrate the importance of
increasing MKT in preservice and novice teachers. As shown in this study, teachers with
higher MKT had higher visions of mathematical tasks as evident on the VHQM, had higher
implementation of mathematical tasks as evident on the IHQMI, and were more sophisticated in their analysis skills. Therefore, in order to improve MKT, teacher preparation programs should focus on engaging preservice teachers in high-quality mathematical tasks and allow them to watch and critique the implementation of mathematical tasks via videotaped lessons (Santagata & Angelici, 2010; Smith & Lane, 1996; Smith & Stein, 1998). Not only will this help to improve the quality of implementation of mathematical tasks, but it will also provide preservice teachers with opportunities to discuss ways to improve lessons for future students.

**A refinement of field placements for preservice teachers.** A second implication is focusing on the field placements during their teacher preparation program. Selecting cooperating teachers that share visions of the teacher preparation program are crucial in supporting preservice teachers’ visions of high-quality mathematical tasks (Mewborn, 1999). Although finding cooperating teachers with similar visions may be challenging in rural areas where the number of preservice teachers exceeds that of cooperating teachers, creating opportunities for professional development with the local university is a way to improve inservice teachers’ visions and align them with that of the university.

Furthermore, preservice teachers need field placements that allow for them to gain horizontal content knowledge, or knowledge of the grade level in which they are a part of while, they are in the classroom (Ball et al., 2008; Shulman, 1986). Oftentimes, preservice teachers enter elementary classrooms once or twice a week in order to fulfill field placement requirements. Due to sporadic visits, preservice teachers struggle to gain a horizontal understanding of the grade level and struggle with students’ understanding when they teach a lesson to the class. It is my suggestion that field placements provide more consecutive days
within a classroom in order for the preservice teacher to gain an understanding of how the day-to-day workings of a classroom run before they reach student teaching. Furthermore, by providing these longer placements, preservice teachers gain horizontal curriculum knowledge of the grade in which they are a part.

**Providing time for reflection during teacher preparation programs.** The third implication of this research is for teacher preparation programs. In order for preservice teachers to make strides in their visions of high-quality tasks, they need time to reflect upon these visions while in their teacher preparation program. This reflection may come through class discussions or even journaling. Through journaling, preservice teachers would have the opportunity to reflect on their visions at multiple times throughout the semester and track growth over time.

Similarly, preservice teachers need time to reflect upon their implementation of mathematical tasks taught to elementary students while they are in their teacher preparation program and in their first year of teaching. Reflection is critical to teacher improvement (Santagata, 2010; Sherin & van Es, 2005). By allowing preservice teachers the opportunity to reflect on video-taped lessons, either with a university instructor or through peer mentoring, preservice teachers will begin to align their visions of high-quality tasks with the implementation of tasks in the classroom. Additionally, these reflections will help novice teachers attend to the students in the classroom, rather than focus solely on the teacher. It is through these reflections that preservice teachers will be able to discuss opportunities for modifications to lessons given to them by cooperating teachers that they might not have noticed on their own.
Limitations of the Study

Although I am a researcher with the original project and helped collect data on the preservice teachers in this study, the data itself was pre-collected and therefore could not be edited to fit with my current study. This is a limitation because I could not ask specific questions that I would want to know now about the novice teachers, nor could I go back and gather information on their teaching practices during their time in the teacher preparation program. Additionally, the participants in the study were interviewed by a variety of Project ATOMS team members throughout their three years in the study. This variety in interviewers can cause issues with the amount of probing that occurred as well as the comfortableness of the participant in responding to the questions asked. Likewise, one of the three participants in the current study, was interviewed by myself during their first year of teaching.

Furthermore, the lessons taught by the teachers during their first year of teaching were mostly focused on review. This is a limitation because it may impact their implementation level on the IHQMI versus a lesson that introduces a topic. By asking teachers to video-record lessons that develop a concept rather than a review lesson, we can understand more about the way they truly implement tasks in their classroom.

Finally, the findings of this study are limited to the three cases presented. Despite themes emerging from the data, these themes cannot be generalized to all preservice teachers. Therefore, more research is needed to understand how larger groups of preservice teachers develop over time in their visions and implementation of mathematical tasks.
Recommendations for Future Research

The research in this study is but a stepping stone in understanding how preservice teachers develop into novice teachers. First, as stated before, scaling this research up in order to understand how preservice and/or novice teachers develop in relation to their visions and implementation of mathematical tasks would be useful to teacher preparation programs. By understanding how more teachers envision good tasks as well as implement tasks in their classroom we could begin to help better align these two components. Additionally, giving teachers the LMT-MKT assessment along with their corresponding visions and implemented lessons, we could begin to track over time the connections between MKT and visions.

Secondly, as important as it is to scale the research up to generalize the findings to more preservice teachers, it is also important to scale the research down to a finer grain to better understand one teacher’s development over time. This would allow for more time points to be analyzed and considered, rather than only six. While six time points allowed for multiple teachers to be compared, by analyzing one teacher over multiple time points (i.e. more than six) we can begin to get a sense for what components in a teacher preparation program have the most impact on their visions as preservice teachers. Through this examination, we can begin to pinpoint pivotal moments that shape the visions of preservice teachers in relation to the tasks and discussions they are completing in their methods coursework.

A third question to examine moving forward would be how preservice teachers develop in their visions and utilization of mathematical discourse in the elementary classroom. This is important because discourse is critical to increasing student learning
(Walshaw & Anthony, 2008) as well as provides support to the mathematical tasks used within a lesson. By examining discourse with preservice and novice teachers, we can understand the components they attend to while facilitating discussions in their classroom and better align them with what they envision to happen.

A fourth question in which to study, is how cooperating teacher’s visions align with that of the preservice teacher and university. By understanding how a cooperating teacher’s visions align, we can begin to understand the lessons implemented by the preservice teacher during their field placement in the cooperating teacher’s classroom. Furthermore, by enacting a professional development aimed at increasing inservice teachers visions of high-quality mathematics we can see how their visions change over time which is important to the development of preservice teachers placed in these teacher’s classrooms.

Fifth and finally, this study should be replicated with a parameter that the video-recorded lessons need to be the teacher’s idea of a “best practice” or focused on introducing a concept. The teachers in this study were told in advance their lesson would be observed by a member of the Project-ATOMS team, however, they did not have criteria placed upon them for what the lesson must look like. By introducing these conditions, we can see whether their implementation would look different from the review-oriented lessons presented in this study.

Conclusion

The first couple of years of teaching are critical in the learning-to-teach process (Feiman-Nemser, 1983). It is my hope that through this research, we, as mathematics teacher educators, can better equip our preservice teachers for their own classrooms. In order to do
so, we must work to strengthen the MKT of preservice teachers as well as the field placements in which they are placed. Additionally, we must provide support to preservice teachers through the guidance of expert teachers by creating communities in which they can be a part of and ask questions related to teaching.

Although only three teachers were observed as they moved through their teacher preparation program and into their first year of teaching, this study allowed for a glimpse into the development of preservice teachers’ visions, implementation, and analysis of mathematical tasks. This work reaffirmed previous studies around the importance of MKT (Carpenter et al., 1989; Copur-Gencturk, 2015; Hill & Charalambous, 2012; Hill et al., 2005; Hill et al., 2008; Ma, 1999), field placements (Darling-Hammond, Chung, & Frelow, 2002; Ebby, 2000; Gallego, 2001; Killian & McIntyre, 1986; Livingston & Borko, 1989; Mewborn, 1999), the role of visions (Hammerness, 2001; 2008), as well as understanding what teachers attend to when analyzing a lesson (Cavanagh & McMaster, 2015; Santagata, Zannoni, & Stigler, 2007; Sherin & van Es, 2005). Therefore, based upon existing research and the current study, mathematics teacher educators should continue to work to increase MKT in preservice teachers in order to increase the quality of instruction in elementary classrooms. Furthermore, by providing field placements with cooperating teachers who share visions similar to the university in which preservice teachers are a part of, we can reinforce the reform-oriented practices being taught to preservice teachers. Finally, by allowing preservice teachers time to reflect on the visions they hold in relation to tasks, allowing them time to align their instruction with these visions, as well as providing opportunities to analyze their
teaching with the help of expert teachers through a community of practice, we are providing preservice teachers the opportunity to grow in their development of learning to teach.
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Appendices
Appendix A: Interview Questions for Fall 2012 - Spring 2015

Project ATOMS

*First letter denotes the year of the study (J = Junior, S = Senior, 1 = 1st year of teaching) and BOY = Beginning of Year, MOY = Middle of Year, EOY = End of Year. Example: J_BOY = Junior, beginning of year interview

<table>
<thead>
<tr>
<th>Visions</th>
<th>Question Asked</th>
<th>Timepoint Asked</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visions</td>
<td>Describe a math lesson in an elementary school classroom that you would consider effective and explain why you consider it to be effective.</td>
<td>J_BOY; J_EOY; S_BOY; S_EOY; 1_BOY; 1_EOY</td>
</tr>
<tr>
<td>Visions</td>
<td>What the teacher does and what the students are doing during mathematics instruction are really important. Describe what you think the teacher should be doing most of the time. Describe what you think the students should be doing most of the time.</td>
<td>J_BOY; J_EOY; S_EOY; 1_BOY; 1_EOY</td>
</tr>
<tr>
<td>Visions</td>
<td>How does a good teacher select the task he or she will use with students?</td>
<td>S_BOY; S_EOY; 1_BOY; 1_EOY</td>
</tr>
<tr>
<td>Visions</td>
<td>How has your vision of instruction in math has changed as a result of your methods courses over the last three semesters?</td>
<td>S_MOY</td>
</tr>
<tr>
<td>Visions</td>
<td>How will you choose which math content to teach?</td>
<td>S_MOY; S_EOY; 1_BOY; 1_EOY</td>
</tr>
<tr>
<td>Visions</td>
<td>How will you choose the best math tasks or activities?</td>
<td>S_MOY; S_EOY; 1_BOY; 1_EOY</td>
</tr>
</tbody>
</table>
Interview Questions (continued).

<table>
<thead>
<tr>
<th>Post-Lesson Analyses</th>
<th></th>
<th>J_BOY; J_EOY; S_EOY; 1_BOY; 1_EOY</th>
</tr>
</thead>
<tbody>
<tr>
<td>So, let’s talk about how the lesson played out. How do you think it went?</td>
<td></td>
<td>J_BOY</td>
</tr>
<tr>
<td>Explain how you addressed the mathematical concept you were teaching and was it effective or ineffective? OR What evidence do you have as to whether the task(s) were appropriate and effective in helping them understand the concept?</td>
<td></td>
<td>J_EOY; S_BOY; S_EOY; 1_BOY; 1_EOY</td>
</tr>
<tr>
<td>What would you have done differently?</td>
<td></td>
<td>J_BOY; J_EOY; S_EOY; 1_BOY; 1_EOY</td>
</tr>
<tr>
<td>What do you think the students learned from the lesson and what makes you think that?</td>
<td></td>
<td>J_BOY; J_EOY; S_BOY; S_EOY; 1_BOY; 1_EOY</td>
</tr>
<tr>
<td>So, you planned and selected particular task(s) for students to complete in the lesson. Comment on your facilitation of the task and your students’ completion of the task during implementation.</td>
<td></td>
<td>J_EOY; S_BOY; S_EOY; 1_BOY; 1_EOY</td>
</tr>
</tbody>
</table>