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ESTIMATION OF THE PARAMETER CONTROLLING SHORT-TERM CREEP CRACK GROWTH UNDER COMBINED PRIMARY AND SECONDARY LOADING

R A Ainsworth[1], D W Dean[2], P J Budden[2] and D G J Hughes[2]

[1] Professor of Structural Integrity, The University of Manchester, Manchester, UK
[2] Assessment Technology Group, EDF Energy, Gloucester, UK

ABSTRACT

Components in Advanced Gas-Cooled Reactor (AGR) plant operate at temperatures where creep is important. However, for some components the temperature is not sufficiently high for steady state creep conditions to be attained for most of the lifetime. Consequently, defect assessments in such components need to consider creep crack growth controlled by transient or short-term creep conditions at the crack tip. In addition, such components are often subjected to combined primary and secondary loading where creep relaxation of secondary stresses is not complete so that the effect of secondary thermal and residual stresses on creep crack growth needs to be considered throughout the lifetime. In this paper, formulae for the estimation of the short-term creep crack tip characterising parameter C(t) are derived for elastic-plastic-creep material response for combined primary and secondary loading. The formulae are shown to be less conservative in their treatment of crack tip plasticity than existing estimates in the literature, particularly at the times which are relevant to some AGR components. The new formulae are compared with existing estimates, theoretical solutions and with finite element data to provide confidence in their application to AGR plant components.

INTRODUCTION

Defect assessment methods for components operating at high temperature are contained in a number of fitness-for-service procedures (EDF Energy, 2014; British Standards, 2013; American Petroleum Institute/ASME, 2007). These use characterizing parameters which have been proposed for creep fracture mechanics analysis as summarised, for example, by Webster and Ainsworth (1994). Under steady state creep conditions, the C^*-integral applies. For small-scale transient creep conditions where the stress and strain are still redistributing, the local crack tip stress and strain rate fields can be characterized by the C(t)-integral. Saxena (1986) proposed another parameter, C_t, based on the expansion rate of a creep zone at the crack tip and this can be related to C(t) as discussed by Oh et al. (2010).

Estimates of C^* and for the time-dependence of C(t) were provided by Riedel and Rice (1980) and Ohji et al. (1980) for elastic-creep material response in components subjected to mechanical loading. Ohji and Kubo (1988) extended the result to materials with power-law plasticity and power-law creep, again for mechanical loading. Further extensions to more general material response, to combined primary and secondary loadings and with allowance for stress redistribution due to crack growth have been presented by Ainsworth and Budden (1990), Ainsworth (1992) and Ainsworth et al. (2011), allowing comprehensive advice to be included in procedures such as R5 (EDF Energy, 2014).

The approaches developed and incorporated in R5 and other codes are based on methods which use the plastic strain and creep strain rate at a reference stress level. However, this describes the bulk component behaviour and has been found to lead to over-conservative estimates of C(t) at short times when bulk behaviour is essentially elastic-creep but crack tip response is also influenced by plasticity. This paper therefore provides an improved estimation formula for C(t), which is derived below. The result is based on the plastic strain and creep strain rate at a stress level which is a multiple, η, of the reference stress. When η=1, the formula reduces to that in previous work. Values of η>1 correspond to the use of a stress level which is higher than the reference stress and more representative of the stress level in the process zone near the crack tip. In order to provide advice on the choice of η, elastic-plastic-creep finite element analysis results for some cracked geometries are presented and compared with the newly developed estimate of C(t).

ESTIMATION OF C(t)

This section provides a general result for estimating C(t) for combined primary and secondary loading when initial loading leads to elastic-plastic response. The derivation is based on a stationary crack present at time t = 0; modifications for significant crack growth and for defects formed in service (t>0) have been given by Ainsworth et al. (2011) and are not repeated here. The underlying approach follows that of Ainsworth (1992), Ainsworth and Budden (1992) and Ainsworth et al. (2011) and assumes that plasticity and creep are described by power-law equations of the form $\varepsilon_p = \beta\sigma^\mu$ and $\dot{\varepsilon}_c = D\sigma^n$, where ε_p is the plastic strain, $\dot{\varepsilon}_c$ is the creep strain rate and β, D, μ and n are material constants. In some cases, it is assumed that $\mu = n$, but the general case of $\mu \neq n$ is also addressed. The result is also written in a more general form which allows other plasticity or creep descriptions to be used. Although the derivation is complex, the main result can be written in a form that appears as a straightforward extension of the earlier solutions. Therefore, only the main steps in the derivation are presented here.

It is shown in Ainsworth et al. (2011) that for $\mu = n$ an estimate of C(t) can be related to an estimate of the time variation of the J-integral through

$$C(t) = \frac{J(t)^{n+1}}{\left[(n+1)\int_0^t J(t')^n dt' + \beta J_0^n / D\right]} \tag{1}$$

where J_0 is the value of J on initial loading at t=0. For combined primary and secondary loading, an estimate of J is

$$J = \sigma_{ref}\varepsilon_{ref}R' \tag{2}$$

where σ_{ref} is the total reference stress for the combined loading, ε_{ref} is the corresponding total (elastic plus plastic plus creep) strain at the time, t, and $R' = (K^p/\sigma^p_{ref})^2$ is a geometrical parameter defined from σ^p_{ref} and K^p, which are the reference stress and stress intensity factor evaluated for the primary loading alone. The initial value of J, J_0, may be written $J_0 = \sigma^0_{ref}\varepsilon^0_{ref}R'$, where $\sigma^0_{ref}, \varepsilon^0_{ref}$ are the initial reference stress and the corresponding total strain, which may be estimated from elastic-plastic fracture mechanics methods such as R6 (EDF Energy, 2015). Following initial loading, stress relaxation is assumed to be described by

$$\dot{\sigma}_{ref} = -(E/Z)[\dot{\varepsilon}_{c,ref} - \dot{\varepsilon}^p_{c,ref}] \tag{3}$$

where $\dot{\varepsilon}_{c,ref}$ is the creep strain rate at the total reference stress and $\dot{\varepsilon}^{p}_{c,ref} = \dot{\varepsilon}_{c}(\sigma^{p}_{ref})$ is that for the primary reference stress only. The factor Z is a measure of the elastic follow-up, with Z=1 corresponding to pure strain controlled relaxation and $Z \to \infty$ corresponding to the case where there is no relaxation and the combined loading behaves like a primary load. The total strain rate is

$$\dot{\varepsilon}_{ref} = \dot{\varepsilon}_{c,ref} + \dot{\sigma}_{ref}/E = \dot{\varepsilon}^{p}_{c,ref} - (Z-1)\dot{\sigma}_{ref}/E \tag{4}$$

There is no plastic contribution to the total strain rate in equation (4) as plastic strains are assumed to be irrecoverable and $\dot{\sigma}_{ref} < 0$. For a secondary creep law (constant creep strain rate), equation (4) can be integrated to give the total strain as

$$\varepsilon_{ref} = \varepsilon^{0}_{ref} + \dot{\varepsilon}^{p}_{c,ref} t - (Z-1)(\sigma_{ref} - \sigma^{0}_{ref})/E \tag{5}$$

Combining this expression with equation (2) and the formula for J_0 leads to

$$\left(\frac{J}{J_0}\right) = \left(\frac{\sigma_{ref}}{\sigma^{0}_{ref}}\right)\left[1 + \frac{\dot{\varepsilon}^{p}_{c,ref} t}{\varepsilon^{0}_{ref}} - (Z-1)\frac{(\sigma_{ref} - \sigma^{0}_{ref})}{E\varepsilon^{0}_{ref}}\right] \tag{6}$$

For use with equation (1), this can be raised to the power n and after manipulations similar to those in Ainsworth et al. (2011) leads to

$$C(t) = \frac{J_0 (J/J_0)^{n+1}}{\dfrac{\Phi(t)\varepsilon^{0}_{ref}}{D(\sigma^{0}_{ref})^{n}}\left[\left(\dfrac{\varepsilon_{ref}}{\varepsilon^{0}_{ref}}\right)^{n+1} - 1\right] + \beta/D} \tag{7}$$

where

$$\Phi(t) = \frac{\dot{\varepsilon}^{p}_{c,ref} - Z\dot{\sigma}_{ref}/E}{\dot{\varepsilon}^{p}_{c,ref} - (Z-1)\dot{\sigma}_{ref}/E} \tag{8}$$

and $\Phi(t)$ is bounded by $1 \le \Phi \le Z/(Z-1)$. In Ainsworth et al. (2011), equation (7) was re-written by normalising by the steady state creep parameter C^*, which depends only on the primary loading, and may be estimated as

$$C^* = \sigma^{p}_{ref}\dot{\varepsilon}^{p}_{c,ref} R' \tag{9}$$

similar to equation (2), and also by writing the ratio β/D in terms of the ratio of plastic strain to creep strain rate at the reference stress. Here, this approach is modified to re-write equation (7) in the form

$$\frac{C(t)}{C^*} = \left(\frac{\sigma_{ref}\dot{\varepsilon}_{c,ref}}{\sigma^{p}_{ref}\dot{\varepsilon}^{p}_{c,ref}}\right)\frac{\left(\dfrac{\varepsilon_{ref}}{\varepsilon^{0}_{ref}}\right)^{n+1}}{\Phi\left[\left(\dfrac{\varepsilon_{ref}}{\varepsilon^{0}_{ref}}\right)^{n+1} - 1\right] + \dfrac{\dot{\varepsilon}_{c}(\sigma^{0}_{ref})}{\varepsilon^{0}_{ref}}\dfrac{\varepsilon_{p}(\eta\sigma^{0}_{ref})}{\dot{\varepsilon}_{c}(\eta\sigma^{0}_{ref})}} \tag{10}$$

where $\eta\sigma_{ref}^{0}$ is a stress level to be determined with $\eta \geq 1$.

An upper bound estimate of C(t) is obtained by setting $\Phi = 1$ and if $\eta = 1$ then the result in Ainsworth et al. (2011) is recovered. If loading is mechanical only and initial response is elastic, then the short-time solution of Ohji et al. (1980) and Riedel and Rice (1980), which shows a variation of $C(t) \propto (1/t)$ for $t \to 0$ is also recovered.

Taking the upper bound estimate of C(t) with $\Phi = 1$, equation (10) may be written

$$\frac{C(t)}{C^*} = \left(\frac{\sigma_{ref}\dot{\varepsilon}_{c,ref}}{\sigma_{ref}^{p}\dot{\varepsilon}_{c,ref}^{p}}\right) \frac{\left(\frac{\varepsilon_{ref}}{\varepsilon_{ref}^{0}}\right)^{n+1}}{\left(\frac{\varepsilon_{ref}}{\varepsilon_{ref}^{0}}\right)^{n+1} - \left[1 - \frac{\dot{\varepsilon}_c(\sigma_{ref}^{0})}{\varepsilon_{ref}^{0}} \frac{\varepsilon_p(\eta\sigma_{ref}^{0})}{\dot{\varepsilon}_c(\eta\sigma_{ref}^{0})}\right]} \tag{11}$$

In this form, with C^* given by equation (9), C(t) may be evaluated for forms of creep and plasticity expressions other than simple power laws, provided an estimate of n can be made. In particular, common plasticity laws which show a distinct elastic region and a yield point can be used. If such plasticity laws are used and σ_{ref}^{0} is below the yield point and $\eta = 1$, then the plastic term in the denominator of equation (11) is zero and the equation reduces to the elastic-creep result which gives $C(t) \propto (1/t)$ for $t \to 0$. However, even for σ_{ref}^{0} below the yield point, there is plastic strain at the crack tip, which will reduce C(t) below the elastic-creep solution. The addition of the parameter η enables this effect to be captured and therefore to reduce conservatisms in previous approaches which assumed $\eta = 1$.

CHOICE OF VALUE OF MULTIPLIER

Validation of the estimate of C(t) for combined loading with $\eta = 1$ was examined in Ainsworth et al. (2011) by comparing predictions of J(t) and C(t) with results from elastic-plastic-creep finite element analyses. This showed that the approach generally leads to conservative predictions of C(t), although predictions are sensitive to the initial value of the total reference stress and the magnitude of the elastic follow-up factor. However, as noted above, the estimate can be over conservative at short times when plasticity local to the crack tip reduces C(t) below the elastic-creep result.

To take advantage of the reduced conservatism in the new approach presented here, it is necessary to input a value for the multiplier η in the estimates of C(t) in equations (10) and (11). Guidance on the choice of the value of η is therefore needed. This guidance may be obtained by examination of solution for power-law materials. With the plastic and creep behaviour described by $\varepsilon_p = \beta\sigma^{\mu}$ and $\dot{\varepsilon}_c = D\sigma^{n}$, as before with $\mu \neq n$ in general, the second bracketed term in the denominator of equation (11) is

$$\left[1 - \frac{\dot{\varepsilon}_c(\sigma_{ref}^{0})}{\varepsilon_{ref}^{0}} \frac{\varepsilon_p(\eta\sigma_{ref}^{0})}{\dot{\varepsilon}_c(\eta\sigma_{ref}^{0})}\right] = \left[1 - \frac{[E\beta(\sigma_{ref}^{0})^{\mu-1}]\eta^{\mu-n}}{[1 + E\beta(\sigma_{ref}^{0})^{\mu-1}]}\right] \tag{12}$$

Now, if the yield stress, σ_y, is defined as the 0.2% proof stress, then $E\beta\sigma_y^{\mu-1} = 0.002E/\sigma_y = \Gamma$, where Γ is a material constant. Then

$$\left[1-\frac{\dot{\varepsilon}_c(\sigma^0_{ref})}{\varepsilon^0_{ref}}\frac{\varepsilon_p(\eta\sigma^0_{ref})}{\dot{\varepsilon}_c(\eta\sigma^0_{ref})}\right]=\left[1-\frac{[E\beta(\sigma^0_{ref})^{\mu-1}]\eta^{\mu-n}}{[1+E\beta(\sigma^0_{ref})^{\mu-1}]}\right]=\left[1-\frac{\Gamma\eta^{\mu-n}(\sigma^0_{ref}/\sigma_y)^{\mu-1}}{1+\Gamma(\sigma^0_{ref}/\sigma_y)^{\mu-1}}\right]=\Psi(\eta,\ L_r^{'};\Gamma,n,\mu) \quad (13)$$

where $L_r^{'}=\sigma^0_{ref}/\sigma_y$ reduces to the normal definition of the limit load ratio, L_r, used in R6 (EDF Energy, 2015) for primary loading only, but is defined from σ^0_{ref} for combined loading. In the limit, $t\rightarrow 0$, equations (11) may then be written

$$\frac{C(0)}{C^*}=\frac{(\sigma^0_{ref}/\sigma^p_{ref})^{(n+1)}}{1-\Psi(\eta,\ L_r^{'};\Gamma,n,\mu)} \quad (14)$$

Finite element results for given material constants (Γ, n, μ) may be used to provide guidance on the choice of η and its dependence on the magnitude of the applied loading, $L_r^{'}$.

The result above holds provided there are plastic strains at all stresses. If elastic response is taken at low stresses with the power law only applicable at high stresses, then the result depends on whether or not there are plastic strains at the stresses σ_{ref} and $\eta\sigma_{ref}$. However, again finite element results for given material descriptions may be used to provide guidance on the choice of η.

FINITE ELEMENT VALIDATION

The new formula given in equation (11) has been validated by comparing C(t) estimates with results from elastic-plastic-creep finite element analyses to provide confidence in its application to AGR plant components. This section describes a comparison of the new C(t) estimates of equation (11) with elastic-plastic-creep finite element analyses of single edged cracked plates subjected to combined primary tension and secondary bending (Lei and Dean, 2013; Lei, 2013). The plates considered were of width, w = 30mm and length, 2L= 500mm and contained edge cracks with normalised depths of a/w = 0.1, 0.2, 0.3 and 0.4.

Material Properties

The tensile properties assumed (Lei and Dean, 2013; Lei, 2013) were described by the following equations:

$$\varepsilon=\begin{cases}\sigma/E & \text{for } \sigma\le\sigma_0\\ \sigma/E+\alpha(\sigma-\sigma_0)^{\mu_0} & \text{for } \sigma_0<\sigma\le\sigma_y\\ \sigma/E+\beta\sigma^{\mu} & \text{for } \sigma>\sigma_y\end{cases} \quad (15)$$

where Young's modulus, E = 200 GPa, and the material constants μ_0=2.8, μ=5, α=2.68x10^{-8} (MPa)$^{-5}$, β=6.25x10^{-15} (MPa)$^{-5}$. The yield (0.2% proof) stress, $\sigma_y=200$ MPa and the limit of proportionality, $\sigma_0=145$ MPa. The creep behaviour of the material was assumed to follow Norton's law

$$\dot{\varepsilon}_c=D\sigma^n \quad (16)$$

with D = 1.0x10^{-16} MPa^{-5}/h and n = 5. Thus $\mu=n$ for these analyses.

Loadings, Reference Stresses and Elastic Follow-up Factors

The secondary bending stress was introduced by applying a fixed rotation of 0.015 radians to both ends of the plate in a direction to cause crack opening. An additional mechanical load was applied in the form of a membrane stress, σ_m, at the ends of the plate with σ_m = 0, 10, 20, 30, 50, 75 or 100 MPa. Note that due to the boundary conditions applied to introduce the secondary stress, the ends of the plate were not able to rotate freely to respond to the membrane stress load. It can, therefore, be treated as a fixed-end plate. Values of reference stress for these load cases were obtained by using applied membrane stress and the reaction moment measured at the ends of the plate in the solution for combined membrane stress and bending moment given in equation (21) of Lei and Dean (2013). The relaxation of the total reference was used in conjunction with the relaxation equation (equation (3)) to determine the reference stress based elastic follow-up factors, Z_{ref} (Lei and Dean, 2013; Lei, 2013) given here in Table 1.

Table 1: Summary of the Finite Element Cases Considered (Lei and Dean, 2013; Lei, 2013) and Optimised η Values

a / w	σ_m (MPa)	σ^0_{ref} (MPa)	Z_{ref}	Optimised η Value	Optimised Stress, $\eta\sigma^0_{ref}$ (MPa)
	0	120.8	1.3	1.55	187.6
	10	121.6	1.4	1.52	184.6
	30	127.4	2.0	1.42	181.5
	50	137.6	2.3	1.31	180.0
0.1	100	174.0	2.7	1.07	186.3
	0	139.0	2.3	1.40	194.2
	10	139.8	2.5	1.36	190.2
	30	146.2	3.4	1.27	185.3
	50	157.6	3.9	1.17	183.6
0.2	100	198.0	4.2	1.01	200.0
	0	167.4	4.7	1.20	200.0
	10	168.4	4.6	1.19	200.0
	30	175.2	5.3	1.14	200.0
	50	187.8	5.7	1.07	200.0
0.3	100	233.2	5.8	N/A#	N/A#

\# N/A identifies cases where $\sigma^0_{ref} > \sigma_y$, for which the C(t) estimates are independent of η. No results are presented here for a/w = 0.4 because $\sigma^0_{ref} > \sigma_y$ even for $\sigma_m = 0$.

Alternative C(t) Normalisation

In Lei (2013) and Lei and Dean (2013), the C(t) values obtained from the elastic-plastic-creep analyses were normalised by $(C^*_0)_m$, the C^* value for the case of pure primary stress with $\sigma^p_{ref} = \sigma^0_{ref}$, rather than the value of C^* used here in equation (9), which is based on the primary reference stress, σ^p_{ref}. If this alternative normalisation is used, equation (11) can be expressed as

$$\frac{C(t)}{(C_0^*)_m} = \left(\frac{\sigma_{ref}\dot{\varepsilon}_{c,ref}}{\sigma_{ref}^0\dot{\varepsilon}_{c,ref}^0}\right)\frac{\left(\frac{\varepsilon_{ref}}{\varepsilon_{ref}^0}\right)^{n+1}}{\left(\frac{\varepsilon_{ref}}{\varepsilon_{ref}^0}\right)^{n+1} - \left[1 - \frac{\dot{\varepsilon}_c(\sigma_{ref}^0)}{\varepsilon_{ref}^0}\frac{\varepsilon_p(\eta\sigma_{ref}^0)}{\dot{\varepsilon}_c(\eta\sigma_{ref}^0)}\right]} \qquad (17)$$

instead of that presented in equation (11). This alternative C(t) normalisation allows pure secondary loading as well as combined primary and secondary loading cases to be considered, noting that the prediction of C(t) is unchanged. It is results with this normalisation that will be presented below. Further, time was normalised using the redistribution time defined by $(t_{red})_m = (\sigma_{ref}^0/E)/[D(\sigma_{ref}^0)^n]$.

Optimising η Values for the Cases Considered

For the cases considered here for which $\sigma_{ref}^0 < \sigma_y$, it has been shown that equation (11) with $\eta = 1$ (or equation (17) with $\eta = 1$) results in overly conservative predictions of C(t) at short times, most notably In the limit $t \to 0$, as shown in the examples presented in Figures 1 and 2. Following Lei (2013) and Lei and Dean (2013), the times presented in Figures 1 to 2 have been normalised using a redistribution time defined by $(t_{red})_m = (\sigma_{ref}^0/E)/[D(\sigma_{ref}^0)^n]$. This over-conservatism in C(t) predictions at short times with $\eta = 1$ is most significant for cases where the initial reference stress, σ_{ref}^0, is less than the limit of proportionality, σ_0, such that the plastic strain at the reference stress, ε_{ref}^p, is zero (see Figure 1, for example). Although the effect is less pronounced for cases where $\sigma_0 < \sigma_{ref}^0 < \sigma_y$, the use of $\eta = 1$ in equation (11) or (17) still results in conservative predictions of C(t) at short times (see Figure 2, for example). For the most highly loaded cases analysed (including that for a/w = 0.3 and $\sigma_m = 100$ MPa and all cases for a/w = 0.4), where $\sigma_{ref}^0 > \sigma_y$, the C(t) predictions of equation (11) or (17) are insensitive to the value of η. For each case where $\sigma_{ref}^0 < \sigma_y$, an optimised value of η was derived, which minimised the difference between the values of C(t)/$(C_0^*)_m$ predicted using equation (17) and those obtained from the finite element analyses, whilst remaining conservative at short times. Optimised η values and hence optimised values of the crack tip stress, $\eta\sigma_{ref}^0$, for the cases considered here are summarised in Table 1. Figures 1 and 2 show some examples comparing the C(t) predictions of equation (11) or (17) using the optimised η value with those using $\eta = 1$ and the finite element results. These results demonstrate that for all cases where $\sigma_{ref}^0 < \sigma_y$, improved predictions of C(t) are obtained when optimised values of η (which are in excess of unity) are used in equation (11) or (17). Figure 2 shows a case where the optimised η value results in $\eta\sigma_{ref}^0 = \sigma_y$ whereas Figure 1 shows a case where the optimised η value results in $\eta\sigma_{ref}^0 < \sigma_y$. However, Figure 1 shows that reasonable predictions of finite element C(t) values are also obtained for $\eta\sigma_{ref}^0 = \sigma_y$ (i.e. $\eta = 1.66$).

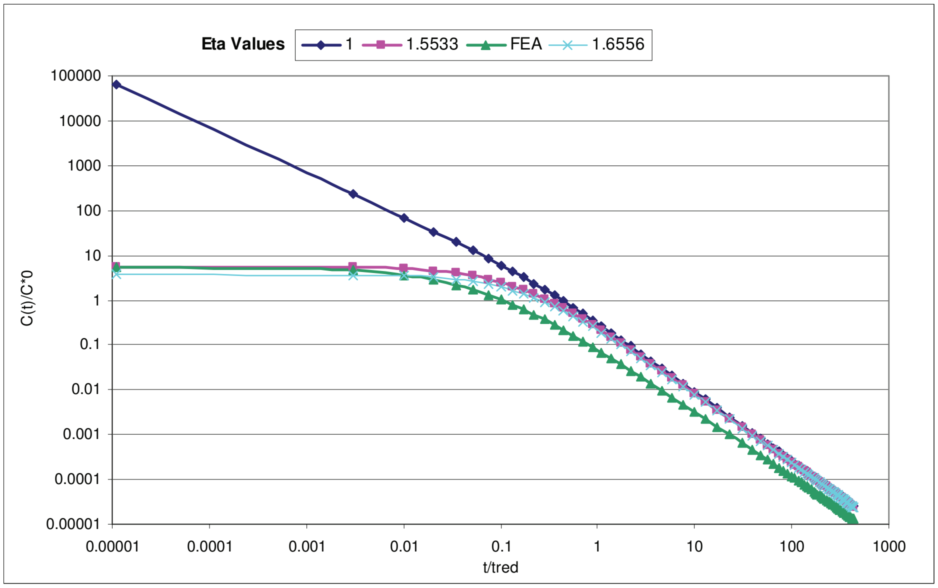

Figure 1. $C(t)/(C_0^*)_m$ plotted against $t/(t_{red})_m$ comparing the finite element results of Lei (2013) for a/w = 0.1 and $\sigma_m = 0$ with predictions of equation (17) for η=1, for the optimised η value and for $\eta\sigma_{ref}^0 = \sigma_y$ (i.e. $\eta = 1.66$). This represents a case where $\sigma_{ref}^0 < \sigma_0 < \sigma_y$.

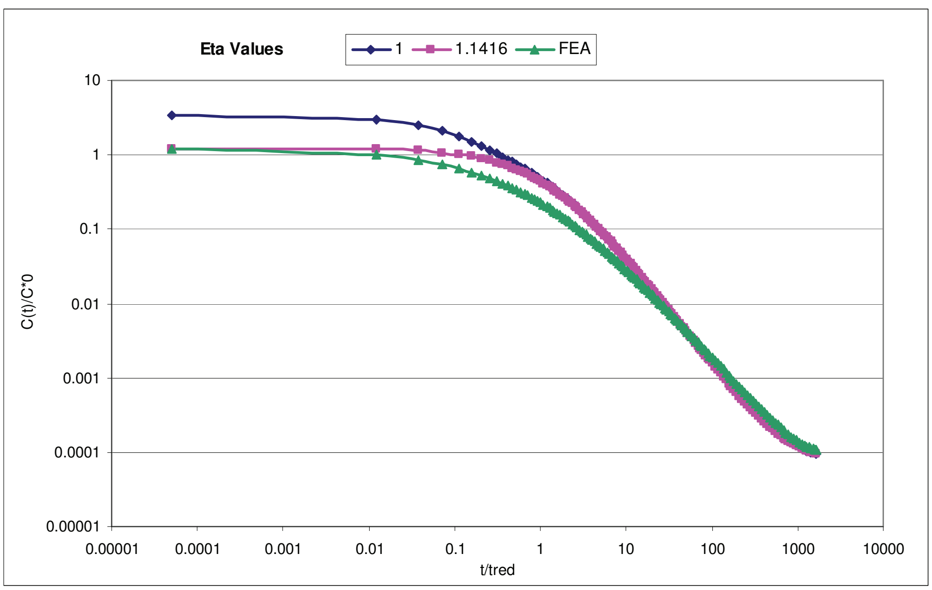

Figure 2. $C(t)/(C_0^*)_m$ plotted against $t/(t_{red})_m$ comparing the finite element results of Lei (2013) for a/w = 0.3 and $\sigma_m = 30$ MPa with predictions of equation (17) for η=1and for the optimised η value. This represents a case where $\sigma_0 < \sigma_{ref}^0 < \sigma_y$

DISCUSSION

The finite element validation presented here using elastic-plastic-creep finite element analyses of single edged cracked plates subjected to combined primary tension and secondary bending (Lei and Dean, 2013; Lei, 2013) has shown that the new C(t) estimation formula given in equation (11) gives improved C(t) predictions compared to existing formulae (Riedel and Rice, 1980; Ohji et al., 1980; Ohji and Kubo, 1988; Ainsworth and Budden, 1990; Ainsworth, 1992; Ainsworth et al., 2011), particularly at short times. This results from the use of a multiple, η, of the reference stress in the new formula, which allows the effects of local crack tip plasticity on C(t) to be taken into account. This is particularly important for situations where elastic response is taken at low stresses with power law plasticity only applicable at higher stresses as in the finite element analyses of Lei and Dean (2013) and Lei (2013). Figure 1 shows a case where the initial reference stress is lower than the limit of proportionality and the yield (0.2% proof) stress ($\sigma_{ref}^0 < \sigma_0 < \sigma_y$). Here, the use of $\eta = 1$ in equation (17) results in a prediction singular behaviour in the limit, $t \to 0$, as the plastic strain at σ_{ref} is zero. However, the use of $\eta > 1$ in equation (17), which reflects the higher stresses present local to the crack tip, allows improved estimates to be made of C(t), particularly in the limit $t \to 0$. It is shown in Figure 1 that improved predictions of C(t) are obtained using the optimised η value and if it is assumed that $\eta\sigma_{ref}^0 = \sigma_y$. Figure 2 shows that qualitatively similar improved C(t) predictions are obtained for a case where the initial reference stress is higher than the limit of proportionality but lower than the yield stress ($\sigma_0 < \sigma_{ref}^0 < \sigma_y$); for this case the optimised η value is such that $\eta\sigma_{ref}^0 = \sigma_y$. The finite element validation presented here is for cases with equal plasticity and creep exponents ($\mu = n = 5$) and further work is required to extend this to consider more general cases including those where $\mu \neq n$. Future finite element validation should also cover a wider range of geometries and loadings, including cases more relevant to real AGR plant components.

CONCLUDING REMARKS

A new formula for the estimation of the short-term creep crack tip characterising parameter C(t) has been derived for elastic-plastic-creep material response for combined primary and secondary loading. This formula has been shown to be less conservative in its treatment of crack tip plasticity than existing estimates in the literature, particularly at the times which are relevant to some AGR components. The new formula uses plastic strain and creep strain rate at a stress level which is a multiple, η, of the reference stress. When $\eta = 1$, the formula reduces to that in previous work. Values of $\eta > 1$ correspond to the use of a stress level which is higher than the reference stress and more representative of the stress level in the process zone near the crack tip. Elastic-plastic-creep finite element analysis results for single edged cracked plates subjected to combined primary tension and secondary bending have been compared with predictions using the new C(t) estimation formula. These comparisons show that for the cases considered with equal plasticity and creep exponents ($\mu = n = 5$), improved predictions of C(t) are obtained using optimised η values that result in crack tip stress, $\eta\sigma_{ref}^0$, values that are close to or equal to the yield (0.2% proof) stress, σ_y. Further work is required to investigate the applicability of these initial findings to a wider range of geometries, loadings and material behaviour.

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