STRENGTH OF RC AND PSC BEAMS WITH HIGH STRENGTH STIRRUPS

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ABSTRACT

Development of high strength steel bars has recently been an important issue due to the increase of large-scale structures such as high-rise buildings, long-span bridges, and nuclear power plants. Specified yield strengths for shear reinforcements for reinforced and prestressed concrete members are different in the national codes of standards. The yield strength of those reinforcements has been limited to as 420MPa in ACI 318-11. While EC2-04 permits the strength up to 600MPa, CSA-04 allows the strength up to 500MPa. The reason to limit the yielding strength of shear reinforcement in the design codes is primarily due to keep shear tension failure and control a diagonal crack width. In the case where a concrete shear contribution is constant and high strength shear reinforcement is employed, the spacing of the shear reinforcement can be larger. Hence, there is a strong possibility that the shear failure modes are changed from the shear tension failure to the shear compression failure and the diagonal crack width is increased. Although there are many studies regarding the behavior of reinforced and prestressed concrete beams subjected to shear, only a limited number of studies on beams regarding the yield strength of shear reinforcement are available. In this study, a total of 244 concrete beam tests incorporating high strength shear reinforcement were analysed and the applicability of the reinforcement was thus assessed. The analytical results indicate that the limitation on the yield strength of shear reinforcement in the design code is somewhat under-estimated and needs to be increased for high-strength concrete beams.

INTRODUCTION

The failure modes of reinforced concrete (RC) or prestressed concrete (PSC) members dominated by shear force are generally classified into four types of modes: minimum shear reinforcement failure, tension failure (under reinforcement failure), balanced failure, and compression failure (over reinforcement failure). These four failure modes are influenced by the compressive strength of concrete and the amount of shear reinforcement in the concrete. Among these failure types, the minimum shear reinforcement failure and over reinforcement failure occur abruptly without sufficient advanced warning (Lee and Hwang, 2008).

The most design codes for shear have several limitations such as a minimum amount of shear reinforcement, the maximum spacing of stirrups, a maximum amount of shear reinforcement, and the yield strength of stirrups. The ACI 318-11 design code (2011) requires a minimum amount, \( \rho_{\text{min}} \), of shear reinforcement in RC and PSC beams to reserve shear strength and to prevent sudden shear failure upon first diagonal tension cracking. The code also requires the maximum spacing, \( s_{\text{max}} \), of the vertical stirrups as the smaller of \( d/2 \) or about 600mm, so that each crack will be intercepted by at least one stirrup.
The ACI 318-99 design code required that the minimum amount of shear reinforcement be \( \rho_{\text{min}} = 0.33/ f_y \) (MPa) for the compressive strength of concrete, \( f_c' \) up to 69MPa. For \( f_c' > 69 \text{MPa} \), the code increased the \( \rho_{\text{min}} \) requirement by multiplying a factor of \( f_c'(\text{MPa})/35 \), but not to a value greater than 1.0MPa where \( f_y \) is the yield stress of stirrups. The requirement of \( \rho_{\text{min}} \) in the ACI 318-99 did not include the influence of \( f_c' \) in the relation when \( f_c' \leq 69 \text{MPa} \) and abruptly changes after 69MPa. The ACI Building code 318-11 avoids the sudden jump at \( f_c' = 69 \text{MPa} \), as stated in the ACI 318-99 code, and proposes an equation considering the compressive strength of concrete. While the equation in the ACI 318-11 or the Canadian code (CSA-04, 2004) is expressed in terms of the compressive strength of concrete, \( f_c' \) and the yield stress of the shear reinforcement, \( f_y \), the Japanese code specifies the \( \rho_{\text{min}} \) as a constant value, 0.002. The requirement of \( \rho_{\text{min}} \) in the European code (EC2-04, 2004) includes the influence of the characteristic compressive strength of concrete, \( f_c' \). For example, when \( f_c' = 69 \text{MPa} \), the \( \rho_{\text{min}} \) calculated by the EC2-04 is much larger than that calculated by the ACI 318-11. When \( f_c' = 40 \text{MPa} \), the Japanese code gives the largest value of \( \rho_{\text{min}} \), which is almost double the five values of \( \rho_{\text{min}} \) calculated by the other codes.

The ACI 318-11 design code also requires a maximum amount, \( \rho_{\text{max}} \), of shear reinforcement in reinforced concrete beams for two reasons, first to ensure adequate reserve shear strength and to prevent possible sudden shear failure due to concrete crushing before yielding of stirrups due to over shear reinforcement and second to reduce unsightly cracking. Primary national codes of standard for reinforced concrete members and structures such as ACI 318-11, EC2-04, CSA-04 and JSCE-02(2002) specify a limited value of the maximum amount, \( \rho_{\text{max}} \), of shear reinforcement. The design equations provided by the aforementioned four codes differ substantially from one another. For \( f_c' = 30 \text{MPa} \), the \( \rho_{\text{max}} \) calculated by the EC2-04 and CSA-04 is much greater than that calculated by the ACI 318-11. For \( f_c' = 20 \text{MPa} \), the Japanese code provides the largest value of \( \rho_{\text{max}} \), which is more than double the value of \( \rho_{\text{max}} \) determined by the ACI 318-11. In addition, the required \( \rho_{\text{max}} \) in the current design codes except CSA-04 do not include other important parameters, such as shear span-to-depth ratio and longitudinal tensile reinforcement ratio. The effect of longitudinal axial strain on the angle of inclination and the factor accounting for shear resistance of cracked concrete is echoed in the General Method of CSA-04; increase in the longitudinal reinforcement ratio reduces the longitudinal strain in the reinforcement resulting in a larger concrete shear contribution (Lee and Hwang (2010)).

Regarding of the yield strength of shear reinforcement used in design calculations the ACI 318-11 code also limits the yield strength, \( f_{sy} \), of shear reinforcement used in shear design to 420 MPa, while the European code based on a variable strut inclination method allows \( f_{sy} \) to 600 MPa. The Japanese code (JSCE-02, 2002) also allows the yield strength of shear reinforcement up to 800 MPa when the compressive strength of concrete is greater than 60 MPa. On the other hand, the shear design in the CSA-04 limits the yield strength, \( f_{sy} \), of shear reinforcement to 500 MPa. The limitations on the yield strength of shear reinforcement provided by the aforementioned four codes differ substantially from one another. For high strength concrete \( f_c' = 60 \text{MPa} \), the maximum \( f_{sy} \) required in the Japanese code is almost double the value of \( f_{sy} \) in the ACI 318-11 code. There are two main reasons why the design codes limit the yield strength of shear reinforcements. The first reason is because of yielding for shear reinforcement: as the yield strength of shear reinforcement is increased, the yield strain of the reinforcement is also increased. This may lead to a concrete compressive failure prior to yielding of the reinforcement. In this case, the specified yield strength of the reinforcement cannot be used in the code equations of shear resistance since the reinforcement does not yield. The first reason is because of controlling a diagonal crack width:
shear can be resisted by either concrete or shear reinforcement. In the case where a concrete contribution is constant and high strength shear reinforcement is employed, the spacing of the shear reinforcement can be larger. Hence, there is a strong possibility that the diagonal crack width is increased. This is supported by ACI 318-11, which specifies that yield strength of shear reinforcement is limited to 420MPa in order to control the diagonal crack width. This is also supported further by the report of ASCE-ACI Committee 445 on Shear and Torsion, describing that the ACI 318 shear design approach is based on a parallel truss model with 45 degree constant inclination diagonals supplemented by an experimentally obtained concrete contribution, and hence it limits the maximum shear contribution of shear reinforcement to prevent diagonal crushing failures of the web concrete before yielding of the shear reinforcement (Lee and Lim (2011)).

Recently, the adaptability of high strength shear reinforcement to RC members has been actively tested as being successful. Lee and Lim (2011) tested RC beams incorporating high strength shear reinforcement and evaluated the crack width at the serviceability loading stage. They concluded that the increase of average crack width is minor in beams with 520MPa and 650MPa high strength shear reinforcement, and thus the measured width satisfies the regulation required by ACI 318-11. Similar research has been conducted in Japan by Aoyama (2001), who tested the applicability of high strength shear reinforcement on RC structures, where the reinforcement is categorized into two types based on yield strength, i.e., 685MPa (USD685A and 685B) and 980MPa (USD980). In this study, while the former reinforcements are designed to yield when a member fails, the latter is designed to excurse only into the elastic range. In addition, Fukuhara and Kokusho (1982) and Iwai et al. (1991) carried out an analysis of the adaptability of high strength shear reinforcement of 750MPa (USD785) and 1,200MPa (USD1275) on RC members. Their results showed that, amongst the RC members incorporating over 600MPa of yield strength of shear reinforcement, yielding occurred in members with over 50MPa of concrete compressive strength when these members failed in shear (Lee et al. 2015).

While various researches have been performed to predict shear strength, only a few studies have been conducted with regard to the assessment of shear behaviour of beams with high strength stirrups. In the present study, shear failure modes and shear strength have been investigated through beam tests that incorporate shear reinforcement of high yield strength.

**EXPERIMENTAL RESULTS OF BEAM TESTS**

The experimental results of the 244 beams (Reineck et al. 2014) reported in the literature were used to check the limitations of the current design codes for shear. The results were selected from the 886 collected test results. The data were not only collected but were also critically reviewed, and criteria were developed to achieve a reliable database for assessing shear design relationships. This database was considerably expanded until 2006 and then even more in the recent years by a joint ACI-DAfStb group.

**Table 1: Range of the material properties of the 244 beams.**

<table>
<thead>
<tr>
<th>Range</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$f'_{c}$</td>
<td>11.2 ( \leq f'_{c} \leq 122.9 ) (MPa)</td>
</tr>
<tr>
<td>$\rho_t$</td>
<td>0.00078 ( \leq \rho_t \leq 0.0387 )</td>
</tr>
<tr>
<td>$f_{ty}$</td>
<td>237.20 ( \leq f_{ty} \leq 820.0 ) (MPa)</td>
</tr>
<tr>
<td>$\rho_t f_{ty}$</td>
<td>0.0032 ( \leq \rho_t f_{ty} \leq 0.244 ) (MPa)</td>
</tr>
</tbody>
</table>
Figure 1. Material properties of the 244 beams.

(Reineck et al. 2014). The yield strength of the stirrups varied from 237.20 MPa to 820.0 MPa, while the concrete compressive strength varied from 11.2 MPa to 122.9 MPa. Different end conditions were used to model different support conditions, such as simply supported, restrained, and continuous beams. The material and geometrical conditions of the 244 beams are listed in Table 1 and are shown in Fig. 1. In Table 1, \( f'_c \) is the compressive strength of concrete, \( \rho_t \) is the ratio of the stirrup, \( f_{ty} \) is the yield strength of the stirrup, \( a/d \) is the shear span-to-depth ratio, \( d \) is the effective depth of section, \( b \) is the width of section, and \( \rho_v \) is the ratio of the longitudinal tensile reinforcement.

**CALCULATION RESULTS AND DISCUSSIONS**

One of the reasons to limit the yield strength of shear reinforcement in the codes is to prevent possible sudden shear failure due to concrete crushing before yielding of stirrups because of the two reasons. The yield strain of steel bars is proportional to the yield strength of the steel bars. Because the high-strength shear reinforcement has greater yield strain, the web concrete may crush before the shear reinforcement arrives at its yield strain. In this case, the shear resistance of stirrups of the beam could not be calculated by substituting the yield strength of stirrup. In addition, if a beam is over shear reinforced, the web concrete crushes before the yielding of shear reinforcement, which leads to brittle shear failure. This failure mode violates the requirement in the design code to calculate the shear contribution of the stirrups \( (V_s) \) because the stress of stirrup does not reach the yield strength of the stirrup \( (f_{ty}) \).

<table>
<thead>
<tr>
<th>Code</th>
<th>Shear reinforcement ratio, ( \rho_t )</th>
<th>Maximum yield strength of shear reinforcement, ( f_{ty} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI318-11</td>
<td>( 0.062 \sqrt{\frac{f'<em>c}{f</em>{ty}}} \leq \rho_t \leq \frac{2 \sqrt{\frac{f'<em>c}{f</em>{ty}}}}{3} )</td>
<td>420</td>
</tr>
<tr>
<td>EC2-04</td>
<td>( 0.08 \sqrt{\frac{f'<em>c}{f</em>{ty}}} \leq \rho_t \leq \frac{1}{2} \sqrt{\frac{f'<em>c}{f</em>{ty}}} \sin^2 \theta )</td>
<td>600</td>
</tr>
<tr>
<td>CSA-04</td>
<td>( 0.06 \sqrt{\frac{f'<em>c}{f</em>{ty}}} \leq \rho_t \leq 0.25 \sqrt{\frac{f'<em>c}{f</em>{ty}}} - \beta \sqrt{\frac{f'<em>c}{f</em>{ty}}} \tan \theta )</td>
<td>550</td>
</tr>
</tbody>
</table>
There are also two possible reasons why the shear reinforcement does not reach its yield strain at the peak load; high yield strain of shear reinforcement and large amount of shear reinforcement \( \rho_{\text{max}} \). If a beam is over-reinforced for shear, the web concrete crushes before the yielding of shear reinforcement, which leads to brittle shear failure. Therefore, the design code requires a maximum amount, \( \rho_{\text{max}} \), of shear reinforcement in beams to ensure adequate reserve shear strength and to prevent possible sudden shear failure due to the concrete crushing before the yielding of stirrups because of over-reinforcement for shear as listed in Table 2 which show the shear limitations in the current design codes. In Table 2, \( \theta \) is the angle of inclination of diagonal compressive stresses to the longitudinal axis of the member, \( v \) is the effective strength reduction factor of concrete defined as \( v = 0.6(1 - f'_{c}/250) \).

Figures 2 through 4 compare the shear strength calculated by the current design codes with the experimental results of the 244 beams. In the determination of the code-specified shear strengths in the figures, safety factors were not taken into account and no limitation is applied regarding maximum reinforcement ratio and yield strength of shear reinforcement. Shear strength by ACI 318-11 (\( V_{\text{ACI}} \)) is calculated using Eqs. (1) and (2) of ACI 318-11 in which effects of both flexural reinforcement and aspect ratios are taken into account. In Eq.(2-1), the limitation, \( 0.29\sqrt{f'_{c}}b_{w}d \) in the ACI 318-11 was not adopted.

\[
V_{n} = V_{c} + V_{s} \\
V_{c} = 0.16\sqrt{f'_{c}} + 17.6\rho_{w} V_{d} M_{w} d \tag{2-1} \\
V_{s} = \frac{A_{f} f_{y} d}{s} \tag{2-2}
\]

where, \( V_{n} \) is the nominal shear strength, \( V_{c} \) is the shear contribution of concrete, \( V_{s} \) is the shear contribution of shear reinforcement, and \( s \) is the spacing of the stirrups.

Regarding the shear resistance of EC2-04 (2004), design value of the maximum shear force sustained by a member \( (V_{d,\text{max}} \text{ in Eq. (3)}) \), limited by crushing of the concrete compression strut should be set higher than the shear force sustained by the yielding of shear reinforcement \( (V_{sd} \text{ in Eq. (4)}) \). Angle of crack inclination is recommended as the expression, \( 1 \leq \cot \theta \leq 2.5 \). However, in the present study, shear resistance by the yielding of shear reinforcement is set as the maximum shear force. Subsequently, angle of crack inclination is calculated when \( V_{sd} \) is equal to \( V_{d,\text{max}} \).

\[
V_{d,\text{max}} = \frac{v \cdot f'_{c} b_{w} \cdot z}{\cot \theta + \tan \theta} \tag{3} \\
V_{sd} = \frac{A_{st}}{s} \cdot f_{y} \cdot \cot \theta \tag{4}
\]

where, \( v \) is the effective strength reduction factor of concrete defined as \( v = 0.6(1 - f'_{c}/250) \).

As for CSA-04 (2004), the shear strengths are calculated using the Eq. (5). In the calculation of shear strength by CSA-04 \( (V_{CSA}) \), the longitudinal strain at mid-depth of the member, \( \varepsilon_{x} \) is assumed to be 0.001.

\[
V_{n} = \beta\sqrt{f'_{c}} b_{w} d + \frac{A_{f} f_{y} d}{s} \cot \theta \tag{5}
\]
where, $\beta$ is defined as $\beta = 0.4 / (1 + 1,500 \varepsilon_s) \cdot 1,300 / (1,000 + s_z c_e)$ and the angle of crack inclination is calculated as:

$$\theta = 29 + 7000 \varepsilon_s$$

Figure 2 compares the observed and calculated shear strengths. In the determination of the code-specified shear strengths in the figure, no limitation is applied regarding maximum reinforcement ratio and yield strength of shear reinforcement. As observed in Fig. 2, shear redundancy (measured/calculated) is reduced as the yield strength of shear reinforcement is increased. The redundancy of some specimens is less than 1. This can be attributed to the fact that the specimens have failed in shear prior to yielding of shear reinforcement. Meanwhile, the redundancy is greater than 1 for specimens undergoing yielding of shear reinforcement. Results regarding the shear redundancy indicate that yield strength of 600MPa can be a threshold value for yielding of shear reinforcement. In short, there is a strong possibility that yielding of shear reinforcement may not occur for specimens with a yield strength of shear reinforcement of more than 600MPa and normal compressive strength of concrete.

Figure 3 shows a comparison between the measured shear strength and that calculated from the design codes. The limitation of the yield strength of shear reinforcement has been used to calculate the shear strength specified in the design codes; 420MPa, 600MPa, and 550MPa for the ACI 318-11, EC2-04, and CSA-04, respectively. Figure 3 shows that the shear redundancy is increased because of the limitation of the yield strength of shear reinforcement. However, the coefficient of variances (COV) of the prediction also increased. The COVs of the ACI318-11, EC2-04, and CSA-04 design codes are 24.1%, 36.5%, and 22.6%, respectively.

Figure 2. Comparisons of the observed and calculated shear strengths – no shear limitations.

Figure 3. Comparisons of the observed and calculated shear strengths – limitation of $f_y$. 
In Fig. 4, the limitation of the maximum amount of shear reinforcement ($\rho_{\text{max}}$) has been used to calculate the shear strength specified in the design codes. The design codes require a maximum amount of shear reinforcement in reinforced concrete beams to ensure adequate reserve shear strength and to prevent possible sudden shear failure due to concrete crushing before yielding of stirrups because of over shear reinforcement and second to reduce unsightly cracking. As shown in Fig. 4, the accuracy of the design codes was increased by limiting the maximum amount of shear reinforcement. The COVs of the ACI318-11, EC2-04, and CSA-04 design codes are 22.6%, 21.0%, and 20.8%, respectively.

![Fig. 4. Comparisons of the observed and calculated shear strengths – limitation of $\rho_{\text{max}}$.](image)

In Fig. 5, the limitations of the maximum reinforcement ratio ($\rho_{\text{max}}$) and yield strength of shear reinforcement ($f_y$) have been used to calculate the shear strength specified in the design codes. The figure shows that the limitations of $\rho_{\text{max}}$ and $f_y$ make the analytical results of the design codes for the shear strength of 244 beams to be conservative. When $f_y$ is greater than about 600MPa, the calculated results of the design code are very conservative.

![Fig. 5. Comparisons of the observed and calculated shear strengths – limitation of $f_y$ and $\rho_{\text{max}}$.](image)

**CONCLUSION**

A total of 244 concrete beam tests incorporating high strength shear reinforcement were analysed and the applicability of the reinforcement was assessed. The conclusions to this study are summarized as follows.

1) Shear strength redundancy (measured/calculated) is linearly reduced as the yield strength of shear reinforcement is increased and the yielding of shear reinforcement is closely associated with the compressive strength of concrete. The maximum yield strength of shear reinforcement may be
600MPa. There is a strong possibility that yielding of shear reinforcement may not occur for specimens with a yield strength of shear reinforcement of more than 600MPa.

2) The accuracy of the design codes to calculate shear strength was increased by limiting the maximum amount of shear reinforcement, while it was decreased by limiting both $\rho_{\text{max}}$ and $f_{ry}$. For specimens with greater than 600MPa of yield strength of shear reinforcement, the calculated results by the design code having the limitations of $\rho_{\text{max}}$ and $f_{ry}$ underestimated the observed shear strength.

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