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INVESTIGATION INTO LOAD ORDER EFFECTS ON THE TREATMENT OF COMBINED PRIMARY AND SECONDARY STRESSES WITHIN R6

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ABSTRACT

A range of methods are available within the R6 defect assessment procedure to describe how primary and secondary stresses combine. Each of these methods has different levels of associated conservatism depending on the level of elastic follow-up assumed, or inherently modelled, when deriving these methods. Approaches that include an elastic follow-up factor, Z, have recently been investigated by Ainsworth and James and included in the latest amendment to R6. However, one additional factor that has not yet been fully considered is the effect that load order has on this interaction.

This work considers finite element analyses of a circumferentially cracked cylinder with two thermal distributions and two crack sizes combined with an axial primary load. These conditions were controlled to manipulate the level of Z. The analyses were performed with either the secondary or primary load applied first to consider load order effects. The work found:

1) The effect of load order has been shown to be linked to the formation of the initial plastic zone by its extent and shape.
2) The recently published approach by Ainsworth generally allowed a very good fit to the results. However, it was not able to conservatively predict the interaction for the case with the greatest Z.
3) The approach derived by James provided a means to capture the peak interaction for all cases. However, the approach was not capable of capturing the entire curve if primary loads were applied first.
4) The Ainsworth and James approaches can be combined to capture any load order effect.

INTRODUCTION

The work presented in this paper considers the combination of primary and secondary stresses on a body containing a crack-like flaw, under conditions that promote elastic follow-up, and investigates the effect of the order in which the loads are applied. This follows from work recently considered where the secondary load was applied first [1]. It is also a result of the recommendations contained within [1] and recent data by Oh *et al* [2] showing that the order in which the loads are applied can have an effect on the combined crack driving force.

A range of methods have been available within R6 [3] to describe how primary and secondary stresses combine, with further approaches in the latest revision [4], which correspond to estimates of the total crack driving force. It is this combined crack driving force that is compared to the material fracture toughness in defect assessment studies. Each of these methods to describe the interaction has different levels of associated conservatism. The main reason for these different levels of conservatism is the underlying theory, or fit to finite element analyses, that have been used to define each respective approach. This is because the effect of elastic follow-up may be significantly different over a range of cases (i.e. geometry, loading and material), which will lead to different levels of plastic enhancement to the contribution of the secondary stress to the total crack driving force. Indeed, the earlier V-Factor approach within R6 [3] to detail the influence of primary and secondary stresses to fracture was generally considered conservative (which has led to further developments such as [5, 6]). However, even for this generally conservative R6 V-Factor approach there have been cases that have shown non-conservatism with excessive levels of elastic follow-up (e.g. [1, 7]). This has led the guidance in R6 for such cases to suggest treatment of secondary stresses with a large elastic follow-up as an additional primary stress.

Recently James [1] considered finite element analyses of a circumferentially cracked cylinder with two different secondary stress distributions and two (shallow) crack depths. These conditions were modified in [1] in order to optimise the level of elastic follow-up in the analyses. This has been achieved by following the advice in the UK Technical Advisory Group on Structural Integrity (TAGSI) report [8] that suggests that “smaller cracks, flatter stress strain curves and lower secondary stresses are cases where elastic follow-up might have a more significant impact on the crack driving force". Therefore, the analyses considered two small crack depths, three stress strain curves and a range of secondary stresses. The work in [1], through comparison of these finite element analyses and comparison to engineering solutions for the R6 V interaction term [3], demonstrated that:

1) A recently published approach by Ainsworth [9] that specifically includes the level of elastic follow-up through the Z term allowed a conservative fit to the finite element results for the linear through wall temperature gradient but was not able to conservatively predict V/V_0 (where V_0 is the plasticity correction terms for secondary loads in isolation) for the step temperature gradient, even if Z was changed to be greater than 50. It was also noted that this approach was not able to accurately match the subsequent reduction in V/V_0 at higher values of primary load.

2) A re-derivation of the Ainsworth approach with the assumption that the secondary self-induced plasticity has little effect on the level of secondary reference stress allows a new estimate of V/V_0. This new estimate of V/V_0, termed the Re-Evaluated approach, followed a similar form to the Ainsworth approach but with the R6 failure assessment curve squared. It was seen to provide an improved description of the finite element results.

One of the recommendations from this work was that the influence of application of the secondary stress after the primary stress on these results should be considered. This recommendation has been taken as the basis for the work presented here. The same analyses as performed in [1] have been re-assessed with the primary load applied before the secondary thermal through wall stress distribution.

BACKGROUND

Combined Loading

There are many cases where an assessment of the structural integrity of a component may need to consider both primary and secondary stresses. Primary stresses are those that contribute to plastic collapse of a structure and generally arise from an external force such as an internal pressure or gravity. Secondary stresses are those that arise from displacements in the material, such as the application of a thermal stress field or contraction during welding, and are therefore removed under extreme levels of plasticity where the additional plastic strain can account for the internal displacements. Under plasticity the added strains are therefore said to redistribute the secondary stress, i.e. allow the stress to become inconsequential to failure of a component.

For a cracked body the interaction can be more complex. The presence of the crack provides a stress intensification factor around the crack tip; where it is assumed that this can be described by the Hutchinson, Rice and Rosengren (HRR) stress field [10, 11], and hence the energy release rate per unit area crack growth, J, or its equivalent elastic-plastic stress intensity factor K_J ($K_J = \sqrt{JE'}$, where $E' = E/\left(1-\nu^2\right)$ in plane strain and E is the elastic modulus and ν is Poisson's ratio). The crack therefore provides a location where added plasticity can develop whilst being surrounded by elastic material. Under such conditions the contribution from the secondary stress to crack initiation, which is a function of both stress and strain, is not necessarily reduced by plasticity. The degree to which the secondary stress may increase or decrease the crack driving force is dependent on the range over which the secondary stress is applied (where a small range will allow some relaxation but a longer range will still induce a remote stress to the region of plastic strain), material considered, geometry, magnitude of secondary stress and the order in which it is applied. Therefore, in combination with a primary stress the secondary stress will redistribute under excessive plasticity (where the plastic zone is large enough that it allows the secondary stress to redistribute) but might also be enhanced under small scale yielding (i.e. the plastic zone is small compared to the remaining ligament and both the increase in strain and stress increase the value of K_J). A number of approaches allow for this inter-dependent redistribution of secondary stresses with primary stresses and its effect on the total crack driving force; some of which are discussed further below. One parameter that describes how likely the influence of secondary stress to fracture is enhanced under added plasticity is the level of elastic follow-up in the structure.

Work considered by Oh *et al* [2] has shown that there may also be an effect of load order on the interaction of primary and secondary stresses. The work demonstrated that if the secondary stress is applied subsequent to the primary stress the resulting value of crack driving force may be greater than if the secondary load is applied first.

Elastic Follow-up

Elastic follow-up is where the secondary load is acting over a sufficiently large length scale that localised relaxation (e.g. in the vicinity of a crack) does not reduce the influence of the remote stresses; this therefore reduces the level to which the secondary stress localised to the crack tip can be redistributed. In fracture, this means that the secondary stress can act more like a primary stress than a secondary stress (only if a primary stress has not induced gross yielding). The effect of elastic follow-up may be captured by a so-called elastic follow-up factor termed Z.

Typically, elastic follow-up is characterised by the three cases shown in Figure 1. In the figure an initially elastically loaded point (A) can result in a number of eventual stress and strain conditions. Under pure displacement (strain) controlled conditions, the strain will not change such that the final position is given by the case $Z = 1$. For the case where the stress behaves as if under load controlled conditions, the stress will not change and the strain will continue until the materials stress-strain curve is met; this indicates large elastic follow-up, $Z \to \infty$. In most cases, however, the actual value of Z will be somewhere between pure load controlled and pure displacement controlled conditions (shown as moderate elastic follow-up, $Z > 1$).

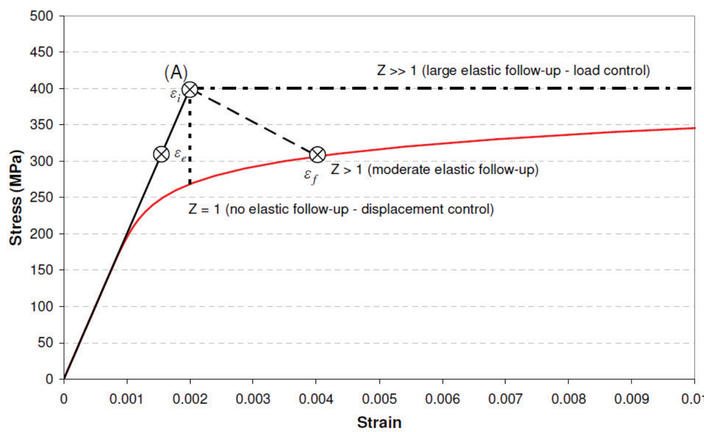

Figure 1 - Example of the effect of elastic follow-up on strain.

An approximate evaluation of Z can be made, for a single point in a structure, by the ratio of the difference between the elastic strain at the relaxed stress level, ε_e, and the final strain, ε_f, to the difference between the elastic strain at the relaxed stress level and that at point (A), ε_i (as shown in Figure 1 for the case of moderate elastic follow-up). This can be more formally written, noting that $\varepsilon_f > \varepsilon_i > \varepsilon_e$ and hence $Z > 1$ as:

$$Z = \frac{\varepsilon_f - \varepsilon_e}{\varepsilon_i - \varepsilon_e} \tag{1}$$

Therefore, displacement controlled stresses, such as pure secondary stresses, will have a value of $Z = 1$, the load controlled stresses will have an infinite value of Z, and most secondary stresses in real structures will have a value of Z between these values (typically less than 5).

However, the actual value of Z can be difficult to ascertain in a real cracked geometry as the level of stress will change throughout the structure and the degree of elastic follow-up will be dependent on the location of the assessed point in relation to the surrounding elastic material. Evaluating elastic follow-up is further complicated by the need to consider long range displacement stresses (such as fit-up stresses or long range thermal stress fields) as load controlled at the region of interest. Such conditions are not easily modelled and are not necessarily included in an assessment, meaning that estimates of Z are often subjective.

BASIC FRACTURE MECHANICS APPROACH IN R6

A conventional R6 assessment considers the criticality of a defect by plotting assessment points on a Failure Assessment Diagram (FAD). The position of the assessment point is defined by the parameters L_r and K_r, where L_r defines the magnitude of applied primary load and is determined from:

$$L_r = \frac{P}{P_L} = \frac{\sigma_{ref}^{p}}{\sigma_y} \tag{2}$$

where P is the applied load, P_L is the plastic limit load for the structure considered, σ_{ref}^{p} is the primary reference stress and σ_y is the material's 0.2% proof stress. The K_r parameter is given by:

$$K_r = \frac{K_I^p + VK_I^s}{K_{mat}} \tag{3}$$

where K_I^p and K_I^s are the elastic stress intensity factors for primary and secondary stresses respectively and K_{mat} is the material's fracture toughness. Different estimates for the plasticity interaction term, V, are provided in R6 [3, 4]; those relevant to the work presented herein are outlined in the following sub-sections. When the assessment point, (L_r, K_r), is plotted against the FAD, defined by a plot of L_r versus $f(L_r)$, the crack is assumed to have initiated if the point lies outside the area of the plot.

R6 Estimate of V

Within R6 [3, 4] there are two overall approaches to estimate V; the "simplified" and "complex" approaches. The more conservative simplified approach provides a bounding relation to the complex approach and, therefore, is not considered further here. Under the complex R6 V-Factor Approach, the estimation of V is provided by $V = V_0\xi$, where $V_0 = K_J^s / K_I^s$ with K_J^s the elastic-plastic secondary stress intensity factor and ξ is provided by look-up tables in R6. These tables are given in terms of L_r and the parameter $K_J^s / \left(K_I^p / L_r\right)$.

Approach of Ainsworth

Recent work by Ainsworth [9], now included in R6 [4], has shown that an estimate of the interaction term V can be provided by Equation (4) below. This approach was derived from detailing how the combined reference stress will be changed with applied primary and secondary stresses when accounting for the effect of elastic follow-up.

$$\frac{V}{V_0} = f(L_r) + \tfrac{3}{4}\frac{Z-1}{Z} L_r(\beta + L_r)[f(L_r)]^2 f(\beta) \tag{4}$$

where $\beta = K_I^s L_r / K_I^p$, which can be assumed to be the secondary reference stress normalised by the yield stress, $\sigma_{ref}^{s} / \sigma_y$, such that it provides a similar term to L_r but for secondary stresses. In this approach the potential for elastic follow-up is clearly provided by the inclusion of the Z term. It is noted that the derivation of Equation (4) adopted the use of an Option 2 FAD but that the approximate use of an Option 1 FAD in its place was argued to remain conservative in [9]. In practice the function f is that used in the overall defect assessment.

Re-Evaluation of the Ainsworth Approach

A re-derivation of the Ainsworth approach is contained within [1] and adopted the assumption that the secondary self-induced plasticity has little effect on the level of secondary reference stress (i.e. $\beta V_0 = \sigma_{ref}^{s} / \sigma_y$ as opposed to $\beta V_0 = \left(\sigma_{ref}^{s} / \sigma_y\right) / f\left(\sigma_{ref}^{s} / \sigma_y\right)$) which allows a new estimate of V/V_0 as described in Equation (5) below. This follows a similar form as the Ainsworth approach but squares the $f(L_r)$ term, which then allows an improved fit to finite element results in Reference [1].

$$\frac{V}{V_0} = [f(L_r)]^2 \left(1 + \tfrac{3}{4}\frac{Z-1}{Z} L_r(\beta + L_r)\right) \tag{5}$$

In this approach the potential for elastic follow-up is clearly provided by the inclusion of the Z term but provides a greater increase in V/V_0 for larger secondary stresses as $f(\beta)$ is not included in the term that includes Z.

FINITE ELEMENT ANALYSES

The finite element analyses are as detailed in Reference [1] but with the primary load applied before the secondary stress to investigate load order effects. The finite element analyses conducted are for a fully circumferential, externally cracked cylinder with a number of load cases. The cylinder adopted in the finite element analysis is modelled to have a mean radius of 200 mm, a thickness, t, of 10 mm and is 1000 mm long. Two fully circumferential defects are included in the range of cases considered. These are 0.5 and 2 mm deep, corresponding to $a/t = 0.05$ and $a/t = 0.2$, respectively.

The finite element model to describe this geometry was created and run within ABAQUS 6.12 [12] and was composed of approximately 23,000 CAX8 axi-symmetric two dimensional elements, with the crack tip modelled as part of a focused region. The crack tip elements were modelled with a 0.01 mm radial extent.

In total, four cases were considered (see illustration of these two gradients shown in Figure 2); (1) shallow crack ($a/t = 0.05$) with a bending secondary stress; (2) shallow crack ($a/t = 0.05$) with a step secondary stress (tensile over the crack balanced by a compressive region away from the crack); (3) deeper crack ($a/t = 0.2$) with a bending secondary stress; (4) deeper crack ($a/t = 0.2$) with a step secondary stress.

For each of these four cases the magnitude of the applied secondary load was modified to optimise the level of elastic follow-up in the analyses. The actual magnitude of the secondary thermal load to apply was determined by comparing the relative magnitude of the elastic and elastic-plastic secondary stress intensity factors, i.e. V_0, whilst adjusting the magnitude of the thermal field, thus changing β. For each case one magnitude of β was used so that the value of V_0 is maximised, therefore increasing the likelihood of elastic follow-up.

To investigate the interaction of the secondary stress with primary stresses a basic end-load was applied to the cylinder. It is noted that the lack of internal pressure was to maximise the enhancement from elastic follow-up; an internal pressure will induce a hoop stress which will then act to redistribute the thermal stress at a lower axial stress, reducing the effect of elastic follow-up. This primary load was applied either before or after the secondary stress, meaning that a large number of individual analyses were required in order to generate one set of results.

Each geometric case and thermal loading was assessed with three different strain hardening coefficients, n, (n = 6, 12 and 18) as part of a Ramberg Osgood relationship, as shown in Equation (6), in addition to an elastic calculation. The material properties adopted can be seen in Table 1.

$$\varepsilon = \frac{\sigma}{E} + 0.002\left(\frac{\sigma}{\sigma_y}\right)^n \tag{6}$$

where ε is the strain associated with stress σ. In the analyses large strain effects have been included. This is to ensure the strain is correctly calculated under redistribution.

Table 1: Material properties adopted

Young's Modulus, E (MPa)	Poisson's Ratio, v	Yield Stress, σ_y (MPa)	Strain Hardening Coefficient, n	Thermal Expansion Coefficient, α ($10^{-6}\,{}^{o}C^{-1}$)
200,000	0.3	300	6, 12, 18	14.56

Comparison of Load Order Effects on V/V_0

A direct comparison of the effect of load order on V/V_0 can be seen in Figure 3 for the thermally induced linear bending stress cases (left), and the step secondary stress field (right), both for the deepest crack ($a/t = 0.2$). Both cases show that the resulting estimate is very similar at low applied primary loads and that the peak value of V/V_0 is the same. There is a difference, however, at higher primary loads where the cases where the secondary stress is applied first show a more rapid decrease in V/V_0 when increasing L_r compared to the cases when the secondary stress is applied after the primary load. This appears to confirm the results shown by Oh *et al* [2] where the differences with load order provide a higher resultant K_J when the primary load is applied first.

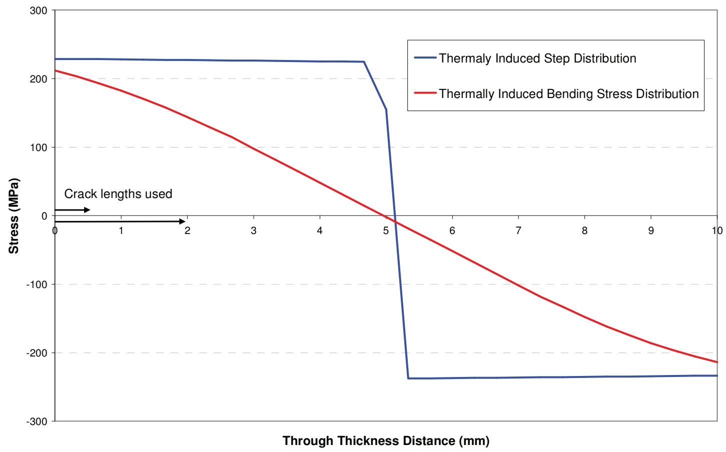

Figure 2 – Step and bend through-wall stress distributions

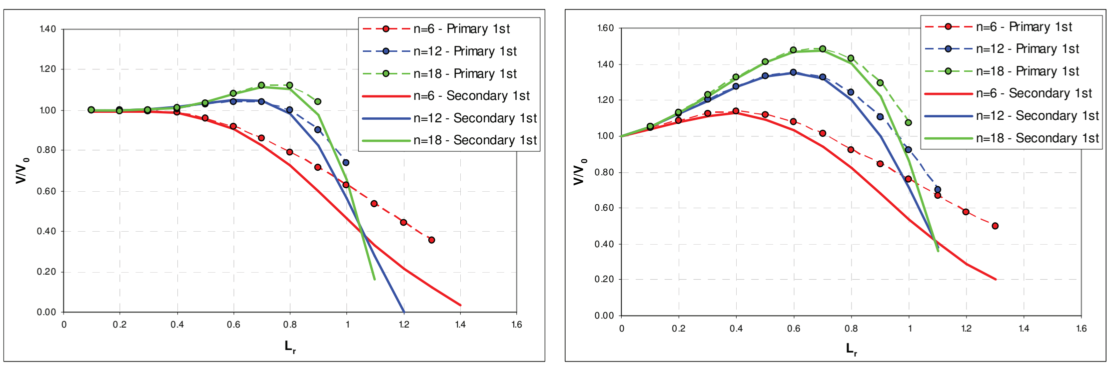

Figure 3 – Finite element estimate of V/V_0 for the bending secondary stress field (left) and the step secondary stress field right, $a/t = 0.2$.

Comparison of Simplified Approaches of V/V_0 to Finite Element Analyses

Comparisons of the Ainsworth Approach of Equation 4 to the finite element results are shown in Figure 4 to Figure 5. Also included is the value of Z that best covers the redistribution in the figures. The results for the bending secondary stress indicate very good agreement with the finite element results, with the peak value of V/V_0 and the progression to this value at lower values of L_r conservative. Unlike the results presented in [1], where the secondary stress was applied before the primary stress and redistribution occurred at an accelerated rate, the estimates of V/V_0 are seen to remain very close to the finite element results (also seen for the step profiles) once the secondary stress is redistributed within the structure. However, for the deep and shallow crack in the stepped distribution, the required value of Z is very large, which was also noted in the previous work [1].

The plots comparing the Re-Evaluated Ainsworth approach of Equation 5 and the finite element results are shown in Figure 6 and Figure 7 with the value of Z that best covers the redistribution observed. It can be seen that the Re-Evaluated approach is capable of capturing the peak interaction but over-predicts the rate of redistribution. This can be seen where the skeletal point for the finite element results is always at a slightly larger value of both L_r and V/V_0. The poor fit for the Re-Evaluated approach here, compared to the results presented previously, is a result of the load-order effect on the rate of redistribution; where redistribution is faster when the primary load is applied to a pre-existing secondary stress field.

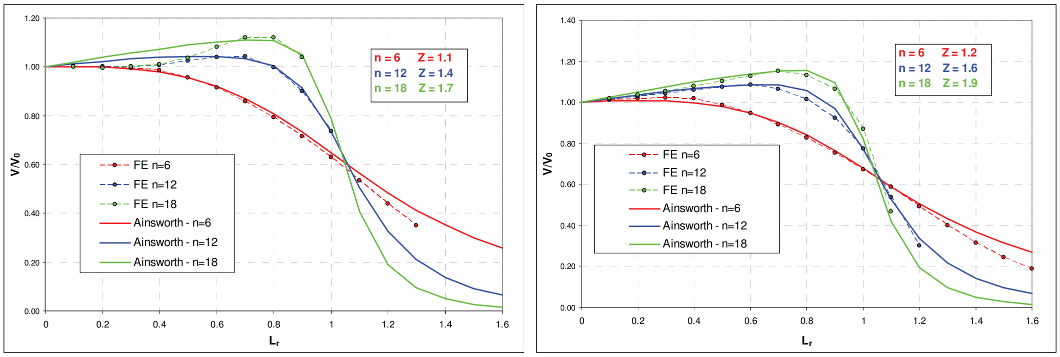

Figure 4 – Approach of Ainsworth applied to the circumferentially cracked cylinder with a through wall bending temperature profile and a deep (left) and shallow (right) crack

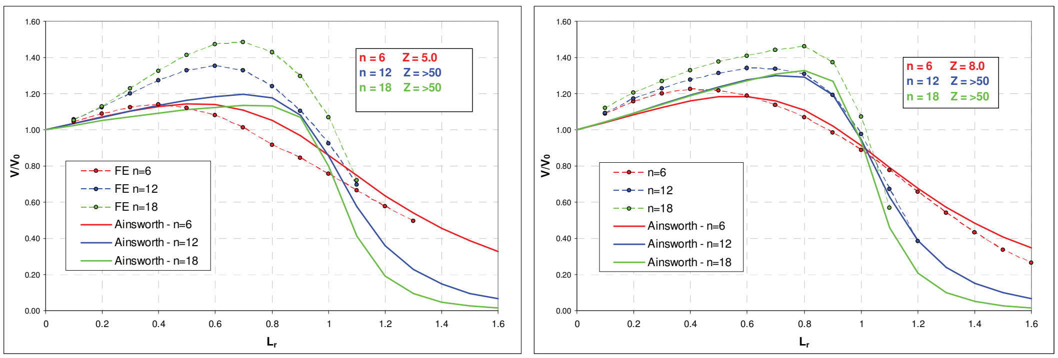

Figure 5 – Approach of Ainsworth applied to the circumferentially cracked cylinder with a through wall step temperature profile and a deep (left) and shallow (right) crack

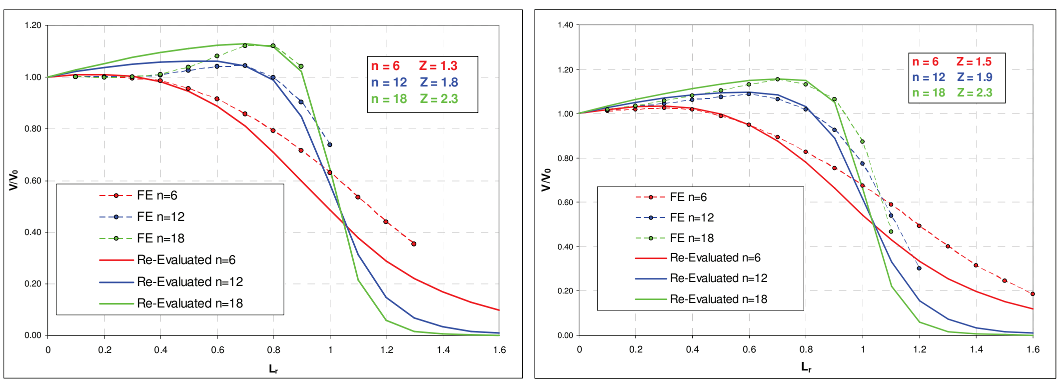

Figure 6 – Re-Evaluated Ainsworth approach applied to the circumferentially cracked cylinder with a through wall bending temperature profile and a deep (left) and shallow (right) crack

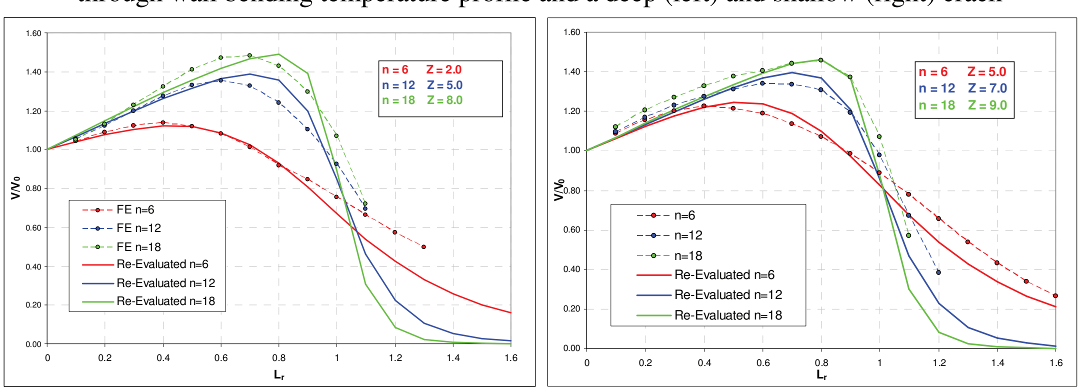

Figure 7 – Re-Evaluated Ainsworth approach applied to the circumferentially cracked cylinder with a through wall step temperature profile and a deep (left) and shallow (right) crack

DISCUSSION

Load-Order Effects on V

The results show that there is a slight difference in the resulting estimate of V/V_0 depending on the order in which the loads are applied. This difference, for these cases, shows as a reduced rate of redistribution with increasing primary load. The difference is not seen to change the peak value of V/V_0 between L_r values of 0.5 and 0.8 depending on the value of n adopted, but does then change the level of redistribution once L_r is greater than 0.9 (i.e. under widespread, primary stress-induced, plasticity). The reason that the peak value of interaction appears unaffected is likely to be due to the fact that the plastic zone, from either the primary stress, secondary stress or when combined, is still under small scale yielding. This means that the resulting stress and strain fields ahead of the crack tip should be the same irrespective of the order in which the loads are applied.

When the primary load is large enough such that widespread plasticity is seen under primary load alone, the order in which the loads are applied then becomes more significant. This can be seen in Figure 8 for the von Mises stress field at the crack-tip for the case where $n = 12$ at $L_r = 1.0$, comparing the results with the primary stress applied first (left) and the secondary stress applied first (right). Here it can be seen that the scenario where the primary stress is applied first has a larger yielded area, and hence higher crack tip stress and strain, than the case where the secondary stress field is applied first. This has the effect of increasing the contribution that the secondary stress field has on J, making the value of V/V_0 at this value of L_r higher, as can be seen in Figure 3. It can also be seen that the compressive stress resulting from the secondary stress means that the zone where the von Mises stress is above yield is contained within the structure (which would not be the case if the primary stress was acting alone). Overall, if a significant plastic strain field is established from the initial load, a different stress field will evolve depending on the order in which the stresses are applied, which can then affect the peak level of interaction and the subsequent rate of redistribution.

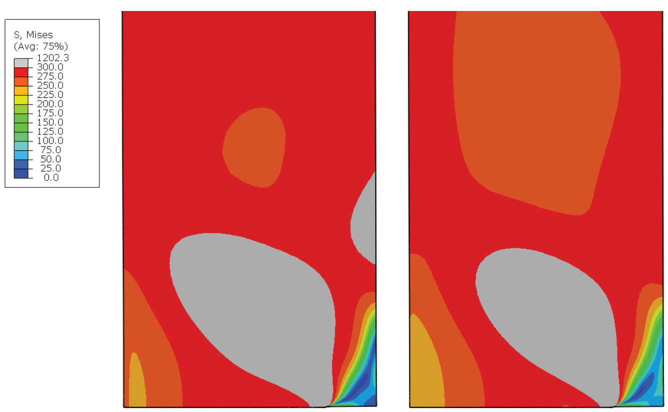

Figure 8 – Von Mises stress field at the bending stress field with the n=12 strain hardening case at L_r = 1.0 comparing the primary stress applied first (left) with the secondary stress applied first (right)

Ainsworth Approach

The Ainsworth Approach of Equation (4) is seen to predict the redistribution of secondary stresses very well for the bending stress fields. For these cases the results predict the peak of the interaction well and also captures the level of redistribution as L_r increases. However, the results have shown that for the stepped distribution the approach can under-predict the interaction, even if the elastic follow-up factor $Z > 50$. This was noted previously [1] which led to the development of the Re-Evaluated approach. It was also noted previously that the Ainsworth approach may not be valid for large values of Z (itself acknowledged in [9]).

Re-Evaluated Ainsworth Approach

The results for the Re-Evaluated Ainsworth approach of Equation (5) show that the approach can predict the peak in the redistribution of secondary stresses very well for all cases but was seen to redistribute too quickly at higher applied primary loads. The estimate of Z required to provide a reasonable fit to the finite element data can be seen to remain low for most cases, with an increase for the stepped profile with the shallow crack. This increase, however, remains less than 10 even for the worst case such that the values are considered more appropriate. A potential reason for the faster rate of redistribution is presented above, with discussion on the effect that the pre-established plastic zone has on the subsequent stress applied.

LINKING THE AINSWORTH AND RE-EVALUATED APPROACHES

The above discussion indicates a number of shared features regarding the Ainsworth and the Re-Evaluated approaches; where the two approaches seem to provide a bound of the results. The following shows how these can be considered in the same methodology.

Modelling the rate of redistribution

The difference between the Ainsworth and Re-Evaluated approaches can be traced back to the basic assumption on the level of enhancement for the secondary stress acting in isolation. Here, this level is considered to be a variable such that the derivation in [1, 9] can be re-expressed as:

$$\beta V_0 = \frac{\sigma_{ref}^{s}/\sigma_y}{\left[f\left(\sigma_{ref}^{s}/\sigma_y\right)\right]^{\alpha}} \tag{7}$$

$$\frac{\left(\sigma_{ref}/\sigma_y\right)}{\left[f\left(\sigma_{ref}/\sigma_y\right)\right]^2} = \frac{L_r}{\left[f\left(L_r\right)\right]^2} + \frac{\left(\sigma_{ref}^{s}/\sigma_y\right)}{\left[f\left(\sigma_{ref}^{s}/\sigma_y\right)\right]^{\gamma}} + \left(Z-1\right)\left[L_r + \sigma_{ref}^{s}/\sigma_y - \sigma_{ref}/\sigma_y\right] \tag{8}$$

where σ_{ref} is the combined primary and secondary reference stress and the exponents to the secondary stress contribution are α for the definition of βV_0, which should have the range -1 to 1, and γ is included in the redistribution equation. If this definition is included in the derivation of V/V_0 as outlined in [1, 9] the estimate of V/V_0 becomes (intermediate steps not included here for brevity):

$$\frac{V}{V_0} = f\left(L_r\right) f\left(\sigma_{ref}/\sigma_y\right)\left[f\left(\sigma_{ref}^{s}/\sigma_y\right)\right]^{(\alpha-\gamma)} + \frac{3}{4}\frac{\left(Z-1\right)}{Z} L_r\left(L_r+\beta\right)\left[f\left(L_r\right)\right]^2\left[f\left(\beta\right)\right]^{\alpha} \tag{9}$$

As such, there is now only the concern on how to approximate the $f\left(\sigma_{ref}/\sigma_y\right)$ term in the first expression on the right hand side. In the approach developed by Ainsworth, α was 1 and γ was 2, meaning that the most appropriate approximation for $f\left(\sigma_{ref}/\sigma_y\right)$ was $f\left(\sigma_{ref}^{s}/\sigma_y\right)$. However, for the Re-Evaluated approach both α and β were 0 meaning it was more appropriate to set $f\left(\sigma_{ref}/\sigma_y\right)$ equal to $f\left(L_r\right)$. As such, to maintain both scenarios and potentially an intermediate case, the above equation was re-written as:

$$\frac{V}{V_0} = f\left(L_r\right)\left[f\left(\sigma_{ref}/\sigma_y\right)\right]^{\lambda}\left[f\left(\sigma_{ref}/\sigma_y\right)\right]^{(1-\lambda)}\left[f\left(\sigma_{ref}^{s}/\sigma_y\right)\right]^{(\alpha-\gamma)} + \frac{3}{4}\frac{\left(Z-1\right)}{Z} L_r\left(L_r+\beta\right)\left[f\left(L_r\right)\right]^2\left[f\left(\beta\right)\right]^{\alpha} \tag{10}$$

where λ defines the effective contribution of the primary load to the redistribution. Hence the first $f\left(\sigma_{ref}/\sigma_y\right)$ can be equated to $f\left(L_r\right)$ and the second to $f\left(\sigma_{ref}^{s}/\sigma_y\right)$. This therefore allows a mixture of contributions, with those in the Ainsworth and Re-Evaluated approaches the two bounding scenarios. This leads to:

$$\frac{V}{V_0} = f\left(L_r\right)^{(1+\lambda)}\left[f\left(\sigma_{ref}^{s}/\sigma_y\right)\right]^{(1-\lambda+\alpha-\gamma)} + \frac{3}{4}\frac{\left(Z-1\right)}{Z} L_r\left(L_r+\beta\right)\left[f\left(L_r\right)\right]^2\left[f\left(\beta\right)\right]^{\alpha} \tag{11}$$

Here it is further noted that for the estimate of V/V_0 to equal unity at $L_r = 0$ it is required that $1-\lambda+\alpha-\gamma=0$. For a simplified method, it is therefore sufficient to adopt:

$$\frac{V}{V_0} = f\left(L_r\right)^{(1+\lambda)} + \frac{3}{4}\frac{\left(Z-1\right)}{Z} L_r\left(L_r+\beta\right)\left[f\left(L_r\right)\right]^2\left[f\left(\beta\right)\right]^{\alpha} \tag{12}$$

where the value of λ can be altered between 0 and 1 depending on the order of the loading applied. It is suggested that; if the primary stress is adding to the plastic strain, i.e. this is applied second, then λ

is set to 1; if the secondary stress is causing the additional plasticity, i.e. the secondary stress is applied second, then the value of λ is set to 0; if it is likely that if the primary and secondary stress are applied in parallel then the value of λ to use is suggested to be 0.5, although this needs to be confirmed by further analyses.

Capturing the peak redistribution

The above work shows that it is not possible for the Ainsworth approach in Equation (4) to accurately describe the peak interaction of the primary and secondary stresses in cases of extreme elastic follow-up. Previously [1] this was linked back to the value of $f(\beta)$ in the second term reducing the effect of the addition for elastic follow-up. In Equation (12) an additional factor α has been applied which allows a different relation. However, as the assumption that $\sigma^s_{ref}/\sigma_y = \beta$ has been made, Equation (7) can be reduced to show that $[f(\beta)]^\alpha = 1/V_0$. From Equation (12), this will lead to an estimate of V/V_0 as:

$$\frac{V}{V_0} = f(L_r)^{(1+\lambda)} + \tfrac{3}{4}\frac{(Z-1)}{Z}\frac{L_r}{V_0}(L_r+\beta)[f(L_r)]^2 \quad (13)$$

This approach will be better able to account for the peak interaction in the primary and secondary stress field, in a similar means to the Re-Evaluated approach, as for most cases V_0 is approximately 1.

The resulting estimate when applying Equation (13) with λ set to 0 can be seen in Figure 9, which is the case that demonstrated the worst fit when applying the Ainsworth approach. Clearly the replacement of $f(\beta)$ by V_0 allows an improved fit to the distribution with more realistic estimates of Z. Other results are not repeated here as they essentially follow those of the Ainsworth approach. Nonetheless, the result does show how the Ainsworth approach can be simply manipulated to remain more conservative for cases of higher levels of elastic follow-up.

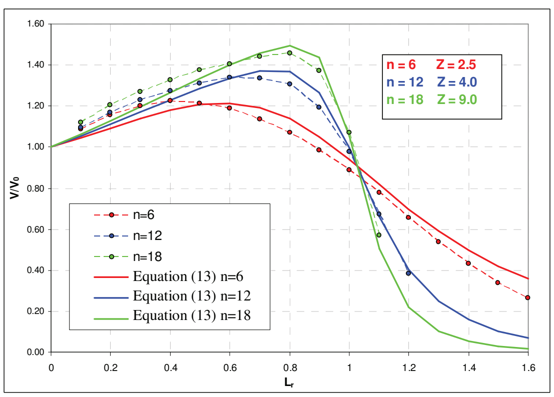

Figure 9 – Comparison of Equation (13) for the shallow crack with the step thermal distribution to the finite element results with the primary load applied first

CONCLUSIONS

The investigation into the effect of elastic follow-up presented above has provided a number of conclusions, summarised below.

1) The effect of load order has been shown to be linked to the formation of the initial plastic zone by its extent and shape.
2) The recently published approach by Ainsworth generally allowed a very good fit to the results. However, it was not able to conservatively predict the interaction for the case with the greatest Z.
3) The approach derived by James provided a means to capture the peak interaction for all cases. However, the approach was not capable of capturing the entire curve if primary loads were applied first.
4) The Ainsworth and James approaches can be combined to capture any load order effect.

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