KABIR, MD ASHFANOOR. High Performance Reluctance Motor Drives with Three-phase Standard Inverter. (Under the direction of Dr. Iqbal Husain).

Electric motors perform electromechanical energy conversion within a motor driven system. These electromechanical systems are the most significant type of electrical loads utilizing around 45% of global energy, the vast majority of which is consumed by the electric motor itself. Efficiency improvement of this largest consumer of electricity is critical for saving both energy and environment. In the industrial sector, induction motors (IMs) dominate the motor market. High cost, supply volatility, and detrimental environmental impact of mining rare-earth permanent magnet (PM) materials and simplicity, robustness of IMs have made them the largest shareholder in the field of electrical loads.

Promoting a competitive motor market transformation towards improved efficiency can lead to significant reduction to energy consumption and greenhouse gas emission. In this direction, International Electrotechnical Committee (IEC) has approved a new international efficiency standard to globally harmonize motor energy efficiency. In addition to revisiting the existing IE3 class efficiency, equivalent to premium efficiency class according to National Electrical Manufacturers Association (NEMA), this new IEC standard defines IE4 (super-premium) and IE5 (ultra-premium) class efficiencies. Moreover, compared to line-fed motors, its variable speed drive (VSD) alternative is advantageous in improving overall system level efficiency under variable loads and VSD fed systems are being widely adopted in newly installed motors. This newer technology trends toward improved efficiency motivates the search for low-cost, rare-earth-free, and higher efficiency alternatives to induction motors.

Realizing the need, this dissertation aims at designing reluctance motors (RMs) that have no PMs or secondary windings in its rotor and operate through the principle of magnetic reluctance. As a result, RMs have cooler rotor without any PM or rotor conduction losses. This provides an opportunity in improving the power conversion efficiency with RM drives. This dissertation presents the design, modeling and performance improvement of two RM topologies utilizing three-phase standard voltage source inverters (VSIs) as its drive.
Performances of the designed motors have been verified with analytical and semi-numerical models, finite element analysis (FEA), and experimental testing.

A novel design of segmented rotor switched reluctance motor (SSRM) has been developed having compact structure with tooth-wound concentrated windings. The designed SSRM achieves its mutual inductance variation through rotor segments enabling the use of three-phase standard VSI. Moreover, with increased contribution from each phase the designed motor has improved torque density compared to conventional SRMs. The major torque ripple sources in SSRM have also been identified and a new rotor segment design is proposed with segmental dip for torque ripple minimization. Finite element analysis (FEA) based multi-dimensional design optimization including mechanical stress and acoustic noise studies are performed and a prototype SSRM is built and tested. With the application of standard VSIs and conventional motor control, proposed SSRM topology overcomes the major challenge of conventional SRM’s commercial adoption.

Design of a synchronous reluctance motor (SynRM) using a new multilayer (ML) distributed winding has been developed. Compared to conventional distributed windings, the ML winding yields a more sinusoidal stator MMF with shorter end-winding length. This translates into the reduction of space harmonics and reduced stator \( I^2R \) losses in the motor. The ML winding is optimized to design an IM (MLIM) and a SynRM (MLSynRM) under a commercial premium efficiency benchmark IM (BMIM) and prototype motors are built. Performance of the test motors are evaluated following the IEEE 112 standard for loss separation and rated efficiency determination. Compared to the premium efficiency BMIM, the designed MLIM and MLSynRM can attain super-premium and ultra-premium efficiencies, respectively under the same frame size and cooling type. This new multilayer winding configuration can be a technology trend in gaining efficiency improvement with low cost non-PM designs under the standard frame sizes.
High Performance Reluctance Motor Drives with Three-phase Standard Inverter

by
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To Ammu & Abbu.
BIOGRAPHY

Md Ashfanoor Kabir received his Bachelor’s and Master’s degrees in Electrical and Electronic Engineering (EEE) from Bangladesh University of Engineering and Technology (BUET), Bangladesh in 2009 and 2012, respectively. From 2009 to 2012, he served the Department of EEE, BUET as a lecturer. He received his Ph.D. degree from the department of Electrical and Computer Engineering at North Carolina State University, Raleigh, NC, USA. Currently he is working as a research scientist at ABB US Corporate Research Center, Raleigh, NC, USA.

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1.8 Dissertation Outline
1.1 Introduction

Electric motors are the single largest category of electricity end use. In the industrial sector, they are the most prevalent electrical loads utilizing approximately 70% of its electricity [1]. Among different types of AC electric motors, induction machines (IMs) hold the largest market share (87% according to the projected revenue share in a report to the European Commission [2]). As a result, efficiency improvement for this major consumer of industrial electricity is highly significant for global energy savings. NEMA (National Electrical Manufacturers Association) premium efficiency or IEC (International Electrotechnical Committee) IE3 efficiency class motors are now mandatory in North America. In the second edition of IEC 60034-30 standard [3], super-premium (IE4 class) and ultra-premium (IE5 class) are being defined. These new regulations in energy efficiency classes of electric machines are motivating industries and academia for research and development for its efficiency improvement.

1.2 Industrial Motor Technology Trend

Among different commercial motors with standard frame sizes, squirrel cage induction machines (SCIMs) can achieve super premium/IE4 efficiency class only for power levels more than 7.5 kW as shown in Figure 1.1(a). For lower power levels (≤ 7.5 kW) that constitute 79% of the total AC motor market [2], IE4 designs are possible with line start permanent magnet (LSPM) machines whose price/kW is 2 to 4 times of that of SCIMs [1]. Price analysis of different commercially available electric machines shows the research gap in finding a low-cost, high efficiency alternative in standard frame sizes. The motivation of this dissertation has been built on this background; the objective is to find a way to minimize the SCIM losses so that higher efficiency (IE4/IE5) designs are possible with standard frame sizes even at the lower power levels (≤ 7.5 kW).

Loss distribution of SCIMs [4] at low power levels (rated power ≤ 7.5 kW) are presented in Figure 1.1(b). Reluctance motors have a cooler rotor due to the absence of permanent magnet or secondary winding and associated losses in them. As a result, rotor \( I^2R \) losses, which is about 20% of the total loss in the considered power range, is not present in reluctance motors.
which translates to gain in power conversion efficiencies. Moreover, for low power levels, stator conduction loss ($I^2R$ loss) has the highest loss contribution ranging from 45% to 55% of the total loss. As a result, design strategy in minimizing stator $I^2R$ loss will be an effective efficiency improvement technique. Induction machine design involves full-pitched phase windings with large end-winding lengths extending over the full pole pitch. These end-windings are necessary for completing the phase windings but they do not contribute towards electromagnetic torque production. Minimization of this end-winding length is a viable way to reduce stator $I^2R$ losses and improve machine efficiency.

![Figure 1.1](image1.png)

Figure 1.1: (a) Price/kW comparison of different commercial motors and (b) Loss distribution of SCIMs for rated power ≤ 7.5 kW

### 1.3 Advantages of Variable Speed Drives

Electric motor work as an integral part of electromechanical systems. Steady state operation of electric motors are determined by the system output parameters which are usually mechanical parameters like pressure, flow etc. The optimal operation of a motor drive system should be evaluated from its system level perspective. Depending on the system demand electric motors can realize different control options. On/off control is a simple and well known control strategy for line fed motors but they reduces system efficiency at partial load condition with increased on/off operations. An alternative is a step wise control where the system consists
of several small units and control requirement is meet with their parallel operation. Large installation cost is a demerit of this control. Both on/off and stepwise controls lacks the flexibility of continuous control over system parameter variation [5].

For continuous operation of line fed motor drive systems mechanical means can be used to control the output parameters as shown in Figure 1.2(a) where a pump is driven by electric motor. The line-fed motor operates at fixed speed and a throttling valve is used to control the output flow rate of this system. High energy loss associated with the mechanical means is a drawback of this system and variable speed drive (VSD) operation as in Figure 1.2(b) can provide a significant energy savings at the system level.

![Figure 1.2: Line-fed versus VSD operation for an electric motor driven pump](image)

Energy savings by VSD operation compare to line-fed drives can be explained with the help of Figure 1.2(c). The nominal operating point of the system is represented by point A where the motor is operating at full speed to provide the required flow-rate, Q1. Now if the flow rate is reduced to Q2, for a line fed system this can be achieved by throttling the valve (Figure 1.2(a) and the new operating point will be at B. However, for the VSD operation (Figure 1.2(b)) flow-rate Q2 can be achieved by reducing the motor speed to 80% of its nominal and the new operating point will be at C. The required and provided motor power are very close at point C without any extra losses over throttling valve. A power savings of $\Delta H \times Q2$
is achieved by adopting VSD which provides a large improvement in the efficiency of the motor drive system.

1.4 Reluctance Machines

Reluctance machines (RMs) have a long history of almost 200 years. In 1838, a patent for “electromagnetic engine” by WH Taylor [6] showed the basic mechanism of switched reluctance motor (SRM) using salient pole soft irons and stationary electromagnets that produced torque based on alignment principle and maintained motion with proper excitation sequence of the electromagnets. Four years later, Davidson demonstrated the first application of SRM [7] by powering a locomotive on the Glasgow to Edinburgh railway. In 1920, stepper motor was invented by Walker [8]. This invention held many features common to modern day reluctance motors. In these early stages, lack of suitable power switching and control devices and improved electromagnetic design restricts SRM developments and electric motor market has been dominated by DC and induction motors (IMs).

The earliest reference to synchronous reluctance machine (SynRM) is from 1923, when Kostko [9] first introduced the concept of reluctance slots, flux guides, and segments. This structure was designed for line-start application including a starting cage. In 1930s, SynRMs with typical IM rotor punching and some teeth cut-out for saliencies were introduced. These early designs of SynRMs provided low-cost and reliable synchronous operations but suffered from low saliency ratios, and low power factor and torque output. Applications of these machines were found in control rod positioning in nuclear reactors and spinning plants [10]. However, SynRMs were assumed inferior to other AC motors for its low power density and poor performance [11].

With the invention of thyristor in 1960s and commercialization of power electronic inverters in 1980s VSD applications of electric machines have increased significantly. Price reduction of semiconductor devices, availability of high speed power electronic switches, powerful microcontrollers and digital signal processors (DSPs) have rekindled reluctance
machines research with VSDs. With continuous design efforts from industry and academia, reluctance motor drives has gained significant improvements over the last half century.

In 1972 Bedford [12], introduced the concept of using control electronics to synchronize the phase currents with rotor positions developing the fundamentals for understanding and developing switched reluctance drives. Following notable contributions in control, design and characterization of SRM drives are by Bausch et al. [13], Lawrenson et al. [14], Byrne et al. [15], Miller [16] and Ray et al. [17]. In the field of SynRMs, axially laminated SynRM rotors were designed [18], [19] achieving saliency ratios of seven or more. Although, these machines performed head to head with IMs, high manufacturing cost associated with the axially laminated rotors made them less favorable for mass production. Multiple flux barriers with transversely laminated rotor and saturable bridges were introduced as an alternative to salient pole designs in the early seventies [20], [21]. These flux barriers were introduced to reduce quadrature axis (q-axis) inductances without reducing the direct axis (d-axis) inductance. These design contributions paved the way towards developing improved variable speed reluctance motor drives competitive to already established AC machine topologies.

1.5 Reluctance Machine’s Advancements

Modern day’s research and developments are continuously improving the characteristics of reluctance motors as VSDs. Some notable contributions in the field of SRMs are by Krishnan et al. on SRM designs [22], [23] and drives [24]; Mecrow et al. on mutually coupled [25], [26] and segmented rotor [27] SRMs; Miler et al. on SRM design [28] and control [29]; Torrey et al. on SRM modeling [30] and control [31]; Ehsani et al. on SRM converter [32], and control [33], [34]; Husain et al. on SRM design [35] and control [36], [37]; Chiba et al. on SRM application as traction drives [38], [39]; Fahimi et al. on double stator [40], [41] and bipolar [42] SRMs.

Implementation of SynRM field oriented control (FOC) [43], [44] is a major milestone in its development. FOC enabled SynRM operation without the starting cages and much of SynRM design improvements has occurred since then. Miller et al. contributed on the rotor
structure optimization in maximizing the saliency [45], [46]. D. Platt presented a rotor design with iron lamination separated by non-magnetic spacers to achieve high saliency [47]. Lipo et al. contributed significantly in control [48], [49] and designing optimum bridge to barrier ratio [50]. Boldea et al. contributed in design [51] and control [52] of SynRM. Vagati et al. have pioneered contributions [53]-[55] in SynRM design including cross-saturation and improvement in its torque ripple performances.

Reluctance machines are advantageous compared to other AC machine counterparts due to its low cost, permanent magnet free designs, cooler rotor without any rotor windings or cages and possible operation at high temperatures. Reluctance motors have copper losses only in its stator which are much easier to cool than the rotor and consequently they are more favorable than induction motors in low speed operations. Moreover, switched [39], [56] and synchronous [57], [58] reluctance motors have robust rotors suitable for high speed operations.

1.6 Reluctance Machine Fundamentals

Both switched reluctance and synchronous reluctance motors depend on magnetic reluctance principle for torque production. However, their operation and characteristics are different from each other. SRMs are doubly salient motor and their phase excitations are of rectangular/ trapezoidal shape synchronized with rotor position. SRMs operate in high magnetic saturation to maximize their torque capability. On the other hand, saliency exist only on the rotor side for SynRMs and it has a cylindrical stator as IMs with distributed windings to generate sinusoidal rotating MMF. Conventionally, SynRMs are excited with sinusoids and its operating regions are well below its magnetic saturation level. The following two subsections will discuss the basic operating principles of SRM and SynRM, respectively.

1.6.1 Fundamental of SRM

A rectilinear representation of SRM has been utilized to explain its operating principle as presented in Figure 1.3. Both stator and rotor of an SRM are salient pole type. Conventional SRM phase windings are concentrated around each stator pole. When stator poles of a certain phase are energized, the closest rotor pole gets attracted by the energized stator and tries to
align itself to the energized rotor in minimizing the reluctance of the magnetic path as presented in Figure 1.3(a). Energized stator pole faces the minimum air-gap length to rotor in this aligned position. Machine phase inductance is maximum in the aligned position and it decreases gradually as the rotor pole moves away from the energized stator pole. When two consecutive rotor poles are symmetrically misaligned with the energized stator, unaligned rotor position occurs. The unaligned position is presented in Figure 1.3(b) and at this point the energized stator pole faces the maximum air-gap length and corresponding phase inductance is minimum. Variation of machine phase inductance as a function of the rotor position is periodic with its maxima and minima equal to its aligned and unaligned values, respectively.

![Diagram](image.png)

Figure 1.3: (a) Aligned and (b) unaligned stator-rotor positions of SRM

Inductance variation of one SRM phase is presented in Figure 1.4. Ideally, the linked flux of one phase is solely carried by the stator pole of that phase so inductance profile of Figure 1.4 is the self-inductance profile of one SRM phase. Phase excitation is synchronized with the positive slope of its inductance for motoring and with the negative slope for generating operation. Electromagnetic performance of an SRM can be explained with its $\lambda - i - \theta$ characteristics as presented in Figure 1.5. Envelop of $\lambda - i - \theta$ is defined by machine aligned and unaligned inductance and their quotient defines the motor saliency. Highest possible saliency ratio is required to achieve maximum torque capability.
The general voltage balance equation of SRM is given by,

$$V_{ph} = iR + \frac{d\lambda}{dt}$$

(1.1)

Here, $V_{ph}$ is the dc bus voltage, $i$ is the instantaneous phase current, $R$ is the phase resistance, and $\lambda$ is the coil flux linkage which is a non-linear function of rotor position and stator current, $\lambda(i, \theta)$. Eq. (1.1) can be re-written as,

$$V_{ph} = iR + \frac{\partial \lambda}{\partial i} \frac{di}{dt} + \frac{\partial \lambda}{\partial \theta} \frac{d\theta}{dt} = iR + L_{inc} \frac{di}{dt} + e\omega$$

(1.2)
Here, $L_{inc}$ is the incremental inductance, $e$ is the current dependent back-MMF coefficient, and $\omega$ is the rotor angular speed. Assuming magnetic linearity ($\lambda = L(\theta)i$), power balance in SRM can be expressed as:

$$P = V_{ph}i = i^2R + \left( Li \frac{dl}{dt} + \frac{1}{2}i^2 \frac{dL}{d\theta} \omega \right) + \frac{1}{2} i^2 \frac{dL}{d\theta} \omega = i^2R + \frac{d}{dt} \left( \frac{1}{2}Li^2 \right) + \frac{1}{2} i^2 \frac{dL}{d\theta} \omega$$

(1.3)

The first term represents the stator winding loss; the second term denotes the rate of change of magnetic stored energy, $E$ while the third term is the mechanical output power. Most effective way of energy conversion for SRM is by maintaining constant current during positive $\frac{dL}{d\theta}$ slope as shown in Figure 1.4. The general instantaneous torque expression is given as,

$$T_{ph}(i, \theta) = \frac{\partial W'}{\partial \theta}$$

(1.4)

Here, $W'$ is the co-energy defined as,

$$W' = \int_0^i \lambda(x, \theta) dx$$

(1.5)

The effectiveness of SRM energy conversion is defined by its energy ratio, $ER$ as,

$$ER = \frac{W'}{W' + E}$$

(1.6)

In case of linear SRM operation as shown in Figure 1.5(a), energy ratio can be at most 50%. This low energy ratio increases converter volt-ampere rating. $ER$ increases in favor of energy conversion if SRM is operated under high magnetic saturation as in Figure 1.5(b). This is the primary reason to always operate SRMs under saturation. The term energy ratio is analogous to power factor for AC machines. SRM instantaneous torque is not constant. For continuous production of motoring torque, SRMs are built with multiple phases and the total electromagnetic torque is given by the sum of each phase torque.

$$T_{EM}(i, \theta) = \sum_{\text{phases}} T_{ph}(i, \theta)$$

(1.7)
1.6.2 Fundamental of SynRM

Torque production principle of SynRM depends on the interaction between rotor reluctance and synchronously rotating magneto motive force (MMF) produced by its sinusoidally distributed stator windings. This torque production concept is explained with a three-phase SynRM with its rectangular two-pole rotor formed by an isotropic magnetic material as presented in Figure 1.6(a). With three-phase balanced sinusoidal excitation, the stator winding produces a synchronously rotating MMF ($\psi$) and it links with the rotor through a small air-gap. Direction of such MMF at a certain instant is presented by dotted arrows in Figure 1.6(b). The rotor is salient pole type having two different reluctances along the $d$-axis and the $q$-axis. Along the $d$-axis, rotor reluctance to stator MMF is minimum and along the $q$-axis it is maximum. When a magnetic field, $\psi$ is applied, the rotor tries to align itself to its minimum reluctance position with the stator MMF ($\delta = 0$). So if there is an angle difference between the $d$-axis and direction of $\psi$, it will produce electromagnetic torque in the direction shown by blue arrows in Figure 1.6. The stator field is rotating in synchronous speed, $\omega_s$ and the rotor is trying to catch up. Under constant load angle, $\delta$ electromagnetic energy will continuously be converted into mechanical energy.

![Figure 1.6: (a) Three-phase two-pole conceptual SynRM and (b) interaction of salient pole rotor with stator MMF](image_url)
Stator current is responsible for producing both magnetization (stator MMF production) and torque production. The stator windings are sinusoidally distributed and air-gap flux harmonics will contribute only to the stator leakage inductance. As a result, Park’s equations, which describe the conventional wound-field synchronous machine, can be used to model the SynRM [59]. However, in SynRM rotor field windings or damper windings are absent. Eliminating both field and damper winding equations from Park’s equation, the basic \( d - q \) equations for SynRM can be written as,

\[
v_{ds} = r_si_{ds} + p\lambda_{ds} - \omega\lambda_{qs} \tag{1.8}
\]
\[
v_{qs} = r_si_{qs} + p\lambda_{qs} + \omega\lambda_{ds} \tag{1.9}
\]

Here, \( v \) and \( i \) represent stator voltage and currents, respectively, \( r_s \) is stator resistance per phase, \( \omega \) is the electrical angular speed of the reference frame, and \( \lambda \) represents flux linkages which can be written as,

\[
\lambda_{ds} = (L_{ls} + L_{md})i_{ds} = L_di_{ds} \tag{1.10}
\]
\[
\lambda_{qs} = (L_{ls} + L_{mq})i_{qs} = L_qi_{qs} \tag{1.11}
\]

Here, \( L_{ls} \) is stator leakage inductance and \( L_{md} \) and \( L_{mq} \) are the direct and quadrature axis magnetizing inductance, respectively. Phasor relationship of SynRM at steady state \( (p\lambda = 0) \) can be obtained by multiplying Eq. (1.8) by \(-j\) and adding it to Eq. (1.9). Vector diagram of SynRM at steady state is presented in Figure 1.7. Here, \( \epsilon \) is the MMF angle or current angle, \( \delta \) is the torque angle, and \( \varphi \) is the power factor angle. The general expression of electromagnetic torque of SynRM is given by,

\[
T_e = \frac{3P}{2} \left[ \lambda_{ds}i_{qs} - \lambda_{qs}i_{ds} \right] \tag{1.12}
\]

Here, \( P \) is the number of poles. In terms of \( d - q \) axis inductances and constant current operation the torque expression can be re-written as,
\[ T_e = \frac{3P}{2} \left( L_{ds} - L_{qs} \right) i_{ds} i_{qs} \]  

(1.13)

**Figure 1.7**: SynRM phasor diagram at steady state

### 1.7 Research Motivations

Since the inception of reluctance motor concept both switched and synchronous reluctance topologies have advanced a lot in its research and development. However, wide acceptance of these topologies in the industry arena are yet to be achieved. SRMs are still not being considered as a strong contender for VSD applications and its market penetration are very limited [60]. SynRMs have made a recent entry in commercial VSD market when ABB first introduced their product line in 2011. The following two subsections discuss certain design challenges of SRM and SynRM that need to be addressed to expedite their commercial adoption as well as find out the research opportunities to improve their performance as VSD drives. Based on these analysis, research objectives of this dissertation has been presented in subsection three.
1.7.1 Design Challenges in SRM

The operation of conventional SRM is discussed in section 1.6.1 that requires unipolar phase currents for continuous torque production. This special phase excitations require independent control of each phase making SRM converter significantly different from other AC machine technologies. Asymmetrical half bridge converter as presented in Figure 1.8 is used for SRM operation.

![Figure 1.8: Asymmetrical half bridge inverter for three-phase conventional SRM](image)

Due to the unconventional configuration of its converter, SRMs have received less attention by the industries despite its advantages as a variable speed drive. Conventional SRM designs cannot operate with the readily available commercial three-phase voltage source inverters (VSIs). Independent phase current control requires more current sensors than that required for an AC machine with line current control. Control of conventional SRM converter is also more challenging compared to conventional vector control of AC machines.

For a three-phase SRM converter, six additional diodes are required as internally packaged body diodes of switching devices cannot be utilized. In addition, freewheeling due to the pulsed nature of the phase currents and the large size of the filter capacitors increase the converter size for conventional SRMs. Unipolar operation in conventional SRMs (CSRM) increases the peak phase current by about 40% than for bipolar operation for the same RMS current value, which increases the current stress of the drive power devices [61]. These drawbacks of
controller and converter complexities motivate research towards designing a switched reluctance machine that would be able to use a standard six-switch voltage source inverter (VSI) as with other AC machines.

Three-phase standard VSI as presented in Figure 1.9 has many advantages. Three-phase full bridge circuits are the most commonly known and readily available topologies. The commercially available packages usually comes with current transducers and controller electronics. With the three-phase VSIs, using current balance, only two current sensors are required to establish current feedback control. Only three motor to converter connections are required compared to six for the SRM converter. Moreover, with unipolar non-overlapping pulses torque ripple and vibrations are higher and can be reduced with shared phase currents using bipolar excitations with standard inverters [62]. Moreover, with increased effective torque contribution from each phase, machine power density will improve with bipolar excitation [63].

![Figure 1.9: Three-phase standard VSI.](image)

Enabling SRM operation with three-phase standard VSI requires simultaneous motoring torque contribution from at least two of its phases. This can be achieved by maintaining a variation in SRM mutual inductance profiles. SRM designs enabling mutual inductance variation are known as mutually coupled switched reluctance machine (MCSRM). B.C.
Mecrow first designed an MCSRM in 1993 [25]. This designed MCSRM incorporate full-pitched windings with higher end-turn length increasing copper losses, weight and manufacturing cost. Moreover, achievable slot fill factor is also low (approximately 40%) with this fully pitched configuration which also degrades machine performance. In order to make the SRM application more attractive with three-phase VSI, machine design improvement is needed to obtain a more compact design with shorter end-winding length and higher fill factor.

### 1.7.2 Design Challenges in SynRM

Following the commercialization of power inverters, research on improving SynRM performance as a VSD rekindled in the nineties. The multi-barrier synchronous reluctance rotor design has been extensively covered in the literature targeting torque density and power factor improvement [64]-[66] or torque ripple minimization [54], [67], [68]. The evolution of SynRM rotor design from its inception in 1923 till present day is presented in Figure 1.10.

![SynRM Rotor Designs](image)

Figure 1.10: Evolution of SynRM rotor designs

Kostko [9] was the first to introduce multiple flux barrier designs as shown in Figure 1.10(a). Later, salient pole rotors were made by removing iron materials from an induction
motor rotor [46]. The rotor construction was simple as in Figure 1.10(b) but the saliency ratio was very poor to give a competitive performance. For precise speed control in fiber spinning multiple barrier rotors as in Figure 1.10(c) were designed in early sixties. These motors were synchronously operated with inverter fundamental frequency set by a crystal controlled oscillator. In mitigating ‘swing’ and ‘pull in/ pull out’ effect rotor designs with saturable bridges as in Figure 1.10(d) were employed [21]. After the commercialization of inverters in the seventies, SynRM s were designed with segmental rotors as shown in Figure 1.10(e). Rotor cage was no longer present and machine was started in synchronism with inverter frequency [10]. Saliency ratio of 5 or more were achievable with these designs such that the same size design as that of an induction motor was achievable. However, poor power factor and efficiency challenged the widespread use of these machines. To overcome these difficulties, rotors with axially laminated steel sheets bent into a ‘V’ or ‘U’ shapes as in Figure 1.10(f) and (g), respectively were designed [18], [19]. Saliency ratio of seven or more were achievable with these designs making them favorably competitive to their IM counterparts. Manufacturing costs were still high with these axially laminated type rotor designs. Transversely laminated SynRM rotors as presented in Figure 1.10(h) have been the most beneficial entry in SynRM designs [53], [55]. Rotor is radially laminated in the same way as other radial flux machines and thin lamination bridges hold multiple flux guides together. Designing these bridge thicknesses is very crucial and is a tradeoff between the mechanical robustness of the rotor versus achieving high saliency ratio with saturated bridges. With intensive research efforts in design and optimization of rotor structure, SynRM performances have been extensively improved over the years and brought it to the spotlight of a new VSD alternative to IMs and permanent magnet synchronous machines (PMSMs).

However, the stator winding configuration remains the same as that of an induction motor in these research efforts. Recent literatures have reported on concentrated wound synchronous reluctance motors [69], [71]. Concentrated windings have the advantages of non-overlapping end-turns, high slot fill factor, and easier manufacturing. However, existing literature on concentrated windings show that these designs are characterized with high space harmonics in
its air-gap MMF. These undesirable space harmonics result in higher torque ripple, increased core loss and decreased power factor [69], [72]. Different three-phase winding configurations are also proposed to obtain lower space harmonics [73], [74]. These designs use 5th order space harmonic for torque production. A multiple layer (layer number ≥ 3) winding is presented in [73] that has additional space harmonics of 7th, 17th, and 19th order. Winding configuration in [74] is of fractional-pitch type with overlapped windings having 17th and 19th order space harmonics. Moreover, machine pole number increased to 10 with 12 slot [73] and 24 slot [74] stators. This higher pole number increases machine’s fundamental frequency increasing core-losses and reducing machine input power factor. Torque ripple of these designs are also higher compared to their distributed winding counterparts [72]. Based on relative design tradeoffs between fully pitched distributed and concentrated/ fractional pitch windings, there is a room for improvement in AC machine winding configuration to minimize their negative impacts while incorporating both of their positive attributes in improving machine performances.

1.7.3 Research Objectives

Design challenges in SRM and SynRM topologies have been identified in last two subsections. A concentrated wound SRM design that can hold the compactness, robustness of SRM and can utilize the established, off-the-shelf, commercially available three-phase VSIs is necessary to expedite the commercial adoption of SRM. Design of a concentrated wound SRM utilizing three-phase standard VSI and its operation with conventional vector control is set as one research objective. SynRM stator winding configurations has not yet been extensively explored. Conventional distributed winding as IMs results in longer end-winding length that translates in higher copper losses compared to its concentrated/ fractional pitch counterparts. However, the concentrated/ fractional pitch designs have large unwanted harmonics in its air-gap MMF resulting in larger core loss, torque ripple and inferior power factor. The author believes that there exists a certain winding configuration in between that can minimize the negative effects of these reported windings to the fullest and further improve SynRM performances. Design of a new winding configuration to optimize SynRM performances is considered as another research objective. In summary, a new topology of SRM utilizing
conventional VSI and a new winding configuration of SynRM that can provide significant efficiency gain (one to two efficiency class higher than IMs) have been considered as the research target in this dissertation. These research focuses drive reluctance motor technology forward to have positive impact and accelerate the adoption of these machines in the industry.

This dissertation covers design and optimization of two novel reluctance machines using three-phase standard VSIs. First, an SRM topology is introduced that utilizes concentrated winding reducing end-winding length to the shortest and improving slot fill factor. Next, a multilayer wound SynRM is presented that can achieve sinusoidal air-gap MMF with shorter end-winding length than its fully pitched counterparts can. Both these design contributions reduces end-winding length and hence stator $I^2R$ losses. Moreover, the cold rotor of reluctance machines eliminates rotor $I^2R$ losses. As a result, the designed machines would provide gain in motors power conversion efficiencies. Moreover, employing VSD has system level efficiency gain over line-fed alternatives.

Motor electricity consumption is the largest in the world, consuming more than twice as lighting, the second largest category. According to the International Energy Agency (IEA) energy efficiency series report on electric motor driven systems [75], by year 2030, annual energy consumption from electric motors is expected to rise to 13360 TWh (trillion watt-hour) with $CO_2$ emission of 8750 Mt and this will result end user expenditure of around USD 900 billion. The considered power level of this research ($P_{Rated} \leq 7.5 \text{kW}$) consumes around 79% of this energy [2]. A contribution towards efficiency improvement will improve global energy savings and reduce release of greenhouse gases into the atmosphere. Moreover, improvement of these non-rare-earth technologies is particularly valuable under current scenario of industrial motors, where price, mining, and supply of rare-earth materials are alarming issues and low-cost, high efficiency alternatives are necessary.
1.8 Dissertation Outline

The organization of this dissertation has been summarized in the following paragraphs.

Chapter 1: Reluctance Machines: A Drive Forward

This chapter presents the drive towards machines efficiency improvement from the context of industrial motors energy consumption along with the possibility of reluctance machines to be a low-cost, high efficiency alternative. It also covers the evolution and inherent advantages of the reluctance motor technology. Operation fundamentals for both SRM and SynRM are discussed next. Current design challenges and technology trend in SRM and SynRM research are discussed which form the research motivations and objectives of this dissertation. The final section discusses the outline of this dissertation.

Chapter 2: SRM with Three-phase Standard VSI

Research contributions in SRM design using three-phase standard VSI are covered in this chapter. First, the reported FP-MCSRM are investigated and analytical inductance model of the machine is developed. Design guideline for mutual torque maximization is developed and an FP-MCSRM is designed for a given design benchmark. Design challenges with this FP-MCSRM are discussed next. A novel concentrated wound segmented rotor SRM is introduced that can overcome the major design challenges of FP-MCSRM. Fundamental operating principle of the proposed topology is elaborated. Design parameter optimization targeting torque maximization are discussed next. Performance of the designed machine is compared with a commercially available SRM. Initial analysis showed high torque ripple which has been addressed next through machine design approach.

Chapter 3: Design of Segmented Rotor SRM

This chapter details the methodology of designing a low torque ripple segmented rotor SRM and optimizing its design for the selected design benchmark. First, a semi-numerical machine inductance model is developed to investigate the sources of torque ripple in SSRM. Based on this analysis a novel shaping of the rotor segment is suggested to reduce torque ripple. Effect of different geometric parameters on torque ripple are studied to suggest design updates.
targeting torque ripple minimization. Based on the design benchmark, a set of design objectives are selected and Multi-dimensional optimizations are performed targeting both single and multiple design objectives. Multi-physics based mechanical stress analysis are performed next on the electromagnetically optimized design. Acoustic noise analysis of the designed machine including its mode frequencies are studied along with its noise performance comparison against commercially available SRMs. A prototype segmented rotor SRM is built and its simulation results are experimentally verified under conventional vector control.

Chapter 4: IM Modeling with Ring Parameter Estimation

The first chapter has elaborated the potential application domain of the designed reluctance motors to be high power density, high efficiency alternatives to line fed IM drives in the selected power level (≤ 7.5 kW). Consequently, a 1 HP IE3/premium efficiency induction motor has been selected as the design benchmark. An accurate machine model is required to properly predict the machine performance during its design stages. Reviewing the IM technology, absence of an accurate IM end-ring model is identified. This chapter details on the development of end-ring modelling for IM. Starting from its analytical background an accurate 3D FEA based end-ring model development is presented in this chapter. Finally, the developed IM model has been verified through experimental analysis. Performance matrices are obtained and set as the designed benchmark for the designed reluctance machines in this dissertation.

Chapter 5: New multilayer winding configuration

This chapter introduces the novel multilayer winding concept that can provide sinusoidal MMF with shorter end-winding length compared to its fully pitched counterparts. First, the research background is developed analyzing different AC winding configurations and their design tradeoffs. Next, the concept of the proposed multilayer winding is elaborated. The designed winding concept is characterized in terms of its winding factor, end-turn length and harmonic content in air-gap MMFs using an analytical model based on winding function theory. Next, the developed analytical model is verified with the finite element analysis (FEA). Finally, the quality of the air-gap MMF is compared with other reported AC windings under the same stator configuration and its performance matrices are quantified.
Chapter 6: SynRM Design with Multilayer winding

The proposed multilayer winding is employed for designing a SynRM under the selected design benchmark. First, the designed winding configuration is detailed for the selected stator structure. Number of turns in phase windings are optimized next based on winding characteristics and machine performance analysis. Next, selection of the multi-barrier rotor design is discussed to obtain a low ripple design. A set of design parameters and design objectives are defined from the design benchmark. Next, FEA based multidimensional optimization of rotor design parameters are performed to achieve the design target. Machine electromagnetic performance along with the effect of rotor skewing are analyzed next. Multi-physics based mechanical stress analysis are performed on the optimized design to evaluate its mechanical robustness. Finally, a prototype multilayer wound SynRM motor is built to experimentally validate the design concept.

Chapter 7: SynRM Controller Development

SynRM controller has been developed based on its direct and quadrature axis \((d-q)\) modelling in this chapter. SynRM performance parameters are explained in terms of its \(d\) and \(q\) axis inductances and currents. FEA based method to obtain \(d-q\) inductances are presented next and a look-up-table based machine model is developed. \(d-q\) inductance values from the FEA analysis are also validated with experimental measurements. Controller simulation is implemented in Matlab Simulink using both table and FEA based machine models. Both torque and speed controllers are developed in Matlab Simulink including space vector pulse width modulation for switching. A five hp dyno test-bed is developed for experimental testing. Vector control of the prototype SynRM is implemented through TI DSP based drive. Finally, machine performances under different control conditions are experimentally evaluated with vector control and simulation results are experimentally verified.

Chapter 8: Performance Analysis

In this chapter, the efficiency improvement concept of the multilayer winding is experimentally validated. Performance of the multilayer wound induction and synchronous
reluctance machines are evaluated and compared against their commercial premium efficiency benchmark. Experimental loss separation of these test motors are performed under IEEE 112 test standard. Both no-load and loaded tests are performed for loss separation. Next, heat-run tests are performed to evaluate the test motors under thermal steady state. Rise by resistance test is performed to determine the ‘specified temperature’ for efficiency calculation according to IEEE 112. Rated performance comparison under thermal steady state showed significant efficiency gain with the designed multilayer wound machines compared to their commercial premium efficiency benchmark under the same frame size and cooling type.

Chapter 9: Summary and Future Work

The final chapter summarizes the research contributions of this dissertation that covers designs of segmented rotor SRM with standard VSI, torque ripple minimization of segmented rotor SRM, FEA based IM end-ring parameter estimation model, new multilayer AC winding concept, and design of SynRM motor using the multilayer AC winding. It also discusses the potential impact of these contributions and some future directions of research inspired by them.
Chapter 2 SRM with Three-phase Standard VSI

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2.4 Summary
2.1 Introduction

This chapter covers two special types of SRM designs that enable the use of three-phase standard VSI as its drive. The first design has the same iron core construction of a conventional SRM (CSRM) but its stator windings are fully pitched to enable machine mutual inductance variation. B. C. Mecrow invented this full pitched mutually coupled SRM (FP-MCSRM) design in 1993 [25]. Analytical inductance modelling, design guidelines for mutual torque maximization and challenges associated with the FP-MCSRM are discussed in this chapter. Next, a new concentrated wound segmented rotor SRM (SSRM) design is presented. The SSRM is designed with compact concentrated windings wrapped around stator poles with rotor segments creating the shortest flux-paths. The designed SSRM is one of the novel contributions of this dissertation. Research background, operating principle and sensitivity analysis on design parameters for torque maximization are discussed. Performance of the design SSRM is also compared with a CSRM to identify its design challenges.

2.2 Full pitched MCSRM (FP-MCSRM)

Compared to CSRM that solely uses the self-inductance of each phase, MCSRM utilizes the mutual inductance between two phases to produce the instantaneous electromagnetic (EM) torque. As a result, unlike conventional SRM (CSRM) where only one phase conducts at a time, two phases need to be conducting at the same time for MCSRM. Electromagnetic torque in MCSRM is produced from the simultaneous excitation of two stator phases. Compared to CSRM, MCSRM has fully pitched winding configuration as shown in Figure 2.1.

2.2.1 Machine Inductance Model

Analytical model for the air-gap flux and inductance profiles of MCSRM has been derived by considering the spatial distribution of its windings as presented in Figure 2.2(a). The magnetic circuit is considered as unsaturated iron in this analysis. The mutual inductance in terms of the geometry is obtained from considering the differential flux linkage as is similarly done for sinusoidal distributed windings in [76]. However, in this case the phase winding distributions are discrete in stator slots creating a square-wave winding distribution as shown
in Figure 2.2(a). Considering each phase winding has $N_s$ equivalent number of turns, the differential flux linkage of phase-A crossing the air-gap at any point can be written as:

$$d\phi = \vec{B} \cdot d\vec{A} = \mu_0 \vec{H} \cdot r d\tau dl$$

or,

$$d\phi = 3\mu_0 \frac{l_A N_s}{g \pi} r \theta (d\tau dl)$$

(2.1)

where $l_A$ is the current through phase-A winding, $g$ is the air-gap length, $r$ is the radius to air-gap, $\theta$ is angular position in radians, $H$ is the magnetic field intensity, $d\tau$ is the differential angular element in Figure 2.2(b) and $dl$ is the length element along iron stack.

![Figure 2.1: Comparison of 6/4 3-phase (a) conventional SRM (CSRM) and (b) mutually coupled SRM (MCSRM) structure](image)

Considering the 6/4 MCSRM in Figure 2.2(a), which is the base machine for a 3-phase MCSRM, the winding distribution between 0 and $\pi$ for a phase (assuming phase A) can be expressed as:

$$n_{As}(\theta) = N_p \ast d\theta, \quad 0 \leq \theta \leq \pi / N_s$$

$$= 0, \text{ elsewhere.}$$

(2.2)

Here, $N_p$ is the conductor density expressed in turns per radian and $N_s$ is the number of slots. Assuming phase-A winding has $N_t$ equivalent number of turns (i.e., $2N_t$ conductors), the integral of the conductor density in between 0 and $\pi$ has a total of $N_t$ conductors which is
\[ N_t = \int_{0}^{\pi} n_{As}(\theta).d\theta = \int_{0}^{\pi/N_s} N_p. d\theta \]

\[ N_p = \frac{N_s N_t}{\pi} \]

Therefore, from (2.2)

\[ n_{As}(\theta) = \frac{N_s N_t}{\pi} \times d\theta, \quad 0 \leq \theta \leq \pi/N_s \]

\[ = 0, \quad elsewhere \]

Applying ampere’s law across the air-gap,

\[ \oint \bar{H}.\bar{d}l = \sum N i \]

Figure 2.2: (a) Phase winding distribution of 6/4 MCSRM and (b) differential flux linkage at angle \( \tau \) relative to \( \theta^o \)

The MMF will cross the air-gap twice, so the magnetic field intensity can be obtained as

\[ 2 \times g \times H(\theta) = I_A \times N_p \int d\theta \]
\[ H(\theta) = \frac{N_t \times N_s \times I_A \times \theta}{2 \times g \times \pi}, \quad 0 \leq \theta \leq \frac{\pi}{N_s} \quad (2.3) \]

Each packet of flux \(d\Phi\) links a different number of turns. From Figure 2.2(b), number of turns linked by \(d\Phi\) located at \(\tau\) relative to an initial angular position on phase winding, \(\theta_0\)

\[ N = \int_{\theta_0 - \tau}^{\theta_0 + \tau} \frac{N_t N_s}{\pi} \, du = \frac{2 N_t N_s \tau}{\pi} \]

The total flux linkage of phase ‘A’, \(\lambda_{Am}\) linked by all such packets of flux \(d\Phi\) between 0 to \(\pi\) can be obtained by integrating the differential flux linkages within a stator phase;

\[ \lambda_{Am} = 3 \mu_0 \frac{I_A N_t}{g \cdot \pi} r \theta \int_0^{L_{stk}} \pi \frac{N_s}{\pi} d\tau = \frac{2 N_t N_s \tau}{\pi} d\tau \]

Or, \(\lambda_{Am} = 3 \mu_0 \frac{I_A N_t^2}{g N_s} L_{stk} \cdot r \theta \quad (2.4)\)

Finally, the magnetizing inductance of MCSRM can be calculated as;

\[ L_m = \frac{\lambda_{Am}}{I_A} = 3 \mu_0 \frac{N_t^2}{g N_s} L_{stk} \cdot \beta \quad (2.5) \]

Here, \(L_{stk}\) is the iron stack length and \(\beta = r \cdot d\theta\) is the length of overlap between rotor and stator teeth through which flux generated by the phase currents flows. The final analytical expression of inductance is similar to that presented by B. C. Mecrow in [25], and hence, similar analysis is performed to quantify the self and mutual inductances of MCSRM with rotor positions. From Figure 2.3, the self-inductance is obtained using \(\beta = \beta_L\) which is the sum of the overlapping lengths \(x\) and \(y\) of two rotor teeth with stator teeth. For mutual inductance, \(\beta = \beta_M\) which is the sum of pole overlaps considering the polarity of their magnetic flux vectors that varies periodically with the rotor position. Positive current in phase ‘A’ produces an MMF in an outward radial direction on stator teeth S3, S4 and S5; only S3
flux links positively with phase ‘B’, while S4 and S5 fluxes link negatively. Thus, for mutual inductance between phases ‘A’ and ‘B’ with rotor position as in Figure 2.3 $\beta_M = x - y$ and for self-inductance $\beta_L = x + y$.

Figure 2.3: Overlapping teeth for self and mutual inductance calculation

From the symmetry of stator and rotor iron structures, the variation of self-inductance occur simultaneously in all three-phases. As a result, if the variation of self-inductance is non-zero then there will be negative torque contribution from the negative slope of self-inductances. It is possible to utilize the positive slope of self-inductance variation to produce positive torque, but that will increase control complexity and limit the use of the standard six-switch inverter. Therefore, in MCSRM the self-inductance is made constant for all rotor positions by making the stator and rotor pole widths equal to stator inter-polar gap such that the summation of pole overlap angles ($x + y$) remains constant for all rotor positions. Inductance profiles for such design are presented in Figure 2.4. Therefore, the EM torque in MCSRM is produced from the variation of mutual inductance only. Assuming magnetic linearity and constant self-inductance as seen in Figure 2.4, the EM torque expression for an MCSRM can be written as [77]

$$T = i_a i_b \frac{dM_{ab}}{d\theta} + i_b i_c \frac{dM_{bc}}{d\theta} + i_c i_a \frac{dM_{ca}}{d\theta}$$  \hspace{1cm} (2.6)
2.2.2 Design Guideline

Different design parameters for the FP-MCSRM are illustrated in Figure 2.5. Here, $\beta_s$ and $\beta_r$ refer to stator and rotor pole angular width, respectively, $LSI$ and $LRI$ are the length of stator and rotor back-irons, $R_3$ is stator outer radius, $R_2$ is radius to stator back iron, $R_1$ is the rotor radius to air-gap, $g$ represents air-gap length, $Fil_s$ and $Fil_r$ are the stator and rotor pole fillet angle, $Tpr_s$ and $Tpr_r$ are the stator and rotor pole tapering angle, and $R_{SH}$ is the shaft radius. The machine was parameterized and designed in 2D FEA. This parametrization helps in analyzing machine performance under different design parameter variation. Based on the analysis from section 2.1.1 and machine flux-path observations, following design guides have been adopted in selecting the initial FP-MCSRM configuration.

- Maintain the same pole widths in stator and rotor for maximization of mutual torque (constant machine self-inductance). Since, $g \ll R_1$ this condition can be maintained by making $\beta_s = \beta_r$.

- During the aligned position half of the pole flux lines are carried by stator and rotor back irons, which is the minimum thickness requirements of $LSI$ and $LRI$ from electromagnetic characteristics. However, to maintain low noise and mechanical robustness the final values of $LSI$ and $LRI$ will be higher.
\[
L_{SI_{\text{min}}} = W_{sp}/2, \text{ where } W_{sp} = (R_1 + \text{Gap}) \cdot \beta_s^c \\
\text{and } L_{RI_{\text{min}}} = W_{rp}/2, \text{ where } W_{rp} = R_1 \cdot \beta_s^c
\] (2.7)

- Machine electromagnetic torque increases with higher values of tapering angles because the magnetic saturation is eased with a wider flux distribution around the pole. However, with larger \( T_{pr_s} \), machine slot-area reduces which negative effect its torque capability. Moreover, with higher \( T_{pr_r} \), rotor contribution to unaligned inductance will increase which tends to reduce its torque.

- Under same number of turns and current density, magnetic saturation is reached at lower excitation levels with lower values of \( L_{SI} \), which will not maximize the EM torque. With higher stator back iron thickness, it is easier to attain magnetic saturation for a specific level of excitation and the EM torque at base speed improves. However, once saturation is achieved this positive effect ceases and the EM torque does not increase anymore with higher \( L_{SI} \).
• Considering the iron stack length $L_{stk}$ as a multiple of $R_1$ and some constant the EM torque can be expressed by $T = k \cdot R_1^3$. However, under a given radial constraint (fixed $R_3$) machine slot area would reduce with larger $R_1$ reducing the torque capability.

Machine design parameters are required to be optimized to meet its benchmark. An FEA based multidimensional optimization would be effective design strategy considering machines nonlinearity, saturation effects and including all design parameters in each design iteration. The design benchmark of this dissertation is presented in Appendix A. The following section will discuss parameter sensitivity analysis on FP-MCSRM along with some discussion on the optimized machine performance.

### 2.2.3 Parametric Analysis on FP-MCSRM

FEA based simulation tool FLUX 2D version 11.2.3 has been utilized to perform the FEA based parametric analysis on the FP-MCSRM. Design strategies discussed in section 2.1.2 are followed in obtaining the initial FP-MCSRM structure and the parametric analysis are performed on it. Following paragraphs will discuss the effect of variation of some design parameters on machine performance.

Effect of stator back iron thickness, $LSI$ variation has been discussed first. Isovalues of the magnetic flux density in $B$ (T) for three different $LSI$ values are presented in Figure 2.6. With lower values of $LSI$, number of turns in stator coil can be increased with larger stator slot area. However, magnetic saturation is reached with lower excitation levels with lower values of $LSI$, which will not maximize torque. With higher stator back iron thickness, it is easier to attain magnetic saturation for a specific level of excitation and the EM torque improves. However, once saturation is achieved this positive effect ceases and the EM torque doesn’t increase anymore with higher $LSI$. Moreover, within the radial constraint, slot area reduces with higher $LSI$, which in turn reduce effective number of turns, and hence, the EM torque after a certain level.
Effect of stator pole height, $L_{SP}$ variation can be studied using the analytical model of
SRM unaligned inductance [78]. The model considers both the contribution of stator and rotor
towards machine unaligned inductance. Stator contribution towards machine’s unaligned
inductance can be written as;

$$L_{us} = \frac{2N_{ser}N_{turn}^2}{N_{par}+L_{SP}*l_{w}} \left[ \frac{2}{3} c_{sy} L_{SP}^2 - c_{sx} \frac{l_{w}}{2} + \sum_{n=1}^{\infty} a_{shn} \left( \cosh \left( \frac{\pi n L_{SP}}{l_{s}} \right) - \frac{l_{s}^2}{(\pi n)^2 + L_{SP} + l_{w}} \sinh \left( \frac{\pi n L_{SP}}{l_{s}} \right) \sin \left( \frac{\pi n l_{w}}{l_{s}} \right) \right) \right]$$

(2.8)

Here, the parameters under consideration are $L_{SP} = R_2 - R_1$, $l_{w} = \frac{R_1(\beta_s - \beta_p)}{2}$, and

$$l_r = R_1(\beta_r - \beta_p) + 2g$$

with $\beta_p$ being the stator inter-polar gap. Since we have already
constrained $\beta_r$, $\beta_s$ and $\beta_p$ to maintain constant self-inductance, the geometric parameter that
will influence the unaligned inductance is the stator pole height $L_{SP}$ in equation 2.8. Increasing
$L_{SP}$ will reduce machine unaligned inductance. The reduction in unaligned inductance helps
in improving machine saliency, and hence, EM torque. Moreover, with larger stator pole
heights, stator slot area will increase and electrical loading will increase. However, since $R_1$
has been reduced by this process under the radial constraint, it may reduce the EM torque. Effect
of LSP variation on machine performance is presented in Figure 2.7(a).

![Figure 2.6: Variation of magnetic flux density, B (T) with LSI](image)
EM torque at base speed increases with higher values of rotor pole tapering angle because the magnetic saturation has been eased with a wider flux distribution around the pole. However, with higher $Tpr_r$, rotor contribution to unaligned inductance will increase as in equation 2.8 which reduces machine saliency and hence this positive effect on torque capability ceases. The effect of $Tpr_r$ variation on FP-MCSRM is presented in Figure 2.7(b).

![Figure 2.7: Effect of LSP and Tpr_r variation on MCSRM](image)

Figure 2.7(a) shows that with variation of parameters that effects machine slot area (like; $\beta_s$, $Tpr_s$, LSP, LSI and Fil_s) number of turns in phase windings, $N$ will also need to be varied maintaining the same slot current density. The effect of $N$ is very significant on machine inductance and torque production according to equation 2.5. Hence, variation $N$ is also considered during the parameter sensitivity and design optimization stage by;

$$N(k) = N(k - 1) \pm \frac{\Delta N}{\Delta x} (x(k) - x(k - 1))$$  \hspace{1cm} (2.9)

Here, $x \in [\beta_s, Tpr_s, LSP, LSI, Fil_s]$ and the $\Delta N/\Delta x$ term adds positively for LSP and negatively for the rest. So far, FEA based ‘one factor at a time’ approach has been practiced to study the sole effect of different design parameters. In order to properly optimize machine performance, different design parameters needs to be considered simultaneously along with multiple design targets like average torque and torque ripple. Hence, multidimensional optimization is preferred than one factor at a time approach. Figure 2.8(a) and (b) show the
FEA model for the initial and optimized designs of FP-MCSRM, respectively. Per unit EM torque profiles for these two designs are presented in Figure 2.8(c). Compared to the initial design a 13.54% reduction to torque ripple was achieved with 9.72% improvement in average torque. The following section will discuss some design challenges in FP-MCSRM that encourages the research on novel segmented rotor SRM topology.

Figure 2.8: (a) Initial and (b) Optimized FP-MCSRM models and (c) comparison of their per unit torque waveforms

2.2.4 Challenges with FP-MCSRM

FP-MCSRM holds the robust structure of conventional SRM and enables the three-phase inverter technology as its drive. However, phase windings of this machine is full-pitched expanding the complete rotor pole pitch. As a result, the end-winding length for the FP-MCSRM is the largest among different winding techniques. These end-windings are necessary to complete the phase coil connections but they do not contribute towards electromagnetic torque production. Hence, much higher copper losses occur with FP-MCSRM compared to its concentrated wound counterparts. Moreover, the manufacturing of full-pitched winding usually achieves low slot fill factor, FF (typically 30% - 40%). This low fill factor reduces machine torque density and increases copper losses. Finally, the axial extent of end-windings are also higher with fully pitched configurations. Usually FP-MCSRM are wound in double layer arrangement as shown in Figure 2.9(a) for easier assembly and to reduce chording cross-sections [79]. End-winding length in axial direction for a concentrated winding is given by;
\[ l_{ax-ew} = \frac{\pi \times (R_1 + g)}{2N_{sp}} \]  

(2.10)

Here, \( N_{sp} \) is the number of stator poles. For FP-MCSRM three such end winding length is present in as shown in Figure 2.9(a). Consequently, the axial extent on end-winding for FP-MCSRM has been much larger compared to that of concentrated windings [77]. This increases machine dimension and reduces its torque density. In other words, a more compact design can be obtained with concentrated wound SRM compared to FP-MCSRM as shown in Figure 2.9(b) and (c). This reduction of axial extent also enables the use of smaller machine frames or increase in machine active length.

![Figure 2.9](image_url)

**Figure 2.9**: (a) Double layer FP-MCSRM windings and side view of (b) FP-MCSRM and (c) Concentrated wound SRM to show their relative sizes

Considering these design challenges associated with FP-MCSRM a concentrated wound topology would be necessary to overcome them. As a result, a concentrated wound SRM topology has been developed in this research that can utilize three-phase standard VSIs as its drive. The next section will discuss in details on the novel concentrated wound segmented rotor SRM topology.
2.3 Concentrated Wound Segmented Rotor SRM (SSRM)

A new topology of three-phase segmented rotor switched reluctance machine (SSRM) that enables the use of standard voltage source inverters (VSIs) for its operation is presented in this section. The designed machine has shorter end-turn length, axial length compared to other SRM topologies that use three-phase inverters; compared to the conventional SRM (CSRM), these new topologies has the advantage of shorter flux paths that results in lower core losses. Details on machine design, operating principle and selection of concentrated winding configuration has been discussed. Performance of the designed SSRM has been compared with that of a CSRM to show its competitive performance and identify its design challenges, which has been addressed in Chapter 3.

2.3.1 Segmented SRM Background

Segmented rotor construction of switched reluctance motors has gained much attention in recent years with improved utilization of the magnetic geometry and higher torque density than conventional salient pole SRMs [80], [81]. Potential applications of this topology includes aerospace [56] and automotive traction [82], [83]. Reported segmented SRM designs in literatures use unipolar phase currents with three-phase asymmetrical half-bridge converters which are unique, requires special controls, additional sensors and of higher cost compared to commercially available integrated VSIs.

Bipolar operations of segmented rotor SRM have also been reported in the literature [84], [89]. Bipolar segmented rotor SRMs reported in [84]-[87] incorporate full-pitched windings with higher end-turn length increasing copper losses, weight and cost. In addition, the excitation topologies in [84]-[88] involved H-Bridge inverters, thereby increasing the number of semiconductor switches and converter cost. In [88], toroidal double windings are presented complicating the machine structure and its manufacturing. In [89], outer rotor, concentrated winding design is presented but with its converter circuit deviating from standard six-switch bridge topologies. In [60], six-phase segmented rotor SRM with concentrated winding is presented for low torque ripple operation. However, the design required six extra diodes in
series with the phase coils and additional current sensors to operate the machine with a three-phase standard VSI. This section presents a new topology of segmented rotor SRM having concentrated stator windings with three-phase bipolar excitations using standard VSIs.

### 2.3.2 Design Details

Rotor poles of the designed segmented rotor SRM is made of magnetic steel lamination segments embedded in non-magnetic support. Two different concentrated winding configurations, one with fractional slot fill and the other with full slot fill, are presented. Detailed 3D structure of the designed fractional-slot segmented rotor SRM (FrS-SRM) and full-slot segmented rotor SRM (FS-SRM) are presented in Figure 2.10(a) and (b), respectively. Compared to the fully pitched designs [84]-[87], the designed concentrated winding topologies can achieve higher fill factor with shorter end-winding extent that can improve machine torque density and efficiency [69].

![Figure 2.10: Detailed 3D Structure of (a) FrS-SRM and (b) FS-SRM](image)

Phase windings of the designed topologies are of concentrated type wrapped around one stator pole. Based on the relative distribution of phase coils, four different configurations are presented and studied to select one fractional slot and one full-slot configuration based on their electromagnetic torque profiles. The fractional slot (FrS) winding configurations are shown in Figure 2.11(a) and (b) and the full slot (FS) configurations are shown in Figure 2.11(c) and (d).
for SRMs with six stator slots and four rotor segments for simplicity. For the unidirectional configurations as in Figure 2.11(a) and (c), winding orientation of two consecutive phases are of the same polarity (+/+ or -/-). The bidirectional configurations of Figure 2.11(b) and (d) have winding orientation of opposite polarities (+/- or -/+ for two consecutive phases. For symmetric winding configuration, minimum number of stator poles required for unidirectional FS-SRM is twelve and that for other topologies are six.
Figure 2.11: Fractional slot (a) unidirectional; (b) bidirectional and full slot (c) unidirectional and (d) bidirectional winding configurations

2.3.3 Principle of Operation

Operating principle of the designed topologies with bipolar excitation is explained using the rectilinear representation of the three-phase FrS-SRM with unidirectional and bidirectional excitations shown in Figure 2.12. For the unidirectional configuration, flux paths for the two consecutive excited phases are in the same radial direction, and hence, the aligned flux is carried through the adjacent unexcited stator poles as shown in Figure 2.12(a). The unaligned condition occurs when a single rotor segments shorts the two excited stator poles and their flux lines oppose each other in the rotor segments as shown in Figure 2.12(b). For the considered
12/8 configuration, position difference is 22.5° mechanical between the aligned and unaligned position of the rotor.

Figure 2.12: Flux paths during aligned and unaligned rotor positions for (a), (b) unidirectional and (c), (d) bidirectional FrS-SRM

For the bidirectional winding arrangement shown in Figure 2.11(b), simultaneous excitation of two consecutive phases in opposite polarities create a flux-path involving the two excited stator poles and the stator back iron connecting them. In the aligned position shown in Figure 2.12(c), a rotor segment will short the magnetic flux path and create the closed loop flux flow. The unaligned condition (Figure 2.12(d)) occurs when air-gap between rotor segments blocks flux lines to flow from one excited stator pole to the next. Since flux lines from two phases are assisting each other in the aligned position, electromagnetic torque is produced from the rate of change of both self and mutual inductances in these FrS-SRM topologies.

The segmented rotor FrS-SRM designs have concentrated windings which reduces end-turn lengths, machine axial length, amount of copper, and mass and cost compared to the full-pitched windings. Unlike conventional SRM (CSRM), aligned flux paths are created between
adjacent poles as shown in Figure 2.12(a) and Figure 2.12(c), which reduces the flux path, and hence, the core loss under the same excitation frequency.

### 2.3.4 Design Specification

Initial evaluations of the SSRM topologies presented in section 2.2.2 are performed under a CSRM specification. A 860 W, three-phase, 12/8 CSRM which has been designed for home appliance applications is selected as the benchmark machine for the performance comparison of the designed concentrated winding segmented rotor SRM topologies. Structure of the benchmark CSRM is presented in Figure 2.13 and parameters for the test machine are presented in Table 2.1. Design optimization for the SSRM topologies targeting the maximization of electromagnetic (EM) torque at the base speed and improving torque and power density, are performed under the same axial and radial dimensional constraints as given in Table 2.1.

![Figure 2.13: Benchmark 12/8 CSRM Structure](image)
Table 2.1: Benchmark CSRM Specification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Outer Diameter, $D_O$</td>
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</tr>
<tr>
<td>Motor Axial Length, $L_{STK}$</td>
<td>46.89 mm</td>
</tr>
<tr>
<td>Shaft diameter, $D_{SH}$</td>
<td>30.48 mm</td>
</tr>
<tr>
<td>Volume, $V$</td>
<td>0.97 L</td>
</tr>
<tr>
<td>Weight, $W$</td>
<td>4.76 kg</td>
</tr>
<tr>
<td>Peak Phase Current, $I_P$</td>
<td>5.5 A</td>
</tr>
<tr>
<td>Rated Torque, $T_{EM}$</td>
<td>8.20 Nm</td>
</tr>
<tr>
<td>Rated Power, $P_{OUT}$</td>
<td>860 W</td>
</tr>
<tr>
<td>Number of turns, $N_{TURNS}$</td>
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</tr>
<tr>
<td>Current Density, $I_{DEN}$</td>
<td>6 A/mm²</td>
</tr>
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<td>Base Speed, $\omega_B$</td>
<td>1000 rpm</td>
</tr>
<tr>
<td>Maximum Speed, $\omega_{MAX}$</td>
<td>2500 rpm</td>
</tr>
</tbody>
</table>

2.3.5 Machine Configuration Selection

The selected CSRM design in section 2.2.4 has stator teeth to rotor segment ratio as $6x/4x$ ($x$ being an integer). Three different FrS-SRM configurations with $x = 1, 2, \text{ and } 3$ are considered under the same volumetric and phase current constraint and there electromagnetic performances are evaluated using FEA. Figure 2.14 shows geometries of the considered FrS-SRM configurations and Table 2.2 shows there performance comparison. The $12/8$ FrS-SRM structure can yield higher torque/ampere ratio with lower $T_{Ripple}$ compared to other two designs and has been selected as the SSRM configuration to optimize. Compared to FrS-SRMs $6x/4x$ configurations, FS-SRM requires a $12x/10x$ structure to produce continuous motoring torque. As a result, $12/8$ FrS-SRM and $12/10$ FS-SRM designs are considered next for selection of their winding configurations.
Figure 2.14 Considered stator/rotor configuration of SSRM (a) 6/4, (b) 12/8 and (c) 18/12

Table 2.2: Performances of the considered SSRM configurations

<table>
<thead>
<tr>
<th>Structure</th>
<th>6/4</th>
<th>12/8</th>
<th>18/12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{OUT}$ (mm)</td>
<td>139.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_S$ (°)</td>
<td>24</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>$LSI$ (mm)</td>
<td>11</td>
<td>6.5</td>
<td>4.68</td>
</tr>
<tr>
<td>$\beta_{SEG}$ (°)</td>
<td>71</td>
<td>35.5</td>
<td>24</td>
</tr>
<tr>
<td>$Tpr_{tip}$ (°)</td>
<td>12</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>$N_{TURN}$</td>
<td>317</td>
<td>220</td>
<td>159</td>
</tr>
<tr>
<td>$A_{SLOT}$ (mm²)</td>
<td>212</td>
<td>147</td>
<td>106</td>
</tr>
<tr>
<td>$L_{STK}$ (mm)</td>
<td>40</td>
<td>46.9</td>
<td>49.2</td>
</tr>
<tr>
<td>$L_{coilend}$ (mm)</td>
<td>13.8</td>
<td>6.9</td>
<td>4.6</td>
</tr>
<tr>
<td>$L_{Axial}$ (mm)</td>
<td></td>
<td>53.8</td>
<td></td>
</tr>
<tr>
<td>$I_{RMS}$ (A)</td>
<td></td>
<td></td>
<td>4.27</td>
</tr>
<tr>
<td>$T_{AVG}$ (Nm)</td>
<td>3.81</td>
<td>6.58</td>
<td>6.51</td>
</tr>
<tr>
<td>$T_{Ripple}$ (%)</td>
<td>+52.08</td>
<td>-</td>
<td>+21.04</td>
</tr>
<tr>
<td>Torque/Ampere</td>
<td>0.89</td>
<td>1.54</td>
<td>1.53</td>
</tr>
</tbody>
</table>
2.3.6 Selection of Winding Configuration

Electromagnetic (EM) torque output and torque ripples are considered for selecting the winding structure from different configurations shown in Figure 2.11. A 12/8 and a 12/10 structure is considered as the selected machine configuration for the FrS-SRM and FS-SRM, respectively. All four topologies are simulated in FLUX 2D under the same RMS phase current and current density levels. Torque profiles from FEA analysis are shown in Figure 2.15. For the FrS configuration, the unidirectional winding arrangement (Figure 2.11(a)) gives similar average torque with a much lower torque ripple compared to its bidirectional counterpart. Moreover, the bidirectional FS configuration (Figure 2.11(d)) shows higher average torque and lower torque ripple compared to its unidirectional counterpart. As a result, unidirectional FrS-SRM and bidirectional FS-SRM are selected for design optimization and performance evaluation in this research.

Figure 2.15: EM torque profiles of different winding configurations

The SSRM topologies work on balanced three-phase bipolar excitations enabling the use of standard VSIs which can accelerate its commercial adoption. The designed machines have improved magnetic utilization compared to CSRM as twice the number of stator poles are involved simultaneously to provide the flux paths. Moreover, effective torque contribution from each phase is at least 33% more than that of CSRM. These features improve the torque
and power density of the designed machine compared to CSRM under the same excitation level. Optimization of the segmented rotor SRM designs for a given design specification has been discussed in details in Chapter 3. The following section compares the performance of the selected FrS-SRM and FS-SRM windings with a CSRM to verify its performance and identify its design challenges.

2.3.7 Performance Analysis

Dynamic simulations were performed for CSRM, FrS-SRM and FS-SRM by coupling the motor controller developed in MATLAB Simulink with the FEA tool. Three-phase currents for the machines are regulated using the three-phase standard inverter for FrS-SRM and FS-SRM, and the asymmetrical half-bridge converter for CSRM. Performance of the designed segmented rotor SRMs are compared with that of CSRM at the base speed $\omega_b$ (1000 rpm) and the results for the same phase current level are given in Table 2.3.

The segmented rotor SRM topologies improve effective torque contribution from each phase by utilizing the mutual inductance between phases for EM torque production. As a result, EM torque at base speed improves by 11% compared to CSRM under the same excitation level. Moreover, the core loss of the segmented rotor SRM topologies are lower than that of CSRM due to the shorter flux paths as shown in Figure 2.12. Results presented in Table 2.3 shows that the designed segmented rotor SRM topologies can improve machine torque density, power density and torque-weight and power-weight ratios while maintaining comparable efficiencies under the same design constraints.

2.3.8 SSRM Design Challenges

Static EM torque for the optimized designs along with that of the CSRM are presented in Figure 2.16(a). Analyzing the static torque profile in Figure 2.16(a), it was found that one electrical cycle is equivalent to 72° mechanical for FS-SRM (10 pole) and 90° mechanical for the FrS-SRM and CSRM (8 pole). Torque profiles from machine dynamic simulations are presented in Figure 2.16(b).
Table 2.3: Performance comparison of the designed SSRM topologies with CSRM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>FrS –SRM (unidirectional)</th>
<th>FS –SRM (bidirectional)</th>
<th>CSRM (unidirectional)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak phase current, $I_P$ (A)</td>
<td>5.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average torque, $T_{AVG}$ (Nm)</td>
<td>9.30</td>
<td>9.31</td>
<td>8.34</td>
</tr>
<tr>
<td>Output power, $P_{OUT}$ (W)</td>
<td>973.89</td>
<td>974.94</td>
<td>877.55</td>
</tr>
<tr>
<td>Machine weight, $W$ (kg)</td>
<td>4.41</td>
<td>4.12</td>
<td>4.76</td>
</tr>
<tr>
<td>RMS Current, $I_{RMS}$ (A)</td>
<td>4.25</td>
<td>3.86</td>
<td>3.88</td>
</tr>
<tr>
<td>Max. Torque density, $T_{DEN}$ (Nm/L)</td>
<td>9.69</td>
<td>9.70</td>
<td>8.73</td>
</tr>
<tr>
<td>Power density, $P_{DEN}$ (kW/L)</td>
<td>1.01</td>
<td>1.02</td>
<td>0.91</td>
</tr>
<tr>
<td>Power-weight ratio, $P/W$ (kW/kg)</td>
<td>0.22</td>
<td>0.24</td>
<td>0.18</td>
</tr>
<tr>
<td>Torque-weight ratio, $T/W$ (Nm/kg)</td>
<td>2.11</td>
<td>2.26</td>
<td>1.76</td>
</tr>
<tr>
<td>Core loss, $P_{Core}$ (% of $P_{OUT}$)</td>
<td>4.0</td>
<td>4.25</td>
<td>5.17</td>
</tr>
<tr>
<td>Copper loss, $P_{Cu}$ (% of $P_{OUT}$)</td>
<td>9.29</td>
<td>8.84</td>
<td>6.86</td>
</tr>
<tr>
<td>Efficiency, $\eta$ (%)</td>
<td>88.27</td>
<td>88.43</td>
<td>89.26</td>
</tr>
</tbody>
</table>

Figure 2.16: (a) Static and (b) Dynamic torque profiles for the considered topologies

The designed SSRM are optimized targeting maximization of EM torque. Torque ripple of these designed SSRM are found to be higher than that of CSRM which is considered as the
next design challenge. From design perspective, updating machine geometry with stator back iron and pole design optimization, updates in rotor segment designs and its optimization, and rotor skewing can be adopted for torque ripple minimization. From the controller side profiling of phase currents for low torque ripple, implementing higher switching frequency, lower hysteresis band current control or predictive current control can help in improving torque ripple. Torque ripple minimization through machine design approach is considered in this dissertation and the detailed machine design steps have been covered in the next two chapters.

2.4 Summary

Characteristics of FP-MCSRM is investigated by developing its analytical inductance model and design details for maximizing its mutual torque is presented. Effect of different design parameters on MCSRM is also investigated to optimize the design under a given benchmark. Design challenges associated with its full-pitched windings are identified and to overcome them a concentrated wound SRM design enabling the use of three-phase standard VSI is considered as an alternative.

Two new concentrated winding segmented rotor SRM topologies were identified through analysis and subsequently designed for torque maximization. The designed machines have shorter end-turn length compared to full-pitched winding SRMs and shorter flux-paths, lower core loss, improved magnetic utilization, and better torque contribution from each phase. Machine efficiencies are comparable to the conventional SRM for the targeted design. Moreover, the designed machines use the three-phase standard six-switch bridge inverters for phase excitations which are advantageous for its commercial adoption. Design parameters were identified and optimized to improve machine performance against a benchmark appliance CSRM. The designed segmented rotor SRM topologies have improved machine torque and power densities compared to CSRM at base speed under the same excitation level. However, there is room for improving the torque ripple performance in these machines which has been addressed in the next chapter.
Chapter 3 Design of Segmented Rotor SRM

3.1 Introduction

3.2 Background on SSRM Torque Ripple

3.3 Development of Semi-Numerical Model

3.4 Torque Ripple Sources in SSRM

3.5 Design of Rotor Segments

3.6 Parameter Sensitivity Analysis

3.7 Design Update for Torque Ripple Minimization

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3.10 Electromagnetic Performance Analysis

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3.14 Experimental Evaluation

3.15 Summary
3.1 Introduction

Preliminary performance evaluation in Chapter 2 showed that SSRM can improve torque density compared to a toothed rotor SRM under the same excitation level but torque ripple of SSRM was high (around 40%) [91]. Minimization of torque ripple is necessary to satisfy the motor drive performance requirements in many applications, and has been considered as the design objective in this dissertation. Starting from an FEA based semi-numerical machine model to identify torque ripple sources, a novel rotor segment shaping is suggested to minimize torque ripple. Next, effect of different design parameters one SSRM performance are studied and Multi-dimensional optimization of the SSRM is performed under a selected benchmark. Mechanical stress analysis on the electromagnetically optimized design is performed using Multi-physics FEA. Acoustic noise analysis on the designed SSRM is performed and its performance is compared against commercial SRMs. A prototype SSRM is also built and FEA results are experimentally validated using conventional vector control.

3.2 Background on SSRM Torque Ripple

Torque ripple minimization of the segmented SRM has been investigated in prior research [84], [60]. The method proposed in [84] increases the machine number of phases up to six deviating from the three-phase standard VSIs. In [60], six-phase segmented rotor SRM is designed requiring six extra diodes in series with the phase coils to operate the machine with standard VSIs for low torque ripple operation. This research focuses on torque ripple reduction of three-phase concentrated wound segmented SRM without any additional converter components to standard VSI or construction complexity. First, the physics behind torque ripple sources in these machines are studied with FEA based semi-numerical model. Next, different rotor segment designs are presented and their performances to mitigate torque ripple are compared. A new rotor segment design is selected and effects of different design parameters are studied. This chapter presents the step-by-step design approach for torque ripple reduction of concentrated wound segmented rotor SRM without any additional construction or circuital complexity using a three-phase standard VSIs.
3.3 Development of Semi-Numerical Model

In order to identify the sources of torque ripple in SSRM its torque production characteristics need to be analyzed. According to the machine fundamental covered in Section 1.6.1, SRM torque production can be explained with its $\lambda - i - \theta$ characteristics that depends on machine geometry and material properties. Mathematical expressions can be obtained for $\lambda - i - \theta$ characteristics based on exponential curve fitting [90] as;

$$\lambda_k(i_k, \theta_e) = \lambda_s[1 - \exp(-i_k f_k(\theta_e))]$$  \hspace{1cm} (3.1)

$$f_k(\theta_e) = \alpha + \beta \cos(\theta_e - \frac{(k-1)2\pi}{m})$$ \hspace{1cm} (3.2)

Here, $\lambda_s$ represents the saturated flux linkage, $\alpha$ and $\beta$ are the tuned constant, $k = 1, 2 \ldots m$ is the phase number, $\theta_e$ is rotor electrical position. This exponential representation of phase flux linkage as a function of $i_k$ and $\theta_e$ is a non-linear approximation and does not accurately explain the machine characteristics. To improve the accuracy in torque prediction, rather than a completely mathematical model, a semi-numerical model based on machine $\lambda - i - \theta$ obtained from FEA has been utilized in this research. Design parameters and results presented in this dissertation onwards are based on a one hp industrial motor benchmark, detailed specification of which is presented in Appendix A. A high resolution $\lambda - i - \theta$ characteristics is obtained from FEA and is presented in Figure 3.1. The corresponding incremental inductance $L_{inc}(i_k, \theta_e)$ can be obtained by;

$$L_{inc}(i_k, \theta_e) = \frac{\partial \lambda(i_k, \theta_e)}{\partial i_k}$$ \hspace{1cm} (3.3)

Electromagnetic (EM) torque $T_{EM}$ of SRM is produced from its inductance variation between the aligned and unaligned positions with synchronized phase excitations. Aligned flux lines for the designed SSRM are carried through both excited and adjacent unexcited stator poles and so electromagnetic torque is produced from the variation of both self and mutual inductances in these machines [91]. Machine inductance profiles are first obtained from its $\lambda - i - \theta$ characteristics. The selected design benchmark has an operating current density of
around 4.5 A/mm² at which machine flux density is mostly around the knee points of its material $B - H$ curve with minimal local saturation near its tooth tip as shown in Figure 3.2. As a result, machine magnetic saturation can be neglected while performing this inductance analysis. Phase inductance variation over one electrical cycle for a 12/8 SRM with typical segmented rotor is presented in Figure 3.3. Assuming magnetic linearity, the analytical expression utilized for SSRM electromagnetic torque production is given by equation 3.4 [77];

$$T_{EM} = \frac{1}{2} \left( i_A \frac{dL_{AA}}{d\theta} + i_B \frac{dL_{BB}}{d\theta} + i_C \frac{dL_{CC}}{d\theta} \right) + i_A i_B \frac{dM_{AB}}{d\theta} + i_B i_C \frac{dM_{BC}}{d\theta} + i_C i_A \frac{dM_{CA}}{d\theta}$$

(3.4)

Figure 3.1: $\lambda - i - \theta$ characteristics from FEA between aligned and unaligned position

Figure 3.2: SSRM flux density distribution during aligned position
Machine dynamic electromagnetic torque is calculated using equation (3.4) and compared with that obtained from FEA and the results are presented in Figure 3.4. As can be observed from Figure 3.4, machine inductance based semi-numerical model can adequately predict torque variation and it has been utilized to study the torque ripple sources in SSRM.

### 3.4 Torque Ripple Sources in SSRM

This section will investigate dynamic pulsation in SSRM torque waveform based on its inductance variations. Analyzing the inductance profiles in Figure 3.3 and corresponding EM torque profile in Figure 3.4 the major torque ripple region is identified as ‘Region 1’. Over one electrical cycle, three such regions will exist for a three-phase machine. Trapezoidal phase currents are used for torque production in segmented SRM [91] and in ‘Region 1’ EM torque is produced from equal and opposite excitation of phase A and C. ‘Region 1’ is divided into three sections to describe torque pulsation in this region.

From Figure 3.3 between line ‘a’ and ‘b’, both LAA and LCC are decreasing with rotor position and this will offset EM torque build up from MCA variation according to equation 3.4. As a result, torque reduces from ‘a’ to ‘b’ as shown in Figure 3.4. Between points ‘b’ to ‘c’ in Figure 3.3, rate of change of LAA and its negative torque share ceases and consequently...
total EM torque increases as shown in Figure 3.4. From ‘c’ to ‘d’ in Figure 3.3, rate of change of MCA and its positive torque contribution ceases which cause reduction in EM torque of Figure 3.4. Between points ‘b’ and ‘d’, machine self-inductance profiles are not smooth (a similar region is marked by a cyan ellipse in Figure 3.3) and it produces a large torque pulsation as shown in Figure 3.4. Design modification of rotor segments to reduce this torque pulsation has been addressed in the next section.

Figure 3.4: Comparison of electromagnetic torque profiles between FEA and calculated from semi-numerical model

3.5 Design of Rotor Segments

Minimization of torque ripple is attempted with rotor segment designs in this research. The initial rotor segment design is presented in Figure 3.5(a). Aligned flux of the segmented SRM is carried by one excited and one un-excited rotor pole shorted by a rotor segment [91]. Analyzing the inductance profiles with stator pole and rotor segment overlap, it was found that the largest torque pulsation region (from point ‘b’ to point ‘d’ in Figure 3.4) occurs when the center of the rotor segment is crossing stator inter-polar gap. Hence, geometry of the center of rotor segment is selected as the design region.

Four different designs of the rotor segment along with the initial one are considered and these are presented in Figure 3.5. Design 1 is the initial segmented rotor [91]. Design 2 of the rotor segment is created by introducing a segmental dip at its center near the air-gap. Self and mutual inductance profiles for designs 1 and 2 are presented in Figure 3.6. With introduction
of the segmental dip, self-inductance profile between points ‘b’ to ‘d’ in Figure 3.3 becomes smoother as indicated by a green circle in Figure 3.6(a) which should reduce the torque pulsation in that region. However, the rate of change of inductance is reduced with design 2 which will reduce machine torque capability. For a toothed SRM, considering unidirectional rotation, modifying the leading edge of rotor poles with fillet helps in reducing torque ripple [92]. Adopting a similar modification in rotor segments of design 2, design 3 is obtained. Design 4 and 5 are two versions of the rotor segments having fillets around the edges.

![Figure 3.5: Initial and considered rotor segment designs](image)

Static torque profiles for the rotor segments of Figure 3.5 with constant excitation of two phases (in opposite polarities) are presented in Figure 3.6(b). As predicted earlier, design 2 has a flat torque profile but the peak torque level is lower than that of design 1. Physically, this is related to the reduction of flux carrying cross-sections through rotor segments. Designs 3 to 5 have smaller flat torque profiles compared to design 2 hence they are not a good choice for reducing torque ripple. Two design considerations have been summarized from static torque profile analysis. First, design 2 has been most effective in torque ripple reduction with the widest flat torque profile; second, segment shape for design 2 is not optimized unlike design 1 which is optimized for maximization of EM torque [91]. In addition, practical bipolar trapezoidal phase currents need to be considered for performance analysis. Next, 2D FEA based dynamic simulations are performed with balanced three-phase trapezoidal currents to optimize the selected rotor geometry for torque ripple minimization [93].
Parameter Sensitivity Analysis

Rotor segment structure is selected as the design domain in this section. A parameterized rotor segment model is presented in Figure 3.7, which is symmetric with respect to its center line. Here, $\beta_{SEG}$, $\beta_{DIP}$ and $\beta_{SEGB}$ refers to segment top angle, angle of segmental dip and segment bottom angle, respectively. $H_{SEG}$ and $H_{DIP}$ are the respective heights of segment and segmental dip. $R_1$, $R_2$ and $R_3$ are three parameters defining fillet radius at the segment edges and $Arc_{DIP}$ is the parameters defining arc angle of the segmental dip. Effect of varying these
different design parameters are obtained from FEA simulations and they are discussed in the next section.

Figure 3.7: Parameterized rotor segment model

The effect of varying the angle of segmental dip, $\beta_{DIP}$ is presented in Figure 3.8(a). Keeping other parameters constant, with the increment of $\beta_{DIP}$ machine average torque reduces as discussed in section 3.4. In addition, there is an optimum value of $\beta_{DIP}$ that provides minimum torque ripple. Results from Figure 3.8(a) shows that the optimum value for $\beta_{DIP}$ is $3.5^\circ$ that provides 16.02% reduction in torque ripple with 7.9% reduction in average torque. Next, the top angle of rotor segment $\beta_{SEG}$ is varied to analyze its effect on machine performance. With the selected value of $\beta_{DIP}$ from previous step, Figure 3.8(b) shows the variation of average torque and torque ripple with $\beta_{SEG}$. The optimum value of $\beta_{SEG}$ is found to be $36^\circ$ at which the torque ripple was further reduced by 2.11% from previous step with similar average torque.
Figure 3.8 Average torque and torque ripple variation with (a) β_{DIP} and (b) β_{SEG}

Sensitivity of average torque and torque ripple with the variation of the angle of bottom segment, β_{SEGB}, is presented in Figure 3.9(a). Maintaining constant values of other design parameters, with the reduction of β_{SEGB} machine average torque initially increases as it helps magnetic saturation but this effect ceases with higher values of β_{SEGB}. Moreover, there is an optimum value of β_{SEGB} that provides minimum torque ripple. Results from Figure 3.9(a) shows that the optimum value for β_{SEGB} is 38°. Next, the height of the rotor segment H_{SEG} is varied to analyze its effect on machine performance. Figure 3.9(b) shows the variation of
average torque and torque ripple with $H_{SEG}$. The optimum value of $H_{SEG}$ is found to be 21.25 mm at which the torque ripple was further reduced by 4.44% from previous step with similar average torque. Performance of the updated design based on this design iterations has been presented in the next section.

![Graph](attachment:image.png)

Figure 3.9: Average torque and torque ripple variation with (a) $H_{DIP}$ and (b) $H_{SEG}$
3.7 Design Update for Torque Ripple Minimization

Going through the design iteration steps for each geometric parameter illustrated in Figure 3.7, an optimum design of rotor segment is obtained. Table 3.1 lists different rotor design and performance parameters before and after the optimization done by ‘one factor at a time’ approach. A total of 22.19% reduction of torque ripple was attained with a reduction of average torque by 6.61%. This ‘one factor at a time’ approach is a good way to study the effect of different design parameters on machine performance which was the main focus of this section. However, design optimization through this method cannot take into account the combined effect of these design parameters during each design stage.

Table 3.1: Parameters before and after one factor based optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Design</th>
<th>Optimized Design</th>
<th>Parameter</th>
<th>Initial Design</th>
<th>Optimized Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Arc_{DIP}$ (°)</td>
<td>10</td>
<td>90</td>
<td>$R_1$(mm)</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>$\beta_{SEG}$ (°)</td>
<td>38</td>
<td>36</td>
<td>$R_2$(mm)</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>$\beta_{SEGB}$ (°)</td>
<td>38</td>
<td>40</td>
<td>$R_3$(mm)</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>$\beta_{DIP}$ (°)</td>
<td>4</td>
<td>7</td>
<td>$T_{AVG}$(Nm)</td>
<td>4.68</td>
<td>4.38</td>
</tr>
<tr>
<td>$H_{SEG}$(mm)</td>
<td>24.25</td>
<td>21.2</td>
<td>$T_{RIPPLE}$(Nm)</td>
<td>1.68</td>
<td>0.60</td>
</tr>
<tr>
<td>$H_{DIP}$(mm)</td>
<td>1</td>
<td>3.45</td>
<td>$T_{RIPPLE}$(%)</td>
<td>35.92</td>
<td>13.67</td>
</tr>
</tbody>
</table>

Figure 3.10 shows both average torque and torque ripple with the variation of $\beta_{DIP}$ and $\beta_{SEG}$ together in 3D plots. The optimum value of $\beta_{DIP}$ and $\beta_{SEG}$ are found to be 3.53° and 37.2°, respectively and an additional 2.7% reduction in torque ripple is possible compared to optimizing each of these parameters separately. Considering multiple design parameters at a time for each optimization stage helps in achieving a better optimized design compared to ‘one factor at a time’ based approach. As a result, FEA based multi-dimensional optimization of the designed SSRM is considered next.
Results from section 3.6 showed that considering multiple design parameters at a time is more effective in optimizing machine performance. This leads towards multi-dimensional

**3.8 Design Objectives**

Results from section 3.6 showed that considering multiple design parameters at a time is more effective in optimizing machine performance. This leads towards multi-dimensional
(MD) optimization of the SSRM structure. Figure 3.7 presents the design parameters that are considered in the multi-dimensional optimization process. Both single objective (SO) and multi-objective (MO) optimization are performed and the details of the optimization process has been presented in the next section. With a target to build the SSRM prototype in an existing frame of its benchmark, the selected radial and axial constraints are set to be 163.5 mm and 151.2 mm, respectively. The prototype SSRM is of totally enclosed non-ventilated (TENV) cooling type, as a result, the current density limit is set as 4.5 A/mm². FLUX 2D FEA tools is coupled with optimization tools GOT-It for the optimization. Transient FEA analysis over an electrical cycle is used for the performance evaluation with each geometric parameters being varied over an acceptable range. Average electromagnetic torque ($T_{AVG}$), torque ripple ($T_{Ripple}$), and slot current density ($J_{Slot}$) results are considered as decision variable on each optimization steps.

### 3.9 SSRM Multidimensional Optimization

The optimization process flow diagram is presented in Figure 3.11(a). Py-FLUX script is utilized to calculated the decision variables on each design iteration. To minimize evaluation time of the candidate solutions, one fourth machine model in FLUX 2D was designed considering periodicity over its pole-pairs as shown in Figure 3.11(b). For the SO optimization, minimization of $T_{Ripple}$ is considered as the optimization goal subject to a minimum $T_{AVG}$ constraint of 4.35 Nm in addition to the volumetric and excitation constraints. For the MO optimization, both minimization of $T_{Ripple}$ and maximization of $T_{AVG}$ are considered as the design objectives.

Meta-heuristic optimization algorithms use a set of candidate solutions (population) which are iteratively modified according to probabilistic rules aiming at finding the global minimum of the chosen objective function. Genetic algorithm (GA) is a popular class of meta-heuristic algorithms where the candidate solutions evolve using the biology-inspired operators of crossover and mutation. GA is selected with a population size of 40 and maximum iteration limit of 600 for the SO. Grid Multi-objective GA (GMGA), which finds the nearest points of
the Pareto frontier on a rounded grid based on GA, with population size of 350 and maximum iteration limit of 500 is utilized to optimize this MO problem. Results from both of these optimization processes are presented in the next section along with their electromagnetic performance comparison against the initial design.

![Diagram](image)

**Figure 3.11:** (a) Optimization process flow (b) Meshed machine model in FEA

Evaluation of the objective function from the MDSO design iterations are presented in Figure 3.12. With 37,352 nodes in 2D FEA based machine model, the MDSO optimization process took around 7 days to complete. The minimization of $T_{Ripple}$ ceases during the final simulation steps. Pareto front for the MDMO process in $T_{Ripple}$-$T_{AVG}$ domain is presented in Figure 3.13. Using the same FEA model in FLUX 2D as MDSO but with higher candidate solution consideration, the MDMO process continued for around 24 days. The top left corner of the Pareto chart, provides the most optimum result with improved $T_{Ripple}$ and $T_{AVG}$ performances as can be seen from Figure 3.13 and the process was stopped with this acceptable set of results in the decision domain.
3.10 Electromagnetic Performance Analysis

Electromagnetic performance of MDSO and MDMO processes are quantified in Table 3.2 along with results from the initial design and ‘1F-Opt’ methods. Figure 3.14 presents the dynamic torque profiles along with the bipolar trapezoidal phase currents for these considered designs. Both multidimensional optimizations are found to be more effective compared to the ‘1F-Opt’ method in improving the overall machine performance. Compared to the initial design, the ‘1F-Opt’ method minimizes $T_{Ripple}$ by 22.19% with 6.61% reduction in $T_{AVG}$. The
MDSO, with single objective of minimizing $T_{\text{Ripple}}$, is found to be most effective in $T_{\text{Ripple}}$ reduction. MDSO minimizes $T_{\text{Ripple}}$ by 27.90% with 1.07% reduction in $T_{AVG}$ compared to the initial design. The MDMO has been most effective in minimizing the overall cost function with 24.79% reduction in $T_{\text{Ripple}}$ with 2.77% improvement in $T_{AVG}$. The multidimensional optimization process are found to be time consuming but very effective in improving machine performance and should be applied to finalize machine’s electromagnetic designs. The MDMO design is selected for the prototype SSRM development and will go through mechanical stress and acoustic noise analysis in the following two sections.

Table 3.2: Electromagnetic performance comparison of different SSRM designs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial</th>
<th>1F-Opt</th>
<th>MDSO</th>
<th>MDMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Arc_{\text{DIP}}$ ($^\circ$)</td>
<td>10</td>
<td>90</td>
<td>35</td>
<td>36.5</td>
</tr>
<tr>
<td>$\beta_{\text{SEG}}$ ($^\circ$)</td>
<td>38</td>
<td>36</td>
<td>38</td>
<td>40.95</td>
</tr>
<tr>
<td>$\beta_{\text{SEGB}}$ ($^\circ$)</td>
<td>38</td>
<td>40</td>
<td>33.6</td>
<td>36.47</td>
</tr>
<tr>
<td>$\beta_{\text{DIP}}$ ($^\circ$)</td>
<td>4</td>
<td>7.0</td>
<td>3.5</td>
<td>6.02</td>
</tr>
<tr>
<td>$H_{\text{SEG}}$ (mm)</td>
<td>24.3</td>
<td>21.2</td>
<td>20.25</td>
<td>19.4</td>
</tr>
<tr>
<td>$H_{\text{DIP}}$ (mm)</td>
<td>1</td>
<td>3.5</td>
<td>3.12</td>
<td>2.81</td>
</tr>
<tr>
<td>$R_1$ (mm)</td>
<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
<td>0.78</td>
</tr>
<tr>
<td>$R_2$ (mm)</td>
<td>0.5</td>
<td>0.1</td>
<td>1</td>
<td>2.26</td>
</tr>
<tr>
<td>$R_3$ (mm)</td>
<td>0.5</td>
<td>0.1</td>
<td>1</td>
<td>1.37</td>
</tr>
<tr>
<td>$L_{\text{stk}}$ (mm)</td>
<td></td>
<td></td>
<td>127.4</td>
<td></td>
</tr>
<tr>
<td>$D_{\text{out}}$ (mm)</td>
<td></td>
<td></td>
<td>163.5</td>
<td></td>
</tr>
<tr>
<td>$T_{AVG}$ (Nm)</td>
<td>4.69</td>
<td>4.38</td>
<td>4.64</td>
<td>4.82</td>
</tr>
<tr>
<td>$T_{\text{RIPPLE}}$ (Nm)</td>
<td>1.68</td>
<td>0.60</td>
<td>0.37</td>
<td>0.53</td>
</tr>
<tr>
<td>$T_{\text{RIPPLE}}$ (%)</td>
<td>35.86</td>
<td>13.67</td>
<td>7.96</td>
<td>11.07</td>
</tr>
</tbody>
</table>
3.11 Mechanical Stress Analysis

After the electromagnetic design is finalized, the next step is to perform mechanical stress analysis on the designed SSRM. Stress is defined as the force per unit area. For an electric machine, two types of forces need to be considered in stress analysis and they are (1) electromagnetic radial force occurring from its excitation and torque production and (2) centrifugal force that is acting on its rotor due to rotational speed. A typical stress versus strain graph [94] is presented in Figure 3.15. In the elastic region, stress is proportional to applied strain following Hooke’s law. The deformed material come back to its original form after the removal of stress in this region. The maximum strain boundary of elastic region is defined by material yield strength ($\sigma_y$). Beyond its elastic limit is the plastic region where Hooke’s law no longer applies and non-recoverable deformation occurs. Elastic region is typically smaller than the plastic region. The analysis of interest is to evaluate SSRM stresses under the rated operating condition to verify if it remains within the elastic region of its material.
3.11.1 Structural FEA Analysis

Mechanical properties for the 29-gauge M19 electrical steel material [95] are presented in Table 3.3. Ansys workbench is utilized for the structural FEA analysis on the SSRM including both electromagnetic radial ($F_{Rad}$) and mechanical centrifugal ($F_{Cen}$) forces. Maxwell 2D has been utilized for the electromagnetic FEA analysis to extract $F_{Rad}$ and import to Ansys mechanical. Both electromagnetic and mechanical FEA tools are coupled in the workbench environment. Figure 3.16 presents the SSRM design is electromagnetic and structural FEA environment. Cyclic symmetry was utilized to minimize simulation time. Results from the stress analysis are presented in the next subsection.

![Figure 3.15: Typical material stress vs. strain curve](image)

![Figure 3.16: (a) Maxwell 2D model (b) static structural model with fixed supports shown](image)
Table 3.3: Electrical steel material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Density</th>
<th>Young’s modulus</th>
<th>Yield strength</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>7850 kgm⁻³</td>
<td>2e+11 Pa</td>
<td>350 Mpa</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### 3.11.2 Stress Analysis Results

The prototype SSRM is targeted to be built with the MDMO design with 100 mm stack length for manufacturing ease. The equivalent von Mises stress distribution on the rotor for only $F_{Rad}$ and $F_{Cen}$ are presented in Figure 3.17(a), (b) and Figure 3.17(c), (d), respectively. The peak von Mises stress results from only $F_{Rad}$ is 85051 Pa and that from only $F_{Cen}$ is 659610 Pa. Results show that $F_{Cen}$ is more dominant stress component compared to $F_{Rad}$. The stator will only experience radial forces and its stress distribution is presented in Figure 3.18(a), (b). In practical scenario, both $F_{Rad}$ and $F_{Cen}$ are present at the same time and rotor stress distribution from their combined effect is presented in Figure 3.18(c), (d). The peak stress is found to be 0.74 Mpa which is much below its material yield strength limit. The maximum deformation under rated condition was 22.3 nm, much lower than the air-gap length of 0.4 mm.

![Figure 3.17: Distribution of SSRM rotor stress; (a), (b) only $F_{Rad}$ and (c), (d) only $F_{Cen}$](image)

![Figure 3.18: SSRM stress analysis on (a), (b) stator; and (c), (d) with $F_{Rad}$ and $F_{Cen}$](image)
Results from this multi-physics (electromagnetic and mechanical) based stress analysis showed that, under rated operating condition, the stress and deformation limits for the designed SSRM is well below its material yield strength limit. As a result, the prototype SSRM is safe to build. The next section will discuss the noise analysis of this SSRM design.

3.12 SSRM Noise Analysis

The main source of acoustic noise in a switched reluctance machine is the interaction between its magnetic radial force ($F_{RAD}$) and the circumferential mode shapes of the stator. Particularly if the excitation frequencies ($f_{EXC}$) of $F_{RAD}$ induce resonance with the stator natural mode frequencies ($f_m$) the acoustic noise become very high. To investigate the noise of SSRM design, first $F_{RAD}$ is obtained from electromagnetic FEA analysis over one electrical cycle. Frequency domain analysis of $F_{RAD}$ provides the proportional intensity level of that force at each excitation frequency. $F_{RAD}$ acts on the stator to ovalize in different circumferential mode shapes (with mode number, $m$) at their own natural mode frequencies. Both analytical and FEA based models are used to identify the natural mode frequencies for the SSRM stator and results were compared. Next, circumferential deflection due to radial force wave, acoustic noise intensity, and sound power level were calculated under both sinusoidal and trapezoidal excitations. Finally, results from this acoustic noise analysis were compared with an industrial SRM [96] to show that the designed SSRM have acceptable acoustic noise level according to OSHA standard.

3.12.1 Mode Shapes

Circumferential mode shapes of a stator for mode numbers 0-5 are shown in Figure 3.19. Radial forces acting on diametrically opposite rotor poles characterize these modes. For small machines, most important circumferential modes for vibrations are $m = 1$ to 4 and higher modes are important for larger machines [96]. The most dominant mode is the second order cylindrical mode or fundamental mode with $m = 2$ [97]. Double ovalization is caused by the fourth order mode ($m = 4$) and the first order mode ($m = 2$) causes a uniform radial pulsating
vibration. Majority of the acoustic noise results from these three circumferential modes \((m = 0, 2 \text{ and } 4)\).

### 3.12.2 Natural Mode Frequencies

Each circumferential mode shape has its own natural mode frequencies. Any particular mode shape is excited when its natural mode frequency matches with any harmonics of its magnetic radial force. Longitudinal vibration modes \((l)\) may also contribute to acoustic noise for simply supported stator like a cylindrical shell. Except for very long stator stacks, the most important longitudinal mode for simply supported stators are \(l = 0\) [98]. Other than the fundamental \((l = 0)\), \(f_m\) for the dominant circumferential modes are too high to contribute to circumferential deflection \((D_{Cir})\) or noise power level. As a result, natural mode frequencies for circumferential modes are only considered in this research.

![Figure 3.19: Circumferential mode shapes of a stator (m=0 to 5)](image)

Mode frequencies are a function of the mass and stiffness. Formula developed by Jordan et al. [98] considers shear, rotary inertia, teeth and winding masses in estimating circumferential mode frequencies and are utilized to calculate the mode frequencies analytically. Mode frequency for pulsating vibration \((m = 0)\) is given by equation 3.5 as,
\[ f_{m=0} = \frac{1}{2\pi R_m} \sqrt{\frac{E_s}{\rho_s \Delta}} \text{ Hz} \]  

(3.5)

Natural mode frequency for unity circumferential (\( m = 1 \)) mode is;

\[ f_{m=1} = f_{m=0} \frac{2}{\sqrt{1+i^2(\Delta/\Delta_m)}} \text{ Hz} \]  

(3.6)

and natural mode frequencies for \( m \geq 2 \) are given by equation 3.7 as;

\[ f_{m=2} = \frac{f_{m=0} \cdot i \cdot m \cdot (m^2 - 1)}{\sqrt{(m^2 + 1) + i^2(m^2 - 1)(4 + m^2 + 2\Delta_m/\Delta + 3)}} \text{ Hz} \]  

(3.7)

Here,

\[ i = \frac{1}{2\sqrt{3}} \frac{y_s}{R_m} \quad R_m = \frac{R_{\text{out}} + R_{\text{sy}}}{2} \quad \Delta = 1 + \frac{W_t}{W_y} \quad W_t = W_p + W_{wi} \]

\[ \Delta_m = 1 + \frac{1.91 \cdot N_{sp} \cdot A_{sp} \cdot h_s^2 \cdot W_t}{R_m l_{stk} y_s^3 W_p} \left\{ 1 + \left( \frac{y_s}{2h_s} \right)^2 \right\} \]

With, \( E_s \) modulus of elasticity of stator material, \( \rho_s \) density of stator material, \( \Delta \) and \( \Delta_m \) are the mass addition factors of displacement and rotation, \( W_y, W_p \) and \( W_l \) are the weight of stator yoke, poles and windings, respectively, \( N_{sp} \) is the number of stator poles, \( l_{stk} \) stack length, \( A_{sp} \) is cross-sectional area of each stator pole. SSRM stator geometric parameters utilized in above equations are shown in Figure 3.20.
Dominant stator vibrations occur with resonance between any excitation frequencies of the radial force and one of the above mode frequencies. The peak of vibration is independent of the damping ratio ($\zeta$) as it only determines the amplitude of the decaying vibrations with time.

### 3.12.3 Mode Frequency Calculation:

Mode frequencies for the designed SSRM stator are determined using both analytical and FEA based models. The analytical model includes stator design, material properties and their mathematical relationships as discussed in the previous section and results from this analytical prediction are also compared against FEA. The FEA model includes the modal analysis of the stator structure in ANSYS workbench. Mode shapes obtained from the FEA analysis are presented in Figure 3.21. Mode frequency results from the analytical and the FEA models are presented in Table 3.4. The analytical model shows good agreement with FEA based results in predicting mode frequencies with their mode frequencies having the similar trends as in literature [99].
Figure 3.21: Mode shapes of the 12/8 SSRM stator obtained from FEA analysis

Table 3.4: Mode frequency results from analytical and FEA based models

<table>
<thead>
<tr>
<th>Mode number</th>
<th>$f_m$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
</tr>
<tr>
<td>0</td>
<td>7600.9</td>
</tr>
<tr>
<td>2</td>
<td>1691.3</td>
</tr>
<tr>
<td>3</td>
<td>4264.4</td>
</tr>
<tr>
<td>4</td>
<td>7184.8</td>
</tr>
<tr>
<td>5</td>
<td>10211</td>
</tr>
<tr>
<td>6</td>
<td>13245</td>
</tr>
</tbody>
</table>

Magnetic radial force wave, as presented in Figure 3.22, was obtained from the electromagnetic FEA analysis of the SSRM prototype for both sinusoidal and trapezoidal excitations using a force sensor at the top of a stator tooth over one-half electrical cycle. Frequency domain analysis of this radial force will provide its frequency contents at excitation frequencies for stator yoke which can be expressed as;
\[ f_{\text{EXC}}(n) = n \cdot f_p = \frac{n \cdot \omega_m \cdot N_{\text{rp}}}{60} \]  

(3.8)

Here, \( N_{\text{rp}} \) is the rotor pole number, \( \omega_m \) is the mechanical speed in rpm, \( f_p \) is the fundamental frequency of phase current in Hz, and \( n = 1, 2, 3, ... \) is the harmonic number. Results from the fast Fourier transforms (FFT) of these radial force waveforms in Figure 3.22 are presented in Figure 3.23(a) and Figure 3.23(b) for sinusoidal and trapezoidal excitations, respectively. Having smoother transition in phase currents than trapezoidal, the sinusoidal excitation yields a lower value of peak radial force as shown in Figure 3.23.

![Figure 3.22: Magnetic radial force wave over half-electrical cycle](image)

![Figure 3.23: SSRM magnetic radial force spectrum (left) sinusoidal and (right) trapezoidal excitation](image)

The next step is to determine stator circumferential deflection (\( D_{\text{Cir}} \)) considering the interaction of its mode frequencies with relative intensities of \( F_{\text{RAD}} \) at each excitation
frequency, stator structure, and material properties. Acoustic noise at any operating condition depends on $D_{Cir}$ whose analytical expression for $m \geq 2$ can be expressed as;

$$D_{Cir}(f_{exc}) = \frac{12F_{RAD}(f_{exc})R_m (R_m)}{m^4 \zeta \sqrt{\left(1 - \left(\frac{f_{exc}}{f_m}\right)^2\right)^2 + \left(\frac{\delta f_{exc}}{2f_m}\right)^2}}$$

(3.9)

Here, $\delta = 2.\pi.\zeta$ is the logarithmic decrement and $F_{RAD}(f_{exc})$ is the amplitude of the radial force wave (per unit area). Spectrum of circumferential deflection is presented in Figure 3.24.

![Figure 3.24: Frequency spectrum of circumferential deflection for (left) sinusoidal and (right) trapezoidal excitation](image)

The emitted sound wave analysis depends on the geometry of the machine along with the frequency and wavelength of that particular medium. Sound radiation factor ($\sigma_{rel}$) is a function of the mode number and excitation frequency. Sound power radiated by electric machines can be expressed as [98], [100];

$$P = 4. \sigma_{rel} \cdot \rho \cdot c \cdot \pi^3 \cdot f_{exc}^2 \cdot D_{Cir}^2 \cdot R_{out} \cdot l_{stk}$$

(3.10)

Here, $P$ is the radiated sound power in $W$, $\sigma_{rel} = k^2/(1 + k^2)$ with $k = (2.\pi. R_{out} \cdot f_{exc})/c$ as the wave number, $c$ speed of sound (m/s) in the medium and $\rho \cdot c = 415\ N.m^3$ for air at 20 degree Celsius. The calculated acoustic noise power ($P$) according to equation 3.10 are presented in Figure 3.25.
The reference for the sound power level is $10^{-12}$ W based on the threshold of human ear sensation. As a result, the acoustic noise power level in decibels becomes:

$$L_w = 10\log\left(\frac{2P}{P_{ref}}\right) \text{ dB} \quad (3.11)$$

For a particular stator structure, the mode frequencies are fixed and the excitation frequency changes with the operating speed and rotor pole numbers. The nominator of deflection equation 3.9 is called the static component, which determines the relative intensity of the magnetic radial force causing acoustic noise. The denominator in the deflection equation is the dynamic component that determines the magnification of deflection. Figure 3.26 presents the frequency domain plots and noise power level ($L_w$) in decibels for both sinusoidal and trapezoidal excitations. The peak noise power level for sinusoidal and trapezoidal excitations at 1800 rpm are found to be 58.40 dB and 59.34 dB, respectively.

Figure 3.25: Acoustic noise power for (left) sinusoidal and (right) trapezoidal excitation

Figure 3.26: Sound power level for (left) sinusoidal and (right) trapezoidal excitation
3.12.4 Noise at Mechanical Resonance

Each excitation frequency contributes to the dynamic circumferential deflection for a particular mode shape and mode frequency. The farthest $f_{exc}$ has the lowest contribution while the nearest $f_{exc}$ has the highest contribution. The contribution reaches maximum when any one of the excitation frequency matches with a natural mode frequency of the stator causing resonance. For the designed SSRM stator, fundamental mode frequencies are presented in Table 3.4. With second order mode frequency at 1691.3 Hz, the noisier speeds will be 6342.38 rpm ($2 \times f_{exc} = f_{m=2}$), 4228.25 rpm ($3 \times f_{exc} = f_{m=2}$), 3171.19 rpm ($4 \times f_{exc} = f_{m=2}$), 2536.95 rpm ($5 \times f_{exc} = f_{m=2}$) and so on. Among these different speeds, the noisiest operating condition was found at 6342.38 rpm with the peak noise level as 63.21 dB and 64.15 dB for sinusoidal and trapezoidal excitations, respectively as presented in Figure 3.27. This operating speed is way beyond (3.5 times) the rated operating condition of SSRM.

![Figure 3.27: Sound levels at 6342.38 rpm for (a) sinusoidal and (b) trapezoidal excitation](image)

Results from the acoustic noise analysis showed that the peak noise level for the designed SSRM at rated operating condition is found to be 59.34 dB. Compared to a conventional 3-phase, 12/8 SRM and 4-phase, 8/6 industry SRM designs [99], the designed SSRM has 18.17 dB and 10.56 dB lower acoustic noise levels, respectively. Moreover, according to occupational safety and health administration (OSHA) recommended safety hearing environment standard, as presented in Table 3.5, the designed SSRM acoustic noise level is well under the permissible noise exposure level. It was concluded that the designed SSRM has acceptable structural and acoustic performance and a prototype SSRM is being built.
Table 3.5: Permissible noise exposure level based on OSHA noise standard

<table>
<thead>
<tr>
<th>Duration (hours/day)</th>
<th>Sound Level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>95</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>105</td>
</tr>
<tr>
<td>0.5</td>
<td>110</td>
</tr>
<tr>
<td>≤ 0.25</td>
<td>115</td>
</tr>
</tbody>
</table>

3.13 Prototype SSRM

A prototype SSRM is built to validate the SSRM design concept. The motor was built under the same frame size as a benchmark 1 hp machine whose details are given in Appendix A. The prototype motor has the same structure as the MDMO design as presented in Section 3.8. Photos from SSRM prototyping stages are presented in Figure 3.28.

![Figure 3.28: Photos from SSRM prototyping stages](image-url)

(a) (b) (c) (d) (e) (f) (g) (h)

Figure 3.28: Photos from SSRM prototyping stages; (a), (b) lamination stacks, (c) segment and support assembly, (d) press fitting the shaft, (e) completed rotor, (f), (g) SSRM stator, and (h) completed prototype
Laminations were laser cut and bonded into stacks of a single piece stator and eight rotor segments as shown in Figure 3.28(a) and (b). The support structure of rotor segments are made with non-magnetic structural steel (SUS 303) through electrical discharge machining (EDM) and the assembled rotor segments with the support is shown in Figure 3.28(c). Next, rotor shaft was press-fit into the support structure as shown in Figure 3.28(d) and the finalized rotor is presented in Figure 3.28(e). The stator assembly inside the motor frame without and with winding are shown in Figure 3.28(f) and (g). Completed prototype SSRM inside the 1 hp machine frame is presented in Figure 3.28(h).

3.14 Experimental Evaluation

Experimental analysis on the SSRM prototype was performed to validate its performance and to show that its operation is possible with standard VSI using conventional vector control. Block diagram of SSRM test setup is presented in Figure 3.29. The prototype SSRM is operated with standard 3-phase inverter under sensed $d-q$ vector control. SSRM rotor position was detected from a shaft-end, 1048 pulse; QEP encoder and phase currents were sensed using the built-in current sensors of TI HV motor controller. The dyno machine was operated using ABB ACS800 3-phase, 208 V AC drive.

Figure 3.30 presents the rotor positions and phase current (green graph) data obtained from the SSRM test. Both electrical (yellow graph) and mechanical (cyan graph) rotor positions are plotted which showed their relationship according to pole number. The phase current is sinusoidal which shows that the SSRM operation is possible with conventional $d-q$ vector control. Average torque under different current commands are obtained using the installed torque transducer with sinusoidal phase excitations on the prototype SSRM. Average torque under different phase currents are presented in Figure 3.31 from both FEA and experimental analysis. Results from Figure 3.31 show that SSRM experimental performance agrees well with its FEA based prediction. At higher excitation level, difference between FEA and test results increases. This larger difference at higher excitation level may occur from the uncertain information of the saturated $B-H$ curve of the laminations.
The prototype SSRM stator was built from single lamination pieces as shown in Figure 3.28(a) and (f). The concentrated stator windings of Figure 3.28(g) are then hand-wound to make the prototype. Winding manufacturing through this method achieves lower slot fill factor (FF) of around 40% compared to segmented stator based approach, which are reported to achieve around 60% fill factor. Moreover, the prototype SSRM is built with 100 mm stack length and additional room is available inside the 56C frame as seen from Figure 3.28(g) which would allow increased motor stack length up to 127.4 mm. Considering these manufacturing improvements, the power and torque density of the prototype SSRM can be further improved.
under the same frame size. With results from Figure 3.31 as the base case, Figure 3.32 presents SSRM performance with higher $FF$ and $L_{stk}$ based on FEA analysis, which are possible to achieve under the same motor frame. Results show that high torque density performance, as predicted during the design stage, is achievable by adopting segmented stator structure during the manufacturing process.

Figure 3.31: Performance comparison under different phase currents ($L_{stk}=100$ mm)

Figure 3.32: SSRM performance under different $FF$ and $L_{stk}$
3.15 Summary

Design of SSRM rotor segment is presented targeting torque ripple minimization. The proposed method is advantageous because there is no requirement of phase current profiling, controller complexity or additional converter components. Starting from FEA based semi-numerical model, SSRM major torque ripple sources are identified. Next, segmental dip at the center of rotor segment is introduced as an effective design update for torque-ripple minimization. Rotor design parameters are optimized using ‘one factor at a time’ approach as well as GA based multidimensional optimization techniques (MDSO and MDMO) and their performances are compared. The MDMO design is selected as it was most effective in minimizing the overall cost of optimization. The electromagnetically optimized design is then studied using Multiphysics based mechanical stress, deformation and acoustic noise analysis. A prototype SSRM has been built to validate the SSRM design concept and its FEA based performances are validated through experimental analysis under vector control using a three-phase standard VSI.
Chapter 4 IM Modeling with Ring Parameter Estimation

4.1 Introduction

4.2 Technological Background

4.3 Induction Machine End-ring Modeling
   4.3.1 Backgrounds of End-ring Models
   4.3.2 Ring Parameter Estimation using 3D FEA
   4.3.3 Comparison among Different Estimation Methods

4.4 Model Verification
   4.4.1 Results from FEA Analysis
   4.4.2 Experimental Setup
   4.4.3 Performance Evaluation

4.5 Summary
4.1 Introduction

Electrical motors are the major consumer of the world’s electrical energy, utilizing approximately 70% of the industrial electricity [1], and the induction motor is the largest shareholder in this field of electrical loads [2]. High cost of permanent magnets (PMs), robustness, and simplicity of squirrel cage induction motors (SCIMs) have enhanced the research focus on this field [112], [113]. Higher efficiency, higher power density alternative VSD drives to SCIM at low power level ($P_{\text{Rated}} \leq 7.5 \text{ kW}$) have been selected as the application filed of this research. Large industrial occupancy by machines at this power level [2] and absence of low cost, high efficiency alternatives is the motivation behind selecting this power range. The research theme in this dissertation now shifts towards the induction motor type, non-rare-earth AC machines with sinusoidal stator, and addressing new design approach from both rotor and stator sides. A one hp commercially available, NEMA premium efficiency (IEC IE3 efficiency class) SCIM is selected as the benchmark machine. This is the highest efficiency induction machine commercially available at this power level. Design details of the benchmark SCIM is presented in Appendix A.

4.2 Technological Background

SCIM starting performance prediction is important for applications that requires frequent ON/OFF operations, for example, the compressor motors used in the HVAC (heating, ventilation and air conditioning) systems. Moreover, providing a reliable nameplate data on starting performance requires time-consuming repeated testing on manufactured products, which can be minimized through accurate performance prediction. To improve SCIM performance prediction, use of 3D FEA can aid development of accurate models considering its end parameters in details [114]. Despite numerous investigations on rotor bar design of SCIM [115]-[119], few studies can be found on accurate end-ring modelling [120]-[122]. An accurate rotor resistance estimation method which incorporates the physical value of the end-ring resistance, $R_{\text{ring}}$ is developed in 3D FEA. Advantages of the proposed model over other reported methods are that the 3D cage model incorporates detailed cage geometry variation.
along axial and radial directions as well as it imposes precise current density distribution from 2D transient field analysis by segmenting the bar-ring contact surface [108]. The model is able to account for non-uniform current distribution in rotor end rings as well as predict the effect of variation of ring geometry parameters on the ring and total rotor resistance.

In this chapter, IM modeling is performed based on the one hp benchmark motor defined in Appendix A. Figure 4.1 shows the cross-sectional diagram of the benchmark one hp induction machine. First, the existing [123], [124] and proposed end-ring resistance estimation methods have been detailed and their results have been compared. Next, the outcome of the proposed ring resistance estimation model have been utilized in 2D FEA based simulation of the benchmark squirrel cage induction machine (SCIM) and both starting (locked-rotor) and rated machine performances have been predicted. Results from the developed model has been compared with that from starting and rated performance experiments of the benchmark machine to evaluate its accuracy. The developed 2D FEA based SCIM model shows really good agreement with the experimental results and this has been utilized for benchmarking the proposed reluctance machine topologies during their design stages.

Figure 4.1: 2D cross-sectional diagram of the benchmark induction machine
4.3 Induction Machine End-ring Modeling

The purpose of this modeling method is to obtain the physical end-ring resistance value as $R_{\text{ring}}$, which can be included in the 2D dynamic FEA simulation models to develop a complete model of squirrel cage induction machine (SCIM). This will replace more time consuming dynamic simulation involving 3D FEA and yet hold the accuracy in predicting machine performance. Typical analytical induction machine model incorporates the end-ring resistance as a reflected element to the stator side in its per-phase equivalent circuit model [125]. Although the analytical model will require less computational time than 2D FEA, performances predicted by these models deviates by 10%-20% compared to the actual outcome [126]. On contrary, FEA based simulations can include the magnetic non-linearity, localized and overall saturation, and slot-leakage effects which makes the performance predicted by these methods to be in good agreement with actual machine behavior [127]. The development of the end-ring resistance model is detailed in the following. First, the concept of ring-resistance calculation and some existing methods to compute $R_{\text{ring}}$ has been discussed. Next, details on the developed loss model in 3D FEA and computation flow diagram of the proposed method has been covered. Finally, results from different $R_{\text{ring}}$ prediction methods has been compared and discussed in terms of their relative performance variations.

4.3.1 Backgrounds of End-ring Models

An accurate Squirrel Cage (SC) model is necessary to properly estimate the end-ring resistance. At first, the joule loss and current in the ring were determined using reported analytical models. Three different analytical models from [121], [123], and [124] were utilized. In these methods, the end-ring is modelled as a solid conductor and rotor bars are replaced by a conducting sheet of metal of uniform thickness with its current flowing in straight lines parallel to the rotor axis. The conducting sheet is assumed to join the end-ring at its outer edge. The first one is the ideal loss estimation model assuming that the ring-current is uniformly distributed over the whole ring-cross section. Losses in both end-rings and the peak current around each pole of the machine can be expressed as in equations 4.1 and 4.2 [123].
\[ P_{\text{ring-loss}} = \frac{p \rho_r h^2 J_m^2 \lambda_p^3}{S_r \pi^2} \]  

(4.1)

\[ I_{\text{peak-ring}} = hJ_m \int_0^{\lambda_p/2} \cos \frac{\pi x}{\lambda_p} \, dx \]

\[ = hJ_m \frac{\lambda_p}{\pi} \]  

(4.2)

Here, \( J_m \) is the peak linear current density of the uniform sinusoidal sheet approximating the rotor bars, \( x \) is distance along periphery from pole mid, \( h \) is the axial thickness or height of the ring, \( \lambda_p \) is pole pitch, \( p \) is number of poles, \( S_r \) is the ring cross sectional area, and \( \rho_r \) is the resistivity of the ring material. These parameters are illustrated in Figure 4.2 with current-voltage contours been showed for more distributed current density considered in Trickey’s calculation method [124].

![Distributed current density, J](image)

**Figure 4.2: 2D end-ring current voltage map**

The second method, based on Alger [123], estimates the ring resistance by considering the effective radius that carries the current at the end-ring outer diameter. Assuming uniform current distribution along ring width, \( w \), this method assumes that the effective ring width is less than \( w \) as the current has been more concentrated towards ring’s outer periphery. The
expression for joule loss in both rings for this case is given in equation 4.3, where \( c \) is the ratio between ring inner \( (R_i) \) and outer \( (R_o) \) radius.

\[
P_{ring-loss} = \frac{2\pi \rho_r I_{peak-ring}^2}{h} \times \frac{1}{1-c} \tag{4.3}
\]

The third analytical model is the end-ring model by Trickey [124] that considers different distributed segments of the ring and the radial \( (j_r) \) and tangential \( (j_\theta) \) current densities along with their loss components. The joule loss expression obtained from this model is given by;

\[
P_{ring-loss} = \frac{p \pi \rho_r I_{peak-ring}^2}{h} \times \frac{(1+c^p)}{(1-c^p)} \tag{4.4}
\]

Along with these three analytical models, a ring model in 2D FEA was developed based on its electric conduction properties where the bar current is set through the ring cross-section through a line current density distribution along outer periphery. Other boundaries of the ring model are also defined by assigning proper boundary conditions based on [121]. A 3 phase, 208 V, 1 hp, 4 pole die-casted motor was selected as the test machine and half of the pole model was designed in 2D FEA as presented in Figure 4.3(a). Figure 4.3(b) presents the resulting electric potential by equipotential lines and the imposed sinusoidal current density distribution by arrows. For imposed current density magnitude, \( J_m \) first bar currents, \( i_{bar} \) are obtained from 2D dynamic simulation of SCIM then ring currents, \( i_{ring} \) are obtained from bar current according to equation. Over a machine pole, the \( i_{ring} \) distribution will have its peak ring current, \( I_m \) once. Finally, \( J_m \) is obtained from \( I_m \) by dividing it with area \( S_r \).
Figure 4.3: (a) End-ring model built in 2D FEA and (b) Corresponding electric potential lines and current density arrow plots for the test case

\[
i_{ring}(1) = \left\{-\left(\sum_{k=1}^{N_b} i_{bar}(k)\right)\right\}/2
\]

\[
i_{ring}(i) = i_{ring}(i - 1) + i_{bar}(i - 1); \quad i = 2, 3, 4 \ldots N_b \quad (4.5)
\]

4.3.2 Ring Parameter Estimation using 3D FEA

The actual rotor and its 3D FEA based cage model are presented in Figure 4.4. At first, transient analysis is done in 2D FEA to capture the bar current density \(J_{bar}\) distribution as shown in Figure 4.5(a) in locked-rotor condition. FEA software flux 2D has been used for these analysis. This software incorporates machines electrical components like; phase conductors, end-winding, end-ring resistance and reactance through a coupled circuit. Information for the end-ring segment between bars are considered as the end-ring parameter in FLUX 2D and so the value of \(R_{ring}\) is divided by total bar numbers to obtain them. Bar regions in both 2D and 3D models are segmented into progressively widening annular portions towards the shaft, as shown in Figure 4.5(b) to capture the current density distribution.
The locked rotor condition has the most variation in the bar current distribution having the highest induced frequency in rotor bars and so the number of segments are selected based on the $J_{bar}$ distribution at locked rotor. The same bar currents from these segments are imposed near the ring-bar interface of the ring modeled in 3D FEA to avoid redistribution of the $J_{bar}$ currents before entering the ring. The 3D FEA model, presented in Figure 4.5(c) consist of the actual end-ring geometry along with a thin bar region to impose the excitations from 2D FEA. The 3D model is solved next to calculate the ring joule loss as:

$$P_{ring-loss} = \iiint P_{joule} \, dV$$  \hspace{1cm} (4.6)

Locked rotor $J$ distribution of end-ring in 3D FEA is presented in Figure 4.5(d). To obtain end-ring resistance the effective (RMS) ring current, $I_{RMS-ring}$ also needs to be calculated. Two different methods are used to validated and cross check values for end-ring current. The first method is from 2D bar currents using equation 4.5. Additionally, ring cross-sections were placed between bar-regions to obtain the ring current flowing through them from the solved 3D FEA. Figure 4.6 presents the current density distribution over such ring cross-section in 3D FEA. Ring cross sectional currents are calculated by integrating these current densities over its area and good agreement was found between these results and that found from equation 4.5.
Both 2D and 3D FEA models are designed for one machine pole over which the ring current have its half electrical cycle. To obtain \( I_{RMS-ring} \), root mean square of the currents at different ring segments are taken. Finally, the ring resistance is calculated by:

\[
R_{ring} = \frac{P_{ring-loss}}{I_{rms-ring}^2}
\]  

(4.7)

Figure 4.5: Different stages from the end-ring modeling (a) \( J_{bar} \) distribution from 2D Transient FEA (b) 3D segmented bar to implement \( J_{bar} \) (c) Developed end-ring model in 3D FEA and (d) \( J_{ring} \) distribution in 3D FEA

Figure 4.6: Current density in ring cross sections from end-ring model in 3D FEA
A model flow diagram of the proposed 3D FEA end-ring resistance estimation method is presented in Figure 4.7. First, a two dimensional FEA simulation model of the induction machine is developed for transient analysis based on machine design data. Rotor bars of the designed machine is segmented in to pieces as presented in Figure 4.5(a) to capture the bar current density distribution. Both current densities and bar currents are obtained from this dynamic 2D finite element analysis. Next, the cage model of the designed machine is developed in 3D FEA having the same segmentation of rotor bars. An automated script was developed for the command tools in FLUX software to extract the bar current density distribution from 2D FEA and properly assign them in 3D FEA model. Next, the 3D FEA model was solved to calculate the total ring loss and ring current. Next, the end-ring resistance, $R_{ring}$ was obtained by equation 4.7. This calculated value of $R_{ring}$ ($R_{ring}(n)$) was then used in the 2D FEA model and the overall process was repeated to calculated $R_{ring}$ value of the next iteration $R_{ring}(n+1)$. A convergence condition was checked as;

$$R_{ring}(n+1) - R_{ring}(n) \leq \delta$$

Where, $\delta$ is set as 2.5% of the difference between the calculated $R_{ring}$ and end-ring DC resistance. If the convergence condition is met then, the $R_{ring}$ value is selected as the estimated end-ring resistance, otherwise the process is repeated back to calculate a new value of $R_{ring}$ from next iteration. This process has been tested for different end-ring dimensions at different power level and typical convergence was reached within 3 to 4 iterations.

The end-ring reactance $X_{ring}$ can be obtained from the 3D FEA model [128] by obtaining the potential difference over the ring section, $V_{ring}$ and calculating the reactive power, $Q_{ring}$ as given by equation 4.8.

$$Q_{ring} = \sqrt{(V_{ring}I_{rms-ring})^2 - P_{ring-loss}^2}$$

$$X_{ring} = \frac{Q_{ring}}{I_{rms-ring}^2}$$

(4.8)
The calculated $R_{ring}$ for motor operation under a certain slip, $s$ bar current densities from 2D FEA are obtained at different instances of transient simulation over one electrical cycle and imposed on 3D FEA model. Ring resistances were estimated for each case using equation 4.6 and 5.7. It was found that, under the same slip condition, calculated $R_{ring}$ values are same. However, $R_{ring}$ values calculated over different slip condition vary and this can be referred as the variation of the induced frequency in rotor conductors which results in variation of rotor AC resistance. At the synchronous speed the rotor and stator MMF are rotating in sync and there would be no relative frequency induced in rotor conductors. End-ring resistance at synchronous speed is hence assumed to be the same as its DC resistance and the AC ring resistance at locked rotor is estimated from the developed model. A linear variation of $R_{ring}$ is assumed between these two points to estimate the ring-resistance in other slip conditions.
4.3.3 Comparison among Different Estimation Methods:

Ring resistance of the SCIM was determined for all five methods mentioned above and the comparison among these estimated values for the one hp machine are presented in Figure 4.8. The first three analytical methods yield similar $R_{\text{ring}}$ values as the selected end-ring can be considered as a narrow ring according to [124] assuming ring currents remains uniform over the whole ring width. The 2D FEA estimates a bit higher values as the ring currents gets concentrated near higher potential region as presented in Figure 4.2. The result with the 3D FEA is slightly higher than other methods since the 3D model accounts non-uniformity in distribution of both bar currents as well as over the cross section of the ring geometry. To predict $R_{\text{ring}}$ for 2D transient simulation, results from the 3D ring model has been considered onwards to predict machine performance and the accuracy of this model has been verified with experimental results. Dynamic simulation models for the benchmark induction machine were developed in 2D FEA. The next section, will detail on the performance of the benchmark SCIM predicted from the FEA based simulation model and compare it with experimental results for verifying the developed model.

![Figure 4.8: $R_{\text{ring}}$ estimation comparison for 1hp machine](image)

Figure 4.8: $R_{\text{ring}}$ estimation comparison for 1hp machine
4.4 Model Verification

Two dimensional FEA model for the benchmark induction machine is developed in finite element software FLUX 2D. The developed model includes machine end-winding and end-ring resistances and inductances as lumped parameters in its circuit. Dynamic FEA simulations of the benchmark SCIM were performed to obtain its performance at both starting and rated conditions. Next, an experimental setup was developed to test the benchmark SCIM. Finally, the machine performance predicted from the FEA model is compared with that from its experimental results to evaluate the accuracy of the developed FEA model.

Figure 4.9: (a) Magnetic flux density distribution and (b) field distribution of the induction machine at rated condition

4.4.1 Results from FEA Analysis

Magnetic flux density distribution of the induction machine at rated condition is presented in Figure 4.9(a) along with the magnetic field distribution in Figure 4.9(b) showing its four-pole behavior. Bar current density distribution for both starting (locked rotor) and rated (3.05% slip) conditions are presented in Figure 4.10(a) and (b), respectively. At starting condition, slip = 1 and the induced frequency in the rotor bars are the same as machine excitation frequency which is 60 Hz. As a result, skin effect is very prominent in the rotor bar as can be seen from the radial gradients in bar current densities. In the rated speed, the induced rotor bar frequency is much less than that at starting and current density distribution over one bar is almost uniform.
in this case. These variations of bar current densities were incorporated in the developed 3D FEA based end-ring model to improve its accuracy in machine performance prediction. The accuracy of the developed model is validated next from comparison with experimental results.

![Figure 4.10: Bar current density distribution at (a) starting and (b) rated speed conditions](image)

### 4.4.2 Experimental Setup

The accuracy of the FEA based end-ring resistance estimation model is verified experimentally for the one hp test machine. A test-bed with five hp dyno as presented in Figure 4.11 is developed. A torque transducer was installed between the test machine and the dyno for real time measurement of the torque. For machine starting condition test under locked rotor, shaft of the 1 hp test machine was mechanically locked and the machine was line connected through a VARIAC. Locked rotor test was performed under different levels of line voltages and torque and rms line currents are obtained. For machine performance evaluation under rated condition, heat run test was performed to get performance parameters at machine’s thermal steady state. The next section compares machine performance from FEA and experimental setup for both starting and rated conditions.
4.4.3 Performance Evaluation

Machine end-ring parameters have significant influences over its starting performance as large slip value is associated during this condition. A properly estimated end-ring parameters will be able to predict machine performance more accurately. Starting torque, $T_{START}$ and RMS line current, $I_{START}$ are obtained from machine locked rotor setup by applying different line-to-line voltages through a VARIAC. Results are also obtained from the 2D FEA by applying the same line-to-line voltages and using the calculated end-ring resistance from the proposed model. Comparisons of the locked-rotor performance are presented in Table 4.1. Results show really good agreement between the simulated and test results which verifies the accuracy of the FEA model.

Accuracy of the 2D FEA model is also experimentally verified with machine’s rated performance, and results are presented in Table 4.2. As with the starting characteristics, machine test results at rated condition agree well with the FEA based prediction. Results presented in Table 4.1 and Table 4.2 show that the developed end-ring model can yield $R_{ring}$ values to properly predict machine performance at both starting and rated conditions. Performance predicted from induction machine model in 2D FEA show good agreement with experimental results. During design stages of the SSRM and multi-layer wound SynRM, this FEA model of the induction machine has been considered as the benchmark design.
Table 4.1: Starting performance comparison for the 1 hp SCIM

<table>
<thead>
<tr>
<th>$V_{LL}$ (V)</th>
<th>$T_{START}$ (Nm) Test</th>
<th>$T_{START}$ (Nm) FEA</th>
<th>$I_{START}$ (A) Test</th>
<th>$I_{START}$ (A) FEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>82.47</td>
<td>0.35</td>
<td>0.27</td>
<td>1.94</td>
<td>1.87</td>
</tr>
<tr>
<td>159.49</td>
<td>1.48</td>
<td>1.4</td>
<td>4.17</td>
<td>4.02</td>
</tr>
<tr>
<td>299.55</td>
<td>6</td>
<td>5.81</td>
<td>8.59</td>
<td>8.59</td>
</tr>
<tr>
<td>362.82</td>
<td>9.03</td>
<td>8.98</td>
<td>10.7</td>
<td>10.11</td>
</tr>
<tr>
<td>459.60</td>
<td>15.16</td>
<td>15.13</td>
<td>13.96</td>
<td>13.82</td>
</tr>
</tbody>
</table>

Table 4.2: 1 hp SCIM Rated performance comparison

<table>
<thead>
<tr>
<th>$T_{rated}$ (Nm)</th>
<th>Test</th>
<th>2D FEA</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>4.07</td>
<td>-0.7</td>
<td></td>
</tr>
<tr>
<td>$I_{rated}$ (A)</td>
<td>1.44</td>
<td>1.44</td>
<td>0</td>
</tr>
<tr>
<td>$PF$ (pu)</td>
<td>0.76</td>
<td>0.75</td>
<td>-1.3</td>
</tr>
<tr>
<td>$\eta$ (%)</td>
<td>85.5</td>
<td>86.3</td>
<td>+0.9</td>
</tr>
</tbody>
</table>

4.5 Summary

An accurate rotor resistance estimation model of SCIM is developed in 3D FEA. Transient analysis was utilized for excitations in the 3D model to improve its accuracy over previous 2D and analytical methods. The estimated end-ring parameter is then used in 2D FEA to predict starting and rated performances. Good agreement is shown between the FEA results and nominal as well as starting performance of the test machine. Having confidence on the developed FEA based SCIM model, this simulation model is utilized for evaluation of the multilayer winding designs that are covered in the following chapters. Moreover, performance of this modelled SCIM is used as the benchmark in the design stages of the proposed AC machine topologies.
Chapter 5 New Multi-Layer Winding Configuration

5.1 Introduction

5.2 Research Background

5.3 Development of the Multilayer Winding

5.4 Winding Characterization
   5.4.1 Analytical Winding Model
   5.4.2 Stator MMF Analysis
   5.4.3 Model Verification

5.5 Comparison with other AC Windings
   5.5.1 Twelve Slot Stator Structure
   5.5.2 Twenty-Four Slot Stator Structure
   5.5.3 Thirty-Six Slot Stator Structure
   5.5.4 Performance Evaluation

5.6 Summary
5.1 Introduction

The research is focused on electric machine applications in the low power level (≤ 7.5 kW) considering its large market share of 79%. Significant stator conductor loss (45% to 55% of total loss) in this power level has been observed in Figure 1.1(b). It can be concluded that the design strategy in minimizing stator $I^2R$ loss will be an effective efficiency improvement technique for these machines. Conventional AC machine design involves full-pitched phase windings with large end-winding lengths extending over the full pole pitch. These end-windings are necessary for completing the phase windings but they do not contribute towards the production of its electromagnetic torque. Minimization of these end-winding lengths is a viable way to reduce stator $I^2R$ losses and improve machine efficiency.

5.2 Research Background

End-winding length reduction has been addressed in the literature with concentrated winding SCIM designs [101], [104]. Concentrated windings have the advantages of non-overlapping shorter end-turns, high slot fill factor, and easier manufacturing. However, existing literature on concentrated winding SCIMs show that these designs are characterized with high space harmonics in its air-gap MMF. These undesirable space harmonics result in average torque reduction [101], and increase in core and rotor $I^2R$ losses [102]. As a result, traditional distributed windings showed superior performances compared to its concentrated winding counterparts. The air-gap MMF quality can be improved with 5-phase concentrated windings [103], but the machine requires non-conventional 5-phase inverters with higher converter cost and its direct on-line application is no longer possible.

Different three-phase winding configurations are proposed as well to obtain lower space harmonics [72], [74]. These designs use fifth order space harmonic for torque production. A multiple layer (layer number ≥ 3) winding is presented in [73] that has additional space harmonics of $1^{\text{st}}$, $7^{\text{th}}$, $17^{\text{th}}$, and $19^{\text{th}}$ order. Winding configuration in [74] is of fractional-pitch type with overlapped windings having $1^{\text{st}}$, $17^{\text{th}}$ and $19^{\text{th}}$ order space harmonics. Moreover, machine pole number increased to 10 with 12 slot [73] and 24 slot [74] stators. This higher
pole number increases machine’s fundamental frequency increasing core-losses and reducing machine input power factor [105]. Torque ripple of these designs are also higher compared to their distributed winding counterparts [72].

Investigating relative advantages and disadvantages of the full-pitched distributed, fractional-pitch overlapped and concentrated windings, a new winding configuration is developed in this research. This new winding configuration has the advantage of shorter end-turn lengths as the fractional pitch ones and it also provides high quality rotating MMF while maintaining same pole number as distributed windings. Design details of the new winding configuration including machine characteristics have been presented in this chapter. Moreover, performance of the proposed winding configuration has been studied and compared with other reported winding configurations from [73], [74], and [101] to quantitatively evaluate its performance for the same stator configuration.

5.3 Development of the Multilayer Winding

The concept of the proposed multilayer winding has been discussed using rectilinear representation of a six slot stator, which is the base stator configuration for a balanced three-phase machines. Figure 5.1(a) shows the full-pitched winding configuration with its end-winding expanding over three slot pitches ($w_p = 3 * s_p$) and Figure 5.1(b) shows the double layer concentrated winding configuration with a fractional winding pitch equal to one slot pitch ($w_p = s_p$). Three-phase radial flux machines under the selected 6/2 combinations will have largest end-winding length for Figure 5.1(a) and the shortest for Figure 5.1(b). The proposed multilayer winding configuration is developed based on these two winding configurations.

Phase winding distribution in Figure 5.1(a) and (b) are presented in way that the stator MMF generated from equal phase currents will add in phase for these two configurations. If the total number of turns per slot is $N$ and a snap of three-phase sinusoidal excitation with $i_a = +i$, $i_b = -i/2$ and $i_c = -i/2$ is taken, stator MMF diagram [106] for the windings of Figure 5.1 can be presented by Figure 5.2, with phase $a$ axis being designated for reference.
Figure 5.1: Winding diagram for six slot two pole (a) fully pitched distributed winding and (b) concentrated winding

Figure 5.2: Stator MMF diagram for six slot two pole (a) fully pitched distributed winding and (b) concentrated winding

Study on the stator MMF distribution diagrams of Figure 5.2 shows that the fully pitched winding links twice the amount of stator MMF compared to its concentrated wound counterpart under the same electrical loading ($Ni$). However, the concentrated windings have one third of the end-winding length compared to its fully pitched wound counterpart which significantly reduces machine end-winding losses. As a result, a relative design trade-off between these two
windings are clear and a combination of these two configurations will inherit the positive features of both windings in a single design. The new winding configuration has the advantage of shorter end-turn lengths as the fractional pitch ones and it provides high quality rotating MMF while maintaining same pole number as distributed windings.

Rectilinear representation of the proposed multi-layer winding for a six-slot stator is presented in Figure 5.3(a). The fully pitched configuration of Figure 5.1(a) is placed on the top row and the concentrated wound fractional pitch winding (Figure 5.1(b)) is place in the bottom two rows with their number of turns being half of that of their initial representation to maintain the total number of turns per slot as \( N \). Considering the same instant of sinusoidal excitation as before the stator MMF diagram for the multilayer winding configuration is presented in Figure 5.3(b).

Figure 5.3: (a) Six slot multi-layer winding configuration (b) corresponding Stator MMF

Considering the stator MMF diagram presented in Figure 5.3(b) the proposed multi-layer winding configuration can be considered as a trade-off design between the fully pitched and fractional pitch concentrated windings. The peak value of stator MMF for the multi-layer winding is in mid-point between that of full-pitched and concentrated windings. Moreover, the equivalent end-winding length of the multi-layer winding is equal to two slot pitches achieving an effective reduction in end-winding losses. Although this six-slot configuration is utilized to explain the development of the multi-layer winding concept, for sinusoidal AC machines in general the number of stator slot is much larger and the advantages of the multi-layer winding
compared to other reported AC windings for different stator configurations has been discussed in section 5.5. The next section will develop an analytical model for the multilayer winding and quantify its stator MMF in terms of fundamental and harmonic contents.

5.4 Winding Characterization

The effective value of the magnetomotive force (MMF) induced in a coil with $n_c$ turns of a sinusoidally distributed flux is given by;

$$E = \frac{2\pi}{\sqrt{2}} f n_c \varphi$$

(5.1)

Here, $f$ is the frequency of excitation and $\varphi$ is the flux linkage. The expression of equation 5.1 is of an ideal MMF based on three main assumptions. First, all of $n_c$ turns of the coils are linked with the same flux, the coil pitch of the phase winding is equal to machine pole pitch and the flux distribution is completely sinusoidal. In practical AC machines these assumptions are not valid as phase coils are distributed among different stator slots achieving a quasi-sinusoidal flux distribution. Moreover, in most AC machines the winding is chorded having a coil pitch less than pole pitch. Accordingly, the effective MMF induced is less than that given by equation 5.1. Two different armature winding factors namely; the distribution factor, $k_d$ and the pitch factor, $k_p$ are defined to characterize these effects in practical windings. The next section will explain these factors in terms of winding configurations.

5.4.1 Analytical Winding Model

The stator winding is distributed in the slots of the stator surface and flux (which is proportional to the current linkage, $\Theta$) does not intersect all windings simultaneously but with a certain phase shift. As a result, winding factor $k_w$ corresponding to its harmonics, $\nu$ are required to characterize the actual winding. Considering a full pitch winding with its coils from a single phase are distributed over 2 stator slots as in Figure 5.4(a), the voltage of the phase is reduced because of this winding distribution by the factor $k_d$. If the coil pitch is shorter than pole pitch, the winding is called short-pitched or chorded winding with phase windings being
placed in multiple layers as in Figure 5.4(b). Short pitching is another reason for phase winding voltage reduction and the factor of this reduction is given by $k_p$. The total winding factor is given as:

$$k_w = k_d k_p$$  \hspace{1cm} (5.2)

Figure 5.4: (a) Fully pitched and (b) short pitched distributed winding of AC machines

The equation to calculate the distribution factor has been derived considering fully pitched windings with coils form a single phase being distributed over 5 stator slots. The magnitude of the induced MMF is the same for all coils but the time phase is not the same as each coils are located in different slots. The time phase shift between the coils is equal to the angle between two slots, $\alpha$ given by:

$$\alpha = \frac{180 \times 2p}{Q} = \frac{180}{mq}$$  \hspace{1cm} (5.3)

Here, $Q$ is the total slot number, $m$ is the number of phases, $p$ is the number of pole pairs and $q$ is the number of slots over which coils from a single phase winding are distributed. The resultant MMF of the coil group is not equal to the algebraic sum of single coil MMFs but to the geometric sum of these MMFs.
For the three-phase windings with \( q = 5, \alpha = 12^\circ \), the voltage polygon of the induced MMFs in the five-coil group is presented in Figure 5.5. Each phasor AB, BC, CD, DE, EF is equal in magnitude and represents the maximum MMF value induced by a coil, \( E_c \). AF represents the resultant MMF, \( E_r \) induced in coil group. The distribution factor for the fundamental component is the ratio between these geometric and algebraic sum of individual coil MMFs given by:

\[
k_d = \frac{\text{geometric sum}}{\text{sum of absolute values}} = \frac{E_r}{qE_c} \tag{5.4}
\]

From Figure 5.5 for the triangle OGF,

\[
sin \frac{q\alpha}{2} = \frac{E_r/2}{R} \text{ or } E_r = 2R \sin \frac{q\alpha}{2}
\]

and for the triangle OAB;

\[
sin \frac{\alpha}{2} = \frac{E_c/2}{R} \text{ or } E_c = 2R \sin \frac{\alpha}{2}
\]

So, the distribution factor can be written as;
\[ k_d = \frac{E_r}{qE_c} = \frac{2R \sin \frac{q\alpha}{2}}{q2R \sin \frac{\alpha}{2}} = \frac{\sin \frac{q\alpha}{2}}{q \sin \frac{\alpha}{2}} \]  \hspace{1cm} (5.5)

Equation 5.5 represents the expression for fundamental distribution factor. Harmonic components of the air-gap flux density are also present and the calculation of the distribution factor for the \(v^{th}\) harmonics can be done by applying the angle \(v\alpha\) as;

\[ k_{dv} = \frac{\sin \frac{vq\alpha}{2}}{q \sin \frac{v\alpha}{2}} \]  \hspace{1cm} (5.6)

With application of short pitching, coil ends have become shorter and it reduces copper consumption. Moreover, a correctly short pitched winding produces a more sinusoidal current linkage distribution than full pitched winding. However, coil flux linkage decreases with short pitching and so the number of coil turns at the same voltage has to be higher than for a fully pitched winding. This reduction in flux linking the coil is accounted using pitch factor, \(k_p\). The calculation of \(k_p\) is performed using Figure 5.6. For short pitched coils or chorded windings, when the coil pitch is less than the pole pitch, the effective MMF induced in the coil has been less than that of a fully pitched winding. It has been equal to the voltage of the full pitched coil times the ratio of the shaded area to the total area of Figure 5.6. That is the pitch factor;

\[ k_p = \frac{\int_{(t-W)/2}^{(t+W)/2} \sin \left(\frac{\pi x}{t}\right) dx}{\int_0^t \sin \left(\frac{\pi x}{t}\right) dx} = \sin \left(\frac{W\pi}{t}\right) \]  \hspace{1cm} (5.7)

Here, \(t\) is the pole pitch and \(W\) is the coil span. The pitch factor for the \(v^{th}\) harmonic can be written as;

\[ k_{pv} = \sin \left(\frac{W\pi}{t}\right) \]  \hspace{1cm} (5.8)

With different number of turns in coils, individual coil MMFs will also have different magnitudes. Accordingly, equation 5.8 needs to be updated with different \(N_{\text{turn}}\) values.
Considering the group of 5 coils each with different \( N_{\text{turn}} \) values (\( N_1 \) to \( N_5 \)) with different slot pitches of \( W_1 \) to \( W_5 \), the modified expression for the pitch factor can be written in terms of weighted sum of individual MMFs as:

\[
k_{p\nu} = \frac{N_1 \sin(v \frac{W_1 \pi}{t}) + N_2 \sin(v \frac{W_2 \pi}{t}) + N_3 \sin(v \frac{W_3 \pi}{t}) + N_4 \sin(v \frac{W_4 \pi}{t}) + N_5 \sin(v \frac{W_5 \pi}{t})}{N_1 + N_2 + N_3 + N_4 + N_5}
\]

or, \( k_{p\nu} = \frac{\sum_{i=1}^{q} N_i \sin(v \frac{W_i \pi}{t})}{\sum_{i=1}^{q} N_i} \) \hspace{2cm} (5.9)

Distribution and short-pitching combination can eliminate certain harmonics. Winding factor, \( k_{w\nu} \) as in equation 5.2 describes the stator MMF harmonics, and with fully pitched single layer winding as reference, the stator MMF expression can be written as equation 5.10.

\[
\vec{F}^\nu_{st} = \frac{3}{\pi v} \cdot \frac{N_{ph} \cdot k_{w\nu} \cdot \hat{s}}{p}
\] \hspace{2cm} (5.10)

For a poly-phase winding the harmonic \( \nu \) is calculated by summation of all the harmonics created by different phases. So for a three-phase machine the harmonic distribution factor is zero if \( \nu \alpha = \pm 2\pi c \) \((c = 0, 1, 2, 3, 4 \ldots)\) as the coil sides are on the same magnetic potential [111]. The winding thus produces harmonics in the order of \( \nu = 1 \pm 2cm \). As a result, a symmetrical three-phase winding will create harmonics as presented in Table 5.1 [111].
Table 5.1: Harmonic order created by balanced three-phase windings

<table>
<thead>
<tr>
<th>c</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>+v</td>
<td>+1</td>
<td>+7</td>
<td>+13</td>
<td>+19</td>
<td>+25</td>
<td>Positive</td>
</tr>
<tr>
<td>-</td>
<td>-5</td>
<td>-11</td>
<td>-17</td>
<td>-23</td>
<td></td>
<td>Negative</td>
</tr>
</tbody>
</table>

For the twelve slot configuration with $2p = 4$, the pole pitch $\tau = 3$. The 12-slot multi-layer winding consists of coil groups of two different pitches as presented in Figure 5.7. The first group (between slots 2-5 and 8-11) has coil pitch equal to its pole pitch and the second group (among slots 3-4, 6-7, 9-10 and 12-1) have coil pitch equal to one-third pole pitch. A weighted sum of different pitch factors based on their number of turns are calculated to obtain the fundamental and harmonic pitch factors. Winding factors up to $z + 2p$ hold significance in stator MMF analysis as larger harmonics will have their amplitude diminished by the ratio of fundamental to harmonic orders [73], and they are calculated and compared with FEA results in the next section for verification.

![Figure 5.7: Twelve slot multi-layer winding layout for a single phase](image)

**5.4.2 Model Verification**

Analytical stator MMF model of [106] is utilized to obtain the winding function distribution over 12 stator slots. In addition, a 2D FEA model of the 12 slot multi-layer stator is created with a solid round rotor made of complete iron material so that the effect of stator winding can be investigated separately (the solid round rotor does not have any reaction to stator MMF). Stator MMF distribution for both analytical and FEA models are presented in
Figure 5.8 which shows a good correlation between FEA and analytical results except the FEA includes the additional effects of slot leakage and slot harmonics.

![Figure 5.8: Air-gap MMF distributions of the designed ML12 configuration](image)

Table 5.2 presents different air-gap MMF harmonic orders calculated from both equation based [111] and waveform based [106] methods. The calculated harmonic orders are close in these methods and the trend of relative harmonic amplitudes are accurately held. Since, the analytical air-gap MMF waveform matches well with the FEA results in Figure 5.8, for further stator MMF harmonic analysis this method has been adopted.

<table>
<thead>
<tr>
<th>Harmonic Number, $\nu$</th>
<th>Relative Amplitude (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equation 5.2</td>
</tr>
<tr>
<td>$p$</td>
<td>100</td>
</tr>
<tr>
<td>$Z - p$</td>
<td>22.43</td>
</tr>
<tr>
<td>$Z + p$</td>
<td>18.28</td>
</tr>
<tr>
<td>$2Z - p$</td>
<td>8.31</td>
</tr>
<tr>
<td>$2Z + p$</td>
<td>8.62</td>
</tr>
</tbody>
</table>
5.5 Comparison with other AC Windings

Different three-phase winding configurations are presented in the literatures to obtain lower space harmonic contents in their air-gap MMFs [72], [74]. These designs use 5th order space harmonic as their fundamental for torque production. A 12 slot multiple layer (layer number ≥ 3) concentrated winding is presented in [73] that has additional space harmonics of 1st, 7th, 17th, and 19th order in its stator MMF. Moreover, for having the fifth order harmonic for torque production pole number for the 12 slot configuration is become 10. This higher pole number increases machine’s fundamental electrical frequency increasing core-losses and reducing machine input power factor [105]. These design challenges can be mitigated adopting the proposed multi-layer winding with 12-slot configuration.

5.5.1 Twelve Slot Stator Structure

The 12 slot rectilinear structure of the multi-layer winding is presented in Figure 5.9 with end-windings shown on one side. For this 12 slot structure. A single-phase winding consists of four concentrated and two overlapped windings with coil span of three slot pitches. Since each stator slot will house windings from all phases, total number of layers for a three-phase machine has been three for this multilayer winding.

![Figure 5.9: 12 slot multi-layer winding layout showing a single phase](image-url)
Slot distribution for the proposed multilayer winding is presented with its rectilinear model in Figure 5.10. Each winding is designated by its phase number, coil span in slot pitches and coil orientation. Thus, “A1M” is referring to a negative coil end of “phase A” concentrated winding with coil span of one slot pitch. Half of the stator slots are presented because the other half is a duplicate of this arrangement. This winding holds minimal number of full-pitched winding to maintain shorter end-turn lengths than conventional distributed windings.

![Figure 5.10: Slot distribution of the 12 slot three-phase multi-layer windings](image)

Analytical air-gap MMF model [106] for the 12 slot multilayer (ML12) configuration is derived and compared with that from the 2D FEA in Figure 5.8. For a $p$ pole-pair machine with total number of slots as $Z$, harmonics in the order of $Z \pm p$ will always exist due to uniform slotting. Additionally, odd space harmonics are present due to known periodicity [73]. Air-gap MMF for the multilayer concentrated winding (MLCW) structure of [73] is obtained from FEA and relative amplitudes of its MMF space harmonics are compared with the MMF distribution of ML12 winding in Table 5.3. The major harmonic components are designated in terms of machine slot and pole-pair numbers and other odd harmonics up to $20^{th}$ order are summed up as ‘odd’ in Table 5.3. THD for the MMF waveforms is calculated by the amplitude of $n^{th}$ harmonic, $A_n$ using equation 5.11 [107].

$$THD = \sqrt{\frac{A_2^2 + A_3^2 + \ldots + A_n^2}{A_1}}$$  \hspace{1cm} (5.11)
Table 5.3: Quantified space harmonics of air-gap MMF for 12 slot designs

<table>
<thead>
<tr>
<th>Harmonic Number, ( \nu )</th>
<th>Relative Amplitude (%)</th>
<th>MLCW [68] ( 2p = 10 )</th>
<th>ML12 ( 2p = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.52</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>27.71</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>74.40</td>
<td>7.62</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.84</td>
<td>14.66</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.79</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>25.67</td>
<td>9.85</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>26.05</td>
<td>2.01</td>
<td></td>
</tr>
<tr>
<td>THD</td>
<td>84.35</td>
<td>33.71</td>
<td></td>
</tr>
</tbody>
</table>

It is evident from the quantified space harmonics distribution of Table 5.3 that the designed ML12 winding has much lower space harmonic contents compared to that of MLCW windings. Moreover, unlike the MLCW winding, the proposed multilayer winding can reduce the space harmonic content with larger stator slot numbers without increasing machines fundamental frequency. This advantage of the multilayer winding is presented next for a 24-slot design.

5.5.2 Twenty-Four Slot Stator Structure

Single-phase layout for the 24-slot winding (ML24) is presented in Figure 5.11. Only 12 slots are shown as the rest are duplicates of that of Figure 5.11. Air-gap MMF from analytical and FEA models for ML24 winding are presented in Figure 5.12, and its space harmonic component is compared with that of the 24 slot configuration of fractional pitch overlapped (FPOL) windings of [74] in Table 5.4.
Results from the space harmonic analysis are quantitatively compared in Table 5.4. For 24 slot machines, multilayer winding designs show lower space harmonic content compared to that of FPOL. It is evident that, the designed multi-layer windings can yield more sinusoidal air-gap MMF with reduced harmonic contents compared to the existing AC winding configurations [73], [74] under the same stator structure. Moreover, at rated operating speed, the fundamental electrical frequency of the designed machine is lower than that of both MLCW and FPOL configurations resulting in lower core loss and higher machine power factor.
Table 5.4: Quantified space harmonics of air-gap MMF for 24 slot designs

<table>
<thead>
<tr>
<th>Harmonic Number, ν</th>
<th>Relative Amplitude (%)</th>
<th>FPOL [69] (2p = 10)</th>
<th>ML24 (2p = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>(Z - p)</td>
<td>26.4</td>
<td>9.21</td>
<td></td>
</tr>
<tr>
<td>(Z + p)</td>
<td>17.2</td>
<td>7.59</td>
<td></td>
</tr>
<tr>
<td>(2Z - p)</td>
<td>11.7</td>
<td>4.47</td>
<td></td>
</tr>
<tr>
<td>(2Z + p)</td>
<td>9.35</td>
<td>3.89</td>
<td></td>
</tr>
<tr>
<td>Odd</td>
<td>45.57</td>
<td>12.21</td>
<td></td>
</tr>
<tr>
<td>THD</td>
<td>57.39</td>
<td>18.07</td>
<td></td>
</tr>
</tbody>
</table>

The proposed winding configurations can obtain much lower space harmonics in its air-gap MMF compared to its concentrated windings and fractional pitch counter parts. Moreover, it can maintain the same fundamental frequency with different stator slot numbers. The next step is to apply this multilayer winding technology in the commercially available standard frame machines to evaluate its usefulness.

5.5.3 Thirty-Six Slot Stator Structure

A commercially available 36 slot stator with double layer distributed winding (DLDW) is selected as the benchmark design [108]. Single-phase winding layout for the benchmark DLDW SCIM and designed multilayer winding (ML36) are presented in Figure 5.13. The designed machine has both concentrated and overlapped windings and analytical model from [109] has been utilized to calculate average conductor length \(L_{cond}\) and end-winding length \(L_{end}\). Then machine phase resistance \(R_{ph}\) is calculated using the following expression:

\[
R_{ph} = \rho_T \frac{2L_{cond} \times N_{turn} \times q}{A_{cond} N_{pp}}
\]  

(5.12)
Here, $A_{\text{cond}}$ is the conductor cross-sectional area, $N_{\text{turn}}$ is the number of turns in the conductor group, $N_{pp}$ is the number of parallel paths, $\rho_T$ is the conductor material resistivity at temperature $T$, $q$ is the number defining slots/pole/phase, and $L_{\text{cond}}$ is the summation of machine iron stack length, $L_{stk}$ and end-winding length, $L_{\text{end}}$ which can be derived as [109]:

$$L_{\text{end}} = k_c \frac{2\pi r_w}{Q_s} \tag{5.13}$$

Here, $r_w$ is the average slot radius, $Q_s$ is the total number of stator slots, and $k_c$ is co-efficient depending on winding type. For overlapped windings $k_c$ is $\pi/2$ times the coils span in slot pitches and for fractional slot concentrated windings the value for $k_c$ is 0.93.

![Figure 5.13: Single phase winding distributions for 36 slot (a) benchmark SCIM (DLDW), (b) designed multilayer winding (ML36)](image)

Table 5.5 presents the average conductor length, end-winding length, and the total phase resistance for both DLDW and the designed ML36 windings with equal number of turns $N_{\text{turn}}$. Machine phase resistance is also measured at the ambient temperature and the difference of the measured and analytical resistance is found to be 3.25%. End-winding length of the designed winding is 12.74% less than that of the benchmark SCIM that reduces the phase resistance by 7.5%.
Table 5.5: Winding Parameter Comparison

<table>
<thead>
<tr>
<th>Winding</th>
<th>DLDW</th>
<th>ML36</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{turn}}$</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>$L_{\text{cond}}$</td>
<td>194.2 mm</td>
<td>179.65 mm</td>
</tr>
<tr>
<td>$L_{\text{end}}$</td>
<td>114.2 mm</td>
<td>99.65 mm</td>
</tr>
<tr>
<td>$R_{\text{ph}}$</td>
<td>7.63 Ω</td>
<td>7.06 Ω</td>
</tr>
</tbody>
</table>

Figure 5.14: Air-gap MMF comparison for DLDW and ML36 windings

Per unit air-gap MMF distributions are obtained from 2D FEA analysis for both DLDW and ML36 windings and presented in Figure 5.14. Space harmonic components for these two MMF distributions are quantified in Table 5.6. The significant harmonics in the stator MMF are lower with ML36 winding compared to the DLDW windings. The designed ML36 winding can obtain more sinusoidal air-gap MMF than DLDW windings with 3.1% reduction in THD.

5.5.4 Performance Evaluation

Squirrel cage induction motor has been selected as the machine topology for initial evaluation of the designed winding. SCIMs do not contain rotor reaction feature and so effect of stator MMF is more directly translated to machine performance through this topology. Two different SCIM stators, one made with DLDW and other with ML36 winding configurations
are studied in this section. Both machines have the same number of rotor bars and iron core geometry so that their performance can be evaluated based on winding configurations only.

Table 5.6: Quantified space harmonics of air-gap MMF for 36 slot stator

<table>
<thead>
<tr>
<th>Harmonic Number, ( \nu )</th>
<th>Relative Amplitude (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DLDW [99] ( 2p = 4 )</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>17</td>
<td>24.81</td>
</tr>
<tr>
<td>19</td>
<td>13.92</td>
</tr>
<tr>
<td>35</td>
<td>13.87</td>
</tr>
<tr>
<td>37</td>
<td>11.64</td>
</tr>
<tr>
<td>Odd</td>
<td>4.89</td>
</tr>
<tr>
<td>THD</td>
<td>34.08</td>
</tr>
</tbody>
</table>

Table 5.7 shows the performance comparison of SCIMs for two different windings at the rated speed of 1751 rpm under the same amount of stator \( I^2R \) loss (in Watt). Phase resistances of both machines were increased to account for a steady state temperature rise of 75° C. The designed machine with ML36 winding can provide higher torque density (7.2% more) compared to the benchmark SCIM under the same stator \( I^2R \) loss. As a result, the designed machine will be cooler in attaining rated power with lower stator \( I^2R \) loss compared to the benchmark SCIM, which will further increase its efficiency than that presented in Table 5.7.

Analysis on different configurations of the designed multi-layer winding showed that it can reduce the harmonic content in air-gap MMF due to its fully distributed nature. This harmonic reduction is very useful in sinusoidal AC machines with a prospect of its improved performance as presented in Table 5.7. Design of a synchronous reluctance machine is one of the research focuses of this dissertation. With a more sinusoidal winding the designed SynRM can have improvement in its torque ripple [54]. Moreover, SynRM has a cold rotor with no
rotor $I^2R$ losses which is an effective way of improving machine power conversion efficiencies compared to SCIMs. The design of a SynRM with proposed multi-layer winding has been detailed in Chapter 6 to achieve a higher efficiency alternative for the targeted application of low power SCIMs in industrial drives.

Table 5.7: Comparison of SCIM performances under the same stator $I^2R$ loss

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winding Type</td>
<td>DLDW</td>
</tr>
<tr>
<td></td>
<td>ML36</td>
</tr>
<tr>
<td>Phase $V_{RMS}$ (V)</td>
<td>319.46</td>
</tr>
<tr>
<td></td>
<td>331.10</td>
</tr>
<tr>
<td>Stator $I^2R$ Loss (% of $P_{OUT}$)</td>
<td>7.65</td>
</tr>
<tr>
<td></td>
<td>7.13</td>
</tr>
<tr>
<td>Rotor $I^2R$ Loss (% of $P_{OUT}$)</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>1.54</td>
</tr>
<tr>
<td>Core loss (% of $P_{OUT}$)</td>
<td>2.77</td>
</tr>
<tr>
<td></td>
<td>2.48</td>
</tr>
<tr>
<td>Input Power, $P_{IN}$ (W)</td>
<td>903.65</td>
</tr>
<tr>
<td></td>
<td>957.86</td>
</tr>
<tr>
<td>Average Torque, $T_{AVG}$ (Nm)</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>4.61</td>
</tr>
<tr>
<td>Rated Speed (rpm)</td>
<td>1751</td>
</tr>
<tr>
<td>Output Power, $P_{OUT}$ (W)</td>
<td>788.65</td>
</tr>
<tr>
<td></td>
<td>845.68</td>
</tr>
<tr>
<td>$\eta$ (%)</td>
<td>87.27</td>
</tr>
<tr>
<td></td>
<td>88.28</td>
</tr>
</tbody>
</table>

5.6 Summary

New winding configuration to attain high quality rotating MMF with shorter end-turn length compared to the conventional distributed winding is proposed. The designed multilayer winding holds its advantage over the fractional pitch and concentrated winding counterparts with near sinusoidal air-gap MMF with much lower space harmonic contents. Performance of the designed winding is evaluated for commercial premium efficiency/ IE3 induction machine design and compared with its benchmark. Results from rated speed analysis showed that the proposed winding topology can provide improvement in both torque density and efficiency while maintaining its thermal limits with standard frame size. The next chapter will discuss the application of this novel multi-layer winding in improving the performance of SynRM.
Chapter 6 SynRM Design with Multi-Layer AC Winding

6.1 Introduction

6.2 Designed Winding Configuration
   6.2.1 Harmonic Characteristics of Multilayer Winding
   6.2.2 Model Verification

6.3 $N_{TURN}$ Optimization
   6.3.1 Analysis on Winding Functions
   6.3.2 FEA Based Performance Evaluation
   6.3.3 Optimization Problem
   6.3.2 Optimization Results

6.4 Rotor Configuration Selection

6.5 Design Variables

6.6 Design Optimization

6.7 Electromagnetic Performance Analysis

6.8 Prototype Manufacturing
   6.8.1 Prototype Stator
   6.8.2 Rotor Skewing
   6.8.3 Mechanical Stress Analysis
   6.8.4 Machine Prototype

6.9 Summary
6.1 Introduction

This chapter will cover the design of the multi-layer wound SynRM (MLSynRM) under the given design benchmark. A prototype SynRM design is targeted having the same 36-slot stator configuration as the benchmark induction machine. At first, the multilayer winding configuration for the 36-slot stator configuration is presented. Next, the winding configuration is evaluated in terms of machine winding factor and end-winding resistance versus $N_{TURN}$ and an optimized $N_{TURN}$ configuration for the designed winding is selected from the comparison of machine performance. SynRM with transversely laminated multi-barrier rotor is selected for prototype in this research. Selection of the proper number of barriers from machine reaction function analysis is presented and optimum number of rotor barriers are selected next. Design objective targeting maximization of machine power conversion efficiency subject to average torque, torque ripple and input power factor constraints are stated. An FEA based multi-dimensional optimization of the SynRM rotor is performed next under the given design benchmark and performance comparison between the considered topologies are presented. Next, skewing is performed on the designed rotor to minimize its torque ripple. Moreover, Multiphysics based mechanical stress analysis is performed on the optimized multi-layer SynRM to evaluate its mechanical robustness. A prototype MLSynRM motor has been built to validate the design concept.

6.2 Designed Winding Configuration

The three-phase ML winding has three layers per stator slot. Each phase winding occupies all stator slots in varying number of turns. For a three-phase, 36-slot machine with two pole pair, the designed ML winding has 9 coil groups, $C_1$ to $C_9$, per pole pair in each phase with 5 different number of turns ($N1$ to $N5$) as shown in Figure 6.1. The harmonic winding factors for a phase have been derived for this layout.
6.2.1 Harmonic Characteristics of Multilayer Winding

Slot conductors 6 to 14 of Figure 6.1 carry current inward and the rest carry current in outward direction with the peak phase MMF occurring between slots 5, 6 and 14, 15. The peak of harmonic MMF due to coil $C_x$ is referred to as $\hat{F}_{C_x}^v$ and these MMFs are superimposed in airgap. MMFs from all coil sets are in phase, making $k_{dv}=1$ for each harmonic and $k_{pv}$ in equation 5.2 needs to be accounted for the winding factor. This coil distribution results in 9 coil sets, $C_1$ to $C_9$, centered on its peaks. Winding pitch to pole ratio, $W/\tau_p$ is $(2x - 1)/9$ for $C_x$ ($x = 1, 2, \ldots, 9$). The derivation of stator MMF is presented in equation 6.1 and the modified pitch factor is given by equation 6.2. Here, $N_{ph}$ is the total number turns/phase as defined in equation 6.3, $p$ is the number of pole pairs and $N_x$ is turn-number in coil group $C_x$. These notations are introduced to maintain consistency with the final MMF expressions of equation 5.10 and equation 6.4. Single phase winding excited with a sinusoidal current produces standing MMF wave while three-phase windings excited with balanced three-phase currents produces a travelling MMF wave. The superposition of the three-phases also
introduces a scaling factor of 3/2. The peak of harmonic waves generated by three-phases can be represented as in equation 6.4.

\[
\hat{F}^v_{\nu} = \frac{2}{\nu \pi} \sin \left( \nu \cdot \frac{2x - 1}{9} \cdot \frac{\pi}{2} \right) \cdot N_x \cdot \hat{s} 
\]

\[
\hat{F}^v = \sum_x \hat{F}^v_{\nu x} = \frac{2}{\nu \pi} \sum_x \sin \left( \nu \cdot \frac{2x - 1}{9} \cdot \frac{\pi}{2} \right) \cdot N_x 
\]

\[
\hat{F}^v = \frac{2}{\nu \pi} \cdot \frac{N_{ph}}{p} \cdot \hat{s} \sum_x \sin \left( \nu \cdot \frac{2x - 1}{9} \cdot \frac{\pi}{2} \right) \cdot \frac{N_x \cdot p}{N_{ph}} 
\]

\[
k_{pv} = \frac{p}{N_{ph}} \sum_x N_x \cdot \sin \left( \nu \cdot \frac{2x - 1}{9} \cdot \frac{\pi}{2} \right) 
\]

\[
N_{ph} = p \cdot \sum_x N_x 
\]

\[
\hat{F}^v_{st} = \frac{3}{\nu \pi} \cdot \frac{N_{ph}}{p} \cdot k_{pv} \cdot \hat{s} 
\]

6.2.2 Model Verification

Stator MMF is calculated analytically for 36 slot multilayer winding distribution and its harmonics are calculated using equation 6.1 [106]. In addition, 2D finite element analysis model of the 36 slot multilayer stator is developed with slot-less stator and solid round rotor to consider only the effect of stator winding excluding slot harmonics. Stator MMF waveforms and its harmonic characteristics for both analytical and FEA based models are presented in Figure 6.2 and Figure 6.3 which show good correlation between them. The derived stator MMF is near sinusoidal and the ‘Triplen harmonics’ (\( \nu = 3n \), \( n \) is an integer) of Figure 6.3 will also be absent under balanced three-phase excitation.
Figure 6.2: Analytical stator MMF model verification

Figure 6.3: Analysis on stator MMF harmonics

6.3 $N_{\text{TURN}}$ Optimization

Each phase of the designed winding has distributed turns over stator slots, $N_{\text{TURN}}$ and its combination can be varied according to design requirements. Stator MMF for the ML winding has been defined in equation 6.4 and machine phase resistance has been defined in equation 5.12. Based on equations 5.12 and 6.4, four different winding parameters: Winding factors ($k_{wv}$), effective fundamental MMF coefficient ($F_1 = k_{w1} \times N_t$), end-winding length ($l_{\text{end}}$),
and end-winding resistance \( (R_{\text{end}}) \) have been analyzed with \( N_{\text{TUR}} \) variation. First, these parameters are obtained for the benchmark double layer (DL) distributed wound machine and presented by the straight dotted lines in Figure 6.4 and Figure 6.5.

**Figure 6.4: Winding factor and MMF coefficient variation (v=1)**

**Figure 6.5: End-winding length and resistance variation**

### 6.3.1 Analysis on Winding Functions

Different \( N_{\text{TUR}} \) combinations for ML design have been analyzed against their DL benchmark. Referring to equations 6.1 and 6.2 on subsection 6.2.1, for coil groups \( C_4 \) to \( C_6 \), the fundamental pitch factor, \( k_{p1} \) and \( F_1 \) are larger than its benchmark but with larger end-winding lengths. For shorter pitch coil groups, \( l_{\text{end}} \) and \( R_{\text{end}} \) are low but it also yields low \( k_{w1} \) and \( F_1 \). The optimum winding configuration can be obtained from their design tradeoff
analysis. The $N_{TURN}$ combinations presented in Figure 6.4 and Figure 6.5 have differences in their total number of turns per slot ($N_S$), as a result the trend in $l_{end}$ and $R_{end}$ are different as presented in Figure 6.5. MLSynRM FEA based performance evaluation using these $N_{TURN}$ combinations have been presented in the subsection.

6.3.2 FEA Based Performance Evaluation

$N_{TURN}$ combinations of Figure 6.4, that yields at least 95% $F_1$ compared to the double layer benchmark, have been selected for FEA based evaluation. MLSynRM designs with the same rotor have been evaluated for these different winding configurations under the same stator $I^2R$ loss. Machine average torque ($T_{AVG}$), torque ripple ($T_{Ripple}$), efficiency ($\eta$), and power factor ($PF_{IN}$) have been selected as the performance parameters. Figure 6.6 and Figure 6.7 presents the results from the FEA based analysis, which show that, under the same stator $I^2R$ loss and rotor configuration, machine $T_{AVG}$ and $\eta$ follow similar trend as $F_1$ of its stator MMF that was determined from its analytical model. $PF_{IN}$ for the selected winding configurations are found to be similar. Results show that the analytical stator MMF model can accurately predict MLSynRM performance trend and has been utilized next in optimizing its $N_{TURN}$ combinations under the selected design benchmark.

![Figure 6.6: Average torque and torque ripple variation](image_url)
6.3.3 Optimization Problem

Stator MMF harmonics for the ML winding are characterized by equation 6.4 and this will be utilized to formulate the objective function in terms of THD and the constraint function in terms of stator fundamental MMF. In addition, phase resistance model for the ML winding has been defined in this section by updating equation 5.12. Formulation of the objective and constraint functions to form the problem statement for the multi-dimensional, multi-objective optimization (MDMO) of the ML winding has been discussed next. Finally, results from the optimization process of the ML winding has been presented in section 6.3.5.

![Figure 6.7: Efficiency and power factor variation](image)

To account for the varying number of turns and coil spans among groups, weighted sum of the average conductor lengths have been considered in phase resistance calculation. For distributed ML windings, coefficient accounting for the end-winding length, $k_{cML}^x$ is defined as equation 6.5 [109], and the end-winding length ($l_e^x$) is defined as equation 6.6 with $r_{Slot}$ being defined as the average slot radius.

$$k_{cML}^x = \frac{\pi}{2} \times W = \frac{\pi}{2} \times (2x - 1)$$  \hspace{1cm} (6.5)

$$l_e^x = k_e^x \frac{2\pi r_{Slot}}{Q_s}$$  \hspace{1cm} (6.6)
Following similar argument of section 6.2.1 in calculating pitch factor for coil groups having the same coil spans, the phase resistance formula has been defined as equation 6.7. Here, $A_{cond}$ is the conductor cross sectional area, $L$ is the lamination stack length, $N_{par}$ and $N_{ser}$ are the number of parallel and series coils, respectively, and $\rho$ is the resistivity of the conductor material.

$$R_{phase} = \frac{2 \times \rho \times N_{ser} \times \left[ \sum x (L + l_x^2) \times N_x \times q_x \right]}{A_{cond} \times N_{par}} \tag{6.7}$$

Minimization of $THD$ is selected as one of the objective functions for the ML winding optimization process. From equation 6.4, $THD$ is defined by

$$THD = \sqrt{\left( \frac{\hat{f}_{5st}}{f_{st}} \right)^2 + \left( \frac{\hat{f}_{7st}}{f_{st}} \right)^2 + \left( \frac{\hat{f}_{11st}}{f_{st}} \right)^2 + \left( \frac{\hat{f}_{13st}}{f_{st}} \right)^2 + \ldots} \tag{6.8}$$

Minimization of $R_{phase}$ is considered as another objective. From the 36 slot ML winding configuration with coil groups having 5 different number of turns ($N_1$ to $N_5$), the following relationships are added to maintain constant total number of turns per slot, $N_S$ and reduce the number of design variables.

$$N_1 = N_S - N_4 - N_3 \tag{6.9}$$

$$N_2 = (N_S - N_5)/2 \tag{6.10}$$

$N_3$, $N_4$ and $N_5$ were varied to optimize the $N_{Trun}$ combination for the multilayer winding with a fixed value of $N_S$ yielding 40% slot fill factor. The purpose of a multi-objective optimizer is to find an approximation of the Pareto frontier of the problem. Grid Multi-objective Genetic Algorithm (GMGA), which finds the nearest points of the Pareto frontier on a rounded grid based on genetic algorithm (GA), was utilized to optimize this multi-objective problem. GMGA belongs to the stochastic algorithm family with three main attributes; population size, maximum
generation (iteration), and parameter rounding mode. Having three independent parameters, \((N_3, N_4 \text{ and } N_5)\) the GMGA based optimization was performed with a population size of 40 and maximum generation of 500 with an integer rounding of its \(N_{\text{Turn}}\) parameters. The optimization problem can be summarized as below;

**Objectives:**

\[
\begin{align*}
\text{Min}(THD)|_{5\%} &= f(N_x)|_{x=3,4,5} \\
\text{Min}(R_{\text{Phase}})|_{2\Omega} &= f(N_x)|_{x=3,4,5}
\end{align*}
\]

Subject to the following constraints:

- Fundamental MMF, \(C_{F_{st}}\): \(\hat{F}_{st}^1 \geq 0.79\) (pu) (floor constraint)

- and End-winding, \(C_{R_{\text{End}}}\): \(R_{\text{End}} \leq 1.6\ \Omega\) (ceiling constraint)

The combined mono-objective cost function based on the problem formulation is defined by equation 6.11.

\[
\operatorname{MonoObj} = \text{Goal}_{\text{Min}} \left\{ \text{Min}(THD) + \text{Min}(R_{\text{Phase}}) + \delta_p \times \left( \max\left[0, C_{F_{st}}\right] + \max\left[0, C_{R_{End}}\right] \right) \right\}
\] (6.11)

Here, \(\text{Min}(THD), \text{Min}(R_{\text{Phase}})\) are objective functions, and \(C_{F_{st}}\) and \(C_{R_{End}}\) are the constraint functions defined to achieve at-least 5\% higher fundamental MMF than the benchmark DL winding with at-most the same end winding resistance, \(R_{\text{End}}\), and \(\delta_p\) is the coefficient of penalty whose default value was set at 500. Minimization of this cost function is performed in the optimization process.

### 6.3.4 Optimization Results

Pareto-optimal set for the 36 slot ML winding multi-objective optimization problem is presented in Figure 6.8. The position of the benchmark DL winding and the selected ML
winding design on the Pareto chart are presented by orange and green circles, respectively. Moreover, the performance parameters of the optimal ML winding have been presented in Figure 6.4 to Figure 6.7 as combination number 7 to show the relative advantages with this optimized ML winding. Compared to the benchmark DL winding the optimized ML winding has 6.5% higher fundamental MMF coefficient, \( \hat{f}_{st1} \) with similar \( R_{End} \). The optimized winding has a THD value of 7.8% that is 9.6% lower compared to its double layer winding benchmark. Results, from FEA based analysis in Figure 6.6 and Figure 6.7 shows that the optimized winding has yielded improved \( T_{AVG} \) and \( \eta \) compared to others. This optimum winding configuration will utilized to build the ML winding prototype.

![Pareto chart of the 36 slot ML winding optimization.](image)

### 6.4 Rotor Configuration Selection

The ML winding design has been optimized for a four pole, 36 slot stator. Next, its transversely laminated multi-barrier rotor design has been optimized. The initial shape of the rotor has been selected for minimum torque ripple. The selection objective is to find a rotor shape that will minimize the interactions between stator and rotor MMF harmonics. Analysis presented in [54] showed that for a 36 slot, 4 pole stator (\( ns = 18 \) slots/pole pair) the optimum number of separation points (\( n_r \)) per pole to reduce rotor MMF and stator ‘slot’ harmonic interaction is \( ns \pm 4 \) or 14 and 22 with 1 virtual separation point along the q-axis. Three barriers
per pole rotor structure are selected for manufacturing ease. Qualitative design of the initial rotor shape, rotor MMF distribution, and its harmonic spectrum are presented in Figure 6.9.

![Image of a rotor design](image)

**Figure 6.9:** (a) 3 barrier rotor design with \( n_r = 14 \) (7 separation points per half pole), (b) rotor MMF distribution and (c) normalized harmonic spectrum.

### 6.5 Design Variables

Rotor geometry is parameterized with the following design variables: angular position of barrier ‘\( x \)’ \( (x = 1, 2, 3) \) from \( d \)-axis along rotor outer radius, \( \theta_{dx} \); position of barrier ‘\( x \)’ along \( q \)-axis, \( D_x \); top width of barrier ‘\( x \)’, \( W_{tx} \); mid width of barrier ‘\( x \)’, \( W_{mx} \); arc angle at barrier top, \( \theta_{tx} \); and two arc angle of the barrier ‘\( x \)’, \( \theta_{x1} \) and \( \theta_{x2} \); central bridge thickness, \( t_{mx} \) \( (x = 1, 2) \); and radial bridge thickness, \( t_x \). These parameters are qualitatively illustrated for \( x = 3 \) in Figure 6.10, and their ranges are given in Table 6.1. Along with these design parameters, the current phase angle \( \gamma \) is also considered as input variable in each design iteration through an FEA based multi-dimensional optimization targeting maximization of power conversion efficiency subject to torque ripple and input power factor constraints. FEA analysis tool FLUX 2D is coupled with the optimization tool GOT-It for the multi-dimensional optimization. Details of the optimization method is presented in the next subsection.
Figure 6.10: Parameterized multi-barrier rotor model

Table 6.1: Selected parameter ranges in FEA based optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min Value</th>
<th>Max Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{mx}$</td>
<td>0.1</td>
<td>0.5</td>
<td>p.u.</td>
</tr>
<tr>
<td>$W_{tx}$</td>
<td>0.05</td>
<td>0.5</td>
<td>p.u.</td>
</tr>
<tr>
<td>$D_x$</td>
<td>3.5+(3-x)×10</td>
<td>14.5+(3-x)×10</td>
<td>mm</td>
</tr>
<tr>
<td>$t_x$</td>
<td>0.5</td>
<td>1</td>
<td>mm</td>
</tr>
<tr>
<td>$t_{mx}$</td>
<td>0.5</td>
<td>1</td>
<td>mm</td>
</tr>
<tr>
<td>$\theta_{x1}$, $\theta_{x1}$</td>
<td>5</td>
<td>80</td>
<td>degrees</td>
</tr>
<tr>
<td>$\theta_{tx}$</td>
<td>10</td>
<td>75</td>
<td>degrees</td>
</tr>
<tr>
<td>$\theta_{bx}$</td>
<td>5</td>
<td>45</td>
<td>degrees</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>40</td>
<td>50</td>
<td>degrees</td>
</tr>
</tbody>
</table>

6.6 Design Optimization

Genetic algorithm (GA), a popular meta-heuristic optimization class, is selected to optimize the multi-barrier rotor design. GA randomly selects individuals from the current population in each iteration and uses them to produce the next generation using crossover and
mutation. Over successive generations, the population "evolves" toward an optimal solution. Input parameters for the optimization problem are listed in Table 6.1. For each FEA evaluation, one machine pole model is utilized with dynamic simulation over one electrical cycle for the same stator $I^2R$ loss. Input and optimization functions are defined by equations 6.12 to 6.16 with $T$ representing electromagnetic torque waveform obtained from FEA in each iteration.

\[
P_{IN} = \text{Mean}\{U(I_A) \times I(A) + U(I_B) \times I(B) + U(I_C) \times I(C)\}
\]

\[
P_{OUT} = T_{AVG} \times \omega_{Rad}
\]

\[
\eta = (P_{OUT}/P_{IN}) \times 100
\]

\[
PF_{IN} = P_{IN}/(3 \times V_{RMS} \times I_{RMS})
\]

\[
T_{Ripple} = \{(T_{MAX} - T_{MIN})/T_{AVG}\} \times 100
\]

Optimization problem:

\[
\max(\eta)^{95.0\%}_{\omega=\omega_{\text{Rated}}} = f(D_x, \theta_{xi}, \theta_{tx}, \theta_{bx}, W_{mx}, W_{tx}, t_x, \gamma)_{x=1,2,3}
\]

Subject to the following constraints,

Torque ripple, $C_{Rip}$ : $T_{Ripple} \leq 5\%$ (ceiling constraint)

Power factor, $C_{PF}$ : $PF_{IN} \geq 0.7$ (floor constraint)

Genetic algorithm (GA) with population size of 50 and maximum iteration limit of 600 has been used for optimization. Results from the optimization steps are presented in Figure 6.11. For the designed synchronous reluctance motor, efficiency improvement is possible with improved torque per Cu-loss. This can be achieved by improving its saliency which improves both torque capability and power factor as shown in equation 6.17 and 6.18. Here, $L_{ds}$ and $L_{qs}$ are $d$ and $q$ axes inductances, respectively, and $\varepsilon$ is the angle between $d - q$ axes currents.
\[ T = \frac{3}{2} P (L_{ds} - L_{qs}) I_{ds} I_{qs} \]  
\[ PF_{IN} = \frac{(L_{ds} - L_{qs}) \cos \epsilon \sin \epsilon}{\sqrt{(L_{ds} \cos \epsilon)^2 + (L_{qs} \sin \epsilon)^2}} \]

Figure 6.11: Results from FLUX-2D and GOT-It coupled optimization using GA.

Since, both \( \eta \) and \( PF_{IN} \) are assertive functions of rotor saliency their variations over different optimization ranks follow similar trends. Based on the objective function and its constraints, the cost function is developed as

\[ MonoObj = Goal_{Min} \{ \max(\eta) + \delta_p \times (\max[0, C_{PF}] + \max[0, C_{Rip}]) \} \]  

Here, \( \max(\eta) \), \( C_{PF} \), and \( C_{Rip} \) are objective and constraint functions defined in the problem statement. Minimization of this cost function is performed in the optimization process (‘MonoObj’). For the same value of \( \eta \), the optimization process improves \( T_{Ripple} \) to minimize
the overall cost. After 100 iterations of the minimization of ‘MonoObj’ ceases, and hence, the optimization process was stopped after 120th iteration. The optimized multi-barrier rotor has minimized its ‘MonoObj’ cost by 75.02%.

The optimized rotor design is presented in Figure 6.12 which has the value of \( n_r \) as 22 compared to its initial design with \( n_r \) value of 14. This increment in rotor peripheral separation points is instrumental towards minimization of torque ripple [54] and results from the optimization analysis in this research agrees well with the existing literature.

![Figure 6.12](image)

Figure 6.12: (a) Finalized rotor design with \( n_r = 22 \) (11 separation points per half pole), (b) analytical MMF distribution and (c) normalized harmonic spectrum.

6.7 Electromagnetic Performance Analysis

Dynamic FEA simulation with the optimized MLSynRM design is performed at the rated frequency of 60 Hz. Moreover, performances of benchmark SCIM and a conventional DLSynRM are also evaluated and Table 6.2 presents their rated performance comparison under the same stator \( I^2R \) loss of 60 Watts. These machines have the same stator core, air-gap length, rotor outer diameter, and iron stack lengths. Both DL and MLSynRMs have the same iron core
geometry to directly evaluate the effect of their windings. All these machines were considered non-skewed axially, and friction, windage losses were not included in the efficiency calculations. Performance analysis results are presented in Table 6.2 and Figure 6.13.

Table 6.2: Rated performance analysis of the considered topologies

<table>
<thead>
<tr>
<th>Machine</th>
<th>SCIM</th>
<th>DLSynRM</th>
<th>MLSynRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{I^2R-ST}$ (% of $P_{out}$)</td>
<td>7.65</td>
<td>7.23</td>
<td>6.89</td>
</tr>
<tr>
<td>$P_{I^2R-RT}$ (% of $P_{out}$)</td>
<td>3.61</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_{Core}$ (% of $P_{out}$)</td>
<td>3.37</td>
<td>2.80</td>
<td>3.04</td>
</tr>
<tr>
<td>$P_{Total}$ (% of $P_{out}$)</td>
<td>14.58</td>
<td>10.04</td>
<td>9.93</td>
</tr>
<tr>
<td>$T_{AVG}$ (Nm)</td>
<td>4.30</td>
<td>4.41</td>
<td>4.63</td>
</tr>
<tr>
<td>$P_{out}$ (W)</td>
<td>788.65</td>
<td>831.83</td>
<td>872.73</td>
</tr>
<tr>
<td>$P_{in}$ (W)</td>
<td>903.65</td>
<td>919.08</td>
<td>959.42</td>
</tr>
<tr>
<td>$\eta$ (%)</td>
<td>87.27</td>
<td>90.51</td>
<td>90.96</td>
</tr>
<tr>
<td>$PF_{IN}$ ($pu$)</td>
<td>0.75</td>
<td>0.67</td>
<td>0.70</td>
</tr>
<tr>
<td>$T_{Ripple}$ (%)</td>
<td>7.82</td>
<td>12.83</td>
<td>8.08</td>
</tr>
</tbody>
</table>

Figure 6.13: EM Torque profiles from FEA at rated condition.

Results presented in Table 6.2 show that the designed MLSynRM, with higher fundamental winding factor than the benchmark double layer winding, can yield higher torque output for the same stator $I^2R$ loss. Consequently, the designed MLSynRM has higher torque per unit
copper loss compared to other machines. The designed MLSynRM with higher $\hat{F}_{st}$ results in higher induced voltage in phase windings causing its core loss percentage to be slightly higher than the DLSynRM. However, the per unit total loss contribution is lower for MLSynRM compared to others and it can attain 3.69% higher efficiency than SCIM.

Application of the ML winding in SynRM design shows that the designed MLSynRM yields much lower torque ripple compared to DLSynRM. With more sinusoidal stator MMF, the ML winding reduces the ‘belt’ harmonics significantly, which in turn yields a much smoother torque profile [54] as presented in Figure 6.13. Moreover, with reduction in space harmonics their contribution in leakage reactance reduces and the ML wound SynRM can achieve higher power factor compared to conventional double layer winding. Based on the above analysis, ML winding can deliver superior performance for SynRMs. Next, a prototype MLSynRM has been built based on the optimized design to validate the design concept and simulation results.

6.8 Prototype Manufacturing

This section discusses the manufactured prototype of MLSynRM. ML winding is built on the same stator geometry as its benchmark SCIM. Skewing is applied on electromagnetically optimized rotor design to further reduce its torque ripple without sacrificing too much torque/Cu loss. Multiphysics based structural FEA analysis has also been performed on the optimized design before manufacturing its prototype for hardware testing.

6.8.1 Prototype Stator

The prototype stator and its phase winding configuration are presented in Figure 6.14. Fabrication process of the ML winding involves the same construction stages as its DL wound counterpart without any additional tooling cost or complexity. Moreover, for typical operating voltage range (240V/480V), phase separators are made of thin insulation papers and the prototype ML winding with one additional phase separator has attained similar slot fill factor as its benchmark. Resistance measurements in Table 6.3 shows that the prototype ML winding
has balanced coil resistances and the measured resistance matches well with its calculated value using equation 5.12.

![Image](image-url)

Figure 6.14: (a), (b) Prototype ML wound stator and (c) winding configuration.

<table>
<thead>
<tr>
<th>Start</th>
<th>Finish</th>
<th>R (Ω)</th>
<th>Start</th>
<th>Finish</th>
<th>R (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>4.6</td>
<td>3</td>
<td>6</td>
<td>4.5</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>4.6</td>
<td>9</td>
<td>12</td>
<td>4.5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>4.5</td>
<td>Eq. R (Ω) Measured</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>11</td>
<td>4.5</td>
<td>Eq. R (Ω) Calculated</td>
<td>2.14</td>
<td></td>
</tr>
</tbody>
</table>

**6.8.2 Rotor Skewing**

Skewing is effective to minimize torque ripple of sinusoidal AC machines. However, machine average torque also reduces with it [129]. To obtain an optimum skew angle, $\theta_{Skew}$ MLSynRM rotor is skewed up to 15° mechanical in one-degree step in FEA and these results are presented in Figure 6.15. The minimum torque ripple is obtained at $\theta_{Skew} = \theta_{Slot}$ (10°), which agrees with existing literature [129]. However, this reduces average torque by 4% with
3.7% torque ripple. $\theta_{\text{skew}}$ for the prototype motor is selected as 4° instead as it reduces torque ripple to 4.83% ($< 5\%$) with only 0.7% reduction in average torque.

![Graph showing effect of skew angle on torque and ripple](image)

**Figure 6.15: Effect of rotor skewing on the designed MLSynRM**

### 6.8.3 Mechanical Stress Analysis

Machine structural analysis is conducted in ANSYS workbench by coupling electromagnetic tool Maxwell 2D to ANSYS mechanical. First, electromagnetic simulation of the designed motor is performed at rated condition and the radial forces ($F_{\text{Rad}}$) are imported to ANSYS mechanical. Next, structural FEA is performed considering both electromagnetic ($F_{\text{Rad}}$) and mechanical (centrifugal, $F_{\text{cen}}$) forces together in its Multiphysics evaluation. Equivalent (von-Mises) stress ($\tau$) and total deformation ($\delta$) results from structural analysis are presented in Figure 6.16 and Table 6.4. Analysis on these results show $F_{\text{cen}}$ to be dominant over $F_{\text{Rad}}$. Moreover, stress analysis is performed at four times rated speed (7200 rpm) to evaluate the impact of forces at extreme condition and results are presented in Table 6.4. The mechanical stresses are found to be well within the structural limit of the lamination material, 29 gauge M19-C5 electrical steel (yield strength = 350 MPa), with acceptable deformation in both rated and extreme operating scenarios. The prototype motor is built based on this design.
Figure 6.16: Stress and deformation analysis performed on the designed rotor.

Table 6.4: Results from stress and deformation analysis in ANSYS

<table>
<thead>
<tr>
<th>Rated Condition</th>
<th>(\omega) (rpm)</th>
<th>(F_{Rad}) (N)</th>
<th>(F_{Cen}) (N)</th>
<th>Peak (\tau) (MPa)</th>
<th>Peak (\delta) ((\mu)m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1800</td>
<td>--</td>
<td>2196.1</td>
<td>7.18</td>
<td>0.464</td>
<td></td>
</tr>
<tr>
<td>1800</td>
<td>632.1 (peak)</td>
<td>2196.1</td>
<td>7.20</td>
<td>0.467</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Extreme Condition</th>
<th>(\omega) (rpm)</th>
<th>(F_{Rad}) (N)</th>
<th>(F_{Cen}) (N)</th>
<th>Peak (\tau) (MPa)</th>
<th>Peak (\delta) ((\mu)m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7200</td>
<td>632.1 (peak)</td>
<td>35137.6</td>
<td>115.15</td>
<td>7.43</td>
<td></td>
</tr>
</tbody>
</table>

6.8.4 Machine Prototype

The manufactured stator and rotor are presented in Figure 6.14 and Figure 6.17, respectively. The prototype motor has optimized winding from section 6.2. Moreover, it has the multi-barrier design optimized in section 6.5 with an applied skew angle of \(4^\circ\), as seen in Figure 6.17(c), according to the discussion in subsection 6.7.2. MLSynRM \(d - q\) modelling, controller development, and experimental evaluation under vector control has been discussed in chapter 8. Chapter 9 will discuss the experimental loss separation, thermal steady state performance and efficiency comparison of the benchmark induction machine (BMIM), multilayer induction machine (MLIM) and MLSynRM prototypes.
6.9 Summary

Design of a synchronous reluctance motor using the multilayer AC winding is discussed in this chapter. The design MLSynRM has the same stator geometry as the benchmark induction motor. First, the multilayer winding configuration is optimized for the 36 slot stator. Analytical stator MMF model is utilized for evaluating different $N_{\text{TURN}}$ combinations and results are verified with FEA. Multi-barrier SynRM rotor is selected with three-barrier per pole configuration and its design is optimized using FEA based multi-dimensional optimization. Mechanical stress and rotor skewing analysis are performed before making the prototype. Finally, a prototype MLSynRM motor is built to validate the design concept and implement $d-q$ vector control.
Chapter 7 SynRM Controller Development

7.1 Introduction
7.2 SynRM $d-q$ Modeling
7.3 Determination of $d-q$ Axis Inductances
  7.3.1 FEA Based Analysis
  7.3.2 Experimental Validation
7.4 Controller Implementation in MATLAB Simulink
  7.4.1 Current Controller with LUT Based Machine Model
  7.4.2 Current Controller with FEA Based Machine Model
  7.4.3 Closed Loop Speed Controller in MATLAB Simulink
7.5 Five hp Dyno Test-bed Development
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  7.6.1 $d-q$ Current ($i_d, i_q$) Control
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  7.6.3 Test Results with Speed Controller
7.7 Summary
7.1 Introduction

Salient rotor ‘synced’ with the stator rotating MMF produces torque to realize SynRM operation. Torque production concept of SynRM is explained in section 1.6.2. Considering a continuously rotating reference frame across rotor $d$-axis, the sinusoidal voltages and currents of a three-phase SynRM are converted into DC quantities in synchronously rotating reference frame. This makes modelling and analysis of SynRM much easier, similar to the DC machines.

This chapter discusses the direct and quadrature axis ($d – q$) modelling of SynRM in explaining its performance parameters in terms of $d – q$ axis inductances ($L_d, L_q$) and currents ($i_d, i_q$). FEA based method to obtain $L_d, L_q$ are discussed to formulate the look-up-table (LUT) based SynRM model. Next, results from the FEA analysis are verified with experimental measurements. SynRM vector control is discussed by developing machine model and controller in Matlab Simulink. Results from both LUT and FEA based machine models are presented for comparison and results are obtained for both torque and speed controls.

A five hp dyno test-bed has been designed and developed for the experimental analysis. Laser alignment of the drive shaft was performed to minimize mechanical friction, windage and improve operation safety. Experimental analysis on the developed MLSynRM prototype is performed next under vector control. MLSynRM closed loop torque control, torque versus $i_d, i_q$ analysis, maximum torque per ampere (MTPA) analysis, and speed control are performed and simulation results are experimentally verified.

7.2 SynRM d – q Modeling

Relationship between $dq$ variables and three-phase $abc$ variables are presented in Figure 7.1(a). Here, $\theta$ is the angle of transformation, which for a synchronously rotating $dq$ reference frame (at speed $\omega = \omega_e$) can be written as;

$$\theta = \int_0^t \omega_e(\tau)d\tau + \theta_0$$  \hspace{1cm} (7.1)
The mathematical relationship between $dq$ and $abc$ variables can be written as:

\[
f_q = \frac{2}{3} \left[ f_a \cos \theta + f_b \cos(\theta - 120^\circ) + f_c \cos(\theta + 120^\circ) \right]
\]

\[
f_d = \frac{2}{3} \left[ f_a \sin \theta + f_b \sin(\theta - 120^\circ) + f_c \sin(\theta + 120^\circ) \right]
\]

The designed MLSynRM is a four-pole machine as presented in Figure 7.1(b). Here, rotor $d$-axis is initially (at $t=0$) aligned with stator $as$-axis. To maintain consistency with equation 7.2, the relationship between rotor mechanical position ($\theta_r$) and $\theta$ is defined as:

\[
\theta = \frac{p}{2} \cdot \theta_r + 90
\]

Having a sinusoidal stator with no rotor damper winding, the simplified Park’s equations of 1.8 to 1.11 and steady state vector diagram of Figure 1.7 are applicable to MLSynRM. Moreover, the torque and power factor expressions of equation 6.10 and 6.11 defines its performance parameters.
7.3 Determination of d – q Axis Inductances

Transferring from abc to dq domain is helpful for modeling AC machines as it simplifies the machine model and controller design. The following two subsections discuss $L_d$, $L_q$ extraction from FEA and development of the LUT based MLSynRM model as well as the verification of simulation results with experiments.

7.3.1 FEA Based Analysis

Vector control method to determine dq axis inductances are adopted in this thesis. First, the input current amplitude ($I_s$) and MMF angle ($\epsilon$) are converted into $i_d$ and $i_q$ using equation 7.4. By changing rotor initial position in FEA, its d-axis is aligned with stator as-axis. Next, abc variables are obtained from dq variables using equation 7.5 and are presented in Figure 7.2(a). Three-phase flux-linkages ($\lambda_a, \lambda_b, \lambda_c$) are obtained using flux-sensors in FEA and $\lambda_d, \lambda_q$ are obtained from them using equation 7.2 and are presented in Figure 7.2(b). Finally, $L_d, L_q$ are determined from $\lambda_d, \lambda_q$ using equation 7.6.

Figure 7.2: abc and dq variables from FEA (a) phase currents and (b) flux-linkages

$$i_d = I_s \cos \epsilon; \quad i_q = I_s \sin \epsilon \quad \text{(7.4)}$$

$$i_a = i_q \cos \theta + i_d \sin \theta$$

$$i_b = i_q \cos(\theta - 2\pi/3) + i_d \sin(\theta - 2\pi/3)$$

$$i_c = i_q \cos(\theta + 2\pi/3) + i_d \sin(\theta + 2\pi/3) \quad \text{(7.5)}$$
\[ L_d = \lambda_d/i_d; \quad L_q = \lambda_q/i_q \]  \hspace{1cm} (7.6)

Both \( L_d \) and \( L_q \) varies with variations of both \( i_d \) and \( i_q \), which shows cross coupling between \( d \) and \( q \) axis. As a results, 3D LUTs are obtained for both \( L_d \) and \( L_q \) and the results obtained from the FEA analysis are presented in Figure 7.3.

![Figure 7.3: Calculated dq inductances from FEA (a) \( L_d \) and (b) \( L_q \)](image)

### 7.3.2 Experimental Validation

Vector control method to determine \( dq \) axis inductances have been studied in the literatures [130], [131]. The system level block diagram for experimental \( L_d, L_q \) measurement is presented in Figure 7.4. In this test, the machine is rotated at a constant speed which is lower than its rated speed through vector control. \( I_s \) is varied up to 1.25 times the rated current while \( \epsilon \) is set as either 0° (only \( i_d \)) or 90° (only \( i_q \)). Phase current is measured with current probe while the line-to-neutral voltage is measured with differential probe. Simultaneous rotor position is also recorded and \( d - q \) voltage and current values are calculated using the 0° and 90° crossing of the as-axis voltage and current during post-processing. \( dq \) flux linkages and inductances are then calculated using equation 7.7 and 8.8.

\[ V_d = I_dR - \omega_e \times \varphi_q; \quad V_q = I_qR + \omega_e \times \varphi_d \]  \hspace{1cm} (7.7)

\[ L_d = \frac{V_q}{\omega_e i_d} \quad i_q = 0; \quad L_q = \frac{V_d}{\omega_e i_q} \quad i_d = 0 \]  \hspace{1cm} (7.8)
Experimental setup for the $L_d, L_q$ measurement is presented in Figure 7.5(a) and the measured rotor position from encoder feedback, phase current, and phase voltage are presented in Figure 7.5(b). The fundamental phase voltage and current from the measured data is plotted in Figure 7.6(a) and the resultant $L_d, L_q$ data are presented in Figure 7.6(b). Figure 7.6(b) also presents $L_d, L_q$ values from FEA under the same phase currents. FEA based prediction agrees well with the experimental measurements. $d$-axis inductance measurement is slightly off (≤ 5% in most cases) which can be relative to the lamination fabrication tolerances of flux guides and the uncertain material $B - H$ characteristics at saturated conditions.

Figure 7.4: Block diagram for experimental $L_d, L_q$ measurement

Figure 7.5: Experimental $L_d, L_q$ measurement (a) test setup and (b) test data
Figure 7.6: Results from $L_d$, $L_q$ measurement (a) fundamental voltage and current; (b) Measured and predicted $L_d$, $L_q$ values

7.4 Controller Implementation in MATLAB Simulink

The controller block diagram for MLSynRM vector control is presented in Figure 7.7. Here the inner loop is for current control and the outer loop is for speed control. Park and inverse park transformations are employed for $dq$ to $abc$ and $abc$ to $dq$ conversion, respectively using rotor position feedback. The MTPA look-up-table holds the required $i_d$, $i_q$ commands for a certain torque command. Both speed and current PI blocks are implemented for the feedback control loops to generate the torque and $dq$ voltage command, respectively. Space vector pulse width modulation (SVPWM) technique is utilized for the gate pulse generation. The controller was developed in Matlab Simulink. For the machine model, both LUT based model as in section 7.2 and FEA based models are utilized and their performances are compared. Both torque and speed control of MLSynRM is investigated in this study and the results are discussed in next three subsections.

7.4.1 Current Controller with LUT Based Machine Model

The first controller simulation model includes both motor controller and machine analytical model developed in Matlab Simulink. The machine model is developed according to the discussion on section 1.6.2 and 7.1.2. MLSynRM electromagnetic performance under vector control is investigated for constant speed, current control mode. Stator $dq$ vector control with
optimum MTPA operating points \( (i_d^*, i_q^*) \) has been applied to investigate current controller performance. The PI controller gains are adjusted to achieve good regulation (minimal overshoot and steady state error) within the voltage constraints.

\[
\begin{align*}
V(a,b,c) &\rightarrow \text{PWM Carrier} \\
\text{PWM Pulse Gen.} &\rightarrow T_a, T_b, T_c \\
&\text{Inverter} \\
&\text{Vdc} \\
&\text{MLSynRM} \\
&\theta_e \\
&\text{Current sensors} \\
&\text{Park} \\
&\text{Inverse Park} \\
&i_d, i_q \\
&v_d, v_q^* \\
&i_d^*, i_q^* \\
&\text{MTPA LUT} \\
&\omega_r \\
&\omega_r^* \\
&\text{Speed PI} \\
&\omega_r \\
&\text{Speed Calc.} \\
&\text{Position sensor} \\
&\text{MLSynRM} \\
&\theta_e \\
&\text{Current} 
\end{align*}
\]

**Figure 7.7:** General control configuration of MLSynRM

Results from current controller simulation with machine analytical model are presented in Figure 7.8. \( dq \) current commands are generated from the torque command using an MTPA LUT. Results in Figure 7.8 are presented for step changes in torque command to rated, 37.5% rated, and 75% rated torque. Results are obtained for rated operating speed of 1800 rpm with 50 \( \mu s \) Simulink trigger block being used to simulated 20 kHz switching frequency. Different gains were applied for \( d \) and \( q \) axis PI controllers (\( k_{pd} = 300, k_{id} = 250; k_{pq} = 200, k_{iq} = 400 \)). Results from Figure 7.8(b) show that phase currents are good sinusoids. Moreover, the \( dq \) current and electromagnetic torque achieves good regulations with a high frequency ripple present from their SVPWM based switching.
Figure 7.8: Current controller response with LUT based machine model with step changes in torque command (a) $dq$ currents (b) $abc$ currents (c) command and developed torque
7.4.2 Current Controller with FEA Based Machine Model

FEA based MLSynRM model is included next in the current control simulation to validate the performance of the LUT based model. FEA based machine models are considered to be accurate for controller development including the inverter ratings and DC link voltage constraints [77]. However, simulation of FEA based models are very time consuming as the required solution of the meshed FEA structure in each time step. Simulation of MLSynRM current controller is performed with both FEA and LUT based model to validate the LUT based one. Next, the LUT based model has been used in section 7.4.3 to perform speed control of MLSynRM as the speed loop has a larger time constant than the current loop.

Results from the FEA and LUT based models with current control under the same torque command is presented Figure 7.9. Results are obtained for the same PI gains and control commands in \(dq\) current controllers. Results from Figure 7.9 show that, performance of both LUT based and FEA MLSynRM models are similar. With detailed machine geometry and material property, FEA based model can capture the torque ripple variation from PWM switching. Moreover, controller dynamic response to step change in command and steady state performances agrees well for both LUT based and FEA based models. In terms of the simulation final time and resolution, LUT based model requires 21 seconds and FEA based model requires 47 hours to solve. This large difference in simulation time makes the LUT based simulation more justified for controller and machine coupled simulation. As a results, for the speed control simulation, which has much larger final simulation time, MLSynRM LUT based model is considered.

7.4.3 Closed Loop Speed Controller in MATLAB Simulink

Speed PI controller is implemented as the outer loop of the current PI controllers. Speed PI generates the torque command which determines the required \(dq\) currents through MTPA LUT. The controller responses are evaluated for change in load torque commands while the speed PI regulates the rated speed at 1800 rpm. Results from closed loop speed control are presented in Figure 7.10.
Figure 7.9: MLSynRM controller response comparison LUT based vs. FEA (a), (b) $dq$ currents; (c), (d) $abc$ currents; (e), (f) command and electromagnetic torque
Load torque is varied to zero, rated, 43.75% rated, and 62.5% rated while the speed controller maintains a constant speed of 1800 rpm. Results from Figure 7.10 show that the designed speed and current PI regulators can maintain the required torque and speed under the given load torque variation.

Figure 7.10: Speed and current controller response under step changes in load torque (a) actual speed and torque with reference (b) per-unit dq currents (c) per-unit abc currents
7.5 Five hp Dyno Test-bed Development

A test-bed with five hp dyno machine has been developed to experimentally validate the performance of the designed machines. Three machine prototypes (a) segmented rotor SRM (SSRM), (b) multilayer induction motor (MLIM) and (c) multilayer synchronous reluctance motor (MLSynRM) have been designed and developed in this dissertation under the selected design specification of a double layer benchmark induction motor (BMIM). The rated torque requirement is 4.0 Nm with a rated speed of around 1800 rpm. Components of the 5 hp dyno test-bed have been selected based on these requirements.

The selected dyno machine is a three-phase, four-pole, 60 Hz induction motor (CEM3218T motor from Baldor electric company) rated at 5 hp with rated speed of 1750 rpm. This makes the test-bed enable to test electric motors up to 4 hp (75% of rated). The selected torque transducer is a rotating high precision digital torque meter (MCTR 48201V of S. Himmelstein and company) that is rated at 5.65 Nm and measure up to 8000 rpm. The dyno motor was driven with a three-phase, ABB ACS800 four-quadrant AC drive supplied from 208V, three-phase AC supply. Shaft couplings, base mounting plates to adjust shaft heights and power/signal cables were the other required components for the dyno test-bed. Mechanical alignment of the drive shaft (dyno machine-torque transducer-test motor) with the assembled components are performed using a laser shaft alignment tool (shaft hog from Vibralign) to minimize mechanical friction due to misalignment and improve operation safety of the drive shaft.

Figure 7.11 presents the photos of the developed test-bed. Additional circuit breakers are installed between the drives and motors for overcurrent protection. A protective cage is installed over the couplings at the center of the drive shafts to provide protections from high speed fly-offs. Torque transducer data (torque, speed) are utilized for mechanical output power and power analyzer data were utilized for the input electrical power and power factor. To maintain consistency of testing environment among different prototypes, only the test motor part is changed in each experiments while maintaining the other components to be the same.
Figure 7.11: Developed 5 hp dyno test-bed for experimental evaluation

7.6 Controller Experimental Evaluation

The SynRM vector control developed in section 7.3 has been experimentally evaluated. Moreover, performance and response of the prototype MLSynRM under different control conditions are evaluated and compared against FEA. To design the motor controller and implement it with a drive circuit, Texas instrument’s high voltage (up to 350V DC) motor control kit equipped with TMS320F28035 fixed point DSP is utilized. Block diagram of the experimental setup is presented in Figure 7.12. Experimental $L_d$, $L_q$ measurements under vector control is discussed in section 7.2.2. $dq$ current control, torque versus $i_d$, $i_q$ map, MTPA analysis, and speed control tests are performed on the prototype MLSynRM. Experimental results from this section verify the vector controller and evaluate MLSynRM design concept under different control conditions.
**Figure 7.12:** Block diagram of experimental setup for MLSynRM vector control

### 7.6.1 $d-q$ Current ($i_d$, $i_q$) Control

Under vector control, first the motor controller is tested with the inner current control loop of Figure 7.7. The dyno motor is rotated at a constant speed and the test motor were given different current commands. Figure 7.13 presents results from current regulation tests under a single step change in current commands. Here, the yellow, magenta, green, and turquoise colored graph represents the rotor position (degree electrical), reference $dq$ current, actual $dq$ current, and $a$-phase current, respectively. $d$-axis current is changed from 0.5A to 3.5A in Figure 7.13(a), the $q$-axis current is changed from 3.5A to 0.5A in Figure 7.13(b), $d$-axis current is changed to 3.5A to 2.0A to 0.5A in Figure 7.13(c), and the $q$-axis current is changed to 3.5A to 2.0A to 0.5A in Figure 7.13(d). Results in Figure 7.13 show good current regulations under step changes in the $dq$ current commands.

### 7.6.2 Torque versus $i_d$, $i_q$ Map and MTPA Analysis

After achieving $dq$ current regulation through vector control, MLSynRM characteristics under $i_d$, $i_q$ variation is obtained. Two standard characteristics of SynRM motors, (a) Torque versus $i_d$, $i_q$ map and (b) MTPA are obtained through a set of experiments. For the average
torque ($T_{AVG}$) vs. $i_d, i_q$ map, $i_d, i_q$ are varied directly and for the MTPA analysis, MMF angle $\epsilon$ is varied from $0^\circ$ to $90^\circ$ under certain values of $i_s$.

Figure 7.13: $dq$ current control with step changes in commands (a), (c) $i_d$ and (b), (d) $i_q$

Experimental $T_{AVG}$(Nm) from MLSynRM tests under different current commands are presented in Figure 7.14 by solid lines. Figure 7.14(a) presents $T_{AVG}$ vs. $i_d, i_q$ maps and Figure 7.14(b) presents the MTPA results. Moreover, FEA analysis on MLSynRM are performed using the same control conditions and the results are presented through dotted lines.

Experimental and FEA results presented in Figure 7.14 show good agreement between them validating the FEA based MLSynRM model. At higher excitation level ($I_{RMS} > I_{Rated}$), larger deviation between FEA and experimental results are observed. These differences are likely to be justified through the tolerances in manufacturing and material properties uncertainties. Small differences in the rotor bridge thicknesses in the fabricated laminations and uncertain knowledge of their saturated $B-H$ curve are possible reasons for differences between the predicted and actual performances at higher excitation levels. MTPA points at
different excitation levels are obtained from FEA and experiments and they are indicated through red circles in Figure 7.14(b). Good agreement between FEA and experimental MTPA lines are obtained which validates the designed MLSynRM performance under different control condition.

![Figure 7.14: Experimental (solid lines) and FEA (dotted lines) results for MLSynRM torque under different current commands (a) $T_{\text{AVG}}$ vs. $i_d, i_q$ map and (b) MTPA analysis](image)

### 7.6.3 Test Results with Speed Controller

The speed controller is implemented by closing the outer speed PI loop as shown in Figure 7.7. Speed controller performance under no-load is evaluated by keeping the dyno machine off and reversing the speed commands. Figure 7.15 shows results from no-load speed control tests for 900 and 600 rpms. Rotary encoder position feedback (electrical degrees) is shown in yellow graph while the phase current is presented in magenta graph. With a reverse command on speed the applied phase currents change its polarity and the direction of rotation changes ($-\nu e$ to $+\nu e$ and vice versa) as can be seen from the encoder feedback.

Performance of the speed controller is also evaluated under loaded conditions. The test motor is run in speed control mode while the dyno machine is controlled to apply different...
load torques ($T_{Load}$). Figure 7.16(a) and Figure 7.16(b) presents MLSynRM rotor position ($\theta_r^e$, in yellow), phase current ($i_a$, in magenta) and phase voltage ($v_a$, in green) changes with load torque increment and decrement, respectively. Figure 7.16(C) and Figure 7.16(d) presents the speed regulation under closed loop speed control while the load torque is varied to rated to 36.6% of rated to rated load and vice versa. Results in Figure 7.16 show that good speed regulation is achieved with tuned PI gains achieving around 8.3% overshoot with minimal steady state error.

![Figure 7.15: MLSynRM no-load speed control test with speed commands being reversed (a) 900 rpm and (b) 600 rpm](image)

![Figure 7.16: Speed controller performance under load. $\theta_r^e$, $i_a$, and $v_a$ variation with $T_{Load}$ (a) increment, (b) reduction, (c), (d) speed regulation with $T_{Load}$ variation](image)
7.7 Summary

SynRM controller development and its performance analysis under vector control is discussed in this chapter. Machine $d - q$ modelling is presented and its performance parameters are obtained in terms of $L_d$, $L_q$. Measurement method for $L_d$, $L_q$ are detailed and the FEA results are verified through experiments. Vector controller of MLSynRM is implemented in Matlab Simulink using both FEA and LUT based machine models and both current and speed controls are investigated. Experimental evaluation of the prototype MLSynRM is performed under vector control to obtain machine characteristics and controller performance under both current (torque control) and speed controls. Results validate the MLSynRM design concept showing good agreements between experimental and FEA based results. Moreover, test results also verify the performance of the developed MLSynRM controller under different control and loading conditions.
Chapter 8 Performance Analysis

8.1 Introduction

8.2 FEA Based Performance Comparison

8.3 Loss Separation of Test Motors
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8.4 Performance Evaluation at Thermal Steady State
   8.4.1 Heat-run Test
   8.4.2 Thermal Steady State of Test Motors
   8.4.3 Heat-run Test Results
   8.4.4 Rotor Temperature Comparison
   8.4.5 Rise by Resistance Test
   8.4.6 Rated Performance Evaluation

8.5 Machine Efficiency and Stability

8.6 Summary
8.1 Introduction

The concept of multilayer winding is presented to minimize the dominating stator $I^2R$ loss component by reducing the end-winding length and resistance compared to its conventional double layer counter parts. In addition, the ML winding samples the stator MMF in each stator slot. Utilizing the large number of slots in distributed stator, the ML winding can also minimize stator MMF harmonics. These end-winding length reduction and more sinusoidal stator MMF helps in minimizing both stator and rotor $I^2R$ losses and core loss.

This chapter presents the validation of the designed ML winding concept for efficiency improvement through experimental results. Performance of MLIM, MLSynRM are evaluated and compared against their benchmark BMIM under the same test environment. First, FEA based rated performance evaluation of the considered topologies are presented under the same stator $I^2R$ loss. Next, experimental loss separation of the considered topologies are performed following IEEE 112 test standard [132]. Figure 7.11(a) and (b) show the testbed utilized for the experimental analysis and Figure 7.11(c) presents the data acquisition system. Both no-load and loaded tests are performed to separate stator $I^2R$ ($P_{St-I^2R}$), rotor $I^2R$ ($P_{Rt-I^2R}$), core ($P_{Core}$), and mechanical friction and windage ($P_{F,W}$) losses and these losses are compared among different topologies. Motor performance at thermal steady state condition are obtained next by performing the heat-run tests according to IEEE 112 standard. Rise by resistance tests are also performed to obtain the ‘specified temperature’ [132] for machine efficiency calculation. Rotor temperatures are captured to verify that SynRMs have cooler rotor compared to IMs. Rated performance comparison at thermal steady state shows that the designed MLIM can attain IE4 class and the designed MLSynRM can attain IE5 class efficiency under the same frame size and cooling type as their IE3 class BMIM.

8.2 FEA Based Performance Comparison

Performances of the designed MLIM and MLSynRM are initially evaluated and compared against BMIM using FEA at rated condition. These machines have the same stator geometry while both induction motors have the same rotor structure to directly evaluate the ML winding
effect. Performance of the designed SSRM from Chapter 3 is also included in this analysis for comparison against other topologies. Rated performances are investigated under the same stator $I^2R$ loss of 60 W. Non-skewed machine geometry is considered and mechanical friction and windage losses were not included for the efficiency calculations. Results from the FEA analysis along with their material cost [133] comparison are presented in Table 8.1.

Table 8.1: FEA based rated performance comparison under the same stator $I^2R$ loss

<table>
<thead>
<tr>
<th>Machine Type</th>
<th>BMIM</th>
<th>MLIM</th>
<th>MLSynRM</th>
<th>SSRM</th>
<th>SSRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Material</td>
<td>M19-29</td>
<td>10JNEX900</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_{\text{Rated}}$ (rpm)</td>
<td>1751</td>
<td>1800</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J_{\text{Slot}}$ (A/mm$^2$)</td>
<td>4.35</td>
<td>4.15</td>
<td>4.15</td>
<td>3.44</td>
<td>3.44</td>
</tr>
<tr>
<td>$P_{I^2R-\text{St}}$ (% of $P_{\text{Out}}$)</td>
<td>7.65</td>
<td>7.17</td>
<td>6.90</td>
<td>6.77</td>
<td>7.18</td>
</tr>
<tr>
<td>$P_{I^2R-\text{Rt}}$ (% of $P_{\text{Out}}$)</td>
<td>3.61</td>
<td>3.07</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{\text{CORE}}$ (% of $P_{\text{Out}}$)</td>
<td>3.37</td>
<td>3.02</td>
<td>3.04</td>
<td>9.35</td>
<td>3.31</td>
</tr>
<tr>
<td>$P_{\text{Total}}$ (% of $P_{\text{Out}}$)</td>
<td>14.63</td>
<td>13.26</td>
<td>9.94</td>
<td>16.12</td>
<td>10.49</td>
</tr>
<tr>
<td>$T_{\text{AVG}}$ (Nm)</td>
<td>4.30</td>
<td>4.61</td>
<td>4.63</td>
<td>4.82</td>
<td>4.54</td>
</tr>
<tr>
<td>$T_{\text{Den}}$(Nm/L)</td>
<td>1.35</td>
<td>1.45</td>
<td>1.46</td>
<td>1.52</td>
<td>1.43</td>
</tr>
<tr>
<td>$P_{\text{Out}}$ (W)</td>
<td>788.47</td>
<td>845.68</td>
<td>872.73</td>
<td>908.55</td>
<td>855.85</td>
</tr>
<tr>
<td>$P_{\text{In}}$ (W)</td>
<td>903.65</td>
<td>957.86</td>
<td>959.42</td>
<td>1054.98</td>
<td>945.67</td>
</tr>
<tr>
<td>$\eta$ (%)</td>
<td>87.27</td>
<td>88.28</td>
<td>90.96</td>
<td>86.12</td>
<td>90.50</td>
</tr>
<tr>
<td>Material cost ($)</td>
<td>28.07</td>
<td>27.79</td>
<td>23.89</td>
<td>30.11</td>
<td>85.77</td>
</tr>
</tbody>
</table>

Using the conventional M19-29 electrical steel for lamination (0.35 mm thick), the designed SSRM can achieve higher torque density compared to other considered topologies in Table 8.1. However, the core loss of the designed SSRM is found higher compared to other topologies. This higher core loss can be related to the higher pole number and unwanted space harmonics associated with the concentrated wound SSRM design. As a result, efficiency of the designed SSRM with M19-steel is lower than other topologies in Table 8.1. To minimize the
core loss and improve SSRM efficiency, 0.1 mm thick, 10JNEX900 electrical steel [39] can be considered for its laminations. Results from Table 8.1 show that the designed SSRM can achieve comparable torque density and efficiency to MLSynRM with 10JNEX900 material. However, the relative cost material cost of 10JNEX900 steel is around 3.5 times compared to conventional electrical steels [134]. Considering the higher lamination cost and lower power factor of SSRM compared to MLSynRM, SSRM efficiency improvement has not been considered in this dissertation. Instead, ML wound designs are considered as the low-cost, higher efficiency alternatives.

Under the same stator $I^2R$ loss, the designed ML wound machines can provide higher torque output compared to BMIM. Moreover, loss distribution comparison of the considered ML wound machines show that they have lower loss shares compared to its benchmark. With absent $P_{I^2R-Rt}$, efficiency gain of the MLSynRM is 3.69% compared to BMIM whereas the designed MLIM can achieve 1.01% higher efficiency than BMIM. These reductions in losses and improvement in overall efficiency validate the design contribution of ML winding.

### 8.3 Technology Readiness Evaluation

According to their system level perspectives, input power factors of both BMIM and MLIM are the same (0.75 pu), which is higher than that of the MLSynRM (0.70 pu) as shown in Table 6.2. A lower power factor will increase the drive size for the same input power requirement. This increment is motor drive size will increase initial cost of the overall VSD system (motor and drive). However, with higher power conversion efficiency, the life cycle cost of the MLSynRM drive will be comparable to its IM counterparts. Compared to MLSynRM, MLIM is a more suitable higher efficiency alternative to BMIM for commercial adaption as the inverter size remains the same. The MLIM can provide efficiency improvement with some modifications in the process of winding manufacturing. Both DL and ML windings have variable turn numbers and coil pitches, and their coil groups can be formed using the same winding tools. Compared to the two coil group insertions of the double layer winding, ML winding requires three coil group insertions and it requires one additional phase separator in its slots. Including these two updates in the winding manufacturing process, the ML winding
can be built using the same tools with some increment in manufacturing time. However, the reduction in stator $I^2R$ loss is significant in ML wound design and machine efficiency has been improved as shown in Table 8.1. Both ML wound designs represent improved efficiency topologies without increasing the overall cost of the motor drive systems, which also makes their commercial adaptability possible. Analysis on machines’ stability margin with improved efficiency alternatives are presented in Appendix B. Experimental validations of the ML wound prototypes and their performance comparison against the premium efficiency benchmark are presented in the next section.

### 8.4 Loss Separation of Test Motors

IEEE 112 standard test procedure [132] discusses the loss separation and efficiency measurement techniques for polyphase induction motors and is followed to experimentally evaluate the designed MLIM and benchmark BMIM motors. Compared to an squirrel cage IM, SynRM (non-PM) does not have rotor $I^2R$ losses and its other loss components can be measured following the same procedures given that the SynRM can be brought into synchronism by gradually increasing its supply frequency at no-load. Both no-load and loaded tests were performed for the considered machines to isolate the losses and calculate efficiency.

#### 8.4.1 No-load Test

Machine no-load tests are performed by mechanically decoupling the test motors from the dyno drive and operating it at rated frequency (60 Hz) with voltages ranging from 125% of the rated voltage down to the point where the input current increases with further reduction of the applied voltage [132]. No-load test (with slip = 0) is utilized to isolate the motor core losses ($P_{\text{CORE}}$) from stator $I^2R$ loss and the mechanical friction and windage loss ($P_{\text{F,W}}$) to identify the relationship between $P_{\text{CORE}}$ and the line-to-line applied voltages. Figure 8.1 presents combined core, and friction, windage losses against square of the applied line to-line voltages.
Figure 8.1: No-load test results and loss identification

Through curve fitting in the low voltage region, where core loss and voltage squared relationship is linear, $P_{F,W}$ is found as the intercept at zero voltage and core loss coefficient ($\alpha_{CORE}$) is found as its linear regression coefficient to line-to-line voltage squared [132]. Results from no-load tests are presented in Table 8.2, which shows that the designed MLIM yields 9.4% lower and MLSynRM yields 30.2% lower value of $\alpha_{CORE}$ compared to BMIM for the same applied voltage and excitation frequency. This demonstrates that the designed ML winding can attain lower harmonics in its airgap MMF compared to DL configuration. However, having a lower power factor than SCIM, SynRM will require higher supply voltage to achieve the same magnetization level. Effect of this on $P_{CORE}$ at rated condition has been discussed in the next subsection. The friction and windage losses for these machines are found to be similar. Results from the no-load test has been utilized next to separate IM rotor $I^2R$ losses and calculate the power conversion efficiency under loaded conditions.
Table 8.2: $\alpha_{\text{Core}} (W/V^2)$ and $P_{F,W}$ results from no-load tests

<table>
<thead>
<tr>
<th>Motor</th>
<th>$\alpha_{\text{Core}} (W/V^2)$</th>
<th>$P_{F,W}$ (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMIM</td>
<td>$5.3 \times 10^{-4}$</td>
<td>7.2</td>
</tr>
<tr>
<td>MLIM</td>
<td>$4.8 \times 10^{-4}$</td>
<td>6.2</td>
</tr>
<tr>
<td>MLSynRM</td>
<td>$3.7 \times 10^{-4}$</td>
<td>7.3</td>
</tr>
</tbody>
</table>

### 8.4.2 Load Test

Motor load test is performed by connecting the test motor to the dyno drive including a torque transducer in the middle as shown in Figure 7.11(b) and running the motor under different loading conditions at rated voltage and rated frequency. Under different load torques and speeds, IM rotor $I^2R$ loss can be calculated from the input power, stator $I^2R$ loss, $P_{\text{CORE}}$ and slip as in equation 8.1

$$P_{I^2R-Rt} = \frac{(P_{\text{INPUT}}-P_{I^2R-ST}-P_{\text{CORE}})}{\text{slip}} \quad (8.1)$$

Rotor $I^2R$ losses for IMs under different loading conditions from the load tests are presented in Figure 8.2, which show that the designed ML wound machine has lower rotor $I^2R$ losses compared to the benchmark DL wound machine. Lower rotor $I^2R$ losses under the same magnetization level also justifies that the ML winding can yield more sinusoidal stator MMF compared to the conventional distributed windings. Compared to IMs, SynRM does not have rotor $I^2R$ loss, which is around 22% of the total losses. This absence of rotor $I^2R$ loss helps to improve SynRM efficiency significantly compared to IMs. Combining this more sinusoidal stator MMF feature with reduction in stator $I^2R$ losses, the designed ML wound motors should provide higher efficiency at the rated operating condition compared to BMIM which has been verified in the next subsection.
8.4.3 Results Comparison

Rated performance of BMIM, MLIM, and MLSynRM are experimentally obtained at rated frequency under the same load torque and these results are presented in Table 8.3. These results are obtained for cold-resistance, when the winding temperature remains within 5°C of the ambient [132]. Results presented in Table 8.3 show that the prototype ML wound motor with higher fundamental winding factor $F_1$ than its benchmark can provide the same torque with 17.55% lower stator $I^2R$ loss which is a significant improvement. The designed ML winding with higher $F_1$ results in higher induced voltage in the phase windings which results in higher $P_{CORE}$ (in Watt) even with lower core loss coefficients as shown in section 8.2.1. The prototype ML wound motor, with more sinusoidal stator MMF and reduced stator $I^2R$ loss, reduces the total motor loss by 8.7% compared to its premium/IE3 efficiency class benchmark.

The prototype MLSynRM, with higher fundamental winding factor $F_1$, can provide the same torque with 6.84% lower stator $I^2R$ loss compared to BMIM. MLSynRM with higher $F_1$ and lower $PF_{IN}$ results in higher induced voltage as shown in Table 8.3. However, with lower core loss coefficients as shown in Table 8.2, MLSynRM $P_{CORE}$ at rated condition is lower than that of BMIM. The prototype MLSynRM motor, with more sinusoidal stator MMF, reduced
stator $I^2R$ loss, $P_{core}$, and absent rotor $I^2R$ loss, reduces the total motor loss by 25.65% compared to its BMIM benchmark at the rated condition.

Table 8.3: Experimental evaluation of motors' rated performance

<table>
<thead>
<tr>
<th>Machine Type</th>
<th>BMIM</th>
<th>MLIM</th>
<th>MLSynRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{Rated}$ (rpm)</td>
<td>1748</td>
<td>1750</td>
<td>1800</td>
</tr>
<tr>
<td>$V_{RMS}$ (V)</td>
<td>206.2</td>
<td>231.97</td>
<td>241.54</td>
</tr>
<tr>
<td>$I_{RMS}$ (A)</td>
<td>3.13</td>
<td>2.61</td>
<td>2.78</td>
</tr>
<tr>
<td>$P_{I^2R-St}$ (W)</td>
<td>55.77</td>
<td>45.98</td>
<td>52.2</td>
</tr>
<tr>
<td>$P_{I^2R-Rt}$ (W)</td>
<td>23.57</td>
<td>21.58</td>
<td>-</td>
</tr>
<tr>
<td>$P_{CORE}$ (W)</td>
<td>22.53</td>
<td>25.83</td>
<td>21.59</td>
</tr>
<tr>
<td>$P_{F,W}$ (W)</td>
<td>7.2</td>
<td>6.2</td>
<td>7.3</td>
</tr>
<tr>
<td>$P_{LOSS}$ (W)</td>
<td>109.07</td>
<td>99.59</td>
<td>81.09</td>
</tr>
<tr>
<td>$T_{AVG}$ (Nm)</td>
<td>3.97</td>
<td>3.97</td>
<td>3.97</td>
</tr>
<tr>
<td>$P_{Out}$ (W)</td>
<td>726.71</td>
<td>727.54</td>
<td>748.46</td>
</tr>
<tr>
<td>$P_{in}$ (W)</td>
<td>835.78</td>
<td>827.13</td>
<td>829.55</td>
</tr>
<tr>
<td>$\eta$ (%)</td>
<td>86.95</td>
<td>87.96</td>
<td>90.22</td>
</tr>
</tbody>
</table>

The prototype IM and SynRM rotors are skewed by $10^\circ$ and $4^\circ$, respectively hence, the FEA results in Table 8.1 have overestimated BMIM torque for the same stator $I^2R$ loss. Moreover, without inclusion of mechanical losses FEA results slightly overestimate the efficiency. However, the performance trends for benchmark BMIM and designed MLIM and MLSynRM motors are consistent in both FEA and experimental evaluations. The designed ML winding with more sinusoidal airgap MMF and improved torque/Cu loss achieves significant reduction in total loss, which improves its power conversion efficiency at rated condition by 1.01% for MLIM and 3.3% for MLSynRM.
8.4.4 Efficiency and Power Factor Maps

Performances of the MLSynRM under different speeds and load torques are obtained from both FEA as well as experiments. Power conversion efficiency and input power factor are considered as the two performance matrices in this section. Figure 8.3(a) and (b) present the efficiency maps obtained from FEA analysis and experimental evaluation of the MLSynRM, respectively. Compared to the FEA based model the prototype MLSynRM was skewed to minimize the torque ripple, and the mechanical friction and windage losses are included in the efficiency calculations. Without inclusion of mechanical losses in electromagnetic FEA, the predicted efficiency is higher than the experimental results. However, the predicted efficiencies from FEA under different loading conditions maintain similar trend with experimental results.

![Efficiency map obtained from FEA and experiments](image)

Figure 8.3: MLSynRM Efficiency map obtained from (a) FEA and (b) experiments under different loading conditions.

Machine input power factor is also obtained from both FEA and experiments and their results are presented in Figure 8.4(a) and (b), respectively. Compared to experimental results the FEA based prediction underestimates the power factor over different loading conditions. The experimental results were obtained in quick successions maintaining the machine temperature similar to the ambient. In thermal steady state, as considered in the FEA by increased phase resistance, it requires higher level of input excitation to achieve similar torque which will reduce the power factor numbers presented in Figure 8.4(b). Moreover, the trend of power factor variation matches well with the experiments under different loading conditions.
Analyzing the results from Figure 8.3 and Figure 8.4 shows good agreements between FEA based prediction and experimental results on MLSynRM performances under different speeds and load torques.

Figure 8.4: MLSynRM input power factor map obtained from (a) FEA and (b) experiments under different loading conditions.

The benchmark induction machine is a continuous duty motor and so its efficiency at rated load should be measured at a ‘specified temperature’ according to IEEE 112 [132]. As a result, evaluation of these considered topologies need to be performed at thermal steady state. Section 8.3 will discuss the heat-run test to achieve thermal steady state at rated operating condition, rise by resistance test to determine the ‘specified temperature’ for each motor, and evaluate performance of these considered topologies at thermal steady state and compare their power conversion efficiencies.

8.5 Performance Evaluation at Thermal Steady State

IEEE 112 standard test procedure recommends that efficiency of a continuously-rated machined should be calculated at a ‘specified temperature,’ $T_S$ that defines its thermal steady state. For a TENV type cooling, determination of $T_S$ requires the machine to be equipped with devices to measure the winding and stator core temperature and temperature rise at 1.0 service factor (rated load) shall be used in calculating machine performance [132].
Thermocouples (J-type) were utilized to measure the temperature of various parts of the machine during the temperature test. Multiple thermocouples are used to measure the winding temperature and all of their temperature measurements were recorded, with the maximum of these values reported as the measured winding temperature. Additionally, rise by resistance test were also performed after the machines were shutdown at thermal steady state condition to measure the winding temperature. Maximum winding temperature rise for each of these test machines is utilized for efficiency calculation as recommended by IEEE 112.

### 8.5.1 Heat-run Test

Figure 8.5 presents the heat-run test setup. Thermocouples are installed on stator core, stator end-winding, motor frame and ambient. Multiple thermocouples were installed on similar regions of the machine to improve measurement reliability. Thermocouples were installed in similar location for each of the test machines (BMIM, MLIM, MLSynRM) to directly compare their temperatures from heat-run tests. Keysight data acquisition switch unit 34972A, as presented in Figure 8.5(a), is utilized to record real time temperature data and a typical temperature measurements from the heat-run test are presented in Figure 8.5(b).

![Figure 8.5: Heat-run test (a) setup (b) recorded temperature from data acquisition unit](image-url)
8.5.2 Thermal Steady State of Test Motors

IEEE 112 standard was followed to maintain loading limits and determine the thermal steady state for each test machine. For continuously rated machines as the test motors, long testing time is required to attain steady temperature at rated load. As a result, during the preliminary heating period, 150% of the rated load is applied in order to shorten the time of test [132]. The overload was removed before the temperature goes above the expected final temperature during each heat-run test. Heat-run test was continued until constant temperature under rated operating condition (service factor 1.0) has been reached. Temperature readings of the thermocouples are updated at an interval of 30 seconds. According to IEEE 112, for continuously rated machines, the test shall continue until there is 1°C or less change in temperature over a measurement interval of 30 minutes (or 0.5°C temperature variation over 15 minutes period) and this provision has been followed for heat-run test of each test motor.

8.5.3 Heat-run Test Results

Measured temperatures from the heat-run tests for the BMIM, MLIM and MLSynRM motors are presented in Figure 8.6, Figure 8.7, and Figure 8.8, respectively. The measurement includes temperature recording from the end-winding, middle of stator lamination stack, motor frame, and the ambient. Each of these test motors has been built under the same 56C frame with the same load torques. Based on the temperature when 50% overload was removed, the final time to achieve thermal steady states are different for each motor. However, thermal steady state operation for each motors were ensured according to IEEE 112.

Measured temperatures from all motor parts (winding, lamination and frame) are compared with their ambient to calculate the temperature rise for each motors and the results are presented in Figure 8.9. Results show that the end-winding temperature rise is the highest for all machines, which is coherent with IEEE 112. Moreover, the prototype MLSynRM is 17.40°C and prototype MLIM is 7.72°C colder than benchmark BMIM that agrees with their cold-resistance efficiencies as presented in Table 8.3. The following three sections will discuss
the measurement of rotor temperature, rise by resistance test to determine specified
temperature and machines’ rated efficiency comparison at thermal steady state.

Figure 8.6: Recorded temperatures from BMIM heat-run test

Figure 8.7: Recorded temperatures from MLIM heat-run test

Figure 8.8: Recorded temperatures from MLSynRM heat-run test
8.5.4 Rotor Temperature Comparison

Non-PM rotors of electric machines have three major loss components; core loss in its lamination, $I^2R$ losses from rotor winding or cage, and friction and windage losses from its rotation. Compare to IMs, SynRM does not have the rotor $I^2R$ losses which is around 20% of the total loss. With the absence of rotor $I^2R$ losses, SynRM has a cooler rotor than IMs which has been verified by directly measuring the rotor temperature through thermal camera during the heat-run test.

Temperature data capturing through thermal camera requires clear line of sight for the considered region. To capture rotor temperature during motor operation an opening was created at the motor end-plate as presented in Figure 8.10(a). Through this end-plate; stator end-winding, rotor core and rotor end-wing (for IMs) are accessible to capture their real-time temperature as shown in Figure 8.10(a). FLIR ONE thermal imaging camera was utilized to capture temperatures. Inclusion of this end-plate opening has changed the TENV cooling nature of the motor frame for that region. However, testing of each of the considered topologies are performed using the same end-plate to ensure that their performance comparison is conducted under the same test environment and cooling type.
Measured rotor temperature are compared for both MLIM and MLSynRM prototypes having the same stator. As a result, rotor temperature can be directly compared based on their rotor structures. Measure temperature data from thermal camera are presented in Figure 8.11. Initial temperatures are captured before starting each test and photos of the static rotor and their thermal images are presented in Figure 8.11(a) and (b) for the prototype MLSynRM and MLIM, respectively. Figure 8.11(c) and (d) presents photos of the rotor in motion and their thermal images captured during the thermal steady state conditions obtain from heat-run tests.

Two temperature markers are placed on each of these thermal images of Figure 8.11, one showing the end-winding temperature and another showing the rotor core temperature. MLSynRM and MLIM end-winding temperature captured at thermal steady state in Figure 8.11(c) and (d) matches well with the final end-winding temperatures presented in Figure 8.8 and Figure 8.7, respectively. This shows that temperature measurements from thermocouples and the thermal camera are consistent. At the initial condition, the motor temperature is uniform as very small temperature differences (0.2 to 0.3°C) are observed in Figure 8.11(a), (b) between ‘Spot 1’ and ‘Spot 2’. At thermal steady state, these motors have achieved a constant elevated temperature with its winding temperature being the highest. Compared to MLIM rotor, that attains a steady state temperature of 76.5°C (4.7°C lower than stator winding) the MLSynRM rotor reaches a final temperature of 46.7°C (27.1°C lower than stator
winding). Compared to MLIM, MLSynRM rotor is found to be 29.8°C colder under the same rated load. Results from rotor temperature measurement show that SynRM has a cooler rotor compared to its IM counterparts and thereby validating their cold rotor concept.

Figure 8.11: Rotor temperature measurement data at (a), (b) initial condition and (c), (d) thermal steady state for the prototype (a), (c) MLSynRM and (b), (d) MLIM motors

8.5.5 Rise by Resistance Test

Determination of the $T_s$ for machine efficiency calculation was performed through rise by resistance test. In this test, the temperature of the stator winding is determined by determining the winding resistance after shutdown [132]. Machine resistance is measured between two line terminals for which a reference value has been measured at a known temperature (initial ambient). Resistance measurement after shutdown requires quickly bringing down the rotor to zero speed and quick application of the leads from the resistance measuring device. For the test motor power level the recommended initial resistance reading time interval is within 30
seconds after shutdown. For the test BMIM, MLIM and MLSynRM motors the initial resistance readings were achieved within 20, 24 and 17 seconds after shutdown, respectively. Additional readings were obtained at intervals of 20-30 seconds for a minimum of 12 readings.

A curve of these measured resistances shall be plotted as a function of time and shall be extrapolated to the initial shutdown time. Semi logarithmic plot with the resistance being plotted in the logarithmic scale is recommended by IEEE 112 standard. Results from the rise by resistance tests are presented in Figure 8.12(a), (b) and (c) for the BMIM, MLIM and MLSynRM motors, respectively.

![Figure 8.12](image-url)

Figure 8.12: Results from rise by resistance test (a) BMIM, (b) MLIM, (c) MLSynRM, (d) final winding temperatures and temperature rise
The extrapolation gives resistance, $R_b$, at motor shutdown from which the winding temperature at shutdown, $T_b$ can be determined by equation 8.2. Here, $R_a$ is the reference value of the resistance measured at a known temperature $T_a$. $k_1$ is 234.5 for Cu according to the international annealed copper standard (IACS).

\[ T_b = \frac{R_b (T_a + k_1)}{R_a} - k_1 \]  
(8.2)

Equation 8.2 has been applied to calculate temperature at thermal steady state ($T_b = T_S$). Figure 8.12(d) presents the final winding temperatures of the test motors along with their temperature rises with respect to the ambient. These temperature rises at thermal steady state have been utilized to calculate the rated efficiency of the test motors in the next section.

**8.5.6 Rated Performance Evaluation**

The temperature obtained by using equation 8.2 is the total temperature of the winding at the time of the test. IEEE 112 recommends determining machine efficiency considering an ambient temperature of 25°C. As a result, the final winding temperature are adjusted by considering ambient temperature readings from section 8.3.3. This adjustment was made by subtracting the test ambient from the calculated total temperature and then adding 25°C to the obtained temperature difference. The temperature tests of BMIM, MLIM and MLSynRM are performed at rated load, and so, the resultant sum is the total winding temperature in a 25°C ambient and is the specified temperature to be used in the rated efficiency analysis [132].

Rated performance of the test motors under thermal steady state condition are presented in Table 8.4. Rated load torques are applied to the test motors and there performances are obtained under rated supply voltage and frequency. Compared to the BMIM, 7.8% loss reduction is possible with MLIM and 25.4% loss reduction is possible with the MLSynRM prototype under their rated operating conditions. Based on machine final winding temperature, the prototype MLIM is 8.73°C cooler and MLSynRM is 18.87°C cooler than the BMIM. According to the IEC600-34-30 standard [3], for standard 60Hz, 4 pole motors, IE4 class efficiency is defined as 85.5% where as IE5 class efficiency is defined as 88.05% considering 20% loss reduction
compared to IE4 class. Results from thermal steady state performance analysis showed that the prototype MLIM can achieve 86.48% efficiency and the MLSynRM can achieve 89.35% efficiency which, compared to the BMIM, are of 1.0% and 3.9% higher efficiency, respectively. The rated efficiency of the MLIM and MLSynRM is 0.98% and 1.3% higher than the IE4 and IE5 class requirements, respectively. Results from the experimental evaluation validate that, under the same frame size and cooling type as IE3 BMIM, the designed MLIM and MLSynRM can attain IE4 and IE5 class efficiencies, respectively.

Table 8.4: Motor performance comparison at thermal steady state

<table>
<thead>
<tr>
<th>Machine Type</th>
<th>BMIM</th>
<th>MLIM</th>
<th>MLSynRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{\text{Rated}}$ (rpm)</td>
<td>1741.2</td>
<td>1743.1</td>
<td>1800</td>
</tr>
<tr>
<td>$P_{I^2R-St}$ (W)</td>
<td>69.74</td>
<td>58.07</td>
<td>65.12</td>
</tr>
<tr>
<td>$P_{I^2R-Rt}$ (W)</td>
<td>26.07</td>
<td>25.50</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_{\text{CORE}}$ (W/V^2)</td>
<td>5.3e-4</td>
<td>4.8e-4</td>
<td>3.7e-4</td>
</tr>
<tr>
<td>$P_{F,W}$ (W)</td>
<td>7.2</td>
<td>6.2</td>
<td>7.3</td>
</tr>
<tr>
<td>$P_{\text{LOSS}}$ (W)</td>
<td>126.02</td>
<td>116.21</td>
<td>94.02</td>
</tr>
<tr>
<td>$T_{AVG}$ (Nm)</td>
<td>4.07</td>
<td>4.07</td>
<td>4.19</td>
</tr>
<tr>
<td>$P_{IN}$ (W)</td>
<td>868.15</td>
<td>859.67</td>
<td>882.87</td>
</tr>
<tr>
<td>$P_{OUT}$ (W)</td>
<td>742.12</td>
<td>743.46</td>
<td>788.85</td>
</tr>
<tr>
<td>$\eta$ (%)</td>
<td>85.48</td>
<td>86.48</td>
<td>89.35</td>
</tr>
<tr>
<td>$PF_{IN}$ (pu)</td>
<td>0.78</td>
<td>0.78</td>
<td>0.72</td>
</tr>
</tbody>
</table>
8.6 Summary

Efficiency improvement concept of the ML winding has been validated through experimental testing. IEEE 112 standard test procedure being followed and the performance of the prototype MLIM and MLSynRM motors are evaluated and compared against their commercial premium efficiency benchmark. Experimental loss separation through no-load and loaded test are performed and efficiency of these test motors are evaluated under thermal steady state condition. Rotor temperatures are also measured through thermal camera and the concept of SynRMs having a cooler rotor than IMs is validated. Final winding temperature has been determined through rise by resistance test and power conversion efficiencies of the test motors has been determined. Results show that the designed MLIM and MLSynRM prototypes can achieve IE4 and IE5 class efficiency, respectively under the same frame and cooling of their commercial IE3 class benchmark. Trend in machine stability margin with higher efficiency motors (IE3 to IE4) are analyzed in the final section using eigenvalues of their state transition matrix to show that efficiency improvement through machine design approach using conventional material has negligible impact on its stability margin.
Chapter 9 Summary and Future Work

9.1 Research Contributions

  9.1.1 SSRM with Standard VSI
  9.1.2 SSRM Torque Ripple Minimization
  9.1.3 IM Modeling with Ring Parameter Estimation
  9.1.4 Multilayer Winding for AC machines
  9.1.5 SynRM Design with Multilayer AC Winding

9.2 Potential Impacts

9.3 Future Work
9.1 Research Contributions

VSD based motor drive system enhances the system level efficiency by a great deal compared to their line-fed counterparts which will result in significant global energy savings. In addition, VSD enables the operation of alternative motor topologies like PMSM or reluctance motors that do not have the direct online starting capability. In terms of machine performance, PMSMs provide improved power density and efficiency than IMs. However, these PMSMs are high cost topologies utilizing rare-earth magnets whose supply volatility also influences the price. Moreover, mining of rare-earth magnets cause different detrimental effect on the environment. This dissertation hence focused on the design of rare-earth-free reluctance motors that can utilize three-phase standard VSIs as its drive. Technical contributions of this dissertation have been summarized in the following subsections.

9.1.1 SSRM with Standard VSI

A novel concentrated wound, segmented rotor SRM design is presented that has the advantages of compact structure, shorter flux paths and possible operation with three-phase standard VSIs. The designed SSRM achieves its mutual inductance variation through rotor segments and, compared to existing mutually coupled SRMs, achieves shorter end-winding lengths and lower core loss. Moreover, with increased contribution from each phase the designed motor has improved torque density compared to conventional SRMs. The proposed SSRM topology utilizes of-the-shelf standard VSIs and conventional motor control, making its more suitable for commercial adoption than conventional SRMs.

9.1.2 SSRM Torque Ripple Minimization

Torque ripple minimization of concentrated wound segmented rotor SRM is achieved through rotor segment designs. Compared to existing ripple minimization techniques the proposed method does not require current profiling, controller complexity or additional converter components. First, an FEA based semi-numerical machine model is developed to identify the torque ripple sources. Next, a new design of rotor segment is presented with segmented dip to effectively minimize torque ripple. Effect of different rotor design parameters
on machine performance are studied. FEA based multi-dimensional optimization using genetic algorithm is utilized to finalize the SSRM design including mechanical stress, deformation and acoustic noise analysis. A prototype SSRM is built to prove the design concept and simulation results are experimentally validated using three-phase standard VSI with conventional $d - q$ vector control.

9.1.3 IM Modeling with Ring Parameter Estimation

An accurate rotor resistance estimation model of squirrel cage induction motors is developed combining 2D and 3D FEA. 2D transient analysis was utilized for excitations in the 3D end-ring model to improve its accuracy over previous 2D and analytical methods. Compared to other existing FEA based methods, the proposed method utilizes detailed 3D geometry and 2D FEA based transient analysis of the IM for its excitation. One hp, four pole, three-phase, premium efficiency induction motor is selected as the benchmark. Results from the developed FEA model has been compared with that from experiments of the benchmark machine. Results shows good agreement for both starting and rated conditions and this developed model has been utilized for benchmarking the proposed reluctance machine topologies during their design and performance evaluation stages.

9.1.4 Multilayer Winding for AC machines

A new multilayer AC winding configuration is presented that can reduce end-winding length and improve the quality of its Stator MMF. Compared to the convectional fully pitched winding the designed winding can yield lower stator $I^2R$ and induced losses to improve the machine power conversion efficiency. Stator MMF harmonics have been characterized with standard winding functions and verified using finite element analysis. The multilayer winding design is optimized for a commercial premium/IE3 efficiency class benchmark machine. Performance evaluation show that the designed machine attain higher torque density and efficiency compared to the design benchmark.
9.1.5 SynRM Design with Multilayer AC Winding

A new SynRM design using the multilayer AC winding is presented to take the advantages of its shorter end-winding length and lower stator MMF harmonics. The multi-barrier rotor configuration is selected targeting exclusion of stator-rotor mutual harmonics and its design is optimized based on FEA using genetic algorithm. Compared to the conventionally wound SynRM, the designed MLSynRM yields higher torque/copper loss, lower torque ripple, and better power factor, thereby improving the overall SynRM performance. A prototype multilayer wound SynRM has been built and its simulation results have been experimentally verified under vector control. Rated efficiency evaluation at thermal steady state shows that the designed MLIM and MLSynRM can attain IE4 and IE5 class efficiencies, respectively under same frame size and cooling type as their commercial IE3 class benchmark.

9.2 Potential Impacts

The designed SSRM topology in this dissertation holds the robust SRM structure with concentrated tooth-wound winding that makes it more fault tolerant compared to its distributed winding counterparts. Performance evaluation showed that the designed SSRM achieves higher power density compared to its commercial benchmark and the reduced ripple design makes it suitable for variable speed and traction applications. Moreover, enabling the application of a standard three-phase VSI with conventional vector control with the SSRM design overcomes the commercial adaptability challenges of the conventional SRM topologies.

The developed multilayer winding concept minimizes end-winding losses and improves the stator MMF quality of AC machines. These features have positive impact on machine’s performance and the effect of the multilayer winding in improving power conversion efficiency has been detailed in this thesis. The multilayer wound IM and SynRM can achieve super-premium and ultra-premium level efficiencies, respectively under the same frame size and cooling type of their commercial premium efficiency conventionally wound benchmark. This new winding configuration can be a technology trend in gaining efficiency improvement with low cost non-PM designs under the standard frame sizes.
Application of multilayer winding to SynRM design reduces its stator ‘belt harmonics’ which helps in minimizing SynRM torque ripple significantly compared to its distributed winding counterparts. As a result, the multilayer wound SynRM will be a suitable design choice for ripple sensitive applications like servo, automotive, and traction application. Moreover, the concept of multilayer winding is valid for AC machines in general and will be suitable for performance improvements of PMAC machines as well.

Electric motor systems are the single largest category of electricity end-use, consuming more than twice as lighting which is the next largest category. In the industrial sector, electric motors consumes around 70% electrical energy. By year 2030, annual energy consumption from electric motors is expected to rise to 13360 TWh (trillion watt-hour) with CO2 emission of 8750 Mt and this will result end user expenditure of around USD 900 billion [75].

Induction motor dominates the industrial motor market with around 87% market share. Considering the commercially available standard frame industrial motors [1], IE4 class efficiency induction motors are available for rated power level $P_N > 7.5$ kW. For the lower power level ($P_N \leq 7.5$ kW), that constitute around 79% of the total market [1], IE4 class efficiency is commercially available with PM alternatives that has two to four times higher Price/ kW compared to induction motors. Considering the above scenario, the impact of 1% efficiency gain with ML wound induction machine and 3.9% efficiency gain with ML wound SynRM in minimizing electricity consumption and Carbon footprint is presented in Table 9.1.

Table 9.1: Potential impact of high efficiency motors using ML winding

<table>
<thead>
<tr>
<th>Parameters</th>
<th>IE3 IM</th>
<th>MLIM</th>
<th>MLSynRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total electricity consumption in TWh</td>
<td>13360</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electricity Consumption ($P_N \leq 7.5$ kW) in TWh</td>
<td>10554.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Loss ($P_N \leq 7.5$ kW) in %</td>
<td>25.05</td>
<td>23.50</td>
<td>19.23</td>
</tr>
<tr>
<td>Average Loss in TWh</td>
<td>2643.88</td>
<td>2480.28</td>
<td>2029.61</td>
</tr>
<tr>
<td>Savings with IE4 and IE5 motors in TWh</td>
<td>-</td>
<td>163.6</td>
<td>614.27</td>
</tr>
<tr>
<td>Average electricity cost ($/kWh)</td>
<td></td>
<td>0.1042</td>
<td></td>
</tr>
<tr>
<td>Annual savings (Billion USD)</td>
<td>-</td>
<td>17.05</td>
<td>64.01</td>
</tr>
<tr>
<td>Reduction in $CO_2$ emission (Mt)</td>
<td>-</td>
<td>11.17</td>
<td>41.92</td>
</tr>
</tbody>
</table>
Considering the average efficiency for different pole/speed combinations of standard frame industrial motors at the selected power level [3], 1% and 3.9% efficiency gain translates into an annual savings of 163.6 TWh and 614.27 TWh of electrical energy, respectively. As a result, high efficiency motors using this multilayer winding technology will save billions of dollars in electricity cost and reduce millions of metric tons in CO$_2$ emission as presented in Table 9.1.

### 9.3 Future Work

The following research directions are suggested that can be the direct evolution or inspired research direction from this dissertation.

- SSRM current profiling for controller based torque ripple minimization. This research can augment with SSRM analytical modelling for faster tuning of its controller.
- $d-q$ modeling and control of SSRM to develop an accurate and robust controller for SSRM using three-phase standard inverter.
- Developing a ferrite PM-assisted segmented rotor to improve SSRM torque density and power factor through the principle of flux focusing.
- Developing an analytical ring parameter estimation model from the 3D FEA based end-ring model presented in this dissertation.
- Develop alternative fractional slot winding with asymmetric teeth/ dummy slots that can further minimize end-winding lengths while maintaining the stator MMF quality.
- Direct online application of MLSynRM with rotor cage assistance at starting which will eliminate the inverter requirement with SynRM topologies.
- PM-assisted MLSynRM with improved power factor. With a high saliency ratio ($L_d/L_q$) this machine will also be suitable to achieve wide constant power speed range.
REFERENCES


[85] Chen, Xiaoyuan; Deng, Zhiquan; Peng, Jingjing; Li, Xiangsheng, "Comparison of two switched reluctance motors with bipolar excitation," 5th IET International Conference on Power Electronics, Machines and Drives (PEMD 2010), pp.1-6, 19-21 April 2010.


[88] Chen, Xiaoyuan; Deng, Zhiquan; Wang, Xiaolin; Peng, Jingjing; Li, Xiangsheng, "New designs of switched reluctance motors with segmental rotors," 5th IET International Conference on Power Electronics, Machines and Drives (PEMD 2010), pp.1-6, 19-21 April 2010.


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APPENDICES
Appendix A

Design Specification of the Benchmark Induction Machine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Outer Diameter, $D_{OUT}$</td>
<td>163.5 mm</td>
</tr>
<tr>
<td>Iron Stack Length, $L_{STK}$</td>
<td>77 mm</td>
</tr>
<tr>
<td>Machine Axial Length, $L_{AXIAL}$</td>
<td>151.2 mm</td>
</tr>
<tr>
<td>Number of poles, $N_{POLE}$</td>
<td>4</td>
</tr>
<tr>
<td>Fundamental frequency, $f$</td>
<td>60 Hz</td>
</tr>
<tr>
<td>Frame size</td>
<td>56C</td>
</tr>
<tr>
<td>Number of stator slots, $N_{SLOT}$</td>
<td>36</td>
</tr>
<tr>
<td>Number of Rotor Bars, $N_{BAR}$</td>
<td>44</td>
</tr>
<tr>
<td>Peak Phase Current, $I_{PEAK}$</td>
<td>4 A</td>
</tr>
<tr>
<td>Rated Power, $P_{RATED}$</td>
<td>746 W</td>
</tr>
<tr>
<td>Rated Speed, $\omega_{RATED}$</td>
<td>1745 rpm</td>
</tr>
<tr>
<td>Cooling type</td>
<td>TENV *</td>
</tr>
</tbody>
</table>

* TENV: totally enclosed non-ventilated cooling
Appendix B

Machine Efficiency and Stability

Trend in motor stability with higher efficiency alternatives has been discussed. Induction motor is selected for case study while considering eigenvalues of its linearized system matrix for stability analysis. The per-unit $d$-$q$ voltage equations for an idealized symmetrical induction machine in synchronous reference frame (with angular speed, $\omega = \omega_b$) can be given by equation B.1 [135].

\[
\begin{bmatrix}
    \nu^e_{qs} \\
    \nu^e_{ds} \\
    \nu^e_{qr} \\
    \nu'^e_{dr}
\end{bmatrix} =
\begin{bmatrix}
    r_s + \frac{p}{\omega_b} X_s & \frac{\omega_e}{\omega_b} X_s & \frac{p}{\omega_b} X_m & \frac{\omega_e}{\omega_b} X_m \\
    -\frac{\omega_e}{\omega_b} X_s & r_s + \frac{p}{\omega_b} X_s & -\frac{\omega_e}{\omega_b} X_m & \frac{p}{\omega_b} X_m \\
    \frac{p}{\omega_b} X_m & \frac{\omega_e-\omega_r}{\omega_b} X_m & r'_r + \frac{p}{\omega_b} X'_r & \frac{\omega_e-\omega_r}{\omega_b} X'_r \\
    -\frac{\omega_e-\omega_r}{\omega_b} X_m & \frac{p}{\omega_b} X_m & -\frac{\omega_e-\omega_r}{\omega_b} X'_r & r'_r + \frac{p}{\omega_b} X'_r
\end{bmatrix}
\begin{bmatrix}
    \nu^e_{qs} \\
    \nu^e_{ds} \\
    \nu^e_{qr} \\
    \nu'^e_{dr}
\end{bmatrix}
\] (B.1)

Here, $X_s = X_{ls} + X_m$, $X'_r = X'_{lr} + X_m$ with $X_{ls}$, $X'_{lr}$ and $X_m$ being the stator and rotor leakage reactance and magnetizing reactance, respectively. $p = d/dt$, $r_s$ is the stator resistance, $r'_r$ rotor resistance, $\omega_b$ is the supply frequency (377 rad/sec). The electromagnetic torque and electromechanical system equation can be written as;

\[
T_e = X_m (i^e_{qs} i'^e_{dr} - i^e_{ds} i^e_{qr})
\] (B.2)

\[
T_e = \frac{2H}{\omega_b} p\omega_r + T_L
\] (B.3)

Here, $H$ is the inertia constant (ratio of the stored kinetic energy at base mechanical speed to the base power) expressed in seconds and $T_L$ is the per unit load torque applied to the machine. The above-mentioned set of five non-linear differential equations describe the behavior of an induction machine. Local stability of the system is studied by linearizing the system about a steady state operating condition [136]. If all the variables in the above mentioned equation are allowed to change by a small amount about the steady state operating point (method of small displacement) and with zero subscript quantities denoting steady state operating point quantities, the resulting system equation can be written as in equation B.4.
\[
\begin{bmatrix}
\Delta v_{qs}^e \\
\Delta v_{ds}^e \\
\Delta T_L
\end{bmatrix}
= \begin{bmatrix}
\omega_e X_s & \frac{p}{\omega_b} X_s & \frac{p}{\omega_b} X_s & \frac{p}{\omega_b} X_m & f_R X_m & 0 \\
-\omega_e X_s & \omega_e X_s & \frac{p}{\omega_b} X_s & \frac{p}{\omega_b} X_m & 0 & 0 \\
\frac{p}{\omega_b} X_m & \frac{p}{\omega_b} X_m & \frac{p}{\omega_b} X_m & \frac{p}{\omega_b} S_0 X_r & \frac{p}{\omega_b} S_0 X_r & \frac{p}{\omega_b} S_0 X_r \\
-\omega_e S_0 X_m & \omega_e S_0 X_m & -\omega_e S_0 X_m & r_r' + \frac{p}{\omega_b} X_r' & \omega_e S_0 X_r & \omega_e S_0 X_r \\
X_m i_{dr0}^e & -X_m i_{dr0}^e & -X_m i_{dr0}^e & X_m i_{qs0}^e & X_m i_{qs0}^e & -2H_p \\
\end{bmatrix}
\] (B.4)

Here, \( S_0 = \frac{\omega_e - \omega_{r0}}{\omega_e} \). A more concise form of equation B.4 can be written as equation B.5.

\[
\begin{bmatrix}
\Delta v \\
\Delta T_L
\end{bmatrix} = \begin{bmatrix}
R & v_{10} \\
v_{20}^T & 0
\end{bmatrix}
\begin{bmatrix}
\Delta i_{\omega_r} \\
\omega_b
\end{bmatrix}
+ \frac{p}{\omega_b} \begin{bmatrix}
X \\
0^T
\end{bmatrix}
\begin{bmatrix}
0 \\
-2H_p
\end{bmatrix}
\begin{bmatrix}
\Delta v \\
\Delta T_L
\end{bmatrix} \quad \text{(B.5)}
\]

Upon solving for the vectors denoting time derivatives of currents equation 8.7 can be written as equation B.6.

\[
\frac{p}{\omega_b} \begin{bmatrix}
\Delta i_{\omega_r} \\
\omega_b
\end{bmatrix} = \begin{bmatrix}
-X^{-1} R & -X^{-1} v_{10} \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta i_{\omega_r} \\
\omega_b
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0^T
\end{bmatrix}
\begin{bmatrix}
0 \\
-1
\end{bmatrix}
\begin{bmatrix}
\Delta v \\
\Delta T_L
\end{bmatrix} \quad \text{(B.6)}
\]

The above equation constitute vector magnetic differential equation of the linearized system. The column vector \([\Delta i^T \Delta \omega_r / \omega_b]^T\) is the state vector containing linearized system state variables. By setting forcing function vector \([\Delta v^T \Delta T_L^T]^T\) equal to zero, the solution to equation B.6 is given by equation B.7.

\[
\begin{bmatrix}
\Delta i_{\omega_r} \\
\omega_b
\end{bmatrix} = e^{A \omega t} \begin{bmatrix}
\Delta i(0) \\
\omega_b
\end{bmatrix} \quad \text{with} \quad A = \begin{bmatrix}
-X^{-1} R & -X^{-1} v_{10} \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
-2H_p
\end{bmatrix} \quad \text{(B.7)}
\]

Here, the right hand side column vector represents the initial conditions. The matrix exponential function \(e^{At}\) represents the unforsrced response of the system known by the state transition matrix \(A\) of the system. Local stability is assured if all elements of the transition matrix approach zero asymptotically as time approaches infinity. This asymptotic behavior of all matrix elements occur when all of the roots of the characteristic equation have negative real parts [136], which in turn refers to having the real part of the eigenvalues of the state transition matrix, \(A\) being negative.
Induction motors are typically lightly damped at low frequencies \( f_R = \omega / \omega_b < 50\% \) [137]. Transient analysis on a symmetrical induction machine model under unforced response is performed under three different frequency ratios \( f_R = 1.0, 0.4, \) and \( 0.2 \) and the results are presented in Figure B.1. It was found that, for \( f_R = 0.4 \) (24Hz) there is a sustained oscillation which occurred with operation inside the region of instability (positive real part of eigenvalues) while the other operating point being stable. Moreover, having the same set of electrical parameters as Figure B.1(a) but with an increase in system inertia constant, the operating point at \( f_R = 0.4 \) become stable as shown in Figure B.1(b). Take away from this analysis is that, based on the system mechanical time constant, a particular motor (with the same electrical parameters) may or may not have unstable operations at certain low frequencies. The following sections focuses on machine stability with its efficiency by considering variation on its electrical parameters only.

![Figure B.1: Region of stability with supply frequency and system parameter changes (a) \( r_s = 0.13 \) pu, \( r'_s = 0.11 \) pu, and \( H = 0.05 \) s; (b) \( r_s = 0.13 \) pu, \( r'_s = 0.11 \) pu, and \( H = 0.1 \) s](image)

In terms of equivalent circuit parameters an induction motor with higher efficiency will have lower resistances \( (r_s, r'_s) \) with minimized losses and higher magnetizing inductance \( (X_m) \)
in achieving same magnetization level with lower excitation. To investigate the effect of these changes in machine parameters, variation of the eigenvalues of the state transition matrix are studied with these parameter changes under different supply frequencies. Eigenvalues of the state transition matrix have repeated roots with widely dissimilar real and imaginary parts. Excluding repeated results, eigenvalues of the one hp, premium efficiency BMIM, with measured per-unit circuit parameters as $r_s=0.050$ pu, $X_m=3.31$ pu, $r'_f=0.03$ pu, $x_{ls'}, x_{lr'}=0.2$ pu, under supply frequency variation are plotted in Figure B.2. Variation of $r_s$ and $X_m$ are considered in the eigenvalue study with parameters of BMIM being the base case and the results are presented in Table B.1 and Table B.2, respectively.

![Figure B.2: Eigenvalue variation of state transition matrix with supply frequency](image-url)
Table B.1: Real part of eigenvalue variation with $r_s$

<table>
<thead>
<tr>
<th>$f_s$ (Hz)</th>
<th>$\text{Real}(EigA(1,2)) \times (-1)$</th>
<th>$\text{Real}(EigA(3,4)) \times (-1)$</th>
<th>$\text{Real}(EigA(5)) \times (-1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_s =$ 0.05pu</td>
<td>0.104555 0.078967 0.053379</td>
<td>0.006744 0.004405 0.001586</td>
<td>0.210202 0.157855 0.10647</td>
</tr>
<tr>
<td>$r_s =$ 0.03pu</td>
<td>0.104576 0.078983 0.05339</td>
<td>0.006873 0.004405 0.001491</td>
<td>0.209902 0.157824 0.106495</td>
</tr>
<tr>
<td>$r_s =$ 0.01pu</td>
<td>0.104716 0.079088 0.053461</td>
<td>0.007154 0.004405 0.001479</td>
<td>0.20906 0.157613 0.106529</td>
</tr>
<tr>
<td>$r_s =$ 0.05pu</td>
<td>0.104948 0.079264 0.05358</td>
<td>0.007249 0.004405 0.001474</td>
<td>0.208406 0.157262 0.106283</td>
</tr>
<tr>
<td>$r_s =$ 0.03pu</td>
<td>0.105271 0.079508 0.05374</td>
<td>0.007288 0.004405 0.001472</td>
<td>0.207681 0.156773 0.105962</td>
</tr>
<tr>
<td>$r_s =$ 0.01pu</td>
<td>0.105685 0.07982 0.053956</td>
<td>0.007307 0.004405 0.001472</td>
<td>0.206816 0.156149 0.105544</td>
</tr>
</tbody>
</table>

Table B.2: Real part of eigenvalue variation with $X_M$

| $f_s$ (Hz) | $|\text{Real}(EigA(1,2))|$ | $|\text{Real}(EigA(3,4))|$ | $|\text{Real}(EigA(5))|$ |
|-----------|---------------------------------|---------------------------------|---------------------------------|
| $X_M =$ 3.3pu | 0.104555 0.104643 0.104711 | 0.006744 0.005879 0.005211 | 0.210202 0.210252 0.210289 |
| $X_M =$ 3.8pu | 0.104576 0.104658 0.104723 | 0.006873 0.005992 0.00531 | 0.209902 0.209996 0.210066 |
| $X_M =$ 4.3pu | 0.104716 0.104765 0.104807 | 0.007154 0.006237 0.005528 | 0.20906 0.209293 0.209464 |
| $X_M =$ 3.3pu | 0.104948 0.104941 0.104945 | 0.007249 0.00632 0.005602 | 0.208406 0.208774 0.209039 |
| $X_M =$ 3.8pu | 0.105271 0.105187 0.105138 | 0.007288 0.006354 0.005632 | 0.207681 0.208214 0.208592 |
| $X_M =$ 4.3pu | 0.105685 0.105502 0.105386 | 0.007307 0.006371 0.005648 | 0.206816 0.207551 0.208066 |

Results presented in Table B.1 and Table B.2 showed that with reduced $r_s$ and increased $X_M$ induction motor operating region of instability expands as the negative real parts of its eigenvalues are reduced. However, variation of these numbers are largely overestimated in Table B.1 and Table B.2 with 40% to 80% variation in $r_s$ and 15% to 30% variation in $X_M$. For practical machines, the variation of circuit parameters are much smaller with higher efficiency motors. For instance, the selected base case in this analysis is the premium efficiency BMIM motor with 85.48% efficiency. In comparison, the super-premium efficiency MLIM with
86.48% efficiency has its measured per-unit parameters as $r_s=0.0479$ pu (4.2% lower), $X_M=3.41$ pu (3.02% higher) with similar $r'_r$, $x_{ls}$, $x_{lr}'$ values having the same rotor.

Variation of eigenvalues for IE4 efficiency class are compared with IE3 class in Figure B.3 for different supply frequencies. Results in Figure B.3 show that, compared to their zero values which corresponds to the region of instability, the changes in eigenvalues for higher efficiency motors are negligible. As a result, even with 10% total loss reduction the stability margin for the IE4 motor remains almost unaltered. Moreover, under saturated conditions the magnetizing inductance decreases in practical machines, which would improve motor stability. Analysis and discussions presented in this section showed that efficiency improvement techniques by machine design approach (using conventional materials) results small changes in machine per unit parameters and has negligible impact in altering its stability margin.

Figure B.3: Variation of eigenvalues for IE3 and IE4 class induction motors