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COMPLIANCE CORRELATION FOR THROUGHWALL CRACKED ELBOW TO MEASURE CRACK GROWTH

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ABSTRACT

The unloading compliance technique is very commonly used to measure crack growth during fracture experiment. The pre-requisite of this technique is an equation correlating crack length with unloading compliance. Such correlations are easily available for small specimens. Recently the present authors developed similar correlations for pipe with throughwall circumferential crack under bending moment. But no such correlation exists for elbows. Development of such a correlation for throughwall circumferentially cracked elbow under in-plane bending moment forms the motivation of the present paper. Unlike conventional specimen geometries, elbow cross section ovalises significantly during deformation which makes the problem non-linear. Considering these complexities, new compliance correlations are proposed based on non-linear FEA for cracked elbow geometry. The proposed correlations are expressed in terms of normalized parameters to make them independent of specific values of geometric dimensions. Elastic perfectly plastic material response has been assumed while developing the correlation. However, effect of actual material stress-strain response on the compliance values has also been studied.

INTRODUCTION

The unloading compliance technique is one of the most commonly used and also the simplest methods to measure crack growth during fracture test of small specimens, e.g. Compact Tension (CT) and Three Point Bend (TPB) etc. (ASTM (1999)). This technique allows crack growth to be inferred from the unloading compliance values at periodic intervals during the fracture tests. The pre-requisite of this technique is the availability of an equation that correlates crack length with unloading compliance. Such correlations are easily available (ASTM (1999)) for small specimens, (e.g. CT and TPB). However, the same is not true for piping components, which are commonly tested all over the world to resolve various fracture mechanics issues. Recently, Chattopadhyay et al (2014) proposed a new unloading compliance correlation for through wall circumferentially cracked (TCC) pipe under bending. Similar equation for elbow or pipe bend is still not available in the open literature. The development of unloading compliance correlation for through wall cracked elbow thus forms the motivation for the present work. As mentioned in Chattopadhyay et al (2014), development of compliance correlation for pipe / elbow has additional challenges due to ovalisation of cross section during deformation. In case of pipe, the degree of ovalisation is very small. However, in case of elbow, the degree of ovalisation is quite significant and nature of ovalisation is even different for in-plane closing and opening bending moment (Chattopadhyay et al (2005a)). Considering all these complexities, an elaborate numerical study has been undertaken to develop unloading compliance correlations for through wall cracked elbow. Nonlinear finite element analysis considering both geometric and material non-linearity has been carried out for 90^o elbows of different *R/t* ratios with various sizes of through wall circumferential cracks at extrados under in-plane closing bending moment and at intrados under in-plane opening bending moment. The basic approach has been same as adopted for pipe (Chattopadhyay et al (2014)). First, an equation correlating initial elastic compliance and crack angle of TCC elbow has been developed. Subsequently, the ratios of compliance at different value of load/deformation and the initial elastic compliance have been evaluated to include the effect of load / deformation on compliance.

FINITE ELEMENT ANALYSIS

The finite element method is used to develop the compliance correlations. Elbow with through wall circumferential crack at extrados / intrados subjected to in-plane closing / opening bending moment (Figs.1a, 1b) is considered for analysis. Each elbow is loaded till around the theoretical plastic collapse bending moment with periodic unloading. Theoretical plastic collapse bending moment is calculated using the equations proposed by Chattopadhyay et al (2009).

The load vs. crack mouth opening displacement (CMOD) has been generated for the entire load range with periodic unloading. The CMOD is calculated at the middle of crack length (where it is maximum) and at mid-thickness. The amount of unloading is around 15% of collapse moment. Stiffness is evaluated from the slope of the load-CMOD curve by linear curve fitting of the middle portion of the unloading path leaving aside the top and bottom 1% of collapse moment. Compliance is evaluated by taking reciprocal of this stiffness evaluated at different stages of loading including the initial elastic portion. Compliance is defined as follows:

$$C = \delta / M \tag{1}$$

where, 'M' is the moment at the cracked cross section of the elbow, δ is the total crack mouth opening displacement and 'C' is the compliance. The initial elastic compliance parameter has been expressed in non-dimensional form as:

$$\lambda_o = \pi R^2 C_o E \tag{2}$$

where, 'λ_o' is the non-dimensional initial elastic compliance, 'C_o' is the initial elastic compliance, 'E' is the Young's modulus of the pipe material and 'R' is the mean radius of elbow cross section. The form of normalization has been based on the proposed CMOD equations for throughwall circumferentially cracked elbow subjected to in-plane bending moment by Zahoor (1989-1991) and Chattopadhyey et al (2005b,2007). Compliance values at higher load levels (C) are normalized with respect to the initial elastic value (C/C_o). Applied moment (M) is also normalized with respect to the theoretical collapse moment as follows:

$$m = M / M_L \tag{3}$$

where, m is the normalized applied moment and M_L is the theoretical plastic collapse moment of elbow containing a throughwall circumferential crack at extrados / intrados. The rationale of normalizing the compliance and load level is to make the equations independent of specific geometric dimensions of elbow. Thus, normalized compliances have been generated for various values of normalized load levels (m) and crack angles (θ/π).

Geometric details

Geometrically, a 90^o TCC elbow is characterized by three parameters, namely, R_b/R, R/t and 2θ. Table 1 shows different combinations of these parameters taken in the study. In general, R_b/R = 3 is assumed in the present study indicating a long radius elbow. However, few short radius elbows with R_b/R = 2 have also been analyzed to study the effect of bend radius on compliance correlations of elbows. The R/t is varied from 5 to 20 covering a wide range for engineering use. Crack angles (2θ) are varied from 30^o – 150^o depending on the mode of bending moment and R/t. It was concluded by Chattopadhyay et al (2009) that there is a threshold crack angle for TCC elbow beyond which crack starts opening up and weakening effect of crack initiates and the value of this threshold angle varies with R/t. Range of crack angles as mentioned in Table 1 takes care of this aspect. In the present analyses, the elbow is connected with straight pipes of length equal to the six times the mean cross sectional radius. It is important to note that this straight pipe length allows free propagation of ovalisation from mean elbow cross section to the end of straight pipe portion where the bending moment is applied. The actual value of mean radius (R) has been chosen as 50 mm, although results do not depend on any specific value of 'R' as they all are expressed in non-dimensional form. To validate this aspect of independence of normalized compliance parameters with respect to specific value of mean radius, few cases with R = 100 mm have also been analyzed.

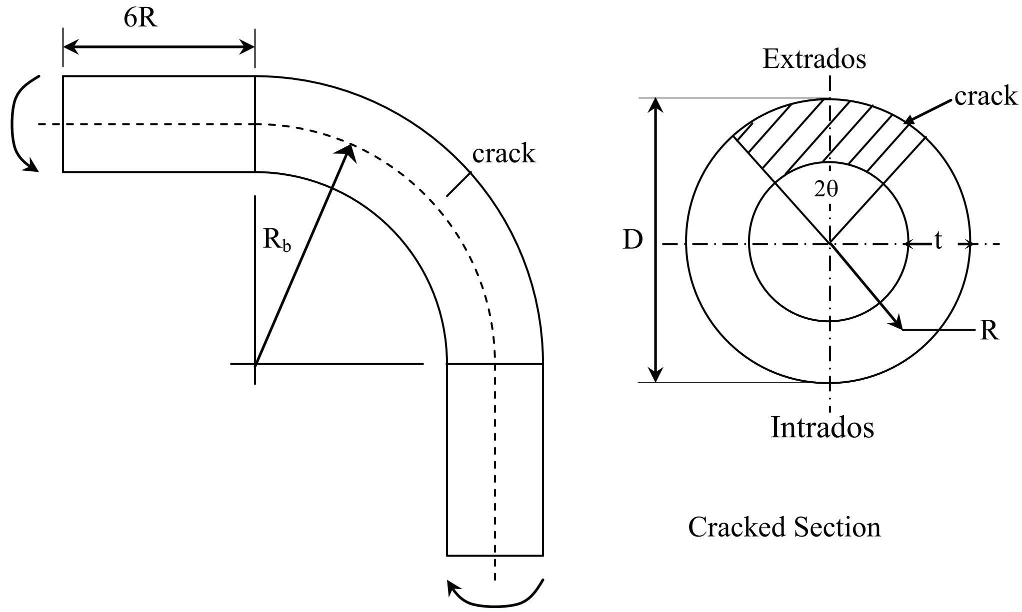


Fig.1a Geometry of elbow with a through wall circumferential crack at extrados under closing moment

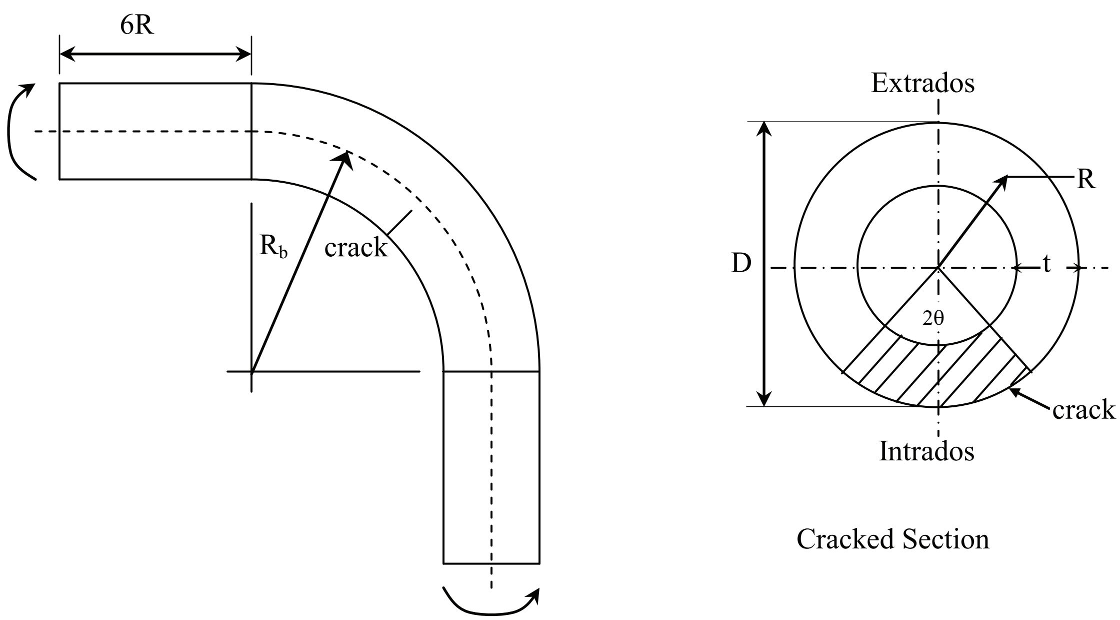


Fig.1b Geometry of elbow with a through wall circumferential crack at intrados under opening moment

Material Properties

Elastic-perfectly plastic material model is used in this study. The other material properties chosen are as follows: Young's modulus (E): 203 GPa, Yield stress: 288 MPa, Poisson's ratio: 0.3. Although, it has been shown later that choice of these specific values does not affect the compliance correlations significantly.

Table 1 Various geometric and loading parameters of analyzed elbows

Bending Mode : Closing R : 50 mm, R_b/R : 3 2θ : 60°,90°,105°,120°,135°,150° (R/t = 5) : 90°, 105°,120°, 135°,150° (R/t = 10) : 120°, 135°,150° (R/t = 15 & 20) No. of cases : 17	Bending Mode : Opening R : 50 mm, R_b/R : 3 2θ : 30°, 60°, 90°, 105°, 120°, 135°, 150° (R/t = 5) : 60°, 90°, 105°, 120°, 135°, 150° (R/t = 10) : 90°, 105°, 120°, 135°, 150° (R/t = 15) : 105°, 120°, 135°, 150° (R/t = 20) No. of cases : 22
Bending Mode : Closing R : 50 mm, R/t : 5, R_b/R : 2, 2θ : 150° No. of cases : 1	Bending Mode : Opening R : 50 mm, R/t : 5, R_b/R : 2, 2θ : 150° No. of cases : 1
Bending Mode : Closing R : 100 mm, R/t : 10, R_b/R : 3, 2θ : 150° No. of cases : 1	Bending Mode : Opening R : 100 mm, R/t : 10, R_b/R : 3, 2θ : 150° No. of cases : 1
Total no. of cases: 43	

Finite Element Model

Twenty-noded solid elements are used to model the elbows. Because of symmetry, only one fourth of the elbow is modelled. The total number of elements varied between 204-221 for closing cases and 216-234 for opening cases with 1 element across the thickness of the elbow, and number of nodes varied between 1576 – 1700 for closing cases and 1665 – 1796 for opening cases. Mesh convergence studies for closing and opening mode of bending moments have been carried out to prove the adequacy of the finite element mesh employed.

RESULTS AND DISCUSSION

Figure 2 shows the typical moment versus CMOD data for 90° elbows having R/t = 5 for opening mode of bending moment. From these data, initial elastic compliance (C_o) has been calculated by linear regression of the initial 5% data. It is then normalized using Eq.(2). Figure 3 shows the plot of crack angle versus normalized initial elastic compliance for opening mode of bending moments. Subsequently, unloading compliance values have been calculated and normalized with respect to initial elastic compliance (C/C_o) at various load levels expressed in normalized form as per Eqn.(3). Figure 4 shows the variation of normalized compliance (C/C_o) with normalized load level (m) for various crack angles for R/t = 5 and 20 subjected to closing bending moment. Figure 5 shows similar variations for opening mode of bending moment for R/t = 5 and 10.

The line C/C_o = 1 indicates the condition when there is no geometric hardening / softening; in other words, it indicates that deformation does not affect unloading compliance. A general trend is observed from Figs.4-5 that deformation effect is most prominent for thinner elbows and it gradually diminishes as the relative thickness (t/R) increases. It is also observed that for a given relative thickness (t/R), the effect decreases with crack angles and increases with load / deformation. Between closing and opening mode of bending moments, it is seen that deformation effect on unloading compliance is more prominent for closing case. Figure 4 shows that except for relatively thicker elbow and larger crack angles subjected to closing mode of bending moment, there is mostly geometric hardening, which leads to decrease of compliance (i.e. C/C_o decreases less than 1) with increase in load or deformation. A different trend is observed in case of opening mode of bending moment (see Fig.5). In this case, mostly geometric softening is observed that leads to increase of compliance (i.e. C/C_o increases more than 1). However, the degree of geometric softening reduces with increase in crack angles and it transforms to geometric hardening at larger crack angles ($2\theta \geq 120°$). This geometric hardening / softening is attributed to ovalization of the circular cross section of the elbow during deformation, which also explains the difference in trend in case of opening and closing mode of bending moments. This is because it is well known (Chattopadhyay et al (2005a,2009)) that the ovalisation pattern is different for these two modes.

PROPOSED COMPLIANCE EQUATIONS

For use of unloading compliance method to measure crack growth during fracture tests, crack length needs to be expressed as a function of compliance values. Therefore, two new correlations (Eqs.4 & 5) expressing crack length (θ/π) as a function of normalized initial elastic compliance ($\lambda_o = \pi R^2 C_o E$) are first proposed for two different modes of bending moments, namely, closing and opening, respectively. Figure 3 shows the typical comparison of FE data points with predictions of the proposed equations 5 for opening mode of bending moment.

Closing moment

$$\frac{\theta}{\pi} = \frac{\left[-7.4999.\lambda_0^{0.5933} + 7.258.\lambda_0^{0.5977}\right]}{\left[-3.851\left(\frac{R}{t}\right)^{0.5938} + 2.125\left(\frac{R}{t}\right)^{0.7264}\right]} \tag{4}$$

Opening moment

$$\frac{\theta}{\pi} = \frac{0.5782\left[0.5233\lambda_0^{0.5339} - 0.3362\lambda_0^{0.5875}\right]}{\left(\frac{R}{t}\right)^{0.2546}} \tag{5}$$

The average error between actual FE data and predictions by proposed equations (4 and 5) is around 0.6% and 1.07% for closing and opening mode of bending moment respectively. Subsequently, equations correlating the ratio of compliance at higher deformation level and the initial elastic compliance *(C/C_o)* with normalized crack angle (θ/π) and load level ($m = M/M_L$) are proposed for various R/t values. It is done through regression analyses of the finite element data. Four different equations (Eqs.6-9) of normalized compliance (C/C_0) have been proposed for R/t = 5, 10, 15 and 20 for closing mode of bending moment. Similarly, another set of four equations (Eqs.10-13) are proposed for opening mode of bending moment. The average error in curve fitting has been around 0.75% and 1.61% for closing and opening mode of bending moment respectively. The proposed equations are as follows:

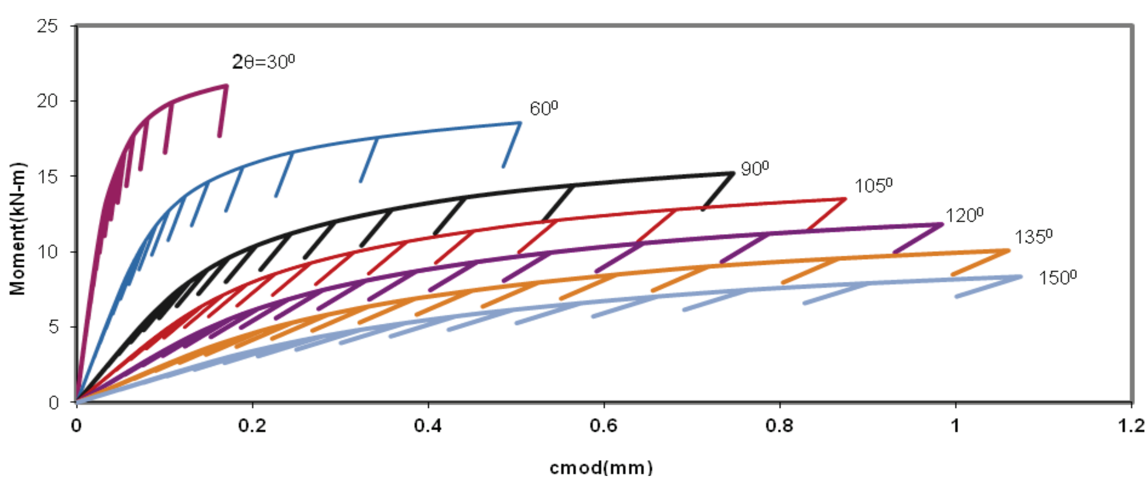


Fig.2 Moment v/s CMOD data for one elbow (R/t = 5) with various crack angles under opening moment

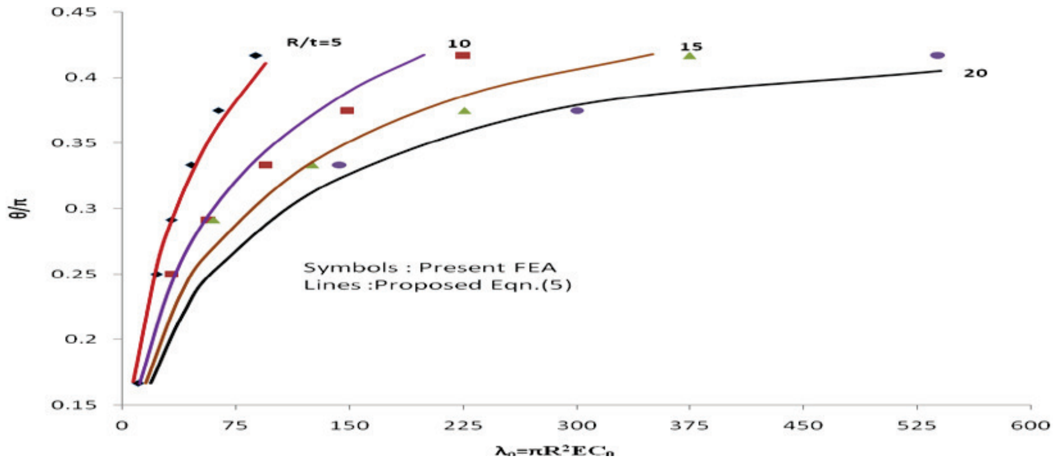
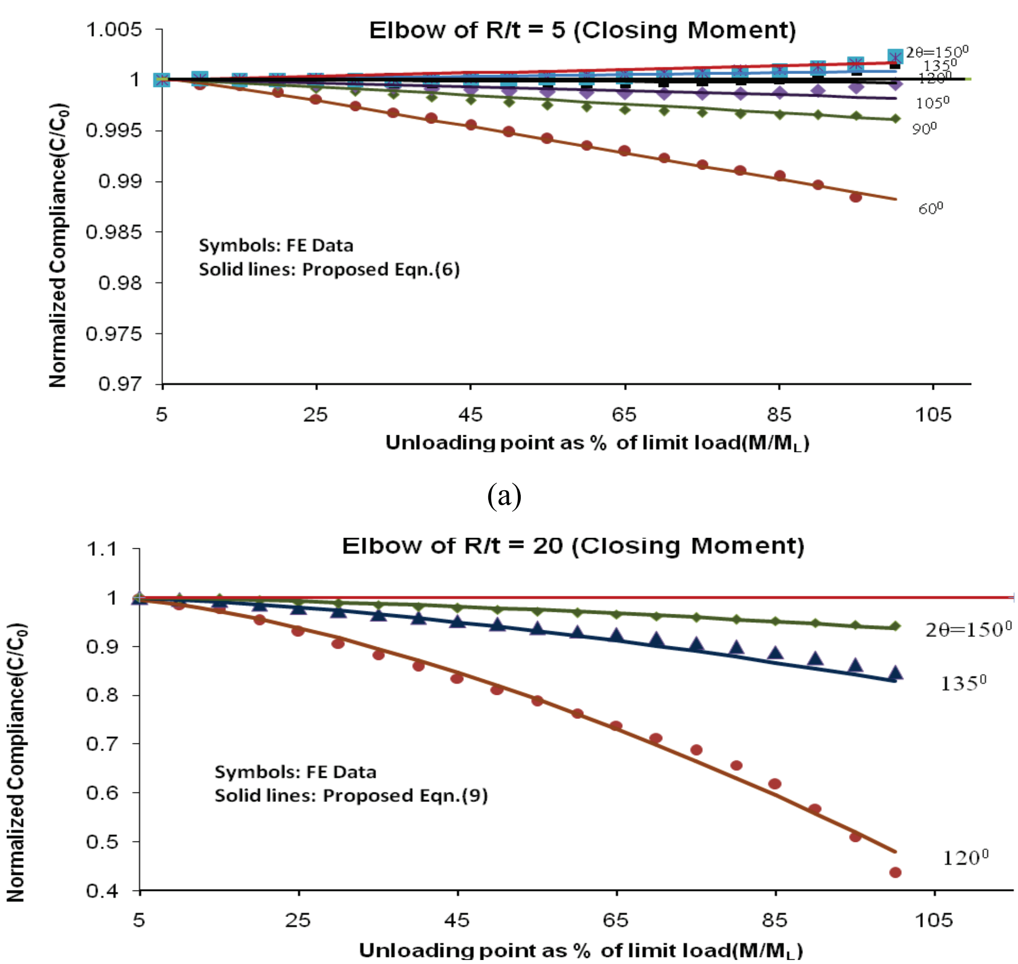


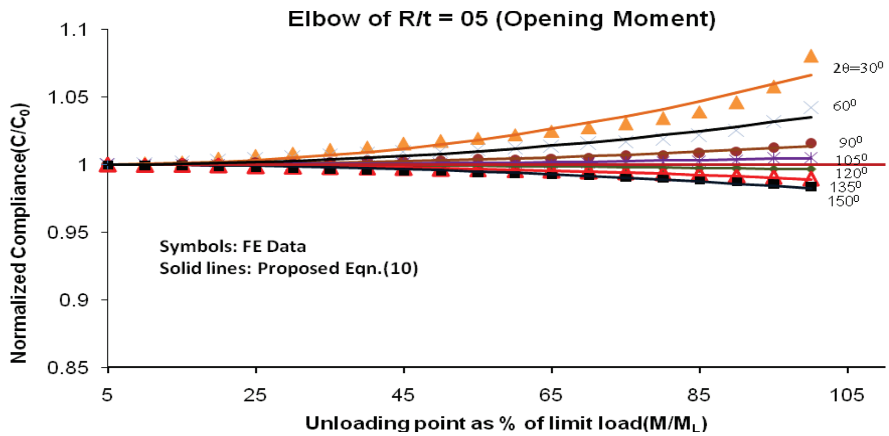
Fig.3 Crack angle versus normalized initial elastic compliance curves for 90^o elbows having different R/t ratios subjected to opening mode of bending moment



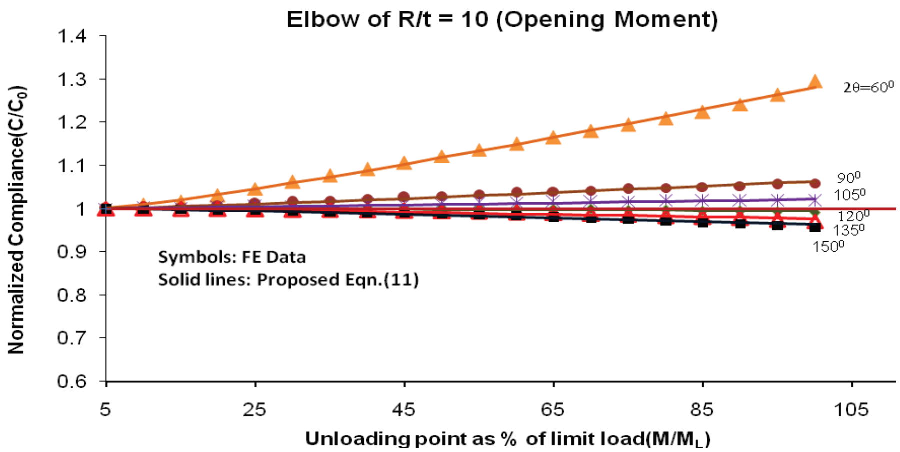
(a)

(b)

Fig. 4 Normalized compliance (C/C_o) v/s normalized load level ($m = M/M_L$) for 90° elbows subjected to closing mode of bending moment with various crack angles for (a) R/t = 5 and (b) R/t = 20



(a)



(b)

Fig. 5 Normalized compliance (C/C_o) v/s normalized load level ($m = M/M_L$) for 90° elbows subjected to opening mode of bending moment with various crack angles for (a) R/t = 5, (b) R/t = 10

Closing Moment

For R/t = 5

$$\frac{C}{C_0} = 1 + \left[-0.9434\left(\frac{\theta}{\pi}\right)^{-0.8139} + 0.9623\left(\frac{\theta}{\pi}\right)^{-0.7958} \right]\left[0.2295m^{1.2076} + 0.0516m^{21.9405} \right] \tag{6}$$

For R/t = 10

$$\frac{C}{C_0} = 1 - \left[0.118\left(\frac{\theta}{\pi}\right)^{-3.0382} - 0.9933\left(\frac{\theta}{\pi}\right)^{-0.3208} \right]\left[0.333m^{1.445} - 0.31661m^{1.4452} \right] \tag{7}$$

For R/t = 15

$$\frac{C}{C_0}=1-\left[0.001341\left(\frac{\theta}{\pi}\right)^{-4.7801}-0.002159\left(\frac{\theta}{\pi}\right)^{-3.9524}\right]\left[1.2811m^{1.3196}+0.1594m^{14.8639}\right] \tag{8}$$

For R/t = 20

$$\frac{C}{C_0}=1+\left[-0.112\left(\frac{\theta}{\pi}\right)^{-9.5726}+0.5029\left(\frac{\theta}{\pi}\right)^{-6.2089}\right]\left[0.0001137m^{1.2307}+0.00003526m^{5.7316}\right] \tag{9}$$

Opening Moment

For R/t = 5

$$\frac{C}{C_0}=1-\left[-0.3646\left(\frac{\theta}{\pi}\right)^{0.1284}+0.3781\left(\frac{\theta}{\pi}\right)^{0.16}\right]\left[4.2337m^{1.3879}+3.0107m^{15.0679}\right] \tag{10}$$

For R/t = 10

$$\frac{C}{C_0}=1-\left[-0.002484\left(\frac{\theta}{\pi}\right)^{-2.5079}+0.07489\left(\frac{\theta}{\pi}\right)^{0.514}\right]\left[-1.7635m^{0.8007}+3.2225m^{0.9874}\right] \tag{11}$$

For R/t = 15

$$\frac{C}{C_0}=1-\left[-0.1882\left(\frac{\theta}{\pi}\right)^{-3.1406}+0.5964\left(\frac{\theta}{\pi}\right)^{-2.1142}\right]\left[2.3292m^{2.1284}-2.2748m^{2.1915}\right] \tag{12}$$

For R/t = 20

$$\frac{C}{C_0}=1-\left[-0.001528\left(\frac{\theta}{\pi}\right)^{-6.6005}+3.1603\left(\frac{\theta}{\pi}\right)^{0.199}\right]\left[0.04151m^{0.9717}\right] \tag{13}$$

EFFECT OF STRAIN HARDENING ON THE COMPLIANCE

The FE analyses to generate the compliance correlations are carried out assuming elastic-perfectly plastic material behavior to simplify the material characterization. However, actual materials show strain hardening behavior. Therefore, a study has been undertaken to check the effect of material strain hardening on compliance values in comparison to postulated elastic-perfectly plastic material behavior. One case for closing mode of bending moment has been taken for study with R/t = 15 and 2θ = 120°. To represent a realistic material property, true stress – true strain data of one nuclear grade piping carbon steel SA333Gr6 material is taken (Chattopadhyay (2005a)). Four sets of results are shown with different material parameters, namely, (i) actual true stress – true strain data, (ii) elastic- perfectly plastic at YP = 288 MPa, (iii) elastic-perfectly plastic at UTS = 420 MPa and (iv) elastic-perfectly plastic at flow stress = 354 MPa (average of YP and UTS). Figure 6 shows the comparison for various material properties. It may be seen from the figure that if actual true stress – true strain material characterization is idealized as elastic-perfectly plastic material behavior at flow stress equal to average of yield and ultimate strength, compliance values are very close to each other. It proves that elastic-perfectly plastic material idealization does not affect significantly the compliance values provided one chooses perfect plasticity at appropriate flow stress.

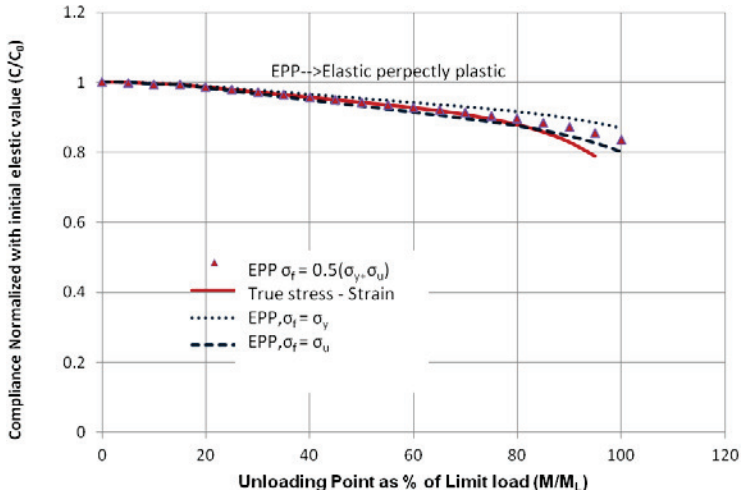


Fig. 6 Effect of strain hardening on compliance values for R/t = 15 and $2\theta = 120^o$ under closing moment

INDEPENDENCE OF NORMALIZED COMPLIANCE WITH RESPECT TO CHANGE OF GEOMETRIC PARAMETERS

To make the compliance value independent of geometric parameters e.g. radius etc., it has been normalized as: $\lambda_o = \pi R^2 C_o E$. A study has been undertaken to investigate the effectiveness of the normalization of compliance by changing the mean radius (R). Both the modes of bending moments, namely, closing and opening are studied. Two values of mean radii (R) of 50 and 100 mm are taken for study with R/t = 10 and $2\theta = 150^o$. Figure 7 shows the plot of normalized compliance (C/C_o) with normalized moment (M/M_L). Average percentage error with respect to standard radius of 50 mm cases is found to be 0.11% and 0.52% for closing and opening moments respectively. This is well within acceptable limit.

Also a study has been undertaken to investigate the effect of mean bend radius of elbow on the normalized compliance (C/C_o). Two values of bend radii (R_b) of 150 and 100 mm are taken for study with R/t = 5 and $2\theta = 150^o$. Standard bend radius of R_b = 150 mm indicate a long radius elbow ($R_b = 3R$) and R_b = 100 mm indicate a short radius elbow ($R_b = 2R$). As in the study of mean radius effect, both the modes of bending moments are analyzed in this case as well. Average percentage error with respect to standard bend radius of 150 mm cases is found to be 0% and 0.12% for closing and opening moments respectively. This is also well within acceptable limit.

These exercises prove the effectiveness of the normalization of the compliance parameters to make the correlations independent of specific geometric dimensions and there is no significant effect of bend radius on these compliance correlations.

HOW TO USE THESE EQUATIONS

Following procedure should be followed to use the above-proposed equations to measure crack growth during fracture tests of TCC elbow:

a) Get unloading compliance (C) value at a particular load level ($m = M/M_L$) from test data.
b) Use flow stress, σ_f equal to average of yield stress and UTS to calculate 'M_L'.
c) Get the initial elastic compliance (C_o) value corresponding to 'm' using the proposed (C/C_o) Eqs.(6-9) or Eqs.(10-13) depending on the mode of bending moment (closing or opening) applied for a particular value of R/t (linear interpolation for intermediate R/t may be adopted).
d) Get the value of (θ/π) from the C_o or λ_o value using the proposed Eq. (4) or Eq.(5) depending on the mode of bending moment.

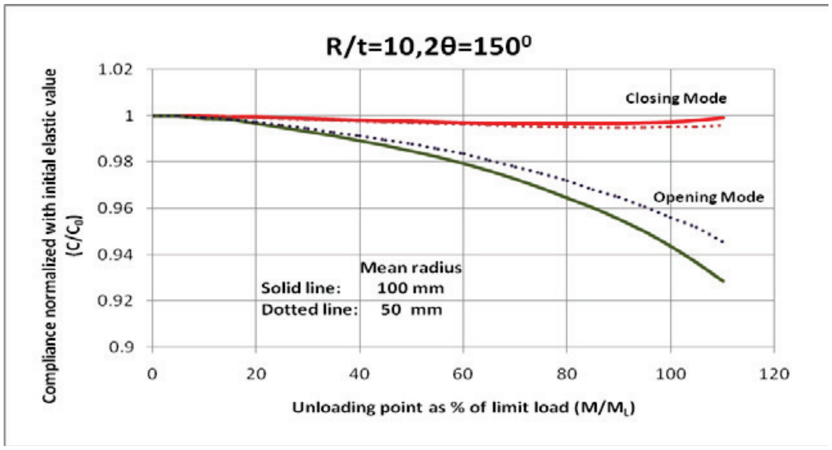


Fig. 7 Effect of change of mean radius on the normalized compliance values for in-plane moment

CONCLUSION

New compliance correlations have been proposed for throughwall circumferentially cracked elbow under in-plane closing and opening mode of bending moment for measurement of crack growth during fracture experiments. Unlike the conventional compliance correlations, the presently proposed ones account for the ovalisation of original circular cross section of elbow during deformation. Elastic-perfectly plastic material behavior has been assumed to characterize the material stress-strain response. However, it has been shown that error due to this approximation with respect to the actual stress-strain behavior is negligible if one chooses flow stress equal to average of yield and ultimate strength. The proposed correlations are expressed in terms of normalized parameters to make them independent of specific values of geometric dimensions.

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