ABSTRACT

KAKUMANU, NAVYATHA. Low-Order Simulation of Unsteady Airfoils Operating in Varying Freestream Velocity Including Reverse Flow. (Under the direction of Dr. Ashok Gopalarathnam.)

The numerical method based on the lumped vortex element (LVE) model was extended to handle simulation of unsteady airfoils in varying freestream velocity including reverse flow. The objective for the current work is to develop an algorithm that can predict the aerodynamic loads of an airfoil in varying freestream and reverse flow. The LVE method is extended and used to simulate flow over airfoil in perching maneuver (pitching plus decelerating airfoil motion) and the results obtained are compared with CFD and theoretical results. The CFD results for a pitching airfoil in reverse flow was used to determine the critical Leading Edge Suction Parameter (LESP) value for a NACA 0012 airfoil in reverse flow. The LESP critical value of an airfoil determines the vortex formation at the leading edge of an airfoil. The LESP critical value obtained, was successfully used to replicate the pitching of NACA 0012 airfoil in reverse flow. The NACA 0012 airfoil in a sinusoidally varying freestream velocity, sometimes including a reverse flow region, was simulated using the modification to the LVE method and the LESP critical value for reverse flow obtained for the above case. The normalized lift values calculated using the current algorithm are compared with experimental and theoretical values. The results from the current method compare well with experimental results and are an improvement over previous theoretical predictions. While further improvements are needed, the current work shows promise for handling varying freestream-velocity flows that include both forward and reverse flow along with arbitrary pitch and plunge motions.
Low-Order Simulation of Unsteady Airfoils Operating in Varying Freestream Velocity
Including Reverse Flow

by
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DEDICATION

To my ever supportive parents and my loving sister.
BIOGRAPHY

Navyatha Kakumanu on May 14th 1993 in Hyderabad, India to Dr.Kaspar Kakumanu and Showri Koya. After completing her undergraduate degree in aeronautical engineering from Institute of Aeronautical Engineering, she was hired as a Quality Assurance Engineer at Tata Advanced Systems Limited (TASL), an aircraft manufacturing plant. After an year at TASL, she moved to US in 2015 in pursuit of higher studies and enrolled at North Carolina State University for a Masters degree in Aerospace Engineering. Shortly after she joined the Applied Aerodynamics Lab under the guidance of Dr. Ashok Gopalarathnam. Navyatha will pursue a PhD. at NCSU upon completion of her Masters.
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Chapter 1

Introduction and Background

An airfoil is said to operate in reverse flow when fluid travels from the geometric trailing edge of an airfoil (generally sharp) toward the geometric leading edge (generally blunt). This phenomenon can be observed in high-speed helicopters. In forward flight, a helicopter rotor blade element experiences a sinusoidally varying freestream due to the rotation of the rotor [2]. This means that the helicopter blade experiences a reduced velocity in the retrieving phase and, depending on the advance ratio, the blade element might experience reverse flow. The reverse flow region grows in size with increasing advance ratio, leading to high drag, negative lift, pitching moment, and unsteady airloads [3]. Modern high-speed helicopters commonly operate with a slowed rotor thus increasing the advances ratio and the size of reverse-flow region which makes high-speed flights challenging due to the unfavorable aerodynamic effects of reverse flow.

One example of the operating regime of helicopter blade elements can be seen by considering the Sikorsky X2 Technology Demonstrator (X2TD), an experimental high speed helicopter operating at speeds as high as 460 km/hr. The contour in Fig. 1(a) shows the distribution of the theoretical relative velocity of the blade of an X2TD rotor disk operating at an advance ratio $\mu = 0.77$ [1]. The velocity component experienced by the blade element in the plane of rotation $U_T$ is shown in Fig. 1(a). Note that the radial flow ($U_R$) and inflow ($U_P$) velocities are not represented in the Fig. 1(a). When we consider the path of a blade element located at
mid-span of the rotor blade, as highlighted in the Fig. 1(a), the value of $U_T$ determines if the blade cross-section or element is in reverse flow. The region along the rotor disc sections, where the $U_T$ value dips below zero, determines the reverse flow region. This region is in the retreating side as shown in the Fig. 1(a). The in-plane velocity profile as the blade element travels around the blade azimuth is shown in Fig. 1(b). As the azimuthal angle changes from $0^\circ$ to $360^\circ$ the blade experiences both forward flow and reverse flow regions. The blade element is in forward flow for $0^\circ \leq \Psi \leq 220^\circ$ and $320^\circ \leq \Psi \leq 360^\circ$, but when the azimuth is $220^\circ \leq \Psi \leq 320^\circ$ it is in reverse flow. Along with the rotation the blade element also experiences sinusoidal (cyclic) pitching. While for the majority of the pitch cycle the blade element operates in forward flow, for a portion of the cycle it operates in reverse flow [3].

There have been several studies examining typical airfoil sections in reverse flow with a sharp geometric trailing edge acting as an aerodynamic leading edge [4, 5, 6, 7]. The typical behavior of the airfoils observed in these studies is that the reverse flow is generally characterized by negative lift, early onset of flow separation (stall), and periodic vortex shedding, all of which contribute to the unsteady aerodynamic loads experienced by a rotor blade [4]. Figure 1(a) shows that the change in airloads due to the reverse-flow region would lead to an unbalanced lateral
lift distribution over the rotor. The X2TD mentioned above is a compound helicopter with coaxial rotors. The advancing blade concept, first proposed by Cheney in 1969 [8] is the basis for the design of the X2TD. The lateral lift is evenly balanced as the advancing blades of this counter-rotating coaxial configuration unload the retreating blades. However, this configuration does not change the torsional loads experienced by the blades and the hub of the rotor in the retreating phase due to dynamic stall in the reverse flow region [9]. A better understanding of reverse flow region is required to improve the design of high-speed-helicopter blade elements.

Operation of other rotor applications such as wind and tidal turbines are also affected by reverse flow. Horizontal axis wind turbine blades experience high angles of attack when the rotor is at a standstill, starting or stopping, or when subjected to an unexpected change in direction of the freestream [10]. In these conditions, the blades act as bluff bodies, resulting in vortex shedding and unsteady airloads[11]. Modern marine hydrokinetic devices (i.e., tidal turbines) operate under constantly switching tidal currents. They are designed to accommodate for this reverse inflow by either yawing by 180 degrees or changing the pitch of the rotor blades [12]. However the complex mechanical design increases the operational and maintenance costs of the turbines thereby reducing their flexibility. An alternative to this complexity is a fixed-pitch rotor, in which flow can pass through the rotor disk in both directions, thereby permitting simple and low-maintenance rotor designs. However, this simplicity requires the blades to operate in reverse flow for one half of a tidal cycle. Hence, the estimation or prediction of the unsteady loads for reverse flow of the blades is essential for design of such tidal/wind rotors.

Flight simulators for helicopters or performance-prediction methods for blade geometry of helicopters and turbines generally use look-up tables developed from CFD and/or experimental data. The lack of such extensive CFD or experimental data makes it almost impossible to apply similar techniques for cases of reverse flow. To try to make up a large enough database for reverse flow using CFD and/or experiments would require significant amount of effort and expense. To develop a physics based lower-order predictive model using currently available CFD and or experimental data to validate the results from such an algorithm would be highly desirable.
Semi-empirical methods can also be important tools in predicting the unsteady airloads for various motion kinematics. These kinds of models are based on a combination of observations made from measured variable (loads) and theoretical considerations relating variables through fundamental principles [13]. Over the years many such models were developed to estimate the unsteady loads over airfoils [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. One of the earliest models that was successful in predicting unsteady aerodynamic loads is the ONERA model [15]. The ONERA model was further developed into two extended models namely ONERA EDLIN (Equations Differentielles Lineaires) [17] and ONERA BH (Bifurcation de Hopf) [26].

Lift, drag, and moment changes are calculated by solving second-order differential equations for the ONERA EDLIN dynamic stall model while, for the ONERA BH model, these values are obtained using first-order and second-order differential equations. Both models use the Kussner’s coefficients that can be extracted from the tables provided by van der Vooren [27]. Another frequently used model to estimate unsteady aerodynamic loads is the Leishmen-Beddoes model [16]. This theory can be applied to both attached flow cases [28, 29, 30] and for flows where dynamic stall can be observed [31, 32]. In 1998, Johnson [33] modified the equations from the above mentioned classic theories to accommodate for reverse flow. He used the new set of equations as part of the CAMRAD II software. However, even though semi-empirical models can be extended and modified to simulate reverse flow regions they are reliant on experimental and CFD data, thereby making them insufficient for rapid design and analysis.

Unlike semi-empirical models, physics-based theoretical models do not need constant calibration using experimental and/or CFD data. Theodorsen’s [34] approach is a good example of such a model. Based on potential-flow theory, this model predicts the unsteady loads on an airfoil undergoing harmonic oscillations in pitch and plunge. Another model that is equivalent to Theodorsen’s model, as established by Garrick [35], is Wagner’s approach [36]. Both these models are applicable only for small-amplitude oscillations. There are other theoretical models which are more flexible and can accommodate for various airfoil motion kinematics, including non-uniform motions and large amplitude kinematics[37, 38, 39]. However, the above mentioned
methods can only simulate inviscid flow over pitching and plunging airfoils. As discussed above, we know that in addition to the pitching and plunging motion, a helicopter blade experiences sinusoidally-varying free stream. Theoretical models that predict the aerodynamic forces for sinusoidally-varying freestream include Issacs’ two-dimensional theory \[40\] which is applicable for helicopter blade sections at constant incidence angle. Greenberg \[41\] later simplified Isaacs model \[40\] assuming that the shed vorticity in the wake is sinusoidal. In the 1994 article by Van derWall and Leishman \[42\], these two methods were compared and it was concluded that Greenberg’s assumption \[41\] of a sinusoidal wake is only valid for small amplitude streamwise disturbances and that Isaacs method \[40\] is more suitable for larger amplitudes. Both of these theories assume attached flow over the rotor blade and do not account for leading-edge vortex(LEV) formation.

In unsteady aerodynamics, when the flow parameters at the geometric leading edge reach a critical value, it leads to LEV formation. According to Jones and Platzer \[43\], when the pressure distribution reaches a critical value, which is independent of motion kinematics, LEV formation initiates. At NC State University, a low-order method named LESP-modulated Discrete Vortex Method (LDVM), \[44\] where LESP stands for Leading Edge Suction Parameter, was developed to obtain fast predictions of low Reynolds number flows over unsteady airfoil motions. The LDVM method can simulate intermittent LEV shedding by implementing a critical LESP value. The foundation for this method is the large-angle unsteady thin airfoil theory from Katz and Plotkin\[45\]. Resembling the classic thin airfoil theory, the vorticity \((\gamma(x))\) along the airfoil is represented by a Fourier Series:

\[
\gamma(\theta, t) = 2U \left[ A_0(t) \frac{1 + \cos \theta}{\sin \theta} + \sum_{n=1}^{\infty} A_n(t) \sin(n\theta) \right] \tag{1.1}
\]

where \(\theta\) is the transformation variable such that

\[
x = \frac{c}{2} (1 - \cos \theta) \tag{1.2}
\]
where \( c \) is the airfoil chord, \( U \) is the airfoil velocity and the Fouries coefficients are \( A_0(t), A_1(t), \ldots, and A_n(t) \).

The key to estimating the exact time when the LEV initiation starts is based on the criticality of the flow at the geometric leading edge. In LDVM, this criteria is based on the inviscid parameter LESP [46], which can be determined at every time step of the unsteady thin airfoil theory. Ramesh et al. [47] showed that the LESP can be directly equated to the \( A_0 \) value of the Fourier series

\[
LESP(t) = A_0(t)
\]  

The instantaneous LESP value exceeding the critical LESP value (determined from CFD or experiments for a given airfoil and Reynolds number) triggers LEV formation. In the LDVM, the LEV shedding and termination was modeled with the premise that, during the LEV shedding process, the LESP is limited to the critical value. The LDVM was extended by Ramesh et al. [48] to simulate perching (varying freestream) and hovering (zero freestream) motions of airfoil.

The underlying principles of the LDVM method were applied to the lumped-vortex-element method by Narsipur et al [49] with the purpose of modeling dynamic stall of airfoils in incompressible flows and with the aim of extending the method to 3D wing analysis. In the current research, the inviscid LVE algorithm with LEV shedding capabilities is adopted as the foundation method to which modifications are made to accommodate for varying free-stream conditions. The inclusion of this feature into the code will allow for us to investigate flows that are decelerating or accelerating. The varying freestream experienced by the airfoil changes the Reynolds number at each time step, which would require for us to use a varying LESP critical value [48]. Additionally, the existing lumped vortex element theory is modified to change the vortex element location from quarter chord to leading edge to achieve a symmetry in the airfoil configuration. The symmetry achieved, would aid the use of the algorithm to predict the unsteady airloads for airfoils in reverse flow. This new configuration would also facilitate the extension of the model to include simulations of reverse flow over three-dimensional wings or helicopter blades, with methods similar to vortex-lattice theory.

In the following Chapters we will discuss the development of the low-order method that
can be used to simulate the sinusoidal varying freestream without the assumption of attached flow over the airfoil and has a built in function to detect otherwise. Chapter 2 would be a brief explanation of the methodology and development of the current model, it would also include the validation of the current model using results from above mentioned LVE and LDVM models. The results are presented in the Chapter 3. The effect of changing the vortex-element location on the calculation of aerodynamic loads are discussed in Section 3.1. The results from the modified LVE method of an airfoil undergoing perching motion are compared with CFD and theoretical data in Section 3.2. The simulation of airfoil in reverse-flow regions is shown in Sections 3.3 and 3.4. Chapter 4 gives a brief explanation of the conclusions made from the current work and the scope of Future work
Chapter 2

Methodology

In this chapter, we discuss the Lumped Vortex Element (LVE) model with extension to account for LEV formation. The algorithm is discussed step by step and is validated by comparing the results with the solutions given by Narsipur et al [49]. The modifications made to expand the code to simulate flows with varying free-stream are explained. The implications of changing the vortex location along the airfoil segment are explored in detail.

2.1 Lumped Vortex-Element Model

The LVE [49, 50] is preferred to LDVM for simulation of unsteady flows over airfoils, mainly to enable the extension of the model to finite wing geometries. The vortex lattice method is the three-dimensional counterpart to the LVE model making numerical methods and solutions developed using LVE model suitable to form the foundation for analysis of unsteady aerodynamics of 3-D wings. In this research effort, the algorithm to predict the unsteady airloads of airfoils is based on the LVE method provided by Katz and Plotkin [45].

For potential flows, an airfoil section can be modeled by a vortex distribution along its camber line. The vortex distribution along a lifting symmetric airfoil is given by,

\[ \gamma(\theta) = 2Q_{\infty}\alpha \frac{1 + \cos\theta}{\sin\theta} \]  

(2.1)
Figure 2.1: (a) Vortex distribution over a flat plate at an angle of attack $\alpha$ (b) Equivalent lumped-vortex configuration with the collocation point.

An illustration of the vortex distribution is shown in Fig. 2.1a. This distribution can be effectively replaced by a single vortex from a far field point of view, which has the strength of,

$$\Gamma = \int_0^c \gamma(x) dx$$  \hspace{1cm} (2.2)

Since the location of the center of pressure for a symmetric airfoil is at quarter chord, the single vortex is placed at that location. If the forces acting on the airfoil are represented at only one location then the zero-normal-flow condition is also applied at only one location i.e., at the three-quarter chord. This point is called the collocation point. The schematic of the lumped vortex element along with the collocation point is shown in Fig 2.1b. This representation of the airfoil is inclusive of Kutta condition at the trailing edge as it is based on results of Thin-airfoil theory. This lumped-vortex-element representation of the airfoil is further extended to the LVE algorithm by modeling the airfoil using many lumped vortex elements along the chord. The following subsections details the algorithm of the LVE method.
2.1.1 Geometric Model and Kinematics of the Airfoil

To construct a numerical solution on the basis of the LVE method, we discretize the camberline of the airfoil. From a given input file of airfoil surface coordinates, the camberline is extracted by finding the mean of the upper and lower surface coordinates. The camberline is then divided into \( N \) number of discrete segments. Each segment consists of a vortex element at 25% of the segment length and a collocation point at 75% of its length. For a given \( i^{th} \) segment, \( n_i \) and \( \tau_i \) are the normal and tangential vectors respectively. For a thin airfoil, the geometric model for the LVE method is as shown in Fig. 2.2. At each time step, as the forces change due to the motion of the airfoil, a wake vortex is shed to maintain the same total circulation value of the system. The location of this wake vortex is at 25% of the total distance traveled by the geometric trailing edge in the last time step. The complete modeling of the wake vortices is discussed in detail in the next section.

The inertial frame of reference represented by \((X, Z)\) is considered to be stationary as the airfoil moves in that frame. Attached to the airfoil is the body frame of reference given as \((x, z)\) with its origin at the geometric leading edge of the airfoil. As this is a time-stepping algorithm
we start with $t=0$, at which instance the origin of both the frames of reference coincide, and as time increases ($t > 0$) the airfoil moves along a specified path. At a given time instance ($t$), depending on the motion kinematics, the location of any point on the airfoil in the inertial reference frame can be calculated using the following transformation matrix,

$$ \begin{pmatrix} X \\ Z \end{pmatrix} = \begin{pmatrix} \cos \theta(t) & \sin \theta(t) \\ -\sin \theta(t) & \cos \theta(t) \end{pmatrix} \begin{pmatrix} x - pp \\ z \end{pmatrix} + \begin{pmatrix} X_0 \\ Z_0 \end{pmatrix} \tag{2.3} $$

where $(x, z)$ are the coordinates of any point on airfoil in body frame of reference, $(X_0, Z_0)$ are the location of that point at $t=0$ with reference to the inertial frame of reference, $\theta(t)$ is the angle of attack or angle between the two frames at time $t$, and $pp$ is the pivot point location along the $x$-axis of the body frame of reference.

### 2.1.2 Influence Coefficients and Circulation Calculation

The zero-normal flow boundary condition needs to be satisfied at each of the collocation points. The normal velocity component at each panel on the camberline consists of self-induced velocities, wake-induced velocities and the kinematic velocities. At any time step, the momentary boundary condition in the body frame of reference is given by,

$$ (\nabla \Phi_B + \nabla \Phi_W - V_0 - v_{rel} - \omega \times r).n = 0 \tag{2.4} $$

The self induced velocity $\nabla \Phi_B$ in the above equation is the velocity induced due to the bound vortices present on the camberline of the airfoil. Consider a bound vortex of strength $\Gamma_j$ on the $j^{th}$ panel of the airfoil. To calculate the induced velocity due to this vortex on the $i^{th}$ panel we use the following equation.

$$ \begin{pmatrix} u_i \\ w_i \end{pmatrix} = \frac{\Gamma_j}{2\pi r_j^2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_i - x_j \\ z_i - z_j \end{pmatrix} \tag{2.5} $$
where

\[ r_j^2 = (x - x_j)^2 + (z - z_j)^2 \]  \tag{2.6}

The normal velocity component induced by a unit strength airfoil vortex element \( j \) on the airfoil segment \( i \) is called the influence coefficient \( a_{ij} \) given by,

\[ a_{ij} = (u, w)_{ij} \cdot n_i \]  \tag{2.7}

The wake potential, \( \nabla \Phi_W \), is also modeled by discrete vortices and represents the wake induced velocity, \( (u, w)_W \) for each segment of the airfoil. Therefore, the total induced velocity \( q_i \) for the airfoil segment due to bound vortices and the wake vortex is given by,

\[ q_i = a_{i1} \Gamma_1 + a_{i2} \Gamma_2 + a_{i3} \Gamma_3 + \ldots + a_{iN} \Gamma_N + a_{iW} \Gamma_W \]  \tag{2.8}

One problem with the discrete-vortex method is that, when the vortices are convecting in the flow and come close to each other, then the estimated induced velocity calculated using Eqn. 2.5 are artificially large. This in turn results in disturbances in the solution. A theoretical vortex has a singularity point at its center while real vortices often have a finite core at its center. Chorin [51] presented the idea of using a vortex blob with finite radii to model realistic vortex distributions. This method works well as long as the vortex core radius was larger that the average spacing between the vortices, as proven by Hald [52]. The second-order vortex-core model proposed by Vatistas et al [53] is used in the current method to modify Eqn. 2.6 as given below,

\[ r_j^2 = (x - x_j)^2 + (z - z_j)^2 + v_{core}^4 \]  \tag{2.9}

where \( v_{core} \) the vortex core radius, is set to be 1.3 times the average spacing between the vortices based on the guidelines provided by Leonard [54]
\[
\frac{v_{core}}{c} = 1.3 \delta t^* \tag{2.10}
\]

Finally, the kinematic velocity term, \((-V_0 - v_{rel} - \omega \times r)\), which represents the normal velocity component due to airfoil’s translation and rotation, can be represented by the following equations for tangential and normal velocities,

\[
U(t) = -\dot{X} \cos \alpha(t) + \dot{Z} \sin \alpha(t) - \dot{\alpha} \eta \tag{2.11}
\]

\[
W(t) = -\dot{X} \sin \alpha(t) - \dot{Z} \cos \alpha(t) - \dot{\alpha} (x - pp) - \frac{\partial \eta}{\partial t} \tag{2.12}
\]

On combining the equations, the zero-normal boundary condition for the \(i^{th}\) segment of the airfoil can be written as,

\[
a_i \Gamma_1 + a_i \Gamma_2 + a_i \Gamma_3 + \ldots + a_{iN} \Gamma_N + a_{iW} \Gamma_W + [U(t) + u_w, W(t) + w_w]_i . n_i = 0 \tag{2.13}
\]

For simplicity we have,

\[
[U(t) + u_w, W(t) + w_w]_i . n_i = R.H.S \tag{2.14}
\]

Another condition applicable to our airfoil in the Kelvin condition. It states that “in an inviscid flow with conservative body forces, the circulation around a closed curve (which encloses same fluid elements) moving the fluid remains constant with time”, Kelvin condition for the current method is modeled using the equation,

\[
\Gamma(t) - \Gamma(t - \Delta t) + \Gamma_{W_i} = 0 \tag{2.15}
\]

Equation 2.15 models the circulation of the wake vortex is the difference between the total circulation of the airfoil at the current time step and the previous time step.

By combining the Eqn. 2.13 for all the segments of the airfoil with Eqn. 2.15, we can get a
system of equations that can be written in a matrix form as follows,

\[
\begin{bmatrix}
    a_{11} & a_{12} & \ldots & a_{1n} & a_{1w} \\
    a_{21} & a_{22} & \ldots & a_{2n} & a_{2w} \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    a_{n1} & a_{n2} & \ldots & a_{nn} & a_{nw}
\end{bmatrix}
\begin{bmatrix}
    \Gamma_1 \\
    \Gamma_2 \\
    \vdots \\
    \vdots \\
    \Gamma_N
\end{bmatrix}
=
\begin{bmatrix}
    RHS_1 \\
    RHS_2 \\
    \vdots \\
    \vdots \\
    RHS_N
\end{bmatrix}
\begin{bmatrix}
    \Gamma(t - \Delta t)
\end{bmatrix}
\]

(2.16)

By solving this set of equations we get the bound circulation over the airfoil and also the shed wake-vortex circulation at any time step. These values can be used to get the aerodynamic loads and coefficients of the airfoil in unsteady motion.

2.1.3 Aerodynamic Coefficient Calculations and Validation

On calculating the bound and wake circulation at each time step, the airfoil’s pressure difference for each panel can be calculated using the following equation,
\[
\Delta p_i = \rho \left[ (U(t) + u_w, W(t) + w_w)l_i \tau_i \frac{\Gamma_i}{l_i} + \frac{\partial}{\partial t} \sum_{k=1}^{i} \Gamma_k \right]
\]  

(2.17)

From the pressure difference obtained above, we can get the normal force coefficient, \( C_N \), as shown below.

\[
C_n = \frac{\sum_{i=1}^{N} \Delta p_i l_i}{\frac{1}{2} \rho U_\infty^2 c}
\]  

(2.18)

As discussed in Chap. 1, the instantaneous value of the LESP plays a critical role in predicting the LEV formation and in calculating the suction force acting at the geometric leading edge. In the LDVM model, the LESP value was set equal to \( A_0 \) Fourier term. The parameter closest to \( A_0 \) is the circulation, \( \Gamma_1 \), of the first segment of the airfoil, but it is dependent on the discretization. To get an LESP(\( t \)) value that is independent of the discretization, Aggarwal[50], equated the normal force over the forward-most panel from the LVE calculation with the normal force from the thin airfoil theory over the same region and got the below given equation which matches excellently with \( A_0(t) \)

\[
LESP(t) = \frac{1.13(\Gamma(t))}{U_\infty(t)c\left[\cos^{-1}(1 - \frac{2l}{c}) + \sin(\cos^{-1}(1 - \frac{2l}{c}))\right]}
\]  

(2.19)

where \( \Gamma_1(t) \) is the strength of the discrete vortex in the first panel, \( l \) is the chordwise length of the first panel, and 1.13 is an empirically-determined correction factor. The suction coefficient, \( C_s \), is calculated as,

\[
C_s(t) = 2\pi LESP^2(t)
\]  

(2.20)

The lift coefficient, \( C_l \) and the drag coefficient, \( C_d \) are given by

\[
C_l(t) = C_n(t) \cos \alpha(t) + C_s(t) \sin \alpha(t)
\]  

(2.21)

\[
C_d(t) = C_n(t) \sin \alpha(t) + C_s(t) \cos \alpha(t)
\]  

(2.22)
Moment coefficient can be calculated using the equation.

\[
C_m = \frac{\sum_{i=1}^{N} \Delta p_i l_i (x_i - x_{ref})}{\frac{1}{2} \rho U^2 \infty c}
\]

(2.23)

The results from the current algorithm are compared with the solution by Narsipur et al. [49] for validation. NACA 0012 airfoil in pitching motion is simulated and compared with the LVE method. The motion kinematics of the airfoil is shown in the Fig. 2.4. The pitching motion follows an Eldredge function [55, 56] with an amplitude of 45 degrees and a non-dimensional pitch rate (K) of 0.5. The LESP critical value considered for this case is 0.2[57]. The comparison of the lift coefficient, drag coefficient, moment coefficient, and instantaneous LESP values is shown in the Fig. 2.5. Figure 2.5 shows that the results obtained match very well with the existing solutions from existing LVE methods. The small variation seen for \( t^* > 3 \) can be attributed to the small differences in the locations of the shed vortices.
2.2 Extension of LVE method to Perching Motion

In this section, the LESP criteria is extended to cases where the freestream velocity is varying. Perching maneuvers employed by birds while landing is one such case. Perching can be modeled as a ramp variation in angle of attack in conjunction with the freestream velocity decreasing to zero. Perching is a special case of a more general set of kinematics involving variable freestream velocity. The low-order method of LDVM was successfully extended by Ramesh et al.[48] to include modeling of unsteady flows in which the freestream is varying. This revised formulation of the LDVM method is used as a guideline for augmentation of the current LVE method described in the above section.
The airfoil circulation and normal flow calculations for varying freestream velocity with no LEV formation, uses the same set of equation, Eqn. (2.3 -2.18), as described in the Sec. 2.2. The changes to equation for LESP critical conditions and suction coefficient calculations are discussed in detail in this section. When the airfoil is at an angle of attack and as the stagnation point moves away from the geometric leading edge a suction peak can be observed at the geometric leading edge. A good measure for this peak is the LESP [57]. As the airfoil thickness approaches zero, the leading-edge radius also approaches zero, giving rise to a theoretically infinite flow velocity at the leading edge. Although the theoretical velocity at the leading edge is infinite, real airfoils have rounded geometric leading edges which can support certain level of suction even when the stagnation point is away from the geometric leading edge[58]. The amount of suction that can be supported is a characteristic of the airfoil and the Reynolds number at which it is operating. For cases that do not include a variation in the free stream a constant LESP value is taken. For the current perching case we would have to implement a varying critical-LESP criterion for LEV formation. Hence, calculation of Reynolds number for each time step is necessary.

In the current case, the freestream velocity is time-varying and can become zero or very small when compared to the other velocity components. Therefore, to consider the Reynolds number of the freestream to estimate the change in the LESP critical value is not ideal as a zero freestream velocity would result in a zero Reynolds number which does not reflect the velocity experienced by the geometric leading edge. As shown by Ramesh et al. [48], a better Reynolds number is one that is calculated from the induced velocities experienced by the geometric leading edge. The equation for this Reynolds number is given by,

\[
Re_{V_{mag}}(t) = \frac{V_{mag}(t)c}{\nu} = \frac{V_{mag}(t)}{U_{ref}}Re_{ref}
\]  

(2.24)

where \(U_{ref}\) is considered to be 1 for the cases considered here and the value of \(V_{mag}\) is given by

\[
V_{mag} = |\vec{U} + \vec{h} + \vec{\alpha ac} + \vec{V_{ind}}|
\]

(2.25)
where \( U \) is the free stream velocity, \( \dot{h} \) is the rate of plunge of the airfoil, and \( V_{ind} \) includes the induced velocities due to wake vortices and LEV from previous time steps. The instantaneous LESP value at each time step is equal to \( A_0 \) of the Fourier series for the LDVM method. In the current method, we used the Eqn. 2.19 for the calculation of instantaneous LESP. The presence of \( U_\infty(t) \) in the denominator of this equation poses a problem as the freestream velocity \( (U_\infty) \) values come closer to zero the method would predict artificially large values of instantaneous LESP in such situations. To avoid this error, we are modifying the equation for LESP as follows,

\[
LESP(t) = \frac{1.13(\Gamma(t))U_{\text{ref}}}{V_{\text{mag}}(t)c\left[\cos^{-1}(1-\frac{2}{c}) + \sin(\cos^{-1}(1-\frac{2}{c}))\right]}
\]  

(2.26)

Subsequently the equation to calculate the suction coefficient, Eqn. 2.20 has been modified to get,

\[
C_s(t) = 2\pi LESP^2(t)\left(\frac{V_{\text{mag}}}{U_{\text{ref}}}\right)^2
\]

(2.27)

Equation calculating the normal force coefficient and moment coefficient also includes the freestream velocity in the denominator as shown in Eqn. 2.18. This results in large errors as the freestream velocity reaches values closer to zero. The freestream velocity \( (U_\infty) \) in that equation is replaced by the reference velocity \( (U_{\text{ref}}) \) to avoid such errors.

\[
C_n = \frac{\sum_{i=1}^{N} \Delta p l_i}{\frac{1}{2} \rho U_{\text{ref}}^2 c}
\]

(2.28)

\[
C_m = \frac{\sum_{i=1}^{N} \Delta p l_i(x_i - x_{\text{ref}})}{\frac{1}{2} \rho U_{\text{ref}}^2 c}
\]

(2.29)

The lift and drag coefficients are calculated using Eqns .2.21 and 2.22 respectively.
2.3 Modification of the Lumped-Vortex Element Location

In this section, we will discuss the reasons for and the effects of changing the vortex element location along the airfoil segments. As shown in Sec (2.1), the lumped vortex element that is effectively replacing the vortex distribution along the airfoil is placed at the 25% chord location. This vortex location was chosen because the center of pressure for a flat plate is at the quarter-chord location and placing the vortex element there would be appropriate. In the current research, the lumped-vortex element is displaced from 25% chord location to the 0% chord location, to place it at the geometric leading edge. In the following paragraphs, we will discuss the change in configuration of the airfoil segments due to the change in vortex element location and the motivation behind this decision. The results of the change in the vortex location on the aerodynamic forces and moments in comparison to the original configuration are shown in Chapter 3.

As the vortex-element location is changed, the corresponding collocation-point location should also be altered. Collocation points are defined as the points where the zero-normal-flow boundary condition is satisfied. Consider the collocation point at $k_c$ distance from the
geometric leading edge, as shown in Fig.2.6. The zero normal flow boundary condition to be satisfied at this location is given by,

\[ \frac{-\Gamma}{2\pi kc} + U_{\infty}\alpha = 0 \]  

(2.30)

where the strength of the lumped vortex element \( \Gamma \) is given as,

\[ \Gamma = \pi c U_{\infty}\alpha \]  

(2.31)

Hence we have,

\[ \frac{-\pi c U_{\infty}\alpha}{2\pi kc} + U_{\infty}\alpha = 0 \]  

(2.32)

The solution to Eqn. 2.32 gives the collocation point when the vortex-element is at 0% chord location:

\[ k = 1/2 \]  

(2.33)

Extending the single lumped-vortex element to a discrete-vortex model for a modified vortex element location, we get the representation of a thin airfoil as shown in the Fig. 2.7a. Figure 2.7b shows the typical discrete-vortex model of an airfoil with the vortex at 25% chord. In the modified configuration we have an additional vortex at the geometric trailing edge of the airfoil.
and a counter clockwise vortex at the same location. For a sharp-trailing-edge airfoil these two vorticies cancel each other out to have a net vortex strength of zero, hence this vortex can be considered as a pseudo vortex. One of the main reasons to change the vortex element location on the airfoil segments is to achieve symmetry in airfoil configuration along the X-axis to better simulate reverse flow over airfoils.

In the current research work, since only airfoils with sharp trailing edges are studied, the strength of the pseudo vortex is taken as zero. However, to simulate flows over blunt or smooth geometric trailing edge cases, a set of equations including the pseudo-vortex element strength needs to be developed. Such a model would solve for the aerodynamic loads of an airfoil operating in unsteady flow including reverse flow regions. A critical aspect of implementing an algorithm like that would be, the treatment of vortex shedding from both the geometric leading edge and the geometric trailing edge with the same criteria. As discussed above the LESP and \( LESP_{\text{crit}} \) are important criterion to predict the vortex formation at the leading edge. Similar to the LESP, an Edge Separation Parameter (ESP) may be adopted to predict the vortex formation at both the geometric leading edge and the geometric trailing edge depending on the geometry of these edges. The same method may be extended to simulate reverse flow over three-dimensional wings and helicopter blades with algorithms that build on methods like vortex-lattice theory for 3-D flows. To construct an algorithm like that, the first step is to verify how the change in the location of the vortex element affects the force calculations, and if so, evaluate if the errors are quantifiable. The results for this are discussed further in the following chapter.
Chapter 3

Results

In this chapter, we first discuss the effects of changing the vortex location along the airfoil segment on the prediction of aerodynamic loads for steady and unsteady cases. We simulate the flow over a NACA 0012 airfoil undergoing the perching maneuver using the extended form of the LVE method and compare the results with CFD and theoretical data. An airfoil pitching in reverse flow for angles of attack from 135 degrees to 180 degrees is simulated and the results are compared with CFD data. The normalized lift values of an airfoil experiencing a sinusoidally varying freestream, including regions of reverse flow, obtained using the current algorithm are compared with experimental and theoretical data.

3.1 Changing Vortex Location

As discussed in Section 2.3, one of the aims of the current work is to check if the change in the vortex location affects the results obtained from the original formulation of the discrete-vortex method. In this section, the current numerical method is used to find the aerodynamic coefficients for steady cases and unsteady cases while changing the vortex location along the airfoil segments. The vortex location is changed from 25% chord to 0% chord and the results from both the methods are compared with each other and with available theoretical data.
3.1.1 Steady Results

In this case the flow over airfoils kept at a static angle of attack is simulated and the steady steady aerodynamic coefficients are studied. The lift and moment coefficients at various angles of attack are evaluated for symmetric airfoils and airfoils with parabolic camber. The values for the coefficients given by the current algorithm at changing vortex locations along the airfoil segment are compared with the theoretical results from thin airfoil theory provided by Katz and Plotkin [45]. The equations to calculate the lift and moment coefficients for a symmetric airfoil at a static angle of attack are,

\[
C_l = 2\pi \alpha \quad (3.1)
\]

\[
C_{m_0} = -\frac{\pi}{2} \alpha \quad (3.2)
\]

\[
C_{m_{\tfrac{c}{4}}} = 0 \quad (3.3)
\]

where \(C_{m_0}\) is the moment coefficient about the geometric leading edge of the airfoil and \(C_{m_{\tfrac{c}{4}}}\) is the moment coefficient about the quarter chord of the airfoil. For a parabolic camber airfoil, according to the thin airfoil theory, the same coefficients can be calculated using the following equations.

\[
C_l = 2\pi \left( \alpha + 2\frac{\epsilon}{c} \right) \quad (3.4)
\]

\[
C_{m_0} = -\frac{\pi}{2} \left( \alpha + 4\frac{\epsilon}{c} \right) \quad (3.5)
\]

\[
C_{m_{\tfrac{c}{4}}} = -\frac{\pi \epsilon}{c} \quad (3.6)
\]

where \(\epsilon\) is the maximum camber that defines the camberline equation of a parabolic camber airfoil, given by

\[
\eta(x) = 4\epsilon \frac{x}{c} \left[ 1 - \frac{x}{c} \right] \quad (3.7)
\]

Figure 3.1 compares the results from current method with the theoretical values. The lift and moment coefficients for various angle of attacks are calculated and plotted for the two airfoils
Figure 3.1: (a) $C_l$ vs. $\alpha$ curve for symmetric airfoil, (b) $C_l$ vs. $\alpha$ curve for parabolic chamber airfoils, (c) $C_{m_0}$ vs. $\alpha$ curve for symmetric airfoil, (d) $C_{m_0}$ vs. $\alpha$ curve for parabolic chamber airfoil, (e) $C_{m_c}$ vs. $\alpha$ curve for symmetric airfoil, (f) $C_{m_c}$ vs. $\alpha$ curve for parabolic chamber airfoils.
with two vortex location, at 0% chord and 25% chord of an airfoil segment. Each of these cases are run with airfoil segment numbers (n) ranging from 5 to 50. For calculation of lift coefficient for a flat plate the change in the vortex location does not affect the results as shown in the Fig. 3.1a. However, for a parabolic camber airfoil, the predicted $C_l$ values are slightly lower than the theoretical value for cases of lower number of airfoil segments. The difference between the lift coefficient values due to change in vortex location is imperceptible as shown in Fig. 3.1b. A more noticeable difference between the various cases considered here can be seen in the calculation of moment coefficients. Figures 3.1c and 3.1d show the variation of moment coefficient about LE ($C_{m_\alpha}$) with change in angle of attack. A clear trend that can be observed here is that the accuracy of the results increase as the number of segments in the airfoil increases. Additionally, for the cases with the same number of airfoil segments, the results for vortex at 25% chord are closer to the theoretical values than for vortex at 0% chord. However as the number of segments increase the difference in prediction due to change in vortex locations decrease considerably. It is observed that for $n \geq 50$ the results from vortex at 0% and at 25% of the airfoil segment are essentially equivalent. The same trends given above can also be observed in the calculation of $C_{m_\alpha}$ as shown in the Figs 3.1e and 3.1f.

From the above results we can infer that as long as the distance between the vortex and the collocation point are maintained at 50% of the length of the airfoil, segment the calculated strengths of the vortices on each of these segments are the same. Hence, the $C_l$ vs. $\alpha$ curves match up for both the vortex locations. The differences observed in the moment calculations is due to the change in the moment arm as the vortex location changes. As discussed above, this difference diminishes as the number of vortices increases.

3.1.2 Unsteady Results

To look at the effects of the change in vortex location in predicting the aerodynamic loads acting on airfoils undergoing unsteady motion we consider the NACA 0012 airfoil undergoing a pitching motion. The pitching motion of the airfoil is as shown in the Fig. 2.4, with a pivot
location at 0% chord of the airfoil. The results are compared with the LVE method as discussed in Section 2.1. As shown in the Fig. 3.2 the values are matching very well. The number of airfoil segments considered for this case is $n = 200$. Note that here after for all the calculation the value of n is set to 200 unless otherwise specified.

Figure 3.2: NACA 0012, Eldredge pitching motion. Comparison of the aerodynamics coefficients and the instantaneous LESP solutions of the current code, for different vortex locations, with the LVE method using elements along the chord
Figure 3.3: Variation of the free stream velocity and the pitch angle for the perching motions

Figure 3.4: Variation of the free stream velocity and the pitch angle for the perching motions
3.2 Perching motion comparison

A classic pitch-ramp motion may be used to approximate a perching maneuver i.e, when the angle of attack of an airfoil changes rapidly over large amplitude. Additionally in perching along with the pitching motion, the airfoil also decelerates to zero freestream velocity. Three perching maneuvers are investigated in this section as shown in the 2013 article by Ramesh et al. [48]. The pitch-ramp motions with freestream velocity decelerating to zero at different rates with a starting freestream velocity $U_{ref}$ that corresponds to $Re_{ref} = 200,000$ are considered here. The motion kinematics of the SD7003 airfoil, analyzed in this section, for the three perching cases are shown in Fig. 3.3. The change in the freestream velocity would result in a change in Reynolds number. Hence, the LESP critical value cannot be considered as a constant for the current cases. Ramesh et al. [48] used CFD data to calibrate the $LESP_{crit}$ against $Re_{mag}$ for the SD7003 airfoil. The determined $LESP_{crit}$ vs. $Re_{mag}$ is shown in Fig. 3.4.

The results for lift coefficient, drag coefficient, Reynolds number, and the LESP critical for the cases P1, P2, and P3 are compared with the available data from CFD and/or the LDVM method as shown in the Figures 3.5, 3.7, and 3.9 respectively. The comparison of flow visualization for four different time steps in P1, P2, and P3 cases are shown in the Figs 3.6, 3.8, and 3.10 respectively. For cases P1, lift and drag coefficient results match well with the results from the lower order LDVM method given by Ramesh et al. [48]. We can observe small inconsistencies with the Reynolds number and LESP critical data, but these does not seem to affect the coefficient calculations. However for the case P2 and P3, while the lift coefficient results are the same as the LDVM method, the same cannot be said about the drag coefficient data. This might be due to the differences in calculation of the Reynolds number. The clear trend we can observe when we consider the Reynolds number variation of all the three cases is that after some time period (1-1.5 $t^*$) the calculated Reynolds number deviates from the LDVM results and overshoots, while the Reynolds number data trend is similar to the LDVM, the difference in prediction can be attributed to the difference in the method for calculating the bound vortex strength of the airfoil in the two algorithms. The vorticity distribution over
Figure 3.5: Case P1. Comparison of lift and drag coefficient with CFD data and the LDVM method. Comparison of Reynolds number and LESP critical values with the results from LDVM.

Figure 3.6: Case P1. Comparison of flow visualization at four different time steps between CFD data, LDVM results and current code results respectively from top to bottom.
Figure 3.7: Case P2. Comparison of lift and drag coefficient with CFD data and the LDVM method. Comparison of Reynolds number and LESP critical values with the results from LDVM

Figure 3.8: Case P2. Comparison of flow visualization at four different time steps between CFD data, LDVM results and current code results respectively from top to bottom.
Figure 3.9: Case P3. Comparison of lift and drag coefficient with CFD data and the LDVM method. Comparison of Reynolds number and LESP critical values with the results from LDVM

Figure 3.10: Case P1. Comparison of flow visualization at four different time steps between CFD data, LDVM results and current code results respectively from top to bottom.
the airfoil, in the LDVM method, was taken as

\[ \gamma(\theta, t) = 2V_{mag}(t) \left[ A_0(t) \frac{1 + \cos \theta}{\sin \theta} + \Sigma_{n=1}^{\infty} A_n(t) \sin(n\theta) \right] \]  

(3.8)

The current algorithm follows the method detailed in section 2.2. It can be observed that the parameter \( V_{mag} \) is only used in the determination of instantaneous LESP and was not used while calculating the bound vortex strength over the airfoil. Hence as the the rates of deceleration change between the three cases (P1 P2 and P3), the deviation from the LDVM method changes. In case P3, the airfoil takes longer time to reach zero velocity hence maximizing the deviation unlike case P1 where the deviation does not have any effect on the drag coefficient calculations. In all the three cases the lift coefficient is unaffected due to the deviations in Reynolds number calculation, making the current algorithm a good tool to simulate decelerating flows over airfoils.

### 3.3 Elderedge Motion in Reverse Flow

As discussed in Chap. 1, the key to modeling LEV formation is to detect if the flow parameters at the geometric leading edge reach a critical value, this parameter is generally related to the pressure experienced by the edge. LESP is a measure of the suction peak at the leading edge. When the LESP value reaches a critical level, it triggers LEV formation. The LESP critical was found to be independent of the motion kinematics for a given airfoil operating at a given Reynolds number [57]. However these studies do not consider airfoils operating in reverse flow. In this section, we simulate a NACA 0012 airfoil pitching in reverse flow with angles of attack varying from 135 degrees to 180 degrees. The results from the CFD simulation are used to find the LESP critical of the round geometric leading edge based on suction force data from CFD simulations. The results from the current algorithm by using the LESP critical of 0.04, are compared with the results from the CFD simulations.

The current section shows the computational results from NCSU’s REACTMB-INS code, developed by Prof. Edward’s CFD group. The CFD simulation was carried out for NACA 0012
Figure 3.11: Variation of the free stream velocity and the pitch angle for the perching motions airfoil pitching with an angle of attack variation from 180-135-180 degrees and a non-dimensional pitch rate of 0.1 as shown in the Fig. 3.11. The pivot point location is at the geometric leading edge of the airfoil. The lift coefficient and the drag coefficient results from the current algorithm are compared with the CFD results in the Fig. 3.12. The trends followed by the CFD results and the solutions from the current code are a good match. Thus we can say that, irrespective of the change in the Reynolds number, the LESP critical value changes for a given edge geometry of an airfoil depending on whether it is experiencing forward flow or reverse flow.

3.4 Streamwise Oscillation into Reverse Flow

Streamwise oscillation of an airfoil can be used to replicate the conditions experienced by a helicopter blade element. An helicopter blade element experiences a constant incoming free stream and additionally a relative velocity due to the rotational velocity. The sum of the two
Figure 3.12: NACA 0012, Eldredge pitching motion. Comparison of the aerodynamics coefficients obtained from the current algorithm with the CFD results.

Figure 3.13: Variation of the normalized velocity with changing $\lambda$. 

35
velocities results in an effective sinusoidal velocity variation experienced by the blade element. The amplitude of this sinusoidal variation depends on the advance ratio of the helicopter blade. We can express the normalized velocity experienced by the blade element as,

\[ \frac{U(t)}{U_\infty} = 1 + \lambda \sin \left( \frac{2kU_\infty t}{c} \right) \]  

(3.9)

The freestream velocity experienced by the airfoil is dependent on the values of \( k \) and \( \lambda \). These cases were studied experimentally in Granlund et al. [59] and form the motivation for the current study. The advance ratio (\( \lambda \)) is one of the important parameters; its value dictates if the airfoil experiences any reverse flow during its rotation cycle. If the \( \lambda \) value is greater than 1, then the blade experiences reverse flow in its retreating phase. As the value of \( \lambda \) increases above 1, the reverse flow region also increases. This can be observed in the velocity profiles shown in the Fig.3.13.

In this section we will simulate the streamwise oscillation of the airfoil for different values of \( \lambda \) at a constant \( k \) value of 0.133. As discussed in Chap.1, Issacs’ [40] two-dimensional theory and Greenberg [41] simplified model of Issacs’ theory can predict the aerodynamic forces of sinusoidally-varying freestream velocities. To simulate the motion shown in Fig 3.13 at 6 degree angle of attack, we consider a high value of LESP critical when the airfoil is in forward flow and switch to an LESP critical value of 0.04, as discussed above, when the airfoil goes into reverse flow. This LESP critical criterion is applied to the geometric leading edge only. The vortex shedding from the geometric trailing edge is governed by the Kutta condition. The results from this simulation are compared with the experimental result given by Granlund et al. [59] and the theoretical results from Issacs’ and Greenberg’s theories.

As shown in Fig 3.14 we can see that the results for the normalized lift data especially in the retreating phase of the airfoil matched well with the results from the current algorithm. The flow visualization shows the presence of LEV (from geometric leading edge) in reverse flow, as can be observed in the Fig 3.15b. This LEV is more prominent in the case of \( \lambda = 1.4 \). The
Figure 3.14: Comparison of Normalized lift values for cases of $\lambda = 1.0$, 1.2 and 1.4 from top to bottom
Figure 3.15: Comparison of flow visualization with experimental data.

changes made to LESP critical value helped achieve these results.
Chapter 4

Conclusions and Future Work

The performance prediction of an airfoil section in reverse flow is essential to understanding the aerodynamic loads acting on blade elements of high-speed helicopters and other rotor applications like wind/tidal turbines. The detrimental affects of reverse-flow regions, generally characterized by negative lift, early onset of flow separation and periodic vortex shedding, are analyzed and well documented by various experiential and CFD studies. However, to rely solely on experimental and/or CFD methods to conduct detailed study of these phenomena is highly undesirable due to the high cost and time investments required for such methods. Hence the development of a low-order method to simulate flows with varying free stream velocities including reverse flow is looked into with interest. The current research effort can be divided into three phases: (1) examining the effect of change in vortex location in the Discrete Vortex Method, to enable the development of new algorithms to simulate both forward and reverse flow over airfoils, (2) modification of the lumped-vortex-element method with LEV formation capabilities to accommodate for varying-freestream-velocity flow conditions and (3) to extend the low-order method capabilities to include reverse flow regions by examining the change in LESP critical values for reverse flow.

In this research effort it was observed that the change in the vortex location from 25% to 0% of the length on the airfoil segment does not effect the aerodynamic load calculations in the
lumped-vortex-element method as long as the distance between the vortex and the collocation point is maintained at 50% of the length of the airfoil segment and the number of airfoil segments is maintained above 50. On the basis of these results we can say that the vortex location can be changed to 0% in lumped-vortex-element method without affecting the results obtained. This kind of an airfoil model/configuration can be used in the development of an algorithm where the vortex shedding criteria for both the geometric leading edge and the geometric trailing edge are the same. Like the LESP parameter that dictates the vortex shedding at the geometric leading edge, an edge separation parameter (ESP) can be defined based on the airfoil edge geometry and can be used to simulate flow over the airfoil edge whether it is experiencing forward or reverse flows.

Based on the LDVM method presented by Ramesh et al [57] for varying freestream flow condition, the current lumped vortex method has been modified to include the simulation of varying freestream conditions. The main feature is to include a varying LESP critical value based on the Reynolds number experienced by the Leading edge of the airfoil. This Reynolds number is derived from the velocity $V_{mag}$ which is defined as the freestream velocity plus the induced velocities due to bound and ejected vortices as experienced by the LE of the airfoil. The modified method works well in predicting the lift coefficient of the airfoils in varying freestream flows like perching maneuvers. There were small discrepancies observed in the calculation of the drag coefficients with this modified method. As discussed before in Chapter 3, these errors can be attributed to the differences in the way the bound vortices on the airfoil are calculated in the LDVM method and the current method. The LDVM method considers the influence of the velocity component $V_{mag}$ in its calculation of vortex strengths, while the current method does not. The current algorithm can be improved further by incorporating the effect of induced velocity at the LE while calculating the bound vortex strength on the airfoil.

The LESP is an important parameter in predicting the LEV formation for an airfoil undergoing unsteady motion. When the LESP value exceeds the LESP critical value, LEV formation is initiated. In previous research efforts LESP critical was shown to be independent of the
motion kinematics for a given airfoil operating at a given Reynolds number. There have not been any studies that investigated the reverse-flow region and its effects on the LESP critical values. In the current work, on the basis of CFD simulations of airfoil pitching in reverse flow, we estimated an LESP critical value of 0.04 for a NACA 0012 airfoil in reverse flow (for geometric leading edge). In forward flow for the same NACA 0012 airfoil the LESP critical value is 0.2 \cite{57}. Hence it can be concluded that the LESP critical value drops by a large amount as the flow goes from forward to reverse flow. The same LESP critical obtained here was used in the simulation of a streamwise oscillating airfoil which goes into a reverse flow region in the retreating phase. By switching the LESP critical value at the flow enters into reverse flow we were successful in simulating the stream wise motion of the airfoil.

The scope of future work is also three-fold as has already been discussed in the above paragraphs. The three potential research works include an improvement to the current algorithm, a parametric study to understand the LESP critical values for reverse flows and development of a stand-alone model to simulate reverse flow regions more effectively. Improvements to the current method are in more effectively predicting the drag coefficients for varying freestream flow conditions. The parametric study suggested here would be to evaluate and analyze the change in LESP critical values for different airfoil edge geometries (smooth, blunt, or sharp), in both forward and reverse flows. The development of a new stand-alone algorithm as discussed before would be to implement an edge separation parameter for sharp or rounded edges that function as aerodynamic leading or trailing edges depending on whether the flow is forward or reversed.
REFERENCES


