

## **ABSTRACT**

ALNIZAMI, REEMA. Math Talk and Representations in Elementary Classrooms: A MLM Exploratory Correlational Analysis. (Under the direction of Temple Walkowiak).

This study examined the math talk and the use of multiple representations in elementary classrooms of 134 beginning teachers, all in their second year of teaching. A quantitative correlational research design was employed to investigate the research questions. The data were collected using a log instrument, the Instructional Practices Log in Mathematics (IPL-M), on which the teacher participants logged approximately 45 mathematics lessons over one academic year. Due to the nested nature of the collected data, a Multilevel Linear Modeling (MLM) methodology was used. Level one is mathematics lessons nested within teachers (level two). Three investigations were conducted based on the collected data. In the first study, the relationship between math talk and the use of representations was investigated. The purpose of study II was to investigate if mathematical knowledge for teaching (MKT) scores of beginning elementary teachers predict opportunities for math talk and use of multiple representations that take place during lessons. In the last study, the effect of the mathematical strand taught on the reported math talk and use of representations was explored. On average, mathematics lessons in grades 3-5 were found to have more opportunities for math talk than lessons in grades K-2. Although teacher use of multiple representations were positively correlated with opportunities for math talk, the correlation between student use of representations and math talk was reduced by accounting for teacher use of representations. Furthermore, the findings indicate content differences in opportunities for math talk and use of multiple representations for a subset of the investigated mathematical content strands. Contrary to what was hypothesized about MKT effect on the two constructs of interest, teachers' MKT scores in number and operations were not

significantly correlated with opportunities for math talk, nor with the use of multiple representations, during lessons with a primary focus on number and operations.

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Math Talk and Representations in Elementary Classrooms of Beginning Teachers: A MLM  
Exploratory Analysis

by  
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## **DEDICATION**

This dissertation is especially dedicated to my beloved children, Rami, Yazan, and Rawan, who inspire me, motivate me, and brighten my days. I also dedicate this effort to my entire family and friends.

## **BIOGRAPHY**

Reema Alnizami will complete the requirements of a Ph.D. in Mathematics Education at North Carolina State University in summer 2017. She earned a Master of Science in Mathematics Education from NCSU in December 2012 and a Bachelor of Science in Mathematics Education (minor Mathematics) from NCSU in 2010. She also earned a Bachelor's degree in Business Administration from Jordan University in 1996.

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## CHAPTER 1

### Introduction

The literature on teaching has highlighted the complexity of learning to teach for novice teachers (e.g., Bailey & Taylor, 2015). Recent educational reform efforts suggest that decomposing the practice of teaching into fine grain-sized practices can help teacher educators in supporting novice teachers' learning of the practice of teaching (e.g., Anthony et al., 2015; Boerst, Sleep, Ball, & Bass, 2011; Grossman et al., 2009). Such focus on finer grain-sized practices can help novice teachers achieve “ambitious teaching” (Forzani, 2014), which can be observed in classrooms where the goal is for all students to “develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning to solve authentic problems” (Bailey & Taylor, 2015, p. 112). To achieve the ambitious instruction called for by the National Council of Teachers of Mathematics' (2000) *Principles and Standards for School Mathematics*, students should be presented with learning opportunities in which they participate in mathematical argumentations, use multiple representations, and make connections between representations (Jackson & Cobb, 2010). Both NCTM's *Principles to Actions* (2014) and the Standards for Mathematical Practice in the CCSS-M (2010) outline the importance of discourse and the use of representations.

### Significance

**Discourse.** The National Council of Teachers of Mathematics (2014) recommend promoting “meaningful mathematical discourse” (p. 10) among students in K-12 classrooms. Furthermore, the establishment of reforms in mathematics education have pushed for launching

rigorous learning environments for all students, not only for a select group of students (Ball & Forzani, 2010). Teacher orchestration of strategies that foster student engagement in rich mathematical discourse can have significant implications for students' mathematical learning (Ball, 1991; Steinbring, Bartolini-Bussi, & Sierpiska, 1998). Furthermore, equity and discourse in mathematics classrooms are fundamentally related (Herbel-Eisenmann, Choppin, Wagner, & Pimm, 2012), as equity requires providing all students with comparable opportunities to learn (Esmonde, 2009). Such opportunities should essentially include students' access to participation in mathematical discourse during lessons (Ardasheva, Howell, & Vidrio Magna, 2016; Moschovich, 2011). Although the need for rich mathematical discourse in classrooms is established as explained above, many teachers find it difficult to orchestrate such discourse in their classrooms (Herbel-Eisenmann, Steele, & Cirillo, 2013; Kazemi & Stipek, 2001; Nathan & Knuth, 2003; O'Connor, 2001; Peterson & Leatham, 2009; Staples, 2007), especially for beginning teachers (Bennett, 2010).

**Representations.** Using representations is an essential aspect of doing mathematics (Dreher & Kuntze, 2015). Research findings highlight the importance of using multiple representations as it relates to students' mathematical understanding (Hiebert, 1984; Lesh, Post, & Behr, 1987). Furthermore, earlier studies illustrate that using one mode of representation of a mathematical object or idea is not enough for helping students develop coherent understanding; meaningful use of multiple representations have agreed upon benefits for students' mathematical learning (Duval, 2006).

Given the value of opportunities for mathematical discourse, particularly talk, and use of multiple representations for students' mathematical learning as highlighted by policy documents and research, the present investigations examining the two dimensions have the potential to inform researchers and teacher educators as they work to advance novice elementary teachers' learning. The instrument used for data collection in the present study attends to teaching practices in these two dimensions in mathematics classrooms as occurring on a daily basis.

**Using log data and MLM design to understand opportunities for student learning.**

Although extensive research has been conducted on mathematical discourse (Herbel-Eisenmann, 2011) and on the use of representations (Dreher & Kuntze, 2015), much of the research has been qualitative in nature (e.g., Ardasheva et al., 2016; Boerst et al., 2011; Cengiz, Kline, & Grant, 2011; Herbel-Eisenmann, Wagner, & Cortes, 2010). Using an instructional log allows for collecting detailed, at-scale data with minimized costs as opposed to other commonly used observational methods; However, according to Kurz et al. (2014) and Walkowiak et al. (under review) instructional logs are not frequently used for measuring instructional practices, which results in lack of at-scale studies of instructional practices. This scarcity is partly “because few measures exist that *directly* measure instructional practice on a large-scale” (Matsumura et al., 2006, p. 2). The need for such instrumentation motivated the use of the instructional log discussed in this study. Studies like the present can contribute to the field by analyzing aspects of instructional practices and students' opportunities to learn obtained from a large sample.

The quantitative data collected using an instructional log instrument is analyzed using multilevel linear modeling design (MLM). MLM is a powerful modeling technique due to its

flexibility in representing in data across multiple levels (Schulenberg & Maggs, 2001). In the present studies, MLM allows for an understanding of variability (and its extent) at both the within-teacher level and between-teacher level. Analyzing between-teacher differences provides insight into characteristics of teachers who report implementing certain practices in their classrooms. On the other hand, understanding within-teacher variability provides understanding on the level of the lesson, i.e., explaining variability in the relationship between the use of representations and math talk during mathematical lessons.

### **Definition of Terms**

Selected key terms used in this dissertation are defined for the goal of establishing consistency and clarity.

**Beginning (novice) teachers.** Beginning teachers have been defined differently in the literature (Schmidt, Klusmann, Lüdtke, Möller, & Kunter, 2017). The range of teaching experience of beginning teachers was defined to be up to two years (e.g., Veenman, 1984), but recently Lavigne (2014) considered teachers in their fifth year of teaching to be beginning teachers. In the present study, *beginning teacher* is used to refer to the participants who were in their second year of teaching.

**Rich (productive) mathematical discourse.** “Interactive and sustained discourse of dialogic nature between teachers and students aligned to content of the lesson that addresses specific student learning issues” (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008, p. 378). Mathematical discourse includes communicating mathematical ideas using talk, writing, and listening.

**Grand mean of a variable.** The mean of the entire level-1 scores on the variable of interest.

**Math talk.** Verbal discourse between a teacher and students and among students with primary focus on mathematical concepts and ideas during whole- or small-group work.

**Mathematical knowledge for teaching (MKT).** Knowledge that “includes both the mathematical knowledge that is common to individuals working in diverse professions and the mathematical knowledge that is specialized to teaching” (Hill et al., 2008, p. 430).

**Opportunities to learn (OTL).** Opportunities created for students to learn during a lesson, which are shaped by a teacher’s implementation of instruction (Walkowiak, Pinter, & Berry, 2017).

**Use of representations.** The use of visual illustrations of mathematical concepts, ideas, or objects, such as symbols, story problems, pictures, diagrams, and tangible materials.

## **Background**

The data analyzed in this study was collected through a larger project, Project ATOMS, a longitudinal study focused on the evaluation of a STEM-focused elementary teacher preparation program. One component of the data collection on instructional practices was teacher-reported data on a daily log, the Instructional Practices Log in Mathematics (IPL-M) (Walkowiak & Lee, 2013a), developed and validated by the Project ATOMS team (Walkowiak, Adams, Porter, Lee, & McEachin, under review). In the present studies, data collected on the use of multiple representations and mathematical discourse in 134 second-year teachers’ mathematics classrooms, as reported by the teachers, was analyzed.

## **Organization of Dissertation**

Chapter 2 of this dissertation brings forth a brief literature review of preceding studies on the topic of discourse and using representations in mathematics classrooms. In chapter 3, the methodology is described in general, including description of participants, data collection, measures, and limitations of the study. Chapters 4, 5, and 6 consist of three quantitative studies that will be submitted for publication in three different academic journals. The purpose of the first study was to explore the correlation between the use of multiple representations and opportunities for math talk. Study II aimed to investigate how teachers' mathematical knowledge for teaching (MKT) in number and operations relates to the use of multiple representations and math talk. The third and last study explored content strand differences in opportunities for math talk and use of representation, all in elementary mathematics classrooms of beginning teachers. Finally, chapter 7 concludes with summary and implications of the combined findings from the three studies.

## CHAPTER 2

### Literature Review

#### Theoretical Assumptions

The theoretical foundation of this study stems from Wertsch's (1991) Vygotskian framework, which indicates that social interaction in classrooms can influence children's cognitive processes. Vygotsky's psychological perspective on individuals' construction of knowledge (Vygotsky, 1978) motivated this study's investigation of math talk, which is one form of mathematical discourse, as the dependent variable of interest in the investigation. Students' learning happens within social interactions that cultivate their construction of meanings (Vygotsky, 1978). Mathematical meanings are "co-constructed" (Ryve, 2011); that is, students understand the meanings of the mathematical language introduced in the classroom within the context of the conversations that are present during instructional settings. This phenomenon implies that teachers' discourse strategies can play an important role in students' learning of mathematical concepts. Informed by the sociocultural perspective (Rogoff, 1997; Vygotsky, 1978), this study attempts to obtain insight into opportunities created for student learning by investigating math talk that takes place in elementary classrooms as related to the use of representations.

Researchers conceptualized the concept of students' opportunities to learn (OTL) mathematics in varying manners. Some researchers illustrated OTL as time spent on certain instructional practices during lessons (e.g., Carroll, 1963). Others considered teacher's MKT, time allocations during lessons, nature of tasks used during lessons, and math talk that takes

place during instruction as indicators of students' OTL (Walkowiak et al., 2017). The present study assumes that student learning can be influenced by exposure and experiences with enacted practices of their teachers during daily lessons, which is conceptualized as students' opportunities to learn (OTL) (Carroll 1963; Cooley & Leinhardt, 1980; Porter, Kirst, Osthoff, Smithson, & Schneider, 1993).

From this perspective, instruction is conceptualized as a series of repeated (i.e., daily) exposures to instruction, and the key measurement problem is to obtain an estimate of the overall amount or rate of exposure to particular elements of instruction occurring over some fixed interval of time (e.g., a school year) (Rowan & Correnti, 2009, p. 120).

The use of IPL-M is informed by the concept of students' OTL. The teachers' responses on the items were numerically scored based on literature in mathematics education on best practices to reflect the creation of OTL. Frequency and quality of occurrence of a particular practice of interest during a mathematics lesson are considered indicators of OTL. Theoretically, a high score represents creating more opportunities to learn and a low score represents less opportunities to learn (Walkowiak et al., under review). However, teacher implementation of certain practices, which are reported in the log, do not automatically imply that student learning will be achieved. IPL-M provides reports on frequency of occurrence of certain instructional aspects (Walkowiak & Lee, 2013a). The log developers recognize that, although high frequency of best practices indicates more opportunities for student learning, frequency does not necessarily equate to quality of instruction (Walkowiak, et al., 2017). Nevertheless, one way to conceptualize OTL is that "certain instructional variables can *impact* the quality of instruction as

evidenced by their relation to student achievement” (Kurz et al., 2014, p. 163). A rationale for this conceptualization of OTL is that particular practices have been found by earlier research to have a positive influence on student achievement, thus such practices that are empirically supported can be thought of as potentially creating opportunities for student learning (Kurz et al., 2014). Motivated by Walkowiak et al’s (2017) conceptualization of OTL as including the dimension of math talk, the present study considers that facilitating opportunities for students to participate in math talk can influence students’ mathematical understanding. The IPL-M includes items that are indicators of students’ OTL based on earlier studies. The items of interest are those measuring the use of representations and others that are indicators of students’ opportunities for participating in math talk. Since literature indicates that students’ participation in math talk can enhance students’ learning (e.g., Jackson & Cobb, 2010), the present study assumes that the IPL-M score for students’ opportunities for math talk is an indicator of students’ opportunities for learning.

### **Beginning Teachers and Practice**

The practice of teaching is complex, particularly for beginning teachers, who face challenges trying to succeed at implementing instructional practices, such as orchestrating mathematical discourse and using representations. They get overwhelmed, and even stressed, with the wide range of demands required of them as they transition from pre-service status to in-service status (Schmidt, Klusmann, Lüdtke, Möller, & Kunter, 2017). Unlike other professions, beginning teachers are usually given more assigned duties than they can handle, and at the same time, are often not provided with adequate support. Add to that their feeling of being on their

own within the walls of their isolated classrooms (Darling-Hammond, 1996). Overall, “beginning teachers have different struggles than their more experienced colleagues” (Neergard & Smith, 2012, p. 4); therefore, evaluation measures of teachers’ instructional practices might be more appropriate if used in differentiation based on teacher experience level (Neergard & Smith, 2012). Neergard and colleagues expressed their concern about evaluating teachers using a uniform measure and score without taking into consideration teacher experience levels and content taught. The present study takes that concern into consideration by using the measure, IPL-M, with a sample of teachers all in their second year of teaching.

The existing literature on beginning elementary teachers is extensive and focuses primarily on challenges that beginning elementary teachers face (e.g., Bailey, 2015; Singer-Gabella, Stengel, Shahan, & Kim, 2016) and on efficacy issues relating to novice elementary teachers (e.g., Brady, 2012; Hart, 2004; Santagata, Yeh, 2016; Jong, 2016). While most of those earlier studies focused on general issues relating to beginning teachers’ practice, what we know about beginning elementary mathematics teachers’ specific practices is largely based upon small-scale observational studies (e.g., Griffin, League, Griffin, & Bae, 2013). The present study extends our knowledge on beginning mathematics elementary teachers’ practices by focusing on two instructional domains deemed important by mathematics education research: math talk and use of representations.

### **Mathematical Discourse**

“Scholars have conceptualized mathematics as a discourse” (Ryve, 2011). The National Council of Teachers of Mathematics (2014) recommended the teaching practice of fostering

meaningful discussions mathematics classrooms. Furthermore, the reforms in mathematics education have pushed for launching rigorous learning environments for all students, not only for a selected group of students (Ball & Forzani, 2010). That ambition for inclusion was aimed at promoting classroom settings in which both teachers and students are active members of the mathematical dialogue. In order for that type of instruction to happen, the teacher should not be the only speaker in the classroom. Students need to develop mathematical knowledge by actively participating in higher-order classroom discourse (Horowitz, Darling-Hammond, & Bransford, 2005; Turner et al., 1998). Discourse strategies that teachers orchestrate play an important role in influencing student engagement in rich mathematical discourse, which can have significant implications for students' mathematical learning (Ball, 1991; Steinbring et al., 1998). Furthermore, teacher orchestration of rich discourse can positively affect students' motivation to learn mathematics (Kiemer, Gröschner, Pehmer, & Seidel, 2015).

Although the need for rich mathematical discourse in classrooms is established, many teachers find it difficult to stage such discourse during instruction (Herbel-Eisenmann et al., 2013; Kazemi & Stipek, 2001; Michaels & O'Conner, 2015; Peterson & Leatham, 2009; Staples, 2007), mainly because establishing it involves considerable time and determination (Li & Ni, 2011, p. 83). Discourse is continually receiving attention in mathematics education research and practice; however, researchers in the field do not uniformly define it (Ryve, 2011). According to Ball (1991), "discourse is used to highlight the ways in which knowledge is constructed and exchanged in classrooms."

**Multi-directional discourse.** Discourse is addressed by research in varied manners (Ryve, 2011), with some overarching ideas. One recurring idea in literature is the distinction between the two types of discourse, “univocal” and “dialogic” (Wertsch & Toma , 1995). Univocal refers to a discourse that is presented by the speaker (teacher) to the listener (student) with the function of conveying the exact meaning to the listener as intended by the speaker. On the other hand, dialogic discourse requires multidirectional communication between the teacher and students, among students, or a mix of both in order for the intended meaning to be delivered (e.g., Peressini & Knuth, 1998; Ryve, 2011; Tofel-Grehl, Callahan, & Nadelson, 2017); Truxaw, Gorgievski, & DeFranco, 2008). Teachers’ listening to students’ mathematical ideas is also valuable for obtaining insight into students’ reasoning (Rowland, Huckstep, & Thwaites, 2005). A productive mathematical discourse during class should encourage students to be active participants in their own learning, which can empower them to think of themselves as doers of mathematics (Herbel-Eisenmann et al., 2013). In such discursive classroom settings, the discussions are multidirectional; students reflect on and question each other’s ideas, and the teacher and students exchange questions and reflect on each others’ ideas. However, the instruction types that are usually found in mathematics classrooms in the US are mostly channeled in one direction from teacher to students, which promote students’ feelings of being less interested in mathematics (Herbel-Eisenmann, et al., 2013).

Brendefur and Frykholm (2000) introduced four hierarchical categories of communications in mathematics classrooms, “uni-directional, contributive, reflective, and instructive communication” (p. 148). *Uni-directional* communication is predominantly led by the

teacher, which indicates the least opportunities for students to communicate their ideas and reasoning. *Contributive* communication happens when students contribute to classroom discourse, however in a limited manner that lacks deep mathematical reasoning and argumentation. Influenced by Cobb and colleagues' (1997) and von Glasersfeld's (1991) conceptions of reflective discourse, Brendefur and Frykhom (2000) explain that *reflective* communication refers to mathematical discourse in which "students reflect on the relationships within the mathematical topics by focusing on other students' and the teacher's ideas, insights, and strategies" (p. 148). (e.g., students discussing the validity of other students' ideas). Finally, *instructive* communication was informed by Steffe and D'Ambrosio's (1995) description of instructional settings during which students and teacher are engaged in multidirectional and repeated discussions that cyclically influence subsequent instructional decisions of the teacher, who would be paying close attention to students' mathematical thinking and any misunderstandings.

Teachers need to know what students understand so that true assessment of students' learning can be achieved; therefore, the need for students to communicate their ideas is inevitable (Sfard, Neshet, Streefand, Cobb, & Mason, 1998). To achieve a rich mathematical communication setting, communications between teacher and students and among students should not only be multidirectional, but should also encourage mathematical thinking (Brendefur and Frykholm, 2000) with a main focus of the discussion on "the mathematics of the task" (Ball, Sleep, Boerst, & Bass, 2009, p. 469). By simply correcting students when producing incorrect answers, or agreeing with students' correct answers—which is called an Initiate-Response-

Evaluate (I-R-E; Mehan, 1979) discourse pattern—the teacher misses on creating rich learning opportunities. On the other hand, requesting justifications from students about their responses and ideas can give teachers insights into the students’ mathematical thinking and foster students’ knowledge about the nature of mathematical justification (Ball, 1991). Ball illustrated an example of different possible classroom discourses when a student conducts a mathematical investigation and arrives at an inaccurate conclusion. A teacher has different options on how to respond to the student. Some of those options include, telling the student that the conclusion is incorrect and providing a correct answer; asking other students to reflect on the student’s initial conclusion and opening a whole-group discussion, in which students compare among their results; or suggesting to the student a different approach for investigating the mathematical problem. Each of these discourse options has different consequences on the way a student grasps mathematical justifications and mathematical knowledge.

**Student argumentation.** The importance of students’ mathematical justifications was emphasized in the Common Core State Standards Initiative (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). “Mathematically proficient students... justify their conclusions, communicate them to others, and respond to the arguments of others” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Mathematical argumentation is a basic practice in mathematics, which needs to be included in lessons to help students’ conceptual understanding (Staples & Newton, 2016). Brendefur and Frykholm (2000) recommended that teachers establish the sociomathematical norms of conjecturing, justifying, and generalizing during mathematical

discussions. Sociomathematical norms are “normative aspects of mathematical discussions that are specific to students’ mathematical activity” (Cobb & Yackel, p. 458) within mathematics classroom settings. Four sociomathematical norms were found by Kazemi and Stipek (2001) to be of importance for fostering students’ conceptual thinking:

- When providing a mathematical explanation, explaining the procedure is not sufficient, instead the explanation should include a mathematical argument;
- “Mathematical thinking involves understanding relations among multiple strategies”;
- “Errors provide opportunities to reconceptualize a problem, explore contradictions, and pursue alternative strategies”; and
- “Collaborative work involves individual accountability and reaching consensus through mathematical argumentation” (p. 78).

Enhancing students’ higher-level thinking within rich classroom discourse can be achieved by implementing what Staples and Truxaw (2010) called “the three pillars of mathematics learning discourse” (p. 29). Those pillars consist of students’ advancement in their effective use of academic language, students’ posing of mathematical justification and argumentation during mathematical discussions, and teachers providing opportunities for all students to be exposed to rigorous mathematics during mathematical lessons.

Yackel (2001) argued based on her involvement in teaching experiments with elementary students that providing opportunities for students to listen to each other’s ideas and get involved in mathematical debates and disagreements with each other can help students’ sense-making and conceptualization. Influenced by Yackel’s work with respect to the value of disagreement,

Barlow and McCrory (2011) investigated ways for creating disagreement opportunities for young students. Based on their work with third-grade students, they suggested three strategies that could create opportunities for discussing mathematical disagreement among students. Those strategies are: (1) pushing students to choose sides when different answers come up, (2) choosing mathematical tasks that highlight and bring out students' misconceptions, and (3) choosing tasks that facilitate arriving at disagreements with earlier taught concepts. Addressing students' misconceptions and errors was also emphasized by Staples and Colonis (2007) by stating that a teacher needs to attend to students' incorrect mathematical ideas as well as their correct ones, otherwise "the mathematics that gets explored is limited, and students whose original ideas were incorrect may hold on to incorrect mathematics" (p. 259).

**Teacher discourse-promoting practices.** A classroom setting in which teachers orchestrate discourse with and among all students, while attending to children's mathematical thinking can lead to what Lampert, Beasley, Ghouseini, Kazemi, and Franke (2010) referred to as "ambitious mathematics teaching." Teachers who implement ambitious instruction do not possess dominant authority in classroom discourse; rather, they provide chances for students to actively participate in creating ideas (Lampert et al., 2010). That ambitious view of mathematics instruction indicates that all students are active participants in mathematical discussions during lessons (Jackson & Cobb, 2010), which was also proposed by the Standards (2000). Jackson and Cobb (2010) proposed a set of social and sociomathematical norms that teachers can initiate in their classrooms to enhance the establishment of ambitious mathematics instruction. The norms include that students explain and justify their mathematical work, provide reasoning for their

choice of solution strategies, and communicate any connections between different solution strategies. Lampert and Graziani (2009) asserted the complexity of orchestrating such ambitious instruction by teachers. However, they acknowledged the contribution of Stein, Engle, Smith, and Hughes (2008) in putting together activity structures “that regularize aspects of instruction in relation to ambitious principles of mathematics content and learning” (Lampert & Graziani, 2009, p. 506). Stein et al. (2008) suggested five teacher practices that relate to students’ responses and foster rich mathematical discourse.

Those practices are: (1) anticipating likely student responses to cognitively demanding mathematical tasks, (2) monitoring students’ responses to the tasks during the explore phase, (3) selecting particular students to present their mathematical responses during the discuss-and-summarize phase, (4) purposefully sequencing the student responses that will be displayed, and (5) helping the class make mathematical connections between different students’ responses and between students’ responses and the key ideas (p. 321).

Stein et al.’s pedagogical practices added to the body of knowledge on teacher knowledge with respect to classroom discourse by influencing studies on mathematics teacher education (e.g., Speer & Wagner, 2009), and studies that focused particularly on teacher’s mathematical knowledge for teaching (e.g., Sztajn, Confrey, Wilson, & Edgington, 2012). Sztajn et al. (2012) asserted that Stein et al.’s five practices “strengthened teacher preparedness for building on student’s voice while attending to mathematical goals” (p. 151). To better understand how to help teachers orchestrate mathematical discourse, researchers recommended investigating the effect of teacher mathematical and pedagogical knowledge on orchestrating mathematical

discourse (e.g., Brendefur & Frykholm, 2000).

Michaels and O'Connor (2015) suggested a set of talk moves to assist teachers in orchestrating rich mathematical discourse, which are the product of about twenty years of research on classroom discourse. They recommended purposeful use of the talk moves to enable students to share their thinking, to listen to each other, to deepen their mathematical reasoning, and to draw on each others' ideas. For example, the *say more moves* (e.g., can you say more?) refers to teachers facilitation of students' elaboration on their ambiguous responses. The "press for reasoning moves (*Why do you think that? What's your evidence? What led you to that conclusion?*)" (p. 334) can help teachers obtain insight into students' reasoning. Furthermore, revoicing students' ideas by the teacher is emphasized by Chapin et al. (2009) as a talk move that can promote rich mathematical discourse among students. They recommend re-stating students' responses by the teacher using appropriate mathematical language. Teacher *wait time* is one particular move that was "the most researched of all talk moves," (Michaels & O'Conner, 2015, p. 337) but was found to be the most difficult for teachers to practice. The value of exercising wait time by the teacher was earlier addressed by Tobin (1986); He concluded that a wait time of three to five seconds by the teacher when waiting for student responses can enhance mathematical discussions, which can positively affect student achievement.

**The role of questions in classroom discourse.** High-quality discourse in which both the teacher and students participate in the creation of mathematical ideas are considered effective in promoting students' higher-level thinking (Lampert, 2001; O'Connor, 2001; Scherrer & Stein, 2013; Staples, 2007). Such high-quality discourse can be obtained by teacher's use of

questioning practices in ways that encourage students to think conceptually and by encouraging students to ask questions to each other as well as to the teacher (Boaler & Brodie, 2004; Nystrand, Wu, Gamoran, Zeiser, & Long, 2003). The National Council of Teachers of Mathematics (2014) recommended that teachers ask students purposeful questions that aim at assessing students' understanding and improving their mathematical reasoning. "Questions from teachers and their reactions to student responses have a tendency to shape the way in which classroom discourse takes place" (Li & Ni, 2011, p. 73). Generally, questioning techniques in mathematics classrooms have received particular attention in research on mathematical discourse (e.g., Gillies, 2011; Herbel-Eisenmann, 2010; Hufferd-Ackles, Fuson, & Sherin, 2004; Males, Otten, Herbel-Eisenmann, 2010; Nathan & Knuth, 2003).

Gillies (2011) pointed out two questioning models that have potential positive effects on students' thinking and problem solving skills. One of the models is "Ask to Think Tell-Why model for transactive peer tutoring" (p. 74), in which teachers establish a norm for their students to address one another with series of questions in a sequential manner for the purpose of pushing students to make connections and develop deeper reasoning. In this model, the types of questions asked should be for the goal of pushing for higher-level mathematical thinking. The other model is "the Cognitive Tools and Intellectual Roles" (p. 75). In the latter, the teacher encourages students to use a set of tools when working on mathematical investigations. The tools consist of "predicting, questioning, summarizing and clarifying" (p. 75), which can be practiced by students working in groups to conduct mathematical investigations. Ideally, students also use these cognitive tools to provide thoughtful feedback to other groups' presentations of their

mathematical investigations. Teacher questioning techniques can help establish norms with respect to student participation and authority ownership during mathematical discourse. Questions that teachers ask in reaction to students' responses have implications for instituting who owns the authority in a classroom during discourse (Li & Ni, 2011). Furthermore, teacher's frequent questions for students to think differently about mathematical problems can help establish the norm that discussing multiple solutions or multiple methods are encouraged (Baxter & Williams, 2009).

Overall, teacher questioning should be geared towards probing students' mathematical thinking and reasoning, rather than leading them to a particular answer or method (Boaler & Brodie, 2004). The value of questioning mathematical ideas when doing mathematics was expressed as early as 1954 by Polya in his well-known book, *Induction and Analogy in Mathematics*. He suggested that when doing mathematics, one should practice the "inductive attitude." This attitude pertains to questioning mathematical ideas and being ready to refute or revise one's own idea when solid evidence supports that. Therefore, establishing an inductive attitude as a norm during mathematical discourse among students can help students become doers of mathematics. The influential study discussed by Lampert (1990) was motivated by Polya's (1954) conception with respect to questioning mathematical ideas. Lampert (1990) conducted action research in which she designed lessons and implemented them with a group of elementary students. The mathematical problems in her lessons were heavily dependent on students answering questions that expected them to justify and prove their thinking and their solution methods. The students also questioned each other's solutions and arguments. The

resulting social interactions in her classroom indicated that her students' views differed from other fifth grade students in that "they put themselves in the position of authors of ideas and arguments" (p. 34)—versus authority of the textbook or teacher. Lampert's work received great attention and was influential for mathematics educational research particularly on the topic of discourse (e. g., Cobb, Boufi, McClain, & Whitenack, 1997).

**Math talk as an important aspect of mathematical discourse.** Ryve (2011) analyzed 108 articles, which focused on discourse in mathematics education, and found that, much of the analyzed studies focused on the talk aspect of mathematical discourse, which was evident in that "a large majority of articles use talk as data when analyzing discourse" (p. 179).

Correspondingly, in the present study math talk is the main discursive aspect of interest. In this section, examples on how talk was investigated by a few previous studies are presented. Math talk has been investigated in varied manners among researchers. To investigate student talk, Hiebert and Wearne (1993) compared the number of words spoken by the teacher to those spoken by the students in six classrooms. They also investigated types of teacher verbal questioning of the students. Whole-group discussions were coded for the occurrence of four types of questions: questions that asked for recall of facts and procedures, questions that asked for describing different solution strategies, questions that asked for formulating a story problem, and questions that asked for explanations. Like Hiebert and Wearne, Rowland et al. (2005) saw value in teacher verbal questioning as an instructional strategy. They investigated the types of follow-up questions that the teacher asked, or did not ask (e.g., probing or eliciting questions) of students to address students' emerging needs. Hufferd-Ackles et al. (2004) investigated "the

building of math-talk community” within elementary mathematics classrooms. Math-talk communities refer to a classroom setting in which participants (students and teachers) help each other learn through mathematical discourse. Classroom observations and teacher interviews were analyzed. Additionally, the researchers facilitated teachers’ meetings throughout, in which the teachers reflected on their teaching. The development of math talk throughout the school year was evaluated with respect to four main discursive components: (a) teacher questioning, (b) level of explanation of mathematical ideas, (c) owner of or producer of the mathematical ideas (teacher or student), and level of students taking responsibility for their learning. Hufferd-Ackles and colleagues describe trajectories of math-talk communities in the investigated classrooms with respect to a case-study of a third-grade teacher, who participated in the study.

**Beginning teachers and orchestrating discourse.** As demonstrated in this review, providing students with opportunities for rich mathematical discourse during instructional lessons have agreed upon benefits for students’ mathematical thinking. However, orchestrating such mathematical discussions as those recommended by literature on best practices is a complex task for teachers, particularly for beginning teachers (Boerst et al., 2011). Boerst and colleagues further stated that, “Teachers draw on substantial knowledge and skill to maintain collective engagement in discussions in order to avoid more simplistic and less effective ‘show-and-tell’ sessions” (p. 2848). Given that teachers need to be equipped with substantial knowledge and experience to be able to facilitate the types of mathematical discussions recommended in literature, for a beginning teacher to provide students with simply opportunities of mathtalk might be appreciated as an indication of good instructional practice. With that assumption of

reasonable expectations for beginning teachers, the present study examines second-year teachers' facilitation of mathematical discourse by investigating the teachers' daily report of their students' engagement in math talk.

### **The Use of Representation During Mathematics Lessons**

Mathematics lessons should include opportunities for using multiple representations and making connections among the representations in ways that contribute to students' deep mathematical understanding of concepts and procedures (National Council of Teachers of Mathematics, 2014). Research findings indicate that using multiple representations and students' ability to translate among different forms of representations and connect between representations and mathematical concepts are related to students' mathematical understanding (Hiebert, 1984; Lesh, Post, & Behr, 1987). Mere use of multiple representations beyond symbolic does not automatically imply students' involvement in problem solving, mathematical reasoning, or high-level thinking. The ways in which different representations are chosen and used matter as well as how connections are made among different representations of the same mathematical concept.

**Defining Representations.** The use of *representations* in mathematics educational research has wide-ranging meanings. For example, Sung, Shih, and Chang (2015) used *representations* to distinguish between different geometric images based on the number of dimensions of the image that is being illustrated (e.g., 2-D representations and 3-D representations). On the other hand, some researchers classified representations into symbolic and non-symbolic (e.g., Lourenco, Bonny, Fernandez, & Rao, 2012; Smedt, Noel, Gilmore, & Ansari, 2013). *Symbolic representations* are representations of mathematical ideas using

characters and images that are formally taught to students in schools (e.g., Arabic numbers and number lines). *Non-symbolic representations*, on the other hand, are illustrations of numbers that are informal, not formally taught in schools, and are considered more inherent to students than symbolic representations (e.g., a visual of a group of items). *Representations* is used among educational research studies in different ways, yet using it to refer to illustrations of mathematical entities seems to be universal among the different studies discussed in this review. Hardy (2001) defined the act of representing as “the translation between symbol systems (verbal, symbolic, and iconic),” (p. 5) indicating that mathematical representations include verbal, symbolic, and iconic representations. Similarly, the mathematical representations that were the focus of a study discussed by Stein and Lane (1996) consisted of symbolic, pictorial, and graphical representations. A more comprehensive view of *representations* was discussed by Lesh et al. (1987). Informed by earlier studies on mathematics education, Lesh and colleagues (1987) highlighted five types of representational systems: (a) story problems that are formulated based on real-world situations; (b) tangible materials that are used for modeling mathematical ideas; (c) static pictures and diagrams; (d) verbal language; and (e) written symbolic representations. In the present study, *representations* refers to illustrations of mathematical concepts, ideas, or objects, such as symbols, story problems, pictures, diagrams, and tangible materials.

**Using representations beyond symbolic.** The use of different representations beyond, and in addition to, symbolic representations (e.g., words, pictures, diagrams, and tangible objects) while working on mathematical tasks has potential value for students’ mathematical thinking and reasoning (Hardy, 2001). For instance, the use of tangible objects (or sometimes

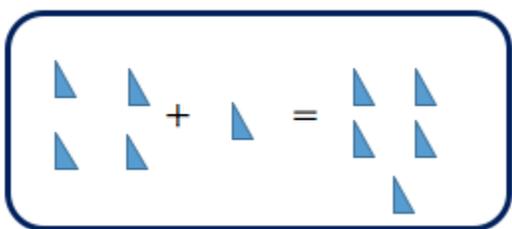
called manipulatives) was found by Baxter, Woodward, and Olson (2001) to aid engagement of students whose achievement is below grade level during mathematical investigations.

Representations, such as tangible objects, should be used as a medium for mathematical thinking; however, at times, the manipulatives themselves become the focus of the class instead of an aid for mathematical thinking (Baxter et al., 2001). Although research indicates value for using tangible objects as representations of abstract mathematics, especially for young students, using tangible objects as representations of abstract mathematics does not automatically imply students' mathematical reasoning. When using manipulatives to aid students' mathematical reasoning, little attention should be given to physical properties of the object, so that students pay more attention to how the object relates to the intended abstract mathematical ideas. On the other hand, when too much attention is given to properties of manipulatives, students seem to lose comprehension of connection between the manipulatives and the intended abstract mathematical idea (Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009).

Hardy (2001) explained that efficient use of different representations can expose structural elements of mathematical concepts, which can provide the learner with points of reference for mathematical discussions. She, however, reminded that while the use of multiple representations can potentially aid students' success in problem solving, tasks that require particular and restricted ways of using representations by students can hinder students' mathematical thinking because this can lead to a procedural solution of the task. She classified mathematical tasks that might include the use of representations into two types based on expected ways of using representations by students; the two types are "Low Structure

Representation (LSR) tasks (open task statement with emergent use of representations) and High Structure Representation (HSR) tasks (explicit use of representations in the task statement)” (Hardy, 2001, p. 4). LSR tasks are tasks that leave it open to the student to choose and/or explore representations for solving the problem. On the other hand, HSR tasks include direct instruction about the types of representations to use and the aspects of the representations to explore (see figure 1). To further clarify the difference between the two categories, following are examples on each type:

- A high structure representation (HSR) task: *Write the equation that represents the picture seen below.*



*Figure 1: Example of a pictorial representation presented to students to illustrate the equation  $4+1=5$ .*

- A low structure representation (HSR) task: *Show that adding two odd numbers makes an even sum and adding an odd number and an even number makes an odd sum.*

Tasks that involve students’ use of representations in an open-ended manner, allowing students to choose representations to use, are associated with higher cognitive processes than tasks that are highly structured by design with respect to the use of representations (Hardy, 2001).

**Making connections among multiple representations.** Previous research has established benefits of connecting among different representations on students’ mathematical

learning (e.g., Hardy, 2001; Hiebert, 1984; Lesh et al., 1987). Making connections between concrete representations (e.g., tangible objects) and symbolic representations--which can be thought of as representing abstract mathematics--is important for helping young students understand more advanced mathematical ideas as they move on to higher grades (Uttal et al., 2009). Benefits of using instructional tasks that involve the use of and connecting among different representations were confirmed by the results of a study that investigated the use of instructional tasks in four middle school classrooms (Stein & Lane, 1996). Stein and Lane defined an instructional task as “an activity engaged in by teachers and students during classroom instruction that is oriented toward the development of a particular skill, concept, or idea” (p. 54). They investigated instructional opportunities for students’ mathematical learning as they relate to students’ mathematical performance, and found that translating between different representation is one of different factors that are related to opportunities for students’ learning, thus influencing mathematical performance.

As illustrated in this review, providing students with learning opportunities of using multiple representations and translating between different representations has documented benefits for students’ mathematical reasoning. With that assumption, the IPL-M included items for which the participating teachers reported on the creation of opportunities for use of multiple representations and translating among the representations (Walkowiak & Lee, 2013a).

### **Representations and Discourse**

Cengiz et al. (2011) concluded that in order to learn about the effects of a particular instructional practice on “extending students’ mathematical thinking,” investigating that

particular teacher practice might not be sufficient. They observed that while a teacher's action proved to be efficient in extending students' thinking at one point of a whole-group discussion, it did not seem to do so at another point. This indicates that any particular instructional action does not work in isolation. Consequently, to obtain more solid results with respect to an instructional action of interest, one should try to "capture the impact of not only single instructional actions, but collections of them as well" (p. 372). Similarly, Hiebert and Wearne (1993) stated that, "It is impossible to isolate specific instructional features and connect them to specific learning outcomes" (p. 420).

Two instructional actions, which might be related to each other, are of interest in this study: the use of multiple representations within instructional tasks and facilitating opportunities for mathematical discourse in elementary classrooms. The use of different representations and orchestrating mathematical discourse are two important aspects of mathematical instruction that have been investigated in association by some researchers (e.g., Cengiz et al., 2011; Hiebert & Wearne, 1993). Hiebert and Wearne (1993) found that "instructional tasks and classroom discourse mediate the relationship between teaching and learning" (p. 420). Therefore, some researchers examined the two constructs of the *use of representations* and *mathematical discourse* together in the same study. Some of them explored the relation between the two constructs (e.g., Anderson-Pence & Moyer-Packenham, 2016; Hardy, 2001), while others explored both constructs as a subset of aspects of instructional actions that relate to students' learning (e.g., Rowland et al., 2005).

**Relation between representations and discourse.** Based on a number of studies in mathematics education, a relation exists between the use of multiple representations and mathematical discourse. In this section, a summary of ways in which some studies investigated this relationship is presented.

Cengiz et al. (2011) claimed based on previous research that aspects of teacher mathematical knowledge for teaching (MKT)—including knowledge in making or exploring alternative representations of mathematical ideas and knowing the appropriate representations to be used in teaching particular mathematical ideas to students—are related to creating opportunities for extending students' thinking during whole-group discussions. Based on that assumption among other assumptions, the researchers investigated video recordings of classroom episodes. Cengiz et al. (2011) investigated aspects of discussions during whole-group work in six elementary classrooms in an effort to understand teacher instructional activities that facilitate “extending student thinking.” *Extending student thinking* refers to creating opportunities for students to build on and develop their mathematical thinking, which was investigated within the context of students' discussions as a whole group. A teacher's MKT was found to be positively related to his or her capability to make decisions during whole-group discussion in ways that can extend students' thinking. Similarly, Rowland et al. (2005) suggested that teacher knowledge and beliefs are related to a teacher's instructional actions, including a teacher's choice of representations to be used during mathematical investigations.

Cengiz et al. (2011) concluded that a teacher's facilitation of using different, connected representations during a whole-group discussion can potentially extend students' mathematical

thinking, particularly by supporting students in producing mathematical explanations and reflections. They observed that when the students in their study were encouraged to use different solution methods and different connected representations, the resulting discourse among the whole group indicated deepening of students' thinking about mathematical structure. Hardy (2001) also emphasized how the appropriate incorporation of representations into mathematics instruction as communicational mediums can “enable students to gain new insights into mathematical structure” (p. 1). The study reported by Hardy (2001) was informed by the socio-constructivist theory, which implies that rich discourse is associated with opportunities for students to construct their own mathematical understanding.

Hardy (2001) investigated the hypothesis that the use of instructional tasks that are low structured with respect to the use of representations (LSR) can potentially produce richer discourse among students than highly structured representation tasks (HSR). Videos of classroom observations of 60 middle grade classrooms from three different countries (Germany, Japan, and USA) were examined for the types of mathematical discourse during resulting from working on the two types of mathematical tasks (i.e., HSR and LSR tasks). In addition to classifying mathematical tasks into HSR and LSR, the study design also had assumptions about what encompasses as high-quality discourse. For instance, a discussion about mathematical meanings between the teacher and students was an indication of high-quality discourse; whereas, a discussion focused on the use of procedures and memorized facts indicated low-quality mathematical discourse. Another example of what was considered an indication of high-quality discourse was the occurrence of incidents of mathematical argumentation.

Hardy (2001) found that most of the talk associated with HSR included less student talk, “shorter sequences,” and less argumentation occurrences than talk that was associated with the implementation of LSR tasks. The distinction between short and long sequences was determined according to the number of meaningfully-connected utterances that were associated with a particular task. In other words, more connected, meaningful utterances imply longer sequence of talk, and less connected utterances imply shorter sequences. Hardy described that longer sequences of math talk—in which patterns of discussion interaction is multidirectional and includes more than just teacher-student-teacher talk turns—give opportunities for students to build deeper mathematical thinking based on the discussed ideas. It is worth noting that the talk that was investigated in this study was limited to talk that occurred during whole-group settings; for example, no small-group discussions were analyzed. Furthermore, the results indicated that there were more instances of using HSR than using LSR tasks in the observed mathematics classrooms.

A recent study by Anderson-Pence and Moyer-Packenham (2016) also investigated the relationship between discourse and representations, but in the context of a computer environment and small-group setting, unlike Hardy’s (2001) study. The subjects in the study by Anderson-Pence and Moyer-Packenham (2016) were three fifth-grade students, who worked in pairs and were observed during nine instructional sessions. The students worked on tasks that were associated with the use of virtual manipulatives (VMs). Virtual manipulatives are digitalized mathematical representations such as graphs, tables, and picture. The quality of discourse in relation to the manipulatives used was assessed on the extent to which the discourse indicated

evidence of generalization of mathematical concepts, justification of students' ideas, and cooperation among the students. The three types were: (1) Linked VMs, which are multiple representations that vary together, allowing the user to observe simultaneous change in symbolic representation that is linked to other forms of representations; (2) Pictorial VMs, which are pictures and diagrams that vary based on the user's manipulation, but are not linked to symbolic representations; and (3) Tutorial VMs, referring to representations that are static highly structured in the feedback provided for the user. While linked and pictorial VMs allow students to choose solution methods, tutorial VMs restrict the choice of solution methods to ones that are proposed by the program. The discourse quality that took place among students while working with linked VMs was found to be significantly higher than discourse connected to the use of pictorial and tutorial VMs. Furthermore, the researchers found that the quality of students' justification, generalization, and collaboration increased throughout a session in which they were using linked VMs. Based on these results, Anderson-Pence and Moyer-Packenham (2016) concluded that students' mathematical investigation of representations that are highly connected, as in the case of linked VMs, "enabled the students to effectively generalize and justify mathematics concepts during problem solving tasks" (p. 23).

The results of the studies summarized in this section indicate that: a teacher's facilitation of using different, connected representations during whole-group discussions can potentially extend students' mathematical thinking (Cengiz et al., 2011); tasks that allow students to choose representations are associated with higher cognitive processes than tasks that are highly structured with respect to the use of representations (Hardy, 2001); and using tools that foster

instantaneous translation among different representations can promote students' generalization and justification of mathematical concepts during small-group investigations (Anderson-Pence & Moyer-Packenham, 2016).

**Representations and discourse as aspects of instruction.** While the main focus of the studies discussed in the previous section was investigating the relationship between mathematical representations and discourse, other studies investigated both constructs as aspects of mathematics instruction that relate to students' learning, but without explicit investigation of the relation between the two constructs (e.g., Hiebert & Wearne, 1993; Kazemi & Stipek, 2001; Stein & Lane, 1996). For example, Stein and Lane (1996) found that providing students with opportunities for making connections among multiple representations and requiring explanations and justifications are, among other task features, associated with higher gains in student learning, as compared to tasks that did not have such features. The value of the use of tasks that require students' justifications was also emphasized by Ball (1991); Ball demonstrated that requesting justifications from students about their responses and ideas can give the teacher insight into the students' mathematical thinking and foster students' knowledge about the nature of mathematical justification (Ball, 1991).

Hiebert and Wearne (1993) explored the frequency of use of certain types of representations with mathematical tasks that were used during whole-group discussion. They categorized the use of representations into use of stories, physical materials, pictures, and symbols. Their main research question was: "How do instructional tasks and classroom discourse influence learning?" (p. 420). To that end, they found indications that spending more time on

each mathematical task, making connections between different representations, and teacher questioning that require students' explanation of the mathematical ideas are related to student mathematical learning.

### **Using Instructional or Teacher Logs**

Kurz, Elliott, Kettler, and Yel, (2014) stated that, "Few, if any, OTL measurement options have been available that can be deployed at scale daily or weekly" (p. 178). One such measure of student opportunities to learn (OTL) is a teacher log. The use of instructional logs is a promising data collection technique for measuring students' opportunities to learn (OTL) (Rowan & Correnti, 2009; Walkowiak et al, under review). Using instructional logs for data collection as an instrument for reporting on teacher enactments of practices on the level of the lesson, researchers have been able to obtain specific details about the lessons (Heck et al., 2012). The use of instructional logs for data reporting on instructional practices compensates for disadvantages of both the relatively-expensive, involved method of classroom observations and the one-time survey method.

Similar to using one-time surveys, instructional logs allow for collecting large-scale data (Rowan and Correnti,2009). However, logs provide finer-grained details about the lesson as opposed to data collected using one-time, end-of-period, surveys (Heck et al., 2012; Kurz et al., 2014). Teacher-reported data collected using one-time survey questions about details of daily instructions have inherent issues, such as memory errors; whereas, instructional logs completed shortly after instruction occurs allow for collecting more accurate reports (Heck et al., 2012). Collecting data using a log allows for analysis of reported events in their natural, spontaneous

setting (Reis, 1994); it can play an important role in reducing the likelihood of retrospection effects of data collection (Bolger, Davis, & Rafaeli, 2003). Although classroom observations allow for collecting real-time and more detailed data than logs about instructional practices, they have their drawbacks and affordances. They are costly and time-consuming, thus typically feasible for only on a small scale (Rowan & Correnti, 2009). Using instructional logs allows for collecting more accurate data than one-time surveys on larger samples than typically possible using observations. It also facilitates obtaining data that allows for examining both between-teacher and within-teacher variability (Nezlek, 2001).

Since the use of log instruments is a relatively new method for data collection about classroom practices, validity and reliability of logs is a concern for researchers using them. This motivated them to follow rigorous processes in validating data collected using instructional logs (e.g., Heck et al., 2012; Kurz et al., 2014; Walkowiak, et al., under review). Validation and reliability usually include a combination of comparing a sample of teacher reported data to classroom observation results; consulting experts about instrument items; training the participating teachers on using the log to increase the fidelity of responses to the log questions as intended by log developers; and conducting statistical techniques to compare results obtained from random sample of logging days to the entire sample of collected log data (e.g., Kurz et al., 2014; Heck et al., 2012; Walkowiak et al., under review). The goal of the last method is to determine a reliable number of lessons to be administered as a representation of the teacher's instruction for the entire instructional period (e.g., academic year).

Researchers developing instructional logs recommended that logs should be easy to use and incorporate within teachers' daily practice without taking much time or effort (e.g., Kurz et al., 2014; Heck et al., 2012). An example of recommended criteria is found in Nielsen's (1994) definition of usability of products and instruments, which includes the following components: "(a) learnability (i.e., ease of logging), (b) efficiency (i.e., logging time once trained), (c) memorability (i.e., ease of reestablishing proficiency after a period of nonuse), (d) errors (i.e., frequency and severity of errors), and (e) satisfaction" (as cited in Kurtz et al., 2014, p. 173). Although the context of usability by Nielsen (1994) was not focused on instructional logs, it can be generalized to log instruments.

### **Summary**

This review set out with the aim of synthesizing classroom-based research on mathematical discourse and the use of representations during mathematics instruction, with particular attention to the relation between representations and discourse. A large and growing body of literature has investigated mathematical discourse in classrooms (Herbel-Eisenmann, 2011). This was reflected in the number of studies that returned from searching for papers to be investigated in this review. Similarly, a large amount of work was found on the topic of mathematical representations. Only a few studies were found with a main focus on the relation between the use of representations and mathematical discourse (e.g., Anderson-Pence & Moyer-Packenham, 2016; Cengiz et al., 2011; Hardy, 2001). Recent evidence suggests the two constructs are related; however, little seems to be known about any significance in the

correlation between the two constructs. Future research that investigates any correlation between discourse and the use of representations is likely of value for the field of mathematics education.

Rich mathematical discourse in the classrooms has observed benefits on students' mathematical learning; However, a simple prescription for making discourse in the mathematics classroom rich and productive for student learning is not available. Indeed, "one cannot prescribe practices, but one can guide practice by means of exploratory frameworks accompanied by data, evidence, and argument" (Confrey, 2006, p. 139). Research cited in this review discussed particular practices that can help teachers in the creation of opportunities for mathematical discussions in their classrooms, such as encouraging students to explain their thinking to the teacher as well as to other students (Michaels & O'Connor, 2015), restate each other's ideas (Chapin et al., 2009), and pose questions to the teacher as well as to other students (Boaler & Brodie, 2004), all within group-discussion settings. Moreover, effective use of representations in association with instructional tasks entails purposefully choosing representations for the goal of extending students' mathematical understanding (Cengiz et al., 2011), using multiple representations including mathematical symbols (Hardy, 2001), and providing opportunities for making meaningful connections among different representations (Stein & Lane, 1996). The three exploratory studies described herein investigate math talk as related to the use of representations in elementary classrooms using data reported by beginning teachers.

**Main research goals.** The main research goals of interest in the present studies are:

- 1) Is there a relationship between use of multiple representations and opportunities for math talk in elementary mathematics lessons of beginning teachers?

- 2) Are there MKT differences in opportunities for math talk and use of multiple representations in elementary mathematics lessons of beginning elementary teachers?
- 3) Are there content-strand differences in opportunities for math talk and use of multiple representations in elementary mathematics lessons of beginning elementary teachers?

## CHAPTER 3

### Methodology

#### Participants

As part of longitudinal evaluation study of a STEM-focused teacher preparation program, logging data was collected from 134 second-year teachers, who responded to 5,170 lesson logs on days of mathematics instruction. The participating teachers taught at public elementary schools in a southeastern state of the U.S. (Table 1 includes demographic data about the participating teachers). About 85% of the participating teachers are white, which is representative of the low level of racial diversity among public school teachers in the U.S. (i.e., about 82% of public school teachers were reported to be white in 2011-2012 academic year (U.S. Department of Education, 2016)). The average age of the participating teachers was approximately 23 years at the time of the study.

*Table 1. Demographic characteristics of participating teachers*

<u>Gender</u>	<u>White</u>	<u>Black</u>	<u>Hispanic</u>	<u>Multiracial</u>	<u>Other race</u>	<u>Total</u>
Female	114	9	3	2	2	130
Male	1	0	0	0	1	2

*Note: Gender and ethnicity data were not reported for 2 of the participants*

#### Data Collection

**Measurement-burst design.** Data collection in this study utilized a measurement-burst design (Sliwinski, 2008), which makes feasible collecting fine-grain details through a daily diary, but also employs benefits of longitudinal designs. In a daily diary design, day-to-day

fluctuations in teacher practices are observed and examined (Nesselroade, 1991). Whereas, a longitudinal design consists of different data collection points over an extended period of time. “The measurement-burst design is a hybrid” (Sliwinski, 2008, p. 247) of daily diary and longitudinal study design. Using a measurement-burst design, the elementary teachers in this study completed the log for three “bursts” of fifteen consecutive days throughout the school year. This design can help differentiate between within-teacher variability from day to day on one hand, and within-teacher variability over a longer time period (e.g., few months).

Since data was also collected on science instructional practices via the Instructional Practices Log in Science (IPL-S) (Adams et al., 2017) and since science is not taught daily in elementary classrooms, the 15-day logging windows were individualized and based upon when a teacher was scheduled to teach science daily. That is, for each of the 15-day bursts, the participants began logging at the beginning of science units of instruction. Although the teachers were expected to log for 45 instructional days total, some teachers logged for less than instructed. The median of the completed mathematical logs was 41 instructional lessons. The majority of the participating teachers, 134 teachers (96%) completed the log for ten or more instructional lessons and the remaining teachers, 5 (4%), completed less than ten instructional logs. The decision to use ten days as a cut point was based on the recommendation by Walkowiak et al. (under review) that 10 days of logging resulted in a reliable estimate of the IPL-M scales. Consequently, a decision was made to drop data from the 5 teachers who completed the log for less than 10 instructional lessons each.

All data management was performed using Stata statistical software for its powerful capabilities, yet ease of use for data management. On the other hand, MLM was conducted using SAS software. PROC MIXED command in SAS software was used to run the MLM model.

**Measures.** Participating teachers completed the IPL-M (Walkowiak & Lee, 2013a) as previously described. Prior to data collection, all participating teachers completed a face-to-face two-hour training on how to use the IPL-M as part of a full-day event that included training on a similar science instructional log (the IPL-S; Adams et al., 2017) and data collection on other surveys. The teachers each received a hard copy of the IPL-M user's guide at the training and an electronic hyperlinked version of the user's guide on every day that they logged. They also received a follow-up online tutorial on their first day of logging to remind them about the interpretations of the response scales on the log.

The development of the IPL-M was informed by literature and CCSS-M, and rigorous procedures to collect evidence of validity and scale reliability occurred (Walkowiak et al., under review). Sources of validity evidence included: content review by experts in the field of mathematics education; cognitive interviews with participants to evaluate thinking processes when responding to items (Willis, 2005); factor analyses to examine each scale's reliability; and comparisons of teachers' IPL-M responses to a trained live observer's responses (Walkowiak et al., under review). Primarily due to the evidence of the IPL-M's validity, the instrument "is a promising and viable tool for researchers and educators interested in measuring mathematics instructional practices" (Walkowiak et al., under review, p. 26).

The IPL-M was designed to measure engagement in the following five main activities during mathematics lessons: problem solving, making mathematical connections, math talk, use of multiple representations, and performing mathematical procedures (Walkowiak et al., under review). In the present study, a subset of the data collected using the IPL-M was analyzed. Particularly, the *math talk* and the *use of multiple representations* dimensions are of primary interest. The variables investigated in the present investigations are listed and briefly described in table 2.

Table 2. List of MLM models' input and output variables

	<u>Variable name</u>	<u>Description</u>	<u>Score</u>
Output of MLM model	Talk (Level 1)	Students' opportunities for math talk	(1-4): higher indicates more frequency
Input of MLM model	TeacherReps (level 1)	<i>Teacher</i> use of multiple representations: (i.e. symbols, physical objects, story problems, or tables or charts)	(1-5): higher indicates multiple representations and connecting between them
	StudentReps (level 1)	<i>Student</i> use of multiple representations: (i.e. symbols, physical objects, story problems, or tables or charts)	(1-5): higher indicates multiple representations and connecting between them
	Pictures (level 1)	<i>frequency</i> of student use of pictures	(1-4): higher indicates more frequency
	Manipulatives (Level 1)	<i>frequency</i> of student use of manipulatives	(1-4): higher indicates more frequency
	Mathematical content strand (level 1)	Each of the 5 contents was entered as a dichotomous variable	0: not primary focus of lesson 1: primary focus of lesson
	MKT (level 2)	Mathematical knowledge for teaching number & operations	Scores standardized to z-scores with national mean High: More Knowledge of Math & Pedagogy
	GradeBand (level 2)	2 grade bands teacher teaches: K-2 & 3-5	0: K-2 1: 3-5

**Math talk.** The *math talk* dimension in the IPL-M was measured by nine items to which the teacher reported on the extent that it occurred, measured in time, on that particular day. As a measure of creating opportunities for math talk, the participants responded to the items included in table 3. The score for each item ranged from 1 to 4, where 1 represents “not today,” 2 represents “little” (made up a relatively small part of the instruction), 3 represents “moderate” (made up a large portion, but not the majority of instruction), and 4 represents “considerable” (made up the majority of today’s instruction) (Walkowiak et al., under review). A high score of an item represents creating more opportunities of the target aspect of math talk. In general, the scores represent the frequency of discussions among students and between the teacher and the students.

*Table 3. Math talk dimension, including the question stem and the sub-items within the question as presented to participants*

<u>Question stem</u>	<u>Items</u>
How much time did the students in the target class:	<ol style="list-style-type: none"> <li>1) explain orally his/her thinking about mathematics problems?</li> <li>2) talk about similarities and differences among various solution methods?</li> <li>3) restate another student’s ideas in different words?</li> <li>4) talk about similarities and differences among mathematical representations?</li> <li>5) make explicit connections between concrete and abstract representations through discussion?</li> <li>6) pose questions to other students about the mathematics?</li> <li>7) pose questions to the teacher about the mathematics?</li> <li>8) discuss ideas, problems, solutions, or methods with other students in small groups or pairs?</li> <li>9) discuss ideas, problems, solutions, or methods in large group? (Walkowiak et al., under review, p. 47)</li> </ol>

A variable (Talk) was generated to represent a score given to each instructional lesson indicating the frequency of opportunities for students to engage in math talk during the lesson.

Since the research question of interest in this study is about the correlation between opportunities for math talk and the use of multiple representations, items on the math talk scale and items on the representations scale used for the analysis needed to be mutually exclusive. Therefore, two of the items on the IPL-M's "math talk" scale—which inquired about both constructs of math talk and representations (the *similarities among mathematical representations* and the *connection between abstract and concrete representations* items)—were not included in the creation of the math talk variable. To test for the empirical strength of the seven items as representation of the math talk scale, a confirmatory factor analysis (CFA) model was conducted using Stata software for the new set of items. The standard coefficient loadings of each of the seven items to the math talk variable ranged from .39 to .66, and were each significant ( $p < .001$ ). Since the item loadings were significant, the Talk variable was generated to represent a score for the creation of opportunities for math talk during the instructional lesson. The average of the seven math talk items was computed, and that average was the score for a lesson given to the created Talk variable.

***Use of multiple representations.*** *The use of multiple representations* dimension in the IPL-M include items that represent *teachers'* use of multiple representations and making connections among different representations. Similarly, replicated items represented *students'* use of multiple representations and connecting among them. To define representations, the researchers used a classification by Lesh et al. (1987), who outlined five representations that can be utilized by teachers and students to make sense of mathematical concepts: pictures, written symbols, oral language, real-world situations, and manipulative models. The IPL-M items

obtained responses from teachers about the modes of representation used during instructional lessons (i.e. symbols, physical objects, story problems, or tables or charts) and about making connections among different representations. To obtain the score for each of the two variables (*teacher use of representations* and *student use of representations*), the participants responded to the dichotomous items included in table 4 with a prompt for “what did the STUDENTS use” and “what did YOU [teacher] use.” Since the six items are dichotomous, each resulted in an outcome of 1 (yes) or 0 (no) for each level-1 observation (lesson). The teachers’ responses on the items were used to compute a numerical score using a number that reflects the learning opportunities for students based on literature in mathematics education on best practices. The inclusion of more opportunities for using multiple representations (Lehrer & Schauble, 2004) and making connections and translations among different representations (Duval, 2006) during a lesson was considered an indication of higher quality in use of representations. This informed the scoring of the two variables, *teacher use of representation* and *student use of representations*.

*Table 4. Use of multiple representations dimension, including the question stem and the sub-items within the question as presented to participants*

<u>Question stem</u>	<u>Sub-items</u>
What did the STUDENTS use to work on the mathematics today?	(a) numbers and symbols; (b) tangible materials; (c) real-life situations or story problems; (d) pictures or diagrams; (e) tables or charts; (f) The students made explicit links between two or more of these representations (Walkowiak et al., under review).

Note: Participants also responded to these same sub-items with the prompt: “what did you (the teacher) use to work on the mathematics today?”

Consequently, the two scores of the teacher use of representations and student use of multiple representations during an instructional day were calculated based on results obtained from the log items (a to f) included in table 4, where 1 represents less opportunities for multiple representations and/or translating among representations and 5 represents the highest opportunities (see table 5). For example, if a teacher responded about student use of representation with *yes* to tangible materials and *yes* to numbers and symbols, but *no* to all other items, the student use of representation score was 2, indicating little opportunities for use of multiple representations since it was not reported that students made connections among representations.

*Table 5. Description of how the scores for the use of representations variables were computed*

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<u>Representation items used to compute the score</u>	<u>Score</u>
if one of a-f items are selected OR one of a-e is selected in addition to item f	1
if two of a-e items are selected without item f	2
If three or more of a-e items are selected without item f	3
if two of a-e items are selected with item f	4
If three or more of a-e items are selected with item f	5

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In addition to a score created for the level of use of multiple representations, IPL-M included items that indicated frequency of use of different types of representations. Given a recognized value of using diagrams (Diezmann, 2006) and tangible materials (Hardy, 2001) on students' problem solving skills and mathematical reasoning, of interest in this study is

frequency of use of diagrams and use of tangible materials within a lesson. Teachers responded to the following two items: (1) *During today's lesson, how much time did the students use pictures or diagrams to represent mathematical concepts?* and (2) *During today's lesson, how much time did the students use hands-on tools to explore mathematical ideas or to solve problems?* The answer choices for each of the two items included: *not today, little, moderate,* and *considerable*, like the aforementioned “math talk” items. The scores correspondingly ranged from 1 to 4.

***Grade band and mathematical content taught.*** GradeBand variable was created as a teacher-level predictor, which is a dichotomous variable with dummy code {0, 1}, where a 0 value represents teachers who taught grades K-2 and 1 represents teachers who taught grades 3-5. One of the items on the IPL-M for which the teachers logged stated, *to what extent was each of the following topics a focus of today's mathematics instruction with the target class?* For each of five mathematical strands, the participants chose among: *not today, secondary focus, and primary focus*. The list of mathematical contents that are included in the log are based on those listed in the *Common Core State Standards for Mathematics* (CCSS-M) (2010): (1) Counting and Cardinality (counting), (2) Number and Operations in Base Ten and/or Operations and Algebraic Thinking (number),<sup>1</sup> (3) Number and Operations- Fractions (fractions), (4) Measurement and Data (data), and (5) Geometry. For simplicity, abbreviations are used when

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<sup>1</sup> These two contents from Common Core Standards were combined as one focus in the log because they are considered highly correlated.

referring to these contents in this study (e.g., number and operations in base ten and/or operations and algebraic thinking is referred to as *number*). One should note that the reported data were not all mutually exclusive—some teachers reported more than one primary focus of the lesson. For example, 218 lessons had both *counting and cardinality* and *number and operations in base ten* as primary foci of each lesson, 20 lessons were reported to have primary focus on both *number and operations in base ten* and *fractions*, and 10 for *number and operations in base ten* and *measurement*. Since there were observations (lessons) that had more than one primary focus (8% of the observations), only observations with a single primary focus were included in the analysis. The reasoning for this decision is to obtain a more accurate effect of the content of the lesson on the reported math talk indicator, use of representations indicators, and on the relationship between math talk and the use of representations. Approximately half of the instructional lessons with one primary focus had a primary focus on number and operations in base ten and/or operations and algebraic thinking. The remaining half of the lessons were split relatively close between the other four content strand as a primary focus of the lesson (ranging from 11% to 16%). Dummy codes of one and zero were created to represent each of the five contents as a distinct primary focus of the lesson. In other words, a dichotomous dummy variable is created for the content of interest, which has the values {0, 1}, where 1 represents lesson with primary focus on the content of interest, and 0 represents other lessons in the sample.

***MKT scores.*** Finally, teachers' scores on Mathematical Knowledge for Teaching Assessment in Number and Operation (MKT-N&O) (Learning Mathematics for Teaching, 2004) were considered in one of the investigations of this study as a teacher-level variable. The

participating teachers completed the MKT-N&O in the online Teacher Knowledge Assessment System (TKAS) during face-to-face sessions in the summer prior to their second year of teaching (Lee & Walkowiak, 2013b). The participating teachers' MKT scores ranged from -1.4639 to 1.954, with a mean of 0.4084 and a median of 0.4488.

### **Multilevel Linear Modeling (MLM)**

Multilevel linear modeling (MLM) was used to conduct the quantitative analyses in the study. The use of MLM has worthwhile advantages over single-level techniques that are traditionally used. As opposed to simpler models that address change at a single level, MLM is a powerful modeling technique due to its flexibility in representing change in data across multiple levels (Schulenberg & Maggs, 2001). MLM was chosen for the present study for multiple reasons.

First, MLM characterizes variability within and between teachers (Hawkins, Guo, Hill, Battin-Pearson, & Abbott, 2001). The data in the present study have a nested structure (i.e., lessons are nested within teachers), thus creating two sources of variability in the dataset; variability between teachers in opportunities for math talk and variability within teachers (between different lessons). Using MLM, we can obtain partitioning measures of variance in opportunities for math talk among level 1 (lessons) and level 2 (teachers) (Raudenbush & Bryk, 2002). MLM takes into consideration that estimates of regression models for each teacher might have similarities across teachers, consequently improving the estimate of a within-teacher regression model for each teacher. Since lessons are grouped within teachers, entries of each teacher are most probably not independent. A traditional linear model cannot be used because the

assumption of independence is violated, but MLM does not require this assumption, rather it accounts for non-independence of the within-teacher observations (Hawkins et al., 2001).

Additionally, one of the strengths in using MLM is that it allows for modeling unbalanced data, thus testing the effect of varying number of observations (i.e., instructional lessons) on the relationship between opportunities for math talk and the use of representations within teachers (Raudenbush & Bryk, 2002). Although the participants in this study were expected to complete the log for a total of 45 mathematics lessons, some of them logged for fewer lessons. MLM takes into consideration imbalances among groups, since it has built-in characteristics that allows for giving less weights for participants who logged for less than other participants. This characteristic of MLM allows for including a larger number of the teacher participants in the analysis, rather than having to drop cases with fewer log entries. Furthermore, using MLM allows for compensating for unbalance at the item level of log entries. That is, partial non-response on an instructional lesson, does not result in disregarding the entire observation for that instructional lesson. Other items that the teacher responded to for that lesson can be included in the analysis, which can maximize the power of the statistical test.

Opportunities for math talk at level 1 (instructional lessons) was modeled as a linear function of lesson predictors within each teacher. Then, the coefficients from level 2 (teachers) models were modeled as a linear function of level 2 (teacher) predictors (Burton, 1993).

**Mean centering.** Centering around the grand mean (CGM) is a statistical method that can be used to improve parameter estimation, particularly with nested data (Kreft, Leeuw, & Aiken, 1995). A variable grand mean is the mean of all level-1 observations. To conduct CGM,

the grand mean was computed for TeacherReps, StudentReps, Diagrams, and Manipulatives variables, then new scores were created for the new centered variables. For each variable, the new scores were computed by subtracting the raw score for each observation from the grand mean of that variable. The decision to use CGM was mainly based on “computational ease and stability” (Kreft et al., 1995, p. 17). First, since the scales have no meaningful zero value, a CGM creates a meaning for a zero value, which represents the data point that is equal to the grand average of the sample. CGM can help create scores that are on the same scale; some of the predictor variables have different scales (e.g., the scale for Diagrams consists of {1, 2, 3, 4}, and the scale for TeacherReps consists of {1, 2, 3, 4,5}, as seen in table 5). Furthermore, Nezlek (2001) stated that, “models with uncentered predictors are more likely than those with centered predictors to experience problems estimating parameters due to high correlations between intercept and slopes” (p. 775); centering predictors can reduce such correlation. It should be noted that centering a variable affects the meaning of the resulting intercept obtained from running MLM. After centering, the intercept obtained from an MLM model is the expected value of the dependent variable when the predictor is the grand mean.

### **Limitations**

This exploratory study does not provide sufficient insight into the quality of the mathematical discourse that actually occurred during the lessons neither does it provide knowledge about the related student learning outcome. Teacher implementation of recommended practices as reported in the log is not directly related to students’ learning outcomes. However, one way to conceptualize the concept of OTL is that “certain instructional variables can *impact*

the quality of instruction as evidenced by their relation to student achievement” (Kurz, et al., 2014, p. 163). Particularly, one should be cautious in interpreting the results obtained on opportunities for math talk using teacher-reported data. Quality of mathematical discourse and its effect on student learning was not the purpose of the present study. Future studies can look into details of the instruction by conducting classroom observations and/or analyzing video recordings of lessons to obtain more insight about the richness of the discourse and the productivity of using representations during the lessons. Obtaining richer description of the quality of discourse that occurred during instruction as associated with particular form of using representations can help better understand the relationship between the two constructs.

Stein and Lane (1996) suggest value in investigating aspects of instructional tasks that might influence discourse in mathematics classrooms. The present study provides knowledge on whether the implementation of particular aspects of tasks and activities (i.e., ways of using representations) in elementary classrooms might affect the reported opportunities for math talk. However, the design of IPL-M does not allow for directly pairing tasks to math talk. Although IPL-M log included items that inquired about the cognitive demand of mathematical tasks that were used during the instructional lesson, the *use of representations* log items were not paired with the *task* items. Therefore, examining within-task correlation between the use of multiple representations and opportunities for math talk was not feasible in this study. Due to a close relationship between the types of task and the use of multiple representations (e.g., Hardy, 2001), more refined log items that measure the extent of use of multiple representations and

opportunities for math talk during each particular task implemented in the lesson might help obtain better understanding of that relationship.

Since the data were self-reported by the participating teachers, one should interpret the results with caution. Respondents might have less than accurate in their reports on some of the items, due to the social desirability bias (Tourangeau et al., 2000). However, the ATOMS team was clear from the beginning in assuring the participants of confidentiality of their responses. Reliance on self-reports is a primary concern for some; however, the decision to use data collected using self-reported data using IPL-M is supported by Desimone's (2009) suggestion that, researchers should avoid "automatic biases either for or against" different forms of data collection, whether they were self-reported or not. Instead, researchers should focus on "evaluations and critiques of measurement instruments on the quality of their design and administration, according to best practice, and on their appropriateness given a study's particular research question" (p. 192). A log with evidence of validity of the resulting data allows for reduction of data collection and analysis costs in comparison to other instrument forms.

The findings of this study are constructed based on logs provided by teachers who worked in public schools in the southeastern U.S.. Although many of the schools where those teachers taught had a reasonable level of diversity with respect to student demographics, other public schools in different areas of the U.S. might provide different school settings. Therefore, generalization to other elementary teachers in other classroom settings should be done with caution.

### **Three Studies for Publication**

Three main investigations were conducted resulting in a paper for publication each. Study I, Study II, and Study III will be submitted for publication in the following three journals respectively:

- 1) *Journal for Research in Mathematics Education* is an academic journal of the National Council of Teachers of Mathematics (NCTM). The focus of its publications is on mathematics educational research on all educational levels (pre-k to college). It is published five times a year.
- 2) *Educational Researcher* is published nine times each year and specializes in publishing educational studies that are general, covering a wide variety of content and research foci.
- 3) *The Elementary School Journal* publishes peer-reviewed studies with primary focus on issues relating to elementary and middle school education.

## CHAPTER 4

### **Study I: Opportunities for Math Talk and Use of Multiple Representations: A HLM Study on Elementary Mathematics Lessons**

#### **Abstract**

This study employed a correlational research design to explore the relationship between using multiple representations and math talk in beginning elementary teachers' classrooms. Data collection was conducted using a teacher log, the Instructional Practices Log in Mathematics (IPL-M). Due to the nested nature of the data, Hierarchical Linear Modeling (HLM) was used, where mathematics lessons (level one) were nested within teachers (level two). Beginning teachers' use of multiple representations was found positively correlated with creating opportunities for math talk; however, the correlation between student use of representations and math talk was mediated by the teacher use of representation. Furthermore, the frequencies of student use of diagrams manipulatives were positively correlated with math talk. Implications for future research are presented.

#### **Introduction**

The National Council of Teachers of Mathematics' (NCTM) landmark publication in 2000, the *Principles and Standards for School Mathematics* called for ambitious instruction (Forzani, 2014) in K-12 classrooms. Ambitious instruction means students are presented with learning opportunities in which they engage in mathematical practices such as participating in mathematical argumentations, utilizing multiple representations, and making connections between representations (Jackson & Cobb, 2010). More recent documents, both NCTM's

*Principles to Actions* (2014) and the Standards for Mathematical Practice in the Common Core State Standards for Mathematics (NGA CBP & CCSSO, 2010), also outline the importance of discourse and the use of representations.

Information on the transformation of classroom practices due to reforms is scarce. This scarcity is partly “because few measures exist that *directly* measure instructional practice on a large-scale” (Matsumura et al., 2006, p. 2). The need for such instrumentation motivated the development of an instructional log, the Instructional Practices Log in Mathematics (IPL-M) (Walkowiak & Lee, 2013), utilized in the current study. Neergaard and Smith (2012) shared a concern that present-day evaluation systems “use the same rating criteria regardless of experience level” (p. 4). The present study takes that concern into consideration by using the measure, IPL-M, with a sample of teachers all in their second year of teaching.

A considerable amount of research has been carried out on mathematical discourse (Ryve, 2011) and on the use of representations in mathematics classrooms (Dreher & Kuntze, 2015). However, much of the research has been qualitative in nature (e.g., Boerst, Sleep, Ball, & Bass, 2011; Cengiz, Kline, & Grant, 2011; O’Connor, 2001). Studies like the present can contribute to the field by analyzing aspects of instructional practices and students’ opportunities to learn obtained from a large sample. Throughout this paper, *math talk* was used to refer to oral mathematical discourse that takes place during whole- or small-group communications in mathematics classrooms. The purpose of this study is to investigate the relationship between use of multiple representations and opportunities for math talk in elementary classrooms based on data reported by teachers.

## **Theoretical Assumptions**

The theoretical foundation of this study stems from Wertsch's (1991) Vygotskian framework, which indicates that social interaction in classrooms can influence children's cognitive processes. That perspective motivated this study's investigation of math talk, which is one form of mathematical discourse, as the dependent variable of interest. Student learning happens within social interactions that cultivate their construction of meanings (Vygotsky, 1978). Mathematical meanings are "co-constructed" (Ryve, 2011); that is, students understand the meanings of the mathematical language introduced in the classroom within the context of the conversations that are present during instructional settings. This phenomenon implies that teachers' discourse strategies can play an important role in students' learning of mathematical concepts. Informed by the sociocultural perspective (Rogoff, 1997; Vygotsky, 1978), this study attempts to obtain insight into opportunities created for student learning by investigating math talk that takes place in elementary classrooms as related to the use of representations.

This study assumes that student learning can be influenced by exposure and experiences with enacted practices of their teachers during daily lessons, which is conceptualized as students' opportunities to learn (OTL) (Carroll 1963; Cooley & Leinhardt, 1980; Porter, Kirst, Osthoff, Smithson, & Schneider, 1993).

From this perspective, instruction is conceptualized as a series of repeated (i.e., daily) exposures to instruction, and the key measurement problem is to obtain an estimate of the overall amount or rate of exposure to particular elements of instruction occurring over some fixed interval of time (e.g., a school year) (Rowan & Correnti, 2009, p. 120).

The teachers' responses on the log items in this study were numerically scored based on literature in mathematics education on best practices to reflect the creation of OTL. Frequency and quality of occurrence of a particular practice of interest—a practice found by earlier literature to have potential positive effects on students' learning—during a mathematics lesson are considered indicators of OTL. Consequently, a high score on one of the IPL-M constructs represents creating more opportunities to learn, and a low score represents less opportunities to learn (Walkowiak, Adams, Porter, Lee, & McEachin, under review). However, student learning is not necessarily a direct result of teacher implementation of certain practices, which are reported in the IPL-M. The log developers recognize that, although high frequency of best practices indicates more opportunities for student learning, frequency does not necessarily equate to quality of instruction (Walkowiak, et al., under review). Nevertheless, one way to conceptualize OTL is that “certain instructional variables can *impact* the quality of instruction as evidenced by their relation to student achievement” (Kurz, Elliott, Kettler, & Yel, 2014, p. 163). One rationale for this conceptualization of OTL is that there are specific practices that have been found by earlier research to have positive influence on student achievement, thus such practices that are empirically supported can be thought of as potentially creating opportunities for student learning (Kurz et al., 2014).

### **Math talk**

NCTM (2014) recommends promoting “meaningful mathematical discourse” (p. 10) among students in K-12 classrooms. Teacher orchestration of student engagement in rich mathematical discourse can have significant implications for students' mathematical learning

(Anderson-Pence, 2017; Ball, 1991; Steinbring, Bartolini-Bussi, & Sierpiska, 1998). Although the need for rich mathematical discourse in classrooms is established, many teachers find it difficult to orchestrate such discourse in their classrooms (Herbel-Eisenmann, Steele, & Cirillo, 2013; Kazemi & Stipek, 2001; Nathan & Knuth, 2003; O'Connor, 2001; Peterson & Leatham, 2009; Staples, 2007). This is particularly challenging for beginning teachers (Bennett, 2010). By rich mathematical discourse, we mean “interactive and sustained discourse of dialogic nature between teachers and students aligned to content of the lesson that addresses specific student learning issues” (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008, p. 378). Teachers can create opportunities for students to learn through the math talk they facilitate during a lesson. By simply correcting students when producing incorrect answers, or agreeing with students' correct answers—which is called an Initiate-Response-Evaluate (I-R-E; Mehan, 1979) discourse pattern—the teacher misses on creating rich learning opportunities. On the other hand, requesting justifications from students about their responses can give teachers insights into the students' mathematical thinking (Ball, 1991).

One useful way that might help categorizing richness of discourse in mathematics classroom, which is deeper than simply categorizing discourse as I-R-E or not, is the hierarchical categories suggested by Brendefur and Frykholm (2000). Those categories are: “uni-directional, contributive, reflective, and instructive communication” (p. 148). *Uni-directional* communication is predominantly led by the teacher, which indicates the least opportunities for students to communicate their ideas and reasoning. *Contributive* communication happens when students contribute to classroom discourse, however in a limited manner that lacks deep

mathematical reasoning and argumentation. Influenced by Cobb, Boufi, McClain, and Whitenacks' (1997) and von Glasersfeld's (1991) conceptions of reflective discourse, Brendefur and Frykholm (2000) explain that *reflective* communication refers to mathematical discourse in which "students reflect on the relationships within the mathematical topics by focusing on other students' and the teacher's ideas, insights, and strategies" (p. 148). (e.g., students discussing the validity of other students' ideas). Finally, *instructive* communication was informed by Steffe and D'Ambrosio's (1995) description of instructional settings during which students and teacher are engaged in multidirectional and repeated discussions that cyclically influence subsequent instructional decisions of the teacher, who would be paying close attention to students' mathematical thinking and any misunderstandings.

However, implementing such highly reflective and productive discourse is not easy for teachers and requires substantial effort (Li & Ni, 2011). To assist teachers in orchestrating productive math talk, Michaels and O'Connor (2015) suggested a set of talk moves, which are the product of approximately twenty years of research on classroom discourse. They recommended purposeful use of the talk moves to enable students to share their thinking, listen to each other, deepen their mathematical reasoning, and draw on each others' ideas. For example, the say more moves (e.g., *can you say more?*) refers to a teacher's facilitation of students' elaboration on their ambiguous responses. The "press for reasoning" move (*Why do you think that? What's your evidence? What led you to that conclusion?*)" (p. 334) can help teachers obtain insight into students' reasoning. Furthermore, revoicing students' ideas by the teacher is emphasized by Chapin O'Connor, and Anderson (2009) as a talk move that can promote rich

mathematical discourse among students. Chapin and colleagues recommend re-stating students' responses by the teacher using appropriate mathematical language. Evidence based on small-scale investigations suggested that teachers' meaningful implementation of the talk moves in their classrooms helped establish classroom norms and environments in which all students felt safe to participate productively in mathematical discussions (Michael & O'Connor, 2015) and, in some cases, improve students' academic achievement (Dudley-Marling & Michaels, 2012).

Providing students with opportunities for rich mathematical discourse during instructional lessons have agreed-upon benefits for students' mathematical thinking. However, orchestrating such mathematical discussions is a complex task for teachers, particularly for beginning teachers (Boerst et al., 2011). Boerst and colleagues further stated that, "Teachers draw on substantial knowledge and skill to maintain collective engagement in discussions in order to avoid more simplistic and less effective 'show-and-tell' sessions" (p. 2848). Given that teachers need to be equipped with substantial knowledge and experience to be able to facilitate the types of mathematical discourse recommended from research, it seems appropriate to expect beginning teachers to simply provide opportunities for students to engage in math talk. With the assumption of reasonable expectations for beginning teachers, the present study investigates math talk in second-year teachers' classrooms by examining the teachers' daily reports of how much their students engaged in math talk.

### **Use of Representations During Mathematics Instruction**

An assumption informing this study is based on research demonstrating a relation between math talk and the use of representations (e.g., Anderson-Pence & Moyer-Packenham,

2016; Hardy, 2001). Informed by earlier studies on mathematics education, Lesh, Post, and Behr (1987) highlighted five types of representational systems: (a) story problems that are formulated based on real-world situations; (b) manipulatives that are used for modeling mathematical ideas; (c) static pictures and diagrams; (d) verbal language; and (e) written symbolic representations. Subsequently, in the present study, *representations* refers to illustrations of mathematical concepts, ideas, or objects, such as symbols, story problems, pictures, diagrams, and tangible materials.

Research findings indicate that using multiple representations, translating among different forms of representations, and connecting between representations and mathematical concepts are related to students' mathematical understanding (Hiebert, 1984; Lesh et al., 1987). Hardy (2001) explained that meaningful use of multiple representations can expose structural elements of mathematical concepts, which can provide the learner with points of reference for mathematical discussions. She, however, reminded that while the use of multiple representations can potentially aid students' success in problem solving, tasks that require particular and restricted (often procedural) ways of using representations can hinder students' mathematical thinking. Making connections between concrete representations (e.g., tangible objects) and symbolic representations—which can be thought of as representing abstract mathematics—is important for helping young students understand more advanced mathematical ideas as they move on to higher grades (Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009).

**Using representations beyond symbolic.** The use of different representations beyond, and in addition to, symbolic representations (e.g., words, pictures, diagrams, and tangible

objects) while working on mathematical tasks has potential value for students' mathematical thinking and reasoning (Hardy, 2001). For instance, the use of tangible objects (or sometimes called manipulatives) was found by Baxter, Woodward, and Olson (2001) to potentially aid the engagement of students whose achievement is below grade level during mathematical investigations. Manipulatives should be used as a medium for mathematical thinking; however, at times, the physical aspects of manipulatives themselves become the focus of instruction instead of being a representation of mathematical concepts or objects (Baxter et al., 2001). When using manipulatives to aid students' mathematical reasoning, little attention should be given to physical properties of the object so that students pay more attention to the manipulative as a representation of the intended abstract mathematical ideas (Uttal et al., 2009). It is important for students to take responsibility for their own learning (Hufferd-Ackles, Fuson, & Sherin, 2004); therefore, student use of representations during class is important for their feeling of ownership of their learning. In the present investigation, student use of representations and teacher use of representations were investigated as separate actions during mathematics lessons.

### **Math Talk and Use of Representations**

As explained in the previous sections, orchestrating rich math talk and using multiple, connected representations during mathematics lessons have implications for students' mathematical learning. This section describes existing research indicating the possibility of a correlation between math talk and use of multiple representations. A teacher's facilitation of using different, yet connected representations during whole-group discussions can potentially extend students' mathematical thinking (Cengiz et al., 2011). Hardy (2001) emphasized how the

appropriate incorporation of representations into mathematics instruction allows students to communicate their mathematical ideas and simultaneously can “enable students to gain new insights into mathematical structure” (p. 1). A recent study by Anderson-Pence and Moyer-Packenham (2016) also investigated the relationship between discourse and representations, but in the context of a computer environment and small-group setting, unlike Hardy’s (2001) study. Specifically, Anderson-Pence and Moyer-Packenham (2016) explored the use of virtual manipulatives (VMs) with three pairs of fifth-grade students, who were observed during nine instructional sessions. Virtual manipulatives are digitalized mathematical representations such as graphs, tables, and pictures. They investigated three types of visual manipulatives: The three types were: (1) Linked VMs, which are multiple representations that vary together, allowing the user to observe simultaneous change in symbolic representation that is linked to other forms of representations; (2) Pictorial VMs, which are pictures and diagrams that vary based on the user’s manipulation, but are not linked to symbolic representations; and (3) Tutorial VMs, referring to representations that are static highly structured in the feedback provided for the user. The quality of discourse in relation to each of the three types was assessed on the extent to which the discourse indicated evidence of generalization of mathematical concepts, justification of students’ ideas, and cooperation among the students. While linked and pictorial VMs allow students to choose solution methods, tutorial VMs restrict the choice of solution methods to ones that are proposed by the program. Based on the results of their study, Anderson-Pence and Moyer-Packenham (2016) concluded that when students’ mathematical investigation of representations are highly connected, as in the case of linked VMs, they “effectively generalize

and justify mathematics concepts during problem solving tasks” (p. 23). Overall, the discourse quality that took place among students while working with linked VMs was found to be significantly higher than discourse connected to the use of pictorial and tutorial VMs.

As illustrated in this section, some preceding studies investigated the relation between the use of representations and discourse; however, they were largely based on small-scale investigations (e.g., Anderson-Pence & Moyer-Packenham, 2016) or based on observational data collected on one or few instructional lessons for each participating teacher (e.g., Hardy, 2001). Learning about the correlation between the two constructs obtained from a large sample of lessons within a large group of elementary teachers can help researchers build on that knowledge to further learn about ways to help teachers improve the quality of discourse.

### **Instructional Log**

Measurement options that allow for rating students’ opportunities to learn (OTL) from a large sample of lessons and teachers on a daily basis are scarce (Kurz et al., 2014). An instructional log is a promising measure of students’ OTL. The use of instructional logs for data reporting on instructional practices compensates for disadvantages of both the expensive, involved method of classroom observations and the one-time survey method. Teacher responses to one-time survey questions about details of instruction have inherent issues, such as memory errors; whereas, instructional logs completed shortly after instruction allow for the collection of more accurate reports (Heck, Chval, Weiss , & Ziebarth, 2012). Although classroom observations allow for collecting real-time detailed data about the quality of instructional

practices, they have their drawbacks. They are costly and time-consuming, thus usually feasible only for small-scale investigations.

Collecting data using a log allows for analysis of reported events in their natural, spontaneous setting (Reis, 1994), and can play an important role in reducing the likelihood of retrospection effects of data collection (Bolger, Davis, & Rafaeli, 2003). It also facilitates obtaining data that allows for examining both between-teacher and within-teacher variability (Nezlek, 2001). Using an instructional log allows for collecting detailed, at-scale data; however, according to Kurz et al. (2014) and Walkowiak et al. (under review) instructional logs are not frequently used for measuring instructional practices, resulting in a lack of at-scale studies of instructional practices.

The hypothesis motivating the present exploratory investigation is that using multiple representations and making connections between different representations in elementary mathematics lessons of beginning teachers is positively correlated with the resulting math talk during the lessons. Consequently, the following research questions are of interest in this study:

- 4) Is there a within-teacher relationship between opportunities for math talk in elementary classrooms and *student use* of multiple representations?
- 5) Is there a within-teacher relationship between opportunities for math talk in elementary classrooms and *teacher use* of multiple representations?
- 6) Are there grade-band (Grades K-2 versus 3-5) differences in the correlation between use of representations and opportunities for math talk in elementary classrooms?

- 7) Is there a within-teacher relationship between frequency of use of diagrams or tangible materials and opportunities for math talk in elementary classrooms as reported by teachers?

## Methodology

**Participants.** The participants are 134 teachers who taught at public elementary schools in a southeastern state of the U.S. (Table 6 includes demographic data about the participating teachers). Most of the participants (85%) are white, which represents the low level of racial diversity among public school teachers in the U.S.; i.e., about 82% of public school teachers were reported to be white in 2011-2012 academic year (U.S. Department of Education, 2016). The participating teachers attended a face-to-face training session on using the IPL-M during summer. During the following school year, they completed log entries using online links that were sent to them by e-mail. Each lesson log was completed shortly after the lesson was implemented.

*Table 6. Demographic characteristics of participating teachers*

<u>Gender</u>	<u>White</u>	<u>Black</u>	<u>Hispanic</u>	<u>Multiracial</u>	<u>Other race</u>	<u>Total</u>
Female	114	9	3	2	2	130
Male	1	0	0	0	1	2

*Note: Gender and race were not reported for 2 participants.*

**Instrument and measures.** The *math talk* dimension in the IPL-M included seven items to which the teachers responded about the extent to which an item occurred during the lesson, measured in time. As a measure of creating opportunities for math talk, the participants

responded to the question stem illustrated in table 7 for each of the seven sub-items of that question. The score for each item ranged from 1 to 4, where 1 represents “not today,” 2 represents “little” (made up a relatively small part of the instruction), 3 represents “moderate” (made up a large portion, but not most instruction), and 4 represents “considerable” (made up most today’s instruction) (Walkowiak et al., under review). A high score of an item represents creating more opportunities of the target aspect of math talk. In general, the scores are considered a representation of the frequency of discussions among students and between the teacher and the students.

*Table 7. Math talk dimension, including the question stem and the sub-items within the question as presented to participants*

<u>Question stem</u>	<u>Sub-Items</u>
How much time did the students in the target class:	1) explain orally his/her thinking about mathematics problems? 2) talk about similarities and differences among various solution methods? 3) restate another student’s ideas in different words? 4) Pose questions to other students about the mathematics 5) Pose questions to the teacher about the mathematics 6) Discuss ideas, problems, solutions, or methods with other students in small groups or pairs 7) Discuss ideas, problems, solutions, or methods in large group (Walkowiak et al., under review, p. 47)

A variable (Talk) was generated to represent a score given to each instructional lesson indicating the frequency of opportunities for students to engage in math talk during the lesson. To test for the empirical strength of the items as representation of the math talk scale, a confirmatory factor analysis (CFA) model was conducted using Stata software for the set of seven of items. The standard coefficient loadings of each of the seven items to the math talk variable ranged from .39 to .66, and were each significant ( $p < .001$ ). Since the item loadings were

significant, the Talk variable was generated to represent a score for the creation of opportunities for math talk during the lesson. The average of the seven math talk items was computed for each observation (lesson), and that average was the score given to the created Talk variable.

*The use of multiple representations* dimension in the IPL-M include items that represent *teachers'* use of multiple representations and making connections among different representations. Similarly, replicated items represented *students'* use of multiple representations and connecting among them. A classification by Lesh et al. (1987) was used to define representations as five modes that can be utilized by teachers and students to make sense of mathematical concepts: pictures, written symbols, oral language, real-world situations, and manipulative models. The IPL-M items obtained responses from teachers about the variety of modes used during instructional lessons and about making connections among the different modes. To obtain the score for each of the two variables (*teacher use of representations* and *student use of representations*), the participants responded to the items included in table 8. Since the six items are dichotomous, each resulted in an outcome of 1 (yes) or 0 (no) for each level-1 observation (lesson). The teachers' responses on the items were used to compute a numerical score using a number that reflects the learning opportunities for students based on literature in mathematics education on best practices. The inclusion of more opportunities for using multiple representations (Lehrer & Schauble, 2004) and making connections and translating among different representations (Duval, 2006) during a lesson was considered an indication of higher opportunities for multiple representations and/or translating among representations.

Table 8. Use of multiple representations dimension, including the question stem and the sub-items within the question as presented to participants

<u>Question stem</u>	<u>Sub-items</u>
What did the STUDENTS use to work on the mathematics today?	(a) numbers and symbols; (b) tangible materials; (c) real-life situations or story problems; (d) pictures or diagrams; (e) tables or charts; (f) The students made explicit links between two or more of these representations (Walkowiak et al., under review).

Note: The sub-items in this table were replicated under the prompt: “what did you (the teacher) use to work on the mathematics today?” to obtain the variable *teacher use of representations*.

Consequently, the two variables, *teacher use of representations* and *student use of representations* were given a value on each lesson based on the above log items demonstrated in table 8 (items a to f), as illustrated in table 9, where 1 represents less opportunities for multiple representations and/or translating among representations and 5 represents the highest opportunities. (e.g., if a teacher responded about student use of representation with *yes* to tangible materials and *yes* to numbers and symbols, but *no* to all other items, the student use of representation score was 2, indicating little opportunities for use of multiple representations since students did not make connections among representations).

Table 9. Description of how the scores for the use of representations variables were computed

Representation items used to compute the two scores	Score representing <i>teacher</i> use of representation or <i>student</i> use of representations
if one of a-f items are selected OR one of a-e is selected in addition to item f	1
if two of a-e items are selected without item f	2
If three or more of a-e items are selected without item f	3
if two of a-e items are selected with item f	4
If three or more of a-e items are selected with item f	5

The standard coefficient loadings of the items in table 8 to the *teacher use of representations* and *student use of representations* variables were statistically significant ( $p < .001$ ) (Walkowiak et al., under review). In addition to a score created for the level of use of multiple representations, IPL-M included items that indicated frequency of use of different types of representations. Given a recognized value of using diagrams (Diezmann, 2006) and tangible materials (Hardy, 2001) on students' problem solving skills and mathematical reasoning, of interest in this study is frequency of use of diagrams and use of tangible materials within a lesson. Teachers responded to the following two items: (1) *During today's lesson, how much time did the students use pictures or diagrams to represent mathematical concepts?* and (2) *During today's lesson, how much time did the students use hands-on tools to explore mathematical ideas or to solve problems?* The answer choices for each of the two items included: *not today*, *little*, *moderate*, and *considerable*. The scores correspondingly ranged from 1 to 4. Descriptive statistics of the representations variables are presented in table 10.

*Table 10. Summary statistics of use of representations variables for all observations (lessons)*

Variable	Mean	SD	Minimum	Maximum
StudentReps	2.89	1.28	1	5
TeacherReps	2.91	1.35	1	5
Pictures	2.4	1.07	1	4
Manipulatives	2.03	1.15	1	4
n = 5175 lessons within 134 teachers				

The quantitative data collected using an instructional log instrument were analyzed using hierarchical linear modeling design (HLM), which is a powerful modeling technique due to its flexibility in representing change in data across multiple levels (Schulenberg & Maggs, 2001). The use of HLM has worthwhile advantages over single-level techniques that are traditionally used. HLM characterizes variability within and between teachers (Hawkins, Guo, Hill, Battin-Pearson, & Abbott, 2001). The data in the present study have a nested structure (i.e., lessons are nested within teachers), thus creating two sources of variability in the dataset; variability between teachers in opportunities for math talk and variability within teachers (between different lessons). HLM allowed for an understanding of variability in opportunities for math talk (and its extent) at both the within-teacher level and between-teacher level. The within-teacher variability on the level of the lesson is of more interest in this study than between-teacher variability. Furthermore, a traditional linear model cannot be used because the assumption of independence is violated, but HLM does not require this assumption, rather it accounts for non-independence of the within-teacher observations (Hawkins et al., 2001).

Although the participants in this study were expected to complete the log for a total of 45 mathematics lessons, some of them logged fewer lessons. HLM takes into consideration

imbalances among groups, since it has built-in characteristics that allows for giving less weights for participants who logged for less than other participants (Raudenbush & Bryk, 2002). This characteristic of HLM allowed for including a larger number of the teacher participants in the analysis, rather than having to drop cases with fewer log entries. Furthermore, using HLM allows for compensating for unbalance at the item level of log entries. That is, observations that had partial non-response on an instructional lesson were not disregarded for that instructional lesson; other items that the teacher responded to for that lesson were included in the analysis, which maximized the power of the statistical test. The majority of the participating teachers, 134 teachers (96%) completed the log for 10 or more instructional lessons and the remaining teachers, five (4%), completed less than 10 instructional logs. Ten days is used here as a cut point based on the recommendation by Walkowiak et al. (under review) that 10 days of logging appears to result in a reliable estimate of all scales on the IPL-M. Consequently, the five teachers who completed the log for less than 10 lessons were not included in the sample.

Centering around the grand mean (CGM) is a statistical method that was used to improve parameter estimation of the study's nested data (Kreft, Leeuw, & Aiken, 1995). The decision to use CGM was mainly based on "computational ease and stability" (Kreft et al., 1995, p. 17). First, since the scales have no meaningful zero value, a CGM creates a meaning for a zero value, which represents the data point that is equal to the mean of all level-1 observations (the grand mean). Furthermore, CGM can help create scores that are on the same scale; some of the predictor variables have different scales (e.g., the scale for Opportunities for Math Talk consists of {1, 2, 3, 4}, and the scale for Teacher Use of Multiple Representations consists of {1, 2, 3,

4,5}). It should be noted that centering a variable affects the meaning of the resulting intercept obtained from running HLM. After centering, the intercept obtained from the model is the expected value of the dependent variable when the predictor equals the grand mean.

A fully unconditional model was conducted as a preliminary step to conducting the HLM conditional analysis (Nezlek, 2001). This model is conducted to obtain the partition variance (between- vs. within-teacher variability) and to obtain a point estimate for the grand mean (to determine if the mean talk is significantly different from zero):

$$\text{Level 1 (lesson): } \text{Talk}_{ij} = \beta_{0ij} + r_{ij}$$

$$\text{Level 2 (teacher): } \beta_{0i} = \gamma_{00} + u_{0i}$$

$\text{Talk}_{ij}$  is the outcome variable representing the score for opportunities for math talk by teacher  $i$  during instructional lesson  $j$ .  $\beta_{0ij}$  represents the expected value of opportunities for math talk for teacher  $i$  and lesson  $j$ .  $r_{ij}$  is the average fluctuation around the within-teacher average of math talk score.  $\gamma_{00}$  represents the sample average of the math talk score, and  $u_{0i}$  stands for the between-teacher fluctuation from the sample average talk.

Results from the null model indicate that the average level of math talk is 1.82, which indicates relatively low opportunities for math talk, but is significantly greater than zero ( $t = 55.86, p < .001$ ). Furthermore, 48% of the variability in math talk was between teachers ( $\tau_{00} = .14, z = 7.91, p < .001$ ) and 52% was within teachers ( $\sigma^2 = .15, z = 50.21, p < .001$ ). Therefore, the fully unconditional model indicates sufficient variability for further analyses. This result is similar to earlier results indicating significant variability of within-teacher practices from day to day (e.g., Rowan & Correnti, 2009). The intraclass correlation coefficient (ICC) was computed

to determine if there is significant variability at each of level 1 and level 2. If so, ICC is used as a baseline to which subsequent models are compared, and the explained measures of variance are explained based on the ICC. The equation  $\rho = \tau_{00} / (\tau_{00} + \tau^2)$  was used to obtain the proportion of variance in Talk that is between teachers ( $\rho = .48$ ); thus,  $1-\rho = .52$  is the proportion of variance that is within teachers (Raundenbush & Bryke, 2002).

### Analysis and Results

**Student and teacher use of representations (research questions 1 and 2).** As a first step in addressing the first and second research questions, model 1 and model 2 were conducted respectively:

Model 1:

$$\begin{aligned} \text{Lesson level: } \text{Talk}_{ij} &= \beta_{0ij} + \beta_{1ij}(\text{StudentReps}) + r_{it} \\ \text{Teacher level: } \beta_{0i} &= \gamma_{00} + u_{0i} \\ \beta_{1i} &= \gamma_{10} + u_{1i} \end{aligned}$$

Model 2:

$$\begin{aligned} \text{Lesson level: } \text{Talk}_{ij} &= \beta_{0ij} + \beta_{1ij}(\text{TeacherReps}) + r_{it} \\ \text{Teacher level: } \beta_{0i} &= \gamma_{00} + u_{0i} \\ \beta_{1i} &= \gamma_{10} + u_{1i} \end{aligned}$$

In model 1,  $\text{Talk}_{ij}$  represents the level of opportunities for math talk for teacher  $i$  during lesson  $j$ .  $\beta_{0ij}$  is the sample mean Talk when StudentReps equals the sample mean (0 represents the grand sample mean since StudentReps was grand-mean centered).  $\beta_{1ij}$  is the unique effect of student use of representations on Talk. Level-1 intercept ( $\beta_{0ij}$ ) and slope ( $\beta_{1ij}$ ) become the outcomes of teacher-level equation.  $\gamma_{00}, \gamma_{10}$  is the average within-teacher Talk and  $\gamma_{20}$  is the fixed effect in the unique effect of student use of representations. Lastly,  $u_{0i}$  and  $u_{1i}$  are the random

effects that represent between-teacher variability in the corresponding average value, and  $r_{ij}$  is the within-teacher residual (a statistic representing within-teacher variability in Talk). The only difference in model 2 from model 1 is that StudentReps is replaced by TeacherReps (the degree to which a teacher uses multiple representations and makes connections among them during a lesson).

For simplicity, throughout this section, the term *high use of representations*, is used to denote using multiple representations and connecting among them; whereas, *low use of representations* is used to refer to little opportunities for using multiple and connected representations. The results from conducting model 1 indicate that lessons with higher scores of student use of representations are those with more opportunities for math talk ( $\gamma_{10} = .07$ ,  $t = 9.57$ ,  $p < .001$ ). Likewise, there a significantly positive correlation between teacher use of multiple representations and opportunities for math talk ( $\gamma_{10} = .08$ ,  $t = 11.96$ ,  $p < .001$ ). (see table 11, model 2).

To obtain a better understanding of the correlations, the covariation among the three variables of interest in the first and second research questions were expressed using model 3, in which all random effects at level-2 were included to control for possible dependence due to repeated measures or level effects:

Model 3:

Lesson level:  $Talk_{ij} = \beta_{0ij} + \beta_{1ij}(\text{StudentReps}) + \beta_{2ij}(\text{TeacherReps}) + r_{it}$

Teacher level:  $\beta_{0i} = \gamma_{00} + u_{0i}$

$\beta_{1i} = \gamma_{10} + u_{1i}$

$\beta_{2i} = \gamma_{20} + u_{2i}$

Talk<sub>ij</sub>, represents the level of opportunities for math talk for teacher *i* during lesson *j*. The lesson-level predictors are StudentReps (student use of multiple representations and connecting among representations) and TeacherReps (teacher use of multiple representations and connecting among representations).  $\beta_{0ij}$  is the sample mean Talk when the two predictors are at the grand mean (StudentRep and TeacherReps were grand-mean centered);  $\beta_{1ij}$  is the unique effect of student use of representations on Talk; and  $\beta_{2ij}$  is the unique effect of teacher use of representations on Talk. Level-1 intercept ( $\beta_{0ij}$ ) and slopes ( $\beta_{1ij}$  and  $\beta_{2ij}$ ) become the outcomes of the teacher-level equation.  $\gamma_{00}$ ,  $\gamma_{10}$ , and  $\gamma_{20}$  are the fixed effects representing the average within-teacher Talk, unique effect of student use of representations, and unique effect of teacher use of representations, respectively. Lastly,  $u_{0i}$ ,  $u_{1i}$ , and  $u_{2i}$  are random effects that represent between-teacher variability in the corresponding average value, and  $r_{ij}$  is the within-teacher residual (a statistic representing within-teacher variability in Talk).

The results illustrated in table 11 (model 3), where both use of representation predictors were included in the model, illustrate a minute positive correlation between TeacherReps and Talk ( $\gamma_{20} = .07$ ,  $t = 8.08$ ,  $p < .001$ ), indicating that lessons with high teacher use of multiple representations are those that include more opportunities for math talk. However, unlike the results obtained from model 1 (with only student use of representations was entered as a predictor), there was no correlation between student use of representations and math talk ( $p = .06$ ). Furthermore, teachers varied significantly in the association between teacher use of representations and opportunities for math talk ( $\tau_{22} = .00$ ,  $z = 2.12$ ,  $p < .05$ ), indicating that the association between teacher use of representations and math talk does not look the same for all

teachers. This model explained 10% of the within-teacher and 13% of the between-teacher variability in creating opportunities for math talk.

When the student use of representations and teacher use of representations were entered as level-1 predictors simultaneously (model 3), the relationship between student use of representations and opportunities for math talk was insignificant ( $P = .06$ ). This indicates that teacher use of representations might be creating a mediation effect (Kenny et al. 2003) on the correlation between student use of representations and opportunities for math talk. In other words, the correlation between teacher use of representations and math talk might be explaining why there was a relationship between student use of representations and opportunities for math talk when teacher use of representations was ignored, but not when teacher use of representation was accounted for in the model.

Comparing the results obtained from models 1 and 2 motivated investigating whether the teacher use of representations was mediating the relationship between student use of representations and math talk. Thus, further analysis was conducted following the method described by Kenny, Karchmaros, and Bolger (2003). First, a null model with the teacher use of representations variable (the potential mediator) as the outcome of the model was conducted to justify further investigation of teacher use of representations as a dependent variable

$$\text{Level 1 (lesson): TeacherReps}_{ij} = \beta_{0ij} + r_{ij}$$

$$\text{Level 2 (teacher): } \beta_{0i} = \gamma_{00} + u_{0i}$$

The results indicated that 42% of the variability in teacher use of representations was between teachers ( $\tau^2 = .75$ ,  $z = 7.86$ ,  $p < .001$ ) and 58% was within teachers ( $\sigma^2 = 1.05$ ,  $z =$

50.16,  $p < .001$ ). This indicates sufficient variability for conducting an analysis with teacher use of representations as the outcome. Subsequently, model 4 was conducted with teacher use of representations (the potential mediator) as the outcome and student use of representations as the level-1 predictor to test if there was a correlation between the two variables:

Model 4:

Lesson level:  $\text{TeacherReps}_{ij} = \beta_{0ij} + \beta_{1ij}(\text{StudentReps}) + r_{it}$

Teacher level:  $\beta_{0i} = \gamma_{00} + u_{0i}$

$\beta_{1i} = \gamma_{10} + u_{1i}$

A significantly positive slope obtained from model 4 indicates that lessons with high scores of student use of multiple representations tend to have high scores of teacher use of multiple representations ( $\gamma_{10} = .76$ ,  $t = 40.45$ ,  $p < .001$ ). In other words, lessons in which students use multiple representations and make connections among them are those during which teachers use multiple representations and make connections among them. Moreover, teachers differed significantly in that slope, indicating that not all teachers have the same correlation in their lessons between StudentReps and TeacherReps ( $\tau_{11} = .03$ ,  $z = 5.33$ ,  $p < .001$ ). This model explained 63% of within-teacher variation and 80% of between-teacher variation in teacher use of representations. Based on the mediation procedure discussed by Kenny et al. (2003), the correlation between student use of multiple representations and opportunities for math talk was significantly reduced after accounting for teacher use of representations as a level-1 predictor (Sobel test = 7.92,  $p < .001$ ).

**Grade band as a teacher-level predictor (Research question 3).** To better understand differences in the use of representations and math talk among teachers, the above investigation

was followed up with a model that included the grade band, at which the teachers taught, as a teacher-level predictor:

Model 5:

$$\text{Lesson level: Talk}_{ij} = \beta_{0ij} + \beta_{1ij}(\text{StudentReps}) + \beta_{2ij}(\text{TeacherReps}) + r_{it}$$

$$\text{Teacher level: } \beta_{0i} = \gamma_{00} + \gamma_{01}(\text{GradeBand}) + u_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{GradeBand}) + u_{1i}$$

$$\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{GradeBand}) + u_{2i}$$

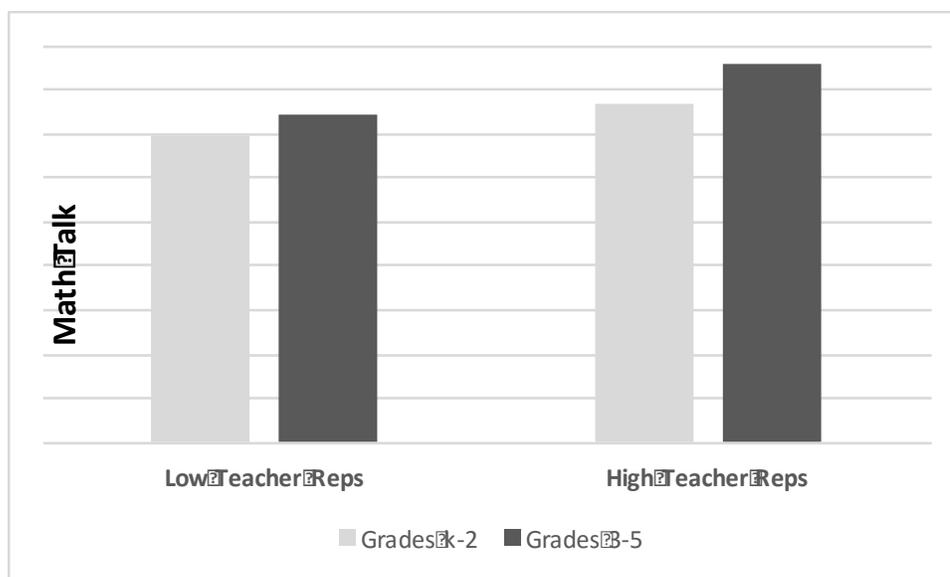
Level-1 equation is the same as the one in model 3; however, the teacher-level GradeBand predictor was added to the teacher-level equation as a dichotomous variable with dummy code {0, 1}—where a 0 value represents teachers who taught grades K-2 and 1 represents teachers who taught grades 3-5. Level-1 intercept and slopes are the outcomes of level-2 equation, where,  $\gamma_{00}$  is the average level of Talk for a teacher who teaches K-2 (Gradeband=0 indicates K-2).  $\gamma_{01}$  tests if the grade band that a teacher teaches is related to the average level of Talk.  $\gamma_{10}$  is the effect of student use of representations on Talk.  $\gamma_{11}$  is the cross-level interaction effect, which tests if there were grade-band differences in the correlation between StudentReps and Talk. Similarly,  $\gamma_{20}$  is the effect of teacher use of representations on Talk.  $\gamma_{21}$  is the cross-level interaction effect, which tests if there were grade-band differences in the correlation between TeacherReps and Talk. Lastly,  $u_{0i}$ ,  $u_{1i}$ , and  $u_{2i}$  are random effects that represent between-teacher variability in each corresponding average value, and  $r_{ij}$  is the within-teacher residual (a statistic representing within-teacher variability in Talk). It is noteworthy that since interaction effects are included in this model, the effects of  $\gamma_{10}$  and  $\gamma_{20}$  are not the unique

effects of StudentReps and TeacherReps because both variables have an effect from the interaction as well.

Running model 5 (see table 11, model 5 for the results) indicated that lessons of teachers who taught 3-5 grade levels had more opportunities for math talk than those of teachers who taught K-2 ( $\gamma_{01} = .22$ ,  $t = 3.74$ ,  $p < .001$ ). Although there were no grade-band differences in the within-teacher relationship between math talk and student use of representations (i.e., the StudentReps X GradeBand effect was not significant), there was a significant interaction effect of TeacherReps x GradeBand on Talk ( $\gamma_{21} = .04$ ,  $t = 2.45$ ,  $p < .05$ ). This denoted the combined effect of TeacherReps and GradeBand on Talk. This model accounts for 19% of the between-teacher variance and 9% of the within-teacher variance in opportunities for math talk.

Since the interaction (TeacherReps x GradeBand) effect was significant (see table 11, model 5), further analysis of simple slopes (grade-band differences in the relation between representations and math talk) and significance of contrasts (representations differences in the relation between grade band and math talk) was conducted (Cohen, Cohen, West, & Aiken, 2003). In this analysis, a high TeacherReps was simulated by the positive standard deviation of TeacherReps, and one negative standard deviation represented a low TeacherReps. Then, the talk score was estimated at each of the TeacherReps values (i.e., positive standard deviation and negative standard deviation). Subsequently, among lessons with high scores on teacher use of representations, those in upper grade levels (3-5) had more opportunities for math talk than those in lower grades (K-2) ( $p < .01$ ). This significant difference is illustrated in the difference between the heights of the grey and black bars in the right of figure 2. Specifically, it was lessons with

high scores of teacher use of representations in grades 3-5 that had the most opportunities for math talk. Furthermore, among lessons within grades K-2, those with high TeacherReps scores had more opportunities for math talk ( $p < .001$ ). Similarly, among lessons within grades 3-5, there was also a positive correlation between TeacherReps and Talk ( $p < .001$ ). This is similar to the result obtained from model 3, where GradeBand was not included, about the positive association between TeacherReps and Talk. What this result adds to the result obtained from model 3 is that when separating the sample lessons into the two grade bands, the positive association between TeacherReps and Talk still holds within each of the two groups of lessons. This last model explained 18% of between-teacher variance and 6% of within-teacher variance in opportunities for math talk.



*Figure 2. Significance of contrasts resulting from the cross-level interaction effect of teacher use of representations and grade band on talk.*

It is noteworthy that when grade band was added to model 3 as a teacher-level predictor, resulting in model 5, student use of representations was significantly correlated with math talk,

which contradicts with results from model 3. But when comparing that correlation (between StudentReps and Talk) obtained from model 5 to the correlation obtained after removing TeacherReps from model 5, that correlation was found to be significantly reduced after accounting for TeacherReps. Explicitly, when conducting Sobel test of the results obtained from model 5 and model 4 using the method explained earlier in this section (Kenny et al., 2003), it was confirmed again that the association between student use of representations and talk was significantly reduced when teacher use of representations was accounted for as a predictor of talk (Sobel test = 4.96,  $p < .001$ ). In other words, in both model 3 and model 5 teacher use of representations reduced the association between student use of representations and talk. But, whereas the association between StudentReps and Talk in model 3 (not accounting for grade band) was not significant, that association remained significant in model 5 (accounting for grade band), but was reduced by .04.

**Frequency of use of pictures and manipulatives (research question 4).** To address the fourth research question, model 6 was conducted with a lesson-level equation consisting of math talk as the outcome and frequency of using pictures and using manipulatives as level-1 input variables:

Model 6

Level 1 (lesson):  $Talk_{ij} = \beta_{0ij} + \beta_{1ij}(\text{Pictures}) + \beta_{2ij}(\text{Manipulatives})$

Level 2 (Teacher):  $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{GradeBand}) + u_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{GradeBand}) + u_{1i}$

$\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{GradeBand}) + u_{2i}$

The results displayed in table 11 (model 6) indicate that lessons with more frequent student use of pictures and diagrams were associated with more opportunities for math talk ( $\gamma_{10}$

$=.04$ ,  $t = 4.12$ ,  $p < .001$ ); similarly, lessons with higher frequencies of using hands-on tools were also those that included more opportunities for math talk ( $\gamma_{20} = .06$ ,  $t = 6.14$ ,  $p < .001$ ). Teachers varied significantly in the correlation between frequency of using pictures and opportunities for math talk ( $\tau_{11} = .00$ ,  $z = 3.73$ ,  $p < .001$ ), indicating that the relationship between frequency of using pictures and math talk does not look the same for all teachers. Similarly, the correlation between frequency of using tangible materials and math talk does not look the same for all teachers ( $\tau_{22} = .00$ ,  $z = 3.60$ ,  $p < .001$ ). However, there were no grade-band differences in the correlation between frequency of using pictures and math talk and between frequency of using tangible materials and math talk. This model explained 18% of the between-teacher variation and 7% the within-teacher variation in opportunities for math talk.

*Table 11. Unstandardized coefficients and standardized errors of multilevel models of use of representations and grade-band differences in opportunities in math talk*

Fixed Effects	Model 1 n=5162 Estimate(SE)	Model 2 n=5170 Estimate(SE)	Model 3 n=5166 Estimate(SE)	Model 4 n=5162 Estimate(SE)	Model 5 5162 Estimate(SE)	Model 6 n=5175 Estimates(SE)
intercept, $\beta_0$						
Intercept,	1.83***(.03)	1.82***(.03)	1.83***(.03)	2.93***(.04)	1.73***(.04)	1.73***(.03)
$\gamma_{00}$						
slope, $\beta_1$						
$\gamma_{10}$	.07***(.01)	.08***(.00)	.017(.01)	.76***(.02)	.02*(.01)	.04***(.01)
slope, $\beta_2$						
$\gamma_{20}$			.07***(.01)		.05***(.01)	.06***(.01)
Grade band,					.22***(.06)	.26***(.06)
$\gamma_{01}$						
Interaction 1,						
$\gamma_{11}$					-.2(.02)	-.01(.02)
Interaction 2,						
$\gamma_{21}$					.04*(.02)	.01(.02)
<b>Random Effects</b>						
intercept, $\tau_{00}$	.12***(.02)	.12***(.02)	.12***(.01)	.14***(.02)	.11***(.01)	.11***(.01)
$\tau_{10}$	.00(.00)	.00(.00)	.00(.00)	.00(.01)	.00(.00)	.00(.00)
$\tau_{11}$	.00***(.00)	.00***(.00)	.00*(.00)	.03***(.01)	.00*(.00)	.00***(.00)
$\tau_{20}$			.00(.00)		.00(.00)	.01**(.00)
$\tau_{21}$			-.00(.00)		-.00(.00)	.00(.00)
$\tau_{22}$			.00*(.00)		.00*(.00)	.00***(.00)
Within- teacher fluctuation, $\sigma^2$	.14***(.00)		.14***(.00)	.52***(.01)	.14***(.00)	.14***(.00)

Note: number of observations(lessons) vary due to missing values for some observations, all within 134 teachers  
 \* $p < .05$ , \*\* $p < .01$ , \*\*\* $p < .001$

## Discussion

**Limitations.** Although the results of this exploratory study can help better understand some aspects of the complex practice of teaching, some limitations should be taken into consideration when building on the findings of this study. One limitation is that the findings neither provide profound insight into the quality of the math talk and use of representations that occurred during the lessons, nor does it take into consideration the related student learning

outcomes. However, the target practices in this study (use of multiple representations and math talk) have been found by earlier studies to have a positive impact on student achievement; thus, the results provide implications about providing students with opportunities to learn.

Another limitation relates to the study measures. The use of representations and talk were investigated in the present study as taking place within a whole lesson, rather than relating to a specific task within a lesson. Although the IPL-M included items that inquired about the cognitive demand of mathematical tasks that were used during the instructional lessons, the participants logged about math talk and use of representations during the entire lesson, not in association with a task. There is a close relationship between the use of representations (Hardy, 2001) and types of tasks and between math talk and task types (Anderson-Pence, 2017; Stein & Lane, 1996). Therefore, examining within-task correlation between the use of multiple representations and opportunities for math talk was not feasible in this study. Subsequently, a follow-up investigation with more refined log items that allow the teacher to log about opportunities for math talk and use of representations for each task within a lesson rather than just logging across the entire lesson might add to the knowledge obtained from this study to better understand within-task correlation.

Reliance on self-reports is a primary concern for some; however, the decision to use data collected using the IPL-M is supported by Desimone's (2009) advice that, researchers should avoid "automatic biases either for or against" different forms of data collection, solely based on whether they were self-reported or not. Instead, researchers should focus on "evaluations and critiques of measurement instruments on the quality of their design and administration, according

to best practice, and on their appropriateness given a study's particular research question" (p. 192). A log with evidence of validity and score reliability, as in the case of the IPL-M (Walkowiak et al., under review), allows for generalizability of the sample and reduction of data collection and analysis costs in comparison to other instrument forms.

A final limitation relates to the homogeneity of the sample of teachers. The participating teachers were recruited for the study based on propensity score matching using pre-college entry variables—e.g., SAT and high school GPA, and they all came from public schools in southeastern U.S. Therefore, this homogeneity of the sample does not allow for generalizability to the population of elementary teachers in general.

**Findings.** It was hypothesized based on synthesis of literature on discourse and use of representations (e.g., Cengiz et al., 2011) that the use of multiple representations in elementary mathematics lessons is correlated to math talk that takes place during lessons of beginning teachers. The results support the assumption that beginning teachers' use of multiple and connected representations is associated with creating more opportunities for math talk in elementary mathematics lessons. It is important to caution that the significant correlation does not guarantee causation—I do not claim that using multiple representations by the teacher produces more opportunities for math talk. However, learning that lessons in which teachers use multiple and connected representations are also those with more opportunities for math talk suggests that one of the two dimensions might be influencing the other, either directly or indirectly. Nonetheless, that correlation between teacher use of representations and opportunities for math talk was found to be minute, yet significant. The same goes for the correlation between

pictures and talk, and manipulatives and talk—the correlation was minute. The small magnitude of the correlations is possibly because differences in opportunities for math talk are associated with many other factors that were not measured nor used as covariates in this investigation, such as ways of facilitating group work (Ball, 1991; Gillies, 2011) and specific types of questions teachers ask students (Hufferd-Ackles et al., 2004; Lampert, 1990). Furthermore, the variables for multiple representations in this study provided a score, representing the extent to which teachers and students used multiple representations and made connections among the representations, as reported by teachers. The use of multiple representations score does not measure other aspects of the use of representations, such as whether the use of representations was structured or unstructured (Hardy, 2001). It would be interesting to take into account aspects of representations beyond the ones included in this study when computing the representations scores. A follow up HLM model that includes such aspects of the use of representations might provide more insight about the correlation between use of representations and math talk.

Interestingly, the correlation between student use of representations and math talk was found to be mediated by teacher use of representations. Put differently, the correlation between student use of multiple representations and math talk was significantly reduced after accounting for teacher use of multiple representations in the HLM model. It was concluded that lessons with student use of multiple and connected representations were also those in which teachers used multiple and connected representations. Consequently, it seemed that the positive correlation that was found between student use of representations and math talk from the HLM model that did not take teacher use of representations into consideration was indirectly, and partially caused by

the correlation between teacher use of representations and talk. This finding of little association between student use of representations and opportunities for math talk in classrooms of beginning elementary teachers is in line with earlier results indicating that beginning teachers find it difficult to focus on the many demands that are expected from teachers, leading them to be overwhelmed (Schmidt, Klusmann, Lüdtke, Möller, & Kunter, 2017). In other words, beginning teachers may tend to focus on a few practices at a time, indicating that during lessons in which they attend to creating opportunities for math talk, they do not provide their students with many opportunities to use and connect among multiple representations. Or they might focus on student use of representations, but not on opportunities for math talk.

Differences between teacher use of representations and student use of representations in opportunities for math talk can be related to differences between students' and teacher's knowledge of how to use and connect among representations. Dreher and Kuntze (2015) explained that when teachers use representations, they usually move from one representation of a mathematical idea to another representation spontaneously and without much attention, given their previous knowledge of the connection among the different representations used. On the other hand, students' transition from one representation to another entails a higher cognitive demand and requires support, thus involves some difficulty, given the students' limited experience and knowledge of how each mode of representation relates to mathematical concepts. Since teachers usually have more knowledge and experience in how different modes of representations relate to mathematical concepts, their use of representations is usually more spontaneous and does not require high cognitive demand, as opposed to student use of

representations. A follow-up investigation of video recordings of lessons can help explain differences in the ways novice teachers differ in their instruction when they are using representations themselves versus how they orchestrate their students' use of representations.

Interestingly, mathematics lessons in upper elementary grades (3-5) were found to have more opportunities for math talk than lessons in the lower grades. To test whether there were grade-band differences in the correlation between use of representations, grade band (K-2 versus 3-5) was taken into consideration as a teacher-level predictor. The findings indicated that although there were no grade-band differences between frequency of using pictures or frequency of using manipulatives and opportunities for math talk, there were grade-band differences in the association between teacher use of representations and opportunities for math talk. That is, lessons with a high score of teacher use of multiple representations in grades 3-5 were reported to have the most opportunities for math talk among the sample lessons (see figure 2). This result could be explained by content differences across the two grade bands; the nature of some mathematics content, that is taught at the upper elementary classrooms, might facilitate creating opportunities for math talk. For example, counting lessons were mainly in lower grades (K-2); only 1% of the sample lessons had a primary focus on counting within teachers in grades 3-5, whereas 10% were within the lower grades (see table 16). An implication of this is the possibility that differences in math talk across upper and lower elementary grade bands are attributed to content differences (e.g., counting or not) among the two grade bands. A follow-up investigation that controls one of the two attributes (content and grade) might reveal more insight about the predictor that is mostly associated with math talk.

Differences in teacher attributes across the two grade bands, such as teachers' expectations of students based on the students' age, is another dimension that might have contributed to the grade-band differences in opportunities for math talk. This dimension was not investigated in the present investigations. Consequently, further studies regarding the role of teacher attribute differences across the two grade bands would be worthwhile in providing a better understanding of grade-band differences in opportunities for math talk. Investigating reasons behind grade-band differences that were found in this study can help researchers better understand reasons why some novice elementary teachers are more able than others to orchestrate math talk and/or use multiple representations and make connections among them in their lessons.

**Implications.** Orchestrating rich math talk indicates teachers putting student thinking at the heart of mathematics instruction. However, factors that contribute to beginning teachers' abilities to centralize student thinking are not completely understood (Singer-Gabella, Stengel, Shahan, & Kim, 2016). Researchers can build on the results of this study to better understand factors that enable beginning teachers to attend to and centralize students' thinking during mathematics lessons. There is great value in orchestrating rich math talk on students' mathematical thinking (Ball, 1991), yet teacher difficulty in creating such opportunities for math talk is acknowledged (Herbel-Eisenmann et al., 2013). Quality of mathematical discourse and its effect on student learning was not the purpose of the present study, rather the opportunities that beginning teachers create for students' math talk. Future studies can investigate instructional implementation through classroom observations to understand the richness of the discourse as

related to the productivity of using representations. The findings raise intriguing questions regarding the differences in teacher use of multiple representations and student use of multiple representations. A next step is to investigate how the teacher use of representations differed from the ways students used representations in their mathematical investigations.

Since this is an exploratory investigation, its implications are not directly towards teacher practice, rather they are primarily for future research. The existing literature on beginning elementary teachers is extensive and focuses primarily on challenges that beginning elementary teachers face (e.g., Bailey, 2015; Singer-Gabella et al., 2016) and on efficacy issues relating to novice elementary teachers (e.g., Brady, 2012; Hart, 2004; Santagata & Yeh, 2016). While most of those earlier studies focused on general issues relating to beginning teachers' practice, what we know about beginning elementary mathematics teachers' specific practices is largely based upon small-scale observational studies (e.g., Griffin, League, Griffin, & Bae, 2013). This larger-scale study confirms earlier evidence of the importance of using figures (Rau, Aleven, & Rummel, 2009) and manipulatives (Baxter et al., 2001) by finding that lessons with more frequency of using pictures and those with more frequency of using manipulatives are associated with more opportunities for math talk. It also extends our knowledge on beginning mathematics elementary teachers' practices by revealing a correlation between teacher use of representations and opportunities for math talk, which are two essential instructional actions for students' mathematical learning

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## CHAPTER 5

### **Study II: Math Talk and Use of Multiple Representations Controlling for MKT: A MLM Study on Elementary Mathematics Lessons**

#### **Abstract**

This exploratory study was conducted to investigate how teachers' mathematical knowledge for teaching (MKT) in number and operations relates to the use of multiple representations and math talk in elementary classrooms. Data was collected from 134 novice elementary teachers via a daily instructional log, the Instructional Practices Log in Mathematics (IPL-M). Due to the nested nature of the data, a multilevel linear model was employed to investigate if there were MKT differences in the relationship between math talk and representations used during mathematics lessons of novice elementary teachers, specifically, lessons with a primary focus on number and operations. The results indicated no MKT differences in the average opportunities for math talk, and no MKT differences in the correlation between use of representations and math talk. Furthermore, there was no association between MKT scores and use of representations in number-and-operations lessons. However, there was a significant interaction effect of student use of representations and MKT on opportunities for math talk, such that among lessons within teachers with high MKT scores, lessons in which students used multiple and connected representations had the highest opportunities for math talk, as reported by the teachers. Implications for future research are presented based on the results of this study.

## Introduction

Students should be exposed to learning environments in which they actively participate in mathematical argumentations (Staples & Newton, 2016), use multiple representations, and connect among different representations (Jackson & Cobb, 2010). Specifically, using multiple representations is an essential component of mathematical learning (National Council of Teachers of Mathematics, 2014; National Governors Association Center for Best Practices [NGA Center] and Council of Chief State School Officers [CCSSO], 2010). Furthermore, NCTM (2014) recommends orchestrating productive mathematical discourse among students in K-12 classrooms. A “powerful” mathematical discourse within classroom settings is one during which students actively participate in their own learning and thus are empowered by believing that they can do mathematics (Herbel-Eisenmann, Steele, & Cirillo, 2013). In such discourses, the channels of discussion are multidirectional, and students and the teacher reflect on and question each other’s ideas. Teachers’ strategies that foster student engagement in rich mathematical discourse can have significant implications for students’ mathematical learning (Ball, 1991; Steinbring, Bartolini-Bussi, & Sierpiska, 1998). However, many teachers face difficulty in including such discourse in their lessons (Herbel-Eisenmann et al., 2013; Kazemi & Stipek, 2001; Michaels & O’Conner, 2015; Peterson & Leatham, 2009; Staples, 2007), as it “requires considerable time and effort” (Li & Ni, 2011, p. 83).

Evidence suggests that elementary teachers’ mathematical knowledge for teaching (MKT), which is defined as “mathematical knowledge used to carry out the *work of teaching mathematics* (emphasis in original)” (Hill et al., 2005, p. 373), plays an important role in teacher

instructional practices (Ball 1990; Ma 2010; Post, Harel, Behr, & Lesh, 1991; Simon, 1993). However, the relationships between teacher knowledge, instruction, and student learning are not completely understood (Hill et al., 2008). This indicates a need to understand such relationships (e.g., the relationship between elementary teachers' MKT and the use of multiple representations). Based on similar arguments, investigating teachers' content knowledge as it relates to orchestrating mathematical discourse is recommended (Brendefur and Frykholm, 2000; Tofel-Grehl, Callahan, & Nadelson, 2017).

There is a large volume of published studies on mathematical discourse (Ryve, 2011) and the use of representations (Dreher & Kuntze, 2015) in mathematics classrooms, but a considerable amount of that earlier research has been qualitative rather than quantitative. Additionally, there is scarcity in the use of large-scale measures that provide information on instructional practices (Matsumura et al., 2006). This motivated the use of an instructional log, the Instructional Practices Log in Mathematics (IPL-M) (Walkowiak & Lee, 2013), to collect teacher-reported data on daily instructional practices. Given the nested nature of the data collected using the instructional log (lessons are nested within teachers), multilevel linear modeling (MLM) was used to investigate MKT differences in the association between use of representations and opportunities for math talk in elementary mathematics lessons. This study can add to the field by analyzing aspects of teachers' instructional practices obtained from a large sample of lessons.

### **Theoretical Assumptions**

This study assumes that social communication in classrooms can have an effect on children's cognitive processes (Wertsch, 1991). This assumption stems from Vygotskian's (1978) psychological perspective with respect to children's construction of knowledge, which suggests that children's learning occurs within social communications that shape their knowledge. The set of communications that happen during mathematics lessons includes math talk, which is the dependent variable of interest in this study. Since students' understanding of the mathematical concepts is influenced by the talk that takes place during instructional settings (Ryve, 2011), teachers' strategies in orchestrating discourse can influence those understandings. . Therefore, a teacher's knowledge (e.g., mathematical knowledge for teaching) can play an important role in extending students' thinking through orchestrating math talk (Brendefur & Frykholm, 2000; Cengiz, Kline, & Grant, 2011).

Students' opportunities to learn (OTL) (Carroll 1963; Porter, Kirst, Osthoff, Smithson, & Schneider, 1993) are influenced by the frequency and quality of practices that repeatedly occur in lessons (Rowan & Correnti, 2009; Kurz, Elliott, Kettler, & Yel, 2014). In the present study, scoring the participants' responses on the IPL-M items was informed by findings from mathematics education literature about the creation of OTL. This study assumes that students' engagement in math talk creates opportunities for their learning. Consequently, *Talk* will be referred to in this paper as *opportunities for math talk*. Similarly, the study assumes that students' and teacher's use of multiple representations and connecting among representations is an indicator of creating students' OTL.

## **Math talk and the Use of Mathematical Representations**

Mathematical discourse and multiple representations are two instructional domains that receive a great deal of attention in literature due to their substantial impact on students' learning (Dreher & Kuntze, 2015; Herbel-Eisenmann, 2011). Orchestrating thoughtful math talk that is multidirectional with and among all students can provide teachers with a window into students' thinking and understanding, which can help teachers achieve true assessment of students' knowledge (Sfard, Nesher, Streefland, Cobb, & Mason, 1998); empower students to believe that they can do mathematics (Herbel-Eisenmann et al., 2013); and promote students' higher-level thinking (Scherrer & Stein, 2013; Staples & Truxaw, 2010) and mathematical argumentation skills (Lampert, 1990). Given the importance of math talk during instructional lessons, it is the main dependent variable of interest in the present investigation.

Another construct of interest in the present study is the use of multiple representations including: (a) story problems that are formulated based on real-world situations; (b) tangible materials that are used for modeling mathematical ideas; (c) static pictures and diagrams; (d) tables; and (e) written symbolic representations. The use of different representations beyond, and in addition to, symbolic representations (e.g., words, pictures, diagrams, and tangible objects) has potential value for students' mathematical thinking and reasoning (Hardy, 2001). For instance, the use of tangible objects (or sometimes called manipulatives) was found by Baxter, Woodward, and Olson (2001) to aid engagement of students, whose achievement is below grade level, during mathematical investigations. Previous research has established benefits of connecting among different representations on students' mathematical learning (e.g., Hardy,

2001; Hiebert, 1984; Lesh et al., 1987). Benefits of using instructional tasks that involve using different and connected representations were highlighted by the results of a study that investigated the use of instructional tasks in four middle school classrooms (Stein & Lane, 1996). They investigated instructional opportunities for students' mathematical learning as they related to students' mathematical performance, and found that translating between different representations was an influencing factor on students' learning opportunities and subsequent performance.

Hiebert and Wearne (1993) concluded that, "instructional tasks and classroom discourse mediate the relationship between teaching and learning" (p. 420). Some aspects of using representations were found to be associated with the math talk that occur during lessons. For example, Hardy (2001) investigated two types of use of mathematical representations: low-structure representation tasks and high-structure representation tasks. Hardy found that most student talk associated with high-structure representation tasks that direct students as to which representations to use and explore, included less student talk and less argumentation occurrences than talk that was associated with low-structure tasks that allow students to choose and/or explore representations for the problem. Subsequently, longer sequences of math talk—in which patterns of discussion interaction is multidirectional and includes more than just teacher-student-teacher talk turns—gave opportunities for students to build deeper mathematical thinking based on the discussed ideas (Hardy, 2001). Another example of a relation between use of representations and math talk was illustrated by Cengiz et al. (2011), who demonstrated that aspects of a teacher's mathematical knowledge for teaching (MKT)—including knowledge in

making or exploring alternative representations of mathematical ideas and knowing the appropriate representations to be used in teaching particular mathematical ideas to students—are related to supporting students in producing mathematical explanations and reflections.

### **Teacher MKT**

Until 1999, research on teacher mathematical knowledge was lacking at-scale studies that investigated mathematical knowledge that elementary teachers need for teaching mathematics competently (Hill, Rowan, & Ball, 2005). Consequently, the *Study of Instructional Improvement* (SII) project was established in response to a need of an at-scale measure of teachers' "specialized knowledge of mathematics" (e.g., knowledge of using a diagram to represent a specific mathematical expression) (Hill et al., 2005). The *Learning Mathematics for Teaching* (LMT) project was initiated as a sister project to SII to continue the initiative to develop measures of mathematical knowledge for teaching (MKT) (Learning Mathematics for Teaching, 2004). The present study used the MKT assessment in Elementary Number and Operations (LMT, 2004) to measure elementary teachers' mathematical content knowledge for teaching number and operations.

A sizable amount of empirical studies have investigated the relation between MKT levels and teacher practices. A study by Cengiz et al. (2011) is an example of such inquiries. They investigated discussions during whole-group work in six elementary classrooms to better understand teachers' instructional activities that provide opportunities for students to extend their mathematical thinking. A teacher's MKT was found to be positively related to his or her facilitation of students' mathematical thinking during whole-group discussions. Similarly,

Rowland, Huckstep, & Thwaites (2005) suggested that teachers' MKT is related to their instructional actions, including their choice of representations during mathematical investigations. Hill et al. (2008) compared instruction across ten teachers with different MKT levels, and found the teachers' instructional practices to be correlated to teachers' MKT. The instructional practices they investigated included teachers': use of multiple and linked representations, mathematical errors (or not) during instruction, responses to students, and use of mathematical language. They concluded a significant positive correlation between MKT and quality of instruction. Similarly, a small-scale investigation of task selection and implementation of two elementary teachers revealed a positive association between the teachers' MKT levels and the cognitive level of their task enactment. The teacher with higher MKT maintained a higher cognitive level of the implemented mathematical tasks during her lessons as compared to the other teacher (Charalambus, 2010). In addition to correlation between MKT and teacher practice, Hill et al. (2005) found a positive correlation between first- and second- grade teachers' MKT and students' achievement.

Teacher content and pedagogical knowledge are important requirements for the complex practice of teaching mathematics (Guberman & Gorev, 2015). Given the assumption that, "content knowledge is immensely important to teaching and its improvement" (Ball, Thames, & Phelps, 2008, p. 404), investigating MKT's effect on teaching can elucidate valuable knowledge for teacher educators (Ball et al., 2008) as well as for researchers. MKT has been documented as having a pronounced impact on mathematics teachers' instructional practices (e.g., Hill et al., 2008). Acknowledging the documented influence of teachers' MKT on instructional practices

(Hill et al., 2008), the present study aims to explore the relation between teachers' MKT and two instructional domains of interest in elementary classrooms (i.e., the use of multiple representations and creating opportunities for math talk) as reported on the IPL-M.

In view of that, the hypothesis driving this investigation is that elementary teachers' MKT in number and operations has an impact on their instructional decisions during lessons focused on that mathematical strand. Such instructional decisions can in turn influence the frequency and quality of the use of multiple representations and the frequency of student math talk reported by teachers to have taken place during the lesson. MLM allows for testing this hypothesis about the effect of a teacher-level variable (i.e., MKT-N&O) on the relationship between lesson-level variables (i.e., talk and representations) (Raudenbush & Bryk, 2002).

Subsequently, the research questions of interest are: (1) Does the within-teacher relationship between *student* use of multiple representations and opportunities for math talk in elementary classrooms, in lessons for which the main focus is number and operations, depend on teachers' MKT?; and (2) Does the within-teacher relationship between *teacher* use of multiple representations and opportunities for math talk in elementary classrooms, in lessons for which the main focus is number and operations, depend on teachers' MKT?

## **Methodology**

Given the nested nature of the data structure (i.e., lessons are nested within teachers), multilevel linear modeling (MLM) was used to conduct the quantitative analyses. The use of MLM has worthwhile advantages over single-level techniques that are traditionally used. MLM characterizes variability within and between teachers (Hawkins, Guo, Hill, Battin-Pearson, &

Abbott, 2001), which helps obtain partitioning measures of variance in opportunities for math talk among level 1 (lessons) and level 2 (teachers) (Raudenbush & Bryk, 2002). As opposed to simpler models that address change at a single level, MLM is a powerful modeling technique due to its flexibility in representing change in data across multiple levels (Schulenberg & Maggs, 2001), and allows for modeling unbalanced data, thus testing the effect of varying number of instructional lessons on the relationship between opportunities for math talk and the use of representations within teachers (Raudenbush & Bryk, 2002). This characteristic of MLM allowed for including a larger number of the teacher participants in the analysis, rather than having to drop cases with fewer log entries. It also allowed including observations with partial non-response on an instructional lesson, which can maximize the power of the statistical test.

**Participants.** The participating teachers were elementary teachers in their second year of teaching in public schools in one state in the southeastern United States. Approximately 85% of the participating teachers are white, which is representative of the level of racial diversity among public school teachers in the U.S. (i.e., around 82% of public school teachers were reported to be white in 2011-2012 academic year (U.S. Department of Education, 2016)). A sample of 134 teachers participated in the study, logging a total of 2935 instructional lessons with a distinct primary focus on number and operations either in base 10 or fraction. This subsample of 2935 observations is used to answer the research questions of interest in this study. Participating teachers completed an average of 22 lessons each with a primary focus on number and operations, with a standard deviation of approximately 11 lessons. The breakdown of the grade levels of those instructional lessons is illustrated in table 12.

*Table 12. Frequency and percentage of lessons with primary focus on number and operations per grade level*

<u>Grade level</u>	<u>Frequency</u>	<u>Percent</u>
K	194	6.61%
1	522	17.79%
2	717	24.43%
3	389	13.25%
4	687	23.41%
5	426	14.51%

The teachers were trained on the use of the IPL-M over the summer during a face-to-face session. The log was completed by the teachers shortly after each instructional lesson using online link sent to them by e-mail. They were expected to each log for a total of 45 instructional lessons throughout their second year of teaching (more details can be found in Walkowiak, Adams, Porter, Lee, & McEachin, under review). Although, elementary teachers are expected to teach a variety of mathematical content besides number and operations (e.g. data, measurement, and geometry), this content-specific measure of MKT was used because much of the elementary mathematics curriculum is focused on number and operations (NGA Center & CCSSO, 2010). This was reflected in the study sample, where more than half (51%) of the observations had number and operations as the distinct primary focus of the lesson. The MKT assessment consists of 16 items that were designed for measuring teachers' mathematical knowledge for teaching number and operations. In addition to completing the IPL-M, the participating teachers completed the Mathematical Knowledge for Teaching Assessment in Number and Operation (MKT-N&O) (Learning Mathematics for Teaching, 2004) in the online Teacher Knowledge Assessment System (TKAS) during face-to-face sessions in the summer prior to their second

year of teaching. The MKT results were reported as IRT scores in standard deviation units from the national mean (LMT, 2004). For simplicity, the MKT acronym is used in this paper to denote Mathematical Knowledge for Teaching Number and Operations. The participating teachers' MKT scores ranged from -1.4639 to 1.954, with a mean of 0.4084 and a median of 0.4488. A score of zero in this measure indicates an MKT level equal to the national mean.

**Lesson-level variables.** A variable (Talk) was generated to represent a score given to each instructional lesson within teachers indicating the frequency of opportunities for students to engage in math talk during the lesson. The IPL-M items that were used to create the Talk variable are found in table 13. The score for each item ranged from 1 to 4, where 1 represents “not today,” 2 represents “little” (made up a relatively small part of the instruction), 3 represents “moderate” (made up a large portion, but not the majority of instruction), and 4 represents “considerable” (made up the majority of today’s instruction) (Walkowiak et al., under review). A high score of an item represents creating more opportunities for math talk. In general, the scores represent the frequency of discussions among students and between the teacher and the students.

*Table 13. Math talk dimension, including the question stem and sub-items within the question as presented to participants*

<u>Question stem</u>	<u>Items</u>
How much time did the students in the target class:	1) explain orally his/her thinking about mathematics problems? 2) talk about similarities and differences among various solution methods? 3) restate another student’s ideas in different words? 4) Pose questions to other students about the mathematics 5) Pose questions to the teacher about the mathematics 6) Discuss ideas, problems, solutions, or methods with other students in small groups or pairs 7) Discuss ideas, problems, solutions, or methods in large group (Walkowiak et al., under review, p. 47)

A confirmatory factor analysis (CFA) model was conducted for testing for the empirical strength of the items as representation of Talk variable. Subsequently, the standard coefficient loadings of each of items in table 13 to Talk variable ranged from .39 to .66, and were each significant ( $p < .001$ ). Consequently, Talk was generated for each observation (lesson) by averaging the seven talk scores.

*The use of multiple representations* dimension in the IPL-M includes items that represent *students'* use of multiple representations and making connections among different representations. Similarly, replicated items represented *teacher's* use of multiple representations and connecting among them. Lesh et al. (1987) outlined five representations that can be utilized by teachers and students to make sense of mathematical concepts: pictures, written symbols, oral language, real-world situations, and manipulative models. Informed by Lesh and colleagues, the IPL-M items obtained responses from teachers about the modes of representation used during instructional lessons (i.e. symbols, physical objects, story problems, or tables or charts) and about making connections among different representations. A numerical score {1, 2, 3, 4, 5} for each of the *student use of representations* and *teacher use of representations* variables was computed for each lesson in the sample based on the participants' responses on representation log items, where 1 represents less opportunities for multiple and connected representations and 5 represents higher opportunities (as described in Study I).

## **Analysis**

**Mean centering.** Centering around the grand mean (CGM) is a statistical method that was used to improve parameter estimation, in which a new centered variable is created by

subtracting the sample mean (grand mean) from the raw score (Kreft, Leeuw, & Aiken, 1995). Level-one predictors (student use of representations and teacher use of representations) were grand-mean centered before running the MLM analysis. Since representations scales have no meaningful zero value—the lowest representations score is 1, CGM created a meaning for a zero value, which represents the data point that is equal to the grand average *use of representations* for the sample of lessons. It should be noted that centering a variable affects the meaning of the resulting intercept obtained from running MLM. That is, the intercept becomes the expected value of Talk when the predictor (i.e., use of representations) equals the grand mean.

**The equations.** The fully unconditional model (also called the null model) was conducted as a preparatory step in the analysis (Raudenbush & Bryk, 2002). This model helps determine if there is sufficient within- and between-teacher variability to justify further multilevel analyses. Level-1 and level-2 linear equations for the null model are:

$$\text{Lesson level: Talk}_{ij} = \beta_{0ij} + r_{ij}$$

$$\text{Teacher level: } \beta_{0i} = \gamma_{00} + u_{0i}$$

Talk<sub>ij</sub> is the dependent variable representing the score for opportunities for math talk by teacher *i* during instructional lesson *j*.  $\beta_{0ij}$  represents the expected value of opportunities for math talk for teacher *i* and lesson *j*.  $r_{ij}$  is variation around the within-teacher average of Talk.  $\gamma_{00}$  represents a constant fixed effect of the sample (the sample average of the math talk score). Finally, the between-teacher fluctuation from the sample average in Talk is represented by  $u_{0i}$ .

The results obtained from the null model indicated that, the average level of Talk for the sample lessons is 1.86 ( $t = 55.40, p < .001$ ). This score indicates low to moderate opportunities

for math talk—the Talk score ranged from 1 to 4, where 1 represents lower opportunities for math talk and 4 represents higher opportunities for math talk. Furthermore, the Intraclass correlation coefficient (ICC) was computed based on statistics obtained from the null model (Raundebush & Bryke, 2002). Subsequently, 48% of the variability in math talk was between teachers ( $\tau_{\text{MKT}} = .14$ ,  $z = 7.55$ ,  $p < .001$ ) and 52% was within teachers ( $\sigma^2 = .16$ ,  $z = 37.44$ ,  $p < .001$ ). Consequently, the fully unconditional model suggests sufficient variability for further analyses.

To answer both research questions, the covariation of the three lesson-level variables as controlled by MKT can be expressed as:

$$\text{Lesson level: Talk}_{ij} = \beta_{0ij} + \beta_{1ij}(\text{StudentReps}) + \beta_{2ij}(\text{TeacherReps}) + r_{ij}$$

$$\begin{aligned} \text{Teacher level: } \beta_{0i} &= \gamma_{00} + \gamma_{01}(\text{MKT}) + u_{0i} \\ \beta_{1i} &= \gamma_{10} + \gamma_{11}(\text{MKT}) + u_{1i} \\ \beta_{2i} &= \gamma_{20} + \gamma_{21}(\text{MKT}) + u_{2i} \end{aligned}$$

Level 1 equation examines the within-teacher relationship between opportunities for math talk (Talk) and student use of representations (StudentReps) and teacher use of representations (TeacherReps). The level of opportunities for math talk for teacher  $i$  during lesson  $j$  (Talk<sub>ij</sub>) is a function of the average level of Talk, frequency of student use of representations during lesson  $j$  ( $\beta_{1ij}$ ), frequency of teacher use of representations during lesson  $j$  ( $\beta_{2ij}$ ), and a residual ( $r_{ij}$ ). The level-2 equations test for between-teacher differences in lesson intercepts and slopes. The intercept and slopes of level-1 equation are outcomes of level-2 equations, with fixed effects  $\gamma_{00}$ ,  $\gamma_{10}$ , and  $\gamma_{20}$  representing the average within-teacher talk intercept, the unique effect of student use of representations on talk, and the unique effect of teacher use of representations, respectively.

The fixed effects  $\gamma_{01}$ ,  $\gamma_{11}$ , and  $\gamma_{21}$  are the between-teacher differences in Talk as a function of MKT, the MKT differences in the within-teacher relationship between student use of representations and Talk, and MKT differences in the within-teacher relationship between teacher use of representations and Talk respectively. Lastly,  $u_{0i}$ ,  $u_{1i}$ , and  $u_{2i}$  are random effects that represent between-teacher variability in the corresponding average value of intercept or slope. That is,  $u_{0i}$  represents the degree to which teachers vary in the sample mean of math talk;  $u_{1i}$  is the degree to which teachers vary from the slope of math talk and student use of representations; and  $u_{2i}$  is the degree to which teachers vary from the slope of math talk and teacher use of representations.

## Results

For simplicity, throughout this section, the term *high use of representations*, will be used to denote using different representations and connecting among representations; whereas, *low use of representations* will be used to refer to little variety in the representations used and lack of connections among representations by either the students or the teacher. The results indicated that, although there were no MKT differences in the average opportunities for math talk scale between teachers ( $p = .84$ ), there was a significant interaction effect of student use of representations and MKT on opportunities for math talk ( $\gamma_{11} = -.03$ ,  $t = -2.42$ ,  $p < .05$ ). This model accounts for 14% of the between-teacher variability and 13% of the within-teacher variability in opportunities for math talk.

Since the interaction was significant (i.e., resulted from adding MKT at the teacher-level equation with  $\beta_{1i}$  as the outcome), further analysis of simple slopes (MKT differences in the

relation between representations and math talk) and significance of contrasts (representations differences in the relation between MKT and math talk) was conducted (Cohen, Cohen, West, & Aiken, 2003). As illustrated in figure 3, the regression model was used to estimate the Talk scores for the four conditions illustrated by the bars, where a high MKT or a high StudentReps score was represented by one positive standard deviation and a low MKT or StudentReps score was represented by a negative standard deviation. Talk was estimated for each of the four conditions using the regression model. The results indicated no MKT differences in the opportunities for math talk among lessons with more student use of representations. Similarly, there were no MKT differences in the opportunities for math talk among lessons with low student use of representations. However, there were contrast differences such that, among lessons within teachers with high MKT scores, lessons with high scores of student use of representations had more opportunities for math talk than those with low scores of student use of representations ( $p < .001$ ). Similarly, among lessons of teachers who had low MKT scores, those with high student use of representations score had significantly higher opportunities for math talk than those with low scores of student use of representations ( $p < .001$ ). As illustrated in figure 3, it was lessons with higher use of representations within teachers with high MKT scores that had the most opportunities for math talk (illustrated by the black bar located far right in figure 3). This model accounts for 21% of the between-teacher variability and 6% of the within-teacher variability in opportunities for math talk.

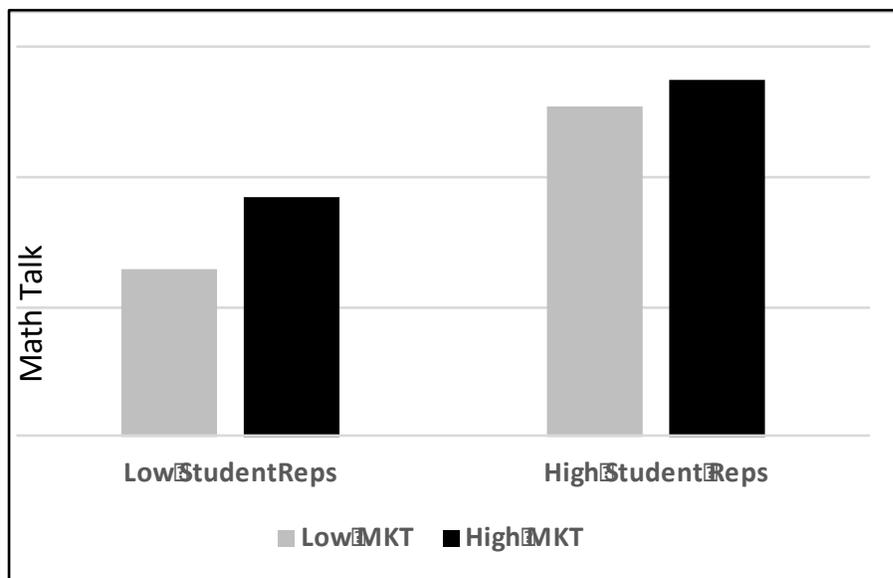


Figure 3. Cross-level interaction of student use of representations and MKT scores.

A follow-up investigation was conducted to test if teachers' MKT scores were correlated with the use of representations using the following models, where *student use of representations* and *teacher use of representations* were the dependent variables and *MKT* was the teacher-level predictor:

Model 1:

Lesson level:  $(\text{StudentReps})_{ij} = \beta_{0ij} + r_{ij}$

Teacher level:  $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{MKT}) + u_{0i}$

Model 2:

Lesson level:  $(\text{TeacherReps})_{ij} = \beta_{0ij} + r_{ij}$

Teacher level:  $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{MKT}) + u_{0i}$

The results indicated no correlation between teachers' MKT scores and student use of multiple representations in lessons with main focus on number and operations ( $p = .20$ ). The same result of no correlation was found for teacher use of multiple representations ( $p = .25$ ). In other

words, teachers' MKT scores did not seem to have a significant effect on the use of multiple representations in the sample lessons neither by students nor by teachers.

## **Discussion**

Before we discuss the findings and their implications, it is important to point out the study limitations. One limitation relates to the sample of teachers who participated in the study. Approximately 70% (93 teachers) of the teachers in the sample had MKT scores that are above the national mean MKT score. If the investigation was conducted with a different sample that is more representative of the population's mean, the findings might not be identical to the ones revealed in this study. Furthermore, the participating teachers were selected based on propensity score matching based on pre-college entry variables, such as SAT and high school GPA, and they all taught at public schools in southeastern U.S. Therefore, this homogeneity of the level-2 sample does not allow for generalizability to elementary teachers in general. Another aspect that might be considered a limitation is the self-report of the data. Memory issues might arise due to self-reported data; however, daily diary designs such as the one used in this study reduces problems relating to memory in reporting data, especially that data were reported shortly after the lesson implementation (Kurz et al., 2014).

This study set out to examine the hypothesis that beginning teachers' mathematical knowledge for teaching affected the resulting opportunities for math talk and teacher instructional decisions resulting in use of mathematical representations. Teachers' knowledge of how to use different modes of representations influences their enactment of the use of representation during lessons (Dreher & Kuntze, 2015). Consequently, teacher's MKT, which

includes their knowledge of using representations, can be related to their use of representations during lessons (Rowland et al., 2005). Furthermore, MKT can relate to some discourse practices that takes place during lessons (Hill et al., 2008). Contrary to what was hypothesized, teachers' MKT scores in number and operations made no significant difference in opportunities for math talk reported in lessons with a primary focus on number and operations. Similarly, teachers' MKT scores had no correlation with student use of multiple representations, nor with teacher use of multiple representations. These findings do not align with previously conducted studies which found positive correlations between teacher MKT and teacher instructional practices and decisions (e.g., Charalambous, 2010; Hill et al., 2008). It is noteworthy here that, unlike the present study, many of those earlier studies were conducted on a small scale and quality of instruction was largely based off observational data. Additionally, the correlation between teachers' MKT and instructional practices can be hindered or reinforced by factors such as teacher professional development, curriculum materials, and teachers' beliefs about teaching mathematics (Hill et al., 2008). A follow-up investigation that controls or takes into consideration such factors might help better understand the correlation between MKT and math talk or use of representations. Another interesting follow-up investigation is to compare the results of this study to results obtained from a sample of experienced teachers.

Interestingly, differences in opportunities for math talk were observed when contrasting the combinations of four groups of lessons with high and low scores of each of the two variables (MKT and student use of representations), as illustrated in figure 3. Specifically, it was lessons with high scores of student use of multiple representations for teachers with high MKT scores

that had the highest opportunities for math talk. This observation uncovers some interesting insight about the relationship between student use of representations and math talk. For the entire sample of lessons on numbers, there was no correlation between student use of representations and math talk ( $p=.08$ ). However, the homogeneity of the lessons with respect to MKT levels indicated a positive relationship between student use of representations and opportunities for math talk. This is an indication that exploring a homogeneous subsample with respect to teacher attributes (i.e., MKT levels) might be insightful and reveal different information.

Contrary to past research (Ball 1990; Ma 2010; Post, Harel, Behr, & Lesh, 1991; Simon, 1993) about a relation between MKT and teacher practices, this study found no correlation between MKT and the two dimensions of interest (i.e., math talk and use of representations). However, the study indicated that when homogeneity of the teacher attribute (i.e., MKT levels) in this sample of beginning teachers was taken into consideration, a positive correlation was found between math talk and student use of representations. The findings of this exploratory study generate several questions for further inquiry about the effect of MKT on teacher practice.

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## CHAPTER 6

### **Study III: Content Effect on Math Talk and Use of Representations: A MLM Study on Elementary Mathematics Lessons**

#### **Abstract**

A Multilevel Linear Modeling (MLM) methodology was used to investigate how the mathematical content taught in elementary grades relates to opportunities for math talk and the use of multiple representations. Data was collected from 134 beginning elementary teachers using a daily instructional log, the Instructional Practices Log in Mathematics (IPL-M). A total of 4,715 logs were investigated in this study. This study highlights content differences in math talk and use of multiple representations. Results from MLM models indicated that lessons with a primary focus on counting and cardinality had less opportunities for math talk than other lessons, but lessons on number and operations (whole numbers) had more opportunities for math talk compared to other sample lessons. There were no content differences in math talk when the lesson level predictor was fractions, data, or geometry. Exploring the content effect on use of multiple representations indicated that lessons with a primary focus on measurement and data included more use of multiple and connected representations by both students and teachers; whereas, geometry lessons were associated with less use of multiple representations. No content differences were found in the use of representations when counting and number were investigated as lesson-level predictors. The need for professional development is illustrated based on the findings.

## **Introduction**

NCTM's *Principles to Actions* (2014) and the Standards for Mathematical Practice in the Common Core State Standards for Mathematics (NGA CBP & CCSSO, 2010) outline the importance of discourse and the use of representations. The importance of mathematical discourse and using multiple representations is observed from the considerable amount of literature that has been published on the two constructs (e.g., Jackson & Cobb, 2010). Some earlier studies investigated mathematical discourse and using representations within the context of a specific mathematical content focus (e.g., Hiebert & Werne, 1993; Panaoura, 2014), while other studies investigated the two dimensions in mathematics classrooms in general—not in relation to a specific content focus of lessons—(e.g., Ardasheva, Howell, & Vidrio Magna, 2016; Baxter, Woodward, & Olson, 2001). The present study provides an exciting opportunity to advance our knowledge of mathematical discourse and using multiple representations by providing insight into mathematical content differences in math talk and use of multiple representations in elementary classrooms.

## **Theoretical Framework**

The idea of students' opportunities to learn (OTL) (Carroll 1963; Cooley & Leinhardt, 1980; Porter, Kirst, Osthoff, Smithson, & Schneider, 1993) motivated investigating two important instructional dimensions (i.e., math talk and use of representations) in beginning teachers' mathematics classrooms as reported using a daily log. Informed by the sociocultural perspective (Rogoff, 1997; Vygotsky, 1978), which suggest that social interaction in classrooms influence students' learning, this study attempts to obtain insight into opportunities created for

student learning of the different mathematical content strands. The theoretical framework motivating this study stems from the assumption that elementary students' opportunities to learn are impacted by their experiences with and exposure to mathematical discourse (Hufferd-Ackles, Fuson, & Sherin, 2004) and use of representations (Hardy, 2001). Frequency and rate of student exposure to practices of interest during a mathematics lesson are considered indicators of OTL. Using data collected using a log allows researchers to obtain measures that represent the "amount or rate of exposure to particular elements of instruction occurring over some fixed interval of time (e.g., a school year)" (Rowan & Correnti, 2009, p. 120). One way to conceptualize the idea of measuring OTL using log data is that since specific instructional dimensions (i.e., math talk and use of multiple representations) have been found by earlier empirical evidence to influence students' learning outcomes, then such practices can be thought of as potentially creating opportunities for student learning (Kurz, Elliott, Kettler, & Yel, 2014).

## **Literature Review**

Rich math talk in classrooms has observed benefits on students' mathematical learning (Tofel-Grehl, Callahan, & Nadelson, 2017); However, research documented teachers' difficulties (Michaels & O'Conner, 2015)—especially beginning teachers (Bennett, 2010; Boerst, Sleep, Ball, & Bass, 2011)—in orchestrating rich math talk in their classrooms. Although there is no simple fix for improving opportunities for discourse during mathematics lessons, earlier research initiatives explored practices that can help teachers create opportunities for mathematical discussions in their classrooms. These practices include, but are not limited to, students' explanation of their thinking to the teacher as well as to other students (Michaels & O'Connor,

2015), establishing talk moves during lessons, such as students' and teacher's restating and rephrasing mathematical ideas (Chapin, O'Connor, & Anderson, 2009), and students' posing of questions to the teacher as well as to other students (Boaler & Brodie, 2004), all within group-discussion settings.

Besides the discourse dimension (math talk), which is of substantial importance for students' mathematical thinking (Carpenter, Frank, & Levi, 2003; Cengiz, Kline, & Grant, 2011), the use of representation is another aspect of interest in the present investigation. In this study, using *representations* refers to using visual tools that stand for mathematical objects, concepts, or ideas. Different modes of representation can communicate different aspects of the represented object and relate to different kinds of thinking about the object. Therefore, "using multiple representations for the same mathematical situation are at the core of mathematical understanding" (Panaoura, 2014, p. 498). A teacher needs to encourage students to attend to modes of representations (e.g., figures), in addition to symbols (Walkowiak, 2014). Furthermore, translating and connecting between different modes of representations can enhance students' learning opportunities (Duval, 2006; Stein & Lane, 1996).

Much of the previous studies on discourse and use of representations in elementary mathematics classrooms have been either on classroom settings within specific mathematical strands or with no specific focus on a target content strand. For example, Hiebert and Wearne (1993) investigated incidents of math talk by both the teacher and students and frequency of use of different representations during whole-group discussions in six elementary classrooms. The unit of analysis in their study was specifically lessons on place value and whole-number addition

and subtraction. They concluded that using multiple representations and teacher questioning that require students' explanation of the mathematical ideas can encourage students' mathematical thinking and reflection on mathematical ideas. Panaoura's (2014) investigation of students' self-efficacy of using mathematical representations was also within the context of a specific mathematical strand (i.e., geometry). The students in her study completed an end-of-year survey about their self-efficacy for using representations to demonstrate their geometric understanding. Additionally, students' geometric knowledge was assessed using an end-of-year assessment. The results presented by Panaoura illustrated a correlation between students' beliefs about using different and connected representations and their geometric understanding.

Math talk and use of representations were also investigated by earlier studies within the context of elementary mathematics classes in general—without particular attention to specific content strands. One such study is a qualitative investigation of the discursive engagement of elementary students, who were recognized as having low achievement levels, during whole-group discussions (Baxter et al., 2001). The results indicated that those students' engagement was supported by the use of manipulatives. However, Baxter and colleagues observed particular challenges for students who had low achievements when presented with reform-based tasks—specifically, challenges that relate to verbal and social demands of the reform-based classroom activities. An example of a study that investigated math talk in elementary mathematics classroom is one by Hufferd-Ackles et al. (2004). They investigated the developmental trajectories in four aspects of math talk; i.e., (a) teacher questioning, (b) level of explanation of mathematical ideas, (c) owner of or producer of the mathematical ideas (teacher or student), and

(d) level of students taking responsibility for their learning. Hufferd-Ackles and colleagues produced developmental trajectories of what they called “math-talk learning communities” according to the four aspects, which are considered main aspects of productive discourse.

The present exploratory study contributes to the field by investigating math talk and use of multiple representations as they relate to the different mathematical content strands in elementary classrooms of beginning teachers. The hypothesis motivating this study is that the use of multiple representations and opportunities for math talk differ among lessons with different strand foci. Some mathematical content strands might lend themselves to using representations in certain manners (e.g., one expects frequent use of diagrams in a lesson with primary focus on geometry). Learning about differences or similarities in the practices that are associated with each strand focus can help researchers better understand ways to help teachers implement instructional practices. The following research questions are of interest in this study:

- 1) Is there a correlation between math talk and content focus of the lesson?
  - a) Are elementary lessons with a certain main mathematical content focus (i.e., number and operations, geometry, fractions, and measurement and data) different from lessons with primary focus on other mathematical content with respect to opportunities for math talk during those lessons?
  - b) How do lower (K-2) and upper (3-5) grades compare in terms of mean use of multiple representations and in terms of the strength of the Content-Talk relationship?
- 2) Is there a correlation between student use of representations and content focus of the lesson?

- a) Are elementary lessons with a certain main mathematical content focus (i.e., number and operations, geometry, fractions, and measurement and data) different from lessons with primary focus on other mathematical content with respect to the student use of multiple representations during those lessons?
  - b) How do lower (K-2) and upper (3-5) grades compare in terms of mean student use of representations and in terms of the strength of the Content-Representations relationship?
- 3) Is there a correlation between teacher use of representations and content focus of the lesson?
- c) Are elementary lessons with a certain main mathematical content focus (i.e., number and operations, geometry, fractions, and measurement and data) different from lessons with primary focus on other mathematical content with respect to the teacher use of multiple representations during those lessons?
  - d) How do lower (K-2) and upper (3-5) grades compare in terms of mean teacher use of representations and in terms of the strength of the Content-Representations relationship?

## **Methodology**

The daily teacher log used for data collection was the Instructional Practices Log in Mathematics (IPL-M), which development was informed by literature and the Standards for Mathematical Practice in the CCSS-M (Walkowiak, Adams, Porter, Lee, & McEachin, under review). In the present study, a subset of the data collected using the IPL-M was analyzed.

Particularly, the mathematical content taught as it relates to *math talk* and the *use of multiple representations* dimensions are of interest.

**Participants.** Participants of this study were 134 elementary teachers in their second year of teaching in public schools in a southeastern state in the U.S. (see table 14 for demographics of the participants). Approximately 85% of the participating teachers are white, which is representative the national racial diversity among public school teachers. (i.e., around 82% of public school teachers were reported to be white in 2011-2012 academic year (U.S. Department of Education, 2016)). The participating teachers attended face-to-face training on the use of the IPL-M during the summer preceding the data-collection school year.

*Table 14. Demographic characteristics of participating teachers*

<u>Gender</u>	<u>White</u>	<u>Black</u>	<u>Hispanic</u>	<u>Multiracial</u>	<u>Other race</u>	<u>Total</u>
Female	114	9	3	2	2	130
Male	1	0	0	0	1	2

*Note: gender and race data is missing for 2 participants*

**Data collection.** Data collection in this study utilized a measurement-burst design with an instructional log (Sliwinski, 2008), which makes feasible collecting details about lessons through a daily diary, but also employs benefits of a longitudinal design. Using a measurement-burst design, the elementary teachers in this study completed the IPL-M log for three “bursts” or time points of fifteen consecutive days throughout the school year. This design can help differentiate between within-teacher variability from day to day on one hand, and within-teacher variability over a longer time period (i.e., the academic year) (Nesselroade, 1991). The logging

time points were determined by the start of science units because science is typically not taught daily and participants were also logging about their science instruction. The median number of logged mathematics lessons was 41 instructional lessons. The log questionnaires were sent to the teachers as online links by email, which they completed shortly after a lesson was taught.

Validity evidence of IPL-M included: content review by experts in the field of mathematics education; cognitive interviews with participants to evaluate thinking processes when responding to items (Willis, 2005); factor analyses to examine each scale's reliability; and comparisons of teachers' IPL-M responses to a trained live observer's responses (Walkowiak et al., under review). The developers of the IPL-M denote that at least 10 days of logging provides a reliable estimate of instructional practices (Walkowiak et al., under review); therefore, participants who logged 10 or more days were included in the analysis for this study. Consequently, 5 teachers were dropped from the initial sample, resulting in a sample of 4715 lessons within 134 teachers (96% of original sample) in this study.

**Measures.** Five variables are of interest in the present investigation: opportunities for math talk (Talk), use of multiple representations by students (StudentReps), use of multiple representations by teachers (TeacherReps), the mathematical content strand taught (Content), and the grade band that the teacher teaches (Grade).

***Opportunities for math talk.*** Seven IPL-M items were considered as a measure of creating opportunities for math talk:

*How much time did the students in the target class:*

- 1) explain orally his/her thinking about mathematics problems?

- 2) talk about similarities and differences among various solution methods?
- 3) restate another student's ideas in different words?
- 4) pose questions to other students about the mathematics?
- 5) pose questions to the teacher about the mathematics?
- 6) discuss ideas, problems, solutions, or methods with other students in small groups or pairs?
- 7) discuss ideas, problems, solutions, or methods in large group? (Walkowiak et al., under review, p. 47)

Each of the seven items was given a score of 1 (not today), 2 (was present in relatively small part of the instruction), 3 (was present in a large portion, but not most of instruction), or 4 (was present most of instruction) (Walkowiak et al., under review). In general, the scores represent the frequency of discussions among students and between the teacher and the students, where a high score on an item represents creating more opportunities for math talk. A confirmatory factor analysis (CFA) resulted in standard coefficient loadings of each of the seven items to the math talk variable to range .39 to .66 ( $p < .001$ ). Subsequently, the Talk variable was generated by averaging the seven items for each observation (lesson) to represent a score for the creation of opportunities for math talk during the lesson.

***Use of multiple representations.*** The IPL-M included items to which the teachers responded about the modes of representation used during their lessons (i.e. symbols, physical objects, story problems, or tables or charts) and about making connections among different modes. To obtain the score for each of the two variables (*teacher use of representations* and

*student use of representations*), the participants responded to the following prompt by choosing among *yes* and *no* for each of the 6 sub-items:

The prompt: *What did the STUDENTS use to work on the mathematics today?*

The sub-items: (a) numbers and symbols; (b) tangible materials; (c) real-life situations or story problems; (d) pictures or diagrams; (e) tables or charts; (f) The students made explicit links between two or more of these representations (Walkowiak et al., under review).

In addition to asking the teachers about student use of multiple representations, they were given the prompt, *what did you (the teacher) use to work on the mathematics today?* The sub-items under this prompt were the same as those used to obtain a score on student use of multiple representations.

*Table 15. Description of how the scores for the use of representations variables were computed*

Representation items used to compute the two scores	Score representing <i>teacher</i> use of representation or <i>student</i> use of representations
if one of a-f items are selected OR one of a-e is selected in addition to item f	1
if two of a-e items are selected without item f	2
If three or more of a-e items are selected without item f	3
if two of a-e items are selected with item f	4
If three or more of a-e items are selected with item f	5

Since the six items (items *a* to *f*) are dichotomous, each resulted in an outcome of 1 (yes) or 0 (no) for each level-1 observation (lesson). The teachers' responses on the items were used to

compute a numerical score using a number that reflects the learning opportunities for students based on literature in mathematics education on best practices. The inclusion of more opportunities for using multiple representations (Lehrer & Schauble, 2004) and making connections and translations among different representations (Duval, 2006) during a lesson was considered an indication of creating more opportunities for students' learning in the use of representations. This informed the scoring of the two variables, *teacher use of representation* and *student use of representations* on a scale of 1 to 5, where 5 represents the highest opportunities for multiple representations and translating among them (see Table 15).

***Mathematical content taught.*** One of the items on the IPL-M for which the teachers logged stated, *to what extent was each of the following topics a focus of today's mathematics instruction with the target class?* For each of five mathematical content strands, the participants chose among: *not today*, *secondary focus*, and *primary focus*. The list of mathematical content strands are based on the *Common Core State Standards for Mathematics* (CCSS-M) (2010): (1) Counting and Cardinality (counting), (2) Number and Operations in Base Ten and/or Operations and Algebraic Thinking (number),<sup>2</sup> (3) Number and Operations- Fractions (fractions), (4) Data and Measurement (Data), and (5) Geometry. Abbreviations, presented in parentheses in the previous sentence, will be used when referring to these content strands. One should note that the reported data were not all mutually exclusive—some teachers reported more than one primary

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<sup>2</sup> These two contents from Common Core Standards were combined as one focus in the log because they are considered highly correlated.

focus of the lesson. For example, 218 lessons had both *counting* and *number* as primary foci of each lesson, 20 lessons were reported to have primary focus on both *number* and *fractions*, and 10 for *number* and *data*. Since there were observations (lessons) that had more than one primary focus (8% of the observations), only observations with a single primary focus were included in the analysis. The reasoning for this decision is to obtain a more accurate effect of the content of the lesson on the reported math talk variable, use of representations variables, and on the relationship between math talk and the use of representations. Table 16 includes the frequencies of lessons that had one distinct primary focus. Approximately one-half of these lessons had a primary focus on number and operations in base ten and/or operations and algebraic thinking. The remaining half of the lessons were split relatively close among the other four content strands (ranging from 11% to 16%).

*Table 16. Frequencies and relative frequencies of primary content focus of lessons within teachers split by grade band (K-2, 3-5)*

<u>Content strand</u>	<u>Number of lessons</u>			<u>Number of teachers</u>		
	K-2	3-5	Total	K-2	3-5	Total
Counting	490 (10%)	68 (1%)	558 (12%)	57 (43%)	12 (9%)	69 (51%)
Number	1,368 (29%)	1,015 (22%)	2,383 (51%)	77 (57%)	55 (41%)	132 (99%)
Fractions	63 (1%)	469 (10%)	532 (11%)	15 (11%)	45 (34%)	60 (45%)
Data	488 (10%)	256 (5%)	744 (16%)	60 (45%)	38 (28%)	98 (73%)
Geometry	291 (6%)	207 (4%)	498 (11%)	48 (36%)	32 (24%)	80 (60%)
Total	2,700	2015	4715			

Dummy codes of one and zero were created to represent each of the five mathematical content strands as a distinct primary focus of the lesson. In other words, a dichotomous dummy variable is created for the strand of interest, which has the values {0, 1}, where 1 represents lesson with primary focus on the strand of interest, and 0 represents other lessons in the sample.

Multilevel linear modeling (MLM) was used to conduct the analyses in the study due to its flexibility in representing change in data across the two levels of lessons within teachers (Schulenberg & Maggs, 2001). The data in the present study have a nested structure (i.e., lessons are nested within teachers), thus creating two sources of variability in the dataset; variability between teachers in opportunities for math talk and variability within teachers (between different lessons). Using MLM, we can obtain partitioning measures of variance in opportunities for math talk among level 1 (lessons) and level 2 (teachers) (Raudenbush & Bryk, 2002). MLM takes into consideration that estimates of regression models for each teacher might have similarities across teachers, consequently improving the estimate of a within-teacher regression model for each teacher. Since lessons are grouped within teachers, entries of each teacher are most probably not independent. A traditional linear model cannot be used because the assumption of independence is violated, but MLM does not require this assumption, rather it accounts for non-independence of the within-teacher observations (Hawkins, Guo, Hill, Battin-Pearson, & Abbott, 2001). MLM takes into consideration imbalances among the teachers, since it has built-in characteristics that allows for giving less weights for participants who logged less than other participants (Raudenbush & Bryk, 2002). This characteristic of MLM allows for including a larger number of the teacher participants in the analysis, rather than having to drop cases with fewer log entries.

Furthermore, using MLM allows for compensating for unbalance at the item level of log entries. That is, partial non-response on an instructional lesson, does not result in disregarding the entire observation for that instructional lesson.

### Analysis

Descriptive statistics for the three lesson-level variables (i.e., Talk, StudentReps, and TeacherReps) were obtained for the subsamples of lessons within each of the five content-strand foci, as illustrated in table 17.

*Table 17. The means and standard deviations of the variables within each of the content strands*

Content strand	Counting	Number	Fractions	Data	Geometry
Mean talk (sd)	1.65 (.47)	1.84 (.52)	1.99 (.55)	1.84 (.51)	1.80 (.53)
Mean StudentReps (sd)	2.62 (1.15)	2.81 (1.27)	3.3 (1.3)	3.19 (1.26)	2.57 (1.3)
Mean TeacherReps (sd)	2.67 (1.23)	2.84 (1.37)	3.34 (1.31)	3.2 (1.30)	2.61 (1.31)

Prior to conducting a conditional MLM model, obtaining a fully unconditional model is recommended as an initial step (e.g., Nezlek, 2001). In this model, only the intercept—which represents the mean dependent variable—is included with no other predictors. The purpose of running the fully unconditional model (also called the null model) is to ensure that there is sufficient within- and between-group variability to justify further analyses. Since we have three dependent variables (i.e., math talk, student use of representations, and teacher use of representations), the following three null models were conducted.

### **The unconditional model for opportunities for math talk.**

$$\text{Lesson level: Talk}_{ij} = \beta_{0ij} + r_{it}$$

$$\text{Teacher level: } \beta_{0i} = \gamma_{00} + u_{0i}$$

TALK<sub>ij</sub> is the dependent variable representing the score for opportunities for math talk by teacher *i* during instructional lesson *j*.  $\beta_{0ij}$  represents the expected value of opportunities for math talk for teacher *i* and lesson *j*.  $r_{it}$  is the average fluctuation around the within-teacher average of math talk score.  $\gamma_{00}$  represents a constant fixed effect of the sample (the sample average of the math talk score). Finally,  $u_{0i}$  stands for the between-teacher fluctuation from the sample average.

Results from the unconditional model indicated that the average level of Talk is significantly different than zero ( $\gamma_{00} = 1.82$ , at  $p < .001$ ). The results of the unconditional model included the statistics of the parameter of between-group variability ( $\tau_{00} = 0.14$ ,  $z = 7.89$ ,  $p < .001$ ) and the parameter representing within-teacher variability ( $\sigma^2 = 0.14$ ,  $z = 47.86$ ,  $p < .001$ ). The two statistics are used to calculate the intraclass correlation coefficient (ICC) using the formulas:  $\rho = \tau_{00} / (\tau_{00} + \sigma^2) = .48$  and  $1 - \rho = .52$  (Raundenbush & Bryke, 2002). Subsequently, there was 48% of variance in opportunities for math talk between teachers and 52% of variance in math talk within teachers. These results on variability within teachers is similar to earlier results indicating that teachers' practices vary from day to day (e.g., Rowan & Correnti, 2009).

### **The unconditional model for student use of representations.**

$$\text{Lesson level: StudentReps}_{ij} = \beta_{0ij} + r_{it}$$

$$\text{Teacher level: } \beta_{0i} = \gamma_{00} + u_{0i}$$

Results from the unconditional model indicated that the average level of student use of representations is significantly different than zero ( $\gamma_{00} = 2.87$ , at  $p < .001$ ). The results of the unconditional model indicated that there was 42% of variance in student use of representations between teachers ( $\tau_{00} = .70$ ,  $z = 7.82$ ,  $p < .001$ ) and 58% of variance was within teachers ( $\sigma^2 = .95$ ,  $z = 47.82$ ,  $p < .001$ ), subsequently justifying further analysis with StudentReps as the dependent variable.

**The unconditional model for teacher use of representations.**

Lesson level:  $\text{TeacherReps}_{ij} = \beta_{0ij} + r_{it}$

Teacher level:  $\beta_{0i} = \gamma_{00} + u_{0i}$

Results from the unconditional model indicated that the average level of teacher use of representations is significantly different than zero ( $\gamma_{00} = 1.90$ ,  $p < .001$ ). Subsequently, there was 55% of variance in teacher use of representations between teachers ( $\tau_{00} = .78$ ,  $z = 7.82$ ,  $p < .001$ ) and 45% of the variance was within teachers ( $\sigma^2 = 1.05$ ,  $z = 47.82$ ,  $p < .001$ ). Therefore, the fully unconditional model indicated that there was sufficient variability for further analyses with TeacherReps as the dependent variable.

**Talk-content correlation: Research question 1.** The MLM model used to investigate the first research question can be expressed using the equations displayed in model 1. For simplicity purposes, the variable *Geometry* is used here and in each of the following models as an illustration of the content predictor. The models will be repeated for each of the five content foci of lessons.

Model 1:

$$\text{Level 1 (lesson): Talk}_{ij} = \beta_{0ij} + \beta_{1ij} (\text{Geometry}) + r_{ij}$$

$$\begin{aligned} \text{Level 2 (teacher): } \beta_{0i} &= \gamma_{00} + \gamma_{01} (\text{GradeBand}) + u_{0i} \\ \beta_{1i} &= \gamma_{10} + \gamma_{11} (\text{GradeBand}) + u_{1i} \end{aligned}$$

Level-1 equation provides a test of the correlation between content focus on geometry and opportunities for math talk.  $\text{Talk}_{ij}$  is level of opportunities for math talk during lesson  $j$  for teacher  $i$ . *Geometry* is a dichotomous variable that indicates whether the main focus of the lesson is on geometry ( $\text{Geometry} = 1$ ) or on other strands ( $\text{Geometry} = 0$ ). Similarly, *GradeBand* is the predictor that determines the grade band that the teacher teaches (0 represents grade K-2 and 1 represents grades 3-5).  $\beta_{0ij}$  is the average talk when the main focus of the lesson is on a strand other than geometry ( $\text{Geometry} = 0$ ).  $\beta_{1ij}$  is the main effect of geometry as a content taught on Talk. Level-1 equation also includes the error term,  $r_{ij}$ , representing how much teacher  $i$  fluctuates in Talk from lesson to lesson.  $\gamma_{00}, \gamma_{10}$  are fixed effects representing the average within-teacher Talk intercept and the average unique effect of geometry as a primary focus on Talk respectively.  $\gamma_{01}$  represents grade-band differences in the average Talk and  $\gamma_{11}$  is the Content-GradeBand interaction effect, which illustrates if the Talk-Content relationship depends on grade-band differences between the teachers. Inputs of level-2 equations also include between-teacher variability in the Talk intercept ( $u_{0i}$ ) and the random effect of the  $i$ th teacher adjusted for whether the focus of the instructional lesson is geometry or not ( $u_{1i}$ ). In this model, lessons that did not have the content of interest as the main focus of the lesson are considered referent lessons in the model. For example, the predictor *Geometry* was assigned a value of one if the main focus of the lesson is geometry, but was coded zero if the main focus was *not* geometry.

Lessons with a primary focus on counting were mainly in lower grades (K-2); only 1% of the sample lessons had a focus on counting within teachers in grades 3-5 (see table 16).

Therefore, all models in this study with Counting as a predictor did not include GradeBand as a teacher-level predictor. Similarly, lessons with primary focus on fractions within K-2 consisted of only 1% of lessons in the study. Therefore, GradeBand was excluded as a predictor in models with Fractions as a lesson-level predictor of Talk.

**Representations-content correlation: Research questions 2 and 3.** Models 2 and model 3 were used to answer the second and third research questions. These models are the same as model 1 with different dependent variables. Model 2 has *StudentReps* (student use of multiple representations) as the dependent variable to investigate the effect of content on students' use of representations. In model 3, *TeacherReps* (teacher use of multiple representations) was entered as the dependent variable to investigate the effect of content on teacher use of representations during lessons.

Model 2:

$$\text{Level 1 (lesson): StudentReps}_{ij} = \beta_{0ij} + \beta_{1ij} (\text{Geometry}) + r_{ij}$$

$$\begin{aligned} \text{Level 2 (teacher): } \beta_{0i} &= \gamma_{00} + \gamma_{01} (\text{GradeBand}) + u_{0i} \\ \beta_{1i} &= \gamma_{10} + \gamma_{11} (\text{GradeBand}) + u_{1i} \end{aligned}$$

Model 3:

$$\text{Level 1 (lesson): TeacherReps}_{ij} = \beta_{0ij} + \beta_{1ij} (\text{Geometry}) + r_{ij}$$

$$\begin{aligned} \text{Level 2 (teacher): } \beta_{0i} &= \gamma_{00} + \gamma_{01} (\text{GradeBand}) + u_{0i} \\ \beta_{1i} &= \gamma_{10} + \gamma_{11} (\text{GradeBand}) + u_{1i} \end{aligned}$$

Similar to model 1, model 2 and model 3 were repeated for each of the other four strands as primary focus of the instructional lessons that teachers reported.

## Results

Running the three models resulted in statistics that are explained in this section. For simplicity, when referring to how much the teacher or students were using multiple representations and translating among them (the variables StudentReps and TeacherReps), the terms *high* use of representations and *low* use of representations will be used. In other words, using different representations and connecting among them indicates a higher score of the use of representations variable, and little variety in the representations used and lack of connections among representations indicate a lower representations score.

**Talk, content, and grade band.** As illustrated in table 18, the results from running model 1 indicates that lessons with a primary focus on counting have an average math talk score that is .07 less than lessons with other content foci ( $\gamma_{10} = -.07$ ,  $t = -2.58$ ,  $p < .05$ ). Neither fraction, data, nor geometry as a primary focus of mathematics lessons was related to opportunities for math talk during the lesson.

Based on the results obtained from the Number model (see table 18), across all lessons, lessons within grades 3-5 had a math-talk scores that is on average .28 higher than lessons in grades K-2 ( $\gamma_{01} = .28$ ,  $t = 4.25$ ,  $p < .001$ ). That positive effect of teaching upper grade levels (3-5) was confirmed by each of data and geometry models as well as seen in table 18 ( $\gamma_{01}$  is significantly positive). Across all lessons, lessons with a primary focus on number had a math talk score that is .08 higher as compared to lessons with other content foci ( $\gamma_{10} = .08$ ,  $t = 3.63$ ,  $p$

<.001). Furthermore, the Content X GradeBand effect on math talk was significant ( $\gamma_{11} = -.11$ ,  $t = -2.95$ ,  $p < .01$ ).

*Table 18. Unstandardized coefficients and standardized errors of multilevel models of content and grade-band differences in opportunities in math talk*

Fixed Effects	Counting Coefficient(SE)	Number Coefficient(SE)	Fraction Coefficient(SE)	Data Coefficient(SE)	Geometry Coefficient(SE)
Math talk level, $\beta_0$					
Intercept, $\gamma_{00}$	1.83***(.03)	1.69***(.04)	1.82***(.04)	1.7***(.04)	1.7***(.04)
Grade band, $\gamma_{01}$		.28***(.07)		.22**(.07)	.24***(.06)
Content slope, $\beta_1$					
Intercept, $\gamma_{10}$	-.07* (.03)	.08***(.02)	.03(.03)	-.05(.03)	-.02(.03)
ContentXgrade, $\gamma_{11}$		-.11**(.04)		.05(.04)	-.06(.05)
Random Effects					
Math talk level, $\tau_{00}$	.14*** (.02)	.14***(.02)	.14***(.02)	.13***(.02)	.13***(.02)
Slope, $\tau_{10}$	-.02(.01)	-.01(.01)	.01(.01)	-.01(.01)	-.00(.01)
Slope, $\tau_{11}$	.02*(.01)	.02***(.00)	.04***(.01)	.02**(.01)	.02**(.01)
Within-teacher fluctuation, $\sigma^2$	.14***(.00)	.14***(.00)	.14***(.00)	.14***(.00)	.14***(.00)

Note: \* $p < .05$ , \*\* $p < .01$ , \*\*\* $p < .001$   
 n = 4715 observations within 134 teachers

To further interpret this significant interaction, an additional model was conducted (Cohen, Cohen, West, & Aiken, 2003). This model tested simple slopes (grade-band differences in the relation between content and math talk) and significance of contrasts (content differences in the relation between grade band and math talk). As illustrated in figure 4, the results indicated that among lessons with a primary focus on number, those in grades 3-5 had more opportunities for math talk than those in grades K-2 ( $p < .01$ ). Among lessons in grades K-2, those with primary focus on number included more opportunities for math talk than those with other content foci ( $p < .05$ ). However, among lessons in grades 3-5, there was no difference in math talk

between lessons with focus on number and lessons with other strands. This model explained 5% of the between-teacher variance and 3% of the within-teacher variance in math talk.

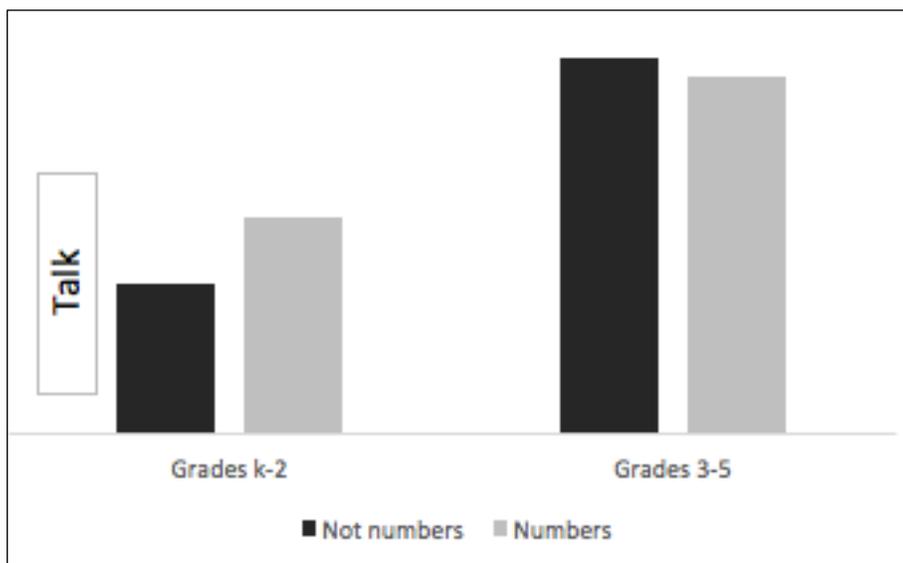


Figure 4. Cross level interaction of content (number and operations) and grade band on talk

**Student use of representations, content, and grade band.** Model 2 was conducted for each of the five mathematical content strands. As illustrated in table 19, each of data, fractions, and geometry had a significant effect on student use of representations as a primary content. Specifically, lessons with data as a primary focus were .33 higher in the student use of representations score than lessons with a primary focus other than data ( $\gamma_{10} = .33$ ,  $t = 4.17$ ,  $p < .001$ ). Lessons on fractions were .39 higher in the score of student use of representations than lessons with other content foci ( $p < .001$ ). On the other hand, geometry lessons were .29 lower in the student use of representations score than other lessons ( $p < .01$ ). For lessons with primary focus on number, data, and geometry, there was no grade-band differences in the relation between content and student use of representations. On the other hand, there was a Content X

GradeBand interaction effect when the primary focus was investigated as number and operations (1) or other (0). Consequently, a test of simple slopes and significance of contrasts was conducted, and the results indicated no grade-band differences in the relation between student use of representations and number as a content focus. However, among lessons within grades 3-5, lessons with primary focus on number have a significantly lower student use of representations score than lessons with content other than number (see figure 5).

*Table 19. Unstandardized coefficients and standardized errors of multilevel models of content and grade-band differences in student use of representations*

Fixed Effects	Counting Coefficient(SE)	Number Coefficient(SE)	Fraction Coefficient(SE)	Data Coefficient(SE)	Geometry Coefficient(SE)
Reps level, $\beta_0$					
Intercept, $\gamma_{00}$	2.87***(.07)	2.92***(.11)	2.8***(.7)	2.84***(.10)	2.83***(.10)
Grade, $\gamma_{01}$		.12(.17)		-.07(.15)	-.05(.15)
Content slope, $\beta_1$					
Intercept, $\gamma_{10}$	-.09(.06)	-.03(.07)	.38***(.10)	.33***(.08)	-.29**(.10)
ContentXgrade, $\gamma_{11}$		-.35**(.12)		.11(.13)	-.13(.15)
Random Effects					
Reps level, $\tau_{00}$	.72***(.09)	.83***(.11)	.68***(.09)	.71***(.09)	.70***(.09)
Slope, $\tau_{10}$	-.11(.06)	-.20***(.06)	-.03(.08)	-.04(.05)	-.03(.07)
Slope, $\tau_{11}$	.07(.05)	.28***(.05)	.37***(.10)	.17***(.05)	.21**(.07)
Within-teacher fluctuation, $\sigma^2$	.94***(.02)	.87***(.02)	.90***(.02)	.92***(.02)	.92***(.02)
Note: * $p < .05$ , ** $p < .01$ , *** $p < .001$ n = 4708 observations within 134 teachers					

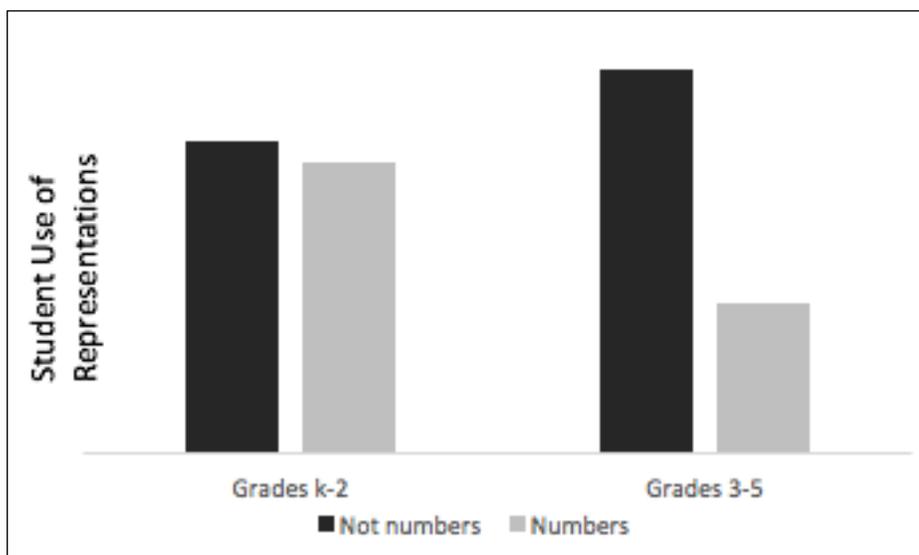


Figure 5. Cross level interaction of content (number and operations) and grade band on student use of representations

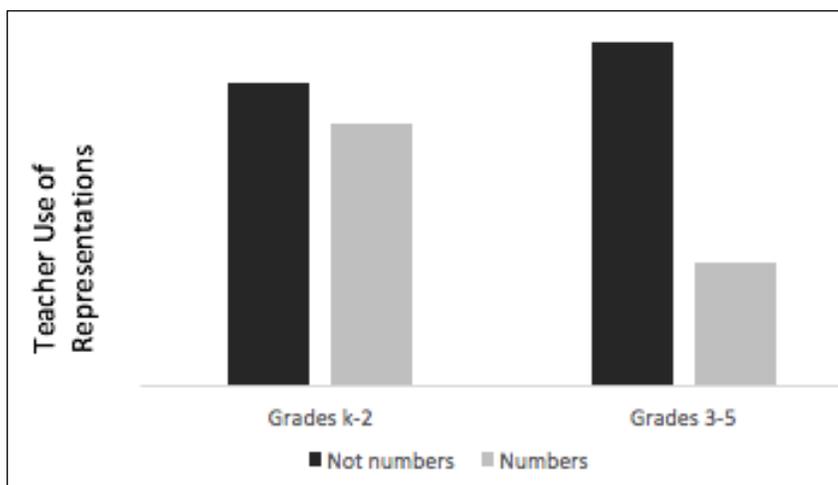
**Teacher use of representations, content, and grade band.** When teacher use of representations was entered as the dependent variable of interest in the MLM model (model 3), the results were similar to those obtained from model 2 (i.e., student use of representations as the dependent variable). Whereas counting and number as content taught had no effect on the teachers' use of representations, data, fractions, and geometry had a significant effect (see table 20). That is, lessons with a primary focus on fractions had a student use of representations score that is .39 higher than other lessons ( $p < .001$ ); those with primary focus on data were .29 higher in the score of teacher use of representations than other lessons ( $p < .001$ ); but geometry lessons had a teacher use of representations score that is .21 lower than lessons with other content foci ( $p < .05$ ).

Table 20. Unstandardized coefficients and standardized errors of multilevel models of content and grade-band differences in teacher use of representations

Fixed Effects	Counting Coefficient(SE)	Number Coefficient(SE)	Fraction Coefficient(SE)	Data Coefficient(SE)	Geometry Coefficient(SE)
Reps level, $\beta_0$					
Intercept, $\gamma_{00}$	2.91***(.08)	2.96***(.11)	2.85***(.10)	2.90***(.10)	2.96***(.10)
Grade, $\gamma_{01}$		.06(.17)		-.08(.16)	-.04(.16)
Content slope, $\beta_1$					
Intercept, $\gamma_{10}$	-.08(.06)	-.05(.08)	.39***(.09)	.29***(.07)	-.21*(.10)
ContentXgrade, $\gamma_{11}$		-.25*(.12)		.08(.12)	-.30*(.15)
Random Effects					
Reps level, $\tau_{00}$	.80***(.10)	.90***(.12)	.77***(.10)	.80***(.10)	.79***(.10)
Slope, $\tau_{10}$	-.12(.06)	-.16**(.06)	-.03(.09)	-.03(.05)	-.03(.07)
Slope, $\tau_{11}$	.07(.05)	.27***(.05)	.29***(.08)	.11**(.04)	.19**(.07)
Within-teacher fluctuation, $\sigma^2$	1.04***(.02)	.98***(.02)	1.01***(.02)	1.03***(.02)	1.02***(.02)

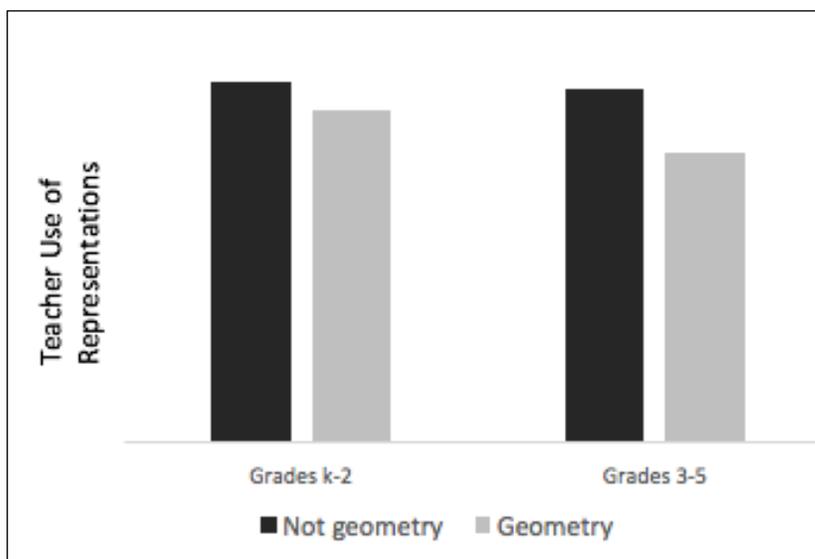
Note: \* $p < .05$ , \*\* $p < .01$ , \*\*\* $p < .001$   
n = 4707 observations within 134 teachers

Although there were no grade-band differences in the teacher use of representations, there was a Content X GradeBand interaction effect on the teacher use of representations in both the number model ( $\gamma_{11} = -.25$ ,  $t = -2.12$ ,  $p < .05$ ) and the geometry model ( $\gamma_{11} = -.30$ ,  $t = -2.02$ ,  $p < .05$ ). Further analysis of the interaction using a model testing for simple slopes and significance of contrasts (Cohen et al., 2003) indicated no grade-band differences in the correlation between number as a content and teacher use of representations; however, among lessons in grades 3-5, number lessons were lower in the score of teacher use of representations than lessons with other content (This is illustrated in the difference between the grey and black columns at the right of figure 6).



*Figure 6. Cross-level interaction of content (number and operations) and grade band on teacher use of representations*

Similarly, there was no grade-band differences in the correlation between geometry as a content and teacher use of representations; however, among lessons in grades 3-5, geometry lessons were lower in the score of teacher use of representations than lessons with other strands (This is illustrated in the difference between the grey and black columns at the right of figure 7).



*Figure 7. Cross-level interaction of content (geometry) and grade band on teacher use of representations*

## Discussion

The goal of this study was to examine mathematical content (i.e., counting, number, fractions, data/measurement, and geometry) differences in opportunities for math talk and the use of multiple representations within a sample of beginning elementary teachers. The results of the investigation are twofold: First, they provide insight into the frequency and duration of the two dimensions of math talk and using representations that take place in elementary mathematics lessons while zooming into each of the content strands. Additionally, the results allow us to explore differences between lower elementary grades (K-2) and upper grades (3-5) with respect to the two dimensions, knowing that grade band is closely related to the mathematical content taught.

It is important to acknowledge limitations of the present investigation before discussing the findings. One obvious limitation of this study is the lack of student achievement data on each

of the mathematical content strands of interest to provide evidence for the effects of using multiple representations and providing opportunities for math talk on student learning. Teacher implementation of certain practices (i.e., orchestrating discourse and using multiple representations), which are reported in a log, do not guarantee that desired learning and teaching outcomes are secured. However, one way to conceptualize the results is that earlier empirical evidence suggests that the two instructional dimensions of math talk and representations can influence the quality of mathematics instruction and student learning, thus can create opportunities for student learning (Kurz et al., 2014).

Another limitation relates to the number of observations (lessons) within each content focus of the lessons. The sample had imbalances at the lesson level with respect to the content focus of the lesson (see table 16). The subsample of lessons with main focus on *number and operations in base ten and/or operations and algebraic thinking* was the largest as compared to other content foci (2,383 lessons). On the other hand, lessons with other content foci each ranged from 498 to 744 lessons. Although these numbers can be considered acceptable to result in reliable results, the large difference in the observation size as compared to lessons with main focus on *number and operations in base ten and/or operations and algebraic thinking* indicates more power in the analysis of the number lessons as compared to the analysis of other lessons.

Finally, there was another limitation that relates to the homogeneity of the level-2 study sample (i.e., teachers). The findings of this study are constructed based on logs provided by second-year teachers who had been recruited based on propensity score matching based on pre-college entry variables—e.g., SAT and high school GPA. Additionally, they all came from

public schools in southeastern U.S. Therefore, this homogeneity of the sample does not allow for generalizability to elementary teachers in general.

**Counting and cardinality.** Lessons with a primary focus on counting and cardinality were associated with less math talk during the lesson in comparison to other content strands. Counting lessons were mainly within lower grades (K-2)—see table 16. Consequently, this difference in math talk could be associated with either content or grade-band. However, there was no differences for counting/cardinality lessons in the use of multiple representations.

**Number and operations in base ten and/or operations and algebraic thinking.** When comparing lessons with a main focus on number—which constitute a large portion of the sample as expected from elementary grades curriculum (NGA Center & CCSSO, 2010)—with other lessons, lessons on number had more opportunities for math talk. There were grade-band differences in opportunities for math talk among lessons with primary on number, such that number lessons in the upper grades (3-5) were reported to have more opportunities for math talk than number lessons in grades K-2. With respect to the use of representations by both the students and teachers in the sample lessons, lessons with main focus on number were not different from lessons with other strands in the use of multiple representations. However, among lessons within grades 3-5, those with a primary focus on number were reported to have less students' and teachers' use of multiple representations than other lessons in the sample (see figures 4 and 5).

**Number and operations (fractions).** Students are expected to think of fractions as numbers starting in third grade; that is, it becomes a content strand within the CCSS-M in third

grade (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). There were no differences in opportunities for math talk between lessons on fractions and other lessons in the sample. With respect to students' and teachers' use of multiple representations in fraction lessons, there was a positive association. In other words, comparing to other lessons in the sample, fraction lessons were reported to have more opportunities for using multiple and connected representations by both students and teachers. This is a promising finding, especially given that earlier empirical evidence suggested that students' learning of fractions was aided by using multiple and connected representations (Rau, Alevan, & Rummel, 2009).

**Data and measurement.** Similar to the findings on lessons with a main focus of fractions, lessons on data and measurement did not differ from other lessons with respect to the reported opportunities for math talk. With respect to the use of multiple representations, both students' and teachers' use of multiple representations were reported to be higher for lessons with main focus on data/measurement compared to lessons on other content strands. It is worth noting that this positive correlation between data/measurement as a content and use of representations varied significantly across teachers. In other words, the relationship between data/measurement as a main focus of the lesson and the use of representations during the lesson did not look the same for all teachers.

**Geometry.** Opportunities for math talk were not significantly different when comparing geometry lessons to lessons with other content foci. When students are presented with a geometric idea by the teacher, their understanding of the concept is often influenced by the

representation (e.g., figure) used than by the verbal explanation. Therefore, the structure of the representation is an important aspect that has implications for students' performance in geometry (Panaoura, 2014). However, lessons with main focus on geometry included less student use of multiple and connected representations compared to lessons with content foci other than geometry. Similarly, teacher use of representations was negatively correlated to geometry as a content, such that, lessons with geometry as a main focus were reported to have less teacher use of multiple and connected representations comparing to other lessons in the sample. It is important for teachers to use multiple and connected representations during lessons (Drehre & Kuntze, 2014); Furthermore, Huffered-Ackles et al. (2004) illustrated the importance of students' taking responsibility for their own learning, which can be in the form of students' meaningful use of multiple, connected representations. This result raises a need for professional development that is specifically geared towards helping beginning elementary teachers use and provide opportunities for using multiple, connected representations during geometry lessons.

## **Conclusion**

This study has three main findings. First, lessons with main focus on counting and cardinality had less opportunities for math talk than other lessons; however, lessons on number and operations had more opportunities for math talk than other lessons. Secondly, each of fraction lessons and data/measurement lessons included more use of multiple, connected representations as compared to other lessons in the sample; however, geometry lessons included less student and teacher use of multiple representations than other lessons. Since there were content differences in math talk and use of representations when comparing each strand to the

remaining four strands, it is important to compare pairs of strands to obtain a better understanding about content differences in the two constructs. A natural next step is to investigate individual content strands to each other in pairs.

Finally, given the close relationship between content and grade band, grade-band differences in the two variables of math talk and use of representations was investigated in this study. Interestingly, lessons within the upper elementary grades were found to have more opportunities for math talk than lower elementary grades. It is important to note that lessons with main focus on counting were those with less opportunities for math talk. Since lessons with main focus on counting were mainly in lower grades, as illustrated in table 16, math talk differences could be attributed to either grade band or content (i.e., counting or other). An interesting follow-up observational investigation is to analyze math talk that took place in classroom episodes within lower elementary grades as compared to upper elementary grades, with special attention to math talk in counting lessons as opposed to other lessons.

Orchestrating mathematical discourse is particularly difficult for beginning teachers; however, providing professional development—particularly mentoring—has been shown to positively impact beginning teachers' orchestration of mathematical discourse (Bennett, 2010). The findings of this exploratory study point to possible professional development needs of beginning elementary teachers, particularly this sample of elementary teachers. Less opportunities for engaging in math talk during lower elementary lessons and less opportunities for using multiple representations during geometry lessons imply a need for professional development with specific foci. There seems to be a need for discourse-specific professional

development geared towards K-2 teachers and professional development opportunities focused on using multiple representations in geometry lessons. More research is needed to determine if these professional-development needs exist for other beginning elementary teachers; nonetheless, the use of instructional log data to identify the professional development needs of teachers is a promising and efficient methodology that can be replicated by others.

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## CHAPTER 7

### Summary and Conclusion

Orchestrating math talk and using multiple, connected representations are important aspects of mathematics instruction as highlighted in literature (e.g., Jackson & Cobb, 2010) and by policy documents (e.g., NCTM's *Principles to Actions*, 2014). Given that beginning teachers are faced with different challenges than more experienced teachers (Neergard & Smith, 2012, p. 4), the present investigations were conducted to better understand beginning teachers' practices based on log data reported by a sample of second-year elementary teachers. Particularly, the findings provide insight into math talk and the use of multiple representations in beginning teachers' elementary classrooms. This chapter provides a summary of the findings obtained from the three studies that were presented in chapters 4, 5, and 6. This summary is organized around the three main research goals of this study.

#### **Research Goal 1: Is There a Relationship Between Use of Representations and Opportunities for Math Talk?**

In study I, it was hypothesized that the use of multiple representations in elementary mathematics lessons might be related to student math talk that takes place during the lessons. The results support the assumption that teacher use of multiple representations and making connections among the different representations is positively correlated with creating opportunities for math talk in elementary mathematics lessons. However, when accounting for teacher use of representations, there was no evidence that student use of multiple representations was correlated with opportunities for math talk. Further investigations into the ways students

used representations might reveal insights into differences between student use of representations and teacher use of representations as correlated with opportunities for math talk. This finding of lack of association between student use of representations and opportunities for math talk in classrooms of beginning teachers is in line with earlier results indicating that beginning teachers find it difficult to attend to the many overwhelming demands that are expected from them (Schmidt et al., 2017). Such feelings of being overwhelmed by beginning teachers might lead them to attend to less practices at a time; for example, when they focus on creating opportunities for math talk during a lesson, they might neglect providing students with opportunities to use multiple representations in the same lesson, and vice versa. However, in lessons that teachers used multiple representations, they also provided opportunities for math talk. Differences between teacher use of representations and student use of representations might be attributed to teacher comfort and authority issues. It might feel less risky for teachers to transition from one representation to another, than to try to facilitate students' use of multiple representations, especially given teachers' ability to spontaneously—and sometimes with little cognitive demand—move from one transformation to another (Dreher & Kuntze, 2015).

### **Research Goal 2: Are there MKT Differences in Math Talk and Use of Representations?**

Given previous empirical evidence that teachers' mathematical knowledge for teaching mathematics influences teachers' practice (Ball 1990; Dreher & Kuntze, 2015; Ma 2010), study II investigated the hypothesis that use of multiple representations and opportunities for math talk during number-and-operations lessons depend on teachers' MKT levels in number and operations. Surprisingly and contrary to what was hypothesized, teachers' MKT scores in

number and operations were not correlated with opportunities for math talk nor with using multiple representations in lessons focused on number and operations. Other teacher-level factors that were not taken into consideration in the present investigation, such as teacher professional development experiences, might have contributed to the differences in math talk and use of representations. Given the nature of instructional logs, the teachers' MKT in action was not captured. A follow-up observational investigation of how MKT plays a role in teacher enactment of the use of representations and math talk can shed some light on how teachers put their knowledge into play.

It is important to note here that caution in interpreting and generalizing the results of this study is advised, given that the mean MKT score for the sample teachers is 0.41 and the median is 0.45, indicating that the sample teachers have relatively high MKT scores compared to the population of elementary teachers—knowing that a score of zero in this measure indicates an MKT level equal to the national mean. A future study of MKT differences in math talk with participants that are more representative of the national sample is recommended.

### **Research Goal 3: Are there Content-Strand Differences in Math Talk and Use of Representations?**

In study III, five investigations of content-strand differences were conducted for each of the dependent variables (i.e., math talk, student use of representations, and teacher use of representations). Differences in each of the three variables was investigated for lessons with a specific strand as compared to other lessons in the sample. Investigating content strand and grade-band differences in math talk and use of representations point to the promise of

instructional log data for identifying the professional development needs of teachers. Providing beginning teachers with professional development opportunities, such as mentoring, was found to assist teachers in improving discourse opportunities in their classrooms (Bennett, 2010). The study's three main findings indicate specific professional-development needs for beginning elementary teachers, particularly for the participants in this study:

1. While lessons on number and operations in base ten included more opportunities for math talk than other lessons, those with main focus on counting and cardinality included less opportunities for math talk than others.
2. Fraction lessons had more student and teacher use of multiple, connected representations than other lessons in the sample. The same conclusion was found for data and measurement. Geometry lessons, however, scored lower on use of representations than other lessons.
3. Lessons in upper elementary grades included more opportunities for math talk than those in lower grades.

Overall, given the exploratory nature of the three large-scale studies presented here, the findings taken together generated future inquiry and research opportunities to expand our knowledge about beginning elementary teachers' practice with specific attention to creating opportunities for math talk and use of multiple representations in elementary mathematics lessons. The investigations presented in this study do not provide any evidence for causation; rather, the aim was to investigate possible association of the different variables of interest. Consequently, follow-up observational investigations to further explain the results presented in

these studies would be interesting and meaningful for teacher educators. Given the different teacher attributes, beyond MKT levels, that can influence instructional actions—e.g., teachers’ self-efficacy (Bennett, 2010)—further studies regarding the role of teacher attribute differences in math talk and use of representations would be worthwhile. Evidence for differences between student use of representations and teacher use of representations in the association with math talk, it is suggested that the associations are further investigated based on observational data. The results obtained in study 3 indicated less opportunities for math talk in counting lessons and less opportunities for use of multiple representations in geometry lessons; therefore, professional initiatives with specific focus on orchestrating opportunities for math talk in counting lessons and incorporating use of multiple presentations in geometry lessons are recommended for the study sample. Consequently, in addition to the log being a promising tool for assessing students’ opportunities to learn (Rowan & Correnti, 2009); Walkowiak et al, under review), instructional logs can also be used by researchers to identify professional-development needs.

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