ABSTRACT

MAROTO VARGAS, ANA PATRICIA. Shifts in Participation and Use of Materials in a Professional Development for Elementary School Mathematics Teachers: A Case Study Using a Situative Perspective. (Under the direction of Dr. Paola Sztajn.)

Quality Professional Development (PD) interventions are important to improve teaching in schools (Desimone, 2009) and students’ achievement (Wayne, Yoon, Zhu, Cronen, & Garet, 2008). Recent research has been focusing on learning using a situative perspective (Greeno, 2006). Within this body of research, this study analyzed elementary school mathematics teachers’ learning during a PD intervention and examined the role of PD materials during the intervention. The PD intervention was conceptualized as a boundary encounter, and the PD materials as boundary objects (Wenger, 1998). The research is comprised of a one-case study (Creswell, 2013), bounded by a cohort of teachers participating in a PD intervention. The study followed an embedded mixed-methods design.

The study characterized main patterns in teachers’ participation in the PD, and identified four shifts in their participation. It also showed that PD materials played an important role in the way teachers participated in the PD and supported the shifts in teachers’ participation. Results from the study added to the research community’s knowledge base about PD features that are important for teachers’ learning (Desimone, 2009; Garet et al., 2001; Penuel, Fishman, Yamaguchi, & Gallagher, 2007) and contributed to research design and PD curriculum development.
Shifts in Participation and Use of Materials in a Professional Development for Elementary School Mathematics Teachers: A Case Study Using a Situative Perspective

by
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DEDICATION

A Melissa y Freddy, quienes compartieron esta aventura conmigo.

A mi mamá Elizabeth y a mis hermanas Rocío, Adriana, Vanessa y Laura, quienes me han apoyado desde antes de iniciar el doctorado. Esta meta no se hubiera cumplido sin su apoyo.

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BIOGRAPHY

Ana Patricia Maroto Vargas was born in 1969 in San Ramón, Alajuela, Costa Rica. The oldest of five sisters, Patricia grew in her hometown. After finishing secondary school, Patricia enrolled in the University of Costa Rica, Occidental Campus. She graduated with a Bachelor’s degree, and a fifth-year college diploma in Mathematics Education. Later she graduated with a Master’s Degree in Education with an emphasis in Educational Leadership.

Patricia’s teaching experience includes elementary, secondary and university levels. She has worked in the University of Costa Rica, Occidental Campus, teaching mathematics courses and as a mathematics educator for elementary and secondary teachers. She was the coordinator of the program in Mathematics Education for secondary teachers, and the coordinator of the Mathematics Section for two years.

She has participated as facilitator in Professional Development interventions for secondary mathematics teachers in Costa Rica.

In the Fall of 2013, Patricia was admitted in North Carolina State University as a Fulbright scholar to pursue a Ph. D. in Mathematics Education. As a graduate student in NC State, she worked as an assistant researcher on the “Learning Trajectory’ Based Instruction” and “All Included in Mathematics” projects under the direction of Dr. Paola Sztajn.
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Chapter 1: Introduction

Teachers have the responsibility to implement reforms in the classroom and must offer students high-quality education. However, research has found that many teachers are not prepared for curricular changes (Garet, Porter, Desimone, Birman, & Yoon, 2001). Professional Development (PD) interventions are important to improve teachers’ knowledge in the context of international educational reform and policy (Koellner, Jacobs, & Borko, 2011). Previous studies have shown that very good PD interventions improve the quality of schools (Desimone, 2009) and students’ achievement (Wayne, Yoon, Zhu, Cronen, & Garet, 2008).

During recent years, much attention has been given to the importance of improving the quality of PD that is offered to mathematics teachers and to the analysis of how teachers learn during those experiences, examining the use of different resources for PD, such as student work (e.g., Jacobs, Lamb, & Philipp, 2010; Kazemi & Franke, 2004), videos (e.g., Sherin, Linsenmeier, & van Es, 2009), or the role of facilitators (Coles, 2013; Linder, 2011; van Es, Tunney, Goldsmith, & Seago, 2014). As part of the efforts to analyze ideas in relation to learning, learning sciences has emerged as an important interdisciplinary field that analyzes teaching and learning visualizing them as cognitive and social processes (Sawyer, 2014). One of the most important questions within the learning sciences is how to design environments that are effective for learning (Sawyer, 2006). Most previous research has focused on student learning, and more research is necessary to understand how teachers learn (Fishman & Davis, 2006; Fishman, Davis, & Chan, 2014).
Part of the research that has been conducted in learning sciences about teachers’
learning is associated with the situative or sociocultural perspective, which focuses on
activity systems, defined as “complex social organizations containing learners, teachers,
curriculum materials, software tools, and the physical environment” (Greeno, 2006, p. 79).
The situative perspective looks at knowledge and learning, thinking about the individual
positioned in a sociocultural context (Evans, Packer, & Sawyer, 2016). Knowledge “is not a
static mental structure inside the learner’s head; instead, knowing is a process that involves
the person, the tools and other people in the environment, and the activities in which that
knowledge is being applied” (Sawyer, 2006, p.5). Learning is understood as “progress along
trajectories of participation, which can involve acting more effectively in contributing more
centrally to the functions of communities and in developing their identities as learning and
knowledgeable people” (Anderson, et al., 2000, p.12). Learning is defined in terms of
changes over time, how and why an intervention or experience change what people do and
how they do it (Greeno, 2006). Learning is both an individual process of coming to
understand how to participate in the discourse and practices of a particular community, and a
community process of refining norms and practices through the ideas and ways of thinking
that individual members bring to the discourse (Putman & Borko, 2000).

Both definitions make a difference in relation to the body of knowledge that has been
previously generated using the cognitive perspective. They provide opportunities to analyze
different variables that the cognitive perspective does not include, such as the analysis of the
social interactions between people, and how those interactions impact learners providing
them with opportunities to learn (Anderson, Greeno, Reder, & Simon, 2000; Greeno, 2006).
In this sociocultural context, the cognitive practice and not the individual is the basic unit of analysis (Spillane, Reiser, & Gomez, 2006). This allows to analyze shared norms, knowledge, practices, language, artifacts, interactions between actors, and other variables that influence how people learn (Spillane et al., 2006). This way of analyzing learning can give more insight about how reform can be implement in the educative system (Spillane et al., 2006).

Within this body of research, one important idea that has been investigated to improve teachers’ knowledge is the creation of communities of practice among them as an opportunity to learn through the relationships they establish with colleagues. When teachers talk to each other and discuss ideas, they have to negotiate new meanings, and this process helps them to learn (Miyake & Kirschner, 2014). Previous research has shown that to bring teachers into a community does not necessarily mean they will learn, and it is important to know what makes a community successful in order to improve the quality of future interventions designed to bring teachers together (Miyake & Kirschner, 2014). Still, studies have shown that collaborative learning between teachers can be a source for building knowledge. For example, Brown, Collins, and Duguid (1989) stated that “within a culture, ideas are exchanged and modified and belief systems developed and appropriated through conversation and narratives, so these must be promoted, not inhibited” (p. 40).

As part of efforts to investigate teachers’ learning in a community, there are important ideas that have not been explored enough and that can contribute to the knowledge about how to improve the quality of PD interventions and therefore teachers’ knowledge and students’ achievement. One example of such topics is the use of a situative perspective to
analyze teachers’ knowledge as “socially, culturally, and historically constructed” (Horn, 2010, p. 228) and to analyze “the specific interactions and dynamics by which professional community constitutes a resource for teachers learning and innovations in teaching practice” (Little, 2013, p. 913). Also more research should be conducted to analyze the relationship between instructional improvement, teachers’ discourse in the PD session, and their professional learning (Horn & Kane, 2015). The relationship between discourse about teaching and the action of teaching deserves greater clarification (Horn & Kane, 2015).

Within the concept of communities of practice there are two important ideas used in the current study: boundary encounters and boundary objects (Star, 2010; Wenger 1998). Boundary encounters are meetings between members of two communities who share knowledge and learn in the interactions between them (Wenger, 1998). Boundary objects are artifacts that are used by members of the communities of practice within the boundary encounters to help them to participate (Wenger, 1998). It is very important to understand how teachers learn new knowledge when they meet in boundary encounters and what is the role of PD materials in those encounters in relation to how teachers learn (Kazemi & Hubbard, 2008). To understand those situations it is important to improve the design of PD interventions.

**The Study**

I conceptualize teachers who participate in a PD intervention as taking part in a boundary encounter (Wenger, 1998). In this study, I was interested in using a situative perspective (Brown et al., 1989; Cobb & Bowers, 1999; Greeno, 1997, 1998; Lave &
Wenger, 1991) to analyze how teachers learned during their involvement in such encounter, that is, a PD setting. As part of the analysis, I was interested in how PD materials that included representations or decompositions of practice helped teachers to learn. In the context of considering PD communities as boundary encounters, these materials represented boundary objects. My goals were to describe how teachers participated in PD conversations, examine shifts in teachers’ participation during a PD, and understand how PD materials helped teachers to learn. In summary, to increase current knowledge on how teachers learn in a PD setting, I examined the following research questions:

- Considering a situative perspective on learning, in which ways do teachers participate in PD interventions? What shifts take place in teachers’ participation during PD interventions?
- What is the role of PD materials as boundary objects in PD interventions?
- In which ways are teachers’ shifts in participation related (or not) to the boundary objects used in the PD?

This study took place in the context of a PD intervention called Project AIM: All Included in Mathematics. This 5-year project has been implemented with different cohorts of teachers over the years. The PD intervention spans one academic year. It includes 40 hours and was designed to help elementary teachers promote high-quality mathematics discourse in the classrooms and develop teachers’ conceptual understanding of mathematics (Sztajn, Heck, & Malzahn, 2013a).
Outline of the Dissertation

This dissertation is divided into six chapters. Chapter 1 is the introduction to the study, and Chapter 2 includes the review of the literature, which was focused on communities of practice and pedagogies of practice. The conceptual framework and the conceptualization of PD as a boundary encounter and PD materials as boundary objects are included in this chapter. Chapter 3 describes the context of the study, the research design, the selection of the case study, how the data was collected, and how it was analyzed. Chapters 4 and 5 present the results of the data analysis, and Chapter 6 includes the conclusions and the final discussion of the study. Implications for research and future research are included in this final chapter.
Chapter 2: Literature Review and Conceptual Framework

This chapter includes the theoretical ideas that support the current study. I introduce ideas about communities of practice, the meaning of learning in a community of practice, boundary encounters and boundary objects. The framework of pedagogies of practice proposed by Grossman and colleagues (Grossman, Compton et al., 2009; Grossman, Hammernes & McDonald, 2009; Grossman, Smagorinsky, & Valencia, 1999; Hatch & Grossman, 2009; Janssen, Grossman, & Westbroek, 2015) is then connected to the concept of boundary objects. A description of the AIM PD as a boundary encounter and a review of the literature on teacher learning in PD from a situative perspective are part of this chapter. The conceptual framework used to analyze the topic of interest is at the end this chapter.

Communities of Practice

Communities of practice or learning communities are an important concept connecting learning to participation (Little, 2003). The concept has been used widely to investigate learning in school classrooms and teacher professional learning (Goos, 2014). Communities of practice are “groups of people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis” (Wenger, McDermott, & Snyder, 2002, p. 4). They emerge when people meet, even if they might not share their daily routines, but participants in the community believe that sharing knowledge has value for them in order to solve problems or to fulfill their needs (Wenger et al., 2002). People who participate in a learning community meet to discuss certain topics. The group can create artifacts or build a common knowledge about a topic. Also, members of the community build relationships and a sense of belonging in their
work with other people. They develop a shared sense of identity. Across this process, they build a community of practice (Wenger et al., 2002).

The boundaries of the community could be ill-defined, but the vital idea is participation in the community, where people share ideas (Lave & Wenger, 1991) and where participants know and learn how to participate in the common practices they share (Greeno & Engeström, 2014). A community can also be seen in relation to other communities and then could be defined as “a set of relations among persons, activity and world over time and in relation with other tangential and overlapping communities of practice (Lave & Wenger, 1991, p. 98), as for example the different groups of children to which a teacher attends or a group of teachers who participates in a professional development intervention.

A community of practice should have three elements: a domain of knowledge, a community of people who are interested in that domain, and a shared practice that they are improving in relation to the domain (Wenger et al., 2002). The domain helps the group to develop identity and a purpose. The domain is the reason for being in the community, a common interest for members. The domain also presses people to participate and to share. The community promotes interactions between members, allows people to ask questions, and imparts a sense of belonging that is connected to learning. The practice is “a set of frameworks, ideas, tools, information, styles, language, stories, and documents that community members share” (Wenger et al., 2002, p. 29). Characteristics that communities of practices have in common are presented in Table 1.
**Table 1**

*Indicators of the Formation of a Community of Practice*

1) Sustained mutual relationships – harmonious or conflictual
2) Share ways of engaging in doing things together
3) The rapid flow of information and propagation of innovation
4) Absence of introductory preambles, as if conversations and interactions were merely the continuation of an ongoing process
5) Very quick setup of a problem to be discussed
6) Substantial overlap in participants’ descriptions of who belongs
7) Knowing what others know, what they can do, and how they can contribute to an enterprise
8) Mutually defining identities
9) The ability to assess the appropriateness of actions and products
10) Specific tools, representations, and other artifacts
11) Local lore, shared stories, inside jokes, knowing laughter
12) Jargon and shortcuts to communication as well as the ease of producing new ones
13) Certain styles recognized as displaying membership
14) A shared discourse reflecting a certain perspective in the world.


**Learning in a Community of Practice**

The term situated learning emphasizes the importance of the context to analyze human relationships and communication among people (Lave & Wenger, 1991). Brown and colleagues (1989) affirmed that “the activity in which knowledge is developed and deployed is an integral part of what is learned. Learning and cognition now are fundamentally situated” (p. 32). In educational settings it is necessary to include the context in order to create useful knowledge. Teachers’ practices are also situated in the culture in which they work, and they constantly need to “negotiate meanings and construct understandings” (Brown et al., 1989, p. 35).
Learning in a situative perspective is conceptualized as changes in participation as part of a community of practice (Cobb & Bowers, 1999; Horn, 2010; Lave, 1996; Lave & Wenger, 1991; Stein & Brown, 1997). It is the construction of identities within the communities (Goos, 2014; Greeno & Gresalfi, 2008; Wenger, 1998) that helps make changes that transform common practices (Cobb & Bowers, 1999).

In this conception of knowing and learning as a social process, Lave and Wenger (1991) offered the concept legitimate peripheral participation to explain the relationship between a learner, who is situated in the periphery, and the expert, who is in the center of the practice. The members of the community who are more experienced are called old timers (Lave & Wenger, 1991). They have three important functions: (a) to encourage engagement with other participants, (b) to take responsibilities as part of the community to negotiate new meanings, and (c) to grow the knowledge they have and that could be used in the practice of the community (Goos, 2014; Wenger, 1998).

Through the process of learning, a newcomer moves from the periphery to full participation as an expert as they become part of the community of practice (Greeno & Engeström, 2014; Lave & Wenger, 1991). To become a legitimate participant in a community “involves learning how to talk (and be silent) in the manner of full participants” (Lave & Wenger, 1991, p. 105).

Learning is “a way of being in the social world, not a way of coming to know about it” (Hanks, 1991, p. 24). In that sense, learning is situated in a social context, it is “an integral part of generative social practice in the lived-in world” (Lave & Wenger, 1991, p. 35).
Learning requires the construction of identities, and “identity, knowing and social membership entail one another” (Lave & Wenger, 1991, p. 53).

According to Lave and Wenger (1991), learning happens in a participation framework—it is not an individual process. It means learning is developed through interactions between participants and with the mediation of the differences of viewpoint among the members. When teachers are working together, they create new meanings and understandings, which “do not exist as abstract structures in the individual participants’ minds; rather they derive from and create the situated practice (or context) in which individuals are coparticipants” (Stein & Brown, 1997, p. 159). Here the levels of expertise teachers bring to the community are important because they affect the process of learning (Stein & Brown, 1997).

For Wenger (1998), every learner in the community should have active participation, and, through this process, each individual builds identities that are common in that particular community. During the process of engagement of each individual in the practices of the community and the interactions among participants, the practices in that organization are reproduced but also transformed because practices are constantly in motion (Cobb & Bowers, 1999; Wenger, 1998). This establishes a “reciprocal relation between persons and practice” (Lave & Wenger, 1991, p. 116). Examples of how members of the community change include how patterns of participation are modified over time (Greeno & Engeström, 2014), how trajectories of participation are built, or how identity grows (Greeno, 1998).

To define learning through the process of participation has consequences for individuals, communities, and organizations. Individuals need to make contributions to the
practice; communities need to integrate newcomers, and organizations need to sustain the connections among members of the communities of practice (Wenger, 1998).

**Boundaries in Communities of Practice**

In the established relationship between different communities of practice there is a space that Wenger (1998) called boundary. In the *boundary*, the “sense of here and there are confounded” (Star, 2010, p. 603). Wenger (1998) defined it as “discontinuities between those who have been participating and those who have not” (p. 103). A boundary refers to the border between being inside or outside, between belonging (or not) to a community. These discontinuities happen because people move across different communities to which they belong (Wenger, 1998). Those boundaries promote learning within the learners of the communities that are connected in that boundary (Wenger, 1998). There are two important concepts associated with the concept of boundaries: boundary encounters and boundary objects.

**Boundary encounters.** The exchange of knowledge between different communities is difficult, but it creates new knowledge (Sztajn, Wilson, Edgington, & Myers, 2014). A community defines ways to communicate and to collaborate, and those mechanisms are boundary encounters (Sztajn et al., 2014). For Sztajn and colleagues (2014), participants from different communities “communicate about, collaborate around, and potentially transform practice” (p. 204). When members of communities meet, “the negotiation of meanings takes place at the same time among members within each practice and across the boundary” (Wenger, 1998, p. 112).
Boundary encounters can take different forms: *one-on-one, immersion,* and *delegation* (Wenger, 1998). The *one-on-one* form happens when one member of each community meets to share. In the *immersion* form, some members of one community visit another community to observe their practices. In this case the relationship between members is more than a one-way relationship. *Delegation* form occurs when members of both communities meet to share practices. The goal with each form of boundary encounter is to advance the boundary relation (Wenger, 1998).

For Wenger (1998), three ways exist in which practice “can offer connections that go beyond boundary encounters” (p. 114). Those ways are defined as *boundary practices*, *overlaps*, and *peripheries*. In *boundary practices* the engagement of members of both communities that are part of the boundary encounter will change across time to become a practice (Wenger, 1998). When an *overlap* practice happens, two different communities with different practices exist, but they have an overlap space in which they share their practices. In the *peripheries* category, the goal is not that members of one community become full members of the other community, but “to offer them various forms of casual but legitimate access to a practice without subjecting them to the demands of full membership” (Wenger, 1998, p. 117).

*Brokers.* In the exchange of knowledge among communities, the role of brokers is important. A broker is a person who sometimes has multimembership, which is useful to members of a community to learn practices of the other. The process of brokering “involves processes of translation, coordination, and alignment between perspectives. It requires enough legitimacy to influence the development of a practice, mobilize attention, and address
conflicting interests” (Wenger, 1998, p. 109). Some people always have a more active role as brokers than others, but normally a small group of people are the leaders (Wenger, 1998).

**Boundary objects.** This term was coined by Star and Griesemer (1989). Their initial aim was to find a term that could help to analyze collaborative work when there is not agreement (Star, 2010). The concept has been used frequently in research about communities of practice and in the cultural historical activity theory on expansive learning. Boundary objects are part of any process of learning, and the concept is associated with the idea of expertise (Akkerman & Bakker, 2011). An object is “something people … act toward and with” (Star, 2010, p. 603). It is important through actions and not because of the materials used to make it.

The concept of boundary object is related to artifacts that make connections between practices that overlap (Akkerman & Bakker, 2011). Boundary objects “are organic arrangements that allow different groups to work together based on a back and forth movement between ill-structured use in cross-site work and well-structured use in local work” (Akkerman & Bakker, 2011, p. 141). A boundary object is “an object that serves to coordinate the perspectives of various constituencies for some purpose” (Wenger, 1998, p. 106). They are artifacts that belong to different intersecting domains. They could be adapted to local requirements and limitations from different worlds using them, but they also should keep collective identity transversely (Star & Griesemer, 1989). In addition, boundary objects “are artifacts that articulate meaning and address multiple perspectives” (Akkerman & Bakker, 2011).
Boundary objects could be defined as $n$-dimensional in the sense they “are at once temporal, based in action, subject to reflection and local tailoring, and distributes throughout all of these dimensions” (Star, 2010, p. 603). Boundary objects can be any kind of reification that is meaningful for the communities of practice (Wenger, 1998). Examples of boundary objects are textbooks, performances, and computer operating systems (Star, 2010).

Akkerman and Bakker (2011) used a teacher portfolio as an example of an object that allows mathematics educators to follow student-teacher learning.

Star (2010) affirmed that in her prior research with Griesemer, they had visualized boundary objects as defined below:

- The object (remember, to read this as a set of work arrangements that are at once material and processual) resides between social worlds (or communities of practice) where it is ill structured.
- When necessary, the object is worked on by local groups who maintain its vaguer identity as a common object, while making it more specific.
- Groups that are cooperating without consensus tack back-and-forth between both forms of the object (pp. 604–605)

When the object scales up and becomes standardized, the boundary moves and starts to be part of the infrastructure of the group (Star, 2010), and then it becomes part of the knowledge that the communities has. Boundary objects have important characteristics:

- Modularity: different views attend to a different part of the object.
- Abstraction: the specific characteristics of each perspective are not included, and the object is viewed as an integration of all of them.
- Accommodation: the boundary object gives something to each activity.
Standardization: the information in the boundary object is complete in the sense that each person can understand how to use it (Wenger, 1998).

In the current study, a pedagogies of practice framework developed by Grosmman and colleagues (Grossman, Compton et al., 2009; Grossman, Hammerness & McDonald, 2009; Grossman Hatch & Grossman, 2009; Janssen et al., 2015) was used to classify the PD boundary objects. Initially, that framework was developed to help educators to think about the teaching of practice in a university setting. They used a situative perspective to analyze practice in complex domains, which they defined as “the orchestration of understanding, skill, relationship, and identity to accomplish particular activities with others in specific environments” (Grossman, Compton, et al., 2009, p. 2059). A PD is a complex domain under this definition. The framework includes three important concepts that overlap: representations, decompositions, and approximations of practice.

The concept of representations of practice “comprise[s] the different ways that practice is represented in professional education and what these various representations make visible to novices” (Grossman, Compton, et al., 2009, p. 2058). Examples of representations in an educational setting are videos or samples of student work. Decomposition of practice is “breaking down complex practice into its constituent parts for the purposes of teaching and learning” (Hatch & Grossman, 2009). The decomposition allow learners to “see and enact elements of practice more effectively” (Grossman, Compton, et al., 2009, p. 2069).

Approximations of practice are experiences provided to learners to engage in specific components of practice, in which they can “experiment with new skills, roles, and ways of
thinking with more support or feedback than actual practice in the field allows” (Grossman, Compton, et al., 2009, p. 2077). This concept is called approximation because the practice could have different grades of authenticity in relation to the real practice (Grossman, Compton, et al., 2009). In the current study two of those concepts are included as part of the conceptual framework: representations and decompositions of practice.

**PD as a Boundary Encounter**

A PD can be considered a boundary encounter because it connects two communities of practice: teacher educators and teachers who meet in a new setting that is not the community of teacher educators and is not the communities of teachers. In the PD, members of both communities meet to negotiate new meanings about teaching and learning, with the goal of transforming teachers’ practices that support students’ learning. In a general model about a PD as a boundary encounter, there are five communities of practice that are related to the PD (See Figure 1). Those communities are: researchers, teacher educators, teachers, the classroom, and students.

For the current study, I focused on the AIM PD as a boundary encounter. This PD was designed by a group of mathematics teacher educators based on previous research. They were interested in supporting teachers to improve quality mathematics discourse in the classroom and their mathematical knowledge for teaching (Sztajn et al., 2013a). The mathematics teacher educators did not participate in the PD. However, through the materials they designed for each session, and with the help of two experienced facilitators, they were communicating a specific message to teachers. Those facilitators were not at the center of the community of
mathematics teacher educators, but were old timers within the community of teachers, experienced teachers and coaches who worked within the same district as participant teachers in the PD. They were trained by the mathematics teacher educators who designed the PD to deliver it. They knew the PD materials well in order to engage teachers during the sessions, to support them to participate, and to negotiate new meanings in relation to their practices in schools. For the current study, the facilitators were the brokers in the boundary encounter. They knew well the content of the PD to support participants while they participate in conversations in the PD sessions. Based on previous characterization, facilitator had multimembership (Wenger, 1998) because they belonged to the community of teacher educators and to the community of teachers.

Using Wenger’s (1998) definitions, this encounter is defined as a periphery type of connection and a delegation type of boundary encounter. The AIM PD was a periphery boundary encounter because the expectation was not that teachers would become experienced researchers or mathematics teacher educators, but to open a boundary in the periphery of their community to experience new practices that have been proven by research to be useful to promote high quality mathematics discourse in the classroom to support students’ mathematical discourse. The Delegation form occurred because members of the community of teacher educators (represented by the facilitators as brokers) met with members of the community of teachers to share practices and negotiate ideas in the PD setting (see Figure 1). In the delegation form “the negotiation of meaning takes place at the same time among members within each practice and across the boundary” (Wenger, 1998). The current PD is a well designed, all the materials were given to the facilitators.
iteration of the intervention, the teacher educators and researchers analyzed the results generated by the experience to redesign the PD for the successive cohorts. Sometimes they had designed new materials, such as strategies handouts that did not exist in the first iteration of the PD; in other opportunities they had increased the details in the documents given to facilitators in order to communicate a clearer message about key ideas in the PD. That practice allows researchers and teacher educators involved in the design of the PD to learn from each experience. This situation supports the community of researchers and the community of teacher educators to negotiate new meanings with the communities of facilitators and with the community of teachers. These multidirectional relationships justify the delegation form of the boundary encounter. For the current study, the focus was on what happened in the PD, and how teachers learn. More research will be needed to understand better the interactions among actors that allow the delegation form of the boundary encounter.

![Figure 1](image_url)

*Figure 1. Representation of the AIM PD and its broader context.* The light green part of the figure was analyzed in the current study. The PD circle represents the boundary encounter (PD intervention), the curve arrows represent the boundary objects, the straight vertical and horizontal arrows represent the delegation type boundary encounter, the sloping arrows inside the boundary encounter represents the periphery type of connection. R: researchers, TE: teacher educators, F: facilitators, T: teachers, C: classroom, S: students. Based on
Boundary Objects in the Context of the AIM PD

The materials designed by mathematics teacher educators to be used in the PD are considered boundary objects. Their aim was to build bridges between theoretical knowledge researchers and mathematics teacher educators have, they had to promote discourse that teachers bring to the PD, and the use of those strategies in mathematics lessons. The PD provided teachers with opportunities to include those materials as part of their repertoire to teach mathematics.

The boundary objects were designed to focus on a specific part of the tasks of teaching and learning with the goal of supporting teachers in developing knowledge about how to improve quality mathematics discourse in the classroom. The materials were designed to be shared with the teacher community with the goal of impacting the student community as well.

The aim of the current study is to determine how those objects were used by participants, and also how they helped them learn. Those objects are based on previous research, were created to summarize important results about theoretical and practical ideas that could be adapted to local conditions teachers have in their classrooms. Because of these characteristics they are n-dimensional as described previously by Star (2010). Those artifacts allow teachers and teacher educators to share ideas about some practices teachers could develop to support quality mathematics discourse in classrooms. The PD materials were designed to be used by teachers during the PD and during their planning time. Across time,
the goal was that teachers would become familiarized with them and then start to use them as part of their routines in their planning and teaching.

Some examples of the boundary objects used in the AIM PD are the *Mathematics Discourse Matrix* (See Appendix B1), the *Mathematics Teaching Guide for Responsive Discourse* (see Appendix B2), and the responsive discourse strategy handouts (see Appendix B3 as an example) that were created for each discourse strategy analyzed in the PD. These materials are further discussed in chapter 3 when I introduce the context for my research. Here, it is important to know that The *Mathematics Discourse Matrix* summarizes four types of discourse (correcting, eliciting, probing, and responsive) in relation to four dimensions (questioning, explaining, listening, and modes of communication). The *Mathematics Teaching Guide* explains the purpose of each phase of a lesson (i.e., plan, launch, explore, discuss, and reflect) and how teachers can promote quality mathematics discourse in each one of them. The responsive discourse strategy handouts explain each strategy analyzed during the PD. In those documents, it is explained in which phase of the lesson the strategy could be used, what type of problems should be selected, considerations for its use in the classroom, support for the use of the strategy in the classroom, key ideas about the strategies, and ideas for modifications that could be implemented.

It is possible to observe in the PD materials the four characteristics defined by Wenger (1998) for boundary objects: modularity, abstraction, accommodation, and standardization. For example, the *Mathematics Discourse Matrix* and the *Mathematics Teaching Guide for Responsive Discourse* were designed to be understood by anybody who reads it (standardization). The documents are useful in all the sessions of the PD, and the
facilitators motivated participants to use them when they are making claims about some idea (accommodation). Also each of those documents attends to different ideas about how to promote responsive discourse (modularity). Finally, each document covers a specific idea about responsive discourse, and all of them are necessary to understand how to reach the goal of improving high-quality mathematics discourse with students (abstraction).

**Teacher learning in a PD setting**

Mathematics teacher education is “an emerging field of study within mathematics education” (Sztajn, 2011, p. 220). In the past years research has been conducted centered on different ideas that could help to understand how to improve teachers’ knowledge about mathematics and about pedagogy. For example, research has analyzed teachers’ learning (Jacobs et al., 2010), and some research has used a situated perspective or the importance of communities of practices to improve teachers’ knowledge (Goos, Dole, & Makar, 2007; Hodges & Cady, 2013; Kazemi & Franke, 2004; Penuel, Sun, Frank, & Gallagher, 2012; Sherin et al., 2009). Some researchers have analyzed how PD interventions could impact teachers’ learning, students’ learning, or both. Researchers have analyzed the impact of PD interventions on student achievement and teacher achievement (Antoniou & Kyriakides, 2013; McMeeking, Orsi, & Cobb, 2012; Saunders, Goldenberg, & Gallimore, 2009). Other studies have examined the use of technology during PD to improve teachers’ ability to teach mathematics (Bennison & Goos, 2010).

One study conducted by Kazemi and Franke (2004) considered teachers’ learning while a group of teachers analyzed student work. They used Cognitively Guided Instruction
(CGI) principles. Their goal was to describe teacher learning through the analysis of shifts of participation. They investigated (a) what teachers learn through collective examination of students’ work and (b) how teacher learning is evident in shifts in participation in discussions centered on student work.

The researchers conjectured that student work is a mediator of teacher learning about student thinking and about mathematics, and it helps teachers to develop their identities as educators. They defined shifts in participation as “transformation of roles and the crafting of new identities” (Kazemi & Franke, 2004, p. 205). They found two important shifts during the process: teachers started to pay attention to details of children’s thinking and the possible construction of instructional trajectories in mathematics due to the reflections teachers made about student thinking. Findings also demonstrated the creation of an intellectual and professional community that allowed teachers to share experiences that helped them to think about instructional trajectories in relation to students learning about place value.

An article by Sherin et al. (2009) analyzed what features of a video clip helped to promote teachers’ exploration of students’ mathematical thinking. One important piece of their theoretical framework was the situative perspective of learning, and learning as a social process. These authors studied “the activity of viewing video within a professional community” (p. 214). The researchers concluded that videos could support teacher learning that was connected to the practice of teaching because teachers devoted more attention to student thinking. They classified the videos according to predefined criteria that included “(a) the extent to which a video clip provides windows into student thinking, (b) the depth of student mathematical thinking shown in the video, and (c) the clarity of the student thinking
shown in the video” (pp. 213-214). For each of these categories they rated the video as low, medium, or high. Researchers were able to identify which type of videos generated less or more productive discussions among teachers.

In another article about the use of video clubs, van Es and Sherin (2008) analyzed teachers’ abilities to notice student work. For them, three important aspects of noticing are “(a) identifying what is important in a teaching situation; (b) using what one knows about the context to reason about a situation; and (c) making connections between specific events and broader principles of teaching and learning” (p. 245). They analyzed shifts in teachers’ participation during the video club meetings.

van Es and Sherin (2008) used five categories to analyze teachers’ changes: actors (student, teacher, or other), topic (mathematical thinking, pedagogy, climate, management, or other), stances teachers adopted (describe, interpret, or evaluate), level of specificity (general or specific), and video-focus (or not). Teachers noticing includes mathematical thinking, pedagogy, climate, and management, whereas stances refer to the way teachers analyzed their practices. One important result of this research is that van Es and Sherin identified three different development paths for teachers’ learning, which they described as direct, cyclical, and incremental. Researchers concluded that at the end of the video club teachers were more focused on the videos to analyze students and also mathematical thinking. Also their interventions during the sessions were more specific and less evaluative than at the beginning of the video club. These results were aligned with the anticipated hypothesis about how teachers would change.
Another study using the situative perspective was conducted by Penuel and colleagues (2012). They analyzed the effects of PD and collegial interactions on teachers’ practices. The researchers used those interactions as indicators of what teachers can learn from PD. In their theoretical framework, they included teacher learning in PD and teacher learning in school-based professional learning communities. The teachers in this study participated in a project called Local Writing Project, which was a network in more than 200 sites. One common characteristic for all the sites was teacher collaboration. The researchers used Social Network Analysis (SNA) to analyze the interactions between teachers and how those interactions can augment what teachers have learned in a PD. To reach those goals, teachers self-reported about their participation on PD during those years, the colleagues who helped them in their teaching, and the frequency with which they engaged in writing processes.

The researchers concluded that previous teachers’ practices and relationships with colleagues were important variables, and the duration of the PD affected in a positive way the level of instruction. This last variable was indirectly impacted for the interaction of teachers with colleagues who had experienced changes due to the PD. Another important conclusion is that teachers who had less instructional experience changed their practices more with the support of the PD than more experienced teachers (Penuel et al., 2012).

Hodges and Cady (2013) conducted a research study in which rural teachers participated in a blended PD program that included online synchronous, online asynchronous, and face-to-face meetings. The goals of the PD were (a) to increase middle-grade mathematics teachers’ content and pedagogical content knowledge (Thames & Ball,
2010), and (b) the construction of communities of practice that could support teachers within the same school or the same school district after the finalization of the PD.

In this research study teachers engaged in activities that included the use of a standards-based curriculum; case analysis to increase teachers’ knowledge; and time spent looking, selecting, and developing tasks to analyze students’ thinking. Instructors used print and video cases and the CGI framework during the courses.

The researchers concluded that teachers did not connect the courses with the problems they faced in the classroom in relation to students who did not have the adequate mathematical level to be in middle school. Also teachers did not appreciate the construction of communities as a way to help them to improve their teaching. The researchers observed that the mechanisms teachers used during the PD to communicate among themselves were no longer kept after the courses finished, and the goal of construction of a community of practice was not reached by some teachers. Finally, the researchers concluded that the use of standards-based curriculum materials gave teachers the opportunity to learn using their own context in their classrooms (Hodges & Cady, 2013).

Using a situated perspective, Goos and colleagues (2007) supported teachers to plan and implement mathematics investigations in relation with a new syllabus in Queensland, Australia. The study aimed to analyze factors that support or hinder teachers’ learning, to describe the PD model and its implementation, and to define implications of the project for policy and practice.
The researchers used the Valsiner’s zone model to study teachers’ learning in complex environments, which extends Vygotsky’s concept of *zone of proximal development (ZPD)* to include the Zone of Free Movement (ZFM) and the Zone of Promoted Action (ZPA). According to Goos and colleagues (2007), ZPD represents teachers’ knowledge and beliefs, ZPA is the PD strategies, and ZFM is the affordances and constraints within the professional context.

The researchers discovered that planning as a team helped teachers to reduce their workload, to learn new teaching strategies, and to open up possibilities for mutual observation and feedback. They concluded that teachers with more experience using the investigations reached better results in relation to the construction of communities. Some of the teachers who participated in the project became leaders because they tried to model new practices for colleagues. Goos and colleagues (2007) believed it to be necessary to work with teachers, heads of departments, principals, and district officers to be successful in improving teachers’ learning, and they recommended including parents and community members as well.

An article by Sztajn and her colleagues (2014) used the concepts of communities of practice, boundary encounters, and boundary objects to present an experience in which teachers and researchers shared their knowledge. For them, both actors are partners during the PD, and boundary encounters are “a mechanism for communities to exchange knowledge” (p. 201). Those authors shared the idea that learning is participation in the community, and the PD meetings are boundary encounters that promote exchange of knowledge and negotiation of meanings. One important idea in this research is that teachers
are researchers who are positioned as learners and as partners depending on the tasks developed during the PD.

Sztajn and colleagues (2014) argued that boundary objects are representations of intersecting knowledge that exist in multiple communities. In their research, they used student work, videos of clinical interviews with students, curricular materials, and a matrix with a representation of a learning trajectory as boundary objects that are shared by researchers and teachers to promote the creation of new knowledge. They concluded that “the use of research artifacts as boundary objects showed that these artifacts carried shared meanings among researchers and teachers” (p. 206). One important boundary object in this study according to the teachers who participated in the PD was the use of PD tasks in teachers’ own classrooms and the discussions they had in the PD meetings after the implementation of the task.

In another study, Horn and Kane (2015) created a research project with three groups of teachers to observe differences in Opportunities to Learn (OTL) among groups. Those authors examined five important ideas: representations, representational practices, problem framing, epistemic claims, and epistemic stances. For them, representations are artifacts to build a shared meaning of practices. Representational practices are defined as “the ways in which people use these artifacts to interpret the world. Standardized representations coupled with common representational practices affix knowledge in ways that support shared interpretations across people, time, and contexts within a profession” (p. 6). The representations are local, and they could not be generalized for all the people in the profession. To analyze representations and representational practices they focused their
attention on curricular artifacts, student work, use of technical language, and classroom talk. In this last category, Horn and Kane (2015) analyzed replays and rehearsals. Replays refer to past experiences teachers had and rehearsals are teachers’ comments about how they anticipate specific events can happen in the classroom. Frames are used to describe how people’s discourse defines themes through activities and exchanges with other people. Finally, an epistemic stance is “a position on what can be known; how to know it and why it matters for teaching, learning, and mathematics” (p. 7). They also believed that epistemic stance could be outlined through discourse. These researchers analyzed epistemic claims that teachers made during conversations to later define epistemic stances teachers had about representations.

During the analysis of data, Horn and Kane (2015) identified three groups of teachers: beginning, emergent, and sophisticated. In the sophisticated group, teachers used two or three types of actors (student, teacher, or mathematics) to make epistemic claims, whereas in the beginning or in the emergent groups the focus was more on only one actor. Also the sophisticated group established more relationships between representations of practice and epistemic claims than the other two groups.

In a study about pedagogies of practice, Sztajn and colleagues (2017) analyzed the design of their own PD program using the Grossman, Compton, and colleagues’ (2009) framework. They included analysis about the three categories defined in that framework: representations, approximations, and decompositions of practice. For that last category, the researchers connected it to nested levels of decomposition using another framework provided by Boerst, Sleep, Ball, and Bass (2011), which contains three levels: domain, organization,
and technique. Sztajn and colleagues concluded that their program was helping teachers with the three levels of decompositions of practice, and they included a new level in Grossman and colleagues’ framework that is useful to analyze work with inservice teachers. That was called controlled implementation of practice, and it is associated with specific ideas that teachers implement in their own classroom under certain controlled conditions provided in the PD setting, with the goal of providing teachers with more knowledge for teaching specific practices.

**Conceptual Framework**

To create the initial framework to guide my data analysis, I used ideas from van Es and Sherin (2008) to support my search for shifts in teachers’ participation and paths in learning in a PD setting. I used the framework proposed by Grossman and colleagues (Grossman, Compton et al., 2009; Grossman, Hammernes & Mc Donald, 2009; Grossman et al, 1999; Hatch & Grossman, 2009; Janssen et al., 2015) to characterize PD materials as representations or decompositions of practices. Finally, I used categories that Horn and Kane (2015) defined in their framework to analyze other representations that are not PD materials but that are part of teachers’ practices, such as curricular materials or representations of classroom talk. In what follows I summarize the variables defined in the prior studies that were used as the initial conceptual framework for my own research.

**Variables to Describe Teachers’ Participation in the PD and Shifts in Participation**

van Es and Sherin (2008) defined five categories to analyze the data they collected both in interviews and during the video club: actors, topic, stance adopted, level of
specificity, and video focus (or not). For this research I used four of them: actors, stances, topic, and level of specificity. The fifth category defined by those authors is not included because it is focused on whether participants’ comments are based on video or not, which is not the focus in the PD analyzed in the current study. Actors refers to the people participants are talking about. They could be students, teachers, themselves, curriculum developers, or others (van Es & Sherin, 2008). Topic comprises five different categories: mathematical thinking, pedagogy, climate, management, or other. In their research van Es and Sherin (2008) used these categories, described in Table 2, in relation to the videos teachers analyzed in their meetings to define what teachers noticed during their conversations. In relation to stances, van Es and Sherin (2008) offered three categories to define how teachers explain their practices: describe, interpret, and evaluate. A teacher comment is labeled as describe if they only recall what they observe in the video. Teachers evaluate when they explain what is acceptable or unacceptable in the video according to their point of view. Also evaluation refers to comments about how to change what they observe. Finally, interpret occurs when teachers share implications about what they have observed in the videos. The level of specificity is attached to teachers’ comments about details they noticed during the video clubs. It could be general or specific depending on teachers’ ideas (van Es & Sherin, 2008).

Table 2

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical thinking</td>
<td>Mathematical ideas and understandings</td>
</tr>
<tr>
<td>Pedagogy</td>
<td>Techniques and strategies for teaching the subject matter</td>
</tr>
<tr>
<td>Climate</td>
<td>Social environment of the classroom</td>
</tr>
</tbody>
</table>
Table 2 Continued

<table>
<thead>
<tr>
<th>Management</th>
<th>Mechanisms of the classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other</td>
<td>Different topic than the four previously defined</td>
</tr>
</tbody>
</table>


**Learning in workgroup conversations.** I used variables from the category representations from Horn and Kane’s (2015) framework. They defined representations as artifacts that could be verbal or physical and that are used to develop a common meaning of practices among teachers. Their framework included four categories for representations: curricular artifacts (e.g., a textbook), student work, use of technical language, and stories of classroom talk. I am not focusing on Opportunities To Learn as those authors were, but I analyzed how curricular artifacts, student work, and stories of classroom talk were included in participants’ conversations during the PD. In stories of classroom talk, they focused their attention on two conversational representations: replays and rehearsals. Teachers’ replays refer to discourse that teachers use to discuss previous experiences in their classrooms; in teachers’ rehearsals they anticipate situations that could happen in their classrooms. In both cases teachers may narrate the situation or act it out (Horn, 2010; Horn & Kane, 2015). Replays and rehearsals “laminated the world of the classroom onto the world of the teacher community, positioning the teachers toward students and classroom situations in ways that were consequential for learning” (Horn, 2010, p. 232).

**Summary**

The initial conceptual framework created based on the literature review is summarized in Table 3. Representations is a common category in Grossman and colleagues
(Grossman, Compton et al., 2009; Grossman, Hammernes & McDonal, 2009; Grossman et al., 1999; Hatch & Grossman, 2009; Janssen et al., 2015) and Horn and Kane’s (2015) frameworks. In this summary, I included the representations provided by Horn and Kane (2015), which are associated with artifacts that teachers mentioned when participating in PD conversations. In Chapter 3 the use of Grossman and colleagues’ framework is explained.

Table 3

<table>
<thead>
<tr>
<th>Summary of Variables Analyzed</th>
</tr>
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<tbody>
<tr>
<td><strong>Category</strong></td>
</tr>
<tr>
<td><strong>Actors</strong></td>
</tr>
<tr>
<td><strong>Topic</strong></td>
</tr>
<tr>
<td><strong>Stances</strong></td>
</tr>
<tr>
<td><strong>Level of specificity</strong></td>
</tr>
<tr>
<td><strong>Representations</strong></td>
</tr>
</tbody>
</table>

*These variables were codified and analyzed but are not reported in results because nothing important was found.

This framework supported the analysis of changes in teachers’ participation in the PD through the detailed examination of conversations that happened in the sessions. I analyzed
the ways in which teachers talked about different actors, topics, stances and what was the level of specificity they used to talk. I analyzed interactions between those variables to establish how teachers’ participation in the PD changed across time. The study used several variables from previous work on noticing and expanded the use of such variables beyond attention to notice to characterize participation more broadly within the PD context. This approach supported the research goal of understanding how teachers learn including several variables that allow for the analysis of the relationship between knowing and the context in which learning happens. As part of the analysis of the context, the role of the PD materials as boundary objects was also included through the analysis of how these materials impacted teachers’ participation in the PD.
Chapter 3: Methodology

In this chapter I first present my refined research questions. Then I describe the context for the study (AIM project), research design, selection of the case, data collected, and how the information was analyzed. Finally, considerations about validity, reliability, and limitations are discussed.

Research Questions

The goals for the current study were to understand how teachers participated during a PD intervention and what they learned in this process. Also the study analyzed how this learning process was impacted by the PD materials. To analyze those ideas, I work with data from project AIM. Based on the framework discussed in the previous chapter, my research questions were defined as follow.

1) In which ways did teachers participate in AIM PD intervention? What shifts took place in teachers’ participation during the AIM PD intervention?

2) What was the role of the AIM PD materials as boundary objects in the AIM PD intervention?

3) In which ways were teachers’ shifts in participation related (or not) to the boundary objects used in the AIM PD?

Context for the Study: Project AIM

The AIM PD and the research associated with it was funded by the National Science Foundation as part of a 5-year project on elementary mathematics education. Participants were second grade teachers who participated in the 40-hour yearlong intervention. This
project focused on promoting quality mathematics discourse in mathematics elementary
classrooms and in improving mathematical knowledge for teaching (Starling, Troki & Sztajn,
2013). The quality mathematics discourse promoted in the AIM PD is called responsive
discourse, and it “supports the development of rigorous mathematics through collective and
assisted participation in content-rich and goal-focused mathematical conversations” (Sztajn,
Malzahn, & Heck, 2012e; see Appendix A1).

The PD was delivered in 13 sessions, 6 of which were in a Summer Institute that was
implemented in 3 full days (see Table 4). During those sessions participants built the
foundational ideas that support the program (Sztajn et al., 2013a). The other 7 sessions were
offered during the school year, and during that time they continued building “on the
programs’ foundational ideas, develop participants’ mathematics knowledge for teaching,
establish additional discourse strategies and moves for teachers to use in their mathematics
instruction” (Sztajn et al., 2013a, p. 2).

In total, nine discourse strategies that were initially used in literacy were analyzed and
rehearsed by participants during the PD sessions or in participants’ own classrooms: All
Talk, Turn and Talk, Think-Pair-Share, Think Aloud, Talk Triangle, Talk Chain, Bet Lines,
Draft and Final Copy, and Probing & Pressing Questions (e.g., see Dick et al., 2016 and
Trocki, Taylor, Starling, Sztajn, & Heck, 2015). For each discourse strategy teachers were
provided with a handout after they rehearsed the strategy (see Appendix A.3 as an example
of a strategy handout for the All Talk strategy).
Table 4

**Description of the AIM PD**

<table>
<thead>
<tr>
<th>Session</th>
<th>Topic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Types of Mathematics Discourse</td>
<td>Building foundational ideas around mathematics discourse</td>
</tr>
<tr>
<td>2</td>
<td>Importance of mathematics knowledge for promoting responsive discourse</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>The Mathematics Teaching Guide for Responsive Discourse and the Think Aloud strategy for the launch phase of the lesson</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>The Talk Triangle strategy for the Explore phase and the Talk Chain strategy for the discuss phase of the lesson</td>
<td>Launch, Explore, Discuss: Discourse Strategies</td>
</tr>
<tr>
<td>5</td>
<td>Using strategies in the launch, explore and discuss phases to engage in responsive discourse around subtraction for teachers</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Cultivating a Responsive Discourse Community in your mathematics class</td>
<td>Establishing Norms</td>
</tr>
<tr>
<td>7</td>
<td>Selecting responsive discourse promoting tasks</td>
<td>Task Selection</td>
</tr>
<tr>
<td>8</td>
<td>Examining different problem types</td>
<td>Problem Types</td>
</tr>
<tr>
<td>9</td>
<td>More strategies: Bet Lines for the launch phase and Draft &amp; Final Copy for the explore phase of the lesson</td>
<td>Launch, Explore, Discuss: Discourse Strategies</td>
</tr>
<tr>
<td>10</td>
<td>Asking Probing and Pressing Questions in the discuss phase of the lesson: pushing for the mathematical goals</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Examining instructional moves for promoting responsive discourse</td>
<td>Instructional moves</td>
</tr>
<tr>
<td>12</td>
<td>Preparing and Planning for Responsive Discourse</td>
<td>Planning</td>
</tr>
<tr>
<td>13</td>
<td>Revisiting foundational ideas and strategies for promoting discourse in mathematics.</td>
<td>Closure</td>
</tr>
</tbody>
</table>

During the PD, teachers were also were with opportunities to improve their Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008) through the use of four different tasks. Two of them were about students’ work. In Isabella’s method task participants had to examine the way in which a student, named Isabella, solved the multidigit subtraction problem 467-148, and in the task Maria’s work participants analyzed the solution that Maria gave to the problem 92-57. In the Ferrero Chocolates Math Problem teachers solved as adult learners a base 3 problem. Teachers also analyzed different story problems to explore different problem types. During the discussion of those tasks, participants discussed deeply how students solved the task, and the facilitator guided them to analyze different representations that could be used to solve the same subtraction problem.

The PD made used of classroom representations in the form of videos and written representations. For each video provided by the PD, a transcript was provided to participants for their use during whole-group discussions. Sometimes the PD only provided a written representation of the classroom and designers called them scenarios, case narratives or readers’ theaters (see Appendix A4). When a written representation of a classroom was provided, it was expected that teachers used them to show evidence of the ideas they wanted to share. All those classroom representations were used with different purposes. Sometimes participants analyzed different patterns of discourse, teacher moves, teacher purpose for that lesson, how the teacher implemented a discourse strategy in the classroom, or how the teacher set up norms with her students. In addition, the written representations were used to prompt teachers to reflect about the importance of mathematical knowledge in order to be
able to both understand students’ mathematical thinking and generate responsive discourse in their classrooms.

The Mathematics Discourse Matrix (See Appendix A1) includes ideas about discourse, decomposing them by type and dimensions. The Mathematics Teaching Guide (See Appendix A2) includes the decomposition of the phases of a lesson, and connects ideas to discourse. The written representations were used to make connections to the foundational documents. Participants analyzed both the dimensions and the types of discourse contained in the Mathematics Discourse Matrix, or phases of a lesson as described in the Mathematics Teaching Guide.

**Research Design**

This current study is defined as a single case study. According to Creswell (2013), a case study is a “qualitative approach in which the investigator explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time through detailed, in-depth data collection involving multiple sources of information, and reports a case description and case themes” (p. 97). Merriam and Tisdell (2016) defined a case study as “an intensive, holistic description and analysis of a single, bounded unit” (p. 232-233). With this approach, the researcher makes sense of the data to understand and describe the case (Merriam & Tisdell, 2016). The analysis includes many variables of interest, and uses theoretical propositions to generate in-depth knowledge about the case (Simmons, 2009) through the analysis of the interaction of many factors in the case (Thomas, 2011).
The current study followed the criteria of embedded mixed-methods design, in which
the researcher adds a quantitative analysis to a qualitative design or vice versa (Creswell &
Plano Clark, 2011). For the analysis of the case of interest, most of it was qualitative, and the
quantitative analysis was used to triangulate data. Both analyses were used to draw
conclusions and to define implications of this case study.

Selection of the Case Study

For the current study I used data from one cohort from Project AIM Year 3 (2012-
2013). It included 20 female teachers from 4 different schools. This cohort was facilitated by
two female experienced teachers and coaches.

The current case study was bounded by the selection of the cohort. To select it, I
watched a few videos of two cohorts in which videos of 13 sessions were available. I selected
the cohort in which the two facilitators provided teachers with more possibilities to
participate in the whole-group work according to the guidelines defined by the AIM PD as
important to generate quality mathematics discourse, such as the group of teachers were
talking to each other, and the facilitators removed themselves from the conversations
providing teachers with opportunities to talk more.

My goal was to describe in details the ways in which teachers participated during the
PD intervention and determine whether there were shifts in teachers’ participation in the PD.
I also analyzed how the PD materials helped teachers to learn. In particular, I observed
teachers’ discourse in whole-group discussions to define shifts in teachers’ participation in
the PD. On the basis of my literature review, I defined my conceptual framework, which
allowed me to define many variables that I used to analyze my case. My goal was to observe
how all those variables were connected to my case to get a holistic view of the situation. My design was flexible enough to be adapted when it was necessary. Following Thomas’ (2011) definition, my case was defined as *diachronic* because my interest is to identify changes over time in teachers’ participation in the PD.

**Data Collection**

The data I used for the current study was previously collected by the Project AIM research team during the implementation of the PD and before I worked as a research assistant on this project. I used videos of the 13 sessions to determine the time stamps for all whole-group conversations participants and facilitators had. I included in my data the whole-group discussions that were planned by PD designers, and the whole-group discussions that were added by the facilitator as they implemented the sessions.

The PD provided participants with opportunities to experience the discourse strategies they were learning, and sometimes the whole-group discussions were organized following a specific discourse strategy. I included all those discussions as part of whole-group conversations. In total there were 71 different whole-group discussions, comprising 10 hours 38 minutes of video (see Table 5).

**Data Analysis**

To analyze the videos, all the whole-group discussions were transcribed verbatim. I coded the transcripts using *idea units*, which are fragments of teachers’ discourse during the PD sessions about a specific idea (Jacobs & Morita, 2002; van Es & Sherin, 2008). An idea unit can also be defined as “distinct shift in focus or change in topic” (Jacobs, Yoshida, Stigler, & Fernandez, 1997, p. 13). The use of idea units allowed me to analyze participants’
ideas within a topic in relation to the variables I defined for my analysis. My goal was to analyze interactions among teachers and between teachers and materials to describe possible shifts in participation.

Each of the 13 PD sessions was examined to identify ideas units, for a total of 282 (see Table 5). In 15 idea units it was impossible to connect what participants said to the whole-group discussion for two reasons: (a) it was inaudible or (b) the comment was something general that the facilitator or a participant said and that was not connected to the discussion. For those situations, the idea unit was coded as Inaudible or Nothing important and they were not included in the current analysis.

Table 5

<table>
<thead>
<tr>
<th>Session</th>
<th>Duration</th>
<th>Total number of idea units</th>
<th>Idea unit coded as Inaudible or Nothing Important</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22:30</td>
<td>13</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1:09:15</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>48:03</td>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>55:21</td>
<td>28</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1:15:10</td>
<td>34</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>1:13:01</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>39:14</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>45:03</td>
<td>31</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>31:49</td>
<td>19</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>31:18</td>
<td>13</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>1:01:41</td>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>1:01:05</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>24:48</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

| Total   | 10:38:18 | 282                         | 15                                            |

After the codification of all whole-group discussions, different ways to organize the data were used to establish findings and to draw conclusions and make recommendations. In
this process, the software Atlas.ti was used. The results of the analysis are reported in the next two chapters. It is important to highlight that the codification is essential because it is the link between the data collected and the meanings that the researcher assigns to it (Saldaña, 2013).

**Development of the Codebook**

The codebook developed to analyze the data in this study includes the list of codes, their definitions, and a set of examples from the current data (DeCuir-Gunby, Marshall, & McCulloch, 2011). The codebook provides an explanation of how the codes were used (see Appendix B). The codebook was developed in three phases.

**Phase 1.** First, I used the initial conceptual framework described in Chapter 2, which was based on previous work (Horn & Kane, 2015; van Es & Sherin, 2008). In the category actors, I followed van es and Sherin’s ideas (2008), and I used dimensions to define the number of actors included in an idea unit. When no actors are included in the conversation, it is defined as zero-dimension; when only one actor is described in the idea unit, it is defined as a one-dimensional actor. If two actors are included it means two-dimensional actors, and a similar way works for three actors (see Appendix B). No dimension was used when the idea unit contains comments about a specific actor without analyzing how many other actors were included in the conversation. For the category Representations, one subcategory was defined as own classroom representations, which included curricular artifacts, student work, and stories of classroom talk (Horn & Kane, 2015; van Es & Sherin, 2008). The categories stances and level of specificity were used as defined by van Es and Sherin (2008).
Phase 2. In the second phase to develop the codebook, I included codes according to the specific characteristic of the AIM PD. For that reason the category PD focus and a new category and subcategory to include the PD materials were added.

The PD focus is the main topic defined in the power point prompt or in the question the facilitator asked participants during that specific conversation. Because of the PD design, some parts of the sessions were clearly directed to specific goals, as for example, making arguments about a specific question using a particular video or transcript, or the focus of the session was mathematics. For this reason, to compare idea units among variables, the idea units were coded according to the topic defined for each specific discussion. The PD focus includes four categories: Discourse Focus, Mathematics Focus, Pedagogy Focus, or Other Focus. The first three categories were identified from the topics of discussion defined by PD designers. The category Other was included because sometimes facilitators asked a question that was not included in the plan of the discussion.

The Discourse category was used when the prompt provided by the facilitator was directly connected to discourse. Examples of this type of focus happened when the facilitator asked for evidence of discourse, or similarities and differences in patterns of discourse in different classroom representations, or connections to the Mathematics Discourse Matrix or discourse strategies. The Pedagogy focus was used when the facilitators were using connections to the process of planning a lesson, to the Mathematics Teaching Guide, or to teacher’s purpose in a classroom representation. In the category Topic, I analyzed the subcategories defined by van Es and Sherin (2008) and I included three more: Discourse, Mathematical Knowledge for Teaching, and Mathematics, to include important topics
analyzed in the PD. I did not find important results about Climate and Management, (van Es & Sherin, 2008), and then I eliminated them for the last version of the codebook included in the current report.

In relation to PD materials, I used two of the categories defined by Grossman and colleagues’ framework (Grossman, Compton et al., 2009; Grossman, Hammernes & Mc Donald, 2009; Grossman et al, 1999; Hatch & Grossman, 2009; Janssen et al., 2015), which are representations and decompositions of practice to describe how participants used PD materials and roles they played in teachers’ participation in the PD. The PD used three types of representations of practice: responsive discourse strategy handouts, mathematics problems sheets, and classroom representations in the form of videos or transcripts. The PD utilized classroom representations in the form of videos and written classroom representations to illustrate teachers’ and students’ work in the classroom or how a specific discourse strategy could be implemented. In addition, the PD uses samples of students’ work or mathematics problems sheets to build participants’ mathematical knowledge. Those PD materials could be characterized as representations of practice. The decompositions of practice are the foundational documents: the Mathematics Discourse Matrix, and the Mathematics Teaching Guide. The PD materials were also coded by dimension: an idea unit was coded as one-dimension when only one PD material was used during conversations in whole group discussions; more than one-dimension was used when two or more PD materials were used by participants during the discussion.

**Phase 3.** In the final phase to development the codebook, I used open coding by constant comparative methods to identify new codes (Creswell, 2013; Merriam, 2009). From
that analysis emerged the code *teaching concerns*. This code was used when teacher made comments about difficulties they have or could have to plan a lesson, implement the lesson or evaluate students’ learning. Sometimes their concerns were associated to curricular artifacts or assessments.

**Qualitative Analysis**

Using a qualitative approach, I analyzed the problem using deductive and inductive methods to define important patterns that were found in the data (Creswell, 2013). In addition, the final report satisfies the characteristics of a qualitative study, which “includes the voices of participants, the reflexivity of the researcher, a complex description and interpretation of the problem, and its contribution to the literature or a call for change” (Creswell, 2013, p. 44).

The PD intervention is defined as natural because it is the regular setting where teachers met to improve their mathematical and pedagogical knowledge to teach mathematics. This is another important characteristic of a qualitative study. In this study, the researcher is a key part of the research, and her interpretations and both inductive and deductive analysis are important for the process. For Creswell (2013) the qualitative research happens in a social, political, and cultural setting, as is the case in a PD setting, in which every participant brings their own background and experiences to the PD discussions. This dissertation reports important conclusions that help mathematics educators understand how teachers learn during PD interventions.

Simmons (2009) pointed out that having a previous theoretical framework could lead the researchers to not observe important unexpected conclusions that could emerge from
data. For this reason, I used constant comparative method analysis as part of the design to be open to new ideas that could emerge from my data. This method is defined as “an iterative and inductive process of reducing the data through constant recoding” (Fram, 2013, p. 3). The constant comparative method allowed me to define a new code that was not included in the initial framework (teaching concerns) and that was identified as an important change in teachers’ participation in the PD.

Quantitative Analysis

The software Atlas.ti was used to analyze the qualitative data, and at the same time to generate summarized quantitative data about the codes that I generated based on the conceptual framework. This quantitative data was used in two ways:

1) To define which variables could be the most important in the data for further qualitative analysis. For example, the quantitative data was analyzed to define which actors were more important during the PD discussions, and those variables were followed with more detail during the qualitative analysis. Also, the software calculated coocurrence of variables, which allowed the researcher to define which combinations of variables were more important.

2) To triangulate qualitative results a Spearman rank correlation test (Hayes, 2009; McDonald, 2014) was used to analyze the strength of the covariation between variables in the current study. The goal was to analyze changes across time. This test determines whether the change was positive, negative, or no change happened. To run the test the data was organized by whole-group discussion (71 in total). The number of times each code occurred in each whole-group discussion was calculated. The test defined the
monotonic association between time in the PD and the number of times a code was observed in each whole-group discussions. I used the null hypothesis of no correlation between time and the code. Results of this data are discussed in the next two chapters.

**Validity and Reliability**

To ensure the validity and reliability of the study it is important to be careful with the analysis and interpretation of data and also with the conceptualization of the study (Merriam, 2009). The internal validity of the research can be built using pattern matching and building explanations (Yin, 2014). The current study used both of those strategies, and examples of data are shown as evidence supporting my conclusions.

To reach validity and reliability it is important to adequately engage in data collection and the researcher should reflect critically about the data (Merriam, 2009). This step was followed in the current study, and this practice was promoted and supported by the dissertation chair.

During the process of coding the data, multiple cycles of coding were necessary as the researcher refined the codes with the support of a second researcher who coded 20% of data for reliability purposes. For each of the codes included in the codebook, the two researchers worked to achieve intercoder agreement of at least 85%. During this process, both researchers analyzed the definitions of the codes in which the inter-coder reliability percentage was below 85% to refine the definitions in the codebook. When the inter-coder reliability reached 85% or higher for each code, the whole set of data was recoded again to
guarantee the correct use of codes. For the second round of coding the whole set of data, I used the videos and transcripts in order to improve the coding of the use of the PD materials.

This process of codification of the information was the first step to analyze and interpret the data. The information was analyzed and reorganized to establish patterns that helped the researcher to understand how teachers learned during the PD, if there were shifts in their participation, and how the materials were used in that process (Saldaña, 2013). The emerging themes and decisions made during this process were supported by my dissertation chair, who is an experienced researcher. She provided constant feedback and also served in the role of auditor, requiring justification and explanations for my themes. She also provided advice about the way the information was codified, organized and about how the results were reported in the current document.

Another important factor to reach reliability is the use the audit-trail method, in which the researcher records all the decisions that were made during the data collection and data analysis (Merriam, 2009). For a case study, Yin (2014) suggested the use of a case-study protocol and case-study database to document all the decisions the researcher makes and all the procedures that the researcher follows during the process. I used the memo manager provided by the software Atlas.ti and a notebook to keep track of all the decisions that were made.

The analysis of this case study aims to contribute to the field of Mathematics Education by providing a more complete view of how teachers learned in a PD and the role of PD materials in the process (Yin, 2014). The single case could give valuable information
to the reader that could be used in other contexts (Merriam, 2009). The quantity of data used
to make the analysis (more than 10 hours of video) collected across 1 year is also a strength
of the study.

**Limitations of the Study**

Qualitative studies in general and case studies in particular also depend on the
previous experience and current situation of the researcher because that person is the
principal instrument for data collection and for the analysis of the data; “the investigator is
left to rely on his or her own instincts and abilities throughout most of this research effort”
(Merriam, 2009, p. 52). I am a novice researcher, and this could be an important limitation,
although it was diminished due to my dissertation chair, who provided support and guidance
during the process.

Another limitation of this study was the quality of the videos. Sometimes it was
difficult to hear what participants said and that limited the quality of the coding. After
Session 6, the camera’s focus was fixed, and frequently it was impossible to identify the
teacher talking. This problem is visible in the quotes provided in the next two results
chapters. When only T is shown instead of T with a number to identify the teacher, it was
impossible to identify the person talking for that data point.
Chapter 4: Results About Teachers’ Participation in the PD

This study examines mathematics teachers’ participation in an AIM PD intervention that focuses on the use of responsive classroom discourse. The findings of this study are reported in two chapters. This chapter addresses Research Question 1: In which ways did teachers participate in AIM PD intervention? What shifts took place in teachers’ participation during the AIM PD intervention? Chapter 5 addresses Research Questions 2 and 3: What was the role of the PD materials as boundary objects during the AIM PD intervention? In which ways were teachers’ shifts in participation related (or not) to the boundary objects used in the AIM PD? Together, the two results chapters suggest ways in which participants learned in a PD setting.

The data on teachers’ conversational participation during the PD revealed actor was the variable with the higher frequencies in the discussions in which teachers participated (see Appendix C). Three actors were important in this process: student, self, and teacher. For that reason, the results in Chapter 4 are presented by actor, connecting them to the variables defined in the codebook: PD focus, topic of discussion, stances, level of specificity, and own classroom representations.

The first section in this chapter analyzes all idea units that included students, encompassing all the combinations of actors associated with students (students, students-self, students-teacher, and students-self-teacher). The second section reports the results regarding teachers’ discussions of themselves and their own teaching. This section focuses only on the results about the actor self that do not include students (self, self-teacher). Finally, the third section presents the results about how participants talked about teachers. It describes only
results about the actor teacher by itself given that all other combinations for this actor are included in previous sections.

In the second part of the chapter, four shifts in teachers’ participation in PD conversations are reported; three are connected to students, and one is about the actor self. Those shifts are related to the way participants talked about: (a) students and themselves in mathematics focused discussions, (b) students and teachers in discourse focused discussions, (c) students (no dimension) and level of specificity, and (d) self (no dimension) in relation to their teaching concerns.

In this chapter, Tables 6 through 11 are included within the text; full version of these tables, as well as additional tables that support the results described here are provided in Appendix C. The most important tables are within the text. When the text is associated with two tables generally they are in the Appendix C. To designate the different locations, tables with a letter C in their titles are in Appendix C, and tables with only a number are within the text.

To guide the reader, after each quote the number of the session in which that conversation happened was added in parenthesis. It is also important to clarify that the majority of quotes provided as examples in this chapter and the next one are segments of idea units and not the whole idea unit. The segments were chosen to illustrate the idea in discussion and the whole idea unit is sometimes not included because they are long, or part of it is not important for the discussion.
Student as Actor

Participants in Project AIM talked about students early on in the PD, and they continued to do so throughout the year. Participants talked more about students as a two-dimensional actor; that is, when talking about students, they included one other actor within the same idea unit, with more connections to themselves than to teachers. The comments about students and teachers were observed more frequently during the Summer Institute than during Sessions 7 through 13 (see Table C1).

During discussions focused on discourse, both combinations (students and self, and students and teachers) were used in an almost equal number of idea units (see Table 6). Participants talked about decisions a teacher made during a lesson they analyzed, or they addressed how a teacher modeled a discourse strategy. They also analyzed students’ mathematical thinking or students’ roles in the classroom. The following excerpt is an example illustrating how the discussion was focused on discourse, and how participants talked about students and the teacher.

F1: What kind of environment do you think that is for students who can do math?
T: Responsive discourse.
F1: Responsive discourse? You think it’s a comforting environment or space? How did they get to [inaudible] the environment though? Do what, T2?
T2: Because with her, she’s allowed them to be able to just talk and discuss and then they feel comfortable with math. Because the atmosphere or the environment she’s created.
T15: Take the ball and encourage them to do the things that she’s modeling, scaffold and encourage them to continue the process (Session 6).

---

1 T without a number is used when it was not possible to identify the teacher talking.
In this excerpt, participants analyzed the role of the teacher to generate conversation in the mathematics classroom. They discussed how the teacher was guiding students to follow what she had modeled for them.

Table 6

*Actor by Dimension and PD Discussion Focus*

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Actors</th>
<th>Discourse (135)</th>
<th>Mathematics (61)</th>
<th>Pedagogy (52)</th>
<th>Other (22)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Student</td>
<td>23</td>
<td>16</td>
<td>6</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>Student-Self</td>
<td>33</td>
<td>22</td>
<td>13</td>
<td>6</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>Student-Teacher</td>
<td>34</td>
<td>4</td>
<td>9</td>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>Student-Teacher-Self</td>
<td>14</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Self</td>
<td>66</td>
<td>35</td>
<td>29</td>
<td>12</td>
<td>142</td>
</tr>
<tr>
<td></td>
<td>Teacher</td>
<td>53</td>
<td>7</td>
<td>20</td>
<td>5</td>
<td>85</td>
</tr>
</tbody>
</table>

*Note:* The number in parenthesis represents the number of idea units for each discussion focus.

Participants talked less about students, teachers, and themselves within the same idea unit than about the combinations of students and teacher or students and themselves.

Conversation including the three actors in the same unit took place more frequently when the focus of the discussion was discourse. In those discussions, participants often talked about what they had noticed in a PD classroom representation, and they made comparisons to their own experiences. Participants also reflected on how the ideas discussed in the PD could be used in their own classrooms or how they would react to specific situations. An example of those types of discussions are below. T24 made that reflection as a member of a small group, and her comment reflected the opinion of other members of the group.

**T24:** We had some of these same conversations within our group, and we were talking about, we wonder if when she was doing the modeling, if she had asked kids to restate what she said, if that would have engaged students some. We’ve talking
about some groups that would have been very engaged the whole time, you know, hanging on to every word you say because they really want to meet your expectations, and we’ve had groups of students that have had a large amount of, you know, students that are far too engaged and they probably would miss a lot, so it kind of just depends on what group that you have. But if you have a group that is more engagement than having them restate (Session 3).

Even though the use of the three actors was the less frequent combination, the above example shows that when participants used the three actors they made connections between what they were experiencing in the PD and their own experiences. This type of reflection happened almost every time they talked about the three actors within the same idea unit. They were thoughtful about how they visualized a specific situation in their own classroom, or something that already had taken place in their own classroom. Sometimes the prompts asked participants to connect to practice, but frequently the prompt was designed to focus on students and the teachers.

**Topic of Discussion**

The most common topics of discussion during the conversations associated with students (no dimension) were discourse, students’ mathematical thinking and pedagogy (see Table 7).

<table>
<thead>
<tr>
<th>Table 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Actor by Dimension and Topic of the Discussion</strong></td>
</tr>
<tr>
<td>Dimension</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
Table 7 Continued

<table>
<thead>
<tr>
<th></th>
<th>Students-Teacher-Self</th>
<th>18</th>
<th>2</th>
<th>13</th>
<th>5</th>
<th>19</th>
<th>6</th>
<th>4</th>
<th>67</th>
</tr>
</thead>
</table>
| No      | Students-
| dimension|                       | 98 | 13| 91 | 41| 87 | 40| 32| 402|

Note: Math is mathematics.

The below excerpt is an example of how participants talked about students. A facilitator and participants were discussing how their mathematics instruction changed due to their participation in the PD. A prompt motivated participants to use their last homework to provide evidence of changes. In that homework participants had planned a lesson, implemented it, and reflected about the experience. During the conversation about the changes they had made, participants underscored different topics within the same idea unit: discourse, mathematical knowledge for teaching, students’ mathematical thinking, and pedagogy. They talked about students and themselves.

F1: Alright. So let’s stay with planning just for a minute, and I’m gonna ask this question. How do you, and you said your planning has changed. I know three people said that, but I think more people, that’s the feeling. How do you think that came about? How did your planning change over the course of this time? How did that happen?

T1: I think the discourse strategies. When we started learning about those and how we were gonna implement those in our lessons, I think, well, for me anyway, I looked at the strategy and thought about thinking about what I wanted to see and then picking out strategies that would help me to that goal, help get me to that point. And so that specifically how it changed me, and that’s what I was putting in my plans, all talk launch, talk triangle, whatever, and I felt like those lessons were a lot more, not that I added more detail in my writing up of the lesson, but I knew exactly where I wanted to go with each part of the lesson, each phase of the lesson because of those strategies.

T15: I thought in the discussion that they have, and when I go around as facilitator and I listen to what they have to say, I can see where breakdown were occurring or where understanding was misguided, and then it can change what I want to do the
next day, or I can say, “okay, wait, we did this lesson,” and I can tell by the way they’re talking to each other and strategies they’re talking about and things that they’re saying to each other, or the questions they’re not asking each other, that they just don’t get it, and so I need to back up in my lesson planning and instead of moving forward to my next day, I need to go back and do that day again. Come up with another word problem, or another whatever, example, that we need to work on so that we can go through this again, so it’s kind of helped me see where the holes are in their understanding when they talk to each other, just through their conversations. T6: You just don’t move forward. You stop to come back and like she said, you reflect a lot more. I think with me, with my planning, I reflected a lot more on where they’re going, what they truly understand, understood, or the grasp from it before I decide where we’re going next (Session 13).

In these reflections, the three teachers showed that they felt more confident about the use of the discourse strategies and how to integrate them into the lesson. They reflected about their mathematical knowledge for teaching when they said they felt more comfortable making decisions during the planning, implementation and reflection of a lesson, and how this impacted their future work with students. They explained how the analysis of how students were thinking and talking during the lesson became important to making decisions about what they would do next. They also talked about pedagogical issues when thinking about their decisions about the planning, implementation, and further reflection about the lesson and their role as facilitators.

**Teaching concerns.** When participants were talking about their teaching concerns, they talked more frequently about students and themselves (see Table 7). During those discussions, participants expressed their questions and doubts about how to implement discourse strategies, how much time they would need to plan a lesson, and how students would react to a specific situation among others. For example, one teacher was hesitant about her preparedness on planning a lesson with the quality that she expected.
T15: I also liked the questions at the end because one of my biggest concerns and questions about this is how do I make sure that I am planning ahead and I am thinking carefully about what I need to do, and then as a learner myself, how do I practice this enough and have it become just more of who I am and what I do on a regular basis, just like I want the kids to do talk triangles like a normal thing, and I worried about the planning part for me being a thing that I can just do and it becomes a natural progression of who I am as a teacher and what I want to do, so I really liked the questions, because I felt that was kind of like a way to scaffold that learning for me and provide an opportunity for me to look at those questions and hopefully give myself a path to make it a more natural occurrence (Session 5).

This quote shows how T15 expressed her interest in integrating the ideas they were discussing in the PD in their work, but at the same time she verbalized the challenges she observed in that process. Her reflection is about how material the PD provided to read as homework the previous day, gave her ideas about how to overcome the challenges.

When participants made comments about students and the teachers, the most common topic in conversations was discourse. This type of conversation was observed when participants and facilitator were analyzing PD classroom representations. They talked about decisions the teacher made during the planning, or during the lesson. They also analyzed the role of the teacher during classroom discussions. In many idea units participants paid more attention to the teacher than to what students were doing. This way of talking changed later, which will be reported in the section about shifts in teachers’ participation in conversations in the PD.

**Stances**

**Describe and interpret.** When participants were talking about students (no dimension) they more frequently described and interpreted. They evaluated less often (see Table C2). When the focus of the conversation was mathematics or discourse, participants described and interpreted students’ mathematical ideas. Participants analyzed what they
watched or read in a PD classroom representation, or their own experiences with their students to talk about how students were thinking mathematically. They talked about how students could solve the task, how students could be limited because of their own knowledge, or level of development. Participants also talked about what students understood or did not understand about a mathematical topic. They made comments about their own expectations or their experiences with second graders or ELL students. For example, T9 reflected about students’ mathematics development.

T9:  We said it felt it was developmental. If you were a young child, and you would crawl and then you would walk and then you would run. It’s the same for them, they have to start with the counting on strategies before they even get to the, you know, instant answer and the subtraction. They just can’t move through it and get to that answer without thinking through the other steps because it’s like scaffolding, it builds upon itself (Session 8).

This comment occurred during a pedagogy focus conversation in which T9 interpreted how she visualized kids learning mathematics by making an analogy through the description of kids’ gross motor development.

As early as in Session 2, when participants were talking about students (no dimension) they interpreted what they could do in their own classroom with the new information they were learning in the PD. They also asked questions such as whether it was a good idea to teach students the mathematical technical vocabulary they were learning in the PD. This way of thinking was preserved across the PD.

Participants commented more on students and themselves when their discussions were focused on pedagogy or mathematics (see Table C2). During mathematics discussions, participants talked in a similar way as they did for students, describing and interpreting what
students were doing in the classroom representation, but they combined this information with their own solution to the task. This type of discussion was promoted by the PD and in particular by the facilitator in order to analyze different representations for the same problem. It was expected that participants would compare students’ solutions to their own work, and to other participants’ work as well. One example of this type of comments happened when participants were discussing a subtraction problem.

T6: It’s the bottom one, what I did was I started at 173, and then I added to 173, I added three to get to 176. Then I counted by tens until I got to 226, which gave me 50, and then I just went to my next hundred.
F2: T11, will you restate what she did?
T11: Started at 173, and added three to get to 176, she skipped counted by tens and to get to 226, and then she added 100 to get 326.
F2: Why would she try to get to 176? She started at 173.
T9: [inaudible]
T: To get to her ones. The final answer has a six in the ones place, whereas most of us – because Isabella went up to 150, got up to the nearest ten, we all automatically, “let’s go up seven and beyond be there.” She saw, oh, I need to be at six at the end, why don’t I just go ahead and get there? So why’d you go to 226?
T6: Because I just added one to skip counting by tens, and I knew once I got to 226, I could just move one big hundred (Session 2).

During the above discussion, the facilitator asked another participant to restate what T6 had said as a way to confirm that everybody was understanding her interpretation of how Isabella solved the problem. T11 and T described what Isabella did, but T also interpreted differences in how participants in the PD solved it in comparison to Isabella’s solution of the problem.

When the discussion was focused on pedagogy, participants reflected about their own experiences in the classroom, explaining what they had implemented in their own classrooms and whether that had worked. They also anticipated how a particular idea could work in their own classroom or the challenges they expected.
**Evaluate.** Almost all the evaluative comments participants used were to make a positive judgment about students (no dimension). Examples of those evaluative comments were: “He did a great job” (Session 1), “That was a good show of his knowledge” (Session 2), “That was actually really powerful for them” (Session 4). The evaluation as a negative judgment was rarely used in participants’ conversation. A few of them were connected to mathematics with phrases like “Guys, this is super hard because there’s zeros as place holders” (Session 5) or “and I think that’s what is hard for kids is when it starts with a big number and having to break it apart and do something with it” (Session 8). When the evaluation was defined in a more negative way, participants concluded the comment with a phrase that included something that they assessed as positive. For example, T9 evaluated her experience with a discourse strategy saying that even though she did not like the way they used the strategy, it helped to promote quality mathematics discourse.

T9: It didn’t seem to flow. It was very static. But then again, it did stop me, a major speaker, from talking, which meant the other people had to talk, so I see it’s [inaudible] but it’s not (Session 4).

From the beginning of the PD, the facilitators created norms to talk about the classroom representations. Those norms established the use of a respectful way to talk about the teacher and students depicted in the representations. The first norm was “treat with respect the teachers who are sharing their practice” (Session 1, Slide 18). The PD asked participants to use the materials to analyze and reflect about certain ideas, but to avoid evaluation or speculation. The rules also asked participants to use the materials to provide evidence to justify their arguments. The previous examples about how participants used the stance *evaluate* showed that participants follow the proposed etiquette when talking in the
PD. The examples suggested participants learned to follow the norms during discussions, and for that reason they talked about the classroom representations thinking more as a function of the positive ways to use the ideas that the material presented.

When participants made evaluative comments about students and themselves, they thought about how a specific idea could work or had worked in their own classroom with comments such as “I can see it being really, really good for those kids that do read those problems and it jumps at them” (Session 9), or “I find too that a lot of times, I’ll go around and I’ll see this awesome discourse going on between the partner groups” (Session 11).

**Describe, interpret, and evaluate.** One example in which the three stances were used in combination is presented below. A participant reflected about what she had noticed in her own classroom when she was implementing a discourse strategy with an ELL student.

**T:** This year, my ELL student is one of my brightest children and I wasn’t gonna use him, but then I decided to because he is, he can be very quiet and, but this process and this strategy of draft and final copy really helped him to be comfortable with sharing when he had to solve it on his own and then he was with his group and he had time to practice and share his strategy with a small group before they got up and presented, so it was very beneficial for him, but I’m used to my ELL students being, you know, just pulling anything I can to get them to participate and he has, once he’s exposed to it, he gets it. He’s a sponge, he just takes it all in and probably one of my brightest children I’ve ever had, which is very different and it’s just, when he’s exposed to it, he gets it, but this was really beneficial for him because it forced him to practice and have that time to understand the vocabulary and be able to express himself in front of the whole class (Session 11).

In the above example, the participant described characteristics of her ELL student. Then, she interpreted and evaluated how the discourse strategy was beneficial for him to support the ELL with his language limitations and to improve his abilities to talk in public. This example also shows how participants observed the benefits of the strategies in relation to
another important idea discussed in the PD: how to generate quality mathematics discourse in the classroom for all students.

**Level of Specificity**

Participants made more specific than general comments during conversations about students throughout the PD (no dimension; see Table 8). All the examples provided in the previous section about describing and interpreting are specific comments participants made during conversations. Participants used specific details to reflect about teaching and learning, in relation to their students or to themselves. For example, in the last quote, T described specific characteristics that her ELL student had, using words such as he was “one of my brightest children I’ve ever had”, “he can be very quiet”, and “he’s a sponge”. All those comments helped her colleagues and the facilitator to understand the ELL student.

Table 8

**Level of Specificity in Relation to PD Focus and Actor by Dimension**

<table>
<thead>
<tr>
<th>PD focus</th>
<th>Math (35)</th>
<th>Discourse (66)</th>
<th>Pedagogy (29)</th>
<th>Other (12)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G  S  G  S  G  S  G  S  G  S</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7  9  7  16 1  5  2  3  17  33</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student</td>
<td>Student-Self</td>
<td>2  20  5  28 0  13 0  6  7  67</td>
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<tr>
<td>2</td>
<td>3  1  5  29 1  8  3  0  12  38</td>
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<tr>
<td>Student</td>
<td>Teacher</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Student-Teacher-Self</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Dim</td>
<td>12 32 18 86 2 33 6 10 38 161</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: Dim: Dimension, Math: Mathematics, G: General, S: Specific.*

In the following example there are no details or explanation given in participants’ comments. They just said one or two words, without deepening in the information. This illustrates a general comment:
So I’m glad you said enrich because this next phase, we’re gonna have a launch phase, this next phase is the explore phase, and explore phase, I love this part of the lesson, because most of your responsive classroom discourse happens not in large groups, but in what?

Ts: [collective answer] Small groups.

And from what I’ve observed from several of the people in here, yesterday especially in the afternoon, they would not say one word in whole-group, but if you put them in small group, they would talk. It happens in your classroom. In a whole-group, they shut down, but in a small group, they start what?

T: Talking (Session 4).

**Own Classroom Representations**

**Curricular artifacts.** When participants talked about students (no dimension) in relation to curricular artifacts, they included the Curriculum Management Application (C-MAPP), which is the district’s daily pacing; the Common Core Standards; the Daily Five, which refers to 5 literacy practices teachers can use; or Math Expressions, the curricular materials they were asked by the school system to use on a daily basis. Many times during those discussions, they were expressing their concerns about how to integrate the ideas about promoting responsive discourse in their own classrooms in relation to the curricular materials they were asked to use to guide their teaching.

T4: I was thinking that this is a lot of best practices in teaching, but then I had a question in my mind of how can I use what C-MAPP is telling me to, to, you know, incorporate? (Session 3)

In that quote, T4 shared her question because she was confused about how to connect the curricular requirements from her school district (C-MAPP) with new ideas they were discussing in the PD.

**Stories of classroom talk.** With regards to stories of classroom talk, participants made comments about students (no dimension) in almost every replay and rehearsal (see
Participants used rehearsals more frequently during the PD Summer Institute and more replays during the school year (see Table C5). This is aligned with the PD design because it was more focused on PD classroom representations during the Summer Institute, and later on more connections to participants’ own practices were promoted.

When participants used rehearsals they made comments about how their own students would solve a mathematical problem, how they would react to a particular situation, or how different students would behave differently to the same situation. For example, T9 reflected about the use of one discourse strategy.

T9: But I think, going back to your old students, even your higher ones are gonna be in that conversation with you but then the artistic students and some of my ESL students, what are they gonna think? Are they gonna be drowning in that language? I think that something, that the vocabulary has to be specifically caught and targeted so I mean, it depends if you’re in a large group or a small group, maybe (Session 3).

This excerpt shows a rehearsal because T9 anticipated how interactions with different students would happen in a real classroom if she were to implement a Think Aloud. She was concerned about what could happen to students with different learning abilities, or language levels. She foresaw some situations that could happen for different students, and at the same time, she proposed to be purposeful with the vocabulary as a way to prevent possible problems when implementing the lesson. T9’s reflection made her think ahead about the implementation of the discourse strategy.

With regard to replays, participants used them to explain how their own students solved mathematical problems, to make connections to literacy settings, to talk about different strategies students used, or to describe how they usually organized the classroom to work with students. When they talked about students, they described mistakes students made
or how kids reacted in specific situations in the classroom. They replayed how kids solved mathematical problems:

T9: One thing I have done, I can see it working here, is I took the document camera and turned it toward the classroom and then played that back to my students so they could see it and that was actually really powerful for them just like it’s powerful for us as adults so that could be one way, so they don’t have to try and remember the discourse, but they actually have it right there (Session 4).

In the quote above T9 was recalling previous decisions that she had made to make students feel more comfortable about their participation in classroom conversations.

When participants expressed ideas about students as a two-dimensional actor, they used replays and rehearsals most often in connections to themselves. Also, those stories of the classroom were more used in conversations about discourse (see Table C4 and Table C6). The second most common way to use stories of classroom practices was in connection to the three actors. In those conversations, participants compared what they had observed in the classroom representation or in their own experiences, in relation to the teacher, students, and themselves. However, the comments were more focused on students and themselves. Their comments about teachers were most often brief. In replays, participants talked about the teachers more in connection to something they experienced in a PD representation. In contrast, when participants utilized rehearsals, they talked about teachers in more general terms. For example, in Session 8, during an idea unit in which teachers used rehearsals, a participant said “Well, you have teachers that are looking for certain types of solution for other students.”

There were a few instances when participants used rehearsals and replays within the same idea units when they were talking about students. In those conversations, participants
compared what they had experienced in their own classrooms to what they could anticipate doing in the future. For example, a few teachers shared ideas about how to provide all kids with more opportunities to talk:

T5: I have something to say. I always have kids that always want to talk and be in charge of everything, [inaudible] have to do this job, so are they allowed to switch or should that be discouraged?

T7: I think that he was obviously stuck and say you wouldn’t want a big group just to sit there and let the talker just sit there and not be able to complete their assignment.

T18: I would discourage the talking if someone can get started another way. Maybe that’s after you ask the question, you go, ohhh, and change the path that maybe you had been… (Session 4).

T5 replayed her previous experiences in the classroom with students who talk a lot, and she asked how she should deal with that situation in the future. T18 rehearsed how she would react to that kind of situation.

**Summary of Results About Students**

Participants talked about students across the duration of the PD. They started talking about this actor in Session 1, and they kept using it as the most common actor in every session, with the exception of Session 13. The most common use of this actor was in connection to other actor, and not as the unique actor in PD whole-group conversations. Participants talked more about students, and students and themselves when the topic of conversation was discourse or mathematics, whereas conversations about students and teachers or about the three actors were found more in discussions about discourse or pedagogy.

When participants talked about the three actors (students, teachers, and themselves), they connected the discussion to their own practices in the classroom. The most frequent
topics of conversations when talking about the three actors were pedagogy and discourse, which were aligned with the way they reflected about how they could implement an idea in their own classroom or when they shared their previous experiences.

Participants’ conversations were more specific than general. They frequently described and analyzed information, while the stance evaluate was less used, and it was more related to constructive ways of processing the information, using pros but also cons of the ideas they discussed.

**Self as Actor**

The second most common actor in participants’ conversations was self (no dimension). Participants frequently talked about themselves in connection to students. This pattern was observed for every focus in PD conversation (see Table 9). It was not common that participants talked about themselves and teachers within the same idea unit, but if the teacher was included in the conversation, they were often talking about the three actors.

Table 9

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Actors</th>
<th>Discussion focus</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Discourse</td>
<td>Mathematics</td>
</tr>
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<td>Self</td>
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<td>Self-Teacher</td>
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<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Student-Teacher-Self</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>No dimension</td>
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<td>44</td>
</tr>
<tr>
<td></td>
<td>Self</td>
<td>66</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Teacher</td>
<td>53</td>
<td>7</td>
</tr>
</tbody>
</table>

There were only a few instances in which participants talked about themselves when they were discussing pedagogy. In discussions about discourse, normally participants made
brief comments without deepening the idea they wanted to share. This situation was observed, for example, when the facilitator asked them how they had used a particular discourse strategy during the literacy lesson.

F2: What sorts of things do you use it for in your literacy classroom?
T: Making predictions or connections.
F2: Making predictions or connections.
T: Showing your metacognition strategies.
F2: Showing your metacognition strategies. Anybody else?
T: We ask questions, like, [inaudible] (Session 3)

Self as a No Dimensional Actor

When participants were talking about themselves (no dimension), the way they talked depended on the focus of the conversation. When the conversation focus was about mathematics, participants used specific details to describe the solution to a mathematical problem or different strategies to solve it, but they also made connections to what their role is as teachers in the classroom, and how they should react to specific situations. The quote on page 60 show what a mathematics focus discussion looks like. There all the participants talked about the mathematics in Isabellla’s method. The quote below illustrates better how participants discussed pedagogical issues when the focus was mathematics. In this comment, T explains her daily experiences during mathematics work in the classroom and how time is a limitation to teach kids more mathematical strategies to solve problems.

T: I think in my classroom, what I’m having the most trouble with this is these kind of, you know, wonderful abstract concepts that, you know, really drive home what mathematics is, I don’t have the time to spend for them to all explain all the different ways that they found their answers, because the ones, when I do, then yes, David gets up and he explains it like this and then he teaches somebody else how to do it like this and so-and-so gets up and they have their little hundreds board and all that, I just think time is such a limitation.
T: I’m lucky I get the full hour in.

When they focused on discourse, their comments were about discourse or pedagogy. Participants also reflected about what they do or could do in the classroom to promote discourse. They talked about how they felt using a specific strategy, they proposed alternatives to promote discourse in the classroom, and they made comparisons to literacy practices. For example, the quote below illustrates this type of conversation. A participant reflected on her experience in the PD during a discourse focus conversation making connections to her own practices in the classroom and to Common Core Standards requirements.

T: I would just like to share that, I mean, this project has really helped me a lot as a teacher and this paired with Common Core has just really integrated and really, I mean, I feel like I’ve grown a lot, and last year, I’ll be honest, it was my second year teaching and it was about how many problems can we practice so we can get better, so that was kind of what I was thinking and through Common Core and this, that has completely changed for me and it’s, now it’s not about how many problems we get done, but how well do they understand it? What is the rigor behind it, the depth of knowledge and can they explain through discussion how they got that answer, so I feel like this and Common Core, I mean, they just really pair together and support each other.

T9: Today, my lesson was one problem and that was an hour, right. Before I would have…

T: We don’t really have to look at this, I mean, it’s so natural now and that happens every day, it’s not like I have to sit down with my lesson plans and type out, okay, my launch will be this, my explore will be that, and we’re a STEM school too, so we’re already using the STEM process, and this is kind of incorporated with that as well, so it’s kind of like all these things are just so connect with each other.

T: The mind meld (Session 12).

Participants started to talk less about themselves (no dimension) after Session 5. There are two exceptions to this situation: Sessions 8 and 12 (see Table C1). In session 8, the conversation was about task selection as a way to promote responsive discourse. Participants
solved different mathematics problems. They also sorted 9 story problems by type (compare, put together/take apart, take from, add to). Because participants were prompted to talk about their ways to solve problems or their justification for the way they sorted the problems, there were more idea units about themselves in Session 8. Again, in Session 12 the main topic of analysis was planning and participants were asked to reflect about their overall experiences in the PD. They also analyzed their experiences in the classroom implementing discourse strategies and other ideas they had learned in the PD. During the first whole-group discussion of that session participants shared their experiences using different instructional moves in the classroom. Later on participants analyzed the discourse instructional tools that they had learned in the PD and how what they had learned impacted their lesson planning. The way the prompts were presented justified that participants talked more about themselves in Sessions 8 and 12.

**Stances**

**Describe and interpret.** When participants talked about themselves (any dimension or no dimension actor) the most common stance was interpret and then describe. The combination of those two stances was also the most frequently used by participants (see Table C2).

In conversations centered on discourse, when participants talked about themselves (no dimension) and they only interpreted, they shared details about how they felt about the implementation of a specific discourse strategy in the PD setting or in their own classrooms. In addition, they interpreted how a certain idea could be implemented in the classroom, or
how students would react to that. For example, the next quote is an exchange between participants in which only the stance interpret was used.

F2: So group two was talking about probing and pressing questions, picked a question that was asked during small group or whole-group discussion and highlighted an important mathematical idea and seemed to enhance the discussion. What was it, what’s your evidence, was it probing or pressing?
T: We talked, somebody had talked about if you could interchange the words distance and difference and how that sparked a whole other conversation about what is the difference between distance and difference and is that confusing to kids, and then you brought up the ruler and the hundreds chart and we decided that was a pressing question. And our evidence was we talked about it for ten minutes [inaudible] about that.
T: We had to come up with an idea. We had to come up with an answer. Is it or is it not? And then probing it, disprobing it, whatever, we had to have that…
T12: It wasn’t looking at something on the poster already and explaining it, it was taking the things we talked about and thought about and then this other concept of, oh, well, if you’re thinking of these two words, how, you know.
T15: And how do they go together and what do they mean in math versus other things?
T12: Right (Session 10).

In this exchange, participants were asked to analyze how they experienced the discourse strategy Probing and Pressing Questions during the session. The teachers explained their interpretations of ideas about that strategy. For example, T interpreted how the concepts distance and difference are confusing for kids, and she interpreted the type of question saying it was a pressing one and the reasons for reaching that conclusion.

Oftentimes, participants described in combination with interpret or evaluate. When they only described, they typically used short comments to illustrate an idea, without many details. One example of this way of talking happened when participants were discussing modification they can do to the Bet Lines discourse strategy.

F2: Okay, so started out with group work and kind of get a feel for it? Anybody else have a modification that they thought may work better next time or they are gonna try?
T7: We did like a turn and talk where they told their neighbor what they thought it was gonna be first and they could say it sometimes before they had to tell the class.
F2: That’s really good for your ESL students, ELL learners, so they have a chance to kind of process it before they have to share.
T: I let them to repeat somebody’s bet.
F2: You what?
T: I let them repeat somebody’s bet the first time.
F2: So kind of like in an all talk where they come to you and you don’t have anything?
T: Kind of like you say, “I agree with this person, and this is what they said, and this is what I agree with them.” Instead of [inaudible] (Session 10).

In that excerpt, T did not provide details about her idea. She only described what they would do in the situation that F2 pointed out, explaining her strategy in the classroom and reading from the text provided by the PD.

**Evaluate.** Similarly, when talking about students, participants used less of the stance evaluation, and when they used it, the conversations had a discourse or pedagogy focus. When participants were talking about themselves and teachers they never evaluated. The way participants used evaluation was similar to conversations about students, using positive statements, with expressions such as “That chart on the last page was really helpful when thinking about planning my lessons” (Session 5; talking about an article they read as homework), “Making the connections was a very exciting experience” (Session 5).

**Topic of Discussion**

When participants talked about themselves, the most important topics of discussion were mathematics and pedagogy (see Table C7). This shows a difference in relation to conversations about students, in which participants talked more about mathematical thinking and discourse. In those discussions participants explained their own thinking when they
analyzed different mathematics tasks. If the focus of conversation was pedagogy or discourse, normally the topic of the conversation was pedagogy. In those conversations participants reflected on planning a lesson, on the importance of the implementation of the lesson in relation to discourse. They also shared information about how their planning and implementation had changed because of the PD experience.

**Level of Specificity**

Overall, there were more specific than general comments in discussions in which participants talked about themselves (no dimension). However, differences were found depending on how many actors were in an idea unit. When participants talked only about themselves there were more general than specific comments. Most of the general comments were found when the focus was discourse (see Table 8). In those comments, participants only gave a short idea or explanation, without including details that allowed others to understand their ideas. The topics they discussed varied. The excerpt below is an illustration of an idea unit that contains only general ideas and in which self is the only actor in the conversation.

F1: Okay. So seeing or hearing them, any thoughts about this so far before we practice and analyze? Anybody want to make a comment or…? Besides that was very entertaining. I loved it.
T: Connect with the sciences.
F1: Could connect with the sciences.
T: Because you have to make predictions. You have to use what you know.
F1: If you didn’t hear, she just said you can connect it to science. How do you think this would help them with their math? In word problems?
T: Kind of think about what the question’s actually asking (Session 9).

Idea units that contained only general comments were associated with tasks in which participants were making a list of ideas about their previous practices or when they were summarizing ideas from an activity they had performed in their classroom or in the PD, as is
the case in the previous example, in which T did not go deeper into her ideas and the comment was very superficial.

That pattern contrasts with discussions focused on mathematics, in which participants used almost exclusively specific comments to talk about themselves. In those discussions, participants provided details to explain their own way of thinking about a problem. This can be exemplified using an excerpt in which participants analyzed the Ferrero Chocolates Math Problem:

T4: Okay, well, in our group, we used division to divide 62 by nine because there’s nine in each box, found out there were six boxes and eight candies left over, and then we were able to split the eight into the two wraps and two candies left over.

T7: You said that you used division to find that there were six boxes of candies and two wraps and two candies. We drew a picture and we just counted the number of boxes that were whole boxes and then figured out there were two whole wraps left and two individual candies (Session 5).

This example shows how participants were able to provide details about their own mathematical thinking, and about the strategies they chose to solve the Ferrero Chocolates Math Problem.

When at least another actor was included in the discussions, the comments were mostly specific, which means the conversations had more details and connections than when participants were talking only about themselves. This holds true for all the different types of PD focus (see Table 10).

Table 10

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*Note:* Math: Mathematics, G: General, S: Specific.

**Own Classroom Representations**

When participants talked about themselves they most often did not use own classroom representations (replays, rehearsals, or curricular artifacts) if those artifacts were not linked to students. Participants used more rehearsals to talk about themselves (no dimension) during the first five sessions of the PD (see Table C5) and more replays later in the PD. This is associated with the PD design because after Session 6 participants were able to implement ideas from the PD in their own classrooms, and then they could talk about their experiences in the classroom in the PD sessions.

**Summary of Results about the Actor Self**

Self was the second most common actor in conversations in the PD. It was used more in connection to the actor student, and it was not frequently connected only to teachers. There were more conversations about themselves when participants were talking about discourse or mathematics. In mathematics discussions, the actor self was common, which happened because participants were provided with opportunities to reflect about their own mathematics knowledge and strategies to solve a problem.

As happened with the actor students, participants used more description and interpretation and less evaluation when talking about themselves. They did not evaluate when
talking about themselves and teachers, which could be evidence they were following the norm about how to analyze classroom representations.

Most of the idea units contained specific comments, but when participants were talking only about themselves the comments were general. In mathematics discussions comments were specific.

**Teacher as Actor**

In this section results about teacher as a one-dimensional actor or no-dimensional actor are included. The combination of teacher with any other actor or combination of actors was already discussed in previous sections.

There were only a few idea units in which teacher was used as one-dimensional actor. Participants did not talk frequently about themselves and other teachers in idea units. The most common combination of actors in which teacher was involved is the pair student-and-teacher, and the majority of the comments about them happened in Sessions 1 through 6 (see Table C1). This pattern is aligned with the PD design in which most of the classroom representations in which a teacher was an actor were analyzed during the Summer Institute. Later in the PD the conversations were focused more on participants’ own experiences in the classroom. For that reason, participants had fewer opportunities to make connections to other teachers’ actions. Participants talked more about teachers (no dimension) when the focus of the conversation was discourse (see Table C2).

Two ways in which participants talked about teachers (no dimension) were observed. First, they discussed a specific teacher who was an actor in a PD classroom representation.
For example, participants described or interpreted what the teacher was doing, or what the teacher’s goal was in the video they had watched. On the other hand, participants used the word “teacher” referring to anybody teaching, without thinking of a specific person. This second option can be seen in the excerpt below:

T2: I also, I see it as it takes a lot of time on the teacher’s end to front load it so that the students are having mathematical discourse and you are with them, so it takes a lot of planning to be able to do that. Just from one little lesson (Session 3).

**Topic of Discussion**

There was no discernable pattern in relation to the topic participants discussed when they talked only about the teacher. More comments were linked to pedagogy, but there was not a large difference in the number of idea units connected to other topics (see Table C7). Comments about teachers (no dimension) were more related to pedagogy and discourse. When participants talked about the teacher and discourse, they discussed decisions the teacher made, how the teacher interacted with students, or strategies the teacher used to generate conversations. During those discussions about teachers, prompts helped participants to focus on specific ideas about teachers with questions such as “What might be the teacher’s purposes for what she does?” (Session 3, Slide 17), but also participants recalled ideas from documents they had already used or read to make comments about this actor. Very few comments were made about participants’ own colleagues in school or other teachers not attending the PD during discussions.

In discussions in which the topic was pedagogy, participants gave their opinions about decisions the teacher made during the lesson, teacher responsibilities in the
organization of the lesson, the importance of making decisions during the class to improve the work, or how to make kids participate. For example, participants discussed the pros and cons of the type of thinking that a teacher demonstrated in two different videos about the same discourse strategy.

F2: Okay, I’m gonna bring us back together to talk about these two think alouds and just think about what are the pros and cons of each of these think alouds and what were you all thinking of the differences between them?

T3: I was thinking in the first one that she’s actually modeling the math sentence for them, and in the second one, she is modeling how to think, how she’s showing the picture and then she is asking them to think and asking them the different ways that they could use to solve it.

T7: I thought in the [inaudible] she demonstrated the thinking, but I liked how in the second one, she said, “I think about this problem,” and I like the gesture like if she is able of thinking, and then I felt like there was more, a little bit more explanation of the thinking.

T: In the second, she really broke the word problem apart and chunked it into little pieces. “I have four large plates, what am I gonna do with four large plates?” instead of trying to see the whole problem at one time, it was apart.

T: The first one, she made it sound like it was the only way to do it. The second one, she let them [inaudible] another way (Session 3).

In this excerpt, four different participants explained their thoughts about the work a teacher did in two different classroom representations. In this discussion participants were talking about pedagogy, analyzing how two different ways of implementing a lesson could lead to different outcomes. They described and interpreted the way the teacher conducted the lesson and how the gestures could help the kids understand the process, they found differences in the Think Alouds, and they could define which experience provided by the teacher gave kids more opportunities to learn how to think about mathematics.
Stances

In a similar way as for students and for themselves, participants interpreted more than described and evaluated within idea units about the teacher. The previous example shows how participants interpreted and evaluated within the same idea unit. In that quote participants described the moves that the teacher in each representation made, how the teacher divided the problem into parts. They even used quotes from the written classroom representations. They also interpreted why the gestures the teacher made were important for students or what was the teacher’s purpose in each think aloud when they commented about what the teacher was modeling in each one. For this particular discussion, the use of those stances was motivated by the prompt participants were given. In relative terms, there were more comments in which participants evaluated than when they talked about students or themselves (no dimension; see Table C2). The half of the idea units in which participants used the stance evaluate, they also talked about teachers, and in almost every instance they talked simultaneously about students. That idea also holds true for the other stances, which means participants connected teachers to students during conversations, and fewer connections were made between teachers and themselves.

Level of Specificity

The number of times participants used general and specific ideas during the whole PD is similar when participants were talking about teachers. Participants made more specific than general comments when talking about teachers (no dimension; see Table C8). Again,
most of the specific comments that contained a teacher as an actor in the conversation were also connected to students, or to all three actors as well.

**Own Classroom Representations**

Participants did not make comments about their own classroom representations when talking about teachers. Also, for teacher (no dimension) there were fewer comments about this topic than for the other actors. For almost all the stories of classroom talk, the three actors analyzed in the current study were included within the same idea unit.

**Summary of Results about the Actor Teacher**

Teacher, as a one-dimensional actor or in connection to the actor self, was not commonly used in discussions participants had during the PD. The actor teacher was more connected to the actor students, and most of the comments happened during the Summer Institute. The majority of comments about this actor were linked to the teachers whom participants watched in a classroom representation, and they rarely made connections to other teachers they knew.

The pattern about the use of stances also is the same for this actor, with more connections to interpret and describe and fewer comments about evaluation. When they used this last stance, normally they included also students in the conversation. In general, participants used similar number of times general versus specific comments to talk about students, but when at least another actor was included in the idea unit, they used more specific ideas.
**Shifts in Teachers’ Participation in the PD**

The PD is seen as a boundary encounter in the current study (Wenger, 1998), and I was interested in analyzing whether participants shifted their way of communicating ideas during the process. This section describes how participants changed the ways they engaged in discussions about different topics during conversations that took place in the 13 sessions of the AIM PD.

**Shifts in Participation When Teachers Talked About Students**

Three shifts in the way participants talked about students during the PD were identified. The first one was observed when they talked about students during mathematics discussions. The second shift was connected to students and teacher during discourse-focused conversations. A third shift was associated with students (no dimension) in relation to the specific level of specificity.

**Shift 1: Students and Self in Mathematics-Focus Discussions** There was a shift in the way teachers participated in project AIM when they reflected about themselves in connection to students in mathematics-focus conversations. Participants discussed mathematical topics in Sessions 2, 5, 8, and 10. In those sessions, participants solved mathematical problems. In Session 2 they worked with the task Isabella’s Method. When participants were analyzing the questions provided in the handout for this task, they focused on the mathematics. One example of this type of conversations was provided on page 60 in the section about stances for the actor student. In that quote, participants focused their
comments on their own mathematical interpretations of how the student Isabella solved a subtraction problem. Similar comments to this quote were made during other whole-group discussions in Session 2.

In Session 5 participants analyzed mathematics content for the second time in the PD. They used a problem designed to help participants understand the meaning of subtraction (AIM Session 5 plan). After solving the problem in small groups, participants used a discourse strategy to present their different solutions and ideas about how to solve the problem. Later on, the facilitator opened the conversation to anybody who wanted to share ideas about different strategies or ideas to solve the problem. During the beginning of this whole-group discussion, participants focused on the mathematical strategies they used to solve the problem as they did in Session 2. After a few minutes in this discussion, a shift in the way participants were participating was observed. T1 asked a question about how the teacher, as the person assessing students’ knowledge, could use discourse strategies that helped the teacher define whether students were understanding what their classmates were explaining to the whole-group. T1 started to think about herself as a student and her role as a teacher.

T1: Like for example, the math group that got up never went over their strategy. As a student I was thinking about how maybe I would have gotten lost when you were explaining it, so as a teacher assessing that, would you have stopped to get other kids to, like we revoiced the last one, but I felt like the last one, I could really see it, and as they were going through it, I think if I could have slowed down and looked at it a little bit more, I would have got it, so would you have asked other students to revoice during that time to see if they were having that issue with the strategy? (Session 5).
Other teachers made suggestions about T1’s ideas. She later asked her colleagues if she needed to have students asking more questions. T1 made suggestions about how to use the strategies they had learned. She said that it is important for students to question each other to verify if they have understood the strategy. It is important to highlight that the prompt provided by the facilitator for that discussion was about how the participants had solved the problem and not about pedagogy. The discussion about the discourse strategies they used while solving this task occurred later in that session.

In Session 8 participants were given nine story problems and they needed to sort them by type (compare, put together/take apart, take from, add to). When they discussed their work as a whole-group, the facilitator asked them to explain their reasoning for the type they chose. During this session there were two comments the participants made that connected the work they were doing with pedagogy and discourse ideas. A participant asked if for teaching purposes it is necessary to define the problem type. Another participant commented that it was not necessary, but it was important that students knew there were different problem types. The facilitator agreed with the idea that it was important idea that students “understand that there are different structures of the operation, and that we need to think about the situation and what’s going on in the problem before we decide how to approach it” (Session 8). Another participant made the comment that follows:

T: That’s why I have a question, there’s a math expressions student book page where they have to read a story problem and match it to the equation. But I always thought it was useless because …
T: So you just throw it away?
T: Yeah.
T2: That’s what I do.
T: Because they could come up with another way to do it a lot of the time. (Session 8).
In Session 10, participants analyzed a new subtraction problem solved by a student, Maria. During this conversation, participants continued making connections to their own practices. They asked questions or made comments on how a second-grade student would think about the problem, or if at that level a child could understand a mathematical idea they were discussing.

In conclusion, over the duration of the PD, participants shifted their mathematics-focus discussions from being only about mathematical ideas to integrating comments about their own practices. They started to think about the implementation in the classroom of the new mathematical ideas they were learning in the PD sessions. They considered how the discourse strategies could be used in their classrooms, what they thought could work, and what were the challenges they anticipated.

**Shift 2: Students and teacher in discourse-focus discussions.** There was a shift in conversations about teachers and students. At the beginning of the PD participants’ discourse more often addressed the teacher’s role in the classroom. Later on participants were able to consider student’s roles in lessons.

In Session 1 participants analyzed a transcript to look for evidence of different types of discourse. Participants commented briefly on student work as a way to justify what the teacher did to scaffold students’ thinking. Participants also talked about how the teacher modeled a specific behavior. They made comments about the type of questions the teacher asked students. Participants also reflected about the materials the teacher chose to work with in class. The second discussion about discourse that included comments on both teachers and
students took place during the second day of the Summer Institute, during Session 3. Participants continued analyzing what teachers did or did not do in the lesson. They analyzed a teacher’s knowledge of their students was important to making instructional decisions. They also noticed how teachers set up a task or the importance of planning carefully. On some occasions participants highlighted what students did, but they quickly came back to talk about teachers. The exchange below is one example of an idea unit in which participants talked about students and a teacher. Participants were analyzing a discourse strategy that can be used to launch a lesson. Participants watched a video twice — a cropped version, and the entire video clip. Later on participants were prompted to talk about the two Think Alouds, and about how they could serve the purpose of modeling thinking. They also reflected about how the strategies could support responsive discourse.

T2: But see, I think she explained it so much more in the Think Aloud that then they were able to explore on their own, when she did launch, at the end of launch the video, they didn’t have enough information and they were like, “okay, well this is how we’re gonna solve it, this is exactly what she said to do, this is what we’re gonna do,” whereas here, I think she gave them so many more ideas that they can just try to attempt in a different way. It’s not comfortable enough is what we…
T7: I kind of like to make them comfortable and I liked how in line 14, she leads them, “how can I compare them,” is how she leaves off and so it’s more like a discovery learning where, and she did show them a method, she did say she could draw it, you know, she said they could draw it whatever shape they wanted, but I felt like it was so open ended that they might go up and have arguments about it. Which would be considered math discourse.
T9: And then she goes back to the second paragraph later…
T5: Hopefully one of the students would have done what she did and lined them top and bottom, they could show that, but maybe someone else did it a different way.
T7: Maybe someone else [inaudible] (Session 3).

This quote is an example of the way participants focused more attention on the role of the teacher than on what the students were doing during the lesson. T2 and T7 even repeated the words the teacher said, and T7 expressed that the teacher’s goal was to make kids feel
comfortable and she highlighted what she liked about how the teacher organized the work. In that quote, three of the four participants focused more on what the teacher was doing.

Even when the prompt or question provided by the facilitator was about students (either about connections between students and the teacher or only to discuss students), participants more frequently made comments about teachers early in the PD. When in these conversations a participant made a comment linked to students, it was observed that the next participant talking shifted to talk again about teachers. The following segment illustrates this situation.

T10: I just had a question, when F2 talked about what a launch is, and I wrote down, “to get them engaged,” there were parts of the launch, but the engagement part was the big question I had: “how engaged should the students be interacting with a teacher?”

T6: In the second question, we were talking about where the student’s doing a response, we talked about how we couldn’t see if they were really engaged or if they were following along because the students we were looking at, you could see them looking at her, but you couldn’t really tell if they were engaged or not or if they were just listening like it was the beginning of direct instruction, and then it looked like she was very engaged and it’s like you see them look at them, but there was never really response to what she was doing from what we could see (Session 3).

The comment below was made by T1 to follow that previous conversation. She changed the focus to the teacher again. From her comment, it is clear that she analyzed T6’s comment related to the teacher’s role and was not focusing on students.

T1: The first time I watched it, that’s exactly what I thought. “She’s having a lesson with herself.” That’s exactly what it seems like, she’s teaching herself, but then the second time, when you think about what her purpose is, it’s almost like, I was thinking, she is modeling what she wants them to be doing when they’re working out the problem, discussing it, talking about it, how they’re going to, how they’re actually working it out, their thinking. And then they find the vocabulary, they’re finding it as they’re working it out. But there was some student interaction there (Session 3).
After that comment, the facilitator prompted all the participants to participate in the discussion. At least 10 participants talked about this topic during the next three idea units, and all of them focused on teachers. There were a few connections to themselves and to students, but they did not talk about students’ role as an actor in the promotion of responsive discourse. Thus, early in the PD, idea units that included both students and teachers were mostly focused on teachers.

Participants started to shift the way they talked in relation to students and teachers in Session 10. In that session, the facilitator asked participants to provide an example “of something that the facilitator did to scaffold or maintain the mathematical focus of the discussion.” She asked how the move used by the teacher helped them to keep the discussion focused on important mathematics ideas. One participant shared her ideas as follows.

T: Well, we talked about how a lot of questions were asked and a lot of them were how questions, “how do you do this?” And they were more thought provoking so that it kept everyone thinking about the mathematics behind, you know, the thought process behind the math that they were doing. And also, with the distance question kind of linked everyone into the discussion so not only was there a check for understanding for the group that was presenting or making sure they were understanding the concept, but it kind of got thrown out to the group to see, a good way for the facilitator to just kind of see who was on the same page and how many people seemed to be understanding the concept and how many people were looking at it like maybe they didn’t understand it. It helped to guide the facilitator, you know, toward maybe asking more questions or going in a certain direction to facilitate more understanding (Session 10).

This example shows how participants changed the way they analyzed the work that a teacher and the students did. The question the facilitator provided was focused on the teacher, but the example provided by the participant showed that she analyzed that situation as a function of the work students did. For example, she talked about the discussions students had and how to observe that situation helps the facilitator to make instructional decisions. Her
way of analyzing the situation shows that she saw the facilitator’s role as a function of students’ work.

In Session 12 there was a discussion to summarize the instructional tools they had learned about during the PD. One of the tools they analyzed was the Mathematics Teaching Guide. T15 connected their ideas about this tool to students even though the document was designed to help participants plan lessons.

T:15 We said that it was a good tool to assist in the formation of lesson plans in that it helps students take responsibility in math for the connectiveness of the discuss and explore phases and how connected those two pieces are, and that it keeps the teacher accountable for and focused on the best practices for students (Session 12).

This comment shows how participants started to think about the important role students had during conversations in the classroom. Later there was a prompt about the teachers’ role, and again participants made comments on how teachers should observe students’ strategies.

In conclusion, participants were able to move their analysis of different PD artifacts from the teacher as the center of the conversation to think more about the role of students in the classroom with the goal of promoting responsive discourse. At the beginning of the PD most of the comments were about teachers and what they do or do not do in the classroom. Later in the PD participants analyzed more the role of students in what happened in the classroom.

**Shift 3: Students (no dimension) and level of specificity.** With regard to the specific level of specificity there was a shift in the way participants talked in the PD conversations. They started early in the PD describing and interpreting what they noticed in a
classroom representation, then they made more connections to practice, and finally they anticipated what could happen in the classroom.

During Session 1 in every commentary made by participants about students, they described and interpreted what they had watched in the video or what they had read in written classroom representations. This situation was promoted partially by the facilitator who was setting a norm to use the PD materials as evidence to talk during the session. Participants made comments about what the teacher did or why she made a decision about something they noticed. They analyzed how a particular student solved a mathematics problem. The only comment in this session in which participants connected ideas to their own practices was promoted by a prompt that asked them to describe the use of discourse in their own classroom and their goals to improve their practices in relation to discourse.

In Session 2 participants started to make connections between the PD materials and their own students, thinking about “advanced” or “low” students, or how in general their own students would react to a problem. For example, T13 said, “More advanced students will take a large jump and then lower students may need to make more smaller jumps.” Participants continued using the details from PD artifacts to build their arguments and to interpret the ideas they were analyzing.

In Session 3 participants made more connections to their own students and their own practices. In this session there was a prompt about pros and cons that participants observed in two different videos about how to launch a lesson using the discourse strategy Think Aloud.
This prompt allowed participants to think about their own classrooms, how their students would react to different ways of launching a lesson, and the role of the teacher.

In Session 4 participants started to think more carefully about how to connect ideas from the PD to their own practices. In one discussion during Session 4, they were analyzing what the teacher in a video was doing to support students’ questioning, explaining, listening, and modes of communication in the discourse strategy Talk Triangle. Participants were asked to provide evidence when they were answering that question. The following segment illustrates how they talked about their previous experiences in the classroom during that discussion.

T5: When you see a group having a discussion, can you kind of replay that skit or the discussion you had, bring them back to what you just had or this and that, you always give them a script using some of their own...  
F1: So I’m just gonna revoice what T5 said, she said if she found a group, and correct me if I’m wrong, that was doing it really well, she would have them go up and model. Is that what you just said, T5? Do you all think that’s a good idea or...?  
T9: One thing I have done, I can see it working here, is I took the document camera and turned it toward the classroom and then played that back to my students so they could see it and that was actually really powerful for them just like it’s powerful for us as adults so that could be one way, so they don’t have to try and remember the discourse, but they actually have it right there.  
T16: I see the vocabulary, like being able to discuss your ideas, you really have to practice that and use that in your think alouds to show them how to get those thought patterns out. You ask them to write it sometimes but it doesn’t, I know [inaudible] ability to discuss it.  
T7: Maybe like some kind of tool for the talker to use to help back them and be able to explain, yeah (Session 4).

The comments made by T16 and T7 showed a new way participants made specific comments because they started to anticipate how they could implement some ideas in the classroom.

This type of anticipation was not shown by participants in earlier sessions. This change in
how participants talked during discussions continued in Session 5 when a specific prompt asked them to connect an article they had read to ideas they had discussed during the previous two sessions about both how to promote responsive discourse and how to establish connections with the Mathematics Teaching Guide. Of course, because of the prompt, there were many specific comments with connections to practice, but the participants anticipated how to use what they were learning. They created connections to the article they read, and they analyzed how to implement the strategies in the classroom. They talked about the process of planning, how to develop a common language in the classroom, and how to deal with different situations in the classroom. In the following exchange, it is possible to observe how participants were anticipating.

T1: I…, when reading the article, one of the things that came up was like what [we] talked about yesterday, when we were going to launch something to a group of students, how to do it without giving them, not sharing the strategy, your strategy, that they would then take back to their seat and copy, getting them to understand the concept and come up with their own strategy is the difficult part that I want to learn more about how I can better launch that in my classroom.

T12: I thought it was really cool to see what that establish purpose, engage, elicit prior knowledge, introduce language, what that actually ended up looking like in a classroom where they had a really difficult task put in front of them and seeing just the way that teacher worded it and kind of make me think that I would be able to use that same sort of thing to launch things in my classroom too.

T8: I was looking at connections to see how I could implement those specific strategies in my classroom, and also just keeping in mind that you have to really make sure that you’re not giving the students the information so they can be more responsible for their own answers (Session 5).

One participant said the questions they read in the article were useful to think about the planning and to make this process more successful. Participants also started to think about how their students would react in a particular situation and the importance of knowing the students in order to be able to help them to solve complex mathematical tasks.
Participants continued making connections to classroom practices during the discussions even when they were talking about mathematics. Sometimes participants made connections to practice by the facilitator’s request, but in many other instances, they did so without a prompt. Participants made comments about their own practices, how the implementation of a new idea went in the classroom, or what students did to solve a mathematic problem.

This way of talking, making connections to practice, and anticipating what could happen in the classroom was maintained during the rest of the sessions. Teachers were wondering how specific students with special abilities would react to a specific mathematic strategy or discourse strategy (e.g., kids who really struggled academically, visual kids), or what type of questions students would ask, how to establish a norm with the kids, how much information to give the kids during a mathematics discussion about a problem during the launch, and so on.

Later in the PD, starting in Session 9 the way teachers talked about connections to practice and to anticipation included more connections to their own experiences in the classroom. An explanation for that situation is that participants had the possibility of implementing ideas in the classroom as part of their responsibilities in the PD, and then they could share suggestions or advice to their colleagues about what they could do in specific situations. Sometimes a participant asked a question and other people gave advice. Also, sometimes the facilitator requested ideas, as is in the example below.

F2: Yeah, and then, you know, there are, more writing content across the areas, maybe we’ll get to a point where this could be some type of write, you know, why do we have to add things into math and we could incorporate it in another direction?
T: You could use it too with the [inaudible] lessons where instead of them going up in front of the whole-group, make a poster, give them all the markers and everything, let them make a poster, pick up your four kids, put them in a corner, have your kids walk around and go through the different centers and that works on your speech and your listening skills on C-MAPP too.

F2: Yeah, one of the strategies, I haven’t ever heard it called this, but the gallery walks, that idea of you get them to the final copy, something they’re proud of, then you display them and maybe this isn’t the one they get to share out.

T: But even those four groups could share, and you send the other class around and they circulate, so they’re repeatedly saying and other kids are hearing it (Session 9).

This way of talking shows the participants were more comfortable with the ideas they were implementing in the classroom, and they could find different options to face specific situations. Also, sometimes the advice was more a function of what they already had done or were doing and that was working well in the classroom. The quote below illustrates that situation.

T2: I always choose, too, a student who can, like, I have one child who can solve it correct every time and uses a new strategy, but he can’t explain at all how he did it, so I choose the student who I feel like can explain it because he gets in a tizzy when you call on him, because he’s like, “uh, uh, well…” but I know he knows it, but he can’t even explain it well enough to even have a discussion, so I think sometimes you have to think about the one child who can explain it. Even with probing questions (Session 13).

To summarize, participants refined their level of specificity across time. They started to talk about the information in the PD materials at the beginning of the PD, later on they started to make connections to their students, then to their own experiences in the classroom, and finally they were able to anticipate what could happen if they were to implement something new in their classrooms, giving hypothetical suggestions to their colleagues or from their previous experiences in the classroom.
Shifts in Teachers’ Participation When Participants Talked About Themselves

Beyond the three shifts discussed in conversations about students, one shift in how participants talked about themselves during PD sessions was found in the analysis of the data. This shift was connected to the teaching concerns, a variable that emerged from the data analysis. This topic was strong in conversations in Session 2 through 4, and later its presence decreased in conversations (see Table C9).

Shift 4: Self (no dimension) and teaching concerns. It is possible to observe a qualitative shift in teachers’ participation in the PD when they talked about themselves in relation to their concerns about the implementation of the new ideas they were learning in the PD. In almost one fourth of the idea units in which participants talked about themselves, they also showed teaching concerns during conversations.

This shift in how participants talked about themselves throughout the PD can be organized into three stages. Initially, participants stated their concerns about how to implement the new ideas to which they were introduced. Then, in Session 5, they started to make connections between their knowledge and the new ideas. Finally, in Session 12 participants made connections that enabled them to talk about how to integrate the new knowledge and their own practices.

Concerns about implementing new ideas. Participants started to show concerns about the implementation of new ideas from Session 1, but the topic was included more in conversations starting in Session 3. During Sessions 3 and 4 participants were unsure about how the AIM framework was connected to the Common Core. They showed concerns about how to plan a lesson using the new discourse strategies, or how much time they need to plan
the lesson. One participant also commented that sometimes it was overwhelming for her to try to fulfill all the requirements they were asked to do by the education system.

**Connecting new ideas to knowledge.** Starting in Session 5, a shift in participants’ discourse was observed. They continued talking about difficulties they anticipated if they were to implement discourse strategies. However, they were able to start talking about advantages of using the PD materials in the classroom. They also said they wanted to try the ideas in the classroom. Another important change was that participants started to connect ideas from different PD materials they were using. For example, T14 connected a list of questions she read in one article to the Teaching Guide. She commented, “that chart on the last page that had all the planning questions was really helpful when thinking about planning my lessons and I noticed it really connected with the diagram.” T15 also talked about this idea by saying

T15: I also liked the questions at the end because one of my biggest concerns and questions about this is how do I make sure that I am planning ahead and I am thinking carefully about what I need to do, and then as a learner myself, how do I practice this enough and have it become just more of who I am and what I do on a regular basis, just like I want the kids to do talk triangles like a normal thing, and I worried about the planning part for me being a thing that I can just do and it becomes a natural progression of who I am as a teacher and what I want to do, so I really liked the questions, because I felt that was kind of like a way to scaffold that learning for me and provide an opportunity for me to look at those questions and hopefully give myself a path to make it a more natural occurrence (Session 5).

Participants started to give positive opinions about the ideas they were analyzing instead of showing doubts about how to integrate them in their practices. T5 made comments about how she liked what she read about cognitive demand and what she should do to reach her goal in relation to this idea. In addition to that, T4 said she had started to reflect about the practices she needed to reach high cognitive demand with her students. Participants started to
make connections between what they were learning in the PD and their practices in Session 5. T16 reflected, “how purposeful and how precise our wording and our intent needs to be when launching that, that just kind of opened up my eyes” (Session 5). It is important to highlight that this session happened before the beginning of the school year.

In Session 9, participants still showed concerns about how much time they needed to plan a lesson, but their ideas also reflected their interest in implementing the Draft and Final Copy discourse strategy with their students. Concerns about time were also a topic of discussion in Session 10, but then in relation to the implementation of the lesson. A participant said she could not spend all the time she would like to show students more ways to solve a problem. However, that participant also was excited about the great job students were doing in class. She described the way a student was using great strategies to solve problems and how he was helping out other classmates during the mathematics class.

**Connecting new ideas to their practice.** In Sessions 12 and 13 there are no idea units in which participants expressed concerns. A major shift in teachers’ participation in the PD was observed in Session 12. Participants discussed what they had in the discourse instructional toolbox. In relation to the Mathematics Discourse Matrix, the discussion was as follows:

T14: Okay, we said it was the be all end all, the foundation of discourse and it opens the door for teach-led to student-led.
F2: What do you mean it’s the be all end all?
T14: I don’t know, this whole thing we’ve done, just … it painted a picture of what mathematics discourse is in the classroom, the guide …
T6: And it leads to what type of question, as you started discussing, not all of your goal isn’t always to have the responsive discourse, sometimes you need to just answer what, or answer what and how, and so it gives you the framework for whatever type of question you’re trying to achieve. It’s there to give you that pathway (Session 12).
In this excerpt it is possible to observe that participants made connections among documents and their practices. Participants seemed confident about the foundational documents they had analyzed. T14 connected the Mathematics Discourse Matrix and the Mathematics Teaching Guide. T6 was able to identify different types of questions depending on her instructional goal. During that conversation, participants also made connections to other curricular materials they used as well as the Common Core Standards. T9 said that year they had a new curriculum, which made the implementation of discourse strategies more challenging. However, she believed that the following year she would be able to pick what would make “a perfect lesson for that.” A significant change in teachers’ participation in the PD is shown in the quote on page 70. This excerpt shows an important change in how participants analyzed the PD materials in comparison to the initial sessions. At the beginning of the PD they did not understand how to connect Common Core Standards and other curricular materials. They were confused about how to integrate the Mathematics Teaching Guide, the Mathematics Discourse Matrix, or the discourse strategies in their classrooms, but later participants were confident about how the materials were connected and about how they could implement ideas in their classrooms. According to the participants’ comments, there is evidence of shifts in their practices as well. Participants discovered they could reach the same mathematical goals using fewer problems. This allowed them to use the discourse strategies. Participants also were able to anticipate better what students would do. They stated that they felt more comfortable making changes in their planning if it was necessary. Participants also were able to connect certain documents with the PD foundational ideas.
To summarize, there are two principal stages in shifts in teachers’ participation in the PD when they talked about themselves. Participants began the PD with many questions about how to connect the implementation of the discourse ideas with the required curricular materials they needed to follow. They also showed concerns about the use of time. In Session 5, they continued talking about their challenges, but they also started to make connections to their practices and find useful the ideas they were analyzing in the PD. In Session 12, when they were trying to summarize what they had learned in the PD, they reported positive experiences with the implementation of the new ideas in their classrooms. Finally, it was easier for them to integrate their teaching practices with the theoretical instruments that were introduced during the PD.

**Spearman Test to Triangulate the Results in This Chapter**

The Spearman test did not show changes across time when the test was corrected for multiple comparisons with Bonferroni ($p = 0.05/19$). This result shows that the observed changes were only qualitative. One example of this situation is the shift in how participants talked about students and the level of specificity was specific. This shift is defined by changes in the way participants discussed their ideas, not the quantity of times they talked.

Table 11

<table>
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<th>Spearman Test for Some Variables</th>
<th>Code</th>
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<tr>
<td>Self</td>
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</tr>
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<td>0.9498</td>
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<tr>
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<td>0.0606</td>
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</tr>
<tr>
<td>LS Specific</td>
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<td>0.8638</td>
<td></td>
</tr>
<tr>
<td>AIM Materials-Curricular artifacts</td>
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<td>0.2827</td>
<td></td>
</tr>
</tbody>
</table>
Table 11 Continued

<table>
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<th>Description</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.8024</td>
</tr>
<tr>
<td>AIM Materials-Replays</td>
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<td>0.1074</td>
</tr>
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</tr>
<tr>
<td>S Evaluate</td>
<td>0.040</td>
<td>0.7429</td>
</tr>
<tr>
<td>S Interpret</td>
<td>0.062</td>
<td>0.6101</td>
</tr>
<tr>
<td>T Teaching concerns</td>
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</tr>
</tbody>
</table>

*Note: n = 69, p-value for multiple comparisons using Bonferroni is 0.05/19 (0.002).*
Chapter 5: Role of the PD Materials in the PD Intervention

This chapter describes the role of the materials used during the PD intervention. Different uses that participants made of them are described and connections to shifts in teachers’ participation in PD conversations are considered. The chapter is divided into two parts: first, results about PD written decomposition of practice, that is, the foundational PD documents, are described. That section includes the Mathematics Discourse Matrix and the Mathematics Teaching Guide. For those foundational materials, a description of the path participants followed to understand the theoretical ideas contained in each one is included. The role that each boundary object played during PD conversations is examined as part of the discussion here.

In the second section, the role of other PD materials used during the intervention are included. In that part, information about how participants used the written classroom representations, mathematics problem sheets, and responsive discourse strategy handouts are included, as well as the role they played during the PD. Finally, connections between the use of all the PD materials and a description of how teachers participated and shifted in conversations during the PD included in the previous chapter are established.

In this chapter, when sharing direct quotes from teachers’ discourse, I underlined the words that came from the PD material analyzed and double underlined the words that came from other PD materials. The goal of such marking of the text is to make more visible when teachers were using specific vocabulary from the PD documents.
PD Foundational Documents

Mathematics Discourse Matrix

The Mathematics Discourse Matrix (see Appendix A1) was designed as a tool to be used by participants and facilitators in every session of the PD. It was introduced to participants in Session 1, when they received a hard copy of it. In addition, it was available in the classroom as a large poster to be used for consultation in every session. As a foundational document, it was intended to be connected to other PD materials and to activities developed across the sessions (Sztajn et al., 2013b). In this context, participants were able to use the Mathematics Discourse Matrix in different ways, as it was intended. My analysis of the data revealed the path teachers followed to understand the theoretical ideas contained in the document, as well as the three roles of this PD material according to the ways in which participants used it: (a) to classify types of discourse, (b) to focus on specific parts of discourse, and (c) to make connections to other PD materials.

Understanding the Mathematics Discourse Matrix. Participants needed time to deeply understand the ideas in the Mathematics Discourse Matrix, and, therefore, there are differences in the ways they discussed this material early in the PD program versus later. The progress in participants’ understanding of the Mathematics Discourse Matrix can be divided in two phases.

Initial understanding of the Mathematics Discourse Matrix. Participants analyzed ideas about discourse starting in Session 1. During that session, the Mathematics Discourse Matrix was introduced to them. Participants watched a video twice to start thinking about the Mathematics Discourse Matrix. During the discussion of those prompts, participants started
using the new vocabulary and introduced words such as *correcting, probing, or responsive* when they were sharing their ideas. They connected ideas about discourse to mathematics. They talked about how important it is for kids to use different strategies to solve mathematics problems. One teacher said it was important to make “connections and comparisons” between the strategies students use to solve a problem.

Even though participants were able to identify important ideas in the Mathematics Discourse Matrix in that session, there is evidence in Session 4 that they were still unclear about some concepts. It was difficult for participants to establish connections between this foundational document and their own practices. When they started to talk about possible connections between the Mathematics Discourse Matrix and their previous literacy practice, they showed their confusion. For example, T4 was disconcerted and could not find connections.

T4: Not sure. I’m really not sure how to fit these into this nice, neat little matrix.
F1: Okay.
T4: It’s kind of perplexing for me, how to transfer them (Session 4).

To start to make sense of ideas, one teacher used the case narrative that they had analyzed on the first day to recreate one example of how they could connect strategies to the matrix. The participant who made the comments noticed that the teacher in the narrative reached responsive discourse when he was working with students in small groups.

Even though it was difficult for participants to start grasping ideas from the Mathematics Discourse Matrix, participants transitioned during that same session 4 to a better understanding of this foundational document. They found examples of types of
discourse using the written classroom representations provided that day. They also noticed connections between students’ mathematical work and types of discourse.

T1: I agree that if one of the students, what you’re saying about how the student in had used the hundred chart by counting by ones, if another student gave the compliment but also make the connection that this is another way that she can move around the 200 chart, I think that one of the closest areas where they did make a connection was in line 58 when she said that she wouldn’t lose, that she put her numbers right there so she wouldn’t lose track, so she kind of explained it, and then there was another area that, actually more so in line 74 where she said, “I pretended that we were just going to put all the seeds together and make one huge plant,” so that, to me, sounded more like getting closer to responsive discourse because she made a connection (Session 4).

**Deeper understanding of the Mathematics Discourse Matrix.** After Session 5, participants showed a more solid understanding of the information in the Mathematics Discourse Matrix because they were using vocabulary from the artifact in their talk and making more connections to ideas such as their classroom experiences in relation to the theory supporting the Mathematics Discourse Matrix. They also made connections to information contained in an article they had read, to discourse strategies, and to phases of the lesson in the Mathematics Teaching Guide. During one whole-group conversation in this session, participants had to connect the discourse strategies that they had analyzed with the Mathematics Discourse Matrix and the Mathematics Teaching Guide. At least eight teachers who participated in the conversation showed they felt comfortable talking about the Mathematics Discourse Matrix and connecting ideas to it. They identified different types of discourse in their own practices or how the types of discourse changed across time in a lesson. They established connections between mathematics and discourse, and they identified types of discourse in their own conversations during the PD sessions.
T15: I think in the whole-group, when we started, we were doing a lot of eliciting where students were sharing their answers and how they got them, and you were accepting all the different, as the person up there, accepting all the different types of answers and we were looking at the answers and then it quickly went into probing and responsive very quickly because questions were starting to be asked, why did you do this or why did you do that? And then the questions went into, what does that look like? What does that look like that she just did what else does that look like, and then we were able to instantly make those connections and do so, it was like one of those things where I felt it started at eliciting, but went quickly into probing and very quickly into responsive as the questions started rolling (Session 5).

Sometimes during discussions, participants used vocabulary or ideas from the Mathematics Discourse Matrix without referring directly to it; other times they also clearly stated that they were looking at the matrix to classify the discourse they observed.

F1: Well, isn’t there different modes of communication though?
T12: I was looking at the listening while you were talking about that and how he’s obviously getting some of it, I was looking through the listening part of the discourse matrix, so, I mean, maybe he’s just, because it looks like he’s progressing, or it sounds like he’s progressing through the listening part, he just is a little bit, well, is lacking in the questioning and the explaining (Session 11).

During Session 12, participants analyzed the most important documents they used in the PD. In participants’ own words, the Mathematics Discourse Matrix was useful to them for many reasons. In addition, participants showed they understood the importance of different types of discourse depending on the goals for the lesson.

Roles of the Mathematics Discourse Matrix in the PD.

**Using the Mathematics Discourse Matrix to classify types of discourse.** Participants used the Mathematics Discourse Matrix to classify their own discourse in the PD sessions or to name the type of discourse they noticed in a classroom representation. This happened quickly in the PD, starting in Session 1. This role is connected to the initial understanding
participants had of the Mathematics Discourse Matrix, in which they focused their attention on the broader categories contained in the document, types and dimension of discourse.

In Session 1, participants classified their classroom discourse and started to use vocabulary from the matrix to make comments. For example, T19 and the small group she worked with defined the type of discourse that they were implementing in the classroom when the PD started.

T19: Our goals were to use more sentence starters, encourage gallery walks with discussion with the children, questioning strategies and moving to the probing-responsive, because we’re more in the corrective and listening discourse right now, some of the challenges, we need to explain a bit more about math talk and teach those questioning strategies for them, challenge now is implementing Common Core with this matrix, because we kind of feel like we’re trying to learn new ways and approaches and having to mix that in. The language barrier that might come across, and reaching all of the students (Session 1).

The Mathematics Discourse Matrix also allowed participants to identify changes in the type of discourse they were using during conversations in the PD. One example of this situation is the example on page 105, T15 identified how the conversation went from eliciting discourse to responsive discourse. She used vocabulary from the Mathematics Discourse Matrix, as it was shown by the underlined words in the quote.

These examples show that the Mathematics Discourse Matrix was a tool that provided participants with new vocabulary and categories that allowed them to understand the types of discourse and how those categories can be interpreted in certain situations. Participants also understood how students’ mathematics work is connected to types of discourse.

Using the Mathematics Discourse Matrix to focus on specific parts of discourse. A second role of the Mathematics Discourse Matrix is associated to with how the Mathematics Discourse Matrix is organized in a way that users can focus on specific parts of the
framework. By contrast with the previous use of the Mathematics Discourse Matrix, here participants were able to identify specific information contained in the artifact to create arguments about a certain type and dimension described in the Mathematics Discourse Matrix. The excerpt below supports this idea. In it, T16 was talking about her experiences with the Talk Triangle discourse strategy and the listening part of probing discourse.

T16: We had what you guys were both saying, but in the listening section of it, during the explore with the talk triangle, how it says the teacher listens for students’ mathematical explanations for their answers and how they got it and why they approached that problem in this way, since we responded and the teacher was kind of out of it, we had to listen to each other’s problems, I had said, “you’re gonna do it my way,” even though I was the questioner and T10 had to do it her way because she was the listener, and she had to do it her way, before we could all get on the same page, we had to kind of look at it our own way so we could come back to listen on how to do it. And let’s see, and how they got it and why they got it and then we’re able to compare our ideas [inaudible] listening for number two and explanations that were given, so in the probing section, on the listening side of it, there was a lot there that we did during our talk triangle (Session 5).

Another example of the way participants talked about specific ideas contained in the Mathematics Discourse Matrix is below. This discussion occurred in Session 12, and facilitator and participants were reflecting about discourse in relation to a PD classroom representation. During that conversation, participants included specific ideas from the categories questioning and listening. With regards to questioning, participants described how the teacher asked the how and why questions. In relation to listening, T12 brought attention to the description of this dimension in connection to the correcting type of discourse when she mentioned students were waiting for the teacher’s verification of their responses. This excerpt also illustrates the first use of the Mathematics Discourse Matrix in which participants used the larger categories in the Mathematics Discourse Matrix (in this case probing and eliciting) when participants were talking.
Okay, so we’ve got some ideas for the goal, but what about the discourse? What discourse was happening?

T: Probing!

F2: Probing? Can you tell me where in the transcript?

T1: Well, I started noting it on the all talk, in line 70, she said, “and what would you do, Omar,” and then in 75, “then how did you count,” and then 81, “how did you count? Show us how you counted,” so it was very, and I felt like more, there was a lot of it too when they were doing their explore phase as well, I just didn’t start noting it until then, and then there was no, there was nothing from the students who listening, there was no discussion there. It was just them sharing, and her getting them to explain the work.

T: I thought there was more of the eliciting discourse, just because she asked how a lot. She asked why a couple of times, but she asked how a lot of the children.

F1: Anybody find in the transcript where she asked how or if she even asked why?

T: There’s one…

T: Line 31, how did you do it?

T: 49 is why.

T12: I was also thinking it seemed like the kids were waiting a lot more for her approval in the listening, that they were waiting for her approval all their procedure was acceptable, because then [the video ends] (Session 12).

This second role evidenced how teachers show a more sophisticated understanding of the theoretical ideas contained in the Mathematics Discourse Matrix because they attended to specific information contained in the foundational document.

Using the Mathematics Discourse Matrix to make connections to other PD materials. The quotes in the previous section include examples of the third role that the Mathematics Discourse Matrix played during the PD: a tool that helps participants to make connections to other PD materials. In the quote on page 107, T16 commented on the roles that different people took during the rehearsal of the Talk Triangle discourse strategy when she mentioned the questioner, and the listener. In addition, T16 made a reference to the explore phase of the Mathematics Teaching Guide. The quote on page 108 also showed how participants used the written classroom representations to generate ideas in connection to the Mathematics Discourse Matrix.
Number of Idea Units in Which Participants Connected the Mathematics Discourse Matrix to Other PD Materials

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<thead>
<tr>
<th>Any other PD material</th>
<th>Written classroom representation or video</th>
<th>Mathematics Teaching Guide</th>
<th>Handout</th>
<th>Article</th>
<th>Mathematics problem sheet</th>
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Table 13

Number of Idea Units that Contain Information from the Mathematics Discourse Matrix

<table>
<thead>
<tr>
<th>Dimension</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
<th>S10</th>
<th>S11</th>
<th>S12</th>
<th>S13</th>
<th>Total</th>
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<td>10</td>
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<td>0</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

Note: Dimension 1 means only one material; dimension more than 1 means use of two or more materials within an idea unit.

In general, the examples provided throughout this section about uses of the Mathematics Discourse Matrix illustrate how participants connected this foundational document to other PD materials. More than half of the idea units in which participants talked about the Mathematics Discourse Matrix included connections to at least another PD material (see Table 12 and Table 13). They made references to discourse strategies or number lines in written classroom representations. The interaction of those materials helped teachers understand concepts, make connections to their practice, and understand the meaning of each type of discourse provided in the Mathematics Discourse Matrix. One example that included different PD materials in the same idea unit is provided below. In this example, participants connected the launch phase from the Mathematics Teaching Guide, the Mathematics
Discourse Matrix, the article they read as homework the previous night, and the written classroom representation they were using to support the discussion. The discussion was about a video they had just watched, and the facilitator asked teachers how they could improve responsive discourse.

T18: I think one of the things that popped up in my head was the girl started with the hundreds chart, what came to mind for me was it actually was linked to the homework we read last night was using that hundreds board and being able to see that when you come down ten and over one, it would have been a faster way instead of going one, two, three, five, seven, eleven, no, I know that it’s plus ten and plus one and seeing that connection, going that step further instead of having to count every single one. I thought that would have [inaudible].

[silence 14s]
T1: I agree that if one of the students, what you’re saying about how the student in launch had used the hundred chart by counting by ones, if another student gave the compliment but also make the connection that this is another way that she can move around the 200 chart, I think that one of the closest areas where they did make a connection was in line 58 when she said that she wouldn’t lose, that she put her numbers right there so she wouldn’t lose track, so she kind of explained it, and then there was another area that, actually more so in line 74 where she said, “I pretended that we were just going to put all the seeds together and make one huge plant,” so that, to me, sounded more like getting closer to responsive discourse because she made a connection.

T12: You just said that you thought line 74 was a good example of a responsive connection and I wanted to add a thought, this was a little bit more metacognitive, on line 48, when she said, “I was going to do tally marks so I did it, but then I realized that wouldn’t work, so I go five, ten, 11, and then had to count on five more.” so she realized that wasn’t quite as effective counting strategy as she had originally thought (Session 4).

**Mathematics Teaching Guide**

The Mathematics Teaching Guide was introduced in Session 3. In a similar way as the Mathematics Discourse Matrix, it is a foundational document in the PD, and teachers had a copy of it and they could use a large poster of it in the classroom in every session. The PD used the Mathematics Teaching Guide in connection to videos and written classroom representations that allowed teachers to reflect on the phases of the model. A second way
participants used the model was in the planning process that participants completed as part of the homework requirements. During whole-group discussions, the Mathematics Teaching Guide was used in relation to only two phases: launch and explore. When the facilitator introduced the Guide for other phases, no whole-group discussion occurred.

As it was made for the Mathematics Discourse Matrix, a description of the way participants understood the ideas in the documents is presented, and then detected uses of the document during the PD are reported. Participants used the Mathematics Teaching Guide in different ways: (a) as a tool to plan, (b) to incorporate new vocabulary, and (c) to make connections to other PD classroom artifacts.

**Understanding the Mathematics Teaching Guide.** Two phases in the way participants talked about the Mathematics Teaching Guide were detected. During Sessions 1 through 5, participants attended to specific vocabulary provided by the document to analyze the phase of a lesson. In contrast, during Sessions 12 and 13, participants used the broad categories to talk about the Mathematics Teaching Guide. There is a gap between sessions 6 and 11 in which no discussions about this foundational document were found. A possible explanation of this situation is provided below.

**Initial understanding of the Mathematics Teaching Guide.** During Session 3 participants started to analyze the Mathematics Teaching Guide. From the time they started to analyze this foundational document, participants demonstrated they were familiar with the cycle shown in the model. For example, one participant explained the reason for not expressing their questions about the model.
T9: I guess that’s why we don’t have as many questions because this is very familiar in a literacy setting. To me, it’s like, yeah, right, makes sense, so it makes sense in math (Session 3).

This comment showed that participants connected the model to their previous practices, and then it was not difficult for them to understand the information contained in it. During discussions, they expressed their concerns in relation to the implementation of the new ideas they were learning in their classrooms, saying they were worried about how much time they will need to plan a lesson, if the whole process shown in the model happens in every lesson, or about how to connect it to the C-MAPP. For example, T2 questioned if she knew enough about “how student learn mathematics in order to do all these phases successfully” (Session 3). In addition, participants were able to make connections to discourse. For example, there was a prompt to make participants think about what the Mathematics Teaching Guide represents. During this discussion, T7 connected the ideas in the document to their experiences in a literacy lesson. Then T24 anticipated what those ideas would look like in a mathematics classroom.

T24: I think it’s also you, in your classroom, you’re setting up a structure in your classroom of how to really elicit this discourse from your students and they’re practicing with language for a while and they’re learning things. And then once they really master that and that’s in their, how they communicate, then you’re gonna take them a step further, you know, you’re just gonna keep modeling that in your classroom (Session 3).

As it was for the Mathematics Discourse Matrix, participants were provided with videos to analyze phases of the Mathematics Teaching Guide. During those discussions, participants used words from the Mathematics Teaching Guide to organize their thoughts about the prompts they were asked to answer. For example, T12 said in response to the first prompt: “They are trying to accomplish a purpose, a foundation, vocabulary, prior
knowledge and expectations” (Session 3). During the second part of that discussion, many teachers shared the connections they made between the video and the Mathematics Teaching Guide, and again, they used words from the model.

T12: So one part that we talked about at the beginning was how effortlessly she sort of introduced the vocabulary into the, to the children, she didn’t ever say they are going to compare, what compare means, she said “I’m going to figure out who has more,” that I’m going to compare, so she sort of filled that in together. Teaching them how to pick out words like some, paying attention to the question marks, all that really stood out to us.

T1: As I was looking at it the first time and looking at the launch portion of the model, I was thinking she is not really, I wondering when she’s going to start engaging the students into talking, and then when I started looking at it more, we were kind of thinking, you know, her purpose isn’t about solving the problem. It’s more about modeling expectations of the discourse.

T8: And I was thinking this was more like a think aloud as we do with a reading, we have expectations set in place for the students and then we would show them specifically what kinds of things you would have for their learning outcomes, so it’s basically a think aloud, just more purpose is going to share how their thought process was working (Session 3).

Those two examples show that at this particular point in the PD teachers were using specific vocabulary from the Mathematics Teaching Guide to understand each phase.

In Sessions 4 and 5, participants made connections to other PD materials. They identified discourse strategies they could use in the launch phase, and they connected ideas from an article they had read as homework with the Mathematics Teaching Guide using vocabulary from the model. For example T12 reflected about the connections she had made.

T12: I thought it was really cool to see what that, establish purpose, engage, elicit prior knowledge, introduce language, what that actually ended up looking like in a classroom where they had a really difficult task put in front of them and seeing just the way that teacher worded it and kind of make me think that I would be able to use that same sort of thing to launch things in my classroom too (Session 5).
**No discussions about the Mathematics Teaching Guide.** During Sessions 6 through 11 (except for Session 8), the Mathematics Teaching Guide was presented to teachers as a reminder of the phases and the discourse strategies they could use in each phase. No whole-groups discussions connected to the Mathematics Teaching Guide occurred during Sessions 6 through 11. This fact could be explained by analyzing the way the sessions were designed. During the work in each one of those sessions, the facilitator made connections to phases on the Mathematics Teaching Guide, while participants learned and rehearsed different discourse strategies. The whole-group discussions were organized more in connection to the discourse strategies and less as a function of the information in the Mathematics Teaching Guide by itself.

**Deeper understanding of the Mathematics Teaching Guide.** In Session 12, discussions about the Mathematics Teaching Guide were part of participants’ reflections about the topics and tools they had learned in the PD. They talked about the value of the Mathematics Teaching Guide as a tool for planning lessons and how that connected to the promotion of responsive discourse.

T15: We said that it was a good tool to assist in the formation of lesson plans in that it helps students take responsibility in math for the connectiveness of the discuss and explore phases and how connected those two pieces are, and that it keeps the teacher accountable for and focused on the best practices for students (Session 12).

This quote makes evident that the tool helped participants to understand the importance of kids making mathematical connections. Another teacher comment showed that they had a deeper understanding of the model and how it is connected to other topics learned in the PD:

T: We don’t really have to look at this, I mean, it’s so natural now and that happens every day, it’s not like I have to sit down with my lesson plans and type out,
okay, my launch will be this, my explore will be that, and we’re a STEM school too, so we’re already using the STEM process, and this is kind of incorporated with that as well, so it’s kind of like all these things are just so connect with each other (Session 12).

The last two examples show that at the end of the PD, participants talked using the larger categories (phases of the lesson) contained in the Mathematics Teaching Guide instead of specific details in it. This way of talking was maintained when they analyzed a PD classroom representation in this session, and in Session 13 as well. Participants talked about the phases, making connections to what they had noticed, but they did not use the specific vocabulary from the model.

F: Okay, so we might have a goal and we agree, does everybody agree she’s eliciting and then let’s talk about what was her launch, explore and discuss. What was her launch?
[multiple answers]
F: Read the problem aloud together. Anything else?
[multiple answers]
T: It’d be interesting if the whole lesson was a launch. You know what I’m saying? It’s just like [inaudible] write it down now, but that would have been…
T: I think what’s interesting is I think we’ve all, from before we started taking this, we saw ourselves in this very, this is how we’re teaching, we felt really good, we’re sharing kids’ work, getting them to share up, they’re actually math talking, but it’s very much like what we did, like what we were doing before, so I think that’s very interesting to think about.
T: I feel like they didn’t go to their explore with any sort of direction, you know, charge, “today, you’re doing this.”
T: Solve this problem, line five six.
[multiple people talking]
T: Now we’re solving story problems every day and it’s sort like every day you have to pick out that focus, and they read it all together and it was like, alright, go get it. It wasn’t started, there wasn’t really, she didn’t sort of hone in on one structure in the problem like I feel like we’ve been doing a lot more of.
F: Okay, that’s the launch (Session 12).
This way of using the Mathematics Teaching Guide contrasts with the use of the Mathematics Discourse Matrix in which participants began by using the larger categories and later on were using specific details from the documents.

**Roles of the Mathematics Teaching Guide.**

**Using the Mathematics Teaching Guide to plan.** The quotes given in previous sections show that participants found it convenient to use the Mathematics Teaching Guide to plan their lessons. In one of the previous quotes from Session 12, on page 114, T said that she was so familiar with the document that she did not need to write down all the details that she was planning to implement in her lesson. That teacher was able to see connections to school requirements as well. Also, teachers defined the changes they had made in their lessons. For example, T7 explained how she had changed the launch phase in her classes.

**T7:** My launch has changed and the evidence I have for that is that I’m not giving away important things in the beginning or solutions to the problem before we even start (Session 13).

**Using the Mathematics Teaching Guide as reference for new vocabulary.** The model gave participants the possibility of using the words provided in the documents for the purposes of each phase of the lesson to talk when analyzing PD materials such as videos or written classroom representations. The example on page 92 showed how T12 used the words: “establish purpose, engage, elicit prior knowledge, introduce language” (Session 5) as they appear in the Mathematics Teaching Guide. This way of talking about the document was common during the first sessions. During later sessions participants used broader categories. One example of this situation is on page 113, when a teacher explained how comfortable she
felt about the planning process of a lesson. She talked about the launch and explore phases in
that quote.

**Using the Mathematics Teaching Guide to make connections with other PD materials.** Participants were able to connect PD materials to the Mathematics Teaching Guide from the introduction of this foundational document. They talked about this document most frequently in connection to other PD materials, with the second most common connection being with the written classroom representations or videos (see Table 14 and Table 15). The examples provided above showed that, from Session 3, participants made connections to discourse and to the PD written classroom representations when they were discussing phases of the Mathematics Teaching Guide. The use of classroom representations made participants reflect deeply about phases of the Mathematics Teaching Guide.

Table 14

<table>
<thead>
<tr>
<th>Any other PD material</th>
<th>Written classroom representations or video</th>
<th>Mathematics Discourse Matrix</th>
<th>Handout</th>
<th>Article</th>
<th>Mathematics problem Sheet</th>
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<td>7</td>
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</table>

Table 15

<table>
<thead>
<tr>
<th>Dimension</th>
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<tbody>
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<td>S2</td>
</tr>
<tr>
<td>More than 1</td>
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<td>0</td>
</tr>
</tbody>
</table>
The quote below occurred in Session 3. Participants watched two videos in which they compared two different Think Alouds. The prompts for that discussion were “What might be the teacher’s purposes for what she does?” and “What are the students doing in response?” Participants engaged in that discussion, and they were able to use the Mathematics Teaching Guide and the written classroom representations to make sense of different ideas.

F2: And my other group, can you guys talk about some of your thoughts as you were going through?
T2: Well, we decided that it wasn’t a launch, that she was teaching them the lesson, because she wasn’t engaging the students. However, she didn’t ever elicit a response from them. When she did the some, she said in line seven, well, it was 89. she said Jane and Ernie have some, and then she just wrote some means we don’t know how many, you know, it was never any, and she didn’t engage prior knowledge by saying, “remember when we talked about some before?” At the ending, she said very quickly, “now how do I figure out who has more, in 14 and 15 she said how can I compare the same way, who has less? Those are the same question, and we questioned was she’s doing or exhibiting prior knowledge, or trying to elicit that? She was real thoughtful in how she introduced things, for example, when she said, “there’s two questions and there’s going to be two answers,” and when she drew out the first set of apples, Jane has six, she drew them out but never counted. When she got to Ernie, she said, okay, I’m going to draw out Ernie’s, and she drew but when she got to six, she went, six, seven, eight, nine, to point out this one has more.
T10: I just had a question, when F2 talked about what a launch is, and I wrote down, “to get them engaged,” there were parts of the launch, but the engagement part was the big question I had: “how engaged should the students be interacting with a teacher?”
T6: In the second question, we were talking about, where the student’s doing a response, we talked about how we couldn’t see if they were really engaged or if they were following along because the students we were looking at, you could see them looking at her, but you couldn’t really tell if they were engaged or not or if they were just listening like it was the beginning of direct instruction, and then it looked like she was very engaged and it’s like you see them look at them, but there was never really response to what she was doing from what we could see.
T1: The first time I watched it, that’s exactly what I thought. “She’s having a lesson with herself.” That’s exactly what it seems like, she’s teaching herself, but then the second time, when you think about what her purpose is, it’s almost like, I was thinking, she is modeling what she wants them to be doing when they’re working out the problem, discussing it, talking about it, how they’re going to, how they’re actually
working it out, their thinking. And then they find the vocabulary, they’re finding it as they’re working it out. But there was some student interaction there (Session 3).

This discussion shows how participants used specific language from the Mathematics Teaching Guide to understand what happened in the videos. Their arguments helped them to assess whether the teachers were doing a launch.

**Conclusions about the Role of the Foundational Documents**

The Mathematics Discourse Matrix and the Mathematics Teaching Guide played a role as frameworks to understand different key concepts decomposed from practice by PD designers. The Mathematics Discourse Matrix helped teachers participate in conversations about different types of discourse and different dimensions of discourse that they could implement in the classroom. The Mathematics Teaching Guide helped teachers participate in discussions about different phases of the lesson. Participants learned the vocabulary contained in both foundational documents, and they participated using it in different contexts. This new vocabulary was part of the common language teachers developed and used as part of the AIM PD community of practice. Participants connected those foundational documents to examples of classroom representations, to their own discourse in the PD, or to their experiences in their own classrooms. This way of analyzing the theory in connection to practice suggests the reason why participants were able to use the stance *interpretation* frequently in PD discussions.

The representation of the discourse framework as a matrix arrangement allowed participants to talk about the broader categories or about specific characteristics for each of them, depending on their goal when talking in the PD setting. A similar organization of the
Mathematics Teaching Guide with five broad categories, one for each phase of the lesson, and verbs that describe the purposes of each phase allows teachers to use technical language as needed. Participants were able to understand the importance of each foundational document and to find connections between them during the process of thinking about classroom representations.

The quotes provided show how teachers participated in PD conversations using details provided in the materials, which helped them to be more specific. The connections to other classroom representations provided participants with opportunities to deepen their understanding of the concepts that the foundational documents contained, and participants discussed key ideas using the examples provided. For example, during the introduction of the launch phase, teachers were exposed to two different Think Alouds, and in the discussion that took place there, they analyzed the purpose of the teacher in each artifact, whether the teacher was launching the lesson. This kind of practice suggests the idea that teachers started to make connections to practices, and to anticipate what could happened when they implement some ideas in the classroom because they were exposed to different representations. They analyzed what kids and the teacher did, what was the role that each one took, how the teacher reacted to certain situations, or how the teacher could plan the lesson.

Participants’ concerns about how to implement the ideas contained in the foundational documents were solved during the PD process, and the comments made at the end of the PD showed they understood how to integrate the new ideas they were learning in the PD with their requirements in school. This conclusion supports the shift in teachers’ participation associated with the topic of teaching concerns. The evidence suggests that the
combination of the analysis using both the foundational documents and the classroom representations helped teachers to interpret the new information and to make sense of the ideas they were learning.

**Other PD Materials**

In this section a description of the use of other PD materials that were not foundational documents is presented: mathematics problem sheets, written classroom representations, and responsive discourse strategy handouts. The mathematics problem sheets can be considered representations or approximations to practice, depending on the design of the task. For example, Isabella’s method was a representation of practice because it simulated how a student solved a mathematics problem (Grossman, Compton, et al. 2009). The responsive discourse strategy handouts are decompositions of practice because they represent pieces of practice that were provided to participants for pedagogical purposes (Janssen et al., 2015).

**Mathematics Problem Sheets**

The four tasks in which teachers developed Mathematical Knowledge for Teaching were Isabella’s Method, the Ferrero Chocolates Math Problem, the Problem Types Task, and Maria’s Problem. Those four PD materials are called mathematics problem sheets in the current study. Isabella’s method and Maria’s work are considered representations of practice, whereas the Ferrero Chocolates Math Problem and the Problem Types Task are approximations of practice. The tasks were designed to analyze student work or to solve a mathematics problem as adult learners. During the analysis of those tasks, participants were
provided with different experiences in order to analyze each one. For example, they analyzed student work individually before the whole-group discussion. The participants frequently used different discourse strategies during the process of analyzing student work or when they were discussing a mathematics problem.

**Use of the mathematics problem sheets in the PD.** When participants were analyzing mathematics problem sheets they were focusing on the problem they were given, and they were not connecting them to other PD materials or foundational documents (see Table 16).

Table 16

<table>
<thead>
<tr>
<th>Dimension</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
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</tbody>
</table>

**Connections to actors and stances.** All the comments made while the mathematics problem sheets were used by participants were connected to students, to themselves, or to both actors. Participants never made connections to teachers when analyzing mathematics problem sheets. This is explained by the fact that the materials were designed to analyze student work, or for participants to solve a problem as adult learners. Also, the materials were shown to be useful in keeping conversations into the specific level of specificity, with teachers using the stance *interpreting* in every idea unit. Mostly all the conversations were focused on students’ mathematical thinking or on mathematics (see Table 17).
Table 17

*Mathematics Problem Sheets in Connection to Other Variables*

<table>
<thead>
<tr>
<th>Category</th>
<th>Subcategories</th>
<th>Number of idea units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actor</td>
<td>Student (no dimension)</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Self (no dimension)</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Student &amp; Self</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Teacher (no dimension)</td>
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</tr>
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<td></td>
<td>Specific</td>
<td>17</td>
</tr>
<tr>
<td>Stances</td>
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<tr>
<td></td>
<td>Evaluate</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Interpret</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>Describe &amp; Interpret</td>
<td>11</td>
</tr>
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<tr>
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</table>

With regards to actors, when participants talked about students they were both describing and interpreting how students solved the problem. In a similar way when participants talked about themselves, they described how they were thinking or how they had solved a problem. The following example shows how T24 described and interpreted Isabella’s method solving a subtraction problem.

T24: I saw her using a number line, this was her description of how she used her number line, she found 467 and the 148 on the number line and then moving up, and so she went her two places to get to the 150 and then she was able to move 17 to get to her, what, 167, and then she just had to do hundreds each time, to look at the jumps (Session 2).  

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Similarly, teachers explained their own solutions to a given problem. For example, a teacher explained how she solved the Ferrero Chocolates Math Problem:

There were six complete boxes that were sold, so that nine, nine times six is 54, and then there were eight additional candies left in that other box that only had one candy left, and so that 54 plus eight is 62 (Session 5).

In both cases the problem sheet and the task helped participants to describe the solution to the problem using specific details and different representations.

An important distinction can be made when participants talked about students and themselves together. Participants continued describing students’ work or their own solutions or interpretations to the problem, but they compared also different ways to solve the problem. They almost never used the stance evaluate, which means they were not comparing which is the best solution, but analyzing different strategies to solve the same problem. This practice of comparing strategies made teachers reflect about their own mathematical knowledge because of the connections they needed to establish. One example of this situation was given by T2. This participant was able to connect her understanding of the problem with Isabella’s way to solve the subtraction problem.

T2: I didn’t do that, but that’s how she did it, but I generally, I don’t know, but I was like, well, okay, she used 150 because that’s how she adds well. She uses those the best, so she said, okay, 67 rounds down to 50, so that’s 17, and 48 rounds up two, so that’s, you know, to 50, so that’s two and 17, so that’s easy to subtract 400 minus 100 is 300, and I tried that on a two digit number instead of three digits and it worked every time.
F2: Okay. And you saw the numbers from a different viewpoint, you were looking at it to your 50s.
T2: Right, I just ignored four, the hundreds places, and just looked at, and she wasn’t rounding, she was just going to the nearest 10, but it was 50. That was the number she chose (Session 2).
**Level of specificity.** This excerpt is also evidence of the specific level of specificity. The materials gave teachers enough details to generate conversations in which they were able to identify precise information that they could use to think about mathematical ideas. It is important to notice that in every idea unit in which participants used mathematics problem sheets, they also used the stance interpreting. That situation shows that teachers not only repeated what they had observed in the mathematics problem sheets, but they were able also to make sense of the information to generate new ideas. This conclusion shows that the materials fulfilled their role as an important support to analyzing mathematical ideas as was planned by the PD designers (Sztajn, Heck, & Malzahn., 2012i).

**PD Written Classroom Representations**

PD written classroom representations were used in nine different sessions during the PD, and participants discussed as many as three different classroom representations in one session (see Appendix A4). More than two thirds of the PD written classroom representations were used during the Summer Institute. It was an explicit expectation for participants to use the written classroom representations as a tool to provide evidence of their arguments.

**Use of the written classroom representations in the PD.** Many previous quotes have shown how participants used written classroom representations during conversations. Participants used them to support their comments and to make sense of the PD foundational documents. The requirement of providing evidence from written classroom representations and videos to make arguments impacted the way teachers talked. One example of how participants used the written classroom representations is provided below. T2 used the
number lines in the written classroom representation to look for evidence of discourse by the facilitator’s request. T2 was talking about a conversation between the teacher and a student called Bernard.

T2: Well, I was just gonna say, when she interacted with Bernard, she did walk him through that, what was an easy way to use those, like, 52, even through 62, she taught him how to put ten together, you know? “How would you make that easier to add?” Then it made it easier for him, you know, because in the beginning, he was like, “I don’t know, I drew these three cards,” he didn’t know where to begin, you know, so she told him, “remember, I used the ten strategy, making a ten, can you do that?” (Session 1)

This quote illustrates how participants were using the number lines from the written classroom representation to make claims about the idea they wanted to highlight from the document. They described important details in the document and interpreted the information. In the example, T2 described part of the transcript, and interpreted how the teacher showed an easier way to the student, who had been confused about what he could do.

Another illustration of how participants used the written classroom representations during the PD can be seen in the example on page 110. In that excerpt, T18 and T1 highlighted the use of the hundreds chart as evidence of a student making mathematical connections, while T12 used the written classroom representation to make connections to discourse, and to the student’s mathematical thinking.

Frequently participants gave their own interpretations of the ideas they were talking about. Sometimes participants also used their own experiences in mathematics or literacy to contribute to the discussion using both descriptions and interpretations (see Table 18). The excerpt on page 113 is an example of this situation, in which participants described what they
noticed in the classroom representation and at the same time gave their own interpretation as to whether that representation was a launch.

Table 18

<table>
<thead>
<tr>
<th>Category</th>
<th>Subcategory</th>
<th>Number of idea units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level of Specificity</td>
<td>General</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Specific</td>
<td>46</td>
</tr>
<tr>
<td>Stances</td>
<td>Describe</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>Evaluate</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Interpret</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>Describe &amp; Interpret</td>
<td>33</td>
</tr>
<tr>
<td>Topic</td>
<td>Discourse</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>MKT</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Mathematical thinking</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Mathematics</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Pedagogy</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>Teaching concerns</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>9</td>
</tr>
</tbody>
</table>

There are three different ways participants made references to written classroom representations. One way occurred when teachers read the exact text in the written classroom representation and indicated the line number in which the text appeared. Another way to use written classroom representations occurred when participants read the text without indicating the line number. Finally, they also commented on ideas from the written classroom representation without reading the idea in the text or providing the line number. These three alternatives were observed across the sessions. In those comments, participants used more specific than general comments independent of the use or not of the number line. Mostly all the general comments happened in Sessions 1 and 2 (see Table 19), which is evidence that
teachers learned to use the written classroom representations to provide details early in the PD. This idea will be analyzed later in this chapter.

Table 19

<table>
<thead>
<tr>
<th>Level of specificity</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Specific</td>
<td>4</td>
<td>11</td>
<td>9</td>
<td>5</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>9</td>
<td>16</td>
<td>11</td>
<td>5</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

*Note: Sessions in which no written classroom representations were used are highlighted gray.

In half of the idea units in which participants were using written classroom representations, they used other PD materials, including foundational documents (see Table 20). When they did, most of the connections were made to the foundational documents. The use of the video provided teachers with opportunities to reflect deeply on key ideas of foundational documents. For example, in Session 3 the group discussed a Think Aloud video. In that discussion, the written classroom representation played an important role in analyzing the Launch Phase definition and its characteristics. Previous examples on page 113 and on page 118 were part of that conversation, in which teachers used the classroom representation to justify whether the interactions they had watched in the video constituted a launch. They used the technical language from the Mathematics Teaching Guide combined with parts of the written representation to justify their ideas.
Table 20

**Number of Idea Units That Contain Ideas about the Written Classroom Representations**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Session</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S1 1</td>
<td>16</td>
</tr>
<tr>
<td>S2 5</td>
<td>S3 0</td>
<td>7</td>
</tr>
<tr>
<td>S4 0</td>
<td>S5 0</td>
<td>0</td>
</tr>
<tr>
<td>S6 0</td>
<td>S7 7</td>
<td>1</td>
</tr>
<tr>
<td>S8 1</td>
<td>S9 0</td>
<td>0</td>
</tr>
<tr>
<td>S10 0</td>
<td>S11 0</td>
<td>0</td>
</tr>
<tr>
<td>S12 0</td>
<td>S13 0</td>
<td>0</td>
</tr>
<tr>
<td>More than 1</td>
<td>4 0</td>
<td>5 0</td>
</tr>
</tbody>
</table>

**Connections to actors.** With regards to actors, the use of the written classroom representations is more linked to students than to teachers. It was less commonly connected to themselves. Most of the connections to each actor happened during Summer Institute, and the justification for that is during that period of time there were more PD classroom representations to be analyzed (see Table 21 and Appendix A4). The use of both actors, student and teacher, within an idea unit occurred more frequently during the Summer Institute as well. Students and a teacher were actors in all the classroom representations used across the PD.

Table 21

**Actors in Connection to PD Written Classroom Representations**

<table>
<thead>
<tr>
<th>Actor</th>
<th>Session</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student (no dimension)</td>
<td>S1 7</td>
<td>15</td>
</tr>
<tr>
<td>Teacher (no dimension)</td>
<td>S2 8</td>
<td>7</td>
</tr>
<tr>
<td>Self (no dimension)</td>
<td>S3 1</td>
<td>1</td>
</tr>
<tr>
<td>Student &amp; Teacher</td>
<td>S4 7</td>
<td>6</td>
</tr>
<tr>
<td>Student &amp; Self</td>
<td>S5 1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Note:** Sessions in which no written classroom representations were used are highlighted gray.
Responsive Discourse Strategy Handouts

Participants were introduced to nine discourse strategies across the PD. To learn about the discourse strategies they were provided with opportunities to try them out during their work in the PD and to implement them in their own classrooms.

Use of the responsive discourse strategy handouts in the PD. In general, participants talked about the strategies using the name for each one and did not provide details from the responsive discourse strategy handouts during discussions. I conjecture that this way of using that PD material occurred because participants were given the document after the tryout and after analyzing the discourse strategy in the session (Sztajn et al., 2012i). For all the strategies, participants started to mention the strategies names after the strategy was introduced (see Table 22).

Table 22
Number of Idea Units in Which Participants Used the Responsive Discourse Strategy Handouts by Name

<table>
<thead>
<tr>
<th>Discourse Strategy</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
<th>Session 4</th>
<th>Session 5</th>
<th>Session 6</th>
<th>Session 7</th>
<th>Session 8</th>
<th>Session 9</th>
<th>Session 10</th>
<th>Session 11</th>
<th>Session 12</th>
<th>Session 13</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Talk (1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bet Lines (9)</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Draft &amp; Final Copy (9)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Probing &amp; Pressing Questions (10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Talk Triangle (4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Talk Chain (4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Think Aloud (3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Think-Pair-Share (2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turn &amp; Talk (1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The number in parenthesis next to the discourse strategy name represents the session in which the strategy was introduced.
One example of this way of talking about the discourse strategies was found in Session 4 when participants had just cited the discourse strategies names.

F1: What we’ve done is the launch this morning, and then the explore was the talk triangle. But I kind of want to go back and we kind of want to go back and go back to launch for a minute and talk about those strategies, maybe name some strategies that we could stick up there on the poster just like we did with the what, how and why connections. So can anyone think of a launch strategy that we’ve done already? Like what did Sidney do this morning?
T4: Well, we did an all talk.
F1: All talk.
T4: I wrote it down.
F1: Hang on. Okay, so all talk could be a launch. What else could be a launch? What?
T6: Think aloud.
F1: Think aloud. Alright, anything else?
T: A turn and talk.
F1: Turn and talk. Good job. Okay. So now that we went from the launch to explore, what could be, remember the launch is more, I wouldn’t say whole-group, but it is kind of whole-group, small group at times (Session 4).

The use of the discourse strategies is linked more to the rehearsals in the PD or implementation of them than to the use of responsive discourse strategy handouts during the PD discussions. There are different ways in which participants reflected about their different familiarities with the discourse strategies. One example of this happened when T16 connected the Mathematics Discourse Matrix, the Mathematics Teaching Guide and the role that she and T10 took during the tryout of the Talk Triangle discourse strategy as the questioner and the listener in the discussion.

T16: We had what you guys were both saying, but in the listening section of it, during the explore with the talk triangle, how it says the teacher listens for students’ mathematical explanations for their answers and how they got it and why they approached that problem in this way, since we responded and the teacher was kind of out of it, we had to listen to each other’s problems, I had said, “you’re gonna do it my way,” even though I was the questioner and T10 had to do it her way because she was the listener, and she had to do it her way, before we could all get on the same page,
we had to kind of look at it our own way so we could come back to listen on how to do it. And let’s see, and how they got it and why they got it and then we’re able to compare our ideas [inaudible] listening for number two and explanations that were given, so in the probing section, on the listening side of it, there was a lot there that we did during our talk triangle (Session 5).

This quote shows another way in which participants talked about the responsive discourse strategy handouts, and it is related to the roles that they took as listener and questioner in the tryout of the Talk Triangle discourse strategy. In this case it is more specific than the discourse strategy name, but it is also related to the tryout and not to the handout in particular.

The possibility of the tryout of the discourse strategy helped participants to reflect about the pros and cons of the implementation of different strategies. For example, T5 described her experience with a discourse strategy.

T5: I felt more comfortable than yesterday, but I still felt that, feel anxiousness about having to repeat what they just said, I still felt with talk triangle was a lot more people, just being in that small group, just me as an adult, I felt I can talk more (Session 5).

The tryout also allowed participants to think about how to implement the discourse strategies in their classrooms. For example, T24 reflected about how she would organized a lesson.

T14: I was wondering if we could just take these questions and start using these questions, I mean, any one lesson, not necessarily small groups, but get them really used to using the questions.
F1: So everything you do, just have the questions available?
T14: Yes. Like make an anchor chart or something similar (Session 4).

Finally the implementation of the discourse strategies in participants’ own lessons, also provided participants with opportunities to reflect about how the strategy worked in the
classroom, pros and cons of the discourse strategies. The quote on page 62 illustrates how a participant reflected about how the Draft and Final Copy discourse strategy is beneficial for English Learners (ELL) students.

Conclusions about the use of other PD materials

Mathematics problem sheets. The data analysis showed that participants focused on students or themselves when using mathematics problem sheets. This suggests that this material helped participants focus more on those two actors when solving mathematics problem sheets. The way in which participants analyzed this material—trying out and reflecting on discourse strategies—could also justify the shift in teachers’ participation in which they integrated the discussion about mathematics with questions about discourse and their own practices. The materials, and the expectations provided to participants promoted the use of different representations to solve the same problem, which could help participants to be more specific and to interpret students’ work.

PD written classroom representations. Written classroom representations gave participants specific information that helped them provide evidence to support their ideas during whole-group discussions. The norm that was established at the beginning of the PD assisted participants in being more specific, as they cited number lines and details from the transcripts.

During whole-group discussions, participants had to interpret information as they connected ideas from the Mathematics Discourse Matrix or from the Mathematics Teaching Guide to the written classroom representations. Evidence showed that participants recalled
previous classroom representations when they were confused about certain ideas as it was the case when they started to make connections to the Mathematics Discourse Matrix and one participant made reference to a Case Narrative they had previously analyzed.

Because the PD provided participants with opportunities to compare classroom representations such as the two different Think Alouds, participants could interpret and use specific information from the transcripts to support their different points of views about different implementations of the discourse strategies. This way of utilizing the classroom representations provided participants with opportunities to reflect about the key ideas in the foundational documents, to connect them to practice, and to talk to each other during whole-group discussions.

The data supports the idea that the implementation of video in connection with the written representations gave participants access to more specific information than if they only had watched a video. This added information gave participants more opportunities to be specific and to interpret information. One conjecture here is that watching the video more than once may have helped teachers to improve their interpretation of certain information.

Finally, in every classroom representation the main actors were students and the teacher, and this could justify why participants made more comments about those two actors when talking.

**Responsive discourse strategy handouts.** The role of the responsive discourse strategy handouts was not possible to analyze using the data. Most of the information found is attached to the tryout of the strategy in the PD or the reflection about the implementation
in participants’ own classrooms. The excerpts used as examples about how participants used responsive discourse strategy handouts evidenced that participants developed a common language that was used during the PD conversations to talk about certain ideas related to the strategies. This language is connected more to the larger category of each strategy (its name) and less to specific details provided in the responsive discourse strategy handouts.

More analysis should be conducted in relation to this material in connection to participants’ practices in their own classrooms because the material was designed more for teacher consultation during planning. Some evidence of how participants were using the discourse strategies in the process of planning lessons were also found in the data. For example, T6 explained her way of including the discourse strategies in her lesson plans.

T1: I think the discourse strategies. When we started learning about those and how we were gonna implement those in our lessons, I think, well, for me anyway, I looked at the strategy and thought about thinking about what I wanted to see and then picking out strategies that would help get me to that goal, help get me to that point. And so that specifically how it changed me, and that’s what I was putting in my plans, all talk launch, talk triangle, whatever, and I felt like those lessons were a lot more, not that I added more detail in my writing up of the lesson, but I knew exactly where I wanted to go with each part of the lesson, each phase of the lesson because of those strategies (Session 13).

**Spearman Test to Triangulate Results in This Chapter**

The Spearman test determined that there were no changes over time when participants talked about PD materials (see Table 23). The justification for this result is that the observed changes are qualitative, in function of the way participants used the materials and not about the number of times they used them during the discussions.
### Table 23

**Spearman Test for PD Materials**

<table>
<thead>
<tr>
<th>Subcategory</th>
<th>Spearman’s rho</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIM Materials-Mathematics Discourse Matrix</td>
<td>0.031</td>
<td>0.7949</td>
</tr>
<tr>
<td>AIM Materials-Mathematics Teaching Guide</td>
<td>-0.080</td>
<td>0.507</td>
</tr>
<tr>
<td>AIM Materials-Mathematics Problem Sheets</td>
<td>-0.107</td>
<td>0.3739</td>
</tr>
<tr>
<td>AIM Materials-Written Classroom Representations</td>
<td>-0.287</td>
<td>0.0152</td>
</tr>
<tr>
<td>AIM-Responsive Discourse Strategies Handouts</td>
<td>0.257</td>
<td>0.0302</td>
</tr>
</tbody>
</table>

*Note:* n = 69, p-value for multiple comparisons using Bonferroni is 0.05/19 (0.002).
Chapter 6. Discussion and Conclusions

The goal of this research study was to analyze teacher learning in a PD setting in the context of a case study. The following research questions were addressed: “considering a situative perspective on learning, in which ways do teachers participate in PD interventions?” What shifts take place in teachers’ participation during PD interventions?”, “what is the role of PD materials as boundary objects in PD interventions?, and “in which ways are teachers’ shifts in participation related (or not) to the boundary objects used in the PD?”.

In the previous two chapters I answered these research questions, describing how teachers in the PD became familiar with certain ways of participation within the PD, when and how they changed their participation in the PD community, and the roles of the PD materials in the PD context. This concluding chapter presents the main contributions the study made to the field of mathematics education. The topics included in this chapter are conclusions and implications about teachers’ learning in the PD and the AIM PD as a boundary encounter. Later, contributions to the research design, and PD design are discussed. In the last part of this chapter, limitations of the study and avenues for future research are discussed.

Teacher Learning

One contribution of the current study is the characterization of how teachers participated in the PD. Previous research has claimed that it is important to know what teachers can learn outside the classroom to improve their practices in the classroom (Little, 2003), and in particular what teachers learn in a PD setting (Borko, 2004; Desimone, 2009).
The current study addressed those needs using a situative perspective that allows the construction of a more complete view of how the social system in which teachers learn impacts their learning. This is reached with the analysis of teachers’ interactions in the PD and the impact of PD materials as boundary objects in the PD intervention defined as a boundary encounter.

In Chapters 4 and 5, descriptions of how teachers participated in the AIM PD during whole-group discussions were reported. During the intervention, teachers became more specific in the comments they made, focusing more on students’ roles than on any other actor, describing and interpreting more than evaluating. They provided evidence from written classroom representations or PD foundational documents to build their arguments. When talking about mathematics, they studied and discussed different representations to solve the same mathematical problem. They became more confident about how to implement discourse strategies and how those tools were connected to their practices and to the requirements from the school district, and they established connections to the theoretical ideas provided in the foundational documents.

This characterization of the teachers’ participation in the PD is the culture developed in the PD, its “composite repertoire created by the interaction, borrowing, imposing, and brokering among its constituent communities of practice” (Wenger, 1998, p. 291). The process of enculturation that happened in the PD, in which participants reflected and constructed new ideas about how to elicit responsive mathematics discourse and how it connects to quality mathematics instruction, helped participants to understand “what it means to teach and to be a teacher, and to do so in this school, with these students and among these
colleagues” (Little, 2003, p. 937). During their participation in the PD, teachers were provided with opportunities to learn new mathematical and pedagogical knowledge through exposure to new concepts and through the opportunities to become familiar with new ways to facilitate mathematics discourse in their classrooms using discourse strategies. Participants experienced mathematics using new ways of reasoning about mathematics. All those experiences were part of the enculturation within the communities in which they were participating (Putnam & Borko, 2000): the PD community, the community of teachers, and with more possibilities to learn new and meaningful knowledge from communities of researchers.

The description about teachers’ participation in the PD presented in this study shows that teachers learned. Greeno and Engeström (2014) emphasized that “an important mechanism leading to change in practices is an expansion of the subject’s understanding of the object” (p. 131, emphasis in the original). In this context, the shifts in teachers’ participation in the PD and the enculturation process described above, evidenced how teachers learned during the PD according to the norms that were defined during the PD. Because of that, teachers participating in the boundary encounter crossed boundaries and moved more towards the center of the community of practice (Lave & Wenger, 1991; Little, 2003), with more knowledge about how to promote quality mathematics discourse and more mathematical knowledge about the concept of subtraction.
Boundary Encounter

The PD was the place where participants met to learn about how to elicit high-quality mathematics discourse and to improve their mathematical knowledge for teaching. How this activity is organized impacts what teachers learned because “how a person learns a particular set of knowledge and skills, and the situation in which a person learns, become a fundamental part of what is learned” (Putnam & Borko, 2000, p. 4). In this context, two important concepts associated with boundary encounters are discussed: the PD materials as boundary objects and the facilitator as the broker in this setting. The role of tools during the process of thinking and learning was documented in previous research as part of the social context that impacts learners (Akkerman & Bakker, 2011; Greeno, Collins, & Resnick, 1996).

Boundary Objects

PD materials played an important role in the way teachers participated in the PD and supported teachers’ learning (Akkerman & Bakker, 2011). The data showed that the way the PD foundational documents, the classroom representations, and the responsive discourse strategy handouts were designed and used afforded opportunities for teachers to participate in the PD and to support shifts in their participation (Lave & Wenger, 1991; Little, 2003). At the same time, those opportunities permeated the way participants talked. The materials were important artifacts that supported participants in learning or in retaining some expected practices, such as being more specific in their conversations using evidence from classroom representations, describing or interpreting information, or promoting the connections.
between ideas in different documents. In this way, the PD materials played an important role as a support for participants’ boundary crossing (Star, 1989).

Wenger (1998) stated that the design of boundary objects is also the design of participation, and the results of the current study supported this claim. The way the PD materials and the sessions were designed provided participants with time and space to reflect about what other teachers and students have done in specific situations, what they themselves had done in their own practices, or how to analyze different representations of the same mathematical problem. Participants also anticipated possible challenges they could face in the implementation of quality mathematics discourse in the classroom.

In the planning and implementation of the PD there must be what Akkerman and Bakker (2011) called a “deliberate target of change” (p. 148) that guides the whole structure. In this process, the PD materials helped to connect different perspectives and practices to fulfill their roles as boundaries objects that support teachers’ boundaries crossings (Akkerman & Bakker, 2011, Wenger, 1998).

**Foundational documents as boundary objects.** The Mathematics Discourse Matrix and the Mathematics Teaching Guide were key boundary objects that provided participants with theoretical ideas that guided them to analyze classroom representations and to make connections to their own practices. The two foundational documents were organized in a way that allowed participants to utilize broader categories or specific information contained in the documents depending on their needs or the way the discussion was organized. Across time, participants learned and used the language described in the documents, and they established
connections between those documents and other PD documents that showed they were interpreting and understanding theoretical ideas.

**Representations of classroom practice as boundary objects.** The written representations of practice were frequently used in the PD, and they provided teachers with specific examples that they connected to theoretical ideas, and to their own experiences in the classroom. This type of PD material provided teachers with enough details about what happened in a classroom to construct arguments during their discussions. The use of this material in connection to the norm of providing evidence when talking gave participants opportunities to be more specific and to describe and interpret information. As concluded by Little (2003), “the teachers employ talk about classrooms to justify themselves and their choices to one another and to rehearse how they will justify their choices to their colleagues” (p. 937). In that sense, the written representations afforded opportunities to reflect about participants’ own practices when they analyzed how they would react to a specific situation in their own classroom, how they would plan the lesson, or how students would solve a problem.

**Mathematics problem sheets as boundary objects.** The mathematical tasks provided during the PD created opportunities to discuss mathematical ideas, and to understand and practice the expectations about how to participate in mathematical discussions. The tasks provided were designed to generate different solutions to the same problem, and that generated teachers’ participation in the discussions during PD sessions. Teachers discussed how to solve the problem, which mathematical or concrete objects could be used in that process, or how their students would react to specific situations.
**Responsive discourse strategies.** The implementation of discourse strategies during mathematical conversations probably supported the shift in teachers’ participation in which they made pedagogical connections during mathematical discussions. In this process, participants learned how to negotiate the meaning of discourse in the mathematics classroom and what it means to teach mathematics in this new environment (Akkerman & Bakker, 2011). One possible impact of this situation is that this new knowledge could support improvement of teachers’ classroom practices because they learned specific abilities that they could translate to their classrooms (Little, 2003).

**Role of the brokers**

Although the roles of the facilitators were not analyzed in the current study, the facilitators supported members of the community of teachers in crossing boundaries. The way facilitators set the expectations and guided participants to keep talking using predefined norms, how they probed for connections among theoretical and practical ideas, and how they guided participants during mathematical discussions were important practices in the PD. Because of the way in which facilitators participated in the boundary encounter with the support of PD materials, they created new opportunities for participants about what it means to promote and engage in high-quality mathematics discourse, what it means to teach mathematics, and how discourse and mathematics are connected (Wenger, 1998). Facilitators enacted the role as brokers of the boundary encounter, giving more access to teachers to the knowledge that researchers shared with them. This conjecture about the important role played by facilitators as brokers in this encounter suggests that further research examining the role of this actor in how they connect participants and boundary objects is needed.
An important practice in the PD was the use of social norms and sociomathematical norms (Bowers, Cobb, & McClain, 1999) to invite participants to share their ideas in the PD sessions using certain criteria. In this process, the facilitators were the actors who promoted them during PD conversations. Evidence provided in the current report supports the idea that the use of defined norms justified in part the way teachers participated in the PD, and then what participants individually and collectively learned (Greeno & Engeström, 2014). To define and keep the use of those norms was part of the PD enculturation process that defined what it meant to participate in the AIM PD conversations (Brown et al., 1989).

The sociomathematical norm using different representations to solve the same mathematical problem provided participants opportunities to think about different mathematical ideas and to make the discussions more specific. The norms distributed responsibilities to participants, who, for example, learned “to develop meaningful explanations of their work, not just procedures that result in correct answers” (Greeno & Engeström, 2014, p. 137) as part of their participation in the PD. This way of talking about mathematics provide participants with a more flexible and broader knowledge about school mathematics, important characteristics of quality PDs (Borko, 2004). The negotiation of those norms between facilitators and the group of teachers supported participants’ learning about what are important values in mathematical discussions (Bowers et al., 1999). This culture of how to generate mathematics discussion may be transferred to the classroom later, and future research should analyze how teachers implement mathematical discussions in the classroom and how they are connected to their experiences in the PD.
The norm that motived participants to use evidence from the transcripts to build arguments to talk about mathematics, discourse, or pedagogy helped them provide details, and in consequence to be more specific. This practice also can justify why participants described and interpreted more frequently during discussions. Previous research has shown that following norms that support high-quality conversation is an important characteristic of learning communities that have been successful (Borko, 2004).

**Designing PD for Inservice Teachers**

The current study supported and extended previous research about what are important characteristics of a high-quality PD that promotes teachers’ learning. Previous research has been based primarily on teachers’ self-reports of their perceptions in surveys, which implies limitations in the interpretation of data (Desimone, 2009; Garet et al., 2001; Penuel, Fishman, Yamaguchi, & Gallagher, 2007). Less research has been conducted on how teachers learn through their participation in a PD intervention and how this process is connected to specific features of the design. The previous sections about the boundary encounter expanded the current knowledge about which characteristics of the PD support teacher learning. This was made through an analysis of the role of boundary objects in teachers’ learning. What follows explains how the current study supports previous conclusions about PD features that are important for teachers’ learning. Four ideas are included in this section: **length of the PD**, **mathematical content**, **coherence**, and **active learning** (Desimone, 2009; Garet et al., 2001). **Length of the PD** refers to the duration of the PD intervention, **mathematical content** refers to the focus of the PD on specific content knowledge, **coherence** represents the connections
between the topics analyzed in the PD and teachers’ practices, and *active learning* is the real engagement of teachers in PD activities such as discussions.

**Length of the PD**

The way participants shifted their discourse during the PD sessions showed they needed time to think carefully about certain ideas about how to elicit high-quality mathematics discourse. For example, the changes in how participants talked about students and teachers started to be more visible in Session 10. Also, shifts in how participants talked about the topic Teaching Concerns were observed in Session 5, but teachers’ comments were changing through different sessions towards using more positive comments about the implementation of new ideas in the classroom, and it was only in Session 12 that participants no longer expressed concerns. This result is aligned with previous research that showed that long-term PDs are more successful for teachers’ learning (Borko, 2004; Garet et al., 2001; Penuel et al., 2007). Further, it shows how from a participation perspective one can understand the importance of allowing teachers time to shift their participation in the PD community and their move toward the center of the community.

During the PD participants had time to discuss ideas, to practice the discourse strategies in the PD setting, to share their experiences implementing discourse strategies in the mathematics classroom, to discuss their mathematical solutions to problems. Participants had opportunities to analyze representations and decompositions of practice, and to establish connections between them and their practices. They were provided with time to plan a lesson that was part of the homework that connected PD ideas to their practices. All those opportunities were possible because the length of the PD intervention. The combination of
those strategies was important in the creation of the PD culture described before, and in the shifts in teachers’ participation in the PD.

Another example that supported the importance of the length of the PD was the path participants followed to understand the foundational documents because teachers needed time to understand the ideas provided in those decompositions of practice. The Mathematics Discourse Matrix summarizes in one page important conclusions of previous research that are key to implementing different types and dimensions of discourse in the mathematics classroom (Sztajn, Heck, & Malzahn, 2013c). The data showed that across time teachers understood better the ideas provided in the documents, and they connected the foundational ideas to their practices and experiences.

**Mathematical Content**

The use of a specific mathematics topic is important to achieve quality in the PD (Desimone, 2009; Garet et al., 2001). The AIM PD had this characteristic because it was focused on subtraction. I suggest that the combination of the established norms and the use of only one topic was important for participants’ learning. The designed tasks made participants think about different representations for the same problem. This way of organizing the work around only one topic supported participants in being more specific because they had to provide concrete solutions to a problem or talk about their experiences teaching that topic or how students think about that content.

One of the identified shifts in teachers’ participation in conversations in the PD is associated with participants being more focused on students than any other actor. According
to Penuel and colleagues (2007), when a PD is focused on content knowledge and on how that content connects to practice, it helps “teachers to focus on what students are expected to know and the nature of common student misconceptions” (p. 930). This characteristic could explain in part this shift.

Another of the identified shifts is associated with how participants talked about their concerns and how later in the PD they felt comfortable implementing new ideas, which showed that participants make connections to their own goals as teachers and see connections to new standards and curricular requirements. This shift may be connected to the opportunities given to teachers to implement the new knowledge in their own classrooms.

**Coherence**

The shift in the way participants talked about their concerns and how later in the PD they reported feeling confident about the implementation of new ideas in the classroom evidences that participants connected ideas about the promotion of quality mathematics discourse in the classroom and the mathematics they were learning with their practices and goals as teachers. They found connections between the new knowledge and the required standards and curricular requirements they had to follow. Participants reported during PD conversations that they were integrating the discourse strategies and new practices to teach mathematics into their planning and in the implementation of their lessons. They also reported how that had been successful. Even though the current study did not analyze directly what happened in their classrooms, the above mentioned shift supports the idea that the implementation of the discourse strategies in their own classrooms as part of their homework helped participants in their process of learning.
Another shift in teachers’ participation reported that teachers started to anticipate how their students would think or react in specific situations. This shift could be associated with the connections between participants’ experiences in the classroom and their work in the PD setting. The use of time to plan during the PD in school teams, the implementation of the discourse strategy in participants’ classrooms, and the reflection of those experiences later during the subsequent PD sessions supported teachers in their reflection about students and their practices in the classroom.

Previous research that highlighted that “longer activities also tend to promote coherence including connections to a teacher’s goals and experiences, alignment with standards, and professional communication with other teachers” (Garet et al., 2001, p. 933). In addition, Penuel and colleagues (2007) had identified the importance of time for planning as a characteristic that promotes teachers’ learning.

**Active Learning**

The AIM PD promoted active learning in every session. The PD materials, the prompts used, the tasks proposed, and the norms that were established made necessary the active participation of teachers in every session. For example, the use of the discourse strategies during PD mathematical discussions (approximations to practice) allowed participants to experience the strategies and to show their concerns in a safe environment prior to implementation in their own classrooms. As part of that active learning, participants connected different PD materials. Theoretical concepts included in the Mathematics Discourse Matrix or in the Mathematics Teaching Guide were analyzed using classroom representations during small- and whole-group discussions in the PD sessions.
The active learning was also promoted during the mathematical discussions. For example, participants solved mathematics problems as adult learners. During those discussions they were motivated to revoice what other participant said and to find different solutions to the same problem. Those practices allowed them to think deeply about the concept of subtraction.

I conjecture that this way of promoting active learning, connecting different activities and materials in the PD, supported shifts in teachers’ participation. For example, the shift in which participants first started to make connections to practice and later began to anticipate what could happen in the classroom when talking about students occurred because they had opportunities to experience, first hand, discourse strategy as learners in the PD and later as teachers with their own students. Teachers learned because they had access to different artifacts and opportunities to learn theory and practical ideas that were connected and integrated in the different experiences they had inside and outside the PD.

**Recommendations for PD designers**

In general, the situative perspective considers the context and activities in which people participate as a fundamental element in what they learn (Greeno et al., 1996). This affirmation is supported by the results from this study that showed that PD materials, the facilitators, and specific practices such as mathematical norms and social norms impacted the way teachers participated in conversations in the PD. This conclusion informs PD designers about specific practices and characteristics that they need to include as they plan a PD intervention. The data showed that the enculturation process impacted teachers’ participation
in the PD, and for that reason PD designers need to define carefully what is the culture they want to promote in their work with teachers, and what opportunities will be given to them to learn in specific ways. For Putnam and Borko (2000) that means to “equipping them with competence in using the concepts and the forms of reasoning and argument that characterize those communities” (p. 5)

In relation to the materials, PD designers should define carefully the purpose of each one, the tasks developed to analyze or use them, how a material is connected to the main ideas analyzed in the PD. For example, the current analysis evidences that the use of written classroom representations to describe the conversations contained in a video provided teachers with information that support them to build arguments using evidence. PD designers should analyze how this combination, video and transcript, can support teachers to learn.

PD designers also need to reflect about the better combination of theory and practice that provide participants with more opportunities to learn. The data analyzed showed that the combination of theory and practice used in the intervention helped teachers to participate and to implement new ideas in their classroom. There was evidence that the presentation of main ideas in the two foundational documents, the Mathematics Teaching Guide as a matrix and the Mathematics Teaching Guide using a model, helped participants to understand details contained in the documents and to connect key ideas to their own practices. To summarize the ideas about types and dimensions of discourse in the Mathematics Discourse Matrix provided participants with a document that they can refer during conversations. They connected the theory to specific ideas they noticed in a video or in their own classroom, and they developed a common language to talk about discourse.
PD designers also need to think about characteristics that high-quality PDs have and that were analyzed previously in this chapter: length, analysis of mathematical content, coherence and active learning. All those four characteristics must be included as key ideas when a PD is designed. For example, the opportunity that teachers had to experience the new ideas in the classroom and the space provided in the following session to reflect about the experience with their colleagues allowed participants to make connections to the theory, to defined which ideas were more useful, and to share experiences with their colleagues. Those experiences provide teachers with real engagement in the PD, which promotes active learning.

**Contributions to Research Design**

The current study analyzed data using a combination of the frameworks provided by Grossman and colleagues (Grossman, Compton, et al., 2009; Grossman et al, 1999; Hatch & Grossman, 2009), Horn and Kane (2015), and van Es and Sherin (2008). This way of analyzing the data contributes to the need of developing theoretical and methodological tools to advance the field (Borko, 2004; Greeno & Engeström, 2014). That combination allowed me to make conclusions about teachers’ participation in the PD, shifts in their participation, and at the same time the establishment of connections to how PD materials and the facilitator supported that participation. This way of analyzing teachers’ participation in a PD setting provides a more complete view of how teachers learn during professional development.
Limitations of the Current Study

The data used for analysis in the study was collected during the school year 2012-2013. For that reason, the researcher did not have any contact with participants, and it was not possible to make decisions about the data that could be collected. This limited the design of the methodology used to analyze the data.

Another limitation was the quality of the videos recorded. During the Summer Institute, the camera that was set up in the classroom was manipulated by a person who moved it to follow participants talking. When the school year started, the PD was located in a school library and the camera was in the same position during the whole session, facing the facilitators and usually a few teachers. In addition, during the whole PD the sound of the video was not high quality, which makes it sometimes difficult or impossible to hear what a person said and more challenging to identify the teacher talking. This limitation did not provide the opportunity of following individual teachers across the PD.

A third limitation of this study was the analysis of only whole-group discussion because it does not give a broad picture of the learning path participants experienced in all PD settings. The interactions during small group conversations and planning time can provide a additional description of the ways teachers participated during conversations in the PD in relation for example to the use of responsive discourse strategy handouts and the Mathematics Teaching Guide. Videos of those additional conversations among teachers were not available for analysis.
A fourth variable that limited the study is related to the use of idea units. To divide the data using idea units does not allow to describe specific details such as the length of each discussion about a topic or how many talk turns happened in each. This is a limitation because sometimes the frequency described in the tables does not capture how rich the discussion was during this period of time. Despite this observation, I believe to divide the data idea using units was also a strength of the analysis because it allows the analysis of participants’ interactions within the same idea in each segment. Each researcher has to make decisions about how to analyze the data, and each method or path defined will determine some pros and cons, as is in this case.

**Future Research**

There is still a long way to go to understand how teachers learn in PD settings. It is important to work in “identifying, conceptualizing, and assessing teacher learning” (Desimone, 2009, p. 191). When a situative perspective is used, the context in which that learning happens is included in that analysis, and then it is more complex to understand how teachers learn. On the basis of results of the current study, I propose here future lines of research that could help to expand my results.

One next step should be to analyze the differences between knowledge and knowing that has been proposed by Kazemi and Hubbard (2008). For them, knowledge refers to “possess” something, whereas knowing is the action or practice, and knowledge is not sufficient to be efficient performing a task. For that reason, future research should understand how knowledge and knowing are connected. For this cohort in particular, data from
participants’ practices in the classroom is available, and my next step as a researcher would be to analyze this data to connect it to the conclusions reported here.

Future research should study the role of small groups as boundary encounters that support what teachers learn and how this practice impacts the way teachers discuss ideas in whole-group discussions. Connections to how participants use boundary objects during small group discussions is a key analysis to be performed to understand better how teachers learn in a PD intervention (e.g., the use of PD materials during planning in the PD session). This information would give PD designers key ideas about how to plan and implement a PD for mathematics teachers, and new features about how to design quality PDs that impact teachers’ learning could be added to the available literature.

It is important to compare teachers’ learning during the Summer Institute with the sessions during the school year. Results showed that teachers changed their talking when the school year started because they had more possibilities to make connections to practice. Future research should analyze the differences between PDs in which connections to practice were promoted as in AIM and other PDs in which teachers do not practice the new knowledge in their classrooms to see if there were changes in their participation. This line of research has already been proposed by Putnam and Borko (2000).

Finally, with the goal of improving the quality of the PD it is necessary to understand which combinations of PD materials and tasks guide teachers to the most productive discussions that lead to teachers’ learning (Desimone, 2009).
REFERENCES


APPENDICES
## Appendix A. PD materials

### A1. Mathematics Discourse Matrix

<table>
<thead>
<tr>
<th>TYPES/DIMENSIONS</th>
<th>Correcting Discourse</th>
<th>Eliciting Discourse</th>
<th>Probing Discourse</th>
<th>Responsive Discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Questioning</strong></td>
<td>1. T asks frequent, short-response questions that attend to students’ answers to the problems</td>
<td>1. T includes open-ended questions that encourage Ss to share thinking about their answers and how they found their answers.</td>
<td>1. T included probing questions that require Ss to mathematically justify their answers, how they found it and why they approached it the way they did.</td>
<td>1. T includes pressing questions that promote Ss’ sharing of their answers, how, why as well as connections between mathematical ideas and representations.</td>
</tr>
<tr>
<td></td>
<td>2. T asks follow up questions to lead to correct answers</td>
<td>2. T asks follow up questions to support sharing of mathematical ideas.</td>
<td>2. T asks follow up questions to press for mathematical depth.</td>
<td>2. T asks follow up questions to check that all students are making sense of the mathematical ideas.</td>
</tr>
<tr>
<td></td>
<td>3. Ss ask T questions to establish correctness of answers</td>
<td>3. Ss ask T “what” and “how” questions to clarify their own thinking about the mathematics in a problem.</td>
<td>3. Ss ask T one another “how” and “why” questions to clarify their own thinking about the mathematics in a problem.</td>
<td>3. Ss ask one another “how” and “why” questions to understand each other’s mathematical thinking.</td>
</tr>
</tbody>
</table>

| **Explaining**   | 1. T explains the procedure to use to solve a problem | 1. T adds to Ss’ presentation of their solution methods for solving a problem | 1. T revoices and extends Ss’ presentations of various solutions methods for solving a problem | 1. Ss restate, extend and make connections across various solution methods presented for solving a problem. |
|                  | 2. Ss provide just their answers when T asks | 2. Ss provide their answers and how they found them when T asks | 2. Ss provide their answers, how they found them, and why they approached a problem as they did when T or other Ss probe | 2. Ss volunteer their answers, how they found them, why they approached a problem as they did, and connections to other ideas. |
|                  | 3. T. corrects Ss’ incorrect answers | 3. T accepts incorrect answers as an indication of Ss’ level of understanding | 3. T probes student thinking around incorrect answers to highlight why they are incorrect. | 3. T and Ss examine incorrect answers so that all Ss can learn from mistakes. |

| **Listening**    | 1. T listens for expected correct answers to a problem | 1. T listens for Ss’ sharing of their answer and how they got it | 1. T listens for Ss’ mathematical explanation of their answer, how they got it, and why they approached the way they did 2. Ss listen to other Ss’ explanations and connections. | 1. T listen for partial and complete understanding of the mathematical ideas in Ss’ explanations and connections. 2. Ss listen to one another’s explanations to make connections across mathematical ideas. |
|                  | 2. Ss listen for T’s responses to verify their answers | 1. T listens for Ss’ mathematical explanation of their answer, how they got it, and why they approached the way they did 2. Ss listen to other Ss’ explanations to consider whether their ideas are similar | 2. Ss volunteer their answers, how they found them, why they approached a problem as they did, and connections to other ideas. 3. T and Ss examine incorrect answers so that all Ss can learn from mistakes. |

| **Modes of Communication** | 1. T and Ss communicate in T-S-T patterns | 1. T and Ss communicate in T-S-T-S patterns. | 1. T and Ss communicate in T-S-T-S or T-S1-S2-S3 patterns | 1. T and Ss communicate with significant S-S patterns |
|                          | 2. T favors the use of verbal or pictorial modes when Ts/Ss share procedures and answers to a problem | 2. T makes verbal, pictorial, or written modes available as Ss communicate the answer and how they got it | 2. T encourages use of verbal, pictorial, written or non-verbal modes as ways for Ss to communicate the answer, how they got it, and why they approached the problem that way 3. T expects comparisons across representations to convey mathematical thinking | 2. Multiple modes (e.g., verbal, written, pictorial, non-verbal, tools) are required, as Ss communicate the answer, how they got it, why and mathematical connections 3. T expects comparisons across representations to develop mathematical understanding 4. T expects use of academic and everyday languages to develop mathematical understanding |
|                          | 3. T provides Ss with representations they need to use to solve a mathematics problem | 3. T accepts all representations as equally effective 4. T allows Ss’ use of academic and everyday language equally as modes to share answers and methods | 3. T encourages Ss’ use of various representations to convey mathematical thinking | 3. T expects comparisons across representations to develop mathematical understanding 4. T expects use of academic and everyday languages to develop mathematical understanding |

### Difference in Breadth

### Difference in Depth

### Difference in Responsibility


A3. Sample of a handout: All Talk Strategy

Thinking about All Talk as a Strategy for Promoting Mathematics Classroom Discourse

All Talk is a strategy that can be used in the Launch, Explore, or Discuss phase of the AIM Mathematics Teaching Guide for Responsive Discourse. Typically, a teacher will use this strategy to ensure that all students have the opportunity to speak. The main purpose is to allow all students to participate in the conversation.

Below are some considerations for using this strategy in your mathematics teaching.

<table>
<thead>
<tr>
<th>Choosing Problems for All Talk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good mathematics questions or problems for All Talk can vary widely. Questions that will help students recall important prior knowledge are good choices. Problems that can be solved using a variety of strategies are also good choices.</td>
</tr>
<tr>
<td>The key in choosing a question or problem is that there are a wide range of brief responses that students might offer. Since all students will be expected to contribute, it is important that the choice of question or problem offers students an opportunity to make many possible contributions that will be helpful for all of the students to hear.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Getting Started with All Talk</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Talk is a quick strategy to help each student share a brief idea.</td>
</tr>
<tr>
<td>All Talk can be used in the Launch, Explore, or Discuss phase of the lesson. In the Launch, for example, the teacher may use it to discuss vocabulary used in an assigned problem. In the Explore, the teacher might ask small groups to do an All Talk before working to solve a problem as a way to keep any single student from dominating the group conversation. In the Discuss, all students may share a strategy for solving the problem, what each found most challenging, or may even share very brief responses of the answer they found.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Supports That Helps</th>
</tr>
</thead>
<tbody>
<tr>
<td>To help students learn the structure of the response in All Talk, and to keep an All Talk from lasting longer than intended, we suggest the following:</td>
</tr>
<tr>
<td>Provide a specific question prompt for students to respond to in the All Talk.</td>
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<tr>
<td>Offer sentence frames:</td>
</tr>
<tr>
<td>o When I think of the word difference, I think of ________.</td>
</tr>
<tr>
<td>o I believe that we should do ________ to get started on this problem.</td>
</tr>
<tr>
<td>o My team and I used the _____________ strategy.</td>
</tr>
</tbody>
</table>
• Model expected responses. Use the sentence frames to demonstrate to students how one might respond.
• Use the way students are seated to create a natural order for the All Talk, so that students take their turns quickly without each being called on to provide a response.
• Some students may benefit from additional support. For example, giving ELL students the prompt and sentence frame a bit in advance can offer them a chance to consider a response before the All Talk begins. Having struggling learners respond early, also with brief advance notice and preparation, may offer them an opportunity to share an idea before other students share the same idea.

**Other Key Ideas**
• All Talk is a step along the way to helping students feel more comfortable in having a voice in the classroom and sharing their thinking.
• If a student doesn’t have anything different to share, the same ideas can be repeated. The goal is for all students to participate in the conversation.

**Ideas for Modifications**
• If using an All Talk during the Discuss phase, the teacher may solicit a response from all groups, but not all individuals.
• An All Talk can be used along with other strategies, such as Think-Pair-Share, with each pair of students contributing an idea.
### A4. List of AIM PD classroom representations

<table>
<thead>
<tr>
<th>Session</th>
<th>PD written representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Adding Three Numbers Class Transcript</td>
</tr>
<tr>
<td></td>
<td>Finding Sums Video Transcript</td>
</tr>
<tr>
<td></td>
<td>Bean Sprouts Video Transcript</td>
</tr>
<tr>
<td>2</td>
<td>MKT Case Narrative</td>
</tr>
<tr>
<td>3</td>
<td>Launch Video Transcript</td>
</tr>
<tr>
<td></td>
<td>Think Aloud Scripts</td>
</tr>
<tr>
<td></td>
<td>Think Aloud Video Transcript</td>
</tr>
<tr>
<td>4</td>
<td>Talk Triangle Video Transcript</td>
</tr>
<tr>
<td></td>
<td>Talk Chain Script</td>
</tr>
<tr>
<td></td>
<td>Seed Packets Video Transcript</td>
</tr>
<tr>
<td>5</td>
<td>None</td>
</tr>
<tr>
<td>6</td>
<td>Setting Norms Video Transcript</td>
</tr>
<tr>
<td></td>
<td>Partial Sums Classroom Transcript</td>
</tr>
<tr>
<td></td>
<td>Planning Expectations?</td>
</tr>
<tr>
<td>7</td>
<td>Task Implementation Scenarios (no hardcopies)</td>
</tr>
<tr>
<td>8</td>
<td>None</td>
</tr>
<tr>
<td>9</td>
<td>Bet Lines Classroom Transcript</td>
</tr>
<tr>
<td></td>
<td>Draft &amp; Final Copy Video Transcript (no hardcopy)</td>
</tr>
<tr>
<td>10</td>
<td>None</td>
</tr>
<tr>
<td>11</td>
<td>None</td>
</tr>
<tr>
<td>12</td>
<td>How Many Trees Video Transcript</td>
</tr>
<tr>
<td>13</td>
<td>Bean Sprouts Video Transcript</td>
</tr>
</tbody>
</table>
Appendix B.

Codebook

General instructions:

1. Every transcript has to be divided in idea units (IU), and for each one of them multiple codes can be assigned.

2. Sometimes one turn talk belongs to two different idea units because the facilitator closes one idea and opens a new one. Normally, the facilitator starts the talk turn closing one idea and at the end of the talk turn she asks a question about a new topic. It would apply for teachers too.

3. The coder needs to focus on teachers comments, not in facilitator’s ideas. The facilitator’s prompt or comments will guide you to define the focus of the conversation, but not to assign a code. For example, if facilitator talks about parents, but teachers don’t, then the code parent is not assigned to that idea unit.

4. You can assign only one code for the categories Level of Specificity and PD focus. For the other categories, you can use as many codes as needed.

5. In the transcript the symbol T# means a teacher is talking, and # represents the number for that specific. When it was impossible to identify which teacher was talking, only T is observed in the transcript.

6. Each facilitator has a coded assigned (F1 or F2)

<table>
<thead>
<tr>
<th>Category</th>
<th>Definition</th>
<th>Description of the subcategories</th>
<th>More details</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD focus</td>
<td>Main topic defined in the</td>
<td>• Discourse  • Mathematics</td>
<td>When the discussion was about the</td>
<td>Example 1:</td>
</tr>
</tbody>
</table>
power point prompt or in the question the facilitator asked the teachers. It is important to think about the PP prompt first, but sometimes the facilitator changed the question during the discussion and then it is necessary to use the facilitator question.

- Pedagogy
- Other

Mathematics Discourse Matrix, it is included in the category Mathematics Discourse. If the discussion was about the Mathematics Teaching Guide the focus is Pedagogy

Discourse focus (Mathematics Discourse Matrix)

F1: Okay, I like that activity because it makes you look at the matrix and it makes you get into that responsive part of the matrix, that really is what we’re striving for, is to go for those responses and the video does lend itself to that area. I’ve noticed that we found quite a lot of evidence for each of the dimensions of the math matrix, but what we’re really gonna get into now is what you did this morning and I’m gonna remove myself from it and you’re gonna discuss this video together as a whole-group and you’re gonna focus on the two questions up there. What specific actions is the teacher doing to support students questioning, explaining, listening and use of modes of communication in the talk triangle? Basically what you just did here, you’re gonna discuss it together now. What evidence do you have that these teacher’s actions are or are not supporting various dimensions of responsive discourse for the student? So make sure you stick to the evidence, just like when I’m giving my teachers all their data from their performance assessments, I’m like, I always stick to the data because numbers do not lie, so the evidence does not lie. Stick to that. Whoever has the first thought,
just start talking and you all discuss the video.

**Example 2:**
**Pedagogy focus**
F1: Alright, so let’s go with that. That was the first time these children had seen that. do you think there’s something she could improve on, or do you think it all worked and you could go in your classroom and do it? What are some of the challenges you think? What are you feeling about, is this doable in your room?

<table>
<thead>
<tr>
<th>Actors</th>
<th>Person who is referred in the comment teacher makes (Horn and Kane, 2015; van Es &amp; Sherin, 2008).</th>
</tr>
</thead>
</table>
|        | • Teacher  
|        | • Students  
|        | • Self  
|        | • Others*  
|        | * These variables were codified and analyzed, but they are not reported in results because nothing important was found. In the examples in the right column, only the categories used in the results chapters are reported.  
|        | Four different dimensions:  
|        | _ zero-dimension: no actor  
|        | _ one-dimension: one actor  
|        | _ Student  
|        | _ Teacher  
|        | _ Self  
|        | _ two-dimension: two actors  
|        | _ Student-Teacher  
|        | _ Student-Self  
|        | _ Self-Teacher  

**Example: Zero-dimension**
F1: And then we go to the explore and we’re talking more small group activities here. This is where most of your responsive discourse is gonna happen, right here in the explore. So what strategy worked best with the explore that we’ve gone through so far? T4: The talk triangle. F1: The talk triangle. So basically that’s where that strategy will fit. But you also could have, what else would be explore? Ts: Turn and talk. F1: Turn and talk would be a good explore. Okay. So that leaves the last phase, which is the what? Ts: Discuss. F1: Discuss, okay.
three dimension: three actors
_Student-Teacher-
Self

In reporting results one more category was used:
Actor no dimension: includes all the idea units about an actor without separating by dimension.

Example: Two-dimension actors (Self-Students):

T: We said in goals, we want students to be responsive and, like everyone else has said, be listeners to other students and make sure that they’re engaged. We also wanted the students to make connections to other strategies. I think that’s important. Students seeing the importance of understanding, and not just getting the right answers, we want them to really delve deep into why they’re getting the answer instead of it’s not just getting the answer. We got, one of the challenges was just getting parents to understand why math talk is so important, so some, discussing, some parents, they just wanted their kids to have the answers and not really explain why they got the answers, so we need to get across to them that this is important. Integrating with language arts, science, and all the other disciplines and faculty, and resources to initiate those type of question to get students to really think, Common Core, just integrating technology more into the process, motivating the students in different ways and making sure that we meet all the different learning styles is a challenge. And also, meeting individual
needs of those **students** that really need to be pushed to advance and also those who need a little bit more help as well.

| Decompositions | Participants used of PD foundational documents. Decomposition is “breaking down complex practice into its constituents parts for the purpose of teaching and learning” (Hatch & Grossman, 2009). | • PD foundational materials:  
  _ Mathematics Teaching Guide for Responsive Discourse  
  _ Mathematics Discourse Matrix | **Example:**  
**Mathematics Discourse Matrix**  
T7: I also, like Annette, noticed that they talked about the common language and I liked that they have similar vocabulary and it would lead to responsive discourse. I looked at my math discourse chart. |
|---|---|---|---|
| Representations | Representations “comprise the different ways that practice is represented in professional education and what these various representations make visible to novices” | **Subcategory 1: materials provided by PD designers**  
  • Responsive discourse strategy handouts  
  • Mathematics Problem Sheets  
  • PD Classroom representations (It includes all Two different dimensions:  
  _ one-dimension: one PD material used in the same IU.  
  _ more than one-dimension: two or more materials used in the same IU. | **Example 1:**  
**Scenario:**  
F2: Okay, I’m gonna bring us back together and I think you guys have some good conversation you’re having here, we’re gonna have a little discussion. I’ve already heard you starting to switch into the next thing we’re gonna be talking about. As far as scenario one goes, what do you think it was that his teacher did that lowered the discourse level, and what were the opportunities for discourse? |
(Grossman, Compton et al., 2009, p. 2058).

<table>
<thead>
<tr>
<th>Material that describe specific details about a lesson or part of a lesson:</th>
<th>Because of the PD design, for every video there was a written classroom representation. If there is no possible to know if the participants talking within the IU were using the video or the written classroom representation, it was coded as written classroom representation. The video was used only when it was very explicit that participants were talking about it.</th>
</tr>
</thead>
</table>

R: [inaudible] [Laughs]
F2: Okay, they said back there that he told them how to solve it.
R: He just kept [inaudible] and stuff.
F2: Okay. Was there a point upon which he was on the right track?
Ts: Uh huh.
T: Posing the question.
T: Maybe just got frustrated and then he started to probe them and then he went a little too far, just basically gave the answer away, did step by step.
T19: I’m curious to know if he used manipulatives to kind of try to solve it on their own from their own discourse or if it was just, “here’s the problem, figure it out.” Because I’m thinking if they had something to play with and try to regroup, maybe they would have made it even further on their own.

**Example 2: Problem Sheet**

| F2: So what did you think? |
| T2: I didn’t do that, but that’s how Crystal did it, but I generally, I don’t know, but I was like, well, okay, she used 150 because that’s how she adds well. She uses those the best, so she said, okay, 67 rounds down to 50, so that’s 17, and 48 rounds up two, so that’s, you know, to 50, so that’s two and 17, so that’s easy to subtract 400 minus |

| F2: Okay, they said back there that he told them how to solve it. |
| F2: Okay. Was there a point upon which he was on the right track? |
| Ts: Uh huh. |
| T: Posing the question. |
| T: Maybe just got frustrated and then he started to probe them and then he went a little too far, just basically gave the answer away, did step by step. |
| T19: I’m curious to know if he used manipulatives to kind of try to solve it on their own from their own discourse or if it was just, “here’s the problem, figure it out.” Because I’m thinking if they had something to play with and try to regroup, maybe they would have made it even further on their own. |
100 is 300, and I tried that on a two digit number instead of three digits and it worked every time.

Example 3: Video
T1: Maybe there was more of the video, more of the discussion we didn’t see, but she didn’t have time to compare strategies, although it was evident in the student’s work that different strategies were used

Example 4: Video and written classroom representation
T7: I agree that she starts off well and she’s going to think about it, but then she keeps showing them, but in lines nine and ten when she’s writing student’s names on the board and then in lines 11 and 12 when she’s actually drawing the circles, she’s, I feel that she’s moving away from thinking and moving into showing them (Session 3).

Artifacts (verbal or physical) that stand in for larger phenomena (Horn and Kane, 2015). | **Subcategory 2: Own classroom representations** | **Example:** Curricular artifacts
---|---|---
Interactions around • curricular artifacts:
teachers make references to curriculum they need to implement, or to tests the students will take at the end of the year.

- student work: this variable was included for data analysis, but there was not important information to report.

### stories of classroom talk
This focuses on two conversational representations:

**Replays:** recall past instructional interactions

**Rehearsals:** anticipate instructional interactions

In the replays and rehearsals teachers should be referring their past of futures experiences in the classroom. Those stories of classroom talk does NOT include commentaries about videos or another AIM materials (responsive discourse strategy handouts, scenarios, etc).

**Example 1:**
**Replay:**
T9: It really would! You could take a group of students who are both stronger in it and you wanted it to be more of a peer lab, then you could get them in the group they could be the ones who teach.

T8: We do that in our reading, students that are stronger than some students, than others [inaudible].

**Example 2:**
**Rehearsal and replay:**
T1: I think it’s the student assessment portion that I’m thinking about because you know, we every day meet with our groups,
we meet with groups of kids for constantly checking their understanding in math, even in small groups, you can’t get to every group to see how they’re doing with the math talk, you can’t ensure that every student’s doing it, well, that’s what I’m thinking about, how to manage that to make sure that they are using the discourse and they’re using it properly and they’re growing, they’re making progress with it.

| Stances | How teachers analyze practice? | Describe: statements that recounted the events that occurred in the clip, the transcription, scenario, in the classroom, etc. | Example 1: Interpret
T2: I didn’t do that, but that’s how Crystal did it, but I generally, I don’t know, but I was like, well, okay, she used 150 because that’s how she adds well. She uses those the best, so she said, okay, 67 rounds down to 50, so that’s 17, and 48 rounds up two, so that’s, you know, to 50, so that’s two and 17, so that’s easy to subtract 400 minus 100 is 300, and I tried that on a two digit number instead of three digits and it worked every time. |
|---|---|---|---|
| Example 2: Evaluate
T1: The difference, so that’s why writing that, going with the equals sign was probably a lot better way of doing that than | Interpret: statements in which the teachers made inferences or deduced ideas from what she had noticed. Teachers made explanations or | | |
find a meaning of what could happen (*The description-interpretation-evaluation, n.d.*).

- **Evaluate:**
  judgement statements in which the teachers commented on what was positive or negative or could or should have been done differently in what she watched or experience (van Es & Sherin, 2008; *The description-interpretation-evaluation, n.d.*).

  the number line. That’s my approach. That wouldn’t be appropriate in the situation.

**Example 3:**

**Evaluate**

T8: I’ll say too that we have, probably knew Mona would get it right [inaudible] Mona had solved a different problem a different way, she probably knew she would get it right, whereas the other students possibly would have a different answer when they solved it exactly the same way, but they just haven’t got a person or that student that you know is going strong and able to explain it without saying, well no, that’s right or ask the teacher. I thought that was pretty good.

<table>
<thead>
<tr>
<th>Level of specificity</th>
<th>Specificity teachers used to discuss events they notice (van Es &amp; Sherin, 2008).</th>
<th>General: there is not specific details or examples in the argument that the teacher is saying.</th>
<th>Example 1: General</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>F1: Anybody else see any evidence of some discourse?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>T: Evidence of it or evidence of [inaudible]?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>F1: Any kind of discourse that you saw, or heard.</td>
</tr>
</tbody>
</table>
**Specific:**
details, evidence and/or examples are used for the teacher to create an argument.

<table>
<thead>
<tr>
<th>T4:</th>
<th>On line 37.</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1:</td>
<td>Okay, everybody go to line 37.</td>
</tr>
<tr>
<td>T4:</td>
<td>She asked Sophia how did she get 15.</td>
</tr>
<tr>
<td>F1:</td>
<td>Okay. Anybody else see anywhere?</td>
</tr>
<tr>
<td>T11:</td>
<td>Asking Kiesha where she got the [inaudible] from there.</td>
</tr>
<tr>
<td>F:</td>
<td>What line is that?</td>
</tr>
<tr>
<td>T11:</td>
<td>Line 63.</td>
</tr>
</tbody>
</table>

**Example 2:**
**Specific**
T9: So maybe he plan the group. (laughs) No, I’m serious, it looks like a plan [people talking at once]
T11: She had been using her banana example I think, you know, you can’t take this many bananas from this many bananas, she was able to use language that they understood to explain it.
T2: I don’t think you planned it in the group because in number one, it says he worked with a small group of students because he had seen that they were struggling with regrouping, and then they needed to regroup, so he pulled those students over.
T9: But it wouldn’t be a bad idea (laughs).
T9: It really would! You could take a group of students who are both stronger in it and you wanted it to be more of a peer
<table>
<thead>
<tr>
<th>Topic</th>
<th>Topic that teachers are talking about in the idea unit (van Es &amp; Sherin, 2008).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Discourse</strong>: it includes comments about discourse, or about the theory contains in the Mathematics Teaching Guide for Responsive Discourse.</td>
</tr>
<tr>
<td></td>
<td><strong>Mathematical knowledge for teaching</strong>: comments about the mathematical knowledge teachers need to teach.</td>
</tr>
<tr>
<td></td>
<td><strong>Example:</strong> F1: On 57, what type of discourse do you think that would be? T8: I’m sorry, I’m just looking… F1: Let’s look at our maps. T: I agree with correcting discourse. F1: What’d you think? T: Correcting discourse when she acknowledged his right answer. F: And then it says, I guess in line 59, “I’m interested to know if there’s anybody else who’d like to explain that one in their own words.” I think that’s what she was trying to get at was line 59.</td>
</tr>
<tr>
<td></td>
<td><strong>Example:</strong> T20: And then you have to, on the fly, come up with another problem and so to me, it’s, you have to be very deliberate and you have to be thoughtful and you have to really know, and you really have to know what you’re talking about with math, and your understanding of math has to be really deep, but you also have to really understand your students. You really have to understand the different ways that they learn and do things and really be very</td>
</tr>
</tbody>
</table>
observant of what they’re doing because, I mean, you have it scripted out, you can have an idea of what you want, but I just know for me, so many times I did small grouped and I think I know which way it’s gonna go and then it doesn’t necessarily go that way anymore and I’m trying to backpedal and come up with something else to do instead of, you know, we just got into a discussion about, you know, how deliberate you have to be and, you know…

<table>
<thead>
<tr>
<th><strong>Mathematical thinking:</strong></th>
<th>Example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>“teachers’ ability to describe their students’ strategies and to develop plausible interpretations of their students’ mathematical thinking” (Dick, 2013, p. 182)</td>
<td>T18: I saw her making a ten group, that the eight plus the two equals a ten, and then her, she was asking, she’s at 40 and she’s gotta get up to 60, so that’s when ten from 17 came to get her to 50, she already made that first ten group to get her to 60. And then the seven remains, and then at 100, and then she had to get to 400, so she added the 300.</td>
</tr>
</tbody>
</table>
it, oh, it was 736 and then that other child could easily, “uh uh, that doesn’t make sense,” whereas you couldn’t do that as quickly with the three digits, you know? Because he had 736 and 732. It was easier to compare when the number was smaller, the one, you know, versus the other one where it was 583, all you could see was the ones place.

T1: You’re going 269 places instead of eight.

T2: Right. And all they could see was the ones place and I don’t know about what the other hundreds are, but this was easier to, you know, because you didn’t have to worry about the tens and the hundreds.

T8: Well, that’s what the error was, was a [inaudible] space, so it’d be a logical place to focus on that one.

Example 2:

T6: And then what Sidney was talking about, it’s not to introduce a new concept, and then she does some, and she says, “some means,” so was some a new concept?

• Pedagogy:
  techniques and strategies for teaching mathematics.

Example 1:

F2: We’re gonna come back together for a [inaudible] do a whole-group discussion now. If one person in each group maybe could share out really quick one thing they
teachers’ decisions about what to do in the classroom, or how to organize the lesson.

noticed, a big aha or the nature of the mathematics discourse. We’ll start with Sally’s group, one thing you’d like to share if you don’t mind.

T5: There was all interested to the teacher and in the small groups there were some students too, but in the class, there was not a lot of interaction.

F1: I don’t mean to put you on the spot, but could you tell me what line? We have to use the evidence from the transcript? Kind of keep that in mind. Or if anybody else can help find the teacher students.

T3: If you take number 40, and he wrote number 40 with blank and he wrote the solution on the board, and then at 46, she says, “I like the way that you did that in asking him questions,” but then she didn’t have another student come up and discuss that and repeat and ask how he, and then talk about how he solved that.

Example 2:

T3: I have several children that have come in and said that when they had done subtraction that they don’t know that’s not the right appearance showed them. And I have said as long as you know how you do it and you can explain it to me, and you can explain it to the class, then that is, that is okay. That they can show that strategy both
### Teaching concerns

Teacher makes comments about difficulties they have or could have to plan a lesson, implement the lesson or evaluate students’ learning. Sometimes their concerns are associated to curricular artifacts or assessments.

#### Example:

T4: To get back on what Alex just said, I think that math goes along with the importance of us knowing where the destination is for our second graders this year, and knowing what they need to be able to do by the end of the year with the new curriculum may be a little challenging.

T5: The new tests that use 21, we don’t exactly know what’s on it and what they’re expecting us to do on that, like [inaudible].

T4: But I’m thinking just in terms of when. I’m having to solve these word problems, what kind of variety of word problems would be expected, you know? Just making sure that they had all the exposure that they needed and that they had all those opportunities to have those rich discussions around how they could solve them.

---

<table>
<thead>
<tr>
<th>Unidentifiable</th>
<th>Impossible to be attached to a code because the inaudibles</th>
</tr>
</thead>
</table>

#### Example:

T17: [13:27 inaudible]

F: Say that again?

T17: Were Es and [inaudible] partners?
| Nothing important | Comments that is not related at all to the discussion. Normally they were made by facilitator | F1: I’m not sure. | Example: F2: Okay, T, I’m just gonna stop for just one second. Don’t forget to talk loud, just because of the video component of this, just so they can hear all our wonderful thoughts. You were fine, I just wanted to remind us all. |

Appendix C. Tables from the analysis of data

Table C1

**Actor by Dimension and Session**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Actors</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
<th>S10</th>
<th>S11</th>
<th>S12</th>
<th>S13</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No Actor</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
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<tr>
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<td>10</td>
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<td>6</td>
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<td>Self-Teacher</td>
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<tr>
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<td>Student-Teacher-Self</td>
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<td>18</td>
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Note: D: Describe, E: Evaluate, I: Interpret, All: Describe, Evaluate, and Interpret.

Table C2

**Actor by Dimension and Stance**

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Note: D: Describe, E: Evaluate, I: Interpret, All: Describe, Evaluate, and Interpret.
Table C3

**Actor by Dimension and PD Discussion Focus**

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<thead>
<tr>
<th>Dimension</th>
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Table C4

**Actors by Dimension and Use of Own Classroom Representations.**

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<th>Stories of classroom Talk</th>
<th>Curricular artifacts</th>
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<td>Rehearsals</td>
<td>Replays &amp; rehearsals</td>
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Table C5

*Stories of Classroom Talk by Session and by Actor*

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Table C6

*Own Classroom Representations in Relation to Student-Self and to the PD focus*

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*Note:* total of IU for each focus and Student-Self actors are in parenthesis.

Table C7

*Actor by Dimension and Topic of the Discussion*

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<td>Teacher</td>
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Note: MKT is Mathematical Knowledge for Teaching, Math is to Mathematics

Table C8

Level of Specificity in Relation to PD Focus and Actor by Dimension

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Notes: Math: Mathematics, G: General, S: Specific. The number on parenthesis represents the number of idea unit for each discussion focus.

Table C9

Variables by Session in Connections with No Actor

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Table C10

*Level of specificity by Session in Connections to the Actor Student*

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*Note: St: Student, G: General, Sp: specific, Sf: Self, T: Teacher*

Table C11

*Stories of Classroom Stories by Session in Connection to the Actor Self*

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<tr>
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