In this dissertation, we develop distributed control designs for oscillation damping in wide-area power system networks. The power system, with a state-feedback communication graph for damping control, is treated as a cyber-physical system (CPS). On this CPS, the following critical cyber-physical constraints are considered - the actuation constraints on the generator excitation controllers (physical constraint), and the communication constraints on the state-feedback architecture for control (cyber constraint). Using the fact that the wide-area oscillations in power systems are caused by the slow inter-area frequency modes, we design online disturbance-aware optimal controllers which promote sparsity in the feedback communication network by only including the most influential generators in its topology, based on the post-disturbance state of the system. Hence the proposed communication topology is adaptive with respect to the location and strength of the incoming disturbance.

In Chapter 2, we first propose a three-step strategy for decentralized state estimation of all generators in the power system, using measurements from strategically placed Phasor Measurement Units in the power grid. In Chapter 3, we design a centralized Model Predictive Controller (MPC) for selective modal damping in the power system. The MPC is designed in frequency domain so that the control energy can be focused on only the most excited inter-area modes. Next, in Chapter 4 we identify the sets of generators which are influential in damping of the most excited inter-area oscillation modes. This is done by analyzing the post-disturbance state of the system, which in turn depends on the unknown strength and location of the incoming disturbance. A sparse feedback communication topology is constructed based in these sets of influential generators.

In Chapter 5, we design a distributed MPC such that both the cyber and physical constraints on the CPS are respected. Using a distributed cloud architecture, details on the implementation of the controllers are provided. In Chapter 6, we relax the actuator constraints on the controllers, and design a sparse Linear Quadratic Regulator (LQR) with guarantees on the closed-loop stability. Sparsity is promoted in the following two ways. First, structural constraints on the feedback matrix are imposed on the LQR problem by involving only the influential generators for feedback control. Secondly, $\ell_1$-regularization of the LQR cost is done to promote further sparsity within the communication links. In Chapter 6, we analyze the closed-loop CPS resiliency to Denial-of-Service (DoS) cyber attacks on the communication network. It is shown that such DoS cyber attacks can be mitigated by redesigning our sparse controller online, i.e. right after the attack is detected.
Online Distributed Optimal Control Designs for Wide-Area Power System Networks

by
Abhishek Jain

A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Electrical Engineering

Raleigh, North Carolina
2018

APPROVED BY:

____________________________________  ______________________________________
H Troy Nagle                                    Edgar Lobaton

____________________________________  ______________________________________
David Papp                                      Aranya Chakrabortty
                                                Chair of Advisory Committee
DEDICATION

To my fiancé Krysta.
BIOGRAPHY

Abhishek Jain was born in New Delhi, India in 1988. He received his B.Tech. degree in Electronics and Communication Engineering from Maharshi Dayanand University, Haryana, India in 2010, and the M.S. degree in Electrical & Computer Engineering from Colorado State University, Fort Collins, CO, USA, in 2013. In 2014, he started his Ph.D. in Electrical & Computer Engineering at North Carolina State University. He is a member of the Institute of Electrical and Electronics Engineers (IEEE). He holds internships from Corning Incorporated, NY, and NovaTech LLC, KS. His research interests include optimal control theory, constrained optimization, wide-area control of power system networks, and applied data science.
ACKNOWLEDGEMENTS

First I would like to thank my advisor Dr. Aranya Chakrabortty for his help and guidance throughout my four years at NC State. His patience, attention to detail, and ability to see the ‘big picture’, had been beneficial to me not only in my thesis research work, but also to become a better engineer. I also want to thank Dr. Emrah Bıyık for his collaboration on nearly every aspect of my dissertation work. His caring demeanor and invaluable career advice has helped me immensely in my PhD journey, for which I will always be grateful.

I am thankful to Dr. Edgar Lobaton, Dr. David Papp, and Dr. Troy Nagle for their service on my doctoral committee. Their comments and suggestions have improved the quality of this thesis. I would also like to thank Dr. Dror Baron, whose wonderful course in Data Science has opened numerous career opportunities for me. I always look forward to our insightful discussions on various topics, which can range anywhere from stock market analysis to socioeconomics.

I thank my FREEDM labmates and friends Nan Xue, Seyedbehzad Navabi, Jianhua Zhang, Souvik Chandra, Mang Liao, Nandini Negi, Kathleen Sico and Sayak Mukherjee for their friendship, support and numerous discussions, technical or otherwise. I will always cherish the memories from our time together at various conferences.

Although not engaged in my dissertation work, my summer internships at Corning Inc., NY have a played a major role in my progression to a well-rounded research engineer. I give special thanks to Dr. Siam Aumi, my supervisor and friend at Corning, for assigning me the most interesting and challenging projects in industrial controls that I could have hoped for.

I thank my parents, Anil and Shail Jain, for their love and unwavering support during these past four years of graduate school. Finally, I thank my lovely fiancé Krysta Jimenez for her love and encouragement. You make my life colorful.
# TABLE OF CONTENTS

## LIST OF TABLES .......................................................... vii

## LIST OF FIGURES ........................................................ viii

### Chapter 1 Introduction ................................................. 1
  1.1 Literature Review .................................................... 1
  1.2 Contributions ......................................................... 3
  1.3 Thesis Organization ................................................... 3

### Chapter 2 Power System Model & State Estimation .................. 5
  2.1 Introduction .......................................................... 5
  2.2 Nonlinear Power System Model ....................................... 7
  2.3 State Estimation ....................................................... 8
    2.3.1 PMU Placement for Observability .............................. 8
    2.3.2 Generator Bus Phasor Estimation .............................. 12
    2.3.3 Generator Dynamics State Estimation ....................... 16
  2.4 Linearized Model ..................................................... 18
  2.5 Conclusions .......................................................... 20

### Chapter 3 Centralized MPC for Wide-Area Control ............... 21
  3.1 Introduction .......................................................... 21
  3.2 Effect of Clustering ................................................ 23
  3.3 Model Predictive Control Design ................................... 25
    3.3.1 MPC Formulation for Damping ................................ 26
    3.3.2 Selective Discrete Fourier Transform ...................... 28
  3.4 Predictive Tuning of SDFT Window ................................ 30
  3.5 Simulation Results .................................................. 30
  3.6 Conclusions .......................................................... 33

### Chapter 4 Online Modal Analysis .................................. 35
  4.1 Introduction .......................................................... 35
  4.2 Post-Disturbance Modal Participation ............................. 35
  4.3 Feedback Communication Topology ................................ 37
  4.4 Conclusions .......................................................... 39

### Chapter 5 Distributed MPC for Wide-Area Control .............. 40
  5.1 Introduction .......................................................... 40
  5.2 Control Objective .................................................... 42
  5.3 Communication Architecture for Control ......................... 43
    5.3.1 Sparse Communication Architecture .......................... 44
    5.3.2 Cyber-Physical Architecture ................................. 46
  5.4 Distributed MPC ...................................................... 47
    5.4.1 Distributed Controller Prediction Modelling .............. 47
## Chapter 6 Sparse LQR for Wide-Area Control

6.1 Introduction .................................................. 64
6.2 Control Objective ............................................ 65
6.3 Construction of Feedback Structure ....................... 67
6.4 Structurally Constrained Optimal Control ............... 68
   6.4.1 Discrete Linear Quadratic Regulator ................ 68
   6.4.2 Generalized Riccati Equation Method ............... 69
   6.4.3 Algorithm for Solution of $P$ ......................... 70
   6.4.4 Simulation Results .................................... 72
6.5 $\ell_1$-Regularized Sparse LQR ......................... 75
   6.5.1 Controller Design ................................... 75
   6.5.2 Simulation Results .................................. 81
6.6 Conclusions .................................................. 85

## Chapter 7 Closed-Loop Resiliency Analysis

7.1 Introduction .................................................. 87
7.2 Cyber-Attack Mitigation ................................... 87
   7.2.1 Closed-loop Network Resiliency ..................... 87
   7.2.2 Resiliency-based Selection of $\mu$ ................ 88
   7.2.3 Post-Attack Controller Redesign ................... 89
   7.2.4 Simulations ......................................... 90
7.3 Conclusions .................................................. 92

## Chapter 8 Conclusions & Future Research

8.1 Conclusions .................................................. 95
8.2 Future Research Directions .............................. 96

## References ..................................................... 97

## Appendices ...................................................... 107

Appendix A Chapter 2 ........................................... 108
   A.1 Construction of $\chi^k, \bar{\chi}^k, z^k, \bar{z}^k$ .......... 108
Appendix B Chapter 3 ........................................... 109
   B.1 Complex Frequency Weighting Matrix ................. 109
   B.2 Linearized Laplacian for Power Systems .......... 110
LIST OF TABLES

Table 2.1 Comparison results for the OPP algorithms for different power system models. 11
Table 3.1 Comparison of 0.633 Hz mode magnitudes (max. FFT stems) for all generators 34
Table 5.1 dMPC simulation results for all case studies. ............................ 58
Table 6.1 Modal coefficients for the MP form of NE system with parallelization coefficients highlighted, with $\mu_1 = 0.76$, $\mu_2 = 1.79$ and $\mu_3 = 3.17$. ........................................ 73
Table 6.2 Linear CL simulation results using the three designed controllers, and the offline controller from [1]. .......................... 75
LIST OF FIGURES

Figure 2.1 NPCC power system network with highlighted utility areas, and PMU buses identified from solving (2.6) in red. .................................................. 10
Figure 2.2 π model of a power system transmission line. .................................. 12
Figure 2.3 4-machine power system model. ..................................................... 14
Figure 2.4 NP-distributions for complex PMU voltage pseudo-measurements noise, for Gens. 1,2. ................................................................. 14
Figure 2.5 Gaussian noise distributions for the estimated polar states at Gens. 1,2. .. 16
Figure 2.6 Decentralized UKF state estimation results, showing the time-evolution of the four estimated states for generator 1. ................................. 18

Figure 3.1 FFT plot for Generator 5 output (PSS-only) ...................................... 24
Figure 3.2 PSS-only output response for all generators ...................................... 25
Figure 3.3 PSS-only output response for all generators ...................................... 26
Figure 3.4 IEEE 39-bus New England power system model ............................... 31
Figure 3.5 SDFT-MPC output response for all generators (rotor speed deviations) .. 32
Figure 3.6 Comparison of post-fault speed deviation for Gen. 4 ........................... 33
Figure 3.7 Comparison of post-fault FFT magnitude of Gen. 4 output ................... 34

Figure 4.1 Modal Decomposition of Wide-Area Power System .......................... 39

Figure 5.1 Architecture of the proposed distributed control system, shown on a five-generator power system example, following (4.6). Subfigure (a) shows the physical interconnections between generators in the Kron-reduced form. CCO receives $\hat{x}_0$ from all generators and using the MP matrix decides the communication architecture, within the time-steps $k = 0:k^*$. CCO then informs all generators about the communication topology. Subfigures (b) and (c) show state and control communications respectively for $k \geq k^*$, with the three dMPC controllers in feedback. The two identified modal areas are highlighted in red and blue. .................................................. 48

Figure 5.2 Cyber-Physical Architecture for dMPC, shown on the five-generator example, following (4.6). For this example, since it is assumed that each generator bus has a PMU installed, the output of the $i^{th}$ PDC will be $\hat{V}_i$ as the generator bus voltage $|V_i|$ and bus angle $\theta_i$, and $\hat{I}_i$ as the vector of all phasor line currents $[|I_1|, \phi_1, |I_2|, \phi_2, \ldots]$ measured on all transmission lines connected to the generator bus. The cloud-to-cloud communication links are given as: $\mathcal{I}_{21}(k) = \mathcal{I}_{32}(k) = \{\hat{x}_2(k), \hat{u}_2(k-1)\}$, $\mathcal{I}_{51}(k) = \{\hat{x}_5(k)\}$, $\mathcal{I}_{12}(k) = \{\hat{x}_1(k), \hat{u}_1(k-1)\}$, $\mathcal{I}_{52}(k) = \{\hat{x}_5(k), \hat{u}_5(k-1)\}$, $\mathcal{I}_{23}(k) = \{\hat{x}_3(k), \hat{u}_3(k-1)\}$ and $\mathcal{I}_{15}(k) = \{u_5(k)\}$. .................................................. 49

Figure 5.3 The one-line diagram of the NPCC model shows the utility areas in background colors, the three faults considered for the three cases at lines connecting buses 10-11, 45-46 and 119-120. Modal areas for Case Study I are also shown, enclosed in dotted boundaries. ................................. 54
Figure 5.4 MP matrices are shown for the three case studies in subfigures (a), (b) and (c) respectively. x- and y-axis represent generator and oscillation mode indices respectively, and z-axis represents modal residues. 38 oscillation modes are chosen such that these modes all have frequencies less than 1.5 Hz, and damping factors less than 0.3.

Figure 5.5 Dynamic state estimation for the five states of Gen. 1 with the decentralized UKF.

Figure 5.6 Open-loop frequency response for all rotor speeds. SDFT windows for control design are highlighted.

Figure 5.7 Subfigure (a) and (b) shows the comparison of rotor speeds for generators 6 and 8 respectively, for the three cases of open-loop, closed-loop with dMPC controller and closed-loop with a centralized MPC controller. Subfigure (c) shows the comparison of control input voltages for generator actuators 6 and 8 respectively, for the two cases of dMPC controller and a centralized MPC controller.

Figure 5.8 (a) Open-loop vs. (b) dMPC closed-loop comparison is shown for electrical power outputs, in p.u., of all generator buses.

Figure 6.1 Timing diagram for solution of $P$.

Figure 6.2 Controller sparsity structures for the NE system obtained using Algorithm 4 for the three parallelized cases.

Figure 6.3 Linearized system output response for OL and CL scenarios with indicated controllers.

Figure 6.4 (a) Open-loop and (b) closed-loop frequency response for all rotor speed outputs of all NE system generators. Excited inter-area oscillation modes are highlighted.

Figure 6.5 Nonlinear simulation response for open-loop and closed-loop system with sparse-optimal feedback $K^* \in \Omega_2$. Subfigures (a)-(b) compare the rotor speeds (in pu) for Gens 1 and 10, and subfigures (c)-(d) compare the generated electric power (in pu) for Gens 3 and 4.

Figure 6.6 Feedback structures for varying values of $\mu$ and $\eta$. It is seen that sparsity increases with an increase in the value of $\mu$ and decreases with an increase in the value of $\eta$.

Figure 6.7 Implementation results for our sparse controller, with sparsity parameters: $\mu = 0.8$ and $\eta = 0.1$. Comparison of open- versus closed-loop responses is shown. The top figures show rotor speed response for generators 4, 5 and 6. The bottom figures show electrical power outputs of generators 9 and 10.

Figure 6.8 Sparsity tuning parameters versus the performance loss index.

Figure 6.9 Comparison of communication topologies for the controller in [1], with our sparse controller.

Figure 6.10 Comparison of control design times for the $H_2$ controller versus the GDARE controller. Note the logarithmic-scaled vertical axis.

Figure 7.1 Plot of $f_0(\mu_i)$ and $f_{RI}(\mu_i)$, parametrized by the unique values of residue threshold $\mu_i$. 
Figure 7.2 Communication topologies for the attacked controller and the redesigned controller, for Case Study I. .............................................................. 93

Figure 7.3 Case Study I: Rotor speed response for all generators, following a three-phase fault at $t = 0$ controlled with $K^I$; DoS attack at $t = 10$ causing system instability; and controller redesign with $K^{III}$ to regain stability at $t = 60$. . . 93

Figure 7.4 Communication topologies for the attacked controller and the redesigned controller, for Case Study II. .............................................................. 94

Figure 7.5 Case Study II: Rotor speed response for all generators, following a three-phase fault at $t = 0$ controlled with $K^I$; DoS attack at $t = 10$ causing system instability; and controller redesign with $K^{III}$ to regain stability at $t = 25$. . . 94
Chapter 1

Introduction

1.1 Literature Review

Over the past decade, significant increase in transmission expansion and renewable integration in the US power grid have forced power system operators to look beyond the traditional mindset of controlling the grid using local control methods, and transition to wide-area control (WAC) using synchronized phasor measurements available from Phasor Measurement Units (PMUs). One of the most commonly known application of WAC is to improve damping of power flow oscillations in small-signal models of power systems by employing state exchange between distant generators through a wide-area communication network. An enormous literature already exists for damping control of synchronous generators [2, 3, 4] using local output feedback via power system stabilizers (PSS) and Flexible Alternating Current Transmission System (FACTS) devices. These controllers are known to damp fast oscillation modes quite satisfactorily, but they often fail to improve the damping of low-frequency inter-area oscillations which arise from clustering of electric power generation units in the grid [5, 6].

Recent papers such as [7, 5, 8] have shown that WAC can be a promising solution to the inter-area oscillation damping problem. WAC employs global signals from the grid (PMU measurements and/or generator states) as feedback to compute supplementary control actions for either the excitation control systems of the generating stations, or the power electronics devices such as the aforementioned FACTS. Modern control theory methods such as optimal control [1, 9, 10] and robust control [4, 11, 12] have been proposed in literature for design of WAC for power systems. The robust control methods usually employ a reduced-order transfer function model of the power system, and hence ignore the transient characteristics of power flows in the system, hence leading to sub-optimal control. On the other hand, the optimal control designs are usually done with the help of a small-signal state-space model, which requires state feedback from potentially thousands of generators in the grid. Since only bus voltages
and currents can be measured with PMUs, the implementation of these designs faces a major roadblock. A centralized state estimator such as the one proposed in [13, 14] is also not practical due to latency and security issues for wide-area communication. In [15], a decentralized state estimator is proposed but requires PMUs to be installed at every generator bus. We tackle this problem by proposing a decentralized state estimation strategy which first gives a PMU placement algorithm such that not all generator buses are required to have PMUs, and then uses the algorithm in [15] to estimate the generator states.

It is noted that the aforementioned optimal control methods are usually done in time-domain to minimize the future state or output trajectories. Hence these methods do not take advantage of the decoupling property of the inter-area modes [3], and waste their control energies trying to damp the entire frequency spectrum, most of which is already taken care of by the installed PSSs. In this thesis we propose control designs that are designed in the frequency domain to specifically damp the inter-area modes. We show that using a Model Predictive Control (MPC) approach, our strategy can result in better damping performance as compared to a traditionally designed MPC. The online nature of the MPC design allows us not only to target the most excited inter-area modes, but also incorporates the actuator constraints in the design.

Another drawback with most of the optimal control designs for WAC in literature is that they usually result in a centralized implementation of the controller, i.e. the cyber graph for feedback communication is a dense graph. For WAC this can prove to be very prohibitive due to latency and network congestion issues. Distributed control strategies for WAC such as [1, 8] have been proposed in literature to counteract this problem. These designs, however, are completely agnostic of the nature and location of the incoming disturbances in the grid, and, therefore, depending on the severity of an event, can lead to both over- and under-sparsification. In this thesis we provide online distributed control algorithms that are designed to specifically target the inter-area modes most excited by the incoming disturbance. We design controllers using the MPC approach for the case when actuator constraints need to be considered, and also the Linear Quadratic Regulator (LQR) approach when this constraint can be relaxed while guarantees on closed-loop stability are more important.

We note that the power system operation in closed-loop with the distributed controllers operating over a communication graph, results in the so-called cyber-physical system (CPS). Hence, just as the effects of a disturbance on the physical aspect of the CPS are considered (such as a three-phase fault), the control designer should also consider the effects of perturbations on the cyber aspect of the CPS. These perturbation can be in the form of cyber attacks such as Denial-of-Service (DoS) attacks on the feedback communication network, and can potentially destabilize the entire system [16]. We thus provide a DoS resiliency analysis of our CPS in feedback with a sparse controller, and provide algorithms to mitigate the effects of such an attack. We show that our proposed algorithm results in regaining closed-loop stability of the
CPS, by taking into account the particular attacked communication link, and redesigning the sparse controller.

1.2 Contributions

The main contributions of this thesis are noted in the following.

1. Decentralized state estimation strategy with limited number of PMUs required to collect measurements.

2. Online identification of generators most influential in damping the excited inter-area modes.

3. Distributed MPC design with explicit constraints on actuators, and limited amount of required communication.

4. Sparse LQR design with structural constraints on the feedback matrix, as well as $\ell_1$-regularization of the closed-loop cost to sparsify the communication links.

5. Cyber attack mitigation for power system CPS with a controller redesign algorithm.

1.3 Thesis Organization

In Chapter 2 of this thesis, we provide the nonlinear power system model, and the state estimation procedure for estimating the dynamic generator states, in a decentralized manner. For state estimation, as a first step, an optimal PMU placement algorithm is first proposed to assure observability of all generator buses in the grid. In the second step, the voltage and current phasors at the generator bus terminals are estimated with an area-wise least squares estimator. For the final state estimation step, the generator states are estimated using a decentralized Unscented Kalman Filter from [15]. We also provide the linearized power system model, used for control design in the subsequent sections.

In Chapter 3, we design a centralized MPC for selective modal damping in power system networks. This is achieved by predicting the output response of the generators after a disturbance, and then constructing the MPC cost function in frequency domain using a selective Fourier Transform approach. This enables the controller to utilize its control energy on only the most excited inter-area oscillation modes. Simulations on the 39-bus power system model show that the proposed MPC approach gives superior results than the traditionally designed MPC in time-domain.

In Chapter 4, we use the post-disturbance state of the power system, along with the linearized model, to carry out online modal analysis of the power system. This enables us to
identify the sets of generators influential in contributing to the most excited inter-area modes, depending on the incoming disturbance. Based on this analysis, we construct our feedback control topology for distributed control in the following sections.

In Chapter 5, we design a distributed MPC for WAC, whose feedback communication topology follows from Chapter 4. MPC enables us to incorporate explicit actuator constraints in its formulation. From closed-loop simulation results with our distributed MPC design on the large 140-bus power system model, we show that for certain fault locations, only a few generators need to participate in damping control. This results in large savings in communication in the feedback network.

In Chapter 6, we follow the same communication topology from Chapter 4, and design a sparse LQR controller with guarantees on closed-loop stability, for the case when the actuator constraints are relaxed. We solve a structurally constrained LQR problem, and use a generalized Riccati equation method to guarantee stability. Further, we sparsify the communication links with an $\ell_1$-regularization on the closed-loop cost. Simulation results on the 39-bus system show the effectiveness of our approach, and compares results with the sparse optimal controller designed in [1].

In Chapter 7, we analyze the closed-loop resiliency of the power system CPS for DoS cyber-attacks. With simulation results on the 39-bus system, we show that the closed-loop system can go unstable if certain feedback links are attacked. We provide an online algorithm which mitigates the effects of such an attack by redesigning the sparse controller, and regaining the closed-loop stability of the CPS.
Chapter 2

Power System Model & State Estimation

2.1 Introduction

In this chapter we first give the nonlinear multi-machine power system model with transient generator dynamics. Next, we provide a decentralized generator state estimation strategy by measuring PMU phasor data from strategically placed minimum number of PMUs throughout the grid.

High-order nonlinear power system models for synchronous generators have been extensively studied in literature [17, 18]. We use the Matlab-based Power System Toolbox (PST) software [19] for simulating the nonlinear power system models. To obtain the small-signal linear time-invariant (LTI) model, linearization is done around the steady-state loadflow operating point [17], also obtained from PST. For a large power system model, this usually results in an LTI system of a very high order, with hundreds of state variables. Moreover, most of the generator states are not measurable directly, and hence have to be estimated for state-feedback control.

The objective of this chapter is to provide a state estimation algorithm for estimating the dynamic generator states in a wide-area power system network, for the subsequent purpose of distributed state-feedback control. PMUs are used for receiving measurements, which allow time-synchronized highly accurate readings of voltage and current phasors at buses where PMUs have been installed. This chapter discusses the possible strategies for choosing which buses PMUs should be installed on for observability, as well as the architecture of the state estimator. It is assumed that the wide-area power system network is geographically divided into multiple non-overlapping Utility Areas, where the utilities inside any area (buses, generators, PMUs, estimators etc.) are operated and monitored by an individual company. Companies can choose to install PMUs at some of the buses inside their area, which can monitor the voltage phasor
on that bus and the current phasors on the transmission lines connecting to that bus. These measurements can be communicated to a local Phasor Data Concentrator (PDC), in real-time, for further processing. The utility companies may or may not be willing to share their data with other areas. The following points detail the steps taken for estimation, and the possible choices for each step considered:

1. **PMU Placement for Observability:** Since measurements are obtained from PMUs located only on a few buses (PMU installation on thousands of buses inside an area will be very expensive), an optimal PMU placement algorithm is needed to identify those buses which provide observability of all generator states. It is shown in [15] that a decentralized state estimation algorithm can be formulated such that the generator states can be estimated from the phasor measurements of the generator bus only. Hence the problem is narrowed to identifying only those buses where PMUs can be placed such that all generator bus phasors are observable. Note that this is different from the usual PSSE (power system state estimation) requirements reported in the literature where observability of all generator buses is needed for monitoring purposes [20]. In Section 2.3.1, an optimal PMU placement (OPP) algorithm is provided by solving an integer programming problem. The two cases compared are:

   - OPP for observability of all buses in the network, as done in [21, 22].
   - OPP for observability of only generator buses, using a centralized as well as the proposed distributed method.

2. **Generator Bus Phasor Estimation:** Taking phasor measurements from some buses following the OPP algorithm, the next step is to estimate voltage and current phasors for all generator buses. In [23, 24], a single maximum likelihood estimation (MLE) problem is solved. Since this method requires inversion of a matrix with transmission line parameters, this method can be numerically difficult for large-scale systems. In [20] an alternating direction method of multipliers (ADMM)-based distributed estimator is proposed, which solves the estimation problem in a distributed manner. For our study, the objective is to only estimate the generator bus phasors, and not all bus phasors. Thus, we propose an area-wise decentralized estimator given in Section 2.3.2.

3. **Generator Dynamic State Estimation:** Once the phasors for all generator buses are estimated, the next step is the estimation of dynamic generator states from their bus phasor estimations. In Section 2.3.3, we implement a decentralized Unscented Kalman Filter (UKF) to estimate the generator states, from [15].
2.2 Nonlinear Power System Model

We consider a power system network with \( m \) synchronous generators. Each generator is modeled by a transient model assuming that the time constants of the \( d \)- and \( q \)-axis flux are fast enough to neglect their dynamics, that the rotor frequency is around the normalized constant synchronous speed, and that the amortisseur effects are negligible. The model of the \( i^{th} \) generator is written as [18]:

\[
\dot{\delta}_i = \omega_i - \omega_s \quad (2.1a)
\]

\[
M_i \dot{\omega}_i = P_{mi} - P_{ei} - d_i (\omega_i - \omega_s) \quad (2.1b)
\]

\[
T_{qi} \dot{E}_{qi} = -E'_{qi} + (\chi_{di} - \chi'_{di}) I_{di} + E_{f_{di}} \quad (2.1c)
\]

\[
T_{di} \dot{E}_{di} = -E_{di} + (\chi_{qi} - \chi'_{qi}) I_{qi} \quad (2.1d)
\]

\[
T_{Ai} \dot{E}_{f_{di}} = -E_{f_{di}} + K_{Ai} (V_{\text{ref},i} - V_i) + u_i(t) \quad (2.1e)
\]

where \( u_i(t) \) is the supplementary control input to the excitation system. Equations (2.1a)-(2.1b) are referred to as the swing equations while (2.1c)-(2.1e) as the excitation equations. The states \( \delta_i, \omega_i, E'_{qi}, E'_{di}, \) and \( E_{f_{di}} \) respectively denote the generator phase angle, rotor velocity, the quadrature-axis internal electromagnetic force (emf), the direct-axis internal emf, and the field excitation voltage. The voltage at the generator terminal bus is denoted in the polar representation as \( \tilde{V}_i(t) = V_i(t) \angle \theta_i(t) \). \( V_{\text{ref},i} \) is the constant set-point for \( V_i \). The generator current in complex phasor form is written as \( I_{di} + jI_{qi} \equiv I_i \angle \phi_i \). \( \omega_s \) is the synchronous frequency, which is equal to \( 120\pi \) rad/sec for a 60-Hz power system. \( M_i \) is the generator inertia, \( d_i \) is the generator damping, and \( P_{mi} \) is the mechanical power input from the \( i^{th} \) turbine, all of which are considered to be constant. \( T_{di}, T_{qi}, \) and \( T_{Ai} \) are the excitation time constants; \( K_{Ai} \) is the constant voltage regulator gain; \( \chi_{di}, \chi'_{di}, \chi_{qi} \) and \( \chi'_{qi} \) are the direct-axis and quadrature-axis salient reactances and transient reactances, respectively. It is assumed that all of these constant model parameters are known. All variables, except for the phase angles (radians), are expressed in per unit. The power flow equations are given by:

\[
P_{ei} = \frac{E'_{qi} V_i}{\chi'_{di}} \sin(\delta_i - \theta_i) + \frac{1}{2} \left( \frac{\chi'_{di} - \chi_{qi}}{\chi_{qi} \chi'_{di}} \right) V_i^2 \sin(2(\delta_i - \theta_i)) \quad (2.2a)
\]

\[
Q_{ei} = \frac{E'_{di} V_i}{\chi_{di}} \cos(\delta_i - \theta_i) + \frac{1}{2} \left( \frac{\chi'_{di} - \chi_{qi}}{\chi_{qi} \chi'_{di}} \right) \frac{\chi'_{di} - \chi_{qi}}{\chi_{qi} \chi'_{di}} \cos(2(\delta_i - \theta_i)) V_i^2 \quad (2.2b)
\]

where \( P_{ei} \) and \( Q_{ei} \) are the active and reactive powers produced by the \( i^{th} \) generator, respectively. Equations (2.1a)-(2.1e) can be written in a compact form as:

\[
\dot{x}_i(t) = g(x_i(t), z_i(t), u_i(t), a_i), \quad (2.3)
\]
where \( x_i = [\delta_i \omega_i E'_{qi} E'_{di} E_{fdi}] \in \mathbb{R}^5 \) denotes the vector of state variables, \( z_i = [V_i \theta_i I_i \phi_i] \in \mathbb{R}^4 \) denotes the vector of algebraic variables, and \( a_i \) is the vector of the constant parameters \( P_{mi}, \omega_s, d_i, T_{qi}, T_{di}, T_{Ai}, M_i, K_{Ai}, V_{ref,i}, \chi_{di}, \chi_{qi}, \chi'_{di}, \) and \( \chi'_{qi} \), all of which are assumed to be known.

The control input \( u_i \) is usually constrained as:

\[
 u_i^{min} \leq u_i(t) \leq u_i^{max},
\]

for all \( t > 0 \), and \( \forall i = 1, \ldots, m \). The definition of the nonlinear function \( g(\cdot) \) follows from (2.1a)-(2.1e).

The model (2.3) is a completely decentralized model since it is driven by variables belonging to the \( i^{th} \) generator only. It is, however, not a state-space model as it contains the auxiliary variables \( z_i \). The states \( x_i \) can be estimated for this model in a completely decentralized way if one has access to \( z_i(t) \) at every instant of time. This can be assured by placing PMUs within each utility area such that the generator buses inside that area become geometrically observable, measuring the voltage and currents at the PMU buses, and thereafter computing the generator bus voltage \( V_i \triangleq \theta_i \) and current \( I_i \triangleq \phi_i \) from those measurements. These steps are described next.

### 2.3 State Estimation

The network is assumed to be divided into \( M \) number of utility areas. All assets including generators, loads, PMUs, controllers, etc. in each area are owned and maintained by the utility company in charge of that area. Each company can choose to install multiple PMUs in its own utility area, but may or may not be willing to share its data with the other areas. Keeping in mind the above restrictions, an area-wise state estimation strategy is formulated in this section for feedback control of the generators.

#### 2.3.1 PMU Placement for Observability

In this subsection an area-wise optimal PMU placement (OPP) algorithm is provided, where the objective is to identify PMU bus locations (inside any area) such that the observability of generator buses is assured. Observability is assured by PMUs due to the following: a PMU placed at any bus will only measure the voltage phasor at the bus, and the current phasors of all transmission lines connecting to that bus. Hence with a PMU placed at an arbitrary bus, the voltage phasor of the adjoining bus can always be estimated using Ohm’s law. Our proposed OPP method differs from the classical OPP methods [24] since our method does not require observability of all buses in the power network, thereby leading to a potentially lesser number of PMUs. We next give OPP algorithms for full observability as well as for only generator-bus observability, and compare results.
OPP for Full Observability

Let the number of buses in the power system be $N$. Let $Y \in \mathbb{C}^{N \times N}$ be the admittance matrix and $\mathcal{Y}$ be the indicator matrix to $Y$, i.e. $\mathcal{Y}_{ij} = 1$ if bus $i$ and $j$ are physically connected, otherwise $\mathcal{Y}_{ij} = 0$. Also, let $P \in \mathbb{R}^N$ be the binary vector indicating the presence or absence of a PMU on a bus, i.e. for an element $p_i \in P$, $p_i = 1$ if bus $i$ has a PMU installed, $p_i = 0$ otherwise. We then solve the following integer programming problem [24]:

$$\min_P \sum_{i=1}^{N} c_i p_i \quad (2.5a)$$

s.t. $\mathcal{Y}P \geq \mathbb{1}_N$ and $p_i \in \{0, 1\}, \forall i = 1, \ldots, N,$

where $c_i$ is the cost of installing a PMU at bus $i$, and $\mathbb{1}_N$ is an $(N \times 1)$ vector of all ones. The solution to the problem (2.5) will assure observability of all buses in the system, and provides the optimal location of PMUs as the non-zero elements of the solution $P^*$. It is noted that since this problem is non-convex, $P^*$ is not unique but rather is one of the possible solutions but with minimum number of PMUs.

OPP for Generator-Bus Observability: Centralized

We can modify the above problem so that only the observability of generator buses is required. This is done since our ultimate goal is to estimate the states of the generators, and for that purpose estimating phasors of all buses in the system is redundant which can lead to more number of PMUs than is essential. Hence for observability of only generator buses, the above problem is modified as:

$$\min_P \sum_{i=1}^{N} c_i p_i \quad (2.6a)$$

s.t. $\mathcal{Y}P \geq \mathbb{1}_N$ and $p_i \in \{0, 1\}, \forall i = 1, \ldots, N,$

where $\mathbb{1}_N$ is given by:

$$\mathbb{1}_{N,i} = \begin{cases} 1 & \text{if bus } i \text{ is a generator bus} \\ 0 & \text{otherwise.} \end{cases} \quad (2.7)$$

The solution of (2.6) assures observability of all generator buses, and the optimal location of PMUs is given by the non-zero elements of $P^*$. This case is called the centralized case since PMU locations are determined irrespective of the distribution of Utility areas. This can lead to
an additional assumption on communication between areas where the identified PMU bus and the corresponding generator bus to be estimated are in different utility areas. This limitation is highlighted with the following case-study.

We consider the Northeast Power Coordinating Council (NPCC) 48-machine, 140-bus power system model as shown in Fig. 5.3, which also shows the assumed geographical distribution of utility areas. The distribution is done considering that clusters of generators (with respect to electrical distance) belong to the same utility company. Problem (2.6) is solved for this system with $c_i = 1$, $\forall i = 1, \ldots, 140$, and the identified PMU buses (24 out of 140 total buses) are highlighted in red. It is seen from Fig. 5.3 that a PMU is needed on bus 99 for estimating the phasors of generator bus 98. Since buses 99 and 98 are in different utility areas, this means that the company monitoring bus 99 will have to communicate its PMU measurements to the company which owns generator 36 (connected to bus 98). This is a limitation of the method and is addressed with a decentralized version of (2.6), explained in the next subsection.

![Figure 2.1: NPCC power system network with highlighted utility areas, and PMU buses identified from solving (2.6) in red.](image)
OPP for Generator-Bus Observability: Distributed

Let $N_\kappa$ be the number of buses in the utility area $\kappa$, and $\mathcal{Y}_\kappa$ is the indicator matrix for the admittance matrix of all buses in area $\kappa$. We propose a simple decentralization of (2.6), for all areas $\kappa$, as:

$$P_{\text{opp}}: \min_{P_\kappa} \ c_\kappa' P_\kappa \quad (2.8a)$$

$$\text{s.t.} \ \mathcal{Y}_\kappa P_\kappa \geq \tilde{1}_{N_\kappa} \quad (2.8b)$$

$$P_{\kappa,i} \in \{0,1\}, \quad (2.8c)$$

where $c_\kappa \in \mathbb{R}^{N_\kappa}$ is the vector of relative costs for installing PMUs, and the elements of vector $\tilde{1}_{N_\kappa}$ are given by:

$$\tilde{1}_{N_\kappa,i} = \begin{cases} 1 & \text{if bus } i \text{ is a generator bus in area } \kappa \\ 0 & \text{otherwise.} \end{cases} \quad (2.9)$$

The NPCC model is also used for solving $P_{\text{opp}}^\kappa$ for all $\kappa = 1, \ldots, 5$, and the results show only one additional PMU being added to the solution of (2.6), at bus 98.

Comparison Results

We solve the integer programming problems (2.5), (2.6) and (2.8) using the method \textit{intlinprog} in Matlab for five different power system models. The results are reported in Table 2.1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Total # of buses</th>
<th># of PMUs from solving (2.5)</th>
<th># of PMUs from solving (2.5), $\forall\kappa$ areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-machine</td>
<td>13</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>10-machine</td>
<td>39</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>16-machine</td>
<td>68</td>
<td>23</td>
<td>16</td>
</tr>
<tr>
<td>48-machine</td>
<td>140</td>
<td>37</td>
<td>24</td>
</tr>
<tr>
<td>50-machine</td>
<td>145</td>
<td>27</td>
<td>12</td>
</tr>
</tbody>
</table>

From Table 2.1 it is clear that decentralization of (2.6) does not add a lot of additional PMUs to the system, while maintaining privacy between areas. An intuitive reasoning for this
is that for observability of only generator buses, the algorithm will essentially try to identify important buses close to generators, increasing the chances of these buses residing in the same utility area. It is also verified that solving (2.5) will result in a lot of redundant PMUs, especially for large systems. It is assumed that the proposed distributed OPP problem is solved offline as part of the system planning.

2.3.2 Generator Bus Phasor Estimation

Next, using the voltage and current measurements from the PMUs located at buses identified from solving $P_\kappa^{opp}$, the phasor voltages and currents for all generator buses inside the area $\kappa$ are estimated, described as follows.

Transmission Line Model

To relate measurements with phasor states, we first give the expression for the transmission line current as a linear function of phasor voltages on connecting buses. We use a $\pi$-model for the transmission line as shown in Fig. 2.2 with a phase-shifting transformer in series (not shown).

![Figure 2.2: $\pi$ model of a power system transmission line.](image)

Using Ohm’s law, the current phasor $\tilde{I}_{ij}$ between buses $i$ and $j$ is given by:

$$\tilde{I}_{ij} = T_{ij} \left[ Y_{ij} \left( \frac{\tilde{V}_i}{T_{ij}} - \tilde{V}_j \right) \right] + \frac{B_{ij}}{2} \tilde{V}_i,$$

(2.10)

where $j = \sqrt{-1}$, $Y_{ij}$ is the admittance of the line, $B_{ij}$ is the line charging (susceptance), and $T_{ij}$ is the phasor tap-ratio of the transformer given by $T = T_R e^{jT_\theta}$, where $T_R$ is the tap-ratio magnitude and $T_\theta$ is the tap-ratio phase. (2.10) can then be written as:

$$\tilde{I}_{ij} = \alpha_{ij} \tilde{V}_i + \beta_{ij} \tilde{V}_j,$$

(2.11)
where $\alpha_{ij} = Y_{ij} + jB_{ij}/2$ and $\beta_{ij} = -T_{ij}Y_{ij}$. Using rectangular co-ordinates, we write (2.11) as the transformed linear equation:

$$I_{ij} = \alpha_{ij}\bar{V_i} + \beta_{ij}\bar{V_j},$$

(2.12)

where $I_{ij} = \text{Re}(I_{ij}) + j\text{Im}(I_{ij})$, and similarly for $\bar{V_i}$ and $\bar{V_j}$.

**Measurement Model**

Let the total number of utility areas by $R$, and let the $k^{th}$ area contains $k_p$ number of PMUs and $k_g$ number of generators. Also, let pseudo-measurements be defined as the complex-valued voltages and currents obtained after applying the co-ordinate transformation on the phasor voltages and currents from PMU measurements, respectively. It is assumed that the phasor PMU measurement noise is Gaussian for all measurements, with a zero mean and a small standard deviation. Since the pseudo-measurements depend non-linearly on the actual measurements (e.g. $\text{Re}(V) = |V|\cos(\theta)$), hence it is clear that the pseudo-measurement noise (for complex-valued $\bar{V}, \bar{I}$) will have a Normal Product Distribution (NP-distribution) due to multiplication of two Normal random variables. For the four-machine power system shown in Fig. 2.3, this distribution is shown in Fig. 2.4, where the noise distributions for $\bar{V_1}, \bar{V_2}$ is shown, with the left two plots showing real components and the right two plots show imaginary components. The $p$-value in this case is a statistical measure of how close the distribution is to Normal, i.e. $p \approx 0$ indicates a non-Normal distribution, and $p > 0.1$ indicates a statistically Normal distribution.
Figure 2.3: 4-machine power system model.

Figure 2.4: NP-distributions for complex PMU voltage pseudo-measurements noise, for Gens. 1,2.
The linear measurement model for the $\kappa^{th}$ area is derived as:

$$\chi^\kappa = H^\kappa z^\kappa + \epsilon^\kappa,$$

where $z^\kappa$ is obtained from a polar-to-rectangular co-ordinate transformation of $z^\kappa$; $H^\kappa$ is a known matrix of transmission line impedences; $\chi^\kappa$ is obtained from polar to rectangular transformation of $\chi^\kappa$; and $\epsilon^\kappa$ is a noise vector with covariance matrix $\Sigma^\kappa = R \Sigma^\kappa R'$ obtained from the linearization of the polar to rectangular co-ordinate transformation equations, as outlined in [25]. Please see the Appendix for details on construction of vectors $\chi^\kappa, \bar{\chi}^\kappa, z^\kappa, \bar{z}^\kappa$. $R$ is a rotation matrix.

For the 4-machine system shown in Fig. 2.3 (with highlighted PMU buses in red), and using (2.12), the measurement model for Area 1 (consisting of the gens $G_1$ and $G_2$) is given by:

$$\begin{bmatrix}
\bar{V}_{10} \\
\bar{I}_{10,1} \\
\bar{I}_{10,20} \\
\bar{V}_{20} \\
\bar{I}_{20,10} \\
\bar{I}_{20,20}
\end{bmatrix} = \begin{bmatrix}
\beta_{1,10} & \alpha_{1,10} & 0 & 0 \\
0 & \beta_{1,10} & -1 & 0 \\
\alpha_{1,20}\beta_{1,10} & \alpha_{1,10} & \beta_{1,20} & 0 \\
0 & \beta_{1,10} & -\alpha_{1,20} & \beta_{1,20} \\
\alpha_{1,10}\beta_{1,10} & 0 & \beta_{1,20} & -1 \\
0 & \beta_{1,10} & -\alpha_{1,20} & \beta_{1,20}
\end{bmatrix} \begin{bmatrix}
\bar{V}_1 \\
\bar{I}_{1,10} \\
\bar{V}_2 \\
\bar{I}_{2,10} \\
\bar{V}_{20} \\
\bar{I}_{2,20}
\end{bmatrix} + \begin{bmatrix}
\epsilon_{V_{10}} \\
\epsilon_{I_{10}} \\
\epsilon_{V_{20}} \\
\epsilon_{I_{20}}
\end{bmatrix}.$$  \hfill (2.14)

**Remark 1.** It is noted that the 4-machine system is chosen only for illustrative purposes since for this system the PMUs can potentially be directly placed at the generator buses.

**Area-Wise Distributed MLE**

The vectors $z^\kappa$ are obtained from the solution of the weighted least squares problem:

$$\mathcal{P}_{\kappa}^{ls} : \min_{z^\kappa} \left[ (\bar{\chi}^\kappa - H^\kappa z^\kappa)^T \bar{\Sigma}^{-1}_\kappa (\bar{\chi}^\kappa - H^\kappa z^\kappa) \right].$$  \hfill (2.15)

The solution is given by:

$$z^{\kappa*} = G^\kappa \bar{\chi}^\kappa,$$  \hfill (2.16)

where the estimation gain is:

$$G^\kappa = (H^\kappa' \bar{\Sigma}^{-1}_\kappa H^\kappa)^{-1} H^\kappa' \bar{\Sigma}^{-1}_\kappa.$$

The voltages and currents in $z^{\kappa*}$ are converted back to polar co-ordinates, and the resulting vector is denoted as $\bar{z}^\kappa$. The noise distributions for these polar states are evaluated and it is observed that the distribution has converted back to a Gaussian. For the 4-machine example used above, the noise distributions for the polar voltages at generators 1 and 2 are shown in Fig. 2.5. It can be clearly seen that these are Normal.
Figure 2.5: Gaussian noise distributions for the estimated polar states at Gens. 1, 2.

The above procedure is done for every utility area \( \kappa \), and at every time instant \( t \), thereby providing the estimate for all \( z_i(t) \) as \( \tilde{z}_i(t) \), \( i = 1, \ldots, m \), in (2.3).

### 2.3.3 Generator Dynamics State Estimation

The continuous-time model (2.3) is sampled using forward Euler transformation, as:

\[
x_i((k+1)T) - x_i(kT) = g(x_i(kT), z_i(kT), u_i(kT), a_i).
\]

We note that discretization by Forward Euler method for power systems has been verified in articles [26, 15]. Re-writing \((k+1)T\) as \(k+1\), and \(kT\) as \(k\), (2.3) gets converted to a discrete-time form:

\[
x_i(k+1) = x_i(k) + Tg(x_i(k), z_i(k), u_i(k), a_i).
\]

Following the input/output strategy proposed in [15] for decentralized state estimation, we choose phasor currents at the generator bus as the outputs, and phasor voltages at the generator bus as inputs to the UKF, i.e. \( y_{ukf}^i(k) = [|I_i(k)|, \phi_i(k)]' + w_i(k) \) and \( u_{ukf}^i(k) = [|V_{ni}(k)|, \theta_{ni}(k)]' \). \( V_{ni}(k) = V_i(k) + v_{1i}(k) \) is the noisy voltage magnitude measurement, and \( \theta_{ni}(k) = \theta_i(k) + v_{2i}(k) \) is the noisy voltage phase measurement. The voltage noise vector is
collected in \( v_i(k) = [v_{1i}(k), v_{2i}(k)]' \), and the current noise vector is \( w_i(k) = [w_{1i}(k), w_{2i}(k)]' \).

Input and output noises are assumed to be Gaussian (with zero means) as well as uncorrelated, and are obtained from the static state estimation algorithm described in Section 2.3.2. The state vector is then augmented with input noise as: \( X_i(k) = [x_i(k); v_i(k)] \), while its covariance matrix is given as:

\[
P_{X_i}(k) = \begin{bmatrix} P_{x_i}(k) & 0 \\ 0 & P_{v_i}(k) \end{bmatrix},
\]

(2.19)

where \( P_{x_i} \) is the covariance of \( x_i \), and \( P_{v_i} \triangleq \text{diag} \left( \sigma_{v_i}, \sigma_{\theta_i} \right) \) is the covariance of \( v_i \). Covariance of the output noise \( w_i \) is given by \( P_{w_i} \triangleq \text{diag} \left( \sigma_{1i}, \sigma_{\phi_i} \right) \). The nonlinear dynamics for the \( i^{th} \) synchronous generator, with inputs and outputs as defined in the last subsection, are written as (dropping the \( i \) notation for readability):

\[
X(k+1) = g(X(k), u^{ukf}(k))
\]

(2.20a)

\[
y^{ukf}(k) = h(X(k), u^{ukf}(k)) + w(k),
\]

(2.20b)

where \( g(\cdot) \) and \( h(\cdot) \) are nonlinear functions from (2.1)-(2.2) in Section 2.2. The UKF algorithm to estimate the state \( \hat{X} \) and the covariance matrix \( \hat{P}_X \) is given in Algorithm 1.

**Algorithm 1** Decentralized UKF algorithm to estimate generator states \([15], \forall k > 0\)

1: if \( k = 0 \) then
2: \( \hat{X}(0) = [\hat{x}(0); 0]' \) and \( P_X(0) = \left( P_x \ P_v \right) \).
3: else
4: \( \hat{v}(k-1) = 0 \) and \( P_X(k-1) = \left( P_x \ P_v \right) \).
5: \( s_l(k-1) = \hat{X}(k-1) \pm (\sqrt{nP_X(k-1)}), \forall l = 1, \ldots, 2n \) \( \triangleright \) Generate sigma points
6: \( \hat{s}_l(k) = g[s_l(k-1), u^{ukf}(k-1)], \forall l = 1, \ldots, 2n \) \( \triangleright \) Predict sigma points
7: \( \hat{X}^-(k) = \frac{1}{2n} \sum_{l=1}^{2n} \hat{s}_l(k) \)
8: \( P_X^-(k) = \frac{1}{2n} \sum_{l=1}^{2n} [s_l^-(k)-\hat{X}^-(k)]'[s_l^-(k)-\hat{X}^-(k)]' \)
9: \( \gamma_l^-(k) = h[\hat{s}_l^-(k), u^{ukf}(k)], \forall l = 1, \ldots, 2n \)
10: \( \hat{y}^{ukf^-(k)} = \frac{1}{2n} \sum_{l=1}^{2n} \gamma_l^-(k) \)
11: \( P_y^-(k) = \frac{1}{n} \sum_{l=1}^{2n} [\gamma_l^-(k) - \hat{y}^{ukf^-(k)}][\gamma_l^-(k) - \hat{y}^{ukf^-(k)}]' + P_{w_i} \)
12: \( P_{XY}^-(k) = \frac{1}{2n} \sum_{l=1}^{2n} [s_l^-(k)-\hat{X}^-(k)][\gamma_l^-(k) - \hat{y}^{ukf^-(k)}]' \)
13: \( K(k) = P_{XY}^-(k) [P_y^-(k)]^{-1} \)
14: \( \hat{X}(k) = \hat{X}^-(k) + K(k)(\hat{y}^{ukf}(k) - \hat{y}^{ukf^-(k)}) \)
15: \( P_X(k) = P_X^-(k) - K(k)P_{XY}^-(k)' \)
16: end if
The 4-machine model shown in Fig. 2.3 is used for verifying state estimation results using the UKF algorithm in Algorithm 1. A decentralized UKF estimator is implemented for all the four machines, and estimation results for Generator 1 are shown in Fig. 2.6.

Figure 2.6: Decentralized UKF state estimation results, showing the time-evolution of the four estimated states for generator 1.

2.4 Linearized Model

We next develop the linearized power system model, to be used for our model-based optimal control designs in subsequent sections. From the generator model provided in (2.1), let the states for all generators be aggregated in vectors as: \( \delta(t) = [\delta_1(t), \ldots, \delta_m(t)]' \), \( \omega(t) = [\omega_1(t), \ldots, \omega_m(t)]' \), and let \( s(t) \) denote the stacked vector of non-electromechanical states, i.e. 
\[
s(t) = [E_{q1}, \ldots, E_{qm}, E_{d1}, \ldots, E_{dm}, E_{fd1}, \ldots, E_{fdrm}]'.
\]
After Kron reduction [27] of equations (2.1)-(2.2) to eliminate the algebraic variables \( z(t) \), and then linearizing about the equilibrium point \((\delta_0, \omega_0, s_0)\) (obtained from load flow), the overall linearized \(n^{th}\)-order dynamic model of the network is given by [5]:

\[
\]
\[
\begin{bmatrix}
\Delta \dot{x} \\
\Delta \dot{\omega} \\
\Delta \dot{s} \\
\Delta x
\end{bmatrix} =
\begin{bmatrix}
0 & I & A_{13} \\
-M^{-1}L & -M^{-1}D & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta \omega \\
\Delta s
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
M^{-1} & 0 \\
B_{31} & B_{32}
\end{bmatrix}
\begin{bmatrix}
\Delta P_m \\
\Delta E_F
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\tilde{B}_2
\end{bmatrix}
d(t) \tag{2.21}
\]

where \( \Delta x \in \mathbb{R}^{n \times 1} \), \( \Delta u \in \mathbb{R}^{m \times 1} \); \( A_c \in \mathbb{R}^{n \times n} \) and \( B_c \in \mathbb{R}^{n \times m} \) represent the continuous-time state and input matrices, respectively; \( D = \text{diag}(D_i); M = \text{diag}(M_i) \); \( \Delta \delta = [\Delta \delta_1 \ldots \Delta \delta_m]^T \), \( \Delta \omega = [\Delta \omega_1 \ldots \Delta \omega_m]^T \), \( \Delta s = [\Delta s_1 \ldots \Delta s_m]^T \), \( \Delta P_m = [\Delta P_1 \ldots \Delta P_m]^T \) and \( \Delta E_F = [\Delta E_{F1} \ldots \Delta E_{Fm}]^T \).

The signal \( d(t) \) denotes a scalar disturbance input to the system that can result from internal faults or extraneous perturbations, while \( \tilde{B}_c \) is in the form as shown in (2.12) since the disturbance can enter the system through either the power flow equations (i.e. through \( \Delta \dot{\omega} \)) or through some excitation failure (through \( \Delta \dot{E} \)). The matrix \( L \) is called the Laplacian matrix since the power system interconnection can be considered as a graph of \( m \) nodes [5], and its detailed expression is provided in Appendix A.2. Since the time constant of \( \Delta P_m \) is much slower than \( \Delta E_F \) [17], we choose the excitation control \( \Delta E_F \) as the control input to the system for every generator.

The continuous-time system (2.21) is first converted to its discrete-time counterpart (with a zero-order hold and a sampling frequency of \( 1/T \)):

\[
\Delta x(k+1) = A \Delta x(k) + B \Delta u(k) + \tilde{B}d(k), \tag{2.22}
\]

where \( k \) is the discrete-time index, and \( C \) is the output matrix. \( \tilde{B} \) is not known prior to the disturbance event but it impacts the residues of the inter-area modes of the electromechanical states \( \Delta \delta, \Delta \omega \). Next, dropping the \( \Delta \)-notation and ignoring the unknown exogenous disturbance term \( \tilde{B}d(k) \), we write the discrete-time LTI power system model (2.22) as:

\[
x(k+1) = Ax(k) + Bu(k), \quad \text{with} \quad x_0 \triangleq x(0), \tag{2.23}
\]

where \( x = [x_1, \ldots, x_m]^T \), \( u = [u_1, \ldots, u_m]^T \), \( A \in \mathbb{R}^{n \times n} \) is the state matrix, \( B \in \mathbb{R}^{n \times m} \) is the control input matrix, and \( x_0 \) is the initial state of the system. It is noted that if the disturbance is assumed to enter the system dynamics at time-step \( k = 0 \), then \( x_0 \) is the \textit{post-disturbance} state of the system.
2.5 Conclusions

In this chapter we provided the transient nonlinear model, a step-by-step decentralized state estimation procedure for the generator states, and the linearized model for an arbitrarily connected power system network. Decentralized algorithms for PMU placement, bus phasor estimation, and finally generator state estimation are proposed. This is done so that the proposed sparse state-feedback distributed control graph (developed in subsequent chapters) does not have to match with the state estimation graph.
Chapter 3

Centralized MPC for Wide-Area Control

3.1 Introduction

In this chapter we develop a model predictive controller (MPC) for WAC in power systems. The controller is centralized in its communication architecture, and is designed so as to specifically target the inter-area oscillation modes. This is achieved by constructing the cost function to be minimized by MPC, in the frequency domain instead of the time domain. We next present a literature review for wide area control and application of MPC in power systems.

Control designs for oscillation damping in power systems using Power System Stabilizers (PSS) have an extensive literature [2, 11, 28]. PSS, however, have been traditionally known to have the highest influence on high frequency oscillation modes or intra-area oscillations that arise due to local interactions between generators within a coherent area of the system, and relatively poor participation in damping of inter-area modes or modes with slower frequencies, that arise from the interactions between the areas themselves [17, 29]. Controllers using the Flexible AC Transmission System (FACTS) such as the static VAR compensator (SVC) and thyristor-controlled series capacitor (TCSC) have been proposed to mitigate this gap [3, 4, 30].

Modern robust control methods have also been proposed via the use of either PSS or FACTS [31, 12, 32] to improve their effectiveness for inter-area damping. However, majority of these designs are done offline, and therefore do not utilize any real-time information about the system model prior to the fault or disturbance. With the increase in transmission capacity, as well as the intrusion of different smart devices in the grid resulting in fluctuating models from one disturbance event to another, and with the increasing probability for disturbances to occur randomly anywhere in the system, such offline designs may under-perform in reality. Moreover, a subtle drawback of these designs is that majority of them are performed in time-domain
using state-space methods, and do not exploit the decoupling property of the oscillation modes (explained further in Section 3.2.1). Therefore, they tend to waste their control energy to a large extent by unnecessarily damping out every mode in the system, including the fast modes that are already damped by the conventional PSS designs.

With this challenge in mind, in this chapter we propose a new predictive control strategy that acts as a supplementary controller on top of a conventional PSS, and provides damping effect on only selected oscillation modes, especially the inter-area modes. The primary tool underlying our design is MPC, which has been shown to be one of the most successful control techniques in various real-world control applications [33, 34, 35]. In recent years, MPC has been also applied to various power system applications in both centralized and distributed architectures [10, 36, 37, 38]. The novelty of our problem, however, lies in the fact that we pose the control design first in frequency-domain using Discrete Fourier Transform (DFT), where the closed-loop damping of selected frequencies can be designed in a decoupled fashion, and thereafter show that this design objective can be stated equivalently as an MPC problem in time-domain. The basic approach is as follows. We first introduce the concept of Selective Discrete Fourier Transform (SDFT) that extracts the energies associated with the inter-area frequency components in the output spectrum of the power system, and uses this information to construct a weighting matrix $Q$. The MPC is then formulated as a quadratic minimization of the outputs using $Q$, resulting in damping only the inter-area modes of interest. In reality, however, the most dominant DFT magnitudes will not be known ahead of time since they are decided by the location of the disturbance. Therefore, we next augment the MPC design by predicting the dominant DFT magnitudes in the desired low frequency range using online measured data, and tuning $Q$ accordingly.

Some related MPC designs with frequency spectrum constraints have been targeted in [39, 40, 41, 42] using short-time fourier transform and bandpass filtering. However, compared to these approaches our method does not involve any extra filter design, and therefore does not introduce any additional state in the system. We also do not assume that the exact value of the frequency mode to be damped is known a priori.

The benefits of our method compared to conventional designs are as follows:

1. With the intrusion of more and more renewable energy as well as new loads such as the plug-in hybrid vehicles (PHEVs) and smart buildings, all of which have their own uncertainty towards the system dynamics as well as on the inter-area modes [43], offline dynamic models of the grid may no longer be reliable in near future. Hence, real-time identification of oscillation modes from measured phasor measurement unit (PMU) data is likely to become a safer alternative. Our proposed controller, therefore, is much more reliable than conventional offline control designs.
2. Since the implementation of the proposed algorithm comes down to a predictive-adaptive tuning of the output trajectory weighting matrix in an online MPC problem, no states are added to the system with this approach.

3. System constraints can be satisfied with the MPC approach which are often overlooked in traditional control designs. As shown in [43], significant renewable penetration can pose more constraints for PSS actuation than normal.

The proposed approach (termed SDFT-MPC) is applied to the IEEE 39-bus power system model and compared with both the PSS-only control architecture and the conventional MPC control architecture (termed nominal MPC). Simulation results show an increase in the damping performance of the identified dominant slow modes.

### 3.2 Effect of Clustering

Referring to the LTI state-space model of the power system (2.12), we note that our objective is to design $\Delta E_F$ so as to damp the slow eigenvalues in $A_c$ that may arise due to clustering in $L$. To further clarify this formulation, we next discuss the impact of clustering on eigenvalues of $A_c$, where these eigenvalues are denoted by $(-\sigma_i \pm j\Omega_i)$, $\sigma_i \geq 0$, where $j \triangleq \sqrt{-1}$.

We assume that the power system is divided into $r$ coherent areas, where the $k^{th}$ area consists of $m_k$ generators. The Laplacian in (2.12) then can be rewritten as in (6.12) (see Appendix B.2) where $\Delta \delta_{i,k}$ denotes the angle of the $i^{th}$ machine in the $k^{th}$ area. $L_{kk}$ shows the connectivity of the $m_k$ generators inside area $k$, and $L_{jk}$ shows the connectivity between the generators in areas $j$ and $k$ in the Kron-reduced form. Due to the assumption of coherency following from the difference between the local and inter-area reactances and inertias, the oscillatory modes of the matrix $A_c$ will be divided into sets of $(r-1)$ inter-area (slow) modes with eigenvalues $(-\sigma_1 \pm j\Omega_1)$ through $(-\sigma_{r-1} \pm j\Omega_{r-1})$. The remaining $(m-r)$ modes will be characterized by intra-area (fast) modes representing the local oscillations inside areas. As a result, the solution of output trajectory $\Delta y_{i,k}(t)$ can be written as:

$$
\Delta y_{i,k}(t) = \Delta y_{i,k}^0(t) + \sum_{l=1}^{r-1} \rho_{il} e^{(-\sigma_l + j\Omega_l)t} + \rho_{r+1}^* e^{(-\sigma_l - j\Omega_l)t} + \sum_{l=r+1}^{m-1} \rho_{il} e^{(-\sigma_l + j\Omega_l)t} + \rho_{m}^* e^{(-\sigma_l - j\Omega_l)t},
$$

(3.1)

where $\Delta y_{i,k}^0(t)$, $\Delta y_{i,k}^s(t)$, and $\Delta y_{i,k}^f(t)$ respectively represent the non-oscillatory, the inter-area, and the intra-area components of $\Delta y_{i,k}(t)$; $\rho_{il}$ denotes the residue of the $i^{th}$ mode in $\Delta y_{i}(t)$, and the superscript (*) denotes the complex conjugate.
The conventional PSS control is designed to damp the intra-area frequency modes and hence is unable to properly damp the inter-area modes in (3.1). In our study, the PSSs are included in the open-loop configuration of the power system (2.12)-(3.2). The resulting control configuration is termed as \textit{PSS-only}. For example, a simple frequency spectrum analysis, with the help of Fast Fourier Transform (FFT), of the post-fault frequency measurements of a generator (which can be represented in the form of (3.1)) is shown in Fig. 3.1. The result follows from the simulation of an IEEE 39-bus power system model, as shown in Fig. 3.4. More details of this simulation will be provided in Section 3.5. Only the output frequency spectrum of a single generator is shown for brevity. The figure shows the inability of the PSS to damp the slow frequency components. Similar characteristics is observed for all the synchronous generators with \textit{PSS-only} configuration. The output response of all the $m = 10$ generators from a three phase fault on the transmission line between buses 3 and 4 for the New England power system (network shown in Fig. 3.4) simulations is as shown in Fig. 3.2.

![FFT plot for Generator 5 output (PSS-only)](image)

Figure 3.1: FFT plot for Generator 5 output (PSS-only)

Even if operators may have some approximate idea about the frequency content of $\Delta y_{i,k}$ in (3.1) from their nominal knowledge of $A_c$, it is impossible for them to know the values of the corresponding residues $\rho_{il}$ prior to the disturbance as that depends on both $\tilde{B}_c$ and $d(t)$, which may be completely different from one disturbance to another. $\tilde{B}_c$, for example, is decided by the location of the fault, and will produce different residues (i.e., different zeros of the transfer
function from $d(t)$ to $y(t)$) for different events. The residues also depend on the choice of $y(t)$. The magnitude of the FFT at any frequency, therefore, will not be known to an operator ahead of time. Because of this, selective modal damping becomes very difficult by traditional offline controllers that are mostly designed using time-domain methods.

To counteract this problem, we next describe our proposed MPC framework, which achieves damping of identified dominant slow modes by minimizing the energy content of those specific modes via a predictive algorithm.

### 3.3 Model Predictive Control Design

A generic overview of the Model Predictive Control (MPC) design strategy is shown in Fig. 3.3, and the steps are listed as follows: MPC is an online prediction-based technique whereby at every time-step $k$:

- Using a dynamical model of the system and the calculated optimal input trajectory from the previous time-step, the future state and output trajectories are obtained throughout the chosen prediction horizon.

- A cost function is then minimized by solving a constrained LQR problem to obtain the optimal future control trajectory.
• The first control input from the calculated optimal input trajectory is applied to the system at the next time-step.

![Figure 3.3: PSS-only output response for all generators](image)

### 3.3.1 MPC Formulation for Damping

In this subsection the basic MPC equations for our linearized power system are provided. The state-space output equation for the discrete-time power system (2.23) is given as:

$$y(k) = \begin{bmatrix} C_1 & C_2 & 0 \end{bmatrix} x(k)$$

(3.2)

where $y$ represents the small-signal measurements of the phase angles and frequencies of selected generators using PMUs at their terminal buses, and $0$ is the zero vector of appropriate size. The rows of $C_1$ and $C_2$ are indicator vectors corresponding to those respective generators. The linearized discrete-time states and, hence, the outputs are moved forward in time to obtain the predicted trajectories. The resulting vector of output trajectories can be written as:

$$\bar{y} = \Lambda x(k) + \Phi \bar{u},$$

(3.3)
where:

\[ \bar{y} = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m]' \]

\[ \bar{u} = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m]' \]

\[ x(k) = [x_1(k), x_2(k), \ldots, x_m(k)]' \]  

(3.4)

with:

\[ \bar{y}_i = [y_i(k+1), y_i(k+2), \ldots, y_i(k+N_p)]' \]  

(3.5)

\[ \bar{u}_i = [u_i(k), u_i(k+1), \ldots, u_i(k+N_c - 1)]' \]  

(3.6)

\[ \bar{y}_i \in \mathbb{R}^{N_p \times 1} \] is the linearized predicted output trajectory for the \( i \)th generator, \( \bar{u}_i \in \mathbb{R}^{N_c \times 1} \) is the linearized future control trajectory for the \( i \)th generator, \( x_i(k) \in \mathbb{R}^{n_i \times 1} \) is the linearized state vector at the current time-step \( k \) for the \( i \)th generator and \( \Lambda \) and \( \Phi \) are appropriate matrices composed of state, input and vector matrices \( A, B, C \) [33]. We assume \( x(k) \) to be available for our design by means of the internal state estimation algorithm provided in Chapter 2. Since the focus of this chapter is on the design of \( u(k) \), we directly pose our design assuming full state availability, and neglect the internal dynamics of the estimator. More details on the MPC formulation can be found in [33]. The mode damping problem is, thereafter, posed as a quadratic minimization of the predicted output trajectory as:

\[ \bar{u}_{opt} = \min_{\bar{u}} J = \min_{\bar{u}} \left( \bar{y}'Q\bar{y} + \bar{u}'R\bar{u} \right) \]  

(3.7)

\[ u^* = \bar{u}_{opt}(l) \]  

(3.8)

subject to:

\[ u_{min} \leq u \leq u_{max} \]  

(3.9)

where \( J \in \mathbb{R} \) is the cost/energy function to be minimized, \( N_p \) is the prediction horizon, \( N_c \) is the control horizon, \( Q \in \mathbb{R}^{mN_p \times mN_p} \) is a block diagonal matrix which serves as the weighting matrix for the output trajectory, \( R \in \mathbb{R}^{mN_c \times mN_c} \) is a block diagonal matrix which serves as the weighting matrix for the control trajectory, \( \bar{u}_{opt} \) is the calculated optimal control trajectory vector and \( u^* \) is the first input from each of the optimal trajectories which will be applied at the next time-step. Rate constraints on the linearized control input \( u \) can be added to (3.9) if needed.

Nominal MPC is then defined as the above formulation with \( Q = I \) (an identity matrix). When applied to our power system problem, this will result in damping of all the modes and therefore lead to unnecessary wastage of control energy since the objective is to damp only
selected modes. We, therefore, next pose the control problem (3.7)-(3.9) by introducing the idea of Selective Discrete Fourier Transform (SDFT).

### 3.3.2 Selective Discrete Fourier Transform

Our objective is to damp certain slow modes in the system output frequency spectrum. To achieve this via MPC, at every time-step $k$, we take the DFT of each of the predicted output trajectories $\bar{y}_i$ to obtain $Y_i[n]$, $\forall n = 0, \ldots, (N_p - 1)$ and $\forall i = 1, \ldots, m$. For the rest of this section and the subsequent one, all derivations and calculations are done for all $m$ subsystems (generator controls), i.e. $\forall i = 1, \ldots, m$.

Next we select only the slow modes of interest from the DFT and hence obtain the SDFT: $\hat{Y}_i[n]$, $\forall n = \alpha_i, \alpha_i + 1, \ldots, \alpha_i + n_{si}$, where $(n, \alpha_i)$ are frequency indices and $n_{si}$ represents the number of interested slow modes. Since by Parseval’s Theorem [44], the energy content of the signal is equal in both time and frequency domain, damping of these slow modes in frequency domain will result in time-trajectory damping of the signal component specific to that frequency.

As a first step in calculating the SDFT, we calculate the DFT of the $i$th generator output trajectory as:

$$Y_i[n] = DFT\{\bar{y}_i\} = \sum_{l=0}^{N_p-1} y_i(k+l+1)e^{-j\frac{2\pi}{N_p}nl}$$

$\forall n = 0, \ldots, (N_p - 1)$ where $n$ is the frequency component index, $N_p$ is the number of samples in the output trajectory $\bar{y}_i$ (also the prediction horizon). Next, (3.10) can be written in the matrix form as:

$$DFT\{\bar{y}_i\} = Y_i = W\bar{y}_i$$

where $Y_i = [Y_i[0], Y_i[1], \ldots, Y_i[N_p-1]]'$ are the DFT frequency components and,

$$W = \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & w & w^2 & \cdots & w^{(N_p-1)} \\
1 & w^2 & w^4 & \cdots & w^{2(N_p-1)} \\
1 & w^3 & w^6 & \cdots & w^{3(N_p-1)} \\
\vdots & & & \ddots & \\
1 & w^{(N_p-1)} & w^{2(N_p-1)} & \cdots & w^{(N_p-1)(N_p-1)}
\end{bmatrix}$$

where $w = e^{-j\frac{2\pi}{N_p}}$ and $W \in \mathbb{R}^{N_p \times N_p}$ is the complex frequency weighting matrix.

Each row of the frequency weighting matrix $W$ in (5.3) represents the weighting of a different frequency mode. To select specific slow modes, we can extract the respective rows of the weighting matrix $W$. Let the range of selected frequency modes be represented by the frequency
indices \((\alpha_i, \beta_i)\) where \(N_p > \beta_i > \alpha_i\) and \(\beta_i = \alpha_i + n_s\). Then the SDFT of the output trajectory is:

\[
\hat{Y}_i = W_{\alpha_i \beta_i}^i \bar{y}_i
\]  

(3.13)

where \(\hat{Y}_i = [Y_i[\alpha_i], Y_i[\alpha_i+1], \ldots, Y_i[\beta_i]]'\) are the selected DFT components and \(W_{\alpha_i \beta_i}^i \in \mathbb{R}^{n_s \times N_p}\) is the corresponding selective frequency weighting matrix, dependent on the variables \((\alpha_i, \beta_i)\).

Next, using Euler’s Formula, the complex matrix \(W_{\alpha_i \beta_i}^i\) can be split into its real and imaginary parts as:

\[
W_{\alpha_i \beta_i}^i = W_{\alpha_i \beta_i}^i_{\text{real}} - jW_{\alpha_i \beta_i}^i_{\text{imag}}
\]  

(3.14)

where the matrices \(W_{\alpha_i \beta_i}^i_{\text{real}}\) and \(W_{\alpha_i \beta_i}^i_{\text{imag}}\) are derived in Appendix B.1 for completeness. Hence from (3.13), the real and imaginary parts of the SDFT \(\hat{Y}_i\) are given by:

\[
\hat{Y}_{i,\text{real}} = W_{\alpha_i \beta_i}^i_{\text{real}} \bar{y}_i, \quad \hat{Y}_{i,\text{imag}} = W_{\alpha_i \beta_i}^i_{\text{imag}} \bar{y}_i
\]  

(3.15)

The energy content of the SDFT of the predicted output trajectory \(\hat{Y}_i\) is given by [45]:

\[
\mathcal{E}_i = \sum_{n=\alpha_i}^{\beta_i} |Y_i[n]|^2 = \sum_{n=0}^{n_{si}} |\hat{Y}_i[n]|^2 = \sum_{n=0}^{n_{si}} |\hat{Y}_{i,\text{real}}[n] - j\hat{Y}_{i,\text{imag}}[n]|^2 = \hat{Y}_{i,\text{real}}' \hat{Y}_{i,\text{real}} + \hat{Y}_{i,\text{imag}}' \hat{Y}_{i,\text{imag}}
\]  

(3.16)

Substituting (3.15) in (3.16), we get:

\[
\mathcal{E}_i = \bar{y}_i' \hat{Q}_i \bar{y}_i
\]  

(3.17)

where,

\[
\hat{Q}_i = W_{\alpha_i \beta_i}^i_{\text{real}} W_{\alpha_i \beta_i}^i_{\text{real}} + W_{\alpha_i \beta_i}^i_{\text{imag}} W_{\alpha_i \beta_i}^i_{\text{imag}}
\]  

(3.18)

To construct an aggregate objective function, we add the individual objectives, i.e.:

\[
\mathcal{E} = \sum_{i=1}^{m} \mathcal{E}_i = \sum_{i=1}^{m} \bar{y}_i' \hat{Q}_i \bar{y}_i = \bar{y}' \hat{Q} \bar{y}
\]  

(3.19)

where \(\hat{Q}\) is a block diagonal matrix, i.e.:

\[
\hat{Q} = \text{blockdiag}(\hat{Q}_1, \hat{Q}_2, \ldots, \hat{Q}_m)
\]  

(3.20)

Comparing (3.19) and (3.7), we observe that the SDFT approach is equivalent to tuning of \(Q\) to \(\hat{Q}\).
3.4 Predictive Tuning of SDFT Window

We next propose an algorithm for online tuning of the matrix $Q$ in (3.7), based on the above derivations, in order to choose the tightest limit on $\alpha_i$ and $\beta_i$ in (3.13) so that the control provides damping to the most dominant frequency component in the selected range of frequencies. The choice of $(\alpha_i, \beta_i)$ may be different for different $i$.

For a pre-fault condition, the nominal-MPC controller ($Q = I$) is proposed since the location and magnitude of the dominant slow modes are not known at that time. Post-fault, the fast modes will be damped by the corresponding PSSs of the generators. A running average of the DFT magnitudes is taken over the next $r$ time-steps (say $r = 10$) to ensure the accuracy of the identified modes. The $\hat{Q}$ weighting matrix, and hence the cost/energy function, is then constructed by the controller based on (3.18)-(3.20). The specific values of $(\alpha_i, \beta_i)$ can be system-specific and can be constrained to be inside a certain tolerance ($\gamma$) around the identified dominant frequency mode, i.e. $\alpha_i = n_{di} - \gamma$ and $\beta_i = n_{di} + \gamma$, where $n_{di}$ is the identified dominant frequency mode. This change in the MPC cost function will emphasize the damping of selected frequency modes, resulting in relatively damped oscillation modes in the actual output frequency spectrum. After the oscillations are settled within a tolerance, the controller may switch back to its nominal version, i.e. $Q = I$.

The procedure is presented in a step-wise format in Algorithm 2. Let $k$ represent the time-step index for discrete-time MPC control, $t = k/f_s$ is the current time (in secs), $\epsilon$ is the settling time tolerance for the predicted trajectory, $\|\cdot\|_\infty$ represents the signal infinity norm, and the notation $\bar{y}_i(k)$ represents the linearized predicted future output trajectory of the $i^{th}$ generator starting from the $k^{th}$ time-step and ending at the $(k + N_p)^{th}$ time-step.

3.5 Simulation Results

In this section we show the performance of the proposed method as applied on a 39-bus New England power system model (Fig. 3.4). The system has 10 synchronous generators which form coherent clusters as shown by studies in [29]. Hence, the system will exhibit slow modes which cause low frequency oscillations in power output of generators. The nonlinear power system simulation is carried out with the help of Matlab [46] and Power System Toolbox (PST) [19] with detailed 13-state-each synchronous generator models. Each generator is equipped with a PSS and the output system performance with the PSS-only configuration is as shown in Fig. 3.2. The sampling frequency of this nonlinear simulation is 100 Hz. A three-phase fault is applied at bus 3 on the transmission line between buses 3 and 4 at the start of the simulation to assess performance of the controllers. The fault is applied at time $t = 0.1$ secs, cleared at bus 3 at $t = 0.15$ secs and cleared at the remote end at $t = 0.18$ secs.
Algorithm 2  SDFT-MPC Window Predictive Tuning \hspace{1em} \forall i=1,\ldots, m

1: Start: $k = 0$
2: Calculate $\bar{y}_i(k)$
3: if $\|\bar{y}_i(k)\|_\infty \geq \epsilon$ then $\triangleright$ Disturbance detection
4: \hspace{1em} $c = 1$ $\triangleright$ Counter
5: \hspace{1em} while $c < r$ do $\triangleright$ Repeat $r$ times
6: \hspace{2em} $\hat{Y}_i = DFT\{\bar{y}_i(k)\}$
7: \hspace{2em} $n_{di} = \max_n\{\hat{Y}_i[n]\}$
8: \hspace{2em} $n'_{di} = \text{running average of } n_{di}$ $\triangleright$ Dominant mode
9: \hspace{2em} $Q = I$ $\triangleright$ Nominal MPC
10: \hspace{2em} $\tilde{u}^{opt} = \min_u J$ $\triangleright$ Eq. (3.7)
11: \hspace{2em} $k \leftarrow k + 1$, $c \leftarrow c + 1$
12: \hspace{1em} end while
13: \hspace{1em} Choose $(\alpha_i, \beta_i)$ $\triangleright$ $\alpha_i \leq n'_{di} \leq \beta_i$
14: \hspace{1em} Construct $\hat{Q}$ $\triangleright$ Eq. (3.20)
15: \hspace{1em} Switch from $Q \rightarrow \hat{Q}$ in MPC cost $\triangleright$ SDFT MPC
16: \hspace{1em} $\tilde{u}^{opt} = \min_u J$ $\triangleright$ Eq. (3.7)
17: else
18: \hspace{1em} $Q = I$ and $\tilde{u}^{opt} = \min_u J$ $\triangleright$ Nominal MPC
19: \hspace{1em} end if
20: $k \leftarrow k + 1$, and go to Step 2

Figure 3.4: IEEE 39-bus New England power system model
To apply the MPC controller, the system is first linearized around the steady-state value and then converted to discrete-time. We apply our SDFT-MPC controller in a centralized architecture, and compare performance in time as well as frequency domain with the nominal MPC ($Q = I$). Except for the output trajectory weighting matrix $Q$ in the cost function, the rest of the parameters for both MPC techniques are kept the same for evaluating performance comparison. Since the prediction horizon has a lower bound due to (3.11), $N_p$ is chosen to be 100, $N_c$ is 10, control input constraints are set as $-0.01 \leq u \leq 0.01$, control trajectory weighting matrix $R$ is $10^{-3}I$, the sampling frequency for applying control signals to the system is 10 Hz. It was observed that when placing the SDFT window in the middle of the low frequency spectrum ($0.1 - 2$ Hz), there was a sharp rise at the 0.1667 Hz mode. Due to this reason, the value of $\alpha_i \forall i = 1, \ldots, M$ is chosen to be 0.1 Hz. The algorithm then chooses the value of $\beta_i$ as per the identified dominant mode, i.e. $\beta_i = n_{di} + \gamma$ where $\gamma$ is chosen to be 0.1 Hz (see Table 3.1).

The closed-loop response of all the synchronous generators for a three-phase fault between buses 3 and 4 with the SDFT-MPC approach is as shown in Fig. 3.5. Comparing it to the response of the PSS-only configuration shown in Fig. 3.2, the increase in performance (damping of slow oscillations) in time-domain is clearly visible. Since the steady-state value of the rotor speed $\omega_{ss}$ is 1 pu (60 Hz), the oscillations settle down to this value.

![Figure 3.5: SDFT-MPC output response for all generators (rotor speed deviations)](image_url)

To compare the performance of the nominal MPC with SDFT-MPC, we plot the output
response of generator 4 in time-domain for both the approaches, shown in Fig. 3.6. A comparison is also drawn in the frequency domain in Fig. 3.7 with the FFT magnitude plot, where the horizontal axis shows only the spectrum upto 3 Hz for clarity. It is seen that the SDFT controller indeed provides a better damping to both 0.3 Hz and 0.633 Hz modes. Similar response results are seen for most of the remaining generators as well; the plots are omitted for brevity.

![Comparison of post-fault speed deviation for Gen. 4](image)

Figure 3.6: Comparison of post-fault speed deviation for Gen. 4

Table 3.1 shows the FFT magnitudes of the identified 0.633 Hz dominant frequency mode for all the generators, and a comparison is drawn between PSS-only, nominal MPC and SDFT-MPC control designs. It is observed that SDFT-MPC damps the dominant 0.633 Hz mode better than both PSS-only and nominal MPC designs. The SDFT window \((\alpha_i, \beta_i)\) is also listed for each of the generators where the value of \(\beta_i\) is determined by the proposed algorithm. The improvement in performance in the frequency domain is reflected in the time domain by Fig. 3.6 for generator 4.

### 3.6 Conclusions

This chapter proposes a novel model predictive control approach to damp specific slow frequency modes in power systems via online tuning of the weighting matrix used for quadratic energy minimization. We develop an algorithm for online prediction of the DFT magnitudes of selected
modes from measured signals, and then distribute the control energy to different generators using MPC. The formulation, in theory, is equivalent to application of an ideal digital bandpass filter to the predicted output trajectories at every time step. Since for the constrained online optimization problem we are only interested in the energy content of the output trajectories and not the predicted waveforms themselves, the use of ‘rectangular window filtering’ is still very effective in damping the dominant mode. Simulation results on a large power system network show effectiveness of this approach.
Chapter 4

Online Modal Analysis

4.1 Introduction

In this chapter, we first present the modal decomposition form for our LTI power system, which results in an online modal participation factor analysis. We then propose a feedback communication topology for the purpose of damping the oscillation modes most excited by a disturbance. The communication topology is adaptive to the fault (disturbance) entering the grid dynamics since it is dependent on the post-disturbance state of the system. Hence, the topology is constructed after a central coordinator (CCO), for instance an Independent Systems Operator (ISO) [47], has detected a disturbance. This strategy forms the basis of all our distributed (sparse) optimal control designs in the subsequent sections. We note that although many works in literature [48, 49] have proposed constructing feedback communication topologies based on participation factor analysis, these types of analysis are generally done offline, and hence do not take into account the unmeasured disturbance characteristics.

4.2 Post-Disturbance Modal Participation

We refer to the discrete-time LTI power system model (2.23), and write our performance output equation and the input constraints, as:

\[ y(k) = \omega(k) = Cx(k), \text{ and } u(k) \in U^m, \tag{4.1} \]

where \( u = [u_1, \ldots, u_m]' \), \( C = \text{blkdiag}(C_1, \ldots, C_m) \in \mathbb{R}^{m \times n} \) is a block-diagonal binary matrix to select the small-signal rotor speeds \( \omega(k) = [\omega_1(k), \ldots, \omega_m(k)]' \), and \( U^m = U_1 \times \ldots \times U_m \) is the \( m \)-times cartesian product of the individual input constraint sets. The model (2.23), and subsequently the output in (4.1), is assumed to be excited by an exogenous but vanishing
disturbance such as a three-phase fault, whose effect is captured by the post-disturbance ‘initial’ state $x_0$. Rotor speeds $\omega(k)$ are chosen as the performance variables $y(k)$, which describe the kinetic energy of the system. Other electromechanical variables, such as paired difference of phase angles between generators describing the potential energy of the system can also be used in the design objective, if needed. It is assumed that $A$ is bounded-input bounded-output stable, and $(A, B)$ is stabilizable.

We first show how the CCO, using $\hat{x}_0$ estimated by various UKFs, estimates the modal residues of $y(k)$, from which it can decide the sparsity structure of the wide-area communication network. From linear system theory it follows that $x(k) = A^kx_0$ for the unforced system (2.23). Equivalently, one may also write the state response in the modal decomposition form as:

$$x(k) = \tilde{M}(\lambda_1^k, \ldots, \lambda_n^k)'$$

where $\{\lambda_j\}$ are the eigenvalues of $A$ (assumed to be distinct); $\tilde{M} = \text{col}(\alpha_1\rho_1, \ldots, \alpha_n\rho_n)$ where $\text{col}(\cdot)$ denotes a matrix of column vectors; $\{\rho_i\}$ are the right eigenvectors of $A$; and $\{\alpha_i\}$ are scalar weights. Let $M = \text{col}(\rho_1, \ldots, \rho_n)$ be the modal matrix, $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_n)$, and $\alpha = (\alpha_1, \ldots, \alpha_n)'$. From (4.2): $x(k) = MA^k\alpha$. Comparing this equation with the state equation in initial value form, and using the identity $A^k = M\Lambda^kM^{-1}$:

$$\alpha = M^{-1}x_0.$$  \hspace{1cm} (4.3)

Also from (6.1b) we have:

$$y(k) = CM\Lambda^k\alpha,$$  \hspace{1cm} (4.4)

which means that the individual output speeds can be written in the modal decomposition form as:

$$y_i(k) = C_i \sum_{j=1}^{n} \alpha_j \rho_j \lambda_j^k = \sum_{j=1}^{n} \tilde{\rho}_{ij} \lambda_j^k.$$  \hspace{1cm} (4.5)

The above expression (4.5) shows that once the CCO knows $\hat{x}_0$ (which depends on the magnitude and location of the unknown exogenous disturbance), it can estimate $\hat{\alpha} = M^{-1}\hat{x}_0$ from (4.3), and therefore, the modal coefficients $\{\tilde{\rho}_{ij}\}$. Note that the implicit assumption here is that the CCO has full knowledge of the matrix $A$. For damping control our design will be focused only on the inter-area modes (i.e. oscillation modes with frequencies between 0.1 and 2 Hz). However, dominance of these modes is defined not based on their frequencies, but on their residues. For example, consider a power system with five generators, with each generator considered as a coherent area in itself yielding 4 inter-area modes. The impulse response of the small-signal
frequencies of the five generators can be written as:

\[
\begin{align*}
G_1 : y_1(k) &= \bar{\rho}_{11}\lambda_1^k + \bar{\rho}_{12}\lambda_2^k + \bar{\rho}_{13}\lambda_3^k + \bar{\rho}_{14}\lambda_4^k + b_1(k), \\
G_2 : y_2(k) &= \bar{\rho}_{21}\lambda_1^k + \bar{\rho}_{22}\lambda_2^k + \bar{\rho}_{23}\lambda_3^k + \bar{\rho}_{24}\lambda_4^k + b_2(k), \\
G_3 : y_3(k) &= \bar{\rho}_{31}\lambda_1^k + \bar{\rho}_{32}\lambda_2^k + \bar{\rho}_{33}\lambda_3^k + \bar{\rho}_{34}\lambda_4^k + b_3(k), \\
G_4 : y_4(k) &= \bar{\rho}_{41}\lambda_1^k + \bar{\rho}_{42}\lambda_2^k + \bar{\rho}_{43}\lambda_3^k + \bar{\rho}_{44}\lambda_4^k + b_4(k), \\
G_5 : y_5(k) &= \bar{\rho}_{51}\lambda_1^k + \bar{\rho}_{52}\lambda_2^k + \bar{\rho}_{53}\lambda_3^k + \bar{\rho}_{54}\lambda_4^k + b_5(k),
\end{align*}
\]  

(4.6)

where \( b_i(k) = y_i^{dc} + \sum_{j=1}^{4} \bar{\rho}_{ij} \lambda_j^k + y_i^f(k) \) with (\(^*\)) denoting complex conjugation since \( \{\bar{\rho}_{ij}\} \) and \( \{\lambda_j\} \) are, in general, complex numbers; \( y_i^{dc} \) represents the DC mode (equal to zero for small-signal rotor speeds); \( y_i^f(k) \) is the high-frequency modal component; and \( \bar{\rho}_{ij} \) is the residue of the \( i^{th} \) mode in the \( j^{th} \) output. Let the residues \( \bar{\rho}_{11}, \bar{\rho}_{21}, \bar{\rho}_{32}, \) and \( \bar{\rho}_{51} \), marked in boldface, be termed as dominant residues. Dominance is defined such that all \( |\bar{\rho}_{ij}| \geq \mu \), where \( \mu \) is a pre-specified threshold. This threshold can be chosen by the CCO in different ways, one possible choice being the arithmetic mean of all residues:

\[
\mu \triangleq \frac{1}{m(m-1)} \sum_{i=1}^{m} \sum_{j=1}^{m-1} |\bar{\rho}_{ij}|.
\]  

(4.7)

In other words, it is assumed that only the inter-area modes \( \lambda_1, \lambda_2 \) are substantially excited by the incoming disturbance while the other inter-area modes have poorer participation in the rotor speed responses. The residue magnitudes are then collected in a so-called modal participation (MP) matrix that shows which generators contribute most to the excitation of which dominant mode. For this example, generators \( G_1, G_2, G_5 \) contribute significantly to mode \( \lambda_1 \), and generators \( G_2, G_3 \) to mode \( \lambda_2 \). Information about this grouping is used to decide the topology of communication. Detailed description of this will be given in Section 5.3.

Note that two different disturbance events can result in two significantly different \( \hat{x}_0 \), and hence, two significantly different MP matrices (see Figs. 5.4(a)-(c) for an example), indicating different sets of generators influencing different combinations of the inter-area modes. It is important for a controller to be aware of this dominance property instead of an offline controller that is agnostic to it.

### 4.3 Feedback Communication Topology

The overall objective of our state-feedback control is to increase performance by decreasing the rate of change of energy of the system closed-loop response from the open-loop response, where energy is formulated by a quadratic function of subsystem states. Hence, considering the
MP matrix, it is sufficient to say that the control objective can now be considered as damping of the excited modes of the system, and hence minimization of their corresponding residues. The structure of the state feedback matrix for control will result in the communication graph between subsystems, where their respective states are to be communicated. Then, from (4.6), the following strategy is proposed for damping both \( \lambda_1, \lambda_2 \):

1. For damping the excited mode \( \lambda_1 \), since the residues \( \bar{\rho}_{11}, \bar{\rho}_{21}, \bar{\rho}_{31} \) are dominant, subsystems \( (G_1, G_2, G_5) \) communicate for supplementary control action.

2. For damping the excited mode \( \lambda_2 \), since the residues \( \bar{\rho}_{22}, \bar{\rho}_{32} \) are dominant, subsystems \( (G_2, G_3) \) communicate for supplementary control action.

The above strategy is justified as follows. It has been shown in [3] that for a large interconnected power system exhibiting inter-area or swing modes of oscillation, the power system model in (2.23) can be written in a multi-modal decomposition form as shown in Fig. 4.1. The decomposed form ‘pulls out’ the \( i^{th} \) modal system (corresponding to the eigenvalue of interest \( \lambda_i \)), and clearly indicates the effects of the rest of the system on the lightly-damped \( i^{th} \) frequency mode. The three transfer functions in Fig. 4.1 are [3]:

- \( K_{ci}(s) \), the controllability function. This function indicates how controllable the \( i^{th} \) mode of interest is by the control signal vector \( u \).
- \( K_{oi}(s) \), the observability function. This function indicates the modal content of the \( i^{th} \) mode in the measured signal \( y \).
- \( K_{ILi}(s) \), the inner-loop function. This function indicates the effects of the controller output on its own input, other than the swing mode of interest.

For our proposed control strategy, \( (u, y) \) will only be associated with the subsystems which have the highest participation in the \( i^{th} \) mode, consisting of their control signals, and measured states respectively. Hence, when evaluated at the frequency \( \omega_i \) corresponding to the interested mode \( \lambda_i \), \( (K_{ci}, K_{oi}) \) will have large values, and \( K_{ILi} \) will have a very small value. This is because we are effectively ignoring the subsystems with low participation in the \( i^{th} \) mode for control.

Various indices are provided in [3] to investigate the damping influence of a wide-area controller on the \( i^{th} \) swing mode, given the choice of vectors \( (u, y) \). We choose the one best suited for our work as the Maximum Damping Influence (MDI) index, given as:

\[
MDI_i = \frac{|K_{ci}(s)| \cdot |K_{oi}(s)|}{|K_{ILi}(s)|},
\]

evaluated at \( s = j\omega_i \). From our choice of \( (u, y) \), it is clearly seen that the index MDI_i will be large. For damping other swing modes of interest, the corresponding MDIs will also be large, and
hence a combination of the communicating subsystems will result in a sparse $K$. An algorithm to construct the proposed feedback structure is given in the next section.

### 4.4 Conclusions

In this chapter we conducted online (i.e. post-disturbance) modal analysis of the power system, based on which the communication topology for feedback communication is proposed. The online modal analysis, and the subsequent construction of the Modal Participation matrix allows the feedback topology to be adaptive to the specific disturbance entering the grid dynamics. In subsequent sections, we will design distributed controllers with this communication topology as a structural constraint.
Chapter 5

Distributed MPC for Wide-Area Control

5.1 Introduction

In this chapter a distributed Model Predictive Control design is presented for inter-area oscillation damping in power systems under two critical cyber-physical constraints - namely, communication constraints that lead to sparsification of the underlying communication network, and actuation constraints that respect the saturation limits of generator excitation controllers.

Ideally, WACs can be designed using standard pole placement techniques and state-feedback controllers such as linear quadratic regulators (LQR) [50] or Model Predictive Controllers (MPC). Compared to the offline optimal control methods such as LQR, MPC exhibits more robustness to load fluctuations and parametric uncertainties in the grid model as it evaluates the control inputs based on the current state of the system at every time-step [33]. It also allows us to explicitly incorporate actuator constraints, which is important for WAC as the margin of variation for excitation voltages in supplementary controllers can be significantly limited [17].

Several works in literature have proposed the use of a single MPC controller in the context of power systems. In [36], an MPC controller is proposed to modulate the reference point of a High Voltage Direct Current (HVDC) controller to damp inter-area oscillations. It is noted that HVDC can only be installed on a fixed transmission line, and hence might be less effective in damping oscillations originating from an electrically distant part of the grid. An adaptive version of centralized MPC is proposed in [51] which solves the problem of simultaneous control and identification of model parameters using subspace methods. Over recent years, MPC has also emerged as a popular choice for frequency regulation and load-frequency control (LFC). For example, in [52] cascaded MPCs are proposed to be deployed for multiple time-scale operations, so as to co-ordinate between the frequency control problem and the long-term power dispatch
problem. This approach does not consider contingency scenarios such as faults on transmission lines, which can cause system instabilities. A constrained LFC problem is solved in [53] with MPC, where the objective is to maintain high-frequency deviations in system frequency, caused due to load fluctuations, within acceptable limits. The effect of low-frequency oscillations is not considered. It is noted that all the above MPC methods applied to power systems are centralized methods, and hence do not consider the communication requirements for control.

Distributed MPC (dMPC), where multiple spatially distributed MPC controllers are used to satisfy a control objective, has also been proposed recently in literature for designing power system controllers. In various works [54, 10, 55, 56, 37, 57], the LFC problem is solved in a distributed/decentralized manner using dMPC. Constraints are usually imposed on the output system frequency for tight regulation. However, these methods are not directly extendable to the WAC problem due to severe computational requirements for large-scale systems and inability to specifically target the inter-area oscillation modes. For instance, in [10] the authors propose multiple iterations for state-feedback communication within a single time-step, whereas in [56] a particle-swarm optimization method is proposed to reach a global solution with adaptive tuning of weights. Both these approaches will be prohibitive when applying these controllers to a WAC problem because of long communication delays and severe computational requirements. In contrast, the dMPC design proposed in this paper promotes communication sparsity for control while also successfully damping inter-area oscillations. A general review of dMPC designs can be found in [58].

To highlight the novelty of our approach, we note that all of the above mentioned control schemes also suffer from either one or both of the following two additional drawbacks. First, they lead to a dense all-to-all communication strategy between the generators amounting to a centralized implementation, and second, they are designed offline based on nominal models of the power system that are most often agnostic of where a disturbance may occur, or how this disturbance may impact the inter-area oscillations. In our recent paper [59] a sparse LQR controller was designed that is devoid of both of these drawbacks. In this paper that design is extended to a completely online MPC strategy that accommodates additional constraints on actuation. A sparse state-feedback controller is developed using excitation control of synchronous generators where the sparsity pattern of the underlying communication network is decided according to the modal residues of the inter-area oscillation modes excited by that disturbance. Generators that share high values of residues corresponding to a certain inter-area mode in open-loop are encouraged to communicate with each other for enhancing the damping of that mode in the closed-loop system. Note that these residues depend on the location and magnitude of the disturbance, and therefore can be different for different events. The choice of the sets of influential generators, and of the resulting communication topology thus becomes completely aware of the disturbance rather than being agnostic. The control signals are computed over
this sparse topology using MPC in a distributed way, and implemented as a supplementary excitation control on top of existing PSS at selected sets of generators.

5.2 Control Objective

The control objective of our problem is formulated by deriving a cost function that reflects all the dominant modes identified by the CCO using the method described in Section 4.2. As in Chapter 3, the concept of SDFT is again used for damping of selected oscillation modes, but this time for distributed control. This is achieved by only targeting the most excited dominant modes with SDFT, where the definition of dominant modes was provided in Chapter 4. Referring to the discrete-time LTI power system model (2.23), let the output trajectory of the $i$th generator, starting at time-step $k$, be $\bar{y}_i(k) = [y_i(k), y_i(k+1), \ldots, y_i(k+N-1)]'$. The $N$-point DFT of $\bar{y}_i$, $\forall i = 1, \ldots, m$, at time-step $k$ can be written as:

$$\mathcal{Y}_i(\tilde{k}|k) = \sum_{k=0}^{N-1} y_i(k)e^{-j2\pi\tilde{k}k/N}, \ \forall \tilde{k} = 0, \ldots, N-1,$$

where $\tilde{k}$ is the DFT frequency index. The above expression can be written in a matrix form as:

$$\tilde{Y}_i[0:N-1|k] = W_N \bar{y}_i(k),$$

where $\tilde{Y}_i[0:N-1|k] = [\mathcal{Y}_i(0|k), \ldots, \mathcal{Y}_i(N-1|k)]'$, and $W_N \in \mathbb{C}^{N \times N}$ is the $N$-point complex DFT matrix:

$$W_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & w & \cdots & w^{N-1} \\ 1 & w^2 & \cdots & w^{2(N-1)} \\ \vdots & \vdots & & \vdots \\ 1 & w^{(N-1)} & \cdots & w^{(N-1)(N-1)} \end{bmatrix},$$

with $w \triangleq e^{-j\frac{2\pi}{N}}$ being the complex exponential. Here, $\mathcal{Y}_i(0|k)$ is the DC component, $[\mathcal{Y}_i(1|k), \ldots, \mathcal{Y}_i(N/2|k)]$ are the positive frequency components, and $[\mathcal{Y}_i(N/2+1|k), \ldots, \mathcal{Y}_i(N-1|k)]$ are the negative frequency components. Since $\bar{y}_i(k)$ is real-valued for our problem, the magnitudes of the positive and negative frequency components are equal in the DFT spectrum. In order to extract a selected range of positive frequency components from (5.2), the appropriate rows of $W_N$ can be chosen to define the $N$-point SDFT as:

$$\tilde{Y}_i[\beta_l^i:\beta_r^i|k] = W_{i,N} \bar{y}_i(k),$$

42
where $[\beta_l^i, \beta_r^i]$ is the desired frequency window with length $\gamma_i \triangleq \beta_r^i - \beta_l^i$, and $W_{i,N} \in \mathbb{C}^{n \times N}$ is a submatrix of (5.3) containing the corresponding rows. $\beta_l^i, \beta_r^i$ represent the left and right window edges respectively. For us the choice of the center of this window is the frequency of any dominant mode, whereas $\beta_l^i, \beta_r^i$ are chosen such that the width is proportional to the steepness of the peaks of (5.1). From (5.4), the modal cost to be minimized is written as:

$$\mathcal{E}_i(k) = \sum_{k=\beta_l^i}^{\beta_r^i} |\mathcal{V}(\hat{k}|k)|^2 = 2\tilde{y}_i(k)'\hat{Q}_i\tilde{y}_i(k),$$

with the SDFT output weighting matrix:

$$\hat{Q}_i = (W_{i,N}^{\text{real}})'(W_{i,N}^{\text{real}}) + (W_{i,N}^{\text{imag}})'(W_{i,N}^{\text{imag}}).$$

The matrices $W_{i,N}^{\text{real}}, W_{i,N}^{\text{imag}}$ contain the real and imaginary parts of $W_{i,N}$ respectively. It is easy to show that for $\beta_l^i = 0$ and $\beta_r^i = N - 1$, i.e. if the selected window encompasses the full frequency spectrum, then $\hat{Q}_i$ becomes an identity matrix, and therefore (5.5) becomes the usual quadratic energy of the outputs with unity weighting.

**Remark 2.** Note that the choice for the center of SDFT window is based on the frequencies of the open-loop eigenvalues of $A$. Minimizing (5.5), however, would require an estimate of frequencies of the dominant eigenvalues of the closed-loop model. Hence, an implicit assumption behind the cost function (5.5) is that the applied control does not alter the frequency of closed-loop modes from their open-loop values to a significant extent. This assumption has been used in previous literature such as in [3], and is verified with simulation results in Section 5.5. If the frequencies of the inter-area modes change from open- to closed-loop, the width of SDFT window can accommodate this shift to some extent.

### 5.3 Communication Architecture for Control

The communication architecture for our controllers is developed next, based on the choice of influential generators for the dominant modes. A few notations are introduced first.

**Notation**

For an arbitrary set $X$, its individual elements are denoted by $X_i$. A power set $\mathcal{P}(X)$ is defined as the set of all its subsets, e.g. for $X = \{X_1, X_2\}$, the power set is the collection (set of sets)
\[ \mathbb{P}(X) = \{X_1, X_2, \{X_1, X_2\}, \emptyset\}. \]

A mapping \( h : \mathbb{P}(X) \mapsto \bar{\mathbb{P}}(X) \) is defined such that:

\[
\bar{P}_i = \begin{cases} 
\bigcap_j P_{i,j} & \text{if } P_i \text{ is a collection,} \\
\mathbb{P}_i & \text{if } P_i \text{ is not a collection,}
\end{cases}
\tag{5.7}
\]

where \( P_{i,j} \) is the \( j^{th} \) element of the collection \( P_i \). Hence, after applying \( h \), no element of \( \bar{\mathbb{P}}(X) \) is a collection. Also, let \( P(X) \triangleq \bar{\mathbb{P}}(X) \setminus \emptyset \). Additional mappings \( g_1, g_2 \) are defined, where \( g_1 \) maps intersecting elements to symbolic forms as \( g_1(\{X_1 \cap X_2, X_3 \cap X_4\}) = \{X_{12}, X_{34}\} \), and \( g_2 \) maps any symbolic form to its set unions, i.e. \( g_2(\{X_{12}, X_{34}\}) = \{X_1 \cup X_2, X_3 \cup X_4\} \). The domain of \( h \) is the set of generators \( \{G_1, \ldots, G_m\} \), and the range of \( h \) is given by (5.7).

### 5.3.1 Sparse Communication Architecture

Let the set of generators that exhibit the greatest influence on the excitation of the \( j^{th} \) dominant mode be denoted as \( \mathcal{A}_j \), referred to as the \( j^{th} \) modal area. For the 5-machine example in Section 4.2, the dominant modes are \( \lambda_1, \lambda_2 \), and the corresponding modal areas are \( \mathcal{A}_1 = \{G_1, G_2, G_5\} \), \( \mathcal{A}_2 = \{G_2, G_3\} \). Two or more modal areas can be overlapping iff any mode in the set of dominant modes are influenced by more than one generator. This is the case for generator \( G_2 \) in the example.

The proposed distributed control strategy then involves designing \( r = p + \tilde{p} \) number of controllers, where \( p \) is the number of dominant modes, and \( \tilde{p} \) is the number of overlappings between modal areas, if any. A single dedicated controller is assigned to each modal area, whereas generators in overlapping modal areas will have an additional controller of their own. The objective of each controller is to minimize the modal cost (5.5) formulated in Section 5.2. The underlying communication infrastructure is defined by:

- **Upstream links**: these links originate from the controller, and transmit control signals to its assigned generators, and
- **Downstream links**: these links originate from the generators, and transmit their states to their respective controllers.

Additionally, controllers may talk to a specific subset of other controllers, where this subset is decided by the overlapping modal areas. The intuition behind this communication strategy is that in order to design a controller \( C_i, \forall i = 1, \ldots, r \), that minimizes a modal cost corresponding to oscillation mode \( \lambda_i \), only those set of generators are important that show dominant participation in \( \lambda_i \) in their predicted output response.

The architecture can then be formalized as follows. Let the set of all identified \( p \) modal areas be: \( \mathcal{A} = \{\mathcal{A}_1, \ldots, \mathcal{A}_p\} \). Since the modal areas can be overlapping, the non-empty intersecting subsets of \( \mathcal{A} \) are: a collection \( P(\mathcal{A}) = h(\mathbb{P}(\mathcal{A})) \setminus \emptyset. \) Let \( P(\mathcal{A}) = P^d(\mathcal{A}) \cup P^o(\mathcal{A}) \), where
\( P^d(\mathcal{A}) \) contains the sets for dominant modal areas, and \( P^o(\mathcal{A}) \) contains the intersecting sets for overlapping modal areas. Then the set of generators which receive their control inputs from a distributed controller \( \mathcal{C}_i \), \( \forall i = 1, \ldots, r \), connected via the so-called upstream communication links, is given by:

\[
\mathcal{C}^u_i = \begin{cases} 
\{ G_l \in P_i \mid G_l \notin P_j \} & \text{if } P_i \in P^d(\mathcal{A}) \\
\{ G_l \in P_i \mid g_1(P_i) \notin g_1(P_j) \} & \text{if } P_i \in P^o(\mathcal{A}),
\end{cases}
\]  

(5.8)

for all \( j = \{1, \ldots, p\}, j \neq i \), and where the generator index \( l \in \{1, \ldots, m\} \). \( P_i \) is the \( i^{th} \) element of the collection \( P(\mathcal{A}) \), and \( g_1(\cdot) \) is the symbolic mapping as defined in Section 5.3. The number of upstream links for the \( i^{th} \) controller is given by \( m_{u,i} = \text{card}(\mathcal{C}^u_i) \). Expression (5.8) conveys assigning controllers to generators such that each controller is minimizing a unique modal cost specific to the generator’s modal area. Finally, the set of generators is constructed for downstream communication to each controller \( \mathcal{C}_i \) as:

\[
\mathcal{C}^d_i = \begin{cases} 
\{ G_l \in P_i \} & \text{if } P_i \in P^d(\mathcal{A}) \\
\{ G_l \in g_2(g_1(P_i)) \} & \text{if } P_i \in P^o(\mathcal{A}),
\end{cases}
\]  

(5.9)

where the mapping \( g_2(\cdot) \) is as defined in Section 5.3. The number of downstream links for the \( i^{th} \) controller is given by \( m_{d,i} = \text{card}(\mathcal{C}^d_i) \). Expression (5.9) conveys assigning generators to the controller \( \mathcal{C}_i \) from where state feedback is needed.

The steps for constructing this communication architecture over time are illustrated in Fig. 5.1 using the 5-machine example in (4.6). The physical states of the generators \( G_1 - G_5 \) are coupled with each other via Kron reduction, and hence the leftmost figure shows all five generators to be connected to each other. Starting from time-step \( k=0 \) (fault-clearing), the CCO uses a small number of time-steps, say \( k^* \), to compute the MP matrix, and determine that in this case the system needs to be divided into two modal areas \( \mathcal{A}_1, \mathcal{A}_2 \) with dominant frequency modes \( \lambda_1, \lambda_2 \), respectively, where \( \mathcal{A}_1 = \{G_1, G_2, G_5\} \) and \( \mathcal{A}_2 = \{G_2, G_3\} \). The power set is \( \mathbb{P}(\mathcal{A}) = \{\mathcal{A}_1, \mathcal{A}_2, \{\mathcal{A}_1, \mathcal{A}_2\}, \emptyset\} \), and \( h(\mathbb{P}(\mathcal{A})) \setminus \emptyset = P(\mathcal{A}) = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_1 \cap \mathcal{A}_2\} = \{\mathcal{A}_1, \mathcal{A}_2\} \cup \{\mathcal{A}_1 \cap \mathcal{A}_2\} \triangleq P^d(\mathcal{A}) \cup P^o(\mathcal{A}) \). As shown, three controllers are then designed with respective sets of upstream and downstream links defined by the sets:

\[
\begin{align*}
\mathcal{C}^u_1 &= \{ G_l \in P_1 \mid G_l \notin P_2, P_3 \} = \{ G_1, G_5 \} & (5.10a) \\
\mathcal{C}^u_2 &= \{ G_l \in P_2 \mid G_l \notin P_1, P_3 \} = \{ G_3 \} & (5.10b) \\
\mathcal{C}^u_3 &= \{ G_l \in P_3 \mid g_1(P_3) \notin \mathcal{A}_1, \mathcal{A}_2 \} = \{ G_2 \} & (5.10c) \\
\mathcal{C}^d_1 &= \{ G_l \in P_1 \} = \{ G_1, G_2, G_5 \} & (5.10d) \\
\mathcal{C}^d_2 &= \{ G_l \in P_2 \} = \{ G_2, G_3 \} & (5.10e) \\
\mathcal{C}^d_3 &= \{ G_l \in g_2(g_1(P_3)) \} = \{ G_1, G_2, G_3, G_5 \}, & (5.10f)
\end{align*}
\]
where \( g_1(P_3) = g_1(A_1 \cap A_2) = A_{12} \), and \( g_2(g_1(P_3)) = A_1 \cup A_2 \). Sets (5.10a), (5.10b), (5.10d), (5.10e) correspond to the condition \( P_i \in P^d(A) \) being true, and the sets (5.10c),(5.10f) correspond to the condition \( P_i \in P^a(A) \) being true. These links can be seen in Fig. 5.1 for all the three controllers \( C_1 - C_3 \). The presence of a third controller is due to the overlap between the modal areas, resulting in controller-to-controller links denoted by \( U_{1,3} \) and \( U_{2,3} \). All controllers are triggered into action starting \( k = k^* \). It is noted that the system shown in Fig. 5.1 is a simple example used for illustrative purposes. For power systems with much larger size, the savings in communication from our distributed controller over a centralized controller will be significant, as will be shown in Section 5.5.

### 5.3.2 Cyber-Physical Architecture

It is noted that since the indices of generators associated with any controller \( C_i \) are not fixed \textit{a-priori} due to the unpredictable nature of disturbances, it is not advisable to install \( C_i \) at any fixed generator site, but rather in a shared computational platform such as a cloud network. Such a cloud-based cyber-physical architecture for wide-area control has been recently proposed in [60, 61], and can be a very promising medium for implementing our proposed distributed controller. This is explained as follows.

Each utility area is assumed to have a Phasor Data Concentrator (PDC). Each PDC is assumed to have access to a local cloud at its local control center, which is part of a larger cloud network referred to as the Internet of Clouds. PMUs in the utility area \( \kappa \) measure the phasor voltages and currents at designated buses, and transmit them to their local PDC. The PDC in area \( \kappa \) then transmits this information to the local cloud. State estimation for all generators in this utility area is done in the local cloud, with an estimator \( E_\kappa \), using the decentralized state estimation procedure detailed in Chapter 2.

This process goes on round the clock. At time \( k = 0^- \) a fault occurs in the system. At time \( k = 0^- + \Delta \), the UKF is assumed to reach steady-state, meaning onwards from \( k = 0^- + \Delta \), \( \hat{x}_\kappa \) is regarded as a statistically close estimate of the true state \( x_\kappa \) for area \( \kappa \). Let \( k = 0^- + \Delta \) be denoted as \( k = 0 \), which is the starting point for the steps needed for computing our control inputs. In reality, the UKFs can be fast enough so that \( \Delta \) is a very small time interval. The state vector \( \hat{x}_\kappa(0) \) is communicated from the local clouds to the CCO. The CCO, after receiving the entire state vector \( \hat{x}_0 = [\hat{x}_1(0), \ldots, \hat{x}_\kappa(0), \ldots, \hat{x}_M(0)]' \), decides the communication topology following the steps outlined in Section 5.3.1. The CCO then informs the controller assignments to the local clouds, where controllers \( C_i \), \( \forall i = 1, \ldots, r \), are created. At every time-step, after receiving the estimated states according to the set \( C^d_i \), these controllers then solve the dMPC optimization problem (to be described in Section 5.4). Finally, the local clouds communicate the computed control inputs to generator actuators according to the set \( C^u_i \). The inter-cloud
information exchange is done via wide-area communication network. Please see Algorithm 3 for step-by-step description of this implementation.

The above procedure is illustrated with the 5-machine example (4.6), as shown in Fig. 5.2. For simplicity, it is assumed that each of the five machines belong to a distinct utility area. Since \( G_4 \) is not contained in any of the modal areas, only four clouds (corresponding to utility areas 1, 2, 3 and 5) are shown. To minimize communication inside the internet of clouds, \( C_1 \) is placed in the local cloud for utility area 1, \( C_3 \) in the local cloud for utility area 2, and \( C_2 \) in the local cloud for utility area 3. Cloud-to-cloud links \( \mathcal{I}_{ij} \), where \( i,j \) are the indices for sending and receiving clouds respectively, are set up for the controllers to exchange necessary information. Their information content is given in the caption of Fig. 5.2. In this figure the previous time-step control trajectory for the \( i^{th} \) generator is denoted by \( \bar{u}_i(k-1) \triangleq [u_i(k|k-1), \ldots, u_i(k+N_c-1|k-1)]' \).

5.4 Distributed MPC

Using the communication architecture defined above, a distributed MPC control problem is formulated next.

5.4.1 Distributed Controller Prediction Modelling

To select a subset of states, outputs and control inputs, let \( z_i = T_{z_i}x, \ v_i = T_{v_i}u, \ w_i = T_{w_i}u, \ \eta_i = T_{\eta_i}y \), where \( T_{z_i} \in \mathbb{R}^{n_{d,i} \times n}, \ T_{v_i} \in \mathbb{R}^{m_{u,i} \times m}, \ T_{w_i} \in \mathbb{R}^{(m_{d,i} - m_{u,i}) \times m} \) and \( T_{\eta_i} \in \mathbb{R}^{m_{u,i} \times 1} \) are indicator matrices whose structures follow from the communication architecture for controller \( C_i \); \( n_{d,i} \) represents the total number of states for the generators associated with the downstream links set \( \mathcal{C}_i^d \); \( z_i \) is the vector of states belonging to generators associated only with the downstream links; \( v_i \) is the vector of optimized control inputs to actuators only associated with the upstream links; \( \eta_i \) is the vector of outputs from generators only associated with the downstream links; \( w_i \) is the vector of communicated control inputs computed at the previous time-step by other controllers, and communicated to \( C_i \). The model for the set of generators associated with \( C_i \), \( \forall i = 1, \ldots, r \), can then be written as:

\[
\begin{align*}
z_i(k+1) &= A_{z_i}z_i(k) + B_{v_i}v_i(k) + B_{w_i}w_i(k), \\
\eta_i(k) &= C_{\eta_i}z_i(k),
\end{align*}
\]

where \( A_{z_i} = T_{z_i}A'_{z_i}, \ B_{v_i} = T_{z_i}B'_{v_i}, \ B_{w_i} = T_{z_i}B'_{w_i} \) and \( C_{\eta_i} = T_{z_i}C'_{\eta_i} \). Outputs \( \eta_i(k) \) thus represent an approximation to the actual generator outputs. Moving (5.11b) forward in time, the output prediction trajectory with horizon \( N \) and control horizon \( N_c \) (with \( N \geq N_c \) can be
Figure 5.1: Architecture of the proposed distributed control system, shown on a five-generator power system example, following (4.6). Subfigure (a) shows the physical interconnections between generators in the Kron-reduced form. CCO receives $\hat{x}_0$ from all generators and using the MP matrix decides the communication architecture, within the time-steps $k = 0 : \tau$. CCO then informs all generators about the communication topology. Subfigures (b) and (c) show state and control communications respectively for $k \geq k^*$, with the three dMPC controllers in feedback. The two identified modal areas are highlighted in red and blue.
Figure 5.2: Cyber-Physical Architecture for dMPC, shown on the five-generator example, following (4.6). For this example, since it is assumed that each generator bus has a PMU installed, the output of the $i^{th}$ PDC will be $\hat{V}_i$ as the generator bus voltage $|V_i|$ and bus angle $\theta_i$, and $\hat{I}_i$ as the vector of all phasor line currents $|I_1|, \phi_1, |I_2|, \phi_2, ...$ measured on all transmission lines connected to the generator bus. The cloud-to-cloud communication links are given as: $I_{21}(k) = I_{32}(k) = \{\hat{x}_2(k), \hat{u}_2(k-1)\}$, $I_{51}(k) = \{\hat{x}_5(k)\}$, $I_{12}(k) = \{\hat{x}_1(k), \hat{u}_1(k-1)\}$, $I_{52}(k) = \{\hat{x}_5(k), \hat{u}_5(k-1)\}$, $I_{23}(k) = \{\hat{x}_3(k), \hat{u}_3(k-1)\}$ and $I_{15}'(k) = \{u_5(k)\}$.
written as:
\[
\hat{\eta}_i(k) = \Lambda_i z_i(k) + \Phi_{v_i} \tilde{v}_i(k) + \Phi_{w_i} \tilde{w}_i(k),
\]
(5.12)
where \( \hat{\eta}_i, \tilde{v}_i, \tilde{w}_i \) denote the trajectories:
\[
\hat{\eta}_i(k) = [\eta_i(k|k)', \ldots, \eta_i(k+N-1|k)']',
\]
(5.13a)
\[
\tilde{v}_i(k) = [v_i(k|k)', \ldots, v_i(k+N_c-2|k)']',
\]
(5.13b)
\[
\tilde{w}_i(k) = [w_i(k|k)', \ldots, w_i(k+N_c-2|k)']',
\]
(5.13c)
and \( \Lambda_i, \Phi_{v_i}, \Phi_{w_i} \) are block matrices easily constructed from \( A_{z_i}, B_{v_i}, B_{w_i}, C_{\eta_i} \). It is noted that due to the distributed nature of the communication architecture, the controller \( \mathcal{C}_i \) does not have access to \( \tilde{w}_i(k) \). Hence, for our control design \( \mathcal{C}_i \) is considered to be the control trajectory computed at the previous time-step by another controller \( \mathcal{C}_j \), and make sure that \( \mathcal{C}_i, \mathcal{C}_j \) communicate so that \( \mathcal{C}_i \) can utilize this trajectory for its local predictions. For this reason the difference between the optimized control trajectory at the current and previous time-steps is penalized in (5.14). Also, because the states are not directly measurable, \( \mathcal{C}_i \) will only receive \( \hat{z}_i(k) \) for feedback instead of \( z_i(k) \). In the subsequent sections the notations for the variables defined in (5.13) are used, but it is assumed that the RHS follows from \( \hat{z}_i(k) \) instead of \( z_i(k) \).

5.4.2 dMPC with Modal Cost

The objective of each dMPC controller \( \mathcal{C}_i \) is to minimize the energy content of the SDFT spectrum of the generators assigned to it, while also respecting both actuation and communication constraints. For a controller \( \mathcal{C}_i, \forall i = 1, \ldots, r \), a cost function using the feedback \( \hat{z}_i(k) \) is formulated as:

\[
J_i(\hat{z}_i(k)) = \hat{\eta}_i(k)'Q_i \hat{\eta}_i(k) + \tilde{v}_i(k)'R_i \tilde{v}_i(k) + \Delta \tilde{v}_i(k)'S_i \Delta \tilde{v}_i(k),
\]
(5.14)
where \( \Delta \tilde{v}_i(k) = \tilde{v}_i(k) - \tilde{v}_i(k-1) \) is the difference between the current and previous time-step optimized control trajectories; \( R_i \) and \( S_i \) are positive definite weighting matrices of size \( m_{u,i}(N_c-1) \); and \( Q_i \) is a semi-positive definite block diagonal output trajectory weighting matrix of size \( m_{u,i}N \) such that \( Q_i = \text{blkdiag}(Q_{i1}, \ldots, Q_{ir}) \) with \( Q_i \) as the SDFT matrix constructed for frequency weighting in (5.6). The dMPC problem for the \( i^{th} \) controller is given by:

\[
\mathcal{P}_i : \min_{\tilde{v}_i(k)} J_i(\hat{z}_i(k))
\]
(5.15a)
\[
s.t. \quad \tilde{v}_i(t) \in U^{m_{u,i}}, \quad \forall t \in [k, \ldots, k+N_c-1],
\]
(5.15b)
\[
\tilde{w}_i(t) = T_{w_i} u(t|k-1), \quad \forall t \in [k, \ldots, k+N_c-1],
\]
(5.15c)
where \( U^{m_u,i} \) is the control input constraint set. From (5.14), \( Q_i \geq 0 \) and \( R_i,S_i > 0 \), hence the cost \( J_i \) is convex. \( P_i \) in the standard Quadratic Programming form is given by:

\[
J_i(\hat{z}_i(k)) = \frac{1}{2} \hat{v}_i(k)\mathcal{H}_i \hat{v}_i(k) + c_i(k)' \hat{v}_i(k) + \epsilon_i,
\]

(5.16)

where \( \mathcal{H}_i = 2(\Phi_i' Q_i \Phi_i + R_i + S_i) \) and \( c_i(k)' = 2[z_i(k)\Lambda_i' Q_i \Phi_i + \bar{w}_i(k)' \Phi_i Q_i A_{zi} - \bar{v}_i(k-1)' S_i] \). The terms collected in the scalar \( \epsilon_i \) are ignored while solving \( P_i \) since they do not contain the optimization variable \( \bar{v}_i(k) \).

**Remark 3.** The horizon length (equal to the SDFT length) \( N \) is a design parameter that should be tuned considering the following trade-off. Since the cost (5.14) is the energy content of a specific frequency window, the SDFT length \( N \) should be long enough to get an adequate frequency resolution \( 1/(TN) \) for selecting the SDFT window. On the other hand, a very large value of \( N \) can result in large prediction errors.

### 5.4.3 Closed-Loop Stability

We next provide *a-posteriori* sufficient conditions for dMPC closed-loop stability of the system. Substituting (5.12) in (5.14), we get:

\[
J_i(\hat{z}_i(k)) = \hat{z}_i'(k) (\Lambda_i' Q_i \Lambda_i) \hat{z}_i(k) + F_i(\hat{z}_i(k)),
\]

(5.17)

where, for the simplified case of a single-step control horizon, \( F_i(\hat{z}_i(k)) \) is quadratic in \( v_i(k), w_i(k), \Delta v_i(k) \), and linear in \( \hat{z}_i(k) \). Substituting (5.11) in (5.17) at time \( k + 1 \), and then subtracting (5.17), we get:

\[
J_i(\hat{z}_i(k+1)) - J_i(\hat{z}_i(k)) = \hat{z}_i'(k) M_i \hat{z}_i(k) + G_i(\hat{z}_i(k)),
\]

(5.18)

where \( M_i = \Lambda_i' Q_i \Lambda_i A_{zi} - \Lambda_i' Q_i A_{zi}, \) and \( G_i(\hat{z}_i(k)) = F_i(\hat{z}_i(k+1)) - F_i(\hat{z}_i(k)) \). Next, we state the following result for closed-loop stability [62, Theorem 2].

**Theorem 1** ([62]). *Considering the matrix \( M_i \) and the scalar-valued function \( G_i(\cdot) \) in (5.18), \( \forall i = 1, \ldots, r \), if the following condition:

\[
x' \left( \sum_{i=1}^{r} T_{zi} M_i T_{zi} \right) x + \sum_{i=1}^{r} G_i(T_{zi} x) < 0
\]

(5.19)

is satisfied \( \forall x \in \mathbb{R}^n \), then (2.23) is closed-loop stable under dMPC.*

It is clear from (5.19) that closed-loop stability of (2.23) will explicitly depend on the sparsity of the communication network. If for a given level of sparsity the test (5.19) fails, then the threshold \( \mu \) in (4.7) can be relaxed, leading to a denser topology until (5.19) is satisfied. In the
Algorithm 3 Algorithm for Implementing Proposed dMPC Controller

1: Location of PMUs is determined by solving the OPP problem (2.8) described in Chapter 2.3.1. This is a planning problem, and can be solved offline by the system operator. PMUs are then installed at these locations.

2: At all times (before, during, after a fault), each local PDC continuously collects voltage and current measurements from all PMUs in area $\kappa$, and sends this data to its local cloud as shown in Fig. 5.2.

3: In each local cloud, this measurement data is fed to a local estimator $E_\kappa$ which first estimates the phasors for all generator buses by solving (2.15), and then estimates the generator states $x_\kappa$ using the Kalman filter (2.20), for all generators in area $\kappa$. These states $\hat{x}_\kappa$ are used for the continuous monitoring of area dynamics.

4: At $k = 0^-$, a disturbance enters the grid, which is subsequently detected by all local clouds.

5: At $k = 0$, i.e. when the estimated state $\hat{x}_\kappa(0)$ is assumed to be statistically close to the actual state $x_\kappa(0)$, all local clouds send their estimated states to the CCO, thereby providing CCO with the full state vector $\hat{x}_0$.

6: Following the control design steps provided in Sections 5.2-5.4 of this chapter, the CCO informs the local clouds to set-up distributed MPC controllers $C_i$, $\forall i = 1, \ldots, r$.

7: For all utility areas $\kappa = 1, \ldots, M$, the control loop is then iterated as follows, for $k \geq 0$.

8: while any local cloud detects significant oscillations in $\hat{x}_\kappa$ do

9: States $\hat{x}_\kappa(k)$ are distributed among the controllers according to (5.9) using wide-area communication links, as shown in Fig. 5.2(a) for the 5-machine example system.

10: Using (5.11)-(5.16), every dMPC controller $C_i$ solves the problem $P_i$, and then sends its control inputs to generators according to (5.8), as shown in Fig. 5.2(b).

11: Advance the control time-step $k \leftarrow k + 1$.

12: end while

13: All wide-area links are terminated, and the local clouds continue monitoring the steady-state state-estimates $\hat{x}_\kappa$. 

52
worst case, this can result in all-to-all communication, where all controllers solve a centralized MPC problem. This situation, however, rarely arises in a practical WAC problem. As will be seen in our simulations, more than 70% sparsity can be retained without destabilizing the system. Also, note that (5.19) is a sufficient condition, not necessary. Hence, in reality one may be able to achieve closed-loop stability with a much sparser controller than the one stipulated by Theorem 1.

5.5 Simulation Results

The 48-machine, 140-bus NPCC nonlinear power system model [63] is used to verify our design, shown in Fig. 5.3. All generators are modeled using transient models with static exciters. Transformers are attached to all generator buses, and the system is open-loop stable. The system is divided into six utility areas based on electrical distances. Fig. 5.3 shows these areas with various background colors.

The system is linearized around its loadflow operating point. Since inter-area oscillations lie in the (0.1-2) Hz range for large power systems [17], the discretized system can only be sampled with a minimum Nyquist frequency of 4 Hz to capture all inter-area modes. For our design $T = 0.1$ secs (10 Hz) is chosen for discretizing the system.

State Estimation

State estimation for the generator dynamics is implemented following the three-step procedure described in Section 2.2. The area-wise OPP problem $P_{\kappa}^{\text{opp}}$ is solved for all $\kappa=1, \ldots, 6$, where the cost of installing PMUs is assumed to be unity for all buses. Locations for 39 PMUs are identified to assure observability of all generator buses. These buses are highlighted in red in Fig. 5.3. Voltage, current, and their phase angle measurements are recorded from these buses. The measurement noise is assumed to be zero-mean Gaussian noise. The noise variance values are obtained from the study in [64], where a signal-to-noise ratio of 45 dB was concluded to be a good approximation for PMU data. This data is used to estimate the phasor currents and voltages at the generator buses. Next, decentralized UKF is used to estimate generator states as detailed in Section 2.4. State estimation results for generator 1, in open-loop, are shown in Fig. 5.5. The results are seen to be consistent with the ones reported in [15].

Post-Disturbance Modal Analysis

The three scenarios considered for post-disturbance analysis on the NPCC model are:

(i) Case Study I: The system is perturbed with a three-phase fault on the transmission line connection buses 10-11. Starting the simulation in steady-state at $t=0$ secs, the fault is applied
Figure 5.3: The one-line diagram of the NPCC model shows the utility areas in background colors, the three faults considered for the three cases at lines connecting buses 10-11, 45-46 and 119-120. Modal areas for Case Study I are also shown, enclosed in dotted boundaries.
Figure 5.4: MP matrices are shown for the three case studies in subfigures (a), (b) and (c) respectively. x- and y-axis represent generator and oscillation mode indices respectively, and z-axis represents modal residues. 38 oscillation modes are chosen such that these modes all have frequencies less than 1.5 Hz, and damping factors less than 0.3.
Figure 5.5: Dynamic state estimation for the five states of Gen. 1 with the decentralized UKF.
at 1.1 secs, which is then cleared at 1.2 secs at bus 10, and at 1.25 secs at bus 11.

(ii) **Case Study II:** System is perturbed with a three-phase fault on the transmission line connecting buses 45-46. The fault is applied at 1.1 secs, cleared at 1.2 secs at bus 45, and at 1.3 secs at bus 46.

(iii) **Case Study III:** System is perturbed with a three-phase fault on the transmission line connecting buses 119-120. The fault is applied at 1.1 secs, cleared at 1.3 secs at bus 119, and at 1.45 secs at bus 120.

Note that the duration of the fault for Case Study I is smaller compared to other events. Since the eastern part of the power network is only connected via two buses (buses 29 and 35) to the rest of the grid, and due to the brief duration of this fault, it can be seen as a relatively ‘localized’ disturbance as compared to the other two events which occur on critical buses for longer durations. Once the fault is cleared at the remote end of the faulted line, the estimated post-disturbance system state $\hat{x}_0$ is used to construct the MP matrix whose elements are shown pictorially in Figs. 5.4(a)-(c). The residues for these different events can clearly be seen to be different from each other. Hence, it is clear that different sets of generators are influencing different sets of inter-area modes depending on the magnitude and location of the fault.

**Distributed Control Design**

For brevity, steps for our control design are provided next with respect to Case Study I only. Control designs for Case Studies II and III are done in a similar manner and their results are summarized in Table 5.1.

From the residues shown in Fig. 5.4(a), $\mu = 1.03$ is calculated for the MP matrix. Two modal areas are then constructed as $A_1 = \{G_1, \ldots, G_9\}$ and $A_2 = \{G_{11}, G_{12}, G_{14}\}$, highlighted in Fig. 5.3 with dotted boundaries. It is noted that for this particular disturbance, only 12 out of 48 generators are influential in the dominant inter-area modes. The remaining 36 generators do not need to participate in wide-area control. Since for this case the intersecting sets between the non-overlapping modal areas are empty, $P_i = A_i$, $\forall i = 1, 2$. Design of $r = 2$ distributed controllers is done for the two modal areas using the SDFT frequency windows (in Hz): $[0.53, 0.73]$ for $C_1$ and $[0.86, 1.2]$ for $C_2$. The open-loop frequency response for all rotor speeds is shown in Fig. 5.6. The windows are chosen by the CCO from the predicted open-loop frequency response. The input constraints enforced on the control signals are $-0.1 \leq u_i(t) \leq 0.1$, $\forall t > 0$, allowing for a maximum of 10% supplementary control effort in the excitation voltage. The optimization problem $P_i$ in (6.22) is solved for the two controllers, with control weightings $R_1 = R_2 = \text{diag}(0.1, \ldots, 0.1)$ for less emphasis on the magnitude of control inputs. Additionally,
constraint set \[ \{ \text{control input voltages, for both generators 7 and 8.} \]

As shown, the control voltages lie in the performance. Fig. 5.7(c) shows the comparison between the centralized MPC and the dMPC time-step. In Figs. 5.7(a)-(b) it is seen that the dMPC performs close to the optimal centralized and send control inputs to all 48 generators, and solves a single optimization problem at every implementation. The centralized MPC receives state estimates of all 48 generators as feedback, and closed-loop results are shown for both a centralized MPC implementation and the dMPC in Fig. 5.7(a)-(b) show the rotor speed output for the generators 7 and 8, in open- and closed-loop. Closed-loop results are shown for both a centralized MPC implementation and the dMPC implementation. The centralized MPC receives state estimates of all 48 generators as feedback, and sends control inputs to all 48 generators, and solves a single optimization problem at every time-step. In Figs. 5.7(a)-(b) it is seen that the dMPC performs close to the optimal centralized performance. Fig. 5.7(c) shows the comparison between the centralized MPC and the dMPC control input voltages, for both generators 7 and 8. As shown, the control voltages lie in the constraint set \([-0.1, 0.1]\).

S_1 = S_2 = \text{diag}(0.01, \ldots, 0.01) \) to obtain a less conservative control policy. The prediction horizon is chosen as \( N = 100 \) keeping in mind the trade-off for unmeasured dynamics and SDFT resolution, as discussed in Remark 3. The control horizon is kept small for lower execution times at \( N_c = 10 \). The optimization toolbox in Matlab is used to solve the constrained QP (6.22) with the interior-point convex algorithm.

**Closed-Loop Results**

Figs. 5.7(a)-(b) show the rotor speed output for the generators 7 and 8, in open- and closed-loop. Closed-loop results are shown for both a centralized MPC implementation and the dMPC implementation. The centralized MPC receives state estimates of all 48 generators as feedback, and sends control inputs to all 48 generators, and solves a single optimization problem at every time-step. In Figs. 5.7(a)-(b) it is seen that the dMPC performs close to the optimal centralized performance. Fig. 5.7(c) shows the comparison between the centralized MPC and the dMPC control input voltages, for both generators 7 and 8. As shown, the control voltages lie in the constraint set \([-0.1, 0.1]\).

A comparison of open-loop versus dMPC closed-loop electrical power outputs for selected generators is shown in Fig. 5.8. It can be clearly seen that the dMPC controller suppresses the oscillation amplitudes successfully for power output of all generators, even the ones not included in the control design. From the closed-loop frequency responses, it is observed that the dominant mode 1 (around 0.6 Hz) show a 40.2% reduction in FFT peak, and dominant mode 2 (around 1 Hz) show a 14.6% reduction. It is also observed that the frequency of the two dominant modes are almost same in open- and closed-loop.

Average controller optimization times are reported in Table 5.1, and for the considered case

<table>
<thead>
<tr>
<th>Case Study</th>
<th>Fault Location &amp; Duration</th>
<th>Open-loop Dominant Modes</th>
<th>No. of Controllers</th>
<th>List of influential generators, and their assignments to controllers ( C_s )</th>
<th>SDFT Windows (in Hz)</th>
<th>Perf. loss Index: ( \xi )</th>
<th>Sparsity Index: ( \theta )</th>
<th>Optim. times (msecs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Line 10-11 for 0.15 secs</td>
<td>-0.638 ± 3.8, 0.419 ± 6.5</td>
<td>2</td>
<td>( C_1^I = {G_1, \ldots, G_9} ) ( C_2^I = {G_{11}, G_{12}, G_{14}} ) ( C_1^I = C_2^I ) ( C_2^I = C_2^I ) ( C_1^I = C_1^I ) ( C_2^I = C_2^I ) ( C_1^I = C_1^I ) ( C_2^I = C_2^I )</td>
<td>[0.53, 0.73], [0.86, 1.2]</td>
<td>0.121</td>
<td>0.250</td>
<td>50 (C_1), 10 (C_2)</td>
</tr>
<tr>
<td>II</td>
<td>Line 45-46 for 0.20 secs</td>
<td>-0.449 ± 5.6, 0.482 ± 8.8</td>
<td>1</td>
<td>( C_1^I = {G_{10}, \ldots, G_{17}, G_{33}, \ldots, G_{38}} ) ( C_2^I = C_2^I ) ( C_1^I = C_1^I ) ( C_2^I = C_2^I ) ( C_1^I = C_1^I ) ( C_2^I = C_2^I ) ( C_1^I = C_1^I ) ( C_2^I = C_2^I )</td>
<td>[0.95, 1.05], [1.35, 1.5]</td>
<td>0.328</td>
<td>0.250</td>
<td>80 (C_1)</td>
</tr>
<tr>
<td>III</td>
<td>Line 119-120 for 0.35 secs</td>
<td>-0.419 ± 6.5, 0.449 ± 5.7</td>
<td>3</td>
<td>( C_1^I = {G_{11}, G_{12}, \ldots, G_{38}} ) ( C_2^I = {G_{11, G_{12}, G_{14}, G_{13}}} ) ( C_1^I = C_1^I ) ( C_2^I = C_2^I ) ( C_1^I = C_1^I ) ( C_2^I = C_2^I ) ( C_1^I = C_1^I ) ( C_2^I = C_2^I )</td>
<td>[0.85, 0.95], [0.85, 1.05]</td>
<td>0.356</td>
<td>0.271</td>
<td>12 (C_2), 10 (C_1)</td>
</tr>
</tbody>
</table>

Table 5.1: dMPC simulation results for all case studies.
Figure 5.6: Open-loop frequency response for all rotor speeds. SDFT windows for control design are highlighted.

studies, observed to be less than the sampling time of 0.1 secs. The optimization times increase with increasing the control horizon $N_c$ and also with increasing the number of generators to be controlled with a single controller. For systems with even larger number of generators per controller, the optimization times can scale up rapidly. In such a case, priorities can be given to only those generators with the highest modal residues in oscillations, or faster computational resources can be provided to the local clouds. It is observed that a longer control horizon does not result in a significant improvement in performance and hence is kept small. For the centralized MPC controller, the average optimization time is observed to be 0.3 secs, which is larger than the considered sampling time of 0.1 secs. All simulations are performed on a 3.8 GHz quad-core Intel i7 processor with 16 GB RAM.

**Performance vs. Sparsity**

To evaluate the trade-off between closed-loop performance and sparsity, a performance metric is defined as:

$$J = \frac{1}{T_{ss}} \sum_{i=1}^{m} \sum_{k=1}^{T_{ss}} |y_i(k) - 1|^2, \quad (5.20)$$

where $T_{ss}$ is the maximum settling time for all outputs, and $y_i$ is the rotor speed of the $i^{th}$ generator, with a steady-state per unit value of 1. Let the open-loop cost, closed-loop centralized MPC cost, and the dMPC closed-loop cost be denoted by $J^{ol}$, $J^{cent}$ and $J^{dMPC}$, respectively.
Figure 5.7: Subfigure (a) and (b) shows the comparison of rotor speeds for generators 6 and 8 respectively, for the three cases of open-loop, closed-loop with dMPC controller and closed-loop with a centralized MPC controller. Subfigure (c) shows the comparison of control input voltages for generator actuators 6 and 8 respectively, for the two cases of dMPC controller and a centralized MPC controller.
Figure 5.8: (a) Open-loop vs. (b) dMPC closed-loop comparison is shown for electrical power outputs, in p.u., of all generator buses.
The dMPC performance loss index is then defined as:

$$\xi = \frac{J_{dmpc} - J_{cent}}{J_{ol} - J_{cent}}.$$  \hspace{1cm} (5.21)

Note that (5.21) normalizes the dMPC cost between $[0,1]$ with 0 representing the optimal centralized cost, and 1 representing the open-loop cost. The closer $\xi$ is to 0, the better is the dMPC performance. A sparsity index is also defined as the ratio between the number of unidirectional communication links required by the dMPC controller, to that required by the centralized MPC. Since a centralized MPC will essentially require communication with all generators, the number of required links will be $2m$. Hence the sparsity index is given by:

$$\theta = \frac{1}{2m} \sum_{i=1}^{r} [\text{card}(C_{d}^{i}) + \text{card}(C_{u}^{i}) + N_{i}],$$  \hspace{1cm} (5.22)

where $\theta \in [0,1]$, and $N_{i}$ is the number of controller-to-controller communication links needed due to possible overlapping modal areas. The closer $\theta$ is to zero, more is the sparsity ($\theta$ to be exactly 0, however, has no physical meaning in this case as that would mean that there is no dMPC controller).

Table 5.1 provides a summary of main results for the three case studies listed in Section 5.5. Since the fault in Case Study I is a ‘localized’ disturbance, a sparsity level of 75\% is achieved (as compared to a centralized controller) for this event using two controllers. The performance loss index is also lowest for this case study since the effect of generators outside the identified modal areas is minimal, as a result of which the unmeasured generator dynamics are close to zero. This clearly shows the advantage of designing our controller online after the fault happens instead of an offline controller that may be agnostic to the localized nature of the fault. Case Studies II and III also show an acceptable trade-off between performance and sparsity. The level of sparsity and performance with Case Study III is lower than the other two case studies due to the longer duration of fault resulting in overlapping modal areas, i.e. multiple generators participating substantially in excitation of same dominant inter-area modes. This necessitates the design of a third controller $C_{3}$ as listed in Table 5.1.

### 5.6 Conclusions

A dMPC design method is presented for damping inter-area oscillations in a large power system network. A set of dominant inter-area modes, resulting from a major disturbance in the network, are first identified. Generators which have highest contribution to these modes are then identified to form the communication topology for distributed control. Energy content of the dominant
modes is extracted from the output open-loop frequency spectrum using the SDFT method. A dMPC problem is then solved for each distributed controller, under communication and actuation constraints. Simulations performed on the NPCC model show effectiveness of the control design by successfully eliminating sustained low-frequency oscillations, while also saving on both communication links and computation times when compared to a centralized control implementation.

In conclusion, we highlight that with our proposed distributed MPC, the improvements with respect to the existing literature on distributed MPC [58] are:

- The communication topology for our distributed controller is constructed in an online manner after detecting the grid disturbance. This allows our distributed MPC to adapt its architecture based on the specific location and magnitude of the incoming disturbance. This is done by constructing the Modal Participation matrix in Chapter 3. In existing literature on distributed MPC, the communication topology is usually fixed and is agnostic to the nature of the incoming disturbance.

- The cost function for MPC is constructed in frequency domain to specifically target the most excited inter-area oscillations modes. This strategy allows our controller to focus most of its control energy on damping the targeted frequency modes. This is shown in Section 5.2 of the chapter with the help of a SDFT operator. Each distributed MPC controller then minimizes a specific excited inter-area mode using its own unique SDFT window. In existing literature on distributed MPC, the cost function to be minimized is usually constructed in time-domain. Since in the case of WAC of large power systems, our objective is to damp only the most excited inter-area oscillation modes, it is advantageous to design the cost function directly in frequency domain.

- We also provide a detailed cyber-physical architecture to implement our distributed MPC for WAC in a large power system network. This is achieved by giving details on the decentralized state estimation so that states can be fed back to the MPC controllers for predictions at every time-step, and also by proposing a cyber-physical cloud architecture for hosting the machines where distributed MPC and the decentralized estimators will reside. These details are provided in Section 5.3 of the chapter, and the implementation steps are provided in Algorithm 1.
Chapter 6

Sparse LQR for Wide-Area Control

6.1 Introduction

In this chapter we propose a sparse LQR design where the sparsity pattern is decided online after a disturbance happens in the grid, based on the dominant residues of the excited modes reflected in the predicted outputs. Identification of the dominant residues corresponding to the most excited modes was done in Chapter 4 using online modal analysis, and the resulting modal participation matrix. Note that many papers such as [65] have designed controllers based on modal participation factors that are only based on the natural dynamics of the power system. Our approach is slightly different from them as residues not only depend on the system poles but also on the input and output matrices of the state-variable model. Hence, depending on where the disturbance enters, the residues of the characteristic modes on the same outputs can be different from one event to another. Thus, the sparsification and the resulting overall structure of the communication network in our design are directly influenced by the physical characteristics of the grid. This is an important feature that separates our design from state-of-the-art control of other cyber-physical systems, where the cyber and the physical infrastructures are often designed independent of each other.

Our objective is to design a state-feedback distributed controller, which is both stabilizing and requires less number of communication links. Hence, we solve a structurally constrained infinite-horizon LQR problem. Since it is assumed that the location and strength of the exogenous disturbance is unknown, we build our communication graph for distributed control using the modal participation matrix (MP) discussed in Chapter 4, and then obtain the structure of the feedback matrix to be used as a constraint in the LQR problem. This problem is solved using an iterative algorithm which assures closed-loop stability of the system. A parallel implementation strategy for computing multiple versions of the controller, depending on different levels of sparsity, is proposed. Analyzing the trade-offs between control performance and the
level of sparsity induced, the most effective controller is chosen.

A subtle limitation of the above approach, however, is that, once designed, every link in the sparse topology is mandated to transmit every state of the corresponding generator. Hence we improve upon this limitation by extending our structurally constrained controller to a two-stage sparse controller, where links are sparsified using the modal constraint, and simultaneously the states in each link are sparsified using $\ell_1$-regularization. We note that the work in [1] also uses an $\ell_1$-regularization based sparsity-promoting optimal control strategy to first determine the control feedback structure, and then optimize the $\mathcal{H}_2$-norm of the system. Using just the $\ell_1$ approach, however, can be quite restrictive as it results in a controller structure that is completely agnostic of the nature and location of the incoming disturbances in the grid, and, therefore, depending on the severity of an event, can lead to both over- and under-sparsification. We perform simulations to compare our results with those published in [1], and show that our controller gives a more sparse controller with comparable closed-loop performance. Our results are validated using simulations on the 39-bus New England power system model. Simulations show that the designed controller is effective in promoting sparsity as well as introducing considerable damping into the system.

6.2 Control Objective

We re-write the discrete-time power system LTI model in (2.23), with a linear state-feedback control, as:

$$x(k+1) = Ax(k) + Bu(k) \quad (6.1a)$$
$$u(k) = Kx(k), \text{ and } x_0 \triangleq x(0), \quad (6.1b)$$

which is sampled with the time-step $T > T_p$ where $T_p$ is the sum of PMU sampling time and state estimation time, where the state estimation procedure was provided in Chapter 2. The matrix $K \in \mathbb{R}^{n \times m}$ is the linear feedback gain for supplementary control. It is to be noted that since the disturbance term $\tilde{B}_c d(t)$ in (2.21) is unknown, the discrete-time model (6.1a) can be excited by the estimated initial state $x_0$ after a disturbance at $k = 0$. It is assumed that the pair $(A, B)$ is stabilizable.

Our control objective is the optimal distributed control of the system (2.23) via a sparse state-feedback control design, in order to reduce the amplitude of oscillations of the closed-loop states. To design a $K$ for (6.1b) which satisfies these objectives, we minimize the discrete-time
infinite-horizon LQR cost:

\[ J_\infty(x_0) = \sum_{k=0}^{\infty} \{x(k)'Qx(k) + u(k)'Ru(k)\}, \quad (6.2) \]

under the structural constraint \( K \in \Omega \), where \( \Omega \in \mathbb{R}^{n \times m} \) is the set of matrices with pre-specified zero locations, and hence enforces the desired sparsity structure. \( Q > 0, \ R > 0 \) are the state and control weighting matrices respectively, where \( Q \) is designed such that the square of the differences of the linearized generator angles (proportional to the power transfer between generators), along with the quadratic energy of the rest of the states are penalized. This translates to the state-weighting term in (6.2) of the form:

\[ x(k)'Qx(k) = \sum_{i=1}^{n} \sum_{j>i}^{n} (\Delta \delta_i(k) - \Delta \delta_j(k))^2 + \sum_{i=1}^{n} \tilde{x}(k)'\tilde{x}(k), \quad (6.3) \]

where \( \tilde{x} \) is the vector of all linearized states except the rotor angle state \( \Delta \delta \). Let the notation \( I_z \) denote an identity matrix of size \( z \), and let the matrix of all ones be denoted by \( \mathbb{1}_{ab} \in \mathbb{R}^{a \times b} \). To obtain the form in (6.3), the state-weighting matrix \( Q \) is constructed with the transformation [66]: \( Q \triangleq \mathcal{T}'\tilde{Q}\mathcal{T} \), where \( \tilde{Q} = (\mathcal{L} I_{m-n}) \) is a block diagonal matrix, \( \mathcal{L} = nI_n - \mathbb{1}_{n1}\mathbb{1}_{n1}' \) is the matrix for difference in angles, and the transformation matrix \( \mathcal{T} \) is as given in [66].

The feedback gain matrix \( K \) in (6.1b) can be designed by a central control authority, which then communicates its individual rows \( K(i,:) \), \( \forall i = 1, \ldots, n \), to the corresponding generator actuators, i.e. \( u_i(t) = K(i,:)x(t) \), \( \forall i = 1, \ldots, n \). Then, depending on the non-zero entries of \( K(i,:) \), the \( i^{th} \) controller can communicate and receive the corresponding states from other generators to calculate its own actuator control input. Hence it is clear that if sufficient number of the \( K \) matrix entries are zero, the communication graph will be sparse, and the control architecture will fall under the category of distributed control. The following section provides modal analysis of the system, which forms a basis for constructing the structure of \( K \).
### 6.3 Construction of Feedback Structure

Following the strategy proposed in the previous section, for the control signal vector \( u = Kx \), the state feedback matrix structure for the MP form (4.6) is thus constructed as:

\[
\begin{pmatrix}
  u_1 \\
u_2 \\
u_3 \\
u_4 \\
u_5
\end{pmatrix}
= \begin{bmatrix}
  \times & 0 & \times & \times & 0 \\
  0 & \times & 0 & 0 & \times \\
  \times & 0 & \times & \times & 0 \\
  \times & \times & \times & \times & 0 \\
  0 & 0 & \times & 0 & \times \\
\end{bmatrix}
\begin{pmatrix}
  x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{pmatrix},
\]

where \((\times)\) indicates an arbitrary scalar value, and \(\Omega\) is the set of matrices with the specific structure with \(K \in \Omega\). The procedure for constructing \(\Omega\) is then generalized as follows.

Let \((\lambda_1, \ldots, \lambda_{n-1})\) be the interested modes of the system which we wish to suppress via a sparse state-feedback control \(K \in \Omega\). Let \(\mathcal{R}\) be the \((n(n-1))\)-tuple of all modal coefficients, i.e. \(|\tilde{p}_{ij}| \in \mathcal{R}, \forall i = 1, \ldots, n, \text{ and } \forall j = 1, \ldots, n - 1\). Then, for each \(\lambda_j, \ j \in \{1, \ldots, n - 1\}\), we obtain the set of indices:

\[
\Xi_j \triangleq \{ i \in (1, \ldots, n) \mid |\tilde{p}_{ij}| \geq \mu, \ \forall |\tilde{p}_{ij}| \in \mathcal{R}\},
\]

where \(\mu \in \mathbb{R}\) is some threshold for dominant coefficient selection (chosen as the arithmetic mean of \(\text{vec}(M)\), and iterated via parallelization as described in Section 6.4.3). Let \(r_j \triangleq \text{card}(\Xi_j)\), where \(\text{card}(\cdot)\) denotes the number of elements of the specified set (cardinality). Construction of the feedback matrix structure is then given by:

\[
\Omega(i,j) = \begin{cases} \times, & \text{if } (i,j) \in \bigcup_{j=1}^{n-1} Z_j \\ 0, & \text{otherwise} \end{cases}
\]

where \(Z_j\) is the set of \(r_j^2\) 2-tuples over \(\Xi_j\). For the example considered above in (4.6),(6.4), the sets/tuples \((\mathcal{R}, \Xi_1, Z_1)\), for the mode \(\lambda_1\), are given as:

\[
\mathcal{R} = (|\tilde{p}_{11}|, |\tilde{p}_{12}|, \ldots, |\tilde{p}_{53}|, |\tilde{p}_{54}|), \text{ with } \mathcal{R} \text{ a 20-tuple,}
\]

\[
\Xi_1 = \{1, 3, 4\}, \text{ with } r_1 = 3,
\]

\[
Z_1 = \{(1,1), (1,3), (1,4), (3,1), (3,3), (3,4), (4,1), \ldots (4,3), (4,4)\}, \text{ with } \text{card}(Z_1) = 9.
\]

The sets \((\Xi_2, Z_2)\) can be determined similarly, and the structure of \(\Omega\) in (6.4) can then be
formed with (6.6) (sets $\Xi_3$ and $\Xi_4$ will be empty). Next, we formulate the optimal control design procedure using LQR control, with the state-feedback structure constructed above as a constraint.

### 6.4 Structurally Constrained Optimal Control

A constrained optimal control formulation for the power system is done by specifying structural constraints on the feedback matrix, and then using a generalized Riccati equation (GDARE) method to obtain a stabilizing constrained $K$.

#### 6.4.1 Discrete Linear Quadratic Regulator

The infinite-horizon quadratic cost (6.2) to be minimized is re-written here for continuity as follows:

$$J_\infty(x_0) = \sum_{k=0}^{\infty} \{x(k)'Qx(k) + u(k)'Ru(k)\}. \tag{6.7}$$

The linear state-feedback control law is given by $u(k) = Kx(k), \forall k > 0$, and the optimal $K$ is obtained from:

$$K = -(R + B'PB)^{-1}B'PA, \tag{6.8}$$

where the unique symmetric matrix $P > 0$ is obtained from the well-known discrete algebraic Riccati equation (DARE):

$$P = A'PA - A'PB(R + B'PB)^{-1}B'PA + Q, \tag{6.9}$$

where the optimal cost can then be given by $J_\infty^* = x_0'Px_0$. In addition to the dynamic system constraint (6.1a), we add the structural sparsity constraint $K \in \Omega$, where $\Omega$ is determined from the modal analysis procedure detailed in Section 4.2. The constrained control problem then becomes:

$$\mathcal{P} : \quad \min J_\infty(x_0) \quad \text{subject to} \quad K \in \Omega. \tag{6.10a}$$

Also, the solution of $\mathcal{P}$ should provide a matrix $K$ such that the closed-loop system matrix $A + BK$ is Hurwitz. It is clear that the nominal solution from (6.8) will no longer necessarily satisfy the structural constraint (6.10b). In the next subsection, we propose an online constrained control design which provides a solution to the above problem.
6.4.2 Generalized Riccati Equation Method

First, we formulate the constraint (6.10b) in an alternative way for use in the proposed method. Let \( I_\Omega \) be the indicator matrix for the structured matrix set \( \Omega \); an example would be to replace the \((\times)\)s with \((1)\)s in the feedback matrix in (6.4). Next, let \( I^c_\Omega \) be the complement of \( I_\Omega \). Then the following identity holds for the structural constraint (6.10b):

\[
F(K) \triangleq K \circ I^c_\Omega = 0,
\]

where \((\circ)\) is the Hadamard product, and \(0\) is the zero matrix of appropriate dimensions. Hence the constraint \( F(K) = 0 \) is equivalent to (6.10b). The following theorem then assures the stability of the discrete-time closed-loop system when the control is obtained using GDARE.

**Theorem 2** ([67]). For the discrete-time system (6.1a) and control law (6.1b), if the state-feedback gain matrix \( K \) satisfies:

\[
K + L = -(R + B'PB)^{-1}B'PA,
\]

where \( L \) is an arbitrary matrix, and the symmetric matrix \( P > 0 \) is the solution of the GDARE:

\[
P = A'PA - A'PB(R + B'PB)^{-1}B'PA + Q + L'(R + B'PB)L,
\]

then the closed-loop system matrix \( A + BK \) is Hurwitz.

The following corollary provides an appropriate choice for the matrix \( L \) so as to satisfy the required structural constraint on the feedback matrix.

**Corollary 1.** To enforce the structural constraint (6.10b), the matrix \( L \) in Theorem 2 can be chosen as:

\[
L = F(\Psi(P)),
\]

where:

\[
\Psi(P) \triangleq -(R + B'PB)^{-1}B'PA.
\]

**Proof.** To see that the choice of \( L \) in (6.14) will give the desired structure in \( K \), we substitute (6.14)-(6.15) in (6.12):

\[
K = \Psi(P) - [\Psi(P) \circ I^c_\Omega] = \Psi(P) \circ [1_{nm} - I^c_\Omega] = \Psi(P) \circ I_\Omega,
\]

where \( 1_{nm} \in \mathbb{R}^{n \times m} \) is a matrix of all ones (since \( K \in \mathbb{R}^{n \times m} \)), and (6.16b) is the result of the
distributive property (over addition) of the Hadamard product [68]. From (6.16c) we see that \( K \in \Omega \), and hence the particular choice of \( L \) in (6.14) gives the desired structure in the feedback matrix.

6.4.3 Algorithm for Solution of \( \mathcal{P} \)

The Geromel’s algorithm [67] is modified to include the constraints (6.10b) by solving the GDARE (6.13). The algorithm is also parallelized so that the problem \( \mathcal{P} \) can be solved with multiple sparsity structures, and the one with appropriate execution time \( \tau \) can be selected (i.e. less than a certain threshold \( \tau_{th} \)). Let \( p \) be the total number of parallelizations, i.e. we obtain \( K_i \in \Omega_i, \forall i = 1, \ldots, p \). For each parallelization index \( i \), a threshold \( \mu_i \in \mathbb{R} \) is calculated to determine the corresponding set of dominant modal coefficients from (6.5). One choice to achieve that is with a ‘running mean’, i.e. taking successive arithmetic means of the modal coefficients vector \( \text{vec}(\overline{M}) \), so as to extract the corresponding sets of relatively large coefficients. These coefficient sets can then be used to construct the sparse feedback matrix structures from (6.5)-(6.6). The above procedure is summarized in Algorithm 4, where \( \overline{\mu}() \) is the arithmetic mean function.

### Algorithm 4 Running Mean Algorithm for Parallelization

1: Start: \( \mathcal{X}_0 = \{\text{vec}(\mathcal{M})\} \) \( \triangleright \) Set of All Coeffs.
2: for \( i \leftarrow (1, \ldots, p) \) do
3: \( \mu_i = \overline{\mu}(\mathcal{X}_{i-1}) \) \( \triangleright \) Arithmetic Mean
4: Construct \( \Omega_i(\mu_i) \) from (6.5)-(6.6)
5: \( \mathcal{X}_i = \{|\overline{\rho}_{ij}| \in \mathcal{X}_{i-1} \mid |\overline{\rho}_{ij}| \geq \mu_i\} \) \( \triangleright \) Dominant Coeffs.
6: end for

From Algorithm 4, we obtain \( \Omega_i, \forall i = 1, \ldots, p \), and attempt to construct the corresponding feedback matrices \( K_i, \forall i = 1, \ldots, p \), by solving the GDARE in (6.13) with the enforced structural constraint \( K_i \in \Omega_i \), and then checking the local convergence of the normalized matrix \( P \) iteratively (see Algorithm 5). Once the algorithm converges corresponding to a small threshold \( \epsilon = 10^{-3} \) in our simulations), the optimal feedback matrix is constructed from (6.12). Parallelization of the algorithm is used to obtain those converged instances which also have convergence times less than a certain threshold time. The feedback matrices which ‘pass’ this test are then stored in a time-feasible matrix set \( \bar{K} \).

From Algorithm 5, a set of time-feasible feedback matrices \( \bar{K} \) is obtained, i.e. all the feedback matrices in this set are computed within a threshold time \( \tau_{th} > 0 \), and hence can be implemented physically. Next, the ‘best’ feedback matrix \( K^* \in \bar{K} \) can be chosen according to the following
Algorithm 5 Modified Geromel’s Algorithm, ∀i = 1, . . . , p

1: Obtain $P_i(0)$ from (6.9) \hspace{1cm} \triangleright \text{Initialize}
2: Start: $k = 0$, $\tau_i = 0$, $\bar{K} = \emptyset$
3: $L_i^{(k+1)} = \Psi(P_i^{(k)}) \circ I_{\Omega_i}$ \hspace{1cm} \triangleright \text{Enforce Structure $\Omega_i$}
4: $P_i^{(k+1)} = A'P_i^{(k+1)}A + A'P_i^{(k+1)}B\Psi(P_i^{(k+1)}) + Q + L_i^{(k)}(R + B'P_i^{(k)}B)L_i^{(k)}$ \hspace{1cm} \triangleright \text{Solve GDARE}
5: if $||P_i^{(k+1)} - P_i^{(k)}||_2^2 < \epsilon$ then \hspace{1cm} \triangleright \text{Check Convergence}
6: $K_i = \Psi(P_i^{(k)}) - L_i^{(k+1)}$ \hspace{1cm} \triangleright \text{Desired Gains}
7: $\tau_i \leftarrow \tau_i^{\text{new}}$ \hspace{1cm} \triangleright \text{Check Execution Time}
8: if $\tau_i < \tau_{th}$ then \hspace{1cm} \triangleright \text{Time-Feasible Set}
9: $\bar{K} \cup K_i \leftarrow \bar{K}$
10: end if
11: else
12: $k \leftarrow k+1$ and goto step 2 \hspace{1cm} \triangleright \text{Re-iterate}
13: end if

engineering design parameters, which reflect the trade-off between the level of sparsity and time-domain performance of the controlled system:

1. Level of block-sparsity: The number of bi-directional communication links in the n-node (generator) network, including self-loops, is given by: $n + n(n - 1)/2 = n(n + 1)/2$. From (6.5), let $r_j \triangleq \text{card}(\Omega_j)$. Hence the number of links corresponding to a specific structure $\Omega_i$ is:

$$g_i = \sum_{j=1}^{n-1} r_j(r_j + 1)/2.$$  

Then the level of block sparsity for the sparse graph, relative to the full graph, can be defined as:

$$\theta_i \triangleq 1 - \frac{2g_i}{n(n + 1)},$$  \hspace{1cm} (6.17)

2. Sub-optimal cost: The sub-optimality index of the system, using a specific $K_i \in \Omega_i$, can be defined by:

$$\xi_i \triangleq (J_{\Omega_i} - J_{\text{opt}})/J_{\text{opt}},$$  \hspace{1cm} (6.18)

where $J_{\Omega_i}$ and $J_{\text{opt}}$ are the optimal costs corresponding to the sparse structure $\Omega_i$, and the unconstrained optimal control problem respectively.

**Remark 4.** Another approach that can be used to obtain a solution of $\mathcal{P}$ is to solve a semi-definite program (SDP) by minimizing an upper bound on the optimal cost $J^*_\infty$ such that several LMI constraints, for ensuring stability and enforcement of the sparsity structure $K \in \Omega$, are satisfied (see [69, 70] for details). We implement the SDP approach in Matlab, and find that since the computational complexity of SDPs do not scale well with increasing order of the problem [71] ($m$ is large for large-scale power system networks), the computation time is on
the order of hours for the considered 39-bus New England system. We conclude that since the solution of $\mathcal{P}$ needs to be solved online (and hence fast), this approach is not viable for our solution.

### 6.4.4 Simulation Results

We verify our algorithm on the New England 39-bus, 10-synchronous generator power system model with a total of 130 states including the exciter states, PSS states, and the turbine governor states. The model data is taken from the Power Systems Toolbox [19] where some transmission line lengths and reactances are modified so that the large-scale open-loop power system (with PSSs attached) exhibits multiple inter-area oscillation modes, as shown in Fig. 6.4(a), while still representing a realistic power transmission grid. Every synchronous generator is equipped with a field voltage exciter to provide the excitation input as a supplementary control on top of the PSS control input. Using the PST software, linear and nonlinear simulations are done to verify the performance of designed controllers.

We also compare our results with the work in [1] where the authors solve a similar problem of designing an optimal sparse feedback controller for a power system network by minimizing the $\mathcal{H}_2$ norm of the system. The weighted sum of Frobenius norms ($\text{blk11}$ in the code available online) is used for obtaining the block-sparse gain structure from [1], with the value of the sparsity promoting parameter chosen as $\gamma = 180$ for comparison with our design. It is to be noted that the design of this controller is done completely offline, and hence will give the same feedback matrix $K_{\mathcal{H}_2}$ irrespective of the location and strength of the exogenous disturbance. In comparison, our design is online (uses the post-disturbance state $x_0$), and hence the structure and values of the matrix $K^*$ is dependent on the location and strength of the disturbance.
The nonlinear power system is excited via a three-phase fault on the line connecting buses 3 and 4, which is cleared after 100 msecs at bus 3, and after 150 msecs at bus 4. Once the fault is cleared at the remote end (bus 4), measurement data (assumed to originate from various PMUs in the grid) is sent to a centralized control authority, which then estimates the post-disturbance initial state of the system $x_0 \in \mathbb{R}^{130 \times 1}$. As shown in Section 4.2, this initial state vector will then contain the information regarding the modal coefficients of the excited modes, and hence can be used to construct the MP form (4.6) of the open-loop system, for which the numerical values of modal coefficients are shown in Table 6.1. Implementation of Algorithms 4 and 5, and the results obtained, are discussed as follows.

Three parallelized control designs are done for the feedback structures $\{\Omega_i\}_{i=1}^3$ obtained from the ‘running mean’ approach, described in Section 6.4.3 and Algorithm 4. Table 6.1 shows the MP form for the linearized e.m. state $\Delta \omega$ of the power system, with the absolute values of modal coefficients in $\{X_i\}_{i=1}^3$ highlighted in various colors as:

- $X_1$: all colors green, orange, red;
- $X_2$: colors orange, red; and
- $X_3$: color red only.

Following the approach in Section 4.2, the sparsity patterns in the feedback controllers for the three structures are shown in Fig. 6.6.

Next, Algorithm 5 is implemented to obtain the block-sparse feedback matrices $\{K_i\}_{i=1}^3$, with the state-weighting matrix $Q$ as given in (6.3), and the control-weighting matrix as $R = 0.1 \times I_n$ for less emphasis on control moves. The convergence parameter is chosen as $\epsilon = 10^{-3}$.

### Table 6.1: Modal coefficients for the MP form of NE system with parallelization coefficients highlighted, with $\mu_1 = 0.76$, $\mu_2 = 1.79$ and $\mu_3 = 3.17$.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$\lambda_1$ (0.12 Hz)</th>
<th>$\lambda_2$ (0.14 Hz)</th>
<th>$\lambda_3$ (0.32 Hz)</th>
<th>$\lambda_4$ (0.63 Hz)</th>
<th>$\lambda_5$ (0.92 Hz)</th>
<th>$\lambda_6$ (1.03 Hz)</th>
<th>$\lambda_7$ (1.07 Hz)</th>
<th>$\lambda_8$ (1.43 Hz)</th>
<th>$\lambda_9$ (1.53 Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \omega_1$</td>
<td>0.05</td>
<td>0.60</td>
<td>0.77</td>
<td>0.14</td>
<td>0.24</td>
<td>0.09</td>
<td>0.09</td>
<td>0.42</td>
<td>4.84</td>
</tr>
<tr>
<td>$\Delta \omega_2$</td>
<td>0.04</td>
<td>0.65</td>
<td>0.63</td>
<td>0.78</td>
<td>0.28</td>
<td>1.01</td>
<td>1.03</td>
<td>2.48</td>
<td>1.27</td>
</tr>
<tr>
<td>$\Delta \omega_3$</td>
<td>0.07</td>
<td>0.64</td>
<td>0.67</td>
<td>0.94</td>
<td>0.37</td>
<td>1.51</td>
<td>1.61</td>
<td>1.41</td>
<td>0.03</td>
</tr>
<tr>
<td>$\Delta \omega_4$</td>
<td>0.06</td>
<td>0.65</td>
<td>0.66</td>
<td>0.79</td>
<td>0.38</td>
<td>0.35</td>
<td>0.10</td>
<td>0.73</td>
<td>0.81</td>
</tr>
<tr>
<td>$\Delta \omega_5$</td>
<td>0.04</td>
<td>0.47</td>
<td>0.46</td>
<td>3.14</td>
<td>1.42</td>
<td>2.33</td>
<td>0.93</td>
<td>0.37</td>
<td>0.21</td>
</tr>
<tr>
<td>$\Delta \omega_6$</td>
<td>0.04</td>
<td>0.51</td>
<td>0.39</td>
<td>2.50</td>
<td>0.49</td>
<td>1.06</td>
<td>1.16</td>
<td>0.23</td>
<td>0.12</td>
</tr>
<tr>
<td>$\Delta \omega_7$</td>
<td>0.04</td>
<td>0.47</td>
<td>0.46</td>
<td>2.45</td>
<td>0.45</td>
<td>0.91</td>
<td>1.01</td>
<td>0.26</td>
<td>0.21</td>
</tr>
<tr>
<td>$\Delta \omega_8$</td>
<td>0.04</td>
<td>0.67</td>
<td>0.18</td>
<td>0.46</td>
<td>0.06</td>
<td>0.08</td>
<td>0.42</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>$\Delta \omega_9$</td>
<td>0.06</td>
<td>0.68</td>
<td>0.51</td>
<td>0.96</td>
<td>3.41</td>
<td>0.67</td>
<td>0.18</td>
<td>0.11</td>
<td>0.64</td>
</tr>
<tr>
<td>$\Delta \omega_{10}$</td>
<td>0.05</td>
<td>0.60</td>
<td>1.01</td>
<td>0.75</td>
<td>0.13</td>
<td>0.07</td>
<td>0.06</td>
<td>0.04</td>
<td>0.20</td>
</tr>
</tbody>
</table>
for accuracy. For a large $\epsilon$, or if the iterations are stopped before convergence, the solution of Algorithm 5 ($K^*$) will still respect the structural constraint but may not be optimal. Since the threshold $\tau_{th}$ will essentially depend on the breaker trip-times in the grid, this value is not specified for Algorithm 5, and all the designed feedback matrices are considered time-feasible for the purpose of simulations. The closed-loop linearized system is then simulated (with the system excited by the received $x_0$ from the nonlinear power system) to obtain the linear predicted closed-loop response of the system, so as to select the ‘best’ $K^* \in \bar{K}$. Results from these linear simulations are shown in Table 6.2 for the three designed controllers with varying levels of sparsity. It is seen that all controllers are stabilizing, and that as the level of sparsity increases, the closed-loop performance degrades along with an increase in execution time. We also see that the controller $K_{H_2}$, designed offline, gives much lower block-sparsity as compared to the three designed controllers, with comparable time-domain performance. Keeping the trade-off for performance and sparsity in mind, we choose $K^* = K_2 \in \Omega_2$ as the best choice for this disturbance scenario.

Time-domain performance on the linear system is shown in Fig. 6.3 where the rotor speed deviations of Gen. 9 ($\Delta \omega_9$) and Gen. 10 ($\Delta \omega_{10}$) are shown, for the following cases:

- open-loop system,
- closed-loop system with centralized optimal gain $K^*_c \in \Omega_{full}$,
- closed-loop system with block-sparse gain matrices $K^* \in \Omega_2$, and

![Sparsity Structure for $K_1 \in \Omega_1$](image1)

![Sparsity Structure for $K_2 \in \Omega_2$](image2)

![Sparsity Structure for $K_3 \in \Omega_3$](image3)

Figure 6.2: Controller sparsity structures for the NE system obtained using Algorithm 4 for the three parallelized cases.
Table 6.2: Linear CL simulation results using the three designed controllers, and the offline controller from [1].

<table>
<thead>
<tr>
<th>$K_i \in \Omega_i$</th>
<th>block sparsity $\theta_i$</th>
<th>exec. time $\tau_i$ (secs)</th>
<th>sub-opt. ind. $\xi_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1$</td>
<td>32.7%</td>
<td>0.42</td>
<td>1.05%</td>
</tr>
<tr>
<td>$K_2$</td>
<td>72.7%</td>
<td>1.33</td>
<td>7.37%</td>
</tr>
<tr>
<td>$K_3$</td>
<td>92.7%</td>
<td>2.68</td>
<td>9.74%</td>
</tr>
<tr>
<td>$K_{H2}$</td>
<td>18.2%</td>
<td>15.15</td>
<td>8.03%</td>
</tr>
</tbody>
</table>

- closed-loop system with block-sparse gain matrix obtained from [1].

A frequency response comparison is shown in Fig. 6.4 where the FFT spectrum for the open-loop system and the closed-loop system with block-sparse $K^* \in \Omega_2$ is shown. It is seen that FFT magnitudes of the excited modes (highlighted) are reduced. Also it can be seen that the frequencies for the closed-loop modes do not differ significantly from the open-loop case, and hence the assumption that the impact of controller on the closed-loop inter-area frequency modes is small remains valid.

Simulations are also conducted on the nonlinear NE power system, with the control feedback gain $K^* \in \Omega_2$. Fig. 6.5 shows the time-domain closed-loop responses (Figs. 6.5(b),(d)) of the generator rotor speeds (frequencies) and the generated electric power for indicated generators, compared with their open-loop responses (Figs. 6.5(a),(c)). We see that the sparse $K^*$ improves the closed-loop performance of the system considerably, while also saving on the communication links as compared to a centralized controller.

### 6.5 $\ell_1$-Regularized Sparse LQR

In this section we extend our structurally constrained LQR controller designed with the help of GDARE in section 6.4, to a two-level sparse controller [72]. The feedback communication links are sparsified with a structural constraint, while the states in each link are also sparsified with the help of $\ell_1$-regularization of the closed-loop LQR objective function.

#### 6.5.1 Controller Design

Sparsity in $K$ is enforced in two ways. First, we enforce a structural constraint $K \in \Omega$, where $\Omega \in \mathbb{R}^{m \times n}$ is the set of matrices with pre-specified zero locations, constructed in (6.6). A second round of sparsity is imposed by using an $\ell_1$-regularization of the states transmitted through
Figure 6.3: Linearized system output response for OL and CL scenarios with indicated controllers.
Figure 6.4: (a) Open-loop and (b) closed-loop frequency response for all rotor speed outputs of all NE system generators. Excited inter-area oscillation modes are highlighted.

each link decided by \( \Omega \). Referring to Theorem 2, we note the following corollary to provide an upper bound on closed-loop cost.

Corollary 2 ([67]). The upper bound on the sub-optimality index \( \xi = (J^*_{\Omega} - J^*_\infty) / J^*_\infty \) is given by:

\[
\xi \leq \beta \|L\|_F^2, \quad \text{for some } \beta \geq 0,
\]

where \( J^*_\infty \) is the optimal solution of the unconstrained problem \( \min_K J_\infty \) using DARE, and \( J^*_{\Omega} \) is the solution of the constrained problem \( \min_{K \in \Omega} J_\infty \) using GDARE. ■

It is noted that Theorem 2 assures the stability of the closed-loop system regardless of the choice of \( L \). Hence the matrix \( L \) can be chosen to satisfy additional objectives on the closed-loop system. In [67], \( L \) is chosen so as to minimize \( \xi \) under the constraint (6.10b) with \( \Omega \) as the decentralized feedback structure. In [59], \( L \) is chosen so as to minimize \( \xi \) under the constraint (6.10b) with \( \Omega \) as the block-sparse feedback structure. To obtain a sparsity-promoting structurally constrained LQR controller with guaranteed stability, we solve the following problem for the optimal choice of \( L \):

\[
P_{\text{card}} : \min_L \left\{ \frac{1}{2} \|L\|^2_F + \eta \ \text{card}(K) \right\}
\]

\[\text{s.t. } K \in \Omega,\]

where \( \eta \geq 0 \) is a constant scalar. The problem \( P_{\text{card}} \) tries to minimize both the upper bound on
Figure 6.5: Nonlinear simulation response for open-loop and closed-loop system with sparse-optimal feedback $K^* \in \Omega_2$. Subfigures (a)-(b) compare the rotor speeds (in pu) for Gens 1 and 10, and subfigures (c)-(d) compare the generated electric power (in pu) for Gens 3 and 4.
the closed-loop sub-optimality index $\xi$ as well as the cardinality of the feedback matrix. These two are conflicting objectives, and hence a scalar weight $\eta$ is used to signify trade-off. Since $P_{\text{card}}$ is non-convex due to the cardinality function, we approximate it with an $\ell_1$-regularization term, and re-write the problem as:

$$\min_L \left\{ \frac{1}{2} \|L\|^2_F + \eta \|K\|_{\ell_1} \right\}$$  \hspace{1cm} (6.21a)

$$\text{s.t. } K \in \Omega,$$  \hspace{1cm} (6.21b)

where the $\ell_1$-norm of a matrix $X$ is defined as $\|X\|_{\ell_1} = \sum_{i,j} |X_{ij}|$ [73]. Using (6.11) and (6.12), the above problem can be again re-written in terms of only the optimization variable matrix $L$ as:

$$P_{\text{sparse}}: \min_L \left\{ \frac{1}{2} \|L\|^2_F + \eta \|L - \Psi\|_{\ell_1} \right\}$$  \hspace{1cm} (6.22a)

$$\text{s.t. } (\Psi - L) \odot I_{\Omega} = 0,$$  \hspace{1cm} (6.22b)

where $\Psi \equiv \Psi(P) \triangleq -(R + B'PB)^{-1}B'PA$. We next present our main result for the solution of $P_{\text{sparse}}$, first presented in [72].

**Theorem 3.** For the sparsity-promoting structurally constrained LQR problem, the optimal solution of $P_{\text{sparse}}$ in (6.22) is given by:

$$L^*_{ij} = \begin{cases} 
\Psi_{ij} I_{\Omega,ij}^c + \eta(I_{\Omega,ij}^c - 1) & \text{if } \eta < -\Psi_{ij} \\
\Psi_{ij} & \text{if } \eta \geq |\Psi_{ij}(I_{\Omega,ij}^c - 1)| \\
\Psi_{ij} I_{\Omega,ij}^c - \eta(I_{\Omega,ij}^c - 1) & \text{if } \eta < \Psi_{ij}, 
\end{cases}$$  \hspace{1cm} (6.23)

\forall i = 1, \ldots, m \text{ and } \forall j = 1, \ldots, n.

**Proof.** Since both the terms in the cost function (6.22a) are convex, and since the constraint (6.22b) is linear (Hadamard product is linear), the resulting problem $P_{\text{sparse}}$ is convex in $L$. We write the Lagrangian function for (6.22) as:

$$\mathcal{L} = \frac{1}{2} \|L\|^2_F + \eta \|L - \Psi\|_{\ell_1} + \text{Tr}\left( \Lambda'[(\Psi - L) \odot I_{\Omega}^c] \right)$$

$$= \frac{1}{2} \langle L, L \rangle + \eta \|L - \Psi\|_{\ell_1} + \langle \Lambda, (\Psi - L) \odot I_{\Omega}^c \rangle$$

$$= \frac{1}{2} \langle L, L \rangle + \eta \|L - \Psi\|_{\ell_1} + \langle \Lambda, \Psi \odot I_{\Omega}^c \rangle - \langle \Lambda, L \odot I_{\Omega}^c \rangle,$$  \hspace{1cm} (6.24)

where $\text{Tr}(\cdot)$ represents the trace of a matrix, $\langle \cdot, \cdot \rangle$ represents the standard inner product, and
Λ is the Lagrange multiplier matrix. The conditions for optimality are:

\[ \frac{\partial L}{\partial L} = 0, \quad \text{and} \quad \frac{\partial L}{\partial \Lambda} = 0. \]  

(6.25)

Differentiating \( L \) with respect to \( \Lambda \) and equating to zero:

\[ \frac{\partial L}{\partial \Lambda} = (\Psi - L) \circ I^c_\Omega = 0 \]  

(6.26a)

\[ \implies \Psi \circ I^c_\Omega = L \circ I^c_\Omega \]  

(6.26b)

\[ \implies \Psi_{ij} I^c_{\Omega,ij} = L_{ij} I^c_{\Omega,ij}. \]  

(6.26c)

Differentiating \( L \) with respect to \( L \) and equating to zero:

\[ \frac{\partial L}{\partial L} = L - (\Lambda \circ I^c_\Omega) + \eta \frac{\partial}{\partial L} \| L - \Psi \|_{\ell_1} = 0. \]  

(6.27)

Since \( \frac{\partial c}{\partial L} \) represents the partial derivative of a scalar with respect to a matrix, it is given by the matrix:

\[ \frac{\partial L}{\partial L} = \begin{bmatrix} \frac{\partial c}{\partial L_{11}} & \cdots & \frac{\partial c}{\partial L_{1n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial c}{\partial L_{m1}} & \cdots & \frac{\partial c}{\partial L_{mn}} \end{bmatrix}. \]  

(6.28)

Next we calculate the subgradients for \( L_{ij} - \Psi_{ij} > 0 \) and \( L_{ij} - \Psi_{ij} < 0 \), denoted by \( g_+ \) and \( g_- \) respectively. Since the Hadamard product in (6.27) is just multiplication in scalar form, we get:

\[ g_+ = \Psi_{ij} - \Lambda_{ij} I^c_{\Omega,ij} + \eta, \]  

(6.29a)

\[ g_- = \Psi_{ij} - \Lambda_{ij} I^c_{\Omega,ij} - \eta. \]  

(6.29b)

Considering \( L^*_{ij} \) as the minimizer for some indices \((i,j)\), we consider the following three possible cases:

- \( L^*_{ij} = \Psi_{ij} \) iff \( g_+ \geq 0 \) and \( g_- \leq 0 \). From (6.29) this implies \( \eta \geq |\Lambda_{ij} I^c_{\Omega,ij} - \Psi_{ij}|. \) Also, from (6.27), \( L^*_{ij} = \Lambda_{ij} I^c_{\Omega,ij} \). Multiplying both sides by \( I^c_{\Omega,ij} \) and comparing with (6.26a), we get:

\[ \Lambda_{ij} I^c_{\Omega,ij} = \Psi_{ij} I^c_{\Omega,ij}. \]

- \( L^*_{ij} - \Psi_{ij} > 0 \) iff \( g_+ \leq 0 \). From (6.29) this implies \( \eta \leq \Lambda_{ij} I^c_{\Omega,ij} - \Psi_{ij}. \) Also, from (6.27), \( L^*_{ij} = \Lambda_{ij} I^c_{\Omega,ij} - \eta \). Multiplying both sides by \( I^c_{\Omega,ij} \) and comparing with (6.26a), we get:

\[ \Lambda_{ij} I^c_{\Omega,ij} = (\Psi_{ij} + \eta) I^c_{\Omega,ij}. \]

- \( L^*_{ij} - \Psi_{ij} < 0 \) iff \( g_- \geq 0 \). From (6.29) this implies \( \eta \leq \Psi_{ij} - \Lambda_{ij} I^c_{\Omega,ij}. \) Also, from (6.27), \( L^*_{ij} = \Lambda_{ij} I^c_{\Omega,ij} + \eta \). Multiplying both sides by \( I^c_{\Omega,ij} \) and comparing with (6.26a), we get:
\[ \Lambda_{ij} I_{\Omega,ij}^c = (\Psi_{ij} - \eta) I_{\Omega,ij}^c. \]

Substituting the appropriate expression for \( \Lambda_{ij} I_{\Omega,ij}^c \) in all cases above to get the corresponding \( L^*_ij \), the required solution (6.23) follows.

The solution \( L^* \) from Theorem 3 provides us with a feasible and optimal value of \( L \) to satisfy the structural constraint, as well as promoting further sparsity in the feedback matrix. To obtain the corresponding \( K \), an iterative method to solve GDARE in (6.13) is used, given in Algorithm 6. The condition (6.23) is represented in matrix form by \( L = h(\Psi, \Omega, \eta) \).

\begin{algorithm}
\begin{enumerate}
\item \textbf{procedure} \textsc{Sparse\_LQR}(A, B, Q, R, \Omega, \eta)
\item Initialize: Iteration \( i = 0 \)
\item Obtain \( P^{(i=0)} \) from the solution of \( \min_K J_\infty \)
\item Get \( L^{(i+1)} = h(\Psi(P^{(i)}), \Omega, \eta) \) from Theorem 3
\item Solve: \( P^{(i+1)} = A'P^{(i+1)}A + A'P^{(i+1)}B\Psi(P^{(i+1)}) + Q + L^{(i+1)}(R+B'P^{(i)}B)\), \( L^{(i+1)} \)
\item if \( ||P^{(i+1)} - P^{(i)}||_2 < \epsilon \) then
\item \( K = \Psi(P^{(i)}) - L^{(i+1)} \)
\item else \( i \leftarrow i + 1 \) and goto step 4
\item end if
\item return \( K \)
\item \textbf{end procedure}
\end{enumerate}
\end{algorithm}

### 6.5.2 Simulation Results

We validate our design using the IEEE 39-bus, 10-machine power system model, shown in Fig. 3.4. The state vector comprises of 130 states in total, including the synchronous generator, turbine governor, exciter and PSS states. A sampling time of 0.1 secs is chosen for discretizing the model. Power System Toolbox software is used to obtain model data and carry out all simulations on the nonlinear power system model. We excite the nonlinear system with a three-phase line-to-ground fault on the transmission line connecting buses 3 and 4. Starting the simulations at \( t = 0 \), we introduce the fault at \( t = 0.1 \) secs, and clear the fault on bus 3 and 4 at \( t = 0.2 \) secs and \( t = 0.25 \) secs respectively.

After the fault is cleared from the remote end of the faulted line, all generator state estimators will send their post-fault states to the ISO. The ISO uses this state vector \( x_0 \) to first construct the MP matrix (4.6), and then identify the dominant inter-area modes: \(-0.04 \pm 3.8j\).
Figure 6.6: Feedback structures for varying values of $\mu$ and $\eta$. It is seen that sparsity increases with an increase in the value of $\mu$ and decreases with an increase in the value of $\eta$.

and $-0.42 \pm 6.5j$ from the dominant residues (see Chapter 4 for details on identification of dominant residues). The dominant modes correspond to the inter-area frequencies of 0.6 Hz and 1.1 Hz. The set of generators participating most in excitation of these modes is captured by the MP matrix. From the MP matrix, $p = 10$ possible values of residue thresholds are obtained: $\{\mu_i\}_{i=1}^{10}$. Corresponding unique feedback structures $\{\Omega_i\}_{i=1}^{10}$ are constructed from (6.6), and Algorithm 6 is run with ten parallelizations to get the corresponding gain matrices $\{K_i\}_{i=1}^{10}$. The $\ell_1$-regularization parameter is chosen to be $\eta = 0.1$. The sparsity indices $\theta_i$ and resiliency indices $RI_i$ are then calculated using $\Omega_i$ and $K_i$ respectively, $\forall i = 1, \ldots, 10$. These are shown in Fig 7.1, where the optimal $\mu_i^*$, calculated empirically from (7.4), is highlighted. The corresponding $K^f = K_5$ is hence used for feedback control. For all simulations $R = \text{diag}(0.1, \ldots, 0.1)$, and $Q$ is a block-diagonal matrix constructed as discussed in (6.3).

Feedback structures for two values of $\mu$ and $\eta$ each are shown in Fig 6.6. It is clearly
seen that increasing $\mu$ will force the optimization problem $P_{\text{sparse}}$ to drop more and more communication links from $\Omega$, hence increasing the level of sparsity. Increasing $\eta$ has little effect on the number of communication links, but encourages sparsity inside the links. To evaluate closed-loop performance, the sub-optimality index $\xi$ in Corollary 2 is modified to a performance loss index as:

$$\tilde{\xi} \triangleq \frac{J^*_{\text{sp}} - J^*_{\infty}}{J^*_{\text{ol}} - J^*_{\infty}},$$

(6.30)

where $J^*_{\text{sp}}$ and $J^*_{\text{ol}}$ are the costs corresponding to the sparse LQR design, and the open-loop system, respectively. The metric $\tilde{\xi}$ lies in $[0, 1]$, and hence the performance is evaluated with respect to both the dense optimal controller and the open-loop case.

From Fig. 7.1, we get $\mu_5 = 0.8$ as the solution of (7.4). Hence we choose $\Omega_5$ as our structural constraint, and use $\eta = 0.1$ to show closed-loop performance with controller $K_5$. The feedback structure is shown in Fig. 6.6(a). A comparison of open-loop versus closed-loop response is shown in Fig. 6.7. It is clearly seen that the designed controller adds oscillation damping to the response of the generators.

In Fig. 6.8 we show the empirical relationship between the sparsity parameters $\mu, \eta$, and the performance loss index $\tilde{\xi}$. From Fig. 7.1, since only the values $\mu = \{0.4, 0.5, 0.6, 0.8\}$ show a high RI, we consider only these values in Fig. 6.8. For each considered $\mu_i$, $i = 1, \ldots, 4$, $\eta$ is varied between 0.01 and 0.2, and the corresponding $\tilde{\xi}$ is evaluated. As expected, the plot shows that increasing sparsity degrades the closed-loop performance. Fig. 6.8 can be used to select the tuning parameter $\eta$ for a good trade-off.

We also compare our sparse control design (referred to as the ‘GDARE controller’) with the sparse $\mathcal{H}_2$ controller designed in [1] using the alternating direction method of multipliers (ADMM) algorithm. For the block-sparse version of [1], a comparison study was done in [59] with our block-sparse controller, and hence we only consider the non block-sparse version of [1] in this paper. Weighted $\ell_1$ norm, with a sparsity factor of $\gamma = 20$, is used to design the controller in [1]. For our control design $\mu_1 = 0.4$ and $\eta = 0.01$ are chosen, resulting in $K_1$. The above values are chosen so that both the performance loss ($\tilde{\xi} \approx 0.03$) as well as the communication sparsity ($\theta \approx 0.45$) are similar for both controllers in closed-loop, for comparison purposes. Figs. 6.9a and 6.9b show the feedback communication topology between generators for the $\mathcal{H}_2$ controller and the GDARE controller, respectively. Each blue circle represents a generator node, and the connecting lines represent bi-directional communication links. It is seen that even though $\theta$ is similar for both cases, the GDARE controller gives a sparser topology in terms of communication links. This implies that the number of states communicated per link will be higher for the GDARE controller, as compared to the $\mathcal{H}_2$ controller, so as to balance the overall sparsity $\theta$. Fig. 6.10 also compares the controller design times for both cases. The time for computing our GDARE controller is faster than that for $\mathcal{H}_2$ as the latter is based on ADMM.
Figure 6.7: Implementation results for our sparse controller, with sparsity parameters: $\mu = 0.8$ and $\eta = 0.1$. Comparison of open- versus closed-loop responses is shown. The top figures show rotor speed response for generators 4, 5 and 6. The bottom figures show electrical power outputs of generators 9 and 10.

Figure 6.8: Sparsity tuning parameters versus the performance loss index.
(a) Comm. topology for the ADMM-based $\mathcal{H}_2$ controller.  

(b) Comm. topology for the GDARE controller $K_1 \in \Omega_1$.

Figure 6.9: Comparison of communication topologies for the controller in [1], with our sparse controller.

that minimizes an exact performance cost while ours only minimizes an upper bound (6.19) for that cost.

6.6 Conclusions

An online approach for suppression of inter-area oscillations in a large power system network, via a sparse LQR, is proposed in this chapter. The structure of the feedback gain matrix is constructed using online modal analysis of the system. The control design is online in the sense that the perturbed ‘initial’ state of the system is estimated by a central control entity immediately after the occurrence of a disturbance. Optimality and stability for the constrained control problem is achieved via a generalized Riccati equation method. Since there is a clear trade-off between the controller performance and the level of induced sparsity in the communication network, a parallelization algorithm is provided for the proposed control design which provides multiple sparse feedback matrices with increasing sparsity. Simulation results on a 39-bus power system model show the effectiveness of this approach. We next improved our structurally constrained LQR controller by designing an $\ell_1$-regularized sparse LQR controller. Sparse communication between generators is achieved by enforcing a structural constraint on the feedback matrix, and by using an $\ell_1$-penalty term on the LQR cost to promote sparsity of data transfer within the communication links. A post-disturbance modal analysis of the physical system is used to
determine the structural constraint which allows communication between the most influential

generators for that particular disturbance event only. Results are validated using the IEEE
39-bus power system model.

Figure 6.10: Comparison of control design times for the $\mathcal{H}_2$ controller versus the GDARE controller. Note the logarithmic-scaled vertical axis.
Chapter 7

Closed-Loop Resiliency Analysis

7.1 Introduction

In this chapter we show that the two-level sparse LQR controller, designed in Chapter 6, is not only advantageous for saving communication and bandwidth costs, but also gives us a way to enhance the resiliency of the closed-loop system against denial-of-service (DoS) attacks [74, 75]. So far cyber attacks in power systems have been considered in wide-area state estimation [76, 77], but the literature on attacks in wide-area control is still sparse. As shown in [16], if a feedback communication link gets deactivated by a DoS attack, the resulting network can become unstable. With our proposed strategy, the sparse controller will redesign itself to regain stability, while also considering the trade-off between performance and sparsity. We propose an algorithm to formalize this redesign considering single-link attacks.

7.2 Cyber-Attack Mitigation

In this section we analyze the relationship between the closed-loop resiliency of the wide-area control feedback network under DoS cyber-attacks, and the amount of sparsity achieved in the communication network with our controller design. We then propose a controller redesign immediately after the cyber-attack so as to regain closed-loop stability.

7.2.1 Closed-loop Network Resiliency

As reported in [78], and also shown with simulations results in this chapter, it is seen that a targeted DoS attack on feedback communication links during closed-loop operation can easily destabilize the system. For our sparse controller, this would mean pushing the closed-loop eigenvalues of $A + B(K \circ I_A)$ outside the unit circle, where $K \in \Omega$, and $I_A \in \mathbb{R}^{m \times n}$ is the
indicator matrix of the symmetric attack link structure $A \in \mathbb{R}^{m \times m}$. The entry $A_{ij} = 0$ indicates an attack on the bi-directional communication link from the $i^{th}$ to the $j^{th}$ generator.

We define a feedback link matrix $\hat{\Omega}$ with respect to $\Omega$, where an element $\hat{\Omega}_{ij}$ represents the existence of a communication link between the $i^{th}$ and the $j^{th}$ generator (i.e. $\hat{\Omega}_{ij}=0$ implies no link between $G_i$ and $G_j$). In this paper we focus only on single link attacks excluding self-loops, i.e. $A_{ij} = 0$ only for scalars $(i,j) \in \{1, \ldots, m\}$, $i \neq j$ and with $\hat{\Omega}_{ij} \neq 0$. Hence we limit the number of possible attack structures to $\binom{m}{2}$, where the notation $\binom{a}{b}$ represents the binomial coefficient for ‘$a$ choose $b$’ combinations. Let the set of all possible attack structures be: $A^1, \ldots, A^{\binom{m}{2}}$. Let a vector $\Pi \in \mathbb{R}^{\binom{m}{2}}$, where each element $\Pi_i$ corresponds to a unique attack structure $A^i$, be defined as:

$$\Pi_i = \begin{cases} 0, & \text{if } |\lambda_{\text{max}}(A + B(K\sigma I_{A^i}))| > 1 \\ 1, & \text{otherwise.} \end{cases} \tag{7.1}$$

In other words, the binary vector $\Pi$ identifies the single-link DoS attack structures which will make the closed-loop system unstable. For a given closed-loop system $A + BK$, we define a metric Resiliency Index (RI) as:

$$RI = \frac{\text{card}(\Pi)}{\binom{m}{2}}, \tag{7.2}$$

where card($\Pi$) is the number of possible attack structures which do not destabilize the system. Thus, higher the value of RI, less is the probability that the system goes unstable after a single-link DoS attack. We next describe how we use RI from (7.2) in the control design to mitigate the effects of a cyber-attack.

### 7.2.2 Resiliency-based Selection of $\mu$

Let $\theta$ be defined as the sparsity index:

$$\theta \triangleq \frac{\text{card}(K_{sp})}{\text{card}(K_\infty)}, \tag{7.3}$$

where $K_{sp}$ is our designed sparse controller from solving (6.12) and (6.22). The dense matrix $K_\infty$ is the solution of the unconstrained problem $\min J_\infty$ with $\text{card}(K_\infty) = mn$. Next, we consider both RI and $\theta$ to make a selection on the choice of the residue threshold $\mu$, and thereby on the choice of the structural constraint $\Omega$. Since increasing the value of $\mu$ sweeps the MP matrix (4.6) to drop the communication links corresponding to the least influential generators with respect to the dominant modes (from (4.7)), it is clear that there are only a finite choices of $\mu$ to obtain a unique set of feedback structures. Let $p$ be the number of choices of $\mu$, given by $\mu_1, \ldots, \mu_p$. Let the corresponding unique feedback structures be given by $\Omega_1, \ldots, \Omega_p$. It is noted
Let the empirical function relating $\mu$ to its corresponding $\theta$ be denoted as $f_\theta$, and the empirical function relating $\mu$ to its corresponding RI be denoted as $f_{RI}$. The optimal selection of $\mu_i$, $\forall i = 1, \ldots, p$, considering both the level of sparsity and the resiliency index follows from:

$$
\mu_i^* = \arg\max_{\mu_1, \ldots, \mu_p} [f_\theta(\mu_i) + f_{RI}(\mu_i)],
$$

(7.4)

where the objective of (7.4) is to maximize both the sparsity as well as the resiliency of the closed-loop system. The threshold $\mu_i^*$ is obtained empirically as shown with simulations in Section 6.5.2. The corresponding $\Omega_i(\mu_i^*)$ is then constructed from (6.6). Finally, Algorithm 6 is executed to obtain a sparse feedback matrix $K'$. 

**Remark 5.** Note that calculating RI using (7.1)-(7.2) involves combinatorial complexity, and can be difficult for a large-dimensional $K$. However, a recent work in [16] proposes a polynomial complexity algorithm to solve for a similar problem as in (7.1), which can be used to obtain RI.

### 7.2.3 Post-Attack Controller Redesign

The choice of $\mu_i^*$ in (7.4) depends on both performance and resilience. Therefore, unless $\mu_i^*$ is chosen exclusively for resilience, it is possible that an attacker may destabilize the closed-loop system by attacking a vulnerable link. To mitigate the effects of such an attack, we propose redesigning the controller immediately after the attack is detected so as to regain closed-loop
stability. The reader is referred to [79] for a survey on DoS attack detection techniques. The procedure for controller redesign is explained as follows.

Let the feedback matrix designed by Algorithm 6 be denoted as $K^I \in \Omega_i(\mu_i^*)$, where $\mu_i^*$ is the solution of (7.4), for some $i \in \{1, \ldots, p\}$. Let the attack structure employed by the attacker to de-stabilize the system be denoted by $A^\kappa$ for some $\kappa \in \{1, \ldots, \binom{m}{2}\}$, with $\Pi_\kappa = 0$ from (7.1). Hence, the post-attack communication structure is given by:

$$\Omega_{att} \triangleq \Omega_i \circ I_{A^\kappa}, \quad (7.5)$$

where $\Omega_{att} \in \mathbb{R}^{m \times m}$. We then propose running Algorithm 6 again, but in this case with input feedback structure $\Omega_{att}$ instead of $\Omega_i$, while keeping $\eta$ at the same value as before. Let the redesigned controller be denoted by $K^{II} \in \Omega_{att}$. It is to be noted that the attacked link in $A^\kappa$ might be very crucial to the stability of the system, and hence Algorithm 6 might fail to converge. In this case, we switch to a lower residue threshold value $\mu_j$, i.e. $\mu_j < \mu_i$, where index $j$ is chosen such that the difference of the costs from (7.4) is minimum. If required, the above procedure can be repeated until a stabilizing controller is found. The steps for the design and redesign of the sparse LQR controller with cyber-attack mitigation are provided in Algorithm 7. It is noted that in Step 7 of Algorithm 7, the calculation of $K_i$ is parallelized to obtain all possible $p$ number of unique designs.

Remark 6. We note that lowering the value of $\mu$ after an attack will usually result in bringing back some of the communication links between generators that were ignored in the initial sparse controller design. This in turn reduces the level of communication sparsity, and can be seen as the trade-off for regaining system stability.

7.2.4 Simulations

In this section we consider two scenarios where a single link DoS cyber attack, in different locations, makes the closed-loop system unstable. For both cases, we show the controller redesign process to regain stability. Similar to our design in the previous section, we choose $\mu = 0.8$ and $\eta = 0.01$ as the sparsity parameters to obtain the controller $K^I = K_5$, for initial deployment, after a fault on the transmission lines connecting buses 3-4. The two attack and redesign case studies are discussed as follows.

- Case Study I: A cyber attack is introduced on the communication link between generators 7 and 9, preventing the generators to exchange state information. This is shown graphically in Fig. 7.2a where the red dotted line represents the attacked link. Once the attack is detected by the ISO, it redesigns the controller using the steps in Algorithm 7. For this case, it was seen that the first redesign ($K^{II}$) fails, and hence the ISO is forced to
Algorithm 7 Implementation algorithm for our sparse LQR controller with cyber-attack mitigation.

1: Time of disturbance: \( k = 0 \).
2: Substations send respective generator states to ISO.
3: ISO receives \( x_0 \), and constructs the MP matrix (4.6).
4: Calculate unique \( \{\mu_i\}_{i=1}^p \) from the MP matrix, and the corresponding feedback structures \( \{\Omega_i\}_{i=1}^p \) (6.6).
5: Select \( \ell_1 \)-regularization parameter \( \eta \), weights \( Q, R \).
6: \textbf{parfor} \( i = 1, \ldots, p \) do
7: \( K_i = \text{Sparse LQR}(A, B, Q, R, \Omega_i, \eta) \) \( \triangleright \text{Algorithm 6} \)
8: \textbf{end parfor}
9: Evaluate \( R_I \) for all \( \{K_i\}_{i=1}^p \) using (7.1)-(7.2) and [78]. \( \triangleright \text{Remark 5} \)
10: Weigh \( f_\theta, f_{RI} \), and select \( \mu_j^* \), and in turn \( \Omega_j(\mu_j^*) \), by solving (7.4), for some \( j \in \{1, \ldots, p\} \).
11: Select the corresponding \( K^I = K_j(\Omega_j) \) from Step 7, and distribute rows of \( K^I \) to respective generators.
12: All generators implement the control law: \( u_l(t) = K^I(l,:)x(t) \), \( \forall l = 1, \ldots, m \).
13: DoS cyber-attack is detected: \( k = k_1 > 0 \), with attack structure \( A^K \) for some \( \kappa \in \{1, \ldots, \binom{m}{2}\} \).
14: ISO constructs \( \Omega_{att} \) from (7.5).
15: ISO designs \( K^{II} = \text{Sparse LQR}(A, B, Q, R, \Omega_{att}, \eta) \) \( \triangleright \text{Algorithm 6} \)
16: if \( |\lambda_{\max}(A+BK^{II})| < 1 \) then
17: Distribute rows of \( K^{II} \) to respective generators.
18: Implement: \( u_l(t) = K^{II}(l,:)x(t) \), \( \forall l = 1, \ldots, m \).
19: else
20: Select index \( j \) for \( \mu_j < \mu_j^* \) such that \( |f_\theta(\mu_j) + f_{RI}(\mu_j) - f_\theta(\mu_j^*) - f_{RI}(\mu_j^*)| \) is minimum.
21: \( K^{III} = \text{Sparse LQR}(A, B, Q, R, \Omega_j \circ I_{A^K}, \eta) \)
22: Distribute rows of \( K^{III} \) to respective generators.
23: Implement: \( u_l(t) = K^{III}(l,:)x(t) \), \( \forall l = 1, \ldots, m \).
24: \textbf{end if}
choose a lower value of $\mu$ to restore some of the communication links. $\mu_3 = 0.6$ is chosen from Step 20 in Algorithm 7, and the corresponding $K^{III} \in \Omega_3 \circ I_{A^K}$ is designed using GDARE, where $I_{A^K}$ is the binary matrix corresponding to the cyber attack. The new communication topology is shown in Fig. 7.2b where the links in green color represent the added communication links. The temporal plot of this whole process is shown in Fig. 7.3 where the time of attack (at $t = 10$ secs) and the time of controller redesign (at $t = 60$ secs) are highlighted. It is seen that the closed-loop system goes unstable after the attack, but stability is restored once the new controller $K^{III}$ is implemented.

- **Case Study II:** A cyber attack is introduced on the self communication link for generators 2. This is shown graphically in Fig. 7.4a where the red dotted line represents the attacked link. Similar to the Case Study I, the ISO detects the attack, lowers the value of $\mu$ to 0.4, and redesigns the controller with a new communication topology. This new topology (shown in Fig. 7.4b) is less sparse than with the original controller, but is successful in bringing back stability. The temporal plot of the attack and redesign process is shown in Fig. 7.5 where the time of attack (at $t = 10$ secs) and the time of controller redesign (at $t = 25$ secs) are highlighted.

It is noted that in Fig. 7.3 and Fig. 7.5, the difference between the attack time and the redesign time is exaggerated for illustrative purposes. In practice these times will be close to each other, and hence the oscillations due to instability might be unnoticeable, assuring a much smoother transition to the new controller.

**Remark 7.** Note that another method to reduce sparsity can be to lower the value of $\eta$ in the controller redesign process. This strategy was seen to not work in simulations. It was observed that reducing $\eta$ only brought back feedback gains of certain non-electromechanical states. Hence, it is conjectured that these states do not have a sufficient level of participation in damping control to re-stabilize the system.

### 7.3 Conclusions

We presented a closed-loop cyber resiliency analysis of the power system, under DoS cyber-attacks. We defined a resiliency index to quantify the measure of closed-loop resiliency of the network, under DoS attacks on the feedback communication links. To mitigate the effects of such an attack on our designed sparse controller in Chapter 6, we propose a controller redesign algorithm so that the stability of the closed-loop system can be recovered. Simulation studies are done with two different attack scenarios on the 39-bus power system model, and show the effectiveness of our algorithm in restoring closed-loop stability.
(a) Case Study I: Comm. topology for $K^I \in \Omega_5$, with DoS attack on link (9,7).

(b) Case Study I: Comm. topology for controller redesign $K^{III} \in \Omega_3 \circ I_{A^k}$.

Figure 7.2: Communication topologies for the attacked controller and the redesigned controller, for Case Study I.

Figure 7.3: Case Study I: Rotor speed response for all generators, following a three-phase fault at $t = 0$ controlled with $K^I$; DoS attack at $t = 10$ causing system instability; and controller redesign with $K^{III}$ to regain stability at $t = 60$.  

93
(a) Case Study II: Comm. topology for $K^I \in \Omega_5$, with DoS attack on link (2,2).

(b) Case Study II: Comm. topology for controller redesign $K^{III} \in \Omega_3 \circ I_{A^*}$.

Figure 7.4: Communication topologies for the attacked controller and the redesigned controller, for Case Study II.

Figure 7.5: Case Study II: Rotor speed response for all generators, following a three-phase fault at $t = 0$ controlled with $K^I$; DoS attack at $t = 10$ causing system instability; and controller redesign with $K^{III}$ to regain stability at $t = 25$. 

94
Chapter 8

Conclusions & Future Research

8.1 Conclusions

In this thesis work we have presented novel control design methods for distributed/sparse control of wide-area power system networks. The controllers are proposed to be designed in an online manner, i.e. right after a disturbance is detected. This allows us to construct a sparse communication graph for feedback control, by involving only those generators in feedback which are influential in damping the most excited inter-area oscillation modes. Hence our controllers are adaptive to the magnitude and location of the incoming disturbance. The sparsity in the feedback communication graph is treated as a cyber constraint, while the generator excitation system actuation limits are treated as physical constraints, on the power system damping problem. We proposed a distributed MPC design such that both the cyber and physical constraints on the CPS are explicitly satisfied. Construction of the MPC cost function was done in frequency domain so that the control energy can be focused on only the most excited frequency modes. We next proposed sparse LQR designs for the power system WAC problem when the physical constraints on actuators can be relaxed, while strong guarantees on closed-loop stability are needed. This was achieved by solving the generalized discrete Riccati equation under structural constraints on the feedback matrix. We employed $\ell_1$-regularization of the closed-loop cost to sparsify the communication links, while losing little in terms of closed-loop damping performance. Finally, we also considered DoS cyber-attacks on the feedback communication graph, and provided an algorithm to mitigate the destabilizing effects of such attacks by redesigning our sparse LQR controller to regain closed-loop stability.
8.2 Future Research Directions

We suggest the following topics for future research.

1. The distributed MPC design given in Chapter 5 only allows those sets of generators to communicate which participate the most in damping of inter-area modes. This strategy thus necessitates ignoring dynamics for some of the (least influential) generators in the grid, for future MPC output predictions. In Chapter 5.4.3 a-posteriori sufficient conditions are provided for closed-loop stability dependent on the level of sparsity. Various simulation case studies show closed-loop stability even when considerable levels of sparsity are enforced on the cyber network. We note that as an extension to the above mentioned efforts, it would be useful to derive theoretical necessary conditions for closed-loop stability. Intuition tells us these stability conditions should depend on not only the level of sparsification in the feedback communication network, but also the physical interconnections among generators. Using reduced-order generator models, the Laplacian structure of the state matrix can potentially be exploited. Techniques in the robust MPC literature [80, 81] can be considered for future research in this direction.

2. For the control designs presented in Chapters 5-7, an implicit assumption is made that the communication traffic on the feedback links is minimal, i.e., all links are available at all times to the control designer. This might not be true if some of the links are congested due to high amounts of traffic on those links. This can potentially lead to one (or both) of the following two scenarios:

   - **Feedback delays**: The data packets being transferred over the congested links might get severely delayed in reaching the actuator controls at their destination. This can lead to performance degradation or even loss of closed-loop stability.
   - **Increased $\$ cost**: In the case of severe network congestion, the network operator might decide to employ a congestion-based pricing of the allocated network bandwidth [82, 83]. For the power system operator, this in turn can substantially increase the monetary cost for feedback communication.

A possible approach to solve the above can be as following. In Chapter 6, the sparse LQR controller is designed with a scalar $\ell_1$ regularization weight $\eta \in \mathbb{R}$ on the cardinality of the feedback matrix. This implies that equal preference is being given to sparsifying all feedback communication links. Instead, it is possible to employ an $\ell_1$ weight matrix $\Lambda \in \mathbb{R}^{m \times n}$ such that each element of $\Lambda$ is a weight on its corresponding feedback link. This approach can make the communication sparsification dynamic to the changes in the network congestion.
REFERENCES


Appendix A

Chapter 2

A.1 Construction of $\chi^\kappa, \bar{\chi}^\kappa, z^\kappa, \bar{z}^\kappa$

Let $I_b^\kappa$ be the index set of all buses in area $\kappa$ that contains a PMU, and let $I_g^\kappa$ be the index set of all generator buses in area $\kappa$. $\chi^\kappa$ is constructed as the ordered vector:

$$\chi^\kappa = \{V_i, \theta_i, \bar{I}_i, \bar{\phi}_i\}, \quad \forall i \in I_b^\kappa. \quad (A.1)$$

Here, $\bar{I}_i$ and $\bar{\phi}_i$ denote the vector of magnitudes and phase angles of the currents leaving bus $i$.

$z^\kappa$ is constructed as the ordered vector:

$$z^\kappa = \{V_j, \theta_j, \bar{I}_j, \bar{\phi}_j\}, \quad \forall j \in I_g^\kappa. \quad (A.2)$$

$\bar{\chi}^\kappa$ is obtained from polar to rectangular transformation of $\chi^\kappa$, i.e.:

$$\bar{\chi}^\kappa = \{V_i \cos \theta_i + j V_i \sin \theta_i, \ldots\}, \quad \forall i \in I_b^\kappa. \quad (A.3)$$

$\bar{z}^\kappa$ is obtained from polar to rectangular transformation of $z^\kappa$, i.e.:

$$\bar{z}^\kappa = \{V_j \cos \theta_j + j V_j \sin \theta_j, \ldots\}, \quad \forall j \in I_g^\kappa. \quad (A.4)$$
Appendix B

Chapter 3

B.1 Complex Frequency Weighting Matrix

The complex matrix $W_{i}^{\alpha_i\beta_i}$ from (3.14) can be split into its real and imaginary parts as follows. From (5.3), we have the complex frequency weighting matrix $W^{ij}$ as being composed of elements which are integral powers of $w = e^{-j\frac{2\pi}{N_p}}$. Let $\theta = \frac{2\pi}{N_p}$, then:

$$w = e^{-j\theta}$$

From Euler’s Formula, the integral powers of $w$ become:

$$w^\gamma = e^{-j\theta\gamma} = \cos(\theta\gamma) - j\sin(\theta\gamma)$$

where $\gamma$ is an arbitrary integer value. Hence we can split the complex matrix $W_{i}^{\alpha_i\beta_i}$ as:

$$W_{i}^{\alpha_i\beta_i} = W_{i,\text{real}} + jW_{i,\text{imag}}$$

where,

$$W_{i,\text{real}}^{\alpha_i\beta_i} = 
\begin{bmatrix}
1 & \cos(\theta\alpha_i) & \cos(\theta2\alpha_i) & \cdots & \cos(\theta(N_p-1)\alpha_i) \\
1 & \cos(\theta(\alpha_i+1)) & \cos(\theta2(\alpha_i+1)) & \cdots & \cos(\theta(N_p-1)(\alpha_i+1)) \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
1 & \cos(\theta\beta_i) & \cos(\theta2\beta_i) & \cdots & \cos(\theta(N_p-1)\beta_i)
\end{bmatrix}$$

$$W_{i,\text{imag}}^{\alpha_i\beta_i} = 
\begin{bmatrix}
0 & \sin(\theta\alpha_i) & \sin(\theta2\alpha_i) & \cdots & \sin(\theta(N_p-1)\alpha_i) \\
0 & \sin(\theta(\alpha_i+1)) & \sin(\theta2(\alpha_i+1)) & \cdots & \sin(\theta(N_p-1)(\alpha_i+1)) \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
0 & \sin(\theta\beta_i) & \sin(\theta2\beta_i) & \cdots & \sin(\theta(N_p-1)\beta_i)
\end{bmatrix}$$
### B.2 Linearized Laplacian for Power Systems

The Laplacian matrix $L$ in (2.12) is shown as:

\[
\begin{bmatrix}
\Delta \delta_{1,1} \\
\vdots \\
\Delta \delta_{m,1} \\
\Delta \delta_{1,2} \\
\vdots \\
\Delta \delta_{m,2} \\
\vdots \\
\Delta \delta_{1,r} \\
\vdots \\
\Delta \delta_{m,r}
\end{bmatrix}
= 
\begin{bmatrix}
L_{11} & L_{12} & \cdots & L_{1r} \\
L_{21} & L_{22} & \cdots & L_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
L_{r1} & L_{r2} & \cdots & L_{rr}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_{1,1} \\
\vdots \\
\Delta \delta_{m,1} \\
\Delta \delta_{1,2} \\
\vdots \\
\Delta \delta_{m,2} \\
\vdots \\
\Delta \delta_{1,r} \\
\vdots \\
\Delta \delta_{m,r}
\end{bmatrix}
\]

where the magnitude of the maximum entry of any of the off-diagonal blocks is much smaller than the minimum magnitude of any of the diagonal block entries, \(i.e.:\)

\[
\forall j \neq k, \quad \max |(L_{jk}(;,:))| \ll \min |(L_{kk}(;,:))| 
\]

where \((i,j)\) are block matrix indices and \((;,:)\) represents an arbitrary element of the corresponding matrix.