ABSTRACT

LUO, SHA. Ordering Strategy, Supplier Incentive and Supply Chain Performance. (Under the direction of Dr. Russell E. King and Dr. Shu-Cherng Fang.)

A supply chain usually consists of different players/companies with conflicting objectives. In this dissertation, we concentrate on a two-echelon supply chain consisting of one retailer and multiple suppliers. We study particularly the interactive effects of the retailer’s sourcing strategy and suppliers’ wholesale pricing decisions. This dissertation consists of three essays, each looking into the supply chain with a different feature.

The first essay considers the retailer’s trade-off between selecting an unreliable, economic supplier and a reliable, expensive supplier with fixed ordering costs. We successfully provide mathematical proofs for an $(s, S)$-like optimal ordering policy in the finite-horizon setting. We also analyze the limiting behavior of the $(s, S)$-like policy to show that the converged policy characterizes the optimal ordering policy for the infinite-horizon setting. Extensive experimental results indicate that the $(s, S)$-like ordering policy is optimal under a wide range of system parameters beyond the conditions required in the optimality proof.

The second essay studies a short-life-cycle product supply chain with a normal supplier and a quick response supplier. As it approaches the selling season, more accurate demand updates become available. The retailer has the option of single sourcing or dual sourcing while the suppliers determine the unit wholesale price under a specific sourcing strategy revealed by the retailer. The retailer’s ordering and the suppliers’ pricing decisions are characterized using game theoretical approach. With an exponentially distributed demand, we find that the retailer prefers single sourcing to dual sourcing. We then consider introducing more QR suppliers and find that contribution benefits the supply chain.

The third essay examines a supplier pricing game with a bounded rational retailer. General wisdom suggests that people fail to reap the highest profit when they suffer from bounded rationality. We employ the classical multinomial logit model to capture the retailer’s bounded
rationality. The Nash equilibrium of the supplier pricing game is characterized. It is found that the suppliers may reduce their wholesale prices to attract the retailer. As such, bounded rationality can lead to an unexpected higher profit for the retailer. We also investigate how the profit of the supply chain is affected by the retailer's bounded rationality and the impacts when bounded rationality is erroneously estimated.
Ordering Strategy, Supplier Incentive and Supply Chain Performance

by

Sha Luo

A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the Degree of Doctor of Philosophy

Industrial Engineering

Raleigh, North Carolina

2017

APPROVED BY:

Dr. Donald P. Warsing, Jr.
Minor of Advisory Committee

Dr. Yunan Liu

Dr. Russell E. King
Co-chair of Advisory Committee

Dr. Shu-Cherng Fang
Co-chair of Advisory Committee
DEDICATION

To my family:
Youyu Zhang
Zhi Luo
Yao Yu
BIOGRAPHY

Sha Luo was born and raised in a beautiful southern city in China, Fuzhou. In 2005, she attended Fuzhou No.1 Middle School. In 2008, she attended Peking University (Beijing, China), where she received her Bachelor of Science degree in Physics and a double major in Economics. After that, she was admitted to the Department of Industrial and Systems Engineering at North Carolina State University (Raleigh, NC) to pursue her Ph.D. degree under the collective supervision of Prof. Russell King and Prof. Shu-Cherng Fang.
ACKNOWLEDGEMENTS

I have been very fortunate to receive a lot of guidance, inspiration and encouragement during my Ph.D. study.

First of all, my deepest gratitude goes to my advisors, Dr. Russell King and Dr. Shu-Cherng Fang. I am very fortunate to have two advisors and get exposed to their different charismata. I am deeply indebted to Dr. King for offering me the opportunity to teach an undergraduate course and his mentorship during the teaching process. I benefited a lot from watching him teach and discussing my research topics with him. Despite his busy schedule, he always gives me the advice and guidance I need. He always has a lot of confidence in me and encourages me to excel myself. Dr. Fang not only serves as an academic supervisor but also a life mentor. Academically, he sets an example for me to be a rigorous scholar. Mentally, he shares his experience with me, enlightens me in daily life, and teaches me to be a caring and wise person by being one. Both Dr. King and Dr. Fang have a profound influence on me. It is truly my privilege to be one of their students.

I am deeply grateful for Dr. Donald P. Warsing and Dr. Yunan Liu for being members of my advisory committee and Dr. Thayer Morrill for serving as my graduate school representative. Especially, I want to thank Dr. Warsing, with whom I had the opportunity to collaborate on one paper. Working with him is indeed an enjoyable learning process.

I would also like to thank Dr. Zhe Liang, for bringing me into the wonderful world of Operations Research and Industrial Engineering, the selection committee for offering me the opportunity to study at NC State, Mr. Edward Fitts and the center for Additive Manufacturing and Logistics for financial support. My sincere thanks and appreciation also go to all the professors, especially those I have taken courses from, in ISE department. I enjoyed all the courses and have benefited a lot from their instructive teaching.

My appreciation is extended to my friends at North Carolina State University. I am also lucky to have many joyful, friendly and helpful officemates: Zhibin Deng, Jian Luo, Ziteng
Wang, Chien-Chia Huang, Tiantian Nie, Mohamed Desoky, Jiahua Zhang, Qi An, Shan Jiang, Yu-Liang Lin, Chi-Yi Chen and many others. We had lunch together every Friday and have shared a lot of happy moments.

The two people I feel I am forever indebted to are my parents. I am very grateful to be raised in a democratic family. Different from many other Chinese parents, my parents never impose their willingness on me and they respect my opinions. They always encourage me to go to a bigger stage to excel myself, even if it means building a long distance between them and me. I could not have been where I am without their constant love and support.

Finally, I would like to thank my boyfriend, Yao Yu, who is also a Ph.D. student in the ISE department of NC State University, for bringing me laughter and making my life colorful. He has the rare ability to make me laugh when I am feeling depressed. I have also received a lot of help and encouragement from him when I am feeling perplexed. We will definitely miss this carefree time at NC State.
TABLE OF CONTENTS

List of Tables ........................................................................ viii
List of Figures ....................................................................... ix

Chapter 1 Introduction .......................................................... 1

Chapter 2 Literature Review .................................................. 8
  2.1 Game Theory in Supply Chain Management ......................... 8
  2.2 Quick Response Supply Chain ........................................... 10
  2.3 Supply Disruption ........................................................... 13
  2.4 Supplier Pricing .............................................................. 15
  2.5 Bounded Rationality in Supply Chain Management ................. 16

Chapter 3 Optimal Policy for a Dual-Supplier System under Disruption 18
  3.1 Introduction .................................................................. 18
  3.2 A Finite Horizon Model ................................................... 23
  3.3 An (s, S)-like Optimal Ordering Policy .............................. 28
  3.4 Extension to an Infinite Horizon Model ............................... 40
  3.5 Numerical Studies .......................................................... 45
  3.6 Concluding Remarks ...................................................... 52

Chapter 4 A Quick Response Supply Chain with Endogenous Wholesale Prices 54
  4.1 Introduction .................................................................. 54
  4.2 Model and Assumptions ................................................... 57
  4.3 Results with Exponential Demand ....................................... 61
    4.3.1 Single Sourcing ....................................................... 61
    4.3.2 Dual Sourcing ......................................................... 63
    4.3.3 Preferred Sourcing Strategy of Each Supply Chain Member 64
  4.4 Effects of Adding Additional QR Suppliers .......................... 67
  4.5 Generalization of Results ............................................... 71
    4.5.1 Single Sourcing ....................................................... 71
    4.5.2 Dual Sourcing ......................................................... 73
    4.5.3 Retailer’s Preferred Sourcing Strategy ......................... 78
  4.6 Concluding Remarks ...................................................... 80

Chapter 5 Pricing Game with a Bounded Rational Retailer ............... 82
  5.1 Introduction .................................................................. 82
  5.2 The Model ................................................................... 86
  5.3 Best Response Analysis .................................................... 90
  5.4 Nash Equilibrium Analysis .............................................. 96
  5.5 Extensions .................................................................... 104
    5.5.1 N-Suppliers ......................................................... 105
5.5.2 Numerical Results for the 3-Supplier Case ......................... 107
5.5.3 2-Suppliers with a No-Purchase Option .......................... 108
5.6 Concluding Remarks .................................................. 110

Chapter 6 Conclusions .................................................... 113
  6.1 Summary and Contributions ........................................ 113
  6.2 Future Research ..................................................... 115

References ................................................................. 118

APPENDICES ............................................................... 128
  Appendix A Appendix for Chapter 3 .................................. 129
    A.1 (s, S) Policy ..................................................... 129
    A.2 The Impact of Beta ............................................. 131
  Appendix B Appendix for Chapter 4 ................................. 138
  Appendix C Appendix for Chapter 5 ................................. 145
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1.1</td>
<td>Key related studies</td>
<td>3</td>
</tr>
<tr>
<td>Table 3.1</td>
<td>Possible permutations and their corresponding cases</td>
<td>37</td>
</tr>
<tr>
<td>Table 3.2</td>
<td>Probability mass function of demand</td>
<td>48</td>
</tr>
<tr>
<td>Table 3.3</td>
<td>Optimal Policy under No Lead Time or One Period Lead time ($h = 0.2, \eta = 1$)</td>
<td>51</td>
</tr>
<tr>
<td>Table 4.1</td>
<td>The impact of the latent QR suppliers on the profit of each supply chain member</td>
<td>69</td>
</tr>
<tr>
<td>Table 5.1</td>
<td>Retailer’s maximum possible profit increase in % under Nash equilibrium</td>
<td>101</td>
</tr>
<tr>
<td>Table 5.2</td>
<td>Numerical results for the equilibrium prices of three-supplier game: $c_1 = 0.2, c_2 = 0.3$</td>
<td>107</td>
</tr>
<tr>
<td>Table B.1</td>
<td>Supply chain profit under dual sourcing with additional QR suppliers</td>
<td>142</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 3.1 Timeline for one period ........................................... 24
Figure 3.2 Minimum of $m_r^* (y_r)$ may not exist .......................... 35
Figure 3.3 Optimal policy over the time as $\alpha$ changes .................... 47
Figure 3.4 Optimal policy over the time as $c_u$ changes ...................... 47
Figure 3.5 Convergence of $(s, S)$ values under all four cases ............... 49

Figure 4.1 The sequence of events in time .................................... 59
Figure 4.2 Equilibrium under single sourcing .................................. 62
Figure 4.3 Preferred sourcing strategy for each supply chain member over different $c_1, c_2$ (“SS”: single sourcing, “DS”: dual sourcing, “=”: indifferent between the two sourcing strategies) ........................................ 67
Figure 4.4 Direction of supply chain profit change under dual sourcing with latent QR suppliers ......................................................... 70
Figure 4.5 Nash equilibrium may (a) or may not (b) exist .................... 76

Figure 5.1 The supplier’s expected profit as a function of his wholesale prices under different $\beta$ when the competitor’s price is known: $w_j = 0.8$ .................. 91
Figure 5.2 Supplier’s best pricing under different $\beta$ given $w_j$ ............. 92
Figure 5.3 Supplier’s best pricing as $w_j$ varies .................................. 93
Figure 5.4 Profit loss as a function of $\beta$ if bounded rationality is ignored .......... 96
Figure 5.5 Profit loss as a function of $w_j$ if bounded rationality is ignored ........ 96
Figure 5.6 Nash equilibrium of supplier pricing game .......................... 98
Figure 5.7 Profit under equilibrium with different levels of bounded rationality and profit when bounded rationality is ignored by suppliers .................. 102
Figure 5.8 Equilibrium price trajectory as bounded rationality level increases: without (blue) and with (red) the no-purchase option ......................... 110

Figure A.1 The impact of $\beta$ on optimal policy: as $\alpha$ varies ............... 132
Figure A.2 The impact of $\beta$ on optimal policy: as $c_u$ varies ............... 133
Figure A.3 The impact of $\beta$ on optimal policy: as $c_r$ varies ............... 134
Figure A.4 The impact of $\beta$ on optimal policy: as $h$ varies ................. 135
Figure A.5 The impact of $\beta$ on optimal policy: as $b$ varies ................. 136
Figure A.6 The impact of $\beta$ on optimal policy: as $K$ varies ................. 137

Figure C.1 Different equilibrium cases based on the ordering of $w_1^* (1; \beta)$ and $\hat{w}_1$: $w_1^* (1; \beta) \leq \hat{w}_1$ (left) and $w_1^* (1; \beta) > \hat{w}_1$ (right) ......................... 150
Chapter 1

Introduction

Supply chains, whether in the manufacturing or service sector, usually consist of players belonging to different companies with conflicting objectives. Optimizing for a single firm and failing to account for interactive strategic relationships among supply chain members usually results in skewed perspectives. In this dissertation, we concentrate on a two-echelon supply chain consisting of one retailer and multiple suppliers. This two-echelon supply chain is able to capture some features of the reality, yet remains amenable to theoretical analysis. We focus our interest on the retailer’s ordering strategy, the suppliers’ pricing decisions, and the performance of the supply chain as a whole. This dissertation is comprised of three essays, each looking into a supply chain with a different feature. Taking into account strategic planning of different supply chain members, we uncover unexpected managerial insights that would otherwise remain hidden.

With the advent of globalization, products can flow unimpeded across national borders, giving rise to global logistics. Many firms are able to move part or all of their manufacturing capacity offshore to take advantage of lower production costs in other regions. It takes longer to ship the products manufactured in the offshore facilities to the domestic market. In addition, production in the offshore developing countries (with lower labor cost) is economically and politically less stable, exposing the firms to a higher risk of supply disruption that impedes
production. Given this, many firms still maintain more costly domestic production facilities in order to better respond to any changes in the supply process and uncertainties in the customer demand. Firms need to determine the order allocation among different supply sources based on purchasing price, reliability, inventory status and the available forecast of market demand. The investment U.S. retailers spend on merchandise and service procurement is enormous. To maintain their competitive advantage, it is very important for firms to remain open to potential outsourcing opportunities and design more innovative and holistic sourcing strategies. These issues raise the need to study the retailer’s ordering strategy in face of multiple suppliers.

In the first essay, we study a single-product inventory system under periodic review. The system consists of two suppliers—one is perfectly reliable while the other offers a more economic price but is subject to possible supply interruptions. The retailer has to consider when it is worthwhile to pay more to guarantee successful delivery of the order. Each supplier is associated with a fixed ordering cost, assumed to be the overhead of generating an order. The existence of fixed ordering cost violates the convex structure of the cost/profit function and therefore significantly enhances computational and analytical difficulty. The stochastic customer demand, the presence of fixed ordering cost, and the consideration of multiple suppliers with unreliable behavior jointly add to the difficulty of the analysis. In such a scenario, it is extremely difficult to consider the actions of the retailer and the two suppliers simultaneously and to optimize the performance of the supply chain globally. Therefore, in this chapter, we limit our attention to the optimal sourcing strategy of the retailer. Ahiska et al. (2013) proposed a discrete-time Markov Decision Process model and conjectured an optimal ordering policy through numerical studies. In this work, we present a theoretical framework with mathematical proofs for an \((s,S)\)-like optimal ordering policy in the finite-horizon setting. Furthermore, we analyze the limiting behavior of the \((s,S)\)-like policy and show that both the optimal cost and ordering policy converge. In this manner, the convergent \((s,S)\)-like policy characterizes the optimal sourcing strategy for the infinite-horizon setting. Through computational studies, we investigate the effects of parameter changes on the optimal policy and demonstrate that the \((s,S)\)-like ordering
policy is optimal under a wide range of system parameters beyond the conditions required in the optimality proof. We provide managerial insights into questions such as how the retailer should determine whether to adopt a single or multiple sourcing strategy; how much to order from each supplier; and how to respond to the changes in reliability level and purchase costs.

Table 1.1 summarizes some selective studies that are most related to our work. To the best of our knowledge, this is the first study that examines the optimality of a dynamic inventory management policy in a multi-period horizon, where there are two suppliers with unreliable behavior, and a fixed ordering cost is charged.

<table>
<thead>
<tr>
<th>Research Paper</th>
<th>Multiple sourcing</th>
<th>Unreliable supply</th>
<th>Fixed ordering cost</th>
<th>Multi-period horizon</th>
<th>Optimality proof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anupindi and Akella (1993)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Özekici and Parlar (1999)</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sethi et al. (2003)</td>
<td>✓</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Fox et al. (2006)</td>
<td>✓</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Y. Wang (2012)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Ahiska et al. (2013)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td><strong>Our work</strong></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

General wisdom suggests that, without the presence of fixed ordering cost, firms should seek as much flexibility and responsiveness as possible. However, if the purpose of engaging multiple suppliers is to promote competition, then the suppliers’ decisions to set the wholesale prices should be considered endogenously. Generally, a retailer with several potential suppliers has two sourcing strategies: single sourcing or multiple sourcing. Multiple sourcing has the advantage of keeping multiple channels of goods and services, promoting innovation, displaying interest in and commitment to continuous growing, etc. Single sourcing, on the other hand, can help promote competition among suppliers. A retailer announcing single sourcing creates a credible threat of ordering from a different sole supplier and hence successfully drives the pool of potential suppliers to offer their best deals. As such, the retailer faces the trade-off between
diversification and price competition effects. Observing two conflicting effects, we are interested in determining which effect takes a dominating role in what scenarios. Therefore, we set out to study a quick response (QR) supply chain, where suppliers make pricing decisions and the retailer decides the order quantity based on suppliers’ quoted prices.

The supply chain we consider in the second essay consists of one retailer and multiple suppliers, similar to our first analysis. In this follow-on study, however, all suppliers are perfectly reliable but vary in their lead times. The suppliers come in two types: the normal supplier and the QR supplier. The normal supplier has a longer lead time compared with the QR supplier. The short lead time of the QR supplier allows the retailer to postpone the ordering decision until more information on market demand is collected. Hence, by ordering from the QR supplier, the retailer is able to reduce demand mismatch costs, i.e., the overage cost when there is any excessive inventory and the stock-out cost when there is any unmet demand. The retailer faces the trade-off between a more accurate demand forecast enabled by shorter lead time and a less expensive ordering cost. We shed light on how the retailer should balance between the initial order and the expedited order. In addition, this work goes one step further to investigate the behavior of the suppliers by allowing the suppliers to set their wholesale prices. When the suppliers are exogenous, it is generally believed that the firms benefit from seeking more flexibility and responsiveness. The involvement of supplier wholesale price setting accounts for the interactive relationship between the retailer and the suppliers and, consequently, can help generate insights that may have been overlooked and explain some seemingly implausible results.

The concept of Nash equilibrium is applied to study the stable outcome of the pricing game in this study. We first consider a benchmark scenario when there is one normal supplier and one QR supplier. The retailer’s ordering strategy and the suppliers’ pricing decisions in equilibrium are characterized. Then we consider introducing more QR suppliers and examine how the profitability of the retailer, the suppliers, and the whole supply chain is influenced by the entrance of additional QR suppliers. We find that under single sourcing, the entrance of
additional QR suppliers affects only the incumbent QR supplier and the retailer, with the QR supplier impaired and the retailer benefited; the normal supplier and the whole supply chain are unaffected. Under dual sourcing, the retailer enjoys higher profit, and both the normal supplier and the QR supplier suffer from the intensified competition; however, the impact on the supply chain depends on the suppliers’ production costs. We find that additional QR suppliers can be advantageous to the supply chain under dual sourcing when the cost of the critical additional QR supplier is not significantly higher than that of the normal supplier.

We contribute to the literature on the competitive effects of quick response by studying the impact of more QR suppliers on the supply chain in a setting where the suppliers make pricing decisions. We not only take care of the relationship between the retailer and the suppliers but also take into account the supplier-supplier relationship by studying supplier competition both within and between different supplier types. We intend to increase the understanding of how supply chain participants behave in an interactive environment by answering two key questions: (i) How does the updated demand forecast affect the suppliers’ pricing decisions and the retailer’s ordering strategy? (ii) How do additional suppliers influence the profitability of the retailer, the incumbent suppliers and the whole supply chain? This work provides managerial insights for the retailers to determine their sourcing strategies and to allocate their orders among different suppliers with different lead times, and for the suppliers to set their wholesale prices.

It should be noted that the additional QR suppliers without a production cost advantage may not have incentives to enter the supply chain without any compensation, since they are not going to receive any orders in equilibrium. Based on this observation, we set out to investigate the scenarios when the retailer will order from the supplier offering a less attractive deal. One possible explanation originates from the behavioral perspectives. Different from the normative approach, the models that take on a behavioral perspective believe that human beings are subject to psychological biases and cognitive limitations and consequently, they do not necessarily make the best choices that maximize their utilities. The inability to optimize can be explained by intrinsic psychological biases that distort decisions. It may also be a result of the limited
information collection and/or limited information processing. Generally, this set of behavioral phenomena is called *bounded rationality* in the literature. Scholars believe that as a new tool, bounded rationality has tremendous potential to generate novel research problems and provide a fresh perspective to provide new insights to mainstream operations management problems (Shen & Su, 2007). Hence, we set out to examine the supplier pricing game through the lens of bounded rationality.

In the third essay, we examine a supplier pricing game with a bounded rational retailer. To exclude irrelevant factors and concentrate our attention on bounded rationality, we assume two identical suppliers providing homogeneous products. Wholesale prices are endogenous variables determined by the two suppliers separately. The retailer makes her ordering decision based on the announced prices of the suppliers. Different from the normative study, in which the retailer always buys from the supplier with a lower price, we model bounded rationality by randomizing the retailer’s decision such that better suppliers are chosen more often. Bounded rationality of the retailer is captured by the well-known multinomial logit model. We analyze the best response of a supplier when the price of his competitor is known and the Nash equilibrium when the two suppliers set their prices simultaneously. We provide a closed form solution for both the best response and the Nash equilibrium analysis. Among the results, one observation is particularly interesting: the retailer receives a higher profit when she is slightly irrational, contradicting the general wisdom that a person suffers from his cognitive limitations. This result holds for both the best response and the Nash equilibrium analysis. In the best response analysis, when the retailer suffers from mild bounded rationality, the supplier is able to receive a higher profit by reducing his price, which increases the supplier’s chance of winning the order. The reason a price cut contributes to completing the transaction is that the retailer is more likely to order from the supplier offering a lower price. Under perfect rationality, an infinitesimal amount of price difference is sufficient to secure the order. However, under bounded rationality, a higher degree of price difference is required to attract the retailer’s order. Interestingly, a similar price reduction is observed in the Nash equilibrium analysis, noting particularly that,
the supplier who provides a price cut is the cost-efficient supplier, from whom the retailer is more likely to make an order. Consequently, the price reduction from the cost-efficient supplier is directly reflected in the retailer’s revenue, and the retailer earns a higher profit when she deviates from perfect rationality. We have also investigated the impact of bounded rationality on the revenue of the retailer and each supplier, as well as the social welfare of the supply chain.

This essay provides a fresh perspective on supplier pricing decisions by taking into consideration human behavior and emotions when facing difficult decision problems. Our analysis suggests that, contrary to conventional wisdom, whereby bounded rationality is always considered to be a performance-degrading impediment, the retailer can actually benefit from her bounded rationality. Thus, the retailer does not have to strive for perfect rationality, and moreover, she could endeavor to project on the suppliers an image of an incomplete rationality. We hope this study enlightens our views on bounded rationality, and in doing so, stimulates future research on behavioral theory in supply chain management.

The rest of this dissertation is organized as follows. In Chapter 2, we review the existing literature that is related to this dissertation. Chapter 3 studies the retailer’s ordering strategy in managing a single-product inventory with one reliable supplier and one unreliable supplier. Chapter 4 examines the retailer’s sourcing strategy and the suppliers’ pricing decisions in a supply chain consisting of one normal supplier and multiple QR suppliers. Chapter 5 considers a supplier pricing game with a bounded rational retailer, focusing on the impact of bounded rationality on the profit of each supply chain member. In Chapter 6, we summarize the contributions of this dissertation and discuss directions for future work.
Chapter 2

Literature Review

Snyder and Shen (2011) and Zipkin (2000) provide a broad overview of interesting topics in supply chain management. In this dissertation research, we are particularly interested in certain methodologies applied in supply chain management (e.g., game theory), certain types of supply chain (e.g., a supply chain with quick response capability and a supply chain subject to supply disruption), certain problems in supply chain management (e.g., supplier pricing problem), and certain characteristics of supply chain participants (e.g., the supply chain members are not perfectly rational). In this chapter, we review these topics that are closely related to this dissertation.

2.1 Game Theory in Supply Chain Management

It is not unusual in the supply chain management that the profit of a corporation is affected by the competing corporations or by the upstream suppliers. Game theory is an effective tool for analyzing the situations when the profit of an agent is affected by the decisions of other agents. As such, game theory studies the interaction of decision makers.

Game theory can be broadly divided into two categories, namely non-cooperative game and cooperative game. The non-cooperative game concentrates on the decision of each player and seeks to predict on the rational outcome of the game. The concept of the Nash Equilibrium
is widely employed to characterize a stable outcome, where no player will unilaterally deviate because such a behavior will result in a lower payoff. However, the cooperative game focuses on the behavior of groups of players, associating the action and payoff to every group, and seeks to find the stable coalition, i.e., groups of people, that will be formed. The concept of the core is applied to refer to a payoff allocation of the grand coalition such that there does not exist a coalition of players that could make all of its members at least as well off. The last twenty decades have witnessed a successful application of non-cooperative game in the field of supply chain management. An excellent review can be found in Cachon and Netessine (2004). Papers that adopt a cooperative game approach are relatively scant, but are becoming more prevalent. This emergence is probably due to the increasing prevalence of bargaining and negotiation in the supply chain. Nagarajan and Sošić (2008) provide a detailed survey of the existing literature on the cooperative game in supply chain analysis.

In the interest of the current work, we lay our attention to the application of non-cooperative games in the supply chain. When the decisions are made simultaneously, the existence and uniqueness of Nash equilibrium is of central importance. Supermodular game is a special class of game in which the players are strategically complement. Supermodular game provides a convenient tool for analyzing the comparative statics of a game. A comprehensive illustration on supermodularity theory is offered by Topkis (1998) and the application in supply chain management can be found in Cachon (2001), Netessine and Shumsky (2005) and Corbett and DeCroix (2001). A large portion of paper has been devoted to inventory competition. Parlar (1988) analyzes an inventory problem with two newsvendors offering substitutable products, where each supplier needs to decide an inventory level. Since each supplier’s demand is the original demand plus the spillover demand from its competitor, game theoretic approach is applied to analyze the existence and uniqueness of the Nash equilibrium. Lippman and McCardle (1997) examine multiple competitive newsvendors when residual demand is allocated among them. Z. Wang (2015) studies a newsvendor game in which the newsvendors may inflate their orders to compete for limited supply and considers the scenarios of without and with inventory transshipment. More
studies on inventory competition can be found in Mahajan and Van Ryzin (2001), Netessine and Rudi (2003) and Caro and Martínez-de Albéniz (2010). Pricing competition is another main stream in the literature. J. Li et al. (2010) study a single-retailer-two-supplier supply chain in which the suppliers compete for the retailer order by setting their wholesale prices. Dai et al. (2005) examine the retailers’ pricing strategies to attract customer orders under capacity constraints. Cai et al. (2009) focus on the pricing competition between the traditional retailer channel and the online direct channel and evaluate the impact of different pricing strategies.

When the players take actions sequentially, a Stackelberg game is applied and the subgame perfect Nash equilibrium (SPNE) is of great significance. In the supply chain management, the interaction between the supplier and the retailer is often modeled by a Stackelberg game. In such a game, the supplier traditionally acts as a leader, setting the wholesale price or other contract parameters, and the retailer acts as a follower, making ordering decision and sometimes setting retail price. Lariviere and Porteus (2001) study a game between the supplier and the retailer where the supplier sets the wholesale price and then the retailer makes the ordering decision based on the quoted wholesale price. An example of a Stackelberg game with the supplier choosing the stocking inventory and the retailer deciding on promotional effort level can be found in Netessine and Rudi (2004). See Lin and Parlaktürk (2012) for a Stackelberg game in the QR supply chain where the supplier sets the wholesale price and then the competing retailers place the regular orders and QR orders sequentially.

2.2 Quick Response Supply Chain

Quick response (QR) has originated from textile and apparel industry in the 1990s and since then it has become a prevalent topic in operations management. The idea of OR is simple: to postpone risky production or procurement decisions till more information on demand is collected. In this way, QR is able to reduce excessive inventory unsold at the end of the selling season.

Fisher and Raman (1996) serve as an archetype of successfully implementing a QR approach
to a fashion ski-wear firm and creates a profit of 60% compared to the existing inventory system. Eppen and Iyer (1997) examine an inventory model where dumping inventory to outlet stores is allowed. They provide an updated newsboy heuristic algorithm when the purchasing decision is only available at the beginning of the first period and the dumping decision can be made at the beginning of each period. Gurnani and Tang (1999) consider a retailer with two ordering instants before the selling season; however, the unit purchasing cost at the second instant is uncertain. They propose a nested newsvendor model for determining the optimal order quantity, discuss the value of information and provide suggestions to the retailer when to postpone the ordering decision. Choi et al. (2003) extend the cost uncertainty form to a general discrete distribution and characterize the optimal ordering policy using dynamic optimization. Fisher et al. (2001) propose a heuristic algorithm to determine the optimal initial and replenish order for a catalog retailer. Their method can also be used to optimize the reorder time and quantify the benefit of lead time reduction. J. Li et al. (2009) examine a similar framework of Fisher et al. (2001), but utilize a quite different approach. Instead of making suggestions on the optimal ordering level, J. Li et al. (2009) shed light on the optimal policy structure for three interacted decisions: the first order quantity, the timing of second order and the second order quantity. The conditions for the problem to be well-behaved is also established. Lau and Lau (1997) also consider a retailer with mid-season replenishment and develop a solution procedure in case of normal demand distribution. Fu et al. (2013) and Choi et al. (2004) consider a retailer having multiple options of expedition and facing the trade-off between cost and responsiveness. Under a Bayesian model of demand update mechanism, Milner and Kouvelis (2002) further examine the best timing of the second order. Several scenarios (demand level, lead time length, demand uncertainty level) are discussed to evaluate the value of information and production timing flexibility. Milner and Kouvelis (2005) study how the value of production quantity flexibility and production timing flexibility is affected by product demand characteristics.

The aforementioned papers assume that there is only one decision maker in the model, i.e., the retailer endowed with QR capability, but do not take into account the interactive relation-
ship between the supplier and the retailer. Iyer and Bergen (1997) study the impact of QR on a two-tier supply system and find that QR benefits the retailer but can be detrimental to the supplier. They further design several contracts such as higher service levels, wholesale price and volume commitments so as to make QR a Pareto improving strategy. Donohue (2000) considers a two-stage newsvendor model with a general demand update rule, and proposes a buyback contract that motivates the retailer to order at a level that coordinates the supply chain. Lariviere and Porteus (2001) consider a manufacturer selling to a newsvendor and develop the regularity conditions, satisfied by many common distributions, for the manufacturer’s decision to be analytically tractable. Weng (2004) examines a QR supply system consisting of one manufacturer and one retailer, where the manufacturer can dictate its wholesale prices. Their focus is channel coordination and a quantity discount policy is proposed to induce the buyer to order the coordinated quantity. Cachon and Swinney (2011) study the interaction between the retailer and the consumers: Consumers decide whether to buy now or wait for the clearance, based on the regular price and their conjecture on the discounted price and the possibility their desired item will last to the clearance. They show that QR helps blunt consumers’ strategic behavior of intentionally waiting for markdowns, because the retailers with QR capability are able to reduce excessive inventory left and therefore reduce the occurrence of markdowns.

Successful implementation of QR requires accurate interpretation of historical demand data and newly collected demand signals. Depending on how market demand is updated, the literature on forecasting and inventory can be divided into three categories: Bayesian based method, Markovian (Martingale) forecast revision and Time series approach. The first approach assumes that the vendor knows demand distribution but is uncertain about specific parameters, and the Bayesian update mechanism is applied to predict on the future demand (see, e.g., H. Scarf, 1959; H. E. Scarf, 1960; Iglehart, 1964; Azoury, 1985; Lovejoy, 1990). The second approach assumes that the demand is modulated by some Markov (Martingale) process. Song and Zipkin (1993) model the demand as a Markov-modulated Poisson process and Heath and Jackson
(1994) innovate a general probabilistic model of demand evolution process, i.e., the Martingale model of forecast evolution (MMFE). The time series approach assumes that demand follows some classical time series (e.g., auto-regressive moving-average process) in which correlation exists among consecutive demand realizations; see Veinott Jr (1965), Johnson and Thompson (1975), and Aviv (2003).

2.3 Supply Disruption

The disruptions in supply process can be attributed to natural factors (such as earthquakes, floods, storms, etc) and intentional or unintentional human actions (such as machine breakdowns, strikes, financial defaults, etc). Recently, an increasing number of scholars have shifted their attention to the randomness in the supply process, resulting in the proliferation of publications. Snyder et al. (2012) provide a comprehensive review of OR/MS models for supply chain disruption.

In the literature, models of uncertainties in supply process can be divided into the following four categories: random yield, all-or-nothing delivery, random lead times and supplier unavailability.

Random yield assumes only a portion of the order is received or a fraction of the order is defective. I refer the readers to Yano and Lee (1995) for a comprehensive review of supply uncertainty with random yield. Federgruen and Yang (2009) analyze a supplier selection and order allocation problem given the set of potential suppliers with random yield factor with a general probability distribution. Two approximation algorithms, based on large-deviations technique and central limit theorem, respectively, are developed, and their asymptotic behaviors are analyzed. Gurnani et al. (2000) consider an assembly system using two components subject to production yield losses, and study the combined ordering and assembly decisions.

All-or-nothing delivery is a special form of random yield, with the successfully delivered fraction being either 0 or 1. Özekici and Parlar (1999) model the status of the supply chain by a Markov chain and the uncertainty is all-or-nothing delivery. Under an assumption on the
fixed cost, an environment-dependent \((s, S)\) policy is shown to be optimal. Model I of Anupindi and Akella (1993) considers a retailer facing a continuous random demand and procuring from two uncertain suppliers with all-or-nothing delivery. It is shown that the optimal ordering policy is no other than three cases: order nothing, order from the less reliable supplier with a lower cost, order from both suppliers. Swaminathan and Shanthikumar (1999) study the same problem but consider discrete demand distribution, and find a different result: ordering solely from the more reliable but more expensive supplier can be optimal under some cases. Babich et al. (2007) study the scenario when the suppliers’ disruption are correlated and allow the suppliers to control their wholesale prices.

Lead time uncertainty is characterized by a random lead time distribution. Model III of Anupindi and Akella (1993) considers delayed delivery as a special form of lead time uncertainty by assuming that the orders not successfully delivered by the unreliable supplier in the current period will definitely arrive in the next period. More discussion on lead time uncertainty can be found in Bagchi et al. (1986), Song (1994), and Y. Wang and Tomlin (2009).

For supplier unavailability, the supplier’s status can go from up to down time to time and orders placed when the supplier is down cannot be delivered successfully. Parlar and Perry (1996) analyze two identical unreliable suppliers in a continuous-review context, where the durations of up/down periods are exponentially distributed. Gürler and Parlar (1997) extend the work of Parlar and Perry (1996) by considering Erlang-\(k\) distributed availability durations and general unavailability durations.

These four categories are capable of capturing a wide range of realistic disruption characteristics. Under this context, Song and Zipkin (1996) propose a model that allows for a general form of uncertainty in the supply system including random lead times, all-or-nothing delivery, and age-dependent deliveries. They show that the optimal policy has the same \((s, S)\) structure as in standard models with a fixed ordering cost, but the parameters are variant to reflect current supply condition.
2.4 Supplier Pricing

In many literatures, it is assumed that the wholesale price that the retailer pays to the supplier for each unit of product purchased is exogenous. Supplier pricing allows the supplier to set the wholesale price and accounts for the interactive relationship between the retailer and the supplier. In the one-supplier-one-retailer supply chains, most of the research that allows for supplier pricing is devoted to designing and constructing different classes of pricing policies, i.e., contracts, to mitigate the double marginalization that leads to supply chain inefficiency. We refer the readers to surveys by Anupindi and Bassok (1999), Lariviere (1999) and Cachon (2003).

When there are multiple retailers or suppliers, supplier pricing can generate very interesting problems due to the coexistence of horizontal retailer (supplier) competition and the vertical supplier-retailer interaction. L. Li and Zhang (2008) consider a supply chain of one supplier and multiple retailers with information sharing: each retailer receives private information on demand and shares with the supplier, and then the supplier determines the wholesale price based on shared information. It is found that under confidentiality (the supplier keeps the information to herself and do not share with other retailers), all retailers have incentives to engage in information sharing, which generates in a lower equilibrium price, and the supply chain profit is also improved. Dong and Rudi (2004) examine the impact of transshipment on the supplier and the retailers considering both exogenous and endogenous wholesale prices. They show that the supplier benefits from the retailers’ transshipment by charging a higher wholesale price, while the retailers are often worse off. Babich et al. (2007) examine a supply chain with one retailer and multiple risky suppliers. Scholars generally believe that the retailer prefers ordering from suppliers with negative default correlation in order to reap a higher diversification benefit. Reversing the traditional intuition, Babich et al. (2007) show that when the wholesale prices are determined by the suppliers endogenously, the competition among the suppliers—which lowers equilibrium prices and therefore benefits the retailer—increases as the default correlation increases, and consequently the retailer prefer a high default correlation. In
the same spirit, Calvo and Martínez-de Albéniz (2015) study a retailer sourcing potentially from one normal supplier and one QR supplier, and shows that different from general wisdom, the retailer would prefer single sourcing to dual sourcing when the suppliers take pricing decisions.

2.5 Bounded Rationality in Supply Chain Management

As demonstrated by empirical experiments, human decision makers do not necessarily make the best choice as theory suggests. In fact, people intend to be rational, but due to cognitive constraint or limited information, they fail in the decision making process. Bounded rationality provides a notion that the decision maker may not be perfectly rational to make the choice that yields the highest level of benefits.

The seminal work of Simon (1955) suggests that instead of performing an exhaustive search, the decision maker will settle for something satisfactory, but less than perfect. Rubinstein (1998) surveys the cognitive limitation that leads to the inherent imperfection of human decision making. Another stream of research has focused on heuristics or rule of thumb (see, e.g., Tversky & Kahneman, 1974), which navigates people in complex decision making process. The review on evolution and development of bounded rationality can be found in Conlisk (1996) and Simon (1982). Boudreau et al. (2003), Bendoly et al. (2006), Loch and Wu (2007), and Gino and Pisano (2008) provide good surveys on behavioral issues in operation management.

The behavioral issue has embraced fruitful applications in economics, finance and marketing but has received relatively less attention in supply chain management. Here we review some relevant works in supply chain management that adopt a behavioral lens. In forecasting, many companies rely extensively on human judgment on historical data to make planning decisions. Lawrence and O’connor (1995) find that people tend to anchor on the latest history value and make insufficient adjustment. Some of the widely known judgmental biases and heuristics in time series forecasting are investigated by Lawrence and O’Connor (1992).

The newsvendor problem is one of the cornerstones of inventory theory. Empirical studies show that decision makers systematically deviate from the optimal newsvendor order fractile
derived by theoretical results. Su (2008) suggests that such behavior can be explained by incorporating “noise” into the newsvendor’s decision process, such that the optimal solution is stochastically preferred by the newsvendor, but not in a deterministic manner. Schweitzer and Cachon (2000) are inclined to the explanation that the newsvendors gain some utility from reducing the left-over inventory, and they anchor their decisions upon the demand average and do not adjust sufficiently towards the optimal ordering quantity.

Prospect theory represents another main stream of modeling bounded rationality in supply chain management. According to prospect theory, the decision maker subconsciously forms a reference point and measures the loss more significantly compared with the same magnitude of gain. As such, prospect theory suggests that utility not only generates from accumulating wealth, but also from losing and gaining wealth. C. X. Wang and Webster (2009) examine a risk-averse newsvendor and find that he/she will order more (less) than a risk-neutral newsvendor when the shortage cost is high (low). An inventory management with two suppliers under supply disruption is studied through a prospect theory lens by Giri (2011). Ma et al. (2012) analyze a QR supply chain with two ordering opportunities and information updates when the buyer is loss-averse. See Wu et al. (2010) for the impact of risk-aversion on manufacturer’s optimal policy in supply contracts. Popescu and Wu (2007) study a dynamic pricing problem when consumer demand is dependent on the pricing history. Prospect theory reflects in how the consumers perceive gains and losses relative to previous prices.

When individuals interact, more psychological issues arise in addition to the cognitive limitation and bias in individual decision making. Normative approach assumes that people care about only monetary payment; however, in reality, many people care not only how much they earn, but also the amount other people earn. People gain disutility when their payoffs are lower than the others’ and when their payoffs are much higher than the others’. Fehr and Schmidt (1999) model the material payoff and equality consideration as a linear function and verify the consistency between theoretic predictions and empirical experiments. Kirchsteiger (1994) incorporates envy into the bargaining model and find it explain experimental anomalies well.
Chapter 3

Optimal Policy for a Dual-Supplier System under Disruption

3.1 Introduction

An average U.S. retailer spends half of its revenue on procuring merchandise and service (Beil, 2010). Procurement cost (including unit cost and fixed ordering cost), product quality and speed of delivery are among the main factors that determine the selection of suppliers. Most of the existing literature has assumed that retailers source from a single supplier, citing benefits such as better price through high volumes, higher quality through continuous improvement and construction of strong and long-term relationship (Goffin et al., 1997). However, retailers employing a single sourcing strategy are susceptible to supply disruptions, in which case companies may not receive the desired quantity at the predetermined time or some of the received order may be defective. These disruptions in the procurement process can result from various reasons such as manufacturers’ machine breakdowns, shortage of raw materials, strikes, bad weather, traffic delays, etc. The implementation of lean manufacturing and just-in-time philosophies has increased a supply chain’s vulnerability to risk disruptions. For instance, just-in-time philosophy keeps little buffer inventory that may not be enough to compensate the impact of any
supply disruption. Additionally, as companies outsource a greater proportion of their manufactured products from low-cost countries that are politically and economically less stable, they become more prone to disruptions.

To mitigate the negative effects of supply disruptions, the simultaneous involvement of two sources in geographically different locations may turn out to be more economical. Some companies employ a form of dual sourcing such that the firm sources exclusively from one supplier when that supplier is available but reroutes to the backup supplier during disruptions (Tomlin, 2006). In addition to providing a back-up source in case of emergency, multiple sourcing is favored by firms for a variety of strategic reasons, such as maintaining competition or meeting customer volume requirements. In particular, a small base of two or three suppliers is favored by firms to reap the benefits of both single sourcing and multiple sourcing yet avoid the disadvantages of purely single sourcing. The selection of suppliers and the optimal size of the supply base is not the emphasis of this work. Instead we are interested in providing insight into the optimal sourcing strategy given a set of two suppliers with different costs and disruption probabilities. Research questions emerge when a supplier, who operates at a high reliability level, but has cost. Sourcing solely from this premium supply agent is expensive and often a non-optimal strategy. On the other hand, due to supply interruptions, purchasing exclusively from the less expensive supplier can likewise be costly.

In this chapter, we consider one retailer who has to decide how it could best utilize two suppliers by properly allocating its order quantity so as to reduce its expected costs. One supplier is perfectly reliable (able to fulfill the amount ordered with certainty at the prearranged time) while the other is not but instead is more economical. Here we define more economical as having either or both of fixed ordering cost and unit ordering cost less than those of the reliable supplier. The unreliable supplier’s state can be either up or down, and we assume that the duration of both the up period and down period are geometrically distributed in our discrete-time model. Therefore, the supply process can be modeled as a discrete-time two-state Markov chain, whose parameters are referred to as the disruption and recovery probability. It is
assumed that the order is either delivered in full or it is canceled depending on its status. This all-or-nothing delivery is valid when, for example, there is a quality problem in production, the production process is interdicted by strike, products are destroyed entirely by fire, or perishable goods deteriorate. The unreliable supplier does not accept any orders when it is down. The supply availability at the beginning of the period does not imply the successful delivery of the products since the state of the unreliable supplier can change from up to down after the order has been made. In this case, the entire order is canceled. Fixed ordering cost is incurred when an order is placed, regardless of whether the order is delivered or not. Unit purchase cost is charged only for the order that is actually delivered.

In this chapter, we first consider a finite-horizon model, where dynamic programming (DP) techniques are used to find the optimal rule for selecting the order quantity for each period given any possible initial inventory level and supply availability status. We apply the concept of $K$-convexity to prove that the optimal policy has an $(s, S)$-like structure. Under an $(s, S)$ policy, an order is placed to raise the inventory level to $S$ when the inventory level reaches or drops below $s$. We start with the analysis of a single period model and demonstrate by mathematical induction that under certain technical conditions, the $(s, S)$-like policy remains optimal for any finite-horizon model. We further refine the proposed $(s, S)$-like policy into four cases depending on the order of the indebted $s, S$ values. The four cases are referred to as

- Case U: order exclusively from the unreliable supplier;
- Case EOB (either or both): either order from both suppliers or order from the unreliable supplier depending on the inventory level;
- Case ENB (either, not both): order from either the reliable supplier or from the unreliable supplier, but not both, based on the inventory level;
- Case R: order exclusively from the reliable supplier.

We show that the optimal policy depends greatly on system parameters. When the parameters change in favor of the unreliable supplier, for instance, disruption probability decreases, reliable
unit cost increases, or backorder cost reduces, the optimal case evolves from Case R to Case ENB to Case EOB, and finally to Case U. Particularly, we are able to distinguish Case U from the remaining cases by a simple computation on the parameters. Next, we extend to an infinite-horizon setting and verify the convergence of the total optimal cost. The convergence of the optimal policy follows from the convergence of the optimal cost, and the resulting limiting policy serves as the optimal ordering policy for the infinite-horizon framework. Characterization of the optimal policy is important because it confines the optimal policy to a certain boundary that facilitates the designing of either exact or heuristic algorithms. In light of the previous literature that focuses on inventory problems that exhibit an \((s, S)\) policy, this is, to the best of our knowledge, the first paper that provides theoretical proof on the optimality of \((s, S)\)-like policy for a multi-period inventory system with two suppliers under supply disruption.

The optimality of an \((s, S)\) policy for a class of inventory problems with a single supplier, single item, random periodic demand and a fixed cost is one of the fundamental results of stochastic inventory theory. The finite-horizon problem is first addressed by H. Scarf (1960) by showing inductively that the \(n\)-period cost \(f_n(\ast)\) is \(K\)-convex. Scarf assumes the penalty function (the sum of holding cost and stock-out cost) to be convex. Veinott (1966) extends the result and points out that the penalty function needs only to be quasi-convex. The proof for the infinite-horizon model is a bit complex. Basically there are two methods to approach this proof. One, which employs the results of finite-horizon setting and the convergence of the \(n\)-period cost \(f_n(x)\) to a limit function \(f(x)\), is established by Iglehart (1963). The other resorts to the renewal theory and relies on the formula of the average cost associated with a given \((s, S)\) policy(Veinott Jr & Wagner, 1965). The optimality proof was elegantly developed by Zheng (1991) by demonstrating that the discounted cost associated with a given \((s, S)\) policy satisfies the optimality function. After the establishment of the optimal \((s, S)\) policy, a host of papers have been dedicated to finding the \(s\) and \(S\) values on the two-dimensional grid, see e.g., Veinott Jr and Wagner (1965) and Zheng and Federgruen (1991). Porteus (1971) examines an inventory model with a concave increasing ordering cost function that encodes quantity discount.
and verifies the optimality of a generalized \((s, S)\) policy through introducing the quasi-\(K\)-convex function. Instead of considering the demand as a pure random variable (most models consider an i.i.d. distribution), Sethi and Cheng (1997) assume that the demand distribution is dependent on a Markov chain, and show that the optimal policy is a state dependent \((s, S)\) policy when there is a fixed ordering cost.

One common strategy to mitigate the negative effects of uncertainty in the supply system is to order from multiple suppliers. The literature on multiple sourcing has focused on the selection of suppliers and determination of order quantity from each supplier. Dada et al. (2007) consider the selection of unreliable suppliers in a newsvendor setting. Dual sourcing is particularly important because it preserves the advantage of single sourcing and greatly reduces risks. Fox et al. (2006) consider trade-offs between a supplier with a lower variable cost but nonzero fixed cost and a supplier with high variable cost and negligible fixed cost. The resulting optimal policy can be generalized into three cases: a base-stock policy for ordering from the high variable cost supplier, an \((s_{LVC}, S_{LVC})\) policy for purchasing exclusively from the low variable cost supplier and a hybrid \((s, S_{HVC}, S_{LVC})\) policy for buying from both suppliers. The result holds for both lost sales and back-order cases. Sethi et al. (2003) consider an inventory system with fast and slow delivery modes (the fast mode is assumed to be more expensive than the slow mode). With the existence of fixed ordering cost and demand information update, a forecast-update-dependent \((s, S)\) type policy is proved to be optimal. However, there is no unreliability issue addressed in the above literature.

There is little in the literature that discusses multiple suppliers with unreliable behavior, especially when there is a fixed ordering cost. Anupindi and Akella (1993) consider purchasing from two unreliable suppliers. Both single-period and multiple-period problems are considered and three modes of disruption risk, including all-or-nothing delivery, random yield, random lead times, are discussed. Swaminathan and Shanthikumar (1999) extend Anupindi and Akella (1993) work by considering discrete demand distribution. However, there is no fixed cost in both of their models, resulting in a convex cost function and a base-stock like optimal policy.
Y. Wang (2012) investigates the optimal policy of ordering from two unreliable suppliers. Taking into account different configurations of the fixed ordering cost including overall, placement and receipt costs, Wang develops the characteristics of optimal policy and its dependence on system parameters. However, only single-period problems are discussed. Ahiska et al. (2013) analyze an inventory model in which a retailer replenishes from one reliable supplier and one unreliable supplier. Through abundant numerical experiments solved by a corresponding Markov Decision Process (MDP), they report that the optimal policy appears to be an \((s, S)\) policy when the unreliable supplier is down, and generalize the policy into four cases when the unreliable supplier is up, depending upon problem parameters.

The rest of this chapter is organized as follows: We state our model, list the notation and provide definitions in §3.2, and prove the optimality of \((s, S)\)-like ordering policy under a finite horizon framework in §3.3. An extension to the infinite horizon scenario is studied and the convergence of the optimal cost function and the optimal policy is presented in §3.4. In §3.5, through computational experiments, the optimality of the proposed policy is verified for a range of cost configurations and different supply reliability levels. Conclusions are provided in §3.6.

### 3.2 A Finite Horizon Model

We consider an inventory control system which is reviewed periodically over a finite horizon. The system consists of one perfectly reliable supplier with higher cost and one unreliable supplier who is more economical, i.e. having either or both of fixed ordering cost and unit ordering cost less than those of the reliable supplier. The reliable supplier always delivers the amount identical to the amount ordered. The unreliable supplier is either functioning normally, corresponding to an up state, or disrupted, referred to as the down state. When the unreliable supplier is up, it behaves like a perfectly reliable supplier and delivers the desired order, but when it is down, the entire order is canceled. The probability that the unreliable supplier will be up in the next period depends only on whether it is in an up state or down state currently. Therefore, the unreliable supplier’s supply process can be modeled as a two-state Markov chain. No order can
be placed on the unreliable supplier when it is down at the beginning of the period. In this case, the only option for the retailer is to order from the reliable supplier. Even if the unreliable supplier is up at the beginning of the period, it does not guarantee that the placed order will be successfully delivered. It is possible that the unreliable supplier goes from up to down after the order is placed, and consequently, the active order is canceled. The retailer needs to decide, based on its inventory level, how much to order and how to distribute the order.

Fixed ordering cost, assumed to be the overhead cost of generating an order, is considered. For each supplier, if an order is placed, a fixed ordering cost is charged whether the order is delivered or not. Unit ordering cost, assumed to be linear in the order quantity, is incurred for the products that are finally received. A unit holding cost is incurred for each unit held in stock at the end of a period. There is a unit penalty cost when the retailer does not have enough inventory to satisfy the demand. It is assumed that fulfilled orders from both suppliers arrive before demand is realized. The retailer fulfills its stochastic demand from the market with on-hand inventory and backlogs any unsatisfied demand. The events in each period are shown in Figure 3.1. Since we consider a finite-horizon model, the overall schedule repeats the daily events.

![Timeline for one period](image-url)

**Figure 3.1: Timeline for one period**
The following notation is introduced:

- $q_r$: order quantity on the reliable supplier;
- $q_u$: order quantity on the unreliable supplier;
- $c_r$: unit ordering cost of the reliable supplier;
- $c_u$: unit ordering cost of the unreliable supplier;
- $K_r$: fixed ordering cost of the reliable supplier;
- $K_u$: fixed ordering cost of the unreliable supplier;
- $\alpha$: probability that the unreliable supplier remains up in the next period when its current state is up, $0 < \alpha < 1$;
- $\beta$: probability that the unreliable supplier becomes up in the next period when its current state is down, $0 < \beta \leq 1$;
- $W$: transition probability matrix for the status of the unreliable supplier, where state 0 is the up state and state 1 is the down state;
- $b$: backorder cost per item per period, $b \geq c_r$;
- $h$: holding cost per item per period;
- $D$: demand distribution, with $f(\cdot)$ being its pdf and $F(\cdot)$ being its cdf;
- $E_D[\cdot]$: expected value w.r.t $D$; and
- $\eta$: period discount factor, $0 < \eta \leq 1$.

It is assumed that all the above costs are non-negative. Demand in each period is independently and identically distributed. The demand and cost parameters are stationary, i.e., do not vary over the time. The value $1 - \alpha$ is referred to as the disruption probability and $\beta$ is the
recovery probability. The assumption \( b \geq c_r \) is made to avoid trivial cases.

The problem can be formulated as a stochastic dynamic programming (DP). The state of the system is characterized as the tuple \((X, J)\), where \(X\) is the initial inventory at the beginning of each period and \(J\) is the state of the unreliable supplier (0-up, 1-down). The retailer’s inventory level is limited to a range \([I_{\text{min}}, I_{\text{max}}]\), where \(I_{\text{max}}\) represents the storage capacity of the retailer and \(I_{\text{min}}\) may be a negative number, indicating the maximum backorder level allowed by the retailer. These limits on the range of the inventory level can be set arbitrarily large (positively or negatively) to reflect unlimited inventory or backorders yet restrict the number of states such that the problem is not excessively difficult computationally. Let \(\theta^j_t(x)\) denote the total discounted cost of operating \(t\) periods when the optimal policy is implemented in each period, given \(X = x\) and the unreliable supplier’s status \(J = j\). The optimal discounted cost of a \(t\)-period problem when the unreliable supplier is up becomes

\[
\theta^0_t(x) = \min_{q_u \geq 0, q_r \geq 0} \{ K_r \delta(q_r) + K_u \delta(q_u) + c_r q_r + \alpha c_u q_u + \alpha g(x + q_r + q_u) + (1 - \alpha) g(x + q_r) \\
+ \alpha \eta \mathbb{E}_D[\theta^0_{t-1}(x + q_r + q_u - D)] + (1 - \alpha) \eta \mathbb{E}_D[\theta^1_{t-1}(x + q_r - D)]\},
\]

(3.1)

and the corresponding cost when the unreliable supplier is down becomes

\[
\theta^1_t(x) = \min_{q_r \geq 0} \{ K_r \delta(q_r) + c_r q_r + g(x + q_r) + \beta \eta \mathbb{E}_D[\theta^0_{t-1}(x + q_r - D)] \\
+ (1 - \beta) \eta \mathbb{E}_D[\theta^1_{t-1}(x + q_r - D)]\},
\]

(3.2)

where \(g(\cdot)\) is the loss function, defined below as the sum of inventory cost and backorder cost:

\[
g(y) = h \int_0^y (y - \xi) f(\xi) d\xi + b \int_y^\infty (\xi - y) f(\xi) d\xi.
\]
\( \delta(x) \) is defined over \([0, \infty)\) as 
\[
\delta(x) = \begin{cases} 
0 & \text{if } x = 0 \\
1 & \text{if } x > 0 
\end{cases}
\]. When \( t = 1 \), there is no future cost, so we arbitrarily assume there is not salvage value for residual inventory and so we define 
\[ \theta_0^t(x) = 0, \forall x \in [I_{\min}, I_{\max}], j \in \{0, 1\}. \]

At the beginning of each period, the retailer observes its inventory level and must decide the amount to order from each supplier. Instead of using the order quantities \( q_r \) and \( q_u \), it is convenient to recast the decision variables to order-up-to levels \( y_r \) and \( y_u \). In practice, the orders on the two suppliers are placed simultaneously. However, for analytical tractability, we consider first ordering from the reliable supplier and so define \( y_r \) as the inventory position after ordering from the reliable supplier and \( y_u \) as the inventory position after ordering from the unreliable supplier under the assumption that the order is delivered successfully. Given the inventory level at the beginning of the period \( X = x \), the following relationship between order quantities and order-up-to levels holds:

\[ y_r = x + q_r, \quad y_u = x + q_r + q_u. \]

The cost function can be reformulated as

\[
\theta_0^t(x) = \min_{y_r \geq x} \{K_r \delta(y_r - x) + K_u \delta(y_u - y_r) + c_r y_r + \alpha c_u (y_u - y_r) + \alpha g(y_u) \\
+ (1 - \alpha) g(y_r) + \alpha \eta E_D[\theta_0^{t-1}(y_u - D)] + (1 - \alpha) \eta E_D[\theta_1^{t-1}(y_r - D)]\} - c_r x; \quad (3.3)
\]

\[
\theta_1^t(x) = \min_{y_r \geq x} \{K_r \delta(y_r - x) + c_r y_r + g(y_r) + \beta \eta E_D[\theta_0^{t-1}(y_r - D)] \\
+ (1 - \beta) \eta E_D[\theta_1^{t-1}(y_r - D)]\} - c_r x. \quad (3.4)
\]

The objective is to choose the order quantities from the reliable and unreliable supplier over the time so as to minimize the cost of the entire time horizon, given any initial inventory level \( x \) and supply availability status \( J \).
3.3 An \((s, S)\)-like Optimal Ordering Policy

In this section, we propose an \((s, S)\)-like policy as follows: when the unreliable supplier is up, order first from the reliable supplier according to an \((s^0_r, S^0_r)\) policy and then order from the unreliable supplier according to an \((s^0_u, S^0_u)\) policy, henceforth referred to as an \((s^0_r, S^0_r, s^0_u, S^0_u)\) policy; moreover, when the unreliable supplier is down, order from the reliable supplier based on an \((s^1_r, S^1_r)\) policy. Optimality of this policy is proven by mathematical induction. We start with a single period case and show the optimal policy in Lemma 3.3.1. Then, for any finite horizon of more than one period, the optimality of an \((s^1_r, S^1_r)\) policy when the unreliable supplier is down and an optimal \((s^0_r, S^0_r, s^0_u, S^0_u)\) policy when the unreliable supplier is up are addressed in Lemma 3.3.2 and Lemma 3.3.3, respectively. Furthermore, we obtain the condition when \(s^0_r\) and \(S^0_r\) do not exist, as presented in Proposition 3.3.4.

To make the objective analytically tractable, we rewrite Eq. (3.3), which is the cost function of \(t\) periods when the unreliable supplier is available, as

\[
\theta^0_t(x) = \min_{y_r \geq x} \{K_r \delta(y_r - x) + (c_r - \alpha c_u) y_r + (1 - \alpha) g(y_r) + (1 - \alpha) \eta E_D[\theta^1_{t-1}(y_r - D)] \} - c_r x. \tag{3.5}
\]

For ease of representation, we define the following expressions:

\[
m^u_t(y_u) = \alpha c_u y_u + \alpha g(y_u) + \alpha \eta E_D[\theta^0_{t-1}(y_u - D)]; \tag{3.6}
\]

\[
L_t(y_r) = \min_{y_u \geq y_r} \{K_u \delta(y_u - y_r) + m^u_t(y_u)\}; \tag{3.7}
\]

\[
m^r_t(y_r) = (c_r - \alpha c_u) y_r + (1 - \alpha) g(y_r) + (1 - \alpha) \eta E_D[\theta^1_{t-1}(y_r - D)] + L_t(y_r). \tag{3.8}
\]

Similarly, we reorganize the cost function when the unreliable supplier is unavailable and denote the inner minimization excluding the fixed cost of Eq. (3.4) as

\[
m^1_t(y_r) = c_r y_r + g(y_r) + \beta \eta E_D[\theta^0_{t-1}(y_r - D)] + (1 - \beta) \eta E_D[\theta^1_{t-1}(y_r - D)]. \tag{3.9}
\]
Thus, the dynamic cost function can be rewritten as

\[ \theta_0^t(x) = \min_{y_r \geq x} \{ K_r \delta(y_r - x) + m_{r}^t(y_r) \} - c_r x, \quad (3.10) \]
\[ \theta_1^t(x) = \min_{y_r \geq x} \{ K_r \delta(y_r - x) + m_{1}^t(y_r) \} - c_r x. \quad (3.11) \]

Let \( x \) be the inventory level at the beginning of a period and denote \( \tilde{y}_r^0(x) \) and \( \tilde{y}_u^0(x) \) as the functions that minimize (3.3), and \( \tilde{y}_r^1(x) \) as the function that minimizes (3.4). We are now ready to analyze the optimal policies in each period.

**Lemma 3.3.1.** In a single period model, if \( g(y) \) is convex and \( K_r \geq K_u \), then we have the following results:

(i) when the unreliable supplier’s state is down:

\[ \tilde{y}_r^1 = \begin{cases} S_{1,r}^1, & \text{if } x \leq s_{1,r}^1, \\ x, & \text{otherwise}, \end{cases} \quad (3.12) \]

with \( S_{1,r}^1 \) being the minimizer of \( m_{r}^1(y_r) \) and \( s_{1,r}^1 < S_{1,r}^1 \).

(ii) when the unreliable supplier’s state is up:

\[ \tilde{y}_r^0 = \begin{cases} S_{1,r}^0, & \text{if } x \leq s_{1,r}^0, \\ m_{1}^t(y_r) \text{ has a minimum}, \\ x, & \text{otherwise}, \end{cases} \quad (3.13) \]

\[ \text{otherwise}, \]

where \( S_{1,r}^0 \) is the minimizer of \( m_{r}^1(y_r) \), if exists and \( s_{1,r}^0 < S_{1,r}^0 \);

\[ \tilde{y}_u^0 = \begin{cases} S_{1,u}^0, & \text{if } x \leq s_{1,u}^0, \\ \tilde{y}_r^0, & \text{otherwise}, \end{cases} \quad (3.14) \]

where \( S_{1,u}^0 \) is the minimizer of \( m_{u}^1(y_r) \) and \( s_{1,u}^0 < S_{1,u}^0 \).

(iii) \( \theta_0^t(x) \) and \( \theta_1^t(x) \) are both \( K_r \)-convex.
Proof. (i) For a single period model, when the unreliable supplier is down, \( m_1(y_r) = c_r y_r + g(y_r) \), and \( \lim_{y_r \to -\infty} c_r y_r + g(y_r) \to \infty \) since \( b > c_r \). Therefore, according to Lemma A.1.3, if \( g(y) \) is convex, there exists \( s_{1,r}^1 \) independent of \( x \) such that

\[
\theta_1^1(x) = \min_{y_r \geq x} \{ K_r \delta(y_r - x) + m_1^1(y_r) \} - c_r x
\]

\[
= \min_{y_r \geq x} \{ K_r \delta(y_r - x) + c_r y_r + g(y_r) \} - c_r x
\]

\[
= \begin{cases} 
K_r + c_r S_{1,r}^1 + g(S_{1,r}^1) - c_r x, & \text{if } x \leq s_{1,r}^1, \\
g(x), & \text{if } x > s_{1,r}^1.
\end{cases} 
\]  

(3.15)

Eq. (3.15) implies that when \( x \leq s_{1,r}^1 \), it is best to order up to \( S_{1,r}^1 \), and when \( x > s_{1,r}^1 \), it is best not to make any order. Hence, Eq. (3.12) follows.

(ii) When the unreliable supplier is up, we have \( L_1(y_r) = \min_{y_u \geq y_r} \{ K_u \delta(y_u - y_r) + \alpha c_u y_u + \alpha g(y_u) \} \). Since \( \alpha c_u y_u + \alpha g(y_u) \) is convex and \( \lim_{y_u \to -\infty} \alpha c_u y_u + \alpha g(y_u) \to \infty \), there exist \( s_{1,u}^0 \) and \( S_{1,u}^0 \) such that

\[
L_1(y_r) = \begin{cases} 
K_u + \alpha c_u S_{1,u}^0 + \alpha g(S_{1,u}^0), & \text{if } y_r \leq s_{1,u}^0, \\
\alpha c_u y_r + \alpha g(y_r), & \text{if } y_r > s_{1,u}^0.
\end{cases} 
\]  

(3.16)

Hence, we have Eq. (3.14). By Lemma A.1.3, it can be shown that \( L_1(y_r) \) is \( K_u \)-convex in \( y_r \), so is \( K_r \)-convex in \( y_r \). Combining with the fact that \( (c_r - \alpha c_u) y_r + (1 - \alpha) g(y_r) \) are both convex, we know \( m_1^r(y_r) = (c_r - \alpha c_u) y_r + (1 - \alpha) g(y_r) + L_1(y_r) \) is \( K_r \)-convex. If \( \lim_{y_r \to -\infty} m_1^r(y_r) \to \infty \), we have

\[
\theta_1^r(x) = \min_{y_r \geq x} \{ K_r \delta(y_r - x) + (c_r - \alpha c_u) y_r + (1 - \alpha) g(y_r) + L_1(y_r) \} - c_r x
\]

\[
= \begin{cases} 
K_r + (c_r - \alpha c_u) S_{1,r}^0 + (1 - \alpha) g(S_{1,r}^0) + L_1(S_{1,r}^0) - c_r x, & \text{if } x \leq s_{1,r}^0, \\
(c_r - \alpha c_u) x + (1 - \alpha) g(x) + L_1(x) - c_r x, & \text{if } x > s_{1,r}^0.
\end{cases} 
\]  

(3.17)
Otherwise, \( m_1^r(y_r) \) does not have a minimum and, consequently,

\[
\theta_1^0(x) = (c_r - \alpha c_u)x + (1 - \alpha)g(x) + L_1(x) - c_r x, \quad \forall x.
\]

(3.18)

Eq. (3.17) and Eq. (3.18) directly imply Eq. (3.13).

(iii) Lemma A.1.2 (iv) implies that \( \theta_0^1(x) \) is \( K_r \)-convex by definition. The \( K_r \)-convexity of \( \theta_1^1(x) \) follows the same logic.

\[\Box\]

Lemma 3.3.2. In the finite horizon model, if \( \theta_0^1(x) \) and \( \theta_1^1(x) \) are both \( K_r \)-convex for \( t = 2, 3, \cdots, T \), then

(i) there exist numbers \( s_{t,r}, S_{t,r} \) with \( s_{t,r} < S_{t,r} \) such that

\[
\bar{y}_{t,r}^1 = \begin{cases} 
S_{t,r}^1, & \text{if } x \leq s_{t,r}; \\
x, & \text{otherwise.}
\end{cases}
\]

(ii) \( \theta_1^1(x) \) is \( K_r \)-convex.

**Proof.** (i) Because \( \theta_0^1(x) \) and \( \theta_1^1(x) \) are \( K_r \)-convex, \( m_1^r(y_r) = c_r y_r + g(y_r) + \beta \eta E_D[\theta_0^1(y_r - D)] + (1 - \beta) \eta E_D[\theta_1^1(y_r - D)] \) is \( \eta K_r \)-convex. Besides, since \( \lim_{y_r \rightarrow -\infty} m_1^r(y_r) > \lim_{y_r \rightarrow -\infty} c_r y_r + g(y_r) \rightarrow \infty \), we have

\[
\theta_1^1(x) = \min_{y_r, \beta x} \{ K_r \delta(y_r - x) + c_r y_r + g(y_r) + \beta \eta E_D[\theta_0^1(y_r - D)] + (1 - \beta) \eta E_D[\theta_1^1(y_r - D)] \} - c_r x
\]

\[
= \begin{cases} 
K_r + c_r (S_{t,r}^1 - x) + g(S_{t,r}^1) + \beta \eta E_D[\theta_0^1(S_{t,r}^1 - D)] + (1 - \beta) \eta E_D[\theta_1^1(S_{t,r}^1 - D)], & \text{if } x \leq s_{t,r}^1; \\
g(x) + \beta \eta E_D[\theta_0^1(x - D)] + (1 - \beta) \eta E_D[\theta_1^1(x - D)], & \text{if } x > s_{t,r}^1.
\end{cases}
\]

(3.20)

Eq. (3.19) can be readily obtained through Eq. (3.20).

(ii) The \( K_r \)-convexity of \( \theta_1^1(x) \) follows from Lemma A.1.3.

\[\Box\]
Lemma 3.3.3. In the finite horizon model, if $\theta^0_{t-1}(x)$ and $\theta^1_{t-1}(x)$ are both $K_r$-convex for $t = 2, 3, \ldots, T$, and $\alpha \eta K_r \leq K_u \leq (1 - (1 - \alpha)\eta)K_r$, then

(i) the following results on $\tilde{y}^0_{t,r}$ and $\tilde{y}^0_{t,u}$ hold:

\[
\tilde{y}^0_{t,r} = \begin{cases} 
S^0_{t,r}, & \text{if } x \leq s^0_{t,r}, \\
x, & \text{otherwise,}
\end{cases}
\text{ if } m^r_t(y_r) \text{ has a minimum,}
\tag{3.21}
\]

where $S^0_{t,r}$ is the minimizer of $m^r_t(y_r)$, if exists and $s^0_{t,r} < S^0_{t,r}$;

\[
\tilde{y}^0_{t,u} = \begin{cases} 
S^0_{t,u}, & \text{if } x \leq s^0_{t,u}, \\
\hat{y}^0_{t,r}, & \text{otherwise,}
\end{cases}
\tag{3.22}
\]

where $S^0_{t,u}$ is the minimizer of $m^u_t(y_r)$, and $s^0_{t,u} < S^0_{t,u}$.

(ii) $\theta^0_t(x)$ is $K_r$-convex.

Proof. (i) When $\theta^0_{t-1}(x)$ is $K_r$-convex, $m^r_t(y_u) = \alpha c_u y_u + \alpha g(y_u) + \alpha \eta E_D[\theta^0_{t-1}(y_u - D)]$ is $\alpha \eta K_r$-convex. It is not difficult to see that $\alpha c_u y_u + \alpha g(y_u) + \alpha \eta E_D[\theta^0_{t-1}(y_u - D)] \geq \alpha c_u y_u + \alpha g(y_u) \to \infty$ when $y_u \to -\infty$. Therefore, if $K_u \geq \alpha \eta K_r$, there exists $S^0_{t,u} > s^0_{t,u} > -\infty$ such that

\[
L_t(y_r) = \begin{cases} 
K_u + \alpha c_u S^0_{t,u} + \alpha g(S^0_{t,u}) + \alpha \eta E_D[\theta^0_{t-1}(S^0_{t,u} - D)], & \text{if } y_r \leq s^0_{t,u}, \\
\alpha c_u y_r + \alpha g(y_r) + \alpha \eta E_D[\theta^0_{t-1}(y_r - D)], & \text{if } y_r > s^0_{t,u}.
\end{cases}
\tag{3.23}
\]

Eq. (3.22) follows from Eq. (3.23). In addition, $L_t(y_r)$ is $K_u$-convex by Lemma A.1.3. Because $\theta^1_{t-1}(x)$ is $K_r$-convex and $L_t(y_r)$ is $K_u$-convex, we know $m^r_t(y_r) = (c_r - \alpha c_u)y_r + (1 - \alpha)g(y_r) + (1 - \alpha)\eta E_D[\theta^1_{t-1}(y_r - D)] + L_t(y_r)$ is $[(1 - \alpha)\eta K_r + K_u]$-convex. Moreover, since $K_r \geq (1 - \alpha)\eta K_r + K_u$, $m^r_t(y_r)$ is $K_r$-convex. However, there is no certain answer to whether the minimum of $m^r_t(y_r)$
exists or not. If \( \lim_{y_r \to -\infty} m^{t}_r(y_r) \to \infty \), then

\[
\theta^{0}_t(x) = \min_{y_r \geq x} \{ K_r \delta(y_r - x) + (c_r - \alpha \upsilon_u) y_r + (1 - \alpha) g(y_r) + (1 - \alpha) \eta \bar{E}_D[\theta^{1}_{t-1}(y_r - D)] \\
+ L_t(y_r) \} - c_r x
\]

\[
= \begin{cases} \\
K_r + (c_r - \alpha \upsilon_u) S^{0}_{t,r} + (1 - \alpha) g(S^{0}_{t,r}) + (1 - \alpha) \eta \bar{E}_D[\theta^{1}_{t-1}(S^{0}_{t,r} - D)] \\
+ L_t(S^{0}_{t,r}) - c_r x, \quad \text{if } x \leq s^{0}_{t,r}, \\
- \alpha \upsilon_u x + (1 - \alpha) g(x) + (1 - \alpha) \eta \bar{E}_D[\theta^{1}_{t-1}(x - D)] + L_t(x), \quad \text{if } x > s^{0}_{t,r}.
\end{cases}
\]

(3.24)

Otherwise \( \lim_{y_r \to -\infty} m^{t}_r(y_r) \to -\infty \), i.e., \( m^{t}_r(y_r) \) does not have a minimum, resulting in \( y^{0}_{t,r} = x \) and

\[
\theta^{0}_t(x) = - \alpha \upsilon_u x + (1 - \alpha) g(x) + (1 - \alpha) \eta \bar{E}_D[\theta^{1}_{t-1}(x - D)] + L_t(x), \quad \forall x. 
\]

(3.25)

Eq. (3.21) follows from Eq. (3.24) and Eq. (3.25).

(ii) From Lemma A.1.3 we know that \( \theta^{0}_t(x) \) is \( K_r \)-convex.

\( \square \)

The above Lemmas establish the optimality of the proposed \((s, S)\)-like policy. We note that \( K_r \geq K_u \) is dominated by \( \alpha \eta K_r \leq K_u \leq (1 - (1 - \alpha) \eta) K_r \). Therefore, the technical condition \( \alpha \eta K_r \leq K_u \leq (1 - (1 - \alpha) \eta) K_r \) alone guarantees the optimality of \((s, S)\)-like policy in each period. Also notice that when \( \eta \) equals to 1, \( \alpha \eta K_r \leq K_u \leq (1 - (1 - \alpha) \eta) K_r \) is reduced to \( K_u = \alpha K_r \), which prescribes a single value of \( K_u \) for any given \( K_r \). The constraint is less rigid when \( \eta \) takes a small number and requires \( K_u \leq K_r \) only when \( \eta \) equals to 0. This constraint may be restrictive to some extent since the discount factor \( \eta \) usually takes a number close to 1. The reason is that in the above proof, we directly apply the \( K \)-convexity properties, such as preservation of \( K \)-convexity under linear and expectation operations. However, the lemmas do not fully reveal the characteristics of \( K \)-convexity. For example, suppose \( f_{1}(x) \) is \( K_1 \)-convex and \( f_{2}(x) \) is \( K_2 \)-convex, then \( \alpha f_{1}(x) + \beta f_{2}(x) \) is \((\alpha K_1 + \beta K_2)\)-convex for all \( \alpha, \beta \geq 0 \) according to Lemma A.1.2. Whereas, in fact, \( \alpha f_{1}(x) + \beta f_{2}(x) \) is usually \( Q \)-convex, for some \( 0 \leq Q < \alpha K_1 + \beta K_2 \). As such, a stricter than necessary condition is needed to guarantee the
optimal policy. However, our results, which do not rely on the demand distribution, provide a sufficient condition for the optimality of the $(s,S)$-like policy when the retailer faces one reliable supplier and one unreliable supplier with fixed costs. In §3.5, we conduct some computational studies to show that the $(s,S)$-like policy remains optimal for a wide range of system parameters (including unit cost, fixed ordering cost, backorder cost, reliability levels, etc) and demand distributions.

It is noted in Lemma 3.3.3 that $m^r_t(y_r)$ may not have a minimum. Next, we evaluate the performance of $m^r_t(y_r)$ and discuss when its minimum does not exist. For the classic single supplier problem, $m^r_t(y_r)$ has a minimum when the penalty cost is higher than unit ordering cost ($b > c$). Otherwise when $b \leq c$, instead of paying the unit cost, the retailer always prefers not to purchase and pay the penalty cost. For this two-supplier problem, the same logic applies when later making orders on the unreliable supplier. It does not make a difference when multiplying by a positive number $\alpha$ (see $m^u_t(y_u)$). Therefore, it is guaranteed that $\lim_{y_u \to -\infty} m^u_t(y_u) \to \infty$. However, the situation becomes different when analyzing the order quantity from the reliable supplier, because the minimum of $m^r_t(y_r)$ may not exist and this happens when $\lim_{y_r \to -\infty} m^r_t(y_r) \to -\infty$. Figure 3.2 shows how the function $m^r_t(y_r)$ behaves under different parameter settings. Note that we are not able to find any $s^0_r$ and $S^0_r$ in the left case of the figure. We generalize our findings on when the minimum of $m^r_t(y_r)$ does not exist in Proposition 3.3.4.
Figure 3.2: Minimum of $m^r_t(y_r)$ may not exist \(^1\)

**Proposition 3.3.4.** $m^r_t(y_r)$ does not have a minimum if and only if $\alpha \geq \frac{b+\eta c_u-c_r}{b+\eta c_r-c_u}$.

**Proof.** The existence of the minimum of $m^r_t(y_r)$ depends on the slope when $y_r \to -\infty$, and $m^r_t(y_r)$ does not have a minimum if and only if $\lim_{y_r \to -\infty} \frac{\partial m^r_t(y_r)}{\partial y_r} \geq 0$. Note that

$$
\lim_{y_r \to -\infty} m^r_t(y_r) = \lim_{y_r \to -\infty} \left[ (c_r - \alpha c_u) y_r + (1 - \alpha) g(y_r) + (1 - \alpha) \eta \mathbb{E}[\theta^1_{t-1}(y_r - D)] + L_t(y_r) \right],
$$

where

$$
\lim_{y_r \to -\infty} \mathbb{E}[\theta^1_{t-1}(y_r - D)] = \mathbb{E}[K_r + m^1_t(S^1_{t-1,r}) - c_r(y_r - D)],
$$

$$
\lim_{y_r \to -\infty} L_t(y_r) = K_u + m^u_t(S^0_{t,u}).
$$

\(^1\)The small bump around $y_r = 10$ that breaks the convexity of the function is caused by orders being placed on the unreliable supplier. This does not convey any special property and is not relevant in showing the minimum of $m^r_t(y_r)$. The dip around $y_r = -50$ is caused by our treatment on the boundary by setting future cost with initial inventory lower than $I_{min}$ equals to that at $I_{min}$. Since the range of inventory level is set large enough, the boundary processing does not affect the finding of $(s, S)$ values.
Consequently, we have
\[
\lim_{y_r \to -\infty} \frac{\partial m^r_t(y_r)}{\partial y_r} = c_r - \alpha c_u - (1 - \alpha)b - (1 - \alpha)\eta c_r.
\] (3.26)

Therefore, the minimum of \(m^r_t(y_r)\) does not exist if and only if
\[
c_r - \alpha c_u - (1 - \alpha)b - (1 - \alpha)\eta c_r \geq 0
\]
\[
\Rightarrow \alpha \geq \frac{b + \eta c_r - c_r}{b + \eta c_r - c_u}.
\] (3.27)

Note that when the function \(m^r_t(y_r)\) does not have a minimum, it implies it is more costly to place an order on the reliable supplier. Therefore, Proposition 3.3.4 actually provides the condition when the optimal policy is to source from the unreliable supplier exclusively, which we define as Case U. We note that the right hand side of (3.27) is increasing in \(b\), decreasing in \(c_r\) and increasing in \(c_u\). It is in accordance with our expectation that the retailer prefers to order from the unreliable supplier when it operates under a small disruption probability, offering a comparatively low unit price, and the stockout consequence due to not stocking enough inventory is not severe. \(\eta\) is the discount factor, measuring how much the retailer values the future expenditure. A high \(\eta\) is associated with a forward-looking retailer who cares about the future cost, while a low \(\eta\) corresponds to a myopic retailer who is more concerned with current spending. It can be verified that the right hand side of (3.27) is increasing in \(\eta\), which suggests that compared with a forward-looking retailer, it is more likely for a myopic supplier to order from the unreliable supplier. Notice that the condition is independent of \(K_u, K_r, \beta\) and \(h\).

When the problem parameters satisfy the condition in Proposition 3.3.4, the optimal policy is to order from the unreliable supplier exclusively according to an \((s^0_u, S^0_u)\) policy. When that condition is violated, an \((s^0_r, S^0_r, s^0_u, S^0_u)\) policy is shown to generate the minimum cost over the entire planning horizon. The \((s^0_u, S^0_u)\) policy in Proposition 3.3.4 can also be classified as
an \((s_r^0, S_r^0, s_u^0, S_u^0)\) policy by treating \(s_r^0 = S_r^0 = -\infty\). Therefore, we conclude that the optimal ordering policy when the unreliable supplier is up can be generalized as an \((s_r^0, S_r^0, s_u^0, S_u^0)\) policy.

Our further explanation provides some managerial insights into the optimal policy and gives answers to questions such as which supplier/suppliers to place orders from and how much to order from each supplier, given specific \(s_r^0, S_r^0, s_u^0, S_u^0\) values. The ordering of the \(s_r^0, S_r^0, s_u^0, S_u^0\) values plays an important role in translating the \((s_r^0, S_r^0, s_u^0, S_u^0)\) policy into practice. Given that \(s_r^0 < S_r^0\) and \(s_u^0 < S_u^0\), there are 6 possible permutations for these four numbers. If we have \(S_r^0 > s_r^0 > -\infty\), then all of the 6 permutations of \(s_r^0, S_r^0, s_u^0\) and \(S_u^0\) can be categorized into three cases, denoted by Case EOB, Case ENB and Case R, as shown in Table 3.1. Together with Case U, these four cases completely characterize the optimal ordering policy when the unreliable supplier is available.

Table 3.1: Possible permutations and their corresponding cases

<table>
<thead>
<tr>
<th>#</th>
<th>Permutation</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(s_r^0 &lt; S_r^0 &lt; s_u^0 &lt; S_u^0)</td>
<td>EOB</td>
</tr>
<tr>
<td>2</td>
<td>(S_r^0 &lt; s_r^0 &lt; S_u^0 &lt; S_u^0)</td>
<td>ENB</td>
</tr>
<tr>
<td>3</td>
<td>(S_r^0 &lt; s_u^0 &lt; S_u^0 &lt; S_r^0)</td>
<td>ENB</td>
</tr>
<tr>
<td>4</td>
<td>(s_u^0 &lt; s_r^0 &lt; S_r^0 &lt; S_u^0)</td>
<td>R</td>
</tr>
<tr>
<td>5</td>
<td>(S_u^0 &lt; s_r^0 &lt; S_u^0 &lt; S_r^0)</td>
<td>R</td>
</tr>
<tr>
<td>6</td>
<td>(S_u^0 &lt; S_r^0 &lt; s_r^0 &lt; S_u^0)</td>
<td>R</td>
</tr>
</tbody>
</table>

The ordering policy for each case can be explicitly stated as follows:

- **When the unreliable supplier is down at the beginning of the period \((J = 1)\),** apply an \((s_r^1, S_r^1)\) policy, where \(s_r^1\) and \(S_r^1\) are the reorder point and order-up-to point for the reliable supplier.

- **When the unreliable supplier is up at the beginning of the period \((J = 0)\),**

  1. When \(\alpha \geq \frac{b+pc_u-c_r}{b+pc_u-c_u}\), Case U: order only from the unreliable supplier based on an \((s_u^0, S_u^0)\) policy.
2. When $\alpha < \frac{b + \eta c - c_r}{b + \eta c - c_u}$,

(i) Case EOB ($s_r^0 < S_r^0 < s_u^0$): order from both suppliers or order from the unreliable supplier, based on the initial inventory level $x$:

$$
\begin{align*}
(q_r, q_u) &= (S_r^0 - x, S_u^0 - S_r^0), & \text{for } x \leq s_r^0, \\
(q_r, q_u) &= (0, S_u^0 - x), & \text{for } s_r^0 < x \leq s_u^0, \\
(q_r, q_u) &= (0, 0), & \text{for } x > s_u^0.
\end{align*}
$$

(ii) Case ENB ($s_r^0 < s_u^0 < S_r^0$): order from only one of the suppliers, based on the initial inventory level $x$:

$$
\begin{align*}
(q_r, q_u) &= (S_r^0 - x, 0), & \text{for } x \leq s_r^0, \\
(q_r, q_u) &= (0, S_u^0 - x), & \text{for } s_r^0 < x \leq s_u^0, \\
(q_r, q_u) &= (0, 0), & \text{for } x > s_u^0.
\end{align*}
$$

(iii) Case R ($s_u^0 < s_r^0$): order only from the reliable supplier based on an $(s_r^0, S_r^0)$ policy.

From the optimal ordering policy, it is noted that orders are made exclusively from the unreliable supplier when it offers a reasonable price at a sufficiently high reliability level. On the contrary, orders are made from the reliable supplier only if the price difference between the two suppliers is not significant and the unreliable supplier is under high risk. For those circumstances in between, the retailer can either (i) order from both suppliers or order only from the unreliable supplier depending on the initial inventory level, which corresponds to Case EOB, or (ii) order from either the reliable supplier or the unreliable supplier depending on the inventory level, but not both, with respect to Case ENB.

To the best of our knowledge, our work is the first to verify the optimality of an $(s, S)$-like policy to a supply system consisting of two suppliers with certain disruption. The existing
literature has shown the optimality of a state dependent \((s, S)\) policy for a supply system whose state is modeled by a Markov chain to reflect randomness in the supply process (Özekici & Parlar, 1999; Song & Zipkin, 1996) and the optimality of an \((s, S)\)-like policy for a system consisting of two perfectly reliable delivery modes with different lead times (Sethi et al., 2003). These results stimulate our interest in the domain where the \((s, S)\)-like policy can be applied. Therefore, we consider two alternative models to better understand the scope of the optimality of an \((s, S)\)-like policy. First, we consider a simpler model where the supply process is Bernoulli, a special case of the two-state Markov model in which the probability of disruption in the next period keeps the same whether the current period is disrupted or not. And it is assumed that the unreliable supplier is open to accept orders in each period but with a fixed nonzero probability the merchandise will not be delivered successfully. Other settings are similar to the previous finite horizon model. In this case, the state at the beginning of each period reduces to just the inventory level, and the \(t\)-period optimal cost can be written as

\[
\theta_t(x) = \min_{y_u \geq y_r, y_r \geq x} \{K_r \delta(y_r - x) + K_u \delta(y_u - y_r) + c_r(y_r - x) + \alpha c_u(y_u - y_r) + \alpha g(y_u) + (1 - \alpha) g(y_r) \\
\alpha \eta \mathbb{E}_D[\theta_{t-1}(y_u - D)] + (1 - \alpha) \eta \mathbb{E}_D[\theta_{t-1}(y_r - D)]\}.
\]

(3.28)

Similar analysis can be conducted and the resulting optimal policy for this model is the same \((s, S)\)-like policy. Next, we investigate an alternative model when both suppliers are unreliable in terms of all-or-nothing delivery. Like the first model, we assume the supply process is Bernoulli. The only difference is that the two suppliers are both unreliable. Suppose that with probability \(\alpha_1\) the order from supplier 1 will be successfully delivered, and with probability \(\alpha_2\) the order from supplier 2 will be successfully delivered, where \(\alpha_1\) and \(\alpha_2\) are independent. This is an extension of (Anupindi & Akella, 1993) by incorporating the fixed ordering cost. We assume that the cost parameters, their generation mechanism and timetable are the same as in
the finite horizon model. The $t$-period optimal cost can be written as

$$
θ_t(x) = \min_{y_2 \geq y_1} \{K_1δ(y_1 - x) + K_2δ(y_2 - y_1) + α_1c_1(y_1 - x) + α_2c_2(y_2 - y_1) + α_1α_2g(y_2)
+ α_1(1 - α_2)g(y_1) + (1 - α_1)α_2g(x + y_2 - y_1) + (1 - α_1)(1 - α_2)g(x)
+ α_1α_2ηED[θ_{t-1}(y_2 - D)] + α_1(1 - α_2)ηED[θ_{t-1}(y_1 - D)]
+ (1 - α_1)α_2ηED[θ_{t-1}(x + y_2 - y_1 - D)] + (1 - α_1)(1 - α_2)ηED[θ_{t-1}(x - D)]\}.
$$

(3.29)

The numerical results (not shown) indicate that the ordering policy is not necessarily $(s, S)$-like.

In summary, the $(s, S)$-like policy remains optimal for a supply system consisting of one reliable supplier and one unreliable supplier, no matter the uncertainty is modeled by Markov chain or Bernoulli distribution. However, such policy is no longer optimal when two unreliable suppliers are present.

### 3.4 Extension to an Infinite Horizon Model

It is pointed out by H. Scarf (1960) that even if the cost parameters and demand distribution are stationary, the optimal policy fails to be stationary due to the presence of a terminal cost incurred at the end of time horizon. This property naturally extends to our two supplier problem. In addition, the case category depends on the values of $s^0_r, S^0_r, s^0_u$ and $S^0_u$, which indicates that even the optimal case is not invariant across the time. Iglehart (1963) shows the convergence of the optimal $(s, S)$ policy under a concave penalty cost. We find the similar convergence behavior in our finite horizon model. In this section, we provide a theoretical proof for the convergence of the optimal ordering policy. In addition, the limiting $(s, S)$-like policy characterizes the optimal ordering policy for the infinite horizon model.

To complete the proof, the discount factor is assumed to be strictly less than one in this section. In the infinite horizon model, the optimal discounted costs, denoted by $θ^0(x)$ and $θ^1(x)$,
satisfy the following Bellman equations:

\[
\theta^0(x) = \min_{y_r \geq x} \left\{ K_r \delta(y_r - x) + K_u \delta(y_u - y_r) + c_r(y_r - x) + \alpha c_u(y_u - y_r) + \alpha g(y_u) + (1 - \alpha)g(y_r) + \alpha \eta \mathbb{E}_D[\theta^0(y_u - D)] + (1 - \alpha)\eta \mathbb{E}_D[\theta^1(y_r - D)] \right\},
\]

\[
\theta^1(x) = \min_{y_r \geq x} \left\{ K_r \delta(y_r - x) + c_r(y_r - x) + g(y_r) + \beta \eta \mathbb{E}_D[\theta^0(y_r - D)] + (1 - \beta)\eta \mathbb{E}_D[\theta^1(y_r - D)] \right\}.
\] (3.30)

(3.31)

In order to obtain the convergence of the optimal ordering decision, we first need to investigate the limiting behavior of the cost function. The proof follows the spirit of Iglehart (1963). Assume the demand distribution and cost parameters are both stationary and let

\[
V^0(y_r, y_u, x, \theta^0_t, \theta^1_t) = K_r \delta(y_r - x) + K_u \delta(y_u - y_r) + c_r(y_r - x) + \alpha c_u(y_u - y_r) + \alpha g(y_u) + (1 - \alpha)g(y_r) + \alpha \eta \mathbb{E}_D[\theta^0(y_u - D)] + (1 - \alpha)\eta \mathbb{E}_D[\theta^1(y_r - D)].
\] (3.32)

Denote \( \tilde{y}^0_{t,r}(x), \tilde{y}^0_{t,u}(x) \) (henceforth denoted by \( y^0_{t,r}, y^0_{t,u} \) for convenience) as the minimized values of \( y_r, y_u \), respectively, given the initial inventory level \( x \), i.e.,

\[
\theta^0_t(x) = \min_{y_u \geq x} \left\{ V^0(y_r, y_u, x, \theta^0_{t-1}, \theta^1_{t-1}) \right\} = V^0(y^0_{t,r}, y^0_{t,u}, x, \theta^0_{t-1}, \theta^1_{t-1}).
\] (3.33)

Since \( \{y^0_{t,r}, y^0_{t,u}\}, \{y^0_{t-1,r}, y^0_{t-1,u}\} \) are minimizing solutions to the \( t \)-period and \( (t - 1) \)-period problems, respectively, we have

\[
V^0(y^0_{t+1,r}, y^0_{t+1,u}, x, \theta^0_t, \theta^1_t) - V^0(y^0_{t+1,r}, y^0_{t+1,u}, x, \theta^0_{t-1}, \theta^1_{t-1}) \leq \theta^0_{t+1}(x) - \theta^0_t(x) \leq V^0(y^0_{t,r}, y^0_{t,u}, x, \theta^0_t, \theta^1_t) - V^0(y^0_{t,r}, y^0_{t,u}, x, \theta^0_{t-1}, \theta^1_{t-1}).
\] (3.34)
Therefore,

\[
|\theta^0_{t+1}(x) - \theta^0_t(x)| \leq \max \left\{ |V^0(y^0_{t+1,r}, y^0_{t+1,u}, x, \theta^0_t, \theta^1_t) - V^0(y^0_{t+1,r}, y^0_{t+1,u}, x, \theta^0_{t-1}, \theta^1_{t-1})|, \right. \\
\left. |V^0(y^0_{t,r}, y^0_{t,u}, x, \theta^0_t, \theta^1_t) - V^0(y^0_{t,r}, y^0_{t,u}, x, \theta^0_{t-1}, \theta^1_{t-1})| \right\}. 
\]  

(3.35)

Substituting the expression of \( V^0(\cdot) \) into Eq. (3.35) yields

\[
|\theta^0_{t+1}(x) - \theta^0_t(x)| \leq \max \left\{ \right. \\
(1 - \alpha) \eta \left| \int_0^\infty [\theta^0_t(y^0_{t+1,r} - \xi) - \theta^0_{t-1}(y^0_{t+1,r} - \xi)] f(\xi) d\xi \right| + \\
\alpha \eta \left| \int_0^\infty [\theta^0_t(y^0_{t+1,u} - \xi) - \theta^0_{t-1}(y^0_{t+1,u} - \xi)] f(\xi) d\xi \right|, \\
(1 - \alpha) \eta \left| \int_0^\infty [\theta^0_t(y^0_{t,r} - \xi) - \theta^0_{t-1}(y^0_{t,r} - \xi)] f(\xi) d\xi \right| + \\
\alpha \eta \left| \int_0^\infty [\theta^0_t(y^0_{t,u} - \xi) - \theta^0_{t-1}(y^0_{t,u} - \xi)] f(\xi) d\xi \right| \left. \right\}. 
\]  

(3.36)

Due to the presence of holding cost, it is never optimal to order up to a very high inventory level, i.e., the sequences of order up to levels \( \{S^0_{l,r}\}, \{S^0_{l,u}\} \) are bounded above. Assume that \( \bar{S}^0_{l,r} \) and \( \bar{S}^0_{l,u} \) are the corresponding upper bounds. Let \( N \) be a sufficiently large number such that \( N \geq \max\{\bar{S}^0_{l,r}, \bar{S}^0_{l,u}\} \). Then, for any arbitrary number \( M < N \), we have \( y^0_{l,r} \in [M, N] \),
$y^0_{t,u} \in [M, N]$, independent of $t$ for all $x \in [M, N]$. Therefore, it follows that

$$
\max_{M \leq x \leq N} |\theta^0_t(x) - \theta^0_{t-1}(x)| \leq \max_{M \leq x \leq N} \left\{ (1 - \alpha) \eta \left| \int_0^\infty [\theta^1_t(x - \xi) - \theta^1_{t-1}(x - \xi)] f(\xi) d\xi \right| 
+ \alpha \eta \left| \int_0^\infty [\theta^0_t(x - \xi) - \theta^0_{t-1}(x - \xi)] f(\xi) d\xi \right| \right\}
$$

$$
\leq (1 - \alpha) \eta \max_{M \leq x \leq N} \left\{ \int_0^\infty |\theta^1_t(x - \xi) - \theta^1_{t-1}(x - \xi)| f(\xi) d\xi \right\}
+ \alpha \eta \max_{M \leq x \leq N} \left\{ \int_0^\infty |\theta^0_t(x - \xi) - \theta^0_{t-1}(x - \xi)| f(\xi) d\xi \right\}
\leq (1 - \alpha) \eta \left\{ \int_0^\infty \max_{M \leq x \leq N} |\theta^1_t(x - \xi) - \theta^1_{t-1}(x - \xi)| f(\xi) d\xi \right\}
+ \alpha \eta \left\{ \int_0^\infty \max_{M \leq x \leq N} |\theta^0_t(x - \xi) - \theta^0_{t-1}(x - \xi)| f(\xi) d\xi \right\}.
$$

(3.37)

When the unreliable supplier is available, the retailer, observing a low initial inventory level at the beginning of a period, will make an order from either the reliable supplier or the unreliable one, i.e., $\theta^1_t(x)$ is linear with slope $-c_u$ or $-c_r$ when $x$ is sufficiently small. Meanwhile, when the unreliable supplier is unavailable, the retailer will always order from the reliable supplier if the initial inventory is low, i.e., $\theta^1_t(x)$ is linear with slope $-c_r$ when $x$ is sufficiently small. Also note that $\xi$ denotes the random demand, which is nonnegative. Therefore, we can find some sufficiently small $M$ such that

$$
\max_{M \leq x \leq N} |\theta^0_t(x - \xi) - \theta^0_{t-1}(x - \xi)| = \max_{M \leq x \leq N} |\theta^0_t(x) - \theta^0_{t-1}(x)|,
$$

(3.38)

$$
\max_{M \leq x \leq N} |\theta^1_t(x - \xi) - \theta^1_{t-1}(x - \xi)| = \max_{M \leq x \leq N} |\theta^1_t(x) - \theta^1_{t-1}(x)|.
$$

(3.39)
Substituting Eq. (3.38) and Eq. (3.39) into Eq. (3.37), we have

\[
\max_{M \leq x \leq N} |\theta_{t+1}^0(x) - \theta_t^0(x)| \leq (1 - \alpha)\eta \max_{M \leq x \leq N} |\theta_t^1(x) - \theta_{t-1}^1(x)| + \alpha \eta \max_{M \leq x \leq N} |\theta_t^0(x) - \theta_{t-1}^0(x)|.
\]

(3.40)

Similarly, when the unreliable supplier’s status is down, define

\[
V^1(y_r, x, \theta_t^0, \theta_t^1) = K_r \delta(y_r - x) + c^r(y_r - x) + g(y_r) + \beta \eta D [\theta_t^0(y_r - D)] + (1 - \beta) \eta D [\theta_t^1(y_r - D)]
\]

(3.41)

and

\[
\theta_t^1(x) = \min_{y_r \geq x} \{V^1(y_r, x, \theta_{t-1}^0, \theta_{t-1}^1)\} = V^1(y_{t-1}^1, x, \theta_{t-1}^0, \theta_{t-1}^1).
\]

(3.42)

Following the same logic, we have

\[
\max_{M \leq x \leq N} |\theta_{t+1}^1(x) - \theta_t^1(x)| \leq (1 - \beta)\eta \max_{M \leq x \leq N} |\theta_t^1(x) - \theta_{t-1}^1(x)| + \beta \eta \max_{M \leq x \leq N} |\theta_t^0(x) - \theta_{t-1}^0(x)|.
\]

(3.43)

Denote \(A_t = \max_{M \leq x \leq N} |\theta_{t+1}^0(x) - \theta_t^0(x)|\) and \(B_t = \max_{M \leq x \leq N} |\theta_{t+1}^1(x) - \theta_t^1(x)|\). Define \(m_1 = 1 - \alpha, n_1 = \alpha\) and \(m_t = m_{t-1}(1 - \beta) + n_{t-1}(1 - \alpha), n_t = m_{t-1}\beta + n_{t-1}\alpha\) for \(t = 2, 3, \ldots, T\).

Consequently,

\[
A_T \leq (m_1 B_{T-1} + n_1 A_{T-1})\eta
= m_1[(1 - \beta)\eta B_{T-2} + \beta \eta A_{T-2}]\eta + n_1[(1 - \alpha)\eta B_{T-2} + \alpha \eta A_{T-2}]\eta
\leq [m_1(1 - \beta) + n_1(1 - \alpha)]\eta^2 B_{T-2} + (m_1\beta + n_1\alpha)\eta^2 A_{T-2}
= (m_2 B_{T-2} + n_2 A_{T-2})\eta^2
\leq \ldots
\leq (m_{T-1} B_1 + n_{T-1} A_1)\eta^{T-1}.
\]

(3.44)
Similarly, define $u_1 = 1 - \beta$, $v_1 = \beta$ and $u_t = u_{t-1}(1 - \beta) + v_{t-1}(1 - \alpha)$, $v_t = u_{t-1}\beta + v_{t-1}\alpha$ for $t = 2, 3, \ldots, T$. We can see that

$$B_T \leq (u_{T-1}B_1 + v_{T-1}A_1)\eta^{T-1}. \quad (3.45)$$

With Eq. (3.44) and Eq. (3.45), it is not difficult to verify that $\lim_{T \to \infty} A_T \to 0$ and $\lim_{T \to \infty} B_T \to 0$. Therefore, we have the next major theorem:

**Theorem 3.4.1.** When $\eta < 1$, $\theta_0^0(x)$ and $\theta_1^1(x)$ converge uniformly for all $x$ in any finite interval, and the limit functions $\theta_0^0(x)$ and $\theta_1^1(x)$ satisfy the optimal Equations (3.30) and (3.31).

Based on the optimal cost equations, it is not difficult to see that the optimal ordering policy also converges, i.e., $\{s_0^0_t, s_1^0_t, s_0^u_t, s_1^u_t, S_0^0_t, S_1^0_t, S_0^u_t, S_1^u_t\}$ converge to $s_0^r, s_1^r, s_0^u, s_1^u, S_0^r, S_1^r$, respectively. Furthermore, the limiting policy characterizes the optimal sourcing strategy for the infinite horizon model.

### 3.5 Numerical Studies

In this section, we conduct numerical experiments to solve the DP formulation in Matlab under different parameter settings. The experiments serve three purposes. First, they demonstrate the optimality of an $(s, S)$-like policy and its convergence and show how the optimal policy is affected by the system parameters. Second, by considering cases where the technical condition on the fixed cost required in Lemma 3.3.3 does not hold, (i.e. $\alpha\eta K_r \leq K_u \leq (1 - (1 - \alpha)\eta)K_r$ does not hold), we demonstrate the proposed $(s, S)$-like policy is still optimal for a wide range of system parameters. Finally, we compare our results with those of Ahiska et al. (Ahiska et al., 2013) for a similar problem but with a one-period lead time in order to provide managerial insights.

In Figures 3.3 and 3.4, we show how the optimal policy is affected by system parameters over the time. We show in the bar-chart the optimal case when there are $t$ periods to go. The base-case parameter settings are $c_r = 1.8, K_u = 2, K_r = 2, h = 0.2, b = 2, \beta = 0.3, \eta = 0.9$ and
the demand distribution follows a discrete triangular demand distribution defined on [0, 10] with a peak and mean at 5 (its PMF is given in Table 3.2 as $P_{D_2}(d)$). Here we present the effects of the reliability level $\alpha$ and unit cost of unreliable supplier $c_u$ on the optimal policy. For Figure 3.3, $c_u = 1$ and the reliability level $\alpha$ is ranged from 0 to 1. It can be clearly seen that as $\alpha$ increases the optimal policy evolves from Case R to Case ENB, Case EOB, and then Case U which is consistent with the results of Ahiska et al. (2013). It is not surprising that the retailer orders more from the unreliable supplier and less from the reliable one as the reliability level of the unreliable supplier increases. For Figure 3.4, $\alpha = 0.5$ and $c_u$ is ranged from 0.6 to 1.8. The figure shows that the retailer relies more on the reliable supplier as the unreliable supplier’s price increases. It is also noticeable in both of the figures that it Case U dominates when there is only one period left. From Proposition 3.3.4 we know that the threshold for Case U reduces to $\alpha \geq \frac{b-c_r}{b-c_u}$ when there is no future cost, which is easier to satisfy than when there are multiple periods to go. The impacts of the other parameters are similar and consistent with our intuition and so not presented here. Interestingly, it is brought to our attention that $\beta$ barely plays a role in the optimal policy. Since $\alpha$ and $\beta$ together define the reliability level of the unreliable supplier, it is tempting to think that $\beta$ plays a similar role in the optimal policy like $\alpha$. However, numerical studies (see Appendix A.2) show the opposite. A further contemplation gives the correct explanation: $\beta$ characterizes the likelihood that the unreliable supplier turns up when it is down while the optimal case is valid when the unreliable supplier it is up, so the $\beta$ resides in the future terms only and therefore plays a rather insignificant role.
Figure 3.3: Optimal policy over the time as $\alpha$ changes

Figure 3.4: Optimal policy over the time as $c_u$ changes

Figure 3.5 shows the convergence of the optimal control parameter values for the base-case scenario for each of the four cases. The $x$-axis is the number of remaining periods, $t$, and the
y-axis shows the corresponding $s_0^r, S_0^r, s_0^u, S_0^u$ values (if they exist). Note that the convergence occurs quickly in about five to six periods. For cases U, EOB and ENB we observe that the converged value of the reorder trigger, $s_u$, is almost always higher than average demand of 5 units and the converged order-up-to level, $S_u$, is above the maximum possible demand of 10 units. This implies that the retailer regards the unreliable supplier as a back-up supplier and takes advantage of its low unit cost. Additionally, the order-up-to level of the unreliable supplier is higher than that of the reliable supplier (see cases EOB and ENB). This is consistent with our intuition since, when ordering from the unreliable supplier the retailer incurs the fixed ordering cost ($K_u$) whether the order is received or not, so she tends to make a large order in order to mitigate the potential loss of the fixed ordering cost.

The technical condition $(\alpha \eta K_r \leq K_u \leq (1 - (1 - \alpha)\eta)K_r)$ provides a sufficient condition for the $(s, S)$-like policy to be optimal, but as we have discussed in §3.3, it may be more restrictive than necessary. To explore this, 192 test scenarios are created by setting $\eta = 1$, $c_u = 1$, $K_r = K_u = K$, and taking all combinations of the following: $c_r \in \{1.1, 1.25\}$, $\alpha \in \{0.7, 0.9\}$, $\beta \in \{0.5, 1\}$, $h \in \{0.1, 0.25\}$, $b \in \{2, 5\}$, $K \in \{1, 10\}$ and $D \in \{D_1, D_2, D_3\}$ (see Table 3.2 for the PMFs). For these scenarios (results not shown), no policy structure other than the characterized $(s, S)$-like policy is observed. This indicates that the optimality of $(s, S)$-like policy is not limited to the range of fixed ordering cost required for the proof of Lemma 3.3.3 and so the results can be extended to a wider range of parameter configurations and demand distributions.

<table>
<thead>
<tr>
<th>$d$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{D_1}(d)$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
<td>0.09</td>
<td>0.12</td>
<td>0.15</td>
<td>0.18</td>
<td>0.18</td>
<td>0.14</td>
</tr>
<tr>
<td>$P_{D_2}(d)$</td>
<td>0.02</td>
<td>0.05</td>
<td>0.08</td>
<td>0.11</td>
<td>0.15</td>
<td>0.18</td>
<td>0.15</td>
<td>0.11</td>
<td>0.08</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>$P_{D_3}(d)$</td>
<td>0.14</td>
<td>0.18</td>
<td>0.18</td>
<td>0.15</td>
<td>0.12</td>
<td>0.09</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Lastly, we compare our experimental results with those of Ahiska et al. (2013). They
Figure 3.5: Convergence of \((s, S)\) values under all four cases

\[ s_{tr}^0, S_{tr}^0, s_{tu}^0, S_{tu}^0 \]
consider a similar problem that consists of one reliable and one unreliable supplier, whose reliability status is modeled a Markov chain. As in our model, there is a fixed probability that the entire order from the unreliable supplier will be canceled with the fixed ordering cost still incurred. The cost structure, as well as its generating mechanism, is similar to ours. However, they considered a lead time of one period while we assume a negligible lead time. In addition, they analyzed an infinite horizon model, while we first evaluate the finite horizon model and generalize the results to the infinite horizon setting. Using an MDP-based approach, Ahiska et al. performed a comprehensive numerical study using different combinations of parameters under a specific triangular distribution. They generalized, without proving, the optimal policy based on the results of thousands of test scenarios. We apply DP to characterize the cost function and provide theoretical proof to an optimal \((s, S)\)-like policy. Realizing that the same logic applies to the case when lead time is one period, we can utilize this approach to prove the optimal policy in Ahiska et al. (2013). Therefore our work complements that of Ahiska et al. by providing theoretical support. We compare the optimal policy under different lead times and demonstrate the result in Table 3.3. The benchmark parameters used are 

c_r = 1.8, c_u = 1.5, K_u = K_r = 1, h = 0.2, b = 2, \alpha = 0.8, \beta = 1, \eta = 1, \text{ and we change one parameter at one time. It is shown that } (s, S)\text{-like policy continues to be optimal when lead time is one period. Also, the optimal case (U, EOB, ENB, and R) is the same in all cases except when } \mu_D \text{ is changed to 4. The existence of a one-period lead time increases both the ordering trigger level and order-up-to level by an amount around the average demand.}

The convergent optimal policy generated by our DP coincides with that obtained by MDP in Ahiska et al. (2013). It is noted that, according to Ahiska et al. (2013), the occurrence of Case U is affected by fixed cost, which contradicts our result in Proposition 3.3.4. We briefly explain the reason as follows. In their MDP approach, steady state is solved and the optimal decision for each recurrent state is reported. They define the recurrent states to be those with stationary probability higher than \(10^{-8}\) under optimal control. Consequently, those states with stationary probability lower than \(10^{-8}\) are treated as transient states and are not reported when
Table 3.3: Optimal Policy under No Lead Time or One Period Lead time ($h = 0.2, \eta = 1$)

<table>
<thead>
<tr>
<th>Lead time=0</th>
<th>Lead time=1</th>
</tr>
</thead>
<tbody>
<tr>
<td>($s_1^1, S_1^1$)</td>
<td>($s_1^1, S_1^1$)</td>
</tr>
<tr>
<td>0.1</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.2</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.3</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.4</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.5</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.6</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.7</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.8</td>
<td>(5,8)</td>
</tr>
<tr>
<td>0.9</td>
<td>(4,8)</td>
</tr>
<tr>
<td>1</td>
<td>(4,7)</td>
</tr>
<tr>
<td>1.1</td>
<td>(4,7)</td>
</tr>
<tr>
<td>1.2</td>
<td>(4,7)</td>
</tr>
<tr>
<td>1.3</td>
<td>(5,8)</td>
</tr>
<tr>
<td>1.4</td>
<td>(5,8)</td>
</tr>
<tr>
<td>1.5</td>
<td>(5,8)</td>
</tr>
<tr>
<td>1.6</td>
<td>(5,9)</td>
</tr>
<tr>
<td>1.7</td>
<td>(5,9)</td>
</tr>
<tr>
<td>1.8</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.2</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.4</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.6</td>
<td>(5,9)</td>
</tr>
<tr>
<td>0.8</td>
<td>(5,9)</td>
</tr>
<tr>
<td>1</td>
<td>(5,8)</td>
</tr>
<tr>
<td>1.5</td>
<td>(5,9)</td>
</tr>
<tr>
<td>1.6</td>
<td>(5,9)</td>
</tr>
<tr>
<td>1.7</td>
<td>(5,9)</td>
</tr>
<tr>
<td>1.8</td>
<td>(5,9)</td>
</tr>
<tr>
<td>1</td>
<td>(5,8)</td>
</tr>
<tr>
<td>2</td>
<td>(5,8)</td>
</tr>
<tr>
<td>2</td>
<td>(5,8)</td>
</tr>
<tr>
<td>3</td>
<td>(5,8)</td>
</tr>
<tr>
<td>4</td>
<td>(5,8)</td>
</tr>
<tr>
<td>5</td>
<td>(5,8)</td>
</tr>
<tr>
<td>6</td>
<td>(5,8)</td>
</tr>
<tr>
<td>7</td>
<td>(5,8)</td>
</tr>
<tr>
<td>8</td>
<td>(5,8)</td>
</tr>
<tr>
<td>9</td>
<td>(5,8)</td>
</tr>
<tr>
<td>10</td>
<td>(5,8)</td>
</tr>
<tr>
<td>11</td>
<td>(5,8)</td>
</tr>
<tr>
<td>12</td>
<td>(5,8)</td>
</tr>
</tbody>
</table>

51
discussing the optimal policy. However, our DP procedure only tries to find the optimal policy and does not tell anything about the stationary probability distribution. When \( s^0 \) is low enough such that \((X = s^0, J = 0)\) becomes a transient state, the MDP concludes with Case U while DP suggests Case U \( \lor \) R. While at first glance our procedure produces a seemingly different result from the MDP, the two approaches culminate in fundamentally identical optimal solutions.

### 3.6 Concluding Remarks

This chapter studies a supply system consisting of one reliable supplier and one unreliable supplier offering a lower unit price. The retailer has to decide how to distribute her order between these two suppliers so as to minimize the expected cost. Our main contribution lies in developing a theoretical framework with mathematical proof to characterize the optimal ordering policy. Dada et al. (2007) and Anupindi and Akella (1993) state that the cost takes precedence over the reliability in the selection of suppliers and a supplier is selected only if all less-expensive suppliers are selected. Different from their results, the optimal policy of our model does not reveal any dominating effect of either cost or reliability. We prove that the optimal ordering policy exhibits an \((s, S)\)-like structure, with the supplier selection and order allocation implied in the optimal \(s\) and \(S\) values. Based on the ordering of those \(s, S\) values, we generalize the optimal ordering policy into different cases and uncover the optimal case change as system parameters change. Particularly, we derive the condition when orders are made exclusively from the unreliable supplier. An extension to the infinite horizon model is established and the convergence of the optimal cost and optimal policy is proved. Knowing the potential optimal policy structure a priori is important and it helps us design heuristic algorithms to determine the optimal policy efficiently. Previous literature has only shed light on the optimality of an \((s, S)\)-like policy under either a single supplier with uncertainty or two suppliers with different costs or lead times. The contribution of this work is that we extend the optimality of an \((s, S)\)-like policy to an inventory system consisting of one reliable supplier and one unreliable supplier.
In the computational experiments, we verify that both the optimal policy and policy parameters change in an intuitive fashion with respect to the changes in the system parameters. The sufficient condition for the optimality of the \((s, S)\)-like policy derived by the DP becomes restrictive when the discount factor is large. We show by numerical studies that, in practice, the \((s, S)\)-like policy remains optimal for a wide range of parameters and demand distributions. In addition, we compare the optimal policy when lead time is equal to one period with that when there is no lead time. We find that the lead time does not play a significant role in the optimal cases.
Chapter 4

A Quick Response Supply Chain with Endogenous Wholesale Prices

4.1 Introduction

The apparel industry is characterized for its fast-changing trends, volatile customer demand and short product life-cycles, all of which have caused high holding costs, frequent stock-outs and frequent markdowns for apparel retailers. The situation becomes worse as today’s consumers seek more fashionable and distinctive apparel to reflect their personalities. In light of the highly unpredictable customer demands, fashion companies have been struggling to manage their supply chains to meet customer demands. Originating in the U.S. apparel industry in the early 1980s, quick response (QR) has been adopted by many companies to respond more promptly to shifting customer preferences (Hunter, 1990; Lowson et al., 1999). Quick response works by compressing the time between product design and appearance on retail shelf, thereby allowing the merchants to respond to more recent demand, provide better service and capture more consumers. From an operational perspective, QR allows items to be produced in small batches and with short lead times. In production, QR relies on near-shore production, which reverses the prevalent off-shoring trend and brings the manufacturing back to the developed countries.
In addition, the implementation of quick response is contingent upon advanced information sharing, such as point of sale data and electronic data exchange, to enable a prompt and accurate capture of the nascent market. Improved logistics including automated warehouses and frequent use of air transportation also contributes to the acceleration of production and transportation processes. QR has attracted attention from academia and industry over the past thirty years due to the following reasons: (i) QR creates for the merchant a replenishment opportunity in the middle of the selling season; (ii) the short lead time enabled by QR allows a firm to postpone production until more demand information can be collected; (iii) by better aligning supply with demand, QR successfully reduces lost sales as well as unnecessary markdowns at the end of the sales window. The Spanish fashion retailer Zara exemplifies how to maintain competitiveness and efficiency by compressing the time of apparel design, production and distribution (Caro & Gallien, 2010). In practical implementation, Zara chooses to work with two suppliers: one off-shore supplier with a low cost and one near-shore supplier with a cost premium. The mass order is placed through the long-leadtime, low-cost channel, and a quick delivery for a nearly identical item is requested if the item sells faster than expected. Similar approaches are adopted by other industries such as toys and electronics.

Considerable research has studied the benefits generated by QR to the supply chain; see e.g., Fisher and Raman (1996), Caro and Martínez-de Albéniz (2010), Cachon and Swinney (2011). However, few papers have realized that QR is a supply chain strategy and its success relies on the behavior of the suppliers and the retailer. The supply chain is a dynamic and interactive system where each of the supply chain members makes its decision and each decision may affect strategies and profits of other supply chain members. Failure to account for the interactive nature of the supply chain can result in serious mistakes in decision-making. General wisdom and existing studies (Cachon & Swinney, 2011; Luo et al., 2016) suggest that involving more suppliers and seeking more flexibility and responsiveness are advantageous to the retailer. However, the underlying pitfall is that these studies consider the uncertain demand but fail to account for the roles of the suppliers. The suppliers will act differently if they learn about the
retailer’s sourcing strategy. When the retailer adopts dual sourcing, lack of exclusivity and less order volume may lead the suppliers to inflate their prices. This implies, when the wholesale prices are set endogenously and the interactive relation between the retailer and the suppliers are accounted for, dual sourcing may not necessarily be beneficial for the retailer.

With this observation, we consider a retailer sourcing from two types of suppliers with different lead times. The normal supplier provides a low sourcing cost but holds a long lead time. The QR supplier offers a short delivery lead time but requires a high procurement cost. The retailer has to make a trade-off between cost and responsiveness. The benefit of responsiveness is captured by having more accurate information that can be used to reestimate the market demand. One example of this is where a retailer previews a new product with selected potential customers to gauge acceptance. We assume that we are able to have a perfect forecast of the demand. There are two types of sourcing strategies that the retailer can choose, i.e., single sourcing and dual sourcing. Under single sourcing, the retailer chooses to order from the supplier that offers an overall cost-responsiveness advantage, while dual sourcing allows the retailer to append an order until more accurate market information can be collected. The suppliers set their wholesale prices, which is contingent on the retailer’s implemented sourcing strategy and will, in turn, influence the retailer’s subsequent ordering decision.

We contribute to knowledge on the interactive QR supply chain by answering the following key questions: How does the retailer make the ordering decision facing an off-shore, slow but economic supplier and a near-shore, responsive but expensive supplier? How does the retailer’s decision affect the suppliers’ pricing strategies? Which sourcing strategy (single sourcing or dual sourcing) yields the retailer a higher profit when wholesale prices are endogenous? We also extend our analysis to the scenario of multiple QR suppliers. We are interested in how additional suppliers, though never winning any order, influence the profitability of the retailer, the incumbent suppliers and the whole supply chain.

The main thesis of this chapter is that the retailer’s sourcing strategy heavily influences the suppliers’ pricing decisions. It is suspected that alleviated competition under dual sourcing
may lure the suppliers to inflate their prices, rendering dual sourcing a less attractive strategy for the retailer. We verify this suspicion with an exponential demand example and find that the retailer is better-off with the single sourcing strategy. Our new findings complement the discoveries of Calvo and Martínez-de Albéniz (2015). In the general analysis with an arbitrary demand distribution, it is shown that the Nash equilibrium may not exist, prompting the suppliers to negotiate their pricings. We are the first to study the Nash bargaining solution. Furthermore, we investigate the scenario of multiple QR suppliers, and find that the entrance of additional QR suppliers changes the retailer’s allocation between the initial order and the expedited order. Moreover, the intensified competition can be beneficial for the supply chain.

The rest of this chapter is organized as follows. We state the key assumptions and present the basic model in §4.2. §4.3 studies the case with an exponential probability distribution for uncertain demand and presents explicit results on the retailer’s ordering decision and suppliers’ pricing strategies. §4.4 introduces more QR suppliers into the supply chain and examines the impact of latent QR suppliers on the profit of incumbent supply chain members. §4.5 extends the results in §4.3 to the general case with an arbitrary demand distribution and investigates the Nash bargaining solution. Conclusions are provided in §4.6. All proofs are provided in Appendix B.

### 4.2 Model and Assumptions

Consider a fashion retailer selling a product at market price $p$, as a price taker. Since fashion products usually have a short sales window, we assume that the demand is realized at one time. Before the sales season starts, it is unknown how many customers are willing to buy the product, but forecasts are available for estimating the market demand. The forecasts become clearer as the selling season comes closer. By the end of the selling season, the excessive inventory, if there is any, is scrapped with no salvage value. As a one-period problem, there is no inventory holding cost. Without loss of generality, the market price is normalized to one, i.e., $p = 1$.

The retailer may choose to source from multiple upstream suppliers, who offer homogeneous
products and services but differ in their production costs and lead times. Based on the lead
time, suppliers can be categorized into two types: normal suppliers and QR suppliers. A normal
supplier has a longer lead time compared with a QR supplier. The retailer may reduce demand
mismatch by ordering from a QR supplier at a higher unit cost because the short lead time
allows the retailer to postpone the sourcing decision until the demand forecasts are updated.

Different from most of the literature that concentrates on the retailer’s decision, we realize
that the suppliers’ responses, such as pricing decisions, are critical to the supply chain. By
allowing the suppliers to set their wholesale prices, we study the interactive outcomes of the
retailer and suppliers. A supplier would like to maximize his own profit, but he also knows that
the retailer will make her ordering decision based on wholesale prices of all suppliers. Therefore,
a supplier anticipates the retailer’s response and offers a price that can potentially maximize
his profit. Since the supplier's profit is determined by his own pricing as well as other suppliers’
pricing decisions, we apply a game-theoretic model to analyze suppliers’ pricing strategies and
seek a Nash equilibrium solution of the game, under which no supplier will unilaterally change
his price.

For simplicity, in the basic model, we assume that there are one normal supplier and one
QR supplier. Before any ordering or pricing decision is made, the retailer decides whether to
allocate her order to both suppliers (dual sourcing) or award the entire order to one supplier
(single sourcing). When the wholesale prices are given exogenously, it is generally believed
that involving more suppliers and having more flexibility are advantageous to the retailer.
However, when supplier incentives are taken into account, the suppliers will act differently in
the face of different retailer sourcing strategies. Specifically, when the retailer adopts a dual
sourcing strategy, lack of exclusivity and alleviated competition may lead the suppliers to ask
for higher wholesale prices. To better understand the suppliers’ incentives and supplier-retailer
interaction in a QR supply chain, we analyze the scenarios under both of the single sourcing and
dual sourcing strategies to shed light on how an equilibrium arises differently under different
sourcing strategies and which sourcing strategy generates a higher profit for the retailer.
Figure 4.1 summarizes the order of events: First, the retailer releases her sourcing strategy (single sourcing or dual sourcing). Then, the suppliers reveal their prices based on the announced sourcing strategy. After learning the suppliers’ prices, the retailer decides her ordering decision. If the retailer adopts the single sourcing strategy, she needs to decide whether to order from the normal supplier or the QR supplier and how much to order. If the retailer adopts the dual sourcing strategy, she needs to decide the ordering quantity from the normal supplier far in advance of the selling season and the ordering quantity from the QR supplier after demand updates become available. At last, the selling season begins.

Having introduced the general framework, we now clarify the assumptions made in order to generate tractable results. First, we consider the extreme case that the lead time of the QR supplier equals to zero. In other words, there is no demand uncertainty when an order is placed on the QR supplier. Consequently, the retailer will have no stock-outs if she places an order from the QR supplier. The zero lead time assumption is a simplification of reality, but it is sufficient to capture the main effects of the demand updates and the reordering opportunity that better aligns supply with demand. Second, we assume that all the information including the
production cost of each supplier and demand update is public knowledge. When the production cost is a supplier’s private information, we need to resort to the auction theory. The demand update is public knowledge, thus suppliers can anticipate the retailer’s ordering strategy.

In §4.4, we extend the analysis to the case of multiple QR suppliers. With perfect information of suppliers’ production costs, the cost-inferior QR suppliers will not be awarded any orders. However, they will change the equilibrium prices and therefore redistribute the profits of the incumbent supply chain members. The aim of §4.4 is to derive additional insights on the impact of additional QR suppliers.

Suppose supplier 1 is a normal supplier and suppliers 2, 3, · · · are QR suppliers ordered by their production costs with supplier 2 being the lowest-cost QR supplier. Let $L_i$ denote the lead time of supplier $i$. Define $c_i$ as the production cost of supplier $i$. The following list provides the notation that will be used throughout the chapter. Some assumptions are given as well.

\[ i : \text{supplier index, with } i = 1 \text{ indicating a normal supplier and } i = 2, 3, \cdots \text{ indicating a QR supplier;} \]

\[ L_i : \text{lead time of supplier } i, L_1 > 0, L_i = 0, \forall i = 2, 3, \cdots ; \]

\[ c_i : \text{production cost of supplier } i, c_1 > 0 \text{ and } c_2 < c_3 < \cdots ; \]

\[ p : \text{market price, } p = 1; \]

\[ D : \text{estimation of market demand at time } -L_1, \text{ with its pdf being } f(\cdot), \text{ cdf being } F(\cdot), \text{ and ccdf being } \bar{F}(\cdot); \]

\[ S/D : \text{single/dual sourcing strategy;} \]

\[ w_S^i/w_D^i : \text{equilibrium wholesale price of supplier } i \text{ under the single/dual sourcing strategy;} \]

\[ w^m_i : \text{monopoly price if supplier } i \text{ were the only supplier in the market;} \]

\[ w^*_i : \text{maximum wholesale price that supplier } i \text{ can ask for to keep other suppliers out of the market;} \]

\[ q_S^i/q_D^i : \text{ordering quantity from supplier } i \text{ in equilibrium under the single/dual sourcing strategy;} \]

\[ \pi^k_S(w_1, w_2) : \text{expected profit of supply chain member } k \text{ given wholesale prices } w_1 \text{ and } w_2 \text{ under the single sourcing strategy, } k = s_1, s_2, r, sc, \text{ where } s_1 \text{ stands for supplier 1, } s_2 \text{ stands for supplier 2, } r \text{ stands for retailer, and } sc \text{ stands for supply chain;} \]
\[ \pi^D_k (w_1, w_2) : \text{expected profit of supply chain member } k \text{ given wholesale prices } w_1 \text{ and } w_2 \text{ under the dual sourcing strategy, } k = s_1, s_2, r, sc, \text{ where } s_1 \text{ stands for supplier 1, } s_2 \text{ stands for supplier 2, } r \text{ stands for retailer, and } sc \text{ stands for supply chain; } \]

\[ d_i : \text{the payoff to supplier } i \text{ if the negotiation fails in the Nash bargaining process; } \]

\[ w_i^{D,b} : \text{Nash bargaining solution of supplier } i \text{ under the dual sourcing strategy; } \]

\[ W_0(\cdot) : \text{principal branch of the Lambert W function.} \]

### 4.3 Results with Exponential Demand

In this section, we first consider a common case in which the demand is exponentially distributed with parameter \( \lambda \) to analyze the retailer’s ordering strategy and the suppliers’ pricing decisions under the single sourcing and dual sourcing strategies in §4.3.1 and §4.3.2, respectively. Then the retailer’s preference over the single sourcing strategy and dual sourcing strategies is studied in §4.3.3.

#### 4.3.1 Single Sourcing

Under the single sourcing strategy, the retailer consolidates her demand in one order. Recall that the basic case involves one normal supplier (Supplier 1) and one QR supplier (supplier 2). Under the single sourcing strategy, the retailer places the whole order from either supplier exclusively. If there were only one supplier in the market, this supplier will set a “monopoly price” that maximizes his profit. When there is more than one supplier, however, the supplier may not be able to set the monopoly price because of the competition. A higher wholesale price leads to a higher unit margin but brings the risk of losing the order. Nash equilibrium (NE) characterizes a stable outcome where each supplier considers his pricing and the competitor’s reaction prudently. The following result shows the existence of a pure strategy Nash equilibrium. Specifically, the winner’s wholesale price and the retailer’s order volume are explicitly identified in the theorem.
Theorem 4.3.1. If the demand is exponentially distributed with parameter \( \lambda \) and the retailer adopts the single sourcing strategy, then there exists a unique pure NE, in which the suppliers’ wholesale prices and the retailer’s order quantity are given below:

(i) When \( c_2 \geq c_1 - c_1 \ln c_1 \), the normal supplier wins the entire order with a wholesale price of \( w_1^S = \min\{w_1^c, w_1^m\} \), where \( w_1^c \in (0, 1] \) and \( w_1^c - w_1^c \ln w_1^c = c_2; w_1^m = \frac{c_1}{W_0(c_1 e)} \); \( W_0(\cdot) \) is the principal branch of the Lambert W function. The retailer orders a quantity of \( q_1^S = -\frac{1}{\lambda} \ln w_1^S \) from the normal supplier.

(ii) When \( c_2 < c_1 - c_1 \ln c_1 \), the QR supplier wins the entire order with a wholesale price of \( w_2^S = w_2^c = c_1 - c_1 \ln c_1 \). The retailer orders a quantity of \( q_2^S = D \) from the QR supplier.

Figure 4.2: Equilibrium under single sourcing

Figure 4.2 illustrates the decisions stated in Theorem 4.3.1. It can be seen how the market is claimed by the normal supplier or the QR supplier as their production costs vary. It is predictable that each supplier keeps reducing his price to make it attractive to the retailer until
one of them hits his production cost. When $c_2 \geq c_1 - c_1 \ln c_1$, the normal supplier has the cost advantage to occupy the market, while when $c_2 < c_1 - c_1 \ln c_1$, the QR supplier captures the market. Especially, we can see the lead time advantage of the QR supplier by the fact that the QR supplier still takes over the entire market when his cost is slightly higher than that of the normal supplier. When a supplier occupies the market, there are two possible prices he could ask for: the monopoly price and the competition price. The monopoly price, denoted by $w_i^m$, is the price a supplier will quote if he were the only supplier in the market, whereas the competition price, denoted by $w_i^c$, is the maximum price a supplier can ask for to keep the other supplier out of the market. Since the QR supplier has zero lead time, the monopoly price of the QR supplier is the market price, which the QR supplier is no longer able to ask for when competition exists. Hence, the QR supplier asks for the competition price $w_2^c$ in equilibrium. When the normal supplier has a dramatic cost advantage over the QR supplier, the normal supplier asks for the monopoly price $w_1^m$. Otherwise, the normal supplier has to lower his price to prevent the order from being stolen by the QR supplier, and consequently, he quotes the competition price $w_1^c$.

4.3.2 Dual Sourcing

Under the dual sourcing strategy, the retailer allocates her order between the normal supplier and QR supplier. While each supplier intends to maximize his profit, each of them also understands that the other supplier’s pricing will influence his profit by affecting the retailer’s order allocation. It is conceivable that, when the normal supplier offers a wholesale price lower than that of the QR supplier, the retailer will first purchase from the normal supplier and replenish from the QR supplier if she has more than expected demand when uncertain demand gets realized. On the other hand, if the QR supplier quotes a price lower than the price offered by the normal supplier, the entire order goes to the QR supplier, who provides the retailer with both a cost and lead time advantage. Therefore, the QR supplier needs to consider whether to ask for a high price to reap the excessive market demand or to ask for a moderate price to
eliminate the normal supplier from the market.

Similar to the single sourcing strategy, we find that there is a unique pure strategy Nash equilibrium in the supplier pricing game. In the next theorem, we explicitly identify the equilibrium prices and the corresponding retailer ordering behavior.

**Theorem 4.3.2.** If the demand is exponentially distributed with parameter $\lambda$ and the retailer adopts the dual sourcing strategy, then there exists a unique pure NE, in which the normal supplier offers a wholesale price of $w_1^D = w_1^m$, the QR supplier offers a wholesale price of $w_2^D = 1$, and the retailer orders a quantity of $q_1^D = \frac{1}{\lambda} \ln \frac{w_2^D}{w_1^D}$ from the normal supplier and a quantity of $q_2^D = (D - q_1^D)^+$ from the QR supplier.

Theorem 4.3.2 shows that there is a unique Nash equilibrium in the supplier pricing game. In equilibrium, the QR supplier asks for the market price (normalized to one) and not surprisingly, the normal supplier asks for the monopoly price as if he were the only supplier in the market. Under this scenario, the retailer satisfies more customer demand, but gains no extra profit because her purchasing price equals her selling price. The offer the retailer sees from the normal supplier does not look good either. This leads to the question whether the dual sourcing strategy continues to be the superior strategy for the retailer when suppliers can leverage their wholesale prices.

### 4.3.3 Preferred Sourcing Strategy of Each Supply Chain Member

In order to investigate whether the dual sourcing strategy continues to be the superior strategy for the retailer, in this subsection, we derive and compare the retailer’s equilibrium profit under single and dual sourcing strategy. In addition, we explore how the profits of suppliers and the supply chain are influenced by the retailer’s choice of single or dual sourcing strategy. To see this, we first need to calculate the profit functions of each member and the supply chain. From the analysis in subsections 4.3.1 and 4.3.2, we know that the equilibrium prices are functions of production costs. Assuming that each player is rational and acts according to the Nash equilibrium, we can calculate the profits of each supply chain member in terms of the
production costs. The profit of the supply chain is the sum of the profits of all supply chain members.

Under single sourcing, the profits of the retailer, supplier 1, supplier 2 and supply chain are given as functions of the wholesale prices:

\[
\pi_{s_1}(w_1, w_2) = \begin{cases} 
\frac{1}{\lambda} - \frac{w_1}{\lambda} + \frac{w_1}{\lambda} \ln w_1, & \text{if } w_2 \geq w_1 - w_1 \ln w_1, \\
\frac{1}{\lambda}(1 - w_2), & \text{otherwise},
\end{cases}
\]

\[
\pi_{s_2}(w_1, w_2) = \begin{cases} 
-\frac{1}{\lambda}(w_1 - c_1) \ln w_1, & \text{if } w_2 \geq w_1 - w_1 \ln w_1, \\
0, & \text{otherwise},
\end{cases}
\]

\[
\pi_{s_{sc}}(w_1, w_2) = \begin{cases} 
\frac{c_1}{\lambda} \ln w_1 + \frac{1}{\lambda} - \frac{w_1}{\lambda}, & \text{if } w_2 \geq w_1 - w_1 \ln w_1, \\
\frac{1}{\lambda}(1 - c_2), & \text{otherwise};
\end{cases}
\]

Similarly, under dual sourcing, the profits of the retailer, supplier 1, supplier 2 and supply chain, in terms of the wholesale prices, are given as:

\[
\pi_{D_r}(w_1, w_2) = \begin{cases} 
\frac{1}{\lambda} - \frac{w_1}{\lambda} \ln w_2 - \frac{w_1}{\lambda}, & w_1 < w_2, \\
\frac{1}{\lambda}(1 - w_2), & w_1 \geq w_2,
\end{cases}
\]

\[
\pi_{D_{s_1}}(w_1, w_2) = \begin{cases} 
\frac{1}{\lambda}(w_1 - c_1) \ln \frac{w_2}{w_1}, & w_1 < w_2, \\
0, & w_1 \geq w_2,
\end{cases}
\]

\[
\pi_{D_{s_2}}(w_1, w_2) = \begin{cases} 
\frac{w_1}{\lambda}(1 - \frac{c_2}{w_2}), & w_1 < w_2, \\
\frac{1}{\lambda}(w_2 - c_2), & w_1 \geq w_2,
\end{cases}
\]

\[
\pi_{D_{sc}}(w_1, w_2) = \begin{cases} 
\frac{1}{\lambda} - \frac{w_1 c_2}{w_2} + \frac{c_1}{\lambda} \ln \frac{w_1}{w_2}, & w_1 < w_2, \\
\frac{1}{\lambda}(1 - c_2), & w_1 \geq w_2.
\end{cases}
\]

From equations (4.1)-(4.8), we can determine which sourcing strategy generates a higher profit
for the suppliers, retailer and supply chain. We show our results in Figure 4.3, for different values of $c_1$ and $c_2$ between 0 and 1. The most interesting observation comes from subfigure (a). It can be seen that the retailer always prefers single sourcing to dual sourcing, if not strictly prefers. The reason the retailer prefers single sourcing is that dual sourcing greatly mitigates the suppliers’ pricing competition and leads the suppliers to inflate their prices. Under dual sourcing, the QR supplier is inclined to quote a high price to reap the excessive benefits. While the normal supplier, free of competition, can likewise ask for a high price. As a result, the retailer has to pay higher wholesale prices to purchase the same products and her ultimate profit is actually damaged by the dual sourcing strategy, notwithstanding the better chance to match supply with demand. At the top left corner of the subfigure (a), the retailer’s profit is the same under both sourcing strategies. The reason is that the normal supplier’s equilibrium price under dual sourcing coincides with his price under single sourcing, and the order quantity is identical because the QR supplier charges the market price. We can see from subfigures (b) and (c) that, for each supplier, if the production costs fall in the range that he has the whole market under single sourcing, he prefers single sourcing. Otherwise, if the production costs fall in the range that a supplier loses to his competitor, the supplier prefers dual sourcing. As show in subfigure (d), for the supply chain, dual sourcing always generates a better profit as it adds more flexibility to the supply chain.
4.4 Effects of Adding Additional QR Suppliers

In the basic model, we consider one normal supplier (supplier 1) and one QR supplier (supplier 2). In this section, we extend the basic model to include multiple QR suppliers to investigate how these QR suppliers may affect the profits of the incumbent supply chain members. To facilitate our analysis, we consider the exponential demand distribution we have analyzed in Section 4.3.

Since these QR suppliers are assumed to be homogeneous, the retailer will only purchase
from the QR supplier offering the lowest price. For a one-time game, each supplier is concerned about his current profit, and hence, none of the suppliers will offer a price below his production cost. It is conceivable that, the QR supplier with the lowest production cost can always offer a lower price than others. Consequently, in Nash equilibrium, the expedited order only flows to the QR supplier with the lowest production cost. Though the additional QR suppliers are not able to share a piece of the “pie”, it is still interesting to see how the equilibrium prices are affected by having additional QR suppliers.

Let suppliers 3, 4, · · · denote the additional QR suppliers and let $c_i$ be the production cost of supplier $i$ such that $c_2 < c_3 < c_4 < · · ·$. Including the incumbent QR supplier, we sequence the QR suppliers by their production costs with supplier 2 being the lowest-cost supplier. As we just argued, supplier 2 can always offer a lower price in order to win the QR order. However, to prevent the orders from being stolen by the additional QR suppliers, supplier 2 cannot ask a price higher than the production cost of any other QR suppliers. In other words, supplier 2 cannot ask for more than $c_3$. We notice from Equations (4.1)-(4.8) that the profits of the retailer, the suppliers and the supply chain are functions of the suppliers’ quoted prices in equilibrium. Therefore, by substituting the equilibrium prices after the entrance of additional QR suppliers into Equations (4.1)-(4.8), we are able to generalize the variations in the profit of each supply chain member. The resulting changes in profit are given by Theorem 4.4.1.

**Theorem 4.4.1.** In a supply chain consisting of one normal supplier and one QR supplier, when the demand is exponentially distributed, the following changes in profit can be observed as additional QR suppliers enter the supply chain:

(i) Under the single sourcing strategy; the retailer’s profit is non-decreasing; the normal supplier’s profit is unaffected; the QR supplier’s profit is non-increasing; and the supply chain’s profit is unchanged.

(ii) Under the dual sourcing strategy; the retailer’s profit is non-decreasing; the normal supplier’s profit is non-increasing; the QR supplier’s profit is non-increasing; and the supply chain’s profit is uncertain.
Table 4.1 is used to summarize Theorem 4.4.1. An uparrow implies a nondecreasing profit and a downarrow indicates a nonincreasing profit. Table 4.1 shows as the competition intensifies, the retailer benefits, no matter which sourcing strategy is adopted. Consequently, the retailer is incentivized to invite more suppliers to participate in the supply chain. However, the introduction of more QR suppliers compromises the profit of the extant suppliers. The QR supplier suffers directly from the entrance of potential QR suppliers and the normal supplier is embroiled. It is interesting to note that introducing more QR suppliers can be beneficial for the entire supply chain when the production cost of the quick response supplier is not significantly higher ($c_2 \leq W_0(c_1 e)$) than that of the normal supplier. More detailed results on how the supply chain profit is affected are presented in the next Theorem.

Table 4.1: The impact of the latent QR suppliers on the profit of each supply chain member

<table>
<thead>
<tr>
<th></th>
<th>retailer</th>
<th>normal supplier</th>
<th>QR supplier</th>
<th>supply chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>single sourcing</td>
<td>↑</td>
<td>→</td>
<td>↓</td>
<td>→</td>
</tr>
<tr>
<td>dual sourcing</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
<td>not certain</td>
</tr>
</tbody>
</table>

**Theorem 4.4.2.** In a supply chain consisting of one normal supplier and one QR supplier, when the demand is exponentially distributed and the retailer adopts the dual sourcing strategy, the following changes in the profit of the supply chain can be observed as additional QR suppliers enter the supply chain:

(i) when $c_2 \leq c_1$, the profit of the supply chain increases;
(ii) when $c_1 < c_2 \leq W_0(c_1 e)$, the profit of the supply chain first increases and then decreases as market competition intensifies;
(iii) when $c_2 > W_0(c_1 e)$, the profit of the supply chain decreases.
We interpret the market competition level by the value of $c_3$, with a lower $c_3$ implying a more competitive market. We make this interpretation because $c_3$ is the lowest production cost of the additional suppliers and a lower $c_3$ poses a higher pressure on the incumbent suppliers. Additionally, as the number of additional QR suppliers increase, the value of $c_3$ tends to decrease, indicating a more competitive market. In Theorem 4.4.1, we do not specify the market competition level, because those profit changes (except the profit of the supply chain under dual sourcing) are certain when additional QR suppliers are present, regardless of the value of $c_3$. Figure 4.4 visualizes the result of Theorem 4.4.2, i.e., how does the profit of the supply chain change as market competition intensifies. Since there is no uncertainty surrounding the market demand at the time of the expedited order, the retailer will replenish if there is a shortage. Consequently, the profit of the supply chain is influenced by the allotment of the first order and the expedited order, which is governed by the ratio of the wholesale prices $\frac{w_1}{w_2}$. The supply chain
is coordinated when $\frac{w_1}{w_2} = \frac{c_1}{c_2}$. Since the equilibrium prices are functions of production costs, the expected profit change of the supply chain may well depend on the production costs. As shown in Figure 4.4, the square is divided into three areas and the arrows in each area suggest the change in the supply chain profit as competition intensifies. When $c_2 \leq c_1$, it is first-best for the supply chain if all the orders are assigned to the QR supplier. Therefore, it is beneficial to the supply chain when the latent QR suppliers pressure the extant QR supplier to reduce his price. When $c_1 < c_2 \leq W_0(c_1 e)$, a moderate level of market competition can reallocate the retailer order in a more efficient manner that heightens the profit of the supply chain, but an overly competitive market may hamper the profit of the supply chain. When $c_2 > W_0(c_1 e)$, the profit of the supply chain deteriorates as more QR suppliers enter.

4.5 Generalization of Results

Having investigated the retailer’s ordering and suppliers’ pricing decisions in §4.3 and the impact of adding additional QR suppliers to the supply chain in §4.4 under the assumption that the demand is exponentially distributed, in this section, we generalize the results for an arbitrary demand distribution. Due to the complexity of the problem, we restrict our attention to the scenario of one normal supplier and one QR supplier.

4.5.1 Single Sourcing

Let $D$ be an arbitrary demand distribution, with its pdf $f(\cdot)$, cdf $F(\cdot)$, complementary cdf $\bar{F}(\cdot)$. $F^{-1}(\cdot)$ denotes the inverse function of $F(\cdot)$. Parallel to Theorem 4.3.1, the next two results, Theorems 4.5.1 and 4.5.2, follow under a general demand distribution. In Theorem 4.5.1, we study the retailer’s optimal ordering behavior given the wholesale prices of the suppliers.

**Theorem 4.5.1.** Under the single sourcing strategy, if the retailer chooses to order from the normal supplier who offers a wholesale price $w_1$, her ordering quantity is $F^{-1}(w_1)$ and expected
profit is \( \int_0^{F^{-1}(w_1)} xf(x)dx \). Moreover, for any \( w_1 \in [0, 1] \), we have

\[
(1 - w_1) \int_0^\infty xf(x)dx \geq \int_0^{F^{-1}(w_1)} xf(x)dx. \tag{4.9}
\]

Theorem 4.5.1 sheds lights on the retailer’s ordering quantity and profit of ordering from the normal supplier. The left-hand side of Eq. (4.9) is the retailer’s profit when ordering from the QR supplier at a wholesale price \( w_1 \). Eq. (4.9) suggests that when the QR supplier and the normal supplier offer the same wholesale price, the retailer will order from the QR supplier, because, by ordering from a more responsive supplier, the retailer can receive a higher profit by postponing her ordering decision and acquiring more accurate market information. Theorem 4.5.2 addresses the supplier’s pricing strategy and the resulting retailer’s ordering behavior in equilibrium.

Theorem 4.5.2. Under the single sourcing strategy, if the demand \( D \) has an increasing generalized failure rate, then there exists a unique pure Nash equilibrium. In equilibrium,

(i) when \( c_2 \geq \frac{\int_0^\infty x f(x)dx}{\int_0^\infty x \bar{F}(x)dx} \), the normal supplier wins the entire order with a wholesale price of \( w_1^S = \min\{w_1^m, w_1^c\} \), where \( w_1^m = \bar{F}(y^m) \), \( y^m \) is determined by \( y^m f(y^m) - \bar{F}(y^m) + c_1 = 0 \); and \( w_1^c \) is determined by \( (1 - c_2) \int_0^\infty x f(x)dx = \int_0^{F^{-1}(w_1^c)} x f(x)dx \). The retailer orders a quantity of \( q_1^S = \bar{F}^{-1}(w_1^S) \) from the normal supplier.

(ii) when \( c_2 < \frac{\int_0^\infty x f(x)dx}{\int_0^\infty x \bar{F}(x)dx} \), the QR supplier wins the entire order with a wholesale price of \( w_2^S = \frac{\int_0^\infty x f(x)dx}{\int_0^\infty x \bar{F}(x)dx} \). The retailer orders a quantity of \( q_2^S = D \) from the QR supplier.

A distribution with pdf \( f(\cdot) \) and complementary cdf \( \bar{F}(\cdot) \) is said to have an increasing generalized failure rate (IGFR) when \( \frac{x f(\cdot)}{\bar{F}(\cdot)} \) is nondecreasing. Literature shows that a large number of distributions have IGFR (Banciu & Mirchandani, 2013). According to Lariviere and Porteus (2001), this problem is well-behaved (the objective of the supplier is unimodal) when the demand has an IGFR. Then the monopoly price \( w_1^m \) can be determined by the first-order condition of the normal supplier’s expected profit. Given two potential suppliers, the retailer will order from the supplier who offers a price that yields her a higher profit. The retailer’s
expected profit of ordering from the normal supplier at \( w_1 \) is provided in Theorem 4.5.1, and her expected profit of ordering from the QR supplier at \( w_2 \) is \((1 - w_2) \int_{0}^{\infty} x f(x) dx\). Each supplier keeps reducing his price to make it attractive to the retailer until one of them hits his production cost. Similar to Theorem 4.3.1, Theorem 4.5.2 specifies the condition under which the market is claimed by the normal supplier and the QR supplier under equilibrium, and the retailer’s ordering and supplier’s pricing decisions. In equilibrium, the winning supplier asks for the minimum of the monopoly price and the competitive price. It is worthwhile noticing that \( \int_{\bar{F}^{-1}(c_1)}^{\infty} x f(x) dx \geq c_1 \), which can be easily verified by Eq. (4.9). This shows the lead time advantage of the QR supplier in that the QR supplier may still capture the market even when his production cost is higher than that of the normal supplier.

4.5.2 Dual Sourcing

In this subsection, we study the supplier pricing game under dual sourcing through the perspectives of the pure strategy Nash equilibrium and the Nash bargaining solution. First, we show that the pure strategy Nash equilibrium may or may not exist. Without a Nash equilibrium, it is difficult to make a rational guess on the autonomous decisions of the suppliers. Therefore, it is meaningful to study the Nash bargaining solution, which provides a mechanism to allow the suppliers to negotiate their prices and reach a win-win situation.

Nash Equilibrium The next result studies the retailer’s ordering strategy when suppliers’ wholesale prices are given. The suppliers’ equilibrium prices under dual sourcing are hardly tractable. We provide an example with the uniform demand distribution to show that the pure Nash equilibrium of suppliers pricing game may or may not exist. Based on this observation, we proceed to investigate the Nash Bargaining solution.

**Theorem 4.5.3.** Under the dual sourcing strategy, given the normal supplier’s wholesale price
\( w_1 \) and the QR supplier’s wholesale price \( w_2 \), the retailer orders a quantity

\[
q_{D^*}(w_1, w_2) = \begin{cases} 
\bar{F}^{-1}(\frac{w_1}{w_2}), & \text{if } w_1 < w_2, \\
0, & \text{if } w_1 \geq w_2.
\end{cases}
\]  

(4.10)

from the normal supplier and a quantity

\[
q_{D^*}(w_1, w_2) = (D - q_{D^*}(w_1, w_2))^+
\]

(4.11)

from the QR supplier. Moreover, when \( w_1 < w_2 \), we have \( \frac{\partial q_{D^*}}{\partial w_1} < 0 \), \( \frac{\partial q_{D^*}}{\partial w_2} > 0 \), and \( \left| \frac{\partial q_{D^*}}{\partial w_1} \right| > \left| \frac{\partial q_{D^*}}{\partial w_2} \right| \).

It can be seen from Theorem 4.5.3 that the order allocation under the dual sourcing depends on the wholesale prices of the suppliers. The total order quantity \( q_{D^*}(w_1, w_2) + q_{D^*}(w_1, w_2) \) equals to the realized demand, which is independent of both \( w_1 \) and \( w_2 \). The retailer makes a larger order from the normal supplier when he offers a lower price or when the QR supplier asks for a higher price. Moreover, compared with the QR supplier’s wholesale price \( w_2 \), the normal supplier’s wholesale price \( w_1 \) has a greater impact on the order allocation.

Prior to the retailer’s ordering decision, the suppliers, knowing that the retailer will allocate the order to the best of her own interest, submit their wholesale prices at the same time. The payoff functions of the normal supplier and the QR supplier are

\[
\pi_{\Delta s_1}(w_1, w_2) = \begin{cases} 
(w_1 - c_1)\bar{F}^{-1}(\frac{w_1}{w_2}), & \text{if } w_1 < w_2, \\
0, & \text{if } w_1 \geq w_2;
\end{cases}
\]

\[
\pi_{\Delta s_2}(w_1, w_2) = \begin{cases} 
(w_2 - c_2)\mathbb{E}\left[\left(D - \bar{F}^{-1}(\frac{w_1}{w_2})\right)^+\right], & \text{if } w_1 < w_2, \\
(w_2 - c_2)\mathbb{E}[D], & \text{if } w_1 \geq w_2,
\end{cases}
\]

respectively. Since a supplier is unable to influence the other player’s price, he can only alter his own wholesale price. The best response is the pricing strategy that generates the highest
profit for the supplier, taking the other supplier’s price as given, i.e.,

\[ w_D(D^*)_1(w_2) = \arg \max_{w_1} \pi_{s_1}(w_1, w_2), \]
\[ w_D(D^*)_2(w_1) = \arg \max_{w_2} \pi_{s_2}(w_1, w_2). \]

The Nash equilibrium is then characterized by solving a system of best responses, i.e.,

\[
\begin{cases}
  w_D^1 = w_D(D^*)_1(w_2^D), \\
  w_D^2 = w_D(D^*)_2(w_1^D).
\end{cases}
\]

(4.12)

Notice that the QR supplier has two options for setting the wholesale price: either quote a high wholesale price to reap excessive market demand or ask for a fairly low price to take over the entire market. These two possible alternatives may cause a jump in the best response of the QR supplier. The QR supplier may prefer a high price to reap excessive demand when the normal supplier asks for a low price, whereas the QR supplier is incentivised to occupy the entire market by matching the price of the normal supplier when the normal supplier quotes a high price. Consequently, the Nash equilibrium under the dual sourcing strategy might not exist. The following are examples for which the Nash equilibrium may and may not exist. Figure 4.5 shows the best response of each supplier under the production costs \( c_1 = 0.3, c_2 = 0.4 \). In Figure 4.5(a), the demand follows \( U(0, 8) \) distribution and the Nash equilibrium is uniquely determined by the intersection of the best responses. While in Figure 4.5(b), where the demand is \( U(1, 9) \) distributed, the Nash equilibrium does not exist. We can see that the payoff function of supplier 2 is not continuous in \( w_2 \), noting particularly that supplier 2 experiences a sudden shrinkage in profit when slightly increasing his price from \( w_1 \) to \( w_1 + \delta \), due to losing a sizable portion of demand to supplier 1. Another observation is that the QR supplier does not necessarily ask for the market price. The reason is that, as the QR supplier raises his wholesale price, portions of the order may be taken by the normal supplier.
Nash Bargaining Solution When the retailer adopts the single sourcing strategy, it is unlikely for the suppliers to cooperate. However, when the retailer decides to allocate the order between two suppliers, it becomes natural for the suppliers to partner to determine their prices that generate a higher level of profit for each supplier. Moreover, it is more likely for the suppliers to cooperate when it is difficult to predict the other supplier’s strategy, e.g., when there are multiple equilibria or no equilibrium in the supplier pricing game. Therefore, we set out to examine the suppliers’ decisions through the lens of the Nash bargaining solution (Muthoo, 1999). Different from Nash equilibrium where the suppliers strategically make their decisions such that no supplier has the incentive to deviate unilaterally, the Nash bargaining game provides a mechanism to allow the suppliers to negotiate and reach a Pareto-efficient result. In our setting, the Nash bargaining solution is the negotiated wholesale price pair \((w_{1}^{D, b}, w_{2}^{D, b})\) that
solves the following optimization problem (P):

\[
\begin{align*}
\max_{w_1, w_2} & \quad h(w_1, w_2) := (\pi_{s_1}(w_1, w_2) - d_1)(\pi_{s_2}(w_1, w_2) - d_2) \\
\text{s.t.} & \quad \pi_{s_1}(w_1, w_2) \geq d_1, \\
& \quad \pi_{s_2}(w_1, w_2) \geq d_2, \\
& \quad w_1 \leq w_2 \leq 1, \\
& \quad \pi_{s_1}(w_1, w_2) = (w_1 - c_1) \bar{F}^{-1}\left(\frac{w_1}{w_2}\right), \\
& \quad \pi_{s_2}(w_2, w_2) = (w_2 - c_2)\mathbb{E}\left([D - \bar{F}^{-1}\left(\frac{w_1}{w_2}\right)]^+\right),
\end{align*}
\]

where \(d_1, d_2\) are the profits of the normal supplier and the QR supplier, respectively, if they do not reach any agreement in the negotiation. In our study, we assume that the disagreement point is exogenous and both suppliers have full knowledge of the disagreement point. Without additional information, we may further assume that \(d_1 = d_2 = 0\). It is not difficult to observe that if \((d_1, d_2) = (0, 0)\), then problem (P) is always feasible. In the next result, we show that if an agreement can be reached, the price of the QR order will be set to the market price.

**Theorem 4.5.4.** In the supplier pricing game under dual sourcing, if problem (P) is feasible, then the Nash bargaining solution is always achieved at \(w_{2D,b} = 1\).

Theorem 4.5.4 says the Nash bargaining price for the QR supplier is always the market price. Using this result, we can further simplify the Nash bargaining price for the normal supplier. Since \(w_{2D,b} = 1\), problem (P) is reduced to \(\max_{w_1 \leq 1} (w_1 - c_1) \bar{F}^{-1}(w_1)(1 - c_2) \int_{F^{-1}(w_1)}^{\infty} \bar{F}(x)dx\).

Therefore, the optimal solution \(w_{1D,b}^{D,b}\) is independent of \(c_2\), and it must satisfy the first-order condition:

\[
(1-c_2) \left[ \bar{F}^{-1}(w_1) \int_{F^{-1}(w_1)}^{\infty} \bar{F}(x)dx - \frac{w_1 - c_1}{f(F^{-1}(w_1))} \int_{F^{-1}(w_1)}^{\infty} \bar{F}(x)dx + \frac{(w_1 - c_1)\bar{F}^{-1}(w_1)w_1}{f(F^{-1}(w_1))} \right] = 0.
\]

It is worthwhile noticing that problem (P) can be infeasible for some disagreement points. In that scenario, one of the suppliers does not want to cooperate but instead prefers the payment at
the disagreement point. Specially, we point out that Theorem 4.5.4 is valid for any disagreement point \((d_1, d_2)\) as long as problem (P) is feasible, but the disagreement point may affect the Nash bargaining price for the normal supplier \(w_1^{D,b}\).

Before we move on to discuss the retailer’s sourcing strategy, we compare the Nash equilibrium and the Nash Bargaining solution under the dual sourcing strategy in Theorem 4.5.5.

**Theorem 4.5.5.** *In the supplier pricing game under dual sourcing, if 1) the Nash equilibrium exists, and 2) distribution of \(D\) has an increasing failure rate, then we have \(w_1^{D,b} \geq w_1^D\).*

Although the Nash equilibrium of the supplier pricing game under dual sourcing may or may not exist, Theorem 4.5.5 asserts that when the Nash equilibrium exists, the normal supplier’s price will be higher under the Nash bargaining solution compared with his price under the Nash equilibrium. The increasing failure rate is required to show that the best response of the normal supplier is increasing with the price of the QR supplier. A distribution with pdf \(f(\cdot)\) and complementary cdf \(\hat{F}(\cdot)\) is said to have an increasing generalized failure rate (IGFR) when \(\frac{f(\cdot)}{\hat{F}(\cdot)}\) is nondecreasing. The IFR is a more restrictive condition than the IGFR. All IFR distributions are also IGFR, but the converse is not necessarily true. Many well-known distributions, including the uniform, exponential, Normal, logistic and Lapalace distributions are IFR.

### 4.5.3 Retailer’s Preferred Sourcing Strategy

General wisdom suggests that the retailer should seek as much flexibility as possible and prescribe a combination of long lead-time and short lead-time sources. In §4.3.3, we saw that when the demand is exponentially distributed, the retailer is always better-off under single sourcing. However, when the demand is characterized by a general distribution, the equilibrium analysis under the dual sourcing strategy is intractable, so it will be difficult to compare the retailer profit directly. Without calculating the retailer profit under equilibrium, the next result provides some insights on the question of when will single sourcing continue to be a superior strategy as demand follows an arbitrary distribution. Its proof is readily obtained from the explanation that follows.
Theorem 4.5.6. In the supplier pricing game under dual sourcing, if the QR supplier charges the market price, then the retailer receives a higher profit under single sourcing than under dual sourcing.

If the pure strategy Nash equilibrium exists and the QR supplier asks for the market price under equilibrium, it can be seen, from the normal supplier’s profit function, that the normal supplier will ask for his monopoly price. Since the QR supplier asks for the market price, the retailer does not add to her profit by sourcing from the QR supplier. Under such a scenario, the retailer profit is equivalent to ordering from a monopoly normal supplier. However, when the retailer adopts the single sourcing strategy with the presence of two suppliers, both suppliers reduce their prices to compete for the retailer order. Therefore, the retailer actually receives more profit under the single sourcing strategy.

In addition to the Nash equilibrium, Theorem 4.5.6 still holds when the suppliers set their prices based on the Nash bargaining solution. When suppliers negotiate to set their wholesale prices, the QR supplier will charge the market price, hence the condition in Theorem 4.5.6 is automatically satisfied. We learn from the previous discussion that under Nash equilibrium, if the QR supplier charges the market price, the retailer is better-off under single sourcing. Theorem 4.5.5 suggests that the normal supplier will charge a higher price under Nash bargaining solution, further hurting the retailer’s profit. Therefore, the retailer receives a higher profit under single sourcing when the suppliers adopt the Nash bargaining solution.

In conclusion, dual sourcing mitigate the competition between two types of suppliers with different lead times, thus the suppliers are able to inflate their prices. Moreover, negotiation creates a mechanism for suppliers to price collaboratively. Consequently, suppliers are able to reach consensus to increase their prices and gain greater profits from the retailer.
4.6 Concluding Remarks

In this chapter, we consider a supply chain with QR capability and endogenized wholesale prices. In the model, we account for not only the vertical relationship between the retailer and suppliers but also the horizontal competition for the retailer order among the suppliers. In the basic model with one normal supplier and one QR supplier, when the demand is exponentially distributed, the retailer’s optimal ordering policy has been characterized. This result provides insights for the retailer on the critical trade-off between cost and responsiveness. A game theoretic approach is applied to study the suppliers’ pricing strategies which can guide the suppliers to set the right wholesale prices in a competitive market. Per the equilibrium prices, the QR supplier will ask for the market price to reap the excessive demand and the normal supplier will ask for the monopoly price. As the suppliers raise their prices and grab more profits from the retailer under dual sourcing, the retailer is actually better-off with the single sourcing strategy.

When additional QR suppliers get involved, we find that the retailer is better-off by inviting more QR suppliers to participate in the supply chain, which results in lower purchase costs for the retailer and thus increasing profit. The suppliers should discourage the introduction of additional suppliers because the intensifying market competition would damage their interests. The impact on the whole supply chain is more interesting. Under single sourcing, the supply chain is unaffected. Whereas, under dual sourcing, the profit of the supply chain changes in every possible way (increases or decreases) depending on the production costs. In particular, when the production cost of the QR supplier is not significantly higher than that of the normal supplier, the introduction of additional QR suppliers can contribute to the supply chain efficiency by mitigating the misallocation between the first order and the appended order. This result, though unexpected, may bode well for the government. The government are encouraged to take actions to promote a competitive market.

We have also extended the analysis to the general case with an arbitrary demand distribution. Under dual sourcing, the QR supplier has two alternatives: either quote a sufficiently
low price to kick the normal supplier out of the market or quote a relatively high price to reap the excessive demand. Consequently, the payoff function of the QR supplier is the maximum of two functions. We find that the pure strategy Nash equilibrium of the supplier pricing game under dual sourcing may not exist. Without a pure strategy Nash equilibrium, the suppliers are incentivised to negotiate to determine their prices collectively. We have applied the concept of Nash bargaining solution and find that the suppliers will always charge the retailer the market price for the QR order. As a result, the retailer will prefer single sourcing to dual sourcing if the suppliers are to negotiate their prices.

A fundamental assumption in this work was that the actual demand is revealed prior to an order from a QR supplier. If this assumption is removed, we believe that, under high-quality demand forecast, our results continue to hold. Specifically, as the QR supplier becomes less competitive, the two lines in Figure 4.2 shift towards the bottom-right corner of the graph. The difficulties lie in finding the Nash equilibrium to the supplier pricing game under dual sourcing. The Nash equilibrium may not exist. Even if the Nash equilibrium exists, the explicit form of the equilibrium prices remains intractable. We haven’t yet explored the scenario when the performance of forecasts degrades, but we think it will be interesting to conduct numerical experiments to obtain the basic insights on the value of forecast quality.
Chapter 5

Pricing Game with a Bounded Rational Retailer

5.1 Introduction

Pricing concerns how a company should set and adjust the price of goods/products in order to maximize its profits. Pricing decisions are ubiquitous and critical determinants of profitability. However, pricing strategies may vary considerably across different industries, areas, and consumers. In general practice, to set prices, companies consider several factors such as production costs, price of similar products, and value to customers. Therefore, to maximize profitability, a company should measure the value its product brings to the customers and survey the customer’s purchasing behavior. In a competitive environment, a prediction bearing on the pricing of its competitors is inevitable.

It is well-known that a buyer’s behavior has a notable impact on the supplier’s revenue. However, most of the pricing models found in the literature assume that the buyer behaves perfectly rationally, which means the buyer always chooses the decision that maximizes his expected profit. In practice, pricing models are not as accurate as they could be. The limitation in human information processing further restricts the accuracy of diagnosis and decision.
Deviations from rational behavior of buyers have been reported. The list includes anchoring where the initial piece of evidence provides a cognitive “anchor” for the decision maker’s belief, salience bias where decision makers selectively process the cues on top of the information display, confirmation bias where the subjects tend to seek information that confirms the chosen hypothesis and reject information that disconfirms it, and availability heuristic where one’s past experience influences the final judgment. Interested readers are referred to Wickens et al. (2015) for the common psychological biases and decision-making heuristics. These limitations lead people to adopt heuristics, or “mental shortcuts”, instead of striving for the optimal solution.

Perfect rationality of buyer’s behavior has also received challenges from different scholars in a variety of empirical settings. Chen (2008) reports the “herd behavior” in purchasing books online. Hausman (2000) measures, both qualitatively and quantitatively, the consumer “impulse buying behavior”. Warren et al. (2011) reveal the consumer preference construction process by showing how value and preference are influenced by goals, cognitive constraints, and experience. Van Osselaer and Alba (2000) find evidence of a learning process that enhances brand equity. These empirical studies indicate that it is hardly realistic to assume that buyers are perfectly rational. Researchers begin to feel the urgency to capture and incorporate human behavior into the standard normative approach.

In this chapter, we study a two-echelon supply chain where a bounded rational retailer sources from multiple suppliers to address the issue on “What decisions will the retailer make when she is constrained by bounded rationality and occasionally makes mistakes?” “How will the suppliers respond if they realize that the retailer is subject to bounded rationality?” and “What is the impact of bounded rationality on the retailer and on suppliers?” To answer these questions, we focus on the model of “decision making with noises” as one possible way of incorporating bounded rationality into the picture. As the noise becomes negligibly small, our model is reduced to the normative case with the retailer being perfectly rational.

The bounded rationality of the retailer can be characterized by the classic quantal choice
model (Anderson et al., 1992; Su, 2008), in which the retailer’s ordering decision is no longer a deterministic but instead a mixed strategy. In other words, when faced with multiple suppliers \( i \in S \) generating utilities \( \{u_i\} \), the retailer does not always choose the utility-maximizing alternative(s) \( i \in \arg \max_i u_i \). Instead, the retailer may randomize over all the suppliers and choose the supplier yielding a higher expected utility with a higher probability. The quantal choice model posits that the probability of supplier \( i \) being chosen is proportional to \( e^{u_i} \). In this logit structure, better alternatives are chosen more often. To measure the extent of bounded rationality, the quantal choice model utilizes a single parameter \( \beta \geq 0 \) with \( \beta = 0 \) representing the perfect rational paradigm. Since the retailer’s profit is contingent upon the pricing of suppliers, we further investigate the suppliers’ pricing decisions when they learn the bounded rationality of the retailer’s ordering behavior. We intend to conduct two different analyses to reveal the suppliers’ pricing strategies. The best response analysis is proposed for studying the scenario when a supplier knows the prices of his competitors while the Nash equilibrium analysis is adopted for studying the pricing game when suppliers quote their prices simultaneously. Moreover, we would like to understand how the retailer’s bounded rationality may impact suppliers’ pricing strategies and the revenue of each supply chain member.

Here, we particularly mention three papers in the literature that are closely related to our work. The first paper is by Su (2008), in which a bounded rational newsvendor problem is studied. The newsvendor is prone to random errors and mistakes in estimating his profit at certain ordering levels. Su (2008) captures this behavior of bounded rationality by the logit choice model and characterizes the ordering decision made by a bounded rational newsvendor. The results of his model successfully explain some anomalies found in empirical experiments. Our work resembles Su (2008) in adopting a similar model to characterize the bounded rational behavior, but we consider a retailer who is imperfect in estimating his utility of purchasing from a supplier. We are also interested in knowing the impact of retailer’s bounded rationality on the suppliers who control the wholesale prices. The second paper is by Calvo and Martínez-de Albéniz (2015), in which a retailer sources from one normal supplier and one quick response
supplier. It is found that when the suppliers make pricing decisions, dual sourcing does not necessarily create a higher profit for the retailer. We are inspired by the problem setting (i.e., wholesale pricing) which generates some interesting and unexpected results. Hence, we adopt the setting and extend it to the scenario when the retailer suffers from cognitive limitation. To focus on the impact of bounded rationality in our model, we assume there are two suppliers who are identical in terms of lead time. The third paper is by F. Li et al. (2015), in which the influence of consumer behavior on the pricing strategy of a dual-channel supply chain is studied. The paper allows the manufacturer to sell the product through a direct online channel and an indirect channel via a retailer. In such a dual channel, both the manufacturer and the retailer decide their prices to maximize their profits. In our work, we consider only the indirect channel but two suppliers are involved in a competition for the retailer’s order. The bounded rational behavior of the customers in F. Li et al. (2015) is characterized by the logit choice model, but too many factors are involved in their model to make the demand function tractable. Consequently, their results can only be derived based on simulation.

In this chapter, we use the logit decision model to provide an explicit expression for the best response function. We prove the existence of the Nash equilibrium with multiple suppliers and fully characterize the Nash equilibrium when there are two suppliers. It is interesting to show in later sections that, as opposed to conventional knowledge, suffering from bounded rationality, the retailer actually receives a higher profit when she is bounded rational, according to both analyses. The reason the retailer may benefit from her bounded rationality is that, with a perfectly rational retailer, an infinitesimal amount of price cut is sufficient to secure the order, while with a bounded rational retailer, a deeper price cut is required to attract the retailer. As a result, the retailer enjoys a higher profit as suppliers offer lower prices. This finding may lend insights to marketing and pricing practitioners to consider more novel pricing strategies.

The rest of this chapter is organized as follows. §5.2 describes the proposed model and states the assumptions made. The best response analysis and the Nash equilibrium analysis are provided in §5.3 and §5.4, respectively. Several extensions are explored in §5.5 and conclusions
are given in §5.6. All proofs are included in Appendix C.

5.2 The Model

Consider a retailer selling a product to consumers at a fixed market price \( r \). The retailer sources the product from two suppliers (\( i = 1, 2 \)) offering a homogeneous product. Before the selling season starts, the retailer must select which supplier to purchase from. The retailer’s utility of ordering from supplier \( i \) at a wholesale price \( w_i \) is given by \( u_i = r - w_i \). This linear utility function is a commonly made assumption in the literature of decision making (Jaffray, 1989; Candeal-Haro & Induráin-Eraso, 1995). When the consumer demand is deterministic and independent of the market price, we can normalize the consumer demand to 1 at will. The retailer is expected to order from the supplier providing a higher utility, i.e., the supplier offering a lower wholesale price. We further assume that the wholesale prices are endogenous variables set by the suppliers. Anticipating the retailer’s ordering strategy, suppliers compete via pricing. The competition drives each supplier to continuously reduce his price. As the supplier with a higher production cost cannot afford to reduce price below his production cost, the more cost-efficient supplier could simply lower his price by an infinitesimal amount to win the order. Theoretically, this infinitesimal amount can be ignored and, we can say that, in equilibrium, the supplier with a cost advantage wins the retailer’s order by quoting the production cost of his opponent.

Under this normative analysis, the retailer will definitely order from the supplier offering a lower price to maximize her utility. In our model, we consider the situation that the retailer lacks the ability to make an informed decision. This inability can result from various reasons including the limited information available to the retailer, the limited time the retailer has to make a decision, and the inherent cognitive and computational deficiency that prevent the optimal decision. The imperfection in rationality suggests that the retailer does not necessarily order from the supplier offering a lower price. Mathematically, this may be represented by adding a stochastic term into the utility of the retailer: \( U_i = u_i + \epsilon_i = r - w_i + \epsilon_i, \ i = 1, 2, \)
where factors caused by bounded rationality, such as limited information, biased perspectives, and cognitive limitations, are encoded in $\epsilon_i$. Then, the retailer is expected to source from the supplier with the highest $U_i$. When the retailer is perfectly rational, we have $\epsilon_i \equiv 0$ for $i = 1, 2$, and the retailer always purchases from the supplier offering a lower price. However, when the retailer is subject to bounded rationality, the retailer may purchase from either supplier, depending on the realizations of those $\epsilon_i$’s. As such, we may describe the retailer’s ordering behavior based on a probability measure. Since the suppliers’ profits depend on the retailer’s ordering strategy, it is foreseeable that the retailer’s behavior of bounded rationality affects the suppliers’ pricing decisions.

In the proposed model, we have made several assumptions. The aim of these assumptions is to derive tractable solutions and generate the basic insights. In what follows, we present the assumptions with explanations and discussions.

**Assumption 5.2.1 (Bounded Rational Retailer Assumption).** The baseline supply chain consists of one retailer with bounded rationality and two perfectly rational suppliers.

In reality, a perfectly rational decision is hardly possible in practice for any decision maker to achieve due to limited information, inherent cognitive constraints, limited amount of time to make a decision, etc. In an attempt to investigate the effect of bounded rationality when the retailer and the suppliers make decisions interactively, in our study, we assume the retailer is subject to bounded rationality in her decision making process but consider the suppliers to be perfect optimizers. It may also be interesting to check the bounded rational behavior of the suppliers. In that case, the Quantal response equilibrium could be introduced in place of the Nash equilibrium, and this research goes beyond the scope of this thesis.

**Assumption 5.2.2 (Multinomial Logit Function Assumption).** The bounded rational behavior of the retailer is characterized by the multinomial logit function such that the probability of supplier $i$ to be chosen is proportional to $e^{u_i/\beta}$, where $u_i$ is the utility generated by supplier $i$ and $\beta$ is the level of bounded rationality of the retailer.
The utility of a bounded rational retailer is modeled to have two parts, a deterministic part \( u_i \) and a random part \( \epsilon_i \). The retailer chooses to purchase from the supplier bringing the highest overall utility \( U_i \). Therefore, the retailer’s decision is uncertain and depends on the realization of the random utility. Assume that the random component \( \epsilon_i \)'s are independent and identically distributed Gumbel random variables with the following distribution function:

\[
F(x) = e^{-e^{-(x/\beta + \gamma)}},
\]

where \( \gamma \) is the Euler constant (0.57722), \( x \in \mathbb{R} \). Its mean is zero, and variance is \( \beta^2 \pi^2 / 6 \). Consequently, the parameter \( \beta \) describes the level of bounded rationality the retailer suffers from. As \( \beta \to 0 \), the probability function takes a value of either 0 or 1, suggesting that there is no uncertainty in the retailer’s purchasing behavior and the retailer always orders from the supplier offering a lower price. As \( \beta \) increases, the retailer becomes less sensitive to prices, such that as \( \beta \to \infty \), the retailer becomes oblivious to price and makes a random choice. Therefore, we refer to the magnitude of \( \beta \) as the level of bounded rationality. Given the distribution of the Gumbel random variable, the probability that the retailer selects supplier \( i \) is

\[
P_i = \mathbb{P}(U_i = \max\{U_1, U_2\}) = \frac{e^{u_i/\beta}}{e^{u_1/\beta} + e^{u_2/\beta}}, \quad i = 1, 2. \tag{5.1}
\]

As noted by Su (2008), the multinomial logit model offers an approach to model bounded rationality. With this logit structure, the best choice need not always be selected but a better choice is selected more often.

**Assumption 5.2.3** (Wholesale Price Only Assumption). The retailer’s purchasing decision depends solely on the wholesale prices quoted by suppliers.

In general, the retailer may consider different factors in selecting a supplier, such as product quality, supplier reliability, service and delivery time. To focus our discussion on bounded rationality (rather than other factors that complicate the supplier selection process), we assume that the wholesale price is the only decision made by the suppliers. Models that allow for
other decision factors such as lead time and reliability can be similarly developed. The key to constructing such models lies in building the deterministic part of the utility function for the retailer, while the multinomial logit function can be used to model the bounded rational behavior in a similar manner.

**Assumption 5.2.4** (Public Information Assumption). *The production cost of each supplier is public information.*

This assumption is needed to derive theoretically tractable results. Similar assumptions are made by scholars in the literature of supplier pricing games; see, e.g., Calvo and Martínez-de Albéniz (2015) and Babich et al. (2007). In the normative approach, when each supplier only know his own production cost, the theory of reverse auction provides the optimal bidding strategy for a supplier. Most of the published research assumes suppliers are symmetric and makes explicit assumptions on their cost distributions.

We conclude this section by describing our model in terms of game theory. The strategic decision makers in the supply chain are called *players*. The *best response* refers to the strategy that produces the most favorable outcome for a player taking other players’ strategies as given. The concept of *Nash equilibrium* is used to represent a stable solution to a game. A set of strategies constitutes a Nash equilibrium if no player has anything to gain by changing his strategy unilaterally. A Nash equilibrium solution can be obtained by a set of best response functions. In our context, players are the two suppliers who determine their own wholesale price. The best response offers a supplier the optimal strategy when he knows his competitor’s price. The Nash equilibrium solution is a set of wholesale prices under which no supplier has an incentive to deviate unilaterally.
5.3 Best Response Analysis

In this section, we present the best response analysis, which provides the best pricing strategy for a supplier when the price of his competitor is known. In our model, we assume the retailer sells to consumers at a fixed market price normalized to 1. As a result, the retailer never purchases from a supplier offering a price exceeding the market price. Knowing this fact, a supplier with a competitive production cost (lower than the market price) will not ask more than the market price. Let $i$ denote any of the suppliers, $j$ be the competitor of supplier $i$, $c_i$ be the production cost of supplier $i$. The profit of supplier $i$ is given by

$$\pi_{s_i}(w_i, w_j; \beta) = (w_i - c_i) \frac{1-w_i}{e^{-\beta} + e^{1-w_i}}, \tag{5.2}$$

where wholesale prices $w_i$ and $w_j$ are implicitly assumed to be within the range of $[0, 1]$. As suggested in Eq. (5.2), a supplier’s expected profit is the multiplication of his net profit given that the order is assigned to him and the probability that he actually receives the order. Figure 5.1 shows a supplier’s expected profit and its two components. Through the probability of being chosen, we can see, by comparing the four panels, how the retailer’s decision is affected by the extent of her bounded rationality. When $\beta \to 0$, the retailer always sources from the supplier offering a lower price. As $\beta$ increases, the retailer becomes less decisive, such that when $\beta \to \infty$, the retailer randomizes his decision among the suppliers despite their quoted wholesale prices. Given the bounded rationality level of the retailer and the price of his competitor, a supplier intends to choose the wholesale price maximizing his profit. It can be seen from Figure 5.1 when $\beta = 0.05$ and $\beta = 0.1$ that the supplier may reduce his price to increase his probability of being chosen when the retailer is not perfectly rational.
Figure 5.1: The supplier’s expected profit as a function of his wholesale prices under different
\( \beta \) when the competitor’s price is known: \( w_j = 0.8 \)

Next, we formalize the supplier’s best pricing strategy when he knows the price of his competitor in the following proposition.

Proposition 5.3.1. For supplier \( i \in \{1, 2\}, j \neq i \), given the competitor’s price \( w_j \ (w_j \leq 1) \) and the retailer’s bounded rationality level \( \beta \), the best pricing strategy of supplier \( i \) is given by

\[
    w_i^*(w_j; \beta) = \min \left\{ c_i + \beta + \beta \cdot W_0\left(e^{\frac{w_j - c_i}{\beta}} - 1\right), 1 \right\},
\]

(5.3)

where \( W_0(\cdot) \) is the principal branch of the Lambert W function.

The Lambert W function (Corless et al., 1996) is a set of inverse functions of \( f(z) = ze^z \). It is useful in combinatorics and commonly seen in solutions to exponential equations. The presence of the Lambert W function conceals the properties of the best pricing strategy. Hence, we plot \( w_i^*(w_j; \beta) \) as a function of \( \beta \) in an example when \( w_j = 0.8 \) in Figure 5.2. It is suggested
that a supplier first reduces and then raises his price as the retailer’s bounded rationality level increases. When the bounded rationality level is sufficiently high, the supplier’s best pricing is bounded by the market price, since the retailer will not accept any price higher than the market price. More properties on the best pricing strategy are provided in the next corollary.

Corollary 5.3.2. (i) There exists some $\tilde{\beta}(w_j)$ such that $w_i^*(w_j; \beta)$ is decreasing in $\beta$ for $\beta \in [0, \tilde{\beta}(w_j)]$ and increasing in $\beta$ for $\beta \in [\tilde{\beta}(w_j), \infty)$. 
(ii) For any $\beta \geq 0$, $w_i^*(w_j; \beta)$ is increasing in $w_j$. 
(iii) $\lim_{\beta \to 0} w_i^*(w_j; \beta) = \begin{cases} c_i, & \text{if } w_j \leq c_i, \\
 w_j, & \text{if } w_j > c_i. \end{cases}$

Throughout this chapter, we refer to “increasing/decreasing” in a weak sense. Part (i) formalizes our observation in Figure 5.2 that a supplier first decreases and then increases his price as the retailer becomes more irrational. Part (ii) suggests that, for any bounded rationality level $\beta$, the best pricing strategy of a supplier is an increasing function of his competitor’s price. Hence, it is recommended for a supplier to raise his price in response to his competitor’s price increase. Part (iii) shows that the consistency between the limiting behavior when $\beta \to 0$ and
the case of perfect rationality with $\beta = 0$. A perfectly rational supplier tries to match the price of his competitor. However, if his production cost is higher than his competitor’s price, he quotes his production cost and quits the market.

Figure 5.3 views the supplier’s best pricing strategy from another angle. It plots $w^*_i(w_j; \beta)$ as a function of $w_j$ for different values of $\beta$. Two observations can be drawn from Figure 5.3. First, it conforms with Corollary 5.3.2 (ii) that a supplier will ask for a higher price when the price of his competitor increases and, this is valid for all bounded rationality levels. Second, fixing $w_j$, we can see that a supplier tends to ask for a higher price when the retailer’s extent of bounded rationality is high. However, when the retailer is mildly affected by bounded rationality, the supplier may reduce his price, as indicated by Corollary 5.3.2 (i).

Figure 5.3: Supplier’s best pricing as $w_j$ varies

It is interesting to notice that a supplier will cut his price when the retailer is mildly affected
by bounded rationality. Figure 5.1 helps illustrate the supplier’s behavior. It is noted that the expected profit of a supplier is the multiplication of the net profit if he receives the order and the probability the retailer ultimately chooses to order from him. When the retailer is entirely rational, the supplier will quote an infinitesimal amount less than his competitor. Whereas, when the retailer is slightly irrational, the supplier is induced to maximize his expected profit by increasing his probability of being chosen by the retailer. An effective approach is to offer a lower price, which implies that the retailer gets a better deal due to his bounded rationality. As such, bounded rationality, traditionally viewed as a performance-degrading impediment, can potentially lead to an unexpected higher profit for the retailer.

Impact of Ignoring Bounded Rationality When the retailer is completely rational ($\beta = 0$), a supplier asks for a wholesale price of $w_i^*(w_j; 0) = \min\{c_i, w_j\}$. Failing to realize the bounded rational behavior of the retailer, the supplier simply quotes $w_i^*(w_j; 0)$ and experiences a potential profit loss. We are interested in knowing the supplier’s profit loss for mistakenly taking the retailer as perfectly rational and hence carry out some numerical studies. In the numerical studies, we fix the production cost $c_i$ to 0.3. In Figure 5.4, we examine the supplier’s profit loss under different levels of bounded rationality $\beta$ when his competitor’s price $w_j$ is given (low and high). In Figure 5.5, we show the profit loss with different prices of the competitor under a specific level of bounded rationality (low and high).

From Figure 5.4, we observe that the profit loss is a multimodal function of bounded rationality level $\beta$. There exists some $\bar{\beta}$ such that the supplier’s best response pricing when bounded rationality of the retailer is accurately accounted for coincides with his pricing when bounded rationality is ignored. Therefore, the supplier incurs no profit loss by ignoring bounded rationality at $\bar{\beta}$. At other levels of bounded rationality, there is a strictly positive profit loss. Moreover, whether a large profit loss is incurred at a moderate level or high level of bounded rationality depends on the competitor’s wholesale price. When a supplier fails to account for the bounded rational behavior of the retailer, he will match the price of his competitor. Two sources of profit loss can result from such blind price following. First, by following the price of
his competitor, the supplier fails to ask for a high wholesale price to reap a high profit margin. This kind of profit loss is significant when the competitor asks for a comparatively low price and when the retailer is subject to a relatively high level of bounded rationality. Second, the supplier could have strategically lowered his price to induce the retailer to order from him. However, an ignorant supplier fails to take this advantage, thus resulting in a profit loss. This kind of profit loss is significant when the other supplier asks for a comparatively high price and when the retailer suffers from a moderate level of bounded rationality. We find that the first kind of profit loss has a prominent effect in Figure 5.4a, while the second kind of profit loss dominates in Figure 5.4b.

Figure 5.5 indicates that the supplier’s profit loss of ignoring bounded rationality is significant when his competitor charges a low price since an ignorant supplier follows his competitor and quotes a low price. In this case, the first kind of profit loss is remarkable. In Figure 5.5a, there exists some $\bar{w}_j$ that ignoring the retailer’s bounded rationality results in the same pricing level as the best response pricing. When the competitor’s price continues to hike, the second kind of profit loss appears and gains its dominance. In Figure 5.5b, we cannot find such a $\bar{w}_j$ such that ignoring bounded rationality does not affect the supplier’s pricing strategy, and the second kind of profit loss never occurs. As a result, the profit loss function is decreasing in the competitor’s price.
5.4 Nash Equilibrium Analysis

The best response analysis provides the optimal pricing strategy for a supplier when he knows his competitor’s price. However, as it usually occurs in practice, a supplier does not know in advance his competitor’s price level. Thus, to analyze the pricing competition, we use the notion of Nash equilibrium. In our setting, a Nash equilibrium, if exists, is a pair of wholesale prices
determined by the two suppliers non-cooperatively. We are interested in the pure strategy Nash equilibrium, assuming that the suppliers are perfectly rational. Without loss of generality, it is assumed that supplier 1 is the more cost-efficient supplier, i.e., \( c_1 \leq c_2 \). We find that there is a unique Nash equilibrium to the supplier pricing game. Let \((w^\beta_1, w^\beta_2)\) denote the Nash equilibrium when the retailer’s bounded rationality level is \( \beta \); we then characterize the Nash equilibrium in the next proposition.

**Proposition 5.4.1.** There exists a unique pure strategy Nash equilibrium for the supplier pricing game when the retailer is bounded rational with parameter \( \beta \). Without loss of generality, we assume that \( c_1 \leq c_2 \); then the Nash equilibrium can be characterized as:

**Case 1:** If \( \beta \leq \beta^b \), where \( \beta^b \) satisfies \( \frac{1-c_2}{2} \leq \beta^b \leq \frac{1-c_1}{2} \) and \( W_0(e^{\frac{1-c_1}{\beta^b-1}}) = \frac{1-c_1}{\beta^b-1} - 1 + \ln(\frac{1-c_2}{\beta^b-1}) \), then \( w^\beta_1 = [\frac{1}{A(\beta^b)} + 1] \beta + c_1 \) and \( w^\beta_2 = [A(\beta) + 1] \beta + c_2 \), where \( A(\beta) \) is determined by the equation \( \ln A(\beta) = [\frac{1}{A(\beta)} - A(\beta)] + \frac{1}{\beta}(c_1 - c_2) \);

**Case 2:** If \( \beta^b < \beta \leq \frac{1-c_1}{2} \), then \( w^\beta_1 = c_1 + \beta + \beta \cdot W_0(e^{\frac{1-c_1}{\beta-1}}) \) and \( w^\beta_2 = 1 \);

**Case 3:** If \( \beta > \frac{1-c_1}{2} \), then \( w^\beta_1 = w^\beta_2 = 1 \).

We notice from Eq. (5.3) that the best response function is piecewise, taking the smaller of the expected profit maximizing solution determined by the first derivative and the market price. Therefore, the determination of the Nash equilibrium depends on which segments of the best responses intersect. When the best responses intersect at the first derivatives, we have Case 1. Under this case, the bounded rationality level of the retailer is mild, and none of the supplier’s best response is restricted by the fact that the retailer does not accept a price higher than the market price. Case 2 is valid when the pricing of supplier 2 is bounded by the market price and intersects the best response of supplier 1 at \( w_2 = 1 \). The bounded rationality level yielding this case is higher than that in Case 1 but still moderate. Case 3 holds when the Nash equilibrium is determined by the market price, and the underlying bounded rationality level is high. The three cases form a complete characterization of the Nash equilibrium because, under the assumption \( c_1 \leq c_2 \), the case that the pricing of supplier 1 is bounded by the market price and meet the best response of supplier 2 at \( w_1 = 1 \) is impossible. In the next corollary, we
present more properties of the Nash equilibrium.

**Corollary 5.4.2.** (i) If \( c_1 = c_2 \), then \( w_1^\beta = w_2^\beta = \min\{1, 2\beta + c_1\} \);

(ii) For supplier \( i \in \{1, 2\} \), \( j \neq i \), if \( c_i < c_j \), then \( w_i^\beta \leq w_j^\beta \), \( \forall \beta \);

(iii) For supplier \( i \in \{1, 2\} \), \( j \neq i \), if \( c_i < c_j \), then \( w_i^\beta - c_i \geq w_j^\beta - c_j \), \( \forall \beta \);

(iv) For supplier \( i \in \{1, 2\} \), \( j \neq i \), if \( c_i < c_j \), then \( \lim_{\beta \to 0} w_i^\beta = c_j \) and \( \lim_{\beta \to 0} w_j^\beta = c_j \).

Figure 5.6: Nash equilibrium of supplier pricing game

(Red line represents the best response of supplier 1 while blue line represents the best response of supplier 2. Nash equilibrium is the intersection of best responses.)

Figure 5.6 illustrates the properties of the equilibrium prices described by Corollary 5.4.2 through examples. The first three figures correspond to part (i) where the two suppliers have the same production cost, while the last three figures visualize parts (ii)-(iv) where the two suppliers
have different production costs. With two symmetric suppliers, Case 2 in Proposition 5.4.1 diminishes. It can be further checked that $A(\beta)$ can be explicitly solved with $A(\beta) = 1$ being the unique solution. Thus there exists a simple characterization of the Nash equilibrium. According to part (i) of Corollary 5.4.2, if the equilibrium price is not bounded by the market price, it is a linear function of the retailer’s bounded rationality level. Considering three different levels of bounded rationality of low, medium, and high, Figures 5.6a-5.6c show how the two suppliers raise their prices as they perceive the retailer to be more irrational. With two asymmetric suppliers, part (ii) suggests that the supplier with a lower production cost will ask for a lower price in equilibrium. However, per part (iii), the supplier with a lower production cost will have a higher mark-up. These imply that in the face of the competition, the supplier with the cost advantage strives to gain a higher profit margin, while he tries to keep the retailer by offering an attractive price. For the cost-inefficient supplier, though the bounded rational behavior of the retailer allows him to share a piece of the “cake” with the cost-efficient supplier, he is not able to take the dominating role in the market. Part (iv) shows the limiting behavior of the equilibrium, which corroborates the normative analysis with a perfectly rational retailer, in which the more cost-efficient supplier wins the retailer’s order by quoting the price of his competitor.

With two asymmetric suppliers, Corollary 5.4.2 Parts (ii)-(iv) show the comparison of the equilibrium price of each supplier and its limiting behavior. However, it is still unclear how the equilibrium prices are affected by the retailer’s level of bounded rationality. In the next corollary, we intend to shed lights on how the equilibrium prices change as the bounded rationality parameter $\beta$ increases.

**Corollary 5.4.3.** If w.l.o.g. we assume $c_1 < c_2$, then

1. There exists some $\tilde{\beta} < \beta^b$ such that $w_1^\beta$ is decreasing in $\beta$ for $\beta \in [0, \tilde{\beta}]$ and increasing in $\beta$ for $\beta \in [\tilde{\beta}, \beta^b]$, and
2. $w_2^\beta$ is increasing in $\beta$ for any $\beta \in [0, \beta^b]$.

Part (i) depicts the equilibrium price of the supplier with a cost advantage. This result
resembles Corollary 5.3.2 Part (i), in which the best pricing strategy of a supplier given the price of his competitor first decreases and then increases as the retailer becomes more irrational. The difference is that the best response pricing strategy continue to rise after $\beta^b$, while the trend of the equilibrium price for the supplier with a cost advantage after $\beta^b$ is uncertain. Part (ii) indicates that the supplier without the cost advantage continuously raises his price as the retailer becomes more irrational.

As predicted by Corollary 5.4.3 part (i), supplier 1 reduces his price as the retailer becomes irrational when the retailer’s bounded rationality level is low. It is reminiscent of the price cut in the supplier’s best pricing strategy in Figure 5.2. A similar price reduction is observed in the Nash equilibrium. More importantly, from the facts that supplier 1 has a cost advantage and that the supplier with a cost advantage asks for a lower price in the Nash equilibrium (Corollary 5.4.2), we can infer that the retailer is more likely to source from supplier 1. Consequently, the price reduction of supplier 1 is directly reflected in the retailer’s revenue and the retailer enjoys a higher profit when she is not perfectly rational and when she successfully delivers this information to suppliers. Taken in sum, we have reached a counter-intuitive conclusion that bounded rationality, commonly viewed as a performance-degrading barrier that must be overcome, can lead to an unexpected superior result for the retailer.

Next, we investigate when the retailer enjoys a higher level of profit boost. Noticing that the retailer’s profit depends on suppliers’ production costs, we consider all possible combinations of production costs of both suppliers from 0.1 to 0.9 with an increment of 0.1. The purport is to find the maximum possible profit increase for the retailer under different combinations of production costs. The results are shown in Table 5.1. We obtain the results by exhausting the range of bounded rationality and report the profit increase at the bounded rationality level that generates the highest profit for the retailer. In other words, Table 5.1 provides an upper bound for the percentage of profit increase induced by the retailer’s bounded rationality. We can see that the retailer embraces a higher level of potential profit increase when the cost difference of suppliers is high. This is because a greater difference in production costs offers a greater leeway
for the more cost-efficient supplier to strategically reduce his price. A deeper price reduction further contributes to the retailer’s profit.

Table 5.1: Retailer’s maximum possible profit increase in % under Nash equilibrium

<table>
<thead>
<tr>
<th>c1</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.00</td>
<td>1.05</td>
<td>2.41</td>
<td>4.21</td>
<td>6.74</td>
<td>10.53</td>
<td>16.85</td>
<td>29.49</td>
<td>67.34</td>
</tr>
<tr>
<td>0.2</td>
<td>1.05</td>
<td>0.00</td>
<td>1.20</td>
<td>2.81</td>
<td>5.05</td>
<td>8.42</td>
<td>14.04</td>
<td>25.27</td>
<td>58.97</td>
</tr>
<tr>
<td>0.3</td>
<td>2.41</td>
<td>1.20</td>
<td>0.00</td>
<td>1.40</td>
<td>3.37</td>
<td>6.32</td>
<td>11.23</td>
<td>21.06</td>
<td>50.55</td>
</tr>
<tr>
<td>0.4</td>
<td>4.21</td>
<td>2.81</td>
<td>1.40</td>
<td>0.00</td>
<td>1.68</td>
<td>4.21</td>
<td>8.42</td>
<td>16.85</td>
<td>42.12</td>
</tr>
<tr>
<td>0.5</td>
<td>6.74</td>
<td>5.05</td>
<td>3.37</td>
<td>1.68</td>
<td>0.00</td>
<td>2.10</td>
<td>5.62</td>
<td>12.63</td>
<td>33.70</td>
</tr>
<tr>
<td>0.6</td>
<td>10.53</td>
<td>8.42</td>
<td>6.32</td>
<td>4.21</td>
<td>2.10</td>
<td>0.00</td>
<td>2.80</td>
<td>8.40</td>
<td>25.27</td>
</tr>
<tr>
<td>0.7</td>
<td>16.85</td>
<td>14.04</td>
<td>11.23</td>
<td>8.42</td>
<td>5.62</td>
<td>2.80</td>
<td>0.00</td>
<td>4.20</td>
<td>16.85</td>
</tr>
<tr>
<td>0.8</td>
<td>29.49</td>
<td>25.27</td>
<td>21.06</td>
<td>16.85</td>
<td>12.63</td>
<td>8.42</td>
<td>4.20</td>
<td>0.00</td>
<td>8.40</td>
</tr>
<tr>
<td>0.9</td>
<td>67.34</td>
<td>58.97</td>
<td>50.55</td>
<td>42.12</td>
<td>33.70</td>
<td>25.27</td>
<td>16.85</td>
<td>8.40</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Impact of Ignoring Bounded Rationality When the retailer’s bounded rational behavior is ignored by the suppliers, the supplier with a comparatively higher production cost lowers his price to his production cost, and the other supplier matches his price. Similar to the best response analysis, we continue to examine the impacts when bounded rationality is ignored by the suppliers in the equilibrium analysis. Notice that the equilibrium prices are contingent on the production costs \( c_1, c_2 \) and the level of bounded rationality \( \beta \). Since we intend to examine the impact of bounded rationality, we vary \( \beta \) in each numerical study. To see the role of production costs, we fixed \( c_1 \) to 0.2, define \( \Delta c = c_2 - c_1 \), and consider two scenarios with a high \( \Delta c \) and a low \( \Delta c \), respectively. The results are shown in Figure 5.7.

Notice that there are some inflection points, causing local changes, in the profit function of each supply chain member. Each inflection point corresponds to a transition of different cases of the Nash equilibrium, which can be ultimately traced back to the assumption that the retailer sells to the consumers at a fixed market price and does not accept a price higher than the market price. Here, we do not discuss these inflection points and local profit changes specifically, but focus on the general trend of the profit change as the bounded rationality level \( \beta \) varies.
Figure 5.7: Profit under equilibrium with different levels of bounded rationality and profit when bounded rationality is ignored by suppliers.

First, we examine how the profit of each supply chain member is affected by the retailer's
bounded rationality level when bounded rationality is correctly considered by suppliers. It can be seen that the retailer’s profit first increases and then decreases as the retailer becomes more irrational. The initial profit increase suggests that the retailer benefits from her bounded rationality when $\beta$ is small, verifying what we have discussed. Moreover, the amplitude of profit increase is contingent on the difference of production costs $\Delta c$. A bigger difference in production costs leads to a larger degree of profit increase. The impact of bounded rationality on the cost-efficient supplier, namely supplier 1, also relies on the difference of production costs. When the bounded rationality level is low, it initially causes a decline in the profit function of supplier 1 due to the synergistic effects of two factors: first, due to the retailer’s bounded rationality, the order goes to the other supplier with a positive probability; second, the supplier strategically lowers his price to entice the retailer to order from him. These two factors collectively contribute to an initial profit decline of supplier 1 when $\beta$ is small. As $\beta$ increases, whether supplier 1 gains a higher profit relies on the difference of production costs. When $\Delta c$ is small, the profit of supplier 1 under perfect rationality is low. Both suppliers inflate their prices when the retailer becomes irrational. Hence, the profit drop of supplier 1 is followed by a bounce. On the other hand, when $\Delta c$ is big, the profit of supplier 1 under perfect rationality is already high. The consequent price inflation is not sufficient to compensate for the initial profit drop. Consequently, supplier 1 suffers from the retailer’s bounded rationality when $\Delta c$ is sufficiently high. The impacts of bounded rationality on supplier 2 and the supply chain are comparatively simple. Supplier 2 benefits from the retailer’s bounded rationality, because he does not receive any order when the retailer is perfectly rational while he is able to share the market with supplier 1 when the retailer becomes irrational. The supply chain efficiency is impaired by the bounded rational behavior of the retailer. The reason is that the retailer always purchases from the cost-efficient supplier when there is no behavioral issue, whereas she may order from the less cost-efficient supplier when the retailer is constrained by bounded rationality.

When the retailer’s bounded rationality is neglected by suppliers, supplier 2 asks for a
wholesale price of $c_2$ and supplier 1 quotes an infinitesimal amount less than $c_2$. Since the infinitesimal amount can be ignored, the retailer’s profit is a constant. Supplier 1 experiences a strict profit loss because, the retailer, confined by her bounded rationality, allocates a portion of the order to supplier 2. The profit of supplier 2 is zero because he lowers his price to his production cost. The supply chain achieves the highest level of profit when the order is always made from the cost-efficient supplier. The profit of the supply chain deteriorates as the retailer orders from the less cost-efficient supplier with a higher probability. The probability distribution of the retailer’s purchasing decision relies on her bounded rationality level and the price difference between two suppliers. When $\beta$ is minor, the retailer regards supplier 1 as a better supplier, despite the inconspicuous price difference. Therefore, the profit loss is trivial. When $\beta$ is significant, the price difference whether bounded rationality is taken into account is nearly zero. Therefore, there is little profit loss. When $\beta$ is moderate, we observe a significant profit loss since ignoring bounded rationality results in a much lower price difference, whose effect can hardly be counteracted by a moderate level of bounded rationality. As a result, the retailer orders from supplier 2 with a higher chance, degrading the supply chain profit.

5.5 Extensions

In this section, we explore our model in several ways. The model we present has two identical suppliers, so we first extend to multiple suppliers in §5.5.1. An explicit expression for the supplier’s best response is provided and the Nash equilibrium under certain special circumstances is investigated. Noticing that the Nash equilibrium is generally intractable, we present in §5.5.2 the numerical results of the Nash equilibrium of the supplier pricing game with three suppliers. §5.5.3 studies the case when the retailer has an option of not purchasing anything from either supplier.
5.5.1 N-Suppliers

In our analysis thus far, we have considered two suppliers competing for the order of a retailer subject to bounded rationality. The two-supplier case allows us to derive explicit solutions and generate basic insights. More generally, there are usually more than two suppliers in the market. Therefore, in this subsection, we extend the supplier base to \( n \) suppliers and intend to answer the following questions: How does a supplier change his pricing strategy when there are more than two suppliers? What are the resulting equilibrium prices? Similar to the two-supplier case, we first study the best pricing strategy of a supplier when he learns his competitor’s price, as shown in the next proposition. Then based on the result of the best response analysis, we proceed to characterize the Nash equilibrium.

**Proposition 5.5.1.** When there are \( n \) suppliers, given the price of other suppliers \( w_j \ (w_j \leq 1) \) for any \( j \neq i \) and the retailer’s bounded rationality level \( \beta \), the best pricing strategy of supplier \( i \) is given by

\[
 w_i^*(w_{-i}; \beta) = \min \left\{ c_i + \beta + \beta \cdot W_0 \left( \frac{e^{-\left( \frac{1}{\beta^2} \right)^{1/2} + 1}}{\sum_{j \neq i} e^{-\frac{w_j}{\beta^2}}} \right), 1 \right\}, \quad (5.4)
\]

where \( W_0(\cdot) \) is the principal branch of the Lambert W function.

We can further derive the following limiting behavior of the best response functions:

\[
 \lim_{\beta \to 0} w_i^*(w_{-i}; \beta) = \begin{cases} 
 \min_{j \neq i} w_j & \text{if } \min_{j \neq i} w_j \geq c_i, \\
 c_i, & \text{otherwise}. 
\end{cases} \quad (5.5)
\]

Eq. (5.5) conforms with the normative analysis in which a perfectly rational supplier matches the current lowest price in the market if his production cost permits. Otherwise, the supplier lacks the competitive advantage and quits the market by quoting his production cost.

As we try to characterize the Nash equilibrium, we find the analysis intractable. In order to acquire the basic insights, we consider the Nash equilibrium under two special scenarios. First, when the bounded rationality level \( \beta \to 0 \), the equilibrium prices can be analyzed. Without
loss of generality, the suppliers can be ranked by their production costs from low to high, i.e., 
$c_1 \leq c_2 \leq \cdots \leq c_n$. It is not difficult to check that $(c_2, c_2, c_3, \cdots, c_n)$ satisfies the best response of each supplier and therefore forms a Nash equilibrium of the $n$-supplier pricing game. This result, consistent with the normative analysis, suggests that, when the retailer is perfectly rational, the most cost-efficient supplier charges the production cost of the supplier with the second lowest cost and the other suppliers quote their own production cost. Second, the symmetric case with $n$ suppliers of the same production cost, i.e., when $c_1 = c_2 = \cdots = c_n$, can be analyzed. In equilibrium, each supplier quotes $w_1^\beta = w_2^\beta = \cdots = w_n^\beta = \min\left\{c_1 + \frac{n}{n-1} \beta, 1\right\}$. It suggests that each supplier asks for a lower price as the number of suppliers increases, echoing the fact that market competition drives suppliers to lower their prices.

Although the equilibrium wholesale prices are generally intractable with the presence of $n$ suppliers, we find that there always exists a pure strategy Nash equilibrium. In fact, the existence of pure strategy Nash equilibrium only requires a general non-increasing utility function on the retailer. The next result summarizes our findings.

**Proposition 5.5.2.** In the wholesale pricing game consisting of $n$ suppliers, if the retailer’s utility function $u(w_i)$ of ordering from supplier $i$ offering a wholesale price $w_i$ is a non-increasing function of $w_i$, then there exists a pure strategy Nash equilibrium.

The assumption that the retailer’s utility is a non-increasing function of her purchase price is commonly made in previous research and widely received in practice. The wholesale price is the retailer’s monetary investment in obtaining the product. Hence, other factors (quality, reliability, after-sales service, etc) being equal, a retailer’s general utility is negatively related with the wholesale price. The non-increasing utility function ensures that a supplier heightens (drops) his price in response to the price increases (decreases) of his competitors. This increasing best response guarantees the existence of a pure strategy Nash equilibrium.
5.5.2 Numerical Results for the 3-Supplier Case

As we have mentioned, the Nash equilibrium is generally intractable with more than two suppliers. In this subsection, we conduct numerical experiments for the three-supplier case with different production costs. The purpose of the numerical experiments is to test whether our findings and insights with two suppliers can be extended to multiple suppliers. In the numerical study, we fix the production costs of supplier 1 and supplier 2 to $c_1$ and $c_2$, respectively, and change the cost of supplier 3 from $\max\{c_1, c_2\} + 0.1$ to 1 with an increment of 0.1. Specifically, the production cost $c_3$ equaling to 1 (the market price) indicates a completely inefficient supplier, resulting in the same equilibrium price for each supplier. For any fixed $c_3$, we then vary the bounded rationality level from 0 to 0.2 and search for the equilibrium price for each supplier.

Table 5.2: Numerical results for the equilibrium prices of three-supplier game: $c_1 = 0.2$, $c_2 = 0.3$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$c_3 = 0.4$</th>
<th>$c_3 = 0.5$</th>
<th>$c_3 = 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$w_3$</td>
</tr>
<tr>
<td>0</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.4000</td>
</tr>
<tr>
<td>0.005</td>
<td>0.2911</td>
<td>0.3053</td>
<td>0.4050</td>
</tr>
<tr>
<td>0.008</td>
<td>0.2901</td>
<td>0.3088</td>
<td>0.4080</td>
</tr>
<tr>
<td>0.01</td>
<td>0.2904</td>
<td>0.3112</td>
<td>0.4100</td>
</tr>
<tr>
<td>0.05</td>
<td>0.3360</td>
<td>0.3721</td>
<td>0.4532</td>
</tr>
<tr>
<td>0.1</td>
<td>0.4043</td>
<td>0.4487</td>
<td>0.5193</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5491</td>
<td>0.5994</td>
<td>0.6635</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$c_3 = 0.7$</th>
<th>$c_3 = 0.8$</th>
<th>$c_3 = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$w_3$</td>
</tr>
<tr>
<td>0</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.7000</td>
</tr>
<tr>
<td>0.005</td>
<td>0.2911</td>
<td>0.3053</td>
<td>0.7050</td>
</tr>
<tr>
<td>0.008</td>
<td>0.2901</td>
<td>0.3088</td>
<td>0.7080</td>
</tr>
<tr>
<td>0.01</td>
<td>0.2904</td>
<td>0.3112</td>
<td>0.7100</td>
</tr>
<tr>
<td>0.05</td>
<td>0.3445</td>
<td>0.3764</td>
<td>0.7500</td>
</tr>
<tr>
<td>0.1</td>
<td>0.4355</td>
<td>0.4694</td>
<td>0.8015</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5969</td>
<td>0.6368</td>
<td>0.9217</td>
</tr>
</tbody>
</table>

It can be seen from Table 5.2 that under a small $\beta \leq 0.01$, the pricing strategies of
supplier 1 and supplier 2 are nearly insusceptible to the introduction of the third supplier. This suggests that adding a cost-inefficient supplier can hardly change the current competitive landscape when the retailer stays relatively rational. However, when the retailer suffers from a considerable level of bounded rationality, the involvement of the third supplier forces the current suppliers to reduce their prices, and the resulting price cut is deeper when the third supplier is more competitive (with a lower production cost). The underlying rationale that competition leads to lower prices is consistent with our result of n symmetric suppliers that the equilibrium price is decreasing in the number of suppliers.

5.5.3 2-Suppliers with a No-Purchase Option

In this chapter, we assume that suppliers know the retailer sells to the customers at a fixed market price normalized to 1. Therefore, in order to win the retailer’s order, suppliers will not ask for more than the market price. The bounded rational behavior of the retailer is reflected in influencing the retailer’s decision of which supplier to source from, based on the condition that the retailer always purchases from one of the suppliers. In fact, we use the term supplier and retailer to refer to upstream and downstream of the supply chain to invoke the flow of materials. Our model might as well be applied to the scenario in which a retailer sells a product to customers. In this case, a customer can leave the store empty-handed without buying anything. In an attempt to extend our current model to the retailer-customer setting, we allow the retailer not to purchase from any of the suppliers, and by doing so, the retailer receives an expected utility of 0 while a random utility is present to complicate the retailer’s choice. In modeling, an option of not purchasing with deterministic utility \( u_0 = 0 \) is available to the retailer. Consequently, the probability that the retailer selects supplier \( i \) becomes

\[
P_i = \frac{\frac{1-w_i}{e^{\frac{1-w_i}{\beta}}}}{\sum_{j \in S} \frac{1-w_j}{e^{\frac{1-w_j}{\beta}}} + 1},
\]

108
and the probability that the retailer does not to purchase from any of the suppliers is

\[ P_0 = \frac{1}{\sum_{j \in S} e^{-\beta} + 1}. \]

Under the two-supplier setting, we compare the equilibrium prices when the retailer can choose not to purchase from any of the suppliers with the equilibrium prices in the original model in Figure 5.8. It can be seen that for a small \( \beta \), the impact of having the no-purchase option on suppliers’ pricing strategies is very small or nearly non-existent. However, with a moderate \( \beta \), both suppliers are forced to reduce their prices due to the additional no-purchase option available to the retailer. Moreover, the price cut on the cost-efficient supplier is more prominent, especially when the price difference between the two suppliers is high.

It is worth mentioning that in the retailer-customer setting, different customers may be subject to different levels of bounded rationality. Therefore, this extension can be applied to the case when all customers are of the same bounded rationality level. In reality, bounded rational behavior is more frequently observed in customers than in retailers. We think this extension of including a no-purchase option is one step towards extending our model to the retailer-customer setting.
5.6 Concluding Remarks

Most theoretical models assume that decision makers are perfectly rational and always select the best choice. This assumption is too restrictive and ignores the cognitive limitations of human beings. Bounded rationality is an observable and proven human behavior associated with decision making. We notice that retailers are substantially restricted by bounded rationality due
to either incomplete information or limited capability in information procession and integration. In this work, we consider a bounded rational retailer in the sense that instead of always making the best decision, she makes better choices more often than those bad choices. The bounded rational behavior of the retailer is characterized by the multinomial logit model, an ingenious framework that utilizes a single parameter to delineate the bounded rationality level of the retailer. We allow suppliers to control their wholesale prices and investigate the impact of the retailer’s bounded rationality on the suppliers’ pricing strategies. We analyze the best response of a supplier when the price of his competitor is known and the Nash equilibrium when two suppliers set their wholesale prices simultaneously. Among the results, we found one particularly interesting observation, i.e., the retailer gains a higher profit when she is slightly irrational, contradicting general wisdom that an individual usually suffers from bounded rationality. In the best response analysis, when the retailer is slightly irrational, reducing price yields the supplier a higher profit through increasing the chance of sealing the order. The reason a price cut can contribute to the chance of completing the order is that under perfect rationality, an infinitesimal amount of price cut is sufficient to secure the order, while under bounded rationality, a higher degree of price difference is required to attract the retailer’s order. Interestingly, a similar price reduction is found in the Nash equilibrium analysis, and moreover, the supplier who cuts his price is the cost-efficient supplier, from whom the retailer is more likely to make an order. Consequently, the price reduction from the cost-efficient supplier is reflected in the retailer’s profit, and the retailer enjoys a higher profit when she slightly deviates from perfect rationality.

There are several implications for each supply chain member on dealing with the retailer’s bounded rationality. The counter-intuitive finding that the retailer can benefit from her bounded rationality provides a fresh perspective on bounded rationality: instead of striving for perfect rationality, the retailer might as well maintain in a state of slight imperfection; and moreover, the retailer could endeavor to project on the suppliers an image of an incomplete rationality. Before converting these results into practice, it is imperative to estimate the retailer’s (or maybe
customers’ bounded rationality level in different product industries. On the part of the suppliers, whether the supplier with a lower production cost benefits or suffers from the retailer’s bounded rationality is contingent on the difference of the production costs, which determines the profit leeway when the retailer is fully rational. The supplier with a higher production cost benefits from the retailer’s bounded rationality, since he receives zero profit when the retailer is fully rational. Therefore, it is advised that suppliers undertake a careful examination into the retailer’s purchasing behavior and take advantage of the retailer’s bounded rationality. Ignorance or erroneous estimation of the retailer’s bounded rationality can result in a serious profit shrinkage. From the perspective of the whole supply chain, the existence of bounded rationality deteriorates the efficiency of the supply chain because, with a strictly positive probability, the order is allotted to the less cost-efficient supplier.

There are three main contributions of this chapter. First, we are the first to model bounded rationality in a pricing game and offer solutions that are analytically tractable. The quantal choice model is prominently general to capture the bounded rationality of the retailer, while remaining extremely parsimonious in that there is only one parameter to be estimated. Second, we provide insights to managers and practitioners for dealing with retailer or customers’ bounded rationality. Ignorance or erroneous estimation of the retailer’s behavior can substantially undermine the company’s profitability. Third, this work complements the behavioral and theoretical rationales in the existing literature, and provides a framework to stimulate future empirical and experimental work for pricing problems and behavioral operations.
Chapter 6

Conclusions

In this chapter, we summarize the research in this dissertation, highlight its contributions and point out the avenues for future research. §6.1 recaps the findings in Chapters 3, 4 and 5, respectively. Future research directions are discussed in §6.2.

6.1 Summary and Contributions

Different entities in the supply chain often have conflicting objectives. For instance, wholesale price is the price a supplier charges a retailer for a product sold in bulk. A supplier favors a high wholesale price to generate a higher unit margin for himself while a retailer prefers a low wholesale price for her own profit. Retailers and suppliers are in direct conflict in the setting of wholesale price. We focused on a two-echelon supply chain consisting of one retailer and multiple suppliers, and we studied the retailer’s ordering policy, suppliers’ pricing strategies, and revenue/social welfare of the supply chain. This dissertation aims to offer decision strategies for supply chain players in an interactive environment and shed some managerial insights that may remain hidden if each supply chain entity is analyzed in isolation.

Chapter 3 examines a single-product inventory system with the presence of fixed ordering cost under periodic review. The system consists of two suppliers. One is perfectly reliable while the other offers a cost advantage but is susceptible to possible interruptions. We presented a
theoretical framework with mathematical proofs to show that, under certain technical condi-
tions, the retailer’s optimal ordering policy follows an \((s, S)\)-like structure. We also analyzed the
limiting behavior of the \((s, S)\)-like policy and showed that both the optimal cost and optimal
policy converge over time. In this manner, the convergent \((s, S)\)-like policy characterizes the
optimal ordering policy for the infinite-horizon setting. Noticing that our technical condition is
restrictive when the discount factor is high, we conducted numerical experiments to show that
the \((s, S)\)-like optimal policy continue to hold for a wide range of system parameters beyond
the conditions required in the optimality proof.

Chapter 4 studies a retailer sourcing from two types of suppliers offering homogeneous
short-life-cycle goods. The normal supplier has a long lead time, requiring orders be placed
far in advance of the selling season, while the quick response supplier offers a replenishment
opportunity after the demand is revealed. The retailer has the flexibility to choose either single
sourcing or dual sourcing strategy, and the suppliers determine the unit wholesale prices given
the specific sourcing strategy of the retailer. With an exponentially distributed demand, we first
studied a benchmark scenario when there is one normal supplier and one quick response supplier.
The retailer’s ordering and the suppliers’ pricing decisions in equilibrium are characterized. We
then introduced more QR suppliers and examined how the profits of the retailer, suppliers and
whole supply chain are influenced by adding additional QR suppliers. We found that under both
sourcing strategies, as more QR suppliers are introduced, the retailer enjoys a higher profit,
but the incumbent normal supplier and QR supplier suffer from the intensified competition.
However, the impact on the supply chain depends on the production cost. Additional QR
suppliers can be advantageous to the supply chain under the dual sourcing strategy if the cost
of the QR supplier is not significantly higher than that of the normal supplier.

Chapter 5 investigates a supplier pricing game with a retailer subject to bounded rationality.
The retailer is bounded rational in that she does not always choose the supplier generating the
highest level of profit but instead chooses a better suppliers with a higher probability. We
adopted the classical quantal choice model to capture the retailer’s bounded rationality. The
best response analysis provides the optimal pricing strategy for a retailer when the competitor’s price is given. The Nash equilibrium predicts a rational outcome of the supplier pricing game when two suppliers submit their wholesale prices simultaneously. One of the most interesting findings of this work is that, contrary to traditional perception, the retailer may benefit from her bounded rationality, since suppliers may reduce their prices to appeal to the retailer subject to a moderate level of bounded rationality. This finding refreshes our perspectives on the effects of bounded rationality. Furthermore, our work lends insights to marketing and pricing practitioners and could, therefore, give rise to novel pricing strategies.

This dissertation provides a dynamic programming framework for proving an \((s, S)\)-like optimal ordering policy and a model of game-theoretic analysis for studying suppliers’ pricing strategies. It complements to the literature by extending the inventory systems that exhibit an \((s, S)\)-like optimal policy to systems of two suppliers under the risk of supply disruption. It also shows the gaming effect of the supplier pricing game on each player. Moreover, the performance of the whole supply chain is investigated. We provide practical suggestions on dealing with the competition among suppliers and the bounded rational behavior of a retailer. In application, this research could contribute to the management of fashion and retail industry and behavioral operations in supply chains.

6.2 Future Research

One of our assumptions in Chapter 3 is that the status of the unreliable is either up or down and the order is either delivered in full or canceled depending on its status. Under most circumstances in practice, only a portion of the order is received or a fraction of the order is defective. Sometimes suppliers are uncertain in their lead times. The literature has successfully captured these forms of uncertainty in supplier delivering behavior. For example, Song and Zipkin (1996) has proved an \((s, S)\)-like policy for a model that allows for a general form of uncertainty including random lead times, all-or-nothing delivery and age-dependent deliveries for a single-supplier system. Studying a general form of supply disruption for multiple-supplier
systems is a much more challenging task. We conjecture that the optimal policy would still be \((s, S)\)-like for a supply system consisting of one reliable supplier and one unreliable supplier with a more generalized form of uncertainty, and this is one area we would like to pursue future research.

One of the limitations of Chapter 3 is that the technical condition on the fixed ordering costs is somewhat restrictive, especially when the discount factor is high. The reason for this restrictive condition is that we apply the properties of \(K\)-convexity in our proofs. However, our results, which do not rely on the demand distribution, provide a sufficient condition for the optimality of the \((s, S)\)-like policy when the retailer faces one reliable supplier and one unreliable supplier with fixed costs. It would be worth relaxing the condition and extending it to a wider range of fixed costs. In this direction, the established properties of \(K\)-convexity cannot be employed directly. The analysis may not be straightforward.

In the QR supply chain in Chapter 4, we assume that the retailer faces a retail price that is market driven. As opposed to price-takers, some retailers are price-setters who are able to control the retail price, but their demands are negatively correlated with the quoted retail price. The price-setting case usually applies to the monopolies such as telecommunication, railway and gasoline companies. It is also applicable to near-monopolies that exist due to geographic restriction or brand recognition, and oligopolies who collude to control the market price. Price-setting equips the retailer with another leverage to mitigate the mismatch between the stocked inventory and the realized demand. Intuitively, the advantage of quick response is undermined under the price-setting case compared with that under the price-accepting case. It is worthwhile to investigate the value of pricing flexibility and QR flexibility, as well as their combined effects on the supply chain.

Chapter 5 has reached a very interesting conclusion that the retailer may benefit from her bounded rationality. However, in our work, we regard the level of bounded rationality as a general parameter and lack empirical study to provide support on the most commonly observed level of bounded rationality in the retailer’s ordering decision. Moreover, as we have mentioned,
our definition of bounded rationality is broadly treated to include all deviations from the traditional rational choice paradigm, and we haven’t yet tailored our analyses to accommodate specific decision biases and mental “shortcuts”. In order to better transform the fruits of this research into practice, it is important to estimate the level of bounded rationality associated with different buyer purchasing behaviors in different product industries and specific mental “shortcuts” employed in human decision making. This could be done in carefully controlled experiments by assigning experimental parameters to subjects and observing their decision-making process and consequent purchasing behavior. Towards this end, we hope to gain a deeper understanding on bounded rationality in pricing games and better align theoretical findings with anomalies in empirical experiments.

Finally, we assume the suppliers in Chapter 5 to be perfectly rational. However, human behavior is inherently uncertain and people realize that the behavior of others is not fully predictable. Hence, another interesting direction of future research is to use the quantal response equilibrium to investigate the scenario when suppliers also exhibit bounded rational behavior.
REFERENCES


Iglehart, D. L. (1963). Optimality of (s, S) policies in the infinite horizon dynamic inventory


APPENDICES
Appendix A

Appendix for Chapter 3

A.1 \((s, S)\) Policy

For a single-product single-supplier finite horizon inventory control problem, the optimal total cost of \(t\) periods is

\[
\theta_t(x) = \min_{y \geq x} \{K\delta(y - x) + c(y - x) + g(y) + \eta E_D[\theta_{t-1}(y - D)]\},
\]

where \(K\) is the fixed cost, \(c\) is the unit cost, \(x\) is the initial inventory, \(y\) is the order-up-to level and \(\eta\) is the discount factor. Let

\[
H_t(y) = cy + g(y) + \eta E_D[\theta_{t-1}(y - D)],
\]

then \(\theta_t(x)\) can be rewritten as

\[
\theta_t(x) = \min_{y \geq x} \{K\delta(y - x) + H_t(y) - cx\}.
\]

Definition A.1.1 (K-Convexity). A function \(f : \mathbb{R}_+ \rightarrow \mathbb{R}_+\) is K-convex for a given \(K \in \mathbb{R}_+\),
If
\[ f(x) + a\left[\frac{f(x) - f(x-b)}{b}\right] \leq f(x+a) + K, \]
holds for all \( x \in \mathbb{R}_+, a \geq 0 \) and \( b > 0 \).

**Lemma A.1.1** (Characterization of \( K \)-convex functions). Let \( f(\cdot) \) be a continuous, \( K \)-convex function, \( S^* \) be its smallest global minimizer and \( s^* \) be the largest \( x \leq S^* \) such that \( f(x) = f(S^*) + K \), then the following properties hold:

(i) \( f(\cdot) \) is non-increasing on \( (-\infty, s^*]\); 
(ii) If \( s^* < x \leq S^* \), then \( f(x) < f(s^*) \); 
(iii) If \( S^* < x_1 < x_2 \), then \( f(x_1) - f(x_2) \leq K \).

Lemma A.1.1 says that a \( K \)-convex function first decreases for a while, up to \( s^* \), then it may increase, but never exceeds \( f(s^*) \), then it reaches its minimum \( S^* \). After passing by \( S^* \), if the function ever decreases, it never decreases by more than \( K \). This property directly implies the optimality of an \((s, S)\) policy. The following properties of \( K \)-convexity are important in the results that follow.

**Lemma A.1.2** (Properties of \( K \)-convexity). (i) If \( f(x) \) is \( K \)-convex, then it is \( L \)-convex for any \( L \geq K \).

(ii) If \( f_1(x) \) is \( K_1 \)-convex, and \( f_2(x) \) is \( K_2 \)-convex, then \( \alpha f_1(x) + \beta f_2(x) \) is \((\alpha K_1 + \beta K_2)\)-convex for all \( \alpha, \beta \geq 0 \).

(iii) If \( f(x) \) is \( K \)-convex and \( Y \) is a random variable, then \( E_Y(f(x-Y)) \) is \( K \)-convex, provided that \( E_Y(f(x-Y)) < \infty \) for all \( Y \).

(iv) If \( f(x) \) is a \( K \)-convex function, then the function \( g(x) = \min_{y \geq x} Q \delta(y - x) + f(y) \) is \( \max\{K, Q\} \)-convex for any \( Q \in \mathbb{R}_+ \).

Using the above properties of \( K \)-convex functions, the optimality of the \((s, S)\) policy can be obtained for the finite horizon setting.

**Lemma A.1.3.** If \( \theta_t(x) \) is continuous and \( K \)-convex, then

(i) \( H_t(y) \) is continuous and \( K \)-convex.
(ii) An \((s, S)\) policy is optimal in period \(t\), with \(S_t^*\) being the smallest minimizer of \(H_t(y)\) and \(s_t^*\) being the largest \(x \leq S_t^*\) such that \(H_t(x) - H_t(S_t^*) = K\).

(iii) \(\theta_t(x)\) is continuous and \(K\)-convex.

The proofs of Lemmas A.1.1, A.1.2, and A.1.3 can be found in H. Scarf (1960).

A.2 The Impact of Beta
Figure A.1: The impact of $\beta$ on optimal policy: as $\alpha$ varies
Figure A.2: The impact of $\beta$ on optimal policy: as $c_u$ varies
Figure A.3: The impact of $\beta$ on optimal policy: as $c_r$ varies
Figure A.4: The impact of $\beta$ on optimal policy: as $h$ varies
Figure A.5: The impact of $\beta$ on optimal policy: as $b$ varies
Figure A.6: The impact of $\beta$ on optimal policy: as $K$ varies
Appendix B

Appendix for Chapter 4

Proof of Theorem 4.3.1. If the retailer purchases from supplier 1, the first-order condition of the retailer profit yields $q_1^S(w_1) = F^{-1}(w_1) = -\frac{1}{\lambda} \ln w_1$. The profit of supplier 1 is $\pi^S_{s_1}(w_1) = -\frac{1}{\lambda}(w_1 - c_1) \ln w_1$. If supplier 1 is the only supplier, he will ask for the monopoly price determined by $\frac{\partial \pi^S_{s_1}}{\partial w_1} = -\frac{1}{\lambda} (\ln w_1 + 1 - \frac{c_1}{w_1}) = 0$, which yields $w_1^m = \frac{c_1}{W_0(c_1e)}$. If supplier 2 is the only supplier, the retailer will order $q_2^S = D$, as long as $w_2 \leq 1$. Therefore, supplier 2 will ask for the market price, i.e., $w_2^m = 1$. With the presence of two suppliers, the retailer chooses to order from the supplier that yields her higher profit. Retailer’s profit of ordering from supplier 1 at $w_1$ is $\pi^S_{s_1}(w_1) = \int_0^{-\frac{1}{\lambda} \ln w_1} x \cdot \lambda e^{-\lambda x} dx + \int_{-\frac{1}{\lambda} \ln w_1}^{\infty} \frac{1}{\lambda} \ln w_1 \cdot \lambda e^{-\lambda x} dx - \frac{1}{\lambda} \ln w_1 = \frac{1}{\lambda} - \frac{w_1}{\lambda} + \frac{w_1}{\lambda} \ln w_1$. Her profit of ordering from supplier 2 at $w_2$ is $\pi^S_{s_2}(w_2) = \frac{1}{\lambda} (1 - w_2)$. If $\frac{1}{\lambda} - \frac{w_1}{\lambda} + \frac{w_1}{\lambda} \ln c_1 \geq \frac{1}{\lambda} (1 - c_2)$, i.e., $c_2 \geq c_1 - c_1 \ln c_1$, supplier 1 has the market. To keep supplier 2 out of the market, supplier 1 sets $w_1^S = \min\{w_1^m, w_1^c\}$, where $w_1^c$ satisfies $\pi^S_{s_1}(w_1^c) = \pi^S_{s_2}(c_2)$. If $\frac{1}{\lambda} - \frac{w_1}{\lambda} + \frac{w_1}{\lambda} \ln c_1 < \frac{1}{\lambda} (1 - c_2)$, supplier 2 has the market. To keep supplier 1 out of the market, supplier 2 sets $w^S_2 = \min\{w_2^m, w_2^c\} = w_2^c$, where $w_2^c$ is determined such that $\pi^S_{s_1}(c_1) = \pi^S_{s_2}(w_2^c)$.

Proof of Theorem 4.3.2. Since demand is fully realized when the retailer orders from supplier 2, then the retailer will order $q_2 = (D - q_1)^+$ from supplier 2 as long as $w_2 \leq p$. The retailer’s profit if she orders $q_1$ from supplier 1, given $w_1$ and $w_2$, is $\pi^D_r(q_1; w_1, w_2) = \int_0^{q_1} \pi^D_r(w_1, w_2; x) dx$. If $\lambda w_1 + \lambda w_2 < 1$, then $\pi^D_r(q_1; w_1, w_2) = \int_0^{q_1} \pi^D_r(w_1, w_2; x) dx$. If $\lambda w_1 + \lambda w_2 \geq 1$, then $\pi^D_r(q_1; w_1, w_2) = \int_0^{q_1} \pi^D_r(w_1, w_2; x) dx$.
$$\mathbb{E}[D - w_1 q_1 - w_2 (D - q_1)^+] = \frac{1}{\lambda} - w_1 q_1 - \frac{w_2}{\lambda} e^{-\lambda q_1}.$$ The first-order condition yields

$$q_1^D(w_1, w_2) = \begin{cases} \frac{1}{\lambda} \ln \frac{w_2}{w_1}, & \text{if } w_1 < w_2, \\ 0, & \text{if } w_1 \geq w_2 \end{cases} \quad (B.1)$$

Supplier 1’s profit function is $$\pi_1^D(w_1, w_2) = (w_1 - c_1) \cdot \frac{1}{\lambda} \ln \frac{w_2}{w_1}$$. Therefore, supplier 1’s optimal pricing given $$w_2$$ can be derived from the first-order condition: $$w_1^D(w_2) = \arg \max_{w_1} \pi_1^D(w_1, w_2) = \max \{ \frac{w_1}{W_0(w_2) e}, c_1 \}$$.

Supplier 2 has two options. If he chooses to quote a high wholesale price to reap the excessive demand, his profit is $$\pi_2^D(w_1, w_2|w_1 < w_2) = (w_2 - c_2) \mathbb{E} \left[ (D - \frac{1}{\lambda} \ln \frac{w_2}{w_1})^+ \right] = (w_2 - c_2) \cdot \frac{1}{\lambda} \cdot \frac{w_1}{w_2} = \frac{w_1}{\lambda} (1 - \frac{c_2}{w_2}).$$ If supplier 2 chooses to quote a low wholesale price to occupy the entire market, his maximum possible profit is $$\pi_2^D(w_1, w_2|w_1 \geq w_2) = (w_2 - c_2) \frac{1}{\lambda}$$. Given $$w_1$$, supplier 2 sets $$w_2^D(w_1)$$ to maximize his profit, i.e.,

$$\max_{w_2} \pi_2^D(w_1, w_2) = \max \left\{ \max_{w_2 \leq w_1} \mathbb{E}[(w_2 - c_2)D], \max_{w_2 > w_1} (w_2 - c_2) \mathbb{E}_D \left[ (D - \frac{1}{\lambda} \ln \frac{w_2}{w_1})^+ \right] \right\}$$

$$= \max \left\{ (w_1 - c_2) \frac{1}{\lambda} \cdot \frac{w_1}{\lambda} (1 - c_2) \right\}$$

$$= \frac{w_1}{\lambda} (1 - c_2). \quad (B.2)$$

Eq. (B.2) indicates that regardless of $$w_1$$, supplier 2 is always better off asking for the market price, i.e., $$w_2^D(w_1) = 1$$. The Nash equilibrium can be derived combining the best responses: $$w_1^D = \frac{c_1}{W_0(c_1 e)}, w_2^D = 1$$. $\square$

**Proof of Theorem 4.4.1.** First we consider single sourcing. For supplier 1, the condition for supplier 1 dominating the market is not changed when additional QR suppliers enter. Furthermore, the equilibrium price remains the same. Therefore, we can conclude that supplier 1 is not affected by the entrance of additional QR suppliers. For supplier 2, The profit of supplier 2 is an increasing function of $$w_2$$: $$\pi_2^S(w_1, w_2) = \frac{1}{\lambda} (w_2 - c_2)$$. And the equilibrium price is lower as more QR suppliers are introduced. Therefore, supplier 2 suffers. For the retailer, when it is more rewarding to order from supplier 1, the retailer’s profit is unchanged after more QR suppliers enter.
suppliers are introduced since the additional QR suppliers do not change $w_1^S$. When it is more rewarding to order from supplier 2, the retailer’s profit increases since supplier 2 asks for a lower $w_2^S$ with a lower $c_3$. In conclusion, the retailer receives higher profit when there are multiple QR suppliers. For the supply chain, as suggested by Eq. (4.4), when supplier 1 is chosen, the supply chain profit only relies on $w_1$, but the additional QR suppliers do not change $w_1^S$. When supplier 2 is chosen, $\pi_{sc}^S(w_1, w_2) = 1 - c_2$ is irrelevant of $w_1$ and $w_2$. Therefore, the supply chain profit is unaffected by the additional QR suppliers.

Under dual sourcing, the additional QR supplier limit the price the current QR supplier can quote, and further influence the price of the normal supplier. In equilibrium, we have

$$w_1^D(c_3) = \max \left\{ \frac{c_1}{W_0(\frac{c_1}{c_3} \cdot e)}, c_1 \right\},$$

$$w_2^D(c_3) = c_3.$$

The expected profit of each supply chain member can be readily obtained by plugging the equilibrium prices into equations (4.5)-(4.8). Denote $\Pi_i^D$ ($i = s_1, s_2, r, sc$) as the profit of $i$ under equilibrium, i.e., $\Pi_i^D(c_3) = \pi_i^D(w_1^D(c_3), w_2^D(c_3))$. Noticing that $\frac{c_1}{W_0(\frac{c_1}{c_3}e)} > c_1$ if and only if $c_3 > c_1$, we can reduce the condition $w_1^D \geq w_2^D$ to $c_3 \leq c_1$ and $w_1^D < w_2^D$ to $c_3 > c_1$. The partial information of the supplier 1’equilibrium price is derived as follows:

$$\frac{\partial w_1^D}{\partial c_3} \bigg|_{c_3 > c_1} = -\frac{c_1}{[W_0(\frac{c_1}{c_3}e)]^2} \cdot \frac{W_0(\frac{c_1}{c_3}e)}{\frac{c_1}{c_3}e[1 + W_0(\frac{c_1}{c_3}e)]} \cdot (-\frac{c_1}{c_3}e) = \frac{\frac{c_1}{c_3}}{W_0(\frac{c_1}{c_3}e)[1 + W_0(\frac{c_1}{c_3}e)]} > 0 \quad (B.3)$$

The equilibrium profit of each supply chain member is

$$\Pi_{s_1}^D(c_3) = \begin{cases} 
\Pi_{s_1}^{D, i}(c_3) = 0, & c_3 \leq c_1, \\
\Pi_{s_1}^{D, ii}(c_3) = \frac{1}{\lambda}(w_1^D - c_1) \ln \frac{c_1}{w_1^D}, & c_3 > c_1,
\end{cases}$$
\( \Pi^D_{s2}(c_3) = \left\{ \begin{array}{ll} \Pi^D_{s2,1}(c_3) = \frac{1}{\lambda}(c_3 - c_2), & c_3 \leq c_1, \\ \Pi^D_{s2,2}(c_3) = \frac{w_1^D}{\lambda}(1 - \frac{c_2}{c_3}), & c_3 > c_1, \end{array} \right. \)
\( \Pi^D_r(c_3) = \left\{ \begin{array}{ll} \Pi^D_{r,1}(c_3) = \frac{1}{\lambda}(1 - c_3), & c_3 \leq c_1, \\ \Pi^D_{r,2}(c_3) = 1 - \frac{w_1^D}{\lambda} \ln \frac{c_3}{w_1^D} - \frac{w_1^D}{\lambda}, & c_3 > c_1, \end{array} \right. \)
\( \Pi^D_{sc}(c_3) = \left\{ \begin{array}{ll} \Pi^D_{sc,1}(c_3) = \frac{1}{\lambda}(1 - c_2), & c_3 \leq c_1, \\ \Pi^D_{sc,2}(c_3) = \frac{1}{\lambda} - \frac{w_1^D c_2}{c_3} + \frac{c_1}{\lambda} \ln \frac{w_1^D}{c_3}, & c_3 > c_1. \end{array} \right. \)

Notice that \( \Pi^D_i(c_3) \), where \( i = s_1, s_2, r \) and \( sc \), is a continuous function of \( c_3 \). To check the monotonicity of a piecewise function, it suffices to check each of its piece. It is not difficult to check that \( \frac{\partial \Pi^D_{s1,1}}{\partial c_3} = 0, \frac{\partial \Pi^D_{s2,1}}{\partial c_3} = \frac{1}{\lambda}, \frac{\partial \Pi^D_{r,1}}{\partial c_3} = -\frac{1}{\lambda} \). Differentiate \( \pi_i^D, i = s_1, s_2, r \), with regard to \( c_3 \) yields

\[
\frac{\partial \Pi^D_{s1,2}}{\partial c_3} = \frac{w_1^D - c_1}{\lambda c_3} + \frac{1}{\lambda} \ln \frac{c_3}{w_1^D} - \frac{w_1^D - c_1}{w_1^D} \frac{\partial w_1^D}{\partial c_3} \\
= \frac{w_1^D - c_1}{\lambda c_3} \geq 0,
\]
\[
\frac{\partial \Pi^D_{s2,2}}{\partial c_3} = \frac{1}{\lambda} (1 - \frac{c_2}{c_3}) \frac{\partial w_1^D}{\partial c_3} + \frac{w_1^D}{\lambda} (1 + \frac{c_2}{c_3}) \geq 0,
\]
\[
\frac{\partial \Pi^D_r,2}{\partial c_3} = -\frac{1}{\lambda} \ln \frac{c_3}{w_1^D} \cdot \frac{\partial w_1^D}{\partial c_3} - \frac{1}{\lambda} \frac{w_1^D}{c_3} \leq 0.
\]

Consequently, we prove that the retailer’s profit is non-decreasing and both suppliers’ profits are non-increasing as additional QR suppliers get involved.

**Proof of Theorem 4.4.2.** From Eq. (4.8), we can see that when \( w_1 < w_2 \), the supply chain profit \( \pi^D_{sc} \) relies on \( w_1 \) and \( w_2 \) only through its fraction \( \frac{w_1}{w_2} \) and, it is concave in \( \frac{w_1}{w_2} \) with its maximum at \( \frac{w_1}{w_2} = \frac{c_1}{c_2} \). When there is an additional QR supplier, we have \( \frac{w_1}{w_2} = \frac{c_1}{c_3 W_0(\frac{c_3}{c_3})} \). The supply chain profit is maximized at \( c_3^* \) such that \( c_2 = c_3^* W_0(\frac{c_3}{c_3}) \). Depending on whether \( c_3^* \) can be obtained in \([c_1, 1]\), we divide into three cases. The trend of \( \Pi^D_c(c_3) \) is shown in Table B.1 and the results follow.
Table B.1: Supply chain profit under dual sourcing with additional QR suppliers

<table>
<thead>
<tr>
<th>$c_2$</th>
<th>(0, $c_1$)</th>
<th>($c_1$, $W_0(c_1)\cdot 1$)</th>
<th>($W_0(c_1)$, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^*_3$</td>
<td>(0, $c_1$)</td>
<td>($c_1$, 1)</td>
<td>[1, $\infty$)</td>
</tr>
<tr>
<td>$\Pi^p_{sc}(c_3)$</td>
<td>$\phi$</td>
<td>$\phi$</td>
<td>$\phi$</td>
</tr>
</tbody>
</table>

Proof of Theorem 4.5.1. The problem of the retailer is a newsvendor problem; her profit of ordering $q_1$ at wholesale price $w_1$ equals to $\pi^{S_1}_r(q_1; w_1) = E[\min(D, q_1)] - w_1 q_1 = \int_0^{q_1} x f(x) dx + \int_{q_1}^{\infty} q_1 f(x) dx - w_1 q_1$. The retailer’s optimal ordering quantity can be determined by the first-order condition: $q^{S_1}_r(w_1) = \bar{F}^{-1}(w_1)$. The expected profit at the optimal ordering level is $\pi^{S_1}_r(w_1) = \int_0^{\bar{F}^{-1}(w_1)} x f(x) dx + \int_{\bar{F}^{-1}(w_1)}^{\infty} q_1 f(x) dx - w_1 q_1^{S_1}_r(w_1) = \int_0^{\bar{F}^{-1}(w_1)} x f(x) dx$. Eq. (4.9) is equivalent to $\int_0^{\infty} y f(x) dx \geq w_1 \int_0^{\infty} x f(x) dx$. Let $\bar{F}^{-1}(w_1) = y$, then it suffices to prove that $\frac{\int_0^y x f(x) dx}{F(y)} \geq \int_0^\infty x f(x) dx$, which can be readily obtained from $\mathbb{E}(X|X \geq y) \geq \mathbb{E}(X)$.

Proof of Theorem 4.5.2. Assume that supplier 1 is the only supplier, supplier 1 will ask for a price that maximizes his own profit: $w_1^m = \arg\max_{0 \leq w_1 \leq 1} (w_1 - c_1) \bar{F}^{-1}(w_1)$. According to (Lariviere & Porteus, 2001), this newsvendor pricing problem is well behaved (the objective is unimodal) when the demand distribution has increasing generalized failure rate (IGFR). Under this scenario, $w_1^m$ can be determined by the first derivative, i.e., $\bar{F}^{-1}(w_1^m) - \frac{w_1^m - c_1}{f(\bar{F}^{-1}(w_1^m))} = 0$. Denoting $y^m = \bar{F}^{-1}(w_1^m)$, we have $y^m f(y^m) - F(y^m) + c_1 = 0$. It is not difficult to check that $c_1 \leq w_1^m \leq 1$. If supplier 2 is the only supplier, supplier 2 will ask for the market price, i.e., $w_2^m = 1$.

Given two potential suppliers, the retailer will order from the supplier who offers a price that yields her a higher profit. The retailer’s expected profit of ordering from supplier 1 at $w_1$
is $\pi^S_{s_1}(w_1) = \int_0^{F^{-1}(w_1)} xf(x)dx$, and her expected profit of ordering from supplier 2 at $w_2$ is $\pi^S_{s_2}(w_2) = (1 - w_2) \int_0^\infty xf(x)dx$. If $\int_0^{F^{-1}(c_1)} xf(x)dx \geq (1 - c_2) \int_0^\infty xf(x)dx$, supplier 1 has the cost advantage and he keeps supplier 2 out of the market by quoting a wholesale price $w_1^S = \min\{w_1^m, w_1^c\}$, where $w_1^c$ is determined such that $\pi^S_{r_1}(w_1^c) = \pi^S_{r_2}(c_2)$. If $\int_0^{F^{-1}(c_1)} xf(x)dx < (1 - c_2) \int_0^\infty xf(x)dx$, supplier 2 occupies the market with $w_2^S = \min\{w_2^m, w_2^c\}$, where $w_2^c$ is determined such that $\pi^S_{r_1}(c_1) = \pi^S_{r_2}(w_2^c)$. Obviously, we have $w_2^c \leq w_2^m$, therefore $w_2^S = w_2^c$.

**Proof of Theorem 4.5.3.** (i) It is obvious that the retailer will order $q_{r_2}^D = (D - q_1^*)$ from supplier 2 as long as $w_2 \leq p$. At time $-L$, the retailer chooses a nonnegative $q_1$ to maximize her expected profit

$$\pi_r^D(q_1; w_1, w_2) = E_D[p \cdot D - w_1 q_1 - w_2 (D - q_1)] = \mu - w_1 q_1 - w_2 \int_{q_1}^\infty (x - q_1) f(x)dx.$$ 

The first derivative $\frac{\partial \pi_r^D}{\partial q_1} = -w_1 + w_2 \bar{F}(q_1) = 0$ and the non-negativity of $q_1$ gives Eq. (4.10).

(ii) Since $q_{r_2}^D$ is determined by $\frac{\partial \pi_r^D}{\partial q_1} = 0$, from implicit function theorem, we have

$$\frac{\partial q_1^D}{\partial w_1} = -\frac{\frac{\partial^2 \pi_r^D}{\partial q_1 \partial w_1}}{\frac{\partial^2 \pi_r^D}{\partial q_1^2}} = -\frac{1}{w_2 \bar{f}(q_1^D)} < 0, \tag{B.4}$$

$$\frac{\partial q_1^D}{\partial w_2} = -\frac{\frac{\partial^2 \pi_r^D}{\partial q_1 \partial w_2}}{\frac{\partial^2 \pi_r^D}{\partial q_1^2}} = \frac{-\bar{F}(q_1^D)}{w_2 \bar{f}(q_1^D)} > 0. \tag{B.5}$$

The inequality in Eq. (B.5) holds because $\bar{F}(q_1^D) = \frac{w_1}{w_2}$ and $w_1 < w_2$. Since $\bar{F}(q_1^D) < 1$, we have $\left|\frac{\partial q_1^D}{\partial w_1}\right| > \left|\frac{\partial q_1^D}{\partial w_2}\right|$. \hfill \Box

**Proof of Theorem 4.5.4.** Suppose the optimal solution to (P) is obtained at some $(w_1^*, w_2^*)$ with $w_2^* < 1$, we prove that we can find another set of wholesale prices $(\frac{w_1^*}{w_2^*}, 1)$ such that $h(\frac{w_1^*}{w_2^*}, 1) > h(w_1^*, w_2^*)$. Notice that $h(w_1, w_2)$ is an increasing function of $\pi_{s_1}(w_1, w_2)$ and $\pi_{s_2}(w_1, w_2)$. Moreover, we have $\pi^D_{s_1}(\frac{w_1^*}{w_2^*}, 1) > \pi^D_{s_1}(w_1^*, w_2^*)$ and $\pi^D_{s_2}(\frac{w_1^*}{w_2^*}, 1) > \pi^D_{s_2}(w_1^*, w_2^*)$. Hence,
we know that \( h(w_1^*, w_2^*) > h(w_1^*, w_2) \), by which we complete the proof.

**Proof of Theorem 4.5.5.** We prove this Theorem by showing that \( w_1^D \leq w_1^{D^*}(1) \leq w_1^{D_b} \).

First we verify the first inequality. According to Eq. (4.12), we know that \( w_1^D = w_1^{D^*}(w_2^2) \). It remains to prove that \( w_1^{D^*}(\cdot) \) is an increasing function of \( w_2 \). From the implicit theorem, we can find its slope as follows

\[
\frac{\partial w_1^{D^*}}{\partial w_2} = -\frac{\partial^2 \pi_s^D}{\partial w_1 \partial w_2}.
\]

Since \( w_1^{D^*} \) is the best response, it must be that \( \frac{\partial^2 \pi_s^D}{\partial w_1 \partial w_2} < 0 \). Then to prove its slope to be positive, it suffices to show that \( \frac{\partial^2 \pi_s^D}{\partial w_1 \partial w_2} > 0 \). Differentiate \( \pi_s^D(w_1, w_2) = (w_1 - c_1)q_1 = (w_1 - c_1)\bar{F}^{-1}(\frac{w_1}{w_2}) \) with respect to \( w_1 \) and \( w_2 \), we have

\[
\frac{\partial^2 \pi_s^D}{\partial w_1 \partial w_2} = \frac{\partial q_1}{\partial w_2} + \frac{\partial^2 q_1}{\partial w_1 \partial w_2} = \frac{\bar{F}(q_1)}{w_2 f(q_1)} + \frac{w_1 - c_1}{w_2 f^3(q_1)} \left[ f^2(q_1) + f'(q_1)\bar{F}(q_1) \right] > 0
\]

The inequality is valid since the \( D \) has an increasing failure rate, i.e., \( \frac{\partial}{\partial q_1} f(q_1) \bar{F}(q_1) \geq 0 \), and \( \bar{F}(q_1) > 0 \). In the following, we show that \( w_1^{D^*}(1) \leq w_1^{D_b} \). From the definition of \( w_1^{D^*}(1) \) and Theorem 4.5.4, we have

\[
w_1^{D^*}(1) = \max (w_1 - c_1)\bar{F}^{-1}(w_1)
\]

\[
w_1^{D_b} = \max (w_1 - c_1)\bar{F}^{-1}(w_1)(1 - c_1) \int_{\bar{F}^{-1}(w_1)}^{\infty} \bar{F}(x)dx
\]

When \( D \) has IGFR, \((w_1 - c_1)\bar{F}^{-1}(w_1)\) is unimodal. Besides, it is not difficult to see that \((1 - c_1) \int_{\bar{F}^{-1}(w_1)}^{\infty} \bar{F}(x)dx\) is an increasing function of \( w_1 \). Therefore, we have our results on the maximizers of the above two functions \( w_1^{D^*}(1) \leq w_1^{D_b} \). □

144
Appendix C

Appendix for Chapter 5

Proof of Proposition 5.3.1. When \( w_i \leq 1 \), we take the derivative of \( \pi_{s_i}(w_i, w_j; \beta) \) with respect to \( w_i \):

\[
\frac{\partial \pi_{s_i}(w_i, w_j; \beta)}{\partial w_i} = \frac{e^{\frac{1-w_j}{\beta} + \frac{1-w_i}{\beta}}}{(e^{\frac{1-w_j}{\beta}} + e^{\frac{1-w_i}{\beta}})^2} \left[ e^{\frac{1-w_j}{\beta} + \frac{1-w_i}{\beta}} - \frac{w_i - c_i}{\beta} e^{\frac{1-w_j}{\beta}} \right] \]

It is not difficult to verify that \( e^{\frac{-w_j}{\beta}} + e^{\frac{-w_i}{\beta}} - \frac{w_i - c_i}{\beta} e^{\frac{-w_j}{\beta}} \) is decreasing in \( w_i \). Therefore, there is at most one root to the equation

\[
e^{-\frac{w_j}{\beta}} + e^{-\frac{w_i}{\beta}} - \frac{w_i - c_i}{\beta} e^{-\frac{w_j}{\beta}} = 0. \quad \text{(C.1)}
\]

Let \( w_i^0 \) be the unique solution to Eq. (C.1). Denote \( x = \frac{w_i}{\beta}, p = e^{-\frac{w_j}{\beta}}, q = -(\frac{c_i}{\beta} + 1)p \) and reorganize Eq. (C.1) as

\[
e^{-x} = p \cdot x + q. \quad \text{(C.2)}
\]
It is known from Corless et al. (1996) that the solution to Eq. (C.2) is
\[ x = -\frac{q}{p} + W_0\left(\frac{1}{p}e^\frac{q}{p}\right), \]
where \( W_0(\cdot) \) is the principal branch of the Lambert W function. Consequently, we have an explicit solution of \( w_i^0 \):
\[ w_i^0 = c_i + \beta + \beta \cdot W_0\left(\frac{e^{-(\frac{q}{p} + 1)}}{e^{-\frac{q}{p}}}\right). \]

It can be checked that \( \frac{\partial \pi_i}{\partial w_i} \geq 0 \) for \( w_i \leq w_i^0 \) and \( \frac{\partial \pi_i}{\partial w_i} \leq 0 \) for \( w_i \geq w_i^0 \). Therefore, when \( w_i^0 \leq 1 \), the maximum of \( \pi_s(w_i, w_j; \beta) \) is determined by its first derivative \( w_i^0 \). Moreover, when \( w_i^0 > 1 \), the maximum of \( \pi_s(w_i, w_j; \beta) \) is achieved at 1. Consequently, we have Eq. (5.3).

**Proof of Corollary 5.3.2.** (i) When \( w_i^* < 1 \), taking the derivative of \( w_i^*(w_j; \beta) \) with regard to \( \beta \), we have
\[
\frac{\partial w_i^*(w_j; \beta)}{\partial \beta} = 1 + W_0\left(e^{\frac{w_j - c_i}{\beta} - 1}\right) + \beta \cdot \frac{W_0\left(e^{\frac{w_j - c_i}{\beta} - 1}\right)}{e^{\frac{w_j - c_i}{\beta} - 1} \left[1 + W_0\left(e^{\frac{w_j - c_i}{\beta} - 1}\right)\right]} \cdot \left(-\frac{w_j - c_i}{\beta^2}\right)
\]
\[
= 1 + W_0\left(e^{\frac{w_j - c_i}{\beta} - 1}\right) - \frac{w_j - c_i}{\beta} \cdot \frac{W_0\left(e^{\frac{w_j - c_i}{\beta} - 1}\right)}{1 + W_0\left(e^{\frac{w_j - c_i}{\beta} - 1}\right)}.
\]

Letting \( e^{\frac{w_j - c_i}{\beta} - 1} = z \), we have \( \frac{w_j - c_i}{\beta} = \ln z + 1 \) and
\[
\frac{\partial w_i^*(w_j; \beta)}{\partial \beta} = 1 + W_0(z) - (\ln z + 1) \frac{W_0(z)}{1 + W_0(z)}. \tag{C.3}
\]

Since the second derivative
\[
\frac{\partial^2 w_i^*(w_j; \beta)}{\partial \beta^2} = \frac{W_0(z)}{z(1 + W_0(z))} - \frac{1}{z} \cdot \frac{W_0(z)}{1 + W_0(z)} - (\ln z + 1) \frac{W_0(z)}{[1 + W_0(z)]^2} \cdot \frac{\partial z}{\partial \beta}
\]
\[
= - (\ln z + 1) \cdot \frac{W_0'(z)}{[1 + W_0(z)]^2} \cdot e^{\frac{w_j - c_i}{\beta} - 1} \cdot \left(-\frac{w_j - c_i}{\beta^2}\right)
\]
\[
= \frac{(w_j - c_i)^2}{\beta^3} \cdot \frac{W_0'(z)}{[1 + W_0(z)]^2} \cdot e^{\frac{w_j - c_i}{\beta} - 1} \geq 0,
\]

146
we know that $w_i^*(w_j; \beta)$ is convex in $\beta$ when $w_i^*(w_j; \beta) < 1$. Notice that

$$
\lim_{\beta \to 0} \frac{\partial w_i^*(w_j; \beta)}{\partial \beta} = \lim_{z \to \infty} \left[ 1 + W_0(z) - (\ln z + 1) \frac{W_0(z)}{1 + W_0(z)} \right] \\
= \lim_{z \to \infty} [W_0(z) - \ln z] \\
= \lim_{z \to \infty} [- \ln \ln z] \\
= - \lim_{\beta \to 0} \ln \left( \frac{w_j - c_i}{\beta} - 1 \right) < 0,
$$

where the third equality follows from the asymptotic behavior of $W_0(z)$ when $z$ is sufficiently large. Hence, $w_i^*(w_j; \beta)$ first decreases and then increases. We obtain $\tilde{\beta}(w_j)$ by setting $\frac{\partial w_i^*(w_j; \beta)}{\partial \beta} = 0$. Using Matlab to solve Eq. (C.3)=0, we have $z^* = 130.3$. The corresponding $\tilde{\beta}(w_j)$ satisfying $\frac{\partial w_i^*(w_j; \beta)}{\partial \beta} = 0$ can be obtained from $\tilde{\beta}(w_j) = \frac{w_j - c_i}{1 + \ln z} \bigg|_{z=130.3} = \frac{w_j - c_i}{1 + \ln 130.3}$.

(ii) Since $W_0(\cdot)$ is the inverse function of $f(x) = x e^x$ defined on $x \in [0, \infty)$, $f(\cdot)$ is an increasing function implies that $W_0(\cdot)$ is also an increasing function. Consequently, the fact that $w_i^*(w_j; \beta)$ is increasing in $w_1$ can be readily obtained by knowing that $e^{\frac{w_j - c_i}{\beta} - 1}$ is increasing in $w_j$.

(iii) When $w_j < c_i$, $\lim_{\beta \to 0} e^{\frac{w_j - c_i}{\beta} - 1} = 0$. Since $W_0(0) = 0$, we have $\lim_{\beta \to 0} w_i^*(w_j; \beta) = c_i$.

When $w_j = c_i$, we have $\lim_{\beta \to 0} w_i^*(w_j; \beta) = \lim_{\beta \to 0} c_i + \beta \cdot W_0(1) = c_i$.

When $w_j > c_i$, $e^{\frac{w_j - c_i}{\beta}}$ goes larger for a smaller $\beta > 0$. From (Corless et al., 1996), we know that, for $x$ being sufficiently large, $W_0(x)$ is asymptotic to

$$
\hat{W}_0(x) = L_1 - L_2 + \frac{L_2(-2+L_2)}{2L_1^2} + \frac{L_2(6-0L_2+2L_2^2)}{6L_1^3} + \frac{L_2(-12+36L_2-22L_2^2+3L_2^3)}{12L_1^4} + \ldots
$$
where \( L_1 = \ln x, L_2 = \ln \ln x \). Therefore, when \( w_j > c_i \),

\[
\lim_{\beta \to 0} w_i^*(w_j; \beta) = \lim_{\beta \to 0} \left[ c_i + \beta \cdot W_0(e^{\frac{w_j - c_i}{\beta}}) \right] \\
= \lim_{\beta \to 0} \left[ c_i + \beta \cdot \left( \frac{w_j - c_i}{\beta} - \ln \frac{w_j - c_i}{\beta} + \frac{\ln \frac{w_j - c_i}{\beta}}{\beta} + \cdots \right) \right] \\
= \lim_{\beta \to 0} \left[ c_i + w_j - c_i - \beta \cdot \ln \frac{w_j - c_i}{\beta} + \frac{\beta^2}{w_j - c_i} \cdot \ln \frac{w_j - c_i}{\beta} \right] \\
= w_j - \lim_{\beta \to 0} \left[ \beta \cdot \ln \frac{w_j - c_i}{\beta} \right] + \lim_{\beta \to 0} \left[ \frac{\beta^2}{w_j - c_i} \cdot \ln \frac{w_j - c_i}{\beta} \right] \\
= w_j.
\]

Proof of Proposition 5.4.1. According to Corollary 5.3.2, \( w_i^*(w_j; \beta) \) is increasing in \( w_j \).

Therefore, if there exists \( w_j \) such that \( w_i^*(w_j; \beta) = 1 \) for a given \( \beta \), then we have \( w_i^*(1; \beta) = 1 \).

In this case, we say the best response function of supplier \( i \) is bounded by the market price.

Notice that the best response of supplier 2 is bounded by the market price when \( c_2 + \beta + \beta \cdot W_0(e^{\frac{1-c_2}{\beta} - 1}) > 1 \), which can be rearranged as \( W_0(e^{\frac{1-c_2}{\beta} - 1}) > \frac{1-c_2}{\beta} - 1 \). Since \( W_0(\cdot) \) is an increasing function with its inverse being \( W_0^{-1}(y) = ye^y \), applying the inverse of \( W_0 \) on both sides yields

\[
e^{1-c_2 - 1} > \left( \frac{1-c_2}{\beta} - 1 \right)e^{1-c_2 - 1}.
\]

(C.4)

It is not difficult to see that Eq. (C.4) is valid when \( \beta > \frac{1-c_2}{2} \). The same logic applies to supplier 1 and the best response function of supplier 1 is bounded by the market price when \( \beta > \frac{1-c_1}{2} \).

When \( \beta \leq \min\{\frac{1-c_1}{2}, \frac{1-c_2}{2}\} = \frac{1-c_2}{2} \), none of the suppliers has a best response bounded by the market price. In this case, the Nash equilibrium is determined by the following equations:

\[
\begin{cases}
  w_2 = c_2 + \beta + \beta \cdot W_0(e^{\frac{w_1 - c_2}{\beta} - 1}), \\
  w_1 = c_1 + \beta + \beta \cdot W_0(e^{\frac{w_2 - c_1}{\beta} - 1}).
\end{cases}
\]

(C.5)
After regrouping and taking the inverse of $W_0$, we have
\[
\begin{cases}
  \left(\frac{w_2-c_2}{\beta} - 1\right)e^{\frac{w_2-c_2}{\beta}-1} = e^{\frac{w_1-c_2}{\beta}-1}, \\
  \left(\frac{w_1-c_1}{\beta} - 1\right)e^{\frac{w_1-c_1}{\beta}-1} = e^{\frac{w_2-c_1}{\beta}-1}.
\end{cases}
\]
(C.6)

Equations of (C.6) can be further simplified as
\[
\begin{cases}
  e^{\frac{w_1-w_2}{\beta}} = \frac{w_2-c_2}{\beta} - 1, \\
  e^{\frac{w_2-w_1}{\beta}} = \frac{w_1-c_1}{\beta} - 1.
\end{cases}
\]
(C.7)

Denoting $A(\beta) = e^{\frac{w_1-w_2}{\beta}}$, we have
\[
w_2 = c_2 + [A(\beta) + 1]\beta
\]
(C.8)

and
\[
w_1 = c_1 + \left[\frac{1}{A(\beta)} + 1\right]\beta.
\]
(C.9)

Notice that $A(\beta)$ can be determined by
\[
\ln A(\beta) = \frac{w_1-w_2}{\beta} = \frac{c_1-c_2}{\beta} + \left[\frac{1}{A(\beta)} - A(\beta)\right].
\]
(C.10)

If $\frac{1-c_2}{2} \leq \beta \leq \frac{1-c_1}{2}$, the best response of supplier 1 is not bounded by the market price while the best response of supplier 2 is. Let $\hat{w}_1$ be the inflection point of supplier 2’s best response, then $\hat{w}_1$ satisfies $c_2 + \beta + \beta \cdot W_0(e^{\frac{w_1-c_2}{\beta}-1}) = 1$. After simplification, we have
\[
\hat{w}_1 = \beta \cdot \ln(\frac{1-c_2}{\beta} - 1) + 1.
\]
(C.11)
The best response of supplier 1 intersects with $w_2 = 1$ at $w_1^*(1; \beta)$. Figure C.1 shows how the specific case of the NE is contingent on the ordering of $w_1^*(1; \beta)$ and $\hat{w}_1$. When $w_1^*(1; \beta) \leq \hat{w}_1$, the Nash equilibrium is determined by Equations (C.5). When $w_1^*(1; \beta) > \hat{w}_1$, the best pricing of supplier 2 is bounded by 1. Consequently, the corresponding best response of supplier 1 is given by $w_1^*(1; \beta)$, and $(w_1, w_2) = (w_1^*(1; \beta), 1)$ forms a Nash equilibrium. Denoting $H(\beta) = \frac{1}{\beta}(w_1^*(1) - \hat{w}_1)$, we have

$$H(\beta) = \frac{1}{\beta} \left[ c_1 + \beta + \beta \cdot W_0(e^{\frac{1-c_1}{\beta}} - 1) - \beta \cdot \ln\left(\frac{1-c_2}{\beta} - 1\right) - 1 \right]$$  \hspace{1cm} (C.12)$$

and

$$\frac{\partial H(\beta)}{\partial \beta} = \frac{-1-c_1}{e^{\frac{1-c_1}{\beta}} - 1 + e^{W_0(\frac{1-c_1}{\beta}) - 1}} + \frac{1-c_1}{\beta^2} - \frac{1-c_2}{\beta^2 - 1}$$  \hspace{1cm} (C.13)$$

$$= \frac{1-c_1}{e^{\frac{1-c_1}{\beta}} - 1 + e^{W_0(\frac{1-c_1}{\beta}) - 1}} + \frac{1-c_2}{\beta(1-c_2 - \beta)} \geq 0.$$
The last inequality holds because $1 - c_2 - \beta \geq 0$, which follows from Equation (5.3) and the fact that $W_0(x)$ is positive for any $x$. Furthermore, notice that

$$H\left(\frac{1 - c_2}{2}\right) = \left[W_0\left(e^{\frac{1-c_1}{\beta} - 1}\right) - \left(\frac{1 - c_1}{\beta} - 1\right)\right]_{\beta = \frac{1-c_2}{2}} \leq 0,$$

(C.14)

$$H\left(\frac{1 - c_1}{2}\right) = -\ln\left(\frac{2(1 - c_2)}{1 - c_1} - 1\right) \geq 0.$$

(C.15)

Equations (C.13)-(C.15) suggest that there is a unique solution to $H(\beta) = 0$ on the interval $\beta \in \left[\frac{1-c_2}{2}, \frac{1-c_1}{2}\right]$. And the unique solution, denoted by $\beta_b$, is the critical point that determines which segments of the best responses intersect. When $\frac{1-c_2}{2} \leq \beta \leq \beta_b$, the Nash equilibrium is determined by Equations (C.5), which is of the same expression that dictates the Nash equilibrium when $\beta \leq \frac{1-c_2}{2}$. Putting $\beta \leq \frac{1-c_2}{2}$ and $\frac{1-c_2}{2} \leq \beta \leq \beta_b$ together forms the condition for Case 1, i.e., $\beta \leq \beta_b$, and the Nash equilibrium is given by Equations (C.8)-(C.10). When $\beta_b < \beta \leq \frac{1-c_1}{2}$, the best response of supplier 1 intersects that of supplier 2 at $w_2 = 1$, hence $(w_1, w_2) = (w_1^*(1; \beta), 1)$ becomes the Nash equilibrium. This corresponds to Case 2.

Case 3 characterizes the scenario when each supplier’s best response function is bounded by the market price. This case is valid when $\beta > \max\{\frac{1-c_1}{2}, \frac{1-c_2}{2}\} = \frac{1-c_1}{2}$. In this case, the best response of each supplier is bounded by 1 and $(w_1, w_2) = (1, 1)$ forms a Nash equilibrium. \(\square\)

**Proof of Corollary 5.4.2.** In order to prove this Corollary, we first prove the following lemma:

**Lemma C.0.1.** (i) If $c_1 = c_2$, then $A(\beta) = 1$ for any $\beta \in [0, \infty)$.

(ii) If $c_1 \neq c_2$, w.l.o.g. we may assume that $c_1 < c_2$. In this case, we have (a) $A(\beta)$ is increasing in $\beta$ for any $\beta \in [0, \infty)$; (b) $0 < A(\beta) < 1$ for any $\beta \in [0, \infty)$; (c) $\lim_{\beta \to 0} A(\beta) = 0$.

**Proof of Lemma C.0.1.** (i) $A(\beta) = 0$ is immediately known from the fact that $x = 0$ is the only solution to $\ln x = \frac{1}{x} - x$. 

151
(ii)(a) Taking derivative of (C.10) with respect to $\beta$, we have

$$\frac{1}{A(\beta)} \frac{\partial A(\beta)}{\partial \beta} = \frac{c_2 - c_1}{\beta^2} + \left[ -\frac{1}{A^2(\beta)} - 1 \right] \frac{\partial A(\beta)}{\partial \beta}.$$ 

Reorganizing the terms, we have

$$\frac{\partial A(\beta)}{\partial \beta} = \frac{c_2 - c_1}{\beta^2} \left(\frac{1}{A^2(\beta)} + \frac{1}{A(\beta)} + 1\right) > 0.$$ 

Consequently, $A(\beta)$ is increasing in $\beta$.

(ii)(b) By its definition, $A(\beta) = e^{\frac{w_1}{\beta} - \frac{w_2}{\beta}} > 0$. Moreover, from (C.10) we know that $\ln A(\beta) - \frac{1}{A(\beta)} + A(\beta) = \frac{\alpha - c_2}{\beta}$. Since $\ln A - \frac{1}{A} + A$ is increasing in $A$ when $A > 0$ and $\left[\ln A - \frac{1}{A} + A\right]_{A=1} = 0$, it is not difficult to see that $A(\beta) < 1$.

(ii)(c) Since $\ln A(\beta) - \frac{1}{A(\beta)} + A(\beta) = \frac{\alpha - c_2}{\beta} \to -\infty$ as $\beta \to 0$, we have $A(\beta) \to 0$ as $\beta \to 0$.

Now we are ready to prove Corollary 5.4.2.

(i) When $c_1 = c_2$, Case 2 of the Nash equilibrium vanishes and $\beta_b$ is reduced to $\frac{1-c_1}{2}$. When $\beta \leq \frac{1-c_1}{2}$, it corresponds to Case 1. Equation (C.10) can be explicitly solved and we have $A(\beta) = 1$. Consequently, the equilibrium can be derived as $w_1^\beta = w_2^\beta = 2\beta + c_1$. When $\beta > \frac{1-c_1}{2}$, it follows from Theorem 5.4.1 (iii) that $w_1^\beta = w_2^\beta = 1$.

To prove (ii) to (iv), without loss of generality, we may assume that $c_1 < c_2$.

(ii) We prove this result case by case:

- Case 1: When $\beta \leq \beta_b$, according to Proposition 5.4.1 (i), we have $w_1^\beta - w_2^\beta = c_1 - c_2 + \left[\frac{1}{A(\beta)} - A(\beta)\right]\beta$. Since $A(\beta) < 1$ from C.0.1 (ii)(b), $\ln A(\beta) = \left\{\frac{1}{A(\beta)} - A(\beta)\right\} + \frac{1}{\beta}(c_1 - c_2) < 0$. Consequently, we have $w_1^\beta < w_2^\beta$.

- Case 2: When $\beta_b < \beta \leq \frac{1-c_1}{2}$, according to Proposition 5.4.1 (ii), we have $w_1^\beta = c_1 + \beta + \beta \cdot W_0(e^{\frac{1-c_1}{2}} - 1)$ and $w_2^\beta = 1$. Hence $w_1^\beta \leq w_2^\beta = 1$ is obvious.

- Case 3: When $\beta > \frac{1-c_1}{2}$, according to Proposition 5.4.1 (iii), we have $w_1^\beta = w_2^\beta = 1$, and of course $w_1^\beta \leq w_2^\beta$.

(iii) Similarly, we prove this result case by case:
Proof of Corollary 5.4.3

- Case 1: When $\beta \leq \beta_b$, according to Proposition 5.4.1 (i), we have $w_1^\beta = c_1 = \left(\frac{1}{A(\beta)} + 1\right)\beta$ and $w_2^\beta = c_2 = (A(\beta) + 1)\beta$. Since $A(\beta) < 1$ when $c_1 < c_2$, we have $w_1^\beta - c_1 > w_2^\beta - c_2$.

- Case 2: When $\beta_b < \beta \leq \frac{1-c_2}{2}$, according to Proposition 5.4.1 (ii), we have $w_1^\beta - c_1 = \beta + \beta \cdot W_0\left(e^{\frac{1-c_1}{\beta}}\right)$ and $w_2^\beta - c_2 = 1 - c_2$. Proving $w_1^\beta - c_1 \geq w_2^\beta - c_2$ is equivalent to proving $W_0\left(e^{\frac{1-c_1}{\beta}}\right) \geq 1 - c_2 - 1$. We know that $e^{\frac{1-c_1}{\beta}} > e^{\frac{1-c_2}{\beta}}$ for $c_1 < c_2$. Therefore, we have $W_0\left(e^{\frac{1-c_1}{\beta}}\right) > W_0\left(e^{\frac{1-c_2}{\beta}}\right)$ because $W_0(\cdot)$ is a strictly increasing function. In addition, because the best response of supplier 2 is bounded by the market price, we have $W_0\left(e^{\frac{1-c_2}{\beta}}\right) > 1 - c_2 - 1$.

- Case 3: When $\beta > \frac{1-c_2}{2}$, according to Proposition 5.4.1 (iii), we have $w_2^\beta = w_2^\beta = 1$.

Consequently, $c_1 < c_2$ directly implies that $w_1^\beta - c_1 > w_2^\beta - c_2$.

(iv) It follows from (C.10) that $\frac{\partial}{\partial \beta} A(\beta) = \beta \ln A(\beta) + \beta A(\beta) + c_2 - c_1$. We know $\lim_{\beta \to 0} A(\beta) = 0$ from Lemma C.0.1 (ii)(c). Using L’Hôpital’s rule, we have $\lim_{\beta \to 0} \beta \ln A(\beta) = \lim_{\beta \to 0} \frac{\ln A(\beta)}{1/\beta} = \lim_{\beta \to 0} A(\beta) = c_2 - c_1$. Plugging $\lim_{\beta \to 0} A(\beta) = 0$ and $\lim_{\beta \to 0} \frac{\partial}{\partial \beta} A(\beta) = c_2 - c_1$ into (C.8) and (C.9), respectively, we have $\lim_{\beta \to 0} w_1^\beta = c_2$ and $\lim_{\beta \to 0} w_2^\beta = c_2$.

Proof of Corollary 5.4.3. When $\beta \leq \beta_b$, the Nash equilibrium is determined by $w_1^\beta = c_1 + \left[\frac{1}{A(\beta)} + 1\right]\beta$ and $w_2^\beta = c_2 + [A(\beta) + 1]\beta$. From Lemma C.0.1, we know that $A(\beta)$ is increasing in $\beta$. Hence, $w_2^\beta$ is increasing in $\beta$ for $\beta < \beta_b$. Taking derivative of $w_1^\beta$ with respect to $\beta$, we have

$$\frac{\partial w_1^\beta}{\partial \beta} = \frac{1}{A(\beta)} + 1 - \frac{\beta}{A^2(\beta)} \frac{\partial A(\beta)}{\partial \beta} = \frac{1}{A(\beta)} + 1 - \frac{c_2 - c_1}{\beta(1 + A(\beta) + A^2(\beta))}. \quad (C.16)$$

We know from Equation (C.10) that $\frac{c_2 - c_1}{\beta} = \ln A(\beta) + A(\beta) - \frac{1}{A(\beta)}$. In addition, $\lim_{\beta \to 0} A(\beta) = 0$, hence we have

$$\lim_{\beta \to 0} \frac{\partial w_1^\beta}{\partial \beta} = \lim_{A \to 0} \left[ \frac{1}{A} + 1 + \frac{\ln A + A - \frac{1}{A}}{1 + A + A^2} \right] = 1 + \lim_{A \to 0} \ln A = -\infty, \quad (C.17)$$

153
where the second equality follows from Lemma C.0.1 (ii). We take the second derivative of $w_1^\beta$ with respect to $\beta$

$$\frac{\partial^2 w_1^\beta}{\partial \beta^2} = \frac{1}{A^2} \frac{\partial A}{\partial \beta} + \frac{c_2 - c_1}{\beta^2(1 + A + A^2)^2} \left[ 1 + A + A^2 + \beta \left( \frac{\partial A}{\partial \beta} + 2A \frac{\partial A}{\partial \beta} \right) \right]$$

$$= \frac{c_2 - c_1}{\beta^2(1 + A + A^2)} + \left[ \frac{\beta^2(1 + A + A^2)^2}{\beta^2(1 + A + A^2)} - \frac{1}{A^2} \right] \frac{(c_2 - c_1)A^2}{\beta^2(1 + A + A^2)}$$

$$= \frac{(c_2 - c_1)^2A^2(1 + 2A)}{\beta^3(1 + A + A^2)^3} > 0. \quad (C.18)$$

Expression (C.18) implies that $\frac{\partial w_1^\beta}{\partial \beta}$ is increasing in $\beta$ and there is at most one solution to $\frac{\partial w_1^\beta}{\partial \beta} = 0$. Letting (C.16) equal to 0 and replacing $\frac{c_2 - c_1}{\beta}$ by $\ln (A + A - 1) A$, we have

$$\left( \frac{1}{A} + 1 \right)(1 + A + A^2) + \ln A - \frac{1}{A} + A = 0. \quad (C.19)$$

Eq. (C.19) can then be solved using Matlab to yield $A^* = 0.0994$. The corresponding $\beta$ that results in $\frac{\partial w_1^\beta}{\partial \beta} = 0$ is $\beta = \frac{c_2 - c_1}{\ln A + A - 1} \bigg|_{A=0.0994} = \frac{c_2 - c_1}{12.2606}$. \qed

**Proof of Proposition 5.5.1.** The proof is similar to the that of Proposition 5.3.1. \qed

**Proof of Proposition 5.5.2.** The expected profit of supplier $i$, who offers a wholesale price of $w_i$, equals to

$$\pi_{si}(w_i; w_{-i}; \beta) = (w_i - c_i) \frac{\frac{u(w_i)}{\beta} e \frac{u(w_i)}{\beta}}{e \frac{u(w_i)}{\beta} + \sum_{j \neq i} e \frac{u(w_i)}{\beta}}.$$

The best response is given by its first derivative with respect to $w_i$

$$\frac{\partial \pi_{si}}{\partial w_i} = \frac{\frac{u(w_i)}{\beta} e \frac{u(w_i)}{\beta}}{e \frac{u(w_i)}{\beta} + \sum_{j \neq i} e \frac{u(w_i)}{\beta}} + (w_i - c_i) \cdot \frac{u'(w_i) \frac{u(w_i)}{\beta} \sum_{j \neq i} e \frac{u(w_i)}{\beta}}{e \frac{u(w_i)}{\beta} + e \frac{u(w_i)}{\beta}}^2$$

$$= \left( \frac{\frac{u(w_i)}{\beta} e \frac{u(w_i)}{\beta}}{e \frac{u(w_i)}{\beta} + \sum_{j \neq i} e \frac{u(w_i)}{\beta}} \right)^2 \left[ \frac{u(w_i)}{\beta} e \frac{u(w_i)}{\beta} + \sum_{j \neq i} e \frac{u(w_i)}{\beta} + (w_i - c_i) \frac{u'(w_i)}{\beta} \sum_{j \neq i} e \frac{u(w_i)}{\beta} \right]. \quad (C.20)$$
Denote the part in the bracket of (C.20) as $F_i(w_1, \cdots, w_n)$. Then supplier $i$’s best response $w^*_i(w_{-i}; \beta)$ is dictated by $F_i(w_1, \cdots, w_n) = 0$. Hence, we have

$$\frac{\partial w^*_i}{\partial w_j} = -\frac{\partial F_i}{\partial w_j}.$$ 

A well-known fact (Cachon & Netessine, 2004) says that non-decreasing best responses guarantee the existence of a Nash equilibrium. Clearly, if $w^*_i$ is the best response, it must be the case that $\frac{\partial F_i}{\partial w_i} < 0$. Hence, it is sufficient to have $\frac{\partial F_i}{\partial w_j} \geq 0$ for the slope to be positive. Note that

$$\frac{\partial F_i}{\partial w_j} = \frac{u'(w_j)}{\beta} e^{\frac{u(w_j)}{\beta}} \left[ 1 + \frac{(w_i - c_i) u'(w_i)}{\beta} \right].$$

From (C.20), we know that $1 + \frac{(w_i - c_i) u'(w_i)}{\beta} = -e^{\frac{u(w_i)}{\beta} - \sum_{j \neq i} u(w_j)} < 0$. Therefore, when $u(w_j)$ is non-increasing in $w_j$, we have $\frac{\partial F_i}{\partial w_j} \geq 0$, which indicates that supplier $i$ has a non-decreasing best response. Notice that $\frac{\partial F_i}{\partial w_j} \geq 0$ is valid for any $j \neq i$. This completes the proof. \qed