ABSTRACT

SHEN, MINAO. A New Hybrid LES/RANS Model with Eddy Viscosity Transport (EVT) Based Outer-layer Length Scale. (Under the direction of Jack R. Edwards.)

A new hybrid large-eddy simulation / Reynolds-averaged Navier-Stokes simulation (LES/RANS) turbulence model is presented in this work. The new model is then compared to earlier models developed at NCSU and the IDDES model as a ‘state-of-art’ competitor for various flow configurations. In common with other hybrid turbulence models developed at NCSU, this new model utilizes a flow-dependent blending function that shifts the turbulence closure from Menter’s baseline(BSL)/shear-stress transport (SST) RANS model to a sub-grid model of choice in the logarithm-layer as the distance from wall increases. This blending function is based on estimates of outer- and inner turbulence length scales. Unlike earlier models that used a problem-specific calibration of a model constant or ensemble-averaged the modeled turbulence kinetic energy (TKE) and specific turbulence dissipation rate, the estimated outer-length scale of the new model is a function of an eddy viscosity that is specifically used for this purpose. A transport equation for this eddy viscosity is solved in addition to other transport equations, with excessive growth of eddy viscosity due to fluctuating strain rates controlled by a destruction term based on the von Kármán length scale. As a result, this new model is completely local. The new model is tested through simulation of zero pressure-gradient flat-plate boundary layers at low and moderate Reynolds numbers, a supersonic flat-plate boundary layer, an airfoil near stall, flow over a wall-mounted hump, and flow over a simplified car body (Ahmed body).

For the low-Re incompressible flat-plate boundary layer (FPBL) case, the results given by the new model are comparable to wall-resolved LES as well as experimental data. For the moderate-Re FPBL case, the effects of model constant values and numerical schemes are discussed, and the model is further tuned. The new model is proven capable of predicting incompressible and compressible FPBLs reasonably well. For the airfoil case, the new model gives good surface pressure and skin friction predictions, as well as separation-bubble predictions near the trailing edge. It is not able to capture a laminar separation bubble near the leading edge. Extensive study on the wall-mounted hump case shows that the tuned new model can do a better simulation than Gieseking’s model and the IDDES model on this case, and its results are close to those of Choi’s model which required pre-calculation calibration. This study also shows that spanwise resolution and dimension and choice of numerical scheme can affect the boundary layer structure and separation predictions produced by the hybrid models. The Ahmed body is a full 3-D test case for the new model, and results are sub-satisfactory due to insufficient surface mesh resolution. Over all, the EVT based new model works reasonably well for a wide range of problems, producing results comparable to other NCSU LES/RANS models while retaining a completely local formulation. Meanwhile, further investigations
relating to the sensitivity and mechanism of impact of numerical methods, 3-D implementations with adequate resolution, and turbulent flows that may not possess a statistically-steady state are needed.
DEDICATION

To my parents.
BIOGRAPHY

Minao Shen was born on March 15th, 1990 in Hangzhou, China, a lovely city known for its beautiful scenery. He graduated from Hangzhou No.2 High School of Zhejiang Province in 2008. He was then interested in seeing more of the world and experiencing western style higher education. Given the chance and support by his father and mother, Minao started to pursue a degree at the University of Pittsburgh in 2009. During his undergraduate study, Minao found that CFD is a magical combination of physics and programming, either of which could have interested him alone. After receiving a Bachelor’s degree in Mechanical Engineering in 2013, and thanks to the recommendation from Dr. Peyman Givi from Pitt, Minao started his "Direct Path" to Ph.D. program in CFD at NC State University under Dr. Jack Edwards. Since then, Minao has been working on applied aerodynamics simulation and hybrid LES/RANS turbulence modeling. He received a Masters degree in Aerospace engineering "en route" to the Ph.D. in May, 2016.

Aside from digging into his graduate school work, Minao enjoys cars, tennis, and swing dancing. He has been on the Formula SAE team of NCSU for four years. Minao likes to listen to classic rock and classical music. He also has a cutey over-weight cat friend named Chloe.
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1.1 The Study of Turbulence

The study of turbulent flows has been one of the fundamental and major area of research in the subject of fluid dynamics since its early days. In 2000, Pope summarized in his book [Pop00] the main motivations for the study of turbulent flows.

Firstly, the majority of the flow we observe or study are turbulent. These flows range from natural phenomena such as water in a river or the buffeting of a strong wind, to highly engineered flow under controlled environment, such as a jet flow from a rocket motor or flows around air, ground, or marine vehicles.

Secondly, turbulence greatly enhances the rates of transport and mixing of matter, momentum, and heat in the flow. For example, the effect of enhanced mixing of matter by turbulence is demonstrated by Reynolds in his experimental study in 1883 [Rey83]. In the study, he reported different mixing rates and patterns of injected dye in a channel flow of water, for laminar flow (slow mixing, dye streak with increasing diameter) and turbulent flow (dye streak jiggled near injector, and fast mixing downstream). Based on this experiment, Reynolds in 1895 established [Rey95] a non-dimensional parameter, now known as the Reynolds number $Re = U L / \nu$, that characterizes the turbulence level of the flow.

The third part of the motivation is that the transport and mixing of matter, momentum, and heat in flows is practically important in many applications. For instance, a faster mixing of reactants
(matter in this case) is usually desired in combustion devices and chemical reactors. Another example would be the demand for a faster heat exchange on the interface of solid-fluid or two-phase fluid in cooling applications. Because turbulence enhances these processes, a better understanding of it becomes vital for fluid dynamics researchers.

1.2 Computational Approaches to Turbulence

In conventional computational fluid dynamics (CFD), the *continuum hypothesis* is used where fluid is treated as a continuous medium despite its discrete molecular nature. In this continuum fluid domain of study, the governing equations, Navier-Stokes equations, are solved. Navier-Stokes equations, to be discussed in detail in Chapter 2, named after Claude-Louis Navier and George Gabriel Stokes, are a set of transport equations that describe the motion of viscous fluids. Although relatively simple, the Navier-Stokes equations describe the governing laws of conservation of mass, momentum, and energy in complete detail. The accurate description of physics from the complete Navier-Stokes equations is powerful, but also a limit to practical implementation in case of turbulent flows. Because the length and time scales of turbulent flows vary greatly, from the characteristic scales of the flow all the way down to the smallest scales of turbulent motions, aka. Kolmogorov scales $\eta$, the information contained in the velocity field can be huge depending on the Reynolds number. As a result, direct approach of solving the Navier-Stokes equations, named direct numerical simulation (DNS), is so computationally heavy that for moderate-to-high Reynolds number flows, it is not a feasible or practical option. Thus, different turbulence modeling strategies come into play to simulate the effects of turbulence at various levels. The two main categories are Reynolds averaged Navier-Stokes (RANS) simulation and large-eddy simulation (LES). In the remainder of this section, the basic ideas and history of DNS, RANS, LES, and some other modeling techniques will be discussed.

1.2.1 Direct Numerical Simulation (DNS)

As its name suggests, direct numerical simulation solves the Navier-Stokes equations with all scales of fluid motion resolved. Because of its simplicity and the lack of closure modeling, the accuracy and level of details described from DNS surpasses other methods. At the same time, because of the high computational cost, DNS was not feasible even for researchers until the 1970s when computers became more powerful. Orszag and Patterson [Ors72] developed a method in 1972 for homogeneous isotropic turbulence in incompressible flow called the pseudo-spectral method. Later in 1981, Rogallo [Rog81] addressed some numerical and computational issues of the original pseudo-
spectral method and extended it to homogeneous turbulence in incompressible flow subject to uniform deformation or rotation. The basic idea of the pseudo-spectral method is to represent a three-dimensional turbulence field by spatially periodic discrete values on a uniform mesh in the physical space, or equivalently by coefficients of a finite Fourier series in the wavenumber space. The transformation between the two spaces is completed by fast Fourier transformation. As summarized by Rogallo [Rog81], there are two basic requirements that a DNS must meet to represent turbulence: scale and sample. While all scales of motion from Navier-Stokes equations need to be adequately resolved, the periodic solution also needs to provide adequate statistical resolution to include all possible fluid motions allowed by the governing equations. However, the volume of a scale is proportional to the inverse of its computational sample size. This contradictory pair of requirements results in the high computational demands for DNS, and thus limits the application of DNS to a small Reynolds number range. With a maximum wavenumber of $\kappa_{\text{max}} \eta = 1.5$, which corresponds to the smallest, viscous dissipative scale in the physical space, determined by Yeung and Pope [YP89]. Pope estimated in his book [Pop00] that the computational time for a DNS is proportional to $(Re_L/800)^3$ or $(Re_\chi/70)^6$, where $Re_L$ is the turbulence Reynolds number based on the length scale characterizing the large eddies, and $Re_\chi$ is the Reynolds number based on the Taylor microscale. With the continuous advances in supercomputer technology, more simulations of simple turbulent flows at moderate Reynolds numbers are carried out using DNS ([Kim87], [Mos99], [Sil13], [LM15]). However, the computational cost of DNS scales with the Reynolds number so rapidly that it is not practical for most high Reynolds number engineering flows.

### 1.2.2 Reynolds Averaged Navier-Stokes Simulation (RANS)

The concept of the time-averaged Navier-Stokes equations is believed to be first introduced by Reynolds [Rey95] in the late nineteenth century. The decomposed velocity field, which is split into mean and fluctuation parts, results in additional terms known as the turbulent stresses. In 1877, Boussinesq tried [Bou77] to solve this closure problem by developing a mathematical description of the turbulent stresses with the assumption that the turbulent fluctuation has a dissipative effect on the mean flow. This dissipative effect was then modeled with a term known as the eddy viscosity or turbulent viscosity. Neither of the two authors above solved the time-averaged Navier-Stokes equations in a systematical manner.

In the early twentieth century, Prandtl proposed the concept of the mixing-length model [Pra25], which is the foundation of the category of models now known as the algebraic or zero-equation models. The mixing-length model received a viscous damping correction from Van Driest [VD56] which is still in use in modern turbulence model according to Celik [Cel99]. Later the model was
refined for use with attached boundary layers by Cebeci and Smith [CS74]. Another alternative algebraic model was introduced by Baldwin and Lomax [BL78] to address issues in defining a turbulence length scale from the shear-layer thickness.

Prandtl also introduced the first one-equation model in 1945 [Pra45] with a differential equation that is solved for $k$ (turbulent kinetic energy, TKE). The eddy viscosity is then approximated using the value of $k$. After that, until the mid 1990s, one equation models were not particularly popular. In 1994, Spalart and Allmaras formulated another one-equation model [SA94], known as the Spalart-Allmaras (S-A) model, that has been widely used in aerodynamic flows.

Kolmogorov in 1941 [Kol41] introduced the first complete two-equation turbulence model aimed at specifying both turbulence length scale and time scale by modeling both $k$ and an additional parameter $\omega$ (specific dissipation rate). This model is named the $k-\omega$ model. The two-equation model is a "complete" model because turbulence length scale can be formed by $k^{1/2}/\omega$ and turbulence time scale can be formed by $1/\omega$, and flow-dependent specifications is not needed. The $k-\omega$ model was then developed over the years, and Wilcox’s version [Wil93] [Wil08] is widely used. Another complete two-equation model is the $k-\epsilon$ model which evolved mainly in the 1960s, thanks to Davidov [Dav61], Harlow and Nakayama [HN68], Hanjalić [Han70], and Launder and Spalding [LS72]. The credit for the most widely used ‘standard’ $k-\epsilon$ model goes to Launder and Sharma [LS74] for their improved values of the model constants. Compared to the $k-\epsilon$ model, the $k-\omega$ model has the following advantages: its treatment of viscous near-wall region is superior, and it accounts for the effect of stream-wise pressure gradient better. On the other hand, because of the choice of the transported variable, a non-zero value of $\omega$ needs to be specified at boundary conditions. For a flow with non-turbulent inlet, this is not only non-physical but the solution is also sensitive to the specified $\omega$ value at the inlet [Wil06]. In 1994, Menter proposed [Men94] a non-standard version of the $k-\omega$ model that tried to combine the best behaviors from the $k-\epsilon$ and $k-\omega$ models. It treats the near-wall region with the $k-\omega$ model and switches to the $k-\epsilon$ model using a blending function when away from walls. This special model is often referred to as the Menter’s BSL (baseline) model. A modification on the BSL model which limits $\omega$ to be no greater than the strain rate is also presented by Menter in the same paper and is called the SST (shear stress transport) model.

Other than the eddy viscosity models above, there is another category of modeling called the second-order or second-moment closure. Also known as the Reynolds-stress closure, this method solves for the individual Reynolds stresses $u'_i u'_j$ and for the dissipation rate $\epsilon$ or $\omega$ rather than using eddy viscosity. Rotta [Rot51] is one of the first to use this method. Because this method introduces seven transport equations (six for each Reynolds stress component, and one for the dissipation rate), its complexity limits its popularity in research and application. Some noteworthy contributors includes Launder et al. [Lau75], Lumley [Lum78], Speziale [Spe87], and Reynolds [Rey87].
In RANS, only wall-normal resolution is required. The computational cost and grid size is much lower than in DNS. Because of its low cost and robustness, RANS has been the 'state of practice' CFD tool for engineering and industrial use. However, since the turbulence models all involve assumptions and not all aspects of the physics are modeled, RANS can be inaccurate and can miss important flow features in certain flow conditions, for example high Reynolds number wall-bounded aerodynamic flows [Ke14] with separation.

1.2.3 Large Eddy Simulation (LES)

As discussed above, DNS resolves all scales of turbulent motions described by the Navier-Stokes equation in a time-accurate manner, while in RANS all turbulent behaviors are modeled using the concept of the eddy viscosity. In large-eddy simulation, only the larger 3D turbulent motions that contain most of the TKE are resolved, while turbulent motions of smaller scales are modeled. A filtering operation is required on the velocity field, decomposing it into a resolved component, which describes the 3D time-accurate motions of the larger eddies, and a subgrid-scale component. A filtered version of the Navier-Stokes equations is derived, with additional terms called the SGS stress tensor, similar to the Reynolds stress tensor, from the SGS component of the velocity field. The SGS stress tensor is then modeled using SGS models, which can be an eddy-viscosity model. Because of the filtering procedure, the computational cost for resolving the small-scales motions is eliminated, thus LES is more practical than the DNS for flows with moderate Reynolds numbers. In wall-resolved LES, the mesh is still required to be fine enough to resolve near-wall motions, which are of the order of $\delta \chi$ (viscous length scale). The number of grid points is then estimated to be proportional to $Re^{1.78}$ [Cha79]. Because of that, LES is still significantly more expensive than RANS, but with the large-scale eddies resolved, it is expected to be more accurate where large-scale unsteadiness is significant in a flow. The relatively high cost of LES for wall bounded flows still limits its application to engineering problems.

Noteworthy early works on LES are from Smagorinsky [Sma63], Lilly [Lil67], and Deardorff [Dea74]. Later development work of LES focuses on isotropic turbulence, (Chasnov 1991 [Cha91]), fully developed turbulent channel flow (Moin and Kim 1982 [MK82] and Piomelli 1993 [Pio93]), and applying LES to more complex geometries (Akselvoll and Moin 1996 [AM96] and Haworth and Jansen 2000 [HJ00]). Other than the original Smagorinsky model [Sma63], which has been, and still is, widely used, different SGS models with decent performance are composed by Germano et al. [Ger91], Lenormand et al. [Len00], Vreman [Vre04], and Nicoud et al. [ND99]. A more recent review of the capabilities and recommended practices on LES is provided by Georgiadis et al. [Geo10].
1.3 Hybrid LES/RANS

RANS can predict flat plate boundary layers and initial separation points well, but its weakness lies in regions with great unsteadiness (e.g. large separation). LES does a better job in the largely separated regions, and its most costly part is the near-wall region. After understanding the strength and weakness of each method, the idea emerges that the two methods can be combined to optimize their advantages. The goal of hybrid LES/RANS methods is to accurately capture unsteadiness with LES through most part of the flow, while reducing grid density, thus computational cost, by modeling near-wall turbulent motions with unsteady RANS. There have been many ideas on how this hybridization can be achieved. The most popular ones and the ones that are related to this work are discussed next.

1.3.1 WMLES, the DES Family, and Other Hybrid Models

Wall-modeled LES (WMLES), also known as the wall-stress-modeling approach, is a category of methods that use a wall model for the near-wall physics in the inner layer of the boundary layer. The philosophy of WMLES is to greatly reduce the near-wall mesh resolution and increase the permissible time step when compared to a wall-resolved LES. There are many approaches towards how the near-wall layer is modeled, from simple models utilizing the law of the wall to applying RANS in near-wall region.

For example, Deardorff [Dea70] and Schumann [Sch75] calculated the local wall stress by assuming that it is proportional to the mean wall stress by a coefficient defined by the ratio of local and averaged stream-wise velocity at the first off-wall grid point. The momentum flux due to the normal diffusion then can be calculated and is fed back to the LES region. However, a value for the mean wall stress is still needed. For well studied simple channel flows, this quantity is easy to get. For other cases, Schumann [Sch75] used the averaged velocity at the first off-wall point and the law of the wall for an estimate of the mean wall stress. As can be seen, there are many strict assumptions in this approach, and thus it is limited to very simple steady flows.

Another widely used wall model is the two-layer model, also known as the zonal model, firstly brought out by Balaras et al. [BB96]. In a two-layer model, a separate set of RANS equations, which is weakly coupled to the filtered Navier-Stokes equations on the LES grid, is solved on a virtual grid in the inner wall layer. The virtual grid extends from the wall to the first off-wall grid point. The LES provides boundary condition at the top of the virtual grid. Through integration, stream-wise and span-wise wall stresses can be obtained and returned to the LES calculation. The inner virtual grid is only refined in the wall-normal direction. The added set of equations are simple, as they
neglect wall-parallel viscous diffusion terms and impose the pressure gradient from adjacent outer condition. As a result, the added computational cost is very low compared to a wall-resolved LES. The two-layer model has been proven acceptable in various flow conditions. Cabot and Moin [CM99] investigated its performance on a channel flow and flow over a backward facing step. Wang and Moin [WM02] modified the two-layer model to incorporate dynamic mixing-length eddy viscosity, which accounts for only the unresolved part of Reynolds stress, and had a successful application to flow over an asymmetric trailing edge. Tessicini et al. [Tes07] applied the two-layer model to a 3-D flow around a hill shaped obstruction with very reasonable results. Later Patil and Tafti [PT12] extended the conventional two-layer model to use local wall coordinates within the inner layer, resulting a further reduction of computational cost especially for body fitted meshes on complex geometries, and no significant loss of accuracy.

Detached-Eddy Simulation (DES), originally proposed by Spalart et al. [Spa97], was aimed at accommodating flows with large separation at higher Reynolds numbers, such as flows over airborne and ground vehicles. Spalart’s version of DES, referred to as DES97, was defined by Travin et al. [Tra00] as "a three-dimensional unsteady numerical solution using a single turbulence model, which functions as a subgrid-scale model in regions where the grid density is fine enough for a large-eddy simulation, and as a Reynolds-averaged model in regions where it is not." A modified S-A turbulence model is used for both RANS and LES closure. The length scale used in the destruction term in the S-A model, wall distance \( d \), is replaced by \( \tilde{d} = \min(d, C_{DES}\Delta) \), where \( \Delta = \max(\Delta x, \Delta y, \Delta z) \). For highly anisotropic near-wall grids, where wall-parallel spacing is much larger than wall-normal spacing, the closure behavior remains RANS as \( \tilde{d} = d \). For regions with large separation or free stream, where grid spacing is often close to isotropic and larger than wall distance, the turbulence closure behaves as a one-equation SGS model, like the Smagorinsky’s SGS model. Subsequent development [Str01] expanded DES’s compatible turbulence models to include Menter’s SST model [Men94] by modifying the length scale used in the \( k \)-transport equation. DES has demonstrated its quantitative robustness as well as good response to grid refinement in many simple ([Tra00], [Mor03], [EM08]) and complicated ([All05], [Spa03], [PH08]) geometries. There are more examples that can be found in the Spalart’s review on DES [Spa09].

One of the most significant issues with DES is the modeled-stress depletion (MSD), followed by grid-introduced separation (GIS). Because DES defines RANS and LES regions based on wall distance (geometry) and grid size, there is a high requirement on the user to generate an appropriate grid for a specific problem, taking into account of preliminary information of the flow. MSD occurs when an ‘ambiguous’ grid spacing is used for DES, where the spacing in certain regions is not small enough to support accurate LES but so small that the eddy viscosity is affected by the activated DES limiter. In this case, modeled Reynolds stress is suppressed due to the eddy viscosity switching to
from RANS level to SGS level, but resolved Reynolds stress cannot keep up because the grid spacing is larger than the scale of the eddy that supposed to be resolved. The resulting low surface friction can cause early separation, termed GIS. GIS was first detected by S. Deck [Spa09] and Menter & Kuntz [MM02]. Menter & Kuntz [MM02] came up with a shielding method with the SST model where the DES limiter is disabled in the boundary layer. Spalart et al. [Spa06] also introduced a modification on the DES limiter that takes into account extra local flow characters, and named it Delayed DES (DDES). DDES can detect the 'ambiguous' grid situation to DES, and extend the RANS region to keep the model in RANS mode, preventing MSD and GIS, and can be applied to most models. The cost of DDES is some more complexity and higher chance of multiple solutions [Spa06]. It is worth noting that both DES and DDES intend to cover the entire boundary layer by RANS closure, different from the WMLES concept in this work which uses RANS type models as a wall model.

Another issue in DES, DDES, and many other hybrid LES/RANS models is the logarithmic-layer mismatch (LLM). An LLM is the misalignment of the two log layers from the RANS region and the LES region. Nikitin [Nik00] first observed this imperfection in his study of using a wall model based on the DES formulation, instead of simulating the entire boundary layer using RANS as in DES97. The application of the wall model was a success on large grid spacings, and responded well to Reynolds number and grid refinement. However, the log layer from the LES has a higher intercept than that of the RANS by about three wall units of velocity $U^+$. As a consequence, the skin friction was reported to be under-predicted by over 15%, and the velocity gradient at the RANS/LES interface was over-predicted by up to 65%. Over the transition region, turbulence activities changes from being completely modeled to mostly resolved, without providing enough real connection and information of resolved content.

A new definition of a reduced subgrid length-scale with steeper variation was introduced by Travin et al. [Tra06] to elevate the instabilities near the wall. They also constructed a blending function with an empirical 'restore' or 'elevating' factor that inhibits extra reduction of Reynolds stress in RANS. This modification on the DDES-based wall-model provides a better transition between the RANS and LES closures and enhanced near-wall stresses, thus partially alleviating LLM. Later Shur et al. [Shu08] composed an improved delayed DES (IDDES) which combines the original DDES with the above LLM-free WMLES. The switch between the two branches is based on the inflow and initial condition of the simulation. DDES-like functionality is active only when the inflow/initial condition does not have any turbulent content, and the improved WMLES is active otherwise. Despite its complexity, the performance of IDDES is favorable – it resolves the LLM issue for channel flows, can be used as DES97 or DDES in external flows with massive separation, and has a better capability in mixed flows with attached and separated regions. Studies from Piomelli et al. [Pio07] and Mockett et al. [Moc07] have had satisfactory results from IDDES. However, because the baseline transition...
blending function of IDDES is based on the ratio of wall distance and mesh spacing ratio (switch from RANS to LES within the range of $0.5h_{\text{max}} < d < h_{\text{max}}$, where $h_{\text{max}}$ is the largest edge length of the cell), changes in the boundary layer scale will require changes in the mesh. Also, the fact that the elevating-function is empirical makes IDDES not able to correct LLM on a physical basis.

One major challenge and solution to LLM for hybrid LES/RANS models that function as a wall-layer closure is to match the outer and inner-layer solutions in an instantaneous manner. Because the instantaneous LES field does not provide an outer-length scale directly, some proposals seeks external input for the measurement of outer-scale information. We refer to these practices as 'constrained LES/RANS'. Keating & Piomelli [KP06] attempted to resolve this issue by generating stochastic forcing to enhance the resolved Reynolds stress at the LES/RANS interface. 'Hybrid filters' or similar concepts have been introduced ([Ger04], [SRM09], [Gir06]) to try to provide hybrid LES/RANS modelling a more solid groundwork. Source terms proportional to the difference between filtered variables and 'RANS' variables are usually required in these approaches. The RANS-type variables are obtained by either ensemble-averaging the LES data or having a RANS running alongside with the main simulation. Some other works ([Uri10], [XJ12], [Che12]) try to use LES closure for the whole domain, but the development of Reynolds stress and eddy structures near the wall is constrained. The forcing source terms are formed via the differences between the ensemble-averaged resolved Reynolds stresses and that from a (supposedly) more accurate near-wall method, usually a RANS-type closure based on the Boussinesq hypothesis. Ensemble-averaged LES flow field data is often used to solve transport equations for the RANS turbulence variables. Because the SGS closure is not directly affected by the hybridization, LLM is not apparent in these methods. Meanwhile, questions about the legitimacy of near-wall eddy structures predicted by these methods arise, since there is no guarantee that the grid resolution in the wall-parallel directions is fine enough to capture the actual eddy structural changes near the wall. Further, the imposition of scale information from an exterior source renders these models difficult to apply for complex flows that may not possess a well-defined statistically steady state.

1.3.2 Previous NCSU Approaches

Here at North Carolina State University, another category of 'constrained LES/RANS' has been under development. The model by Edwards, Choi, and others presented in [Edw08] and [Cho09] uses a blend of the eddy viscosity provided by a RANS closure and a Smagorinsky model. The blending function for the hybrid eddy viscosity uses a modeled Taylor micro scale together with a pre-calibrated model constant to fix the transition from RANS to LES at the outer extent of the logarithmic layer. The pre-calibration is based on the equilibrium flat-plate boundary layer theory,
and needs to be performed for each case of simulation. Results from their Mach 5 compression
corner case with this model was satisfactory [Edw08]. The disadvantage of needing a pre-calibration
is also obvious – extra work from users is required, and there is no guarantee that it will work for
flows with unsteady mean boundary layers.

An improved model, composed by Gieseking, Edwards and others ([Gie11a], [GE11]), removes
the requirement for the case-by-case pre-calibration. Instead, time-averaged values of modeled
and resolved TKE and specific dissipation rate are used to estimate the outer length scale in the
LES region. These averaged quantities are calculated using weighted exponential averaging of the
unsteady flow fields, meaning that more recent information would have a heavier weight in the
averaging process. Neither Choi’s model nor Gieseking’s model is dependent on the filter width to
determine the position of hybridization, in contrast to the DES-type methods introduced earlier.
In the case of having a mesh that is too coarse to capture the outer scale, the LES flow field could
be under-resolved. In order to prevent this issue, a ‘protection’ function based on mesh scale was
added later ([SE14]) to limit the outer scale. Because both the inner and outer scale information of
Gieseking’s model comes from the flow field, it can adjust to flows with strong local non-equilibrium
and can respond to large changes in boundary-layer thickness without problem-specific adjustment.
This model has been applied in simulations of shock/boundary layer interactions ([GE11], [Gie11b],
[GE12]), scramjet combuster calculations ([Ful14], [PE15], [Ram15]), and airfoils undergoing static
stall ([KE13]) and dynamic stall([KE15], [Ke14]). Static-stall predictions are quite reasonable and
similar to WRLES by Mary and Sagaut ([MS02]). However, the prediction on the dynamics stall cases,
based on experiment of Pruski, et al. ([PB13]), never achieved this level of success. While other
factors certainly influenced predictive capability, extra uncertainty is introduced by implementing
the ensemble-averaging strategies of Gieseking’s model, which may not function properly on a
pitching airfoil.

1.4 Motivation

In the previous sections, we talked about the application considerations and limitations of both RANS
and LES, and how hybrid LES/RANS can be a viable method to simulate flows with important eddy
structure for a reasonable computational cost. As mentioned above, the major problem in designing
an LES/RANS hybrid wall-layer closure model is that the LES solution, which is instantaneous,
does not directly provide an outer-layer length-scale that can be used to match the inner-scale.
‘Constrained LES/RANS’ approaches take external inputs to estimate this length-scale, but the best
way to obtain this information is still unclear. Another issue is that the structure of turbulence
boundary layers can change drastically due to external disturbances such as intrusion of shock...
waves and changes in wall curvature. A desirable hybrid models should be able to adjust to these structural changes in the boundary layers. To make the models applicable to more engineering problems, a reasonable prediction in non-equilibrium regions is also preferred. In addition, because these models are usually designed based on flat-plate scaling arguments, there is no guarantee that they are still valid in wall-bounded, separated regions, nor is it clear that how such models should respond in wall-bounded, separated regions. To summarize, future developments of 'constrained LES/RANS' should aim at satisfying the following three characteristics:

1. the models must revert to a viable SGS model far away from solid surfaces (thus the hybridization of RANS to LES)

2. the models, if applied within a boundary layer, must preserve (in a time-averaged sense) the composite structure of a turbulent boundary layer and must be responsive to boundary-layer structural changes.

3. the models must not employ non-local or non-instantaneous information to achieve the second characteristic. Examples of such information include precursory RANS simulations, problem-specific calibrations, ensemble-averaging, concurrent RANS simulations, and other means.

The first two characteristics are straightforward to understand, and the reasoning for the third characteristic is explained here. Injecting RANS/Boussinesq field data (or in the form of pre-calibration, from precursory or concurrent RANS simulation) directly into the LES framework implies that these models are 'better' with their known limitations. This assumption may be true for simple thin-shear flows when considering near-wall resolution limits, it is not proven to be the case in general. Also, there is a high risk of the RANS component overly constraining the LES response. While ensemble-averaging the hybrid LES/RANS solution can be a better alternative than directly using RANS/Boussinesq information, it can be very inconvenient if the averaging operation is applied to many variables and to calculate various stress, diffusion, and conduction terms in, for example, a multi-component reactive flow. Ensemble averaging is also not applicable to situations where the boundary layer does not have a well-defined, statistically-stationary state (for example a wing undergoing pitch-up and pitch-down motion). As a result, we even can hardly consider ensemble averaging methods meeting the second characteristic above.

Introduced in this dissertation is a new pathway towards the development of new LES/RANS models that satisfies these three characteristic requirements. The framework of RANS/LES hybridization used in this work is originated from Baurle et al. [Bau03], but the concepts described here should not be hard to apply to other LES/RANS models. The formulation of this new LES/RANS approach
is presented in Section 2.4.4. Results of the new model compared to other turbulence closures as well as experiments are discussed in Chapter 4, 5, and 6. The following section will introduce the simulation cases used in this study.

1.5 Simulation Cases

To test the capability and performance of the newly developed model, it is applied in simulations of different cases of flow with wall-bounded turbulent boundary layers. From simple flat-plate boundary layers to more complicated three-dimensional flows, each case has its unique characteristics that will challenge the new model. Data from experiments and other comparable turbulence models are used to assess the performance of the new hybrid LES/RANS model. The following is a brief description of each tested cases.

1.5.1 Subsonic Flat-Plate Boundary Layer with Low to Moderate Reynolds Number

The first case chosen is the flat-plate boundary layer, which is simple yet still challenging to many hybrid models. This case is based on the experiments conducted by DeGraaff and Eaton [DE00]. In the experiment, zero pressure-gradient turbulent boundary layers at five Reynolds numbers based on momentum thickness, ranging from $Re_\theta = 1430$ to $Re_\theta = 31000$, were mapped using a high-resolution laser-Doppler anemometer. The test facility includes a lab-scale wind tunnel enclosed in a pressure vessel. The large range of Reynolds number comes from a combination of changing the ambient density (by a factor of 8) and the free stream velocity (by a factor of 3). Streamwise mean velocity and various stress and stress production data are available from the experiment for all $Re_\theta$ cases.

In this study, cases with $Re_\theta = 2900$ and $Re_\theta = 13000$ are chosen to perform simulations. The reason for choosing the $Re_\theta = 2900$ case is that it is low enough to conduct a wall-resolved LES to be added to the comparison, while it is high enough to reveal a characteristic turbulent boundary layer structure. For the moderate Reynolds number ($Re_\theta = 13000$) case, it is very expensive and nearly impractical for a wall-resolved LES, but it is suitable for hybrid LES/RANS models to unveil its boundary layer structure, including a very well-defined logarithmic layer and stress distributions. The purpose of the two flat-plates cases are to test the new model’s ability to "revert to a viable SGS model far away from solid surfaces" and "preserve the composite structure of a turbulent boundary layer" as stated in the Motivation section.
1.5.2 Supersonic Flat-Plate Boundary Layer

The second case chosen is also a flat-plate boundary layer case. This supersonic flat-plate boundary layer experiment was conducted by Elena and Lacharme [EL88]. The free stream Mach number for this case is 2.32, with the Reynolds number based on momentum thickness being 4700. The experimental measurements include velocity fluctuations, Reynolds shear stresses, skewness and flatness factors, and the intermittency factor. This case was tested by previous generations of NCSU’s hybrid models, i.e. Choi’s model ([Cho09]) and Gieseking’s model ([Gie12]), with reasonable results being obtained.

1.5.3 Aérospatiale ’A-Airfoil’

The ’A-airfoil’ is designed at Aérospatiale and tested in two wind tunnels at ONERA [Gle88] over various Reynolds numbers, Mach numbers, and angles of attack. Experimental data is available in terms of skin-friction, surface pressure distribution, and velocity profiles and Reynolds stress components measured by laser Doppler velocimetry (LDV). The chord is 0.6 m long, and for this study, the case with the free-stream Mach number of 0.15, \( Re_c = 2.1 \times 10^6 \), and an angle of attack of 13.3 degrees is used.

This case was previously tested using Choi’s model and Gieseking’s model ([Ke14]). Other studies on this case also include an LES test by Mary and Sagaut [MS02].

1.5.4 NASA Wall-Mounted Hump

The next case is flow over a wall-mounted hump. This hump has similar geometry employed by Seifert and Pack [SP02] but the exact set up was initially introduced at the NASA CFDVAL2004 Workshop (Case 3) as a flow validation case for turbulence models. It is now also a part of the ERCOFTAC Database as case C.83. The geometry consists of a hump model mounted on a splitter plate and between two glass endplates. The experiment has two tests, one with a blowing (steady and oscillatory) from a slot near the separation point, and the other with no slot. We only conducted simulation on the case without a slot and plenum. Figure 1.1 shows the setup of the experiment. The details of the nominally 2D experiments (although with side-wall effects from the end plates) can be found in [Gre06a], [Gre06b] and [Nau06].

Since this case is a CFD validation test, there are numerous simulations available, including using RANS closures ([Cap05], [Rum07], [CM13] etc.), hybrid or WMLES closures([Lyo09], [Avd09], [Shu14], [Dil14], [IM16], etc.), and a few WRLES studies([You06], [FC10], [UM17]). Many of the currently available RANS models fail to simulate the non-equilibrium effects in the separated region, resulting
Figure 1.1 Experimental setup of the wall-mounted hump. Source: NASA CFDVAL2004 Workshop

in a delayed reattachment of the flow and an up to 35% longer separation bubble, except for the $k-\epsilon$ model, reported by Cappelli [CM13] (however the skin friction level on the hump is low for the $k-\epsilon$ model.) For WMLES and hybrid LES/RANS simulations, the results are mixed. The quality of the results relies on the accuracy of the wall model and other aspects of the simulations (e.g. grid refinement, inlet boundary condition and turbulence generation mechanism, forced separation etc.) Among the three WRLES mentioned above, two of them are likely under-resolved near the surface due to insufficient mesh refinement. In the study of You et al. [You06], a mesh with only 7.5 million grid points was used, and the report of viscous spacing being $\Delta x^+ < 50$ and $\Delta z^+ < 25$ was incorrect and should be $\Delta x^+ < 1440$ and $\Delta z^+ < 112$ according to the reported physical spacing (confirmed by private communication between Iyer and You mentioned in [IM16].) In the study of Franck and Colonius [FC10], they reported a near-wall viscous spacing of $\Delta x^+ = 94$, $\Delta y^+ = 8.7$ and $\Delta z^+ = 31$. The stream- and span-wise spacing from You et al. and the wall-normal spacing from Franck and Colonius can be considered too large compared to the recommended values of Georgiadis et al. [Geo10] for LES without any near-wall modeling. Though the results in these two studies are encouraging, the validity of the under-resolved simulations is questionable. In a recent study by Uzun and Malik [UM17], they applied an 'overset-grid' system to add adequate near-wall resolution and obtained decent results, capturing the relaminarization near the start of the hump (characterized by a plateau in $c_f$) that most hybrid and RANS models fail to predict.
1.5.5 Flow Around a Simplified Car (Ahmed Body)

Another case that is tested with our new hybrid LES/RANS model is flow over a simplified car model, also known as the 'Ahmed body.' This model was designed by Ahmed et al. [Ahm84] and was used as a benchmark case in the 9th and 10th ERCOFTAC/IAHR Workshops on Refined Flow Modelling ([Jak01], [MB02].) The geometry of the model consists of a car-shaped body with a blunt front and a slanted rear top surface, as shown in Fig. 1.2. The slant angle $\varphi$ is the main variance in the model. Flow over the Ahmed body presents many of the basic features of ground vehicle aerodynamics, including laminar-to-turbulent flow transition, flow separation, vortex shedding, and their interactions which dominates the aerodynamic forces on the body.

![Figure 1.2 The Ahmed body model. The slant angle $\varphi$ is variable and $\varphi = 25$ deg is used in this work.](image)

The first experimental measurements were taken by Ahmed et al. [Ahm84] for various slant angles, and very complex wake flow structures were observed. Almost 85% of the body drag was found to be pressure drag, which varies with the slant angle and wake structure. A maximum value of drag is at a critical slant angle of 30 deg. Above the critical angle, the strong adverse pressure gradient detaches the flow completely over the slant from the beginning. Below 30 deg, the vortices from lateral edges help the flow reattach to the slant. Further experimental investigations were conducted by Lienhart et al. [Lie02] for two slant angles $\varphi = 25$ deg and $\varphi = 35$ deg, which are below and above the critical angle, respectively. In this experimental study, mean velocity fields and turbulence statistics are taken at a Reynolds number of 768,000 (based on height of the body), using a laser Doppler velocimetry measurement.

A number of simulation studies have been conducted on this benchmark case for validation...
of turbulence models. Various RANS simulations ([Gil99], [Han89], [Cra01], [Gui08], etc.) tend to predict the 35 deg case relatively successfully, but their results for the 25 deg case are not satisfactory. It is hard for RANS models to capture the small scale structures cased by the partial separation at the top of the slant, which promotes the three-dimensionality and mean-stream momentum transfer. Meanwhile, LES studies ([HP02], [KD05], [Min08]) show improvements of the solutions over RANS by capturing the topology and unsteady phenomena of the flow as well as level of turbulent stresses and turbulent kinetic energy. Because of the high Reynolds number, the thickness of the boundary layer is small. A wall-resolved LES is still very computationally expensive, and various wall-modeled and hybrid methods were also applied to this case. DES/IDDES approaches ([Kap03], [Gui16], [Ser13]) show improved results compared to RANS and are able to resolve the vortical structures, but only the IDDES model can predict the partial reattachment on the 25 deg slant.
2.1 Navier-Stokes (N-S) Equations

The Navier-Stokes equations are a set of transport equations that describes the conservation law of mass, momentum, and energy for fluids in continuum mechanics. The mass equation makes sure that no mass is either created or destroyed. The momentum equations are derived by applying Newton's second law of motion to a Newtonian viscous fluid, for which the dynamic viscosity is a function of temperature, and does not change with strain rate. The energy equation governs the conservation of all energy (internal, kinetic, potential etc.) There is no body force or addition of heat in all the cases studied in this work, so terms correspond to effects caused by body forces and external heat sources are neglected. In tensor notation, the Navier-Stokes equations for compressible flow read as

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0 \quad (2.1)
\]

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \quad (2.2)
\]

\[
\frac{\partial \rho E_0}{\partial t} + \frac{\partial \rho H_0 u_j}{\partial x_j} = \frac{\partial}{\partial x_j} (u_i \tau_{ij} - q_j) \quad (2.3)
\]

where \( E_0 \) is total energy, \( H_0 \) is enthalpy, \( \tau_{ij} \) is the shear stress tensor. The definitions of these
Note that Eq. 2.2 is a vector equation that consists of three scalar equations representing the three coordinate directions, while Eq. 2.1 and Eq. 2.3 are scalar equations. Source terms of each conservation equation are on the right-hand side. The source term of Eq. 2.1 is zero because mass cannot be created or destroyed. Source terms of Eq. 2.2 include a pressure effect term and a viscous force effect term. Source terms in Eq. 2.3 account for effects of heat generation due to viscous work and internal heat conduction. Note that the effect of pressure work is already included in the enthalpy term $H_0$.

Since the medium in this work is assumed to be Newtonian, its molecular viscosity can be calculated by Sutherland’s formula as a function of temperature:

$$\mu = \mu_{ref} \left( \frac{T}{T_{ref}} \right)^{3/2} \frac{T_{ref} + s}{T + s}$$

where $\mu_{ref}$ is the viscosity at a reference temperature $T_{ref}$. The reference values used are $T_{ref} = 273.15 K$ and $\mu_{ref} = 1.716^{-5} kg/m \cdot s$ with a constant $s = 110.4 K$.

To close this compressible N-S equations system, some assumptions are needed. The Prandtl number in Eq. 2.4 is set to $Pr = 0.71$ for air. Ideal gas law is sufficient to be used as the equation of state:

$$P = \rho RT$$

and energy can be assumed to be a linear function of temperature:

$$e = C_v T$$

$$h = C_p T$$
The N-S equations can also be written in what is referred to as the conservative vector form:

\[
\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{F}}{\partial x} + \frac{\partial \vec{G}}{\partial y} + \frac{\partial \vec{H}}{\partial z} = \vec{J}
\]  
(2.9)

where the conserved variables are contained in \( \vec{U} \):

\[
\vec{U} = \begin{bmatrix}
\rho \\
\rho u_1 \\
\rho u_2 \\
\rho u_3 \\
\rho(e + \frac{u_i u_i}{2})
\end{bmatrix}
\]  
(2.10)

and the fluxes, including inviscid and viscous fluxes, are:

\[
\vec{F} = \vec{F}_{inv} - \vec{F}_{visc} = \begin{bmatrix}
\rho u_1 \\
\rho u_1 u_1 + p \\
\rho u_1 u_2 \\
\rho(u + \frac{u_i u_i}{2})u_1 + p u_1 \\
\rho(u + \frac{u_i u_i}{2})u_1 + p u_1
\end{bmatrix}
\]  
(2.11)

\[
\vec{G} = \vec{G}_{inv} - \vec{G}_{visc} = \begin{bmatrix}
\rho u_2 \\
\rho u_1 u_2 \\
\rho u_2 u_2 + p \\
\rho(u + \frac{u_i u_i}{2})u_2 + p u_2 \\
\rho(u + \frac{u_i u_i}{2})u_2 + p u_2
\end{bmatrix}
\]  
(2.12)

\[
\vec{H} = \vec{H}_{inv} - \vec{H}_{visc} = \begin{bmatrix}
\rho u_3 \\
\rho u_1 u_3 \\
\rho u_2 u_3 \\
\rho u_3 u_3 + p \\
\rho(u + \frac{u_i u_i}{2})u_3 + p u_3
\end{bmatrix}
\]  
(2.13)

The term \( \vec{J} \) on the right side of Eq. 2.9 is a zero vector in this work.

### 2.1.1 Simplifications for Incompressible Flows

In the majority of this work, the Mach number of the flow is low \( (M \leq 0.1) \), thus the flow can be assumed to be incompressible. Density then can be considered a constant and moved out of
any partial derivative operations. Heat conduction in these cases is also neglected, so the energy equation can be dropped. With these two assumptions, the N-S equations (Eq. 2.1 and 2.2) can be simplified for incompressible flow as follows:

\[
\frac{\partial u_j}{\partial x_j} = 0 \quad (2.14)
\]

\[
\rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ji}}{\partial x_j} \quad (2.15)
\]

Note that the continuity equation (2.14) is decoupled from the momentum equations (2.15) because of the absence of any pressure or density term. To overcome this difficulty, the numerical solution to this set of incompressible N-S equations is obtained using an implicit artificial compressibility (AC) method, which will be discussed in Section 3.4.

2.2 RANS Equations and Turbulence Closure

The RANS equations are derived by splitting instantaneous variables in the flow field into a time-averaged component and an instantaneous fluctuating component. This procedure is called Reynolds decomposition, first introduced by Osborne Reynolds [Rey95] in 1895. For an arbitrary variable \(a\), Reynolds decomposition is described by Eq. 2.16, where \(\langle a \rangle\) is the mean (time-averaged) component and \(a'\) is the fluctuating component:

\[
a = \langle a \rangle + a' \quad (2.16)
\]

\[
\langle a \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0+T} a(\vec{x}, t) \, dt \quad (2.17)
\]

\[
\langle a' \rangle = 0 \quad (2.18)
\]

Applying Reynolds decomposition to variables in the incompressible N-S equations (2.14 and 2.15), and taking average of the entire equations, we get the Reynolds-averaged Navier-Stokes equations:

\[
\frac{\partial \langle u_j \rangle}{\partial x_j} = 0 \quad (2.19)
\]

\[
\rho \frac{\partial \langle u_i \rangle}{\partial t} + \rho \frac{\partial \langle u_i u_j \rangle}{\partial x_j} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial \langle \tau_{ji} \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \quad (2.20)
\]

Note that in this case \(\rho = \langle \rho \rangle\) is a constant. This RANS equation system is not closed because of the
introduction of the Reynolds stress term $-\rho \langle u'_i u'_j \rangle$. The Reynolds stress can be decomposed to an anisotropic part which is responsible for momentum transfer, and an isotropic part:

$$R_{ij} = -\rho \langle u'_i u'_j \rangle + \frac{2}{3} \rho k \delta_{ij} \quad (2.21)$$

where $k$ is the turbulence kinetic energy (TKE)

$$k = \frac{\langle (u')^2 \rangle + \langle (v')^2 \rangle + \langle (w')^2 \rangle}{2} \quad (2.22)$$

The Boussinesq approximation is used to model the anisotropic part $R_{ij}$ of the Reynolds stress term, and an additional variable called turbulent eddy viscosity ($\mu_t$) is introduced:

$$R_{ij} = \mu_t \langle S_{ij} \rangle \quad (2.23)$$

$$-\rho \langle u'_i u'_j \rangle = \mu_t \langle S_{ij} \rangle - \frac{2}{3} \rho k \delta_{ij} \quad (2.24)$$

where $\langle S_{ij} \rangle$ is the mean rate of strain

$$\langle S_{ij} \rangle = \frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} - \frac{2}{3} \frac{\partial \langle u_k \rangle}{\partial x_k} \delta_{ij} \quad (2.25)$$

The isotropic part of the Reynolds stress can be absorbed in a modified mean pressure. As a result, the mean momentum Eq. 2.20 can also be written as:

$$\frac{\partial \langle u_i \rangle}{\partial t} + \frac{\partial \langle u_i \rangle \langle u_j \rangle}{\partial x_j} = -\frac{1}{\rho} \frac{\partial}{\partial x_i} \left( \langle p \rangle + \frac{2}{3} \rho k \right) + \frac{1}{\rho} \frac{\partial}{\partial x_j} \left[ (\mu + \mu_t) \langle S_{ij} \rangle \right] \quad (2.26)$$

The effect of momentum transfer caused by turbulent eddies is modeled by the turbulent eddy viscosity. Different models have been developed to calculate the eddy viscosity, targeting at a more physical response. The RANS eddy viscosity models used in this work are two-equation models by Menter ([Men94]), named Menter’s baseline (BSL) and shear-stress transport (SST) models.

### 2.2.1 Turbulence Closure – Menter’s BSL and SST

Menter’s two equation models (BSL and SST) are used in this study. The BSL model is based on Wilcox’s $k - \omega$ model ([Wil06]) and the $k - \epsilon$ model. The philosophy of the BSL model is to maintain the robustness and accuracy of the Wilcox $k - \omega$ model near the wall and to utilize the advantage of free-stream independence from the $k - \epsilon$ model in the outer part of the boundary layer. The combination of the two model starts with transforming the $k - \epsilon$ model in to a $k - \omega$ formulation.
Formulation of the original $k-\omega$ model is as follows (note that variables in this subsection are all Reynolds-averaged quantities):

\[
\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) = R_{ij} \frac{\partial u_i}{\partial x_j} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_{k1} \mu_t) \frac{\partial k}{\partial x_j} \right]
\]

(2.27)

\[
\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_j}(\rho u_j \omega) = \frac{\gamma_1}{v_t} R_{ij} \frac{\partial u_i}{\partial x_j} - \beta_1 \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_{\omega1} \mu_t) \frac{\partial \omega}{\partial x_j} \right]
\]

(2.28)

and the transformed $k-\epsilon$ formulation is:

\[
\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) = R_{ij} \frac{\partial u_i}{\partial x_j} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_{k2} \mu_t) \frac{\partial k}{\partial x_j} \right]
\]

(2.29)

\[
\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_j}(\rho u_j \omega) = \frac{\gamma_2}{v_t} R_{ij} \frac{\partial u_i}{\partial x_j} - \beta_2 \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_{\omega2} \mu_t) \frac{\partial \omega}{\partial x_j} \right] + 2 \rho \sigma_{\omega2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}
\]

(2.30)

Note that the differences between the transformed $k-\epsilon$ equations and the original $k-\omega$ equations include different model constants ($\phi_1$ and $\phi_2$) and an additional cross-diffusion term.

The next step is to multiply the original $k-\omega$ equations (Eq. 2.27 and 2.28) by a blending function $F_1$, and multiply the transformed equations (2.29 and 2.30) by $(1 - F_1)$. The resulting equations are added together and giving the formulation of the BSL model:

\[
\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) = P_k - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_{k1} \mu_t) \frac{\partial k}{\partial x_j} \right]
\]

(2.31)

\[
\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_j}(\rho u_j \omega) = P_\omega - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_{\omega1} \mu_t) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \rho \sigma_{\omega2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}
\]

(2.32)

In application in this work, the term $R_{ij} \frac{\partial u_i}{\partial x_j}$ in both $k$ and $\omega$ equations' production term is replaced by $\Omega^2/\mu_t$, where $\Omega$ is the the magnitude of mean vorticity field:

\[
P_k \equiv \mu_t \Omega^2
\]

(2.33)

\[
P_\omega \equiv \gamma \rho \Omega^2
\]

(2.34)

\[
\Omega = |\epsilon_{ij} \frac{\partial u_j}{\partial x_i} e_k|
\]

(2.35)

22
where \( e_{ijk} \) is the permutation symbol and \( \vec{e}_k \) is a unit vector in each coordinate system direction. The constants \( \phi \) in equations 2.31 and 2.32 are calculated from the constants \( \phi_1 \) and \( \phi_2 \), as follows:

\[
\phi = F_1 \phi_1 + (1 - F_1) \phi_2 
\]

(2.36)

where the values of set 1 (\( \phi_1 \), Wilcox) and set 2 (\( \phi_2 \), standard \( k-\epsilon \)) are tabulated in table 2.1, and the blending function \( F_1 \) is given by:

\[
F_1 = \tanh(\text{arg}_{BSL,1}^4) 
\]

(2.37)

\[
\text{arg}_{BSL,1} = \min \left[ \max \left( \frac{\sqrt{K}}{0.09 \omega y}; \frac{500 \nu}{y^2 \omega} \right); 10.0 \right] 
\]

(2.38)

where \( y \) is the distance to the next wall. The first argument in Eq. 2.38 is about 2.5 in the log layer and goes to zero towards the edge of the boundary layer. The second argument makes \( F_1 \) unity in the sublayer, and this argument goes to zero in the log region and above. The third argument, which is different from Menter’s original formulation but serving the same purpose, is a safeguard against the freestream-dependent solution.

The boundary condition of \( \omega \) at a solid surface is:

\[
\omega = \frac{60 \nu}{\beta_1 (\Delta y_1)^2} \quad \text{at} \quad y = 0 
\]

(2.39)

In Menter’s BSL model, the turbulence eddy viscosity is given by

\[
\nu_t = \frac{k}{\omega} 
\]

(2.40)

However, this term is slightly modified in this work to protect the laminar sub-layer:

\[
\nu_t = \frac{a_1 k}{\max(a_1 \omega; F_{sub} \Omega)} 
\]

(2.41)

\[
F_{sub} = \tanh(\text{arg}_{BSL,2}^2) 
\]

(2.42)

\[
\text{arg}_{BSL,2} = \min \left( \frac{500 \nu}{y^2 \omega}, 10.0 \right) 
\]

(2.43)

By using this formulation, the turbulent shear stress in the laminar sub-layer is switched to a form with Bradshaw’s assumption, which models the shear stress to be proportional to the TKE:

\[
\tau_t = \rho a_1 k 
\]

(2.44)
This is also the idea of the SST model, and the difference is that for SST model, the whole boundary layer uses Bradshaw’s assumption (Eq. 2.44) by applying a different control factor ($F_2$). For the SST model, the eddy viscosity is calculated as:

$$
\nu_t = \frac{a_1 k}{\max(a_1 \omega; \Omega F_2)} \quad (2.45)
$$

$$
F_2 = \tanh(\arg_{SST}^2)
$$

$$
\arg_{SST} = \max \left( \frac{2 \sqrt{k}}{0.09 \omega y}; \frac{500 \nu}{y^2 \omega} \right) \quad (2.47)
$$

Some model constants for the SST model are different from the BSL model and are listed in table 2.1.

<table>
<thead>
<tr>
<th>Model Constant</th>
<th>Inner, $\varphi_1$ $(k-\omega)$</th>
<th>Outer, $\varphi_2$ $(k-\epsilon)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>For BSL</td>
<td>For SST</td>
<td></td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>0.5</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.0750</td>
<td>0.0750</td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5532</td>
<td>0.5532</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.31</td>
<td>0.31</td>
</tr>
</tbody>
</table>

2.3 Filtered N-S equations and SGS models

LES computes the dynamics of the larger-scale motions, which are usually geometry dependent and not universal, and represent effects of smaller scales by using simple models. There are generally four steps in LES. The first is to decompose the velocity field ($u_i$) into a filtered (or resolved) component ($\overline{u}_i$) and a residual (or subgrid-scale, SGS) component ($u'_i$). The filtering operation is defined by ($[\text{Leo75}]$):

$$
\overline{u}_i(\vec{x}, t) = \int G(\vec{r}, \vec{x}) u_i(\vec{x} - \vec{r}, t) d\vec{r}
$$

$$
(2.48)
$$
where the integration is over the entire domain, and \( G \) is the specified filter function that satisfies the normalization condition:

\[
\int G(\vec{r}, \vec{x}) = 1
\]  

(2.49)

Some of the most commonly used filters include the box filter, the Gaussian filter, and the sharp spectral filter etc. For this work, the grid itself is acting like a box filter:

\[
G(r) = \frac{1}{\Delta} H\left(\Delta^2 - |r|\right)
\]  

(2.50)

\[
H(n) = \begin{cases} 
0, & n < 0, \\
1, & n \geq 0
\end{cases}
\]  

(2.51)

The filter width (\( \Delta \)), or cut-off scale, is considered to be

\[
\Delta = \left(\Delta x \Delta y \Delta z\right)^{1/3}
\]  

(2.52)

This way, \( \overline{u}_i \) is simply the average of \( u_i(\vec{x}_c) \) in the cell at \( \vec{x}_c \) if the cell is close to isometric. The residual component is what is left after the filtering:

\[
u''_i \equiv u_i - \overline{u}_i
\]  

(2.53)

This decomposition looks similar in forms to the Reynolds decomposition. However, the differences are that \( \overline{u}_i \) is still a random field with fluctuations, and the filtered residual is generally not zero (\( u''_i \neq 0 \)). By applying the operation of (spatially uniform) filtering and decomposition to the incompressible N-S equations (Eq. 2.14 and 2.15), we get the filtered N-S equations:

\[
\frac{\partial \overline{u}_i}{\partial x_i} = 0
\]  

(2.54)

\[
\rho \frac{\partial \overline{u}_i}{\partial t} + \rho \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \tau_{ij} - \tau^R_{ij} \right)
\]  

(2.55)

where \( \tau^R_{ij} \) is the SGS (or residual) stress tensor. Analogous to the Reynolds stress (\( R_{ij} \)), the SGS stress term comes from the difference between he filtered product \( \overline{u}_i \overline{u}_j \) and the product of the filtered velocities \( \overline{u}_i \overline{u}_j \):

\[
\tau^R_{ij} \equiv \overline{u}_i \overline{u}_j - \overline{u}_i \overline{u}_j
\]  

(2.56)

The SGS stress term represents the small scale motions that cannot be resolved by the provided mesh and need to be modeled for the closure of the equation system. Similar to modeling the Reynolds
stress, it can also be divided into an isotropic part and an anisotropic part ($\tau_{ij}^f$) which contributes to momentum transfer.

$$\tau_{ij}^f \equiv \tau_{ij}^R - \frac{2}{3} k_r \delta_{ij} \quad (2.57)$$

where $k_r$ is the residual TKE. The Boussinesq approximation is used to model the anisotropic part by introducing a variable called the residual (or SGS) eddy viscosity ($\mu_r$):

$$\tau_{ij}^R = \mu_r S_{ij} \quad (2.58)$$

where $S_{ij}$ is the filtered strain rate

$$S_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \quad (2.59)$$

As a result, the filtered momentum equation (Eq. 2.55) can also be written as

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial}{\partial x_i} \left( \bar{p} + \frac{2}{3} \rho k_r \right) + \frac{1}{\rho} \frac{\partial}{\partial x_j} \left[ (\mu + \mu_r) S_{ij} \right] \quad (2.60)$$

The main SGS model used in this work is the 'mixed-scale' model by Lenormand et al. ([Len00]). Two other models, naming Vreman’s model ([Vre04]) and the WALE model ([ND99]) are also tested in a WRLES case.

### 2.3.1 Mixed-Scale SGS Model

Lenormand and Sagaut ([Len00]) proposed a family of models with one parameter ($\alpha$) that combines the second invariant of strain rate, the characteristic length scale $\Delta$, and the kinetic energy of the highest resolved frequencies $k_c$:

$$\mu_r = \rho C_m |S|^{\alpha} k_c^{(1-\alpha)/2} \Delta^{1+\alpha} \quad (2.61)$$

This series of models is named the "Mixed-Scale" (MS) model ([Len00]). In this work, value of $\alpha = 0.5$ is kept, and the constant $C_m = C_m(\alpha)$ is set to 0.06. In this original formulation, the second invariant of strain rate is used

$$|S| = \sqrt{S_{ij} S_{ij}} \quad (2.62)$$

In this work, the magnitude of vorticity $\Omega$ is used in the place of $|S|$. The filter width $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$ is used as the characteristic length, and $k_c$ is an approximate evaluation of the residual kinetic energy $k_r$. This variable is obtained by introducing a test filter, which can be recognized as a second-order
approximation of the Gaussian filter

\[
\tilde{u}_{i,j,k} = \frac{1}{12} \left[ (\bar{u}_{i-1,j,k} + 2\bar{u}_{i,j,k} + \bar{u}_{i+1,j,k}) + (\bar{u}_{i,j-1,k} + 2\bar{u}_{i,j,k} + \bar{u}_{i,j+1,k}) + (\bar{u}_{i,j,k-1} + 2\bar{u}_{i,j,k} + \bar{u}_{i,j,k+1}) \right]
\]

(2.63)

Then \(k_c\) is evaluated by

\[
k_c = \frac{1}{2} \left( \bar{u}_k - \tilde{u}_k \right)^2
\]

(2.64)

The resulting form for the SGS eddy viscosity is

\[
\mu_r = \rho c \Omega^{0.5} \kappa_c^{0.25} \Delta^{1.5}
\]

(2.65)

### 2.3.2 Vreman's SGS Model

The Vreman's SGS model [Vre04] is a relatively simple model expressed in first-order derivatives that does not use explicit filtering, averaging, or clipping procedures. Dissipation in transitional and near-wall regions is designed to be low. The residual eddy viscosity \(\langle \mu_r \rangle\) in this model is defined as

\[
\mu_r = \rho c \sqrt{\frac{B}{\alpha_{i,j}}} \alpha_{i,j}
\]

(2.66)

where \(c = 0.07\) is a model constant, and

\[
\alpha_{i,j} = \frac{\partial u_j}{\partial x_i}
\]

\[
B = \beta_{11} \beta_{22} - \beta_{12}^2 + \beta_{11} \beta_{33} - \beta_{13}^2 + \beta_{22} \beta_{33} - \beta_{23}^2
\]

\[
\beta_{ij} = \Delta_i^2 a_{mi} a_{mj}
\]

and \(\Delta_i\) is the cell width in the \(i\) direction. This model is rotationally invariant for isotropic filter widths (when \(\Delta_i = \Delta, \beta = \Delta^2 \alpha^T \alpha\)).

Note that in practice, when the residual eddy viscosity is small, machine precision errors may contaminate the calculation of \(B\) from floating point operations. As a result, \(B\) is set to zero (and so is \(\mu_r\)) if the calculated value of \(B\) is less than \(10^{-8}\).
2.3.3 The WALE Model

The WALE model, or Wall-Adapting Local Eddy-viscosity model ([ND99]), is a subgrid-scale eddy viscosity model based on the square of velocity gradient tensor as the invariant-of-choice that accounts for not only the strain but also the rotation rate of the smallest resolved turbulent fluctuations. The SGS model is formulated as follows.

\[ \mu_r = (C_w \Delta)^2 \frac{OP_1}{OP_2} \]  

(2.67)

where

\[ C_w^2 = C_s^2 \left( \frac{\sqrt{2} (S_{ij}^d S_{ij}^d)^{3/2}}{S_{ij} S_{ij} \overline{OP}_1 / \overline{OP}_2} \right) \]

\[ \overline{OP}_1 = (S_{ij} S_{ij})^{3/2} \]

\[ \overline{OP}_2 = (S_{ij} S_{ij})^{5/2} + (S_{ij}^d S_{ij}^d)^{5/4} \]

\[ S_{ij} = \frac{1}{2} (\overline{g}_{ij} + \overline{g}_{ji}) \]

\[ S_{ij}^d = \frac{1}{2} \left( \frac{\overline{g}_{ij}^2 + \overline{g}_{ji}^2}{3} \delta_{ij} \overline{g}_{kk} \right) \]

\[ \overline{g}_{ij} = \frac{\partial \bar{u}_i}{\partial x_j} \]

The constant \( C_s \) is approximated by \( C_s = \frac{1}{\pi} \left( \frac{3 C_K}{2} \right)^{-3/4} \), and \( C_K \) is the Kolmogorov constant \( C_K \approx 1.4 \), thus \( C_s \approx 1.8 \). More details of assessing the value of \( C_w \) can be found in [ND99], and a resulting value of \( C_w = 0.5 \) is used in the LES study in this work.

2.4 Hybridization of LES/RANS

Previous efforts on developing a hybrid LES/RANS turbulence closure here at NCSU's CFD/hypersonics lab resulted in a class of hybrid models where transition between RANS and LES is controlled by a blended eddy viscosity, calculated as

\[ \mu_t = \rho \nu_t = \rho \left[ \Gamma \nu_{t,RANS} + (1 - \Gamma) \nu_r \right] \]  

(2.68)

where \( \nu_{t,RANS} \) is defined by Eq. 2.41 or 2.45, depending on the RANS closure, and \( \nu_r \) is defined by Eq. 2.65. The blending function \( \Gamma \) is a time-dependent variable that bridges the closure between LES
and RANS. \( \Gamma \) goes to unity when unsteady RANS is needed, and goes to zero for pure LES. How this blending function is prescribed is the core of this class of hybrid methods. Before introducing the new formulation for this blending function, two earlier models of this class, along with the IDDES, are described because they will be compared to the new model.

### 2.4.1 Choi’s Model

The blending function \((\Gamma)\) in Choi’s model ([Edw08],[Cho09]) is defined as a function of the ratio of the wall distance to an estimated Taylor microscale:

\[
\Gamma = \frac{1}{2} \left( 1 - \tanh \left[ \frac{5}{\sqrt{C_\mu}} \eta^2 - 1 \right] - \phi \right) 
\]

\( \eta = \frac{d}{\alpha_1 \chi} \)

where \(C_\mu\) is a constant set to 0.09, and \(\kappa\) is the Von Kármán constant, set to 0.41. The Taylor microscale is defined as:

\[
\chi = \sqrt{\frac{\nu}{C_\mu \omega}}
\]

The constant \(\phi\) is set to \(\tanh^{-1}(0.98) = 2.2975599\) so that the balancing position where \(\frac{\kappa}{\sqrt{C_\mu}} \eta^2 = 1\) is located at \(\Gamma = 0.99\). The constant \(\alpha_1\) can be represented as a function of the wall coordinate \(d^+ = u_\tau d / \nu\) by substituting the log-law expression for the specific dissipation rate \(\omega = u_\tau / (\sqrt{\beta_\star \kappa d})\) into Eq. 2.71. At the balancing location \((\frac{\kappa}{\sqrt{C_\mu}} \eta^2 = 1)\), the above derivation results in \(\alpha_1^2 = d^+\) if the turbulence model constant \(\beta_\star\) is set to \(C_\mu\).

At this point, \(\alpha_1\) (or the \(d^+\) chosen) needs to be calibrated using a particular inflow boundary layer in order to enforce the transition of RANS to LES near the end of the log layer where the wake law starts to take over. The calibration procedure is as follows. First, a predicted equilibrium boundary layer should be specified (usually by running a 2D RANS) with given freestream properties and a specified wall condition. A value for the boundary layer thickness is also needed, and can be calculated from Coles’ law of the wall/wake with the van Driest transformation:

\[
\frac{u_{\nu d}}{u_\tau} = \frac{1}{\kappa} \ln(d_{w}^*) + C + 2 \kappa \sin^2 \left( \frac{\pi d^+ \delta}{2 \delta} \right) 
\]

\[
d_{w}^* = \frac{u_\tau d}{\nu_w} 
\]
\[
\begin{align*}
\nu_d &= \frac{u_\infty}{A} \left\{ \sin^{-1} \left[ \frac{2A^2u/u_\infty-B}{\sqrt{B^2+4A^2}} \right] + \sin^{-1} \left[ \frac{B}{\sqrt{B^2+4A^2}} \right] \right\} \\
A &= \sqrt{\frac{\gamma-1}{2}Pr_i M_\infty^2 T_\infty}{T_w} \\
B &= \left[ 1 + Pr_i^{1/2} \frac{\gamma-1}{2} M_\infty^2 \right] T_\infty}{T_w} - 1
\end{align*}
\]

An estimate for the outer extent of the log layer is defined by setting the log-law contribution to the velocity profile to 98% in Cole's law of wall/wake:

\[
\frac{(1/\kappa)\ln(d_w^+)}{\nu_d/u_\tau} + C = 0.98
\]

The corresponding value of \(d^+\) to this value of \(d_w^+\) can be found using Walz's [Wal69] formula for the static temperature distribution within the boundary layer:

\[
\frac{T}{T_\infty} = \frac{T_w}{T_\infty} + \left( \frac{T_{aw} - T_w}{T_\infty} \right) \frac{u}{u_\infty} - r \frac{\gamma-1}{2} M_\infty^2 \left( \frac{u}{u_\infty} \right)^2
\]

For high Mach number flows, the target value of \(d^+\) will differ significantly from \(d_w^+\) because the kinematic viscosity is a function of temperature and the temperature at the wall is different from inside the boundary layer.

In case of incompressible flow, this procedure can be greatly simplified because \(d_w^+ = d^+\) and \(u\) can be used in place of \(\nu_d\) in eq.2.75. As a result, Eq. 2.75 can be written as following for incompressible flow applications:

\[
\frac{(1/\kappa)\ln(d^+)}{u/u_\tau} + C = 0.98
\]

From the target boundary layer, if one can find a position \(y\) where \(u(y)\) and \(d^+(y)\) can satisfy Eq. 2.77, \(\alpha_1^2\) can then be set to \(d^+(y)\).

### 2.4.2 Gieseking's Model

The next generation of NCSU's hybrid RANS/LES model aimed at eliminating the procedure of pre-calibration in Choi's model. In Gieseking's ([Gie11a],[GE11]) model, the blending function \(\Gamma\) has an alternative form by introducing a ratio of estimated an outer-layer length scale over inner-scale length scale. The idea is that at higher Reynolds number, outer-layer statistics should be independent from molecular viscosity and Reynolds number, thus using Taylor microscale may not...
be appropriate. Instead, a local estimate of outer-layer turbulence length scale is needed. Equating the outer- and inner-scales can give a reasonable estimate of the outer part of the log-layer and a location where one may wish to switch to LES from RANS. The blending function is in the form

$$\Gamma = \frac{1}{2} \left( 1 + \tanh \left[ C_s \left( \frac{1}{\lambda_N^2} - 1 \right) \right] \right)$$

(2.78)

where \( C_s \) is a sharpening factor and is set to 15.0, and \( \lambda_N \) is the ratio between outer- and inner-scales:

$$\lambda_N \equiv \frac{l_{outer}}{l_{inner}}$$

(2.79)

$$l_{outer} = C_N \sqrt{\frac{10 \langle \omega \rangle + \langle k \rangle + \langle k_R \rangle}{C_{1/2} \langle \omega \rangle \omega}}$$

(2.80)

$$l_{inner} = \kappa d$$

(2.81)

where \( C_N \) is a model constant and is set to 1.5 by numerical experiments on supersonic flat plate cases [Gie12]. Here, \( \langle \omega \rangle \) is the ensemble-averaged turbulence frequency, \( \langle k \rangle \) and \( \langle k_R \rangle \) are ensemble-averaged modeled TKE and resolved TKE, respectively, and \( d \) is the distance to the nearest wall. The inclusion of resolved TKE is to make the outer-length scale less directly depend on the placement of the blending function in the boundary layer. The combination of instantaneous and averaged data allows the blending function to fluctuate near a mean value, which comes from the ensemble-averaged flow. The required ensemble-averaging is taken using the following exponentially time-weighted procedure:

$$\langle Q \rangle^n = (1 - A)\langle Q \rangle^{n-1} + AQ^n$$

(2.82)

where \( A = \Delta t / \tau \), and the time scale \( \tau \) is defined as

$$\tau = \begin{cases} 
\min(t, t_{res}), & t < 4t_{res} \\
\min(t - 3t_{res}, t_{res}), & t \geq 4t_{res}
\end{cases}$$

(2.83)

$$t_{res} = \frac{L}{U_\infty}$$

(2.84)

where \( t_{res} \) is the characteristic time (aka. residence time), \( L \) is a representative length scale, and \( U_\infty \) is the free-stream velocity.

In its basic form, Gieseking's model does not depend on the filter width (or mesh scale), unlike the DES-type methods. A later development by Salazar et al. ([SE13]) added a limiting factor based on the mesh scale in the case that the mesh is insufficient to resolve the outer-layer scale. The
limiting factor \( g(l_{outer}) \) is multiplied to the outer turbulence length scale.

\[
g(l_{outer}) = \min \left[ 10, \max \left( 1, \frac{1}{2} \frac{\Delta_{max}}{l_{outer}} \right) \right] \\
\Delta_{max} = \max(\Delta_x, \Delta_y, \Delta_z)
\]

2.4.3 The IDDES Model

The improved delayed detached eddy simulation (IDDES) [Shu08] is a combination of the DDES and a wall-modeled LES, with the switching controlled by a blending function. Though IDDES was originally developed with the S-A RANS model, it was adapted to the SST framework by Gritskevich [Gri12]. Compared to the original BSL/SST model, the TKE equation (Eq. 2.31) is modified as follows:

\[
\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) = P_k - D_k + \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_k \mu_t \right) \frac{\partial k}{\partial x_j} \right]
\]

(2.86)

The constants in Eq. 2.86 are calculated just as in BSL/SST model by combining the constants of the \( k-\epsilon \) and \( k-\omega \) model via a blending function \( F_1 \) defined in Eq. 2.37 and 2.38. \( P_k \) and \( D_k \) are the production and destruction terms, and read as follows:

\[
P_k = \min(\mu_t \Omega^2, 10\mu \rho k \omega)
\]

(2.87)

\[
D_k = \frac{\rho k^{3/2}}{l_{IDDES}}
\]

(2.88)

The IDDES length scale (\( l_{IDDES} \)) is defined as:

\[
l_{IDDES} = \tilde{f}_d (1 + f_d) l_{RANS} + (1 - \tilde{f}_d) l_{LES} \\
l_{LES} = C_{DES} \Delta \\
l_{RANS} = \frac{\sqrt{K}}{C_{\mu} \omega} \\
C_{DES} = C_{DESI} F_1 + C_{DES2} (1 - F_i)
\]

(2.89)

and \( \Delta \) here is the LES length-scale, defined as:

\[
\Delta = \min \{ C_w \max[d_w, h_{max}], h_{max} \}
\]

(2.90)

where \( h_{max} \) is the maximum edge length of the cell.
The function $f_d$ in Eq. 2.89 is an empirical function, computed using the following relations:

$$f_d = \max\{(1-f_{dt}), f_b\}$$

$$f_{dt} = 1 - \tanh\left([C_{dt1} r_{dt}]^{C_{dt2}}\right)$$

$$r_{dt} = \frac{\nu_t}{\kappa^2 d_w^2 \sqrt{0.5(S^2 + \Omega^2)}}$$

$$f_b = \min\{2\exp\left(-9\alpha^2\right), 1.0\}$$

$$\alpha = 0.25 - d_w/h_{max}$$  \hspace{1cm} (2.91)

And the elevating function $f_e$ in Eq. 2.89 reads as:

$$f_e = f_{e2} \max\{(f_{e1} - 1.0), 0.0\}$$

$$f_{e1} = \begin{cases} 2\exp(-11.09\alpha^2), & \alpha \geq 0 \\ 2\exp(-9.0\alpha^2), & \alpha < 0 \end{cases}$$

$$f_{e2} = 1.0 - \max(f_t, f_l)$$

$$f_t = \tanh\left([C^2 t r_{dt}]^{3}\right)$$

$$f_l = \tanh\left([C^2 l r_{dt}]^{10}\right)$$

$$r_{dt} = \frac{\nu}{\kappa^2 d_w^2 \sqrt{0.5(S^2 + \Omega^2)}}$$  \hspace{1cm} (2.92)

The model constants are listed in Table 2.2.

**Table 2.2 Model constants for the SST-IDDES model**

<table>
<thead>
<tr>
<th>Model Constant</th>
<th>Value</th>
<th>Model Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_\mu$</td>
<td>0.09</td>
<td>$\kappa$</td>
<td>0.41</td>
</tr>
<tr>
<td>$C_{DES1}$</td>
<td>0.78</td>
<td>$C_{DES2}$</td>
<td>0.61</td>
</tr>
<tr>
<td>$C_{dt1}$</td>
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<td>$C_{dt2}$</td>
<td>3</td>
</tr>
<tr>
<td>$C_w$</td>
<td>0.15</td>
<td>$C_l$</td>
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</tr>
<tr>
<td>$C_t$</td>
<td>1.87</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.4.4 New Eddy Viscosity Transport (EVT) Based Model

Different from Choi’s model and Gieseking’s model, the estimate of the local outer-layer length scale developed in the new model does not come from external input in the form of either a calibration or use of ensemble-averaged data. One may consider it possible to simply use the instantaneous data in the outer-scale expression, such that

\[ l_{outer} = \alpha \sqrt{\frac{10 \nu \omega + k}{C_{\mu}^{1/2} \omega^2}} \]  

but this method does not work in practice, at least not with the blending eddy viscosity framework. The hybrid eddy viscosity expression (Eq. 2.68) acts as a sink term for modeled turbulent kinetic energy because of the inclusion of the residual eddy viscosity, driving the outer length scale to small values and forcing the blending function to move towards the wall. The results will be very sensitive to the initial conditions, and there is no way to calibrate the model either. As an alternative, it is possible to rewrite Eq. 2.93 using an eddy viscosity:

\[ l_{outer} = \alpha \sqrt{\frac{10 \nu + \nu_{t,outer}}{C_{\mu}^{1/2} \omega}} \]  

This eddy viscosity (\( \nu_{t,outer} \)) should be used solely to define the outer-layer length scale and not directly coupled to the modeled turbulent variables (\( k \) and \( \omega \)). In order to provide this eddy viscosity, ‘eddy viscosity transport’ (EVT) equations are considered in this work since they are the simplest self-contained turbulence model and have achieved success in wide range of applications. The benefits from the additional degree of freedom and the ability to self-adjust to changing boundary layer states may justify the burden of solving an extra transport equation. Though there are many mature EVT models available ([SA94], [Men97], [Shu95]), the one chosen in this work is one of the simpler versions, developed by Edwards and Chandra ([EC96]):

\[ \frac{\partial \rho \tilde{v}}{\partial t} + \frac{\partial \rho \tilde{v} u_j}{\partial x_j} = \rho f_p \sqrt{\tilde{v} \tilde{v}_i S} + \sigma_R (\mu + \mu_t) \frac{\partial^2 \tilde{v}}{\partial x_j \partial x_j} - \sigma_\epsilon \frac{\mu_t \tilde{v}}{d^2} \]  

(2.95)
where

\[ \mu_t = \rho \tilde{\nu}_t = \rho C_\mu \tilde{\nu} f_\mu \]

\[ f_\mu = D_1 D_2 \]

\[ f_p = (D_1 D_2)^{1.5}(1 + \nu/\tilde{\nu}_t) \]

\[ D_1 = 1 - \exp(-T/A_1), \quad A_1 = 19 \]

\[ D_2 = 1 - \exp(-T/A_2), \quad A_2 = 10 \]

\[ T = (C_\mu \tilde{\nu}/\kappa + 0.005|\vec{V}|d)/\nu \]

One characteristic of this model is that its convection, production, diffusion, and destruction components are clearly separated, which is makes it relatively easy to integrate into existing CFD codes. This model works reasonably well for wall-bounded flows, and the transported quantity \( \tilde{\nu} \) varies linearly with wall distance in the logarithmic layer and below.

One potential problem of directly applying this model is that, like any other EVT or two-equation model, integration of this model using unsteady velocity field with a lot of fluctuations will result in a large amplification of eddy viscosity production due to the fluctuating strain rates. In the current context, this will cause the transition location of RANS to LES to move away from the wall, and eventually lead to suppression of turbulent fluctuations. To fix this problem, a modified form of Eq. 2.95 is used, which utilizes the von Kármán length scale to enhance the destruction term in regions of high turbulence activity:

\[
\frac{\partial \rho \tilde{\nu}}{\partial t} + \frac{\partial \rho \tilde{\nu} u_j}{\partial x_j} = \rho f_p \sqrt{\tilde{\nu} \mu} S + \sigma_R (\mu + \mu_t) \frac{\partial^2 \tilde{\nu}}{\partial x_j \partial x_j} - \sigma_\epsilon \mu_t \tilde{\nu}_{\text{max}} \left( \frac{1}{d^2}, \frac{C_{\nu K}}{L_{\nu K}^2} \right) 
\]

(2.96)

where

\[ \sigma_R = 1.64 \sigma_\epsilon \]

\[ \sigma_\epsilon = \frac{\sqrt{C_\mu}}{k^2} \]

\[ \frac{1}{L_{\nu K}^2} = \left( \frac{\partial S}{\partial x_j} \frac{\partial S}{\partial x_j} \right) S^2 \]

The values of \( 1/d \) and \( 1/L_{\nu K} \) are compared in Fig. 2.1 for two cases, with decoupled implementation of EVT above to an LES and a RANS simulation, both for the Elena-Lacharme flat plate boundary layer ([EL88]) case. Wall-resolved LES predictions for this case were reported by Edwards in [Edw14]. Inverse of the filter width for LES is also shown in the figure, suggesting that for resolved turbulence,
the von Kármán scale is approximately on the same scale as the mesh scale, in agreement with Davidson's observation ([Dav07]). From Fig. 2.1, it is obvious that the inverse wall distance $1/d$ is much smaller than the inverse of the von Kármán length scale, and including the von Kármán length scale in the destruction term will limit the growth of the EVT eddy viscosity.

![Figure 2.1 Inverse length scales within a turbulent boundary layer](image)

Some encouraging results using this approach with $C_{yK} = 0.1$ are shown in Fig. 2.2. In this case, an LES is used to drive the decoupled EVT equation (Eq. 2.96), and the vertical axis in the figure is the instantaneous normalized EVT eddy viscosity level in the boundary layer in a statistically-steady state. EVT eddy viscosity is amplified when unconstrained, compared to the RANS value, and the inclusion of the von Kármán length scale brings its level down to be similar to the RANS implementation. The purpose of the controlled transport-equation approach is to maintain the mean RANS length-scale behavior while allowing fluctuations about the mean since RANS models are adequate in predicting turbulence length scales for zero pressure-gradient flat-plate boundary layers.

With the application of the EVT equation approach, the outer-layer length scale in the new
model is defined as:

\[ l_{outer} = \alpha \sqrt{\frac{10 \nu + \nu_t \text{outer}}{C_{\mu}^{1/2} \omega}} = \alpha \sqrt{\frac{10 \nu + \nu_t \text{outer}}{C_{\mu}^{1/2} \omega}} \] (2.97)

where the model constant \( \alpha \) started to be 1.5, the same as in Gieseking’s model, but later re-calibrated to be set to 1.75 for some of the cases. Details will be discussed in Section 4.3 and 5.2.3. Another way to modify the blending function other than changing the model constant \( \alpha \) is to use the Von Kármán length scale to modulate the outer length scale, providing an intermittency effect to the LES/RANS eddy viscosity. Two formulations are tested in the moderate Reynolds number FPBL case. They can be written as the follows.

\[ \tilde{T}_\text{outer}^2 = l_{outer}^2 \left( 1 + l_{outer}^2 \max \left[ \frac{1}{d^2}, C_{int} \frac{1}{L_{VK}^2} \right] \right) \] (2.98)

and

\[ \tilde{\tilde{T}}_\text{outer}^2 = \bar{T}_\text{outer}^2 \left( 1 + \tilde{T}_\text{outer}^2 \max \left[ 0, C_{int} \frac{1}{L_{VK}^2} - \frac{1}{d^2} \right] \right) \] (2.99)

where \( d \) is wall distance, and \( C_{int} \) is a model constant regulating the intermittency effect, set to the value of \( C_{VK} = 0.1 \) when tested. \( \tilde{T}_\text{outer} \) is used in Eq. 2.79 in place of \( l_{outer} \). The first formulation (Eq. 2.98) is more far-reaching and will shift as well as broaden the blending function \( \Gamma \), while the second formulation (Eq. 2.99) primarily broadens the distribution.

In summary, the new model solves the Menter BSL/SST turbulence transport equations, with the eddy viscosity closed by Eq. 2.68. The blending function is determined by 2.78 and 2.79, using
specific dissipation rate \((\omega)\), the decoupled transported quantity \((\tilde{\gamma})\) in EVT model (Eq. 2.96), the wall distance \((d)\), and the kinematic viscosity \((\nu)\).
3.1 Finite Volume Discretization

A finite volume method is utilized for the solver involved in this work. Computational domains are divided into discrete volumes (cells), and averaged variables of each cell are stored in the cell center. Each cell is considered a control volume, and the volume integral of the conservative vector form of the Navier-Stokes equations (Eq. 2.9) can be written as

$$\int_{CV} \left( \frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{F}}{\partial x} + \frac{\partial \vec{G}}{\partial y} + \frac{\partial \vec{H}}{\partial z} \right) dV = \vec{J} \quad (3.1)$$

Part of this volume integral can be rewritten in the form of a surface integral over cell boundary by applying Green-Gauss Theorem:

$$\int_{CV} \left( \frac{\partial \vec{U}}{\partial t} \right) dV + \int \left( \vec{F}_{inv,i} n_i \right) dA_{CV} + \int \left( \vec{F}_{visc,i} n_i \right) dA_{CV} = \int_{CV} \vec{S} dV \quad (3.2)$$

where $\vec{F}_{inv}$ is the inviscid flux vector, and $\vec{F}_{visc}$ is the viscous flux vector, and $i = 1, 2, 3$ in the subscript denotes the three coordinate directions. The surface integral can be treated as a summation of integrals on each cell face, 6 in total for structural meshes, and the integral on each face represents the fluxes of the conservative variables through that face. The fluxes then are function of the cell-centered values of conservative variables on each side of each face. As a result, given that the volume
of the cell does not change, the change of cell-averaged conservative variable in a cell is an effect of combined fluxes and a source term \( \vec{S} \):

\[
V_C \frac{\Delta \vec{U}}{\Delta t} + \sum_{k=\text{faces}} \left( \vec{F}_{\text{inv},i} n_i \right)_k A_{CV,k} + \sum_{k=\text{faces}} \left( \vec{F}_{\text{visc},i} n_i \right)_k A_{CV,k} = \vec{S} V_C
\]  

(3.3)

The source terms \( \vec{S} \) for the Navier-Stokes equations are zero, but the Menter BSL/SST and the EVT models include non-zero source terms. For an incompressible flow with Menter’s BSL/SST and the EVT model, below is a summary of the vectors in Eq. 3.3:

\[
\vec{U} = \begin{bmatrix}
\rho \\
\rho u_1 \\
\rho u_2 \\
\rho u_3 \\
\rho k \\
\rho \omega \\
\rho \tilde{\gamma}
\end{bmatrix}, \quad \vec{F}_{\text{inv},i} = \begin{bmatrix}
\rho u_i \\
\rho u_1 u_i + \delta_{1i} P \\
\rho u_2 u_i + \delta_{2i} P \\
\rho u_3 u_i + \delta_{3i} P \\
\rho u_i k \\
\rho u_i \omega \\
\rho u_i \tilde{\gamma}
\end{bmatrix}, \quad \vec{F}_{\text{visc},i} = \begin{bmatrix}
0 \\
-\tau_{i1} \\
-\tau_{i2} \\
-\tau_{i3} \\
-(\mu + \sigma k_1 \mu_t) \frac{\partial \rho k}{\partial x_i} \\
-(\mu + \sigma \omega_2 \mu_t) \frac{\partial \omega}{\partial x_i} \\
-\sigma_R(\mu + \mu_t) \frac{\partial \tilde{\gamma}}{\partial x_i}
\end{bmatrix}
\]

\[
\vec{S} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
P_k - \beta^* \rho \omega k \\
P_\omega - \beta \rho \omega^2 + 2(1 - F_1) \rho \sigma \omega_2 \frac{1}{\rho \omega} \frac{\partial \omega}{\partial x_i} \frac{\partial \omega}{\partial x_i} \\
\rho f_p \sqrt{\gamma \rho T} S - \sigma_e \mu_i \tilde{\gamma} \max \left( \frac{1}{\rho \omega}, C_{\text{VK}} \frac{1}{\sqrt{\gamma \rho}} \right)
\end{bmatrix}
\]

### 3.2 Flux Construction

The flux terms, in Eq. 3.3, at each faces, are needed to solve the system. Flux splitting approach is used to evaluate interface fluxes, taking into account of the direction of information propagation. The viscous fluxes (\( \vec{F}_{\text{visc},i} \)) are computed using a second-order central difference, while the inviscid fluxes (\( \vec{F}_{\text{inv},i} \)) are splitted using an incompressible version ([Cas09]) of the ‘Low diffusion flux splitting scheme’ ([LDFSS-2001], [Edw01]). LDFSS is a member of the ‘AUSM-family’ [Lio01] given that it is based on the Van Leer/Liou ([Lio95], [VL82])Mach number and pressure-splitting polynomials, and a common interface sound speed is used to define Mach numbers to the left and right of the face. Different from other AUSM-family schemes, it does not use advective upwinding directly to remove
excess numerical diffusion at a stationary contact discontinuity, and additional 'pressure-diffusion' mechanisms are utilized to improve the scheme's response. As a result, this method is suitable for capturing strong shocks, while maintaining accuracy of flux-difference type methods when dealing with shear layers.

The inviscid flux vector in Eq. 3.3 can be split as:

\[ \vec{F}_{inv,i} = \vec{F}_{inv,i}^c + \vec{F}_{inv,i}^p = \rho u_i \vec{E}^c + p \vec{E}^p \] (3.4)

\[ \vec{E}^c = \begin{bmatrix} 1 \\ u_1 \\ u_2 \\ u_3 \\ k \\ \omega \\ \gamma \end{bmatrix}, \quad \vec{E}^p = \begin{bmatrix} 0 \\ n_x \\ n_y \\ n_z \end{bmatrix} \]

where \( \vec{F}_{inv,i}^c \) is contribution from convective components and \( \vec{F}_{inv,i}^p \) is the pressure contribution. The interface value \( \vec{E}_{1/2}^c \) then can be determined by combining the left (subscript L) and right (subscript R) states in a way such that

\[ \vec{E}_{1/2}^c = a_{1/2}[\rho_L C^+ \vec{E}_L^c + \rho_R C^- \vec{E}_R^c] \] (3.5)

In the case of incompressible flow, \( \rho_L = \rho_R = \rho \). \( \tilde{a}_{1/2} \) is the 'numerical sound speed,' defined as:

\[ \tilde{a}_{1/2} = \sqrt{\hat{\mathbf{u}}_{1/2}^2 + 4 V_{ref,1/2}^2} \] (3.6)

where

\[ \hat{\mathbf{u}}_{1/2} = \frac{1}{2} (\hat{\mathbf{u}}_L + \hat{\mathbf{u}}_R) \] (3.7)

\[ \hat{\mathbf{u}}_{L/R} = n_{x,i+1/2} u_{1,L/R} + n_{y,i+1/2} u_{2,L/R} + n_{z,i+1/2} u_{3,L/R} \] (3.8)

\[ V_{ref,1/2} = \frac{1}{2} (V_{ref,L} + V_{ref,R}) \] (3.9)

\[ V_{ref,L/R} = \sqrt{\max\{|\hat{\mathbf{u}}_{L/R}|^2, u_{1,ref}^2, \max(u_{1}^2, u_{2}^2, u_{3}^2, u_{ref}^2)|G|, u_{ref}^2\}} \] (3.10)
and

\[ C^+ = M_L^+ - M_{1/2} \max \left( 0.0, 1.0 - \frac{p_L - p_R + |p_L - p_R|}{2 \rho V_{ref,1/2}} \right) \]  
\[ C^- = M_R^- - M_{1/2} \max \left( 0.0, 1.0 + \frac{p_L - p_R - |p_L - p_R|}{2 \rho V_{ref,1/2}} \right) \] (3.11)

\[ C^- = M_R^- - M_{1/2} \max \left( 0.0, 1.0 + \frac{p_L - p_R - |p_L - p_R|}{2 \rho V_{ref,1/2}} \right) \] (3.12)

where

\[ M_{L/R}^\pm = \pm \frac{1}{4} (M_{L/R} \pm 1)^2, \quad M_{L/R} = \frac{\hat{u}_{L/R}}{\hat{a}_{1/2}} \] (3.13)

\[ M_{1/2} = \frac{1}{2} \left( M_L^+ - \alpha_L^+ M_L - M_R^- + \alpha_R^- M_R \right) \] (3.14)

\[ \alpha_{L/R}^\pm = \frac{1}{2} \left( 1.0 \pm \text{sign}(M_{L/R}) \right) \] (3.15)

The pressure at the interface is split as:

\[ p_{1/2} = \frac{1}{2} \left( p_L + p_R + (p_L^+ - p_R^-)(p_L - p_R) \right) + \rho V_{ref,1/2}^2 \left( p_L^+ + p_R^- - 1.0 \right) \] (3.16)

\[ p_{L/R}^\pm = \frac{1}{2} (1 \pm M_{p,L/R}) \] (3.17)

\[ M_{p,L/R} = \frac{\hat{u}_{L/R}}{\hat{a}_{1/2}} \] (3.18)

\[ (3.19) \]

### 3.3 Higher Order Extension

The basic first-order flux-splitting scheme utilizes the data at the cell-center directly to calculate the flux at the cell interface, with the assumption that the value of variables does not change over the cell. For the system to achieve a higher order, this assumption can be replaced by piecewise parabolic method (PPM) or fourth-order central difference reconstruction. PPM reduces to fourth-order central differencing scheme in a smooth enough region on uniform meshes, and is at least 2nd order accurate. The fourth-order central differencing scheme reads as:

\[ \vec{V}_{L,i+1/2}^C = \vec{V}_{R,i+1/2}^C = \frac{7}{12} (\vec{V}_{i} + \vec{V}_{i+1}) - \frac{1}{12} (\vec{V}_{i+2} + \vec{V}_{i-1}) \] (3.20)
where \( \vec{V} \) is the transported variables

\[
\vec{V} = [p, u_1, u_2, u_3, k, \omega, \nu]^T
\]  

(3.21)

Another monotone scheme is also used in some cases (Ahmed body) in this study. This reconstruction scheme reads as:

\[
\vec{V}^C_{L,i+1/2} = \vec{V}^C_{R,i+1/2} = \frac{1}{2} (\vec{V}_i + \vec{V}_{i+1}) + \frac{1}{6} [DMD(\Delta \vec{V}_i, \Delta \vec{V}_i) - DMD(\Delta \vec{V}_i, \Delta \vec{V}_{i+1})]
\]  

(3.22)

where

\[
DMD(a, b) = \begin{cases} 
2a, & \text{if } |a| < |b| \quad \text{and } 2|a| < \frac{a+b}{2} \quad \text{and } ab > 0 \\
2b, & \text{if } |b| < |a| \quad \text{and } 2|b| < \frac{a+b}{2} \quad \text{and } ab > 0 \\
\frac{a+b}{2}, & \text{if } \frac{|a+b|}{2} < 2|a| \quad \text{and } \left| \frac{a+b}{2} \right| < 2|b| \quad \text{and } ab > 0 \\
0, & \text{if } ab \leq 0
\end{cases}
\]

Sometimes the value at the interface interpolated by using Eq. 3.20 or Eq. 3.22 will be outside the bounds of the two adjacent cells. A second step is used for PPM to preserve the monotonicity of the function, presented below.

\[
\vec{V}^M_{L,i+1/2} = \begin{cases} 
\vec{V}_i, & \text{if } (\vec{V}^C_{R,i-1/2} - \vec{V}_i)(\vec{V}_i - \vec{V}^C_{i+1/2}) \leq 0 \\
3\vec{V}_i - 2\vec{V}^C_{R,i-1/2}, & \text{if } -CC > DC \\
\vec{V}^C_{L,i+1/2}, & \text{else}
\end{cases}
\]  

(3.23)

\[
\vec{V}^M_{R,i-1/2} = \begin{cases} 
\vec{V}_i, & \text{if } (\vec{V}^C_{R,i-1/2} - \vec{V}_i)(\vec{V}_i - \vec{V}^C_{i+1/2}) \leq 0 \\
3\vec{V}_i - 2\vec{V}^C_{L,i+1/2}, & \text{if } DC > CC \\
\vec{V}^C_{R,i-1/2}, & \text{else}
\end{cases}
\]  

(3.24)

where

\[
C = \left( \vec{V}^C_{L,i+1/2} - \vec{V}^C_{R,i-1/2} \right)
\]  

(3.25)

\[
D = 6 \left[ \vec{V}_i - \frac{1}{2} \left( \vec{V}^C_{L,i+1/2} + \vec{V}^C_{R,i-1/2} \right) \right]
\]  

(3.26)

The extra numerical dissipation introduced by Eq. 3.23 and 3.24 may help control undesired numerical oscillations, but in LES of highly turbulent flows, some of the oscillatory behaviors may be physical and desirable. To improve accuracy at local extrema, two modified versions of PPM
are used, naming low-diffusion PPM (LD-PPM) and essentially non-oscillatory PPM (ENO-PPM) [Nor09]. The following two subsections describe these two reconstruction methods.

### 3.3.1 LD-PPM

A blending function, based on pressure curvature, that switches between the low-diffusion scheme (Eq. 3.20 or 3.22) and PPM is used in this incompressible version of REACTMB. The blending function switches to the more dissipative PPM scheme where more pressure fluctuation occurs and uses the 4th-order central difference scheme elsewhere. The implementation of this switch is described as follows:

\[
\hat{V}_{L,i+1/2} = \hat{V}_{L,i+1/2}^C + s \left( \hat{V}_{L,i+1/2}^M - \hat{V}_{L,i+1/2}^C \right) \tag{3.27}
\]

\[
\hat{V}_{R,i+1/2} = \hat{V}_{R,i+1/2}^C + s \left( \hat{V}_{R,i+1/2}^M - \hat{V}_{R,i+1/2}^C \right) \tag{3.28}
\]

where the switch \(s\) is defined as:

\[
s = \max(p_{dif}, 0.0) \tag{3.29}
\]

\[
p_{dif} = 1.25 \times \max \left( \frac{p_d}{p_d + p_{avg}}, 0.2 \right) - 0.2 \tag{3.30}
\]

\[
p_d = \left| p_{i-1} - 2p_i + p_{i+1} \right| \tag{3.31}
\]

\[
p_{avg} = \frac{1}{4} \left| p_{i-1} + 2p_i + p_{i+1} \right| \tag{3.32}
\]

This modified reconstruction scheme is called 'Low-Diffusion Piecewise Parabolic Method' (LD-PPM).

### 3.3.2 ENO-PPM

Originally called "modified piecewise parabolic method (M-PPM)" [Nor09], this essentially non-oscillatory PPM utilizes a different blending function that is a function of "jump severity indicator", \(S\), which is defined as:

\[
S \equiv \left| \frac{\Delta \hat{V}_{i} - |\Delta \hat{V}_{i+1}|}{|\Delta \hat{V}_{i}| + |\Delta \hat{V}_{i+1}| + \epsilon} \right| \tag{3.33}
\]

where

\[
\Delta \hat{V}_{i} = \hat{V}_{i} - \hat{V}_{i-1} \tag{3.34}
\]
is an approximation of the derivative of the variable of interest. This indicator $S$ is supposed to give an estimate of the second derivative, showing how abruptly the first derivative changes. Then the switch function, modified from the original proposed formulation, which is discontinuous, reads as follows in this work:

$$s(S) = S^{6-3S}$$  \hspace{1cm} \text{(3.35)}

The reconstruction is the same as shown by Eq. 3.27 and 3.28.

### 3.4 Time Integration and Solution Update

Time-accurate simulations are realized by performing a temporal integration and updating the the domain after the fluxes are calculated. Explicit methods are simpler to implement, but the CFL number would be limited to a very low value for stability. It would take an impractically large number of iterations to solve a problem with an explicit method. On the other hand, implicit methods are more stable and can take a much larger CFL number, resulting a faster solution evolution, though efforts need to be put into solving large matrix systems and storage requirements are higher. A second-order three-point backward-differencing method, along with a pre-conditioned subiteration procedure is used for incompressible flows in this work. The discrete governing equations (Eq. 3.3) can be reformulated as:

$$V_C \frac{\Delta \hat{\theta}}{\Delta t} + \sum_{k=i-face} (\hat{E}_i n_i)_k A_k + \sum_{k=j-face} (\hat{E}_i n_i)_k A_k + \sum_{k=k-face} (\hat{E}_i n_i)_k A_k = \hat{S} V_C$$  \hspace{1cm} \text{(3.36)}

where $\hat{E}$ is the combined flux

$$\hat{E}_i = \hat{F}_{inv,i} + \hat{F}_{visc,i}$$  \hspace{1cm} \text{(3.37)}

Then the residual vector $\hat{R}$ can be defined as:

$$\hat{R} = \sum_{k=i-face} (\hat{E}_i n_i)_k A_{CV,k} + \sum_{k=j-face} (\hat{E}_i n_i)_k A_{CV,k} + \sum_{k=k-face} (\hat{E}_i n_i)_k A_{CV,k} - \hat{S} V_C$$  \hspace{1cm} \text{(3.38)}
This expression can be simplified by using fluxes at the interfaces of cells, defined as:

\[
\begin{align*}
\vec{E}_{i\pm\frac{1}{2}} & \equiv (\vec{E}_{i+1/2})_{i\pm\frac{1}{2},j,k} A_{i\pm\frac{1}{2},j,k} \\
\vec{E}_{j\pm\frac{1}{2}} & \equiv (\vec{E}_{j+1/2})_{i,j\pm\frac{1}{2},k} A_{i,j\pm\frac{1}{2},k} \\
\vec{E}_{k\pm\frac{1}{2}} & \equiv (\vec{E}_{k+1/2})_{i,j,k\pm\frac{1}{2}} A_{i,j,k\pm\frac{1}{2}}
\end{align*}
\]

As a result, the steady residual \( \vec{R} \) can be written as:

\[
\vec{R} = (\vec{E}_{i+1/2} - \vec{E}_{i-1/2}) + (\vec{E}_{j+1/2} - \vec{E}_{j-1/2}) + (\vec{E}_{k+1/2} - \vec{E}_{k-1/2}) - \vec{S}_{CV}
\]

(3.39)

The time iteration will be denoted by a superscript \( n \). At the current time step, with the field of primitive variables \( \vec{V}^n \) known, the steady residual can be calculated. The unsteady residual \( \mathfrak{R} \), which is the difference between the current and the next time step, is defined as follows.

\[
\mathfrak{R}^{n+1} = -V_{CV} \frac{3\hat{U}^{n+1} - 4\hat{U}^n + \hat{U}^{n-1}}{2\Delta t} + \vec{R}(\vec{V}^{n+1})
\]

(3.40)

Since the field of primitive variables at the next time step \( \vec{V}^{n+1} \) is not known, it is necessary to conduct a sub-iterative procedure which would lead to the next time level. The sub-iterative steps is denoted by \( k \). By substituting Eq. 3.39 into Eq. 3.40, we can get

\[
\mathfrak{R}^{n+1,k+1} = -V_{CV} \frac{3\hat{U}^{n+1,k+1} - 4\hat{U}^n + \hat{U}^{n-1}}{2\Delta t} + (\vec{E}_{i+1/2} - \vec{E}_{i-1/2})^{n+1,k+1} + (\vec{E}_{j+1/2} - \vec{E}_{j-1/2})^{n+1,k+1} + (\vec{E}_{k+1/2} - \vec{E}_{k-1/2})^{n+1,k+1} - \vec{S}^{n+1,k+1} V_{CV}
\]

(3.41)

To solve the equation \( \mathfrak{R}^{n+1,k+1} = 0 \), the data at level \( n+1, k+1 \) need to be obtained using current physical time level \( n \) and current pseudo-time level \( k \). A first-order Taylor expansion with respect to the pseudo-time is used to linearize these terms as follows.

\[
\vec{E}^{n+1,k+1} = \vec{E}^{n+1,k} + \left( \frac{\partial \vec{E}}{\partial \vec{V}} \right)^n_{(i,j,k)\pm1/2} \Delta \vec{V}^{n+1,k+1}_{(i,j,k)\pm1/2}
\]

(3.42)

where

\[
\Delta \vec{V}^{n+1,k+1}_{(i,j,k)\pm1/2} = (\vec{V}^{n+1,k+1} - \vec{V}^{n+1,k})_{(i,j,k)\pm1/2}
\]

(3.43)
Because the operation of the above equation is executed on each of the six cell faces, the flux Jacobian \( \left( \frac{\partial \vec{E}}{\partial \vec{V}} \right)_{i,j,k+1/2} \) needs to be expressed using cell-centered data. As a result, the flux Jacobian is approximated using split flux Jacobians:

\[
\begin{align*}
\left( \frac{\partial \vec{E}}{\partial \vec{V}} \right)_{i+1/2}^n \Delta \vec{V}_{i+1/2}^{n+1,k+1} &= (A^+) \Delta \vec{V}_{i,j,k}^{n+1,k+1} + (A^-) \Delta \vec{V}_{i+1,j,k}^{n+1,k+1} \\
\left( \frac{\partial \vec{E}}{\partial \vec{V}} \right)_{i-1/2}^n \Delta \vec{V}_{i-1/2}^{n+1,k+1} &= (B^+) \Delta \vec{V}_{i-1,j,k}^{n+1,k+1} + (B^-) \Delta \vec{V}_{i,j,k}^{n+1,k+1} \\
\left( \frac{\partial \vec{E}}{\partial \vec{V}} \right)_{j+1/2}^n \Delta \vec{V}_{j+1/2}^{n+1,k+1} &= (C^+) \Delta \vec{V}_{i,j,k}^{n+1,k+1} + (C^-) \Delta \vec{V}_{i,j+1,k}^{n+1,k+1} \\
\left( \frac{\partial \vec{E}}{\partial \vec{V}} \right)_{j-1/2}^n \Delta \vec{V}_{j-1/2}^{n+1,k+1} &= (D^+) \Delta \vec{V}_{i,j-1,k}^{n+1,k+1} + (D^-) \Delta \vec{V}_{i,j,k}^{n+1,k+1} \\
\left( \frac{\partial \vec{E}}{\partial \vec{V}} \right)_{k+1/2}^n \Delta \vec{V}_{k+1/2}^{n+1,k+1} &= (E^+) \Delta \vec{V}_{i,j,k}^{n+1,k+1} + (E^-) \Delta \vec{V}_{i,j,k-1}^{n+1,k+1} \\
\left( \frac{\partial \vec{E}}{\partial \vec{V}} \right)_{k-1/2}^n \Delta \vec{V}_{k-1/2}^{n+1,k+1} &= (F^+) \Delta \vec{V}_{i,j,k}^{n+1,k+1} + (F^-) \Delta \vec{V}_{i,j,k}^{n+1,k+1}
\end{align*}
\]

The split flux Jacobians are evaluated once at the start of each physical time step and kept through out the sub-iteration procedures. They are defined as:

\[
\begin{align*}
A^+ &= \frac{\partial \vec{E}_{i+1/2}}{\partial \vec{V}_{i,j,k}} & A^- &= \frac{\partial \vec{E}_{i+1/2}}{\partial \vec{V}_{i+1,j,k}} \\
B^+ &= \frac{\partial \vec{E}_{i-1/2}}{\partial \vec{V}_{i-1,j,k}} & B^- &= \frac{\partial \vec{E}_{i-1/2}}{\partial \vec{V}_{i,j,k}} \\
C^+ &= \frac{\partial \vec{E}_{j+1/2}}{\partial \vec{V}_{i,j,k}} & C^- &= \frac{\partial \vec{E}_{j+1/2}}{\partial \vec{V}_{i,j+1,k}} \\
D^+ &= \frac{\partial \vec{E}_{j-1/2}}{\partial \vec{V}_{i,j-1,k}} & D^- &= \frac{\partial \vec{E}_{j-1/2}}{\partial \vec{V}_{i,j,k}} \\
E^+ &= \frac{\partial \vec{E}_{k+1/2}}{\partial \vec{V}_{i,j,k}} & E^- &= \frac{\partial \vec{E}_{k+1/2}}{\partial \vec{V}_{i,j,k-1}} \\
F^+ &= \frac{\partial \vec{E}_{k-1/2}}{\partial \vec{V}_{i,j,k}} & F^- &= \frac{\partial \vec{E}_{k-1/2}}{\partial \vec{V}_{i,j,k}}
\end{align*}
\]

Similarly, the vector of conservative variables and the vector of source terms are linearized as well,
written as follows.

\[
\vec{U}^{n+1,k+1} = \vec{U}^{n+1,k} + \left( \frac{\partial \vec{U}}{\partial \vec{V}} \right)^n \Delta \vec{V}^{n+1,k+1}
\]

(3.44)

\[
\vec{S}^{n+1,k+1} = \vec{S}^{n+1,k} + \left( \frac{\partial \vec{S}}{\partial \vec{V}} \right)^n \Delta \vec{V}^{n+1,k+1}
\]

(3.45)

Substituting the linearized vectors into Eq. 3.41 will result a linear system:

\[
\begin{bmatrix}
V_{CV} P - 3V_{CV} \frac{\partial \vec{U}}{\partial \vec{V}} - V_{CV} \frac{\partial \vec{S}}{\partial \vec{V}} + (A^+ + C^+ + E^+ - B^- - D^- - F^-) \Delta \vec{V}^{n+1,k+1} + \\
A^- \Delta \vec{V}_{i+1,j,k}^{n+1,k+1} + B^+ \Delta \vec{V}_{i,j,k}^{n+1,k+1} + \\
C^- \Delta \vec{V}_{i,j+1,k}^{n+1,k+1} + D^+ \Delta \vec{V}_{i,j,k}^{n+1,k+1} + \\
E^- \Delta \vec{V}_{i,j,k+1}^{n+1,k+1} + F^+ \Delta \vec{V}_{i,j,k}^{n+1,k+1} = -\Theta^{n+1,k}
\end{bmatrix}
\]

(3.46)

where \(\frac{V_{CV} P}{\Delta \tau}\) is an auxiliary term from the artificial compressibility method, and the preconditioning matrix \(P\) is

\[
P = \frac{\partial \vec{U}}{\partial \vec{V}} + \frac{1}{V_{ref}^2} \begin{bmatrix}
1 \\
u_1 \\
u_2 \\
k \\
\omega \\
\nu
\end{bmatrix}
\begin{bmatrix}
1, 0, 0, 0, 0, 0
\end{bmatrix}
\]

(3.47)

while \(\Delta \tau\) is the sub-iteration time step, determined by the acoustic eigenvalues of \(P^{-1} \frac{\partial \vec{E}}{\partial \vec{V}}\) and a prescribed CFL number. \(V_{ref}\) is defined by Eq. 3.10 for the current cell. Eq. 3.46 is a linear system in form of \(Ax = b\) that can be solved for the correction vector, \(\Delta \vec{V}^{n+1,k+1}\). The correction vector contains the information to propagate the primitive variables towards the next step of the sub-iteration. Different methods can be used to solve this linear system, among which the incomplete lower-upper (ILU) factorization method is used in this work. The primitive variables are updated as

\[
\vec{V}^{n+1,k+1} = \vec{V}^{n+1,k} + \Delta \vec{V}^{n+1,k+1}
\]

(3.48)

and the step towards the next physical step is taken at the end of the last sub-iteration:

\[
\vec{V}^{n+1} = \vec{V}^{n+1,k_{max}}
\]

(3.49)
where \( kmx \) is the preset maximum number of sub-iterations, usually set to 8. After the primitive variables are updated to the new physical time step, they are used to calculate fluxes, conserved variables, source terms, and Jacobians for the new current step and the cycle continues until the desired number of total iterations is reached.

### 3.5 Initialization and Inlet Turbulence Generation

In this study, LES and hybrid RANS/LES simulations are started by firstly carrying out a RANS simulation on the same grid. LES and RANS/LES simulations are initialized from the RANS solution, superposed with fluctuations extracted and rescaled from another set of flat plate LES simulation data to start the unsteady structures. A recycling/rescaling method is used when large-scale turbulent structures are needed from the inflow. The recycling plane is a crossflow plane located several boundary layer thicknesses (generally \( \geq 8\delta \)) downstream from the inlet. Velocity fluctuations at the recycling plane is separated from the time-averaged data as

\[
\bar{u}' = \bar{u} - \bar{u}
\]

(3.50)

where \( \bar{u} \) is the mean velocity at the recycling plane

\[
\bar{u} = \frac{1}{(t_f - t_i)} \int_{t_i}^{t_f} \bar{u} \, dt
\]

(3.51)

Because of the development of the boundary layer, the recycled properties and fluctuations need to be rescaled down following the similarity laws of the boundary layer [Xia03]. Before the rescaled fluctuations \( f = \bar{u}_{resc} \) is added to the inlet profile, it is further modified by a function based on \( y/\delta \). This location-dependent modification reduces the effect of the recycled fluctuations on the inlet profile away from the boundary. As a result, the inflow velocity can be noted as

\[
\bar{u}_{inl} = \bar{u}_{RANS} + f \bar{u}_{resc}
\]

(3.52)

For other recycled quantities \( (k, \omega, \mu_t, \Gamma \text{ etc.}) \), a similar rescaling procedure is taken

\[
q_{inl} = (1 - f)q_{RANS} + f q_{resc}
\]

(3.53)
where \( q \) is the property recycled and rescaled. Eq. 3.53 also applies to the mean velocity field when recycling the mean velocity is desired. The blending function \( f \) is defined as

\[
f = \left[ 1 + \left( \frac{y_{in}}{1.6\delta_{in}} \right)^6 \right]^{-1}
\]

(3.54)

The rescaled velocity fluctuations \( \vec{u}'_{\text{resc}} \) and rescaled properties \( q_{\text{resc}} \) are calculated differently for outer and inner parts of the boundary layer, with another blending function, \( W \), using the raw rescaled quantities \( q_{\text{resc}0} \) and \( \vec{u}'_{\text{resc}0} \):

\[
q_{\text{resc}} = q_{\text{resc}0} \left( \frac{y_{in}}{R_\beta} \right)(1 - W) + q_{\text{resc}0}(y_{in}R_\delta)W
\]

(3.55)

\[
\vec{u}'_{\text{resc}} = \vec{u}'_{\text{inn}}(1 - W) + \vec{u}'_{\text{out}} W
\]

(3.56)

where

\[
W = \min \left( \frac{1}{2} \left( 1 + \frac{1}{\tanh(4)} \tanh \left[ \frac{4(\eta - B)}{(1 - 2B)\eta + B} \right] \right), 1.0 \right)
\]

\[
\vec{u}'_{\text{inn}} = \vec{u}'_{\text{resc}0}(u_z)_{\text{inn}}
\]

\[
\vec{u}'_{\text{out}} = \vec{u}'_{\text{resc}0}\eta
\]

\[
\eta = \left( \frac{y}{\delta} \right)_{\text{in}}
\]

Here the constants \( R_\beta \) and \( R_\delta \) are the ratios of friction velocity and boundary layer thickness, respectively, at the inlet and recycling plane. These constants are calculated before the simulation using RANS data. Constant \( B \) is the location of the transition point from the inner to outer layer, and is set to 0.2. The recycle plane is usually chosen to be located at around \( 8\delta \) from the inlet plane.
In this chapter, the simulation results on two subsonic flat-plate cases (in accordance to DeGraaff and Eaton’s experiment [DE00]) and a supersonic flat-plate case (in accordance to Elena and Lacharme’s experiment [EL88]) are presented. These case studies serve purposes of validating the flow solver in a large-eddy simulation and determining the new model’s capability of capturing the composite structure of a turbulence boundary layer.

4.1 Simulation Setup: DeGraaff-Eaton FPBL

The computational domain for the flat plate case is represented by a structured hexahedron mesh, shown in Fig. 4.1. The domain has a length(X) of 0.456 m, height(Y) of 0.2 m, and span/width(Z) of 0.2 m. It has $240 \times 200 \times 320$ ($x \times y \times z$) cells, or 15.36 million cells in total. The entire domain is split into 320 blocks, with each mapped to a processor for calculation. Spacing in x- and z-directions is uniform, with $\Delta x = 1.9 \times 10^{-3}$ m and $\Delta z = 6.25 \times 10^{-4}$ m. Spacing in wall-normal direction (y-direction) has a hyperbolic-tangent stretching with $\Delta y_{\text{wall}} = 5.0 \times 10^{-6}$ m. When translated to inner scale coordinates, this set of spacing turns out to be $\Delta x^+ = 49.7$, $\Delta y_{\text{wall}}^+ = 0.131$, and $\Delta z^+ = 16.4$ for the $Re_\theta = 2900$ case, and $\Delta x^+ = 238.4$, $\Delta y_{\text{wall}}^+ = 0.627$, and $\Delta z^+ = 78.4$ for the $Re_\theta = 13000$ case. Some reference values and conditions of the two selected cases from the experiments are listed in Table 4.1.

In order to locate the plane-of-interest where data will be compared to the experiments and to
Table 4.1 Profiles for DeGraaff-Eaton FPBL cases.

<table>
<thead>
<tr>
<th></th>
<th>(Re_\theta)</th>
<th>(U_\infty) (m/s)</th>
<th>(\delta) (mm)</th>
<th>(\theta) (mm)</th>
<th>(u_\tau) (m/s)</th>
<th>(\nu/u_\tau) ((\mu m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2900</td>
<td>9.83</td>
<td>37.93</td>
<td>4.54</td>
<td>0.403</td>
<td>38.2</td>
<td></td>
</tr>
<tr>
<td>13000</td>
<td>14.36</td>
<td>34.56</td>
<td>3.78</td>
<td>0.512</td>
<td>7.97</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.1 The mesh for the flat plate case. (a): View of the x-y plane. (b): View of the y-z plane.

impose the correct inflow profile, a 2D RANS simulation of a developing boundary layer was carried out before all the 3D simulations. The 2D mesh has the same spacing as the 3D grid in wall-normal (Y) direction, and the resulting boundary layer profiles are compared to the experiments. When the position-of-interest where the momentum thickness (\(\theta\)) matches the experiment value is located, the boundary layer profile 0.342 m (9\(\delta\), or \(\frac{3}{4}X\)) upstream is taken as the inlet profile for the 3D simulations. In this way, we know that the plane-of-interest in the 3D domain should be 0.342 m from the inflow, and data there can be extracted, examined, and compared to the experiments. At the same time, the inflow utilizes the recycling/rescaling methods to keep the unsteady flow structures going. The recycling plane is put 8\(\delta\) downstream from the inlet. The boundary conditions on the two side boundaries (\(z_{min}\) and \(z_{max}\)) are periodic, meaning that they are essentially connected. The boundary condition on the top wall is symmetry, and the outlet has an extrapolated boundary condition with a fixed pressure.

To take statistics, a time average of the flow field was taken for 15 flow-through times (\(t_{tr} = \frac{X}{U_\infty}\)) after running for a transient period of 5 flow-through times. The physical time step (\(\Delta t\)) is \(5.0 \times 10^{-5}\) s.
4.2 Low Reynolds Number: DeGraaff-Eaton FPBL

The goals of the lower Reynolds Number case simulations \((Re_\theta = 2900)\) are to compare the effects of different SGS models and to examine the effects of numerical dissipation in a wall-resolved LES (WRLES) content. The new hybrid model will also be compared to Gieseking’s hybrid model and the WRLES results. The cases mentioned in this section are tabulated in table 4.2.

Table 4.2: Simulation cases for DeGraaff-Eaton Flat-plate boundary layer case with \(Re_\theta = 2900\)

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Reconstruction Method</th>
<th>Hybrid Model</th>
<th>SGS Model</th>
<th>RANS Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Basic PPM</td>
<td>-</td>
<td>Mixed-Scale</td>
<td>-</td>
</tr>
<tr>
<td>A2</td>
<td>ENO-PPM</td>
<td>-</td>
<td>Mixed-Scale</td>
<td>-</td>
</tr>
<tr>
<td>A3</td>
<td>LD-PPM</td>
<td>-</td>
<td>Mixed-Scale</td>
<td>-</td>
</tr>
<tr>
<td>A4</td>
<td>LD-PPM</td>
<td>-</td>
<td>Vreman's</td>
<td>-</td>
</tr>
<tr>
<td>A5</td>
<td>LD-PPM</td>
<td>-</td>
<td>WALE</td>
<td>-</td>
</tr>
<tr>
<td>A6</td>
<td>LD-PPM</td>
<td>-</td>
<td>'0' Model</td>
<td>-</td>
</tr>
<tr>
<td>A7</td>
<td>LD-PPM</td>
<td>Gieseking's</td>
<td>Mixed-Scale</td>
<td>Menter SST</td>
</tr>
<tr>
<td>A8</td>
<td>LD-PPM</td>
<td>New Model</td>
<td>Mixed-Scale</td>
<td>Menter SST</td>
</tr>
</tbody>
</table>

4.2.1 Effects of Numerical Schemes

For high-fidelity simulations like WRLES, the level of numerical dissipation can be critical to the quality of the results because the physical dissipation in the resolved small eddy structures can be at a similar scale. Cases A1, A2, and A3 (Table 4.2) are used to compare the effects of basic PPM, ENO-PPM, and LD-PPM.

The mean stream-wise velocity profiles from these cases are shown in Fig. 4.2, along with the experimental data. The reason that outer scales \((u_\infty \text{ and } \delta)\) is used for normalization instead of inner scales \((u^+ \text{ and } y^+)\) is that skin-friction information is needed for normalizing using wall units, but this information may not be consistent between cases. The skin-friction levels will be discussed separately. Fig 4.2 suggests that LD-PPM, which is the least dissipative of the three, gives a closest profile to the experimental data. The basic PPM deviates from the experimental data the most. The PPM limiter degrades the reconstruction accuracy down to first-order at local extrema to preserve monotonicity. This can be a problem because there are fluctuations near the grid-scale in this WRLES case, and damping out these fluctuations results a less energetic boundary layer that has
an inadequate momentum transfer to the wall, thus the incorrect velocity profile. The ENO-PPM result sits between the other two, as it has a blend of the limited PPM value and 4th-order value, like LD-PPM, but the blending function allows more limited PPM weight than the LD-PPM to achieve non-oscillatory behavior.

![Image](image.png)

**Figure 4.2** Stream-wise average velocity profiles, from simulations with different reconstruction schemes. DeGraaff-Eaton FPBL, \( Re_\theta = 2900 \).

Fig. 4.3 shows the responses of the SGS eddy viscosity to the numerical scheme. Observation from Fig. 4.3 can be concluded that more dissipative the numerical scheme is, the lower the SGS eddy viscosity goes. This is understandable in the sense that a higher value of strain rate and approximated resolved TKE can be calculated from a flow field with more fluctuation, which is a result of less numerical dissipation. The TKE plot, (b) in Fig. 4.3, shows that the low-dissipation schemes gives a lower resolved TKE in the log-layer, but a higher value for both resolved and modeled TKE in the sub-layer.

Skin-friction coefficient \( (c_f \equiv \frac{x_w}{0.5 \rho u_\infty^2}) \) is another important measurement of how good a boundary layer simulation is. In Fig. 4.4, skin-friction coefficient is plotted against stream-wise distance for the three cases. The experimental value is located at 3/4 of the domain, indicated by the square dot with error bars. From Fig. 4.4, it is obvious that Case A1 (PPM) and Case A2 (ENO-PPM) are predicting
Figure 4.3 Turbulence predictions from simulations with different reconstruction schemes. (a) Residual (SGS) eddy viscosity profiles. (b) Turbulent kinetic energy profiles. Solid lines: total TKE; Dashed lines: modeled TKE; Dotted lines: resolved TKE. DeGraaff-Eaton FPBL, $Re_\theta = 2900$.

a much lower $c_f$ than the experimental value, while Case A3 (LD-PPM) is close to the experimental value, within its uncertainty. Similarly, this is caused by the incorrect amount of momentum transfer from the free-stream all the way down to the wall in Cases A1 and A2, resulting in a lower velocity value at the first cell and shear stress at the wall. This can also cause different wall-unit scaling as mentioned earlier, because the shear velocity is defined as a function of the wall shear stress: $u_t a u \equiv \sqrt{\tau_w}. \rho$. Thus no inner-scales are used when comparing velocity and stresses.

The mean-square of velocity fluctuations (Reynolds normal stresses), normalized by free-stream velocity, is presented in Fig. 4.5 and 4.6. Again, Stream-wise fluctuation level is predicted the best by LD-PPM, although slightly lower than the experimental value before the peak ($y/\delta < 0.015$) and slightly higher after the peak. The other two methods (PPM and ENO-PPM) predict an even lower fluctuation near the wall and higher peak value at a location further away from the wall. The wall-normal fluctuation level is predicted low by LD-PPM over the entire boundary layer. Case A1 and A2 has a better agreement with the experimental data at the outer layer, but their prediction is even lower than Case A3 in the inner layer when $y/\delta < 0.1$.

Reynolds shear stress level is plotted in Fig. 4.7 for the cases A1-A3. The stress level is lower for Cases A1 and A2 when $y/\delta$ is less than about 0.3 when compared to the experimental value and the prediction from Case A3. Case A3 maintains a good agreement with the experimental data until
Figure 4.4 Comparison of averaged skin-friction coefficients for the three cases with different reconstruction schemes. DeGraaff-Eaton FPBL, $Re_\theta = 2900$.

Figure 4.5 Comparison of stream-wise mean-square velocity fluctuations in the BL for different reconstruction schemes. DeGraaff-Eaton FPBL, $Re_\theta = 2900$. 

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Figure 4.6 Comparison of wall-normal mean-square velocity fluctuations in the BL for different reconstruction schemes. DeGraaff-Eaton FPBL, $Re_\theta = 2900$.

$y/\delta$ is below about 0.02, where it predicts slightly lower value of Reynolds stress.

After investigating Fig. 4.5 to 4.7, some conclusions can be drawn. The more dissipative numerical schemes (PPM, then ENO-PPM) tend to damp out the fluctuations deep inside the boundary layer, creating a deficient momentum transferring rate between the inner and outer layer of the BL. The lack of fluctuation energy and momentum transfer in the inner layer then results in low mean velocity gradient near the wall (and low $\tau_w$), and the entire boundary layer profile is altered, as observed in Fig. 4.2. More accurate solutions can be obtained by switching to a less dissipative reconstruction scheme, for example LD-PPM used in Case A3.

4.2.2 Effects of Residual (SGS) Models

Another potential critical factor in a WRLES is the subgrid-scale model, which is used to emulate the effects of turbulence structures smaller than the resolution of the mesh. Cases A3, A4, A5, and A6 are used to compare the effects of different SGS eddy viscosity models. The models used are Lenormand's "Mixed-Scale" (MS) model [Len00], Vreman's model [Vre04], Nicoud's "Wall-Adapting Local Eddy-viscosity" (WALE) model [ND99], and a "0" model where the SGS eddy viscosity is set to zero, imitating a "DNS" type simulation. Fig. 4.8 shows the different responses of each the SGS
eddy viscosity models (except "0" model) when applied to this FP case. The eddy viscosities ($\mu_r$) are normalized by the molecular viscosity $\mu$. Some observations from Fig. 4.8 are that WALE model and Vreman's model goes to (approximately) zero at the wall, while the MS model remains at a higher level, about $0.05\mu$. Away from the wall, the WALE model has the most obvious and highest peak near $y/\delta = 0.4$, but the MS model stays around a lower value for a larger portion of the boundary layer. Vreman's model behaves between the other two, having a shorter but higher plateau than the MS model but not as high of a peak as the WALE model.

The stream-wise average velocity profile for Cases A3-A6 are plotted with experimental data in Fig. 4.9. From the plot, we can tell that most of the velocity profiles are close to the experimental data. A closer look will reveal that the MS model predicts lower velocity in the sub- and buffer-layer, while Vreman's model and the WALE model slightly over-predicts the stream-wise velocity in the short log-layer. The "0" model, without using any eddy viscosity, also shows a very reasonable prediction. When comparing the velocity profile against the eddy viscosity plot in Fig. 4.8, we can realize that the slope of velocity in the sub-layer has a positive correlation with the level of SGS eddy viscosity. Case A3, with the highest level of eddy viscosity, has the lowest growth rate in velocity, while the "0" model has the highest velocity growth rate there, even slightly higher than the experimental data.
However, in the buffer- and log-layer and above, this correlation seems not valid any more because the resolved fluctuations are playing a more important role and the eddy viscosity is responsible for less momentum transfer.

Figures 4.10 and 4.11 show the Reynolds normal stress in stream-wise and wall-normal directions, respectively. The MS model under-predicts the fluctuation near the wall, but the other models have decent prediction. The "0" model has a slightly higher fluctuation level near the wall. All models, except the MS model, have a higher peak near \( y/\delta = 0.02 \) than the experimental data, while the MS model's peak is shifted away from the wall. In the region where \( y/\delta > 0.1 \), all models under-predict the fluctuation level, with "0" model being the closest to the experimental data. Fig. 4.11 is simpler. All models under-predict the wall-normal fluctuation throughout the boundary layer, but all still maintain a very similar shape of the profile when compared to the experiment. The "0" model has an obviously higher level of fluctuation than other models, but still under-predicts the experimental data.

The Reynolds shear stress level is presented in Fig. 4.12 for the tested eddy viscosity models. All models provide decent predictions, most within the uncertainty of the experimental data. The MS model again predicts the lowest stress level in the sub-layer, but has good agreement with other models and the experimental data in the log-layer \( (y/\delta > 0.05) \). Note that the "0" model is above all other models in predicting the Reynolds stress level, higher than the experimental data in most
Figure 4.9 Stream-wise average velocity profiles for simulations with different SGS eddy viscosity models. DeGraaff-Eaton FPBL, $Re_\theta = 2900$.

Figure 4.10 Stream-wise mean-square velocity fluctuation profiles, for simulations with different SGS eddy viscosity models. DeGraaff-Eaton FPBL, $Re_\theta = 2900$. 
extent of the boundary layer as well. This phenomenon, as well as the over-prediction in fluctuation levels in stream-wise and wall-normal directions, can be a consequence of the lack of dissipation from the SGS eddy viscosity.

![Figure 4.11 Wall-normal mean-square velocity fluctuation profiles for simulations with different SGS eddy viscosity models. DeGraaff-Eaton FPBL, $Re_\theta = 2900$.](image)

The skin-friction coefficient profile predicted by the different SGS eddy viscosity models are shown in Fig. 4.13. Except the "0" model, the three other models have very close prediction of the level of $c_f$ near the experimental data location, though slightly higher than the measured value within the uncertainty. The "0" model gives an distinctively higher level of skin-friction coefficient.

From comparing the results of simulations utilizing different residual (SGS) eddy viscosity models, several conclusions may be drawn. Among all the eddy viscosity models, the MS model produces the highest SGS eddy viscosity in the sub-layer near the wall, and Vreman's model is the second highest but orders of magnitude lower than the MS model. This behavior of the MS model limits the level of fluctuation in the sub-layer through the extra dissipation introduced by the eddy viscosity. The amount of fluctuation then has an effect on the velocity profile, making it slightly lower than the experimental value and other models. Going away from the wall, the behavior of all three SGS models are quite similar, giving low mean-square velocity fluctuations in the stream-wise
Figure 4.12 Reynolds shear stress profiles for simulations with different SGS eddy viscosity models. DeGraaff-Eaton FPBL, $Re_\theta = 2900$.

Figure 4.13 Skin-friction coefficient profiles along the computational domain for simulations with different SGS eddy viscosity models. DeGraaff-Eaton FPBL, $Re_\theta = 2900$. 
and wall-normal directions, but good prediction in the Reynolds shear stress. The "0" model allows higher than normal levels of fluctuations (except $v'v'$). Considering its adequate performance in the log-layer and higher, and the ease of programming, MS model is chosen to be used with the hybrid models in the following studies. In addition, the last two studies have shown the capability of the incompressible REACTMB code in performing WRLES calculations.

4.2.3 Hybrid LES/RANS

For the low Reynolds number case ($Re_\theta = 2900$), the turbulence level is relatively low and the log-layer is relatively short. Though it may not reveal potential problems in flows with a higher level of turbulence, the new hybrid model is tested (Case A8) on this case, compared to WRLES (Case A3) and Gieseking's model (Case A7). The model constant $\alpha$ in Eq. 2.97 is set to 1.5 for this case.

The mean stream-wise velocity profiles are plotted in Fig. 4.14. The velocity profiles obtained by the two hybrid models are not vastly different from each other, and stay closer to the experimental data than the WRLES. In the short log-layer, the velocity is a little higher than the experimental data, but almost all within the uncertainty. The blended eddy viscosity profiles from the hybrid models are presented in Fig. 4.15, together with the SGS eddy viscosity from Case A3. Also shown in Fig. 4.15 is the hybrid blending function $\Gamma$, represented by the dotted line. From this figure, it can be seen that the blended eddy viscosity reaches a higher level in the RANS region before switching to LES closure by the blending function near $y/\delta = 0.04$. After completely switching to LES closure ($\gamma = 0$), the eddy viscosity level is essentially the same as the WRLES in the outer layer. The profiles of the blending function between the two hybrid models are also very similar, with Gieseking's model producing a sharper transition, bringing down the level of eddy viscosity earlier than the new model.

Stream-wise and wall-normal mean-square velocity fluctuations are shown in Fig. 4.16 and 4.17, respectively. Shown plots are from averaged resolved mean-square velocity fluctuations. In the outer layer, both $u'u'$ and $v'v'$ values predicted by the hybrid models are lower than the experimental data, but closer than the WRLES's prediction. The difference between the two hybrid models is minimal in terms of these two mean-square velocity fluctuations. The averaged Reynolds shear stress $u'v'$ is plotted in Fig. 4.18. Solid lines are for total stress, dashed for resolved stress, and dotted lines for modeled stress using Boussinesq eddy viscosity assumption. The resolved part has good agreement with the experiment in the outer layer, being fairly close to the WRLES prediction too. With the modeled stress, both hybrid models give a close match to the experimental data in the inner layer too, all within the uncertainty of the data. Again, the two hybrid models do not show large discrepancy between each other. The new hybrid model gives a slightly lower total stress level in the inner- and buffer-layer, but similar in the outer layer.
Figure 4.14 Stream-wise average velocity profiles for WMLES and LES/RANS simulation with different hybrid models. DeGraaff-Eaton FPBL, $Re_\theta = 2900$.

Figure 4.15 Eddy viscosity ratio for hybrid simulations and WRLES, with hybrid blending function $\Gamma$ shown in dotted lines. DeGraaff-Eaton FPBL, $Re_\theta = 2900$. 
Figure 4.16 Averaged resolved stream-wise Reynolds stress from hybrid models and WRLSE. DeGraaff-Eaton FPBL, $Re_\theta = 2900$.

Figure 4.17 Averaged resolved wall-normal mean-square velocity fluctuations from hybrid models and WRLSE. DeGraaff-Eaton FPBL, $Re_\theta = 2900$. 
This part of the study shows that the new hybrid model is comparable to Gieseking’s model in providing an accurate composite boundary layer velocity profile at $Re_\theta = 2900$, given that the hybrid blending functions are similar. It predicts a reasonable level of stream-wise mean-square velocity fluctuation and Reynolds shear stress, similar to Gieseking’s model. The wall-normal mean-square velocity fluctuation is under-predicted by the new model, as is expected, because fluctuation content in the buffer layer and below is significantly attenuated by the high eddy viscosity.
4.3 Moderate Reynolds Number: DeGraaff-Eaton FPBL

The moderate Reynolds number case \((Re_\theta = 13000)\), based on DeGraaff-Eaton FPBL experiment, consists of a boundary layer with a fuller composite structure, i.e. a more distinguished sub-layer, buffer-layer, log-layer, and wake-like region. The Reynolds number is too high for the same grid to perform a WRLES, and it can potentially pose more challenge to the hybrid model. There are three main focal points for this part of the study: an investigation of the effects of different RANS models, a calibration of the model constant \(\alpha\) (as in Eq. 2.97), and comparisons to other WMLES hybrid models. A list of cases considered in this section is presented in Table 4.3.

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Hybrid Model</th>
<th>RANS Model</th>
<th>Intermittency Correction</th>
<th>Model Constant ((\alpha))</th>
<th>Reconstruction Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>New Model</td>
<td>BSL</td>
<td>-</td>
<td>1.5</td>
<td>LD-PPM</td>
</tr>
<tr>
<td>B2</td>
<td>New Model</td>
<td>BSL</td>
<td>Form #1</td>
<td>1.5</td>
<td>LD-PPM</td>
</tr>
<tr>
<td>B3</td>
<td>New Model</td>
<td>BSL</td>
<td>Form #2</td>
<td>1.5</td>
<td>LD-PPM</td>
</tr>
<tr>
<td>B4</td>
<td>New Model</td>
<td>SST</td>
<td>-</td>
<td>1.5</td>
<td>LD-PPM</td>
</tr>
<tr>
<td>B5</td>
<td>New Model</td>
<td>SST</td>
<td>Form #1</td>
<td>1.5</td>
<td>LD-PPM</td>
</tr>
<tr>
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<td>SST</td>
<td>Form #2</td>
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</tr>
<tr>
<td>B7</td>
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<td>BSL</td>
<td>-</td>
<td>1.6</td>
<td>PPM</td>
</tr>
<tr>
<td>B8</td>
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<td>-</td>
<td>1.6</td>
<td>LD-PPM</td>
</tr>
<tr>
<td>B9</td>
<td>New Model</td>
<td>BSL</td>
<td>-</td>
<td>1.75</td>
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<td>-</td>
<td>1.75</td>
<td>LD-PPM</td>
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<tr>
<td>B11</td>
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<td>-</td>
<td>1.75</td>
<td>ENO-PPM</td>
</tr>
<tr>
<td>B12</td>
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<td>-</td>
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<tr>
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<td>BSL</td>
<td>-</td>
<td>-</td>
<td>LD-PPM</td>
</tr>
<tr>
<td>B14</td>
<td>IDDES</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>PPM</td>
</tr>
<tr>
<td>B15</td>
<td>IDDES</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>LD-PPM</td>
</tr>
</tbody>
</table>

4.3.1 Effects of RANS Models and Intermittency Corrections

Two RANS models, Menter’s BSL and SST, are used with the new hybrid method as wall models to test their effects. Each RANS model is then tested with the ‘intermittency correction’ in two forms (Eq. 2.98 and Eq. 2.99). Fig. 4.19 shows the comparison of the different behaviors of velocity, eddy viscosity, and hybrid blending function \(\Gamma\), resulting from implementing the BSL and SST model.
Average stream-wise velocities are normalized by free-stream velocity, and eddy viscosity values are normalized by molecular viscosity. The velocity plot of Fig. 4.19 suggests that SST creates a more severe log-law mismatch (LLM) than BSL, with an obvious ‘hump’ in the log-region deviating from the experimental data. Also, the hybrid blending function from the case with SST model switches from unity to zero closer to the wall, with a broader transition. Due to this early transition and the fact that the SST model limits the value of $\omega$ to be greater than (a factor of) the strain rate $\Omega$ when calculating the eddy viscosity (Eq. 2.45), which occurs frequently in an unsteady velocity field, the resulting eddy viscosity from SST is much lower than BSL, also shown in Fig. 4.19. On the other hand, BSL model predicts a closer profile to the experimental data, but a LLM still can be observed near the start of the LES region, where $\Gamma$ gets close to zero. In addition, while $\omega$ is filtered by the action of convective and diffusive components in its transport equation, the strain rate is highly intermittent in space. This can result in a higher variance in eddy viscosity for SST model, and its influence on the modeled shear stress and the predicted velocity profile more spread out.

![Figure 4.19](image.png)

**Figure 4.19** Comparison of hybrid results from different RANS closure models. Solid lines: average stream-wise velocity; Dashed lines: eddy viscosity; Dotted lines: hybrid blending function $\Gamma$. DeGraaff-Eaton FPBL, $Re_\theta = 13000$. 
Fig. 4.20 shows the effects of the intermittency corrections (IC) on stream-wise velocity, eddy viscosity, hybrid blending function, and shear stresses. With the same constant $C_{int} = 0.1$, applying the two forms of IC shifts and broadens the blending function to the right and away from the wall. Eddy viscosity values are increased in the log-layer. Form #1 of IC creates more shift, and form #2 gives a stronger broadening effect while not as significant of a shift as #1. The introduction of intermittency corrections to BSL formulation brings the velocity profile too low in the buffer region and the inner part of the log-layer, when compared to experimental data and the RANS solution. The total shear stress predictions are good for the three cases shown in Fig. 4.20(b) in the sublayer and buffer layers, while they tend to over-predict the value in the log-layer. Further away from the wall, the predictions with IC enabled tend to under-predict the resolved shear stress, while the RANS simulation gives a higher value and the hybrid model without IC has close predictions comparing to the experiment. There is a noticeable peak for each total shear stress profile near the LES/RANS transition, with a normalized stress value greater than unity. The localized over-prediction is where the modeled and resolved stresses cross over. It comes from the fact that the modeled Reynolds stress is constructed from time- and span-averaged data

$$\langle \tau_{ij,mod} \rangle \equiv \tau_{ij,mod} \left( \langle \mu_t \rangle, \frac{\partial \langle u \rangle}{\partial y} \right)$$

instead of being averaged directly

$$\langle \tau_{ij,mod} \rangle \equiv \langle \tau_{ij,mod} \left( \mu_t, \frac{\partial u}{\partial y} \right) \rangle.$$  \hspace{1cm} (4.2)

This error was found and fixed in the later simulations.

The effects of intermittency correction are also studied for SST model, as shown in Fig. 4.21. The changes that the intermittency corrections bring to the hybrid blending function are very similar to the BSL case, with form #1 mainly shifting the transition while form #2 shifting less but broadening more. Eddy viscosity level is also increased with the transition from RANS to LES further away from the wall. Considering the low level of eddy viscosity that SST model originally provides, this increase in $\mu_t$ actually brings the velocity closer to the experimental data with less of LLM’s. The resulting velocity profiles are still lower than the experiment and RANS predictions in the buffer and the viscous sub-layer. In terms of total shear stress, the prediction is good in the sub-layer, and the IC terms bring up the profile in the outer log-layer closer to the experimental data. In the inner log-layer where the RANS/LES transition takes place, the prediction of total stress is high, similar to the over-estimation in BSL case but more broadly distributed. This again is due to the incorrect averaging process mentioned above.
Figure 4.20 Comparison of results from BSL + new hybrid model with different intermittency correction (IC) forms. (a) Solid lines: average stream-wise velocity; Dashed lines: eddy viscosity; Dotted lines: hybrid blending function $\Gamma$. (b) Solid lines: total shear stress; Dashed lines: molecular + modeled turbulent shear stress; Dotted lines: resolved Reynolds shear stress. DeGraaff-Eaton FPBL, $Re_\theta = 13000$. 


Figure 4.21 Comparison of results from SST + new hybrid model with different intermittency correction (IC) forms. (a) Solid lines: average stream-wise velocity; Dashed lines: eddy viscosity; Dotted lines: hybrid blending function $\Gamma$. (b) Solid lines: total shear stress; Dashed lines: molecular + modeled turbulent shear stress; Dotted lines: resolved Reynolds shear stress. DeGraaff-Eaton FPBL, $Re_\theta = 13000$. 

4.3.2 Effect of the Model Constant and Numerical Schemes

Another way to modulate the response of the hybrid blending function other than incorporating the intermittency correction is to directly change the model constant $\alpha$ in Eq. 2.97. The original value of 1.5 is taken from Gieseking’s model and may not be the best choice for the new model. Several other model constant values are tested using the BSL model since it showed a better response than SST in the study in Subsection 4.3.1.

Fig. 4.22 shows the results of this test of various model constants, $\alpha = 1.5, 1.6,$ and 1.75. As can be seen in Fig. 4.22 (a), a greater value of $\alpha$ shifts the blending function away from the wall, by increasing the outer-to-inner length scale ratio. This in turns delays the transition from RANS to LES and gives a higher eddy viscosity value. The peak of eddy viscosity also shifts to the right with a greater value of $\alpha$. The velocity profile in the RANS region goes lower with a higher $\alpha$, alleviating the LLM in the outer log-layer to some extent but underpredicting the velocity values in the buffer-layer and inner part of the log-layer. The jump between $\alpha = 1.6$ and 1.75 is much greater than the difference between $\alpha = 1.5$ and 1.6, although the change in $\alpha$ is not the same.

![Figure 4.22](image_url)

**Figure 4.22** Comparison of results from BSL + new hybrid model with different model constant $\alpha$ values. (a): Solid lines: average stream-wise velocity; Dashed lines: eddy viscosity; Dotted lines: hybrid blending function $\Gamma$. (b): Solid lines: total shear stress; Dashed lines: molecular + modeled Reynolds shear stress; Dotted lines: resolved Reynolds shear stress. DeGraaff-Eaton FPBL, $Re_\theta = 13000$. 
Plot (b) in Fig. 4.22 shows the shear stresses (normalized by wall shear stress) obtained from using different values of the model constant. The result from hybrid blending function shift is obvious – the crossing of modeled and resolved shear stress moves to the right as the blending function moves to the right. The smoother total shear stress line for Case B10 is a result of fixing the averaging process from Eq. 4.1 to 4.2. It is close to the experimental data in the inner- and buffer-layer. Comparing cases B1 and B8, the higher value of model constant lowers the resolved Reynolds shear stress.

The effect of numerical reconstruction scheme is also investigated for $\alpha = 1.6$ and 1.75. PPM and LD-PPM are tested on both model constant values, and ENO-PPM was also tested on the case $\alpha = 1.75$. Fig. 4.23 shows how the stream-wise average velocity, eddy viscosity, and hybrid blending function change with numerical schemes. From both (a) and (b) of Fig. 4.23, it can be seen that switching to a scheme with higher numerical dissipation makes the hybrid blending function broader (slower transition). It does not have an effect on the starting location of the RANS/LES transition. The velocity profiles are higher for the higher-dissipation schemes. The eddy viscosity levels in the RANS region are very similar among the cases, but the SGS eddy viscosity in the LES part for the LD-PPM cases is higher than the other two schemes. The trend of SGS eddy viscosity is similar to the numerical scheme investigation in the low-Reynolds-number WRLES study (Sec. 4.2.1 and Fig. 4.3-(a)), and is a result of more resolved-eddy content near the mesh scale.

However, in the WRLES study, LD-PPM resulted in a velocity profile that matches the experimental data best, whereas Fig. 4.23 shows the opposite. A further investigation is carried out in terms of shear stress and TKE level. Fig. 4.24 shows the shear stress profiles. Aside from the discrepancy resulted from incorrect averaging in Fig. 4.24-(a), the change of numerical schemes does not seem to have a large impact on modeled shear stress. Schemes with lower dissipation gives a lower level of resolved Reynolds shear stress, likely because of the elevated SGS eddy viscosity. Fig. 4.25 shows the TKE profiles, both modeled and resolved, from utilizing different numerical schemes. The modeled part of TKE is not very sensitive to the change of numerical methods, but the large difference in resolved TKE level is causing some issues. From the previous WRLES study (Fig. 4.3-(b)), it is expected that the resolved TKE would be lower for LD-PPM than PPM going into the log-layer, then both would reduce to a similar level after the peak near the start of the buffer layer. However, in the case of the new model, the RANS contribution, which is much less sensitive to the numerical scheme, kicks in in the middle of the log-layer, leaving the gap between the schemes. Meanwhile, the modeled level of TKE is almost insensitive to the choice of scheme, causing under-resolved TKE especially for the low-dissipation schemes like LD-PPM. As a result, the momentum transfer is not sufficient, causing the lower velocity profile for LD-PPM.

After looking at the influence of the model constant $\alpha$ and numerical schemes, Case B9 ($\alpha = 1.75$ and PPM) is believed to give a better prediction of the composite structure of the FPBL, among cases
Figure 4.23 Comparison of results from BSL + new hybrid model with different numerical schemes. (a): Cases with $\alpha=1.6$; (b): Cases with $\alpha=1.75$. Solid lines: average stream-wise velocity; Dashed lines: eddy viscosity; Dotted lines: hybrid blending function $\Gamma$. DeGraaff-Eaton FPBL, $Re_\theta = 13000$.

Figure 4.24 Comparison of shear stress profiles from BSL + new hybrid model with different numerical schemes. (a): Cases with $\alpha=1.6$; (b): Cases with $\alpha=1.75$. Solid lines: total shear stress; Dashed lines: molecular + modeled Reynolds shear stress; Dotted lines: resolved Reynolds shear stress. DeGraaff-Eaton FPBL, $Re_\theta = 13000$. 

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Figure 4.25 Comparison of TKE profiles from BSL + new hybrid model with different numerical schemes. (a): Cases with $\alpha=1.6$; (b): Cases with $\alpha=1.75$. Solid lines: total TKE; Dashed lines: modeled TKE; Dotted lines: resolved TKE. DeGraaff-Eaton FPBL, $Re_\theta=13000$.

with the new hybrid model. Although it has room for improvement in terms of utilizing a numerical scheme with lower numerical dissipation (and usually higher accuracy), Case B9 will be used as the 'benchmark' case for the new model when comparing to other hybrid models in the next section.

4.3.3 Comparison to other Hybrid Methods

How the performance of the new model compares to other hybrid WMLES models under the same solver is intrinsically interesting to researchers working on this project. Here two other hybrid WMLES approaches, naming Choi’s model (see Sec. 2.4.1) and the IDDES model (see Sec. 2.4.3) are examined, and the results are presented in the following part.

Fig. 4.26 shows average stream-wise velocity profiles predicted by the hybrid models tested, along with the eddy viscosity and blending function $\Gamma$ (for (a) only). From the Fig. 4.26-(a), we can tell that the new model, with the constant $\alpha$ set to 1.75, behaves very similar to Choi’s model. The blending function from Choi’s model is sharper at the start of the transition, but is very close to the new model’s blending function otherwise. This difference causes the eddy viscosity from the new model to be lower than Choi’s model near the transition. In addition, from the same figure, we can see that Choi’s model suffers from similar velocity under-prediction in the inner part of the
log-layer down to the viscous sub-layer when LD-PPM is chosen. Response from IDDES is shown in Fig. 4.26-(b), where PPM (Case B14) gives a velocity under-prediction and LLM while LD-PPM (Case B15) results in an improved velocity prediction. Compared to the new model (Case B9), IDDES with LD-PPM (Case B15) predicts lower velocity in the sub- and buffer-layer, while is closer to the experimental data in the upper-part of the log-layer. Both methods have slight LLM, but in different locations of the log-layer. LLM in IDDES happens closer to the wall than that of the new model. Because the two models use different hybrid approach (modified $k$ equation for IDDES vs. blending of $\nu$ for the new model), the shapes of the eddy viscosity profiles are not similar. In the outer part of the log-layer, IDDES produces a much higher value of $\nu_t$ than the SGS component of the new model. In the inner part of the log-layer, where the new model has a high peak of eddy viscosity, the IDDES model predicts a lower value before increasing a little near the buffer-layer, creating an 'M' shaped profile.

**Figure 4.26** Comparison of boundary layer results from different hybrid RANS/LES models. (a): new model and Choi’s model; (b): new model and IDDES model. Solid lines: average stream-wise velocity; Dashed lines: eddy viscosity; Dotted lines: hybrid blending function $\Gamma$, (a) only. DeGraaff-Eaton FPBL, $Re_\theta = 13000$.

Fig. 4.27-(a) shows the shear stress response of Choi’s model when using PPM and LD-PPM compared to Case B9. Both modeled and resolved stresses from Choi’s model are close to the new
model, showing the shifting of the blending function to the right from the new model. Numerical scheme does not appear to play an important role here, except near the outer edge of the boundary layer where LD-PPM under-predicts the resolved stress a bit. Fig. 4.27-(b) contains plots of shear stress responses by the IDDES model under different numerical reconstruction schemes. The shape of the resolved and modeled stress distributions are different from the new model, and they cross at different locations. Using LD-PPM lowers the resolved stress (hence total stress) near the outer layer, while using PPM for IDDES increases the shear stress level to be higher than the new model and experimental data in the log-region.

![Figure 4.27](image)

**Figure 4.27** Comparison of shear stress results from different hybrid RANS/LES models. (a): new model and Choi’s model; (b): new model and IDDES model. Solid lines: total shear stress; Dashed lines: molecular + modeled Reynolds shear stress; Dotted lines: resolved Reynolds shear stress. DeGraaff-Eaton FPBL, $Re_\theta = 13000$.

Fig. 4.28-(a) shows the resolved, modeled, and total TKE levels from Choi’s model using PPM and LD-PPM, compared to the new model. The trend of the effect of numerical methods is similar between the new model and Choi’s model – switching from PPM to LD-PPM lowers the resolved TKE quite a bit in the log-layer, while modeled TKE is hardly affected. The total TKE distribution is thus lowered. Meanwhile, Fig. 4.28-(b) compares the TKE level of IDDES using PPM and LD-PPM to the new model. Using LD-PPM instead of PPM also reduces the resolved TKE content at the outer part.
of the log-layer, but the inner-part and sub-layer is not affected. The modeled part is not affected by schemes too much. The result is that the outer-part of the log-layer has a slightly lower TKE profile for IDDES, but the inner part stays pretty much the same, while the new model loses TKE through out the boundary layer.

![Comparison of turbulence kinetic energy from different hybrid models with various numerical schemes. (a): New model and Choi's model; (b): new model and IDDES. DeGraaff-Eaton FPBL, $Re_\theta = 13000$.](image)

Fig. 4.29-(a) includes the plots of resolved Reynolds normal stress $<u' u'>$ from IDDES and the new model. Overall IDDES produces a higher level over the new model, and IDDES with PPM has a big spike which is a lot higher than the experimental data. In Fig. 4.29-(b), resolved wall-normal Reynolds stress $<v' v'>$ from IDDES simulations is compared to that of the new model. All models show a lack of resolved $<v' v'>$ according to the experimental data. Fig. 4.29-(c) shows the resolved span-wise Reynolds stress from IDDES and the new model. Unfortunately, no experimental measurement is available. Overall, IDDES gives higher level of $<w' w'>$ in the inner part of the log-layer and below than the new model, and numerical scheme seems to have a larger impact on the new model than IDDES, lowering the stress level even further when switching to LD-PPM for the new model.
Figure 4.29 Comparison of resolved mean-square velocity fluctuations from new model and IDDES with different numerical schemes. (a): Stream-wise; (b): wall-normal; (c): span-wise. DeGraaff-Eaton FPBL, \( Re_\theta = 13000 \).
4.4 Simulation Setup: Elena-Lacharme FPBL

The supersonic boundary layer (Elena-Lacharme FPBL) case utilizes a mesh of size $301 \times 201 \times 192$ ($X \times Y \times Z$), and an adiabatic wall condition is enforced. The mesh spacing in the stream-wise direction is $1/30^{th}$ of the boundary layer thickness of 0.01 m. Wall-normal spacing is clustered towards the surface with a minimum wall coordinate value of less than one ($\Delta y^+ < 1.0$). The Reynolds number based on $\theta$ is $Re_\theta = 4700$, and the free-stream Mach number is set to 2.32.

Two RANS simulations (BSL and SST) and four hybrid LES/RANS simulations are carried out for this case. The variations include using BSL and SST closures with the new model and implementing the intermittency correction form #1 (Eq. 2.98) on both BSL and SST based hybrid runs. The model constant used in this case is $\alpha = 1.5$, and LD-PPM is utilized as the reconstruction scheme. The next section will discuss the observations on how the new model responds to the supersonic boundary layer case.

4.5 Results on Elena-Lacharme FPBL

Fig. 4.30 plots averaged stream-wise velocity, hybrid blending function, and normalized eddy viscosity profiles for the BSL and SST LES/RANS runs and RANS simulations. The experimental data of $\langle u \rangle / u_\infty$ is also shown in the plots. From Fig. 4.30, it is shown that the inclusion of the intermittency correction term, form #1, increases the level of eddy viscosity for both BSL and SST formulations, because the blending function is shifted away from the wall and is broadened, allowing a larger region with RANS closure. For the BSL formulation, when compared to the RANS solution, the intermittency effect has an adverse effect, serving to reduce the velocity in the inner log- and buffer-regions. For the SST formulation, the effect of the intermittency term is similar to BSL formulation in the inner part of the boundary layer, but the increased eddy viscosity in the outer part strengthens the velocity profile as the profile without the intermittency is a bit lower. As a result, including the intermittency correction leads to a better agreement with the RANS prediction. No model is able to capture the velocity profile provided by the experimental measurements near the buffer region.

Fig. 4.31 compares the total shear-stress profiles from the simulations with experimental measurements, and its separated two components: resolved shear stresses and modeled stress (including molecular shear stress). All simulations over-predict the level of total stress over the experimental data, which does not approach the expected value of unity towards the wall. The experimental data levels out at a value of around 0.8. The over-estimation in total shear stress, especially for BSL near the crossover of modeled and resolved stresses, can be observed again (see discussion of Fig. 4.20.
Figure 4.30 Comparison of (a): BSL and (b): SST on velocity, eddy viscosity, and blending function predictions. Elena-Lacharme FPBL.
and 4.21).
Figure 4.31 Comparison of (a): BSL and (b): SST on modeled, resolved, and total shear stress predictions.

Elena-Lacharme FPBL.
5.1 Aérospatiale "A-Airfoil"

5.1.1 Simulation Setup

In this case, a fine, three-dimensional, C-type structured mesh with a blunt trailing edge was used to discretize the domain, shown in Fig. 5.1. The chord length $c$ is 0.6m, and the span is 2 percent of the chord size. The number of cells in the mesh is $3020 \times 180 \times 72$, totaling 39.2 million cells. Stream-wise, the upper surface of the airfoil has more resolution than the lower surface, with clustering near leading edge (up to $0.25c$), and near the trailing edge. Minimum spacing stream-wise is $0.0002c$, and maximum stream-wise spacing on the airfoil is $0.001c$. Wall-normal spacing clusters towards surface with a minimum size of $5 \times 10^{-6}m$ at the geometry. Span-wise spacing is uniform. The angle of attack is set to 13.3 degrees, and the free-stream velocity is 35 m/s. The flow is assumed to be incompressible. The new model with BSL and PPM is tested on this case with the model constant set to 1.5. Results from the new model are compared to previous generations of NCSU's hybrid models – Choi’s model ([Edw08],[Cho09]) and Gieseking’s model([Gie11a],[GE11]).

5.1.2 Results

Fig. 5.2 shows snapshots of the blending function, normalized eddy viscosity, transported quantity in the EVT model, and span-wise velocity from the new model simulation. The action of the blending
function (Fig. 5.2-(a)) reduces the eddy viscosity (Fig. 5.2-(b)) outside the near-wall part of the boundary layer from RANS level to SGS level. The transported quantity $\tilde{\nu}$ (Fig. 5.2-(c)) responds to the flow and directly affects the profile of the hybrid blending function.

Fig. 5.3 compares the averaged predictions of surface pressure and skin-friction coefficients from the new model to the two previous generations of the hybrid model, as well as the experimental measurements. The pressure coefficient predictions, shown in Fig. 5.3-(a), are close from model to model, and have good agreement with the experimental data, except that $c_p$ is slightly lower near the leading edge. Skin-friction coefficient distribution (see Fig. 5.3-(b)) given by the new model is close to Choi’s model, but slightly lower than Gieseking’s model in the second half of the airfoil. In general the models have good agreement with experimental data except near the leading edge of the airfoil, where a laminar separation bubble was reported in the experiment. No model is able to capture this laminar separation bubble.

Fig. 5.4 shows the stream-wise averaged velocity near the top surface of the airfoil. Velocity profile predictions from the new model are close to Choi’s model, which requires an initial calibration to estimate the transition location as a function of distance along the chord. Predictions from Gieseking’s model, which uses ensemble-averaging to estimate the outer-layer scale, are somewhat worse than the other two models. It tends to over-predict the velocity magnitude in the boundary layer and delay the onset of trailing edge separation.

The RMS of stream-wise velocity fluctuations is shown in Fig. 5.5. The new model’s response is close to Choi’s model again and is closer to the experimental data than Gieseking’s model. However, no model is able to predict the correct level of turbulence intensity near the trailing edge where the flow is about to separate, or in the separation region.
Figure 5.2 Snapshots of turbulence and flow properties near the trailing edge. (a): Blending function $\Gamma$; (b): normalized eddy viscosity $\mu_t/\mu$; (c): transported quantity in EVT model, $\mathcal{T}$; (d): span-wise velocity $w$.

Aérospatiale "A-Airfoil".
**Figure 5.3** (a): Pressure coefficient and (b): skin-friction coefficient distributions. Aérospatiale "A-Airfoil".

**Figure 5.4** Stream-wise velocity \(< u > / u_\infty\) profiles on the upper surface of the Aérospatiale "A-Airfoil". (a): First half of the airfoil; (b): second half of the airfoil.
5.2 NASA Wall-Mounted Hump

5.2.1 Simulation Setup, Cases, and Experimental Data

The domain for the wall-mounted hump case is also discretized as simple structural hexahedron meshes, shown in Fig. 5.6. The top wall has a change in shape above the hump, reproducing the blocking effect of the side plates in the experiment. There are a total of three meshes for this study, described in Table 5.1. The chord of the hump is \( c = 0.42 \) m. In the x-direction (stream-wise), the computational domain extends from \(-2.14c\) to \(4.0c\), with the hump sitting at \( x = 0 \) to \( x = c \). The height is \( Y = 0.909526c \), and the span is \( Z = 0.2186c \) for the 'fine-narrow' configuration, and \( Z = 0.4373c \) for two 'fine-wide' and 'coarse-wide' configurations. The 'fine-narrow' mesh is split into 1536 blocks, and 3072 for the 'fine-wide' configuration. The 'coarse-wide' mesh is split into 816 blocks. Each block is mapped to a core for computation. The x-spacing is close to uniform for the 'coarse-wide' mesh before it relaxes when \( X > c \). For the other two fine meshes, \( \Delta x \leq 1.66 \times 10^{-3} \) m upstream from the hump, and clustered to \( 6.3 \times 10^{-4} \) m near the start of the hump and \( 4.2 \times 10^{-4} \) m near the 'dip' on the hump. \( \Delta x \) then relaxes to \( 4.2 \times 10^{-3} \) m at \( x = 2c \), and relaxes more near the far-field. Fig. 5.7 shows local stream-wise spacing of the 'coarse' and 'fine' meshes using skin-friction results from Case C10 and C11 (see Table 5.2). Wall normal spacing for all the meshes are the same,
following a hyperbolic-tangent stretching function with $\Delta y_{wall} = 5.0 \times 10^{-6}$ m, resulting a near wall $\Delta y^+ < 0.7$.

<table>
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<th>Span Size (×10⁻² m, $3\delta_{in}$)</th>
<th>$&lt;\Delta X&gt;^1$ (×10⁻⁵ m)</th>
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</thead>
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<td>$9.18 \times 10^{-2}$ m</td>
<td>1.123</td>
</tr>
<tr>
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<td>$1536 \times 216 \times 304$ (100.8 mil.)</td>
<td>6.04</td>
<td>$1.836 \times 10^{-1}$ m, $(6\delta_{in})$</td>
<td>1.123</td>
</tr>
<tr>
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<td>12.08</td>
<td>$1.836 \times 10^{-1}$ m, $(6\delta_{in})$</td>
<td>1.420</td>
</tr>
</tbody>
</table>

Table 5.1 Mesh configurations for the NASA wall-hump case

The two sides (spanwise) of the channel are set to periodic boundary conditions, while the top surface is a symmetry boundary. The outlet is extrapolated far-field with prescribed pressure. When doing calculation, the hybrid solution is relaxed to the RANS solution starting at $x = 2.5c$. The purpose of this procedure is to prevent pressure wave reflections from the downstream boundary. The recycling plane for this case is located $0.3069$ m ($\approx 10\delta_{in}$) from the inlet plane, whose profile is determined from a 2D RANS simulation of a boundary layer developed at the same flow condition. Inlet boundary layer thickness $\delta = 0.30609$ is used when for normalizing lengths and the freestream velocity $u_\infty = 34.6$ m is used to normalize velocities and stresses. The physical time step for calculations is $1.5 \times 10^{-5}$ s, and turbulent statistics are taken for $40.78$ flow-through times based on chord ($t_{thr} = \frac{c}{u_\infty}$) after a transient period of $13.59t_{thr}$ starting from a 3D RANS solution. The Reynolds number based on the chord is $Re_c = 9.36 \times 10^5$.

A list of cases discussed in this section is presented in Table 5.2. This series of cases include change of models and reconstruction schemes on different meshes.

The inlet condition of the 3D RANS simulations (Case CR1 and CR2) are extracted from a 2D RANS simulation of a FPBL. The location to extract the profile from the 2D FPBL was picked based on matching $\theta$ for Case CR1 but $\delta$ for Case CR2. The RANS inlet profiles for cases CR1 and CR2 are shown in Fig. 5.8, with data points from the experiment shown as well. In the figure, the velocity profiles from the case CR1 is close to the experiment, while CR2 shows a slightly thicker boundary layer because it was extracted at a different but nearby location on the 2D FPBL. This observation suggests that a matching momentum thickness suits better as a criteria when choosing the profile from a 2D simulation. Both profiles are considered close enough as the inlet condition for the 3D simulations in this study. The hybrid RANS/LES cases are started from 3D RANS simulations with

\[1^\text{Average } \Delta X \text{ from inlet to 1 chord downstream from the hump.} \]
Figure 5.6 The X-Y spacing for the ‘fine’ meshes of the wall-mounted case. (a): Overview of the x-y plane of the entire domain. (b): Closed-up view of the x-y plane near the hump.
their corresponding RANS models and meshes, and the recycling-rescaling method is used for maintaining fluctuations. Cases C1-7 are started from Case CR1, while C8 and C9 are started from CR2. Case C10 is started from C3 with its solution duplicated and shifted to make up the doubled span size. Case C11 uses a separate 3D RANS (not shown in Table 5.2) as a starting point, which employs the inlet condition of CR1 and the coarse mesh of CR2.

For the reference of the simulation cases, the PIV measured flow field is presented here. Fig. 5.9 shows flooded contour plots of averaged X- and Y-velocity field, averaged Reynolds normal and shear stresses, and the streamlines from the experiment by Greenblatt et al. [Gre06a]. Comparable contour plots for each simulation case can be found in Appendix A. Fig. 5.10 shows the iso-surface of the 'swirling strength criterion' [Zho99], a method for vortex identification, at a level of 500 on the hump. The iso-surfaces are colored with velocity magnitude. This illustration (from Case C11) helps visualize the resolved eddy structures in the incoming boundary layer before the hump, on the hump, as well as in the wake region.
Figure 5.8 Velocity profiles at the inlet ($x = -2.14c$). NASA wall-mounted hump case.
Figure 5.9 Experimental measurements of time-averaged properties for the NASA wall-mounted hump case. 
(a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure 5.10 Iso-surfaces of ‘swirling strength’ at level of 500, colored by velocity magnitude for the NASA wall-mounted hump case. (a): looking downstream; (b): looking upstream.
Table 5.2 Simulation cases for NASA wall-mounted hump

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Model Used</th>
<th>Reconstruction Scheme</th>
<th>Mesh Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR1</td>
<td>BSL RANS</td>
<td>PPM</td>
<td>Fine-Narrow</td>
</tr>
<tr>
<td>CR2</td>
<td>BSL RANS</td>
<td>PPM</td>
<td>Coarse-Wide</td>
</tr>
<tr>
<td>C1</td>
<td>New Model (α = 1.5, w/ BSL)</td>
<td>PPM</td>
<td>Fine-Narrow</td>
</tr>
<tr>
<td>C2</td>
<td>New Model (α = 1.5, w/ BSL)</td>
<td>LD-PPM</td>
<td>Fine-Narrow</td>
</tr>
<tr>
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<td>New Model (α = 1.75, w/ BSL)</td>
<td>PPM</td>
<td>Fine-Narrow</td>
</tr>
<tr>
<td>C4</td>
<td>New Model (α = 1.75, w/ BSL)</td>
<td>LD-PPM</td>
<td>Fine-Narrow</td>
</tr>
<tr>
<td>C5</td>
<td>IDDES</td>
<td>LD-PPM</td>
<td>Fine-Narrow</td>
</tr>
<tr>
<td>C6</td>
<td>Choi’s (w/ BSL)</td>
<td>PPM</td>
<td>Fine-Narrow</td>
</tr>
<tr>
<td>C7</td>
<td>Gieseking’s (α = 1.5, w/ BSL)</td>
<td>PPM</td>
<td>Fine-Narrow</td>
</tr>
<tr>
<td>C8</td>
<td>New Model (α = 1.5, w/ BSL)</td>
<td>PPM</td>
<td>Coarse-Wide</td>
</tr>
<tr>
<td>C9</td>
<td>New Model (α = 1.5, w/ BSL)</td>
<td>LD-PPM</td>
<td>Coarse-Wide</td>
</tr>
<tr>
<td>C10</td>
<td>New Model (α = 1.75, w/ BSL)</td>
<td>PPM</td>
<td>Fine-Wide</td>
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<tr>
<td>C11</td>
<td>New Model (α = 1.75, w/ BSL)</td>
<td>PPM</td>
<td>Coarse-Wide</td>
</tr>
</tbody>
</table>

5.2.2 Effects of Model Constant and Numerical Schemes

Similar to the Eaton and DeGraaff FPBL case, the effects of the model constant α and numerical reconstruction scheme are tested on this hump case. Values of α tested are 1.5 (Case C1 and C2) and 1.75 (Cases C3 and C4). Schemes tested are PPM (Case C1 and C3) and LD-PPM (Case C2 and C4). All four cases are carried out on the ‘fine-narrow’ mesh. Fig. 5.11 shows the averaged stream-wise (plot (a)) and wall-normal (plot (b)) velocity profiles from these cases of various model constant and reconstruction schemes. From Fig. 5.11-(a), the two α = 1.5 cases have an under-energized boundary layer prior to separation (at x/c = 0.65) compared to the two cases with α = 1.75 and the experiment, likely causing an premature separation. The cases with low model constant also have lower stream-wise velocity in the wake region compared to their corresponding high-model-constant pairs, resulting a larger separation. When looking at numerical schemes, cases with LD-PPM give lower stream-wise velocity profiles compared to their PPM pair with the same model constant, also resulting in reattachment further downstream. Fig. 5.11-(b) helps verify these observation where not enough ‘downwash’ is presented at x/c = 0.65 for Case C1 and C2 (α = 1.5) indicating that the flow is not as attached as Case C3, Case C4, or the experimental data. The lack of ‘downwash’ is more severe for LD-PPM cases than PPM cases, which agrees with the observations found in Fig. 5.11-(a). Among the four cases, Case C3 can be regarded as the case whose prediction is the closest to the experimental data. Despite the better agreement than other cases from Case C3, it still suffers from
a slightly lower velocity profile (compared to experimental data) in the wake region.

The Reynolds normal and shear stresses for Case C1 to C4 are presented in Fig. 5.12. Cases with $\alpha = 1.5$ give a ‘spike’ of $<u'u'>$ near the wall at $x/c = 0.65$ while $\alpha = 1.75$ cases follow the experimental trend pretty well at that location. In the wake region, $\alpha = 1.5$ cases give a higher stress levels than their corresponding pairs further away from the wall, but lower when getting closer to the wall. This difference grows smaller when moving downstream further away from the hump. The effect of $\alpha$ values differs for different stress components: $\alpha = 1.5$ works better for $<u'u'>$ because $\alpha = 1.75$ takes the stress value too high, while $\alpha = 1.75$ works better for $<v'v'>$ because $\alpha = 1.5$ gives too elevated values. $\alpha = 1.75$ gives higher $<u'v'>$ values near the wall but the difference is not as significant compared to the other two stresses. As for numerical schemes, PPM cases tend to give a higher value of stresses, which is usually higher than the experimental value away from the wall, for example $<u'u'>$ at $x/c = 0.8$. LD-PPM cases under-predict most of the stress values in the recirculation region near the wall, where PPM cases follow the experimental data well. Most discrepancies from the effect of various $\alpha$ values and schemes occur in the recirculation region. After the flow reattaches (at about $x/c = 1.1$), the difference diminishes, and near-wall stress levels from all cases are close to the experiment. In general, PPM cases follow the experimental data better overall compared to LD-PPM.

The surface friction and pressure coefficients are presented in Fig. 5.13 and Fig. 5.14, respectively. In the region of $x/c = 0.1 - 0.4$, all skin-friction predictions are higher than the experimental data because the model is not capable of capturing the ‘re-laminarization’ effect. Near the top of the hump, cases with $\alpha = 1.5$ (C1 and C2) give lower values of $c_f$, and these drop faster than Case C3 and C4, indicating that the momentum transferred to the surface is insufficient compared to the experiment. Cases C3 and C4, on the other hand, stay near the experimental value for longer (close to $x/c = 0.6$), but Case C3’s prediction is somewhat higher. Before the separation, PPM cases (C1 and C3) give slightly elevated $c_f$ values compared to LD-PPM cases, although the trend is more affected by the chosen $\alpha$ value. After separation from $x/c = 0.7 - 1.0$, all simulations give $c_f$ of smaller magnitude compared to the experiment, but values from PPM cases are closer to the experiment. The choice of $\alpha$ does not seem to make a big difference in this region. The reattachment point prediction from each case can be easily read from the $c_f$ plot, and it is where negative $c_f$ becomes positive. All models give larger recirculation region than the experiment, but Case C3 with $\alpha = 1.75$ and PPM is the closest to the data. PPM cases gives smaller recirculation areas than LD-PPM, and cases with $\alpha = 1.75$ predict earlier reattachment than cases with $\alpha = 1.5$.

In the $c_p$ plot (Fig. 5.14), all models follow the experimental fairly well until near $x/c = 1.3$. $c_p$ from LD-PPM cases grows slower than PPM cases near the top of the hump, and drops earlier too. Cases with $\alpha = 1.75$ follows the experimental data closer than cases with $\alpha = 1.5$. In the pressure
Figure 5.11 Normalized average velocity profiles in the wake of the hump, for cases with $\alpha$ value and scheme variations. (a): Stream-wise velocity, each shifted by multiples of 1.5 units; (b): wall-normal velocity, each shifted by multiple of 0.2 unit.
Figure 5.12 Normalized mean-square velocity fluctuation (Reynolds stress) profiles in the wake of the hump, for cases with $\alpha$ value and scheme variations. (a): Stream-wise stress, each shifted by multiples of 0.15 unit; (b): wall-normal stress, each shifted by multiple of 0.075 unit; (c): shear stress, each shifted by multiples of 0.075 unit.
recovery region \((x/c = 1.0 - 1.5)\), Case \(C3\) has the only distribution that is reasonably close to the experimental data, while Case \(C2\) is the furthest off. Again, similar trend follows: PPM gives a better result than LD-PPM, and a higher value of \(\alpha\) helps too.

![Figure 5.13 Skin-friction coefficient \(c_f\) distribution for cases with \(\alpha\) value and scheme variations. NASA wall-mounted hump.]

To see what the model constant really changes, contours of normalized eddy viscosity \(\mu_t/\mu\) distribution on the hump are presented in Fig. 5.15. At this color-scaling level, the numerical methods does not seem to have a big effect on eddy viscosity other than a slight increase in its value in the LES region for LD-PPM (plot (b) and (d)) when compared to PPM (plots (a) and (c)). The change of \(\alpha\) from 1.5 to 1.75 gives a thicker RANS region where \(\mu_t\) is high. As discussed above, the boundary layer profile for Case \(C1\) and \(C2\) at \(x/c = 0.65\) is under-full, indicating that not enough momentum is transferred to the inner layer. This insufficient momentum transfer must not start right at \(x/c = 0.65\), but from somewhere upstream. Unfortunately, no experimental measurements were taken for the flow-field from the inlet \((x/c = -2.14)\) to this point \((x/c = 0.65)\) to be compared with for the simulations. With this hypothesis of insufficient momentum transfer, which may come from under-resolved eddy structures near the wall on the hump, expanding the region of RANS increases the eddy viscosity in the troubled layer, boosting the momentum transfer, and thus produces a more energized BL.

Fig. 5.16 shows the resolved TKE distribution for Cases \(C1-C4\), and Fig. 5.17 presents the contour plots of the total TKE (resolved TKE + modeled TKE). The resolved TKE level is lower in the boundary layer for cases with LD-PPM when compared to cases using PPM. The lower resolved TKE in the
outer layer prevents the correct amount of momentum being transferred to the inner layer, where the modeled TKE value is similar for different numerical schemes. As a result, the total TKE levels (shown in Fig. 5.17) for cases with LD-PPM are lower, resulting in under-full boundary layer profiles (shown in Fig. 5.11) and thus larger recirculation bubbles (shown in Fig. 5.13). The reason for the initial deficiency in resolved TKE can possibly be related to the SGS eddy viscosity, which is shown in Fig. 5.18. Similar to Fig. 5.15, contour of $\mu_t/\mu$ is plotted in Fig. 5.18, but the color scaling is set so that it represents SGS eddy viscosity better. In addition, because the eddy viscosity in the RANS region is much higher than SGS eddy viscosity, the color is cut off at the value of 1.01 for easier visualization. It is now obvious that cases with LD-PPM carry a higher SGS eddy viscosity in the outer boundary layer, no matter whether $\alpha$ is set to 1.5 or 1.75. The higher SGS eddy viscosity for LD-PPM cases could come from extra numerical fluctuations in the outer layer or even in the wake-like region of the boundary layer. The higher SGS eddy viscosity can in turn retard the fluctuations going into the boundary layer, creating a lower level of resolved TKE. Further investigation may be needed to have a conclusive explanation on this phenomenon that a (supposedly) more accurate scheme (LD-PPM) produces worse boundary layer profile than a more dissipative one.

**Figure 5.14** Surface pressure coefficient $c_p$ distribution for cases with $\alpha$ value and scheme variations. NASA wall-mounted hump.
Figure 5.15 Normalized eddy viscosity $\mu_t/\mu$ contour on the hump, for cases with $\alpha$ value and scheme variations. (a): Case C1 with $\alpha = 1.5$ and PPM; (b): Case C2 with $\alpha = 1.5$ and LD-PPM; (c): Case C3 with $\alpha = 1.75$ and PPM; (d): Case C4 with $\alpha = 1.75$ and LD-PPM.
Figure 5.16 Averaged resolved TKE contour near the top of the hump, for cases with $\alpha$ value and scheme variations. (a): Case C1 with $\alpha = 1.5$ and PPM; (b): Case C2 with $\alpha = 1.5$ and LD-PPM; (c): Case C3 with $\alpha = 1.75$ and PPM; (d): Case C4 with $\alpha = 1.75$ and LD-PPM.
Figure 5.17 Averaged total TKE contour near the top of the hump, for cases with $\alpha$ value and scheme variations. (a): Case C1 with $\alpha = 1.5$ and PPM; (b): Case C2 with $\alpha = 1.5$ and LD-PPM; (c): Case C3 with $\alpha = 1.75$ and PPM; (d): Case C4 with $\alpha = 1.75$ and LD-PPM.
Figure 5.18 Normalized turbulent eddy viscosity contour near the top of the hump, for cases with $\alpha$ value and scheme variations. $\mu_t/\mu$ in RANS region is high, so value above 1.01 is cut off for easier visualization. (a): Case C1 with $\alpha = 1.5$ and PPM; (b): Case C2 with $\alpha = 1.5$ and LD-PPM; (c): Case C3 with $\alpha = 1.75$ and PPM; (d): Case C4 with $\alpha = 1.75$ and LD-PPM.
5.2.3 Effects of Models

From the study of Section 5.2.2, it is discovered that Case C3 with PPM and $\alpha = 1.75$ produces the best results of the four, and hence Case C3 is picked to be compared to other turbulence models. Here in this section, the new model, presented by Case C3, is compared to RANS simulations, previous generations of NCSU’s hybrid model (Choi’s and Gieseking’s models), as well as IDDES.

Fig. 5.19 shows the distribution of skin-friction coefficient from RANS simulations on the two meshes (Case CR1 and CR2), with the benchmark case for the new hybrid model. As shown in the figure, the RANS solutions are very close to each other, suggesting that they are not sensitive to span spacing and size. The value of $c_f$ is predicted lower near the start of the hump ($x/c = 0$) by RANS cases than Case C3, which is still slightly below the experimental value. On the hump, the RANS cases do not capture the ‘relaminarization’ either, but get closer to the experimental data near the top of the hump ($x/c = 0.5$) because Case C3 is over-predicts the level of skin-friction. In the recirculation region, the RANS cases produce a higher-magnitude $c_f$ profile than the experimental data and Case C3. The location of reattachment point predicted by RANS simulations is closer to the experimental value than the hybrid case.

![Figure 5.19 Skin-friction coefficient $c_f$ distribution for 3D RANS cases, compared to Case C3. NASA wall-mounted hump.](image)

Fig. 5.20 contains the plot of surface pressure coefficient predicted by the RANS simulations (Case CR1 and CR2) as well as Case C3. All models here have a reasonable $c_p$ prediction up until the separation point near $x/c = 0.65$. From here, RANS cases are not able to produce a correct level of pressure coefficient in the separated region, while the hybrid model follows the experimental
data well, except a slightly high prediction near the reattachment point \((x/c = 1.1)\), where RANS predictions come back to the experimental value.

**Figure 5.20** Surface pressure coefficient \(c_p\) distribution for 3D RANS cases, compared to Case C3. NASA wall-mounted hump.

Fig. 5.21 compares the averaged stream-wise velocity (plot (a)) and wall-normal velocity (plot (b)) predicted from different hybrid models in the wake region of the hump. The prediction of IDDES (Case C5) and Gieseking’s model (Case C7) are very similar in terms of stream-wise velocity. Both have an under-energized boundary layer on the hump which likely also leads to premature separation like Case C1 and C2. For Gieseking’s model, it is understandable because the original new model with \(\alpha = 1.5\) was tuned to act like Gieseking’s model. On the other hand, Choi’s model (Case C6) and the new model with \(\alpha = 1.75\) (Case C3) have very similar behavior – less recirculation and quicker reattachment. In Fig. 5.21-(b), it can be seen that IDDES actually has a better prediction of \(\langle v \rangle\) than Gieseking’s model, especially near the wall. Again, Choi’s model and the new model are closer to the experimental data, but still under-predict the ‘downwash’ between \(x/c = 0.9 – 1.1\).

Fig. 5.22 include plots of normal and shear Reynolds stresses from the prediction of different hybrid models. In plot 5.22-(a), we can see the ‘spike’ of \(\langle u' u' \rangle\) at \(x/c = 0.65\) from IDDES and Gieseking’s model, but otherwise they have good agreement near the wall with the experimental data. Choi’s model and the new model gives close but slightly higher prediction of \(\langle u' u' \rangle\) near the wall. All models have high prediction of \(\langle u' u' \rangle\) away from the wall in the recirculation region. The wall-normal stress prediction is a little different. In the recirculation region, all models predict it high similar to \(\langle u' u' \rangle\), but IDDES gives the highest \(\langle v' v' \rangle\) down the stream while Choi’s
Figure 5.21 Normalized average velocity profiles in the wake of the hump, for cases with different hybrid models. (a): Stream-wise velocity, each shifted by multiples of 1.5 units; (b): wall-normal velocity, each shifted by multiple of 0.2 unit.
model and the new model stays together close to the experimental data between the other two models at $x/c = 0.9 - 1.0$. Further downstream, the new model and Choi’s model stays next to the experimental data but IDDES and Gieseking’s model is still giving high values. For the Reynolds shear stress $<u'v'>$, all models over-predict its level away from the wall, but give closer predictions near the wall, if not a little low. Gieseking’s model gives the lowest value near the wall.

When looking at the surface friction coefficient $c_f$, we can tell that IDDES gives the lowest prediction of the four on the hump, while Gieseking’s model predicts the latest reattachment point. The new model and Choi’s model gives higher $c_f$ values on the hump, and give the smallest recirculation region. Also, a closer look reveals that between the separation ($x/c = 0.65$) and the end of the hump ($x/c = 1.0$), only IDDES follows the experimental data almost exactly. None of these models is able to capture the ‘dip’ of $c_f$ due to ‘relaminarization’. Fig. 5.24 shows the surface pressure coefficient from the four models. Again, IDDES and Gieseking’s model stays fairly close until the separation, and gives smaller of a magnitude than the new model and Choi’s model on the hump but too big of a magnitude in the recirculation region. The new model gets closer to the experimental data in the separated region, but otherwise Choi’s model is the best of the four.

It is not surprising, after observing the similarity between Choi’s model and the new model (Case C3), that the distributions of the eddy viscosity are actually also very similar, as shown in Fig. 5.25. They both have a strong and thick RANS region with high value of $\mu_t$. On the other hand, Gieseking’s model has a thin RANS layer, similar to new model with $\alpha = 1.5$ (Case C1). Likely the under-resolved structures near the wall affect the results of Gieseking’s model as well, while the new model (Case C3) and Choi’s model have an elevated value of eddy viscosity to support more momentum transfer. Oh the other hand, the IDDES model produces a very different distribution of eddy viscosity on the hump compared to other models. The value of $\mu_t$ predicted from the IDDES model is higher in the outer part of the boundary layer where other models have low values from their SGS model. In the inner layer closer to the wall, IDDES does not provide an elevated eddy viscosity level, but a lower band before reaching sub-layer.

The total turbulence kinetic energy (resolved + modeled TKE) contour plots are shown in Fig. 5.26 to compare the four hybrid models. Choi’s model, again, behaves similar to the new model. IDDES has the lowest TKE level.
Figure 5.22 Normalized mean-square velocity fluctuation (Reynolds stress) profiles in the wake of the hump, for cases with different hybrid models. (a): Stream-wise stress, each shifted by multiples of 0.15 unit; (b): wall-normal stress, each shifted by multiple of 0.075 unit; (c): shear stress, each shifted by multiples of 0.075 unit.
Figure 5.23 Skin-friction coefficient $c_f$ distribution for cases with different hybrid models. NASA wall-mounted hump.

Figure 5.24 Surface pressure coefficient $c_p$ distribution for cases with different hybrid models. NASA wall-mounted hump.
Figure 5.25 Normalized eddy viscosity $\mu_e/\mu$ contour on the hump for different hybrid models. (a): Case C3, the new model; (b): Case C5, IDDES; (c): Case C6, Choi’s model; (d): Case C7, Gieseking’s model.
Figure 5.26 Averaged total TKE contour near the top of the hump, for cases with different hybrid models. (a): Case C3, new model; (b): Case C5, IDDES model; (c): Case C6, Choi’s model; (d): Case C7, Gieseking’s model.
5.2.4 Effects of Span Size and Resolution

Case C1 through Case C7 are simulations conducted on the ‘Fine-Narrow’ mesh which has a span of about three times the boundary layer thickness of the inflow ($\delta_{in}$). For a turbulent boundary layer, 3-D effects are significant, and too small of a span may be at the risk of overly constraining the span-wise eddy structures. Cases C8-C11 represent cases with the ‘Coarse-Wide’ mesh and the ‘Fine-Wide’ mesh. The ‘Coarse-Wide’ mesh doubles the span of the narrow mesh but keeps the number of cells, meanwhile the ‘Fine-Wide’ mesh doubles the span of the narrow mesh but keeps the spacing (thus twice the number of cells). The model constant of $\alpha = 1.5$ was tested on the ‘Coarse-Wide’ mesh with PPM and LD-PPM schemes, and $\alpha = 1.75$ was tested on the ‘Fine-Wide’ and ‘Coarse-Wide’ mesh with PPM scheme.

Fig. 5.27 shows the axial and ground-normal averaged velocity profiles in the wake of the hump for Cases C8-C11. Case C9 with LD-PPM still suffers from an under-energized incoming boundary layer and produces the lowest velocity magnitude for both $<u>$ and $<v>$. Case C8 and Case C10 predicts very similar velocity profiles, though C10 is slightly closer to the experimental value. Case C11 shows the best match of $<u>$ compared to the experimental measurements, but a more positive $<v>$ especially near $x/c = 0.9$. When compare C8 to C1, and C10/C11 to C3 (averaged velocities of C1 and C3 are shown in Fig. 5.11), it can be observed that a wider span improves the velocity prediction. The improvement is greater for cases with $\alpha = 1.5$ even though the span-wise spacing is doubled. For cases with $\alpha = 1.75$, the solution (C3) was better than cases with lower model constant (C1), and the improvement of C10/C11 over C3 is not as significant.

Averaged resolved Reynolds normal and shear stresses are shown in Fig. 5.28. From a general observation compared to the narrow span cases (Fig. 5.12), increasing the span size results in an increase in most Reynolds stresses. For $<u'u'>$, this boost from doubling the span leads to an over-prediction of stress level for most cases, especially for Case C10 where $\alpha = 1.75$. At the same time, Span-spacing reduces $<u'u'>$ level when comparing Case C11 to Case C10. $<v'v'>$ values are also brought too high except for Case C9 which was predicting a low stress level on the narrow mesh and Case C11 which is the closest to the experimental data for $<v'v'>$. Moreover, the model was under-predicting Reynolds shear stresses on most cases on the narrow mesh, and the effect of doubling the span brings the near-wall shear stress level closer to the experiment value. Overall, the stress levels are similar for C8 and C10, except for $<u'u'>$, and LD-PPM again predicts lower stress at the start of the separation. Doubling the span increases stress predictions, and smaller spacing further increase them.

Fig. 5.29 shows the skin-friction coefficient for cases using wide meshes. Cases C8, C10 and C11 are predicting very close distributions of $c_f$, showing high but reasonably well results on the hump.
Figure 5.27 Normalized average velocity profiles in the wake of the hump, for cases with wide meshes. (a): Stream-wise velocity, each shifted by multiples of 1.5 units; (b): wall-normal velocity, each shifted by multiple of 0.2 unit
Figure 5.28 Normalized mean-square velocity fluctuation (Reynolds stress) profiles in the wake of the hump, for cases with wide meshes. (a): Stream-wise stress, each shifted by multiples of 0.15 unit; (b): wall-normal stress, each shifted by multiple of 0.075 unit; (c): shear stress, each shifted by multiples of 0.075 unit.
This was not the case on the narrow mesh (Fig. 5.13), where Case C3 showed higher skin-friction over low prediction of C1 and improves the recirculation region size (still slightly large though). Regarding the size of the recirculation region, the three predictions from PPM scheme both catch the reattachment location very well according to the experimental measurement. This is a large improvement over the narrow-mesh cases where the recirculation region was predicted too large for most models. LD-PPM case still over-predicts the recirculation region size and a lower incoming \(c_f\) value. Surface pressure coefficient distributions are shown in Fig. 5.30. Again cases with different model constant values agree better than on the narrow mesh, and Case C8, C10 and C11 give very good agreement with the experimental data. Values of \(c_p\) in the recirculation region is over-predicted by Case C9, which uses LD-PPM. In short, doubling the span size improves both \(c_f\) and \(c_p\) profiles, and \(c_f\) and \(c_p\) distributions are not sensitive to span-spacing.

![Graph](image)

**Figure 5.29** Skin-friction coefficient \(c_f\) distribution for cases with wide meshes. NASA wall-mounted hump.

Contour plots of normalized eddy viscosity are shown in Fig. 5.31. This figure can be compared to Fig. 5.15. Value of eddy viscosity is higher for ‘Coarse-Wide’ cases (C8 and C9) than ‘Fine-Narrow’ cases (C1 and C2), and the thickness of the RANS region is increased for the Case C8 and C9. This change of hybrid blending function profile is most likely an effect of the span-wise mesh spacing, which can also be observed comparing Case C10 and C11 (C11 has a thicker RANS region than C10). Span size does have significant impact on the blending function profile, as the contour of C3 and C10 are very similar.

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Figure 5.30 Surface pressure coefficient $c_p$ distribution for cases with wide meshes. NASA wall-mounted hump.
Figure 5.31 Normalized eddy viscosity $\mu_t/\mu$ contour on the hump, for cases with wide meshes. (a): Case C8, ‘Coarse-Wide’ mesh, PPM, $\alpha = 1.5$; (b): Case C9, ‘Coarse-Wide’ mesh, LD-PPM, $\alpha = 1.5$; (c): Case C10, ‘Fine-Wide’ mesh, PPM, $\alpha = 1.75$; (d): Case C11, ‘Coarse-Wide’ mesh, PPM, $\alpha = 1.75$. 

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6.1 Simulation Setup: Ahmed Body

The mesh for the Ahmed body is a structured grid, with a combination of orthogonal and c-type meshing strategies. A C-type mesh is wrapped around the blunt body, whereas an orthogonal-type mesh fills the space further away from the body. The slant angle studied in this work is at 25 degrees. Fig. 6.1 shows the mesh for the Ahmed body. The four struts used to support the body in the experiment are not included in the mesh in this study.

The Ahmed body has a length \( L = 1.044 \text{ m} \). The domain length \((X)\) extends \(2L\) in front of the body, and \(5L\) downstream from the body. The height of the domain \((Z)\) is \(1.4\) m, and the width \((Y)\) is \(1.87\) m with the body located in the center. The wall-normal spacing starts at \(\Delta_{wall} = 5.0 \times 10^{-6} \text{ m}\) for both the body and the floor. The surface mesh has a spacing of \(\Delta \leq 5\) mm, and is more clustered near corners. The cell size is kept under 6 mm in the wake region, which extends to \(1.0L\) behind the body before the spacing starts to relax. The total number of cells is 28.07 million, and the mesh is broken into 750 blocks, with one CPU assigned to each block for calculation. The floor is a non-slip wall, while the sides and top boundaries are represented as slip-wall (symmetry). The inlet has a prescribed uniform velocity, and the outlet is an extrapolation far-field with constant pressure. The flow is assumed to be incompressible. PPM is used as the reconstruction schemes, and the hybrid model constant \(\alpha\) is set to 1.75. Physical time step of \(\Delta t = 1 \times 10^{-5}\) s is used for time advancement. For each hybrid RANS/LES case, statistics were taken for 10 flow-through times based on body
Figure 6.1 The mesh for the Ahmed body case. (a): Mesh on the center-plane near the body. (b): Surface, floor, and stream-normal mesh near the front of the body. (c): Surface, floor, and stream-normal mesh near the slant of the body. Note that extra non-structured lines are artifacts from the nonparallelism between grid and visualization slices.
length after a 5 flow-through transient time of calculation.

### 6.2 Results: Ahmed Body

Fig. 6.2 shows the streamlines on the center plane near the slant and in the wake of the Ahmed body, based on averaged velocity. The experimental data is also shown in the figure. When compared to the experimental measurement, the RANS simulation predicts too large of a recirculation region, even with two separate vortices near the ground, which is not observed in the experiment. There is no large separation on the slant, only a very light separation bubble. On the other hand, the new model gives too small of a recirculation region, and the flow separates from the slant from the beginning. The new model with the 'g-function' (Eq. 2.85) gives a longer attachment on the slant than the new model only, while the recirculation region in the wake is larger than experiment but smaller than RANS prediction.

![Figure 6.2 Streamlines on the center planes of Ahmed body. (a): Experiment [Lie02]; (b): RANS; (c): new model (avg. flow); (d): new model with 'g-function' turned on (avg. flow).](image)
Fig. 6.3 presents the mean vortex structures predicted by BSL RANS, visualized by the dimensionless ‘swirl-strength’ (SS) criterion for the Ahmed body at SS = 120. Fig. 6.4 presents the instantaneous vortex structures predicted by the new hybrid model, with and without the ‘g-function’, also visualized by the dimensionless ‘swirl-strength’ criterion on a higher level of SS = 600. The figure shows the different sizes and locations of the separation bubbles in each case. The new model with ‘g-function’ in has a delayed separation and less resolved eddies near surfaces than the original new model.

Fig. 6.3 Mean vortex structures around the Ahmed body from RANS simulation. Visualized by the swirling-strength criterion (SS = 120).

Fig. 6.5 compares the averaged stream-wise velocity from the two hybrid cases to RANS and experimental measurements. Fig. 6.5-(a) shows the velocity profiles in the boundary layer on the slant. It shows that the profile is not full for the case with the new model on the top of the body before reaching the slant, and an early and massive separation follows. Including the ‘g-function’ improves the result on the first half of the slant but fails to predict the reattachment of the flow on the second half of the slant. RANS follows the experimental data better than the two hybrid cases on the slant. Fig. 6.5-(b) zooms out and gives the velocity profiles in the wake of the body. In the wake, the hybrid cases follows the experimental trend better than RANS, which over-predicts the recirculation size.
Figure 6.4 Instantaneous vortex structures around the Ahmed body from hybrid LES/RANS simulation. Visualized by the swirling-strength criterion ($SS = 600$). (a): New model; (b): New model with Salazar’s fix (‘g-function’).
and velocity magnitude near the bottom wall. The averaged ground-normal velocity profiles on the slant and in the wake are presented in Fig. 6.6-(a) and Fig. 6.6-(b), respectively. RANS is still closer to the experimental data than the hybrid cases on the slant, and the new model with ‘g-function’ works better than the new model only at the start of the slant before the delayed separation happens. In the wake region, RANS failed to predict the correct profile of the smaller separation vortex from the bottom of the body, while the new model over-predicts the ‘downwash’ while new model with ‘g-function’ does not have enough vertical velocity.

The Reynolds normal stress levels, represented by RMS velocity fluctuations, are shown in Fig. 6.7 and Fig. 6.8. The resolved stream-wise RMS velocity fluctuations \( u_{\text{RMS}} \), shown in Fig. 6.7, is too low near the wall for the new model case on and even before the slant. The new model with ‘g-function’ shows good agreement with experiment at the start of the slant but over-predicts the measured level further downstream due to too its level being too high down-stream in the separation region. As for \( w_{\text{RMS}} \), both hybrid models under-predict its level on the top of the body and at the start of the slant, as shown in Fig. 6.8-(a). In the wake region, the predictions from the two hybrid models are in adequate agreement with the experiment, with the case with ‘g-function’ behaving more ‘flat’ than the new model only. Fig. 6.9 compares the Reynolds shear stress \( \langle u'w' \rangle \) from the two hybrid models to RANS modeled quantity and experimental measurements. On the slant, both hybrid models give too low of a shear stress near the wall, but too high away from the wall (because of incorrect predictions of the size of the separation bubble). The RANS model gives too little shear stress in the initial separation bubble, but follows the experimental value better than the hybrid models in the reattachment region. In the wake region, the new model with ‘g-function’ agrees better with experimental data in the profile near the ground, while over-predicting the peak fluctuation value. The new model only predicts the top peak better but the stress level is too high near the bottom part. RANS again failed to give a reasonable prediction in the bottom of the wake.

As we can see from previous observations, the new model by itself is not able to produce a well-developed, fully-energized turbulent boundary layer at the top surface of the Ahmed body. A closer look reveals that the surface meshing in \( x \) and \( y \) direction is in the same order of the boundary layer thickness \( \delta \) on the top of the body. In other words, the mesh on the Ahmed body, away from the corners (where the mesh is more clustered), is too coarse to resolve the turbulent fluctuations at the level the new hybrid model expects it to. This under-resolved flow leads to incorrect momentum transfer and thus a less energized boundary flow that is subject to early separation. This mechanism is similar to ‘grid-induced separation’ (GIS). The ‘g-function’ acts as a guarding factor on the outer-layer scale to help move the transition of \( \Gamma \) further away from the wall, helping to alleviate this situation. Fig. 6.10 compares the hybrid blending function distribution on the center plane on the top of the Ahmed body from the case without (a) and with (b) the ‘g-function’. The RANS region is
Figure 6.5 Averaged stream-wise velocity profiles (a): on the rear slant, and (b): in the wake.
Figure 6.6 Averaged ground-normal velocity profiles (a): on the rear slant, and (b): in the wake.
Figure 6.7 Resolved stream-wise RMS velocity fluctuation profiles (a): on the rear slant, and (b): in the wake.
Figure 6.8 Resolved ground-normal RMS velocity fluctuation profiles (a): on the rear slant, and (b): in the wake.
Figure 6.9 Reynolds shear stress profiles (a): on the rear slant, and (b): in the wake. Resolved stress for hybrid LES/RANS cases and modeled stress for RANS.
thicker and extends more into the boundary layer, where eddy length scales are expected to be larger and easier to resolve. This modification also adds a higher level of eddy viscosity to the inner-layer to promote more momentum transferred to the sub-layer, as shown in Fig. 6.11 where the normalized eddy viscosity level from the two hybrid cases are compared. The result of the 'g-function' is a more attached flow over the slant, and the separation is delayed.

Although the solution with 'g-function' can be considered improved in some aspects (recirculation size, BL profile, etc.) over the unmodified case, it is still not at an level that can be treated as a accurate simulation compared to the experiment. A more reasonable way to improve the solution of the new hybrid model should be to build a mesh with refined surface spacing to the degree that most eddy structures in the prescribed LES region can be resolved and sustained by a turbulence-generation method. For example, the surface mesh can be in the order of $\delta/10$. However, for a structured mesh, this refinement can easily generate a mesh that is one to two orders of magnitude larger than the current one. Meanwhile it can still be feasible to build an unstructured mesh with adequate surface resolution and reasonable total cell count for a hybrid LES/RANS simulation.
Figure 6.10 Hybrid blending function $\Gamma$ distribution on the body. (a): New model; (b): new model with the ‘g-function’.
Figure 6.11 Normalized eddy viscosity distribution on top of the body. (a): New model; (b): new model with the ‘g-function’.
In this study, a new hybrid large-eddy simulation / Reynolds-averaged Navier-Stokes (LES/RANS) turbulence model has been presented. Assessment and tests of the new model have been conducted on different categories of cases. The reason for a hybrid LES/RANS model is to reduce the near-wall computational cost of LES, while resolving unsteady turbulence structures elsewhere. The goal of the new model development is to come up with a hybrid LES/RANS model that reverts to a viable SGS model far away from surfaces, preserves the composite structure of a turbulent boundary layer, and responds to changes in turbulence length scales while using only local and instantaneous information. To determine the transition point between RANS and LES regions, an estimated outer-layer length scale information is needed. While other models obtain the outer-length scale in LES region through case-specific pre-calibration, ensemble averaging, or other non-local or non-instantaneous means, the new model utilizes a one-equation RANS-type eddy viscosity transport (EVT) model based on the unsteady flow field to estimate the outer-length scale. The use of this transported ‘EVT’ quantity is solely for the determination of the hybrid blending function, thus there is no risk of over-constraining the LES field with a RANS component. Moreover, the amplified production of EVT eddy viscosity due to high fluctuating strain rates is compensated by a modification of the destruction term based on an estimated von Kármán length scale.

Three flat-plate boundary layer cases have been tested to assess the new model. Two are subsonic (assumed incompressible) flows with low and moderate Reynolds numbers, and one is a supersonic flow with a Mach number of 2.32. For the low Reynolds number incompressible flow case, a wall-
resolved LES study was also conducted to evaluate the effects of numerical schemes and different SGS models. The study on numerical schemes reveals that LD-PPM gives a better boundary layer profile than the more diffusive PPM scheme and ENO-PPM scheme. The difference between Lenormand's 'Mixed-Scale' (MS) model [Len00], Vreman's model [Vre04], and Nicoud's 'Wall-Adapting Local Eddy-viscosity' (WALE) model [ND99] is mainly in the viscous sub-layer, whereas in the hybrid framework, the near-wall turbulence effect is modeled by a RANS closure. As such, Lenormand's MS model is then chosen to be used with the new model for its simplicity. The new model is able to predict a reasonable boundary layer profile when compared to the experimental data, and responds similar to Gieseking's LES/RANS model (an earlier NCSU formulation). For the moderate Reynolds number case, more parametric studies on the new model were carried out, including effects of RANS models and intermittency corrections, effects of model constant and numerical schemes, and comparison to other hybrid models. The intermittency correction terms broaden and shift the position of the hybrid blending function, improving the response of the SST model but making the viscosity level too high for the BSL model. A larger model constant $\alpha$ value (1.75) is found to improve the results from the original value (1.5) taken from Gieseking's model. PPM gives better predictions than LD-PPM for the new model, which underestimates TKE in the inner part of the log-layer and below. An earlier generation of NCSU's hybrid model that requires a pre-calibration (Choi's model) also suffers from the same issue of low TKE level when using LD-PPM and provides results similar to the new model. On the other hand, IDDES gives better response with LD-PPM, while giving too high of a TKE spike near transition when using PPM. The new model with PPM, Choi's model with PPM, and IDDES with LD-PPM all give reasonably close predictions of the boundary layer compared to experimental data. For the supersonic FPBL, the new model is tested with BSL and SST RANS closures, along with one form of an intermittency correction. Similar to the incompressible FPBL results, the intermittency correction shifts and broadens the hybrid blending function distribution, causing too low of a velocity profile for BSL but improving predictions when using SST. The simulations do not follow the experimental data in the buffer layer, but have good agreement in the upper log-layer and above. Normalized shear stress predictions are not entirely in agreement with the experimental measurement, which does not approach unity towards the wall.

A couple of statistically 2-D flows on non-FP geometries have been simulated using the new model. For the Aérospatiale "A-Airfoil" case, the new model is compared to Choi's model (which requires case-specific pre-calibration) and Gieseking's model (which needs ensemble averaging), as well as experimental measurements. Surface pressure and friction coefficient predictions from the new model are close to Choi's model and agree well with experiment. Stream-wise velocity predictions from the new model are close to those from Choi's model, and closer to the experimental profiles than those from Gieseking's model. The axial Reynolds stress prediction is low for all models
near separation. For the NASA wall-mounted hump case, the response from the new model with a combination of $\alpha = 1.75$ and PPM surpasses that with $\alpha = 1.5$ and/or LD-PPM. Mesh resolution is considered one of the reasons that a thicker RANS region (form using a higher $\alpha$) helps the boundary layer profile. When looking at other models, RANS simulations cannot get the surface pressure in the separation zone close to the experimental data. Both Gieseking's model and IDDES provide an under-energized boundary layer before separation and yield too large of a separation bubble. Choi's model is the closest to the new model. No model is able to catch the 'relaminarization' on the first half of the hump. Other than the default mesh, two meshes of a wider-span are tested for the effect of span size and spacing. The wide-span cases provide a better velocity, $c_f$, and $c_p$ profiles than their corresponding narrow-span cases. Normal Reynolds stress prediction is too high for the wide-span cases, but the near-wall Reynolds shear stress profiles are improved. A coarser spacing in the span direction is likely to move the hybrid blending function away from wall and produce a higher eddy viscosity distribution.

The Ahmed body is a full 3-D flow case, for which RANS, the new model, and the new model with Salazar's modification ('g-function') are tested. The 'g-function' modifies the estimated turbulent length scale in the situation where the mesh is too coarse to resolve outer-layer eddies. The BSL RANS simulation provides good agreement with experiment on the slant in terms of separation bubble size and reattachment, but in the massively separated region, it gives incorrect vortex structures and the recirculation region is too large. Flow from the new model separates from the start of the slant without reattaching, and the recirculation region is too small. The pre-mature separation is thought to come from the under-resolved near-wall eddy structure on top of the body. Adding the constraint from the 'g-function' delays the transition from RANS to LES, and does improve the boundary layer profile. However, the separation on the slant is delayed, and occurs on an incorrect part of the slant. In the wake region, both hybrid simulations show improved results, relative to RANS.

In conclusion, a newly developed hybrid LES/RANS turbulence model has been presented in this work. It only uses local and instantaneous flow field information to determine the transition between RANS and LES formulations. From the tested FPBL and statistically 2-D cases, the performance of the new model is at least as good as that of previous generations of NCSU’s hybrid models which require either case-specific pre-calibration or ensemble averaging, and is also comparable to a current ‘state-of-art’ hybrid model, the IDDES model. Though meshing requirement is relaxed from WRLES for hybrid LES/RANS, it still needs to be taken into consideration to have a reasonable simulation with the new model.

There are a few research areas that the author thinks may help further the understanding of the capabilities of the new model. The first one is the effect of numerical methods. For the WRLES case and for IDDES on the moderate Reynolds number FPBL case, a scheme with lower numerical
diffusion (LD-PPM) produces a better result than PPM. The opposite happens to the new model. Also the prediction from IDDES with LD-PPM on the hump case is not as good as the new model with PPM (they were close on the FPBL case). A study can be done comparing LD-PPM and PPM on IDDES for the hump case too. Because of the different impact of numerical methods on LES and RANS regions, we need to determine if the transition location can influence the effect of numerical methods, and if there is an optimum transition point where we can take more advantages of the accuracy of low-diffusion methods. Secondly, the new model, in theory, is able to respond to boundary-layer structural changes, and a test on a case with a time-varying ensemble-mean boundary-layer is necessary to demonstrate this capability of the new model. One example is an airfoil undergoing pitching up and/or down motion. A previous study investigated the performance of Gieseking’s model on NACA 0012 airfoil under conditions of dynamic stall [Ke14]. The new model may be expected to outperform the previous model as it eliminates the need for ensemble averaging. Last but not least, the Ahmed body case in this study suffers from an under-refined surface mesh on the body. The overall mesh size (number of cells) is the main constraint because REACTMB uses structured meshes, in which it is not easy to apply isotropic regional refinement parallel to the object's surface normal. To implement WMLES type hybrid LES/RANS models, like the new model, on a full 3-D geometry with enough resolution, an unstructured mesh with prism near-wall layers may be a better option. If such a case is set up, it will also be possible to assess the performance of the new model on unstructured meshes, which has never been done before.
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APPENDIX A

Streamlines and Reynolds Stress Contour Plots for the Hump Case
Figure A.1 Simulation results of Case C1 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.2 Simulation results of Case C2 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.3 Simulation results of Case C3 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.4 Simulation results of Case C4 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.5 Simulation results of Case C5 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.6 Simulation results of Case C6 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.7 Simulation results of Case C7 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.8 Simulation results of Case C8 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.9 Simulation results of Case C9 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.10 Simulation results of Case C10 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.
Figure A.11 Simulation results of Case C11 for the NASA wall-mounted hump case. (a): Streamline; (b): stream-wise velocity; (c): ground-normal velocity; (d): stream-wise Reynolds stress; (e): ground-normal Reynolds stress; (f): Reynolds shear stress.