Abstract The standard housing allocation problem consists of a group of single students applying for a group of rooms where each room has enough space for one student. While this simple version has led to various theoretical implications it does not necessarily represent the real-world problem university housing faces. In an attempt to more accurately depict the issue university housing faces I have modified the standard housing problem to increase the number of available beds in a room as well as divide the students into groups. One group of students, noisy, is only interested in getting the room highest on their rankings. The other group, quiet, is concerned with who their roommate is in addition to the room. Under this framework, I show that the standard mechanisms do not achieve the desired properties and I introduce a dynamic mechanism with the result being Pareto efficient and strategy proof.
Housing Problem with Multiple Preference Types

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To Kali, Po, Sam, my Parents and Family.
BIOGRAPHY

William was born on Long Island in Oceanside, NY. He attended Herricks High School where he played lacrosse and found his interest in Economics. After graduating high school, he attended the University of North Carolina at Wilmington and graduated with a degree in Operations Management and Economics. William was actively involved in working with his professors doing research outside of the classroom. Upon graduation from the University of North Carolina at Wilmington, he interned at Universal McCann in New York City. He then accepted a full time position doing research related to advertising campaign strategy. William then decided to pursue a Master’s degree at North Carolina State University. He achieved his Master’s of Science in Economics and will continue his studies in the Economics PhD program in the Fall of 2018.
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0.1 Introduction

When a student is accepted into a tertiary learning institution there are many activities they must participate in. These activities are varied; and including freshman orientation to graduation, and everything in between. One task is the annual selection process of on-campus housing. Typically first year students must live their first two semesters in on-campus housing. Depending on the institution’s policy, first year students may be allowed to select their roommate. Second year students may or may not be required to live on or be provided with housing on-campus. If housing is available, a standard form is usually filled out including a request for roommates. These applications are then entered into a lottery for student room selection and assignment. This paper will explore a mechanism for student housing requests where some students have a room preference only and others have a preference for a roommate rather than a room.

At North Carolina State University there are more than 9,000 students that live on-campus. According to the university website, 55% of the on campus community are freshmen. The remaining 45% consist of sophomores, juniors, seniors, transfer students, and graduate students [3]. The student choices for housing include 20 residence halls with varied styles of living and 4 apartment communities. Prior to the Fall 2017 semester, North Carolina State incoming freshmen were not required to live on campus. The university recently changed this policy and now requires all incoming freshmen students to live on campus for at least their first year [4]. Transfer students are exempt from this policy. First year students submit an application to University Housing. Rooms are assigned in the order the applications are received [5]. University Housing attempts to fulfill specific roommate requests and building preference whenever possible, but they do not guarantee room assignment requests. Roommate and building request fulfillment is dictated by housing styles available at the time applications are received. For students other than freshman, housing assignments for the following year are completed in the spring semester prior to the enrollment year. The process for these students to select housing differs from the first year student process in that students have the opportunity to select their room and their roommate. Students register with University Housing and then housing will assign each student an order number in a lottery to select a room. When it is the student’s turn to select she will have access to a portal where she can view all of the current available rooms and beds. She then has the opportunity to select the bed she wants. If she is part of a group looking to live together, her roommates who have higher numbers in the lottery are able to choose with her at this time.

In Abdulkadiroglu and Sonmez [1], they analyze a house allocation model where there are both new and existing tenants living in a community. Their paper makes reference to a model for new applicants by Hylland and Zeckhouser [6] and the model with only existing tenants by Shapley and Scarf [7]. Abdulkadiroglu and Sonmez [1] combine these two models to create a
model that better represents real world scenarios. When they applied their model to an actual problem, their results were Pareto efficient, individually rational, and strategy proof [1].

Currently schools implement some form of the random serial dictatorship with squatting rights. While this mechanism is strategy proof it is not individually rational and therefore not Pareto efficient [8]. A possible reason to implement a mechanism other than the random serial dictatorship with squatting rights is due to the low participation rate. If a student wants to obtain a room she prefers better than her own she must give up the current room and risk getting a less desirable one. For most students the risk is not worth the chance to choose the better room and therefore students will chose not to participate. The top trading cycle proposed by Abdulkadiroglu and Sonmez [1] is not only strategy proof but also individually rational and Pareto efficient [8]. Chen and Sonmez [8] attempt to show that when the theory of the top trading cycle is implemented this mechanism will perform better than the random serial dictatorship with squatting rights [8]. Their experiments found that the the top trading cycle was significantly more efficient [8].

In Guillen and Kesten [9] they compare the top trading cycle mechanism and the Gale-Shapley deferred acceptance mechanism to New House 4 (NH4) [9]. In their first laboratory experiment, NH4 and the top trading cycle mechanism were tested head to head. They found that NH4 was the superior mechanism in both participation rates and efficiency [9]. In the second laboratory experiment, NH4 and Gale-Shapley were tested head to head. In this experiment there was no significant difference found between the NH4 and Gale-Shapley in terms of participation, truthful preference revelation, or efficiency [9]. This result occurred because the underlying procedure of NH4 is an agent-proposing deferred acceptance procedure, similar to the Gale-Shapley deferred acceptance. It is important to note that in both Abdulkadiroglu and Sonmez [1] and Guillen and Kesten [9] rooms are assigned to a single person and they are concerned with only an individual’s strict preferences over the housing type.

The stable marriage assignment first introduced by Gale and Shapley [10] demonstrated stable matching exists for their set of parameters. In their paper they make reference to the roommate problem. The roommate problem is where a single set containing an even number of elements want to create pairs of roommates [10]. A set of pairing is stable if under it there are no two individuals who are not roommates and who prefer each other to their actual roommates [10]. Gale and Shapley [10] show that using their method a stable matching does not always exist in the roommate problem. In Irving [11] he takes this idea and attempts to create an efficient algorithm that will find a stable roommate matching. The paper is able to describe a two phase algorithm that will identify if a stable matching exists, and if so, will find a matching [11].

These papers do not address the parameters associated with the North Carolina State University housing problem. While the students in these papers are concerned with room as-
signments, they differ in that housing currently has tenants in place and each room only has one bed. In an attempt to better represent a real campus scenario I will modify the housing problem by increasing the room capacity. In my paper each room will have two beds; therefore a person’s roommate can effect their room assignment. One student group has strict preference for their room. This subset is titled the noisy people. The other group consists of students with a weak preference for their room and a preference for their roommate. This subset is titled quiet people. A student’s level of satisfaction will be measured by the goal of the individual group. The students who are in the noisy group will be more satisfied with their room and are indifferent to their roommate. The students in the quiet group will be more satisfied having a quiet roommate rather than their choice of room.

I will show that the standard serial dictatorship mechanism is no longer Pareto efficient and strategy proof when applied to this scenario. In an effort to adjust the standard model for these parameter changes I propose a dynamic model where the set of quiet students will update their preferences as a room is selected by a student in the noisy set. I found that when the planner is aware of the student’s true type, noisy or quiet, and the noisy students are given priority, by the planner the dynamic model is both Pareto efficient and strategy proof.

0.2 The Model

I will explore a basic scenario where students are selecting rooms and there is no secondary market. The students are divided into two groups, noisy students and quiet students. Let \( S = \{s_1, s_2, \ldots, s_{k+m}\} \), where \( S \) is the set of all students looking for housing. Students either prefer a quiet housing assignment or they do not care. If they are indifferent between the two groups they will be assigned to the noisy group. Let \( H = \{h_1, h_2, \ldots, h_k\} \), where \( H \) is the set of all quiet students looking for rooms. Let \( L = \{l_1, l_2, l_3, \ldots, l_m\} \), where \( L \) is the set of noisy students. The sets \( H \) and \( L \) are disjoint, i.e., \( S = H \cup L \) and \( H \cap L = \emptyset \). Let \( T = \{t_1, t_2, t_3, \ldots, t_n\} \), where \( T \) is the set of rooms with available beds. Let \( q_t \) be the number of available beds in room \( t \in T \).

For the sake of simplicity I consider an environment composed of rooms with capacity two. All else being equal each student \( s \in S \) has ranking over the rooms denoted by \( \succ_s \). Student preferences over pairs, \((t,s)\), are determined by their ranking over the rooms and their types.

Each \( h \in H \) with ranking \( \succ_h \) over \( T \) has preference, \( P_h \), over pairs in \( T \times S \) as follows:

1. Any pair \((t,s)\) is preferred to any pair \((t',s')\) where \( s \in H \) and \( s' \in L \).
2. If \( s \in H \) and \( t, t' \in T \) then \((t,s)P_h(t',s)\) if and only if \( t \succ_h t' \).
3. If \( s \in L \) and \( t, t' \in T \) then \((t,s)P_h(t',s)\) if and only if \( t \succ_h t' \).
4. If either \( s, s' \in L \) or \( s, s' \in H \) then \( h \) is indifferent between \((t,s)\) and \((t,s')\).

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1A secondary market is where before the mechanism is run the students can enter into a contract to manipulate the outcome.
On the other hand for each $l \in L$ I define their preferences as $(t, s) P_l(t', s')$ if and only if $t \succ_l t'$ because members of set $L$ only care about the room. Let $R_s$ be the related at-least-as-good-as relation related with $P_s$ for each student $s \in S$.

A house allocation problem is a list $(H, L, T, P)$. In the rest of the paper, I fix $H, L, T$ and denote a problem with $P$.

A matching is a function $\mu : S \rightarrow T$ such that $|\mu^{-1}(t)| \leq q_t$ for all $t \in T$. A matching $\mu$ Pareto dominates another matching $\lambda$, if for all $s \in S\mu(s) R_s \lambda(s)$ and for some $s' \in S\mu(s) P_s \lambda(s)$. A matching $\mu$ is Pareto efficient if it is not Pareto dominated by any other matching.

A mechanism $\psi$ is a procedure which selects a matching in any problem. For problem $P$, I denote the outcome selected by mechanism $\psi$ with $\psi(P)$ and I denote the match of student $s \in S$ with $\psi_s(P)$.

A mechanism $\psi$ is Pareto efficient if for every $P \psi(P)$ is Pareto efficient. A strategy proof mechanism is one where lying about ones preferences will not make them better off. Formally, mechanism $\psi$ is strategy proof if for every student and every preference profile $P$, $\psi_s(P) R_s \psi_s(P', P_{-s})$ where $P_{-s} = (P'_{s'})_{s' \in S \setminus \{s\}}$. If the agent were to lie about which choice is their highest preference then the result would be a sub optimal choice. Our goal of this paper is to design a mechanism that is Pareto efficient and strategy proof.

### 0.2.1 Serial Dictatorship

In the standard housing allocation problem each student selects a room and each room has space for a single student. The standard housing problem uses a serial dictatorship mechanism where each student in the mechanism is assigned an order. The students choose according to that order one at a time. When it is a student’s turn to choose, she will select her highest ranked available option. The standard form of the serial dictatorship is both strategy proof and Pareto efficient. In the standard form of the serial dictatorship the students only have preference over the houses. In this paper I explore a housing allocation solution where each student selects a room, but each room houses two students. Students must consider room preference and also roommate preference. A student must consider not only their best room choice but also which roommate they want to live with. This standard model corresponds to a special case of the model proposed by this paper where everyone is either a member of the noisy set or the quiet set. This mechanism is strategy proof because the student is selecting their best option or highest preference from the available choices. If the student were to lie about which choice is their highest preference then the result would be sub optimal. This means that any student that went prior also made their selection based on their highest available preference. For a student with a lower pick to trade with someone who picked before them would cause the person who picked earlier to be worse off. The serial dictatorship mechanism requires the student to be
truthful in order to achieve their optimal choice. The serial dictatorship is also Pareto efficient because the student is selecting their highest preference from the remaining choices available.

In this paper I compare the standard and dynamic serial dictatorship. The dynamic model differs from the standard model as students have the opportunity to update their rankings over the room. When a student reports what room she wants, she can also report her type, quiet or noisy. Once a student reports her type, she reveals her complete preferences. In each case the students will pick following the same mechanism as stated below. Each case will be picking under order \( \gamma \). By keeping the mechanism consistent and changing \( \gamma \) I show which order creates a matching with the highest level of satisfaction for the students. By changing \( \gamma \) I attempt to find a priority order in which the resulting match is both Pareto efficient and strategy proof. By changing the way students report their preferences and using the basic serial dictatorship mechanism the result is no longer Pareto efficient or strategy proof.

0.2.2 Results

I will introduce the standard serial dictatorship and give the negative results related with the standard serial dictatorship.

Step 0. Each student in \( S \) is assigned a random order to select a room. Each student submit their rankings over the rooms.

Step 1. The first student in set \( S \) will select the room highest on their ranking over the houses. After selecting a room, \( q_t \) is reduced by one. If \( q_t = 0 \), then room \( t \) will be removed from the set of available rooms.

Step \( k \). The next ordered student in \( S \) will select the room highest on their list of rankings. After selecting a room, \( q_t \) is again reduced by one. If at this time \( q_t = 0 \), then room \( t \) will be removed from the set of available rooms. This process will repeat until there are no more objects in set \( S \).

**Proposition 1** The standard serial dictatorship is neither strategy proof nor Pareto efficient.

**Proof.** I prove this proposition by example. Let \( S = \{s_1, s_2\} \), where \( L = \{s_2\} \) and \( H = \{s_1\} \), and \( T = \{t_1, t_2\} \), \( q_t = 2 \) for all \( t \in T \). Let the selection order be \( s_1 - s_2 \), that is \( s_1 \) picks first. Students ranking over the rooms are given as:

\[ t_1 >_{s_1} t_2 \]
\[ t_1 >_{s_2} t_2 \]

When the mechanism is applied \( s_1 \) picks \( t_1 \) and then \( s_2 \) picks \( t_1 \). However, \( s_1 \) will be better off by moving to \( t_2 \). Hence, the mechanism is not Pareto efficient. Moreover, \( s_1 \) can get \( t_2 \) by ranking it as a top choice. Hence, the mechanism is not strategy proof. ■

One can think the negative result in Proposition 1 comes from the fact that quiet students are picking first. In the following Proposition I show this is not the case.
Proposition 2 When the set of noisy students is assigned priority over the set of quiet students on the selection order, by the planner, the standard serial dictatorship is not Pareto efficient and not strategy proof.

Proof. Let \( S = \{s_1, s_2\} \), where \( L = \{s_2\} \) and \( H = \{s_1\} \), and \( T = \{t_1, t_2\} \), \( q_t = 2 \) for all \( t \in T \). \( s_1 \) and \( s_2 \) have the same rankings of the rooms, \( t_1 \succ t_2 \). Under the students true rankings, when \( s_2 \) chooses she will choose \( t_1 \) and when \( s_1 \) chooses she will choose \( t_1 \). But if \( s_1 \) reports \( t_2 \) or \( t_1 \) then she will benefit. ■

At North Carolina State University there are a number of resident hall options. Certain residence halls have restrictions or requirements. Bagwell Hall is an example. This residence is reserved for students who are in the Honors program. Another example is Welch Hall. This residence is the only female housing on campus. The intention for creating housing for a specific type of student is to enhance the student’s experience by allowing them to live with others having similar interests.

Definition 1 A room is reserved when the students must be part of a group in order to select that room.

I propose the unreserved rooms are open to anyone. Using Proposition 1 I show that the standard serial dictatorship mechanism is neither strategy proof nor Pareto efficient. If I further assume the unreserved rooms are not open for quiet students then I achieve strategy proofness but not Pareto Efficiency.

Proposition 3 Suppose a group of rooms is reserved for quiet students and the rest is reserved for others. Then the resulting matching is strategy proof but is not Pareto efficient.

Proof. I prove the second part of this proposition by example. Let \( S = \{s_1, s_2\} \), where \( L = \{s_2\} \) and \( H = \{s_1\} \), and \( T = \{t_1, t_2\} \), \( q_t = 2 \) for all \( t \in T \) with \( t_1 \) reserved as a quiet room. Let the selection order be \( s_1 - s_2 \). Students ranking over the rooms are given as:

\[
\begin{align*}
t_2 & \succ_{s_1} t_1 \\
t_1 & \succ_{s_2} t_2
\end{align*}
\]

When the mechanism is applied student \( s_1 \) will be assigned to \( t_1 \) and \( s_2 \) will be assigned \( t_2 \). Because \( t_1 \) is reserved as a quite room and \( s_2 \in L \), \( s_2 \) can never be assigned to room \( t_1 \) even if they may prefer. This creates a Pareto inefficiency.

The strategy proofness on this proposition comes from the fact that the market has been divided into two exclusive markets and therefore the strategy proofness of the serial dictatorship mechanism is preserved in this case. ■

In the years following a student’s freshman year the student is allowed to choose not only their room but their roommate. This process typically happens in the spring prior to the school
year housing is required. The process involve two students coming together and agreeing to live together. Then the order is assigned for all of the students to select their room. When it is time for one of the students to select their room they will pick a room and temporarily hold the other bed for their friend. It is then up to the friend to select the room when it is their turn or opt to select another room. If they chose to not select the room then temporary reservation is lifted and the bed becomes available. I will take this example and apply it to our current model. Instead of the room being temporarily reserved for a single person it will be for a single group.

**Definition 2** A temporary reservation is a reservation that will occur if a quiet student is the first to select the room then they will reserve the other bed for a quiet student unless there are no other available beds for the noisy student selecting.

**Proposition 4** Using the basic serial dictatorship, but allowing students in the quiet set to temporarily reserve a bed is neither Pareto efficient nor strategy proof.

**Proof.** I prove this proposition by example. Let $S = \{s_1, s_2, s_3\}$, where $L = \{s_3\}$ and $H = \{s_1, s_2\}$, and $T = \{t_1, t_2\}$, $q_t = 2$ for all $t \in T$. Let the selection order be $s_1 - s_2 - s_3$. The students rankings over the rooms are given as:

- $t_1 \succ s_1 t_2$
- $t_2 \succ s_2 t_1$
- $t_1 \succ s_3 t_2$

When the mechanism is applied $s_1$ will be assigned to $t_1$ and $s_2$ will be assigned $t_2$. Since both $s_1$ and $s_2$ are in set $H$ the open bed is reserved for another member of $H$ unless their are no other available options. Because there are no other options for $s_3$ the reservations drop and $s_3$ may pick according to their preferences. As a result, $s_3$ will select $t_1$. Since $s_3$ is in set $L$ and $s_1$ would prefer a quiet roommate then they will switch to $t_2$ creating a Pareto inefficiency. Because $s_1$ could achieve the same result by simply listing $t_2$ as their top priority this proposition is not strategy proof. ■
0.3 Modified Model

The results from the previous model have shown that I cannot find a Pareto efficient and strategy proof order by using the standard serial dictatorship mechanism. This is due to the standard serial dictatorship using only the information about the ranking of the rooms and not accounting for roommate preference. I will introduce a modified version in which full information is considered.

Step 0. Each student is assigned an order to select a room. Each student will list their rankings over roommate pairs.

Step 1. The first student in the order will select the room highest on their list of rankings. If that student is part of set $L$, then the remaining students in set $H$ will reorder their list of preferences moving the room to the bottom. After a room is selected $q_t$, where $t$ is the room number, is reduced by one.

Step $k$. The $k^{th}$ ordered student will select from the set of remaining rooms. They will select the room highest on their list of rankings. If this student is a member of set $L$, then the remaining members of set $H$ will reorder their list of rankings moving that room to the bottom. If all of the remaining rooms are occupied by a member of set $L$ then the remaining members of set $H$ will select based on the room highest on their list of rankings. If at this time $q_t = 0$, then room $t$ will be removed from the set of available rooms. This process will repeat until there are no more objects in set $S$.

**Example.** Let $S = \{s_1, s_2, s_3, s_4\}$, where $L = \{s_1, s_4\}$ and $H = \{s_2, s_3\}$, and $T = \{t_1, t_2\}$, $q_t = 2$ for all $t \in T$.

Starting with Step 0. The students in set $S$ will be ordered as $s_4 - s_2 - s_1 - s_3$ with the following rankings:

<table>
<thead>
<tr>
<th></th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_2$</td>
<td>$t_2$</td>
<td>$t_1$</td>
<td>$t_1$</td>
<td></td>
</tr>
<tr>
<td>$t_1$</td>
<td>$t_1$</td>
<td>$t_2$</td>
<td>$t_2$</td>
<td></td>
</tr>
</tbody>
</table>

Step 1. Since $s_4$ has the highest priority she will select the room highest on their list of rankings, $t_1$. Since $q_1 = 2$, $q_1$ will be reduced by one and $t_1$ returns to the set of available rooms. Since $s_4$ is a member of set $L$, the members of set $H$ will adjust their preferences resulting in:

\[ t_2 \succ s_2 t_1 \]

\[ t_2 \succ s_3 t_1 \]

Step 2. The next student to select is $s_2$. She will select the room highest on their updated list of rankings, $t_2$. Since $q_2 = 2$, $q_2$ will be reduced by one and $t_1$ returns to the set of available rooms. Since $s_2$ is a member of set $H$, the remaining members of set $H$ will not adjust their preferences.
Step 3. The next student, $s_1$, will select the room highest on their list of rankings, $t_2$. Since $q_2 = 1$, $q_1$ will be reduced by one and $t_1$ is removed from the set of available rooms. Since $s_4$ is a member of set $L$, the members of set $H$ will adjust their preferences resulting in:

$t_1 \succ s_3 t_2$

Step 4. Student $s_3$ will select solely based on their highest preferred room. They will select room $t_1$. Since $q_1 = 0$, $q_1$ will be removed from the set of available rooms. At this time $T = \emptyset$ and the mechanism terminates with matching $\mu$.

$\mu = \begin{array}{cc} t_1 & t_2 \\ s_4 & s_2 \\ s_3 & s_1 \end{array}$

By our assumption of the preferences it suffices to know the type of student and her ranking of the rooms. I first look at the case where the types are known by the planner.

**Proposition 5** The dynamic serial dictatorship is not strategy proof nor Pareto efficient.

**Proof.** I prove this proposition by example. Let $S = \{s_1, s_2\}$, where $L = \{s_2\}$ and $H = \{s_1\}$, and $T = \{t_1, t_2\}$, $q_t = 2$ for all $t \in T$. Let the selection order be $s_2 - s_1$. Students rankings over the rooms are given as:

$t_1 \succ s_1 t_2$

$t_1 \succ s_2 t_2$

The resulting matching is strategy proof and Pareto efficient. However, if the order is changed where $s_1$ picks first. $s_1$ will be assigned to room $t_1$ and $s_2$ will also be assigned $t_1$. This assignment is unacceptable to $s_1$, so $s_1$ will lie about their initial rankings. Therefore, the mechanism is not strategy proof. ■

Now I will consider a picking order where noisy students are given a higher priority than quiet students. I will also assume that the planner knows the student’s true type. In such a case, it is sufficient for students to report their ranking over the rooms.

**Proposition 6** When the students report their true type and the set of noisy students is assigned priority over the set of quiet students, by the planner. The dynamic serial dictatorship mechanism is Pareto efficient and strategy proof.

**Proof.** All of the students in set $L$ will select their highest ranked room before any of the students in set $H$. While the students in set $L$ are selecting rooms the students in set $H$ are updating their rankings over the rooms. Once all of the students in set $L$ have been assigned a room the students of set $H$ will select. If there are rooms that are unoccupied by a student
in set $L$ they will first be filled by students in set $H$. Then when the only remaining rooms are occupied by members of set $L$, the remaining students will be selecting rooms based only on the room. In each step students are picking their best possible room and their ranking will not be affected by the subsequent students. Therefore, Proposition 6 holds. ■

Again I will consider a picking order where noisy students are given a higher priority than quiet students. Different from the previous case, I will also assume that the planner does not know the students true type.

**Proposition 7** When the set of noisy students is assigned priority over the set of quiet students, by the planner, but the type of the students is not known and they need to reveal it, the dynamic serial dictatorship is Pareto efficient and not strategy proof.

**Proof.** I will start with Pareto efficiency. Since I evaluate the efficiency under submitted preferences, by Proposition 6 this is Pareto efficient.

I now show by example students might have incentives to misreport their types. Let $S = \{s_1, s_2, s_3\}$, where $L = \emptyset$ and $H = \{s_1, s_2, s_3\}$, and $T = \{t_1, t_2\}$, $q_t = 2$ for all $t \in T$. Let the selection order be $s_1 - s_2 - s_3$. Students rankings over the rooms are given as:

- $t_1 \succ_{s_1} t_2$
- $t_1 \succ_{s_2} t_2$
- $t_1 \succ_{s_3} t_2$

When I run the mechanism the resulting matching will have $s_1$ and $s_2$ in room $t_1$, and $s_3$ in room $t_2$. Since $s_3$ would prefer $t_1$ over $t_2$ they could recognize there are no student in $L$ and say they are in $L$. This would then give them the highest priority and place them in room $t_1$. Therefore, Proposition 7 is not strategy proof. ■
0.4 Further Discussion

The real world application of this problem is more complex than what I have modeled here. In this model, I assumed all students are independent of one another. In the real world, a pair of students may want to live together. In this case I would have to adjust my model to include this parameter. I have shown the basic serial dictatorship did not meet the requirements under the previous parameters. I will work only with the dynamic serial dictatorship mechanism proposed in this paper. The previous mechanism application is used, but students in the quiet set having the same starting rankings will be paired as roommates. I will introduce a new set $G$. Let $G = \{g_1, g_2, g_3, ... , g_i\}$ where $G$ is the set of paired students. When the paired students select, the student with the higher order number will select for the pair. In the event that the remaining rooms only have a single bed remaining the pairs will be separated and the students will select as individuals. The following example will help to illustrate this scenario.

**Example.** Let $S = \{s_1, s_2, s_3, s_4\}$, where $L = \{s_3, s_4\}$ and $H = \{s_1, s_2\}$, and $T = \{t_1, t_2\}$, $q_t = 2$ for all $t \in T$.

Starting with Step 0. The students in set $S$ will be ordered as $s_4 - s_2 - s_3 - s_1$ with the following rankings:

<table>
<thead>
<tr>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_2$</td>
<td>$t_2$</td>
<td>$t_1$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>$t_1$</td>
<td>$t_1$</td>
<td>$t_2$</td>
<td>$t_1$</td>
</tr>
</tbody>
</table>

Since students $s_1$ and $s_2$ are members of set $H$ and they have the same rankings over the room they will form $g_1$.

Step 1. Since $s_4$ has the highest priority she will select the room highest on their list of preferences, $t_1$. Since $q_1 = 2$, $q_1$ will be reduced by one and $t_1$ returns to the set of available rooms. Since $s_4$ is a member of set $L$, the members of set $H$ would adjust their rankings. Though, since neither of them has $t_1$ ranked first their preferences will remain the same.

Step 2. The next student to select is $s_2$. She will select the room highest on her updated list of preference, $t_2$. Since she is a member of $g_1$, she will select the room for all the members. Since $q_2 = 2$, $q_2$ will be reduced by two and $t_2$ is removed from the set of available rooms. Since $s_1$ is a member of set $g_1$ and is assigned to a room, they are removed from set $S$.

Step 3. Student $s_3$ will select solely based on their highest preferred room. Since her top choice is no longer available she will select room $t_1$. Since $q_1 = 0$, $q_1$ will be removed from the set of available rooms. At this time $T = \emptyset$ and the mechanism terminates with matching $\rho$.

$$\rho = \begin{pmatrix} t_1 & t_2 \\ s_4 & s_2 \\ s_3 & s_1 \end{pmatrix}$$
When I apply this application to Proposition 6 the mechanism is still Pareto efficient and strategy proof. A redundancy may occur when it is time for the quiet students to select rooms and there are no double beds available. The formation of pairs would be unnecessary. I attempt to resolve this issue by adjusting the priority order: giving the pairs the highest priority, followed by the noisy set and then the single quiet students.

**Proposition 8** When the students report their true type and the set of paired students is assigned priority over the set of noisy students and single quiet students, by the planner. The dynamic serial dictatorship mechanism is Pareto efficient but not strategy proof.

**Proof.** All of the students in set $G$ will select their highest ranked room before any of the students in set $L$ or the remaining students in set $H$. Since each $g \in G$ has two members when a room is selected it is immediately removed from set $T$. Next all of the students in set $L$ will select their highest ranked room before any of the remaining students in set $H$. While the students in set $L$ are selecting the students in set $H$ are updating their rankings over the rooms. Once all of the students set $L$ have been assigned a room the students of set $H$ will select. If there are rooms that are unoccupied by a student in set $L$ they will first be filled by a student in set $H$. Then when the only remaining rooms are occupied by members of set $L$, the remaining students will be selecting rooms based only on the room. In each step students are picking their best possible room and their ranking will not affect the subsequent student. Therefore, Proposition 8 is Pareto efficient.

I will now prove by example that Proposition 8 is not strategy proof. Let $S = \{s_1, s_2, s_3\}$, where $L = \{s_3\}$ and $H = \{s_1, s_2\}$, and $T = \{t_1, t_2\}$, $q_t = 2$ for all $t \in T$. Let the selection order be $s_3 \prec s_1 \prec s_2$. The students rankings over the rooms are given as:

- $t_1 \succ s_1 \succ t_2$
- $t_2 \succ s_2 \succ t_1$
- $t_2 \succ s_3 \succ t_2$

When the mechanism is applied, $s_1$ and $s_2$ can recognize that if they adjust their preferences to match they can form $g_1$ and pick before $s_3$ to take room $t_2$. Therefore, Proposition 8 is not strategy proof.

**0.5 Conclusion**

The standard serial dictatorship is both strategy proof and Pareto efficient under the condition where one room is occupied and chosen by one student. Many on-campus housing situations are not conducive to a one-to-one choice. Most residence halls combine varied styles of communal living. In an attempt to make the standard model applicable to a real world scenario I increase
the number of students per room. By adding the additional capacity to each room, I have created a scenario where students may not only care about their room but their roommate. I have shown that that the standard serial dictatorship is no longer strategy proof and Pareto efficient. In an attempt to achieve results that are strategy proof and Pareto efficient I introduce a dynamic model where the students in the quiet set can update their rankings as the noisy set selects a room. When the planner knows the student’s type and the students in the noisy set are given room choice priority, the mechanism is both Pareto efficient and strategy proof. The dynamic model can accommodate the increased bed parameter. To make the problem more applicable to a real world scenario I introduced a model where students in the quiet set form pairs if their initial rankings are the same. This model’s outcomes are Pareto efficient and strategy proof. It could become an unnecessary step because there could be no double beds remaining after the noisy set has selected. While I have shown that there is a dynamic version of the serial dictatorship mechanism that is Pareto efficient and strategy proof, further work will be required to incorporate more varied room types.
REFERENCES


APPENDIX
Appendix A

Mechanisms

A.1 Top Trading Cycle[1]

The top trading cycle was designed to entice people who currently have a house to enter the housing market to obtain a more desirable home. In the first step all of the currently occupied houses will point to the person currently occupying it. Then each resident will point to their highest ranked house. The unoccupied houses will point to the person with the highest priority. While the individual without a residence will point to the house highest on their list of rankings. If a cycle occurs the person will be assigned to the house they are pointing to and both will be removed from the set of available houses and people. This process will continue until there are no more remaining elements in the set of houses and the set of people.

A.2 Deferred Acceptance[2]

The deferred acceptance mechanism is used to create a stable matching when one group is proposing to another. This mechanism is easily shown in the marriage market. During the first round each man will propose to their highest ranked women. If a woman is proposed to by more than one man she will temporarily accept the man highest on her list of rankings and reject the rest. In the following rounds, the men who were rejected will then propose to the next highest ranked women on their list. If the woman he has proposed to has temporarily accepted a man then the woman will see if the new proposal is ranked higher. She will then accept the man ranked higher on her list. This process will continue until all of the men have proposed to all acceptable women. At this point the mechanism terminates and all the women who have temporarily accepted proposals will accept.