ABSTRACT

WILLIAMS, DEREK ANDREW. Student Experiences in Community College Precalculus: A Mixed Methods Study of Student Engagement and Understanding. (Under the direction of Dr. Karen Hollebrands).

The relationship between student engagement and achievement in mathematics classrooms has been investigated in K-12 settings. Whereby, student engagement has been empirically linked to mathematical growth, academic achievement, and learning. However, the literature lacks studies of student engagement in mathematics classrooms at the undergraduate level, where community college mathematics classrooms are particularly under-represented. Further, calls for research on the effects of teaching on student achievement in undergraduate mathematics are increasingly abundant. This research uses mixed methods and flow theory to describe the nature of student engagement in community college precalculus, characteristics of any relationships between student engagement and understanding of precalculus concepts, and the effects of community college precalculus instructors’ teaching approaches on student engagement and the engagement-to-understanding relationship. Fifteen community college instructors and 101 of their students from two community colleges participated in this study during the Fall of 2016. Weekly throughout the semester, student participants completed a survey designed to capture indicators (interest, enjoyment, and concentration) and facilitators (e.g., perceptions about their instructor’s care for their understanding) of student engagement for each week of the study. In addition to Likert-type items, the weekly survey also prompted students to briefly describe their experiences in precalculus from the previous week as if they were making a social media post. Highly engaged students were observed during three class meetings and participated in two interviews.
Quantitative and qualitative results show that student engagement varies within- and between-students from week-to-week, and is associated with students’ perceptions about teaching approaches, psychological needs fulfillment, and personal mathematical ability. Further investigation into the nature of student engagement of highly engaged students reveals the importance of relevant and challenging tasks on student engagement. Additionally, highly engaged students discuss group work as particularly engaging.
Student Experiences in Community College Precalculus: A Mixed Methods Study of Student Engagement and Understanding

by

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BIOGRAPHY

Derek Andrew Williams was born on July 31, 1986 in San Bernardino, California to mother, Robin Williams and father, U.S. Air Force Captain (ret.) Daniel Williams. Derek is the eldest of three. Derek and his family moved from California to Germany in 1989, and returned to the U.S. in 1992, where they settled in Raleigh, North Carolina.

Derek attended grade-school in Raleigh and moved to Greenville, North Carolina to attend East Carolina University. While there, Derek obtained a Bachelor’s of Science in secondary mathematics education and a Master’s of Arts in mathematics. Following undergraduate graduation, Derek married his high-school sweat-heart and long-time girl-friend, Caitlin. Derek and Caitlin now have three children, Hailey (5), Tristan (2), and Landon (infant).

Derek taught high school mathematics for three years, where he also acted as the men’s and women’s varsity soccer coach. Derek accepted a position at Wake Technical Community College in January of 2012, and continued to work at Wake Tech as an assistant professor of mathematics while pursuing a Doctorate of Philosophy in mathematics education at North Carolina State University. As a doctoral student, Derek also served as the preservice manager for the Noyce Project, an NSF-funded scholarship and research project focused on preparing preservice mathematics teachers to teach in high needs school districts.

Following graduate school, Derek would like to acquire a tenure-tracked position at a research institution.
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Chapter 1: Introduction

The intent of this chapter is to provide the reader with background information from the field in which this study is situated, theoretical underpinnings and personal experiences undergirding the study, and an overview of the study including its general purpose.

Background of the Study

In their chapter of *The Second Handbook of Research on Mathematics Teaching and Learning* (National Council of Teachers of Mathematics [NCTM], 2007) Hiebert and Grouws (2007) brought issues related to the teaching and learning of undergraduate mathematics to the forefront, indicating that measuring the effects of teaching on student learning in undergraduate mathematics is an unanswered question. Shortly after this call, numerous research studies in the arena of undergraduate mathematics education were conducted. For example, Carlson and colleagues (Carlson, Oehrtman, & Engelke, 2010) began a research endeavor to identify and measure students’ understanding of precalculus concepts (e.g., covariation and function). The Mathematical Association of American (MAA) orchestrated a national longitudinal study on the state of first semester Calculus education at the undergraduate level (Bressoud, Mesa, & Rasmussen, 2015), and Mesa and colleagues (2014) have begun research on community college instructors’ pedagogies (i.e. teaching approaches). This work is paramount to understanding the effects of teaching on student learning of undergraduate mathematics.

Unfortunately, community college students and teachers are frequently under-represented populations included in research conducted in this arena (Mesa, Wladis, & Watkins, 2014; Mesa, in press; Sitomer et al., 2012). This is peculiar because community colleges are teaching institutions (Mesa, Celis, et al., 2014) that serve approximately half of all undergraduate students (Dowd et al., 2006) in the United States and roughly half of all undergraduate mathematics
students (Rodi, 2007). However, researchers of community college settings are paying close attention to student success (Baldwin, 2012); instructional practices (e.g., Mesa, Celis, et al., 2014); and especially for mathematics education, there is a strong focus on the effectiveness of remedial courses – not-for-credit mathematics courses that students are required to take prior to enrolling in college level mathematics courses (e.g., Attewell, Lavin, Domina, & Levey, 2006; Aycaster, 2001; Bailey, 2009). This work is important, and has just cause, as a considerable amount of federal and state funding is used to support remediation programs at community colleges (Aycaster, 2001; The Institute for Higher Education Policy, 1998). However, mathematics education research on entry level college credit courses, such as precalculus, at community colleges seems absent in the literature (Mesa, Celis, et al., 2014).

Historically, community college students enrolled in precalculus struggle to pass the course on their first attempt (Barnes, Cerrito, & Levi, 2004). However, this has not sparked a research interest in investigating the nature of community college students’ experiences and engagement while taking precalculus. In general, research on student experiences is less prevalent in the literature than studies focusing on teachers or teaching (Knapp, 1997). This is especially true for community college mathematics students (Mesa, Celis, et al., 2014). Kuh (2003) has made tremendous ground in measuring students’ engagement at their institution on a macro-level through a quantitative survey. McClenney (2007) furthered this effort to examine community college students’ engagement. This work is beneficial to gaining an understanding of community college students’ experiences as community college students – or university students in the case of Kuh’s work. However, a deeper examination through mixed methods inquiry would allow for a more detailed investigation into students’ experiences and engagement as students of a particular classroom. This study extends work prompted by the call from Hiebert &
Grouws (2007) while addressing some of the gaps in the research literature in this area. In particular, the research questions examined in this study are:

1. What is the nature of community college students’ engagement during precalculus, and what role do teaching approaches have on these experiences?

2. Is there a relationship between student engagement and understanding of precalculus topics, and if so,
   a. What are characteristics of this relationship?
   b. Is there an association between this relationship and teaching approaches?

**Theoretical Framework**

This section provides a discussion of the theoretical underpinnings and personal experiences guiding my perspective and approach to inquiry. These theories and experiences are relied upon to gain a better understanding of community college students’ engagement while studying precalculus.

**Inquiry worldview.** This work is informed by my philosophical assumptions about the nature of reality (ontology) and knowledge acquisition (epistemology). Subjective realities are constructed through the lived experiences of individuals as they interact with each other. Thus, reality is constructed by the subjective understandings developed from social and cultural experiences. In this regard, learning takes place through investigating the processes of interactions among members of a social group. These beliefs about ontology and epistemology are consistent with a social constructivism worldview. This worldview guides the formation of research questions and procedures for data collection and analysis.

**Subjectivity statement.** As an undergraduate, I studied secondary mathematics education with aspirations of becoming a high school mathematics teacher. I always enjoyed
studying mathematics and working with others who struggled to understand the connections between mathematical topics. During my tenure as an undergraduate, I learned that I was passionate about teaching but overjoyed by studying mathematics. To pursue my interest, I entered a graduate program in mathematics. As a master’s student, I was a teaching assistant for college algebra and tutored in the university’s math center. This is where I first became cognizant of the issue my research intends to study – college students’ struggle with precalculus.

After earning a master’s degree, I became a high school mathematics teacher. During this time, I taught a variety of algebra-based courses ranging from introductory pre-algebra to honors precalculus. Through this experience, I learned that college students were not the only ones struggling to understand precalculus concepts, such as covariation and function. In fact, most of the courses that I taught covered these ideas, and many topics were repeated from one course to the next. For example, quadratic, exponential, and logarithmic functions were in the curriculum for many of the high school mathematics courses I taught. After four years of teaching high school, I took a position at a nearby community college where I continue to teach precalculus (and many of the same topics). My high school and college teaching experiences have furthered my interests in understanding students’ struggles with precalculus; particularly community college students, who have potentially seen the materials numerous times over their academic careers.

Now, I am full-time doctoral student studying mathematics education and continue to teach community college mathematics full-time. As an instructor, I am reminded daily of the struggle that many students encounter while taking precalculus, and as a doctoral student, I am engrossed in the literature surrounding recommendations about the teaching and learning of mathematics (Council of Chief State School Officers, 2010; NCTM, 1989, 2000, 2014). I believe
that students learn best from teachers who have high expectations of them and compassion for them as people. Learning is a social activity that requires productive struggle, confusion, challenge, and sometimes frustration while in the confines of a community established to capitalize on productive struggle in order to promote progress (Boaler, 2015). I am a researcher who is deeply interested in community college students’ experiences and how they interact with the ontological and epistemological assumptions held by their instructors. As such, my approach to research is informed by my social constructivist worldview.

**Foundational research and theories.** In conjunction with my inquiry worldview and personal experiences, this study is also informed by many research studies and theories. To consider the role of teachers’ pedagogies in the social environment created in every mathematics classroom, *teaching approaches* (Mesa, Celis, et al., 2014) must be investigated. According to Mesa and her colleagues, teachers employ a combination of three approaches to teaching: traditional, meaning-making, and student-support. Instructors who utilize a traditional approach to teaching tend to “privilege the content and instructor’s authority in the classroom” (Mesa, Celis, et al., 2014, p. 132). The primary focus of these instructors’ lessons is content coverage, sometimes at the expense of student understanding. Instructors employing the meaning-making approach strive to promote deeper mathematical understanding and to make connections between mathematics and real-world contexts. Also, meaning-making instructors assimilate their role to that of an athletic coach, where successes and failures of the group are shared and overall movement towards a common goal (understanding mathematics) is the objective for each lesson. Finally, instructors adopting the student-support approach do not place content coverage or mastery at the forefront of every lesson. Instead, these instructors focus on student affect, and
aim to improve students’ mathematical self-efficacy. All instructors draw upon each of these primary teaching approaches, but most tend to adopt one predominant approach to teaching.

More specific instructional approaches can be categorized by one or more of the teaching approaches identified by Mesa et al. (2014). For example, according to Rasmussen & Kwon (2007), inquiry oriented instruction

…enables teachers to construct models for how their students interpret and generate mathematical ideas. Second, it provides opportunities for teachers to learn something new about particular mathematical ideas, in light of student thinking. Third, it better positions teachers to build on students’ thinking by posing new questions and tasks (p. 190).

Thus, teachers employing inquiry oriented instruction in this way are utilizing a meaning-making teaching approach.

A second theory that informs the proposed study is Flow Theory (Nakamura & Csikszentmihalyi, 2009). Flow theory conceptualizes student engagement through emotional (i.e., interest and enjoyment) and cognitive (i.e., concentration) aspects, and argues that student engagement is facilitated by a delicate balance of perceived skill versus task challenge (Csikszentmihalyi, 1990, 1997; Nakamura & Csikszentmihalyi, 2006; Shernoff, Csikszentmihalyi, Schneider, & Shernoff, 2003). Through flow theory, student engagement can be measured at the classroom level and intra-individual changes can be documented (e.g., Shernoff et al., 2003). Thus, flow theory provides a focused lens through which to view student engagement and how engagement fluctuates over time in a social learning environment such as a mathematics classroom.
Additionally, students’ understanding of precalculus concepts will be described according to documented conceptualizations in the literature (e.g., Carlson, Jacobs, Coe, Larsen, & Hsu, 2002). An example of one such framework describes students’ understanding of covariation, which is an important concept of precalculus. In their framework, Carlson et al. (2002) present a developmental progression with five levels of covariational reasoning. Thus, the final component of the substantive content theory informing this research study is the literature surrounding student conceptions of precalculus concepts (e.g., Carlson et al., 2002; Carlson, Oehrtman, & Engelke, 2010; Castillo-Garsow, Johnson, & Moore, 2013; Monk, 1992; Moore & Thompson, 2015; Stevens & Moore, 2016; Thompson, 2011).

**Purpose Statement**

This section provides a description of the overall purpose of this study as well as an overview of procedures. This section is only an overview; all elements of this section will be elaborated on in detail in the methodology chapter (chapter 3). The intent here is to relate the study to background literature and to provide readers with a general idea of its scope and significance to the field of undergraduate mathematics education.

**Statement of the problem.** Historically, community college students enrolled in precalculus struggle to pass the course on their first attempt. Although this has been documented (e.g., Barnes et al., 2004) it has not sparked a research interest in determining the nature of community college students’ experiences while taking precalculus nor is there work focused on pedagogies used in engaging first-time precalculus students at community colleges. In fact, research in undergraduate mathematics education is aimed at higher level mathematics courses such as calculus (Bressoud et al., 2015; Oehrtman, 2008) or is addressing concerns pertaining to
student success in remedial courses (e.g., Aycaster, 2001). As a consequence, poor success rates for community college students enrolled in precalculus persist.

**Goals of the research.** This research project has two goals aimed at addressing the call stated above. The first goal is to better understand the lived experiences of students enrolled in precalculus courses at a community college. Student experiences are connected to their instructor’s pedagogies; thus, in order to completely address the first goal of this project it is necessary to investigate the nature of teaching in precalculus at community colleges. Therefore, the second goal of this project is to examine the nature of pedagogical practices utilized by community college instructors teaching precalculus courses. Specifically, this project focuses on identifying and characterizing teaching approaches employed by instructors, how teaching approaches elicit student engagement during class time, and the associations with student understanding.

**Mixed methods definition and rationale.** A mixed methods approach is used for this study. Mixed methods research is defined by Currall as research that “… involves the sequential or simultaneous use of both qualitative and quantitative data collection and/or data analysis techniques” (as cited in Johnson, Onwuegbuzie, & Turner, 2007, p. 119). Mixed methods are used for this study because of the complexity of students’ lived experiences. In order to investigate this complexity, both qualitative and quantitative data collection and analysis techniques will be used. A mixed methods approach allows for student understanding (measured quantitatively) to be interpreted in conjunction with interviews, responses to weekly electronic diaries, and field observations – capturing student engagement and instructor pedagogies. Thus, mixed methods research provides an opportunity to report on students’ engagement in precalculus more completely than with qualitative or quantitative methods of inquiry alone.
**Significance of the study.** This study is important to the field of undergraduate mathematics education for numerous reasons. To begin, there is a gap in the literature on community college precalculus and on students’ experiences learning (Mesa, Celis, et al., 2014). This study will serve to address both of these gaps. Second, an identified problem with student success in precalculus exists (Barnes et al., 2004), and more information needs to be gathered on why this is the case as well as used to inform strategies for solving this issue. The overarching purpose of this study is to investigate the community college precalculus performance issue from a virtually unexplored angle – through student engagement.

Results from this study can be used to inform professional development of community college instructors on best practices for engaging students in precalculus, community college precalculus course design, or student orientation seminars on being successful in community college precalculus. Moreover, results from this study have been acquired based on student-reported data. In this regard, results can be interpreted as students’ collective voice as they inform us (community college precalculus instructors and undergraduate mathematics education researchers) of their stance on what constitutes an engaging learning environment.
Chapter 2: Literature Review

As a construct studied in its own right, student engagement has been an interest to the fields of behavioral and educational psychology since the late 1980s. Research on student engagement originates from work by educational psychologists who were interested in explaining the causes of students dropping out of high school and the events leading them to decide to withdraw from school prior to graduation (Fredricks, Blumenfeld, & Paris, 2004; Reschly & Christenson, 2012). These early researchers discovered that engagement, motivation, participation, involvement, alienation, disaffection, and other sometimes interchangeably used terms were linked to students’ decisions to withdraw from school. Further, researchers determined that these constructs were malleable (Finn & Rock, 1997; Finn & Zimmer, 2012; Fredricks et al., 2004; Newmann, 1989; Newmann, Wehlage, & Lamborn, 1992; Reschly & Christenson, 2012; Skinner, Furrer, Marchand, & Kindermann, 2008; Skinner, Kindermann, & Furrer, 2009), and that teachers and schools have the capabilities of designing environments that could result in positive student experiences, which in turn could lead to lower dropout rates (Finn, 1989; Fredricks et al., 2004; Newmann, 1989; Newmann et al., 1992; Skinner & Belmont, 1993). In addition to engagement being malleable and connected to social contexts, early engagement researchers were also faced with the issue that race/ethnicity and socioeconomic status (SES) were the existing hypothesized factors associated with dropout rates in the United States (Finn, 1989; Finn & Rock, 1997; Fredricks et al., 2004; Newmann, 1989; Reschly & Christenson, 2012). Thus, the construct of engagement provided hope for action to address (or at least explain) the dropout rates in the US, as the alternative predictors of race/ethnicity and SES are impossible (or extremely difficult) to change.
Over time, many different conceptualizations of student engagement emerged; however, agreement on a definition and the composition of engagement is still pending (Fredricks et al., 2004; Reschly & Christenson, 2012). In his seminal theory (Reschly & Christenson, 2012), Finn (1989), aligns engagement with participation, and conceptualizes the construct on a continuum of participation accompanied by a cyclical relationship with school identification (Finn & Cox, 1992; Finn & Rock, 1997; Finn & Zimmer, 2012). Newmann (Newmann, 1989; Newmann et al., 1992), considers student engagement through participation, attachment, and integration in social contexts and through authentic work (Newmann et al., 1992). Skinner and her colleagues (Skinner & Belmont, 1993; Skinner et al., 2008, 2009), operationalize engagement on a pair of continua of emotional and behavioral engagement versus disaffection. Self-determination theory (SDT) aligns student engagement with motivation, positing that engagement is the observable manifestation of motivation (Deci & Ryan, 2008; Deci, Vallerand, Pelletier, & Ryan, 1991; Reeve, 2012; Ryan & Deci, 2000). Finally, Flow theory conceptualizes student engagement through emotional (i.e., interest and enjoyment) and cognitive (i.e., concentration) aspects, and argues that student engagement is facilitated by a delicate balance of perceived skill versus task challenge (Csikszentmihalyi, 1990, 1997; Nakamura & Csikszentmihalyi, 2006; Shernoff et al., 2003).

In the next section, these different perspectives of student engagement will be discussed and then compared and contrasted in order to shed light on how student engagement is conceptualized in the psychology literature. Then, focus is narrowed to student engagement in mathematics classrooms, where empirical work investigating engagement in that setting is synthesized.
Perspectives of Student Engagement

**Participation-identification model.** Considered a seminal theory of student engagement (Reschly & Christenson, 2012), Finn’s participation-identification model conceptualizes the construct along a continuum of participation which interacts cyclically with students’ need to identify with their school and classroom (Finn, 1989). Finn first conceived of this model as a means for explaining why students withdraw from school prematurely, stating,

> While the model does not explain all instances in which youngsters leave school before graduation, it does portray the total withdrawal of some individuals as a process of disengagement over time, and not as a phenomenon that occurs in a single day or even in a single school year (p. 133, emphasis in the original).

The underlying assumption of the model is that participation in school activities is paramount for student success and achievement. In this sense, participation is defined as the level of commitment or involvement that students exhibit within their school or classroom. Finn (1989), suggests that participation is subdivided into four levels: level-one consists of students following instructions from their teachers, attending class regularly and on time, and being prepared; level-two participation is described as students displaying enthusiasm towards learning by completing extra assignments, initiating classroom conversations by asking questions, or spending additional time in the classroom; level-three participation is characterized through students’ involvement with school-based extracurricular activities such as athletics, school

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1 Finn’s initial interest in explaining student disengagement and ultimate choice to withdraw from high school prior to graduation influences his decision to include attendance (or absenteeism) as a primary component of participation. While others agree that attendance is a necessary facilitator of engagement (Newmann, 1989; Newmann et al., 1992), it is not typically thought of as an indicator of engagement because students can be present in class but withdrawn from activities (Finn & Rock, 1997).
performances, or clubs; and lastly, level-four participation is described as participation in school-wide goal setting, decision making, and establishing a disciplinary system. Finn operationalizes students’ level of engagement with their school and classroom through participation, and he uses the two terms interchangeably (Finn, 1989; Finn & Cox, 1992; Finn & Rock, 1997). Students who participate more frequently and at higher levels are more engaged and tend to have higher achievement ratings, even with students identified as “at risk” (Finn & Cox, 1992; Finn & Rock, 1997).

Students who identify with their school have an internalized sense of belongingness and value success in school-related goals, where success is defined within the environment. Thus, identification consists of two components, belongingness and value (Finn, 1989; Finn & Zimmer, 2012). Identification can be viewed positively as a sense of relatedness or attachment, and negatively as alienation or withdrawal. Attachment and relatedness have been associated with success and achievement (Finn & Cox, 1992; Finn & Rock, 1997), while alienation and withdrawal have been linked to failure and dropping out (Bronfenbrenner, 1986).

Finn believes that young children have an initial disposition to participate in classroom activities by following rules and attending regularly (level-one participation). This low level of “participatory behavior continues as long as the individual has the minimal ability level needed to perform the required tasks, and as long as the instruction is clear and appropriate” (Finn, 1989, p. 129). As students progress through school, their opportunities to participate at higher levels increase because older students are more autonomous. When increased levels of participation are met with more experiences of success, students’ align their values with those of their school and have a heightened sense of belongingness (i.e., increased sense of identification), which motivates students to continue to participate. Under ideal circumstances, students’ abilities allow
them to experience enough success early in their education to make them resilient to inevitable experiences of failure or frustration. Thus, participation leading to experiences of success leads to identification, which in turn results in continued or increased levels of participation over time. Additionally, quality of instruction and student ability play a vital role in whether or not students’ initial participation results in academic success (figure 1). Finn does not expand on his thoughts about what constitutes “quality instruction;” however, he alludes to the importance that students’ sense of autonomy has on their participation (Finn, 1989). Thus, it can be inferred that autonomy support is at least one aspect of quality teaching.


Finn and colleagues have used this model as a means for explaining student engagement (i.e., participation) in fourth grade (Finn & Cox, 1992) and longitudinally as students progress through high school (Finn & Rock, 1997). In both studies, students were placed into one of three groups based on their teachers’ reports of their level of participation: active participants, passive participants, and nonparticipants. Active versus passive participants were delineated by the
amount of teacher perceived effort that students were exhibiting on their coursework.

Characteristics of active participants align with the description of level-two participation, and passive participation fits at level one. Nonparticipants were students who had poor attendance, were disruptive during class, and were rarely prepared. Results from both studies have indicated that the achievement and participation differences between active participants and nonparticipants increase over time, on average female students were judged by their teachers to be more engaged, and level-one and level-two participation tended to yield more academic success than nonparticipation. Further, statistical results demonstrate the cyclical relationship between participation and identification.

In summary, participation and identification are reciprocally related to one another, such that increased participation yields a heightened sense of identification. For students where identification is positive, increased engagement and academic success follow. On the other hand, when identification is negative, feelings of alienation occur and students tend to be less successful and more prone to disengage from school activities (Finn, 1989; Finn & Cox, 1992; Finn & Rock, 1997; Finn & Zimmer, 2012). The link between participation and identification is hypothesized by Finn (1989) to be experiences of success during learning activities, which he associates with quality of instruction and student ability.

**Authentic work.** Newmann describes engagement as “more than motivation” involving “participation, connection, attachment, and integration in particular settings and tasks” (Newmann, 1989, p. 34). Further, he expounds that student engagement is not observable because it is a construct describing students’ inner quality of concentration and effort to learn. Student engagement is a commitment of effort towards learning, comprehension, and mastery of concepts and procedures. The level to which students engage in their work can be inferred from
the way they complete tasks. Students exhibiting effort tend to spend time, concentrate, are enthusiastic about their work, and take pride in their results (Newmann, 1989; Newmann et al., 1992). Similar to Finn (1989), Newmann’s interest in studying student engagement stems from motivation to explain processes leading to students’ decisions to withdrawal from school. As a consequence, Newmann examines facilitators of student engagement; whereas, Finn and his colleagues (Finn, 1989; Finn & Cox, 1992; Finn & Rock, 1997; Finn & Zimmer, 2012) investigated indicators of engagement (i.e., level of participation). Additionally, Newmann attempted to inform school reform efforts to establish environments that would nurture student engagement, and therefore increase success and decrease dropout rates (Newmann, 1989; Newmann et al., 1992).

The essential aspects of an engagement nurturing environment described by Newmann et al. (1992) are school membership and authentic work, both of which are facilitated by students’ psychological need for perceived competence (figure 2). Here, school membership consists of social interactions with teachers, administrators, and peers that identify clear purposes for learning activities, are fair, offer personal support, provide opportunities for experiencing academic success, and are caring. Authentic work is described as tasks that are relevant and worthy of concentrated effort. Authentic work is connected to the real world, challenging, interesting, provides students with a sense of ownership of their knowledge, is fun, and when completed by students who commit effort should result in an external reward (e.g., high grades or admission to college).
Newmann and his colleagues (1992) suggest that engagement can be measured according to participation, concentration, and interest in school activities. However, the presence of these indicators could also imply that students are willing to comply with routines commonly associated with school environments. Thus, Newmann acknowledges that care needs to be taken when inferring about students’ overall level of engagement when examining their behaviors.

Figure 2. Facilitators of student engagement (Newmann et al., 1992, p. 18).

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2 Interest, particularly intrinsic interest, is often used interchangeably with motivation in psychology literature (Bergin, 1999; Hidi, 1990). The relationship between engagement and motivation is an element of research on both constructs that is currently disagreed upon. This point is further elaborated in the comparison section.
Additionally, Newmann (1989) suggests that motivation and engagement are different but related constructs. He argues that students’ motivation to invest concentration and effort (i.e., engage) comes from a psychological need for competence.

To conclude, Newmann (Newmann, 1989; Newmann et al., 1992) conceptualizes student engagement as an internal commitment to devote concentrated effort towards learning. This investment is motivated by a need for competence, is facilitated by school membership and authentic work, and manifests through behaviors of participation, concentration, and interest. Unfortunately, schools are at an initial disadvantage in establishing an environment that nurtures engagement when defined in this way because teaching and instruction are inherently coercive (Newmann, 1989; Newmann et al., 1992). However, they can foster students’ sense of school membership by making their purpose clear, establishing fair systems of reward and punishment, providing students with personal support (e.g., mentors), giving students ample opportunities to experience success on challenging work, and being caring. Lastly, students’ sense of competence yields a desire to participate in authentic work.

**Self-systems processes model.** Skinner and her colleagues (Skinner & Belmont, 1993; Skinner et al., 2008, 2009) conceive of engagement through motivation, and her work extends research on the influence that teacher behaviors have on student motivation/engagement and the motivation-to-achievement relationship. Psychology literature suggests that students’ motivation to learn stems from their evaluation of the effectiveness to which their classroom environment meets basic psychological needs of belongingness, autonomy, and competence (Bye, Pushkar, &

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3 Newmann et al. (1992) suggest that grade inflation or reducing rigor will not result in increased school membership. This is based on the assumption that students’ need to feel competent with valued work. Thus, success on routine work or unearned grades for mediocre work inhibits students’ sense of success.
Conway, 2007). “The extent to which children’s basic psychological needs are met or ignored in the school context is reflected in their self-system processes (attitudes and beliefs about the self)” (Skinner & Belmont, 1993, p. 572). This self-systems processes model of motivation is utilized to conceptualize student engagement. An underlying assumption of this model for motivation is that students’ self-evaluations are predictors of their motivation to learn. Using a motivational lens, engagement is described as the outward manifestation of motivation (Reeve, 2012; Skinner & Belmont, 1993; Skinner, Wellborn, & Connell, 1990). Thus, student engagement can be identified by examining students’ perceived fulfillment of their basic psychological needs and teacher reports about student behavior (Skinner & Belmont, 1993; Skinner et al., 2008, 2009).

Skinner and Belmont (1993) suggest that a motivational perspective of student engagement consists of emotional and behavioral components. They continue to describe engaged students as those who,

…show sustained behavioral involvement in learning activities accompanied by positive emotional tone. They select tasks at the border of their competencies, initiate action when given the opportunity, and exert intense effort and concentration in the implementation of learning tasks; they show generally positive emotions during ongoing action, including enthusiasm, optimism, curiosity, and interest. The opposite of engagement is disaffection (p. 572).

From this perspective, engagement or disaffection can be indicated through students’ emotions and behaviors, which are the observable indicators of students’ motivation to learn. In fact, within this characterization of engaged students are elements of autonomy (i.e. “they [students] select tasks…”) and competence. Further, behaviors such as taking initiative and being enthusiastic align with Finn’s (1989) level-two participation, which yields belongingness.
Although subtle, the facilitators of motivation are present in Skinner and Belmont’s (1993) description of engaged students.

In a series of quantitative studies, Skinner and her colleagues (Skinner & Belmont, 1993; Skinner et al., 2008, 2009) used path analysis and multiple regression to investigate their hypotheses about the components of student engagement, the influence of teacher behaviors on engagement and the relationship between motivation and achievement, and the changes in student engagement over time (measured once per semester during an academic year). All of these studies used teacher and student reports about each student’s emotional and behavioral engagement and disaffection to measure engagement, and addressed achievement through student performance on standardized tests.

Results from path analyses suggest that emotional engagement, emotional disaffection, behavioral engagement, and behavioral disaffection are unique components of engagement. Further, students who perceived of their relationship with their teacher as warm and supportive of autonomy tended to be more engaged and less disaffected both emotionally and behaviorally. On the other hand, students who felt alienated by their teachers tended to report less engagement and more disaffection. Teacher reports on both groups of students (engaged and disaffected) corroborated student reports. Additionally, there is a reciprocal relationship between student engagement and teachers’ behaviors, such that positive and negative student-teacher relationships are amplified by behavioral engagement and disaffection, respectively. Also, student engagement tends to decrease and disaffection tends to increase as students transition from middle to high school (Skinner et al., 2008). Students’ perception of autonomy support provided by their teachers was the biggest predictor of changes in all four components of student engagement over time. Finally, engagement versus disaffection, tended to correlate with
achievement. These results are summarized by figure 3.

**Figure 3.** Self-systems processes model of student engagement (Skinner et al., 2008, p. 768).

In sum, the self-systems processes model of student engagement suggests that engagement is an emotional and behavioral manifestation of students’ motivation, which is facilitated through their perceptions about their student-teacher relationship (Skinner & Belmont, 1993; Skinner et al., 2008, 2009). Belief that teachers support autonomy, establish structure in their classroom, and provide a sense of belongingness, is associated with emotional and behavioral engagement. Conversely, students who feel alienated and controlled tend to be emotionally and behaviorally disaffected. In either case, there are reciprocal effects on student-teacher relationships. Conceptualizing student engagement as these four components suggests
that students could be emotionally engaged but behaviorally disaffected or vice versa. Skinner et al. (2008) indicate that research on the sources and outcomes of such states of engagement is needed in order to obtain a richer understanding of engagement and motivation. They hypothesize that boredom, anxiety, and frustration might be emotions worth considering as sources of mixed states of engagement.

Measuring student engagement, conceptualized as four components, is difficult. Skinner et al. (2009) found that teachers’ reports of student behaviors correlated with students’ self-reports of emotion. Accordingly, it is more feasible to consider engagement consisting of emotional and behavioral components that both lie on continua ranging from engagement to disaffection even though a four aspect model was justified empirically (Skinner et al., 2009).

**Self-determination theory.** Deci and Ryan’s Self-determination theory (SDT) considers motivation through fulfillment of sense of competence, relatedness, and autonomy (Deci & Ryan, 1985, 2008; Deci et al., 1991; Ryan & Deci, 2000). SDT operationalizes student engagement through motivation, where motivation is moderated by students’ perceived satisfaction of these psychological needs. SDT suggests that motivated behaviors vary in the degree to which they are autonomous versus controlled. In this sense, autonomous behaviors are only influenced by an individual’s internal self – intrinsic motivation. On the other hand, controlled behaviors are those which are coerced by external factors. For example, students who complete their assignments because they want good grades are being motivated by an external reward system (Deci et al., 1991). Hence, in this situation students are under a degree of control. However, students who complete their assignments because they achieve a sense of satisfaction and competence by doing so are acting autonomously. Controlled behaviors are determined to be outcomes of external motivation.
The theory argues that behaviors can move from being externally motivated to being internally motivated through a process of internalization. Deci and Ryan and their colleagues (Deci et al., 1991) suggest that internalization itself is an externally motivated process, such that social-cultural settings influence internalization,

We believe (a) that people are inherently motivated (out of the three basic needs) to internalize and integrate within themselves the regulation of uninteresting activities that are useful for effective functioning in a social world and (b) that the extent to which the process of internalization and integration proceeds effectively is a function of the social context (Deci et al., 1991, pp. 328–329).

Therefore, SDT posits that students will be engaged if they are intrinsically motivated to learn, and engagement in classroom activities is determined by the degree to which students’ psychological needs are satisfied. Further, the social setting of a classroom can influence the process of internalization by demonstrating that certain activities, which may be uninteresting to individual students, are important. If the classroom environment supports students’ needs for feeling competent, related, and autonomous then students will become intrinsically motivated to learn (i.e., engaged).

Foundationally, SDT considered student engagement as consisting of behavioral, emotional, and cognitive outcomes associated with intrinsic motivation (Deci & Ryan, 2008; Deci et al., 1991; Ryan & Deci, 2000; Vallerand, 2000). In this regard, behavioral engagement is described as time on-task, attention, concentration, and persistence. Emotional engagement considers student interest and enthusiasm, and an absence of boredom, anxiety, worry, and frustration. Finally, students demonstrate cognitive engagement through use of self-regulated learning strategies (e.g., reflection, planning) and “seeking to develop conceptual understanding
over surface knowledge” (Reeve, 2012, p. 151). The degree to which students engage in educational activities is informed by their source of motivation (internal vs. external), and can be determined through their behaviors (e.g., active participation), emotions (e.g., enjoyment), and cognitive effort (e.g., using sophisticated solution strategies).

Additionally, Reeve and Tseng (2011) argue that a fourth component to student engagement is agency, which they refer to as agentic engagement. Agentic engagement is defined as “students’ constructive contribution into the flow of the instruction they receive” (Reeve & Tseng, 2011, p. 258). Reeve (2012) elaborates that “agentic engagement is the process in which students proactively try to create, enhance, and personalize the conditions and circumstances under which they learn” (p. 161).

When conceptualizing engagement as emotional, behavioral, cognitive, and agentic, Reeve and Tseng (2011) were able to completely explain the motivation-to-achievement relationship through mediation analysis and structured equations modeling. That is, with a sample of 365 high school students in Taiwan considering their educational experiences in all classes up to the eighth week of the school year, Reeve and Tseng (2011) determined that students’ motivation (measured by perceived psychological need satisfaction) was a statistically significant predictor of student achievement (measured through an aggregate of class grades). Then, when engagement (measured as a composite of emotional, behavioral, cognitive, and agentic engagement) was included in the model, the main effect of motivation was reduced to zero. Only partial mediation was achieved when agentic engagement was not considered in the engagement measure. Reeve and Tseng (2011) concluded that all four aspects of engagement should be considered if one wishes to examine achievement outcomes associated to motivation.
In conclusion, SDT, like the self-systems processes model described above, is a theory of motivation. Both SDT and the self-systems processes model conceptualize student engagement as the observable aspect of motivation, which is nurtured by social environments that satisfy students’ basic psychological needs. Researchers using self-systems processes (e.g., Skinner et al., 1990) or SDT (e.g., Reeve & Tseng, 2011) believe that engagement at least partially mediates the motivation-to-achievement relationship, and often use the terms “motivation” and “engagement” interchangeably. Also, both conceptualizations acknowledge the influence that social settings have on student engagement. However, as previously discussed, the self-systems

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4 Skinner et al. (1990) and Reeve & Tseng (2011) have statistically demonstrated that engagement mediates the main effect of motivation on achievement through mediation analysis. Both groups of researchers are careful to indicate that they believe motivation and engagement are different constructs, yet they refer to engagement as the public display of underlying motivation.
processes model for motivation considers student engagement to consist of emotional and behavioral components on a pair of engagement-disaffection continua, while SDT views student engagement as containing three (Reschly & Christenson, 2012) or four (Reeve, 2012; Reeve & Tseng, 2011) interrelated components (figure 4).

**Flow theory.** Flow is “the holistic sensation that people feel when they act with total involvement” (Csikszentmihalyi, 1975, p. 36); it is a multifaceted construct comprised of three components. To experience flow, students need to be presented with challenging tasks to which they believe they have sufficient skills to complete. Tasks with an inappropriate challenge versus skill balance result in boredom (i.e., low challenge and high skill), apathy (i.e., low challenge and low skill), worry or frustration (i.e., slightly high challenge and low skill), or anxiety (i.e., high challenge and low skill). Second, flow is often described as a state of total investment accompanied by a loss of self-consciousness and elapsed time (Csikszentmihalyi, 1990, 1997). For example, professional athletes tend to report being completely immersed in their craft (Nakamura & Csikszentmihalyi, 2009). Finally, experiencing flow is highly enjoyable. Thus, flow consists of cognitive, emotional, and motivational aspects. Accordingly, engagement and flow have overlapping conceptualizations, yet they are decidedly different constructs (Csikszentmihalyi, 1990; Steele & Fullagar, 2009).

Although flow and engagement are different constructs, they are conceptually similar in three key ways (Steele & Fullagar, 2009). The first comparison between flow and engagement was mentioned above; both flow and engagement are described according to cognitive, emotional, and motivational components (Csikszentmihalyi, 1990; Deci et al., 1991; Skinner et al., 2009). Additionally, both flow and engagement are described as states of intense concentration and investment in a task or activity (Csikszentmihalyi, 1997; Newmann et al.,
Lastly, flow and engagement are both intrinsically motivating (Jackson & Marsh, 1996; Nakamura & Csikszentmihalyi, 2009; Ryan & Deci, 2000). In light of these similarities, Steele and Fullagar (2009) argue that flow theory is a valid framework for conceptualizing engagement.

Flow is commonly studied in the contexts of performing arts, athletics (Csikszentmihalyi, 1975; Nakamura & Csikszentmihalyi, 2006), and video gaming (Faiola, Newlon, Pfaff, & Smyslova, 2012); however, some have considered flow states of students in high school (Shernoff et al., 2003) and college settings (S. E. Peterson & Miller, 2004; Steele & Fullagar, 2009). Results from this body of work have showed that flow states are correlated with higher perceptions about performance (i.e., competence) and self-efficacy (i.e., emotional engagement).

Some studies have considered student engagement from the perspective of flow (e.g., Shernoff et al., 2003; Steele & Fullagar, 2009). Steele and Fullagar (2009) used a survey instrument with 137 college students to determine some facilitators of engagement from classroom environments. They concluded that students who perceived of their instructor as autonomy supporting individuals who clearly define roles and outcomes for students and provide useful feedback tended to be more engaged during class. Further, they also found that engagement was positively associated with personal well-being (i.e., emotional engagement) and intrinsic motivation, which others have associated with student learning (e.g., Bye et al., 2007; Sonnert, Sadler, Sadler, & Bressoud, 2014). Finally, they operationalized student engagement through flow and found statistically significant correspondence between the two constructs.

Shernoff et al. (2003) utilized a common technique for measuring flow, the experience sampling method (ESM), to capture high school students’ engagement throughout the school day. As a data collection technique, ESM consists of participants being paged at random times throughout the day at which time the participant must stop what they were doing and fill out a
questionnaire about their experience in that moment. In this study, 526 high school students were paged eight times daily at random moments for one week. Accordingly, the researchers collected 3,630 responses, which they used to measure student engagement (i.e., concentration, enjoyment, and interest) and its associations to challenge-skill balance (i.e., flow, apathy, boredom, anxiety, and frustration), instructional relevance, autonomy, and classroom activities (e.g., group work, listening to lecture, taking an exam).

Shernoff and colleagues (2003) found that students’ perceptions of their challenge-skill balance were associated with engagement, such that situations of high challenge and high skill (i.e., flow) resulted in the highest levels of engagement and low challenge and low skill (i.e., apathy) resulted in the least amount of engagement. Additionally, they found that classroom activities such as collaborative group work resulted in high levels of engagement, while listening to lecture resulted in the lowest amount of engagement (on average). Finally, students across the sample tended to report the least amount of engagement during mathematics class than during any other school subject, as mathematics was the subject reported to be least interesting and enjoyable. Students reported high levels of concentration during mathematics; however, low interest and enjoyment resulted in low engagement. These results indicate that flow theory can be used to conceptualize engagement through interest, concentration, and enjoyment; that flow theory allows for engagement to be measured at an individual level; and that classroom activities and engagement are associated.

5 However, Shernoff et al. admit that their choice of statistical analysis of their “beep-level” data (ANOVA) resulted in conflated standard errors, which inflated their significance levels. Multiple level modeling would have been a more appropriate analytical tool for their data structure (Raudenbush & Bryk, 2002).
To conclude, flow theory has been used to conceptualize engagement as consisting of emotional (e.g., intrinsic motivation, interest, and enjoyment), behavioral (i.e., intense concentration and attention), and cognitive components ([e.g., challenge-skill balance]; e.g., Csikszentmihalyi, 1997; Shernoff et al., 2003). Flow has also been used as an operationalization of student engagement (Steele & Fullagar, 2009). However, more research is needed to determine if flow is a component of, or a facilitator for student engagement. This uncertainty is represented by the Venn diagram in figure 5. On the other hand, researchers studying engagement and flow in education highlight the importance of classroom environments that support autonomy and competence, have clearly stated learning goals, utilize activities that are relevant to students’ interests and are intrinsically motivating, and provide useful feedback for students’ self-regulation. Results from this work have demonstrated the influence of student engagement and flow on students’ positive well-being, self-efficacy, and academic achievement (S. E. Peterson & Miller, 2004; Shernoff et al., 2003; Steele & Fullagar, 2009).
The next section offers a comparison of the five perspectives of student engagement selected for this review.

**Comparing and Contrasting Perspectives**

**Common themes.** Many themes about the composition of, and facilitators for student engagement emerged during the review of these selected perspectives. First, an assumption about students’ psychological need to feel and demonstrate competence while working on challenging/relevant tasks undergirds most of these perspectives (Csikszentmihalyi, 1975; Deci & Ryan, 1985; Newmann, 1989; Skinner & Belmont, 1993). Second, motivation and engagement are intertwined. In fact, Skinner and her colleagues (Skinner & Belmont, 1993; Skinner et al., 2008, 2009), Deci and Ryan (1985, 2008), and Reeve and Tseng (2011) conceive of engagement as the observable manifestation of students’ inner motivation. On the other hand, Newmann (1989) and Csikszentmihalyi (1975, 1990, 1997) view student engagement as being unobservable and consisting of more than motivation. Third, student engagement is influenced by social aspects of schools and classrooms. Finn (1989) considers the emotional component of
feeling a sense of belonging as an indicator of student engagement. Others consider a sense of belonging to be a facilitator of student engagement (e.g., Deci et al., 1991; Newmann et al., 1992). Fourth, autonomy is positively related to student engagement; however, in some perspectives autonomy is a facilitator of student engagement (i.e., authentic work and flow), while others would consider a lack of autonomy to be an indicator of disengagement (i.e., SDT).

Fifth, challenging and interesting activities which promote collaboration are generally associated with higher levels of student engagement (Csikszentmihalyi, 1975, 1997; Newmann, 1989), especially when students experience success (Finn & Rock, 1997; Skinner & Belmont, 1993) and are rewarded appropriately (Newmann et al., 1992). Finally, student engagement is associated with academic achievement.

Although student engagement is linked to achievement, disengaged students can still succeed by investing a minimal amount of “required” effort (Finn & Rock, 1997; Newmann, 1989; Shernoff et al., 2003). This connection between engagement and success is partially a consequence of the conventional design of schools and high stakes testing (Newmann, 1989). However, that students can succeed without being engaged is not an issue with the construct, but does suggest difficulties in measuring it. Finn (1989) explained that participation at level-three and level-four would be difficult to observe in a study; moreover, he calls for more research on those degrees of engagement. Skinner et al. (2009) indicated that students’ reports of their own emotional engagement were highly correlated with teachers’ reports of students’ behavioral engagement. Fredricks et al. (2004) considers student engagement to be a *metaconstruct*, where each of its components contains constructs which are heavily researched individually. For example, interest is an indicator of emotional engagement, which is heavily researched (Bergin, 1999; Hidi, 1990; Hidi & Renninger, 2006). Researchers must decide which sub-components to
consider when studying student engagement leaving room for errors in validity. Finally, the ever-changing environment that is a classroom contributes further to difficulties in capturing student engagement. For example, during a semester, students might experience several shifts in their degree of interest, motivation, participation, and active involvement in the course.

Overall, there seems to be agreement that student engagement is a metaconstruct consisting of an array of components (Fredricks et al., 2004). Additionally, from the components addressed by the five perspectives of student engagement described above, there appears to be consensus that engagement is composed of at least emotional, behavioral, and cognitive components. Where, emotional engagement encapsulates positive and negative affect (e.g., self-efficacy beliefs) and interest in classroom activities and relationships. Behavioral engagement includes participation, attendance, and time on-task. Cognitive engagement considers concentration and attention, use of sophisticated learning strategies (e.g., self-regulation), and content mastery (Fredricks et al., 2004; Reschly & Christenson, 2012). Table 1 considers components of the five perspectives of engagement previously discussed and aligns them to these three constructs, which are indicators of the metaconstruct of student engagement.
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<th>Components/Indicators</th>
<th>Alignment</th>
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<td>Participation - Attendance &amp; Rule following, Enthusiasm towards learning &amp; Taking initiative, Involvement in extracurricular activities, School-wide decision making Identification, Belongingness, Value</td>
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<td>Authentic Work(^a)</td>
<td>Motivation, Participation, Connection &amp; Attachment, Concentration &amp; Effort, Interest</td>
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<td>Flow Theory</td>
<td>Concentration, Interest, Enjoyment, Challenge-Skill Balance (Flow)</td>
<td>Cognitive &amp; Behavioral, Emotional, Behavioral, Emotional</td>
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Notes:
\(^a\) Newmann et al. (1992) consider school membership and authentic work to be facilitators of engagement, not components of engagement.
\(^b\) Although Skinner & Belmont (1993) describe engagement as emotional and behavioral, according to the definitions used constructing table 1 behavioral engagement contains behavioral (participation) and cognitive (concentration) elements.
\(^c\) Terms used in the middle column are original to Reeve (2012). The third column organizes terms, based on Reeve’s descriptions, according to definitions used in constructing table 1.
More on flow theory. As indicated above, each of the perspectives selected for this review considers engagement to consist of emotional, behavioral, and cognitive components. Additionally, the importance of students’ basic needs fulfillment to engagement is also discussed. However, whether basic needs are indicators or facilitators is a matter of debate. For example, Finn’s participation-identification model (1989) posits that a sense of belongingness is an indicator of engagement, while Newmann (1989) suggests that social membership is a facilitator of student engagement. Similar contrasting views exist about competence (e.g., an indicator in flow theory and a facilitator as authentic work) and autonomy (e.g., an indicator in SDT and a facilitator as authentic work). Further additional indicators such as disaffection from self-systems processes model, and facilitators such as authentic work seem unique to their perspectives.

Flow theory allows for a robust conceptualization of student engagement that contains the common indicators and facilitators of the other perspectives discussed. Further, the construct of flow as a facilitator of student engagement allows for Newmann’s (1989) concept of authentic work and Skinner and Belmont’s (1993) disaffection (e.g., boredom and anxiety) to be considered. Additionally, Reeve’s (2012) agentic engagement is also encapsulated by the flow construct. Newmann et al. (1992) argued that engagement is more than motivation\(^6\), which is a point echoed by Csikszentmihalyi (1997). Researchers using flow theory have measured student engagement and flow several times within a single day (e.g., S. E. Peterson & Miller, 2004; Shernoff et al., 2003), so a framework exists for reliably recording student engagement and flow on a weekly basis, for example. This would allow for an examination of intra- and inter-

\[^6\] Some argue that intrinsic interest is synonymous to motivation (e.g., Bergin, 1999; Bye et al., 2007; Hidi, 1990; Hidi & Renninger, 2006), so flow theory addresses the role of motivation by containing interest as an element.
individual changes in student engagement over the duration of a course. Therefore, some of the previously mentioned measurement concerns are reduced when conceptualizing student engagement from the perspective of flow theory.

Finally, flow is a construct that shares conceptual similarities with student engagement (Steele & Fullagar, 2009), and flow theory acknowledges the influences that students’ psychological needs have on academic performance (Nakamura & Csikszentmihalyi, 2009). Specifically, autonomy and belongingness are considered facilitators and competence an indicator of flow and student engagement (Csikszentmihalyi, 1990; Shernoff et al., 2003). Thus, flow theory offers a robust perspective for conceptualizing student engagement, which contains common elements of other perspectives and utilizes some of the uniqueness about others. For this study, student engagement will be conceptualized from a flow theory perspective such that student engagement consists of interest, concentration, and enjoyment (i.e., Shernoff et al., 2003).

Conclusion on Perspectives of Student Engagement

Student engagement is an attractive construct to educational researchers because it repeatedly explains variation in student achievement outcomes (Deci et al., 1991; Finn & Zimmer, 2012; Newmann et al., 1992; S. E. Peterson & Miller, 2004; Skinner et al., 2008). Additionally, student engagement is malleable in social contexts unlike race/ethnicity and SES. Therefore, policy makers can use results to inform institutional change and school reform efforts (e.g., Davis & McPartland, 2012; Newmann et al., 1992). Student engagement is a metaconstruct (Fredricks et al., 2004) consisting of emotional, behavioral, and cognitive components. Engagement is closely related to motivation and flow and is influenced by student-teacher
relationships; classroom structure; and students’ sense of belongingness, autonomy, and competence.

In the first part of this review, five different perspectives of student engagement were discussed and compared. Finn’s (1989) participation-identification model conceives of student engagement on a continuum of participation and students’ identification with school. Newmann and his colleagues (Newmann, 1989; Newmann et al., 1992) consider student engagement as a result of school membership and authentic work. Skinner and her colleagues and self-determination theory align engagement and motivation; however, they do so in different ways. Skinner et al. (Skinner & Belmont, 1993; Skinner et al., 2008, 2009, 1990) view engagement as engagement-disaffection continua of emotions and behaviors. On the other hand, SDT argues that students will be engaged when they are intrinsically motivated to learn (Deci & Ryan, 1985; Deci et al., 1991; Ryan & Deci, 2000). Finally, flow theory conceptualizes student engagement as an intrinsically motivating state of deep concentration and total immersion in academic activities (Csikszentmihalyi, 1975, 1990; Steele & Fullagar, 2009).

**Student Engagement in the Mathematics**

This section considers mathematics education literature pertaining to student engagement in mathematics classrooms for students in K-12. Unfortunately, there is little research on engagement of post-secondary students in mathematics. However, there is a pervasive attempt at studying university students’ engagement through the National Survey of Student Engagement ([NSSE]; Kuh, 2001, 2003, 2009) and community college students’ engagement through the Community College Survey of Student Engagement ([CCSSE]; McClenny, 2007)7. These

7 Readers are directed to their websites (www.nsse.org and www.ccsse.org) for a full review of these instruments and information about findings.
projects primarily address college students’ sense of belonging at their institution and do not offer much insight on the inner-workings of particular classrooms. Accordingly, recent studies (cf. P. L. Peterson & Fennema, 1985) from K-12 mathematics classrooms are synthesized here. Additionally, effort was made to consider research that reported on student engagement as an outcome because a connection between student engagement and achievement has already been established in the previous section. There appears to be cross-curricular agreement on the importance of student engagement on learning (Fredricks et al., 2004; Reschly & Christenson, 2012).

This section will be broken into three subsections in which research on elementary, middle, and high school students’ engagement in mathematics is synthesized. Each section will be concluded with implications for teaching and learning and suggestions for future research on student engagement in the corresponding grade band. Following the last section, overall trends from all grade-bands will be discussed, implications for teaching and learning will be provided, and routes for future research will be suggested.

**Elementary students’ engagement in mathematics.** Based on teacher reported survey data, behavioral engagement for students as young as Kindergarten is positively associated with math achievement gains for poor, low-income, and non-poor students, such that high levels of behavioral engagement were especially beneficial for students’ math achievement from poor and low-income groups (Robinson, 2013). Further, behavioral engagement mediated the effect of poverty on achievement gains in a nationally representative sample of nearly 12,000 students. This means that the statistical effect of including behavioral engagement in a model predicting achievement gains nullified the negative effect that poverty had on mathematics growth of Kindergarteners (Robinson, 2013).
Lan et al. (2009) used observations to measure first grade students’ behavioral engagement (student participation and attentiveness) at the classroom level. These researchers coded 70 30-second time intervals during 35 minutes of mathematics time to determine classroom behavioral engagement. If at least 70% of students demonstrated on-task behaviors for at least 20 seconds of the time interval, then the classroom was coded as behaviorally engaged for that time period. They found that compared to classrooms in Beijing, China, first grade classrooms in the US demonstrated higher behavioral engagement at the beginning of designated math time. Both sets of classrooms experienced declines in overall behavioral engagement; however, behavioral engagement from US classrooms decayed significantly faster. Additionally, these researchers found that US classrooms displayed more variability in behavioral engagement than classrooms in Beijing. Experiencing declines in behavioral engagement in US classrooms is not unique to first graders. In fact, students tend to display overall decreases in behavioral engagement in mathematics as they progress from Kindergarten to third grade, according to teacher reports (Bodovski & Farkas, 2007).

Concerned about achievement differences between male and female students, Peterson and Fennema (1985) investigated behavioral engagement of fourth grade students by observing six male and six female students in each of 36 different classrooms. They found that male and female students tended to be behaviorally engaged (participating, attentive, and on task) for about 75% of mathematics lessons observed. They also found that both groups were given very limited opportunity for autonomy during mathematics instruction, as their teachers directed activity roughly 99% of the time. These researchers further concluded that most of the activities conducted were of a low cognitive level, which they hypothesized led to lower levels of behavioral engagement. Finally, teacher sanctioned socializing during transitional time or group
work had a more negative effect on the behavioral engagement of fourth grade girls than boys. However, Rimm-Kaufman et al. (2015) used teacher and student reported data which indicated that teacher sanctioned socializing in fifth graders enhanced behavioral engagement and students’ sense of belonging.

These two studies took place nearly 30 years apart, and note that Peterson and Fennema’s (1985) study predates Fredricks et al.’s (2004) work clarifying definitions of different components of student engagement. Peterson and Fennema considered student engagement to entail participation and attentiveness, which would be classified as behavioral engagement according to Fredricks et al. Further, Peterson and Fennema conceptualized cognitive engagement through the difficulty of tasks on which students worked. Their understanding of cognitive engagement in this way is more consistent with what Stein, Grover, and Henningsen (1996) referred to as cognitive demand. On the other hand, Rimm-Kaufman et al. 2015), used definitions of behavioral, emotional, and cognitive engagement provided by Fredricks et al.

Overall, elementary students tend to have high levels of behavioral engagement, as indicated by observational and teacher reported data (Lan et al., 2009; P. L. Peterson & Fennema, 1985; Rimm-Kaufman et al., 2015). Though, behavioral engagement levels decrease daily (Lan et al., 2009), over the course of the school year (Rimm-Kaufman et al., 2015), and throughout elementary school (Bodovski & Farkas, 2007). Teachers who contribute to establishing a structured learning environment, where expectations are clear and modeled, have students who are behaviorally engaged (Bodovski & Farkas, 2007; Lan et al., 2009). Also, behavioral engagement is contagious; students in classrooms with higher average classroom behavioral engagement are more behaviorally engaged and “classroom behavioral engagement was [is] associated with achievement gains independent of children’s individual behavioral
engagement” (Robinson, 2013, p. 38). First grade teachers in Beijing were able to slow (relative to US classrooms) the decay of their students’ behavioral engagement during lessons by providing clear instructions about student behavior prior to launching group work activities. On the other hand, US teachers were more likely to react to behaviorally disengaged students during group work, and their classrooms experienced comparatively steep declines in behavioral engagement (Lan et al., 2009).

Behavioral engagement of elementary students is a significant contributor to students’ mathematical growth (Bodovski & Farkas, 2007; Lan et al., 2009; P. L. Peterson & Fennema, 1985; Rimm-Kaufman et al., 2015; Robinson, 2013). Behavioral engagement is especially important for students whose families are experiencing economic hardship (Robinson, 2013), boys (P. L. Peterson & Fennema, 1985), and students with poor prior achievement in mathematics (Bodovski & Farkas, 2007; Robinson, 2013). Further, students’ behavioral engagement in mathematics is more beneficial to students’ achievement during early elementary school than the amount of instructional time spent on learning mathematics (Bodovski & Farkas, 2007). In other words, the quality of instruction (e.g., structure, clear goals, feedback, challenging work, student collaboration) is more beneficial than the quantity of instruction. Taken together, the results presented in this section speak to the importance of classroom structure and organization, clearly defined behavioral and learning outcomes, and teachers’ classroom management skills for keeping elementary students behaviorally engaged.

Rimm-Kaufman et al. (2015) were the only authors, whose work was reviewed for this section, to consider emotional and cognitive engagement and use student reported data. They found that fifth grade students tended to report higher levels of cognitive and emotional engagement when they perceived of their teachers as establishing an emotionally supportive and
structured learning environment. Classroom structure was particularly beneficial to male students’ cognitive and emotional engagement. On the other hand, female students did not report such an association, meaning that classroom structure did not have statistically significant effects on girls’ cognitive or emotional engagement. The authors did not consider how teachers’ gender or race related to that of the students, or if this relationship had any effects on student-reported cognitive or emotional engagement. Future research could investigate how similarities and differences in the demographics of students and teachers are associated with student reported engagement in mathematics. Additionally, based on student reported data, all three aspects of student engagement tended to decline over time. The importance of well-organized classrooms on all aspects of student engagement was corroborated through observations and teacher reported data.

The literature reviewed for this section suggests that researchers studying student engagement in elementary classrooms tend to use quantitative methods to primarily examine behavioral engagement (cf. Rimm-Kaufman et al., 2015) in terms of participation, time on-task, and attentiveness (e.g., raising hands and maintaining eye contact with the teacher) based on observational and teacher reported data. These data collection methods are appropriate for examining young students because student reported data may be particularly unreliable given their age. However, Robinson (2013) indicated that teachers were more inclined to report negatively about the behavioral engagement of poor and low-income students and that this bias was “largely owed to achievement performance and to a lesser extent to students’ racial/ethnic background” (p. 38). Thus, it is possible that teachers are reporting about students’ academic performance; not their engagement. Additionally, Skinner and Belmont (1993) found that teachers’ reports about students’ behavioral engagement tended to align with students’ report
about their emotional states. Thus, teachers may not be the most reliable source for extracting data about students’ behavioral engagement.

Of the studies reviewed, only Rimm-Kaufman et al. (2015) indicated using a specific perspective of student engagement (self-systems processes). Similar to the previous section, these authors also aligned Skinner’s language to conventional terms of behavioral, emotional, and cognitive engagement. This study was the only one to report on all three components of student engagement. Further, authors of this study were the only ones to incorporate student-reported data to measure emotional and cognitive engagement – they also included observational and teacher reported data. The other studies only considered behavioral engagement and relied on either teacher reported or observational data. Peterson and Fennema (1985) considered what they termed *cognitive engagement*; however, they conceptualized cognitive engagement based on the difficulty of tasks. Thus, their use of the phrase “cognitive engagement” is different from the definition presented by Fredricks et al. (2004). Bodovski and Farkas (2007) and Robinson (2013) used teacher reported data from a survey to measure students’ behavioral engagement. In both studies, behavioral engagement is defined as in Fredricks et al. (2004); however, Bodovski and Farkas (2007) used “behavioral engagement” and “engagement” interchangeably throughout their report. Lan et al. (2009) also only considered behavioral engagement, but they considered behavioral engagement to be a class-level variable.

Future research about elementary students’ engagement in mathematics should consider approaches to address biases in teacher reporting and emotional and cognitive engagement. Using teacher reports or observations to measure student engagement limits what is being called “engagement” to behavioral engagement. Future work should consider including student interviews to examine emotional and cognitive engagement. Task-based interviews could give
researchers a chance to capture students’ voices about their own engagement in mathematical tasks. Although concentration will likely be high in a task-based interview setting, other indicators of cognitive engagement (e.g., metacognition and sophistication of strategies) could be explored. Finally, a debriefing period upon completion of the task could shed light on students’ emotional engagement while working on the task. Studies such as this would help our understanding of classroom and instructional factors that elementary students believe promote their engagement in mathematics.

**Middle grades students’ engagement in mathematics.** Unfortunately, the trend that student engagement declines in school continues as students proceed through middle school. Martin and his colleagues (2015) conducted a cross-sectional longitudinal study following three cohorts of Australian Catholic school students for an academic year. Their findings suggest that student engagement (emotional, behavioral, and cognitive aspects) is highest at the beginning of the school year, and decreases over time, based on student reported data. Additionally, they determined that older students experienced greater declines than younger students in all three aspects of engagement.

However, Seaton and Carr (2005) observed that seventh graders from their sample did not experience declines in behavioral engagement in their regular mathematics course during a period in which they participated in a supplementary mathematics and science program on Saturdays8. In fact, seventh graders in this sample became more behaviorally engaged in their regular mathematics classroom during the time of the ancillary program. Unfortunately, sixth graders in this sample did not experience similar effects. Instead, these younger students were

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8 The Science Engineering Mathematics Aerospace Academy (SEMAA) program is designed to compensate for deficiencies in daily instruction for middle and high school students.
observed to be less behaviorally engaged in their regular mathematics classroom during a time period in which they were attending the Saturday program. Interestingly, once the Saturday program completed, behavioral engagement from both groups returned to pre-intervention levels (Seaton & Carr, 2005).

Seaton and Carr (2005) concluded that, “the nature of the instruction and classroom environments appeared to be chief determinants of student engagement rates” (p. 429) because the Saturday program did not have lasting effects on their students’ behavioral engagement during their regular mathematics courses. The authors explained that teachers in their study lectured and assigned individual work consistently. Further, “neither inquiry-based student investigations nor cooperative structures were observed. Instructional objectives… tended toward procedural rather than conceptual knowledge” (p. 429). Similarly, Martin et al. (2015) attributed gains in all aspects of student engagement to classroom environment as well. They determined that class averages for student planning, effort, homework completion, and enjoyment predicted individual student engagement respectively. Once again, overall student engagement in a classroom appears to affect individual student engagement in corresponding ways. For example, students who would typically fail to practice self-regulatory learning skills such as planning are more inclined to do so when their classmates exhibit this behavior. This same phenomena was noted for elementary students as well (Robinson, 2013).

Interestingly, classroom aggregated achievement was negatively associated with individual students’ engagement (Martin et al., 2015). Perhaps, students’ need for sense of competence serves to inhibit behavioral and emotional engagement in environments perceived to be full of high achievers? Future research is needed to investigate this question; however, it seems reasonable to hypothesize that students with low math self-efficacy would be more likely
to disengage behaviorally. In addition to classroom factors, school-wide factors also contribute to changes in student engagement during middle school. For instance, school-average achievement and student-teacher ratios are associated with declines in mathematics engagement, while school size and school SES are associated with growth in student engagement (Martin et al., 2015).

Middle grades students tend to experience declines in emotional, behavioral, and cognitive engagement as they transition from elementary to high school (Martin et al., 2015), even for students participating in extra-curricular mathematics learning programs (Seaton & Carr, 2005). However, teachers and policy makers can take actions to combat this issue by clearly stating learning objectives, providing opportunities for collaborative group work, and adopting curriculum materials that focus less on procedural skills (Martin et al., 2015; Seaton & Carr, 2005). Further, policy makers should work to keep student-teacher ratios low (Martin et al., 2015).

Seaton and Carr (2005) and Martin et al. (2015) indicated that declines in middle grades student engagement was hypothesized to be attributed in part to adolescence and transitioning from elementary to high school. Martin et al. (2015) concluded that transitioning was not the sole factor; instead, school, classroom, and student factors contribute to declines in all aspects of student engagement through middle school. Future research in this area should focus on determining why student engagement is negatively associated with classroom-average achievement. As stated above, it is possible that self-efficacy and students’ psychological need for competence partially explain this phenomenon. However, gender differences, race/ethnicity, and goal orientations could also be explored.

**High school students’ engagement in mathematics.** Changes in student engagement in mathematics at the high school level are slightly different from the changes documented from
elementary and middle school students. For example, urban high school students across the US were able to maintain their initial level of behavioral and emotional engagement during mathematics class for roughly 45 minutes before experiencing steady declines for the remainder of the class period (Uekawa, Borman, & Lee, 2007). Additionally, urban students reported being more engaged at times when they were not drowsy or tired. These instances tended to occur in class periods immediately prior to lunch time and later in the week – Monday mornings were reported to be times when students were most tired and least engaged.

Once again, instructional practices and classroom structure have significant effects on high school students’ engagement in mathematics. Trained instructors using technology rich Realistic Mathematics Education (RME) approaches were described by tenth graders from Ireland to be more emotionally engaging (i.e., interesting and enjoyable), which led to higher levels of behavioral engagement (Bray & Tangney, 2016). Technology use by students was also a factor contributing to high school students’ emotional and behavioral engagement in Greece (Barkatsas, Kasimatis, & Gialamas, 2009). Interestingly, collaborative group work negatively affected students’ behavioral engagement, particularly when students perceived the workload to be unfair (Bray & Tangney, 2016). However, this was not the case for urban students in the US, especially Latino students, who reported that collaborative group work was more emotionally and behaviorally engaging (Uekawa et al., 2007). Finally, classrooms with desks arranged in rows and columns where students are assigned their seat were associated with higher levels of behavioral engagement (Uekawa et al., 2007).

Many student-level factors (e.g., achievement, attitudes, and confidence) were related to student engagement. For example, Barkatsas et al. (2009) found gender differences in Greek students based on student reported emotional and behavioral engagement, comfort with
technology, confidence with mathematics, attitude towards learning mathematics with technology, and achievement. They used exploratory factor analysis (EFA), cluster analysis, and correspondence analysis to determine clusters of characteristics which were significantly different from each other and more likely to consist of male or female students. Table 2 summarizes their results. Based on the factors indicated by their EFA results, it is possible that students with achievement and confidence levels that contradict with their emotional and behavioral engagement were not experiencing fulfillment of their psychological need for competence. Future research could consider the role that needs fulfillment has on the relationship between mathematical achievement and student-reported levels of emotional and behavioral engagement.

Table 2: Summary of results from Barkatsas et al. (2009, pp. 568-569).

<table>
<thead>
<tr>
<th>Math Achievement</th>
<th>Math Confidence</th>
<th>Emotional Engagement</th>
<th>Behavioral Engagement</th>
<th>Computer Use</th>
<th>Attitude learning math with computers</th>
<th>Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
<td>Confident</td>
<td>Positive</td>
<td>Male</td>
</tr>
<tr>
<td>Average</td>
<td>Average</td>
<td>Neutral</td>
<td>Average</td>
<td>Not confident</td>
<td>Very negative</td>
<td>Female</td>
</tr>
<tr>
<td>Excellent</td>
<td>Very high</td>
<td>Very high</td>
<td>Very high</td>
<td>Very confident</td>
<td>Very positive</td>
<td>Male</td>
</tr>
<tr>
<td>Average</td>
<td>Average</td>
<td>Neutral</td>
<td>Positive</td>
<td>Average</td>
<td>Very positive</td>
<td>Female</td>
</tr>
<tr>
<td>High</td>
<td>High</td>
<td>Very low</td>
<td>Very low</td>
<td>Not confident</td>
<td>Positive</td>
<td>Female</td>
</tr>
<tr>
<td>High</td>
<td>Very high</td>
<td>Very high</td>
<td>Very low</td>
<td>Very confident</td>
<td>Very negative</td>
<td>Male</td>
</tr>
<tr>
<td>Very low</td>
<td>Very low</td>
<td>Very low</td>
<td>Very low</td>
<td>Not confident</td>
<td>Negative</td>
<td>Female</td>
</tr>
</tbody>
</table>

Additionally, Uekawa et al. (2007) found that urban students were more engaged when the content being covered was known to the students. Specifically, students who were unable to
determine if the material being taught was new to them were significantly less engaged emotionally and behaviorally. Also, students were more engaged (i.e., cognitively, emotionally, and behaviorally) when content being covered was relevant to students’ everyday lives or determined to be important for upcoming assessments. Finally, students were substantially more engaged when they were involved in academic conversation with their teachers than during peer conversation, moments of silence (e.g., lecture), and during transitions. Overall, Uekawa et al. (2007) found that African American students in their sample were highly engaged during all classroom activities; Latino students were least engaged, but were more engaged during group work than any other activity; Asian students were more engaged during individual work than other activities; and lecture was found to be the least engaging activity for all students in their sample. The researchers did not report on the demographics of teachers and staff at their sites. It would be interesting to know if teacher demographics affect students’ sense of belongingness or school membership, which could affect their engagement (e.g., Finn, 1989; Newmann et al., 1992). Future research considering the association between faculty demographics and student engagement is warranted.

In sum, researchers studying student engagement at the high school level have reported about classroom activities that are reported (by students) to be engaging during mathematics instruction. Lecture is least engaging to most students (Uekawa et al., 2007), while working in groups on relevant and interesting activities which utilize technology are substantially more emotionally (Barkatsas et al., 2009; Bray & Tangney, 2016), behaviorally (Barkatsas et al., 2009; Bray & Tangney, 2016), and cognitively engaging (Uekawa et al., 2007). Students reported being more behaviorally engaged when desks were not arranged in tables and their teachers assigned seats (Bray & Tangney, 2016; Uekawa et al., 2007). Finally, differences between male
and female students (Barkatsas et al., 2009) and ethnic groups (Uekawa et al., 2007) were investigated. In this regard, female students, even those with above average mathematics achievement and confidence, tended to feel emotionally disengaged (see table 2; Barkatsas et al., 2009), and Latino students are particularly engaged during group work (Uekawa et al., 2007).

**Implications and Future Research**

Students of all ages appear to be more behaviorally and emotionally engaged during group work than during other classroom activities, especially lecture. This has been observed in elementary (e.g., Lan et al., 2009) and self-reported by middle (e.g., Martin et al., 2015) and high school students (e.g., Bray & Tangney, 2016; Uekawa et al., 2007). However, group work also presents students with opportunities to digress off-task and participate in non-academic conversation with their peers. Such activity is reported by teachers to be detrimental to behavioral engagement in elementary schools (Lan et al., 2009), especially for female students (e.g., P. L. Peterson & Fennema, 1985). Further, high school students reported that time for socialization leads to decreased levels of behavioral and emotional engagement (Barkatsas et al., 2009), and that seating arrangements can be a contributing factor (Uekawa et al., 2007).

Taken together, classroom management, organization, and structure are key teacher-level factors for establishing high levels of student engagement. Use of relevant, interesting, and fun activities can facilitate student engagement at all levels (e.g., Barkatsas et al., 2009; Bodovski & Farkas, 2007; Bray & Tangney, 2016; Martin et al., 2015), and technology use for learning mathematics is emotionally and behaviorally engaging to high school students (e.g., Barkatsas et al., 2009; Bray & Tangney, 2016). These are implications about teaching that mathematics teachers can utilize daily to elicit higher levels of emotional and behavioral engagement from their students.
Although much has been learned about facilitators of student engagement in mathematics, one component of the metaconstruct is noticeably absent from the literature – cognitive engagement. Future research should focus on this aspect of student engagement, particularly in elementary and middle grades students. Next, of the literature reviewed for this section, only one study was not purely quantitative (Bray & Tangney, 2016). As a result, students’ voices about their experiences and all aspects of engagement with mathematics are being neglected, especially at the elementary and middle school levels. More qualitative and mixed methods inquiry considering students’ perspectives is justified, particularly because of the roles that students’ senses of belongingness, autonomy, and competence play on engagement (Csikszentmihalyi, 1990; Finn, 1989; Newmann, 1989; Reeve, 2012; Skinner & Belmont, 1993).

Many of the studies reviewed consisted of large samples examining the effects of student-level, classroom-level, and school-level factors on student engagement in mathematics through multiple level modeling (e.g., Barkatsas et al., 2009; Bodovski & Farkas, 2007; Lan et al., 2009; Martin et al., 2015; Rimm-Kaufman et al., 2015; Robinson, 2013; Uekawa et al., 2007). However, only one considered intra-individual fluctuations in student engagement (Uekawa et al., 2007). Specifically, these researchers indicated that 39% of the variation in student engagement was within-student variation – compared to 51% between students and 10% at classroom-levels. This means that student engagement varies within individual students almost as much as between students. Presently, research has focused on between-student and classroom-level factors contributing to student engagement. In doing so, nearly 40% of variation in student engagement is being unexamined. Consequently, future research should consider the affects that time (week in the school year), content, and other factors which change daily (or even within a single lesson) have on individual students’ engagement. Researchers conceptualizing student engagement...
engagement through flow theory have provided a model for investigating student engagement such that intra- and inter-individual variation can be examined (e.g., S. E. Peterson & Miller, 2004; Shernoff et al., 2003). Results from similar work in mathematics education would allow researchers to make implications about best practices for engaging students while learning about specific content (e.g., fractions, functions, and rate of change).

Finally, most of the studies considered for this review neglected to report on a theoretical perspective being used to conceptualize student engagement. Specifically, Rimm-Kaufman et al. (2015) indicated their use of Skinner’s self-systems processes model (e.g., Skinner et al., 2009); Martin et al. (2015) explained how their survey instrument was grounded in work by Bronfenbrenner; and Uekawa et al. (2007) attribute their use of ESM (experience sampling method) to its uses by flow theorists (e.g., Nakamura & Csikszentmihalyi, 2009) – though it is unclear if the authors conceptualized engagement from through flow theory. However, many of the authors did reference Fredricks et al. (2004) when establishing a definition of student engagement or its components (Barkatsas et al., 2009; Bodovski & Farkas, 2007; Lan et al., 2009; Martin et al., 2015; Rimm-Kaufman et al., 2015; Robinson, 2013). Accordingly, it appears that mathematics education researchers are conceptualizing student engagement as comprising of emotional, behavioral, and cognitive components, but are infrequently considering theoretical perspectives through which to view these constructs. Future researchers should specify which theoretical perspective is being used to conceptualize emotional, behavioral, and cognitive components of student engagement.

**Conclusion on Student Engagement**

Student engagement is a broad construct, consisting of multiple components. The number and nature of components is subject to debate. Finn (1989) conceptualizes engagement through
two components of participation and identification. Newmann (1989) argues the importance of school membership and authentic work. Skinner and her colleagues (Skinner & Belmont, 1993; Skinner et al., 2008, 2009) conceive of student engagement as emotional engagement, behavioral engagement, emotional disaffection, and behavioral disaffection on two continua. Self-determination theory views engagement as the observable manifestation of motivation (Ryan & Deci, 2000), and some researchers adopting this view posit that engagement is comprised of four components: emotional, behavioral, cognitive, and agentic engagement (Reeve, 2012; Reeve & Tseng, 2011). Csikszentmihalyi and his colleagues (Csikszentmihalyi, 1975, 1990; Shernoff et al., 2003; Steele & Fullagar, 2009) have operationalized student engagement through flow, which consists of emotional, behavioral, and cognitive components as well, but also considers a challenge-skill balance. Regardless of how it is conceptualized, there is agreement about the importance of students’ basic psychological needs for autonomy, belongingness, and competence as indicators or facilitators of student engagement. Finally, student engagement is vital for academic achievement and success.

Research on student engagement in the teaching and learning of mathematics has shed light on the importance of group work, classroom management, organization, clarity and feedback, relevant tasks, and fun for fostering students’ behavioral and emotional engagement. However, limited work on cognitive engagement exists in the literature. Recently, numerous large-sample statistical methods have been used to examine student engagement in mathematics quantitatively. These studies have highlighted student-, classroom-, and school-level factors that contribute to student engagement. However, within-student variation in engagement has only been explored minimally (e.g., Uekawa et al., 2007), and students’ perspectives about their own engagement in mathematics is largely neglected (cf. Bray & Tangney, 2016). Future research on
emotional, behavioral, and cognitive aspects of student engagement in mathematics that utilize mixed methods approaches are warranted. This is particularly true at the college level where the most pervasive attempts at studying college student engagement (e.g., Kuh, 2001, 2003, 2009; McClenney, 2007) are investigating students’ sense of belonging at their institution, and are not considering student engagement in specific classrooms.

One goal of this study is to identify and describe any relationships between community college precalculus students’ engagement and understanding of precalculus concepts. The next section briefly discusses literature on students’ understanding of precalculus concepts relevant to data collection and analysis.

**Background of Research on Students’ Understanding of Precalculus Concepts**

Research on students’ understanding of precalculus concepts such as covariation (e.g., Carlson et al., 2002), quantity/quantizing (e.g., Saldanha & Thompson, 1998; Thompson, 1994, 2011), and function (e.g., Dubinsky & Harel, 1992; Oehrtman, Carlson, & Thompson, 2008) has showed that robust understandings of these concepts are paramount for success in future undergraduate mathematics courses. Consequently, assessing students’ covariational reasoning, quantitative reasoning, and conceptions of function is important for student and program evaluation purposes. The Precalculus Concepts Assessment ([PCA], Carlson, Oehrtman, & Engelke, 2010) is such an assessment, consisting of 25 multiple-choice items with three subscales (function, covariation, and computation). Carlson and colleagues (2010) indicate that precalculus students who score at least 13 (out of 25) are more likely to experience success in calculus than their peers with lower scores. Items on the PCA were created and refined through qualitative studies (e.g., Carlson et al., 2002) to establish a valid and reliable instrument, which is important for evaluating large groups of students’ understanding of concepts central to
precalculus and more advanced undergraduate mathematics courses. Though, it remains important to continue qualitative research (e.g., teaching experiments) to further investigate students’ covariational and quantitative reasoning and how these manifest is different contexts – for example while constructing graphs.

Such work has been done; in a review of literature on students’ understanding of functions and graphs (and the ways in which these topics have been taught), Leinhardt, Zaslavky, and Stein (1990) describe various understandings demonstrated by students. Their review suggests that students may often focus solely on specific points (e.g., maximums or minimums) when interpreting and constructing graphical representations and may ignore covariational relationships between two quantities suggested by continuous curves. Students may also view the graph of a situation as a picture (i.e. iconic interpretation). For example, a graph to represent the height of a roller coaster over time may be understood as an image of the ride. These iconic interpretations may manifest in various ways. For instance, students may construct a graph as the path an object travels (which may imply traveling back in time), or they may interpret intersections of two graphs as locations where one object overtakes (or even collides with) another. This synthesis of research on students’ understanding (interpretation and construction) of graphical representations demonstrates how research attending to students’ construction and interpretation of graphs can be used to infer about covariational and quantitative reasoning.

Moore and Thompson (2015) consider students’ reasoning about graph construction and graphical representations, which they term shape thinking, as an approach to investigating students’ covariational and quantitative reasoning. Moore and Thompson describe students’ shape thinking as static or emergent, where “static shape thinking involves operating on a graph
as an object in and of itself” (p. 784); and “emergent shape thinking involves understanding a graph simultaneously as what is made (a trace) and how it is made (covariation)” (p. 785, emphasis in original). In much earlier work investigating students’ understanding of functions generated by physical situations, Monk (1992) identified *iconic translations* as a way of thinking in which perceptual features from a situation are associated with the shape of a graph for that situation. For example, a student who draws a vertical line to represent an elevator’s height over time might be associating the shape of the situation (i.e. travelling vertically in an elevator) to the shape of a corresponding graph. Also, Thompson (2015) explains that students may utilize *thematic associations* in their ways of thinking about graphs and associated situations. In this regard, students associate features of the situation as being necessary features of the corresponding graph. For example, a student who draws a vertical line to represent an elevator’s height over time while emphasizing specific points to indicate stops might be associating physical phenomena with necessary features of its corresponding graph. Stevens and Moore (2016) elaborate that both iconic translations and thematic associations are examples of static shape thinking because both ways of thinking rely on perceptual features of an event for reasoning about a graphical representation for the event.

Many other researchers have investigated students’ quantitative and covariational reasoning (e.g., Carlson et al., 2002; Carlson, Oehrtman, & Engelke, 2010; Castillo-Garsow et al., 2013; Monk, 1992; Saldanha & Thompson, 1998; Thompson, 1994, 2011), and their work informed the recent work by Moore and his colleagues (e.g., Moore & Thompson, 2015; Stevens & Moore, 2016), which directly informed data collection and analysis methods for this study (see chapter 3).
Chapter 3: Methodology

Research Questions

The purpose of this study is to examine the effects of teaching on student engagement and understanding in community college precalculus, specifically aiming to address the following research questions:

1. What is the nature of community college students’ engagement during precalculus, and what role do teaching approaches have on these experiences?
2. Is there a relationship between student engagement and understanding of precalculus topics, and if so,
   a. What are characteristics of this relationship?
   b. Is there an association between this relationship and teaching approaches?

This chapter describes how these research questions will be addressed using a mixed methods research design. For convenience, the reader is reminded of the definition of mixed methods research being used and then a description of the specific mixed methods approach to inquiry is provided.

Mixed Methods Definition and Rationale

While many different definitions of mixed methods research exist, the definition that was selected to guide the current study is that provided by Currall in a paper written by Johnson, Onwuegbuzie, and Turner (2007) who were trying to establish a working definition of mixed methods research. Currall defines mixed methods research as an approach to inquiry that involves the use of both qualitative and quantitative data collection and analysis techniques either sequentially or simultaneously. A decision to pursue a mixed methods approach to inquiry is justified by the complexity of students’ lived experiences situated within a dynamic learning
ecology. Learning environments consist of multiple interconnected student-peer and student-teacher relationships that ebb and flow in a complex fashion. In order to develop a better understanding of this complex dynamic system, both quantitative and qualitative data collection and analysis techniques are warranted. A mixed methods approach to inquiry for this study will allow for student understanding (measured quantitatively and qualitatively) to be understood in terms of student engagement (measured quantitatively and qualitatively) as afforded to them through their instructors’ pedagogies. Therefore, a mixed methods approach to inquiry provides an opportunity to more thoroughly investigate students’ experiences than would be possible by invoking qualitative or quantitative methods alone.

Mixed Methods Design

For this project a convergent parallel design was used in order to best answer research questions pertaining to community college students’ experiences with different teaching approaches while taking precalculus and their effects on student engagement and understanding. A convergent parallel design is a mixed methods approach where both quantitative and qualitative data are collected concurrently, analyzed separately, and then blended together during interpretation (Creswell & Plano Clark, 2011; Decuir-Gunby & Schutz, under contract). Accordingly, quantitative data collected through surveys and a post assessment and qualitative data collected through surveys, interviews, and observations are gathered for the duration of the study and analyzed separately. Results from each dataset are used to address research questions, and their stories are merged during interpretation. Note that with this design it is possible that the two datasets could serve to corroborate or contradict each other.
Setting and Participants

This study took place during the Fall of 2016. Community college instructors teaching precalculus and their students from two community colleges were invited to participate. Permission to work with these instructors and their students was gained through emails sent to mathematics department chairs. These emails contained a brief synopsis of the purpose for the study, why the study is important, and the potential benefits that could be gained by the instructors and students who participate. Additionally, a copy of an approval/exemption letter from the North Carolina State University Institutional Review Board (IRB) was provided (Appendix A). Access to students was acquired through gained access to their instructors following approval from site IRBs where necessary and department chairpersons.

A short video was created to recruit student participants. The video discussed details about the importance of their responses, potential benefits that they might receive (e.g., students were asked to recall their week from class possibly forming self-reflection and meta-cognition habits), what is being asked of them as participants, and a discussion of the rewards system utilized as incentive for their prolonged participation throughout the semester (i.e. monthly raffle for $100, where eligibility was determined by response rates on weekly surveys). Participating instructors posted a web-link for this video to their learning management system (e.g., BlackBoard, Moodle) during the first week of the semester.

Fifteen precalculus instructors from two community colleges and a collection of 110 of their students agreed to participate in the study. Of the fifteen instructors, fourteen are employed by one community college, and the fifteenth instructor is employed by the second community college. The first community college is very large and located in the Southeastern United States. This community college serves nearly 70,000 students, has nine campuses, and more than 40
full-time mathematics faculty across the nine campuses. The fourteen instructors who participated in this study from this community college taught at three of the campuses.

The second community college is much smaller, located in a rural area of the Southeastern United States. At the time of the study, this community college served fewer than 5,000 students with four small satellite campuses. The instructor from this institution is the only mathematics faculty member, full-time or otherwise, teaching at one of these sites. Both community colleges offer precalculus as a two-course sequence, where course one focuses on polynomial, rational, radical, exponential and logarithmic functions, equations, and applications. Course two focuses on trigonometry. Participants in this study were instructors or students of course one.

Overall, eight instructor participants are female, seven are male (female-associated titles and pronouns will be used with all pseudonyms). Their experience teaching community college mathematics ranges from first semester as full-time faculty to more than 30 years. Moreover, the range of experience teaching precalculus is similar; for some instructors this was their first semester while others have taught the course for more than 30 years.

Collectively these instructors had 538 students enrolled in their precalculus courses at the beginning of the study, 110 of which consented to participate. Had all 110 students participated completely for 15 weeks, 1650 diary responses (see Data Collection) would have been acquired. However, from this sample of 110 students, 101 provided sufficient data for quantitative analysis methods to be employed. These 101 students represent the sample of student participants. Together, they submitted 829 diary responses, which were used for multiple level modeling (MLM) and various qualitative analyses.

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9 This is not to suggest that the precalculus course has not adapted over that span of time.
The student sample consists of students self-identified as African (3 F, 1 M), Asian (6 F, 3 M), Black (9 F, 4 M), Hispanic (9 F, 3 M), White (40 F, 13 M), or Other (9 F, 0 M), with one male student choosing not to identify his race/ethnicity. Altogether, 76 student participants were female and 25 students were male. Almost all students were enrolled in the course for the first time - 67 females and 19 males (86%). Nearly three-quarters did not take remedial courses prior to enrolling in precalculus – 55 females and 19 males (74%). Many of the students (78%) were enrolled full-time during the Fall 2016 semester (62 F, 16 M), and most students (82%) worked either part-time (49 F, 18 M) or full-time (10 F, 5 M). One female and four male students were veterans, and five females and one male were parents. Table 3 partitions the number of female and male students based on their academic intentions.

Table 3: Student reported academic intent by gender.

<table>
<thead>
<tr>
<th>Academic Intent</th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associates of Arts (AA)</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Associates of Science (AS)</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Transfer to earn BA without earning an AA or AS</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Transfer to earn BS without earning an AA or AS</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Transfer with AA to earn AA</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Transfer with AA to earn BS</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>Transfer with AS to earn BA</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Transfer with AS to earn BS</td>
<td>27</td>
<td>11</td>
</tr>
</tbody>
</table>

Data Collection

Multiple methods of data collection took place during this study to achieve triangulation and to generate qualitative and quantitative datasets that thoroughly document experiences and constructs relevant to the study. Briefly, student data was collected from two survey instruments, an assessment, through interviews, and through observations. Instructor data was collected through interviews and observations.
Recall that the definition of student engagement used for this study indicates that engagement is comprised of three components, behavioral engagement, emotional engagement, and cognitive engagement. Flow theory posits that student engagement consists of interest, enjoyment, and concentration (Shernoff et al., 2003). In this regard, interest and enjoyment are elements of emotional engagement, while concentration is both behavioral and cognitive according to definitions provided by Fredricks et al. (2004). Accordingly, data collection processes are intended to capture student reported data about their interest, enjoyment, and concentration in precalculus; as well as, information about potential facilitators of student engagement described in the literature review (e.g., perceived needs fulfillment, challenge-skill balance, relevance of work, teaching approach).

Data collection procedures and instrumentation. This section provides a chronological account of data collection procedures and instrumentation used in the study. At the end of the section, table 4 summarizes data collection procedures and instrumentation and indicates the purposes for each mechanism. Additionally, table 4 also serves to remind readers that quantitative and qualitative data are collected concurrently in a convergent parallel mixed methods design.

Instructor interviews. In the first two weeks of the study instructor interviews were conducted to identify teaching approaches utilized by the fifteen instructors in the sample. These interviews were completed using the same protocol as Mesa, Celis et al. (2014). These researchers utilized the protocol to determine community college instructors’ teaching approaches (i.e. traditional, meaning-making, or student-support). The authors identified the teaching approaches of instructors from their study by analyzing interview transcripts. Their analysis had strong inter-rater reliability and was corroborated through extensive classroom
observations. These researchers concluded that “either interviews or classroom observations can be used to describe teaching approaches” (p. 141). Permission to use their interview protocol for this study was granted. Their protocol can be found in Appendix E.

Instructor interviews were video recorded and transcribed. Transcripts were then analyzed (discussed below) to characterize the teaching approaches used by instructors in the sample. Results from this analysis were used to purposefully select three instructors based on their self-reported teaching approaches – Mrs. Jackson, Mrs. Harper, and Mrs. Smith\(^\text{10}\) – whose classrooms were each observed three times.

_Initial demographic survey._ Also in the beginning of the semester, instructors were asked to post two web-links to their course learning management system. One, a link for the recruitment video, and the second link to a student consent form and initial demographic survey (see Appendices B & C). In total, these links were made available to 538 students. The survey is web-based and responses were recorded to a spreadsheet.

The initial survey gathered demographic information such as age, gender, and race/ethnicity; course information such as when their section met; and academic information such as remedial mathematics courses taken, number of semesters as a college student, traditional or non-traditional student, status (e.g., full-time), and program of study. Essentially, the purpose of this instrument is to collect time-invariant student data, which will be referred to below as student-level data.

_Weekly diaries._ At the end of each week, students were sent a web-link to the weekly diary instrument (Appendix D) via text message or email, whichever was preferred by the student (sent via Remind). This diary is composed of multiple Likert-type items and one open-

\(^\text{10}\) All participant names are pseudonyms.
ended prompt. The Likert-type items asked students to report on their perceived level of engagement (through constructs from flow theory), autonomy, competence, belongingness, the relevance of work, and challenge-skill balance. The open-ended prompt asked students to report briefly on their experiences in class during each week of the study as if they were making a social media post. Likert-type items on this instrument have been adapted from a similar instrument used by Shernoff et al. (2003), who utilized the experience sampling method in a diary design study to investigate student engagement from the perspective of flow theory. Items from the original instrument were adapted by including the phrase “during precalculus this week” where appropriate. The weekly diary instrument collects time-variant student data, which will be referred to below as diary-level data.

Responses to the weekly diaries were collected electronically and recorded to spreadsheets. Unique spreadsheets for each instructors’ sections of precalculus being studied were created. By the end of the semester, each student was asked to complete 15 weekly diaries (829 total submissions were recorded).

*Classroom observations.* Classroom observations of Mrs. Jackson, Mrs. Harper, and Mrs. Smith’s classrooms took place three times throughout the semester to cover the span of the term, in early October, early November, and after Thanksgiving. All observations were video recorded and extensive field notes were taken (Appendix G). The video camera was positioned at the front of the room and focused on students to record their behavioral engagement during the observation. Field notes partitioned observations into five-minute intervals, where instructor actions were briefly noted and students’ participation, questions, body language, interest, enjoyment, and concentration were extensively documented.
Classroom observation data serves multiple purposes: First, video data and field notes were used to analyze students’ behavioral engagement during class time. Second, video recordings allowed for thick descriptions of classroom environments to be written, and third, video data can be used to compare instructors’ interview responses to what transpires in their classrooms. In all, 9 classroom observations were conducted and more than 700 minutes of video data were collected.

Student interviews. Two rounds of student interviews were conducted over the duration of the study. Highly engaged students in Mrs. Jackson’s, Mrs. Harper’s, and Mr. Smith’s classes were identified from the first five weeks of diary data. Then students were purposefully selected to participate in interviews based on their responses to the initial demographic survey, responses to the first five weekly diary prompts (posts), and, in some cases, based on the first classroom observation. After identifying highly engaged students, efforts were made to select a sub-sample which captured the diversity of the entire sample of students in the study (e.g., at least one part-time student was selected). Three students from each of Mrs. Jackson’s, Mrs. Harpers’, and Mrs. Smith’s classes were selected. All 9 students participated in the first round of interviews, 8 of whom also participated in the second round. Students tending to report high levels of engagement were identified from their responses to the first five weekly diaries.

The first interview was used to give the selected students opportunities to elaborate on their weekly diary posts and to describe their engagement during precalculus in more detail. The second interview was task-based. This interview was used to delve into these students’ understandings of precalculus concepts of quantitative reasoning (Thompson, 2011), covariational reasoning (Carlson et al., 2002), and graphing (Moore & Thompson, 2015). The Taking a Ride task (Moore, n.d.; Stevens & Moore, 2016) and accompanying interview protocol
(Appendix H) were used during the task-based interview, which was followed by a set of debriefing questions to investigate students’ engagement while working the task (Appendix I). All interviews were video recorded and transcribed.
Precalculus concepts assessment. Lastly, during the week prior to final exams, all instructor participants administered the Precalculus Concepts Assessment ([PCA], Carlson, Oehrtman, & Engelke, 2010) to all their students – participants and nonparticipants. Instructors were informed that the PCA could serve as a course review, and were asked to give students credit accordingly. The PCA is a validated assessment that has been used extensively by its developers to measure undergraduate students’ – including community college students – understanding of covariation, function, and computational reasoning. Items on the PCA were developed through a series of qualitative studies (e.g., Carlson et al., 2002). As a result, Carlson and her colleagues have established a reliable and valid instrument which they still actively use in ongoing research endeavors (Carlson, personal communication, April 2015). Accordingly, a complete copy of the PCA is not available in the appendices\(^{11}\); however, a few items are shared in Appendix F. A hard copy of the assessment can be provided upon request. The PCA was administered to students once in the final weeks of the study, so the data acquired will be considered time-invariant\(^{12}\) student-level data.

Information on proper administration of the PCA was provided to instructors to ensure that all students took the assessment in similar environments. Students recorded their responses on bubble-sheets, and then their responses and scores were entered into a spreadsheet.

Table 4 summarizes the multiple methods of data collection employed during this study. Also, included in table 4 is the type of data collected with each instrument and the overall

\(^{11}\) A copy of the PCA was acquired for this study through personal correspondence with its developers, who requested that the assessment not be published for security reasons.

\(^{12}\) In this sense, time-invariant means that changes in students’ understandings cannot be documented with PCA results obtained during this study. Students’ understandings of precalculus concepts measured by the PCA are not time-invariant in the literal sense of the phrase.
purpose of the data acquired. Consistent with a convergent parallel mixed methods design, two separate databases were generated during data collection – one qualitative and one quantitative.

Table 4: Summary of data collection timeline and procedures.

<table>
<thead>
<tr>
<th>When</th>
<th>Data Collection Method/Instrument</th>
<th>Type</th>
<th>Participant(s)</th>
<th>Purpose</th>
</tr>
</thead>
</table>
| First 2 weeks of semester | Instructor Interviews             | Qualitative     | Instructors    | - Identify teaching approaches  
- Purposefully select instructors                                           |
| First week of Semester    | Initial Student Survey            | Qualitative     | Students       | - Describe sample
- Purposefully select students                                              |
| Throughout (Weekly)       | Weekly Diary                      | Qualitative     | Students       | - Measure engagement
- Gather students’ experiences on a weekly basis
- Purposefully select students                                              |
| Throughout (early Oct,    | Classroom Observations            | Qualitative     | Instructors    | - Purposefully select students
- Corroborate teaching approaches
- Document student behavioral engagement                                     |
| early Nov, late Nov/early Dec) |                                |                 | Students       |                                                                          |
| Middle and end of semester| Student Interviews                | Qualitative     | Students       | - Detailed descriptions of selected students’ engagement and understanding |
| End of semester            | PCA                               | Quantitative    | Students       | - Student understanding                                                  |
Data Analysis

This section discusses the processes by which the data were organized, cleaned, and analyzed.

**Purposefully selecting instructors.** Video recordings from instructor interviews were imported into the qualitative data analysis software, NVivo. Using NVivo is beneficial because it allows for all data to be stored in a central location (electronically), and NVivo projects can be updated as data are collected. Video records of instructor interviews were transcribed and coded using NVivo to determine a profile for the teaching approaches used by each instructor (see Mesa, Celis, et al., 2014). Mesa and her colleagues identified types of speech that instructors use while reporting on their teaching practices. They provided elaborate definitions of each of the three teaching approaches discussed (table 5), and explained that instructors likely utilize an array of teaching approaches in their practice. Using these definitions as an analytical framework, interview transcripts acquired with the instructor interview protocol (Appendix E) were coded to identify a teaching approaches profile for each instructor. This process took place early in the semester to purposefully select instructors for classroom observations and to obtain instructor-level data for the quantitative dataset.
Table 5: Definitions of teaching approaches (Mesa, Celis, et al., 2014).

<table>
<thead>
<tr>
<th>Teaching Approach</th>
<th>Defining Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>“Instructors using the traditional approach privilege the content and the instructor’s authority in the classroom” (p. 132). These instructors believe that students need to see all required content, regardless of the pace of instruction. Instructors have a “perception that they are the authority for presenting knowledge” (p. 134).</td>
</tr>
<tr>
<td>Student-support</td>
<td>The main goal of instruction is not content mastery. Instead, these instructors aim to improve self-confidence, math self-efficacy, and student-teacher relationships. “…prioritize students’ needs over covering the content” (p. 136).</td>
</tr>
<tr>
<td>Meaning-making</td>
<td>“…instructors seek to promote deeper learning and to connect mathematics to real-world contexts” (p. 134). Emphasis on understanding over grades, but acknowledge that grades are important to students for affective reasons.</td>
</tr>
</tbody>
</table>

Passages from interview transcripts were coded per the definitions in table 5. A passage is a complete thought provided by an instructor during their responses to interview items. A response to a single item from the interview protocol can contain more than one passage. For example, an excerpt from one instructor’s transcripts (table 6) is a response to a single question from the interview protocol, but contains multiple passages.
### Table 6: Passages from a response provided by an instructor.

<table>
<thead>
<tr>
<th>Passage Number</th>
<th>Passage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passage 1</td>
<td>Yeah, well so for 171 the, the, I think the consensus is that the textbook and the course deviate as far as our expectations of the students. So I use the textbook to guide me um [sic] before, for each unit I have, and will continue to, go through and do all of the MyMathLab [online homework system] homework before we start the unit so that I can make sure that I’m structuring my PowerPoints appropriately for the lecture part of class.</td>
</tr>
<tr>
<td>Passage 2</td>
<td>Um, I, the so far – I mean we are on unit one – but the test is available already, so I look at the test as I’m going through as well. So for each lesson make sure that I’m covering the topics that are going to be tested.</td>
</tr>
<tr>
<td>Passage 3</td>
<td>Um, I like to use PowerPoints; um, I didn’t used to, but I do now. I mean this is like a really recent change in my life. I’ve, I um, I think it makes it easier for you to stay on point as an instructor. So I used to sit down and handwrite notes, and kind of carry those around with me, and hang on to them as I’m writing on the board. And um, but then, I, invariably you run off on some tangent because somethings crossed your mind and you talk about that, and you deviate from your notes and then, you know, it’s twenty minutes later and you realize that class is ending and you didn’t cover what the kids need to know. So I like the PowerPoints to keep me on track.</td>
</tr>
<tr>
<td>Passage 4</td>
<td>I think it’s great that I can email those to the students, that’s I think helpful for them. I know it surely doesn’t hurt them to have the notes emailed to them, but um I do encourage them to take notes. I don’t say, “keep this PowerPoint, and never take notes again.” Um, so, you know I want to make sure they’re doing that.</td>
</tr>
<tr>
<td>Passage 5</td>
<td>I use the document camera for things, right, “watch me work out this problem.” In the PowerPoints I put step-by-step solutions or examples but it’s nice to have me working out a problem so you can actually watch me physically handwriting it out and doing it. I think that’s great as well.</td>
</tr>
<tr>
<td>Passage 6</td>
<td>Um, I’m a big fan of MyMathLab, I encourage them to use the resources up there so they’ll use help me solve this or view an example um, they have the videos for the topics, so I think those are really great too, and I used to not be a big fan of online homework and you know, the last couple of years, I’ve really changed my opinion of that as well. I think it’s super helpful for the students and I try to incorporate it um – I don’t really have a choice here – but I would try to incorporate it either way.</td>
</tr>
</tbody>
</table>
Only passages referencing the instructors’ teaching practices were coded for teaching approaches. Thus, not all passages required a code. Also, some passages were coded for more than one approach. For example, passage 4 from table 6 was coded as meaning-making and student-support. The code of meaning-making is dictated by the instructor’s coaching role for expecting and encouraging his students to continue to take notes even though his slides are made available electronically. This passage also requires a code of student-support because the instructor is considering students’ needs while structuring his course by making his slides available to them electronically.

**Purposefully selecting students.** Spreadsheets containing the first five weeks of diary responses, students’ initial survey responses, and instructor-level data were created to purposefully select highly engaged students to participate in student interviews. Thus, the multiple spreadsheets containing initial survey data and weekly diary data needed to be merged to create two spreadsheets: one with quantitative data and another with qualitative data. To do so, instructors, students, and weeks were assigned identification numbers (IID, SID, and WID, respectively), such that the responses from each week could be mapped to the student author who could then be matched with their instructor. In other words, quantitative and qualitative spreadsheets were created such that weekly-diary responses were paired with a week-ID, student-ID, and instructor-ID. Table 7 depicts the structure of the qualitative spreadsheet containing students’ weekly diary posts. Additional variables are contained in other columns (not shown in table 7). These posts were provided by student 74 in the first five weeks of the study; student 74 was in instructor 13’s class. The quantitative spreadsheet has a similar structure.
Table 7: Example from the qualitative spreadsheet depicting the data's structure.

<table>
<thead>
<tr>
<th>IID</th>
<th>SID</th>
<th>WID</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>74</td>
<td>1</td>
<td>It was difficult, but I understood the last bit a lot.</td>
</tr>
<tr>
<td>13</td>
<td>74</td>
<td>2</td>
<td>Precalc is getting better. Still tough, but I’m figuring it out.</td>
</tr>
<tr>
<td>13</td>
<td>74</td>
<td>3</td>
<td>I’m getting it :D</td>
</tr>
<tr>
<td>13</td>
<td>74</td>
<td>4</td>
<td>I’ve still got it!</td>
</tr>
<tr>
<td>13</td>
<td>74</td>
<td>5</td>
<td>I’m losing my grip on Precalc again. Gotta start reviewing and really focusing.</td>
</tr>
</tbody>
</table>

The quantitative spreadsheet was used to identify students from Mrs. Jackson’s, Mrs. Harper’s, and Mrs. Smith’s classes who reported high levels of interest, enjoyment, and concentration (i.e. engagement) during the first five weeks of the study. Then demographic information was used to reduce the sample of students to three from each selected instructor’s class to be purposefully selected to participate in interviews. Demographic information was used to assist in selecting a subsample of students who were highly engaged (relative to their classmates) and represented various demographics of the overall student sample.

The remaining 10 weeks of diary responses were merged into the master spreadsheets intermittently throughout the semester using the same process outlined above. The qualitative spreadsheet was imported into NVivo for analysis, while quantitative responses were analyzed with Excel and SAS.

**Analyzing students’ posts.** Students’ weekly posts were coded multiple times; first, as being indicative of students’ interest, enjoyment, or concentration or opposites of these indicators of student engagement. Then posts were coded for facilitators of engagement that emerged in the literature (e.g., autonomy, competence, and belongingness) as well as open-codes stemming from the students’ own words. Coding was carried out in NVivo, where posts were read, themes were
noted, and then posts were coded. All posts were coded at least two times to establish reliable use of the codebook (table 8).

Following coding, themes were developed to describe groups of codes, which were then used to investigate relationships between themes and student engagement. Overall, 31 codes were used, including the seven indicators of student engagement. These codes were used a total of 3047 times; however, not all posts warranted codes. Three categories of codes were identified to describe groups of codes for facilitators of student engagement as referencing students’ challenge-skill balance, needs fulfillment, and perceptions about teaching or course structure. Table 8 provides a list of codes and definitions used in this coding process. Codes for interest, enjoyment, and concentration were developed a priori while other codes were established during coding (note the ‘~’ should be interpreted as “not” or “low”).

Student and instructor-level information were also attributed to posts. This allowed for correlations and associations between student-reported experiences and other variables to be investigated.
### Table 8: Codebook for coding weekly posts.

<table>
<thead>
<tr>
<th>Category</th>
<th>Indicator</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Engagement</td>
<td>Interest</td>
<td>Student expresses interest in precalculus class from the previous week. This may manifest as interest in the content, acknowledging that the content is relevant, or indicating that their instructor is interesting.</td>
</tr>
<tr>
<td></td>
<td>Enjoy</td>
<td>Student expresses having enjoyed precalculus class from the previous week. This may manifest as having fun, excitement about learning, or making connections from/to other courses.</td>
</tr>
<tr>
<td></td>
<td>Concentration</td>
<td>Student expresses having to concentrate during precalculus from the previous week. This may manifest through claims about the class being challenging, requiring a lot of attention, or being difficult.</td>
</tr>
<tr>
<td></td>
<td>~Interest</td>
<td>Student expresses a lack of interest or fails to see the relevance of their work in precalculus from the previous week.</td>
</tr>
<tr>
<td></td>
<td>~Enjoy</td>
<td>Student indicates that they are not having fun during precalculus or they are not excited about attending class.</td>
</tr>
<tr>
<td></td>
<td>Enjoy Neutral</td>
<td>Student expresses that class was bearable, tolerable, straightforward, “ok” or “not that bad.” This may manifest as belief about teacher’s methods.</td>
</tr>
<tr>
<td></td>
<td>~Concentration</td>
<td>Student indicates that they already understood the material being covered based on prior experiences, or the student indicates difficulty with concentrating due to external factors such as being tired or hungry.</td>
</tr>
<tr>
<td>Need Fulfillment</td>
<td>Anxiety</td>
<td>Student expresses feeling anxious or nervous about upcoming assessments or attending class. Praying to do well or hoping to pass.</td>
</tr>
<tr>
<td></td>
<td>Boredom</td>
<td>Student expresses having been bored during class from the previous week. May manifest through comments such as, “Ehh.” Or “Zzzz.”</td>
</tr>
<tr>
<td></td>
<td>Confidence</td>
<td>Student expresses feeling confident in their ability to succeed. This may manifest as student stating that materials make sense.</td>
</tr>
<tr>
<td></td>
<td>Confused</td>
<td>Student indicates that they were confused or lost during precalculus class.</td>
</tr>
<tr>
<td></td>
<td>Easy</td>
<td>Student states that precalculus class was easy. They may reference an assessment that they found easy.</td>
</tr>
<tr>
<td></td>
<td>Hard</td>
<td>Student states that precalculus class was hard. They may reference an assessment that they found difficult.</td>
</tr>
<tr>
<td></td>
<td>~Autonomy</td>
<td>Student indicates that they were not able to take control of their own learning during precalculus class.</td>
</tr>
<tr>
<td></td>
<td>~Belong</td>
<td>Student indicates that they do not feel as though they are welcome in class.</td>
</tr>
<tr>
<td></td>
<td>~Competent</td>
<td>Student indicates that they do not feel competent in precalculus class. May manifest as students stating they are not capable of understanding material from class, or that they performed poorly on an assessment.</td>
</tr>
<tr>
<td></td>
<td>Autonomy</td>
<td>Student expresses a sense of being able to take control of their own learning during precalculus class.</td>
</tr>
<tr>
<td></td>
<td>Belong</td>
<td>Student expresses a sense of belongingness while in precalculus class.</td>
</tr>
<tr>
<td></td>
<td>Competent</td>
<td>Student expresses a sense of competence while in precalculus class. This may manifest as having done well on previous assessments.</td>
</tr>
</tbody>
</table>
Table 8 Continued

<table>
<thead>
<tr>
<th>Category</th>
<th>Code</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perceptions of Teaching</td>
<td>Fast-Paced</td>
<td>Student indicates that the course is moving too quickly for them to comprehend the material or keep up with assignments.</td>
</tr>
<tr>
<td></td>
<td>Slow-Paced</td>
<td>Student indicates that the course is moving too slowly for them.</td>
</tr>
<tr>
<td></td>
<td>Group work</td>
<td>Student specifically mentions having worked in groups during the previous week in precalculus class.</td>
</tr>
<tr>
<td></td>
<td>No group work</td>
<td>Student specifically mentions not working in groups during the previous week in precalculus class.</td>
</tr>
<tr>
<td></td>
<td>Teaching Positive Instructor Cares</td>
<td>Student indicates a feeling that their instructor cares about their personal success in the course.</td>
</tr>
<tr>
<td></td>
<td>Teaching Negative Prepared</td>
<td>Student references their instructor or his/her instructional practices in a negative way. Student expresses feelings of preparedness for upcoming assessments, or that they spent time in the previous week preparing for upcoming assessments.</td>
</tr>
<tr>
<td></td>
<td>~Prepared</td>
<td>Student indicates that they were not prepared for an assessment. This may manifest by them stating they should have studied more or worked harder.</td>
</tr>
<tr>
<td></td>
<td>Test</td>
<td>Student mentions an assessment in their post.</td>
</tr>
<tr>
<td></td>
<td>Too long</td>
<td>Student indicates that the class periods are too long.</td>
</tr>
<tr>
<td></td>
<td>Too short</td>
<td>Student indicates that the class periods are too short.</td>
</tr>
</tbody>
</table>

**Classroom observations.** Field notes and video data from classroom observations were used to describe students’ behavioral engagement while in class. Five minute-intervals were open-coded based on students’ body language, participation, and activity (e.g., taking notes, multi-tasking, sleeping). Then each interval was coded for the instructor’s teaching approaches. These analyses allowed for associations between teaching approaches and students’ behavioral engagement to be investigated and described. Coding observation video and field notes for the instructor’s teaching approaches also allows for a comparison between instructors’ reported and enacted approaches to teaching.

**Student interviews: round one.** Recall that student interviewees were purposefully selected based on their first five weeks of diary responses, and were chosen because they tended
to report being highly engaged relative to their classmates. Thus, the goals in analyzing interview transcripts were to: first, identify themes in these students’ experiences associated with high levels of engagement; and second, identify student- and instructor-level variables frequently reported as factors of these students’ engagement. In this regard, transcripts were coded to describe the nature of student engagement from the perspective of purposefully selected, highly engaged students in greater detail than what was provided in the weekly posts. Transcripts were coded using the codebook established while coding weekly posts to identify facilitators of interest, enjoyment, and concentration discussed by the interviewees. Then, themes were developed to tell the broader story of these students’ lived experiences as community college precalculus students and student- and instructor/classroom-level variables associated with their engagement.

**Student interviews: round two.** The second round of student interviews had two purposes. First, to delve into the selected students’ understanding of precalculus concepts elicited by the *Taking a Ride* task (Moore, n.d.; Stevens & Moore, 2016), and then to use a set of debriefing questions to investigate the students’ engagement while working on the task.

**Student understanding.** The *Taking a Ride* task consists of two videos of a Ferris wheel. In the first video, the Ferris wheel rotates continuously at a constant angular speed and interviewees are asked to “graph the relationship between a rider’s total distance traveled around the wheel and the rider’s distance from the ground” (emphasis in the original). Then, a second video in which the Ferris wheel stops periodically is shown to the interviewee who is prompted to discuss the relevance of their previous graph to the new video. Transcripts from the task-based portion of these interviews were coded per the students’ *shape thinking* with codes for static
shape thinking: iconic translation, static shape thinking: thematic association, and emergent shape thinking (table 9).

Table 9: *Codebook for analyzing students' reasoning from task-based interviews.*

<table>
<thead>
<tr>
<th>Code</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static shape thinking: iconic translation</td>
<td>Student justifies the shape/construction of a graph through perceptual features of a situation <em>and</em> “operates on a graph as an object in and of itself” (Moore &amp; Thompson, 2015, p. 784).</td>
</tr>
<tr>
<td>Static shape thinking: thematic association</td>
<td>Student justifies the shape/construction of a graph <em>and</em> “operates on a graph as an object in and of itself” (Moore &amp; Thompson, 2015, p. 784).</td>
</tr>
<tr>
<td>Emergent shape thinking</td>
<td>Student justifies the shape/construction of a graph through “understanding a graph <em>simultaneously</em> as what is made (a trace) and how it is made (covariation)” (Moore &amp; Thompson, 2015, p. 785).</td>
</tr>
</tbody>
</table>

*Student engagement.* Transcripts from the debriefing portion of the interview were coded for indicators (i.e. interest, enjoyment, and concentration) and facilitators of interviewees’ cognitive and emotional engagement using the same codebook established for coding posts and the first round of student interview data (table 8). Also, rich descriptions of how each interviewee participated during the task are used to describe their behavioral engagement during the interview. Together, the analyses of interviewees’ understanding and engagement while working in the interview are used to discuss associations between student engagement and understanding of precalculus concepts (research question two).

*Quantitative analyses.* Student responses and overall scores on the PCA were appended as columns to the quantitative spreadsheet containing weekly diary responses using student identification numbers. Basic descriptive statistics of PCA results describe the overall
understanding of precalculus concepts for students in the sample, quantitatively. These results were also disaggregated to report on the sample’s performance on sub-scales measuring covariational reasoning, computational reasoning, and conception of function.

The quantitative spreadsheet of weekly diary responses contains 15 rows for each student and columns for instructor-level (level 3) information (e.g., teaching approach), student-level (level 2) information (e.g., initial survey data and PCA scores), and diary-level (level 1) information (e.g., weekly diary data). This form of nesting is necessary for multiple level modeling ([MLM], Raudenbush & Bryk, 2002). This spreadsheet was imported into SAS (quantitative analysis software) so that SAS PROC MIXED can be used for MLM analyses (e.g., Bell, Ene, Smiley, & Schoeneberger, 2013).

The structure of quantitative data is such that weekly observations (level 1) are nested within students (level 2) which are nested within instructors (level 3). Accordingly, any data collected on a weekly basis can serve as level 1 variables (e.g., engagement), student data collected at one time-point can serve as level 2 variables (e.g., PCA score), and instructor data collected at one time-point can serve as level 3 variables (e.g., teaching approach). Procedures for MLM analyses in diary design studies were followed (e.g., Neupert, 2012; Neupert, Almeida, Mroczek, & Spiro, 2006; Neupert, Patterson, Davis, & Allaire, 2011). The structure of these data is captured in figure 6. Note, each instructor had a different number of students from their classes participate and each student submitted a different number of weekly diaries. Though, MLM is still applicable for this type of unbalanced data (Raudenbush & Bryk, 2002).
To begin quantitative analysis, a composite variable for weekly student engagement was created by averaging student-reported levels of interest, enjoyment, and concentration during precalculus from each week. Accordingly, each student has reported on their engagement at most 15 times over the duration of the study (an average of 8.2 responses per student). To analyze the nature of student engagement (research question 1), an unconditional model (a model without any predictor variables) was constructed. This model determined the amount of variation in student engagement present at each level. Additional models were then constructed attempting to explain the variance of student engagement present at each level.

All level 1 predictor variables were group-mean centered, meaning that values used in models are the student-reported deviations in that variable from their personal semester average value. In other words, students’ semester average values for predictor variables are taken to be zero. This form of centering allows for meaningful interpretation of results, and produces better fitting models (Raudenbush & Bryk, 2002). This analysis in conjunction with qualitative analyses are used to address the first research question (i.e. what is the nature of community
college students’ engagement during precalculus, and what role do teaching approaches have on these experiences?).

Secondly, an unconditional model (a model without any predictor variables) will be constructed to determine the amount of instructor- and student-level variation in student understanding (PCA scores) present in the sample. Understanding is a student-level (level 2) dependent variable because PCA scores were collected at one time-point for each student. Accordingly, diary-level variables are not suitable for predictors in models with understanding as the dependent variable. However, semester average values for these variables can be used as student-level data (Raudenbush & Bryk, 2002). For example, the average of reported values for student engagement from all 15 weeks of diary data can be used as a student-level predictor to quantitatively investigate how student engagement is associated with student understanding. Thus, semester average values can be used to “convert” diary-level variables into student-level variables so that they may be used as predictors in models with understanding as the dependent variable. Models with understanding as the dependent variable and independent variables accounting for teaching approaches (level 3), semester average student engagement (level 2), their cross-level interaction variable, and various other predictors were constructed to quantitatively investigate the relationship between student engagement and understanding (research question two). Mediation analysis (Baron & Kenny, 1986) was used to determine the role of teaching approaches on this relationship (research question two).

Integration of different data sources. During interpretation and presentation of results, findings from the separate analyses of qualitative and quantitative data were merged using a narrative approach (Decuir-Gunby & Schutz, under contract). With this approach, the qualitative and quantitative findings are woven together to present a phenomenological account of
community college students’ engagement in precalculus. During this process, results from each database are compared and contradicting and corroborating results are featured.

**Role of theoretical framework.** The role of my theoretical framework has been implicitly embedded throughout the discussions of data collection and analysis. This section will make the roles of my inquiry world view (social constructivism) and adopted theories (flow theory, precalculus conceptions, and teaching approaches) during data collection and analysis more explicit.

Under social constructivism, individuals’ lived experiences combine to form the lens through which they interact with the world. Thus, collecting students’ descriptions of their lived experiences and speaking with students during semi-structured interviews allows me to pull heavily from participants’ views of the nature of their engagement as community college precalculus students. The purpose of theme development during data analysis is to develop a co-constructed understanding of students’ lived experiences and to document student engagement inductively (Creswell, 2013).

Elements of engagement from the perspective of flow theory are measured quantitatively through weekly diaries Likert-type items and coded qualitatively from open-ended prompts on weekly diaries, student semi-structured interviews, and field notes from classroom observations. Additionally, flow theory guided the development of the weekly diary instrument (Shernoff et al., 2003). Task-based interviews and PCA results allow for student conceptions of precalculus concepts to be situated in the inventory of conceptions that exists in the research literature (e.g., Carlson et al., 2002; Carlson, Oehrtman, & Engelke, 2010; Moore & Thompson, 2015; Stevens & Moore, 2016; Thompson, 2011). Lastly, teaching approaches adopted by community college precalculus instructors were identified through instructor interviews using a protocol designed
specifically for that purpose (Mesa, Celis, et al., 2014). Thus, my inquiry worldview and adopted analytical frameworks have guided data collection and analysis.

**Validity and reliability.** Multiple methods of data collection were conducted to achieve triangulation for teaching approaches, student engagement, and student understanding of precalculus concepts. With respect to teaching approaches, instructor interviews were conducted with a protocol that has been used to accurately identify teaching approaches employed by community college mathematics instructors (Mesa, Celis, et al., 2014); video records and field notes from classroom observations, and students’ reports on weekly diaries were used for triangulation.

In terms of student engagement, student interviews, field notes, and weekly diaries are used for triangulation. Diaries serve as complete life documents that “document life as it is lived” (Allport, 1942, p. 56), thus, they are appropriate for capturing student experiences. Further, diary designs have been used to measure students’ engagement according to flow theory (Nakamura & Csikszentmihalyi, 2009), and importantly, this has been conducted in high school (e.g., Shernoff et al., 2003) and college settings (S. E. Peterson & Miller, 2004). The weekly diary instrument has been adapted from a validated instrument, which has been used in diary design studies to reliably measure student engagement from the perspective of flow theory (Shernoff et al., 2003).

Additionally, student understanding has been measured quantitatively via the PCA and qualitatively through task-based interviews. The PCA is a validated instrument for measuring students’ understanding of precalculus concepts (Carlson, Oehrtman, & Engelke, 2010).

All qualitative data was coded at least twice in NVivo and codebooks have been generated during analysis.
Pilot Work

Prior to conducting this study, a pilot study was conducted as a trial of the weekly diaries, and in particular how they were delivered to students regularly. This pilot work shed light on potential limitations for this study. In the pilot study, response rates to weekly diaries were drastically low – a response rate of roughly 5% was observed with a much smaller sample than the one used here. To combat this problem, the reward system for participation was put in place such that for every weekly diary submitted, students were entered for a raffle with a monetary prize. Drawings were held monthly. Additionally, MLM can handle large datasets with missing data, especially if the missing data occur at lower levels (Raudenbush & Bryk, 2002). Second, student responses to weekly diaries are self-reported so response bias is possible. In the case of pilot testing, student interviewees were enrolled in my future course, which was believed to have resulted in inauthentic responses to weekly diaries and interviews.
Chapter 4: Teaching Approaches

This chapter shares results of instructors’ self-reported teaching approaches, and purposefully selected instructors are introduced. Then, classroom observation data are used to provide rich descriptions of each selected instructor’s established classroom environment. Information and results shared in this chapter have been separated from the chapters on quantitative and qualitative results because these results are used as predictor variables in both quantitative and qualitative analyses. Further, these results were obtained early in the study so that groups of purposefully selected instructors and students could be selected, so chapter organization also demonstrates the chronology of the study.

Teaching Approaches

Interviews were conducted with each of the 15 precalculus instructors participating in the study. Lengths of these interviews ranged from 25 to 55 minutes, with a typical interview taking about 45 minutes. Thirteen of the 15 interviews were completed face-to-face. One interview needed to be completed virtually because the instructor needed to teach prior to completing the interview. Scheduling conflicts arose such that the final questions from the interview protocol needed to be completed via email with this instructor. Their written responses were appended to interview transcripts and coded accordingly. Instructor interview transcripts were analyzed to determine each instructor’s self-reported teaching approaches profile. These profiles were needed to address portions of the research questions pertaining to the effects of teaching approaches on the nature of student engagement (i.e. research question one) and any engagement-to-understanding relationship, should one exist (i.e. research question two).

Teaching approaches of each of the 15 precalculus instructors participating in the study are described based on the analysis of their interview transcripts. As stated in the previous
chapter, passages (i.e. a complete thought about their teaching) from interview transcripts were coded as indicating traditional (i.e. instructor is the authority in the classroom; privilege the content), meaning-making (i.e. emphasis on understanding and making connections), or student-support (i.e. students’ affect is more important than content mastery) approaches. Some passages were coded for more than one teaching approach. For example, the following passage is a portion of Mrs. Baker’s response to an interview question,

[Um] They have for 171 they just have homework and tests. I try to do quizzes and labs but that just took up extra time. And since we have so many chapters to cover and not a lot of time I have decided to take out the quizzes and the labs and just focus on the tests. Um, and also the homework. But now in not having that actual labs turned in for a grade um our lab days we turned that into days to cover extra material or to review and also for them to work on their homework in class and so that is another way if they have a particular question they bring it on our lab days and I can go individually again and so whenever they actually have a question with I can help them with it instead of you know I don’t want to talk in class or be embarrassed.

In this passage, Mrs. Baker privileges the content when referencing time constraints for covering all content; thus, she is identified as using a traditional approach. Though, she is also demonstrating use of a student-support approach when describing how she uses lab time give individual students’ attention and considering students’ feelings about asking questions during class time. A consequence of some passages being coded for more than one teaching approach, the number of codes used on each set of transcripts is potentially greater than the number of passages provided by instructors’ responses. Teaching approach profiles were created as triples
reflecting the proportion of codes for traditional, meaning-making, and student-support out of the
total number of codes used from each instructor’s interview transcripts. Transcripts generated an
average of 30 codes with a range from 17 to 42.

Instructor’s teaching approach profiles were then used to classify their overall profile.
Profiles with greater than 40% of codes in a single category were classified as heavy on that
category. This benchmark was chosen because it appears as a natural delineation in the data.
Except for Mrs. Baker, this taxonomy system established clear categories. In her case, the high
proportion of student-support codes were used to characterize her profile. In this regard, four
categories of teaching approach profiles were developed: traditional, meaning-making, student-
supportive, and balanced (table 10).

Table 10: Teaching approach profiles for instructors in the sample (* selected instructors).

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Traditional</th>
<th>Meaning-Making</th>
<th>Student-Support</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Traditional Profile</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harper*</td>
<td>0.69</td>
<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>Nugent</td>
<td>0.68</td>
<td>0.12</td>
<td>0.21</td>
</tr>
<tr>
<td>Brubaker</td>
<td>0.60</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>Holcomb</td>
<td>0.59</td>
<td>0.29</td>
<td>0.12</td>
</tr>
<tr>
<td>Miles</td>
<td>0.55</td>
<td>0.17</td>
<td>0.29</td>
</tr>
<tr>
<td><strong>Meaning-Making Profile</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gross</td>
<td>0.38</td>
<td>0.48</td>
<td>0.14</td>
</tr>
<tr>
<td>Sanders</td>
<td>0.25</td>
<td>0.46</td>
<td>0.29</td>
</tr>
<tr>
<td>Applegate</td>
<td>0.24</td>
<td>0.50</td>
<td>0.26</td>
</tr>
<tr>
<td>Geoffrey</td>
<td>0.22</td>
<td>0.44</td>
<td>0.33</td>
</tr>
<tr>
<td>Jackson*</td>
<td>0.22</td>
<td>0.49</td>
<td>0.29</td>
</tr>
<tr>
<td><strong>Balanced Profile</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flowers</td>
<td>0.35</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>Seager</td>
<td>0.35</td>
<td>0.26</td>
<td>0.39</td>
</tr>
<tr>
<td>Newsome</td>
<td>0.35</td>
<td>0.38</td>
<td>0.27</td>
</tr>
<tr>
<td>Smith*</td>
<td>0.27</td>
<td>0.33</td>
<td>0.40</td>
</tr>
<tr>
<td><strong>Student-Supportive Profile</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baker</td>
<td>0.48</td>
<td>0.08</td>
<td>0.44</td>
</tr>
</tbody>
</table>
These results were then used to purposefully select instructors to take part in classroom observations. Mrs. Harper, Mrs. Smith, and Mrs. Jackson were the three selected instructors. Mrs. Harper was selected because of the high proportion of a traditional approach present in her profile, Mrs. Smith was chosen due to her balanced teaching approach profile, and Mrs. Jackson was selected because of the relatively high proportion of a meaning-making approach in her profile.

In addition to her teaching approach profile, Mrs. Harper was selected for classroom observations because she taught an all-women’s section of precalculus. Her community college offers multiple women’s-only sections of mathematics courses each semester, and this detail along with her teaching approach profile contributed to her being selected over other traditional instructors. At the time of the interview, Mrs. Harper had 6.5 years of college teaching experience, and had taught precalculus multiple times. Mrs. Smith had 10 years of college teaching experience and 9 years of high school teaching experience at the time of our interview. She also has taught precalculus multiple times. Mrs. Jackson had 4 years of college teaching experience with limited high school teaching before taking her position at the community college. She had limited prior experience teaching precalculus to community college students. All three instructors have graduate degrees in mathematics education and are among the full-time faculty at the larger community college.

Classroom observations of Mrs. Harper’s, Mrs. Smith’s, and Mrs. Jackson’s classrooms were primarily used to document behavioral engagement of students in their classes; however, they also serve as an additional data source to report on teaching approaches for these three instructors. Altogether, roughly 713 minutes of classroom observations were conducted (219 minutes with Mrs. Harper, 223 minutes with Mrs. Smith, and 271 minutes with Mrs. Jackson).
Following the classroom observation protocol (Appendix G), instructor and student activities were described for five-minute intervals during each observation. The following paragraphs provide descriptions of the overall structure of each instructor’s classrooms and their typical activity while in class.

Mrs. Harper (traditional approach profile) begins each class period going over administrative items such as the calendar of upcoming assignments, assessments, and topics, and then outlines the agenda for the day. She posts skeleton notes for students to print prior to each class to the course’s learning management system, and uses those notes to guide her daily lectures. Following administrative items, Mrs. Harper begins taking notes on her own copy of the skeleton notes, projecting them for the students to see through use of a document camera. Mrs. Harper works an example or two, explaining her work slowly and clearly, and occasionally asking students if they have any questions about what she is doing. Then she assigns students a similar problem in, “… what I [she] call[s] a you try” (Mrs. Harper, instructor interview, emphasis added). Usually, students are neither encouraged nor deterred from working through practice problems with their peers; though, during the last observation, Mrs. Harper specifically directed students to work in groups of four. While students work through the problem(s), Mrs. Harper circulates the room to answer questions and monitor progress. Then Mrs. Harper presents her solution to the practice problems using the document camera and prompts students to ask questions sporadically throughout her explanations. In one instance, Mrs. Harper solicited volunteers to present their work on the document camera. This cycle (I do, you do) is then repeated until the end of the lesson, where Mrs. Harper assigns an exit ticket consisting of three or four problems for students to complete and have checked prior to leaving. At the end of the first observation, students were asked to turn in their exit ticket as a graded assignment. After
each lesson, Mrs. Harper posts her completed copy of the notes to the learning management system for students’ future reference.

Mrs. Smith’s (balanced teaching approach profile) class typically started with her providing students an opportunity to ask questions about problems they encountered on their homework (the same web-based homework system is used for all precalculus courses at this community college, but instructors may create their own assignments). At the beginning of the second and third observation, Mrs. Smith spent time informing students about the calendar for the remainder of the semester, reminded students about the date and time for the final exam, and informed students about her office hours during exam week and the hours which the college’s tutorial center had devoted specifically for assisting precalculus students. Administrative items were not discussed during the first observation.

Following students’ questions about homework and administrative items, Mrs. Smith presented a problem on the whiteboard. She would work through the problem, frequently asking students questions, and responding to students’ questions as they arose. Usually the class would spend a large portion of the time-period on this first example. In fact, during the first observation, only one example was completed and then Mrs. Smith used student questions to guide the remainder of the lesson while referencing other examples from a handout which she displayed using a document camera. The entire first lesson was conducted in this forum. I did not observe students working problems until 45 minutes into the second observation, which lasted roughly five minutes while Mrs. Smith circulated to answer students’ questions. Almost all 223 minutes of observations conducted in Mrs. Smith’s class were devoted to lecture. Throughout this time, Mrs. Smith used multiple representations (e.g., computer-generated graphs, handwritten tables, etc.), rarely asked questions of students, made connections to previously
covered material and foreshadowed upcoming topics, and welcomed questions from students. Although most of each observation was spent with Mrs. Smith lecturing, she used students’ questions to guide her discussion.

Mrs. Jackson (meaning-making approach profile) began her lessons discussing the agenda, reciting learning objectives for the day, and reminding students of upcoming deadlines. She used PowerPoint presentations, which were made available to students in advance on the learning management system, to present notes and examples. Mrs. Jackson circulated the room while going over notes and frequently asked questions, usually directed to specific students (e.g., “[Student], what does that definition mean to you? How might we apply it to what we have been learning?”). There did not appear to be an order to how she selected students to call on. When an example was encountered on the presentation, Mrs. Jackson would work the problem on the whiteboard while concurrently displaying the solution on the slides (step-by-step). During this time, she continues to pose questions for students (e.g., “[Student], what would you do next?”). Following notes, she would prompt students to work practice problems, where students were encouraged to work in groups. While students worked, Mrs. Jackson circulated the classroom and would pose probing questions to students (e.g., “What do we know about finding domains for rational functions, and what could that tell us about their behaviors?”) in addition to addressing their questions. To end lessons, Mrs. Jackson would either ask students to present their solutions on the whiteboard or she would display their work with the document camera. She would facilitate a discussion making connections across various solution strategies, and continued to pose specifically directed questions to students. This was the pattern observed during the first two observations, the final lesson observed entailed a web-based quiz game used as a review for an upcoming assessment.
The teaching approaches reported by Mrs. Harper and Mrs. Jackson aligned with what was observed during classroom visits. On the other hand, observation data from Mrs. Smith’s class do not completely corroborate the teaching approaches described in the analysis of her interview transcripts. Specifically, observational data suggest her teaching approach profile to be traditional, whereas analysis of her interview transcripts indicated a balanced profile. There are many departmental policies imposed on the instructors at the larger community college, which may have contributed to the discrepancies in the results of these two analyses. For example, faculty of the precalculus course are required to administer common assessments during specific testing windows. Policies such as this may impose time-constraints on instructors, which could yield differences in their reported teaching approaches and those that are observed or enacted.

**Looking Ahead**

This chapter described the student and instructor participants, presented results about instructors’ teaching approaches profiles, introduced Mrs. Harper, Mrs. Jackson, and Mrs. Smith, and provided more details about their classrooms and teaching approaches based on observational data. Information about the sample of students and teaching approach profiles are used as predictor variables in both quantitative and qualitative analyses. The next chapter focuses on results and findings of the quantitative analyses, which is followed by a chapter presenting qualitative results. Then, chapter 7 compares and contrasts the quantitative and qualitative results. Finally, a concluding chapter situates findings obtained in this study within the existing literature and utilizes theory to explain results obtained.
Chapter 5: Quantitative Results and Findings

This chapter elucidates the findings from the quantitative analyses conducted using student responses to Likert-type items on the weekly diary instrument, student demographic information, instructors’ teaching approaches profiles, and PCA scores. These data are used to address the two research questions quantitatively. This chapter is organized as follows: first, the model building process is explained, where examples of different types of hierarchical linear models are provided; then results addressing the first research question (i.e. what is the nature of student engagement and what role do teaching approaches have on these experiences?) are shared; and finally results addressing the second research question (i.e. is there a relationship between student engagement and understanding of precalculus topics?) are presented.

Recall that 829 responses were collected from 101 students over 15 weeks during the Fall 2016 semester. Thus, on average students submitted about 8 weekly diaries, though not all students participated the same amount. Further, each instructor had a different number of students participating. Multiple level modeling (MLM) is appropriate for this type of nested, yet unbalanced data (Raudenbush & Bryk, 2002). Many groups of models were conducted to sufficiently analyze the data and thoroughly address research question one (i.e. what is the nature of student engagement and what role do teaching approaches have on these experiences?). Research question two (i.e. is there a relationship between student engagement and understanding of precalculus topics?) calls for specific analyses (i.e. mediation analysis, [Baron & Kenny, 1986]), so less exploration was required. Results of these two sets of analyses are presented below; first, a description of the MLM model building process is provided.
Model Building Process

The model building process for each group of models was the same. First, a completely constrained model (no random effects\textsuperscript{13}) was conducted (Model A), then a completely unconstrained model (random effects included; Model B), and finally a model containing only the random effects that were significantly different from zero in the unconstrained model (Model C) was conducted. Examples of each type of model constructed are provided in Models A, B, and C, respectively, to help readers conceptualize the structure of these different types of models.

Note, for these models, students’ senses of competence (COMP), belongingness (BEL), and autonomy (AUTO) are weekly predictors. Students’ semester average perceived skill to task challenge relationship was used to characterize students into groups\textsuperscript{14} (GRP1, GRP2, and GRP3\textsuperscript{15}; these groups are described in detail below, see sections on challenge-skill balance). These groups were dummy coded and used as student-level predictors of student engagement.

The results of this analysis are provided below (see Challenge-skill balance as student-level predictor). The intent of displaying equations here is for readers to visualize the structure and differences of models conducted during analysis. Not all results presented required models containing both weekly- and student-level predictors as the example equations do.

\textsuperscript{13} Random effects estimate residual and student-level variation in the intercept and slopes of predictor variables.

\textsuperscript{14} Parameter estimates for the referent group are those indicated by the intercepts.

\textsuperscript{15} These labels are only used for this example, when these groups are described in the sections below, more appropriate variable names are utilized.
Model A: *Completely constrained model*

Level 1: $Engage_{it} = \beta_{0it} + \beta_{1it}(COMP) + \beta_{2it}(BEL) + \beta_{3it}(AUTO) + r_{it}$

Level 2:

$\beta_{0i} = \gamma_{00} + \gamma_{01}(GRP1) + \gamma_{02}(GRP2) + \gamma_{03}(GRP3) + u_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(GRP1) + \gamma_{12}(GRP2) + \gamma_{13}(GRP3)$

$\beta_{2i} = \gamma_{20} + \gamma_{21}(GRP1) + \gamma_{22}(GRP2) + \gamma_{23}(GRP3)$

$\beta_{3i} = \gamma_{30} + \gamma_{31}(GRP1) + \gamma_{32}(GRP2) + \gamma_{33}(GRP3)$

Model B: *Completely unconstrained model*

Level 1: $Engage_{it} = \beta_{0it} + \beta_{1it}(COMP) + \beta_{2it}(BEL) + \beta_{3it}(AUTO) + r_{it}$

Level 2:

$\beta_{0i} = \gamma_{00} + \gamma_{01}(GRP1) + \gamma_{02}(GRP2) + \gamma_{03}(GRP3) + u_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(GRP1) + \gamma_{12}(GRP2) + \gamma_{13}(GRP3) + u_{1i}$

$\beta_{2i} = \gamma_{20} + \gamma_{21}(GRP1) + \gamma_{22}(GRP2) + \gamma_{23}(GRP3) + u_{2i}$

$\beta_{3i} = \gamma_{30} + \gamma_{31}(GRP1) + \gamma_{32}(GRP2) + \gamma_{33}(GRP3) + u_{3i}$

Model C: *Significant random effects from Model B.*

Level 1: $Engage_{it} = \beta_{0it} + \beta_{1it}(COMP) + \beta_{2it}(BEL) + \beta_{3it}(AUTO) + r_{it}$

Level 2:

$\beta_{0i} = \gamma_{00} + \gamma_{01}(GRP1) + \gamma_{02}(GRP2) + \gamma_{03}(GRP3) + u_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(GRP1) + \gamma_{12}(GRP2) + \gamma_{13}(GRP3) + u_{1i}$

$\beta_{2i} = \gamma_{20} + \gamma_{21}(GRP1) + \gamma_{22}(GRP2) + \gamma_{23}(GRP3)$

$\beta_{3i} = \gamma_{30} + \gamma_{31}(GRP1) + \gamma_{32}(GRP2) + \gamma_{33}(GRP3) + u_{3i}$

In all three cases, the intercept and slopes of weekly-level predictors appear at level 1, which become the outputs at level 2. In the level 2 equation for the intercept ($\beta_0$), $\gamma_{00}$ is the grand-mean level of student engagement controlling for all other variables in the model. In the remaining level 2 equations, $\gamma_{vi}$ represents the sample average slope for the respective weekly-level predictor (i.e. main effect of the weekly-level predictor). All other $\gamma$'s in the level 2
equations represent the effects of student-level predictors on the $\beta$ being estimated. In other words, these are the main effects of the student-level predictors on student engagement ($\gamma_{0*}$) and the effects of student-level predictors on the relationships between weekly-level predictors and student engagement (cross-level interactions).

The differences between these models is in their random effects, or the amount of variation between student variation in the slopes of level 1 predictors on student engagement ($u_{si}$). Model A restricts this variation to zero, suggesting an assumption that effects of weekly-level predictors on student engagement do not vary from student to student. On the other hand, Models B and C account for this variation and estimate it. In Model C, variables that are not experienced differently between students have their random effects removed because it was determined that the student-level variation in the slopes of these predictors was not significantly different from zero in Model B.

Finally, Singer’s method (1998) was used to determine which type of model best fit the data. In all groups the completely constrained models (e.g., Model A) were deemed to be the best-fitting. However, in many cases Model B suggested significant random effects, indicating that significant between student variation in the effects of weekly-level predictors on student engagement is present. Thus, a decision was made to report on the appropriate unconstrained models (e.g., Model B or Model C) whenever possible. In some cases, SAS was unable to attain convergence requirements for unconstrained models due to the number of random effects considered. In these instances, results from the completely constrained model are reported (e.g., Model A). The model type used is noted in table headers throughout the chapter. Finally, in some models composite variables were used as predictors. For these instances, variable construction is described prior to the results of those models being presented.
Nature of Student Engagement

The quantitative dataset contains 829 weekly diaries nested within 101 students nested within 15 instructors. Thus, an unconditional 3-level model is warranted. Unconditional models have no predictor variables. They are used to discuss the partitioning of variance in the dependent variable across the levels of nesting and to provide the grand-mean for that variable at the lowest level (i.e., average weekly engagement). Figure 7 displays the results of this model.

![Variance in Student Engagement](image)

**Figure 7.** Partitioning of variance in student engagement across all three levels.

Figure 7 indicates that 33.8% of the variation in student engagement from this sample occurs within students ($\sigma^2 = 0.63, z = 19.09, p < 0.0001$), 62.7% of variation occurs between students ($\tau_{00} = 1.17, z = 5.91, p < 0.0001$), and 3.5% of variation occurs between instructors/classrooms ($\phi^2 = 0.07, z = 0.64, p = 0.26$). However, for this sample the portion of variance in student engagement attributed to differences in instructors/classrooms is not significantly different from zero. Accordingly, no further 3-level analyses were conducted.
Instead, instructor-level variables were attributed to the students from their classes and serve as time-invariant student-level variables for 2-level models.

As with the 3-level analysis, it is appropriate to conduct an unconditional model to begin the 2-level analyses (figure 8).

**Unconditional model.** The 2-level unconditional model indicates that 34% of variation in student engagement occurs within students, while 66% occurs between students ($\sigma^2 = 0.63, z = 19.09, p < .0001; \tau_{00} = 1.23, z = 6.38, p < .0001$). Each of these values are significantly different from zero. The average of weekly student engagement reported was 2.81 on a scale from 0 to 5 ($\gamma_{00} = 2.81, t = 24.08, p < .0001$). Recall that the variable for student engagement is determined as the average of student reported interest, enjoyment, and concentration. This average is computed weekly for each student. This model indicates that the average across all weeks for all students is 2.81.
Instructor reported teaching approaches as student-level predictors. To examine the effects of instructor reported teaching approaches on student engagement, teaching approach profiles were constructed, attributed to students, and used as student-level predictors. This model did not fit the data, and thus returned inconclusive results on the effects of instructor reported teaching approaches on student engagement. Table 11 presents the parameter estimates for this model. This model explains 0% of the between student and 0% of within student variance in student engagement.

Table 11: Parameter estimates for teaching approaches (Model A).

<table>
<thead>
<tr>
<th>Fixed effects</th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept, $\gamma_{00}$</td>
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<td>0.46</td>
</tr>
<tr>
<td>Traditional, $\gamma_{10}$</td>
<td>-18.20</td>
<td>0.54</td>
</tr>
<tr>
<td>Meaning Making, $\gamma_{20}$</td>
<td>-20.43</td>
<td>0.50</td>
</tr>
<tr>
<td>Student Support, $\gamma_{30}$</td>
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<td>0.52</td>
</tr>
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</table>

<table>
<thead>
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<th>Random Effects</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept, $\tau_{00}$</td>
<td>1.21</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>Residual, $\sigma^2$</td>
<td>0.63</td>
<td>&lt; .0001</td>
</tr>
</tbody>
</table>
It is not surprising that this model is not a good fit for the data because of the wide range in the number of students from each instructor’s class who agreed to participate in the study, the relatively small sample of instructors, and the fact that instructor reported teaching approaches were originally instructor-level predictors that are being supplanted to the student-level.

**Three psychological needs as weekly predictors.** On average, students in the sample reported a level of engagement of 2.79 out of 5 \( (\gamma_{00} = 2.79, t = 23.92, p < .0001) \) when controlling for students’ weekly perceived fulfillment of senses of competence, belongingness, and autonomy. Predictor variables for these needs were group-mean centered, no interaction terms were considered, and no student-level variables were used.

In weeks when students reported higher than personal average perceived fulfillment of their need for senses of competence \( (\gamma_{10} = 0.29, t = 7.66, p < .0001) \), belongingness \( (\gamma_{20} = 0.22, t = 6.45, p < .0001) \), and autonomy \( (\gamma_{30} = 0.08, t = 2.02, p = .04) \) they also tended to report higher levels of engagement (table 12). There were significant interindividual differences in the effects of weekly senses of competence \( (\tau_{11} = 0.04, z = 2.90, p = 0.002) \) and autonomy \( (\tau_{33} = 0.04, z = 2.59, p = 0.005) \) on student engagement, suggesting the potential for student-level predictors to explain the variability in these slopes. This model explains 45% of within student variation in student engagement.
Table 12: Parameter estimates for three psychological needs (Model C).

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<th>Estimate</th>
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</tr>
</thead>
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<td>Intercept, $\gamma_{00}$</td>
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<td>&lt; .0001</td>
</tr>
<tr>
<td>Competence, $\gamma_{10}$</td>
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<td>&lt; .0001</td>
</tr>
<tr>
<td>Belongingness, $\gamma_{20}$</td>
<td>0.22</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>Autonomy, $\gamma_{30}$</td>
<td>0.08</td>
<td>0.04</td>
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</table>

<table>
<thead>
<tr>
<th>Random Effects</th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept, $\tau_{00}$</td>
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<td>&lt; .0001</td>
</tr>
<tr>
<td>Competence, $\tau_{11}$</td>
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<td>0.002</td>
</tr>
<tr>
<td>Belongingness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autonomy, $\tau_{22}$</td>
<td>0.04</td>
<td>0.005</td>
</tr>
<tr>
<td>Residual, $\sigma^2$</td>
<td>0.36</td>
<td>&lt; .0001</td>
</tr>
</tbody>
</table>

These effects are significant and demonstrate the importance for instructors (and colleges) to attend to precalculus students’ needs fulfillment throughout the semester to support engagement. However, effect sizes for these results should be contextualized. On average, a 1-point increase in the level of student engagement, would require students to perceive of their needs for senses of competence, belongingness, and autonomy to be 3, 5, and 12 points (respectively) above their personal average needs fulfillment. In the case of students’ reported sense of autonomy, this is not possible as data were collected on a 6-point scale (0 to 5); and for senses of competence and belongingness this is highly unlikely. In other words, establishing a learning environment in which precalculus students feel competent, welcome, and a sense of autonomy on a weekly basis is important for student engagement, but needs fulfillment is only part of establishing an engaging environment for community college precalculus students. Moreover, the presence of significant random effects suggests that the sensitivity of student engagement to students’ weekly senses of competence and autonomy differs between students.

To continue investigating the effects of students’ perceived needs fulfillment on student engagement models were constructed using group-mean centered variables for competence (COMP), belongingness (BEL), autonomy (AUTO), and their four interaction terms.
On average, the level of student engagement was 2.80 out of 5 ($\gamma_{00} = 2.80, t = 23.80, p < .0001$) when controlling for students’ perceived needs fulfillment. In weeks when students felt above their personal average fulfillment of competence ($\gamma_{10} = 0.28, t = 7.25, p < .0001$) and belongingness ($\gamma_{20} = 0.21, t = 5.83, p < .0001$) they also tended to report significantly higher levels of engagement. Students’ perceived fulfillment of autonomy was not associated with the level of student engagement ($\gamma_{30} = 0.05, t = 1.37, p = 0.17$); however, this variable remained in the model because of marginally significant interactions.

The interaction between students’ perceived senses of competence and autonomy is marginally significant ($\gamma_{50} = -0.05, t = -1.71, p = 0.087$) such that in weeks where students reported higher than average sense of competence tended to be more engaged than weeks where students reported a lower than average sense of competence, though the role of autonomy on engagement in either case was the same (figure 9). Additionally, there is a marginally significant three-way interaction with these variables ($\gamma_{70} = 0.03, t = 1.73, p = 0.084$) such that in weeks were students report higher than their personal average sense of competence experienced different effects from autonomy based on their perceived fulfillment of belongingness (figure 10b). In this regard, increases in autonomy negatively affected engagement in weeks when students reported below personal average sense of belongingness. Like the results obtained without using interaction variables, the relationships between competence ($\tau_{11} = 0.04, z = 2.91, p = 0.002$) and autonomy ($\tau_{33} = 0.04, z = 2.63, p = 0.004$) on student engagement differ from student to student. Table 13 presents the parameter estimates for this model. This model explains 43% of within student variation in student engagement.
Table 13: Parameter estimates for three needs and their interactions (Model C).

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<td>Competence, $\gamma_{10}$</td>
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<td>&lt; .0001</td>
</tr>
<tr>
<td>Belongingness, $\gamma_{20}$</td>
<td>0.21</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>Autonomy, $\gamma_{30}$</td>
<td>0.05</td>
<td>0.17</td>
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<tr>
<td>COMP*BEL, $\gamma_{40}$</td>
<td>-0.01</td>
<td>0.80</td>
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<tr>
<td>COMP*AUTO, $\gamma_{50}$</td>
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<td>0.087</td>
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<td>BEL*AUTO, $\gamma_{60}$</td>
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<td>0.17</td>
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<td>COMP<em>BEL</em>AUTO, $\gamma_{70}$</td>
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<td>0.084</td>
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<tr>
<th>Random Effects</th>
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<td>Competence, $\tau_{11}$</td>
<td>0.04</td>
<td>0.002</td>
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<tr>
<td>Belongingness, $\tau_{22}$</td>
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<td>0.004</td>
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<td>Autonomy, $\tau_{33}$</td>
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<td>0.004</td>
</tr>
<tr>
<td>Residual, $\sigma^2$</td>
<td>0.36</td>
<td>&lt; .0001</td>
</tr>
</tbody>
</table>

**Figure 9.** Explanation of COMP*AUTO on student engagement.
Figure 10. Explanation of COMP*BEL*AUTO on student engagement.

Three needs as predictors of weekly changes in student engagement. The diary design of this study structures data such that weekly reports were recorded for each student. This allows for engagement from the previous week to be controlled for, and the effects of independent variables on weekly changes in the average level of student engagement to be investigated (at the
expense of one week’s data for each student). In this regard, heightened senses of competence ($\gamma_{20} = 0.29, t = 10.29, p < .0001$) and belongingness ($\gamma_{30} = 0.18, t = 4.99, p < .0001$) were associated with increased levels of engagement from one week to the next. In other words, students’ perceived fulfillment of their senses of competence and belongingness are related to weekly changes in student engagement (table 14). Students’ perceived sense of autonomy was not associated with changes in student engagement in this sample ($\gamma_{40} = 0.06, t = 1.49, p = 0.14$). However, significant random effects for all three predictors suggest interindividual differences in the effects of perceived weekly needs fulfillment on weekly change in student engagement ([COMP]$\tau_{11} = 0.03, z = 2.35, p = 0.009$; [BEL]$\tau_{22} = 0.05, z = 1.78, p = 0.04$; [AUTO]$\tau_{33} = 0.04, z = 2.43, p = 0.008$). This model explains 52% of within student variation in engagement.

Table 14: Parameter estimates for effects of needs on weekly change in engagement (Model B).

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<th>Estimate</th>
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<td>ENGPW, $\gamma_{10}$</td>
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<td>COMP, $\gamma_{20}$</td>
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</tr>
<tr>
<td>BEL, $\gamma_{30}$</td>
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<tr>
<td>AUTO, $\gamma_{40}$</td>
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<td>0.14</td>
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<td>COMP, $\tau_{11}$</td>
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<td>0.009</td>
</tr>
<tr>
<td>BEL, $\tau_{22}$</td>
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<td>0.04</td>
</tr>
<tr>
<td>AUTO, $\tau_{33}$</td>
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<td>0.008</td>
</tr>
<tr>
<td>Residual, $\sigma^2$</td>
<td>0.39</td>
<td>&lt; .0001</td>
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**Challenge-skill balance as weekly predictor.** A key concept of flow theory is the relationship between students’ perceived skills and level of challenge of tasks/materials (Csikszentmihalyi, 1975, 1990, 1997). This section presents different models using students’ challenge-skill balance as weekly and student-level predictors of engagement. Items on the
weekly diary instrument prompted students to report on how skilled (SKL) they felt they were and how challenging (CHL) they perceived material from the week to be. Responses to these items were used to generate four groups, which were then analyzed with dummy variables: high skill and low challenge (HS-LC; $CHL - SKL < -2.5$), low skill and high challenge (LS-HC; $CHL - SKL > 2.5$), slightly over skilled (S-OS; $-2.5 \leq CHL - SKL \leq 0$), and slightly over challenged (S-OC; $0 < CHL - SKL \leq 2.5$). Figure 11 provides a visualization for the challenge-skill balance for each of these characterizations.

![Figure 11. Visualization of challenge-skill characterizations](image)

Using students’ challenge-skill balance as a weekly predictor means that students were dummy coded as being HS-LC, LS-HC, S-OS, or S-OC for each week based on their responses to that week’s diary. Challenge and skill variables were group-mean centered prior to characterizing students’ challenge-skill balance group. The group of HS-LC students was used as the referent group, so the parameter estimate for the intercept indicates the average level of
engagement for HS-LC students. Main effects for the other dummy codes indicate the average difference in engagement reported by students in those groups compared to that of students in the HS-LC group. Table 15 displays these main effects.

Table 15: Parameter estimates for weekly challenge-skill balance groups (Model B).

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<td>LS-HC, ( \gamma_{10} )</td>
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</tr>
<tr>
<td>S-OC, ( \gamma_{20} )</td>
<td>-0.81</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>S-OS, ( \gamma_{30} )</td>
<td>-0.47</td>
<td>0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random Effects</th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept, ( \tau_{00} )</td>
<td>1.83</td>
<td>0.001</td>
</tr>
<tr>
<td>LS-HC, ( \tau_{11} )</td>
<td>0.92</td>
<td>0.04</td>
</tr>
<tr>
<td>S-OC, ( \tau_{22} )</td>
<td>0.41</td>
<td>0.04</td>
</tr>
<tr>
<td>S-OS, ( \tau_{33} )</td>
<td>1.10</td>
<td>0.002</td>
</tr>
<tr>
<td>Residual, ( \sigma^2 )</td>
<td>0.50</td>
<td>&lt; .0001</td>
</tr>
</tbody>
</table>

In weeks when students reported challenge-skill balance characterized them as HS-LC (high skill and low challenge) the average level of student engagement was 3.48 out of 5 (\( \gamma_{00} = 3.48, t = 14.98, p < .0001 \)). In weeks when students’ challenge-skill balance characterized them as LS-HC (\( \gamma_{10} = -1.44, t = -5.27, p < .0001 \)), S-OC (\( \gamma_{20} = -0.81, t = -4.06, p < .0001 \)), and S-OS (\( \gamma_{30} = -0.47, t = -2.13, p = 0.03 \)) they tended to report significantly lower levels of engagement than HS-LC students. The effects of being in each group, based on weekly challenge-skill balance, on student engagement are experienced differently from student to student, as indicated by significant random effects (table 15). This model explains 21% of within student variation in student engagement.

**Challenge-skill balance as student-level predictor.** Students’ semester average challenge-skill balance allows these groupings to be used as student-level predictors of engagement based on the semester average challenge-skill balance across all weeks. Similar to above, students reported difference between perceived level of challenge of materials and
personal skills were used to group students as HS-LC (high skill and low challenge), LS-HC (low skill and high challenge), S-OS (slightly over-skilled), or S-OC (slightly over-challenged) through dummy codes. However, instead of dummy codes being constructed for each student each week, they were constructed once based on students’ semester average difference between perceived challenge and skill. As with before, the HS-LC group is used as the referent. Table 16 presents the results of the analysis where challenge-skill balance groups are used as student-level predictors of weekly engagement.

### Table 16: Parameter estimates for semester average challenge-skill balance groups (Model A).

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Fixed Effects</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{00}$</td>
<td>3.09</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>LS-HC, $\gamma_{10}$</td>
<td>-1.64</td>
<td>0.0004</td>
</tr>
<tr>
<td>S-OC, $\gamma_{20}$</td>
<td>-0.46</td>
<td>0.16</td>
</tr>
<tr>
<td>S-OS, $\gamma_{30}$</td>
<td>0.27</td>
<td>0.43</td>
</tr>
<tr>
<td><em>Random Effects</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\tau_{00}$</td>
<td>0.98</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>Residual, $\sigma^2$</td>
<td>0.63</td>
<td>&lt; .0001</td>
</tr>
</tbody>
</table>

The level of student engagement for students whose semester average challenge-skill balance characterized them as being HS-LC was 3.09 out of 5 ($\gamma_{00} = 3.09$, $t = 10.83$, $p < .0001$). Students whose semester average challenge-skill balance characterized them as being LS-HC ($\gamma_{10} = -1.64$, $t = -3.64$, $p = 0.0004$) tended to report significantly lower levels of engagement than students characterized as being HS-LC. Students characterized as being S-OC ($\gamma_{20} = -0.46$, $t = -1.41$, $p = 0.16$) and S-OS ($\gamma_{30} = 0.27$, $t = 0.80$, $p = 0.43$) did not report significantly different levels of engagement than students characterized as HS-LC based on their semester average challenge-skill balance. This model explains 20% of between student and 0% of within student variation in student engagement.
Using students’ semester average challenge-skill balance groups as student-level predictors allows for inclusion of weekly-level predictors without overcrowding models. Accordingly, it is possible to examine the effects of students’ perceived needs fulfillment on student engagement for students in each semester average challenge-skill balance group. Table 17 presents the parameter estimates of this analysis.

Table 17: Parameter estimates for three needs by challenge-skill balance groups (Model A).

<table>
<thead>
<tr>
<th>Fixed Effects</th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>HS-LC, ( \beta_{0i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, ( \gamma_{00} )</td>
<td>3.08</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>COMP, ( \gamma_{01} )</td>
<td>0.17</td>
<td>0.03</td>
</tr>
<tr>
<td>BEL, ( \gamma_{02} )</td>
<td>0.23</td>
<td>0.02</td>
</tr>
<tr>
<td>AUTO, ( \gamma_{03} )</td>
<td>0.22</td>
<td>0.0004</td>
</tr>
<tr>
<td>LS-HC, ( \beta_{1i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main Effect, ( \gamma_{10} )</td>
<td>-1.63</td>
<td>0.0004</td>
</tr>
<tr>
<td>COMP, ( \gamma_{11} )</td>
<td>-0.09</td>
<td>0.44</td>
</tr>
<tr>
<td>BEL, ( \gamma_{12} )</td>
<td>-0.01</td>
<td>0.94</td>
</tr>
<tr>
<td>AUTO, ( \gamma_{13} )</td>
<td>-0.01</td>
<td>0.94</td>
</tr>
<tr>
<td>S-OC, ( \beta_{2i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main Effect, ( \gamma_{20} )</td>
<td>-0.47</td>
<td>0.15</td>
</tr>
<tr>
<td>COMP, ( \gamma_{21} )</td>
<td>0.16</td>
<td>0.06</td>
</tr>
<tr>
<td>BEL, ( \gamma_{22} )</td>
<td>0.01</td>
<td>0.93</td>
</tr>
<tr>
<td>AUTO, ( \gamma_{23} )</td>
<td>-0.16</td>
<td>0.03</td>
</tr>
<tr>
<td>S-OS, ( \beta_{3i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main Effect, ( \gamma_{30} )</td>
<td>0.28</td>
<td>0.42</td>
</tr>
<tr>
<td>COMP, ( \gamma_{31} )</td>
<td>0.26</td>
<td>0.007</td>
</tr>
<tr>
<td>BEL, ( \gamma_{32} )</td>
<td>-0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>AUTO, ( \gamma_{33} )</td>
<td>-0.14</td>
<td>0.07</td>
</tr>
</tbody>
</table>

| Random Effects | | |
| Intercept, \( \tau_{00} \) | 1.02 | < .0001 |
| Residual, \( \sigma^2 \) | 0.43 | < .0001 |

On average, the level of student engagement for precalculus students whose semester average challenge-skill balance characterized them as HS-LC was 3.09 out of 5 \((\gamma_{00} = 3.09, t = 10.84, p < .0001)\). Additionally, these students’ perceived senses of competence \((\gamma_{10} = \ldots)\).
0.17, 𝑡 = 2.13, 𝑝 = 0.03), belongingness (𝑦_{20} = 0.23, 𝑡 = 2.33, 𝑝 = 0.02), and autonomy (𝑦_{30} = 0.22, 𝑡 = 3.53, 𝑝 = 0.0004) were all positively associated with their engagement.

Students whose semester average challenge-skill balance characterized them as LS-HC tended to report being significantly less engaged on a weekly basis than students characterized as HS-LC even when controlling for needs fulfillment (𝑦_{01} = −1.63, 𝑡 = −3.65, 𝑝 = 0.0004). Further, perceived needs fulfillment does not dampen the effect of being LS-HC on student engagement ([COMP] 𝑦_{11} = −0.09, 𝑡 = −0.77, 𝑝 = 0.44; [BEL] 𝑦_{21} = −0.01, 𝑡 = −0.07, 𝑝 = 0.94; [AUTO] 𝑦_{31} = −0.01, 𝑡 = −0.08, 𝑝 = 0.94).

Students whose semester average challenge-skill balance characterized them as S-OC tended to report similar weekly levels of engagement as HS-LC students when controlling for needs fulfillment (𝑦_{02} = −0.47, 𝑡 = −1.44, 𝑝 = 0.15). However, S-OC students’ perceived needs fulfillment effected their weekly engagement differently than students whose semester average challenge-skill balance placed them in other groups. Specifically, S-OC students’ sense of competence has marginal positive effects on weekly engagement (𝑦_{12} = 0.16, 𝑡 = 1.88, 𝑝 = 0.06), their sense of belongingness had no effects on weekly engagement (𝑦_{22} = 0.01, 𝑡 = 0.09, 𝑝 = 0.93), and their sense of autonomy was negatively associated with weekly engagement (𝑦_{32} = −0.16, 𝑡 = −2.21, 𝑝 = 0.03).

Finally, students whose semester average challenge-skill balance characterized them as S-OS tended to report similar weekly levels of engagement as HS-LC students when controlling for needs fulfillment (𝑦_{03} = 0.28, 𝑡 = 0.82, 𝑝 = 0.42). Though, S-OS students experienced the effects of needs fulfillment on their weekly engagement differently than students in other groups. S-OS students’ sense of competence was positively associated with weekly engagement (𝑦_{13} = 0.26, 𝑡 = 2.71, 𝑝 = 0.007), their sense of belongingness was not associated with weekly engagement (𝑦_{23} = 0.01, 𝑡 = 0.12, 𝑝 = 0.90).
engagement ($\gamma_{23} = -0.16, t = -1.32, p = 0.19$), and their sense of autonomy has marginal negative effects on weekly engagement ($\gamma_{33} = -0.14, t = -1.79, p = 0.07$). This model explains 17% of between student variation and 32% of within student variation in student engagement.

It is important to note that whether considering students’ weekly challenge-skill balance or semester average challenge-skill balance, students characterized as HS-LC tended to report higher levels of engagement. That is, weeks when students perceived of themselves as highly skilled compared to the level of difficulty they associated with materials being learned were reported to have the highest levels of engagement, on average. And, students who typically reported feeling highly skilled for under challenging work across the semester tended to report higher levels of engagement on a weekly basis than their peers whose perceptions about the relationship between their skills and the difficulty of materials was different.

Another important commonality from these results is the effect size of LS-HC on students’ level of weekly engagement. In the three analyses presented in this section, students whose challenge-skill balance characterized them as LS-HC were about 1.5 points (on a 6-point scale) less engaged than their HS-LC peers. Further, this adverse experience is not lessened by LS-HC students’ perceived needs fulfillment. Based on this result, community colleges (and students) should take care in placing students into precalculus. Also, community college precalculus instructors should consider methods for reducing students’ stress from feeling under-skilled for challenging material (as perceived by students), as once students believe they are under skilled and over challenged their level of engagement in class is drastically lower than that of their peers. Perhaps the focal difference between the four challenge-skill balance groupings is confidence. Students characterized as LS-HC are the only group from this sample who did not
tend to experience at least marginally significant positive effects from their sense of competence on their weekly engagement. Further, two groups of students share the perception that their skills are not up to par with the difficulty they attribute to topics being learned – LS-HC and S-OC. However, S-OC students’ sense of competence is positively related to their level of weekly engagement, and on a weekly basis S-OC students tended to report similar levels of engagement as students who believed they were over skilled and under challenged by the materials being learned.

Students’ perceptions about teaching. Although instructor reported data resulting in teaching approaches profiles did not serve as an appropriate predictor of student engagement with this sample, it is still possible to examine effects of teaching on student engagement through student reports about teaching. Items on the weekly diary instrument prompted students to rank the extent to which they agreed that their instructor: used their ideas in class (MyIdeas), moved too fast for comprehension (MovedFast), slowed down to accommodate confidence (Slowed), cares about them personally (Cares), helped make connections between mathematics and the world (Connections), and cares about their understanding (MyUnderstanding).

When these variables are used as weekly predictors of student engagement, the level of student engagement is 2.80 ($\gamma_0 = 2.80, t = 24.00, p < .0001$). Students’ weekly perceptions about teaching are all significantly associated with their level of engagement such that students tended to report lower levels of engagement associated with a faster than average pace and higher levels of engagement associated with the other variables (table 18). This model explains 22% of within student variation in student engagement.
Table 18: Parameter estimates for effects of student reports about teaching on engagement (Model A).

<table>
<thead>
<tr>
<th>Fixed Effects</th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept, $\gamma_{00}$</td>
<td>2.80</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>MyIdeas, $\gamma_{10}$</td>
<td>0.15</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>MovedFast, $\gamma_{20}$</td>
<td>-0.08</td>
<td>0.0004</td>
</tr>
<tr>
<td>Slowed, $\gamma_{30}$</td>
<td>0.08</td>
<td>0.006</td>
</tr>
<tr>
<td>Cares, $\gamma_{40}$</td>
<td>0.12</td>
<td>0.0002</td>
</tr>
<tr>
<td>Connections, $\gamma_{50}$</td>
<td>0.09</td>
<td>0.0028</td>
</tr>
<tr>
<td>MyUnderstanding, $\gamma_{60}$</td>
<td>0.08</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random Effects</th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept, $\tau_{00}$</td>
<td>1.26</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>Residual, $\sigma^2$</td>
<td>0.49</td>
<td>&lt; .0001</td>
</tr>
</tbody>
</table>

When students’ weekly perceptions about teaching are averaged for the semester and used as student-level predictors, the sample average level of student engagement is 0.78 ($\gamma_{00} = 0.78, t = 2.32, p = 0.02$; see Model 1 in table 19). Student reported semester average accounts of instructors using their ideas in class ($\gamma_{01} = 0.06, t = 0.52, p = 0.60$), moving too quickly for comprehension ($\gamma_{02} = -0.07, t = -1.02, p = 0.31$), slowing down for confidence ($\gamma_{03} = 0.07, t = 0.61, p = 0.54$), demonstrating care for students personally ($\gamma_{04} = 0.13, t = 1.13, p = 0.26$), and helping students make connections between mathematics and the world ($\gamma_{05} = 0.15, t = 1.36, p = 0.18$) are not associated with weekly level of student engagement in the sample (see Model 1 in table 19). However, students’ semester average agreement that their instructor cares about their understanding is positively associated with their level of engagement on a weekly basis ($\gamma_{06} = 0.35, t = 3.33, p = 0.0012$). Model 1 explains 63% of between student variation and 0% of within student variation in student engagement.

The average level of engagement for students who strongly disagreed that their instructor cared about their understanding for the duration of the semester was 0.69 ($\gamma_{00} = 0.69, t = 3.19, p = 0.0019$; see Model 2 in table 19). Increases in semester average perceptions about instructor care towards student understanding are positively associated with student engagement.
\( \gamma_{06} = 0.67, t = 10.73, p < .0001; \) see Model 2 in table 19. Model 2 explains 59\% of between student variation and 0\% of within student variation in student engagement. Table 19 presents the parameter estimates for both models.

Table 19: Parameter estimates for semester average of student reports about teaching on engagement (both Model type A).

<table>
<thead>
<tr>
<th>Variables</th>
<th>Model 1</th>
<th></th>
<th></th>
<th>Model 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, ( \gamma_{00} )</td>
<td>0.78</td>
<td>0.02</td>
<td></td>
<td>0.69</td>
<td>0.0019</td>
</tr>
<tr>
<td>MyIdeas, ( \gamma_{01} )</td>
<td>0.06</td>
<td>0.60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MovedFast, ( \gamma_{02} )</td>
<td>-0.07</td>
<td>0.31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slowed, ( \gamma_{03} )</td>
<td>0.07</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cares, ( \gamma_{04} )</td>
<td>0.13</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connections, ( \gamma_{05} )</td>
<td>0.15</td>
<td>0.18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MyUnderstanding, ( \gamma_{06} )</td>
<td>0.35</td>
<td>0.0012</td>
<td></td>
<td>0.67</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>Random Effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, ( \tau_{00} )</td>
<td>0.46</td>
<td>&lt; .0001</td>
<td></td>
<td>0.50</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>Residual, ( \sigma^2 )</td>
<td>0.63</td>
<td>&lt; .0001</td>
<td></td>
<td>0.63</td>
<td>&lt; .0001</td>
</tr>
</tbody>
</table>

**Student Understanding of Precalculus Concepts**

This section reports on the overall sample of students’ understanding of precalculus concepts as measured by their performance on the precalculus concepts assessment (PCA) and for three subscales measuring students’ process view of function, covariational reasoning, and computational abilities. In this regard, 16 items on the assessment measure process view of function (compared to action view of function), 5 items measure covariational reasoning, and 9 items measure computational abilities. The PCA is a 25-item multiple choice assessment, so some items are included in multiple subscales.

On average, students scored 8.61 (34.4\%) with a standard deviation of 4.35, where the lowest score in the sample was a 0 and the highest score was a 20. With respect to students’ process view of function, the sample average was 4.95 (40.1\%) with a standard deviation of 2.65. The sample minimum for this subscale was 0 and the sample maximum was 12 (out of 16). The
sample average score on the covariational reasoning subscale was 1.83 (36.6%) with standard
deviation 1.39. The sample minimum for this subscale was 0 and the maximum was 5 (out of 5).
Finally, the sample average score on the computational ability subscale was 3.08 (34.2%) with
standard deviation 1.63. The sample minimum here was 0 and maximum was 7 (out of 9). To put
these values in context, Carlson and her colleagues (2010) report that students scoring at least 13
on the PCA prior to entering first-semester calculus were more likely to be successful in calculus
than their peers scoring 12 or lower.

**Relationship between student engagement and understanding.** Like the variation in
student engagement, there was not a significant source of variation in PCA scores at the
instructor level ($\tau_{00} = 0.004, z = 0, p = 0.50$). This is not to suggest that instructors do not
make a difference in students’ understanding of precalculus concepts (or student engagement),
but means that for this sample, the distribution of variation in PCA scores was not significantly
different from one instructor’s students to another’s. The lack of variation across classrooms in
performance on the PCA renders MLM inappropriate for further analysis. Thus, simple
mediation analysis (Baron & Kenny, 1986) was conducted to determine if there was a
relationship between PCA scores and students’ semester average engagement and if teaching
approaches mediated this relationship (i.e. research question two).

Following procedures outlined by Baron & Kenny (1986) for conducting simple
mediation analysis, linear regression was used to examine the relationship between students’
semester average engagement and PCA scores (figure 12). Results of this analysis indicate little
to no quantitative relationship between students’ semester average engagement and
understanding ($\beta_1 = 0.63, t = 1.45, p = 0.15$). Without a statistically significant relationship
between engagement and understanding there is no justification for continuing the mediation analysis process for determining the role of teaching approaches.

Figure 12. Relationship between student engagement and understanding.

Summary of Quantitative Results

In response to the first research question (i.e. what is the nature of student engagement and what role do teaching approaches have on these experiences?), quantitative findings indicate that student engagement varies within students from week to week as well as between students. These results also provide empirical evidence associating factors of community college precalculus students’ perceptions of needs fulfillment, challenge-skill balance, and teaching with their engagement, and suggest that different groups of students (e.g., HS-LC, LS-HC, S-OC, and S-OS) experience the effects of these factors on their weekly engagement differently. With respect to instructor reported teaching approaches, data obtained for this study were not sufficient to make conclusion about the role of teaching approaches on the nature of student engagement. More research to investigate this question is required.
In response to the second research question (i.e. is there a relationship between student engagement and understanding of precalculus concepts?), quantitative results from this sample do not indicate a relationship between community college precalculus students’ engagement and understanding. However, it should be noted that students self-selected to participate in this study, and were only eligible if their instructor had already agreed to participate. Consequently, this sample is likely not representative of precalculus students enrolled at the two community colleges; thus, results presented should be verified in future research and the second research question should be considered open. Results presented here pertaining to the nature of student engagement (research question one) should also be considered with caution; though there is alignment with results documented in the literature on student engagement. A discussion situating the results of this study in the existing literature is provided in the final chapter. First, qualitative results are shared.
Chapter 6: Qualitative Results and Findings

This chapter reports on the results obtained through the various qualitative analyses conducted with the weekly posts, student interviews, classroom observations, and utilizes previously shared results on teaching approaches. To begin, the first research question (i.e. what is the nature of student engagement and what role do teaching approaches have on these experiences?) is addressed using students’ weekly posts. This provides a response to the first research question established from the entire student sample. Then, a deeper investigation into this question is provided through findings from the first round of student interviews. Finally, the second round of student interviews are used to respond to the second research question qualitatively (i.e. is there a relationship between student engagement and understanding of precalculus concepts?).

Nature of Student Engagement

In general, students tended to report on only one indicator of engagement in their posts, occasionally referring to two, and very rarely made posts warranting codes from all three indicators of engagement. In this regard, students mostly reported on their level of enjoyment or indicated concentration as opposed to low concentration. Further, posts tended not to reveal whether students were interested in precalculus class. In other words, students were very vocal about their emotional engagement during precalculus, frequently spoke about needing to be cognitively engaged, and rarely discussed their level of interest in the course. Table 20 enumerates a cross-tabulation of posts coded with one or two indicators of engagement.
Table 20: *Cross-tabulation of engagement codes from weekly posts.*

<table>
<thead>
<tr>
<th></th>
<th>~Concentrate</th>
<th>~Enjoy</th>
<th>~Interest</th>
<th>Concentrate</th>
<th>Enjoy</th>
<th>Enjoy Neutral</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Concentrate</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~Enjoy</td>
<td>2</td>
<td>195</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~Interest</td>
<td>6</td>
<td>13</td>
<td>47</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concentrate</td>
<td>0</td>
<td>22</td>
<td>2</td>
<td>172</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Enjoy</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>24</td>
<td>144</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Enjoy Neutral</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>18</td>
<td>1</td>
<td>29</td>
</tr>
</tbody>
</table>

In addition to providing insight into the general nature of student engagement as written by students on a weekly basis, posts also captured the variation in student engagement between students and within individual students over time. Table 21 shares the posts from three students for the duration of the semester along with the indicators of engagement coded for each post. Reading down the columns provides evidence for how student engagement changes weekly for individual students. All three students exemplify internal variation in their level of engagement. Further, comparing the codes for engagement for different students each week demonstrates differences in individual students’ experiences.
Table 21: *Posts demonstrating within and between student variation in student engagement qualitatively.*

<table>
<thead>
<tr>
<th>Week</th>
<th>Student 24 Engagement Codes</th>
<th>Student 37 Engagement Codes</th>
<th>Student 81 Engagement Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I'd say I take precalc at a time of day when I'm most tired so it's hard for me to concentrate and that I should do more of the homework as practice</td>
<td>~Concentrate</td>
<td>Precalculus is so trying and I can't believe I signed up to take this three times a week 😩😩</td>
</tr>
<tr>
<td>2</td>
<td>Had a good review for test. Teacher wants to make sure we fully understand what's gonna be on the test &amp; can tell he wants us to all do well.</td>
<td>~Concentrate, Enjoy Neutral</td>
<td>Why did I sign up for this class again ?</td>
</tr>
<tr>
<td>3</td>
<td>Math class this week was very straightforward and easy to understand; I feel more confident in class and in my math abilities.</td>
<td>~Concentrate, Enjoy Neutral</td>
<td>I never thought I'd say this, but math was lit this week .</td>
</tr>
<tr>
<td>4</td>
<td>I'm beginning to like calculus more lately &amp; I find it easier to understand topics; I feel more confident within myself and my success in this class.</td>
<td>Concentrate, Enjoy</td>
<td>Math was lit this week</td>
</tr>
<tr>
<td>5</td>
<td>I like precalc more and more and it's making more sense to me now. I can tell that my teacher genuinely cares about us and truly wants us to learn.</td>
<td>Concentrate, Enjoy</td>
<td>I can't remember what we did, and I couldn't tell you if that's bad or not . Hashtag lol</td>
</tr>
<tr>
<td></td>
<td>Sometimes I feel like we go too slow in class so I get bored...but I feel prepared for the upcoming test.</td>
<td>~Interest</td>
<td>Super ready for break 😅😅</td>
</tr>
<tr>
<td>---</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>----------</td>
<td>----------------------------</td>
</tr>
<tr>
<td>7</td>
<td>We only had class 1 day this week and we took a test. I felt very well prepared for the test &amp; thought it was fair. I studied more than usual for it.</td>
<td>Concentrate</td>
<td>Pre calc is definitely not my fave class, but this week was tolerable</td>
</tr>
<tr>
<td>8</td>
<td>I didn't think math this week was too bad; I thought we spent too much time going over the test instead of learning new material, though.</td>
<td>Enjoy Neutral</td>
<td>Pre calc was ok, I improved my test grade by 10 points so that had me slightly more motivated</td>
</tr>
<tr>
<td>9</td>
<td>Precalc hasn't been too difficult...having a hard time paying attention in class. I still think the classes are too long to maintain sufficient focus.</td>
<td>~Concentrate</td>
<td>I can't remember what was talked about bc it went by in such a blur</td>
</tr>
<tr>
<td>10</td>
<td>The material in precalc this week was a little more difficult so I found myself having to ask more questions and concentrate more to understand.</td>
<td>Concentrate</td>
<td>I would be content with a W at this point</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>We didn't have much class time learning new material because of the schedule, but I felt prepared for the test &amp; new material is straightforward.</td>
<td>Ready for thanksgiving break but this week was cool</td>
<td>Enjoy, Interest</td>
</tr>
<tr>
<td>12</td>
<td>My teacher mostly lectured this week but I think did a better job incorporating our ideas into the lesson than usual...made for better class time.</td>
<td>Enjoy Neutral</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>This week was very short; however, the information was pretty interesting. I think I should do some extra practice, though.</td>
<td>Concentrate, Interest</td>
<td>This week was awesome because it was only one day! This impacted my ability to concentrate greatly in a positive manner!</td>
</tr>
<tr>
<td>14</td>
<td>This is the hardest unit for me because of the content; however, my teacher does a good job of answering all our questions. I need to study more...</td>
<td>Concentrate</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Math class this week was pretty straightforward--nothing too hard going on...I think we should have done more final review, though.</td>
<td>Enjoy Neutral</td>
<td></td>
</tr>
</tbody>
</table>
The remainder of this section is devoted to presenting results of themes emerging from posts coded for different facilitators and indicators of student engagement and unpacking the enumeration provided in table 20. First, themes of the relationships between categories of facilitators of engagement with indicators of engagement are described. Then, themes for posts reflecting two indicators of engagement are presented. Table 20 will be used as a guide for this discussion. Finally, the very rare examples of posts coded with all three indicators of student engagement are shared. These posts did not occur frequently, so themes are not suggested.

**Relationship between needs fulfillment and student engagement.** In their posts, students very rarely wrote about perceived fulfillment for autonomy or belongingness. In fact, no posts were coded as indicating the student felt a lack of autonomy, and only one post was coded as demonstrating a sense of autonomy. This is not to say that students felt autonomous (or not), only that they did not write about autonomy in their posts. On the other hand, students posted frequently about the extent to which their need for demonstrating competence was met. Figure 13 shares a matrix of themes that emerged from posts that were coded for various levels of needs fulfillment and indicators of engagement. In some cases, themes are simply other codes that were also prevalent in each collection of posts.
<table>
<thead>
<tr>
<th>~Concentration</th>
<th>~Belong</th>
<th>~Competent</th>
<th>Autonomy</th>
<th>Belong</th>
<th>Competent</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Enjoy</td>
<td>~Competent</td>
<td>Eminent failure</td>
<td></td>
<td></td>
<td>Test → Easy</td>
</tr>
<tr>
<td></td>
<td>Anger</td>
<td>Fast-paced</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Failed test</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~Interest</td>
<td>Concentration</td>
<td>“Not for me”</td>
<td>Discouraged</td>
<td></td>
<td>Enjoy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>~Prepared</td>
<td></td>
<td></td>
<td>Teaching positive</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Test</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Confidence</td>
</tr>
<tr>
<td>Enjoy</td>
<td></td>
<td></td>
<td>Confidence</td>
<td>Teaching positive</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Instructor Cares</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Confidence</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Enjoy Neutral</td>
<td></td>
<td>Teaching positive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td></td>
<td></td>
<td>Confidence</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Enjoy</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 13.** Themes emergent between needs fulfillment and student engagement.

The empty cells in figure 13 provide information about the relationships between students’ perceived needs fulfillment and engagement. Specifically, students’ written posts do not suggest that high levels of enjoyment, interest, or concentration are associated with low levels of belongingness and competence. Further, low levels of interest, enjoyment, and concentration tend not to be written about concurrently when students’ senses of belongingness or competence are met. Additionally, in the case of low perceived needs fulfillment and low...
engagement, students’ perceptions about the fast-paced structure of their precalculus course appears more frequently than other perceptions about teaching or course structure. On the other hand, in the case of perceived needs fulfillment and higher engagement, students reflect on positive aspects of their instructor’s teaching or remark about their instructor’s caring about their success. Tables 22 and 23 share example posts of the themes identified in figure 13.
Table 22: Examples of themes emergent in posts indicative of low needs fulfillment and low engagement.

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Competent</td>
<td>Guess I’m dropping pre calc. Guess I should just drop out of school now if I can’t pass a basic math class</td>
<td>51</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>precalc is making me question my choice of attending college in the first place 🙃 🙃</td>
<td>78</td>
<td>1</td>
</tr>
<tr>
<td>Anger</td>
<td>literally........... get me tf(^{16}) out of precalc please asap im dying</td>
<td>102(^{17})</td>
<td>12</td>
</tr>
<tr>
<td>“Not for me”</td>
<td>math at this level… it just isn’t for me. #crawlingtothefinishline</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>Eminent failure</td>
<td>SURVIVE! <em>Dies</em> Would you like to Re-Spawn. No…</td>
<td>60</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>I would be content with a W(^{18}) at this point</td>
<td>37</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Even if I work hard, I probably won’t get it #precalckills</td>
<td>73</td>
<td>4</td>
</tr>
<tr>
<td>Fast-paced</td>
<td>A lot of material that comes fast but need more practices and confidence before mobbing on, like sitting in front of fire hydrant</td>
<td>106</td>
<td>6</td>
</tr>
<tr>
<td>Failed test</td>
<td>Failed that last test :(</td>
<td>39</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Just failed another test 🙃</td>
<td>72</td>
<td>6</td>
</tr>
<tr>
<td>Discouraged</td>
<td>Tears+anxiety=precalc</td>
<td>103</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Went to 4 study sessions and I still didn't do well on test #disappointing</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>We had a test this week I'm not totally sure if I passed. If I don't I'll be really discouraged.</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>~Prepared</td>
<td>why is it that i can understand everything in class and when it gets to the test everything is so much harder than we've been doing</td>
<td>98</td>
<td>2</td>
</tr>
</tbody>
</table>

\(^{16}\) TF is slang for “the f***.”

\(^{17}\) Student 102 was very expressive of anger in posts, most of them used explicit language.

\(^{18}\) A W is issued to students who withdraw from the course.
Table 23: Examples of themes emergent in posts indicative of needs fulfillment and higher engagement.

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test → Easy</td>
<td>Math is going nicely. I did well on the last test. New material is stuff I am familiar with so it should be easy. Can't believe it's almost over</td>
<td>68</td>
<td>11</td>
</tr>
<tr>
<td>Enjoy</td>
<td>This week I was more focused in class. And I even got recognition for being one of about a hand full of student that got a bonus question right. 😁</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>Teaching positive</td>
<td>Math was pretty good this week. Nothing too hard. My teacher is makes me feel comfortable and willing to learn.</td>
<td>68</td>
<td>2</td>
</tr>
<tr>
<td>Confidence</td>
<td>I got 96 on past precalculus exam. I became more confident by solving questions over and over. I got this!</td>
<td>35</td>
<td>8</td>
</tr>
<tr>
<td>Instructor cares</td>
<td>I like precalc more and more and it's making more sense to me now. I can tell that my teacher genuinely cares about us and truly wants us to learn.</td>
<td>24</td>
<td>5</td>
</tr>
<tr>
<td>Test</td>
<td>I aced my test! studying pays off!</td>
<td>44</td>
<td>11</td>
</tr>
</tbody>
</table>

Two overarching themes transcend these results: tests and confidence. In general, students write about tests frequently, and their performance on tests seems to be frequently described in conjunction with references to their emotional engagement. Also, there appears to be a positive association between students’ confidence and needs fulfillment-to-engagement relationship. This is evidenced by posts indicating low levels of engagement and needs fulfillment also depicting the student’s lack of confidence in their ability to be successful. Conversely, posts reflecting positive states of engagement and needs fulfillment also demonstrate student’s confidence. Finally, the relationship between needs fulfillment and engagement appears to be associated with students’ perceptions of their instructor’s teaching in posts reflecting higher levels of these facilitators and engagement. This theme was especially prevalent when engaged students’ posts reflected fulfillment of their need for a sense of competence.
**Relationship between challenge-skill balance and student engagement.** Students’ challenge-skill balance manifested in their weekly posts in the form of anxiety, boredom, confidence, confusion, perceiving of content studied as easy, or perceiving of content studied as hard. Note that in this chapter, *anxiety* and *boredom* are defined colloquially, and were selected as codes based on students’ language. For example, student 27 describes being bored because of familiarity with the content from previous math courses, “I’ve already learned this math unit, so class is boring” (week 12). Or student 42 reporting feeling nervous about an upcoming assessment, “so nervous about this next exam!” (week 6). Figure 14 displays themes emergent in posts coded for challenge-skill balance and indicators of student engagement.

Elements of figure 14 shed light on relationships between students’ challenge-skill balance and engagement as reported in their weekly posts. Posts expressing students’ anxiety and boredom typically appear in posts also coded for low interest, enjoyment, or concentration. This trend can especially be seen for boredom, where it is only found with low interest, enjoyment, and concentration. Students do not attach positive emotional engagement to confusion, but write about needing to concentrate or experiencing higher levels of concentration when confused. Once again, confidence transcends the array of engagement indicators; however, what students attribute their experience of a confidence-engagement relationship to differs slightly for each indicator of engagement. This relationship is discussed in more detail below. Finally, the level of difficulty students perceive precalculus to be is also correlated to their level of engagement. Moreover, confidence and boredom appear to have important roles in this relationship.
<table>
<thead>
<tr>
<th>Anxiety</th>
<th>Boredom</th>
<th>Confused</th>
<th>Confidence</th>
<th>Easy</th>
<th>Hard</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Concentration</td>
<td>Material covered is already known</td>
<td>Confused</td>
<td>Teaching positive Course structure Material covered is already known</td>
<td>Boredom Confidence</td>
<td></td>
</tr>
<tr>
<td>~Enjoy</td>
<td>~Prepared</td>
<td>Math is boring</td>
<td>Teaching negative Fast-paced ~Prepared</td>
<td>“I don’t like math, but…”</td>
<td>Boredom ~Competent Dropping course Specific content</td>
</tr>
<tr>
<td>~Interest</td>
<td>Too long and too slow</td>
<td>Content irrelevant</td>
<td></td>
<td>Boredom Content irrelevant Content irrelevant</td>
<td></td>
</tr>
<tr>
<td>Concentration</td>
<td>Overwhelmed Test</td>
<td>Prepared Hard work Competence</td>
<td>Enjoy</td>
<td>Confidence Teaching positive Anxiety ~Competent</td>
<td></td>
</tr>
<tr>
<td>Enjoy</td>
<td></td>
<td>“I understand!” Competence Enjoy the material Teaching positive Hard work</td>
<td>Teaching positive Confidence Belong</td>
<td>Confidence</td>
<td></td>
</tr>
<tr>
<td>Enjoy Neutral</td>
<td></td>
<td>“Not so bad!” Tests</td>
<td></td>
<td></td>
<td>Interest</td>
</tr>
<tr>
<td>Interest</td>
<td></td>
<td>What’s next? “I understand!”</td>
<td></td>
<td></td>
<td>Confidence</td>
</tr>
</tbody>
</table>

**Figure 14.** Themes emergent in posts relating challenge-skill balance to indicators of engagement.
Students did not post about enjoying precalculus in weeks when they felt anxious or nervous. Further, feeling unprepared for upcoming assessments emerged as a factor in how anxiety and enjoyment interact. When students write about feeling anxious in posts that are also coded for concentration, they tend to write about an upcoming assessment they foresee to be difficult or feel overwhelmed by the amount of material being assessed. Table 24 shares example posts reflecting these themes.

Table 24: *Examples of emergent themes in posts relating anxiety and indicators of engagement.*

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>~ Enjoy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~Prepared</td>
<td>not looking forward to this math test on monday</td>
<td>27</td>
<td>10</td>
</tr>
<tr>
<td>Concentration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overwhelmed</td>
<td>I would say that I felt like I learned a lot of challenging concepts that I don't understand, rushed and overwhelmed with an upcoming quiz</td>
<td>58</td>
<td>1</td>
</tr>
<tr>
<td>Test</td>
<td>PreCalculus is getting more difficult. I hope I pass this unit.</td>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>

Posts stating that the student was bored during precalculus from the previous week tend also to reflect low levels of interest, enjoyment, or concentration. Posts of this nature frequently related the student’s boredom and level of engagement to being familiar with content covered from previous mathematics courses, believing that mathematics in general is a boring subject to study, or feeling that their course meets for too long and progresses at a slow pace. Table 25 provides example posts demonstrating these themes.
Table 25: *Examples of emergent themes in posts relating boredom and indicators of engagement.*

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Concentration</td>
<td>Precalculus was kind of boring this week. We learned two new units that I have already covered in high school.</td>
<td>53</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>i’ve already learned this math unit, so class is boring</td>
<td>27</td>
<td>12</td>
</tr>
<tr>
<td>~Enjoy</td>
<td>Math is either too easy it’s boring or too complicated it’s boring, no inbetween</td>
<td>23</td>
<td>6</td>
</tr>
<tr>
<td>~Interest</td>
<td>Can’t keep my eyes open during each 2 hour math lectures!</td>
<td>55</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Sometimes I feel like we go too slow in class so I get bored…but I feel prepared for the upcoming test.</td>
<td>24</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>my math class is too long and too slow</td>
<td>27</td>
<td>1</td>
</tr>
</tbody>
</table>

Students do not write about enjoying their precalculus course when they are confused. In posts expressing students’ confusion that were coded for low enjoyment, students also reflect negatively on their instructor’s teaching, express concerns about the fast-paced nature of the course, or comment on feeling unprepared for upcoming assessments. Finally, questioning the relevance of content covered in precalculus class emerged as a theme in posts capturing students’ confusion and indicated low interest. Table 26 provides example posts for these themes.
Table 26: *Examples of emergent themes in posts relating confusion to indicators of engagement.*

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Enjoy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teaching negative</td>
<td>Mrs. Jackson always skips some steps in her explanation leaving me lost</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>I wish my pre-calc teacher was more organized in writing notes &amp; I wish he would writes steps for problems.</td>
<td>19</td>
<td>2</td>
</tr>
<tr>
<td>Fast-paced</td>
<td>I went to a tutoring session, and it's amazing how much everything makes sense when someone actually takes the time to show you how problems are done</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>~Prepared</td>
<td>Let's hope I didn't fail this test! Studied but none of this material made sense to me. Wish my instructor made sense when he teaches things!</td>
<td>39</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Cramming for another big test! I need to do MML. These lectures aren't enough to understand any of this</td>
<td>84</td>
<td>3</td>
</tr>
<tr>
<td>~Interest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Content irrelevant</td>
<td>Augmented matrix is a very weird way to solve a problem and get solutions. I don't know what the person who invented it was thinking.</td>
<td>22</td>
<td>9</td>
</tr>
</tbody>
</table>

Posts presenting the student as confident illustrated a range of indicators of student engagement. Generally, posts of this nature had positive tones and frequently reflected higher levels of interest, enjoyment, and concentration. In particular, posts reflected the student’s preparedness, hard work, and sense of competence as themes relating confidence with concentration. Many themes arose in posts suggesting confidence and enjoyment, such as excitement about understanding, sense of competence, enjoying the content being learned, positive reactions towards teaching, and hard work. The level of enjoyment appeared to downgrade slightly when posts mentioned assessments, but still reflected positive affinity towards the previous week’s experiences. Finally, posts coded for both confidence and interest
demonstrated students’ curiosity about upcoming material to be learned and excitement about understanding. Table 27 shares example posts of themes associated with the relationship between confidence and higher levels of student engagement.

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prepared</td>
<td>Getting ready for midterms, it is hard but I got this.</td>
<td>35</td>
<td>6</td>
</tr>
<tr>
<td>Hard work</td>
<td>It's getting much harder but I can do well as long as I ask questions and open myself up to help</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Precalculus is starting to get challenging but I like a challenge.</td>
<td>86</td>
<td>2</td>
</tr>
<tr>
<td>Competence</td>
<td>I got 96 on past precalculus exam. I became more confident by solving questions over and over. I got this!</td>
<td>35</td>
<td>8</td>
</tr>
<tr>
<td>Enjoy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“I understand”</td>
<td>I'm beginning to like calculus more lately &amp; I find it easier to understand topics; I feel more confident within myself and my success in this class.</td>
<td>24</td>
<td>4</td>
</tr>
<tr>
<td>Competence</td>
<td>I took my first precalculus test and it went fairly well. I'm enjoying the class more than I thought I would.</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Enjoy material</td>
<td>Math was interesting this week. I enjoy working with logarithms. Challenging but not impossible. Confident for the next test</td>
<td>68</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>The materials that we have covered in class was something that I have never done, but it was a pleasure to learn something new!</td>
<td>76</td>
<td>4</td>
</tr>
</tbody>
</table>
Table 27 Continued

<table>
<thead>
<tr>
<th>Teaching positive</th>
<th>Before taking this class, I thought it would be a piece of cake. That wasn't the case but with the help of my teacher, I think it is getting easier!</th>
<th>73</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I think math is getting easier for me to understand And I love the teacher.</td>
<td>66</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Thanks to the teacher now I know my stuff.👍😎</td>
<td>94</td>
<td>2</td>
</tr>
<tr>
<td>Hard work</td>
<td>math is hard but fun</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>I feel like passing this class will be a hard yet exciting challenge</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>challenging but fun</td>
<td>81</td>
<td>5</td>
</tr>
<tr>
<td>Enjoy Neutral</td>
<td>“Not so bad!” Exponential Applications are not that bad!! Just a lot of hard work!!</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>Tests</td>
<td>Math was good this week. I was stressed about the test but I think I did well. Can’t believe there is only 1 more chapter left.</td>
<td>68</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>This week wasn't bad and i really hope the best on m test next week</td>
<td>89</td>
<td>10</td>
</tr>
<tr>
<td>Interest</td>
<td>What’s next? Interesting topic this week, building on what I have learned so far. Can’t wait for what’s next.</td>
<td>107</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Almost there- it’s starting to work out and I’m getting excited about what’s next.</td>
<td>48</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>“I understand” math this week was amazing and interesting because i understood most of them</td>
<td>89</td>
<td>7</td>
</tr>
</tbody>
</table>
On the other hand, in the few cases (7 out of 105 posts coded for confidence) where posts suggested low levels of concentration, students wrote positively about their instructor’s teaching, the course structure, or explained that they had studied the material being covered in previous mathematics courses. Posts capturing students’ confidence and coded for low enjoyment tended to begin with the phrase, “I don’t like math, but…” suggesting that the source of low enjoyment is studying mathematics, not necessarily specific elements of their precalculus course. These posts were generally completed with positive outlooks towards understanding, learning, or assessments. Table 28 provides example posts of the themes associated with the relationship that confidence has on low levels of student engagement.

Table 28: Examples of themes emergent in posts relating confidence to low engagement.

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Concentration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teaching positive</td>
<td>Lesson in pre-calculus this week was easy to comprehend. My teacher explained how to solve the problems very easily. I can't wait to go back!</td>
<td>76</td>
<td>2</td>
</tr>
<tr>
<td>Course structure</td>
<td>Math has been smooth sailing lately. Most of the material is just a review for me. I like having labs to work on homework so I don't forget about it.</td>
<td>68</td>
<td>8</td>
</tr>
<tr>
<td>Material covered is already known</td>
<td>This class is almost like study hall for me because I have already taken pre-calculus and know most of the material already.</td>
<td>21</td>
<td>2</td>
</tr>
<tr>
<td>~Enjoy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“I don’t like math, but…”</td>
<td>I dont really like math but its getting easier and it is easier then the first time i took this class.</td>
<td>66</td>
<td>1</td>
</tr>
</tbody>
</table>

The level of difficulty students attribute to precalculus content being learned also appears to be related to their levels of interest, enjoyment, and concentration. In posts where students
report about the course content being easy they also tend to report being bored, particularly when also coded for indicators of low engagement. Although, instances where students post about the course content being easy which are also coded for indicators of higher engagement do occur. In these cases, students tend to write about their enjoyment, sense of belonging, confidence, or reflect positively on their instructor’s teaching. Alternatively, when students post that the material of their precalculus course is difficult they also tend to demonstrate confidence, particularly when also coded for indicators of higher engagement. However, cases where students write about the course being difficult that are also coded for indicators of low engagement do occur. Posts of this nature clearly express the student’s intentions of withdrawing from their precalculus class or indicate that their need for a sense of competence has not been met.

In other words, students’ posts demonstrate a relationship between indicators of engagement and the level of difficulty attributed to content learned in precalculus. Moreover, when content is perceived as easy, boredom and indicators of low engagement tend also to occur unless the student associates easy-to-learn content to their instructor’s teaching. On the other hand, when content is described as difficult, confidence and indicators of higher engagement tend also to be present. The absence of confidence, particularly when students do not feel able to demonstrate competence, is especially detrimental to student engagement when studying difficult content. In extreme cases, students post about a desire to withdraw from precalculus or community college altogether.

In sum, there appears to be a relationship between students’ challenge-skill balance and level of engagement present in weekly posts. On the surface, when students write about being bored, anxious, or confused during precalculus class they also tend to evince low levels of
interest, enjoyment, and concentration. Posts expressing students’ confidence tend to portray higher levels of interest, enjoyment, and concentration. Further investigation exposed additional roles that confidence, boredom, and students’ perceptions of teaching have on the relationship between challenge-skill balance and student engagement. Specifically, boredom appears especially detrimental to student engagement when students perceive of the material being learned in precalculus to be easy, unless the degree of difficulty is attributed to good teaching. Finally, confidence appears especially important to engagement for students who perceive of precalculus content being learned as difficult. Further, it is possible that confidence reduces the likelihood of students deciding to withdraw from precalculus even when they find the course difficult. A post capturing this sentiment was provided by student 29, stating, “Math: my greatest enemy...I WILL CONQUER YOU!!!!” (week 1).

The previous sections shared results about the interactions between facilitators and indicators of student engagement based on analyzing students’ weekly posts. In the next sections, more results on the nature of student engagement are presented, where attention is given to posts coded for multiple indicators of engagement. Recall that elements off the main diagonal of table 20 display the number of posts which were coded for two indicators of engagement.

**Low concentration.** Posts associated with low concentration during precalculus class from the previous week were not very frequent, but table 20 sheds light on what students are not saying when they do provide posts warranting a code for low concentration. Specifically, students are not necessarily interested or enjoying when they find the content from class to be easy or from previously learned material (i.e. low concentration). In fact, students make their boredom with these experiences clear: “precalculus was kind of boring this week. We learned
two new units that I have already covered in high school” (student 53, week 1) and “my math is so boring, I learned all of this last year” (student 27, week 4).

On the other hand, one post suggested that a student found reviews to be interesting, “Did great on test. Going back to domains and ranges though, but still interesting. Seems like a lot of reviews” (student 10719, week 2). Further, when posts did suggest enjoyment and low concentration, they were often associated with students’ sense of confidence or positive thoughts about the instructor or course structure. As examples, “math has been smooth sailing lately. Most of the material is just a review for me. I like having labs to work on homework so I don’t forget about it (student 68, week 8) and “lesson in pre-calculus this week was easy to comprehend. My teacher explained how to solve the problems very easily. I can’t wait to go back!” (student 76, week 2) demonstrate how students write about enjoying precalculus even when the material may not be challenging or new based on their perceptions about their instructor’s teaching or established course structure.

**Low enjoyment.** Students were very vocal about their level of enjoyment in posts, this section focuses on posts reflecting low levels of enjoyment. Posts of this nature were never associated with interest. In other words, students did not write about being interested and experiencing low levels of enjoyment concurrently in their posts. On the other hand, a few posts reflected low enjoyment and low interest. These posts also indicated students’ boredom, failure to see relevance in their work, lack of confidence, or perceptions that assessed material is difficult (table 29). Moreover, posts portraying low enjoyment and low interest also demonstrated students’ sense of being “#overit” (student 4, week 13). Very few of these posts

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19 Student identification numbers were assigned based on their initial agreement to participate in the study. Thus, students 102 – 110 might be quoted. No posts from students outside of the 101 students in the sample were analyzed, nor will their posts appear in this chapter.
provide information about students’ perceptions of teaching, but those that did demonstrate that the student feels unprepared for assessments, “my math lab is nothing like the exams and I do not understand why we are forced to do for homework. We should be forced to use lab manual” (student 39, week 13).

Table 29: Examples of themes emergent in posts displaying low enjoyment and low interest.

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boredom</td>
<td>Hardest class I’ve ever had to sit through and not because of the course material</td>
<td>23</td>
<td>15</td>
</tr>
<tr>
<td>Lack of relevance</td>
<td>What am I going to use precalculus for? And why do we have to have to take it? [sic]</td>
<td>106</td>
<td>4</td>
</tr>
<tr>
<td>Lack of confidence</td>
<td>Writing paper in math are stupid. I’m barely passing English and you won’t [sic] me to take my barely passing writing skills and put it in math. Smh</td>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>Difficult assessments</td>
<td>I understand the things I learn in class. I can do the homework fine. Can I do well on the tests and quizzes? Nope.</td>
<td>98</td>
<td>5</td>
</tr>
<tr>
<td>Done</td>
<td>I can’t even with polynomials. I’m done #overit</td>
<td>9</td>
<td>7</td>
</tr>
</tbody>
</table>

Though low enjoyment reflects low levels of emotional engagement, this situation was not always paired with low cognitive engagement (i.e. concentration). In fact, many posts coded for low enjoyment were also coded for concentration. Themes emerging in posts of this nature suggest that students perceived of the material being learn in precalculus class was difficult and they experienced feeling incompetent. Additionally, in these posts, students were more vocal about their thoughts on their instructor’s teaching or the structure of the course. Specifically, students indicate that they are struggling to learn difficult material through their instructor’s
presentation style, the fast pace of the course, or believe that they were not being prepared for upcoming assessments (table 30).

Table 30: *Examples of themes emergent in posts displaying concentration and low enjoyment.*

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difficult material</td>
<td>Class is getting hard and it’s making me discouraged because I might fail this class again. I can not [sic] wait to finish this section of precalc… not for me #harasmess</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>Low competence</td>
<td>I studied for 6 hours and cried for 2 this class sucks Went to 4 study sessions and I still didn’t do well on test #disappointing</td>
<td>103</td>
<td>6</td>
</tr>
<tr>
<td>Teaching style</td>
<td>Mrs. Jackson always skips some steps in her explanation leaving me lost This week pre calc was very challenging because the sub we had didn’t do a good job</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Fast pace</td>
<td>Precalculus is rough and very fast paced. It’s a very frustrating class to take!</td>
<td>31</td>
<td>1</td>
</tr>
<tr>
<td>Under-prepared</td>
<td>Why is it that i can understand everything in class and then when it gets to the test everything is so much harder than we’ve been doing</td>
<td>98</td>
<td>3</td>
</tr>
</tbody>
</table>

The role of students’ perceptions about how challenging the material being learned is and their low sense of competence on student engagement exhibited in these posts is interesting (and depressing). Particularly, posts of this nature demonstrate the importance of attending to students’ need for perceived competence in regular class settings outside of assessments, especially while working on materials that students deem difficult. The frequency with which students reflect on their poor performance on an assessment in these posts provides evidence for
how important it is that community college precalculus instructors consciously attempt to boost students’ sense of competence through means other than tests and quizzes.

This phenomenon was explored further by filtering posts coded for hard and either anxiety or low competence to examine the effects on student engagement with these facilitators present. Results of this search yielded posts where students clearly associated low fulfillment of their sense of competence through poor performance on assessments to low enjoyment. Further, these posts provide evidence of the student’s intentions to withdraw from the course or realizations of eminent failure.

Table 31: Examples of the negative effects of anxiety or low competence on student engagement.

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor performance</td>
<td>This class is killing me, I can’t even get bonus questions right.</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>I thought it was easy… then I saw the quiz</td>
<td>72</td>
<td>8</td>
</tr>
<tr>
<td>Eminent failure</td>
<td>This class is getting harder. Im failing and I will have to retake this class</td>
<td>104</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Class is getting hard and it’s making me discouraged because I might fail this class again.</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Still failing precalc. Dropping out. Bye.</td>
<td>51</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Prepare to take MAT 171 twice. Just do it.</td>
<td>20</td>
<td>14</td>
</tr>
</tbody>
</table>

**Concentration.** Students rarely provided evidence to express their level of interest along with indications of concentration in the same posts. This is not to suggest that students did not find concentration interesting; it simply means that students did not write about these two indicators during the same week. However, students do frequently express their level of enjoyment in posts that reflect concentration. The previous section discussed themes in posts
representing low enjoyment and concentration, so focus here is on themes emergent in posts demonstrating higher levels of enjoyment with concentration. Posts of this nature demonstrate the student’s confidence, fulfillment of a sense of competence, reflect positive perceptions about their instructor’s teaching, or mention an assessment. Table 32 shares example posts for these themes.

### Table 32: Examples of themes emergent in posts reflecting concentration and enjoyment.

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence</td>
<td>I got 96 on past precalculus exam. I became more confident by solving questions over and over. I got this!</td>
<td>38</td>
<td>8</td>
</tr>
<tr>
<td>Competence</td>
<td>This week I was more focused in class. And I even got recognition for being one of about a hand full of student that got a bonus question right. 😁</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>Teaching positive</td>
<td>I like precalc more and more and it’s making more sense to me now. I can tell that my teacher genuinely cares about us and truly wants us to learn.</td>
<td>24</td>
<td>5</td>
</tr>
<tr>
<td>Test</td>
<td>I aced my test! studying pays off!</td>
<td>44</td>
<td>11</td>
</tr>
</tbody>
</table>

**Enjoyment and Interest.** There were not many instances in which students wrote about their level of interest in weekly posts; however, when this was the case, more than half of posts provided evidence that the student also enjoyed precalculus during the week. This suggests a relationship between enjoyment and interest. Posts reflecting these two indicators of student engagement frequently demonstrated the student’s confidence in their ability to be successful as well as perceptions of a sense of competence. Posts indicating interest and enjoyment did not often divulge information about students’ perceptions of teaching, but posts that do reflect positively on the instructor’s teaching. Table 33 presents example posts for each of these themes.
Table 33: *Examples of themes emergent in posts reflecting interest and enjoyment.*

<table>
<thead>
<tr>
<th>Related theme</th>
<th>Example Posts</th>
<th>Student ID</th>
<th>Week ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence</td>
<td>Went over matrices, but couldn’t think of a good use for them until I read a 3D game programming book. Now I am happy I am learning about matrices</td>
<td>107</td>
<td>12</td>
</tr>
<tr>
<td>Competence</td>
<td>I really like rational functions and the assessment of them.</td>
<td>48</td>
<td>8</td>
</tr>
<tr>
<td>Teaching</td>
<td>The professor was not here this week, but subs made the class interesting and relevant to we had already been learning!</td>
<td>25</td>
<td>3</td>
</tr>
</tbody>
</table>

**Posts reflecting levels of interest, enjoyment, and concentration.** Only three posts were coded for all three elements of engagement (table 34). Interestingly, all three of these posts reflect student interest, instead of boredom or perceived lack of relevance. Further, these posts tend to reflect enjoyment and suggest the authors concentrated on mathematics during the week (e.g., “…Challenging but not impossible…” [student 68, week 9]).

Table 34: *Posts coded for all three indicators of engagement.*

<table>
<thead>
<tr>
<th>Student ID</th>
<th>Week ID</th>
<th>Post</th>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>107</td>
<td>2</td>
<td>Did great on test. Going back to domains and ranges though, but still interesting. Seems like a lot of reviews</td>
<td>~Concentrate, Enjoy, Interest</td>
</tr>
<tr>
<td>68</td>
<td>9</td>
<td>Math was interesting this week. I enjoy working with logarithms. Challenging but not impossible. Confident for the next test</td>
<td>Concentrate, Enjoy, Interest</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>This week was kinda challenging but my teacher made it interesting so it wasn’t so bad…</td>
<td>Concentrate, Enjoy Neutral, Interest</td>
</tr>
</tbody>
</table>

**The role of teaching approaches on student engagement.** The previous sections shed light on students’ perceptions of teaching and how those perceptions interact with student engagement and the relationships between indicators and facilitators of student engagement. In this section, the nature of student engagement is investigated for each of the four teaching
approach categories discussed earlier in the chapter. Recall these categories were derived based on analyzing instructor interview transcripts, and thus represent instructor reported teaching approaches as opposed to students’ perceptions of teaching. To complete this investigation, students’ posts were filtered according to their instructor’s teaching approaches category. Then, posts coded for each of the indicators of student engagement were examined for themes (figure 15).
<table>
<thead>
<tr>
<th></th>
<th>Balanced</th>
<th>Meaning-Making</th>
<th>Student-Supportive</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Concentration</td>
<td>(23) Boredom</td>
<td>(8) Confidence</td>
<td>(3) Easy</td>
<td>(6) Boredom</td>
</tr>
<tr>
<td>~Enjoy</td>
<td>(60) Confused</td>
<td>(90) Confused</td>
<td>&quot;I don't like math&quot;</td>
<td>(36) -Competent</td>
</tr>
<tr>
<td></td>
<td>Anxiety</td>
<td>~Competent</td>
<td></td>
<td>Test</td>
</tr>
<tr>
<td></td>
<td>~Competent</td>
<td>~Belong</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hard</td>
<td>Fast-paced</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Teaching</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~Interest</td>
<td>(30) Boredom</td>
<td>(12) ~Enjoy</td>
<td>(0)</td>
<td>(5) ~Enjoy</td>
</tr>
<tr>
<td></td>
<td>Too Long</td>
<td></td>
<td></td>
<td>~Prepared</td>
</tr>
<tr>
<td>Concentration</td>
<td>(63) Confidence</td>
<td>(68) Confidence</td>
<td>(10) Confidence</td>
<td>(31) ~Enjoy</td>
</tr>
<tr>
<td></td>
<td>Prepared</td>
<td>Enjoy</td>
<td>Test</td>
<td>Enjoy</td>
</tr>
<tr>
<td></td>
<td>~Prepared</td>
<td>~Enjoy</td>
<td>Test</td>
<td>Test</td>
</tr>
<tr>
<td></td>
<td>Teaching</td>
<td>Test</td>
<td>Confused</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Positive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Enjoy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>~Enjoy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Enjoy</td>
<td>(37) Competent</td>
<td>(52) Confidence</td>
<td>(21) Test</td>
<td>(34) Competent</td>
</tr>
<tr>
<td></td>
<td>Confidence</td>
<td>Competent</td>
<td>Confidence</td>
<td>Test</td>
</tr>
<tr>
<td></td>
<td>Teaching</td>
<td>Test</td>
<td>Belong</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Positive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Concentrate</td>
<td>Competent</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Interest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Enjoy Neutral</td>
<td>(12) Teaching</td>
<td>(13) Confidence</td>
<td>(2) Test</td>
<td>(7) Test</td>
</tr>
<tr>
<td></td>
<td>Positive</td>
<td></td>
<td>Confidence</td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>(6) Enjoy</td>
<td>(9) Confidence</td>
<td>(3) Enjoy</td>
<td>(11) Enjoy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Enjoy</td>
<td></td>
<td>Confidence</td>
</tr>
</tbody>
</table>

**Figure 15.** Nature of student engagement by teaching approach category (number of posts).
The distribution of indicators of student engagement for all four teaching approach categories appears to be consistent with the overall distribution described by table 20 at the beginning of the qualitative findings section, in that within each teaching approach category most posts reflect students’ level of enjoyment. Except for students whose instructors reported a balanced teaching approach profile, where there is a comparatively large number of posts reporting low concentration associated with boredom. Further, there do not appear to be differences in the distribution of posts coded for each indicator of student engagement between teaching approach categories except for low concentration for students whose instructors’ teaching approaches were characterized as balanced. Accordingly, the nature of student engagement reflected in students’ posts does not appear to differ based on teaching approaches of instructors. However, themes associated with various levels of student engagement do appear different between teaching approach categories.

Specifically, students whose instructors reported balanced teaching approach profiles wrote about being bored in conjunction with posts coded for low concentration and low interest. Boredom did not appear as a theme for the other teaching approach categories. With respect to low enjoyment, students with instructors adopting balanced approaches associated confusion, anxiety, low fulfillment of a sense of competence, and tended not to enjoy when they perceived of the material being learned in precalculus to be difficult. In terms of concentration, these students described confidence and reflected positively on their instructor’s teaching; although there were mixed feelings about the level of enjoyment and preparedness associated with concentration. Further, students with these instructors posted about positive feelings towards instructors’ teaching when posts were coded for higher levels of enjoyment. Confidence and competence were also associated with higher levels of enjoyment in students’ posts.
In contrast to students whose instructors reported other teaching approach profiles, students in classes with instructors reporting meaning-making profiles attribute confidence to every indicator of student engagement. Interestingly, confidence even arises as a theme with posts coded for low concentration, suggesting that students with these instructors write about being confident in their abilities with materials that they perceive to be prior knowledge or easy. With regards to low enjoyment, these students post about being confused, feeling low sense of competence, not belonging, or perceive of the course as fast-paced. Additionally, these students tend to experience low enjoyment together with low interest. Like students with instructors reporting balanced teaching approach profiles, students here also made posts reflecting mixed levels of enjoyment associated with concentration. Here, students also mention tests or confidence in posts coded for concentration. Students whose teachers reported meaning-making profiles were the only ones to have all three indicators of engagement emerge as themes in the same set of posts. Specifically, students with these teachers wrote posts coded as indicating enjoyment that were associated with concentration and interest. This is not to say that all three indicators were present in all posts of this nature from these students (recall only three posts warranted codes for all three indicators simultaneously); however, it is worth noting that students of instructors with high proportions of a meaning-making approach in their profiles made posts reflecting this level of engagement. This suggests an association between meaning-making teaching approaches and student engagement.

Mrs. Baker was the only teacher whose profile was characterized as student-supportive, but recall that the proportion of traditional codes in her profile was also above the benchmark for classifying her profile as traditional. This has two consequences. First, all themes for this profile emerged in her students’ posts, and second, claims made about this teaching approach category
should take into consideration the small sample size. Interestingly, Mrs. Baker’s students made posts that associate confidence with concentration and higher levels of enjoyment, a theme which is not present in posts for the same indicators of engagement written by students of other instructors whose teaching approaches are characterized as traditional. This suggests that the proportion of a meaning-making approach present in Mrs. Baker’s profile might interact with her students’ engagement through her ability to foster confidence. More research is needed to investigate this conjecture.

Perhaps the most contrasting theme present in posts written by students of instructors with traditional teaching approach profiles compared to students of other instructors is tests. In fact, students of instructors with balanced approach profiles rarely mentioned tests in posts that were also coded as indicative of any indicator of student engagement, and students from classes where the instructor’s teaching approach profile is categorized as meaning-making mention tests in posts reflecting concentration and enjoyment20. Students of instructors whose teaching approach profile was characterized as traditional wrote about tests consistently in their posts, and this emerged as a theme with posts coded as indicative of low concentration, concentration, enjoyment, and enjoy neutral. The presence of a sense of competence with test and lack of a sense of competence with test in posts coded for enjoyment and low enjoyment, respectively, suggests that students with teachers in this teaching approach category may use feedback on assessments as barometers for their sense of competence. Which in turn is also related to their level of enjoyment.

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20 Almost every post coded for enjoyment and written about a test included information to suggest that the student performed well on a previous test.
Regardless of teaching approach profiles, confidence appears consistently in posts coded for higher indicators of student engagement. It is interesting that boredom appears as a theme for low concentration and low interest in students’ posts for students whose instructors reported balanced teaching approach profiles, but does not appear as a theme associated with student engagement in posts among students with instructors whose teaching approaches are in other categories. Although confidence transcends most teaching approach categories and indicators of student engagement, it is especially prevalent in posts written by students of instructors who reported a more meaning-making approach. Additionally, as a collective, students of these instructors associated interest, enjoyment, and concentration together along with confidence in their posts. Tests appear as common talking points in posts written by students whose instructors’ teaching approaches are categorized as traditional. Mrs. Baker’s students associate confidence and a sense of belonging when they write about enjoying her precalculus course. It is possible that Mrs. Baker’s students’ confidence and sense of belonging are attributable to her student-supportive teaching approach profile, though more research is needed to investigate this association as Mrs. Baker was the only instructor in this study to exhibit such a teaching approach profile.

**Summary of results from posts.** Therefore, in response to the first research question (i.e. what is the nature of community college students’ engagement in precalculus, and what roles do teaching approaches have on these experiences?), qualitative data provided by students’ weekly posts suggest that the nature of student engagement can be described as emotionally polarized and consisting of relatively high levels of cognitive engagement. In this regard, posts generally reflect either high or low levels of interest or enjoyment – as opposed to being emotionally neutral – and tend to indicate high levels of concentration. Qualitative data also
demonstrate that the level of student engagement varies between students and within individual students on a weekly basis. Further, evidence supports the existence of a positive relationship between community college precalculus students’ psychological needs fulfillment and engagement. With respect to students’ posts, challenge-skill balances are related to student engagement such that boredom and anxiety tend to be written about in posts coded for low levels of interest, enjoyment, and concentration, while confidence appears consistently in posts coded for higher levels of interest, enjoyment, and concentration. Broadly speaking, these experiences are not different based on instructor reported teaching approaches. However, there is evidence to suggest that students of instructors with traditional teaching approach profiles tend to relate their sense of competence to performance on tests; students of instructors with meaning-making teaching approach profiles tend to be confident; and boredom appears more frequently in students whose instructors reported a balanced teaching approach profile, which was also associated with low concentration. Thus, although the overall nature of community college precalculus students’ engagement appears similar across students experiencing the course through different teaching approaches, the presence of factors related to the level of student engagement differ among students experiencing the course from different teaching approaches. Stated differently, regardless of instructor teaching approach profiles, community college precalculus students tend to post similarly about their levels of interest, enjoyment, and concentration; however, factors associated with levels of engagement appear to be different among students experiencing the course from different teaching approaches.

Further Investigation

This section presents qualitative results from analyzing data specific to the group of purposefully selected students. Nine students, three each from of Mrs. Harper’s (Michelle,
Beverly, and Marianne), Mrs. Smith’s (Sally, Suzy, and James), and Mrs. Jackson’s (Patricia, Paula, and Richard), classes, were asked to participate in two rounds of interviews based on their relatively high levels of engagement (determined quantitatively) reported during the first five weeks of the study. These students participated in two rounds of interviews: the first round of interviews was used to delve into student engagement from the students’ perspectives and to identify factors associated with high levels of student engagement (at least for the beginning few weeks of the semester); then, the second round of interviews was used to examine any relationship between student engagement and understanding of precalculus concepts. In other words, data from the first round of interviews are used to further address research question one (i.e. what is the nature of community college student engagement in precalculus?) from the perspective of highly engaged students. Data from the second round of interviews are used to address research question two (i.e. is there a relationship between student engagement and understanding of precalculus concepts) qualitatively for a subsample of highly engaged students.

A description of how students were purposefully selected from Mrs. Harper’s, Mrs. Jackson’s, and Mrs. Smith’s classes and an introduction to the students is provided prior to presenting results. Then, the two research questions are addressed in order.

**Purposefully selecting students.** Once instructors had been selected for classroom observations, three highly engaged (compared to their classmates) students from each of Mrs. Harper’s, Mrs. Smith’s, and Mrs. Jackson’s classes were sought after using the results from the first five weekly diaries and the first round of classroom observations. Responses to the quantitative items asking students to rank their level of interest, enjoyment, and concentration (see Appendix D, items 6a, 6b, and 6c) were used to determine a level of engagement for each week (computed as the average of interest, enjoyment, and concentration). In addition to
searching for highly engaged students, efforts were made to select students that represented the various subgroups from the overall sample of students. To be eligible for selection, students needed to have completed at least three of the first five weekly diaries\textsuperscript{21}. Finally, recall that instructor interview results suggest Mrs. Harper’s teaching approaches profile to be traditional, Mrs. Jackson’s profile to be meaning-making, and Mrs. Smith’s profile to be balanced. Accordingly, these nine students experienced the course with instructors who reported different teaching approaches profiles.

Using these criteria, nine students were selected to take part in two rounds of interviews. These nine students were: Patricia, Paula, and Richard from Mrs. Jackson’s class; Sally, Suzy, and James from Mrs. Smith’s class; and Michelle, Beverly, and Marianne from Mrs. Harper’s class. Figure 16 displays the average levels of engagement, interest, enjoyment, and concentration reported by these students for the first five weeks of the study.

\textsuperscript{21} This coincided with the drop/add period for students to make schedule changes.
Figure 16. Average reported engagement for selected students during the first five weeks.

Compared to their classmates these students reported being among the highest engaged students in their class for the first five weeks. Patricia was not originally selected based on the results displayed in figure 16. Patricia was selected due to her behavioral engagement during the first classroom observation of Mrs. Jackson’s class even though her reported level of engagement was lower than that of other students in her class. Table 35 presents these students’ self-reported information on the initial demographics survey (Appendix C).
Table 35: Demographic information for purposefully selected students.

<table>
<thead>
<tr>
<th>Student</th>
<th>Ethnicity</th>
<th>Academic Intent</th>
<th>Status</th>
<th>Remedial Courses</th>
<th>First Attempt</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mrs. Jackson’s students</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Richard</td>
<td>African</td>
<td>Transfer with AS to earn BS</td>
<td>Full-time</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Paula</td>
<td>Black</td>
<td>Transfer to BS without graduating</td>
<td>Full-time</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Patricia</td>
<td>Black</td>
<td>Transfer with AS to earn BS</td>
<td>Part-time</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Mrs. Smith’s students</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sally</td>
<td>White</td>
<td>Earn AA</td>
<td>Full-time</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Suzy</td>
<td></td>
<td>Transfer with AS to earn BS</td>
<td>Full-time</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>James</td>
<td>White</td>
<td>Transfer with AA to earn BS</td>
<td>Full-time</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Mrs. Harper’s students</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Michelle</td>
<td>White</td>
<td>Transfer with AS to earn BS</td>
<td>Full-time</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Beverly</td>
<td>Hispanic</td>
<td>Transfer with AS to earn BS</td>
<td>Full-time</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Marianne</td>
<td>White</td>
<td>Transfer with AA to earn BS</td>
<td>Full-time</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

None of the selected students were veterans or parents at the time of the study. The purpose of this section was to report on which students were selected to take part in interviews, and the describe them with information reported on the initial demographic survey. Now, results about their engagement in community college precalculus using data from the first round of interviews are presented.

**More on the nature of student engagement.** This section delves deeper into the nature of student engagement by presenting results from the first round of student interviews. The
interview protocol prompts students to think about their classroom environment, their perceptions about instructor activities (i.e. what does your instructor do in a typical class?), the types of activities that they personally participate in during a typical class, and their interests, enjoyment, and concentration (i.e. engagement) during class time. Nine students, three each from Mrs. Harper’s (Michelle, Beverly, and Marianne), Mrs. Smith’s (Sally, Suzy, and James), and Mrs. Jackson’s (Patricia, Paula, and Richard), classes, were asked to participate in two rounds of interviews based on their relatively high levels of engagement (determined quantitatively) reported during the first five weeks of the study. The purpose of this section is not to re-establish the nature of student engagement as already described qualitatively through students’ weekly posts. Instead, results are shared that extend what has already been presented to explore any nuances in the nature of highly engaged students’ engagement which may set their experiences apart from their peers’, as well as to further explore factors associated with interest, enjoyment, and concentration from these students’ perspectives.

The first round of student interviews took place during the sixth and seventh weeks of the study (near the beginning of October). The average length of these interviews was about 24 minutes, ranging from 16 minutes (Michelle) to 30 minutes (James and Sally). Interview transcripts were coded with the same codebook as students’ posts. From this analysis trends associated with these students’ descriptions of their interest, enjoyment, and concentration during precalculus class emerged. Specifics about these themes are described below.

In depth analysis of students’ interest. This section is organized to describe what these nine students find interesting about their precalculus course, what factors they associate with their level of interest, and what they suggest might make the course more interesting. This, and the following two sub-sections, are used to further investigate the nature of student engagement
in community college precalculus (i.e. research question one). These highly engaged precalculus students appear to be interested in learning in general and the applicability of mathematics in understanding their world. Beverly declares her interest in knowledge and mathematics,

    I think math in like general [is interesting]. I think it is crazy how these things work out, but… all these things you don’t think about. I think knowledge in general, you want to know like what else am I missing out on because there are so many things. Even though some of it is challenging, I don’t like it… [but] like in real life you can actually I guess incorporate it.

Similarly, Sally states, “the interconnectedness that math… like that is basically math… real world examples and how everything kind of fits together… I find that interesting.” Other students find some elements of precalculus interesting, while disliking others. For example, Patricia does not “like graphing at all,” but does “like um more algebra based things… like finding \( x \) or… finding the term.” Additionally, Patricia relates her interest in these concepts to her confidence and understanding, “I feel more confident in that, so I like doing it… I don’t like graphs at all, but anything that has to do with solving for a term, I think is interesting because I understand it.” Paula’s interests in precalculus bring together Sally’s thoughts about the connectedness of mathematics with Patricia’s feelings about different topics, which implicates importance for making connections across concepts. Paula said,

    …when you have an equation and like you try to break down the pieces in order to get the graph. I find that interesting. Like rational functions, breaking down all the pieces in order to get the \( x \)-intercepts, \( y \)-intercepts, um asymptotes, and things like that. I find that pretty cool… tedious, but… it is worth it in the end because you get the graph. That’s what I like.
Beverly, Sally, Patricia, and Paula all associate enjoyment with interest – a theme which is repeated by other students as well. This is evinced by their use of the phrase, “I like…” when discussing what they find interesting in their precalculus course. In addition to associating interest with enjoyment, these students also imply a relationship between understanding and interest. Though, not all students express experiencing this relationship with the same directionality. For example, Patricia associates a positive relationship between understanding and interest (through confidence); however, Michelle finds interest in concepts that had previously caused her confusion,

I’ve always been really bad at rationals, rational functions and anything fraction related. I’ve been really bad at them, but we just started working on those the last couple of weeks and for some reason the way she teaches I picked it up really, really well and I actually don’t dislike fractions anymore… I find them really interesting now, and I spent a lot of time on my own trying to really understand fractions and rational equations because I could never do it. But now I can because of [Mrs. Harper] her (Michelle).

Although Patricia associates high understanding with high interest while Michelle relates low understanding with high interest, both students imply a positive outlook on their abilities, which demonstrates the importance of confidence on the relationship between understanding and interest.

In addition to particular topics, when prompted about their interests in precalculus, these students also offered information about the structure of classroom norms and their instructor’s teaching. For instance, Suzy compared her experience taking precalculus this semester to a previous attempt. She states that she does not “like math on a daily basis,” but indicates that her
instructor does “a pretty good job at making the class very interesting” and “has gotten me [her] a lot more interested in the class than I have experienced before.” Suzy attributes Mrs. Smith’s ability to make the course more interesting to the way in which she explains everything, and her willingness to answer any students’ questions. This point is echoed by James,

[Mrs. Smith] lectures a lot but it is an interesting- it is more of a conversation than a lecture because [she] is open to interrupting… at every step of the way, even absurdly simple steps… like [Mrs. Smith] will show you how to add 2 and 2 if you ask.

Despite James’ initial feelings about Mrs. Smith’s conversational lectures, his weekly posts from later in the semester suggest that James became increasingly less interested in precalculus over time. In week 8, James wrote, “is it me or does math also make you so bored you question everything you’ve done in life up to this point.” Similarly, in week 15 he writes that precalculus is the, “hardest class I’ve ever had to sit through and not because of the course material.”

When asked, highly engaged precalculus students can describe elements of their course that they find interesting; however, many stated that in general, “math is not that interesting” (Patricia). Providing a unique perspective, Richard speaks about interest in a course as being irrelevant,

I mean, it might be a different answer, but like, like, I don’t know it’s like, I don’t really care if stuff interests me or not. As long as it’s going to get me to where I’m trying to get, I go for it. Like if I’m supposed to get an A in this class to go and do the next math, I don’t care what it is, like I’m going to get an A. I don’t care whether it interests me or not. I just going to do what it takes, get an A, go on to
the next course and just move like that. Like interest doesn’t really come to me…Um, the content like, eh, like some of it like not all of it, but like some course, or some topics that I really kind of like, and some you know, I just don’t really think it’s unnecessary or whatever. But because you got to take it in order to you know get a better grade, I just [you know] stuck in it and just go for it. I don’t really care whether I’m interested in it or not.

Richard’s account may be unique among this group of students, but his thoughts suggest the importance for precalculus instructors to discuss vertical alignment of content. Perhaps Richard, and students with a similar perspective, would find precalculus more interesting if they were consistently made aware that topics discussed will be built upon in future mathematics courses. Richard’s statement also demonstrates his motivation – earning good grades. Perhaps other students motivated by extrinsic rewards, such as grades, share Richard’s perspective on interest? This is a question for future research.

Other students were more direct about how precalculus could be more interesting. Specifically, group work, hands-on activities, and challenging examples emerge as themes commonly described during interviews. Patricia states it would be more interesting “if we…did more hands-on things… I wouldn’t mind more group work if it was more hands on.” Other students indicated that they would find precalculus more interesting if it were more challenging, and utilized examples that “grounds it into reality instead of just fictionalizing it” (James). Sally elaborates,

For me kind of like if [Mrs. Smith] gave more real world examples that would be better. I know teachers don’t always do that but um because then I know how I can apply it in real life. Because if you just give me an equation and you are like
solve this, I am like why? How is that relevant? But if I have a relevant concept to put to it, it will make me more interested.

Thus, highly engaged community college precalculus students describe learning, the connectedness of mathematics, understanding, and their instructor’s teaching as interesting. Though, the level of interest they attribute to their instructor’s teaching may decline over time, as demonstrated by James’ posts from later in the semester. Additionally, Beverly suggests changing instructional methods as a means for making precalculus more interesting, “…switch[ing] it up, maybe not do just notes because that’s boring.” Further, instructors may be able to increase student interest by capitalizing on the connectedness of mathematics through discussing vertical alignment of topics, utilizing group work and hands-on activities, and posing real world examples. Finally, confidence appears to have an important role in the relationship between students’ understanding and interest.

In depth analysis of students’ enjoyment. Like the previous section on interest, this section is guided by three foci: first, describing what highly engaged community college precalculus students enjoy about their course; second, identifying factors they associate with their level of enjoyment; and finally, what they suggest might make the course more enjoyable. With respect to enjoyment, these highly engaged precalculus students discussed the importance of group work, test preparation, relevant work, working problems, and their instructor’s teaching. Also, as indicated above, students associate interest and enjoyment. In fact, when prompted to discuss elements of the course that she found enjoyable, Sally responded, “Enjoyable?... I don’t want to be repetitive… Just like those eureka moments and knowing how everything works together.” This question follows immediately after prompts about interests in the interview protocol, to which Sally reported the interconnectedness of mathematics to be interesting.
Additionally, Michelle makes connections between interest and enjoyment while speaking about online homework assignments, “…she [Mrs. Harper] makes her homework’s very interesting. I find myself right after class looking to see if the homework is up then because it is probably the class I enjoy doing homework for the most.”

Opportunities to work problems in small groups appears to be pivotal for most of these students’ enjoyment. Except for Richard, all eight other students spoke about enjoying class time devoted to working with peers. Group work’s relationship with enjoyment encapsulates many of the other themes stated in the opening paragraph to this section. Specifically, these highly engaged students speak about test review, projects, relevant work, and working problems as being associated with enjoyment and describe how working in groups during each of these activities enhances their experiences. For instance, Marianne, enjoys team-based review games such as Jeopardy because they are, “exciting… it just makes you feel, you know, your abilities are measurable instead of just sitting and listening every day.” Beverly also discussed her enjoyment for review games,

Last week …we did a round of Jeopardy, so that was fun. So, you chose your team and the smart girls won, but everybody won and got points. I think they got four extra points and I think we got two extra points, so it goes like that… It was fun, I found it interesting… It had a math question in each one, so some were like, oh this problem 500, and people were like, oh sorry everyone, I don’t know why I chose that one. But everyone got to work it out and we got points, and whoever got the most points won…. That was fun and it still had math in it.

Beverly’s statement brings to light many elements related to enjoyment (and interest) described by other highly engaged students during interviews, and by others in their weekly
posts. Specifically, her appreciation for extra credit made available to all students (i.e. “everybody won and got points”) speaks to the inclusive nature of her classroom, which establishes a connection between sense of belongingness and enjoyment. She also highlights the group aspect of the game (i.e. “…sorry everyone, I don’t know why I chose that one.”), and perhaps more importantly, explains that it was enjoyable to work problems (i.e. “that was fun and it still had math in it.”).

In addition to being a form of group activity, reviewing for tests emerged as a theme related to student enjoyment. Patricia expresses her appreciation for Mrs. Jackson’s willingness to spend class time for test review, and explains that she enjoys test review because it allows her to hear her classmates’ thought processes,

I like it when we do our review just because we are kind of going over it as a class and I can kind of hear other people’s opinions and thought processes. It goes a little faster than I would like, again because I write everything out so I am a little slow. Um, but I do like doing reviews. I know that not all teachers have to spend an entire class period before the test to review, so I appreciate that and I feel like it gives me an idea of what I need to look over before the test. So, I think I enjoy that the most.

Patricia’s statement establishes a link between enjoyment and concentration by referencing her enjoyment of hearing her classmates’ thought processes as a means for helping her prepare for upcoming assessments. This quote also suggests a relationship between students’ perceptions of teaching on their emotional engagement (i.e. enjoyment and interest).

Apart from group-based test review games, students also enjoy “when we [they] just hold small group discussions and like projects that are interesting” (Paula) because “she [Mrs.
Jackson] puts it in like a real-world situation in which you find a solution to something. So, I guess that is kind of fun because you are trying to [like] come up with something” (Paula). Here, Paula relates enjoying group projects to Mrs. Jackson’s ability to assign work that is meaningful to real-world situations (i.e. “that is… fun because you are trying to… come up with something”). Paula also explains that she finds working on group projects enjoyable “because like I said she [Mrs. Jackson] mostly like lectures. I don’t find that enjoyable.” Thus, group projects offer students opportunities to participate in relevant mathematics and provide variety to classroom activities, which was associated to interest in the previous section.

In fact, working problems, either in groups or individually, was deemed enjoyable by most of these students. Specifically, James associates his enjoyment of precalculus to learning and understanding through working problems,

I just really love like doing the problems. Like learning how to do a problem and doing a problem are very different. Um, and I just think you do it better when you are doing it hands on… So, in that sense I think it is a really good way to learn especially math. Even though sometimes it maybe takes a couple equations before you get it, ah, but 9 times out of 10 like I get it… I have an epiphany, so it is nice.

James’ response suggests a multi-faceted relationship between highly engaged students' enjoyment of precalculus and understanding (which James uses interchangeably with learning). In this regard, James highlights differences between watching and doing mathematics, and implies that persistence may be an important element of an engagement-understanding relationship.

To conclude, results from the previous two sections suggest that highly engaged community college precalculus students describe positive associations between their emotional
engagement (i.e. interest and enjoyment) during class time and group work, understanding, and relevant work (e.g., preparing for assessments, vertical alignment with/to future mathematics courses, etc.). Further, these data suggest a reciprocal relationship between interest and enjoyment, such that students enjoy what they are interested in and are interested in what they enjoy. Accordingly, it is important for students’ emotional engagement that community college precalculus instructors establish a learning environment that capitalizes on this relationship by providing students with opportunities to participate in group activities where they are exposed to the thinking of their peers while working on relevant (to future coursework, assessments, or the world) concepts and problems. Additionally, these highly engaged students shed light on the importance of variety in learning experiences. Finally, it is important to reiterate the positive relationship between group work and emotional engagement as described by these precalculus students. In fact, as is discussed in the next section, group work emerges as paramount for cognitive engagement (i.e. concentration) as well as emotional engagement.

**In depth analysis of students’ concentration.** These highly engaged community college precalculus students reported higher levels of concentration on mathematics during group work than during any other classroom activity because they are simultaneously in tune with their own understanding of the understanding of their classmates. Interestingly, each of these students report that their instructors do not specifically designate time for working collaboratively on a regular basis, “most of it is individual except for those rare times, or she is like check with your partner” (Beverly). Yet, when given a choice, these students worked independently at first and then elected to collaborate with their peers. Moreover, there seems to be gender differences in these students’ motivations for working together and in how group work associates with their concentration. In this regard, female students concentrate on how they might explain their
thinking to their peers to help others understand, while male students appear to concentrate more on using the explanations of others for self-evaluation. These ideas are voiced by James,

So, she [Mrs. Smith] will fill out the equation [um] and what you want to do and you can work with the people beside you or work alone. Um, personally I just sort of just mix it up, ah I usually just work along and then just confer with them to see what we got and why we got different answers if that happens… So, ah, if it is a question I really want to work at I’ll pretty much just work alone um and then that will just be that, and if I have time I’ll see what my neighbors got. And me and the guy to the left of me are on the same page. Um, we usually get the same ones wrong and right, and then the guy beside me is pretty cool because like we are never there. It is cool seeing because we just operate very differently so we never have the same answers. So, it is cool seeing him explain it, and hearing him explain it correctly or wrongly [sic] sort of helps me a lot because if I am able to identify that he is wrong then that means that I am probably good. I hope so at least. I’ll have to confirm after this next test.

Here, James sheds light on how he participates in group work, and what he focuses on while conferring with his neighbors. James’ statement provides evidence to suggest that he uses his peers’ explanations to evaluate his own understanding, which gives insight into his concentration on mathematics during group work. Further, James enjoys “seeing… and hearing him [classmate] explain it” because it “helps… a lot” when James reflects on his own understanding of problems while evaluating the reasoning of his peers. Richard describes some similar characteristics for how he participates in group collaboration, but with a more grade-centric focus,
I usually turn to [classmate], she’s right by me and just turn and ask her what she got on her paper. Uh, you know, what answer it is. Sometimes I pretty much already know what it is so I’m just doing it because it is something she [classmate] wants to do. Um, so I just ask her, you know, what it is and just go through it and see what ideas she got. If I don’t have an answer, and I ask her and she do, and I ask her like, you know, how did you get that answer? You know, she can go ahead and explain how she did it and I can kind of know what it is. Or if I have an answer and she don’t then I can tell her how I did it. Event though, you know, it’s not right or whatever, just the steps in the process and how we got to the answer… I’m not really like specific on what you should do, but like I usually like [say] do yours and I do mine. But, like if you bring yours either way, I’m still going to have to go over it because I’m not sure, so I kind of go over it, you can go over mine if you like, you know, but I just still go over it… I just don’t want to have like a crazy grade or whatever because somebody [else].

On the other hand, highly engaged female students concentrate on how to explain their work to their “friend[s]” (Sally, Patricia, & Paula) and worry about their understanding, What I do is I’ll work the problem out and then my friend and I will check… she [Mrs. Smith] never specifically says get into groups, I mean you can if you want, I do that myself… I want to make sure that they know how to do it too if we are working in a small group because I think that is kind of the point of group work. For everyone to kind of comprehensively understand everything. So, um, not only making sure that I understand but making sure they understand it. And they say if
you teach someone something you have like a higher retention rate of like remembering it and like understanding it, so that kind of helps you too (Sally).

Here, Sally describes how she elects to work in groups when given a choice, and that she is concerned about the collective understanding of everybody in the group because “… that is kind of the point of group work.” Sally’s idea about capitalizing on opportunities to explain mathematics to her classmates for the personal benefits is also expressed by Michelle,

I’m fairly good at math so I tend to be kind of the one where people are like, I don’t know what to do, and I’m like, Oh well it is this…. Most of the time in small groups I find myself focusing on helping other people understand. I tend to take the lead on that because I have a background in teaching\textsuperscript{22}, so I find that I learn better if I teach. So, it’s like I can learn it all day long but if I can’t explain it to somebody else then I don’t really understand it.

Here again, another female student indicates her focus during group work is on assisting her peers. In Michelle’s case, confidence in mathematical abilities manifests as a characteristic of the relationship between group work and concentration. Though, confidence is not always the motivator for female students to care about their peers’ understanding. When prompted about what she focuses on while working in small groups, Marianne said,

Helping the other girls mostly. Um, I don’t want to be overbearing or anything, but if they have a question they ask um if I have the answer I’ll tell them. So, I feel like once I am there I can coach other people through it since I have been there before, the struggle area, and I don’t want people to feel like that because math is hard.

\textsuperscript{22} Michelle taught equestrian prior to returning to college.
Marianne demonstrates care for her classmates’ mathematical well-being as well as for their understanding. Both Michelle and Marianne are confident in their mathematical abilities; however, confidence is not always what prompts students’ high level of concentration while working in small groups,

I am probably like focusing on the problem too, but I am also hoping that I am correct in what I am saying so I am not leading someone else a stray too. So, I think that *I am more focused when I am working with somebody else* because I am like, OK I don’t want to be the weakest link and not contribute but I also don’t want to contribute something incorrect, so I feel like *I am even more focused* on getting it right because someone else’s – not grade, but now there are more people that you have to think about. So, *I think I am even more focused then* (Patricia, emphasis added).

Patricia’s statement provides direct evidence demonstrating high concentration while working in small groups, while other students infer about needing to concentrate on their understanding and the reasoning and explanations of their peers. Further, statements given by these highly engaged community college precalculus students shed light on potential gender differences in concentration during group work such that male students utilize the time to evaluate their own understanding while female students attend to helping their classmates. Also, confidence has a role in the relationship between confidence and group work. In this regard, confident female students (e.g., Michelle and Marianne) are willing to explain concepts to their peers and capitalize on personal benefits from the opportunity. On the other hand, less confident female students (e.g., Patricia) tend to pay careful attention to their own understanding to make sure they are not, “leading someone else astray” (Patricia). Confidence did not manifest in
statements provided by James and Richard pertaining to their concentration during group work; however, both students indicate being able to evaluate the work of their peers, which can be interpreted as confidence. Considering only two male students were included in this group, more research is required to further investigate the relationship between concentration and how male community college students collaborate during precalculus class.

In addition to reporting high concentration during group work, these students also present common thought processes during note taking or watching their instructor complete examples. Specifically, “thinking about other cases like… what if she was to use another number or another sign… So, I’m looking at her [Mrs. Jackson] to get the concept, but I’m also thinking about… the next problem” (Richard). Patricia explains how she also considers upcoming examples, “I really try … [to] use the steps we used in our examples and apply it to this example, and what is the same and what is different. So, I try to make connections.” Similarly, Suzy concentrates on working through problems along with Mrs. Smith, “when she is solving the problem I try to think of it, I try to write it down and try to solve it myself while she is solving it and see the difference.” Considering extreme cases, additional examples, and attempting to work problems with an instructor are behaviors indicative of these students’ concentration on mathematics during precalculus class.

This section has shed light on the nature of student engagement from the perspectives of highly engaged community college precalculus students, and thus addresses the first research question (i.e. what is the nature of student engagement?). Evidence presented suggests that highly engaged students associate many factors to their level of emotional (i.e. interest and enjoyment) and cognitive (i.e. concentration) engagement during class time. As a theme transcending each of these indicators of engagement, group work appears especially important.
Further, these students suggest a relationship between student engagement and understanding, particularly in descriptions of their concentration during group work. These results have direct implications for community college precalculus instructors and students. Specifically, to provide an engaging classroom environment, instructors should incorporate group work regularly, and should provide prompts for students to consider while communicating their work with their peers (e.g., while listening to your classmate’s explanation(s), think critically about their reasoning and compare it to your own.). These prompts could help students develop reflective thinking, which appears as a factor in the relationship between concentration and understanding. Additionally, community college precalculus students should remain open-minded about working collaboratively with their peers.

**Student Understanding of Precalculus Concepts**

In the chapter on quantitative results (chapter 5), it was established that student engagement was not associated with understanding of precalculus concepts as measured quantitatively via weekly diaries and the PCA. This section examines the relationship between student engagement and understanding from a different methodological perspective — qualitatively during task-based interviews. This section is organized as follows: description of the task, presentation of themes in students’ thinking while responding to the task, and then a comparison of student engagement while working the task to engagement reported on posts (i.e. during class). Note, Michelle opted not to participate in this round of interviews.

**Description of task.** The *Taking a Ride* task (Moore, n.d.; Stevens & Moore, 2016) was used in the second round of student interviews to investigate students’ understanding of precalculus concepts such as covariational reasoning (Carlson et al., 2002), quantitative reasoning (Thompson, 2011), and shape thinking (Moore & Thompson, 2015). In this regard, the
task first prompts students to view an animation of a Ferris wheel perpetually rotating clockwise. The ride contains six carts, five red and one green. Participants are then prompted to “graph the relationship between a rider’s total distance traveled around the wheel and the rider’s distance from the ground” (emphasis in the original, see Appendix I). Following this, participants view a second animation of a Ferris wheel; however, in this animation the ride stops periodically. Participants are then prompted to discuss the relevance of their original graph to this new situation.

Briefly, recall (from chapter 3) Moore & Thompson (2015) describe students’ shape thinking as static or emergent, where “static shape thinking involves operating on a graph as an object in and of itself” (Moore & Thompson, 2015, p. 784). Stevens & Moore (2016) further suggest that static shape thinking includes iconic translations (i.e. associating perceptual features of a situation and graph [Monk, 1992]) and thematic associations (i.e. associating the graph an situation phenomena [Thompson, 2015]). On the other hand, “emergent shape thinking involves understanding a graph simultaneously as what is made (a trace) and how it is made (covariation)” (Moore & Thompson, 2015, p. 785, emphasis in original).

**Students’ responses to the task.** This task prompts students to graph a relationship between two covarying quantities; thus, students’ shape thinking (Moore & Thompson, 2015) provides a useful approach for presenting composites of results from analyzing these highly engaged students’ responses to this task. In other words, detailed descriptions of how each individual student worked the task are not provided. Instead, results of the understanding of groups of students who demonstrated similar thinking on the task are presented together. Table 36 provides a summary of these students’ reasoning on the task, their performance on the PCA, and score on the covariational reasoning subscale.

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<table>
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<tr>
<th>Student</th>
<th>PCA Score</th>
<th>Covariational Reasoning Subscale</th>
<th>Shape-thinking from Interview</th>
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<tr>
<td>Richard</td>
<td>11/25</td>
<td>2/5</td>
<td>Static shape thinking: Iconic translations</td>
</tr>
<tr>
<td>Paula</td>
<td>5/25</td>
<td>0/5</td>
<td>Static shape thinking: Thematic associations</td>
</tr>
<tr>
<td>Sally</td>
<td>8/25</td>
<td>1/5</td>
<td>Static shape thinking: Thematic associations</td>
</tr>
<tr>
<td>Beverly</td>
<td>10/25</td>
<td>3/5</td>
<td>Emergent shape thinking: Invoking time</td>
</tr>
<tr>
<td>Suzy</td>
<td>7/25</td>
<td>3/5</td>
<td>Emergent shape thinking: Invoking time</td>
</tr>
<tr>
<td>James</td>
<td>8/25</td>
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<tr>
<td>Marianne</td>
<td>10/25</td>
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<td>Emergent shape thinking: Invoking time</td>
</tr>
<tr>
<td>Patricia</td>
<td>6/25</td>
<td>1/5</td>
<td>Emergent shape thinking: Parameterizing</td>
</tr>
</tbody>
</table>

Three of these eight students exhibit static shape thinking: Richard, whose thinking has been coded as iconic translation; Paula and Sally, whose thinking were coded as thematic associations.

*Static shape thinking: iconic translations.* Richard, a student in Mrs. Jackson’s class, scored an 11 out of 25 on the PCA and responded correctly to 2 out of 5 subscale items for covariational reasoning. On the task, his thinking about the graph was coded as iconic translation. Richard interpreted the image of the Ferris wheel as a coordinate system and explained how he envision such a coordinate system working. Richard explains that, to him, the image/graph was structured with x’s that were all zero “because the fact that every um lines over there [*pointing to the image in the animation*] kind of direct me straight to the middle [of the Ferris wheel].” He used the image of the Ferris wheel to establish radial coordinates (like those of a conventional polar coordinate system), which he described as heights. The following vignette provides Richard’s own explanation of his interpretation of a graph from the image of a Ferris wheel.
Richard: So, that’s four, it’s 1, 2, 3, that’s 4 quadrants right here [counting sections established by the Ferris wheel’s rigging, see figure 17]. So in the middle is 0 over here [pointing to the center of the ride] and then down here that’s 30 if I counted by 5s, so at 0 you’re at 30. So 0 and 30 would be um the two points you can use to calculate the um or calculate the, the rider’s distance from the ground, or get a graph, or even find an equation. Um if it was like over here now, this is 30, 35, 40, I would still go with 0 because the fact that every um lines over there kind of direct me straight to the middle, it kind of gives me an idea that’s a zero. Well if we have like a line over here [pointing to the outer edge of the circle above and to the left of center], then just coming down here [traces a line segment straight down to the horizontal axis, remaining left of center], then I would probably say that’s a negative – you know count these as a negative two or a negative five or, you know, whatever it corresponds with. But the fact that everything goes in the middle, I’m just assuming that that’s a zero, and then whatever um lines there are, you know travelling around just counting by 5s, so at 0 it’s 30 and then you just keep going around to get the whole coordinate.

Interviewer: Ok, with that graph, could you point to where like (5, 20) would be?
Richard: [Repeating] (5, 20), um so that’s 0 over here, 1, 2, 3, 4, 5, I’m just assuming that’s 5 right here [counting from the center of the image along the horizontal structural line to “5”], and then 5, 10, 15, 20 – oops – 5, 10, 15, 20 [counting radially around the image, beginning at vertical and progressing clockwise], so basically on my graph like the whole line over here, it’s 20 [pointing to the entire radial line at “20” extending from the center of the image outward]. But like I’m not just counting like on a regular graph and stuff where you stop over here, like the whole line just going with 20, so at (5, 20) it’ll be, 0, 1, 2, 3, 4, 5, and then just down to 20 over here [rescaling “5” along the horizontal so that when he uses the radial 20 he arrives at the bucket on the edge of the ride].
Near the end of his time spent working the task, Richard clarified that his coordinate structure was the same for both parts of the task such that time was represented by the horizontal axis and height along the vertical axis. This structure is also depicted by a graph he drew while explaining his thoughts (figure 18, “RB” denotes “Red Bucket”).

Interviewer: And in both of them [the animations], you said time was a constant at 0?

Richard: It was at 0 because everything point at 0. I didn’t draw that [the radial lines], but everything point at 0.
To conclude, Richard relied heavily on perceptual cues associated with the image(s) of a Ferris wheel to construct a coordinate system from the image(s). Accordingly, Richard’s thinking exemplifies iconic translation because he associated “visual features of the situation and graph” (Stevens & Moore, 2016, p. 645). Further, not much else can be inferred about Richard’s covariational reasoning from his response to this task because he assigned a value of zero for all time-coordinates in his system, thus, he was not coordinating changes in one quantity with changes in another.

Static shape thinking: thematic association. Paula and Sally both exemplified static shape thinking in the form of thematic associations with their thinking on this task. Paula, a student of Mrs. Jackson’s class, scored 5 out of 25 on the PCA overall and did not respond correctly to any of the items on the covariational reasoning subscale. Sally, a student in Mrs. Smith’s class, scored an 8 out of 25 overall and correctly answered 1 out of 5 covariational reasoning items.

Moore and Thompson (2015) argue that shape thinking provides perspective for describing students’ covariational reasoning in the context of graphs.
Each of these students sketched graphs that were circular depicting the path of the rider on the Ferris wheel. The following example demonstrates Paula’s thinking, which was like Sally’s.

Paula: he’s not going straight up or like going straight to the side, he’s going in a circular motion so that is why I put it like that…

Interviewer: So, how would that change [pointing to Paula’s graph] if the wheel were rotating the other way?

Paula: If it were rotating the other way it would start there [pointing to the right-most side of the wheel, figure 19a], and then go around that way [tracing around the wheel counterclockwise, figure 19]. So, it would go this way [tracing the same path on her graph, figure 20]. The points on the graph would still be the same but like the context would be different.

Interviewer: Ok, can you talk more about that?

Paula: Yeah, because like no matter which way you’re going you’re still going to reach the same height and you are still going to reach like the same distance from the ground. But, like it just depends on which way the ride is going. That makes no sense. like it doesn’t matter which way the ride is going because the height is still going to be the same and the distance from the ground is still going to be the same, which is height. Ugh! And then, it just is like how the person, not the person, but like how it’s perceived maybe?
This vignette from Paula’s interview demonstrates how she thought about the association between her graph and the animation of the Ferris wheel. For Paula, the rider’s distance traveled around the wheel was irrelevant, and never discussed (apart from when she was reading the prompt). Paula settles on the graph of a circle depicting the path a rider would take while on the ride (figure 20). Similarly, Sally also sketched a graph reflecting the path a rider would take on the Ferris wheel (figure 21). Like Richard, Paula and Sally demonstrate static shape thinking, only for them, the graphed object resembles the event in the animation as opposed to interpreting the situation as a graph.
Emergent shape thinking: invoking time. Beverly, James, Suzy, Marianne, and Patricia all exhibited emergent shape thinking while working the task. Except for Patricia, they all also invoked time as a quantity into the situation, though most in slightly different ways.

Beverly, a student in Mrs. Smith’s class, scored a 10 out of 25 on the PCA overall and correctly responded to 3 out of 5 covariational reasoning items. Beverly describes her graph as depicting the rider’s position at a given time during the ride. However, Beverly interprets the horizontal axis to be “distance from the middle” or how she describes the rider’s lateral displacement from a starting point. She constructs her vertical axis in a similar way to reflect the rider’s height above the ground. In this regard, she is coordinating simultaneous vertical and lateral displacement in a bounded space, both with respect to time, to produce her graph as the path the rider travels around the Ferris wheel (figure 22).

24 Suzy and James demonstrated similar thinking.
Suzy and James demonstrated similar thinking on the task; both are Mrs. Smith’s students. Suzy scored a 7 out of 25 on the PCA, where she correctly responded to 3 out of 5 covariational reasoning items. James scored an 8 out of 25 on the PCA. He correctly answered 4 out of 5 items on the covariational reasoning subscale. Each of these students interpreted the prompt from the task to require two graphs: one for the relationship between a rider’s distance traveled around the wheel over time, and second for the relationship between the rider’s height over time. James’ graph(s) in figure 23 reflects how both students thought about this task. James sketches both relationships on the same coordinate plane, where his green graph depicts the rider’s total distance traveled over time and the purple graph reflects the rider’s height over time. The black graph labeled, “theoretical” was used by James to explain that the purple and green graphs may not have the same slope. The “theoretical” graph is depicting the rider’s total distance traveled.
Marianne, a student in Mrs. Harper’s class, scored a 10 out of 25 on the PCA overall, and correctly responded to 1 out of 5 items on the covariational reasoning subscale. Almost immediately, Marianne sketches a “sine or cosine curve” to represent the situation (figure 24a), and continues to interpret the prompt to be about the relationship between a rider’s height over time. Marianne does not ignore the rider’s total distance traveled around the wheel as she works through the task, but this quantity is not reflected in her graphs. Marianne explains the shape of her graph, demonstrating emergent thinking.

I think, just because the way graphs are, like we don’t need them to move the same way um like the visual does because we are looking at it in a different way. And I feel like that is why it is so confusing to me. Um, because I feel like um if you see it on paper it kind of has to be left to right, so you have to figure out how...
this ![pointing to the animation] is moving to put it on paper. And it wouldn’t
move the same way on paper.

The previous quote demonstrates Marianne’s emergent shape thinking through her use of
the word “move” when discussing the differences in the shape of her graph and the path that a
rider travels on a Ferris wheel. Further, she portrays her thinking of her graph as a trace of
the rider’s changing height over time (i.e. “… left to right, so you have to figure out how this is
moving to put it on paper.”). Later in the interview, Marianne explains how different graphs are
produced (figure 24b) from the two animations,

It does stop. Um, I was like wondering if it was like freezing or stopping. But um,
I think that would be a good point ![responding to a probing question about a
hypothetical students’ suggestion]. Um, I think- So it stops about every like
quarter of the way, so you would just have to like scrap your graph where it stops
and draw a straight line. Um, but still have it like connect to the curve. So, I guess
um it would stop like here, so you would just straight line and it would resume.
And then it would stop here, so straight line and keep going.
Figure 24. Marianne’s graphs from the two animations.

*Emergent shape thinking: parameterizing.* Patricia, a student in Mrs. Jackson’s class, scored a 6 out of 25 on the PCA overall, and correctly responded to 1 out of 5 covariational reasoning items. While working through the task, Patricia coordinates changes in the rider’s total distance traveled around the wheel with distance from the ground through time. In this regard, she acknowledges that her graph should not reflect time because “time really isn’t a factor in this,” but still uses time as a common independent variable for the two graphed quantities. In other words, she graphs a relationship of the rider’s total distance traveled around the wheel and the rider’s distance from the ground by thinking about how both quantities vary with time. Thus, she parameterizes the situation. The following vignette from Patricia’s transcripts demonstrates how Patricia explains her thinking.

Patricia: I’m still trying to- It’s really throwing me off that time’s not in there because then I just remember it again that- I feel like, OK back to what I initially said, it wouldn’t matter if it [the ride] stopped because that doesn’t affect the distance from the ground and the distance traveled. Or, hold up, the distance from the ground and the distance traveled [reading the prompt]. If you pause, if you pause there, then you are stopping, that doesn’t- Um, OK. I’m
going back to what I initially said. I feel like it [the stopping] doesn’t affect your graph because I feel like unless we had time as a variable then you would draw a different graph and it would have. But I feel like we are still just talking about distance, so your stops shouldn’t affect your graph. Because you are just talking about how much you traveled.

Interviewer: So, a second ago you were tracing out on your original picture and watch the screen. Can you do that carefully and talk about what you are thinking and how you came to that conclusion?

Patricia: So, I was trying to think, OK, if we are traveling now, and traveling, traveling, [tracing her graph with her pencil while watching the animation, figure 25] and I pause [ceases to trace her graph]. I’m like, it doesn’t affect, like I’m not still going straight with my distance from the ground, and I’m not going down with my distance traveled because I am just standing there. Like I’m still, but then I keep going [continuing to trace her graph, following with the animation]. So, it’s, like I guess if you, no, you are not moving time wise. If you are pausing, then you are just pausing on the graph. Like you don’t have to draw anything else. We are not moving further away or closer to the ground and we are not increasing our distance traveled. So, you just pause on the graph.
The previous example demonstrates how Patricia includes time in her thought processes while thinking about the relationship between the rider’s total distance traveled and the distance above the ground. Further, this example shows her thinking of the graph as an emergent trace generated by the relationship between these two covarying quantities.

**Engagement-understanding relationship.** Debriefing sessions immediately followed the task-based portion of the second round of student interviews. The purpose of these debriefings was to gather data on these eight students’ self-reported interest, enjoyment, and concentration (i.e. engagement) while working the task, allowing for an examination of any engagement-understanding relationships in the setting of a task-based interview.

**Concentration during task-based interview.** To begin, the average amount of time spent working the task was about 48 minutes, ranging from about 26.5 minutes (Patricia) to 76 minutes (Sally). This persistence with explaining their thinking about a single task evidences high levels of concentration. Further, information about what these students concentrated on while working the task can be obtained from the previous section. For example, Richard concentrated on explaining the coordinate system he interpreted the image to be providing; Paula and Sally, concentrated on explaining their thinking about the relationship between their quantities and
perceptual cues from the Ferris wheel context; and the remaining students concentrated on how a trace emerged as two (or more) quantities varied in tandem.

When asked about their concentration during the post-task debriefing, two themes emerged in their responses, regardless of shape thinking: personal points of struggle, and concentrating on their explanations of their thinking. First, students explained their focus to reflect what they were grappling with while working the task. In other words, students were concentrating on their personal points of struggle (or confusion) while working on the task. For instance, in the previous example Patricia (emergent shape thinking) makes it clear that the task would have been easier had one of the quantities been time. In the following vignette from our post-task debriefing Patricia continues to explain how time’s role (or lack thereof) affected her thinking.

Patricia: So, I mean it’s not like any numbers in it, but like I just didn’t like having to, like I wish it would have told me what was my x and what was my y. That would have made it a lot, I guess more, and I guess it doesn’t necessarily matter, but I feel like if it were a lot more specific I would have maybe enjoyed it more. But I feel like I just had a sentence to go off of so I didn’t really know what to do.

Interviewer: Ok, so if there were more specifics, what would you want in there?

Patricia: Time. Um, time, how many times it went around, like how many times the ride completes itself.

Later in our debriefing, Patricia ranks the level of difficulty she associated with the task, “I guess like a 6 on a scale of like 1 to 10,” where she explains that a 1 on this scale would be to solve a simple linear equation and a 10 would be something “intricate and like a lot of steps.” As
another example to demonstrate that these students concentrated on their points of struggle, James (emergent shape thinking) reflects on the role that the circumference of the Ferris wheel played on his graph of distance traveled over time. Recall from figure 23 that James sketched two possible curves for this relationship, which he labeled “theoretical” and “distance around.”

I wish I would have done circumference because that makes more sense, yea, that does make more sense. But, I’m not completely wrong with how I did. I think I did ok, it’s looking at it, it looks logical for what I did. Um, it looks [uh] correct, I just wish I did circumference, that’s the only difference. I can’t even think of how I would do that right now… I did it the way easier way… I at least think I thoroughly explained what I meant to do in the occasion that I didn’t do it correctly.

The last sentence in James’ statement demonstrates how students were also concentrating on their own thinking and explanations while working the task. Paula (static shape thinking) exemplifies this theme by reflecting on how evaluating her thoughts inhibited her work on the task, “just like the fact that maybe I was just wrong, so I would think about something and then I’d be like, no that is wrong don’t say that.” Students concentrating on their own thinking is possibly due to the task-based interview setting, but it does reflect high levels of concentration while working a mathematical task.

*Interest and enjoyment during task-based interview.* All eight students did not describe similar feelings towards interest and enjoyment on the task. In fact, there appear to be differences in the self-reported levels of interest and enjoyment among groups of students whose shape
thinking was categorized differently. Specifically, emergent shape thinkers tended to enjoy the task because it was challenging, promoted problem-solving and thinking, and the context was relatable; these students also tended to find the task interesting because it was challenging, promoted problem-solving, and allowed for autonomy.

However, not all emergent shape thinkers in this group expressed high levels of enjoyment and interest. For example, Marianne indicated that she enjoyed the task because she “enjoyed thinking things out, like trying to make sense of the wheel and drawing it on paper.” However, she mentioned that the open-ended nature of the task “put[ting] me back in that place where like I was unsure of myself.” Further, Patricia found the task uninteresting and unenjoyable, both of which she attributed to the open-ended nature of the task prompt and not knowing what to do. Thus, Marianne and Patricia associated low confidence while working on the task to lower levels of enjoyment (and interest in Patricia’s case).

On the other hand, the students who demonstrated static shape thinking while working on this task did not enjoy the task but tended to find it interesting. They associated their lack of enjoyment with finding the task challenging, confusing, and allowing for too much autonomy. Paula and Sally also discussed low confidence being associated with their low level of enjoyment. For example, when asked about her performance on the task Paula stated, “pretty bad… because I think it is just all wrong… Especially with this one [referring to the first animation] because I never saw this before, like the whole circle in just one little section.” However, these students did report that working the task was interesting because it was challenging and open-ended. Sally explicitly states this apparently contradictory result “so

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25 Readers should interpret the phrases emergent shape thinkers and static shape thinkers as only referencing these students’ thinking on this task. I am not making claims about these students’ shape thinking in general.
interesting because difficult; not enjoyable because difficult.” Figure 26 presents a visualization of this relationship between students self-reported engagement while working this task and their understanding. Tables 37 and 38 provide quotes exemplifying themes presented in figure 26.

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<th>Static Shape Thinking</th>
<th>Emergent Shape Thinking</th>
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<tbody>
<tr>
<td>~Enjoy</td>
<td>Challenging</td>
<td>Low confidence (Patricia &amp; Marianne)</td>
</tr>
<tr>
<td></td>
<td>Confusing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low confidence</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Too much autonomy</td>
<td></td>
</tr>
<tr>
<td>~Interest</td>
<td>Too much autonomy (Patricia)</td>
<td></td>
</tr>
<tr>
<td>Enjoy</td>
<td></td>
<td>Challenging</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Promotes problem solving</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Relatable context</td>
</tr>
<tr>
<td>Interest</td>
<td>Challenging</td>
<td>Challenging</td>
</tr>
<tr>
<td></td>
<td>Open-ended</td>
<td>Promotes problem solving</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Promotes autonomy</td>
</tr>
</tbody>
</table>

**Figure 26.** Relationship between engagement and understanding from task-based interviews.
Table 37: *Examples of themes in student engagement among static shape thinkers.*

<table>
<thead>
<tr>
<th>Theme</th>
<th>Quote</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Enjoyment</td>
<td>Being interested vs. enjoyment is a relationship there, but yeah. I think it was more interesting but not enjoyable because I think it</td>
</tr>
<tr>
<td>Challenging</td>
<td>Well you just, it’s just spinning around and it kind of like you know gets to your nerves and stuff, the way your eyes just keep going around. But the second one, you can kind of know that like it’s probably a game or something you can uh get an answer to; whereas, the first one just like wandering around. (Richard)</td>
</tr>
<tr>
<td>Confusing</td>
<td>Ah, because every time I thought I had it right, I was just like, oh wait but it could also be this. But no, and then I would go back to the other theory and it was like no wait it could be something like that. (Paula)</td>
</tr>
<tr>
<td>Low confidence</td>
<td>I don’t really see the math in it because there weren’t numbers, if that makes sense. I mean I know what distance from the ground and distance around is, you know mathematical. But the fact that there weren’t numbers, I didn’t like. (Sally)</td>
</tr>
<tr>
<td>Too much autonomy</td>
<td>I don’t really see the math in it because there weren’t numbers, if that makes sense. I mean I know what distance from the ground and distance around is, you know mathematical. But the fact that there weren’t numbers, I didn’t like. (Sally)</td>
</tr>
<tr>
<td>Interest</td>
<td>Being interested vs. enjoyment is a relationship there, but yeah. I think it was more interesting but not enjoyable because I think it</td>
</tr>
<tr>
<td>Challenging</td>
<td>Like I don’t think it would be interesting if it was like super easy [Ok] because you don’t have to think about it as much. (Sally)</td>
</tr>
<tr>
<td>Open-ended</td>
<td>Um because it gives you an opportunity to like test and make analysis and see results and do trial and error to see, and just kind of like talk about it basically and see what it is. (Richard)</td>
</tr>
</tbody>
</table>
Table 38: Examples of themes in student engagement among emergent shape thinkers.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Quote</th>
</tr>
</thead>
<tbody>
<tr>
<td>~Enjoy</td>
<td><strong>Low confidence</strong> I feel like it is putting me back in that place where like I was unsure of myself… (Marianne)</td>
</tr>
<tr>
<td>~Interest</td>
<td><strong>Too much autonomy</strong> Like when you have too much information it is hard to narrow down what you should use. So, in general word problems just throw me for a loop. I don’t like them. (Patricia)</td>
</tr>
<tr>
<td>Enjoy</td>
<td><strong>Challenging</strong> Uh, like I enjoyed thinking things out, like trying to make sense of the wheel and drawing it on paper was OK. It was enjoyable. (Marianne)</td>
</tr>
<tr>
<td></td>
<td><strong>Promotes problem solving</strong> And so just figuring out the little things out as we go. Like how it isn’t you know, like that. But yea, I, the process I guess. (Beverly)</td>
</tr>
<tr>
<td></td>
<td><strong>Relatable context</strong> And um, also the real life, how you could apply this. Well I’m thinking Ferris wheel the whole time so, it’s relatable. (Beverly)</td>
</tr>
<tr>
<td>Interest</td>
<td><strong>Promotes problem solving</strong> Um, it makes a person think and I think this is very important to try to think logically to try and solve problems in real life. (Suzy)</td>
</tr>
<tr>
<td></td>
<td><strong>Promotes autonomy</strong> Uh, yea, it was fun to sort of uh you, this, I did more of this myself than a traditional math problem. Um, like if this was a normal thing like I would expect on a test it probably would have had a graph already, or had numbers already. Uh, so having something that um, even though you don’t necessarily have creative freedom because there is right or wrong, but you have to make decision making. That was interesting and engaging. So yea, it was, I liked it. (James)</td>
</tr>
</tbody>
</table>

**Summary of engagement-understanding relationship during task-based interviews.** To conclude, there appears to be a relationship between student engagement and understanding based of results obtained during task-based interviews and debriefing sessions with highly engaged community college precalculus students. Regardless of students’ understanding of covariation and shape thinking, these students demonstrated and discussed high levels of concentration. However, static shape thinkers and emergent shape thinkers described different
levels of interest and enjoyment, such that static shape thinkers tended to be interested while working the task but experienced low enjoyment. On the other hand, emergent shape thinkers tended to find the task both interesting and enjoyable (except for Patricia).

Thus, this section has shed light on a relationship between student engagement and understanding in the context of task-based interviews (research question two). In this regard, variations in student understanding are associated with differences in enjoyment. Specifically, static shape thinkers from this group of highly engaged students tended not to enjoy working on this task, which they associated with it being challenging, confusing, promoting too much autonomy, and to their lack of confidence. Emergent shape thinkers from this group, tended to enjoy working on this task, which they associated with it being challenging, promoting problem solving, and having a relatable context. It should be further elaborated that all students in this sample, regardless of their understanding of precalculus concepts, exhibited and described high levels of cognitive engagement (i.e. concentration). This is evidenced by these students’ persistence and statements made during debriefing sessions.

**Chapter Summary**

This chapter has elucidated the findings from qualitative data analysis pertaining to the nature of student engagement and the role(s) that teaching approaches have on these experiences. Thus, qualitatively addressing the first research question. Then, data from one round of interviews with a purposefully selected group of highly engaged students were used to delve deeper into the nature of student engagement from these students’ perspectives, further responding to the first research question. Finally, data from a second round of interviews with the same group of highly engaged students (without Michelle) were used to examine the relationship between student engagement and understanding of precalculus concepts, which
qualitatively addresses the second research question from the perspectives of highly engaged community college precalculus students. The next chapter merges these results with quantitative results to succinctly respond to both research questions.
Chapter 7: Integration of Quantitative and Qualitative Findings

This chapter considers both the quantitative and qualitative results and provides responses to the two research questions based on both sets of findings. In this regard, findings that both sets of analyses have in common as well as those that are unique to either approach are featured to thoroughly address the two research questions. First, these results are used to describe the nature of community college student engagement in precalculus class and the role that teaching approaches have on those experiences (research question one). Then, these results are used to describe any relationships between community college student engagement and understanding of precalculus topics (research question two). Further, results from the previous two chapters are used to discuss the role that teaching approaches have on any engagement-understanding relationships in the setting of community college precalculus (research question two).

Nature of Student Engagement

Quantitative and qualitative analyses used weekly diary data to determine the overall level of community college student engagement during precalculus class for the duration of the study for all student participants. Quantitatively, it was determined that the sample average for student engagement across the semester was 2.81 out of 5. Interpreting this value from the quantitative results alone is not straightforward because 2.81 is an average of weekly averages of values on a 6-point scale, where 0 corresponds to “strongly disagree” and 5 corresponds to “strongly agree.” However, Qualitative results from weekly posts suggest an interpretation of this value (see table 20). For instance, the sample of community college precalculus students reported high levels of cognitive engagement (i.e. concentration) and fluctuating levels of emotional engagement (i.e. interest and enjoyment) over the duration of the study with their
weekly posts. In other words, students’ posts tended to reflect relatively high levels of concentration, but either demonstrate high or low levels of interest and enjoyment – as opposed to being emotionally neutral.

Both quantitative and qualitative analyses also shed light on variation in student engagement occurring within students, between students, and to a minimal extent between classes. Quantitatively it was deemed that roughly one-third of variation in student engagement occurs within individual students on a weekly basis while the remaining two-thirds occurs between individuals. Considering multiple students’ posts for all 15 weeks demonstrates this finding qualitatively, where fluctuations in individual students’ engagement and contrasting differences in engagement between students can be observed in their writing (see table 21 for example). Multiple level modeling (MLM) analysis determined that the amount of variation in student engagement between classes was not significantly different from zero (see figure 7). However, different themes in factors associated to students’ interest, enjoyment, and concentration emerge from posts written by students whose instructors reported different teaching approaches profiles (i.e. balanced, traditional, meaning-making, or student-supportive; see figure 15). This suggests that there may be variation in student engagement between classes that was not picked up by MLM, which is potentially due to the relatively small sample of instructors (15) or the range in the number of student participants from each instructor’s class.

Regarding students’ perceived psychological needs fulfillment, MLM results revealed that in weeks when students felt above their personal average fulfillment of senses of competence, belongingness, and autonomy they also tended to report higher levels of engagement. Further, the extent to which students experience competence-engagement and autonomy-engagement relationships are different between students (i.e. interindividual
differences). Positive relationships between needs fulfillment and student engagement also manifest in weekly posts, especially regarding students’ perceived sense of competence. Moreover, themes emerging in posts associating students’ sense of competence to indicators of engagement (i.e. interest, enjoyment, or concentration) offer potential student level variables to explain interindividual differences in this relationship. For instance, confidence appears as a theme associated with perceived competence and all indicators of student engagement (see figure 13). This suggests that confident students may be experiencing the effects of perceived competence on their engagement differently than less confident peers.

Students rarely posted about perceived fulfillment of autonomy or belongingness, so quantitative results about the relationships between these factors and engagement cannot be expanded upon with qualitative data. This is not to say that students feel autonomous (or not) or that they belong (or not); instead, it means that they did not write about these feelings in their weekly posts. However, further quantitative analysis considering needs fulfillment as predictor variables reveal more about their effects on community college student engagement in precalculus. First, the presence of marginally significant interactions between perceived autonomy and competence fulfillment as well as the three-way interaction on student engagement explain how different students experienced different relationships between needs fulfillment and engagement. Recall that students reporting higher than personal average sense of competence experience the effects of autonomy on their engagement differently depending on their perceived sense of belonging. Figure 10b depicts that increased autonomy has adverse effects on student engagement in weeks when students reported higher than personal average sense of competence and below average belongingness. Second, MLM results also indicate that community college precalculus students’ perceived fulfillment of competence and belongingness
were positively associated with changes in student engagement from one week to the next. Therefore, quantitative and qualitative results reveal positive relationships between students’ perceived needs fulfillment and engagement, and each set of results can be used to further unpack the nature of these relationships.

An important concept of flow theory is the balance between students’ perceptions of task difficulty and their self-evaluated skill level for completing tasks (Csikszentmihalyi, 1975, 1990, 1997). Quantitatively, students’ weekly challenge-skill balance was used to generate dummy codes distinguishing four categories of this balance: HS-LC (highly skilled and under challenged), LS-HC (highly challenged and under-skilled), S-OS (slightly over skilled), and S-OC (slightly over challenged). When used as weekly predictors of student engagement, MLM reveals that LS-HC, S-OS, and S-OC students reported significantly lower levels of engagement than HS-LC students. On the other hand, when used as student-level predictors of engagement (by averaging students’ weekly challenge-skill balance across the semester), MLM results indicate that students whose semester average challenge-skill balance characterized them as HS-LC, S-OS, and S-OC tended to report similar levels of engagement, while LS-HC students reported drastically lower levels of engagement. Further, students’ perceived needs fulfillment does not dampen the negative effect of feeling under-skilled for overly challenging work (LS-HC) on student engagement.

Qualitative results from students’ weekly posts also indicated relationships between community college precalculus students’ challenge-skill balance and engagement. Specifically, posts indicating boredom tended to also indicate low levels of interest, enjoyment, or concentration, suggesting low levels of student engagement. Also, boredom appears especially detrimental to community college precalculus student engagement when students perceive of
course content to be easy (i.e. low challenge), unless the degree of difficulty is attributed to good teaching.

Alternatively, for students who perceive the content being studied in precalculus to be difficult (i.e. high challenge), confidence appears to be especially important to student engagement. Moreover, the absence of confidence in posts reflecting students’ perceptions of difficult material tend to concurrently indicate lower levels of interest, enjoyment, or concentration. In other words, in posts reflecting student perceptions of difficult course content and higher levels of engagement, confidence emerges as a theme. On the other hand, confidence is absent in posts reflecting student perceptions of difficult course content and lower levels of engagement. In general, posts expressing students’ confidence tend to indicate higher levels of interest, enjoyment, and concentration. Additionally, the group of purposefully selected, highly engaged students reported higher levels of engagement associated with perceiving tasks as challenging.

Therefore, with respect to students’ challenge-skill balance, quantitative and qualitative results suggest positive relationships between perceived high skill (or confidence) and student engagement, even when perceiving of content as challenging. This is demonstrated quantitatively through MLM results, which suggested similar levels of engagement for students whose semester average challenge-skill balance characterized them as HS-LC, S-OC, or S-OS. Posts reflecting high challenge and confidence tended to be associated with indicators of higher levels of engagement; thus, demonstrating this positive relationship qualitatively. Similarly, posts reflecting high challenge and low skill (i.e. lack of confidence) are associated with lower levels of engagement. In this regard, both quantitative and qualitative results divulge that feeling under-
skilled for high-challenge work is negatively associated with community college student engagement during precalculus class.

Students’ perceptions about teaching were found to be important to student engagement from both sets of analyses. Quantitatively, students’ perceiving of their instructor as caring about their understanding tended to report higher levels of engagement. Qualitatively, students posting positively about their instructor’s teaching emerges as a theme associating students’ perceived needs fulfillment (figure 13) and students’ challenge-skill balance (figure 14) with indicators of engagement (i.e. interest, enjoyment, and concentration).

With respect to instructor reported teaching approaches, quantitative analysis did not detect significant effects on student engagement. However, qualitative results reveal different themes associated with indicators of student engagement emerging in posts written by students experiencing the course through different teaching approaches. Taken together, the overall level of student engagement in this sample is consistent across teaching approaches, but the ways in which this level of engagement is nurtured differs based on teaching approaches (see figure 15). Interestingly, confidence emerges as a theme associated with higher levels of student engagement for students whose instructor’s teaching approaches profiles were categorized as balanced, meaning-making, and student-supportive. On the other hand, confidence only appeared as theme associated with interest for students whose instructor’s teaching approaches profiles were categorized as traditional. Although confidence transcends most teaching approach categories and indicators of student engagement, it is especially prevalent in posts written by students of instructors who reported a more meaning-making approach. Further, the group of highly engaged community college precalculus students suggest another group work as a theme transcending teaching approaches profiles.
In summary, there are many factors associated with student engagement as evidenced by quantitative and qualitative results presented in the previous two chapters. Additionally, students experience the effects of these factors on their level of engagement differently. In general, instructor teaching approaches do not have significant effects on the level of student engagement; however, themes associated with the level of student engagement are different based on instructor teaching approaches profiles. In other words, regardless of their instructors’ teaching approaches profiles, students tended to report similar levels of engagement, but the ways in which this level of engagement was attributed to various themes differs based on teaching approaches profiles.

**Relationships Between Engagement and Understanding**

Quantitative analysis of students’ scores on the precalculus concepts assessment (PCA) did not reveal significant differences in student understanding of precalculus concepts between classes. Consequently, simple mediation analysis was conducted to determine if there was a relationship between student engagement and understanding and if teaching approaches mediated this relationship. This process did not find an association between student engagement and understanding of precalculus concepts for this sample of students (see figure 12). However, qualitative analysis of task-based interview data conducted with the group of purposefully selected, highly engaged students revealed a relationship between student engagement and understanding. In the context of task-based interviews, differences in students’ understanding (i.e. shape thinking) were associated with different levels of emotional engagement, specifically enjoyment. Each of these highly-engaged community college precalculus students described the task as challenging, which emergent shape thinking tended to find enjoyable. This was not the case for static shape thinkers, who related the task’s difficulty to interest but not enjoyment. High
levels of concentration were demonstrated and described by all students, regardless of their understanding of precalculus concepts.

To conclude, the relationship between student engagement and understanding uncovered by analyzing task-based interview data leaves open the possibility of such a relationship existing. However, analyses from the larger sample of student data from this study do not reveal one. Consequently, the lack of a relationship from the larger sample renders it impossible to determine whether teaching approaches mediate the effects of student engagement on understanding.
Chapter 8: Discussion and Conclusion

There are several findings from this study that are significant to understanding community college precalculus students’ experiences, which may be leveraged to inform community college precalculus instruction, student orientation seminars, and future research. This chapter situates results obtained from this study in existing literature, and suggests implications for instructors, students, and researchers.

Implications for Instructors

First, student engagement varies significantly within individuals. In this study, roughly 34% of the overall variation in student engagement is attributed to intra-individual fluctuation. Uekawa et al. (2007) used experience sampling methods (ESM) and MLM to investigate urban high school students’ engagement in mathematics and science courses and found that nearly 40% of variation in student engagement occurs within students. Intra-individual variance in student engagement is otherwise unexplored in mathematics education research, even in large quantitative studies (e.g., Lan et al., 2009; Martin et al., 2015; Rimm-Kaufman et al., 2015; Robinson, 2013). Understanding that an individual student experiences changes in his or her level of engagement is important, as it suggests that variety in classroom activities (e.g., note-taking, group work, presentations, etc.) during each week may promote higher levels of student engagement. This suggestion was also voiced by many highly-engaged students in the current study during interviews.

Second, positive associations between needs fulfillment and engagement are posited by many different theories, including flow theory. For example, Finn’s participation-identification model of student engagement explains the importance of autonomy and belongingness for student engagement (Finn & Cox, 1992; Finn & Rock, 1997; Finn & Zimmer, 2012).
Newmann’s authentic work model posits that student engagement hinges upon students’ sense of belonging (1989; Newmann et al., 1992). Skinner and her colleagues (Skinner & Belmont, 1993; Skinner et al., 2008, 2009) and Deci and Ryan (Deci & Ryan, 1985, 2008; Deci et al., 1991; Ryan & Deci, 2000) conceptualize engagement as the outward manifestation of motivation, which they demonstrate to be positively correlated with students’ perceived needs fulfillment. Results from this study demonstrate the positive relationship between needs fulfillment and interest, enjoyment, and concentration (i.e. engagement). Results from MLM suggest that individual students experience the benefits of needs fulfillment on their engagement differently, and qualitative results uncover themes students commonly associate with needs fulfillment and indicators of engagement. Therefore, this study provides empirical evidence for the importance of establishing community college precalculus classroom environments that support students’ psychological needs for competence, belongingness, and autonomy.

Establishing a learning environment in which students feel competent, welcome, and autonomous on a weekly basis is important for student engagement, but supporting needs fulfillment is only part of establishing an engaging environment for community college precalculus students. Another essential element to consider is the nature of work being assigned to students. Csikszentmihalyi (1975, 1990, 1997) theorized the importance of students’ challenge-skill balance on student engagement, where he posits that perceptions of high challenge and skill are accompanied by high engagement. This has been validated empirically in high school students (e.g., Shernoff et al., 2003). Results from this study reveal the importance of students’ challenge-skill balance, where students who feel challenged or skilled tend to be more engaged. Further, task-based interviews demonstrated that perceived challenge has a role in the relationship between highly-engaged students’ understanding of precalculus concepts and
engagement. However, regardless of students’ understanding, all students can engage with tasks which they perceive as difficult. On the other hand, students who feel under-skilled to perform tasks which they believe are difficult (i.e. LS-HC) tend to be significantly less engaged than their peers. In fact, the adverse experience of these feelings on student engagement is not lessened when controlling for needs fulfillment. Also, students’ confidence appears to have a vital role on the relationship between challenge-skill balance and engagement.

Considering the results obtained by this and other studies of student engagement where students’ challenge-skill balance is a factor (e.g., Nakamura & Csikszentmihalyi, 2009; S. E. Peterson & Miller, 2004; Shernoff et al., 2003), community colleges should take care in placing students into precalculus because of the detrimental effects of feeling highly challenged and under-skilled (i.e. LS-HC) on student engagement. Also, community college precalculus instructors should consider methods for reducing student anxiety during class time. This is not to say that community college precalculus instructors should assign less challenging tasks, as students are capable of high levels of engagement on tasks which they perceive to be difficult – regardless of their understanding. Also, students related low fulfillment of their sense of competence through poor performance on assessments perceived as challenging to low enjoyment in their posts. Posts of this nature tended to demonstrate the student’s intentions of withdrawing from the course or accepting failure. Accordingly, community college precalculus instructors should consider methods for celebrating student successes during class time and in settings other than through feedback on assessments to support competence and instill confidence.

In addition to assigning challenging work, community college precalculus instructors should also encourage student collaboration to promote high levels of engagement. Highly-
engaged students described being most engaged while working in groups, and voluntarily participating in group work was a theme in the way most highly-engaged students became involved during precalculus class. The beneficial effects of group work on student engagement in K-12 mathematics classrooms have been reported by other researchers (Bray & Tangney, 2016; Lan et al., 2009; Martin et al., 2015; Uekawa et al., 2007). Group work and task design/selection can be structured to further promote student engagement. In this regard, tasks should be structured such that follow up questions put students in positions to think about their own thinking. Further, group norms should be established to encourage peers to probe the thinking of others.

**Implications for Students**

The previous section is devoted to discussing how results obtained in this and other studies on student engagement can be used to inform community college precalculus instruction. However, there are also results which may be of interest to students. First, results from this study suggest that the negative effects of feeling highly challenged and under-skilled (i.e. LS-HC) on student engagement are severe, and other studies have showed positive relationships between student engagement and academic achievement (Fredricks et al., 2004; Reschly & Christenson, 2012). So, students should carefully consider their readiness for precalculus when enrolling for courses.

Second, students should remain open-minded about collaborating with their peers during precalculus class. Choosing to work in groups when given the opportunity emerged as a theme described by highly-engaged students from the current study during interviews. Also, these students reported being more engaged during group work than other classroom activities, and described how working collaboratively allowed them to think about their own thinking (and the
thinking of their peers). This form of self-reflective behavior (metacognition) has been associated with positive achievement outcomes (Finn & Zimmer, 2012; Wolter & Taylor, 2012).

Implications for Future Research

Flow theory offers a useful conceptualization of student engagement as the amalgamation of interest, enjoyment, and concentration (Csikszentmihalyi, 1975, 1990, 1997; Nakamura & Csikszentmihalyi, 2006). This lens informed methods of data collection and analysis for this study, and allowed for results to be interpreted to address the two research questions. In doing so, additional questions and suggestions for future research were uncovered. First, the existence of a relationship between student engagement and understanding uncovered during task-based interviews leaves open the question about the nature of this relationship on a larger scale.

Second, given the amount of within- and between-student variation in student engagement, more mixed methods research is required to investigate factors other than needs fulfillment and classroom activities (e.g., group work) which may explain this variation. One potential starting place in this endeavor is to further investigate students’ confidence as a facilitator of student engagement, as confidence was a theme commonly associated with students’ challenge-skill balance and engagement in posts.

Third, results from this study suggest that indicators of student engagement (i.e. interest, enjoyment, and concentration) are correlated. This is particularly evident for interest and enjoyment, which were used almost-interchangeably by highly-engaged students during interviews. Also, students’ posts rarely reflected level of interest, and almost never concurrently described students level of interest and concentration. Finally, Richard offered a unique perspective of the importance of interest, by suggesting it was irrelevant, yet demonstrated and described high levels of concentration during a task-based interview. Consequently, more
research needs to be done that illuminates students’ state of interest for various levels of concentration and vice versa. Also, it is important to note that researcher’s conceptualizing student engagement through self-determination theory (SDT) would conclude that Richard’s external motivation (i.e. grades and advancing through coursework) is indicative of lower levels of engagement than the level suggested here (Deci & Ryan, 1985). Accordingly, researchers pursuing this investigation are advised to conceptualize engagement through other theories, or to consider interest as intrinsic motivation (Hidi & Renninger, 2006), adopt SDT, and investigate any relationships with concentration.

Fourth, it would be significant to the scientific community and community college precalculus instructors to further investigate gender differences in the relationship between cognitive engagement and participation in group work. Results from interviews with highly-engaged students in the current study suggest that male students may view group work as a means for evaluating their own understanding by considering the reasoning of their peers. On the other hand, female students described group work as an opportunity to assist their peers, and thus focused their concentration on their own thinking to provide useful help. However, both male and female students reported high levels of concentration during group work. Obtaining a better understanding of how male and female community college precalculus students experience cognitive engagement benefits from group work could be used to further inform best-practices for designing groups and group tasks.

Finally, more research is needed to investigate the role that student-supportive teaching approaches have on students’ confidence, sense of belonging, and engagement. Only one instructor from the current study – Mrs. Baker – reported a student-supportive teaching approach profile, but recall that the proportion of traditional codes in her profile was also above the
benchmark of 40% for classifying her profile. However, her students made posts that associated confidence with concentration and higher levels of enjoyment, a theme which was not present in posts for these same indicators of engagement written by students of other instructors whose teaching approaches profiles were characterized as traditional. This suggests that the proportion of a student-supportive approach in her profile might interact with her students’ engagement through her ability to foster confidence in different ways than those experienced by students of instructors with different teaching approaches profiles.

Limitations

There are some limitations of this study which are important to address. First, results from this sample suggest that there is not significant instructor level variation in student engagement. This contradicts results from Uekawa et al. (2007), who used MLM and determined that nearly 11% of variation in student engagement could be attributed to differences in instructors (or classrooms). The relatively small sample of instructors; wide range in the number of students from each instructor’s classes participating; and departmental policies dictating course pacing and testing windows at the larger community college all offer possible explanations for this discrepancy. Second, students self-selected to participate in the study, which likely decreases the representativeness of the student sample. Further, there was a range in the number of weekly diaries submitted by each student; however, MLM is appropriate for unbalanced groups (Raudenbush & Bryk, 2002). Finally, evidence from classroom observations suggests that instructor reported teaching approaches from interviews may differ from those enacted for the instructors of this sample. This contrasts with conclusions offered by Mesa et al. (2014). Once again, departmental policies at the larger community college may explain this difference. In terms of generalizability, results about the sample of instructors’ teaching
approaches cannot be generalized (e.g., proportion of community college precalculus instructors who employ specific approaches) due to the small number of participants. Further, the small number of instructor participants also affects the statistical power of quantitative analyses. From a statistical standpoint, my sample size is only equal to the number of participants at the highest level of analysis (15 instructors).

**Significance**

Foundational theories of student engagement originated out of necessity for explaining students’ experiences in school, and particularly as means for understanding factors of students’ decisions to dropout (Fredricks et al., 2004; Reschly & Christenson, 2012). In this regard, examining community college student engagement may shed light on alternative avenues for approaching the issue of poor performance in community college precalculus documented in the literature (e.g., Barnes et al., 2004) – which is the overarching purpose of this study. This argument is supported by the existing literature associating student engagement with achievement outcomes (e.g., Finn & Rock, 1997; Hidi & Renninger, 2006; S. E. Peterson & Miller, 2004; Shernoff et al., 2003; Skinner et al., 2009; Steele & Fullagar, 2009), and by the task-based interview results suggesting the existence of a relationship between student engagement and understanding. Despite some limitations in this research, findings obtained are significant and may be used to inform community college precalculus instruction, professional development (including graduate programs focused on preparing community college mathematics instructors), and student orientation seminars.

**Conclusion**

Historically, community college precalculus students have struggled to pass the course on their first attempt (Barnes et al., 2004). Existing literature on students’ quantitative reasoning
(e.g., Thompson, 2011) covariational reasoning (e.g., Carlson et al., 2002; Carlson, Oehrtman, & Engelke, 2010), conceptions of function (e.g., Asiala, Cottrill, & Dubinsky, 1997), and shape thinking (e.g., Castillo-Garsow et al., 2013; Monk, 1992; Moore & Thompson, 2015; Stevens & Moore, 2016) has shed light on students’ ways of thinking about topics and concepts central to precalculus and calculus. This body of work has informed the creation of the precalculus concepts assessment (Carlson et al., 2002) and curricula (e.g., Carlson, Oehrtman, & Moore, 2010) intended to evaluate and develop precalculus students’ understanding of these concepts. Knowledge gained through this work is paramount for teacher education, curricula development, and understanding potential learning trajectories for students as they progress into and through precalculus.

The overarching purpose of this study was to approach the problem of students’ poor performance in community college precalculus from a different perspective – through student engagement – and use results acquired as means for informing alternative avenues for addressing this issue. Further, results obtained from this study stem almost exclusively from student-reported data about their experiences as community college precalculus students. I interpret these results as these students’ collective voice for ways in which we (community college instructors, teacher educators, and mathematics education researchers) can establish engaging learning environments. From this lens, results from this study suggest that community college precalculus students are asking for variety of instructional methods, needs fulfillment, incorporating challenging problems with relevant context, opportunities to work collaboratively with peers, confidence-supportive environments, and instructors to communicate that they care about each students’ understanding.


Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social


E. Dubinsky (Eds.), *The concept of function: Aspects of epistemology and and pedagogy* (pp. 175–193). Washington D.C.: Mathematical Association of America.


https://doi.org/10.1007/BF01273664


Appendix A: IRB Approval Letter

From: Jennifer Ofstein, IRB Coordinator
North Carolina State University
Institutional Review Board

Date: July 13, 2016

Title: Student Experiences in Community College Precalculus: A Mixed Methods Study of Student Engagement and Understanding

IRB#: 7954

Dear Karen Hollebrands,

The research proposal named above has received administrative review and has been approved as exempt from the policy as outlined in the Code of Federal Regulations (Exemption: 46.101. b.2). Provided that the only participation of the subjects is as described in the proposal narrative, this project is exempt from further review. This approval does not expire, but any changes must be approved by the IRB prior to implementation.

NOTE: This committee complies with requirements found in Title 45 part 46 of The Code of Federal Regulations. For NCSU projects, the Assurance Number is: FWA00003429.

Any changes to the research must be submitted and approved by the IRB prior to implementation.

If any unanticipated problems occur, they must be reported to the IRB office within 5 business days.

Please forward a copy of this letter to your faculty sponsor, if applicable.

Thank you.

Sincerely,

Jennifer Ofstein
NC State IRB
Appendix B: Student Informed Consent Form

North Carolina State University  INFORMED CONSENT FORM for RESEARCH
Title of Study: Student Experiences in CC Precalculus: A Mixed Methods Study of Student
Engagement and Understanding
Principal Investigator: Derek Williams  Faculty Sponsor: Karen Hollebrands

What are some general things you should know about research studies?
You are being asked to take part in a research study. Your participation in this study is voluntary. You have the right to be a part of this study, to choose not to participate, or to stop participating at any time without penalty. The purpose of research studies is to gain a better understanding of a certain topic or issue. You are not guaranteed any personal benefits from being in a study. Research studies also may pose risks to those that participate. In this consent form you will find specific details about the research in which you are being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you have questions about your participation, do not hesitate to contact the researcher(s) named above.

What is the purpose of this study?
This study will investigate how teaching methods employed by community college precalculus instructors affect student engagement and understanding.

What will happen if you take part in the study?
If you agree to participate in this study, you will be asked to complete weekly online diaries and a post-test. The weekly diaries will take approximately 2-3 minutes and can be completed on your mobile device. The post-test will take approximately 50 minutes. The total duration of the study will last approximately 16 weeks – one semester. A researcher may videotape approximately 5 hours of your class at random times throughout the semester. Further, you may be asked to participate in a sequence of three interviews, which will last roughly 30-45 minutes each. You may agree to participate in the overall study, but choose not to participate in any interviews.

Risks
Participation in this study will involve an assessment and possibly being videotaped during class. Nothing reported during this study will have an impact on your GPA or affect any grades in any of your courses. Further, interviews are voluntary, and if asked to participate, you may decline.

Benefits
Participating in educational research allows our educational system to grow and serve our students better in the future. As a participant in this study, you would be providing valuable information to the scientific community of educational researchers, and are contributing to help improve and adapt community college mathematics education experiences for future students.

Confidentiality
The information in study records will be kept confidential to the full extent allowed by law. Data will be stored securely in password protected computers/files, as well as maintained in locked filing cabinets. No reference will be made in oral or written reports which could link you to the study.

Compensation
Monthly drawings for monetary compensation ($100) will be conducted in September, October, November, and December. Each electronic diary submission will include your name as a potential winner of these drawings. Drawings will take place every 4 or 5 weeks, so you may have 4 or 5 chances to win each drawing.

What if you are a student?
Participation in this study is not a course requirement and your participation or lack thereof, will not affect your class standing or grades at the institution.

What if you have questions about this study?
If you have questions at any time about the study or the procedures, you may contact the researcher, Derek Williams, at dawilli6@ncsu.edu.

What if you have questions about your rights as a research participant?
If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919/515-4514).

Consent To Participate
“I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may choose not to participate or to stop participating at any time without penalty or loss of benefits to which I am otherwise entitled.”
Appendix C: Initial Demographic Survey

1. Informed Consent (Yes/No).

2. What is your email address (e.g., JohnDoe@email.com)?

3. Did you enter your current college immediately after high school?
   a. Yes
   b. No

4. Is this your first time taking precalculus in college?
   a. Yes
   b. No

5. What gender do you associate with?
   a. Male
   b. Female
   c. Other

6. What is your date of birth?

7. What is your race/ethnicity?

8. Did you take any remedial mathematics courses in college prior to precalculus?
   a. Yes
   b. No

9. Are you a veteran or currently serving in the military in any capacity?
   a. Yes
   b. No

10. What is your enrollment status?
    a. Full-time
b. Part-time

11. What is your work status?
   a. Full-time
   b. Part-time

12. Are you a parent?
   a. Yes
   b. No

13. What is your intended major?

14. What is the first day of the week in which your precalculus class regularly meets?
   a. Monday
   b. Tuesday
   c. Wednesday
   d. Thursday
   e. Friday
   f. Online

15. At what time does your class end (provide the time for the last meeting of each week)?

   Online students, enter 00:00.
Appendix D: Weekly Diary

1. Enter your email address (e.g., myname@email.com)

2. Are you still enrolled in your precalculus course? (Y/N)

3. Please rank each of the following statements (0-Strongly Disagree – 5-Strongly Agree)
   a. During precalculus this week my instructor used my ideas about the material during lessons
   b. During precalculus this week my instructor moved too fast for me to comprehend all of the material
   c. During precalculus this week my instructor slowed the pace to make sure I felt confident with the material
   d. During precalculus this week my instructor made me feel like he/she cares about me as an individual
   e. During precalculus this week my instructor helped me make connections between mathematics and the world
   f. During precalculus this week my instructor made it clear that my understanding of mathematics was important

4. Please rank each of the following (0-Strongly Disagree – 5-Strongly Agree)
   a. I felt capable of understanding the material being covered during precalculus this week
   b. I felt welcome in class during precalculus this week
   c. I had choices that allowed me to direct my own learning during precalculus this week
   d. Activities I worked on in precalculus this week were relevant to my future goals
   e. Activities I worked on in precalculus this week were relevant to upcoming assessments
   f. I wanted to attend precalculus this week
5. How frequently did each of the following take place during precalculus this week (0-Never, 1-Rarely, 2-Sometimes, 3-Frequently, 4-Very Frequently, 5-Always)?
   a. My instructor led whole-class discussions
   b. I worked individually on class assignments
   c. I worked with small peer-groups
   d. My instructor lectured
   e. My instructor gave me opportunities to explain my thinking

6. Please rank each of the following statements (0-Strongly Disagree – 5-Strongly Agree)
   a. Precalculus this week was interesting
   b. I enjoyed precalculus this week
   c. I concentrated on mathematics during precalculus this week
   d. I found the material from precalculus this week to be challenging
   e. During precalculus this week I was skilled enough to be successful

7. If you were going to make a social media post about precalculus this week, what would you say? (150 characters or less)
Appendix E: Instructor Interview Protocol

Pre-observation interview with the instructor

Teaching practices

1. How do you use different forms of instruction such as lecture, small groups, worksheets, technology? (Note responses to similar question on Instructor/Class Information Form.)

2. How do you measure what the students are learning?

3. What do you do to motivate your students?

4. What do you do to help your students in and out of the classroom?

5. What kinds and uses of materials, e.g., textbooks, do you employ?

6. What kinds of assignments do you give?

7. Do you have writing assignments? What types?

Conceptions and philosophies of teaching (reflect on what they do)

8. How do you define good teaching?

9. What are your goals for the students?

10. How have you arrived at your views?

Personal influences on teaching

11. How have outstanding instructors (you have had) influenced how you teach?

12. What impact does the surrounding business and/or disciplinary community have on how and what you teach?

Instructor perceptions of students

13. Why are your students there? Why do they enroll in your course?

14. What kinds of students show up in your class?
15. What are some important issues with your students? What external forces impinge upon your students? What do your students need from you as an instructor?

16. What kinds of attitudes do your students exhibit toward your class and toward education in general when they enter your class? Are they prepared for class (having done all the assignments)? How motivated are they for your class?

17. What kinds of cognitive or affective changes do you see in your students during the semester?

18. What is it in your teaching that causes these changes?

**Collegial influences on teaching**

19. What kinds of exchanges do you have with your colleagues in your department? In other departments?

   Probe: Do you talk about teaching?

   Follow-up: What exchanges occur between academic and vocational and remedial disciplines?

20. What exchanges do faculty as a whole have with one another? How frequently?

   ** What would improve the climate of collegiality?

21. Have you ever taught collaboratively? How common is it in your department? In other departments?

   If yes:

   What are the pros and cons of collaborative teaching compared with solo teaching?

   What kinds of results do you see with collaborative teaching?

22. What forms of contact do you have with other departments?
Institutional influences on teaching

23. What resources and facilities are available for you to use in your teaching?

24. How does the administration influence effective or innovative teaching? For you? For colleagues? For the institution as a whole?

25. What are the significant factors/criteria used in hiring new instructors? In promoting instructors? In acquiring sabbaticals, travel/conference funding?

—— (If teaching isn't mentioned:)

—— Is teaching skill a criteria for hiring?

26. Are you aware of campus-wide programs related to teaching?

27. Have you participated in faculty/staff development activities? If yes, what difference has it made for you, other faculty, the campus as a whole? How do faculty feel about the existing faculty/staff development programs?

28. What additional steps might the administration take to support innovation and effective teaching?

29. What are some important issues going on in the institution or the community at large that affect your teaching?
Appendix F: PCA Items

3) To the right are drawings of a wide and a narrow cylinder. The cylinders have equally spaced marks on them. Water is poured into the wide cylinder up to the 4th mark (see A). This water rises to the 6th mark when poured into the narrow cylinder (see B).

Both cylinders are emptied, and water is poured into the narrow cylinder up to the 11th mark. How high would this water rise if it were poured into the empty wide cylinder?

a) To the 7 1/2 mark
b) To the 9th mark
c) To the 8th mark
d) To the 7 1/2 mark
e) To the 11th mark

The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are traveling in the same direction.) Use this information and the graph below to answer item 8.

8) What is the relationship between the position of car A and car B at \( t = 1 \) hr.?

a) Car A and car B are colliding.
b) Car A is ahead of car B.
c) Car B is ahead of car A.
d) Car B is passing car A.
e) The cars are at the same position.

9) Use the graphs of \( f \) and \( g \) to solve \( g(x) > f(x) \).

a) \( 2 < x < 5 \)
b) \( 1 < y < 4 \)
c) \( x < 4 \)
d) \( 2 < y < 5 \)
e) \( 1 < x < 4 \)
Appendix G: Observations & Field Notes Protocol

Teacher:

Date/time:

Topic Covered:

Make Sketch of classroom to record voluntary student responses and individually elicited responses

Describe the classroom:

Instructions: Watch the class for 5 minutes and then describe the various teacher and student activities. Address the following questions:

1. What is the teacher doing during this time period?

2. What elements of behavioral engagement are present?
   a. Are students taking notes?
   b. Are students participating in the class?
   c. Are students off task?
   d. Are students asking questions?
   e. Do students appear to be concentrating?
   f. Do students appear to be interested (no heads down, on task, not on cell phone, discussing mathematics)?
   g. Do students appear to be enjoying the class (smiling, laughing)?
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<tr>
<th>Time</th>
<th>Teacher Activity</th>
<th>Student Activity</th>
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Appendix H: Student Interview Protocol 1

Environmental Questions

1. Do you feel welcome or have a sense of belonging in your classroom?
   a. If so, what are some things that your instructor does to make you feel welcome?
   b. If not, what are some things that your instructor could do to make you feel welcome?

2. Do you feel competent or that you have the ability to be successful in your classroom?
   a. If so, what are some things that your instructor does to make you feel welcome?
   b. If not, what are some things that your instructor could do to make you feel welcome?

3. Do you feel as though you have choices in how you learn or how you are assessed in your classroom?
   a. If so, what are some things that your instructor does to make you feel welcome?
   b. If not, what are some things that your instructor could do to make you feel welcome?

4. Describe a typical day in class. What do you do? What does your teacher do?

5. During class do you ever have chances to work individually or in small groups on the material being covered?

Engagement Questions

1. Describe some elements of the course so far that you have found interesting.
   a. What could your instructor do to make the course more interesting to you?

2. Describe elements of the course that you find enjoyable.
a. What could your instructor do to make the course more enjoyable to you?

3. Discuss your concentration during class.
   a. What do you find yourself focusing on while the instructor is working examples?
   b. What do you find yourself focusing on while you are working individually?
   c. What do you find yourself focusing on while you are working in small groups?

4. Do you find the content of the course so far to be challenging?
   a. What aspects have been challenging for you?
   b. Do you feel as though you are capable of meeting any challenges you face?

**Elaborate on Weekly Posts**

These are the posts that you have made on the weekly survey (in order), expand on these posts with regards to the questions that we have discussed so far today.
Appendix I: Taking a Ride Task Protocol

Taking a Ride
Version: Implementation

Do not cite or use without author permission.
Contact: kvcmoore@uga.edu

This material is based upon work supported by the National Science Foundation under Grant No. DRL-1350342. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.
Taking a Ride Part I

Graph the relationship between the total distance the rider has traveled *around* the Ferris wheel and the rider’s distance from the ground.

*Interviewer Notes:*
Taking a Ride Part II

Watch the video, which depicts a second rider’s trip around the Ferris wheel. Is the previous graph relevant? Explain.

Interviewer Notes:
Problem Notes

To open the problem (Part I), do not give them this sheet. Instead:

1) Open ‘Taking a Ride 1.mov’.
2) Play the video (ensuring the video is on loop).
3) State, “Tell me about the situation. What’s going on here?”
4) If they identify quantities, ask them to say more. But do not push in ways that aid their clarifying quantities or avoiding quantity conflation.
5) State, “OK, So what I want you to do is graph two quantities.” Slide over the sheet.
6) Request, “I would like you to think aloud as you construct the graph, describing how you are thinking about the situation and the graph.”

Suggested follow-up questions for Part I:
- Say more. I’m not sure what you are trying to say. Could you say that again?
- Can you explain how you drew your graph?
- Why is your graph (curved, straight, etc.)?
- What is meant by distance from the ground? Can you show me on the video?
- What is meant by total distance traveled around the Ferris wheel? Can you show me on the video?
- What does a point represent on the graph? How do you know it represents that on the graph?
- How does that point correspond to the situation? Can you show me on the situation?
- The cart initially moves to the left. But the graph moves to the right. How can that be?
- The car initially moves to the left. How would the graph change if the car moved to the right?
- Ask what they mean by slope if they raise the idea.

For Part II:
1) Open ‘Taking a Ride 2.mov’.
2) State, “This video represents a different rider’s trip on the same Ferris wheel.”
3) Play the video (ensuring the video is on loop) and pass them the Part II.

Suggested follow-up questions:
- Say more. I’m not sure what you are trying to say. Can you say that again?
- If the interviewee claims the graph will be the same: Why is the graph the same as the previous graph?
- If the interviewee claims the graph will be the same: A student argued, “Since the ride stops, we need to draw straight lines to show this.” How would you respond to this student?
- If the interviewee claims the graph will be different: Can you draw the graph that captures this rider’s ride? Why is it different than the previous graph?
- Why is your graph (curved, straight, etc.)?
- What does a point represent on the graph? How do you know it represents that on the graph?
How does a portion of your graph correspond to the situation? Can you show me on the situation? This question is especially useful if the interviewee draws portions of the graph that imply a quantity’s value remains constant.
Appendix J: Task-Based Interview Debriefing Questions

Circle appropriate words in parentheses.

1. When you first watched the video, what did you anticipate you would be doing?
   a. Were you planning ahead for how to do [that]?
   b. What were you paying attention to while watching the movie? Why did you think this was an important element?
   c. If you had to give this task a title, what would it be?

2. Can you describe/explain how you approached this task?
   a. What was important for you to think about while working on the task?

3. If your teacher used this task in your MAT 171 class, what concepts could they teach?
   a. What about this task made you think of [that concept/topic]?

4. Do you find this task interesting? Please explain why (or why not).
   a. (If Yes) What about the task captured your interest?
   b. (If No) Why do you think the task wasn’t very interesting to you?

5. Was working on this task enjoyable? Please explain why (or why not).
   a. (If Yes) Did you enjoy the context, the mathematics, or both? Why?
   b. (If No) Did you dislike the context, the mathematics, or both? Why?

6. How challenging did you find this task? Why?

7. How do you think you did on the task? Why?
   a. Do you think you answered the questions completely?
   b. What (supported or inhibited) your response?