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APPLICATION OF PROPORTIONNAL "GHOST" DAMPING FOR SLIDING OR YIELDING STRUCTURES IN TIME HISTORY DYNAMIC ANALYSES

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ABSTRACT

When studying the response of a structure, system or component to a dynamic excitation, the modelling of energy dissipation is sometime challenging to the engineer. Different methodologies were set up for different kinds of problems, with various degrees of representativeness and complications. The present paper describes a methodology specifically set in place to model the energy dissipation in a linear structure which is sliding and/or rocking on its support or mounted on a yielding support. This methodology was developed to overcome the drawbacks of a standard proportional "Rayleigh" damping. It is primarily intended for engineers facing seismic justification problems and in need for a simple and efficient representation of energy dissipation, suited for application in commercial finite element codes. It has been successfully applied for seismic analyses of handling cranes sliding on their rails, fuel racks sliding and rocking at the bottom of a pool and buildings installed on nonlinear seismic isolation systems.

CONTENT OVERVIEW

The first part of the paper gives a quick overview of the most usual representations of energy dissipation in dynamic analyses, with a particular focus on the use of a proportional Rayleigh damping and its drawbacks when applied to a linear but not linearly supported structure. The second part describes the "ghost" damping methodology set in place to overcome those drawbacks. The third part presents tests case basis on which the "ghost" damping was checked, including simple examples representative of a handling crane, a fuel rack and a seismically isolated building.

PART 1: REPRESENTATION OF ENERGY DISSIPATION IN USUAL ENGINEERING PRACTICES

Energy dissipation in a structure, system or components generally originates in a variety of local and global phenomena, which realistic representation is out of reach for the engineer facing actual dynamic justifications problems. These phenomena include localized yielding of metallic parts or reinforcements, frictional dissipation at interfaces between different parts (typically in bolted junctions), localized cracking (for buildings and anchorages) but also loss of energy from the structure at hand, even though this energy is not necessarily dissipated in the form of heat. Typical loss of energy include the so-called "radiation damping" in soil structure interaction problems and elastic waves induced in a third media after an impact. It also includes the energy transmitted by an immersed component to its surrounding fluid in the form of turbulence (kinetic energy).

From the engineer's point of view though, the question is generally not how the energy is dissipated during a dynamic excitation but how the structure response is affected by this dissipation. For this purpose, different analytical representations of energy dissipations were developed and are commonly used in dynamic analyses. These include friction coefficients, for sliding parts, restitution coefficients, for impacting parts, and modal damping values for linear structures, systems or components. Modal damping values allow the replacement of all dissipative phenomena with a single numerical variable for each structural mode. Furthermore, apart from simplifying the analysis, the use of friction coefficients, restitution coefficients and modal damping values is also justified by the relative simplicity of identifying these values through adequate tests programs.

Values for modal damping are found in numerous seismic design codes around the world and they constitute the energy dissipation modelling basis for most applications in the industry (see references [IAIE28], [ASME], [RG1.61], [ASN] among many others). They depend on the structure's materials (steel, reinforced concrete, pre-stressed concrete…), on the connections of the structure to its surroundings (bolted, welded, embedded…) and sometimes on the interaction with its surroundings (soil, fluid…). The use of modal damping values is extremely efficient when a dynamic problem is to be solved on a modal basis, since such representation of damping does not couple the dynamic equations of the different modes. From the engineer's point of view, the fact that these values can be extracted from existing codes and standards is sometimes even more valuable. For this reason, it has become common practice to try to apply these modal damping values to dynamic problems that are not solved on a modal basis. To do so with a finite element model, the most widely applied method consists in building a viscous damping matrix [C] as a linear combination of the mass matrix [M] and the stiffness matrix [K], as described in equation (1), α and β being referred to as the Rayleigh coefficients. This method is referred to as the Rayleigh method, after the work of Baron Rayleigh in the 1870s (see [Strutt] in the references).

$$[C] = \alpha[M] + \beta[K] \qquad (1)$$

For a linear system, since both the mass and stiffness matrices are diagonal when transformed in the modal basis, any linear combination of the two is also diagonal. The viscous damping term associated to any mode *i* is related to the modal mass m_i and the modal stiffness k_i by equation (2). The equivalent modal damping can then directly be related to α, β and f_i, the frequency of mode *i*, as described in equation (3).

$$c_i = \alpha.m_i + \beta.k_i \qquad (2)$$

$$\xi_i = \frac{1}{2}\left(\frac{\alpha}{2\pi.f_i} + 2\pi.f_i.\beta_i\right) \qquad (3)$$

This equation shows that by modifying α and β, it is possible to create a [C] matrix that produces a required given modal damping value, although for only two structural modes at most. The other modes will inherit of whatever damping is given to them by equation (3). A first clear drawback of this method is that for a three dimensional system, with distinct modes in the three directions, it is impossible to control the damping simultaneously, even for the main mode in each direction. The usual practice is to define α and β so that the required damping value is attributed to the modes with the first and the last significant frequency, which leads to under-damping all modes with frequencies in between. This approach can induce significant and sometime undesired conservatisms, as illustrated by the black solid line on Figure 1 for a 5% damping target between 3 and 30 Hz.

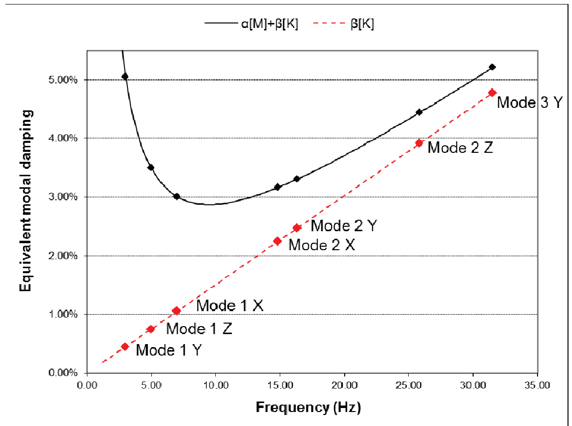


Figure 1. Effect of Rayleigh damping on the modes of a 3D structure

Another strong limitation of the proportional Rayleigh damping is that its use is theoretically restricted to linear structures, whereas engineers tend to use it when confronted with non linearities in their models making the problem non-solvable on a modal basis. For the case of linear structures, with a non linear supporting system, which will be the main focus of this paper, an unwanted effect of using a Rayleigh damping, even only on the linear part of the model, is that it produces damping forces that will spuriously limit, and ultimately stop the motion of the structure relative to its support basis. Indeed, the α[M] part of the Rayleigh damping matrix is predominantly diagonal. These diagonal terms, when multiplied by the velocity vector, produce damping forces proportional to the velocity of each node relative to the calculation referential, as illustrated by equation (4) for a two mass system, with a single degree of freedom for each node (U_X is the displacement of node X relative to the calculation referential, M_X the mass attached to this node).

$$\left\{F_{\alpha\_damping}\right\} = \alpha \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} = \begin{Bmatrix} \alpha M_1 \dot{U}_1 \\ \alpha M_2 \dot{U}_2 \end{Bmatrix} \tag{4}$$

Any rigid body motion of this two mass system relative to the calculation referential gives rise to non intended resisting damping forces. This phenomenon will be referred to as the “spurious damping of rigid body motions” in this paper.

On the other hand, the β[K] part of the Rayleigh damping produces forces that are only proportional to the relative velocities between nodes connected by an element. As an example, if a spring of stiffness K1 connects the two masses of the simple system described earlier, the resulting damping forces are illustrated by equation (5).

$$\left\{F_{\beta\_damping}\right\} = \beta \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 \end{bmatrix} \begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} = \begin{Bmatrix} \beta K_1 (\dot{U}_1 - \dot{U}_2) \\ \beta K_1 (\dot{U}_2 - \dot{U}_1) \end{Bmatrix} \tag{5}$$

These forces do represent dissipation of energy because of structural deformation, which is generally the objective assigned to Rayleigh damping.

The inadequacy of using the α[M] part of the Rayleigh damping is known and some references can be found in regulatory documentations such as [ASME] and [ASN]. As a consequence, cautious engineers will only use the β[K] part and therefore loose the ability to properly damp the lower frequency modes of the structures. Such approach invariably results in overly conservative estimations of the structural responses, as illustrated by the dotted red line on Figure 1: the first mode in each direction has an equivalent modal damping value far lower than the targeted 5%.

PART 2: DESCRIPTION OF THE "GHOST" DAMPING METHODOLOGY

The so-called "ghost" damping methodology has been specifically developed for the case of linear structures on a non linear support. Its objective is to overcome the two drawbacks of the Rayleigh proportional damping method identified in Part 1: controlled modal damping value on only two modes and spurious damping of rigid body motions. In the case of non linearly supported linear systems, the non linear support is generally explicitly modelled and is itself a source of energy dissipation, through friction coefficients (case of a handling crane) or modelling of a hysteretic material behaviour (case of a seismically isolated building). The linear part is modelled by linear stiffness and mass matrices, [K] and [M], which remain constant throughout the calculations.

The "ghost" methodology aims at producing damping forces on the linear structure which are only proportional to the structure deformation velocities. This is what happens with the usual Rayleigh damping for purely linear systems. To achieve this goal, the rigid body motion of the linear part of the model is subtracted from its overall movement when constructing the damping forces vectors. The form of these desired damping forces is illustrated in equation (6) for the two-masses-one-stiffness system described earlier.

$$\left\{F_{\alpha\_damping}\right\} = \begin{Bmatrix} \alpha M_1 (\dot{U}_1 - \dot{U}_{g1}) \\ \alpha M_2 (\dot{U}_2 - \dot{U}_{g2}) \end{Bmatrix} \text{ and } \left\{F_{\beta\_damping}\right\} = \begin{Bmatrix} \beta K_1 (\dot{U}_1 - \dot{U}_2) \\ \beta K_1 (\dot{U}_2 - \dot{U}_1) \end{Bmatrix} \tag{6}$$

The β[K] part is of the same nature as the proportional Rayleigh damping β[K] matrix. The α[M] part results in forces proportional to the node velocity minus the rigid body motion velocity at the node location. In this equation, U_{gX} represents the displacement of node X due only to the rigid body motion.

In a finite element time history analysis, the rigid body motions may not be directly accessible to compute the damping forces as expressed in equation (6). Therefore a rigid "ghost" of the structure is modelled additionally to the actual structure. The "ghost" features one node gX for each actual structural node X. Both nodes X and gX are initially located at the same point of the working space. The "ghost" is animated with movements in translation and rotation defined as a function of the displacement of some key nodes selected on the actual structure. For a simply supported sliding and rocking structure, selecting a few non-aligned key nodes located on the sliding plane is usually enough to make the "ghost" follow the rigid body motion in a satisfactory manner. Once the "ghost" is set in place, the displacements U_{gX} of its nodes as well as their velocities can be easily retrieved at each time step and subtracted from the actual node velocity to produce a damping force at each actual structural node that will reproduce the effect of the α[M] part of the Rayleigh damping without damping out the rigid body motions. This is, in essence, similar to defining the structural velocities relative to a moving referential, instead of relative to the calculation referential, but only for calculation of the damping forces.

Moreover, since the application of the "ghost" damping requires direct application of nodal damping forces onto the structure, or alternatively the implementation of a homemade [C] matrix that reproduces the same effect, nothing prevents the engineer from applying different values of α and β for different parts of the model or for different degrees of freedom of a same node. Such approach is useful for structures having noticeably different behaviour in different directions.

Finally, Figure 2 gives an overview of the different steps to apply the ghost damping methodology to a structural finite element model. In this example, the simple two-masses-one-stiffness system is represented and it is assumed that node 1 serves as the key node prescribing the ghost's displacement.

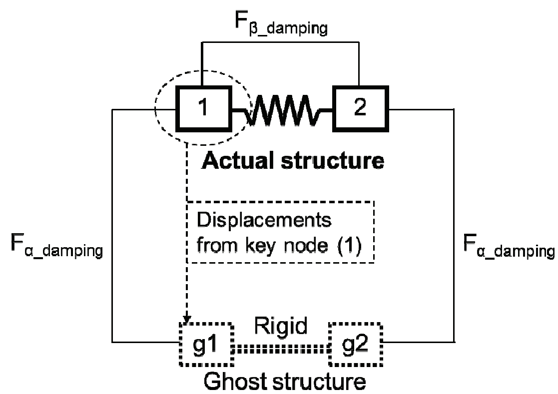


Figure 2. Overview of the "ghost" damping methodology

PART 3: TESTS CASE BASIS FOR THE "GHOST" DAMPING METHODOLOGY

This part presents the application of the "ghost" damping methodology on three simple cases representative of more complex structures. The first structure is representative of a bridge crane, the second of a fuel rack and the third of a seismically isolated building.

Bridge crane model

A simplified bridge crane is modelled with beam elements and illustrated on Figure 3. The model is supported on four nodes representing the wheels. The four wheels are either considered all clamped or all free to roll (equivalent to slide without friction) in the Y direction of Figure 3. Tests are done with using either the "ghost" methodology or with simply applying Rayleigh coefficients. α and β are taken constant in both cases and calibrated to achieve 5% damping at 3 and 30 Hz (similar to Figure 1).

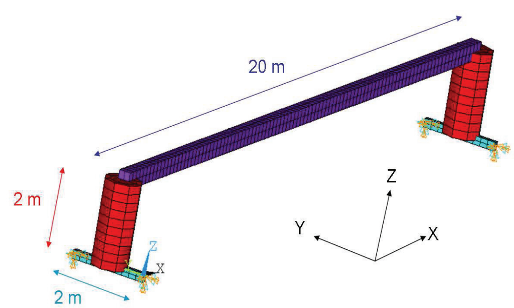


Figure 3. Overview of the bridge crane model

If the wheels are clamped, the bridge crane model becomes purely linear. The structural predominant modes are then calculated at approximately 3 Hz in the Y direction, 5 Hz in the Z direction and 7 Hz in the X direction, as illustrated on Figure 4. Damped modal analyses with each damping methodology give the same modal damping values (see Table 1). This step validates the equivalence of the “ghost” damping methodology to the Rayleigh coefficient methodology if applied on a purely linear system.

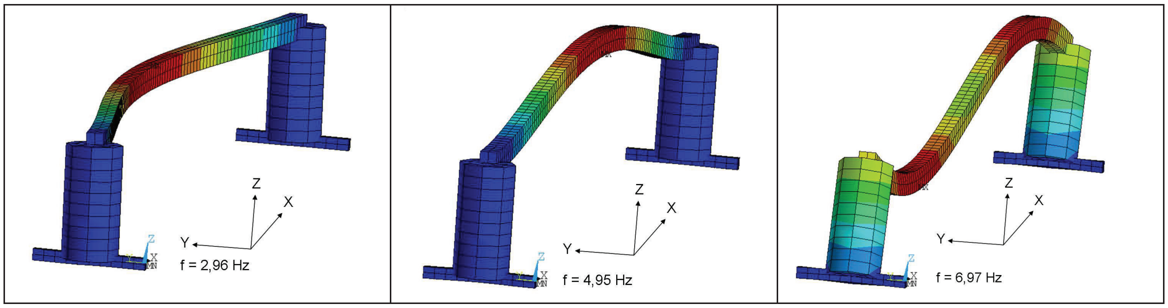


Figure 4. Main modes of the linearly clamped bridge crane model

Mode	Frequency (Hz)	Damping Rayleigh coefficients	Damping “ghost” methodology	Fraction of Mass X	Fraction of Mass Y	Fraction of Mass Z
1	2.96	5.05%	5.05%	0%	75%	0%
2	4.95	3.50%	3.50%	0%	0%	69%
3	6.97	3.01%	3.01%	57%	0%	0%
4	14.81	3.17%	3.17%	0%	16%	0%
5	16.33	3.31%	3.31%	36%	0%	0%
6	25.83	4.45%	4.45%	0%	0%	15%
7	31.51	5.21%	5.21%	0%	3%	0%

Table 1: Frequencies, participating masses and associated damping for linearly clamped bridge crane model

If the wheels are free to roll in the Y direction, the bridge crane model can be defined as being a non-linearly supported linear system. Friction or any other non-linear phenomenon could be modelled between the wheels and their support. It has not been done here for the sake of clarity. A transient analysis is carried out on this model, with the loading being defined as an initial impulse force applied at the center of the structure in the Y direction. The structural response is observed on a 2s time frame. The Y displacement of the middle node of the main beam is plotted against time on Figure 5, with a comparison between the “ghost” methodology and the Rayleigh methodology.

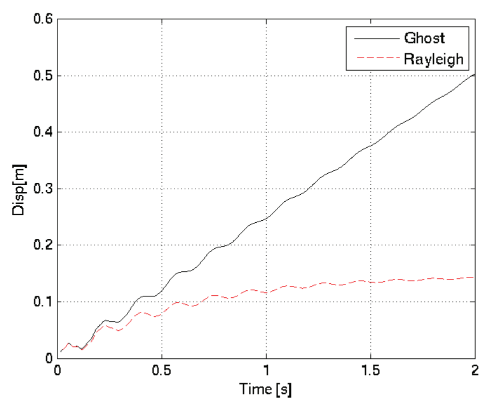


Figure 5. Overview of the Y displacement of the bridge middle node with initial impulse loading along Y

This figure shows clearly that when using standard Rayleigh methodology, the bridge crane will be practically stopped after only 1.5s, even though no friction at all is modelled. On the other hand, when using the "ghost" methodology, the rigid body motion is not damped. The natural oscillations decay, due to the damping applied to the modes of the linear part of the model (here the mode at 3 Hz in the Y direction), appears to be similar in both cases.

Figure 5 demonstrates that the first drawback of using the Rayleigh proportional damping methodology, namely the spurious damping of rigid body motions, is overcome with the "ghost" damping methodology. Figure 6 shows the advantages, from a justification point of view, to use different α coefficients for different directions. This figure shows the equivalent modal damping achieved for the clamped model and is to be compared to Figure 1.

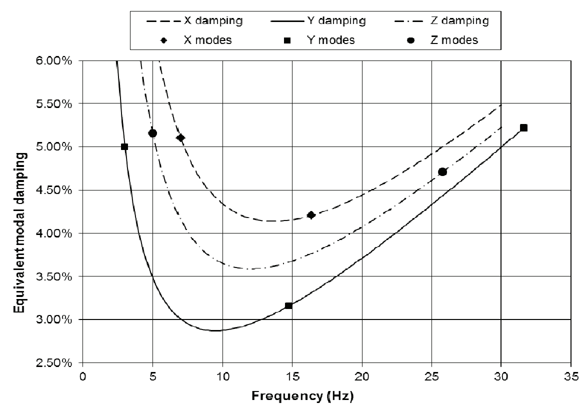


Figure 6. Effect of "ghost" damping on several modes of a 3D structure when using different α coefficients for different directions

A seismic transient analysis is then carried out with the wheels free to roll in the Y direction. Figure 7 shows extracts of the analysis results in terms of absolute acceleration in directions X, Y and Z at the centre of the bridge crane. In the Y direction, it can be seen that the spurious damping forces induced by the proportional Rayleigh damping produce unrealistic accelerations of the structure, whereas the excitation should be almost completely filtered by the no-friction condition at the interface, as it is the case with the "ghost" damping. In the X and Z directions, the advantages of using different α coefficients in the different directions is made clear by the response reduction observed with the "ghost" damping relative to the one obtained with classic Rayleigh damping.

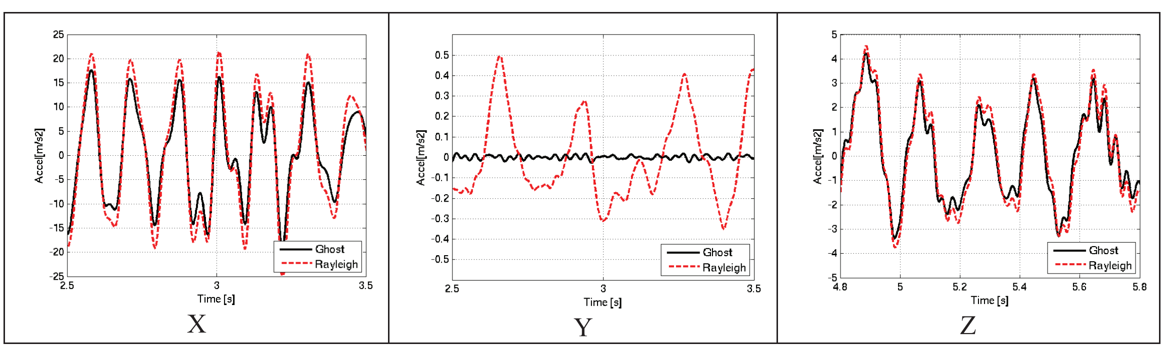


Figure 7. Absolute acceleration in directions X, Y and Z at the centre of the bridge crane under seismic excitation

Fuel rack model

A simplified model of a free standing structure is shown on Figure 8. The model is unsymmetrical so that a horizontal excitation induces a rotation around the vertical (Z) axis. It is vaguely representative of a fuel rack not fully loaded or submitted to non-homogeneously distributed fluid forces during an earthquake. It is in contact with the floor on four nodes at its base. A friction coefficient of 0.2 is considered between these nodes and the floor, when the structure slides horizontally. This example is selected to qualify the "ghost" damping behaviour for rotational motions.

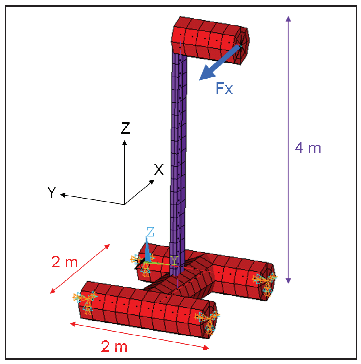


Figure 8. Overview of the fuel rack model

Figure 9 represents the comparative rotational displacement induced by an impulsive load *Fx* on the fuel rack model with proportional Rayleigh damping and with "ghost" damping.

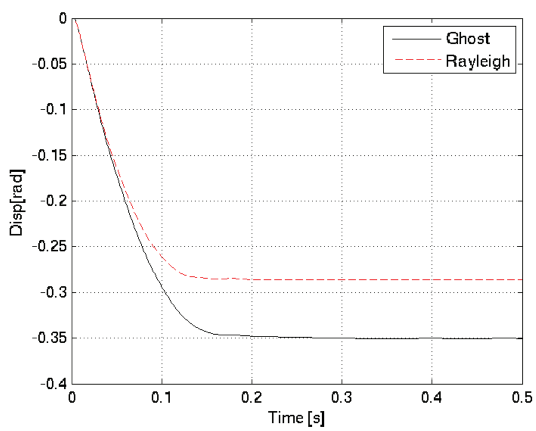


Figure 9. Rotation displacement around Z of the fuel rack model with impulsive loading at the top

As for the bridge crane model, the use of the ghost damping removes the spurious damping of rigid body motions. Its effect in this case is less obvious than on Figure 5 because the friction coefficient itself ultimately stops the rigid body motion.

Seismically isolated structure model

A specificity of seismically isolated buildings using nonlinear isolation systems (lead plug rubber bearings, friction bearings or others…) is that the isolated superstructure remains mostly linear during an earthquake whereas the isolators may undergo significant yielding or sliding. Therefore, a seismically isolated building can enter into the category of nonlinearly supported linear structures. A simplified example of building is illustrated on Figure 10. This model is supported on an isolation system represented by linear springs and viscous dampers and is submitted to a 3D seismic excitation. Practically, any kind of nonlinear isolation model could replace the springs and dampers.

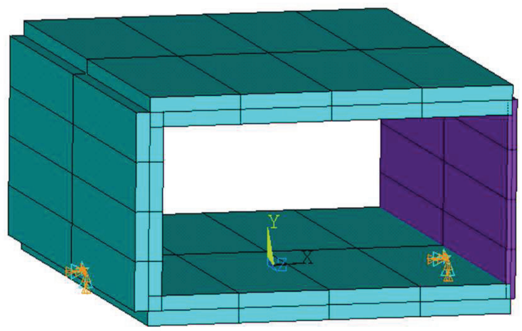
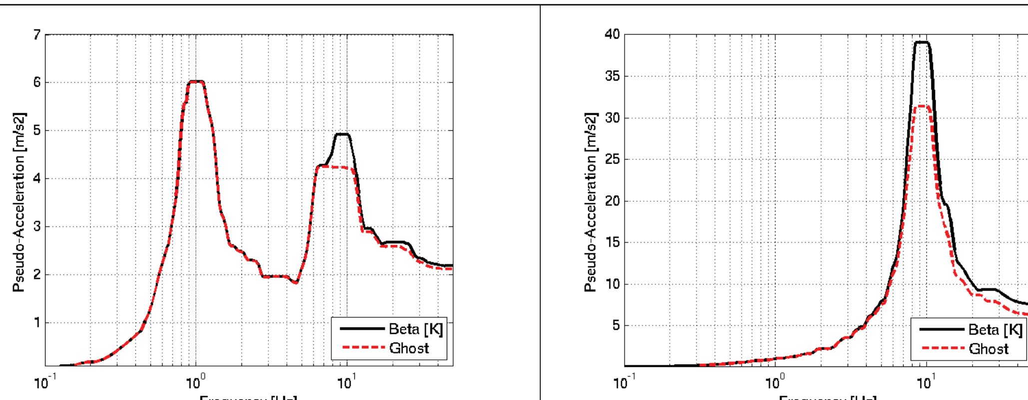


Figure 10. Overview of the building model

Two modelling of the damping are compared. The first one is the modelling chosen by the careful engineer, knowing about the drawbacks of using proportional Rayleigh damping in such case, and consists in applying only the β[K] part, which is always conservative and acceptable. The second modelling consists in applying a "ghost" damping to the structure. Both methods are employed with the same equivalent modal damping target of 7% and both methods are conservative regarding this value. Figure 11 gives a comparison of typical floor response spectra obtained for both cases in the horizontal and vertical direction. The horizontal spectra exhibit one peak at low frequency, typical of an isolated building and due to the isolation system. This peak is not impacted by the damping modelling for the superstructure. It would have been spuriously reduced if an α[M] part of a proportional Rayleigh damping had been used. The horizontal spectra exhibit other peaks at higher frequencies due to the vertical excitation (see [Moussallam & Vlaski] for detailed explanation). The vertical response spectra mainly exhibit one peak corresponding to the first vertical frequency of the isolated building. From this figure, the benefit of applying the "ghost" damping for the justification of systems and components is clear: in-structure response spectra are reduced by approximately 20%.



(a) (b)

Figure 11. Compared floor response spectra (a) horizontal (b) vertical

CONCLUSION

Two drawbacks of using a classic proportional Rayleigh damping methodology in the dynamic analysis of non-linearly supported linear structures were highlighted: spurious damping of rigid body motions and under-damping of some major modes located in between the first and the last mode frequencies of interest. These drawbacks are known and the cautious engineer will usually make conservative assumptions to avoid the first one. The present paper describes a methodology to suppress the spurious damping of rigid body motion by using a “ghost” structure within a FE model and generate damping forces proportional to the actual structure velocities relative to its “ghost”. At the same time, different Rayleigh coefficients are used for different directions, in order to bring the equivalent modal damping values of the linear part of the model closer to the targets in each direction, often defined in the codes. This method was qualified with three test cases, approximately representing a bridge crane, a fuel rack and a seismically isolated building. Some results were presented in this paper, demonstrating both the adequacy of using this method and its efficiency in decreasing unwanted methodological conservatisms.

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