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J CALCULATION FOR A CRACK IN A WELDING RESIDUAL STRESS FIELD FOLLOWING A FE WELDING SIMULATION

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ABSTRACT This paper addresses the calculation of J for a crack in a welding residual stress field using the modified J approach previously developed by the author and ABAQUS Version 6.11 and above. J evaluated using the modified J definition and the new ABAQUS version may become path-dependent when it is evaluated for a crack tip in a welding residual stress field following a numerical welding simulation. This is because the plastic strains and the plastic strain energy density accumulated during the welding simulation are irrelevant to the J-integral and the initial strains cannot be correctly extracted from the results of stress analysis. A "mapping stress" method is proposed in this paper, which includes mapping the uncracked body residual stresses and equivalent plastic strains into a stress-free uncracked model as initial conditions, introducing a crack, applying mechanical/thermal loads and then calculating J. A plate with a slit-weld is analysed for a crack in the weld under mechanical loading. The results show that the J calculated using the recommended method shows good path-independency and can well correlate the crack tip stress field.

INTRODUCTION

For a cracked structure under combined residual stress and other primary and secondary loads, J evaluated using the Rice definition [Rice (1968)] may become path-dependent due to the initial strains corresponding to the residual stresses and the non-proportional stressing conditions. Lei et al. (2000) introduced a new J definition for general initial strain problems and developed methods for the evaluation of initial strains in finite element (FE) analyses for general residual stress types. This J definition has been extended to initial strain problems with non-proportional stressing by Lei (2005). The J definition given in [Lei (2005)] is referred to as "modified J". Software as a post processing program for the commercial FE software ABAQUS [e.g. ABAQUS (2010), (2014)] has been developed to evaluate the "modified J", which is path-independent for a crack tip in a residual stress field, for 2-D and axisymmetric problems. In the early versions of ABAQUS (v6.10 and lower) [ABAQUS (2010)], the Rice J definition [Rice (1968)] with a correction to the thermal load is embedded, which is path-dependent for general residual stress problems. From its version v6.11 and above [e.g. ABAQUS (2014)], ABAQUS has implemented a similar J definition to that given in [Lei et al. (2000)] with an assumption of proportional loading and enables J to be evaluated for cracks located in residual stress fields for 3-D crack problems. However, the J procedure in the current ABAQUS version [e.g. ABAQUS (2014)] should be used with caution because it may not correctly extract initial strains for some types of residual stresses and correct J values can be obtained only when the "residual stress step" is correctly identified. This issue will not be discussed in this paper but the J values obtained from ABAQUS v6.14 [ABAQUS (2014)] and presented in this paper are reliable.

In a FE J-analysis, several methods can be used to simulate self-equilibrated welding residual stresses, but more complex and realistic welding residual stress distributions are normally obtained using a FE welding simulation. Results from recent analyses have shown that for a crack introduced in a welding residual stress field following a FE welding simulation both the modified J method and the J procedure in the later versions of ABAQUS [e.g. ABAQUS (2014)] are strongly path-dependent. Some results from the FE analyses of this paper can be found later in Fig. 7(a). In this paper, the problem will be investigated and a method proposed for evaluating J reliably and correctly following a FE welding simulation. A case of a crack in a slit-weld in a plate will be analysed to validate the proposed method.

BACKGROUND TO *J* CALCULATION

In this section, the theory of the modified *J*-integral and the new version of the ABAQUS *J* are briefly reviewed to provide a background for the methodology used to evaluate the *J*-integral following a FE welding simulation.

J definitions

Lei (2005) modified the general *J*-integral definition given by Shih et al. (1986) into a path-independent integral allowing non-proportional stressing for general initial strain problems. When body force and crack face traction are absent, this can be expressed as

$$J = \int_{\Gamma}\left(W\delta_{1i} - \sigma_{ij}\frac{\partial u_j}{\partial x}\right)n_i\, ds + \int_{A}\left(\sigma_{ij}\frac{\partial \varepsilon_{ij}}{\partial x} - \frac{\partial W}{\partial x}\right)dA \tag{1}$$

where σ_{ij} and u_j are components of stress and displacement, respectively, in Cartesian coordinates, Γ is a curve surrounding the crack tip which begins at the lower face of the crack and ends at the upper one, n_i is the unit outward vector normal to Γ, ds is the arc length along Γ (see Fig. 1), δ_{ij} is the Kronecker delta tensor, *A* is the area enclosed by Γ and the strain energy density, *W*, is defined as mechanical strain energy density by [Lei (2005), (2006)]

$$W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}^{e} + \int_{\varepsilon_{ij}^{p}|_{ucb}}^{\varepsilon_{ij}^{p}}\sigma_{ij}d\varepsilon_{ij}^{p} \tag{2}$$

where ε_{ij}^{e} and ε_{ij}^{p} are the elastic and plastic mechanical strains, respectively, $\varepsilon_{ij}^{p}\big|_{ucb}$ are the uncracked body plastic strains and the total strains are the sum of mechanical strains and initial strains, ε_{ij}^{0}, i.e. [Lei (2006)]

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} - \varepsilon_{ij}^{p}\,|_{ucb} + \varepsilon_{ij}^{0} \tag{3}$$

Note that $\varepsilon_{ij}^{p}\big|_{ucb}$ and thermal strains are treated as part of the initial strains, ε_{ij}^{0}. Here, the term "initial strain" represents any "non-mechanical strain" recognised in fracture mechanics.

Equation (1) reduces to the Rice *J* [Rice (1968)], which is embedded in the early versions of ABAQUS [e.g. ABAQUS (2010)], when proportional loading holds for all material points in the integration domain and initial strains do not exist. The definition expressed in Equation (1) is valid for homogeneous elastic and elastic-plastic materials under combined primary and secondary stresses, such as residual and thermal stresses. However, for elastic-plastic materials, the monotonic stressing condition should be satisfied in the near crack-tip region.

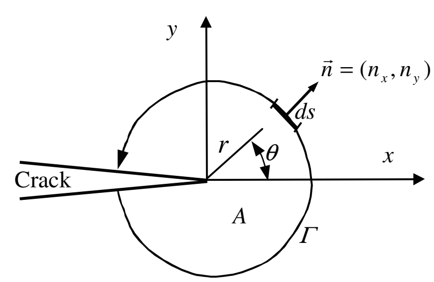


Figure 1 Contour integration path around crack tip

In the later ABAQUS versions [e.g. ABAQUS (2014)], the *J*-integral is defined, based on the assumption of proportional loading, as

$$J = \int_{\Gamma} \left(W\delta_{1i} - \sigma_{ij} \frac{\partial u_j}{\partial x} \right) n_i \, ds + \int_A \sigma_{ij} \frac{\partial \varepsilon_{ij}^{th}}{\partial x} dA + \int_A \sigma_{ij} \frac{\partial \varepsilon_{ij}^{0}}{\partial x} dA \tag{4}$$

where ε_{ij}^{th} represents thermal strains and the strain energy density is defined as

$$W = W^e + W^p = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}^e + \int_0^{\varepsilon_{ij}^p} \sigma_{ij} d\varepsilon_{ij}^p \tag{5}$$

where W^e and W^p are the elastic and plastic strain energy densities, respectively.

Note that Equation (4) is very similar to the *J* definition given in Lei (2000) but the initial strain definitions are different. In Lei (2000), thermal strains are included in the initial strains but in ABAQUS [e.g. ABAQUS (2010), (2014)] they are treated separately (see Equation (4)). Another difference is in the definition of strain energy density. In Lei (2000), the plastic strain energy density accumulated before the crack is inserted into the residual stress field is excluded but in the later versions of ABAQUS [e.g. ABAQUS (2014)] the total value is used in the calculation.

For proportional loading, Equation (1) is identical to Equation (4), noting that the thermal strains are included in the initial strains in Equation (1). However, the definitions of strain energy density are different, as discussed above.

***J* and the crack tip fields**

For power-law hardening materials, *J* is explicitly linked to the crack tip fields via the HRR formulations [Hutchinson (1968), Rice and Rosengren (1968)], as

$$\frac{\sigma_{ij}}{\sigma_0} = \left(\frac{1}{\alpha\varepsilon_0 I_n \bar{r}} \right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta, n) \tag{6}$$

$$\frac{\varepsilon_{ij}}{\varepsilon_0} = \alpha \left(\frac{1}{\alpha\varepsilon_0 I_n \bar{r}} \right)^{n/(n+1)} \tilde{\varepsilon}_{ij}(\theta, n) \tag{7}$$

where

$$\bar{r} = r/(J/\sigma_0) \tag{8}$$

when the material stress-strain relationship is described by the Ramberg-Osgood equation

$$\varepsilon^p/\varepsilon_0 = \alpha(\sigma/\sigma_0)^n \tag{9}$$

where σ and ε^p are stress and plastic strain, respectively, α and n are constants, σ_0 is a normalising stress and ε_0 is a normalising strain with $\varepsilon_0 = \sigma_0/E$ (*E* is Young's modulus). In Equations (6)-(7), I_n is a non-dimensional function of *n* and $\tilde{\sigma}_{ij}(\theta, n)$ and $\tilde{\varepsilon}_{ij}(\theta, n)$ are non-dimensional functions of *n* and θ (θ and *r* are polar co-ordinates centred at the crack tip (see Fig. 1)).

From Equation (6), the normalised stress distribution reduces to a single curve independent of load level when plotted against non-dimensional distance, $\bar{r}$. Also, for any two load levels, Case 1 and Case 2, where the J values are $(J)_1$ and $(J)_2$, respectively, in view of Equation (6), the following equation holds for a given θ and stress values, σ_{ij} , in the J dominant zone:

$$\frac{(J)_1}{(J)_2} = \frac{(r)_1}{(r)_2} \qquad \text{or} \qquad (\bar{r})_1 = (\bar{r})_2 \tag{10}$$

where $(r)_1$ and $(r)_2$ are the distances between the crack tip and the points where the stresses are equal to the given value for Case 1 and Case 2, respectively.

For materials with non-power-law stress-strain relationships, Equations (6) and (7) do not apply. However, Equation (10) still holds and can be used to validate J values obtained from numerical analyses. Note that Equation (10) only defines the ratio between $(J)_1$ and $(J)_2$. A calibration has to be performed to determine the absolute J values.

METHOD FOR *J* CALCULATION FOLLOWING A FE WELDING SIMULATION

Recent *J*-analyses have shown that for a crack introduced in a welding residual stress field following a FE welding simulation or generated by using the ABAQUS “map solution” method from the FE welding simulation, both the modified *J* and the ABAQUS *J* in its later version [e.g. ABAQUS (2014)] may be strongly path-dependent. Some results from FE analyses of this paper can be found later in Fig. 7(a). The problem will be investigated in this section and a method will be proposed for correctly evaluating *J* following a FE welding simulation.

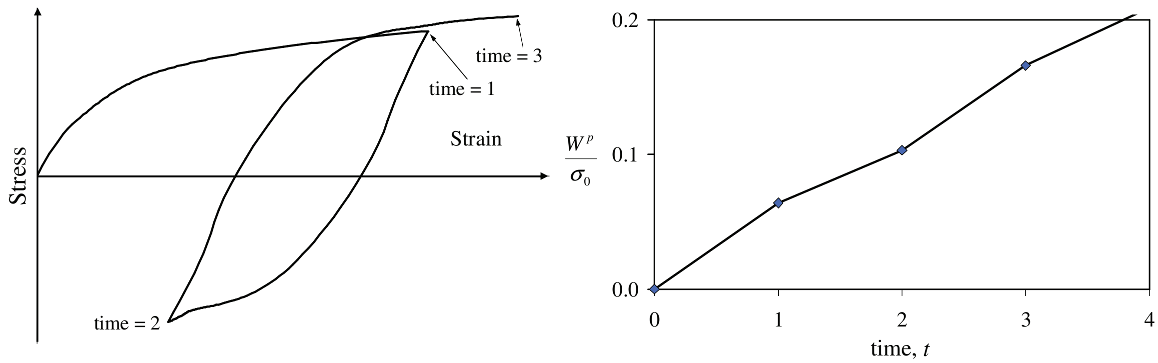
Factors contributing to the path-dependency of J

From Equations (1)-(5), the plastic strains, ε_{ij}^{p} , due to the crack insertion into a residual stress field or due to the crack tip loading, contribute to the *J*-integral via the evaluation of plastic strain energy density, W^{p} . For the modified *J*-integral, they also have a direct contribution to the total strains and, therefore, the *J*-integral. These contributions are necessary and should be correctly evaluated in the *J* calculation. For some reasons, plastic strains may be created in the uncracked body before the crack is inserted due to the loading history. These plastic strains may play a role as initial strains and, therefore, should be correctly extracted and considered in the *J* calculation. However, they should not contribute to the plastic strain energy density because they have nothing to do with the crack tip loading.

In ABAQUS [e.g. ABAQUS (2010), (2014)], the plastic strain energy density is evaluated in an accumulative way for all the loading history for elastic-plastic materials. Figure 2 shows the stress-strain history of a material point in an element and the corresponding plastic strain energy density, as an example [Lei (2003)]. From Fig. 2(b), the plastic strain energy density at a material point, W^{p}, defined by the second part of Equation (5) is a monotonically increasing function of time when the material at this point is loaded, unloaded and loaded again (Fig. 2(a)). This value of W^{p} could be very significant when the uncracked body has undergone some number of mechanical or thermal cycles.

Now look at the calculation of plastic strain and plastic strain energy density during the welding simulation process using ABAQUS [e.g. ABAQUS (2010), (2014)]. A material point in the weld undergoes intense thermal loading leading to large plastic strains and therefore plastic strain energy density. When the temperature of the material reaches the melting point of the material, all effects of the stress-strain history should physically vanish. This is fulfilled by “annealing” in ABAQUS [e.g. ABAQUS (2010), (2014)]. However, in the current ABAQUS code, only the equivalent plastic strain is “annealed” at the melt point, that is set to zero, leaving the plastic strain components and the accumulated plastic strain energy density unchanged. In the region adjacent to the weld, such as the heat affected zone

(HAZ), the metal temperature is lower than its melting point during the welding process. However, the material in such a region also undergoes intense thermal loading, which may lead to large plastic strains and accumulate a certain amount of plastic strain energy density. These data are irrelevant to the crack tip field and may affect the correct evaluation of *J* if they are directly used in the calculation.



(a) Stress-strain hysteresis loops at a material point

(b) Normalised plastic strain energy density vs. time

Figure 2 The stress-strain history and strain energy density, W^p, at a material point in the near crack tip region for a power-law hardening material [Lei (2003)]

In the modified *J* formulations, the plastic strain energy density created in the uncracked body is removed (see Equation (2)). However, in the later versions of ABAQUS [e.g. ABAQUS (2014)], the total plastic strain energy density is used in the *J* evaluation. The plastic strains produced in the welding simulation should contribute to the initial strains which directly affect the *J* calculation for a crack tip in the residual stress field. However, the methods used in the modified *J* scheme [Lei (2005)] and in the later versions of the ABAQUS *J* function [e.g. ABAQUS (2014)] might not extract/separate the correct part of the plastic strains as initial strains due to the complex thermal loading history in the welding simulation. This can directly affect the *J* calculation.

Methodology for solving the problem

The *J*-integral is a measure of the crack-tip stress and strain field intensity for non-linear and elastic-plastic materials. It should be determined by the near crack tip stress, strain and displacement fields due to the crack formation. Any plastic strains and plastic strain energy density for the uncracked body should not contribute to the crack tip stress and strain field intensity and, therefore, the *J*-integral. This means that the uncracked body stress fields are very important data for determining the crack tip *J*-integral. In this case, to avoid the difficulty in identifying the effective initial strains from the plastic strains for uncracked body analysis, an easy method is to map the uncracked body residual stress fields into a new model and then continue the analysis for *J* calculation using the modified *J* [Lei (2005)] or later versions of ABAQUS [e.g. ABAQUS (2014)]. In order to transfer the degree of strain hardening at the material points, the equivalent strain should also be mapped to the new model. The modified *J* may be evaluated by following the step-by-step instructions below.

(i) Map the uncracked body residual stresses and equivalent plastic strains into a stress-free uncracked/cracked model as initial conditions.

(ii) Introduce a crack into the residual stress field as required when mapping to an uncracked body was performed in step (i).

(iii) Apply mechanical or other type of loads.

(iv) Calculate J by following the normal procedures using the modified J definition or ABAQUS version v6.11 and above.

Note that the J values obtained by following this method are for the final residual stress fields. For a pre-existing crack adjacent to the weld, this method may still be used when the final residual stress is judged to be the strongest loading compared with all the thermal loads which the crack tip endured during the welding process. It should also be pointed out that numerical difficulties might be encountered when mapping the residual stress to a cracked body when the residual stress field is strong.

This method is validated and discussed in the remaining parts of this paper.

FINITE ELEMENT VALIDATION

To validate the J calculation method described in the above section, a case of a slit-weld in a plate analysed by Goldthorpe (2005) is re-analysed in this paper using both the old and new methods. The mapping method is used to reproduce the welding residual stresses in the new method. J is evaluated using both the modified J [Lei (2005)] and ABAQUS [ABAQUS (2010), (2014)] in both the old and new methods. The correlation between the obtained J values from the new method and the crack tip stress field is investigated. The details of the welding FE simulation are omitted because the purpose of this paper is to develop a J calculation method for cracks in welding residual stress fields, rather than the welding simulation itself.

A thin plate of width $2w$, thickness B and length $2L$ with a slit-weld of length $2l$ and width $2d$ located in the middle of the plate (Fig. 3) was considered by Goldthorpe (2005). After welding, a centre crack of length $2a$ ($a = l$) was then introduced into the middle of the weld and remote tension perpendicular to the crack plane applied at the ends of the plate (Fig. 3). The cracked plate may be idealised as a centre cracked plate (CCP) with a crack to plate width ratio $a/w = 0.2$ under combined residual stresses and remote tension. The dimensions of the specimen are as follows: $w = L = 250$ mm, $l = a = 50$mm, $d = 2.5$ mm and $B = 5$ mm.

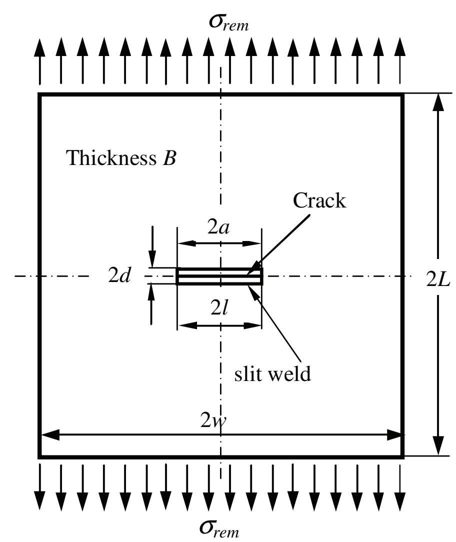


Figure 3 The geometry of a centre cracked plate (CCP) with a crack in a slit-weld

The plate is idealised by 4-noded 2-D elements. Only one quarter of the specimen is modelled with the appropriate boundary conditions on the planes of symmetry. ABAQUS [ABAQUS (2010), (2014)] is used in the analysis. The ABAQUS element type DC2D4 is used for thermal analysis and plane stress element type CPS4 is used for mechanical analyses. J is evaluated on 20 domains around the crack tip. The radius of the first domain was about $1.2\times10^{-4}a$ and that of the 20th domain was about $0.04a$. The modified J is calculated using the post-processing program [Lei (2005), (2006)] and the ABAQUS J [ABAQUS (2010), (2014)] is calculated using its in-built J-integral function.

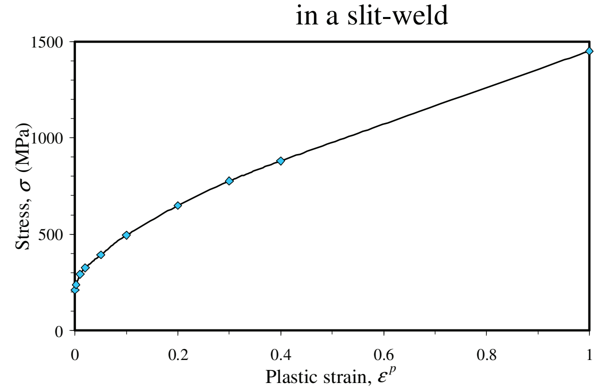


Figure 4 True stress-strain curve of the material at 23°C

To simplify the problem, the weld material is assumed to be the same as the plate material. 316L stainless steel properties are used in the analysis. J is evaluated at 23°C and the material properties at this temperature are as follows. The true stress-true strain curve is shown in Fig. 4 with the last point (at $\varepsilon^p = 1$) in the curve being estimated according to the cross section reduction in the tensile test. The 0.2% plastic strain proof stress, σ_0, is 238 MPa. The Young's modulus, *E*, and Poisson's ratio, ν, are 195400 MPa and 0.294, respectively. In the analyses, small strain isotropic hardening is used with the Mises yield criterion.

The mechanical load applied to the plate is remote tension acting on the direction perpendicular to the crack plane. Following R6 [R6 (2015)], the mechanical load level is measured by L_r, the ratio of applied load per unit thickness, *P*, to the limit load per unit thickness, P_L, as follows

$$L_r = \frac{P}{P_L} = \frac{\sigma_{rem}}{(1-a/w)\sigma_0} \tag{11}$$

where σ_{rem} is the applied remote stress defined by $\sigma_{rem} = P/(2wB)$ and the limit load $P_L = 2wB(1-a/w)\sigma_0$ from R6 (2015) has been adopted.

FE analyses

The analysis starts from the welding simulation. Firstly, a thermal analysis is performed according to the heat input during the welding and the bead size. Secondly, a mechanical analysis is performed to obtain stress, strain and displacement fields under thermal load due to the welding. Finally, a cracked body analysis is performed at room temperature, 23°C. In this third step, a crack is introduced into the middle of the weld by changing the boundary conditions along the horizontal symmetry plane. The remote tensile stress is then applied. Both the modified *J* and the ABAQUS *J* are evaluated in this step. The crack is introduced by releasing all the nodes along the crack plane simultaneously.

Two types of analyses are considered in the analysis of the third step for comparison. Type 1 is a continued analysis (the old method), i.e., a crack is introduced into the weld in the original model after the thermal and stress analyses of the welding simulation have completed and mechanical load is then applied. Note that, in Type 1 analysis, *J* is evaluated using the stress, strain, displacement and strain energy density inherited from the welding simulation history. Type 2 is the new method proposed in the previous section. In Type 2 analysis, only the stress fields and the equivalent plastic strain field from the analysis of the second step (welding simulation) are mapped into a new stress-free model as initial conditions. A crack is then introduced into this new model and, finally, the mechanical load is applied to the cracked model. Note that mapping the equivalent plastic strain field is for inheriting the strain hardening state after the welding simulation. This does not affect the initial plastic strain components in the new model.

FE RESULTS AND DISCUSSION

Residual stress distributions in the uncracked body and in the ligament of the cracked body

The y-direction welding residual stress, σ_{yy}, along the horizontal symmetry plane of the plate obtained from the welding simulation at the end of Step 2 is plotted in Fig. 5, against the normalised distance from the centre of the plate, *x*/*w*. Also included in Fig. 5 is the y-direction stress distribution after the welding residual stresses are mapped into a new model as initial conditions, obtained from the Type 2 analysis described above. Comparing the two distributions in Fig. 5, the mapping method accurately reproduces the original residual stress field in the new model.

The opening crack stresses, σ_{yy}, along the crack ligament due to residual stresses, obtained from both the Type 1 and Type 2 analyses after the cracks are introduced into the residual stress fields are shown in Fig. 6, plotted against the distance from the crack tip, *r*. Comparing the opening stress distributions along the crack ligament obtained from the two types of analyses in Fig. 6, excellent agreement between the results from the Type 1 and Type 2 analyses is seen. Therefore, in view of Equation (6), the *J* evaluated from the

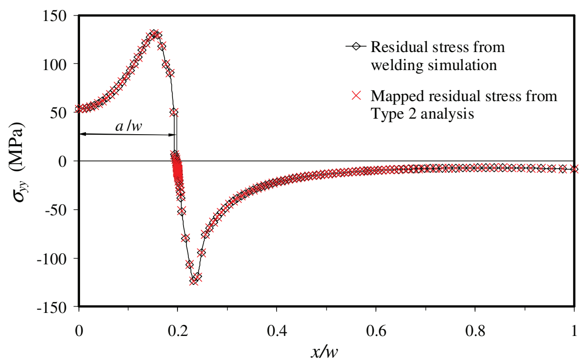


Figure 5 Comparison between welding residual stresses before and after mapping (x: the distance from the centre of the plate)

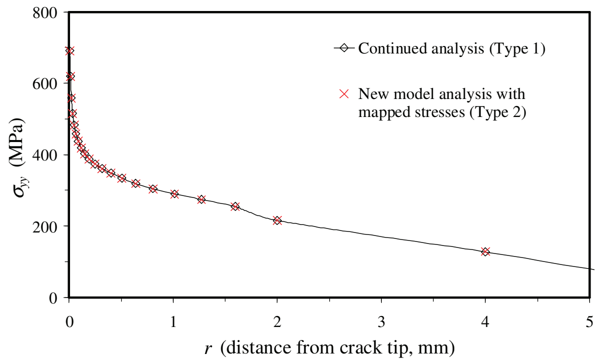
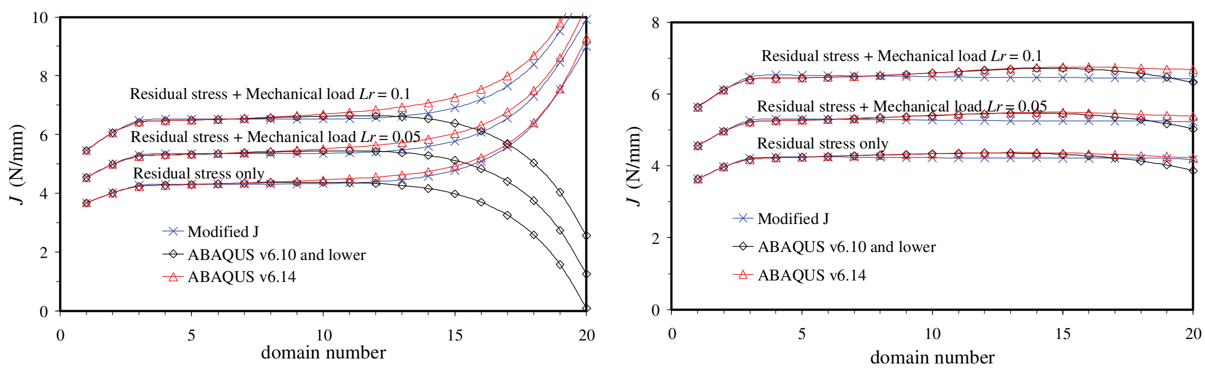


Figure 6 Comparison of crack tip stress distributions due to residual stresses between results from continued analysis and new model analysis with mapped residual stresses

stress, strain and displacement fields in the Type 2 analysis is also a measure of the crack tip fields for the Type 1 analysis.

***J* values**

The FE J values obtained from both the Type 1 and Type 2 analyses are shown in Fig. 7(a) and (b), respectively. From Fig. 7(a) for the Type 1 analysis, both modified J and ABAQUS v6.14 J are strongly path-dependent when the area of the domain increases to include far-field data. Results obtained using the old version of ABAQUS [ABAQUS (2010)] are also strongly path-dependent but the far-field trends are very different from those obtained using ABAQUS v6.14. For the Type 2 analysis, from Fig. 7(b), the modified J shows very good path-independency except for the first two domains very close to the crack tip. The results obtained using both ABAQUS v6.14 and the old version show reasonably good path-independency. It is also seen from Fig. 7(b) that the J values obtained in the Type 2 analyses using the new version of ABAQUS [ABAQUS (2014)] are very close to those obtained using the modified J method.



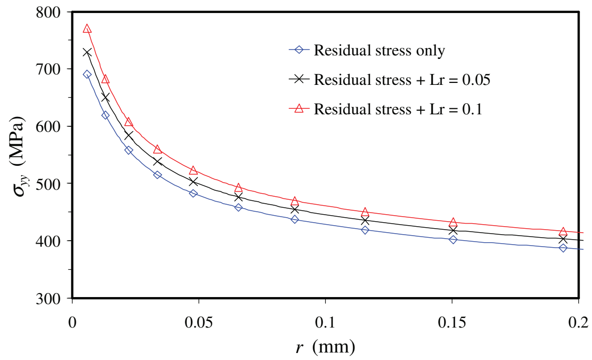
(a) Type 1 analyses

(b) Type 2 analyses

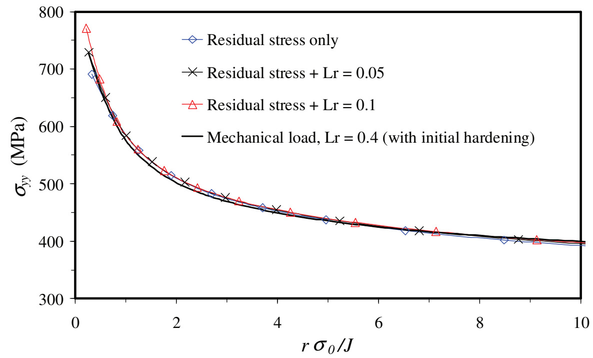
Figure 7 Modified J and ABAQUS J obtained from FE analyses

Crack tip stress fields

To validate the modified J values from the FE analyses, the corresponding crack opening stresses along $\theta = 0$ are extracted from the Type 2 FE results. The crack opening stresses, σ_{yy}, are presented in Fig. 8(a). In Fig. 8(a), the crack opening stresses are plotted against the distance from the crack tip, r. They are replotted in Fig. 8(b) against the normalised distance from the crack tip, $r/(J/\sigma_0)$, where modified J values have been used in the normalisation.



(a) Opening stresses vs. the distance from crack tip



(b) Opening stresses vs. normalised distance from the crack tip

Fig. 8 Crack-tip stress fields ($\theta = 0$) under various loads and therefore modified J levels from Type 2 analysis

From Fig. 8(a), the near tip crack opening stress is singular for residual stress loading only. A non-zero J value is expected to describe such a stress field. From the figure, the crack opening stress increases with increasing mechanical load, implying that a higher J value than that for residual stress only should be used to describe the crack tip stress field due to the combined load. When the crack opening stresses are plotted against the normalised distance from the crack tip, $r/(J/\sigma_0)$, see Fig. 8(b), the three curves in Fig. 8(a) collapse to one. Therefore, the modified J values satisfy Equation (10) and are relevant to the crack tip stress field.

In single parameter fracture mechanics, J is a measure of the intensity of the crack-tip fields in the J-dominant zone. For a power-law hardening material, the crack-tip stress and strain fields are of HRR type (Equations (6) and (7)) and comparing the computed near crack-tip stress/strain field with the HRR field offers an alternative method for checking numerical solutions of J. For non-power-law materials, Equation (10) may be used to check the correlation between J and the crack tip fields and provide the correct ratio between two J values. To validate the absolute values of J, an independent calibration has to be performed.

A pure mechanical load case of the plate is analysed to validate the J values obtained for residual stress only and for combined residual stress and mechanical load. Remote tension corresponding to L_r = 0.4 is applied to the stress-free plate containing a crack of the same size as that in the cases of residual stress loading. The material strain-hardening history is considered by inputting initial equivalent plastic strains obtained from the welding simulation. Note that doing so in ABAQUS does not introduce plastic strain components and stresses because the equivalent plastic strain only defines the “degree of yielding” and is not involved in stress calculation. Both the ABAQUS v6.14 J and the modified J obtained from this FE analysis show good path-independency and their values are, as expected, very close to each other, about 5.33 N/mm. This value is then used to normalise r and the opening crack stresses are plotted in Fig. 8(b) as a thick solid curve, denoted by “Mechanical load, L_r = 0.4 (with initial hardening)”. From Fig. 8(b), this curve is close to all the three curves for the residual stress and combined loading cases. This confirms that the absolute modified J or ABAQUS later version J values obtained from the analyses of residual stress are correct and the validity of the calculated J as a crack tip field parameter is, therefore, proved.

CONCLUSIONS

J calculation for a crack in a welding residual stress field following a welding simulation has been addressed. The J evaluated using the modified J definition or ABAQUS version v6.11 and above may become path-dependent when it is calculated following a welding simulation for a crack tip in the welding residual stress field. This is because the plastic strain energy density accumulated during the welding simulation is irrelevant to the crack tip singular fields and the initial strains may not be correctly extracted from the plastic strains created in the welding simulation. In order to overcome the problem, a "mapping stress" method is proposed in this paper, which includes mapping the uncracked body residual stresses and the equivalent plastic strains into a stress-free uncracked model as initial conditions, introducing a crack, applying mechanical/thermal loads and then calculating *J*.

The proposed method has been validated using a FE welding simulation of a slit-weld in a plate followed by cracked body *J* analyses for residual stresses and combined residual stresses and mechanical load. The correlation between *J* and the crack tip stress field has also been investigated. The conclusions drawn from the results are as follows:

1. The stress mapping method can accurately reproduce both the uncracked body residual stress distribution and the crack tip stress distribution when a crack is introduced into the residual stress field.
2. Both the modified *J* and ABAQUS v6.14 *J* are path-independent and agree very well when the above stress mapping method is followed.
3. The crack tip stress fields can be well correlated by the *J* obtained using the modified *J* method or ABAQUS v6.14.

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