ABSTRACT

FORTUNE, NICHOLAS CHARLES. Supporting a Mathematician’s Instructional Change in Undergraduate Mathematics Through Faculty Collaboration. (Under the direction of Dr. Karen Keene.)

To reform their instruction by using more student-centered approaches, research has shown that faculty benefit from support and collaboration (Henderson, Beach, & Finkelstein, 2011; Henderson, Dancy, & Niewiadomska-Bugaj, 2012; Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007). In this qualitative instrumental case study I examined the ways in which a mathematician’s instruction unfolded during his participation in a faculty collaboration geared towards reforming instruction and aligning it with inquiry oriented instruction (Kuster, Johnson, Keene, & Andrews-Larson, 2017; Rasmussen & Kwon, 2007). This research provides a detailed analysis of a faculty’s instructional practice, something that is currently underreported in literature (Speer, Smith, & Horvath, 2010). Further, his participation in the faculty collaboration was analyzed, with the ultimate goal of relating his experiences in the faculty collaboration to his instructional practice. Results indicated that the faculty collaboration did support the mathematician in achieving his goal of instructional reform. Further, results indicated that the mathematician’s active participation was integral to successful instructional change and how the mathematician’s mathematics research influenced his participation in the faculty collaboration and his instructional practice. Lastly, when considering instruction, a tension was observed between the agenda of the mathematician in a mathematics classroom and inquiry instruction, as well as a tension between inquiry instruction and anticipating student thinking. Implications from this work highlight how the model used within of faculty collaborations holds promise for successfully supporting instructional change in undergraduate mathematics. Namely, this research
highlights changes that can be made to the model in future iterations of faculty collaborations of all types. Conversing about the complex act of instruction, something that is atypical of a mathematician, provided support that this mathematician desired to reform his instructional practice.
Supporting a Mathematician’s Instructional Change in Undergraduate Mathematics Through Faculty Collaboration

by
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DEDICATION

I dedicate this to all those who have pushed me.

“Never half-ass two things. Whole-ass one thing.” – Ron Swanson

I dedicate this to all those who helped me own my confidence.

“I have no idea what I’m doing, but I know I’m doing it really, really well.” – Andy Dwyer

I dedicate this to all those who have continuously made me laugh.

“Time is money; Money is power; Power is pizza; Pizza is knowledge!” – April Ludgate

I dedicate this to all those who have kept me grounded.

“We need to remember what’s important in life: friends, waffles, work. Or waffles, friends, work. Doesn’t matter, but work is third.” – Leslie Knope

I dedicate this to all those who love me for who I am.

“Treat yo self.” – Tom Haverford and Donna Meagle
BIOGRAPHY

Nicholas Charles Fortune was born in Albany, NY on October 22, 1990 to Ellen Gaffney and Christopher Fortune. Nicholas also has two other parent figures in his life, his step-mother Oksana Lanova Fortune and his mother’s partner John Carfagno. Nicholas has one older step-brother, Vanya, and one younger sister, Natalie.

Nicholas grew up in Red Hook, NY for most of childhood where he ultimately graduated from Red Hook High School in 2009. Nicholas was awarded the Rensselaer Polytechnic Institute (RPI) Medal from his high school. The RPI medal was a generous scholarship to attend RPI. Consequently, Nicholas relocated to Troy, NY to attend RPI. Nicholas participated in the co-terminal program at RPI, obtaining his B.S. and M.S. at the same time. He graduated in 2013 with a B.S. in Mathematics and an M.S. in Applied Mathematics.

In 2014, Nicholas started at North Carolina State University to pursue a doctorate in mathematics education. During his time at North Carolina State University, Nicholas taught Calculus for Elementary Teachers and was a graduate research assistant for the Teaching Inquiry-oriented Mathematics: Establishing Supports (TIMES) project, led by Dr. Karen Keene. Nicholas’ dissertation stemmed from his work on TIMES.

Upon graduation, Nicholas will be moving to Bowling Green, KY. He has accepted an Assistant Professor position in the Department of Mathematics at Western Kentucky University. While there he will teach mathematics courses to preservice teachers and the general population of students as well as continue to conduct research on instructional change in undergraduate mathematics.
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“Turns out it’s not where but who you’re with that really matters.”

I must acknowledge the many people who have been with me throughout my life that have led me to this momentous occasion.

To my family (my actual family): Mom and Dad, I am who I am today because of how you raised me. Thank you for providing for me, supporting me, and loving me. To all of my family members, thank you for being with me, even when we are all across the country. I especially would like to thank my sister, Natalie. Natalie, you once told me why you would much rather be an empathetic person than a sympathetic person. You said “an empathetic person doesn’t just feel for someone, they feel with someone.” I never forgot that, and I have tried to live that way ever since. I so look forward to the day where our kids can be best friends. I so look forward to all that you are going to accomplish in your life. I am very proud of you. I love you. Thank you for being my sister.

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Chapter 1: Introduction

Over the last decade there have been numerous calls for reform in undergraduate mathematics education (National Research Council [NRC], 2013; President’s Council of Advisors on Science and Technology [PCAST], 2012). These calls for reform draw on research that has shown the benefits of student-centered instruction for students (Freeman, Eddy, McDonough, Smith, Okorafor, Jordt, & Wendoroth, 2014; NRC, 2000, 2001). To address these calls, change is needed in the instruction of undergraduate mathematics.

The Conference Board of Mathematical Sciences (CMBS) is a large organization consisting of seventeen professional mathematics associations. Five of these professional associations, the Mathematical Association of America (MAA), American Mathematical Society (AMS), American Statistical Association (ASA), American Mathematical Association of Two-Year Colleges (AMATYC), and the Society for Industrial and Applied Mathematics (SIAM), focus in part on undergraduate mathematics education. In 2015, the MAA spearheaded a report from a collaboration of these five associations entitled A Common Vision for Undergraduate Mathematical Sciences Programs in 2025 (MAA, 2015b). While offering recommendations for mathematics programs as a whole, a subset of the recommendations synthesized in A Common Vision, and other reports, are specifically focused on reforming instruction (MAA, 2015b).

A Common Vision gave a general call that instruction should move away from traditional lecture as the sole instructional method in undergraduate mathematics (MAA, 2015b). In another report, Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College, AMATYC (2006) highlighted the need for student-centered and activity-based instruction in undergraduate mathematics. Additionally, the ASA also called
for more lab-centered instruction for undergraduate statistics (2005). Further, the *Curriculum Guide to Majors in the Mathematical Sciences*, encouraged departments to reward the efficacy of faculty teaching more than departments currently tend to (MAA, 2015a). There are also calls for departments and faculty members to collaborate more on the teaching and pedagogy aspect of their job (MAA, 2015a). The *Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra (CRAFTY)* report calls for the promotion of professional development for undergraduate mathematics instructors (2011).

Given these calls for instructional reform, faculty are making changes to their instruction. However, research has shown that even when working with research-supported curricular materials (Rasmussen, Keene, Dunmyre, & Fortune, 2018), faculty are often unprepared to undertake the challenge of introducing changes to their instruction (Henderson, 2005; Henderson et al., 2011, 2012; Speer & Wagner, 2009; Wagner et al., 2007). Many efforts are underway to provide mathematicians and instructors with the support they need to change their instruction. One research-based method of support (i.e., professional development) is the facilitation of faculty collaboration geared towards collectively improving instruction (e.g., Nadelson, Shadle, & Hettinger, 2013).

In this movement of reforming undergraduate instruction, much work is still needed to know how best to support mathematics faculty as they embark on this journey. Faculty collaboration is a viable option of support for this intended reform. This study explored the experiences of a mathematician who participated in one such faculty collaboration geared towards changing instruction that addressed the numerous calls for reform in undergraduate mathematics education and instruction.
Statement of the Problem

Scholars have argued that lecturing dates back over 5000 years (Abaté & Cantone, 2005). Additionally, despite the aforementioned calls for reform, recent surveys show that teacher-centered instruction is still the most common mode of instruction in undergraduate STEM education (Bressoud, Mesa, & Rasmussen, 2015; Fukawa-Connelly, Johnson, & Keller, 2016a, 2016b, Henderson et al., 2011, 2012). Additionally, faculty typically teach the way they were taught (Oleson & Hora, 2013), thus continuing the predominance use of teacher-centered instruction in undergraduate education.

Alternatively, current trends in undergraduate STEM education such as student-centered instruction, the use of contextual problems, rich discourse between faculty and students in the classroom, using group work, and writing in the classroom are gaining traction among some instructors at the university level (Abdulwahed, Jaworski, & Crawford, 2012). There is evidence that using this active learning can improve student achievement and attitudes. Freeman and colleagues conducted a meta-analysis of studies that examined active learning (i.e., student-centered instruction) and its impact on student outcomes. They found overwhelming evidence that student-centered instruction in STEM classrooms yields better student outcomes than teacher-centered classrooms (Freeman et al., 2014). In their conclusion Freeman et al. (2014) made the analogy if their study was a medical study and lecture was the treatment group, they would have had to stop the treatment group for being unethical. They continue and say that we as scientists, scholars, and faculty, should use findings from research on the importance of student-centered instruction and its impact on student outcomes to make our instructional decisions. That is, we should be teaching based on scientific evidence rather than based on tradition (AMATYC, 2006; Freeman et al., 2014;
However, there has been little research conducted on collegiate faculty’s instructional practice, specifically in mathematics (Speer et al., 2010). Even though since 2010, more research has begun to study mathematics faculty’s instruction (e.g., E. Johnson, 2013; E. Johnson & Larsen, 2012), more research is needed on mathematics faculty’s instructional practice, as well as research on ways to support instructional change in undergraduate mathematics education.

Research from undergraduate STEM education has shown that faculty collaborations are a way to support instructional change and develop reflective teachers (Demir, Czerniak, & Hart, 2013; Nadelson et al., 2013). That is, through collaboration with colleagues, mathematicians can focus on collectively reforming instruction by supporting each other. More research could be useful on faculty collaborations as a means to support instructional change in undergraduate mathematics education. Particularly, I propose that the mathematics education community needs to know how mathematicians and mathematics faculty come to reform their instruction to include research-based instructional strategies in the context of faculty collaboration.

**Purpose Statement and Research Questions**

The purpose of this study was to focus on one participant who engaged in the unique experience of a faculty collaboration as a means to reform instruction; and further, to use his experience to begin to shed light on the overall problem of how to best support mathematics faculty as they reform their instruction. The study examined how a mathematician’s instructional practice unfolded over one academic semester while he was part of a faculty collaboration for undergraduate mathematics instructors on inquiry oriented differential
equations; it explored the relationship between the faculty collaboration and his instruction. The faculty collaboration, facilitated by mathematics educators, consisted of mathematics faculty who wished to improve their instructional practice and begin using inquiry oriented instruction (Kuster et al., 2017; Rasmussen & Kwon, 2007).

This research study employed case study methods (Creswell, 2013; Yin, 2013). In this study, the case is the participant going through this experience (i.e., reforming instruction through collaborating with colleagues). Qualitative data consisting of classroom videos, videos of an online faculty collaborations, and interviews were used to characterize the mathematician’s instructional practice and his participation in a faculty collaboration on inquiry oriented differential equations over one academic semester. The study addressed the following research question, with sub research questions:

1. In what ways does one mathematician’s experiences in an online faculty collaboration on inquiry oriented differential equations relate to his instructional practice?
   a. How does his instructional practice unfold over his first implementation of inquiry oriented differential equations and in what ways does it align with inquiry oriented instruction?
   b. How does his participation unfold in the online faculty collaboration?

**Significance of the Study**

As discussed, research has shown that mathematicians have difficulties implementing a new curriculum without support (Speer & Wagner, 2009; Wagner et al., 2007). Further, research has shown how faculty collaboration, summer workshops, and online forums aid mathematicians in sustaining instructional change (Hayward, Kogan, & Laursen, 2015). On the other hand, new mediums exist in which faculty collaboration can take place (e.g., online
video meetings). This research explored the use of a new and way to engage mathematicians and support them in reaching their goal of instructional change. The study addressed a gap in the research because little is known about the effectiveness of these new support structures (e.g., [online] faculty collaborations) for mathematicians who desire to reform their instructional practice to more student-centered approaches. The intention of this study was to zoom in on one mathematician to study the relationships that exists between his experiences in a faculty collaboration and his classroom instructional practice. By concentrating on one mathematician and taking a deep dive into his experience, I will add to what we know in this area, thus contributing something new to the research.

**Overview of Methods**

**Case study research.** Research studies using a case study methodology can vary widely in their interpretation and execution (Creswell, 2013). However, general agreement among the research community is that case study research is a subset of qualitative research in that a case is studied in a real-life context to develop an in-depth understanding of the case or cases (Creswell, 2013; Denzin & Lincoln, 2005; Merriam, 2009; Yin, 2013). This study was bounded by the participant’s experiences in his classroom and participation in the online faculty collaboration during one academic semester and explores the data with descriptive analyses and thus aligns with the qualitative analysis method of case study (Yin, 2013). This study is specifically an instrumental case study (Stake, 1995). This means that this case was not the sole interest (i.e., as in intrinsic case study), however, this case was chosen to describe aspects of this scenario that can add to the undergraduate mathematics education knowledge base. In this instance, the case was defined as the participant’s participation and experiences through this online faculty collaboration.
Why case study methods? As minimal research has been conducted on the relationship between faculty collaboration and instructional practice at the undergraduate level, rich descriptions of these scenarios are necessary. A rich descriptive account of this case provides insight into the impact that faculty collaborations have on instructional change and instructional practice.

Data sources. Data came from observations of the online faculty collaboration, classroom observations, and audio recordings of two interviews. Analyses of the classroom observations used an existing framework that aligns with the intended direction of instructional change. Furthermore, analyses of the online faculty collaboration transcripts used extant frameworks to describe participation in the faculty collaboration (situated in previous research from this context or outside existing literature). Lastly, analysis of interview transcripts was open in nature (Strauss & Corbin, 1998; Yin, 2013) to capture the participant’s perspective on his experiences through this study.

Positionality Statement

In qualitative research, it is important to reveal how my position plays into my research. In this section, I briefly provide a subjectivity statement and then discuss my theoretical views which underpin this dissertation.

Subjectivity statement. I am now near the end of my pursuit of a doctorate in mathematics education from North Carolina State University; however, education was not always my intended domain. I received my Bachelor’s and Master’s degrees in applied mathematics from Rensselaer Polytechnic Institute in Troy, NY; both in 2013. While there, and in that “hard science” discipline, I came to appreciate education by working with students on their mathematics coursework. I became fascinated by the different ways
students learn, believe in themselves, and form mathematical identities. Furthermore, I became very interested in mathematics instructors and the pedagogical choices they make and innovative courses they enact. Ultimately, it caused me to alter my discipline slightly to mathematics education.

All of my mathematics instruction at my previous institution was direct instruction (i.e., lecture); however, I work to ensure students are afforded the opportunity of innovative instruction through my own teaching and my own research. Consequently, I may have an unintentional bias towards reform instruction. This does not imply that lecture is poor instruction; many professors can be engaging even in lecture, however, that was not my experience.

I have a particular proclivity for a view of learning rooted in Realistic Mathematics Education. I now briefly describe this construct and how it, and other related constructs, are connected to this dissertation.

**Theoretical underpinnings.** There are two theoretical underpinnings of this study and my own view of the learning and teaching of mathematics: Realistic Mathematics Education (RME) (Freudenthal, 1991) and the emergent perspective of learning and teaching (Cobb & Yackel, 1996).

**Realistic Mathematics Education.** RME states that mathematics is a human activity. Crucial to RME is the design of instructional sequences that challenge students to reinvent and organize key subject matter at one level in order to produce new understandings at a higher level (Freudenthal, 1991; Gravemeijer & Doorman, 1999). This reinvention demands students to mathematize their own mathematical activity. Horizontal mathematization is when one describes the context of a problem in mathematical terms, whereas, vertical
Mathematization is when one reaches a higher level of mathematics (Gravemeijer & Doorman, 1999).

The mathematization process is embodied in the core heuristics of guided reinvention and emergent models. Guided reinvention speaks to the need to locate instructional starting points that are experientially real to students and that take into account students’ current mathematical ways of knowing (Rasmussen & Kwon, 2007, p. 191).

Another aspect of RME, highlighted by the above excerpt, is that tasks in curriculum are designed to be experientially real for the students (Gravemeijer & Doorman, 1999; Rasmussen & King, 2000; Rasmussen & Kwon, 2007), meaning students can utilize their existing ways of reasoning to make progress on problems.

The construct of RME aligns with how I believe mathematics should be taught. RME influenced the design of many curricula (e.g., inquiry oriented differential equations) and connects to many other theories (e.g., the emergent perspective).

**Emergent perspective.** The emergent perspective, coined by Cobb and Yackel (1996), is a particular version of social constructivism that coordinates symbolic interactionism (one sociocultural perspective) (Blumer, 1969) with the individual cognitive perspective of constructivism (von Glasersfeld, 1995). In this view, mathematical development is seen as a combination of individual construction of mathematical ideas and enculturation into mathematics (Cobb & Yackel, 1996).

I briefly summarize constructs of the emergent perspective and its associated interpretive framework. Cobb and Yackel (1996) outlined the crucial constructs to the emergent perspective, through a framework that they defined as the interpretive framework.
The interpretive framework coordinates both the individual psychological and social perspectives. Figure 1 is a representation of how I view the interpretative framework.

![Figure 1. Interpretive framework. Based on Cobb & Yackel (1996).](image)

Each row in the interpretative framework relates the social and individual perspectives of particular classroom constructs. Classroom social norms characterize collective social activity in a such a way that cannot be characterized individually (Cobb & Yackel, 1996; Yackel & Cobb, 1996). Sociomathematical norms are shared agreements among teachers and students in a classroom about particular mathematical vernacular and representation (Cobb & Yackel, 1996; Yackel & Cobb, 1996). The first two rows in the psychological perspective are the individual perspectives of self and mathematics. To elaborate, the individual has beliefs about their own role, the role of others, and a general nature or understanding of mathematical activity. Further, they possess mathematical beliefs and values that contribute to overall mathematical development for the individual and collective. Extant literature has discussed how classroom social norms, sociomathematical norms, and self and mathematics beliefs are operationalized (see Yackel & Cobb, 1996).

**Connection to study.** Both of these theories, then, support my view of and undergird the context of this study, and particularly the type of mathematical inquiry discussed.
Realistic Mathematics Education and the emergent perspective influenced the design of inquiry oriented mathematics (Rasmussen & Kwon, 2007), the inquiry oriented differential equations curriculum (Rasmussen, 2001; Rasmussen et al., 2018; Rasmussen & King, 2000), and inquiry oriented instruction (Kuster et al., 2017). These constructs will be elaborated on in Chapter 2; and Chapter 3 will begin with a conceptual framework relating these theoretical underpinnings to the methods used in this study.

**Definition of Terms and Abbreviations**

To help the reader, a list of important terms and abbreviations throughout the dissertation are listed here, as well as definitions of those terms and abbreviations.

**Differential Equations [DE or DEs]** – the course titled Differential Equations or a singular differential equation (mathematical equation that relates some function with its derivatives); DEs are multiple differential equations

**Faculty Collaboration** – a form of professional development [PD]; this choice of words was used as the professional development is a collaboration among mathematics faculty from universities across the United States

**Inquiry Oriented [IO]** – a view of the mathematics classroom in which students are inquiring into the mathematics, while the teachers are inquiring into the students’ individual and collective mathematical thinking (Rasmussen & Kwon, 2007)

**Inquiry Oriented Abstract Algebra [IOAA]** – abstract algebra taught with an IO perspective

**Inquiry Oriented Differential Equations [IODE]** – differential equations taught with an IO perspective

**Inquiry Oriented Linear Algebra [IOLA]** – linear algebra taught with an IO perspective

**Mathematician** – a mathematician is someone who has a PhD in Mathematics or a related
field and tends to not have formal teacher training other than any graduate teaching they did during graduate school; additionally, referred to as mathematics faculty or sometimes mathematics instructor

Online Faculty Collaboration [OFC] – one of the support structures for TIMES, weekly online lesson studies

Teaching Inquiry-oriented Mathematics: Establishing Supports [TIMES] – Research project where data from this study comes from; explained in further detail in methods section

**Organization of Dissertation**

Following this introduction, this dissertation provides an overview of the relevant literature for this study in Chapter 2. Then in Chapter 3, the methods that were used for this project are discussed in detail. Following this, Chapter 4 highlights results and provides an answer to research question 1a and Chapter 5 highlights results and provides an answer to research question 1b. Subsequently, Chapter 6 is a discussion of the findings, situated in existing literature, and provides an overall answer to the research question. Additionally, Chapter 6 presents the conclusions and implications of this work and sheds light on the direction of possible future research.
This study examined how the instructional practice of a mathematician teaching with inquiry oriented differential equations materials developed while he participated in a semester-long online faculty collaboration supporting his first implementation. Furthermore, the study related his experiences in the faculty collaboration to his classroom instructional practice. In this section I first elaborate on the need for the instructional change called upon in Chapter 1. Further, relevant research on instructional change at the undergraduate level is discussed, including professional development literature and relations between professional development and instructional practice. Finally, the particular approach to mathematical inquiry used in this study is presented.

The Necessity of Change

Before I discuss what the mathematics education research community knows about instructional change of undergraduate mathematics, I believe it is important to briefly highlight why this change is of the utmost importance. Recently, numerous professional mathematics associations have joined forces to address calls for reform in undergraduate mathematics education from the NRC (2013) and PCAST report (2012). This joint effort is rooted in the idea that to achieve impactful, significant, and long-lasting change a coordinated effort, that is supported by major players from all existing arenas, is necessary (Kania & Kramer, 2011). The MAA spearheaded a report further elaborating on these calls for reform. In this synthesized report, A Common Vision, they concluded, “the status quo is unacceptable” (MAA, 2015b, p. 1).

If the status quo is unacceptable, then what is the current status quo? A Common Vision highlighted previous research that found startling results in undergraduate
mathematics education. For example, about 50 percent of students earn a grade of A-C in college algebra (MAA, 2011). Additionally, failure rates under traditional lecture are 55 percent higher than the rates observed under more active (note, this is broadly defined here) approaches to instruction (Freeman et al., 2014). In terms of underrepresented groups in mathematics, findings indicate that women are nearly twice as likely, compared to men, to choose not to take mathematics classes beyond Calculus I, even when Calculus II is a requirement for their intended major (Bressoud, 2011). Further, in 2012, the National Science Foundation (NSF) found that 19.9 percent of all BS degrees were awarded to underrepresented minorities, however, only 11.6 percent of mathematics BS degrees were awarded to underrepresented minorities (NSF, 2014). These results only just begin to highlight the need for deep rooted change in undergraduate mathematics education.

This necessary change may appear in many forms. For instance, change is needed on the recruitment and placement of students into STEM disciplines (Bressoud et al., 2015), student retention rates (Bressoud, Carlson, Mesa, & Rasmussen, 2013), faculty development (MAA, 2011), instructional strategies (Bressoud et al., 2015; MAA, 2011, 2015b; Mesa, Burn, & White, 2015), assessment (Larsen, Glover, & Melhuish, 2015), course coordination (Bressoud et al., 2015), and much more. These change endeavors are heavily intertwined. Achieving successful change in one category cannot be done without coordination between and among other categories.

In this work, I focused on two of these areas: instructional change, specifically, supporting instructional change through faculty development as a contribution to addressing the inequities that currently exist in undergraduate mathematics education. In the next section, I provide an overview of instructional change, and then discuss barriers to
instructional change and how to facilitate and support instructional change.

**Overview of Instructional Change**

As this study focuses on the development of instructional practice, research literature is discussed on what is known about how, why, and when faculty make changes to their instruction. I am defining *instructional change* to be a change in faculty’s instruction, specifically *from* teacher-centered *to* student-centered instruction. This definition aligns with Henderson et al. (2011). The degree to which the change can happen varies widely and many examples of change are discussed.

**Barriers to Instructional Change in Undergraduate STEM Education**

Despite the plethora of research on the effectiveness of active learning environments on student outcomes (e.g., Freeman et al., 2014), oftentimes, faculty are unprepared to undertake the challenge of introducing changes to their instruction (Henderson, 2005; Henderson et al., 2011, 2012; Speer & Wagner, 2009; Wagner et al., 2007). Unsurprisingly, change efforts are often met with resistance and barriers that need to be overcome (Dancy & Henderson, 2010). In this section I discuss the common barriers that are expressed by faculty during the instructional change process. In a similar fashion to Henderson and Dancy (2007) and their discussion of barriers, I categorize this section into individual and situational barriers. Note that some barriers span both individual and situation contexts (Henderson & Dancy, 2007).

**Individual barriers.** Scholars have discussed how a scientist’s professional identity is often the first barrier to instructional change (Brownell & Tanner, 2012; Kensington-Miller, Sneddon, & Stewart, 2014). Faculty’s individual identities often do not include instruction as an important component (Brownell & Tanner, 2012). This is viewed as the
hardest barrier to overcome because without faculty valuing the teaching aspect of their job, not just the research aspect, they may not be receptive to instructional change. There is a need for a disciplinary shift that adds value to instruction in academia (Brownell & Tanner, 2012; Kensington-Miller et al., 2014). Due to instruction not being viewed as part of faculty’s identity, they typically feel minimal satisfaction or need to change their courses or their instruction (Fukawa-Connelly et al., 2016a). This in turn yields a misalignment between faculty’s instructional beliefs and reform endeavors that change agents (ones conducting and/or researching instructional change) hope to propagate into certain environments (Henderson et al., 2011; Nadelson et al., 2013).

Another barrier to instructional change is faculty’s knowledge for teaching with student-centered instructional strategies. Research has shown that some faculty lack the necessary skills to enact student-center instruction (Hayward et al., 2015), sometimes because they lack specialized content knowledge relating to instruction and being prepared to respond to student questions productively (E. Johnson, Caughman, Fredericks, & Gibson, 2013; Speer & Wagner, 2009; Wagner et al., 2007).

Speer, Wagner, and colleagues examined mathematicians implementing inquiry oriented differential equations material. In one study, they explored the knowledge needed by a mathematician to provide analytical scaffolding in his classroom (Speer & Wagner, 2009). They found that he needed knowledge other than content knowledge, that is, specialized content knowledge (Ball, Thames, & Phelps, 2008), to productively engage with his students’ mathematical reasoning (Speer & Wagner, 2009). He consistently noted that he wanted a roadmap to know where the course was going, but wanted a level of detail that was not possible for the curriculum designers to give (Rasmussen, personal communication, Jan 7,
2017; Speer & Wagner, 2009). In another study, they explored a different mathematician and his implementation of inquiry oriented differential equations and specifically discussed the challenges that arose for the mathematician (Wagner et al., 2007), such as lack of skill to productively respond to student questions which he did not have to do in his teacher-centered instruction classroom. Results indicate that support structures are needed for faculty to be successful in instructional change (Speer & Wagner, 2009; Wagner et al., 2007). This dissertation highlights an example of one such support structure.

In addition to a lack of knowledge specific to reform instruction, faculty’s knowledge of resources (Dancy & Henderson, 2010; Walczyk, Ramsey, & Zha, 2007) or lack thereof (Hayward et al., 2015), is often another barrier that needs to be overcome to achieve instructional change. Faculty feel curricula using research-based instructional strategies are not readily available (Dancy & Henderson, 2010). Contrariwise, there are numerous resources, most of which are open-source and online, which provide research-based curricula that focus on the use of research-based instructional strategies. For example, resources exist for inquiry oriented differential equation (https://iode.wordpress.ncsu.edu/) (Rasmussen et al., 2018), inquiry oriented linear algebra (https://iola.math.vt.edu/) (Zandieh, Rasmussen, Wawro, & Andrews-Larson, 2016), and inquiry oriented abstract algebra (http://www.web.pdx.edu/~slarsen/TAAFU/home.php) (Larsen & Johnson, 2016).

**Situational barriers.** One of the largest situational barriers, that could also be viewed as an individual barrier, is the conflict between what is valued in higher education and instructional change (Hayward et al., 2015; McDuffie & Graeber, 2003; Walczyk & Ramsey, 2003; Walczyk et al., 2007). What is currently being valued in instructional change research and practice is not what is valued by tenure committees or institutes of higher education.
That is, there are few recognitions, rewards, or incentives for changing instruction to align with what instructional change research has suggested (Brownell & Tanner, 2012; Henderson et al., 2011; Henderson & Dancy, 2007; Turpen, Dancy, & Henderson, 2016). Furthermore, institutional (Hora & Anderson, 2012; McDuffie & Graeber, 2003) and departmental (Henderson & Dancy, 2007) norms and policies rarely support innovation in instructional or pedagogical changes. Inquiry and critical reflection, which are necessary components of instructional change, are often overshadowed by the high demands placed on faculty from their departments and institutions (McDuffie & Graeber, 2003). Depending on the norms and policies at their university, faculty are required to publish a certain amount of work in peer reviewed journals. In some instances, the time and work it takes to do that (which is important and necessary work in academia) takes away from efforts faculty could make in changing their instruction. A better system should be in place that rewards instructional change (McDuffie & Graeber, 2003; Walczyk et al., 2007).

Not only are norms potential barriers to instructional change, but administrative policy can also be a barrier at times. Top-down policies or one-size-fits-all policies do not work to change instruction (Henderson et al., 2011). This is because each department within a university has its own pre-established norm structure and policies from change agents who are unaware of that norm structure (or broad policies) typically act as a hindrance to instructional change instead of igniting it (Henderson et al., 2011).

Oftentimes training on instruction comes from department, college, or university groups. Yet, research has shown that there is a lack of training or support for faculty when it comes to instructional change (Brownell & Tanner, 2012; Henderson et al., 2011; Henderson
& Dancy, 2007; Turpen et al., 2016). Administration, while they do place a crucial role in the success of universities, often dampen instructional change efforts before they get off the ground.

Another group of barriers to instructional change are environmental barriers. Less common but frequent reasons faculty claim to use teacher-centered instruction is either the layout of the room is not conducive for student-centered instruction or the size of their class is too large (Henderson & Dancy, 2007). Oftentimes, undergraduate mathematics classes are in large lecture halls. Additionally, faculty have stated that student resistance, lack of student buy-in, and student attitudes of school are reasons why they do not use student-centered instruction (DeLong & Winter, 1998; Hayward et al., 2015; Henderson & Dancy, 2007; A. Johnson et al., 2009). Similar to faculty teaching the way they were taught, students tend to prefer consistency across their classes (Oleson & Hora, 2013), that is if only one of their classes uses student-centered instruction, faculty believe students may be resistant to that.

The most often cited environmental reason by faculty to not use research based instructional strategies or student-centered instruction is how much more time it takes than teacher-centered instruction (Brownell & Tanner, 2012; Dancy & Henderson, 2010; Henderson et al., 2011; Henderson & Dancy, 2007; Turpen et al., 2016). Likewise, faculty say they stray away from student-centered instruction because they have a certain amount of material that needs to be covered over the course of one semester (Hayward et al., 2015; Henderson & Dancy, 2007; Turpen et al., 2016). Faculty say a large reason they stop doing student-centered instruction after one attempt is because of how much time it takes and how much material they were not able to cover (Henderson et al., 2012).

This dissertation’s data is from a large NSF-funded research project where these
barriers to instructional change were considered during the design of the study. Research has provided insights into how to facilitate and sustain instructional change, especially when considering these barriers. In the following section I discuss those approaches to facilitate and sustain instructional change.

**Facilitating and Sustaining Instructional Change**

Henderson et al. (2011) outline four categories of instructional change. I briefly describe these categories. Then I focus on two change strategies which best connect to this dissertation and how those change strategies address the aforementioned barriers to instructional change. The four categories of changes strategies from Henderson et al. (2011) are disseminating curricula and pedagogy, developing reflective faculty, enacting policy, and developing a shared vision. Figure 2 is a simplified version from Henderson et al. (2011).

![Figure 2. Categories of instructional change. Based on Henderson et al. (2011).](image)

Henderson et al. (2011) define three groups of change agents who are initiating, conducting, and/or researching change initiatives: STEM Education Researchers (SER), Higher Education Researchers (HER), and Faculty Development Researchers (FDR). The first category of change strategies is *disseminating curriculum and pedagogy*. SER and FDR change agents are often involved in disseminating curricula and pedagogy in which they...
inform faculty about new (or not so new) instructional practices and encourage their use.

These tend to be strategies that are focused on individuals and focused on prescribed outcomes (i.e., the change agent goes in with an intended plan of what is to change and how it will change). A second category of change strategies is to develop reflective faculty. SER and FDR typically are involved in design-based research or action research in which they encourage faculty to develop new teaching conceptions and a reflective instructional practice. This is similar to disseminating curricula and pedagogy in that the intended target of the change is the individual; however, the intended outcome is emergent (i.e., the change agent does not go in with an intended outcome and one typically emerges from the needs of the faculty in the given environment).

A third change category is enacting policy. HER and FDR change agents prescribe change efforts that they either encourage (or require) faculty to adopt. The targeted population is beyond just an individual and is focused more on either the department level or institution level. The final change category identified by Henderson et al. (2011) is the least frequently studied or used: developing a shared vision. HER change agents attempt to scale up the enactment of policy but in a manner, such that the faculty are part of the emergent change. Borrego and Henderson (2014) elaborated on these four categories of change by defining eight more specific descriptions of change that fit within the framework. This dissertation focused on the first two change categories that I discuss here, disseminating curricula and pedagogy and developing reflective teachers, and their associated change elaborations.

**Disseminating curricula and pedagogy.** First, one way dissemination of curricula and policy occurs is diffusion. The underlying logic of diffusion is “instruction will be
changed by altering the behavior of a large number of individual instructors. The greatest
influences for changing instructor behavior lie in optimizing characteristics of the innovation
and exploiting the characteristics of individuals and their networks” (Borrego & Henderson,
2014, p. 228). Diffusion innovations are designed in one location, then used by others during
a multi-stage adoption process.

A second change elaboration used to disseminate curricula and pedagogy is
implementation. Implementation is a “set of purposeful activities [that] are designed to put
proven innovations into practice in a new setting” (Borrego & Henderson, 2014, p. 226). The
underlying logic is “STEM undergraduate instruction will be changed by developing
research-based instructional ‘best practices’ and training instructors to use them. Instructors
must use these practices with fidelity to the established standard” (Borrego & Henderson,

Professional development is a specific strategy that change agents can use to foster
instructional change that disseminate curricula and/or pedagogical techniques. First,
professional development can help build faculty’s knowledge for teaching (E. Johnson, 2013;
E. Johnson & Larsen, 2012; Speer & Wagner, 2009; Wagner et al., 2007). Second,
professional development efforts can focus on spreading awareness of student-centered
instruction resources to faculty (Dancy & Henderson, 2010; Hayward et al., 2015).
Additionally, professional development can aid in sustaining instructional change by keeping
faculty in the instructional change process longer (Henderson et al., 2012).

From K-12 literature, enacting reform instruction requires access to expertise and
ongoing collegial support for instructors (Coburn, Russell, Kaufman, & Stein, 2012).
Impactful professional development programs and supports need to be ongoing and sustained
(Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Gallucci, 2008; Garet, Porter, Desimone, Birman, & Yoon, 2001; Hill, 2007; Kazemi & Hubbard, 2008), be focused on subject matter, in terms of the mathematics, the students’ mathematical thinking, and the instructional goals (Hill, 2007; Kazemi & Franke, 2004; Kazemi & Hubbard, 2008), be integrated into teacher’s daily work (Darling-Hammond & McLaughlin, 1995; Franke, Kazemi, & Battey, 2007; Putnam & Borko, 2000) and provide teachers with feedback (Elmore, 2002).

Workshops are one form of professional development. Workshops vary widely in terms of their goals, executions, and results (e.g., Hayward et al., 2015; Henderson et al., 2011) and typically focus on helping faculty engage students in mathematical thinking through either watching videos of classrooms, reading and discussing research articles, listening to plenary talks, participating in panel discussions, or developing materials (Hayward et al., 2015; Hayward & Laursen, 2016; Kogan & Laursen, 2012; Laursen, 2016; Walczyk et al., 2007). Professional development that is long-term and ongoing has always shown to be more effective than workshops that only last a few days (Gormally, Evans, & Brickman, 2014; Hayward et al., 2015; Henderson et al., 2011, 2012; Wieman, Deslauriers, & Gilley, 2013).

There are other tactics for facilitating instructional change beyond professional development that align with dissemination. It is important for change agents to acknowledge barriers that are present (Henderson & Dancy, 2007; Turpen et al., 2016) and be aware of the initial political climate (Laursen, 2016). It is equally important for change agents, when entering an environment that is oftentimes not one they are a part of, to not “lecture-shame” (Turpen et al., 2016) as that can be an immediate deterrent to faculty. Faculty are receptive if
they know the innovation or change initiative works (Walczyk et al., 2007) so it is the job of the change agent to effectively relay that information during the diffusion and implementation process.

Environmental barriers are the most often cited reason for not using student-centered instruction (e.g., Henderson & Dancy, 2007). Some environmental barriers are difficult to alter (e.g., room layout or class size). However, change agents are able to improve faculty perceptions about time and coverage of material. For example, when disseminating curricula or pedagogy change agents should articulate potential problems, reasonable expectations, and essential features of RBIS (Henderson et al., 2012) so faculty are aware of what to expect during instructional change.

Instructional change is most successful when it aligns with faculty’s instructional beliefs (Nadelson et al., 2013). As many faculty do not consider instruction to be part of their professional identity (Brownell & Tanner, 2012), change agents need to first focus on changing that mindset (Henderson et al., 2011; Hug, Thiry, & Gates, 2015; Rattan, Good, & Dweck, 2012); this requires them to be aware of initial cultural and political climates (Laursen, 2016). One strategy to gauge the initial climate that change agents can use when they begin new change initiatives is social networking analysis (Hug et al., 2015; Quardokus & Henderson, 2015). For instance, using social networking analysis (SNA) would allow change agents to know who the key players in the department are that would be most beneficial to work with on change initiatives.

**Developing reflective faculty.** Borrego and Henderson (2014) provide two strategies that change agents can use to develop reflective faculty: scholarly teaching and faculty learning communities (Borrego & Henderson, 2014). Scholarly teaching is when “individual
faculty reflect critically on their teaching in an effort to improve” and faculty learning communities are when a group of faculty come together and “support each other in improving teaching” (Borrego & Henderson, 2014, p. 227).

Scholarly teaching oftentimes is characterized by the use of data or research to influence instructional decisions (Borrego & Henderson, 2014). To be a reflective scholarly teacher, faculty must first acknowledge dissatisfaction with student outcomes, for example, in their classroom. At that point, faculty can research within their own classroom ways to self-improve to ultimately improve classroom outcomes. Then, through this adoption of scholarly teaching, faculty will be more prepared to deal with student-buy in to student-centered instruction (DeLong & Winter, 1998; A. Johnson et al., 2009).

Faculty learning communities (collaboration) provide a place for shared reflection, new learning, and opportunities to negotiate new identities (Kensington-Miller et al., 2014; Nadelson et al., 2013). Furthermore, through the creation of faculty collaboration change agents can foster supportive departments (Kensington-Miller et al., 2014; Nadelson et al., 2013), which is crucial if departmental barriers, such as norms, are to be overcome (Henderson & Dancy, 2007; McDuffie & Graeber, 2003). According to Coburn et al. (2012) in order to create sustainable change in instructional reform, it is important to develop an ongoing social network. It is important for that professional development setting to show instructors they have supports at-the-ready (Borko, Jacobs, Eiteljorg, & Pittman, 2008).

In summary, these two strategies work together to improve instruction in faculty learning communities or faculty collaborations is to reflect on instruction with peers (i.e., scholarly teaching). Change initiatives need to move from being focused on individual faculty towards collective groups and from prescribed change to emergent change (Borrego
& Henderson, 2014; Henderson et al., 2011). Yet, before such change initiatives can happen we must know how individuals participate in these faculty learning communities and how best to support their emergent change.

One example of a faculty collaboration is modified Japanese lesson study. A Japanese lesson study is where faculty design, teach, observe, analyze, and revise lessons in collaborative settings (Demir et al., 2013). This typically can be done within departments; however, with new video chat features (e.g., Skype, Google Hangouts), collaboration can also take place online (Kuster et al., 2016). These communities of faculty collaboration all can work together to facilitate and sustain instructional change through the development of faculty who are reflective about their instruction. The chosen case of this dissertation focused on one specific faculty collaboration that is geared towards facilitating and sustaining instructional change. One particular strategy, is the development of reflective teaching through the use of sharing instructional video, used in this study.

**Relationship between PD and Instructional Practice**

In this section I elaborate on professional development’s impact on instructional practice. As mentioned, professional developments are successful tactics for supporting instructional change. The larger research project where this study’s data came from is a form of a professional development (i.e., a faculty collaboration). Characteristics of successful professional developments have been discussed, however, further discussion is necessary on what is known about the impact of professional development on instructional practice. Most literature of this kind is from K-12 settings, but implications are still important for the undergraduate arena.

In a large longitudinal study Desimone, Porter, Garet, Yoon, and Birman (2002)
explored the features of professional developments and which ones led to changing instructional practices. They found professional development that is focused on specific instructional practices increases teachers’ use of said instructional practices in their classroom. Additionally, their results indicate that active learning environments have potential to increase how and to what degree professional development impacts instruction. Similarly, other research has shown the most reported impact on practice from professional development, apart from knowledge, is the “extent to which individual programs provide many opportunities for active learning and reflection on practice” (Ingvarson, Meiers, & Beavis, 2005, p. 14). Further, a strong professional community was found to be a mediating variable between knowledge acquisition and change in practice (Ingvarson et al., 2005). Lastly, continuous feedback on instruction was shown to effect instructional practice (Ingvarson et al., 2005). How instruction changed or developed in each of these studies and the studies they synthesized was operationalized in very different ways. The ways in which instructional practice will be operationalized in this study is heavily tied to the theory driving the study and the specific form of instructional change being pursued, that is, inquiry oriented instruction.

**Inquiry Oriented Mathematics**

In the aforementioned research, instructional change is operationalized in varying ways. In this study, a particular instructional strategy was the foci. The faculty collaboration in this study centered on inquiry oriented mathematics. Here, I elaborate on the constructs of inquiry oriented mathematics, inquiry oriented differential equations, and inquiry oriented instruction.

**Inquiry oriented mathematics.** Rasmussen and Kwon (2007) elaborate on the
common understanding of *inquiry* in undergraduate mathematics by defining it as learning to communicate mathematically via discussion and conjecturing, by noting that inquiry is more than just a process, and by noting that the teacher plays a pivotal role in the development of the shared understanding of mathematics. They define inquiry oriented (IO) learning as teaching where students are inquiring into the mathematics, while the teachers are inquiring into the students’ individual and collective mathematical thinking. As stated, IO has its foundational tenets in Realistic Mathematics Education (RME) (Freudenthal, 1991; Gravemeijer & Doorman, 1999) and the emergent perspective (Cobb & Yackel, 1996).

**Inquiry oriented differential equations.** IO curricula have been fully developed for three content areas: differential equations (Rasmussen et al., 2018), linear algebra (Zandieh, Rasmussen, et al., 2016), and abstract algebra (Larsen & Johnson, 2016). Studies have been conducted using these curricula and examining their efficacy in differential equations (e.g., Kwon, Rasmussen, & Allen, 2005; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006), linear algebra (Wawro, 2015; e.g., Wawro, Rasmussen, Zandieh, Sweeney, & Larson, 2012; Zandieh, Wawro, & Rasmussen, 2016) and abstract algebra (E. Johnson, 2013; e.g., E. Johnson et al., 2013; E. Johnson & Larsen, 2012). For this dissertation, all work specifically focused on inquiry oriented differential equations (IODE).

**IODE curriculum.** The early phases of IODE stemmed from Blanchard’s (1994) early work on DE courses (Rasmussen & Kwon, 2007). The IODE curriculum invite students to engage in challenging problems that provide an opportunity for them to create their own analytical, graphical, and numerical approaches. As mentioned, the IODE materials are adopted from the instructional design theory of RME (Freudenthal, 1991) and rooted in the emergent perspective (Cobb & Yackel, 1996). Thus, each of the tasks that make up units are
designed to *experientially real* for the students (Gravemeijer & Doorman, 1999; Rasmussen & King, 2000; Rasmussen & Kwon, 2007), meaning students can utilize their existing ways of reasoning to make progress through tasks.

Studies have been conducted on the effectiveness of the IODE curriculum related to student understanding (Kwon et al., 2005; Rasmussen et al., 2006). Rasmussen et al. (2006) compared students in traditional differential equations courses to students in an IODE course. Their assessment contained routine procedural problems and more conceptual problems. There was no difference between the two groups on the routine problems; however, IODE students significantly outperformed the traditional group on conceptual problems. Further, Kwon et al. (2005) conducted a follow up study on the retention of the concepts from a subset of both groups of students from Rasmussen et al. (2006). Unsurprisingly, they found no significant difference in retention between the two groups on the procedural oriented items. However, in retention of conceptual knowledge, a significant positive difference was shown in favor of the IODE students compared to students in the traditional counterpart (Kwon et al., 2005).

**Inquiry oriented instruction.** In inquiry oriented mathematics, it is clear that the role the teacher plays is important for advancing the mathematical agenda (Rasmussen, Zandieh, King, & Teppo, 2005). Most recently, researchers from the large project where the data from this dissertation came from have begun to define IO instruction (IOI). That is, they have defined focal components of IO instruction and local practices of IO instruction rooted in extant literature from both undergraduate mathematics education and K-12 research as well. I first describe the focal components then the local practices. Each aspect was used in this study. Further, I discuss the relationship between focal components and local practices of IO
Kuster et al. (2017) define the four **focal components of inquiry oriented instruction** as generating student ways of reasoning, building on student contributions, developing a shared understanding, and connecting to standard mathematical language and notation (figure 3). The components of instruction are broad terms of what instructors do in an IO classroom. It must be noted that what is *standard language and notation* in differential equations may be very different in, for example, abstract algebra. Further, it is important to note that the four focal components very rarely occur independently; oftentimes, these components overlap and occur in the complexities of an IO classroom. Additionally, the components are arranged in such a way in figure 3 as to not imply any hierarchy and further signify the complexities of IO instruction.

<table>
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<tr>
<th><strong>Generating Student Ways of Reasoning</strong></th>
<th><strong>Developing a Shared Understanding</strong></th>
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<tbody>
<tr>
<td>Facilitating student engagement in meaningful tasks and mathematical activity related to an important mathematical point, eliciting student reasoning and contributions, actively inquiring into student thinking</td>
<td>Engaging students in one another’s thinking and bringing the class to a common understanding of the mathematical content</td>
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<th><strong>Building on Student Contributions</strong></th>
<th><strong>Connecting to Standard Mathematical Language and Notation</strong></th>
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<tr>
<td>Being responsive to student contributions and use student contributions to inform the lesson, guiding and managing the development of the mathematical agenda</td>
<td>Teachers introducing language and notation when appropriate, teachers supporting formalizing of student ideas/contributions</td>
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*Figure 3. Focal instructional components (Kuster et al., 2017).*

The **local practices of inquiry oriented instruction** are an elaboration on the four focal components of inquiry oriented instruction (table 1). While the focal components are the ways of composing and discussing IO instruction, the local practices are specific actions that instructors do in an IO classroom. Note, the citations in the description column of Table
I are not necessarily cited in the references in this study as this table is just a replication of part of the full inquiry oriented instruction framework (see Kuster et al., 2017 for details).
Table 1. *Inquiry oriented instructional local practices (Kuster et al., 2017).*

<table>
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<tr>
<th>Practice</th>
<th>Description</th>
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<tr>
<td>1. Teachers facilitate student engagement in meaningful tasks and mathematical activity related to an important mathematical point.</td>
<td>This practice is characterized by student engagement in mathematical activity related to an important mathematical point, this activity is likely to be supported with meaningful, cognitively demanding tasks. (Jackson et al., 2013; Hiebert, 1997; Speer &amp; Wagner, 2009). However, we are not evaluating the tasks, rather the quality of the mathematical activity. Stein’s “doing mathematics” includes conjecturing, argumentation ... all of the things we call engaging in authentic mathematical activity.</td>
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<td>2. Teachers elicit student reasoning and contributions</td>
<td>Teachers prompt students to explain their reasoning and justify their solution strategies, with the focus on the reasoning the students utilized during the task as opposed to solely focusing on the procedures used. Research on instructional quality indicates that the type of contributions teachers elicit is directly related to the students’ opportunities to learn. Thus, it is important that teachers elicit thinking and reasoning that “uncover the mathematical thinking behind the answers” (Hufferd-Ackels, Fuson &amp; Sherin, 2004, p.92). Leatham et al. note that foundational to building on student thinking is the presence of student mathematical thinking that “is likely to advance students’ development of important mathematical ideas—whether the student thinking is mathematically significant” (p. 92). Thus, in order to build on student thinking, rich contributions must be elicited.</td>
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<tr>
<td>3. Teachers actively inquire into student thinking.</td>
<td>This practice means that instructors purposely and intently inquire into student thinking for the purposes of determining if and how student generated ideas can be utilized to promote a more sophisticated understanding of the mathematics. The questions asked by teachers not only direct student investigations and provide the teacher with insight into student thinking, they also help students refine and reflect on their own thought process (Borko, 2004; Hiebert &amp; Wearne, 1993; Rasmussen &amp; Kwon, 2007). By asking them questions about how they are thinking - you are supporting them in further engaging in their thinking. Teacher inquiry serves many functions and roles throughout a lesson (see Hufferd-Ackles, Fuson, &amp; Sherin, 2004; Johnson, 2013; Rasmussen &amp; Kwon, 2007). With regard to building on student contributions, teacher inquiry allows teachers to form models of student thinking and understanding, reconsider important mathematical ideas in light of those models, and formulate questions and tasks which enable the students to build on those ideas (Rasmussen &amp; Kwon, 2007).</td>
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<td>4. Teachers are responsive to student contributions, using student contributions to inform the lesson</td>
<td>Rasmussen and Marrongelle (2006) state that, “an important part of mathematics teaching is responding to student activity, listening to student activity, notating student activity, learning from student activity, and so on” (p. 414). By doing so, the teacher can generate instructional space where “the nature of student mathematical thinking might compel one to take a particular path because of the opportunity it provides at that moment to build on that thinking to further student mathematical understanding” (Leatham et al., 2015, p. 118). Rasmussen: IO informs questions and tasks that enable students to build important math ideas. When teachers are responsive to student contributions they can create new instructional space (Johnson and Larsen, 2012). In regards to this component, the instructional space is created for the purpose of developing a shared understanding within the classroom community.</td>
</tr>
<tr>
<td>5. Students are engaged in one another’s thinking or reasoning</td>
<td>Stein and colleagues (2008) provide several examples of how teachers can support students in making mathematical connections between differing student contributions and important mathematical ideas. Some of these examples include asking students to reflect on the contributions of other students, assisting students in drawing connections between the mathematics present in solution strategies and the various representations that may be utilized, and facilitate mathematical discussions about different student approaches for solving a particular problem. Doing so can prompt students to reflect on other students’ ideas while evaluating and revising their own (Brendehur &amp; Frykholm, 2000; Engle &amp; Conant, 2002). By engaging with one another’s thinking, students are able to deepen their thinking, generate new ideas, and make mathematical connections. As discussed by Jackson et al. (2013), “the teacher plays a crucial role in mediating the communication between students to help them understand each other’s explanations” (p. 648).</td>
</tr>
<tr>
<td>6. Teachers guide and manage the development of the mathematical agenda</td>
<td>Teacher need to actively guide and manage the mathematical agenda and can do so by: identifying and sequencing student solutions to “ensure that the discussion advances his or her instructional agenda” (Jackson et al., 2013, p. 648); utilizing Pedagogical Content Tools “to connect to student thinking while moving the mathematical agenda forward” (Rasmussen &amp; Marrongelle, 2006, p. 389); or by refocusing the class towards the use of certain student generated ideas, marking important student contributions, and assigning tasks meant to clarify and build on students’ ideas/questions. In these ways, teachers can guide and manage the development of the lesson.</td>
</tr>
</tbody>
</table>
while building on student contributions, developing mathematical ideas in directions commensurate with the mathematical agenda, and maintaining the student ownership of the mathematics.

<table>
<thead>
<tr>
<th>7. Teachers introduce language and notation when appropriate and support formalizing of student ideas/contributions.</th>
<th>Formal notation is introduced after the students have generated an understanding of what is being notated and a need for it has been established. “In contrast to more traditional teaching in which formal or conventional terminology is often the starting place for students’ mathematical work, this teacher [one implementing an inquiry oriented curriculum] chose to introduce the formal mathematical language only after the underlying idea had essentially been reinvented by the students” (Rasmussen, Zandieh, Wawro, 2009, p. 203).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical language and notation does not only serve as a way to mark important mathematical ideas. In an inquiry oriented classroom, the introduction of language and notation also serves as a way for teachers to support the use of a common way of thinking about a mathematical idea, which then serves a launching point for the construction of more formal mathematics. Rasmussen and Marrongelle (2006) indicate that inquiry oriented instructors often notate student thinking in ways that allow the class to then solve new problems.</td>
<td></td>
</tr>
<tr>
<td>In inquiry oriented instruction, as the students reinvent the mathematics, their reinventions build to be commensurate with formal mathematical ideas. The instructor must be able to promote the students’ ability to connect their mathematical ideas to more formal mathematics. “The teacher plays a crucial role … in supporting students to link student-generated solution methods to disciplinary methods and important mathematical ideas” (Jackson et al., 2013, p. 648).</td>
<td></td>
</tr>
</tbody>
</table>
In this dissertation, I used both the focal components and local practices (in separate coding methods, to be discussed in Chapter 3). However, there is a clear relationship between the focal components and local practices. As noted, an IO classroom is very complex, thus the relationship between the focal components and local practices is just that, a relationship, not a function. There are many overlapping features with the four focal components and seven local practices but according to the design of the framework, local practices can fall under different focal component categories; it just depends on the context. Table 2 includes the relationship; note that local practices appear under multiple focal components.

Table 2. Relationship between focal components and local practices.

<table>
<thead>
<tr>
<th>Focal Components</th>
<th>Local Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generating student ways of reasoning</td>
<td>1. Teachers facilitate student engagement in meaningful tasks and mathematical activity related to an important mathematical point</td>
</tr>
<tr>
<td></td>
<td>2. Teachers elicit student reasoning and contribution</td>
</tr>
<tr>
<td></td>
<td>3. Teachers actively inquire into student thinking</td>
</tr>
<tr>
<td>Building on student contributions</td>
<td>2. Teachers elicit student reasoning and contribution</td>
</tr>
<tr>
<td></td>
<td>3. Teachers actively inquire into student thinking</td>
</tr>
<tr>
<td></td>
<td>4. Teachers are responsive to student contributions, using student contributions to inform the lesson</td>
</tr>
<tr>
<td></td>
<td>6. Teachers guide and manage the development of the mathematical agenda</td>
</tr>
<tr>
<td>Developing a shared understanding</td>
<td>4. Teachers are responsive to student contributions, using student contributions to inform the lesson</td>
</tr>
<tr>
<td></td>
<td>5. Students are engaged in one another’s thinking or reasoning</td>
</tr>
<tr>
<td></td>
<td>7. Teachers introduce language and notation when appropriate and support formalizing of student ideas/contributions</td>
</tr>
<tr>
<td>Connecting to standard mathematical language and notation</td>
<td>7. Teachers introduce language and notation when appropriate and support formalizing of student ideas/contributions</td>
</tr>
</tbody>
</table>

For example, an instructor can actively inquire into student thinking (practice 2) for the purposes of generating student ways of reasoning or building on previous student
contributions. Additionally, an instructor may introduce appropriate language and notation (practice 7) as a means of developing a **shared** understanding or as a means to formalize mathematics by **connecting** to standard disciplinary language.

It is worth noting that the components and practices each serve a unique purpose. The focal components provide a broad view of how to discuss inquiry oriented instruction, for example in a faculty collaboration setting geared towards improving inquiry oriented instructional strategies; while the local practices provide rigorous detail to describe inquiry oriented instruction in action in an undergraduate mathematics classroom. A faculty collaboration and an undergraduate mathematics classroom are the setting of this study. A participant was chosen to explore the relationship between his participation in the inquiry oriented differential equations faculty collaboration and his unfolding and ever complex inquiry oriented instructional practice.
Chapter 3: Methods

This study analyzed the experiences of one mathematician who participated in a faculty collaboration for undergraduate mathematics instructors. The development of his instructional practice and participation in the faculty collaboration were explored as well as their relationship to one another. In terms of instructional practice, I conceptualized it to be composed of the local instructional practices of inquiry oriented instruction as described above. By this I mean that examining the development of his instructional practice is exploring the execution of the specific instructional local practices and associated focal components (Kuster et al., 2017). Oftentimes in K-12 literature, instructional practice may include the choice of materials, but in this study the mathematician implemented all of the IODE materials. What was studied then, was his ability to implement said materials.

Case study methods were used to provide an in-depth analysis of the case. The study addresses the following overarching research question with additional research sub questions:

1. In what ways does one mathematician’s experiences in a faculty collaboration on inquiry oriented differential equations connect to and affect his instructional practice?
   
   a. How does his instructional practice unfold over his first implementation of inquiry oriented differential equations and in what ways does it align with inquiry oriented instruction?

   b. How does his participation unfold in the online faculty collaboration?

In this chapter I describe aspects of this study’s design, data collection, and analysis methods. Additionally, I address the reliability and trustworthiness procedures pertinent to the study.
Study Design and Conceptual Framework

This study focused on one exemplar participant from an inquiry oriented differential equations online faculty collaboration. This study was bounded by the participant’s participation in the faculty collaboration and his classroom teaching and explored the data with descriptive analyses and thus aligned with the qualitative analysis method of case study (Yin, 2013), more specifically an instrumental case study (Stake, 1995). This means that this specific case is not the sole interest (i.e., as in intrinsic case study), however, this case was chosen to describe aspects of this scenario that can add to undergraduate mathematics education knowledge base. Here the case is not the participant as an individual but rather his collaborative experience in the faculty collaboration and implementation of the IODE materials.

Figure 4 shows the conceptual framework for this study. I used this framework to conceptualize connections between the many components of this study into one cohesive vision. Recall Realistic Mathematics Education guided the mathematical inquiry of this study. Following the figure, I describe the specifics of the conceptual framework and also how the theory relates to the conceptual framework.
Realistic Mathematics Education (Freudenthal, 1991) influenced the construct of inquiry oriented mathematics (Rasmussen & Kwon, 2007), which influenced the inquiry oriented instruction framework (Kuster et al., 2017). This study is part of a larger study (i.e., a professional development) that supports the implementation of inquiry oriented mathematics (Rasmussen & Kwon, 2007), particularly differential equations (Rasmussen et al., 2018; Rasmussen & King, 2000) in the classroom. One support structure from this professional development is an online faculty collaboration. This online faculty collaboration’s goal was to support the implementation of inquiry oriented differential equations in the classroom.

This study aims to describe the participant’s inquiry oriented instructional practice (research question 1a, dark grey box) and participation in an online faculty collaboration (research question 1b, light grey box) over one academic semester. The overall goal of the study, visualized by the dotted line, is to describe the relationship, if one exists, between the participant’s experiences in the faculty collaboration and its affect or impact on his
instructional practice (overall research question).

**Context**

Teaching Inquiry-oriented Mathematics: Establishing Supports, commonly and hereafter called TIMES, is an NSF funded (DUE #143195, DUE #1431641, DUE #1431393) project. TIMES supports the development and refinement of a set of instructional supports to aid university mathematics faculty in shifting their practice towards an IO practice. The main research goal of TIMES is to study how to support undergraduate instructors as they implement changes in their instruction. Additional goals of TIMES are to understand how best to support undergraduate mathematics instructors in effectively implementing inquiry oriented instruction, understand the relationships and interactions between instructional supports, instructors, and instructional practices, characterize and measure inquiry oriented instruction, and assess student learning in inquiry oriented instructional settings.

TIMES conducted pilot studies in Spring 2015, and collected data from participants in Fall 2015, Fall 2016, and Fall 2017. To gather participants for TIMES the lead researchers sent out emails to listservs across the country and to fellow researchers and representatives at colleges and universities. Specific groups such as Project NExT from the MAA and the Special Interest Group of the MAA on Inquiry Based Learning were targeted. We sought participants with a desire to improve their teaching, use IO materials, enhance their ability to expand student thinking in their classroom, and be part of an online faculty collaboration. TIMES focuses on three content areas: differential equations, linear algebra, abstract algebra. There were 18 participants in Fall 2015, 23 participants in Fall 2016, and 6 participants in Fall 2017. This study focused on a participant from Fall 2015.

The following subsections elaborate on the three aspects of support offered by
TIMES: the IODE materials, a summer workshop, and weekly online faculty collaboration meetings. The mathematician in the study thus participated in all of these aspects.

**Materials.** Inquiry oriented differential equations (IODE) materials were developed by Rasmussen in the early 2000’s under NSF CAREER #9875388 and have been modified by him and Keene over the last decade. The materials are open source materials that are available on a website ([https://iode.wordpress.ncsu.edu/](https://iode.wordpress.ncsu.edu/)) for any instructor to download and use in his or her class (Rasmussen et al., 2018). TIMES also has materials for IOLA, developed under NSF #0634074 and #0634099 by Zandieh & Rasmussen and under NSF #1245673, #1245796, and #1246083 by Wawro, Zandieh, & Rasmussen, and IOAA, developed under NSF #0737299 by Larsen, but they are not the focus of this study.

The IODE curricula invite students to engage in challenging problems that provide an opportunity for them to create their own analytical, graphical, and numerical approaches to solving differential equations. These problems are sets of tasks that make up particular units. There are currently 14 units related to three introductory differential equations topics: (1) solving first order differential equations analytically, graphically, and numerically (2) linear systems of differential equations, and (3) second order differential equations (Rasmussen et al., 2018). They are different than traditional DE materials because they focus on dynamic approaches to solving DEs and ascribe to using multiple approaches, whereas traditional DE classes tend to focus on analytical solutions.

**Example from IODE materials.** I briefly include examples from the first day of the IODE course to give the reader a glimpse into the IODE curriculum. Before defining what a differential equation is, students solve the following task (figure 5).
Bees and Flowers

Often scientists use rate of change equations in their study of population growth for one or more species. In this problem we study systems of rate of change equations designed to inform us about the future populations for two species that are either competitive (that is, both species are harmed by interaction) or cooperative (that is, both species benefit from interaction).

1. Which system of rate of change equations below describes a situation where the two species compete and which system describes cooperative species? Explain your reasoning.

(i) \[ \frac{dx}{dt} = -5x + 2xy \]
\[ \frac{dy}{dt} = -4y + 3xy \]

(ii) \[ \frac{dx}{dt} = 4x - 2xy \]
\[ \frac{dy}{dt} = 2y - xy \]

Figure 5. Unit 1 / Task 1 of IODE materials.

In this task, students ascertain which system is cooperative and which is competitive. Here, system (i) is cooperative as both species have positive rates of change for the interaction of the two species. Following this, students focus on scenarios with only one species (figure 6), and then in small groups and as a whole class they conjecture what that DE would look like (figure 7). In this example, the DE is autonomous (does not explicitly depend on time) and thus only depends on the population. During this conjecturing process, the instructor can generate student ways of reasoning, build on student contributions, and begin to develop a shared understanding. After this point is when a DE is formally defined, thus the students’ mathematics is then connected to standard language.
A Simplified Situation

The previous problem dealt with a complex situation with two interacting species. To develop the ideas and tools that we will need to further analyze complex situations like these, we will simplify the situation by making the following assumptions:

- There is only one species (e.g., fish)
- The species has been in its habitat (e.g., a lake) for some time prior to what we call $t = 0$
- The species has access to unlimited resources (e.g., food, space, water)
- The species reproduces continuously

2. Given these assumptions for a certain lake containing fish, sketch three possible population versus time graphs: one starting at $P = 10$, one starting at $P = 20$, and the third starting at $P = 30$.

(a) For your graph starting with $P = 10$, how does the slope vary as time increases? Explain.

(b) For a set $P$ value, say $P = 30$, how do the slopes vary across the three graphs you drew?

3. This situation can also be modeled with a rate of change equation, $\frac{dP}{dt} = \text{something}$. What should the “something” be? Should the rate of change be stated in terms of just $P$, just $t$, or both $P$ and $t$? Make a conjecture about the right hand side of the rate of change equation and provide reasons for your conjecture.

Figure 6. Unit 1 / Task 2 of IODE materials.

Figure 7. Unit 1 / Task 3 of IODE materials.

Summer workshop. The TIMES project team held a three 3-day workshop in the summer before Fall 2015 when this study took place. In this workshop, many logistical considerations the participants would have to consider were discussed, as well as,
introductions to the team and their fellow participants. Additionally, participants began to familiarize themselves with the IO materials (differential equations, linear algebra, or abstract algebra) they would be using in their class. The four focal components of IO instruction, previously discussed, were the focus of many whole group discussions at the summer workshop. Video was used a means to generate discussion on when the participants saw the instructor in the video exuding the focal components of IO instruction. Each content area (IODE, IOLA, and IOAA) had breakout groups and in each breakout group, focus was given to the mathematics of the materials. In IODE participants went through one of the units’ tasks and followed that up with a discussion on the potential student thinking that could arise from that unit. The participant in this study attended the summer workshop as his introduction into the project and IO instruction.

**Online faculty collaborations.** The third component of TIMES was a weekly online faculty collaboration meeting, hereafter referred to as online faculty collaboration or OFC. The participants from each content area met weekly during the semester they were teaching IO materials, virtually via Google Hangouts, because participants were located across the country. They met to participate in modified Japanese lesson studies (Demir et al., 2013; Lewis, Perry, & Hurd, 2009) led by the respective principal investigator or other facilitator. The main goals of the OFC were to: 1) aid faculty in making sense of the instructional IO materials 2) thinking through the sequences of tasks; how students might approach the tasks; how to structure instruction around the tasks to support student learning, 3) assist faculty in developing and enhancing their instructional practice, and 4) provide a general supportive and safe community for faculty. Again, in line with suggestions from Desimone et al. (2002) focus is on specific instructional practices; in this case, the four focal instructional
components.

Each OFC was one hour long. The general structure for each OFC was as follows. For approximately 20 minutes, the facilitator asked participants to share general info on how their week has been going, how classes are going, or if they had pertinent issues or concerns that they wanted to discuss. This open general share out became very important to the participants, in particular, in relation to how it helped build a supportive community. For the remaining 40 minutes, the OFC engaged in the lesson studies (Demir et al., 2013; Lewis et al., 2009). The weekly objective of the lesson study varied. Typically, for the first week of a lesson study participants would work together and actually do the math from a chosen unit of the materials. In this instance, they were asked to do the math as themselves, not “pretend to be students” as our previous research has shown it is important for them to ground their discussion in their own understanding of the mathematics as the IO representation of the mathematics may be different than the representations they have seen in their careers (Andrews-Larson, Peterson, & Keller, 2016). The next week participants would go further and discuss potential student thinking that may arise from that unit and what instructional strategies they anticipate using. Noteworthy here is that oftentimes, separating the mathematics from the pedagogy was not possible so weeks 1 and 2 often merged in practice. Following this, the OFC spent 2-3 weeks watching video of the instructors teaching the unit of focus of the lesson study. Each participant would film their unit (more details discussed in the data collection section) and then upload those videos for the OFC to have access. The participants self-selected their own clips and led a discussion around their videos. The facilitator prompted participants to focus on the focal instructional components or areas in which they wished to improve their instruction or highlight an exciting classroom interaction.
It was important to the OFC that participants self-selected their own clips (cf. Sherin & van Es, 2009; van Es, Tunney, Goldsmith, & Seago, 2014) as it became a reflective practice to look at their instruction (Nadelson et al., 2013). This lesson study structure was implemented two times over the course of the semester (i.e., two units had lesson studies conducted for them).

In addition to the lesson study and the inquiry oriented materials themselves, oftentimes the facilitators would bring in research or practitioner articles for participants to read and discuss or GeoGebra applets to explore. Here the OFC is not collaborating to develop a set of materials, rather they are collaborating on pedagogical considerations related to their first use of this research-based curriculum.

The focus of this study is one participant from the Fall 2015 IODE OFC. That OFC consisted of the facilitator (Dr. KK), two graduate research assistants (GRA1 and GRA2), four faculty teaching the materials for the first time (Drs. DM, AB, PR, CD), and one previous participant from the pilot who was not teaching IODE at the time (Dr. ST).

**Participant**

One participant, Dr. DM, from the IODE cohort was chosen to be the participant of this study. The sampling of Dr. DM was purposeful in nature (Creswell, 2013). “This means that the inquirer selects individuals and sites for study because they can purposefully inform an understanding of the research problem and central phenomenon of the study” (Creswell, 2013, p. 156). There were several reasons for the choice of Dr. DM. First, he was and is passionate about his participation in the project and to this day continues with inquiry oriented instruction in his differential equations classroom. Second, he became a facilitator for the project in future semesters following his participant experience. Additionally, joining
the project as a facilitator shows his continued commitment to inquiry oriented instruction and instructional change. Furthermore, Dr. DM filmed every class of the semester, which was more than was expected of the other TIMES participants, affording a plethora of possible data sources and a semester-long look at instruction.

At the time of data collection, Dr. DM was an Assistant Professor of Mathematics at a public student-centered teaching and learning institution located on the east coast. Recent data shows enrollment of around 5500 students, with the university offering 44 undergraduate degrees, 17 graduate degrees, and 1 doctoral degree. Prior to joining the project, Dr. DM had used Inquiry-based learning ([IBL], e.g., Laursen, 2016; Laursen et al., 2014), another student-centered form of instruction, in his classroom, such as flipped classrooms. Dr. DM’s research interests center around dynamical systems of differential equations. However, prior to joining the project he started to become more interested in the teaching and student learning aspect of his job. Dr. DM attended TIMES’s summer workshop in 2015 and participated in an OFC during the Fall 2015 semester. As noted, Dr. DM became very interested in inquiry oriented instruction and transitioned to a facilitator for the project. Thus, in Fall 2016 and Fall 2017 he ran his own OFC. Additionally, he taught inquiry oriented differential equations again in Fall 2016, Spring 2017, Fall 2017, and Spring 2018. However, his participation in Fall 2015 is the focus of this dissertation.

Data Collection

In this section, I present the data collection methods in a table in chronological order. Along with each component will be what research question it will aid in answering and a brief statement of the purpose of the data. Following Table 3 I provide expanded details on how each of the data components was collected.
Table 3. Summary of data collection components.

<table>
<thead>
<tr>
<th>When</th>
<th>Data Collection Component</th>
<th>Purpose</th>
<th>Research Question</th>
<th>Amount of Data</th>
</tr>
</thead>
</table>
| Early Fall 2015 | Audio recording of entrance interview with Dr. DM | -Determine Dr. DM’s expectations for TIMES  
- Determine his teaching beliefs  
- Determine his mathematical beliefs | 1a, 1b | -Approx. 45-minute interview. -Transcribed. |
| Fall 2015 | Video recordings of select IODE classes Dr. DM taught | -Describe the development of Dr. DM’s instructional practice  
- Explore Dr. DM’s instructional practice alignment with IOI | 1a | -Unit 1-2, Unit 6, Unit 9, Unit 12.  
- Units 6 and 9 were the lesson studies from the OFC.  
- Totaling approx. 9 hours of class video, but closer to 7-8 hours will be analyzed (the remaining is the small group work that is not audible). |
| Fall 2015 | Video recordings of a weekly OFC | -Explore participation in the faculty collaboration | 1b | -9 1-hour OFCs.  
- Totaling approx. 9 hours of video (1 hour per meeting). -Transcribed. |
| Late Fall 2015 | Audio recording of exit interview with Dr. DM | -Capture important contextual features, from his point of view, about TIMES and his overall experiences | 1a, 1b | -Approx. 45-minute interview. -Transcribed. |

Classroom data. Video data from Dr. DM’s classroom was collected in Fall 2015.

As part of being a TIMES Fellow, Dr. DM was asked to film the two units of focus for the lesson studies in the OFC; however, Dr. DM opted to film all days of his class. Note, the first two days are missing because IRB approval had not been obtained. To capture the video, his undergraduate research assistant (URA) positioned an iPad in such a way that it captured as much of the room as possible with Dr. DM being the focus. For example, when Dr. DM was
at the board the camera was zoomed in on him. Dr. DM was told to project his voice to ensure capture of audio. When Dr. DM was working with a small group of students, the camera panned to just him and the group he was working with. However, during small group work time it oftentimes becomes difficult to hear Dr. DM, and thus a majority of the classroom video was of whole class discussions facilitated by Dr. DM. However, if small group conversations with Dr. DM were audible they were included in the data. The classroom video data was very powerful in answering the overall research question and sub research question 1a, as the focus was the development of Dr. DM’s instructional practice while being part of the OFC.

Even though every day was filmed, only select units were analyzed. Classroom video data was chosen to match the units covered in the OFC lesson studies. The OFC in 2015 discussed Unit 6 (autonomous differential equations) and Unit 9 (systems of linear equations). In addition to those units, Unit 1-2 (qualitative, graphical, and numerical approaches) as an introductory unit and Unit 12 (eigentheory applied to linear systems) were analyzed. The beginning of Unit 1 was not filmed; so, what was filmed lasted only one day. For a longer unit, Unit 2 was combined with Unit 1. However, part of Unit 2 data was corrupted. Thus, Unit 1-2 consisted of 2 days of instruction; the end of Unit 1 and the beginning of Unit 2. Unit 6 lasted approximately two days, Unit 9 lasted approximately five days, and Unit 12 lasted approximately two days. Units 1, 6, 9, and 12 were chosen also so instruction from the beginning, middle, and end of the semester could be analyzed. In Chapter 4, I consider the units based on amount of total time of instruction; Unit 1-2 was roughly 87 minutes, Unit 6 was 104 minutes, Unit 9 was 244 minutes, and Unit 12 was 129 minutes. See Appendix A for more details of the chosen video clips.
**OFC data.** All OFCs occurred via Google Hangouts and were all approximately one hour long. Each OFC was screencast using software (QuickTime on iMac). I was present in all OFCs (except week 6) and I was in charge of the data collection of the OFC. All video was uploaded to a secure server. All OFC videos were transcribed so they could be analyzed in Atlas.ti. All transcriptions were analyzed for the fullest possible description of Dr. DM’s participation in the OFC (overall research question and sub research question 1b).

**Interview data.** The interview data served as a third data source to relate Dr. DM’s experiences in the faculty collaboration to his instructional practice. Furthermore, this data offered Dr. DM’s personal perspective on being part of a faculty collaboration. Entrance and exit semi-structured interviews were conducted (Appendices B-C). Both interviews were conducted by other members of the TIMES research team. The entrance interview was designed to capture expectations and baseline beliefs of instruction of the participants. The exit interview was designed to capture the holistic experience of Dr. and perspectives on his participation in TIMES. All interviews were conducted via Google Hangouts and were audio recorded. All interviews were transcribed so they could be analyzed in Atlas.ti. Each interview lasted approximately 45 minutes.

**Timeline of data collection.** For convenience of the reader, Figure 8 includes a timeline of all of the data that was collected for this analysis. The purpose of Figure 8 is to highlight an even distribution of data that was collected throughout Dr. DM’s participation in TIMES (i.e., data is not clumped in certain parts of the semester). Thus, the use of these data provides a thorough look at Dr. DM’s experiences in TIMES and how his instruction relates to his participation in the OFC. Note that Unit 1-2 Class 2 and OFC 6 data were corrupted and Dr. DM was absent from OFC8.
Data Analysis

Table 4 summarizes the data analysis procedures that were employed for this study.

Following the table are detailed elaborations on each aspect of data analysis for this study.

Table 4. Summary of data analysis methods.

<table>
<thead>
<tr>
<th>Collected Data Component</th>
<th>Research Question</th>
<th>Data Analysis / Instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early Fall 2015 Audio recording of entrance interview with Dr. DM</td>
<td>1a, 1b</td>
<td>Open code transcript</td>
</tr>
<tr>
<td>Fall 2015 Video recordings of select IODE classes Dr. DM taught</td>
<td>1a</td>
<td>Code videos using IOI framework, specifically the Local Practices (Kuster et al., 2017)</td>
</tr>
<tr>
<td>Fall 2015 Video recordings of a weekly OFC</td>
<td>1b</td>
<td>Code transcripts using production design and reception design from Krummheuer (2007, 2011) to gauge kind and level of participation - Code transcripts for conversation topic (including focal instructional components) using Keene, Fortune, &amp; Hall (under review) as a priori codes</td>
</tr>
<tr>
<td>Late Fall 2015 Audio recording of exit interview with Dr. DM</td>
<td>1a, 1b</td>
<td>Open code transcript</td>
</tr>
</tbody>
</table>

Classroom data. Recall from the discussion in Chapter 2 that the inquiry oriented instruction (IOI) framework (Kuster et al., 2017) was designed to capture inquiry oriented instruction in action. Consequently, I used the framework as an a priori analytical framework for coding Dr. DM and his classroom instructional practice and to answer research question 1a. In particular, I used the local practices of inquiry oriented instruction. In so doing, I am
able to report on how his instructional practice consisted of the local practices of inquiry oriented instruction. This approach allowed me to discuss his instruction throughout one semester and also compare it to his experiences in the OFC. A further elaboration of the Local Practices [LP] of IOI can be seen in table 5. Specifically, table 5 includes an ‘evidenced by’ column (part of the full framework) that was used to code Dr. DM’s instructional practice. The ‘evidenced by’ column became the codes that I used to code that were then collapsed to their respective LP. Thus, an overall frequency of each LP was generated for each unit (with the exception of Local Practice 1 [LP1], see after table 5).

Table 5. Elaboration on IO instruction Local Practices (Kuster et al., 2017).

<table>
<thead>
<tr>
<th>Local Practice</th>
<th>Evidenced By</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Teachers facilitate student engagement in meaningful tasks and mathematical activity related to an important mathematical point</td>
<td>• Smith and Stein (1998)</td>
</tr>
<tr>
<td></td>
<td>o Doing mathematics</td>
</tr>
<tr>
<td></td>
<td>o Doing procedures with connections</td>
</tr>
<tr>
<td></td>
<td>o Doing procedures without connections</td>
</tr>
<tr>
<td></td>
<td>o Memorizing</td>
</tr>
<tr>
<td>2. Teachers elicit student reasoning and contributions</td>
<td>• Explicitly asking for contributions</td>
</tr>
<tr>
<td></td>
<td>• Students volunteer information</td>
</tr>
<tr>
<td></td>
<td>• Students sharing their thinking/ reasoning</td>
</tr>
<tr>
<td>3. Teachers actively inquire into student thinking</td>
<td>• Asking clarifying questions</td>
</tr>
<tr>
<td></td>
<td>• Questions or requests for individuals or groups to explain their thinking or ideas</td>
</tr>
<tr>
<td></td>
<td>• Says things like “Tell me more about that.”</td>
</tr>
<tr>
<td></td>
<td>• Questions that imply the teacher is trying to build model of student understanding:</td>
</tr>
<tr>
<td></td>
<td>o “It sounds like you are saying _______, am I interpreting you correctly?”</td>
</tr>
<tr>
<td>4. Teachers are responsive to student contributions, using student contributions to inform the lesson</td>
<td>• Posing new tasks</td>
</tr>
<tr>
<td></td>
<td>• Posing questions to the whole class</td>
</tr>
<tr>
<td></td>
<td>• Shifting focus to students’ ideas</td>
</tr>
<tr>
<td></td>
<td>• Engaging with unexpected contributions</td>
</tr>
<tr>
<td></td>
<td>• Following/exploring student contributions</td>
</tr>
</tbody>
</table>
Table 5 (continued).

| 5. Students are engaged in one another’s thinking or reasoning | • Teacher asked students to reflect on the contributions of others  
| | • The teacher rephrases or revoices student contributions  
| | • Teacher encourages students to utilize and make sense of others’ thinking  
| | • Students’ ideas are made public  
| | • Students inquire about other students’ contributions  
| 6. Teachers guide and manage the development of the mathematical agenda | • Marking and recording publicly – “That’s really important”  
| | • Refocusing the students on the important ideas  
| | • Following productive (from their perspective) ideas and shifting focus away from unproductive (from their perspective) ideas  
| 7. Teachers introduce language and notation when appropriate and support formalizing of student ideas/contributions | • Students translate the language and notation  
| | • Language and notation serves to notate student ideas  
| | • Asking how to formalize  
| | • Asking how to generalize a specific case to a more general case  
| | • Transformational record  

LP1 was not coded for unique observable instances in the data. Rather, a holistic categorization of LP1 was given based on Smith and Stein’s (1998) mathematical task framework. Note, the task itself is not what is being evaluated, rather, Dr. DM’s execution of the task in relation to how his students engaged in his class (Stigler & Hiebert, 2004). In Chapter 4, I begin each unit summary with a holistic categorization of Dr. DM’s instruction based on LP1 and then include overall frequencies of LP2-7. For example, a possible characterization is procedures with connections (Smith & Stein, 1998). This indicates that “while the students were provided opportunities to do mathematics, the majority of the mathematics during the discussion was performed by the teacher; however, the teacher used the tasks to engage the students in the conceptual ideas” (Kuster et al., 2017, p. N/A). Another important note on the practices relates to LP2. I did not evaluate the richness
of student contributions anything beyond the fact that it if it was simple. For example, “2+2=—” did not get coded as LP2 but if students responded with a response that contained some level of detail on their thinking LP2 was coded (and the corresponding evidence). This practice on its own is not unique to inquiry oriented instruction. Rather this is quite typical of basic instruction (compare to I-R-E from Mehan, 1979). However, it is still an important piece to the IOI framework because of how it relates to other practices and other probes into students thinking.

Analysis logs were created as opposed to analyzing transcripts (Knoblauch, Tuma, & Schnettler, 2014). This approach also matches the approach to coding video that the larger research team had used. These analysis logs contained timestamps and instances of local practices 2-7 of inquiry oriented instruction. Table 6 is an example analysis log with the different aspects of the coding process and an example to illustrate how an analysis log may look. Each unit of instruction received a unique analysis log. A new row was generated in the analysis log when a new practice was seen.

Table 6. Example analysis log for coding classroom instruction.

<table>
<thead>
<tr>
<th>Timestamp</th>
<th>Local Practice</th>
<th>Evidenced By</th>
<th>Detailed Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>This column will include time for each specific row</td>
<td>This will include one of the eight practices from the framework</td>
<td>This will be specific details of evidence of the practice, from the framework</td>
<td>This will be overall notes describing this specific time segment</td>
</tr>
<tr>
<td>5:37</td>
<td>5. Students are engaged in one another’s thinking or reasoning</td>
<td>DM asked students to reflect on the contributions of others</td>
<td>Dr. DM specifically pointed out a strategy that Group1 used and asked Student1 to explain their strategy but then have Student2 from Group2 explain what Group1’s main point was</td>
</tr>
</tbody>
</table>

**Example of classroom instruction coding.** To identify the local practice, I would first use the codes from the evidenced by column of the framework. Those are specific things an
instructor does to achieve one of the local practices. While the authors note the list is not necessarily complete (Kuster, Rupnow, & Johnson, 2018, personal communication), I never had to add a code to the evidenced by column. The unit of analysis was something said or done by Dr. DM (or a student) that was one of the codes from the evidence by column. Specific attention was given to Dr. DM when he would do or say something. Not everything he did or said would be coded as a practice. If a code from the evidences column was seen that would be written down, with a description of the environment and discussion at that point and then reference to the local practice was given so all codes could be collapsed relative to their local practice. Figure 9 is an example of an analysis log (Unit 6).

<table>
<thead>
<tr>
<th>Timestamp</th>
<th>Local Practice</th>
<th>Detailed Notes</th>
<th>Evidenced By</th>
</tr>
</thead>
<tbody>
<tr>
<td>6:20</td>
<td>2</td>
<td>Student volunteers information on P(0)=5.1</td>
<td>Student volunteers information on their thinking</td>
</tr>
</tbody>
</table>
| 6:50      | 3              | DM asks "And that's based on what assessment?"  
Student responds with his thinking | DM asks clarifying question                       |
| 7:30      | 6              | DM says "that's interesting"                                                   | DM publically marks something as important         |
| 7:34      | 5              | DM asks "Anybody refute this?" A student doesn't refute it but he adds on to it saying it will never actually reach 8 if it starts out between 5 and 8 | DM asks students to reflect on the contributions of others |
| 8:20      | 5              | DM rephrases the word "reach" to be "equal" - "it will never equal 8"         | DM rephrases student contribution                  |
| 9:50      | 6              | DM brings back the helicopter problem and asks "Wasn't this one before able to reach 0 even though dh/dt was 0?" | DM refocuses students on important ideas           |
| 11:45     | 3              | DM asks "Oh so you're saying this kind of graph"                             | DM asks clarifying question                       |
| 12:00     | 3              | DM follows up with asking "And that's based on what?"                        | DM asks clarifying question                       |
| 12:40     | 2              | Student volunteers information on graphing dP/dt v P and see if it is undefined anywhere (i.e., uniqueness theorem) | Student volunteers information on their thinking   |
| 14:55     | 5              | Student engages in another student's thinking by saying "no you are right Allen, it is decreasing towards 8" | Student inquires about another student contribution |
| 15:30     | 6              | DM focuses students on important idea "So we had the uniqueness theorem"     | DM refocuses students on important ideas           |

Figure 9. Example of analysis log for coding classroom instruction.

After the first round of coding, I went back again and revisited analysis logs and make adjustments to the coding as necessary. In this step, I looked for emergent themes from the data. As indicated, I did not have to add codes to the IOI framework itself but I was
looking for instructional themes that were not specifically related to the IOI framework, as well as themes that may be connected to the OFC. One such theme that emerged was how Dr. DM talked about his own research in his course (i.e., on multiple occasions he discussed his own personal mathematics research and indicated his bias towards certain views of mathematics; which was important to note to relate that belief to inquiry oriented instruction).

**OFC data.** In this section, I discuss the analysis of the OFC data. To analyze Dr. DM’s participation in the OFC I coded the transcripts with specific a priori codes and frameworks: the role of the speaker (production design from Krummheuer, 2007), the role of the listeners (reception design from Krummheuer, 2011), and the conversation including the focal instructional components (Keene, Fortune, & Hall, under review). These constructs will be elaborated on within this section.

To answer research question 1b (and ultimately the overall research question), the development of Dr. DM’s participation in the OFC was explored. Similar to how previous researchers has used the emergent perspective (Cobb & Yackel, 1996) to analyze participation in classrooms (Rasmussen, Wawro, & Zandieh, 2015), I used Krummheuer’s constructs of production design (2007) and reception design (2011). These constructs allowed me to classify Dr. DM as he enacts different speaking and listening roles in the discussion. Every time Dr. DM spoke a speaker code was attributed to him. Every time the facilitator, Dr. KK, spoke a listener code was attributed to Dr. DM. The decision was made to focus on Dr. KK (for the listener code) and not the other participants, as how Dr. KK treats Dr. DM is representative of his participation in the OFC. It is representative because, as the facilitator, Dr. KK was the most involved person in the conversation. Consequently, focusing
on her ensures a broad coverage of possible listener codes for Dr. DM.

During analysis, it became clear that the code names and code definitions from Krummheuer were not exactly right for this context and this analysis. Here I first describe those frameworks but I then provide details on how the code names changed to be more representative for this specific context.

In production design, individual speakers take on various roles, which depend on the content origination and the type of statement (Krummheuer, 2007). In reception design, individual listeners take on various roles, which depend on who is the content originator and the type of statement (Krummheuer, 2011). Table 7 provides a combination of production design roles and reception design roles with descriptions. Note, descriptions are modified from Rasmussen et al. (2015) interpretations. In the table, ‘the content’ is what is being said and ‘the formulation’ is how it is being said.

Table 7. Production and reception design (Krummheuer, 2007, 2011).

<table>
<thead>
<tr>
<th>Role</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Production Design</strong></td>
<td></td>
</tr>
<tr>
<td>Author</td>
<td>Responsible for both the content and formulation of a statement (i.e., what is being said and how it is said)</td>
</tr>
<tr>
<td>Relayer</td>
<td>Not responsible for the originality of either the content nor the formulation of a statement (i.e., responsible for neither content nor form)</td>
</tr>
<tr>
<td>Ghostee</td>
<td>Takes part of the content of a previous statement and attempts to express a new idea (i.e., is responsible for content but not form)</td>
</tr>
<tr>
<td>Spokesman</td>
<td>Attempts to express the content of a previous statement in their own words (i.e., is responsible for form but not content)</td>
</tr>
<tr>
<td><strong>Reception Design</strong></td>
<td></td>
</tr>
<tr>
<td>Conversation partner</td>
<td>Listener to whom the speaker seems to allocate the subsequent talking turn</td>
</tr>
<tr>
<td>Co-hearer</td>
<td>Listeners who are also directly addressed but do not seem to be treated as the next speaker</td>
</tr>
<tr>
<td>Over-hearer</td>
<td>Listeners who seem tolerated by the speaker but do not participate in the conversation</td>
</tr>
<tr>
<td>Eavesdropper</td>
<td>Listeners deliberately excluded by the speaker from conversation</td>
</tr>
</tbody>
</table>
The first change I made was to remove the Eavesdropper code. In an online video call it is not possible for someone to be deliberately excluded from the conversation thus the code does not apply in this context. Further, the terms ghostee, spokesman, and relayer also did not match the style of faculty collaborating together. Ghostee was changed to pivoter, spokesman was changed to partaker, and relayer was changed to phatic responder. Ghostee was more applicable in mathematics settings where one could take part of a mathematical argument and change it. However, in this setting, I changed it to pivoter to indicate that the speaker pivoted the conversation to a similar conversation or within the same conversation category, yet they were not authoring new content. Similarly, spokesperson was more applicable to mathematics settings, namely, rephrasing the mathematics of what someone says. I needed a code that could be attributed to when one answers a question. Thus, spokesperson was changed to partaker as one partakes in existing conversation, not authoring new content nor pivoting to semi-related content. Lastly, I found the relayer definition to not be specific enough so I attributed it to phatic responses (responses that move the conversation forward yet without being substantive) and thus renamed it a phatic responder. Finally, I made the definitions, both speaker, and listener, specific to this context. Table 8 includes the list of codes that were used to code all speaking and listening roles in the OFC. Note, the order is not the same for the speaking roles. It has been reordered with highest participation at top and lowest participation at bottom (for both speaking and listening roles).

Table 8. Speaking and listening roles (adapted from Krummheuer, 2007, 2011).

<table>
<thead>
<tr>
<th>Role</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Speaking Roles</strong></td>
<td></td>
</tr>
<tr>
<td>Author</td>
<td>Dr. DM is responsible for both the content and formulation of a statement (i.e., what is being said and how it is said)</td>
</tr>
<tr>
<td>Pivoter</td>
<td>Dr. DM takes part of the content of a previous statement and attempts to express a new idea (i.e., the content stays the same but</td>
</tr>
</tbody>
</table>
Table 8 (continued).

<table>
<thead>
<tr>
<th>Role</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partaker</td>
<td>Partake means to join in. Thus, here Dr. DM forms ideas that are not new content but are a continuation of the previous conversation (oftentimes by answering a question). This can also be seen while Dr. DM is attempting to express the content of a previous statement in his own words (i.e., is responsible for form but not content).</td>
</tr>
<tr>
<td>Phatic Responder</td>
<td>Phatic means of, relating to, or being speech used for primarily social or emotive purposes rather than for communicating information. Possible statements are “yes” or “I agree.” Such statements move the conversation forward without being substantive.</td>
</tr>
</tbody>
</table>

**Listening Roles**

<table>
<thead>
<tr>
<th>Role</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conversation partner</td>
<td>Occurs when Dr. KK responds directly to Dr. DM or when Dr. DM is a listener to whom Dr. KK seems to allocate the subsequent talking turn.</td>
</tr>
<tr>
<td>Co-hearer</td>
<td>Occurs when Dr. DM is a listener who is not explicitly addressed by Dr. KK and chooses to participate in the subsequent conversation.</td>
</tr>
<tr>
<td>Over-hearer</td>
<td>Occurs when Dr. DM is a listener who is not explicitly addressed by Dr. KK and chooses not to participate in the subsequent conversation. It is being assumed that Dr. DM heard all things said by Dr. KK.</td>
</tr>
</tbody>
</table>

I note that in these codes that the difference between co-hearer and over-hearer is subtle. In both cases Dr. DM would not be explicitly addressed as the next speaker but the difference is if he chooses to speak then he was a co-hearer in that moment. This means that his classification as a listener is representative of what conversations he chose to include himself in. For example, if Dr. KK said, “what does everyone thing about that?” Dr. DM would not be the conversation partner as he was not explicitly addressed. If Dr. DM responded then what Dr. KK said would have been classified as “co-hearer” and if he did not it would have been classified as “over-hearer” to Dr. KK’s statement. Additionally, the assumption is being made that when coded as an over-hearer one still heard the conversation and chose not to participate (whereas it is possible that Dr. DM “zoned out” and thus did not...
participate because of that).

In addition to the role Dr. DM plays in the OFC, what is being discussed is important to know as well. Consequently, the conversation was analyzed. The coding of conversation had a priori codes from Keene et al. (under review). Part of those codes consisted of the four focal components of inquiry oriented instruction. The purpose of the OFC was to help participants in understanding inquiry oriented instruction and implementing inquiry oriented differential equations. Consequently, denoting when the focal components are discussed is imperative to explore Dr. DM’s understanding of inquiry oriented instruction. For the OFC, I used the focal components of inquiry oriented instruction as opposed to the local practices because from previous research (Keene et al., under review) we found that participants rarely referenced focal components by name. Further, this coding would not be evidence of the focal component actually happening in the classroom but more about what components were discussed in the OFC.

In previous research, the research team analyzed conversation during select OFC exclusively when participants were sharing video (Keene et al., under review). I used categories of conversation that emerged from that analysis as a priori codes so the codes come directly from the same context (see table 9). Note this previous work was influenced by research on video use in professional development K-12 settings (Sherin & van Es, 2009; van Es et al., 2014). This coding scheme served as a foundation for codes. As the scope of this study was larger than our previous work, new codes emerged (see table 10). Any conversation code with an asterisk (*) emerged from this analysis. Note some of the codes are conversation topics (e.g., mathematics, pedagogy) whereas some are things that happen in conversation (e.g., appraisal). The purpose of coding for conversation was to be able to report
what Dr. DM was discussing during the OFC.

Table 9. *A priori* conversation categories (Keene et al. (under review)).

<table>
<thead>
<tr>
<th>Codes</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedagogy</td>
<td>Teacher instructional moves or ideas about teacher instructional moves. Note oftentimes this will often be picked up with the focal instructional components thus this code will be treated as a general pedagogy code</td>
</tr>
<tr>
<td>Student Mathematical Thinking (SMT)</td>
<td>Evaluating and/or discussing students’ thinking about mathematical concepts</td>
</tr>
<tr>
<td>Mathematical Content (Math)</td>
<td>Discussion of mathematics, IODE material, difficulty of mathematics</td>
</tr>
<tr>
<td>Social and Sociomathematical Norms (Norms)</td>
<td>Discussion about getting students used to participating in group work; includes quality of group work and interaction</td>
</tr>
<tr>
<td>Environment</td>
<td>Classroom behavior, classroom setup, access to tools and technology</td>
</tr>
<tr>
<td>OFC Logistics</td>
<td>How to record videos; what clips to share/select; what to watch; remembering video</td>
</tr>
<tr>
<td>Orienting</td>
<td>Setting the stage for the video that will be shown</td>
</tr>
<tr>
<td>Appraisal</td>
<td>Participants give value judgments about events from the videos, instructional moves, not students’ behavior or mathematical thinking</td>
</tr>
</tbody>
</table>

Table 10. Conversational codes (adapted from Keene et al. (under review)).

<table>
<thead>
<tr>
<th>Code (* indicates emerged from this analysis)</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advice/Feedback*</td>
<td>Participants seek out advice/feedback or advice/feedback is given</td>
</tr>
<tr>
<td>Appraisal</td>
<td>Participants give value judgments about events from the videos, instructional moves, not students’ behavior or mathematical thinking</td>
</tr>
<tr>
<td>Community*</td>
<td>Comments about the community aspect of the OFC and its participants, including hospitality and greetings</td>
</tr>
<tr>
<td>Environment</td>
<td>Classroom behavior (not including the norm of student participation, e.g., student tiredness), classroom setup, student demographics, access to tools/technology (not in OFC but in participants’ classrooms)</td>
</tr>
<tr>
<td>Focal Instructional Components (FIC)</td>
<td>Mentioning Focal Instructional Components in general</td>
</tr>
<tr>
<td>FIC_Generating</td>
<td>Mentioning Generating student ways of reasoning (directly / indirectly)</td>
</tr>
<tr>
<td>FIC_Building</td>
<td>Mentioning Building on student thinking (directly / indirectly)</td>
</tr>
<tr>
<td>FIC_Shared</td>
<td>Mentioning Developing a shared understanding (directly / indirectly)</td>
</tr>
</tbody>
</table>
Table 10 (continued).

<table>
<thead>
<tr>
<th>FIC_Connecting</th>
<th>Mentioning Connecting to standard mathematical language and formal notation (directly / indirectly)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials*</td>
<td>Comments on the IODE materials or course materials (e.g., homework problems)</td>
</tr>
<tr>
<td>Math</td>
<td>Discussion of mathematics or doing mathematics (not coded when conversation is exclusively about students’ mathematics)</td>
</tr>
<tr>
<td>Norms</td>
<td>Discussion about getting students’ participation; includes quality of group work and interaction</td>
</tr>
<tr>
<td>Orienting</td>
<td>Setting the stage for the video that will be shown</td>
</tr>
<tr>
<td>OFC_Manage</td>
<td>Management of OFC (e.g., someone is running late, discussing Google Drive folders, going over agenda, IRB things, plan for OFC, picking units)</td>
</tr>
<tr>
<td>OFC_TechSupport</td>
<td>OFC Tech Issues or considerations (e.g., internet connectivity)</td>
</tr>
<tr>
<td>OFC_Video</td>
<td>OFC Video – how to record videos, what clips to share/select, what to watch, remembering video</td>
</tr>
<tr>
<td>Pedagogy</td>
<td>Teacher instructional moves or ideas about teacher instructional moves. Note oftentimes this will be picked up with FICs thus this code may be used in tandem with those codes</td>
</tr>
<tr>
<td>Resources*</td>
<td>Reference to specific resources used exclusively in OFC and brought in by facilitator (e.g., technology, scholarly article)</td>
</tr>
<tr>
<td>Student Mathematical Thinking_Describe</td>
<td>Discussing students’ thinking about mathematical concepts either seen in video or that happened in class or that they’ve seen before</td>
</tr>
<tr>
<td>Student Mathematical Thinking_Desired*</td>
<td>Discussing the desired student mathematical thinking (e.g., what you want your students to think about)</td>
</tr>
<tr>
<td>Student Mathematical Thinking_Evaluate*</td>
<td>Evaluating students’ thinking about mathematical concepts</td>
</tr>
<tr>
<td>Update*</td>
<td>The facilitator asks for an update in class. Participant provides response</td>
</tr>
</tbody>
</table>

Note that OFCLogistics was broken up into OFCManage, OFCTechSupport, and OFCVideo in this analysis. Further, Student Mathematical Thinking (SMT) was subdivided in this analysis to Describe, Desired, and Evaluate.

**Example of OFC coding.** In this section I include an excerpt from an OFC transcript to demonstrate how coding looked in Atlast.ti. I note two important comments about the
coding schemes. The first important piece to note here is how speaking and listening roles were coded. As mentioned when Dr. DM was speaking a speaking role was assigned to him and when Dr. KK was speaking a listening code was assigned to Dr. DM. As indicated sometimes to properly code a listening role to what Dr. KK said I had to read ahead and see how Dr. DM reacted and then retroactively apply listener codes. This is because the listener codes, particularly the difference between co-hearer and over-hearer, were influenced on how Dr. DM would respond to Dr. KK speaking. Additionally, a chunk of text received a new code when the speaker or listener role changed. Note in figure 10 how codes were grouped based on the speaker or listener (e.g., the 19:27 block has two code chunks when Dr. DM changed his speaking role, and only Dr. DM and KK have codes assigned to them).

| 19:21 | FR | Okay, look, try quickly. How are they in terms of prerequisites? Do you know? |
| 19:27 | DM | I haven’t done a close survey of them. I’ve got some issues with a couple of students who had DE with me before and failed. And so, they are very quick to remember separable equations. So, although, it’s actually, sorry I’m not answering PR’s question anymore but I have a question. Is there a place where we can record interesting student thoughts that then might end up in your teacher materials in the next iteration? |
| 20:07 | KK | That’s a great idea. |
| 20:09 | DM | Because when tasked with the question, is this a solution to the DE? This group of students did something I hadn’t really seen before. Which is, they plugged in everywhere where y was in the equation. They plugged in the function, except for the dy/dt. So, they plugged in only on the RHS. And then, integrated it, the RHS, and then recovered y=whatever it was, t=1. And then they said oh I guess c has to be 1 for this to work. And... |
| 20:53 | ST | They are integrating with respect to what? |
| 20:55 | DM | With respect to t. So, the equation was like y’=2-1 over t+1 or something like that. And so, when you check to see if it’s solution, you plug in the RHS and you check to see if the derivative matches the simplified version. So instead they simplified the RHS so they got dy/dt = 1 and then they integrate and they get okay that’s t+c, okay yes of course, this is a solution, look I can make it work. |
| 21:27 | KK | That’s funny because one of the questions on the assessment... there’s this piece of me that’s going, oh, that’s too bad. And there’s this other piece of me that’s going yes! Because that shows what we’re thinking is true. |

Figure 10. Example of OFC coding.

The second important thing to note is about the conversation. As seen here in Figure 10 it is possible for a chunk of text to have more than one conversation code. The way in which code chunks were subdivided was based on the speaking and listening role. Here, in the 19:27 block, Dr. DM completely changes the conversation to author a new code about
recording interesting student ideas. Thus, a new code was created and it got author, SMT Describe, and OFC Manage. In Atlas.ti, I used the cross tabular feature where I could find instances of speaking or listening roles and conversation topics coded at the same time. However, in this instance, author could be double counted. Table 11 is a fake table to illustrate this only including the chunk of text in the 19:27 row in Figure 9.

Table 11. *Fake cross tabular table.*

<table>
<thead>
<tr>
<th></th>
<th>Author</th>
<th>Partaker</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>OFC Manage</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>SMT Describe</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Totals</td>
<td>2</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Here, summing across the rows does give an accurate representation of the conversation discussed. However, summing down the column, actually double counted the one chunk of text coded as author. The grey cell with 2 really should be a 1. Thus, all tables in Chapter 5 do not sum down the column. Rather, I went into Atlas.ti and got the exact count manually.

**Interview data.** Transcripts of both interviews were open coded (Strauss & Corbin, 1998; Yin, 2013). As the interview data was to add Dr. DM’s perspective on his experiences, I did not want to prescribe my a priori codes on the data. Thus, open coding the data was most appropriate. The most important and noteworthy themes that emerged were Dr. DM’s teaching philosophy, beliefs about learning, and beliefs and knowledge about mathematics based on his mathematical research. The interview data was added to the data to the classroom and OFC data to describe Dr. DM’s experience as his instructional practice developed over the course of the study. The interview data will be used to contribute to answers to the overall research question.

**Merging of results.** Research question 1a and 1b are focused on each aspect of this
study; classroom instruction and faculty collaboration. The overall research question was answered by merging results and interpretations of results from research question 1a and 1b, with support from the interview data. To tie all analyses into one coherent understanding of this case, I looked for relationships (or lack thereof) between what Dr. DM is seen doing in the faculty collaboration (i.e., his participation) and his classroom instructional practice. Chapters 4 and 5 discuss answers to research questions 1a and 1b, respectively, whereas an answer to the overall research question will be discussed in Chapter 6.

### Reliability and Trustworthiness

In order to conduct a reliable and trustworthy study, multiple methods were used to show consistent findings and interpretations. Multiple sources of data were collected to achieve triangulation (OFC, classroom, interview) and reliable data (Creswell, 2013). Additionally, as I have been working with TIMES for over 3 years now, I am in a prolonged engagement with the project (Creswell, 2013) which supports trustworthiness in research. Coding of the OFC data and classroom data each received their own reliability measures.

**Classroom data.** During the analysis of the classroom video data for this study, I participated in a reliability training for using the Inquiry Oriented Instructional Framework (Kuster et al., 2017) with the rest of the coders of the project team as we coded all of our ~50 participant videos during the summer of 2017. Note, we only coded one unit of each instructor and I did not code any of Dr. DM’s videos. The training was a week-long training led by one of the developers of the IOI framework. We read literature on the development of the framework and became familiar with the framework itself. On day 1 the 6 coders and the lead developer all coded 3 different videos together (synchronously online) that had different frequencies of IOI local practices so we could see a good spread of LPs. Day 2 we coded
videos in pairs. And on days 3-5 we coded videos individually, and between each one conferred with the lead coder to ensure reliability. Once training was over, during coding, we individually met with the lead coder to ensure we stayed reliable. This coding reliability system was for a different study, but was all part of the umbrella of the TIMES project. The context was the same thus I ensured reliability.

**OFC data.** To achieve trustworthiness, I gave a colleague two OFC transcripts to code. Thus, a peer review process was conducted to ensure data was reliable and a trustworthy set of codes were generated (Creswell, 2013). After the colleague coded both OFC transcripts I compared our codes and we had ~80% reliability (a block of text was calculated as a match if more than half of the codes matched). We then met and during that process, code disagreements were worked out to ensure I was reliable with my coding, and we reached 100% agreement on both transcripts. To further ensure I was reliable I then picked a different transcript to re-read and verify all codes matched the discussion between me and my reliability coder.
Chapter 4: Results from Instructional Practice

This chapter provides analyses of and in-depth insights into Dr. DM’s experiences as a participant in TIMES. In particular, I first provide a general description of what Dr. DM’s class looked like. Second, results from each of the units of instruction are presented. This includes details on the mathematical goals and content of each unit, the duration of the unit, and the frequencies of inquiry oriented instruction local practices for each unit.

For each unit, I first present the holistic characterization of LP1 (facilitation of engaging mathematical tasks) for each unit. Recall LP1 is not a practice coded for unique observable instances in classroom instruction, rather, it is a holistic categorization of the facilitation of the class specifically related to how students were engaged by the teacher. Integrated into IOI LP1 is Smith and Stein’s (1998) mathematical tasks framework of doing mathematics, doing procedures with connections, doing procedures without connections, and memorization. Here, this categorization is not reflective of the task itself, rather it represents how students were engaged by Dr. DM’s facilitation of the task (Stigler & Hiebert, 2004).

Then I discuss the frequencies of local practices: eliciting student contributions [LP2], actively inquiring into student thinking [LP3], being responsive to student contributions and using them to inform the lesson [LP4], engaging students in one another’s thinking [LP5], guiding the mathematical agenda [LP6], and connecting to standard mathematical language and formal notation [LP7].

I next discuss Dr. DM’s overall analysis of instructional by looking across the semester at all four units of instruction through the lens of the inquiry oriented instruction local practices. Finally, I take a closer look at instruction and three instructional themes are discussed that emerged across the totality of analyzed instruction. These themes will include
excerpts from classroom instruction and the interviews with Dr. DM for support.

**Description of Dr. DM’s class**

In Fall 2015, Dr. DM had 27 students (25 males, 2 females). Each day he randomly assigned his students into groups of 3 or 4 (using a random number generator in a spreadsheet). Students worked in those groups for all small group discussions. Each group had a whiteboard that they would lay across a desk or multiple desks as a shared work space. When presenting small group work to the whole class the groups would typically stay at their groups area and either talk or hold up their board. Very rarely did Dr. DM have students come to the board. During small group work Dr. DM circulated the room talking to different groups often sitting down at an empty seat with a group to be at the students’ eye level. During whole class discussion Dr. DM was at the front of the room and would write numerous things down on the front whiteboard. Sometimes he projected from the class computer onto the whiteboard. When calling on people in whole class discussion he sometimes would call on individual students by name, ask for volunteers, or call on a specific group that he wanted to share. Students tended to jump right in the conversation (i.e., they did not necessarily need to be called on by Dr. DM to speak up).

**Dr. DM’s Instruction Unit**

**Unit 1-2.** As discussed in Chapter 3, neither unit was complete. Thus, the video was from the end of Unit 1 and the beginning and end of Unit 2. Together, these made up the first two days of data available for analysis, which resulted in approximately 88 minutes of instruction.

Unit 1 covers qualitative and graphical approaches to differential equations (if the reader is interested in referencing the materials in detail they can go to
https://iode.wordpress.ncsu.edu/materials/student-materials/). The mathematical goals of Unit 1 are to introduce students to: (1) qualitative ways of reasoning about differential equations (i.e., rate of change equations), (2) the fundamental concept of what differential equations are and what solutions to differential equations are, and (3) slope fields. In the unit, students figure out if a differential equation from a certain context depends explicitly on the population, on time, or both. Here, students can argue that the slope is always the same for a given population, regardless of time, and thus the differential equation does not explicitly depend on time thus a proper form of the differential equation is $dP/dt=kP$. The word explicitly is emphasized because while $P$ is a function of $t$, the differential equation does not explicitly depend on time. Following this, students match slope fields to differential equations and sketch in multiple qualitatively different, yet correct, solutions. Ultimately, students come to the conclusion that a solution to a differential equation is a family of functions.

Unit 2 covers a numerical method for studying solutions to differential equations, Euler’s method; with the mathematical goal of the unit begin a fluent conceptual understanding of the algorithm. Students stitch together tangent vectors to approximate a solution to the differential equation and verify their finding with a GeoGebra applet. They then compare two students’, José and Julia, approaches to the numerical perspective on the problem, and finally generalize their understanding of Euler’s method. This ultimately gives students meaning behind Euler’s method algorithm: $y_{\text{next}} = y_{\text{now}} + (dy/dt)_{\text{now}} \cdot \Delta t$.

**LP1 categorization.** Unit 1-2 happened during the first days of Dr. DM using inquiry oriented instruction and the IODE materials. He had in the past used forms of inquiry-based learning and had taught differential equations many times before. In terms of a holistic
categorization of LP1, Dr. DM was categorized as *procedures with connections* (Smith & Stein, 1998) since that is what the video showed he was most supporting in the class. As noted in Chapter 3, Smith and Stein’s mathematical tasks framework is integrated into LP1. According to the IOI framework this means that “while the students were provided opportunities to do mathematics the majority of the mathematics during the discussion was performed by the teacher; however, the teacher used the tasks to engage the students in the conceptual ideas” (Kuster et al., 2017). This is most evident when Dr. DM would answer his own question. For example, in one instance he asked if \( P=10 \) was a solution to the differential equation but went on to explain that it is by himself.

**Local Practices 2-7.** Table 12 indicates the frequencies of LP2-7.

Table 12. *Frequencies of IOI Local Practices 2-7 for Unit 1-2.*

<table>
<thead>
<tr>
<th>Practice</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>21</td>
<td>6</td>
<td>6</td>
<td>15</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Across Unit 1-2, the most common inquiry oriented instruction local practice used by Dr. DM was LP2, eliciting student reasoning and contributions. Dr. DM frequently asked “why?” questions. He asked questions such as “how did you approach this?” questions that necessitated students to provide detailed reasoning/justification, which they usually did. This practice was coded in 21 instances. In addition to eliciting student reasoning and contributions, there was evidence (15 total instances) of Dr. DM using LP5: engaging students in one another’s reasoning (6 instances), rephrasing student contributions (7 instances), and students themselves inquiring into other students’ thinking (2 instances). This was evidence of two main instructional techniques. First, he often asked students to reflect on another student contribution by asking questions such as “do you agree?” or “what do you think about ___’s solution?” Additionally, Dr. DM aided students in engaging in other’s
thinking by rephrasing student contributions to be more accessible to the entire class. This often occurred while he would be writing something down that the students said, as they were saying it, and slightly rephrasing it in written form on the board.

Dr. DM was coded using LP3 (actively inquiring into student thinking) and LP4 (being responsive to student contributions and using them to inform the lesson) 6 times each. Overall, this is noteworthy in its comparison to LP2. Dr. DM’s most often used practice was eliciting contributions from the class. Less often did he actively inquire into those contributions (LP3) by asking a clarifying question (4 instances) have students expand more on their thinking (2 instances) and use a student contribution to inform his lesson (LP4). One instance of incorporating student work into the lesson was when he saw multiple students including arrows indicating backwards time and he brought that up for discussion, saying, “I’ve seen some graphs with arrows at both ends. Do we need both? What does the arrow signify to you?”

This discussion on backwards time is also an example of LP6 (guiding the mathematical agenda) in that he took time to address a student confusion, yet, he was still at the helm of the class. Typically, however, his instruction involved asking students lots of questions, but not always using the answers to those questions to inform the lesson. In addition, for LP6, he did often take time (5 instances) to note something in the public space as an important contribution or important thought. Lastly, there was one instance of formalizing mathematics (LP7) when students provided their possible functions when crafting Euler’s method algorithm. Dr. DM took their work and combined the answers into the formalized form of Euler’s method.

**Unit 6.** Unit 6 covers an introduction to autonomous differential equations,
predominantly through the introduction of the phase line (if the reader is interested in referencing the materials in detail they can go to
https://iode.wordpress.ncsu.edu/materials/student-materials/). The mathematical goal of the unit is to familiarize students with the phase line and the classification of equilibrium solutions based on the long-term behavior of nearby solutions. Prior to Unit 6 students have seen qualitative and graphical approaches to solving differential equations, as well the numerical method Euler’s method, and the analytic methods, separation of variables and the reverse product rule (integrating factor). They also have seen the uniqueness theorem. In Dr. DM’s class, Unit 6 lasted for 104 minutes of analyzed instruction.

Students first analyze the long-term behavior of solutions near certain starting populations for a given differential equation, without using technology. They then generalize this solution space in a 1-D representation. Following this, students develop the vertical representation of the phase line and then classify the different points on the phase line, which end up being different types of equilibrium solutions.

LP1 categorization. In terms of Dr. DM’s holistic categorization of LP1 and in line with Smith and Stein (1998), students were doing mathematics (however, near the end of the unit they shifted to procedures with connections). In the majority of the case of Unit 6, students were deeply engaged in the mathematics, forming conjectures, and testing hypotheses about autonomous differential equations, and the authority of the classroom rested with them.

Local Practices 2-7. Table 13 indicates the frequencies of the remaining LP2-7.

Table 13. Frequencies of IOI Local Practices 2-7 for Unit 6.

<table>
<thead>
<tr>
<th>Practice</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>22</td>
<td>10</td>
<td>7</td>
<td>11</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>
Similar to Unit 1-2, the most common IOI Local Practice used by Dr. DM was eliciting student contributions (LP2; 22 instances). Many of his questions were ones asking “what does this mean?” or “what does that mean to you?” Very frequently students responded to these prompts with deep and thorough descriptions of their thinking.

Dr. DM was coded 10 times and 7 times for LP3 and LP4, respectively. These occurred more frequently than they did in Unit 1-2. He asked questions such as “how can you make that assessment?” and “what do you mean the graphs show the same things?” In those instances, he was actively inquiring into the students’ thinking. While there was minimal evidence of him trying to build a model of student thinking, he was actively trying to figure out the mathematical contributions and decide if they were useful. In that sense, he was also informing his lesson based on student contributions (LP4). At one point, he overheard two students having a discussion about the phase line and pushed the question to the whole class. He asked, “so, this group was just talking about this, for what ODEs [ordinary differential equations] are phase lines appropriate [to use]?”

When considering LP5, Dr. DM was coded 11 instances. Those 11 instances varied in the specific code they were given (i.e., the code from the evidenced by column in the IOI framework). Specifically, Dr. DM did encourage students to engage in other’s thinking by putting questions to the class to respond to claims or arguments just posed by their fellow classmates (LP5; 3 instances initiated by Dr. DM, 6 instances were student initiated). Dr. DM rephrased student contributions given to the class (LP5; 2 instances). Similarly, LP6 was coded 7 times but the specific evidence took on two forms. Dr. DM frequently made student contributions publicly known as important (LP6; 4 instances). Additionally, when students were confused by the question prompt about the phase line being a one-dimensional
projection of all two-dimensional solutions, he spent time clarifying the mathematical concept of a projection (LP6; 3 instances). Lastly, there were 6 recorded instances of Dr. DM connecting to formal mathematical language based on student contributions and student work (e.g., phase lines, stable/unstable/semi-stable equilibrium solutions). For instance, he said “in mathematics we call this one [pointing to solution on phase line] stable and this one [pointing to other solution on phase line] unstable.”

While not a practice, Dr. DM was coded 1 time as referencing his own research, when saying that the phase plane (the extension of the phase line and the content of Unit 9) shows up a lot in his research and is very important to him.

**Unit 9.** Unit 9 covers an introduction to systems of differential equations (if the reader is interested in referencing the materials in detail they can go to https://iode.wordpress.ncsu.edu/materials/student-materials/). Unit 9 was six days of instruction, however, there was a substitute between days 3 and 4 where students continued working with the next part of the unit. As Dr. DM was not teaching, that data was not collected. Longer than any other unit, Unit 9 lasted approximately 245 minutes of recorded instruction.

The goal of the unit is for students to grasp the concept of what a solution is to a system of differential equations and what projections can be used to ascertain that information (i.e., the phase plane). The unit begins by giving students a nonlinear system that they cannot analytically solve and therefore it necessitates a qualitative approach to analyze the system. Following this, students transition to using pipe cleaners to explore solutions to a system of differential equations (i.e., a solution to a system of differential equations is a curve in 3-space). After additional questions students develop vector fields, and the notions
of nullclines to help in understanding the long-term behavior to systems of differential equations.

**LP1 categorization.** In terms of Dr. DM’s holistic categorization of LP1 and in line with Smith and Stein (1998), students were doing procedures with connections. In the case of Unit 9 Dr. DM reverted to being the mathematical authority, as he was in Unit 1-2.

According to the IOI framework this means that “while the students were provided opportunities to do mathematics the majority of the mathematics during the discussion was performed by the teacher; however, the teacher used the tasks to engage the students in the conceptual ideas” (Kuster et al., 2017). For instance, in this unit Dr. DM’s mathematical understanding of the topic became the driving force in the class, rather than his students’ understanding. This will be most evident by looking at the other Local Practices.

**Local Practices 2-7.** Table 14 indicates the frequencies of LP2-7.

Table 14. Frequencies of IOI Local Practices 2-7 for Unit 9.

<table>
<thead>
<tr>
<th>Practice</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>66</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

In addition to the practices, there were 6 unique instances during this unit where Dr. DM made reference to how his mathematical research, and in turn preference of solving systems of differential equations, is heavily tied to the phase plane (the culminating mathematical goal of this unit). Therefore, his instruction oftentimes put him as the mathematical authority and students less frequently engaged in each other’s thinking. This is most noteworthy when looking at LP5 (14 instances; students engage in one another’s thinking) and LP6 (6 instances; guiding the development of the mathematical agenda). Those practices indicate a use of student work to advance the mathematical agenda and students collaborating with each other. However, a majority of Dr. DM’s questions were questions that he was interested in
asking, not ones that were formed from student contributions.

That being said, he did ask numerous questions, most of which were asking for high level contributions. This can be seen in the large number of observations of LP2 (66 instances). Yet as noted, most of the questions (coded under LP2) were ones that were of his own generation. Instances of LP3 (actively inquiring into student thinking) came about as students themselves were providing high level contributions without prompting. Dr. DM only asked a few clarifying questions. Students answered questions and Dr. DM would move on and not put anything back to the students (i.e., see above about the low instances of LP5 and LP6). Lastly, very infrequently did Dr. DM do any formalizing of student contributions (LP7).

**Unit 12.** Unit 12 covers the eigentheory approach to solving systems of differential equations (i.e., the eigenvalue first method) with the mathematical goal of understanding equilibrium solutions for systems of differential equations (if the reader is interested in referencing the materials in detail they can go to [https://iode.wordpress.ncsu.edu/materials/student-materials/](https://iode.wordpress.ncsu.edu/materials/student-materials/)). Unit 12 was approximately 130 minutes of recorded instruction. Unit 12 is a very atypical unit of the IODE materials in that there is typically far less student exploration and much more guided instruction on algebraically solving for solutions to systems of differential equations. Because of this, scores from this unit are in stark contrast to previous instructional units. Additionally, this was the last unit of the IODE materials that Dr. DM used in his class.

The unit begins by the students developing criteria, based on parameters in the system, which can be used to ascertain the type of equilibrium solutions that a given system has. Subsequently, students generalize this approach to find the eigenvalue first of a given
system then find the corresponding eigenvector. Note, in Unit 10 (not analyzed), students first found the eigenvector then the corresponding eigenvalue. This approach is a more intuitive geometric approach, while the eigenvalue first method is a heavy procedural algebraic approach.

**LP1 categorization.** Due to the fact that the unit was an atypical IODE unit, Dr. DM’s scores looked different. His holistic categorization of LP1 was doing *procedures without connections*. Oftentimes, the justification for doing a particular step in the algorithm was lost or was brought out of his own accord not generated from student contributions. Students in the class were following his lead, doing the procedure, and little was connected to previous units.

**Local Practices 2-7.** Table 15 indicates the frequencies of LP2-7.

Table 15. *Frequencies of IOI Local Practices 2-7 for Unit 12.*

<table>
<thead>
<tr>
<th>Practice</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>14</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Dr. DM most used IOI Local Practice was eliciting student ways of reasoning (LP2; 14 instances). After that all other practice frequencies ranged from 1-4 instances. During this unit, Dr. DM frequently would ask a question but immediately answer it (i.e., that does not get associated with any practice). Consequently, all scores are lower across the board.

**Dr. DM’s Overall Analysis of Instructional**

In this section, I provide a thorough look at the frequencies of all IOI LPs across the entire semester and discuss trends that existed when looking across all units. Due to the fact that each unit lasted a different amount of time, comparing raw frequencies of IOI LPs is not helpful. Therefore, Table 16 indicates the conversion factor that frequencies were scaled by to be able to compare units (all were scaled to the same amount of time as Unit 9; for
example, Unit 6 was 104 minutes then multiplying that by 2.3530 yields 244.7167 minutes
which is the same as Unit 9) and Table 17 indicates the updated and scaled frequencies,
rounded to the nearest whole number.

Table 16. *Conversion factor to compare instructional units.*

<table>
<thead>
<tr>
<th>Unit</th>
<th>Minutes</th>
<th>Conversion Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>87.9833</td>
<td>2.7813</td>
</tr>
<tr>
<td>6</td>
<td>104.0000</td>
<td>2.3530</td>
</tr>
<tr>
<td>9</td>
<td>244.7167</td>
<td>1.0000</td>
</tr>
<tr>
<td>12</td>
<td>129.7333</td>
<td>1.8863</td>
</tr>
</tbody>
</table>

Table 17. *Updated, scaled, and rounded Local Practice frequencies.*

<table>
<thead>
<tr>
<th>Practice</th>
<th>Unit 1-2</th>
<th>Unit 6</th>
<th>Unit 9</th>
<th>Unit 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>58</td>
<td>52</td>
<td>66</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
<td>24</td>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
<td>16</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>26</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>16</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>14</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

The largest takeaway from Table 17 is that LP2, eliciting student ways of reasoning
and contributions, is Dr. DM’s most used local practice across all analyzed units of
instruction. Dr. DM asked numerous questions during whole class discussion beyond just
ones listed on the task sheets. LP3-6, while not consistent across each unit, were consistent in
that they occurred roughly the same (i.e., LP3, LP4, LP5, and LP6 all occurred about the
same each unit relative to each other with the exception of LP5 in Unit 1-2). LP7 varied
widely across all of the units. The IODE materials have less formalized mathematics (e.g.,
there is only one theorem in the whole course, the uniqueness theorem) than other upper level
courses such as linear algebra or abstract algebra. Consequently, less work was needed by
Dr. DM to formalize his students’ work. However, when he would formalize student
contributions it was often in relation to providing the standard mathematical term for
whatever concept the class just unearthed (e.g., phase line).

I remind the reader that the richness of student contributions was not considered (only simplistic responses were not included, but general answer to questions were). Further, I note that LP 2, eliciting student ways of reasoning and contributions, is not specific to IOI (e.g., Mehan, 1979) so does not significantly represent the uniqueness of IOI teaching; however, how this practice relates to others is important to IOI and will be discussed further in this chapter. To further illustrate how Dr. DM’s instruction centered around eliciting student ways of reasoning and contributions I provide two excerpts of classroom dialogue. The first excerpt is from the last day of Unit 6, specifically the last question of the unit, where the task asks students to discuss how the phase line can be seen as a one-dimensional projection of all solutions to the differential equation. Within the excerpts, there will be a **bolded** bracket notation indicating that what just occurred was coded as a Local Practice (e.g., [LP2]).

Physical actions performed by Dr. DM or a student will be noted in *italics*. Lastly, **DM** in the excerpt indicates Dr. DM is speaking and **SA, SB, etc.** indicates a student is speaking (note that within a certain excerpt a student identifier will be unique, however, they will be reused across excerpts).

1. **DM**: What do you observe about this one’s shadow? *(pointing to one solution curve)*. It’s quite close. It just starts there instead. Okay. So, what do these shadows kind of tell us [LP2]?
2. **SA**: If it’s increasing or decreasing and also where it starts.
3. **DM**: Okay, so we can see where it starts and kind of where it goes. Right.
4. What if I had a phase line that looked like that?
5. **DM draws phase line on board with one unstable and one stable equilibrium solution, with the stable one corresponding to the higher dependent variable value.**
6. **DM**: And if I have some initial condition in there *(draws x between two equilibrium solutions)*. What happens to that initial condition [LP2]?
7. **SB**: It increases, gets closer and closer to that point but never reaches it.
8. **DM**: It gets closer and closer to that point and never reaches it. So, if I were to graph this against time. What would that look like [LP2]?
**Sb:** It would asymptote.

**DM:** So, it would asymptote kind of like that? What would that shadow look like? It would be all of this, right? The shadow would be all of this but never reach it. So, the phase line can tell us where it is going, that’s one thing. Let’s go back to this discussion. What’s the difference between these two [LP2]? **DM points to two drawn solution curves on the board.**

**DM:** At some point, they have the same shadow, after some point. After there, they are kind of indistinguishable.

**SC:** It’s the rate of change.

**DM:** Mhm. So, if I look at these two separate initial conditions, we talked last time about how they will grow apart. Because this one starts slower because its derivative is smaller because it’s closer to the critical point. But, if I ask you for this point on the phase line which one of these two solutions did that come from [LP2]?

**SD:** We don’t know.

**DM:** We don’t know. Okay. So, we can’t tell which solution it was on, because that is part of that projection that is something that you lose when you go from two dimensions to one.

In this excerpt, Dr. DM had a clear goal getting the students to realize that while the phase line is a powerful tool in understanding solutions to differential equations, all information from the time variable is lost. In the study of autonomous differential equations, which uses phase lines, this is not a problem, as the differential equation does not depend on time (i.e., autonomous). Consequently, ignoring the time variable can still inform the learner a lot about the solution space of the differential equation. In this excerpt, Dr. DM asked multiple questions to push his students’ thinking, but the questions were always generated by him (e.g., lines 16-19). It is important to note that this is not an inappropriate instructional strategy for this class in the sense that Dr. DM is the one guiding students to reinvent the mathematics (Freudenthal, 1991) and thus he has a mathematical goal in mind for the lesson, yet sometimes the curated mathematics was teacher-driven rather than student-driven.

This was similar to the following excerpt from Unit 9. In this excerpt from the first day of Unit 9, the class is discussing what it means for a system of differential equations to be in equilibrium. In this problem, the functions of study in the differential equation represent
rabbit and fox populations.

1  \textbf{DM:} So, what does it mean to be in equilibrium [LP2]?
2  \textbf{SA:} Rate of changes are 0.
3  \textbf{DM:} Rate of change (emphasized plurality of rates and changes) are 0. Does it have to be both?
4  \textbf{SA:} Yes.
5  \textbf{DM:} Can I get away with just one?
6  \textbf{SA:} No.
7  \textbf{SB:} No.
8  \textbf{DM:} Really? Because we were all on board here when we said the foxes are at equilibrium and the rabbits are reproducing like rabbits.
9  \textbf{SB:} The whole system would not be in equilibrium, only one part.
10 \textbf{DM:} Okay. So, when we talk about equilibrium, we have to talk about the whole system [LP6]. We want to talk about the whole system. Now, in a couple of days we are going to talk about how useful it is to study scenarios where only one part of the system is in equilibrium, or has this 0 rate of change because it’s not truly equilibrium. But if we are going to talk about equilibrium it has to be the entire system. Whole thing. Okay. So, but we didn’t finish this part of the discussion about what happens if there are no rabbits. I want to come back to that. So, no rabbits means \( R = 0 \).
11 \textbf{SC:} So, the population of foxes would decrease.
12 \textbf{DM:} So, \( dF/dt \), you say it will decrease.
13 \textbf{SC:} Because it’s negative \( F \).
14 \textbf{DM:} Because it’s negative \( F \). Right, but if the foxes go down but then are the rabbits then not suppressed? So, the rabbits go up if there are less predators out there.
15 \textbf{SA:} But we are saying there are no rabbits at all. So, there is nothing for them to increase from.
16 \textbf{DM:} Okay, so mathematically can we demonstrate that [LP2]?
17 \textbf{SC:} \( dR/dt \) is 0.
18 \textbf{DM:} \( dR/dt \) is 0…So, that is 0. Okay. So, at the extremes we have, if there are no foxes we have unlimited growth for our rabbits. And if we have no rabbits, we have decay to extinction for our foxes. So, what happens in the general case? The general case is a lot more tricky. Can you think about the scenario when there are almost 0 foxes? What happens to the rabbit population do you think [LP2]?
19 \textbf{SE:} It goes up.
20 \textbf{DM:} It’s got a lot of room to grow. Practically unlimited resources. But if I have a boom in the rabbit population what can you tell me about the foxes?
21 \textbf{SA:} They are going to increase.
22 \textbf{DM:} They are going to increase. What happens when the foxes have a boom in their population then?
23 \textbf{Smultiple:} Rabbits decrease.
24 \textbf{DM:} The rabbits have to go down. So, once the rabbits go down what happens to the foxes?
They go down. And I’m back in a situation where I have almost no foxes. The question is, is there an extinction event of one species or the other in there?

No.

Is it possible for foxes to eat all of the rabbits [LP2]?

I’m going to try to apply the uniqueness theorem and say that if $F=0$ and $R=0$ then both $dF/dt$ and $dR/dt = 0$ then the uniqueness theorem would apply to tell us that they cannot ever go extinct.

In this excerpt we see that, similar to the previous excerpt, Dr. DM engages his students by asking many questions. These questions are aimed at getting students to consider what the solution to this system might look like. Dr. DM frequently asked students for high level contributions (e.g., line 50), which they provided (e.g., lines 51-53). However, when looking closely to what Dr. DM did after every time a student responded; he oftentimes repeated exactly what was said then moved on to the next question (e.g., lines 39-40). The students who responded were on board and Dr. DM got the answer he was looking for and moved on to the next question. This could imply that either 1) Dr. DM knew exactly why his students were saying what they were saying, or 2) he was not interested in asking them why because he got the answer that he needed to move the class forward or something else. This issue of interpretation of his guiding questions emerged during the whole analysis and will be discussed in the discussion.

A Closer Look at Dr. DM’s Instructional Practice

Consistent across Dr. DM’s instruction was that his most used local practice was LP2, eliciting student ways of reasoning and contributions. However, while this trend existed across the totality of his instruction, there were still other notable differences when comparing units. For instance, when considering the local practices, particularly LP2-5 as those practices are specifically about student thinking; eliciting student ways of reasoning
and contributions [LP2], actively inquiring into student thinking [LP3], being responsive to student contributions and using them to inform the lesson [LP4], and engaging students in one another’s thinking [LP5], we see many disparities between units (Figure 11). This is important because it provides more details on how Dr. DM used student thinking in his class. Notice, for example, how Unit 12’s frequencies are lower across the board. Further, notice how Unit 9’s LP2 frequency is highest, while the other three LPs in that unit are significantly lower. Lastly, notice, how LP5 is high in Unit 1-2 but that is not the same in other units.

Figure 11. Frequencies of LP2-5 by unit.

This section elaborates on three themes that emerged from analysis that can be used to explain differences in each of the units’ frequencies of local practices. Namely, I discuss 1) when Dr. DM engaged with student thinking, his frequencies of local practices took a certain form; 2) how the tasks within a unit affected the frequencies of local practices; and 3) how Dr. DM’s mathematical beliefs and his research influenced how he used student thinking and his frequencies of local practices.

**Engaging with student thinking.** To illustrate how Dr. DM engaged with student thinking I highlight an excerpt from Unit 6. In the following excerpt, whole class discussion had been happening for about 4 minutes on the first problem in Unit 6 (Figure 12) prior to the
start of the excerpt. Students are discussing the long-term behavior of the different initial conditions posed by the problem.

Analyzing Autonomous DEs: Spotted Owls

A group of biologists are making predictions about the spotted owl population in a forest in the Pacific Northwest. The autonomous differential equation the scientist use to model the spotted owl population is

\[
\frac{dP}{dt} = \frac{P}{2} \left( 1 - \frac{P}{5} \right) \left( \frac{P}{8} - 1 \right),
\]

where \( P \) is in hundreds of owls and \( t \) is in years. The problem is that the current number of owls is only approximately known.

1. Suppose the scientists estimate that currently \( P \) is about 5 (i.e. there are currently about 500 owls in the forest). Since 5 is only an estimate, they make long-term predictions of the owl population for the initial conditions \( P = 4.9 \), \( P = 5.0 \), and \( P = 5.1 \). Without using a graphing calculator or other software, determine the long-term predictions for these initial conditions based on the differential equation. Are they similar or different? That is, will slightly different initial conditions yield only slightly different long-term predictions, or will they be radically different? Carry out a similar analysis if the current number of owls is somewhere around 8.

Figure 12. Problem 1 from Unit 6 (Rasmussen et al., 2017).

In this particular example, Dr. DM is very responsive to student contributions.

Specifically, a majority of this discussion spurs from a statement generated by a student (\( S_B \)) about how the solution will never actually reach the equilibrium point if the initial condition is beneath it.

1. \( DM \): Let’s talk about 5.1.
2. \( S_A \): The population would be increasing until it reaches a value of 8 \([LP2]\).
3. \( DM \) begins to write what \( S_A \) just said on board.
4. \( DM \): So, you say that the population increases until it reaches \( P=8 \). And that’s based on what assessment \([LP3]\)?
5. \( S_A \): Based on the equation, once it reaches 8, you’ll have 0, so 8 over 8 is 1, and 1 minus 1 is 0 so the whole \( \frac{dP}{dt} \) becomes 0. So, the population can’t exist at \( P=5 \) or \( P=8 \).
6. \( DM \): So, the pop---
7. \( S_A \): Well no, not the population, but the rate of change will be 0 at \( P=5 \) and \( P=8 \).
8. \( DM \): Okay so the rate of change is 0 there. That’s interesting \([LP6]\). Anybody refute this \([LP5]\)?
9. \( S_B \): The only thing that I do have to say about it is, if \( P \) is ever in between 5 and 8, we won’t ever actually reach 8.
10. \( DM \) writes on board what \( S_B \) just said.
11. \( DM \): If \( P \) starts between 5 and 8, it will never reach 8? \([LP4]\)
12. \( S_B \): Yes.
DM: That's a bold statement.
SB: It will go--
DM erases the word “reach” and replaces it with “equal”.
DM: Never equal 8 [LP5].
DM erases the word “it” and replaces it with “P”.
DM: And by it I mean P.
...
SD: If you were to graph this equation you would have an asymptote at 5 and at 8 and 0.
DM: What do you mean by graph the equation [LP3]?
SD: Um, dP/dt on a Cartesian coordinate system. You will have an asymptote at 0, 5, and 8.
DM draws axes on the board and labels them as dP/dt versus P.
DM: So, this graph? Is this the one you want?
SD: No, sorry, population versus time, P versus t.
DM: Oh, oh, oh, oh, oh. So, that graph, you’re saying would have horizontal asymptotes [LP3]?
SD: Yes. P=8, P=5, and P=0.
DM: And that’s based on what [LP3]?
SA: That makes the differential equation 0.
DM: Okay, so that makes dP/dt 0. And that’s what our experience doing direction fields kind of indicates to us, right? But, we haven’t fixed this problem.
DM puts box around dh/dt=-h^{1/3}.
DM: Here was an equation that has that 0 property and yet this didn’t have an asymptote there, it landed.
SB: So, couldn’t we possibly find out by doing the graph of dP/dt versus P and then see if at any point if the slope of that is undefined [LP2]?
DM: So, this graph, right?
DM points to dP/dt versus P graph that was still written on board.
DM: So, what does this graph look like? What’s the highest term?
SE: Cubic.
DM: It’s cubic. And is the leading coefficient negative or positive?
SA: Negative.
...
DM leads students through graphing the equation.
...
DM: Does this have any vertical tangent lines?
SB: Not from the way that you drew it, no.
DM: No, it’s a nice cubic function, it’s continuous and its derivative is continuous.
SB: So, I’m wrong.
DM: Well, um, so this---
SF: No, you’re right SB, because it’s increasing to some value between 5 and 8. And starts decreasing as you get closer to 8 [LP5].
SB: However, the question at hand was if the population could ever hit 5 or 8
and as we discussed about 3 classes ago, we said that that occurs when the slope of \(dP/dt\) versus \(P\) graph becomes undefined.

\[
DM: \text{Okay, so some time ago, we had the uniqueness theorem, right? And that said what? Roughly speaking, if the graph of } dP/dt \text{ versus } P \text{ has no vertical tangent lines then what?}
\]

\[
SA: \text{P is defined at all points.}
\]

\[
DM: \text{Okay, so we have a solution that is defined at all points and? Solutions corresponding to different initial conditions, what is true about them?}
\]

\[
SB: \text{They do not cross.}
\]

\[
DM: \text{They do not cross \textbf{[LP6].}}
\]

Overall, we can see that once \(SB\) mentioned that the solution will never reach 8 (lines 14-15), discussion for the next 8 minutes focused on that one claim. During that time students were responding directly to each other [LP5] or prompted to do so by Dr. DM [LP5] (e.g., lines 12-13). Dr. DM asked clarifying questions [LP3] such as “and that assessment was based on what?” Dr. DM focused his students on important ideas [LP6] pertaining to the uniqueness theorem. In this case, \(SD\) suggested looking at the \(P\) versus \(t\) graph and \(SB\) suggested looking at the \(dP/dt\) versus \(P\) graph as a means to use the uniqueness theorem to prove that it will never reach 8. Dr. DM was responsive to student contributions and used them to inform his lesson [LP4].

This excerpt is one example of how Dr. DM’s instructional looked in Unit 6. His instruction also looked similar in Unit 1-2 where Figure 11 highlighted a large number of instances of students engaging in one another’s thinking [LP5]. In these instances, Dr. DM was fully engaged in inquiring into his students’ thinking, while his students were inquiring into the mathematics (Rasmussen & Kwon, 2007). Critical to inquiry oriented instruction is the use of student thinking. In instances where Dr. DM was open to inquiring into his students’ thinking, that led to his students having opportunities and experiences of engaging in each other’s thinking more often.

**Local practices tied to the task.** Table 17 indicated that LP2 was the most frequent
local practice across all units. Notably, this was consistent throughout the semester. However, the overall frequencies, even after scaled, differed, in particular LP3-5 (see Figure 11). One unsurprising explanation for the frequencies of Dr. DM’s IOI LPs are the tasks in the unit itself. As indicated, Unit 1-2 was a very open-ended unit focused on ultimately defining what a differential equation is. Contrariwise, Unit 12 was a more prescriptive unit about eigentheory applied to linear systems. The IODE teacher materials support this characterization of the materials (Rasmussen et al., 2018). In particular, Unit 12 provides scaffolded questions for students to be led through the process of finding the eigenvalues of a system of differential equations. The process itself it very algebraic and thus the questions were worded in ways like this: “rearrange the statement to look like x.” As the process is very specific, the possible student exploration that can happen is less than many of the other IODE units. Collectively, this sheds light on how, when analyzing instruction, it is important to take into consideration the task itself. Ultimately, Dr. DM had less opportunity to probe his students’ thinking as the task shackled him to a particular prescriptive method.

To illustrate how the IOI LPs are tied to the content of the task, I highlight two excerpts, one from Unit 1-2 and one from Unit 12. The first excerpt from Unit 1-2 focuses on two parts to one question, matching two slope fields to six possible differential equations (figure 13).
8. Below are seven rate of changes equations and three different slope fields. Without using technology, identify which differential equation is the best match for each slope field (thus you will have four rate of change equations left over). Explain your reasoning.

\[
\begin{align*}
(i) \quad \frac{dy}{dt} &= t - 1 \\
(ii) \quad \frac{dy}{dt} &= 1 - y^2 \\
(iii) \quad \frac{dy}{dt} &= y^2 - t^2 \\
(iv) \quad \frac{dy}{dt} &= 1 - y \\
(v) \quad \frac{dy}{dt} &= t^2 - y^2 \\
(vi) \quad \frac{dy}{dt} &= 1 - t \\
(vii) \quad \frac{dy}{dt} &= 9t^2 - y^2
\end{align*}
\]

\[ \text{Figure 13. Problem 8 from Unit 1 (Rasmussen et al., 2017).} \]

DM: So, let’s talk about, what’s the most obvious feature of (a) you identified? Or how did you approach this, maybe I should ask it that way [LP2]?

SA: Um, it has an asymptote. The graph of y does.

DM: Okay, the graph of, in the y – what do you mean the graph of y does [LP3]?

SA: dy/dt was there was so at y=1, the graph of the function that is described has an asymptote there.

DM: Okay, so it looks like we have an asymptote at y=1. Certainly, I can imagine following these arrows into that. Okay, so which one of these, I guess
there is six of them, which one of these six equations did we identify as they 
might have that asymptote?

**SB:** The fourth one. \( \frac{dy}{dt} = 1 - y \)

**DM:** So, how did you make that determination [LP3]?

**SB:** Well, the asymptote is at 1 when \( y = 1 \) you are going to have the rate of 
change equal to 0 and since there is no slope there, it’s the one that made
sense to me.

**DM:** So, at \( y = 1 \), \( \frac{dy}{dt} = 0 \).

**DM writes what he just said on the board, rephrasing what the student said** [LP5].

**DM:** Okay, so you’re saying if I try something like \( y = -2 \) then the first

equation, \( \frac{dy}{dt} = 1 - (-2) = 3 \), which is positive, but for the second equation you’re
telling me \( \frac{dy}{dt} = 1 - (-2)^2 = 1 - 4 = -3 \), which is negative. And here is -2, it’s hard to
see here, and it’s a positive slope which means it can’t be this one [LP5].

That’s a really good way to check this. I like that a lot [LP6]. What other
feature would this one have that is kind of dead giveaway [LP2].

**SB:** The slope in the positive would be the same as the same in the negative y
direction.

**DM:** Ah I see. Because it’s squared you lose that sign information, s-i-g-n

[LP5]. So, there should be a certain symmetry to that. I agree with that. You
picked this one out. Why? You said it’s because of that asymptote. Would this
one \( \frac{dy}{dt} = 1 - y^2 \) have another asymptote [LP2]?

**SE:** Yeah.

**DM:** Where?

**SE:** -1.

**DM:** It would have another one at -1. So, that’s another dead giveaway, in my
mind, in ruling this one out. That’s the one I’m most trained to see first, is
looking for where these equations are equal to 0 but we are going to get to that
later. Okay, how about I, let’s put (b) on hold for a minute. Which one did you
identify for (c) or how did you think of (c)?

**SA:** It was the last one.

**DM:** So, \( \frac{dy}{dt} = 1 - t \). Okay. Why [LP2]?

**SB:** So, if you look at the two equations, the only difference is the last one the
slope goes negative as \( t \) gets larger while the first one, the other one that
would fit, the slope gets positive as \( t \) gets larger. So, you just have to see at
higher values which is positive or negative on that graph.

**DM:** Okay. So, everybody has made one leap that I’m not willing to make yet.

Why is it down to these two [LP2]?

**SF:** Because the slopes aren’t dependent on \( y \), they are only dependent on \( t \).

**DM:** Oh, so, if we look at this. What you are saying is that, if I pick any
random value of t that I want and I look up and I look down, all of the slopes
are the same \([\text{LP5}]\). Okay, so we are between these two because slope doesn’t
depend on y. And these are the only two that do not have a y in their equation.
Okay. So, now I believe you, that we are between these two. Now can you
reiterate what you said \([\text{LP3}]\)?
\[S_B: \text{The only difference between those two is the direction of the slope at the}\]
higher-level values of t. Either under or above 1. So, if it’s the first equation
then as \(t\) gets larger the slope should turn more and more negative. While with
the second equation as \(t\) gets larger the slope becomes more and more
positive.
\[DM \text{ writes down exactly what student just said on the board relating to both}\]
differential equations.
\[DM: \text{I agree} \ ([\text{LP6}]).\]

In this exchange, Dr. DM frequently rephrased his students’ contributions \([\text{LP5}]\) (e.g.,
lines 18-20), asked questions to probe further in their thinking \([\text{LP3}]\) (e.g., lines 60-61), and
oftentimes marked something as publicly important to the mathematical agenda \([\text{LP6}]\) (e.g.,
line 69). Even though his most used local practice for the unit was LP2, there were instances,
such as the one highlight above, where Dr. DM used other practices. However, this happened
far less frequently in Unit 12 when Dr. DM was guiding students to reinvent how to use
eigentheory to solve systems of differential equations algebraically. In the following excerpt,
the class is discussing the mathematics highlighted in Figure 14.
Equilibrium Solutions for Linear Systems

\[ \frac{dx}{dt} = ax + by \]
\[ \frac{dy}{dt} = cx + dy \]

1. For each part below, use two different ways (one algebraic and one geometric using nullclines) to figure out the number and location of equilibrium solutions.

\[
\begin{align*}
\frac{dx}{dt} &= 3x + 2y \\
\frac{dy}{dt} &= -2y \\
\frac{dx}{dt} &= 4x - 2y \\
\frac{dy}{dt} &= -2x + y
\end{align*}
\]

2. Is it possible to find values of \(a, b, c, d\) such that the system of differential equations

\[
\begin{align*}
\frac{dx}{dt} &= ax + by \\
\frac{dy}{dt} &= cx + dy
\end{align*}
\]

has exactly two equilibrium solutions? Explain why or why not.

3. Develop criteria (in terms of the parameters \(a, b, c,\) and \(d\)) that tell us about the number and location of equilibrium solutions for systems of differential equations of the form

\[
\begin{align*}
\frac{dx}{dt} &= ax + by \\
\frac{dy}{dt} &= cx + dy
\end{align*}
\]

**Figure 14.** Problems 1-3 from Unit 12 (Rasmussen et al., 2017).

1. **DM:** What was the necessary condition here [LP2]?  
2. **SA:** Wasn’t it like a matrix?  
3. **DM:** Yeah, it looks like the determinant of a matrix. So, we had the condition \(ad-bc=0\). That was our, for this to be – if we have that then these equations are dependent. So, coming back to this, what do we find? We found that, I can rearrange this. \((\lambda-a)x-\lambda y=0\) and this is \(-\lambda x+(\lambda-d)y=0\). Right? If I rearranged these equations into this form where I just have one coefficient of \(x\) and \(y\) in each equation. What do we get? What do these equations tell me?  
4. **DM:** What does this condition tell me [LP2]?
\[ SB: \text{That } ad=bc. \]

\[ DM: \text{So, I need } ad \text{ and } bc, \text{ right. What is my } A \text{ here? What is my capital } A? \]

It’s lambda-a. So, this looks like. (Lambda-a) – So, the result is a quadratic equation in lambda. Which we know how to solve quadratic equations.

*Multiple students inaudibly respond.*

\[ DM: \text{Okay. We can’t factor it. We go back to our good friend, the quadratic formula. What can happen in the quadratic formula that hasn’t happened yet? What is the quadratic formula?} \]

*Student answers with the quadratic formula.*

\[ DM: \text{Okay, these } a \text{'s and } b \text{'s are now different from those } a \text{'s and } b \text{'s. Before, we were able to get two lambdas. We got like lambda equals 2 and lambda equals 5. That guy [the determinant], right. Before it was the stuff of nightmares. What happens if that [the determinant] is negative? I get an imaginary number. We say, there is no solution. But in fact, it’s better to call them complex numbers, not necessarily imaginary. But with these complex numbers we can still talk about the solutions to the differential equation, because what was the solution to this? If I have } \frac{dx}{dt}=\lambda(x), \text{ what’s the solution? I need some function that when I take the derivative of it I get lambda times itself. What is that function?} \]

\[ SC: \text{Exponential function.} \]

\[ DM: \text{Exponential function. Right. } x \text{ of } t \text{ is } e \text{ to the lambda } t \text{ times some constant and } y \text{ of } t \text{ is some other constant times } e \text{ to the lambda } t. \text{ Okay. So, there is one more piece that we need. So, if this is misbehaved, what I would call beautiful, what happens if lambda is imaginary or there is an imaginary component to it? Can we still talk about } e \text{ to the imaginary number?} \]

*Multiple students say yes.*

\[ DM: \text{We can now, right. } e \text{ to the } (i)(t). \text{ There’s that thing called Euler’s identity or Euler’s formula. What is that?} \]

*Student responds with Euler’s formula.*

\[ DM: \text{Yeah. And just very briefly, do you take this for granted?} \]

\[ DM \text{ goes on an aside with a quick proof of Euler’s formula.} \]

Here we see that Dr. DM answers many of his own questions (e.g., lines 19-27) and when students did answer questions they were oftentimes very procedural and not high-level contributions (e.g., supplying what the quadratic formula is). In an attempt to develop the eigenvalue first method for solving linear systems of differential equations, Dr. DM had to do significant hand holding. In this unit, all of the IOI LPs were less used, and one possible explanation for this is the content of the task itself.
Influence of Dr. DM’s mathematics research.

DM [interview]: And so, um I see DEs, like that’s my goal is for students to be able to start to see that. And for that reason, I have to push that kind of phase plane agenda to start to be able to talk about that. … By viewing myself as the curator of their discussion and just picking apart things and building towards my mathematical agenda allowed me to inject a lot of my personality back into the course and talk about things that I’m really passionate about.

Dr. DM received his PhD in mathematics with a specialized focus on mathematical neuroscience. This research oftentimes consisted of using differential equations to model neuron behavior. Consequently, Dr. DM’s strongest mathematical content knowledge could be viewed as where his research lies. Note, this is not based on assessment of his content knowledge but rather on the fact of where his mathematics research lies and the fact that he would be considered an expert in that field. One tool that he often would use in such research, which he indicated to his students in the above quote from an interview, was the phase plane. As he was very knowledgeable about the phase plane he oftentimes brought in more background knowledge than was provided by the IODE teacher materials or other members in the online faculty collaboration.

When mathematicians bring their own work into the classroom it can lead to students seeing new and beautiful ways in which mathematics emerges in our daily lives. Dr. DM was very passionate about his research and passionate about getting his students to see a research perspective on differential equations. However, in this particular case, and I posit in the general case, when Dr. DM was very knowledgeable about the content, he knew exactly what he wanted to ask and exactly what he wanted his students to learn. In those cases, he was less open to student exploration of the mathematics. As stated, this phenomenon, rather specific to Dr. DM, relevant to what many mathematics instructors do, is just more amplified when that instructor is purposefully trying to include student ideas in the classroom but his/her
mathematical interest can stifle such student exploration.

In this subsection, I highlight excerpts from the entrance and exit interview, as well as some from class discussions, where Dr. DM specifically makes reference to his research. It should be noted that this particular phenomenon emerged in Unit 9, where the concept of the phase plane is developed. Referring back to Figure 10, Unit 9 had the largest disparity between LP2 and LP3-5. Dr. DM asked numerous questions but did not always follow up. In addition to the content itself, the mathematical research and associated beliefs of Dr. DM influenced which LPs appeared in his instruction in a given unit.

When Dr. DM was asked about his agenda in his differential equations course during the entrance interview he remarked the following:

1. DM: And that agenda is largely because of the way I see DEs used in my research. Uh, I want students to have a taste of that.
2. Interviewer: Can you say a little more about that? I think that’s really interesting.
3. DM: Yeah, so, um, my research is in mathematical neuroscience. And so, the models that we study are DE models based on the Hodgkin-Huxley formulation. Um which is sort of a classic one with lots of bifurcation structures and lots of things to study and the problem is as you start to build these models for specific systems, um they get out of control, kind of quickly. So, for instance, one neuron that I study for respiration is a 7-dimensional set of equations. So that’s too much, right? But there is a technique called fast-slow-composition or geometric singular perturbation theory. And the idea is that if you have a fast and a slow subsystem then you can study them independently with the idea that the slow subsystem becomes a drifting parameter which changes the bifurcation structure of the fast subsystem. And so, under that kind of analysis, you take this 7-dimensional monster and you reduce it down to a 2-dimensional phase plane. And so, suddenly, all of the things that you learn about, you know, this is a stable spiral, and that is an unstable node; all of those things come into view in a short period of time until the parameter has changed. Uh so, it’s a very elegant theory for that reason.
4. Interviewer: That’s so cool! I didn’t know that haha.
5. DM: Yeah. So, it lets you take these monster equations and study them in simpler terms. 2- and 3-dimensional phase space. And of course, in 2-dimensions you have a lot more power. Um so I see these things in my research on a daily basis. Um and it’s not just, you know, it’s a whole field of
Clearly, Dr. DM is knowledgeable about the field of differential equations. He has a particular view of the subject, based on his own research in the field, and that is exactly how he wants his students to think about differential equations. During the exit interview, he further noted:

\textbf{DM:} So, I want to minimize that time [early units] so that we can spend a lot more time on the phase plane and so, now that I have a sense of how all of these materials play out I will be much better at getting us to that point.

\textbf{DM:} By viewing myself as the curator of their discussion and just picking apart things and building towards my mathematical agenda allowed me to inject a lot of my personality back into the course, and talk about things that I’m really passionate about, in ways that I wasn’t able to, in past semesters, when I flipped my classroom, so they were reading a textbook and they would do HW problems and they would have to present the HW problems. That didn’t leave a whole lot time for me to talk about how exciting it was. And certainly, my student evaluations said that I was passionate about the subject, so they can see it.

His passion for the course is very important. He noted that his students appreciated his passion. Even in class discussion he would outright mention his passion and bias for certain aspects of differential equations:

\textbf{DM:} This is my home; phase planes are where I live. … All of my research is based in the phase plane, in phase space. … That is a sufficiently strong hint that says I will allow my bias to show and I will promise you many questions on the phase plane on the next celebration of knowledge [Dr. DM’s tests]. I can’t help it. I find it exciting.

\textbf{DM:} …this group of scientists wants to graphically display predictions for many different non-negative initial conditions. Which view might they use? I’m biased. I’m going to say that they use this is (F, R) plane. Because it shows the relation between lots of different solutions.

Dr. DM himself noted that his goal for some of the units was to get his students to
think a specific way that aligned with his view of differential equations. Consequently, when looking at frequencies of local practices of IOI LPs, we see that in relation to how much he elicited from his students [LP2], he less often actively inquired into their thinking [LP3], used their contributions to push the class forward [LP4], or worked towards developing a shared understanding among the class [LP5], rather it was his understanding that was the focal point of his instruction.

Summary of Answer to Research Question 1a: Instructional Practice

How did Dr. DM’s instructional practice unfold over his first implementation of inquiry oriented differential equations and in what ways did it align with inquiry oriented instruction?

To answer this question, I talk first about the second part of the question, in what ways did it align with inquiry oriented instruction. Central to inquiry oriented instruction is the facilitation of mathematics where students are actively inquiring into the mathematics while the teacher is actively inquiring into the students’ individual and collective thinking (Rasmussen & Kwon, 2007). The Inquiry Oriented Instruction framework (Kuster et al., 2017) consists of four broad focal instructional components as well as seven local practices of inquiry oriented instruction. These local practices were used to determine how Dr. DM’s instruction aligned with the tenets of inquiry oriented instruction (i.e., the particular type of instruction central to the focus of the online faculty collaboration).

Recall LP1 was a holistic look at the facility of engaging mathematical tasks. Dr. DM was coded as doing mathematics in Unit 6, procedures with connections in Units 1-2 and 9, and procedures without connections in Unit 12. These characterizations are about how Dr. DM facilitated student engagement around the mathematical task. Unit 12 was coded as low demand, which can also be seen in the frequencies of LP2-7 in the fact that they were all
lower for that unit. Other units were considered high, yet disparities between them are noteworthy and can be discussed in terms of LP2-7.

Recall the other Local Practices: LP2 (eliciting student ways of reasoning and contributions), LP3 (actively inquiring into student thinking), LP4 (being responsive to student contributions and using them to inform the lesson), LP5 (engaging students in one another’s thinking), LP6 (guiding the development of the mathematical agenda), and LP7 (connecting to standard mathematical language and formal notation). Dr. DM most frequent practice was eliciting contributions from his students (LP2), and less often actively inquired into why his students were making such contributions (LP3). Even less often he used contributions to push the agenda forward (LP4), and had students engage in one another’s thinking (LP5; although this happened frequently in Unit 1-2).

This is very telling of Dr. DM’s instruction. He was very interested in generating student contributions. He very frequently, when eliciting these contributions, would write them down on the board and oftentimes put a “?” after them to create a level playing feel of all contributions from his students, whether correct or incorrect. Additionally, while some of the questions asked were ones from the IODE tasks themselves, he often would ask his own questions in his own way as a means to address something that he wanted to focus on or have his students think about. While students had opportunities to engage in others’ contributions as they were written on the board, they less often had opportunities to engage in others’ thinking, as Dr. DM did not tend to follow up with questions to have students elaborate on their thinking.

Essentially, after students made contributions, Dr. DM would more often move on because he got an answer that he wanted rather than probe further as to why a student may be
thinking what they were thinking. It is entirely possible (yet unknowable to me as the researcher), however, that Dr. DM was so in tune with the students in his class and the mathematics itself that he did actually know why his students were thinking along certain lines. However, LP3 and LP4 are about making explicit to the rest of the class such thinking and thus Dr. DM’s LP frequencies were reflective of the fact that the thinking was not made explicit to the class.

I now will answer the first part of the research question, “how did Dr. DM’s instructional practice unfold over his first implementation of inquiry oriented differential equations?” While I cannot make any claim that Dr. DM’s instruction changed from the beginning of the study to the end of the study, I can claim that his instructional change was supported by TIMES. As discussed across the totality of Dr. DM’s instruction his most frequent LP was LP2, eliciting student ways of reasoning and contributions. However, when comparing the four units of analyzed instruction there were stark contrasts between the frequencies of local practices. Namely, the way Dr. DM’s instruction unfolded was tied to 1) how and when he used student thinking in his class, 2) the mathematical content itself, and 3) his mathematical beliefs, rooted in his mathematical research arena.

First, in Unit 1-2, Dr. DM frequently (more often than any other unit when comparing across scaled time) engaged students in one another’s thinking. In particular, this was the unit where his students’ thinking was most at the forefront of the class and he oftentimes used that thinking to advance the mathematical agenda. When student thinking was made prevalent to the rest of the class, Dr. DM’s frequencies of local practices reflected that.

Second, as a mathematics course typically for sophomores or juniors, there is a standard of mathematical rigor associated with IODE. By that I mean much of the
mathematics is regarded as advanced mathematics, oftentimes being prerequisites for future courses. When considering Dr. DM’s frequencies of local practices in relation to the mathematical content of each unit, it was observed that Dr. DM’s instruction was influenced by the tasks presented in the units. Specifically, if a unit had more scaffolded tasks with limited options for student exploration (e.g., Unit 12), the questions that Dr. DM would ask were limited in scope and thus he used less IOI LPs in those units. In this particular case, Unit 12 was a very algebraic unit where students, being led by the teacher, develop an understanding of how to find the eigenvalues of a system of differential equation and use that information to find the associated eigenvectors and in turn the solution to the system of differential equations. Unit 10 was a similar unit (i.e., covered the same topic) but took a geometric approach to find the eigenvectors first then the eigenvalues. Consequently, when the task was more algebraic than geometric in nature, the questions Dr. DM could ask and the probing he could do was significantly impacted; specifically, all IOI LPs occurred less often in such units.

Third, when the mathematics of the unit was associated with Dr. DM’s mathematical research interests he would focus on getting students to get to “the way [he] views the mathematics” rather than having his students’ work or ideas at the center of the development of the mathematical agenda. Unit 9 dealt with the development of the phase plane which was a crucial tool in Dr. DM’s research. That unit had the highest amount of eliciting student ways of reasoning and contributions (LP2) and in comparison, a very low frequency of LP3-5 (the other practices associated with student thinking). Dr. DM was very interested in the mathematics and very knowledgeable as well. Many of the questions that he asked were of his own accord and not generated from the whole class discussion. Because he knew the
mathematics so intimately, he was most interested in getting students to see the mathematics the way he does, rather than letting the mathematics emerge from the students, while he simply guides the reinvention process (Freudenthal, 1991).

In summary, the answer to RQ1a is that Dr. DM’s instruction focused mostly on eliciting student ways of reasoning and contributions but differences across the units can be attributed to how and when he used student thinking in his class, the mathematical content itself, and his mathematical beliefs, rooted in his mathematical research arena.
Chapter 5: Results from Participation in Online Faculty Collaboration

In this chapter, I provide detailed summaries of each online faculty collaboration (OFC) that Dr. DM participated in (9 in total). This includes the intended topic for each OFC and a characterization of his role each week through his role as a speaker and listener as well as what conversations he participated in. The following section takes an overall look at Dr. DM’s participation in the OFC, including overall code frequencies of coded roles and conversations. After that I take a closer look at themes that emerged from the totality of his participation in the OFC.

Recall that OFC1 and OFC2 were introductory, OFC3-5 were lesson study 1, OFC7-10 were lesson study 2, and OFC11 was a debrief meeting. Also, recall the four speaking roles (author, pivoter, partaker, or phatic responder) and three listening roles (conversation partner, co-hearer, or over-hearer), adapted from Krummheuer (2007, 2011), that Dr. DM may assume (see Table 18). As discussed in Chapter 3, changes made to the original frameworks emerged as a means to adapt to the specific context of an online faculty collaboration (e.g., it wasn’t possible for Dr. DM to be an eavesdropper in this context).

Table 18. Speaking and listening roles (adapted from Krummheuer, 2007, 2011).

<table>
<thead>
<tr>
<th>Role</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Speaking Roles</strong></td>
<td></td>
</tr>
<tr>
<td>Author</td>
<td>Dr. DM is responsible for both the content and formulation of a statement (i.e., what is being said and how it is said)</td>
</tr>
<tr>
<td>Pivoter</td>
<td>Dr. DM takes part of the content of a previous statement and attempts to express a new idea (i.e., the content stays the same but is pivoted to a different idea)</td>
</tr>
<tr>
<td>Partaker</td>
<td>Partake means to join in. Thus, here Dr. DM forms ideas that are not new content but are a continuation of the previous conversation (oftentimes by answering a question). This can also be seen while Dr. DM is attempting to express the content of a previous statement in his own words (i.e., is responsible for form but not content)</td>
</tr>
<tr>
<td>Phatic Responder</td>
<td>Phatic means of, relating to, or being speech used for primarily</td>
</tr>
</tbody>
</table>
social or emotive purposes rather than for communicating information. Possible statements are “yes” or “I agree.” Such statements move the conversation forward without being substantive.

### Listening Roles

<table>
<thead>
<tr>
<th>Conversation partner</th>
<th>Occurs when Dr. KK responds directly to Dr. DM or when Dr. DM is a listener to whom Dr. KK seems to allocate the subsequent talking turn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-hearer</td>
<td>Occurs when Dr. DM is a listener who is not explicitly addressed by Dr. KK and chooses to participate in the subsequent conversation</td>
</tr>
<tr>
<td>Over-hearer</td>
<td>Occurs when Dr. DM is a listener who is not explicitly addressed by Dr. KK and chooses not to participate in the subsequent conversation. It is being assumed that Dr. DM heard all things said by Dr. KK</td>
</tr>
</tbody>
</table>

Further, recall that many conversation topics emerged from previous analysis of the OFCs (Keene et al., under review). Those conversation topics centered on the use of video in an online faculty collaboration. The current analysis includes all OFCs, including those centered on video and those not, and thus new codes emerged (see Table 19 for a priori and emergent codes).

### Table 19. Conversation codes (adapted from Keene, Fortune, & Hall (under review)).

<table>
<thead>
<tr>
<th>Code (* indicates emerged from this analysis)</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advice/Feedback*</td>
<td>Participants seek out advice/feedback or advice/feedback is given</td>
</tr>
<tr>
<td>Appraisal</td>
<td>Participants give value judgments about events from the videos, instructional moves, not students’ behavior or mathematical thinking</td>
</tr>
<tr>
<td>Community*</td>
<td>Comments about the community aspect of the OFC and its participants, including hospitality and greetings</td>
</tr>
<tr>
<td>Environment</td>
<td>Classroom behavior (not including the norm of student participation, e.g., student tiredness), classroom setup, student demographics, access to tools/technology (in participants’ classrooms)</td>
</tr>
<tr>
<td>Focal Instructional Components (FIC)</td>
<td>Mentioning Focal Instructional Components in general</td>
</tr>
<tr>
<td>FIC_Generating</td>
<td>Mentioning Generating student ways of reasoning (directly / indirectly)</td>
</tr>
</tbody>
</table>
Table 19 (continued).

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIC_Building</td>
<td>Mentioning Building on student thinking (directly or indirectly)</td>
</tr>
<tr>
<td>FIC_Shared</td>
<td>Mentioning Developing a shared understanding (directly or indirectly)</td>
</tr>
<tr>
<td>FIC_Connecting</td>
<td>Mentioning Connecting to standard mathematical language and formal notation (directly or indirectly)</td>
</tr>
<tr>
<td>Materials*</td>
<td>Comments on the IODE materials or course materials (e.g., homework problems)</td>
</tr>
<tr>
<td>Math</td>
<td>Discussion of mathematics or doing mathematics (not coded when conversation is exclusively about students’ mathematics)</td>
</tr>
<tr>
<td>Norms</td>
<td>Discussion about getting students’ participation; includes quality of group work and interaction</td>
</tr>
<tr>
<td>Orienting</td>
<td>Setting the stage for the video that will be shown</td>
</tr>
<tr>
<td>OFC_Manage</td>
<td>Management of OFC (e.g., someone is running late, discussing Google Drive folders, going over agenda, IRB things, plan for OFC, picking units)</td>
</tr>
<tr>
<td>OFC_TechSupport</td>
<td>OFC Tech Issues or considerations (e.g., internet connectivity)</td>
</tr>
<tr>
<td>OFC_Video</td>
<td>OFC Video - how to record videos, what clips to share/select, what to watch, remembering video</td>
</tr>
<tr>
<td>Pedagogy</td>
<td>Teacher instructional moves or ideas about teacher instructional moves. Note oftentimes this will be picked up with FICs thus this code may be used in tandem with those codes</td>
</tr>
<tr>
<td>Resources*</td>
<td>Reference to specific resources used exclusively in OFC and brought in by facilitator (e.g., technology, scholarly article)</td>
</tr>
<tr>
<td>Student Mathematical Thinking_Describe</td>
<td>Discussing students’ thinking about mathematical concepts either seen in video or that happened in class or that they’ve seen before</td>
</tr>
<tr>
<td>Student Mathematical Thinking_Desired*</td>
<td>Discussing the desired student mathematical thinking (e.g., what you want your students to think about)</td>
</tr>
<tr>
<td>Student Mathematical Thinking_Evaluate*</td>
<td>Evaluating students’ thinking about mathematical concepts</td>
</tr>
<tr>
<td>Update*</td>
<td>The facilitator asks for an update in class. Participant provides response</td>
</tr>
</tbody>
</table>

**Dr. DM’s Participation in Each Online Faculty Collaboration**

**Online faculty collaboration 1.** The main objective of OFC1 was to re-introduce everyone to each other after they met during the summer workshop. Further, the goal was to describe how the OFC will function, address any current issues/concerns, review inquiry
oriented instruction, and decide on which units will be filmed for the lesson study.

A majority of the discussion during the OFC was done by the facilitator, Dr. KK. This is unsurprising as this was the first week where the facilitator was setting goals, norms, and expectations for the OFC. Most discussion considered the management aspect of the OFC (e.g., picking which units will be filmed) or tech issues of the OFC (e.g., audio issues).

However, when discussion was not on OFC management, logistics, or tech issues, Dr. DM was a frequent pivoter of conversation topics, meaning, he did not change the topic completely from the original author comment, but he changed the direction of the conversation slightly to add his own take. For instance, when prompted to discuss the focal instructional components, Dr. KK started by asking Dr. DM to begin the discussion. In this instance Dr. KK included Dr. DM as her conversation partner. Dr. DM focused the discussion in on one particular components of inquiry oriented instruction (connecting to standard mathematical language and formal notation). Other participants chimed in and added their opinions on the necessity of this component. Another example was when he turned the discussion to a different mathematical example when discussing connecting to standard language:

1  **DM:** And sometimes we even run into things like the bifurcations when we say the saddle node bifurcation is meaningless when you first introduce it.
2  And some people prefer to call it a fold bifurcation. So, you’re right, the language hasn’t been settled in a lot of these things.

There were instances of Dr. DM being a phatic responder or partaker but more often than not, in this week, they focused on OFC management, logistics, and tech issues.

In terms of being a recipient of discussion in this OFC, Dr. DM was a conversation partner 13 times that Dr. KK was facilitating. The most common role he assumed was as an over-hearer. Again, as this was the introduction week, Dr. KK was doing a lot of general
explaining how the OFC would look which did not necessitate participant response.

**Online faculty collaboration 2.** OFC2 was planned to be the start of lesson study 1. However, because the summer workshop took place 2-3 weeks before this OFC, and they did the math of Unit 6 during that workshop, there was not a need to do the math again in the OFC. Therefore, this OFC focused on participants sharing how their classes are going and discussing an article they read, Rasmussen & Marrongelle (2006) about pedagogical content tools. Dr. KK made the conscious choice to have the participants do something different and in this case, chose to have them read an article that used examples of pedagogical content tools from inquiry oriented differential equations.

Dr. DM took the role of author 6 times, compared to 0 in week 1. He was the typical responder to the facilitator when she asked questions about the article that they all read. Other participants noted parts of the article went over their head, as an “education-jargon” article. However, Dr. DM noted multiple instances of how his reading of the article was useful in the teaching of Unit 1 for him (this will be elaborated on later in the chapter).

A majority of the times that Dr. DM was the author of the conversation in week 2 was when he was describing student mathematical thinking and/or discussion that occurred in previous days in his class. In one instance, he was evaluating that student thinking but more often than not he was describing what happened rather than evaluating or interpreting his students’ thinking.

Whereas in week 1 he focused on the instructional component of connecting, this week he focused on the instructional component of generating. The concept of generative alternatives (Rasmussen & Marrongelle, 2006) was in the back of his mind as he noted that he was pulling ideas from the class and constantly writing things down on the board. He
noted that he placed a question mark at the end of student claims as to create a level playing field of all claims and leave the opportunity open for them to be discussed.

Dr. DM was a pivoter of conversation in 5 instances in week 2, and a partaker in 25 instances. The topics were spread of a varying degree of topics. As a recipient (listener) in the discussion Dr. DM was treated as the conversation partner in 15 instances. The topics in which he was a conversation partner were varied across all topics. Dr. DM was an over-hearer less often than week 1, but this is unsurprising as the “introductory talk” that Dr. KK had to give did not take place in week 2.

**Online faculty collaboration 3.** OFC3 was the official start of lesson study 1. The facilitator asked participants over the week to work on what they consider the goals/rationale of Unit 6 and to also consider student thinking that could occur during Unit 6. There was also a good amount of logistical discussion (e.g., discussion about the assessment that was being developed for research purposes that would be administered at the end of the semester).

Dr. DM authored 4 conversation topics about pedagogy in general and student mathematical thinking. He discussed his students’ extreme desire to integrate a differential equation and how he dealt with that in his class. He was a partaker 16 times, most of which about OFC considerations (e.g., video, management). His role of a pivoter was similar to week 2 in that the conversation varied when he pivoted and there were 0 instances of him being a phatic responder in the conversation. Overall, in terms of speaking in the OFC, Dr. DM, this week had a quieter role than normal.

As Dr. DM was doing less speaking, he was doing more listening. Dr. DM was a very frequent over-hearer in the conversation. This is largely because two other participants Drs. AB and PR had a conversation that lasted a significant amount of time, just between the two
of them, with Dr. KK chiming in every so often, making Dr. DM an over-hearer in that conversation. Dr. DM did not participate in that conversation as Drs. AB and PR were sharing stories of student misconceptions and Dr. DM’s students did not share those misconceptions. Further when Dr. KK was speaking, Dr. DM was an over-hearer most frequently about OFC management.

**Online faculty collaboration 4.** Dr. DM left this OFC early (by 20 minutes) and so the conversation is only 45 minutes and all takes place before Dr. AB, a fellow participant, shared video. This week introduced the new participant, Dr. CD, and so much of the time was spent introducing him to everyone and giving him an opportunity to ask questions. However, Dr. DM was very vocal in responding to Dr. CD’s queries.

Dr. DM’s week 4 OFC, in terms of his role as a speaker in the conversation, was very similar to that of week 3. Dr. DM’s most common role in Week 3’s OFC was partaker, followed by author. He authored 5 conversation topics, some of which were about norms in his classroom. In particular, he was discussing how in IODE he has not been having students come to the board which is atypical of his instruction. His role as a partaker was mostly due to the fact that he was asked questions by other participants so he was not authoring content but responding to questions. In those cases, the topics were about mathematics but more specifically describing how his students have been thinking about the content of the course. For example, he was discussing the salty tank problem (Unit 4) and said “Yeah, and that’s where we finally started making progress, is when finally, somebody said wait a minute, what are the units of this. And then that got us into a concentration times volume per time type idea.”

As Dr. DM left early, he was very infrequently an over-hearer. If he has stayed he
likely would have been an over-hearer when Dr. AB was sharing video from her class. During the time that Dr. DM was there he was a conversation partner of Dr. KK 11 times and a co-hearer 7 times. Topics ranged from norms, to pedagogy, to OFC management. Half of the times of a co-hearer, Dr. DM was listening to pedagogy conversations. His role as a co-hearer implies that he while he was not necessarily the conversation partner, he was active in conversations centering on pedagogy.

**Online faculty collaboration 5.** This week Drs. DM and PR shared video from their classes. Thus, during the general share out in the beginning, only Dr. AB shared out. Drs. CD and ST were both late. During the time that Dr. PR was sharing video, Dr. DM did not talk much but unsurprisingly when he was sharing video from his class he led the majority of the discussion.

Dr. DM authored 9 discussion points, pivoted 6 discussion points, and was a partaker during 14 instances. Thus, this indicates that he was involved in a large portion of the discussion. There were 4 instances of Dr. DM being a phatic responder (usually when appraising something someone else said). Many of the instances of Dr. DM being an author were when he was orienting participants to what they were about to watch in the videos from his classroom. Also, when he was discussing student mathematical thinking he varied between describing it, saying what he desired his students to know or think about, and evaluating it. When discussing focal instructional components, he discussed building on student contributions twice. Dr. DM was most frequently an over-hearer. This is most likely because he did not participate in the conversation (for the most part) when Dr. PR was sharing his video. Dr. DM’s other listening roles providing no discernible patterns noteworthy of discussion.
Online faculty collaboration 6. Week 6’s OFC data was corrupted. During that meeting they began lesson study 2 and started doing the math for Unit 9.

Online faculty collaboration 7. During Week 7 OFC, they continue the discussion of doing the math of Unit 9 and how students may approach some of the problems. The meeting began with Dr. DM asking two questions, one about education research terms he just read and was not sure what they were. Second, he asked the group what they are doing with homework as the homework structure in this class has been difficult for him. There was also a discussion on one of the DE applets made in Java.

Dr. DM’s role differed this week. During the beginning when Dr. DM asked about the two aforementioned questions, they were instances of him being an author. For the remainder of the OFC, he was a frequent partaker in the discussion of mathematics. As the conversation centered on the math he did not change the topic significantly, rather, he partook in the discussion, often discussing his students, or pedagogical considerations of the Unit 9 math. Of the coded instances of DM speaking, 18 were about pedagogy and 22 were about mathematics, often coded together. As the focus of the OFC was on math that dramatically impacts the role that Dr. DM takes in the OFC.

Similarly, Dr. DM was coded 40 times as a co-hearer. This was because when solving the math tasks, Dr. KK would frequently ask the whole group to respond to her questions (i.e., Dr. DM could have responded to many of her questions). Likewise, similar to his role as a speaker, his listening conversation topics were similar as the focus of the meeting was on the math. 28 of the 40 co-hearer instances were about mathematics. Whereas 10 out of the 11 coded instances of conversation partner had to deal with pedagogy, meaning that when Dr. KK treated Dr. DM as conversation partner it was more likely to be about general pedagogy.
as opposed to when Dr. DM was a co-hearer when it was more likely to be about mathematics.

**Online faculty collaboration 8.** Dr. DM was out of town and not able to attend OFC8 (where the OFC watched Dr. AB’s videos). This absence also caused him to miss one day of instruction in the middle of Unit 9.

**Online faculty collaboration 9.** During Week 9’s OFC the OFC discussed video of Dr. DM’s and Dr. CD’s instruction. Video was watched prior to the OFC (based on a suggestion of the participants). Dr. KK was late to the OFC (she attended approximately half of the meeting).

Even though Dr. DM was sharing his videos this week, he only authored 3 unique conversation topics. Dr. DM’s most common topic of conversation was general pedagogy. He was seeking advice on his instruction because at this point in the semester he feels that his pacing needs to speed up. He asked for advice on where he could speed up his instruction. Dr. KK, who had visited Dr. DM’s classroom recently, noted that he includes many topics that she has never included as that is his area of research. This matches what was seen in the video of his instruction where he focuses the mathematical agenda on his personal experiences with the mathematics and sometimes that supersedes the students’ mathematics.

In OFC9, Dr. DM discussed on multiple occasions the focal instructional component developing a shared understanding. This was addressed because Dr. AB noted his students would oftentimes respond directly to him rather than engage with each other’s thinking. As Dr. KK was late, Dr. DM was coded less frequently as a listener and there were no discernable patterns in his listening role.

**Online faculty collaboration 10.** During Week 10’s OFC, participants watched Dr.
PR’s videos. They also discussed an article that Dr. KK had emailed to them during the week. The article, Rota (1997), was about ten lessons from teaching differential equations and the participants discussed if in 2015 the issues were still present and if they applied to IODE.

Overall, Dr. DM spoke less than he did in previous weeks (this was one of his “quietest” weeks). However, 13 out of the 30 coded instances of Dr. DM’s discussion were about pedagogy. His continued focus is on his (and others’) instruction. Again, in this OFC there is reference to how Dr. DM uses his mathematical beliefs about differential equations to influence his teaching. In response to the Rota article he said,

1 DM: I was going to say, so I have to object on not teaching the uniqueness of solutions theorem. Existence theorem, I am fine with not teaching it, I usually don’t, but the uniqueness theorem is how I view all phase planes and phase lines. My students make it a point to try and say uniqueness theorem every day just to make me happy. So, I can’t agree with getting rid of it. Because it is too much of a conceptual piece.

There is a similar spread to the previous workgroup in that Dr. DM role was almost even as author, pivoter, and partaker, and rarely as a phatic responder. In terms of a listening role, Dr. DM was very frequently an over-hearer, which is consistent with the fact that much of the OFC was Dr. KK speaking with Dr. PR about his videos.

**Online faculty collaboration 11.** OFC 11 was the final OFC of the semester and served as a debrief for the participant’s experiences. All participants were individually interviewed about their experiences with TIMES overall, so the debrief for the OFC focused solely on the OFC. Dr. KK sought out suggestion on how to improve IODE and the OFC in future years to come on the grant.

As Dr. KK was leading most discussion, Dr. DM never authored conversation topics. Rather he was a pivoter and partaker in the conversation (3 and 26 times, respectively).
Consistent with a majority of the semester, he talked about pedagogy and mathematics, but in this case, he focused particularly on the IODE materials providing suggestions on possible ways to restructure the homework. His listening roles also occurred during the same conversation topics; he was a conversation partner 12 times, a co-hearer 6 times, and an over-hearer 24 times.

**Dr. DM’s Overall Participation in the Online Faculty Collaboration**

In this section, I describe Dr. DM’s overall participation in the online faculty collaboration. Namely, I discuss how his participation was consistent throughout the semester. Dr. DM joined TIMES eager to learn about new pedagogical practices he could use in his IODE classroom and thus his participation was reflective of that. Following this section, I take a closer look at his participation.

Table 20 includes the overall code frequencies for speaker and listener roles taken on by Dr. DM in each OFC (note Phatic Responder was shortened to Phatic and Conversation Partner is abbreviated CP).

Table 20. *Dr. DM’s overall code frequencies for speaker and listener.*

<table>
<thead>
<tr>
<th>Role</th>
<th>OFC1</th>
<th>OFC2</th>
<th>OFC3</th>
<th>OFC4</th>
<th>OFC5</th>
<th>OFC7</th>
<th>OFC9</th>
<th>OFC10</th>
<th>OFC11</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Speaker</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Author</td>
<td>0</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>9</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Pivoter</td>
<td>6</td>
<td>5</td>
<td>7</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Partaker</td>
<td>17</td>
<td>25</td>
<td>16</td>
<td>13</td>
<td>14</td>
<td>20</td>
<td>10</td>
<td>4</td>
<td>26</td>
</tr>
<tr>
<td>Phatic</td>
<td>18</td>
<td>8</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td><strong>Listener</strong></td>
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<tr>
<td>OverHearer</td>
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<td>48</td>
<td>22</td>
<td>6</td>
<td>17</td>
<td>7</td>
<td>11</td>
<td>12</td>
<td>24</td>
</tr>
</tbody>
</table>

Dr. DM’s most frequent speaking role was when he was a partaker in the conversation. Being a partaker means that he continued on with existing conversation, either changing the form of what was said but not the content or was responding directly to a question. Dr. DM’s most
frequent listening role was an over-hearer in the conversation. This makes sense as Dr. DM was not the only participant in the OFC the facilitator, Dr. KK, could have spoken to other participants and in those instances, it was likely that Dr. DM was coded as an over-hearer.

In addition to these roles, Dr. DM participated in various conversations. Figure 15 includes all conversation codes. Note, focal instructional component was abbreviated FIC and student mathematical thinking was abbreviated SMT. Further, the darker the cell that means the higher the frequency of coded instances, for ease of observation. Lastly, recall that there is no total summing down a column because that would double code instances of being an author, pivoter, etc.
<table>
<thead>
<tr>
<th>Conversation Topic</th>
<th>Speaker</th>
<th>Listener</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Author</td>
<td>Pivoter</td>
</tr>
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<td>AdviceFeedback</td>
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<td>Appraisal</td>
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<td>3</td>
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<td>Community</td>
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<td>0</td>
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<td>Environment</td>
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<td>0</td>
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<tr>
<td>FIC</td>
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</tr>
<tr>
<td>FIC_Generating</td>
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<tr>
<td>FIC_Building</td>
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<td>FIC_Shared</td>
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<td>FIC_Connecting</td>
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<td>Materials</td>
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<td>Norms</td>
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<td>OFCTechSupport</td>
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<tr>
<td>SMT_Desired</td>
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<td>5</td>
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<td>SMT_Evaluate</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Update</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

*Figure 15. Dr. DM’s conversation codes by speaking and listening roles.*
Here we see that pedagogy and mathematics were conversations that Dr. DM participated the most in by speaking, and pedagogy, materials, math, and OFCManage were conversations that Dr. DM listened the most to. This figure will be discussed in further detail in the following section and in particular the conversation codes will be categorized into four themes.

**A Closer Look at Dr. DM’s Participation in the Online Faculty Collaboration**

In this section I take a closer look at Dr. DM’s participation in the online faculty collaboration through the use of excerpts from the OFCs. First, I describe how Dr. DM’s participation was overwhelmingly active throughout the semester, rather than passive. Namely he was frequently an active participant (which I am considering as author, pivoter, partaker, conversation partner, or co-hearer) and less often a passive participant (which I am considering as phatic responder or over-hearer). Second, I look at what he was talking about when speaking or listening in his varying roles. Specifically, I address the four overall conversation themes and how Dr. DM’s participation looked across those conversations.

**Active and passive participation.** As noted I am defining active participation to be the speaking roles of author, pivoter, or partaker and the listening roles of conversation partner and co-hearer. By their definitions they imply that the participant, Dr. DM, made conscious decisions about his participation (e.g., authoring or pivoting conversation, partaking in existing conversation, or being a co-hearer and inserting himself into the conversation). Contrariwise, passive participation consists of solely phatic responses or over-hearing and not choosing to participate in the conversation. In this subsection, I provide frequencies of active and passive participation and bring in data from the OFC and interviews for support.
To hone in on an overall characterization of Dr. DM’s participation, Figures 16 and 17 combines the author, pivoter, and partaker frequencies as well as conversation partner and co-hearer into active speaking and listening roles, respectively. Meanwhile, phatic responder and over-hearer are classified as passive speaker and listener roles. These figures clearly indicate an overwhelmingly active participation in the OFC by Dr. DM.

**Figure 16.** Dr. DM’s active versus passive speaking roles.

**Figure 17.** Dr. DM’s active versus passive listening roles.

A few anomalies in Figures 16 and 17 are in the listening roles, OFC7 has a higher active role than any other week. That week was the one week where the participants were
actually doing mathematics, so most of those frequencies were co-hearer roles of Dr. DM doing mathematics with his fellow participants. In the same figure for passive roles, OFC1 was very high but as noted that was an introductory week so Dr. KK was doing a lot of introduction into what the OFC would look like which did not require active participation on the part of Dr. DM. Similarly, while OFC2 was Dr. DM’s most active speaking week, it was a passive listening week because a new participant joined that week so there were many conversations that he was simply an over-hearer in.

To illustrate what Dr. DM’s active participation looked like, below are two excerpts of active speaking and listening. The first excerpt is from OFC2. It begins with Dr. KK reminding everyone of the article they were supposed to read, Rasmussen and Marrongelle (2006), and asking Dr. DM to start. Recall all participants in the OFC are Dr. DM, Dr. AB., Dr. CD., Dr. PR, and DR. ST; the facilitator was Dr. KK and there were two graduate research assistants, GRA1 and GRA2. Similar to previous excerpts names will be noted in bold and italics (e.g., DM) and in bolded brackets will be the coded role of Dr. DM.

1    KK: So, we can take these [questions] one question at a time or if you have something you want to say about it, just jump in. I know that DM, I’ll you to start, just because I know you said that you actually used one of these tools when you were doing that, the second page of the first unit. Do you want to share? [Active; Conversation Partner]
2    DM: Yeah, so, uh, one of the classroom examples for this was the second page of the first set of materials on what do the graphs of the populations look like? And, um, they used that, I had just read this, so that really helped, and it really helped get the conversation going in a good way because I had them draw the graphs and then as I walked around, um, there were two major themes that I saw. One was about half the groups had kind of the right graph where on the t=0 axis they had different slopes going up. But then a lot of those kind of joined together at some mysterious asymptote. And the other groups had like what I’m looking at in figure 1 where they were flat initially. And kind of rose at similar rates. And, so, I put those on the board, I said, okay I’m seeing graphs that look like this and like this and I put them on the same graph and I had a conversation about can anyone defend these sets, right? And had them go back and forth a little bit on that. And they got kind of
stuck at some point because they kind of both felt, like the whole class could see, they kind of refused to agree on one. They agreed to agree on well maybe both are possible. And that got us into the conversation of what exactly do these requirements on the population, continuous reproduction, and unlimited resources, what do those really mean? And, eventually, we got to the point where I was using this idea of just writing up their thoughts and just leaving them on the board. [Active; Author] … But this idea of having just their thoughts and then continuing to update that section of the board. And then as their ideas built up about what that really means and is time really a factor? Does it matter when a population reaches 30, in terms of what the rate of change will be? Once a student asked that question, right, looking at trying to differentiate between the graphs that really helped catapult us forward. It was kind of like that question unlocked something. [Active; Pivoter]

KK: So, some of your students actually said does time really matter? Wherever it gets to 30, same rate of change. [Active; Conversation Partner]

DM: Yup! [Passive; Phatic Responder]

KK: That’s nice. [Active; Conversation Partner]

DM: That came about because I kept, I wrote, I surveyed them for some suggestions about what things ought to look like I guess. And what features would they expect. And they eventually settled on that. So, I was very happy with that. [Active; Partaker]

In this excerpt, Dr. DM began by authoring his experience of reading the article and the impact that it had on his instruction. He also continued the conversation through pivoting (the … indicated transcription from other participants not included) and partaking in the conversation set up by Dr. KK. In this second excerpt, from OFC7, participants are doing the math of Unit 9 led by Dr. KK. We see here, while Dr. DM’s role is still active, when the conversation focuses on math, his specific roles differ slightly.

KK: For each of the equilibrium solutions in the previous problems, create your own terms to classify the equilibrium solutions and briefly explain your reasons behind the terms. So, this is that exact activity that we did in the other class. Those of you who teach this, what is this called? Do you know. [Active; Co-Hearer]

DM: It’s a center. [Active; Partaker]

KK: It’s a center, right. So, if you have a situation where you have one point in the middle and then any other place that you start this. Let’s change these to 2 to get a different view. It’s a center because you have all of these solutions that are laying around the center. I like this view. [Active; Co-Hearer]

AB: My students called it an orbit.

KK: Orbit. That’s nice. What about the other one? The (0,0) one? So, if you
put the (0,0) on here and generate that approximation and plot it. So, (0,0) is
down here, it is green. Oh boy, you’re never going to be able to see green. Try
to find different color. Oh well. It is green. So, what did you call that
one? [Active; Co-Hearer]

AB: I didn’t focus on that one as much.

KK: Yes. Okay. Anybody know what that one might be called? [Active;  
Co-Hearer]

DM: That’s a saddle. [Active; Partaker]

KK: It’s a saddle. Why? [Active; Co-Hearer]

DM: Yeah, because there is one trajectory, there is one, if I start with, 0
rabbits then the fox population is just going to decay down to that 0, but if I
start with 0 foxes, I’m going to leave it. So, I have solution coming into it and
one solution leaving it. We call that a saddle. But I don’t know how they
would ever come up with that. [Active; Partaker]

KK: So, that’s an example of another solution. So, one thing that I like, and if
you could do this with AB with yours too, is that students can kind of see
about this. Again, you may choose to eventually name these, and if you aren’t
comfortable of letting them play with it. I think I had students call this a
repeller, the (0,0), whenever something got close to it, it repelled, and they
were trying to use the same words again. [Active; Co-Hearer]

PR: So, I remember my students saying unstable.

KK: I think unstable is a good word for it. [Active; Co-Hearer]

PR: Because I remember they kept on saying stable and unstable, so, I guess
the (0,0) is unstable.

DM: What’s a really fun phenomenon of this model, I’ll probably have to
shoehorn in, you can say them what do you think happens if I start with a very
small number of rabbits and a very small number of foxes? [Active; Pivoter]

KK: Nice, that’s a good question. [Active; Conversation Partner]

DM: Like .01 and .01. [Active; Partaker]

KK: Okay, so I’m going to do that just for the heck of it. I’m going to clear the
graphs and we will put .01 and .01. Oh, look at that! Now, can you see what
happened when I did it? [Active; Co-Hearer]

AB: Yeah.

DM: Yeah. So, you get these huge oscillations. [Active; Partaker]

In the above excerpt, we see most of Dr. DM’s active participation is partaking the
classroom conversation about the math, which also led him to be coded as a co-hearer because while he
was not explicitly addressed he oftentimes joined in on the conversation about mathematics.

In one instance, he pivoted to an analogy of a saddle point and shoehorn (lines 37-39). In the
exit interview, which took place approximately two months after the final OFC, Dr. DM
mentioned how much he missed having those weekly conversations and how much he got out of them, further indicating his desire for active participation.

DM: Yeah. The weekly conversations were very important, in fact I miss them. … I had a [strong] connection with PR, AB, and ST. That really helped in watching the videos. But on the other hand, my first experience with IBL was at Academy for IBL Workshop, where they use videos quite a lot. And these are videos of professors that you’ve never met. Just having that to see how the classroom was structured was really helpful for me the first-time learning, so, I think that I would get something out of it but I think having that once a week component really, really helped. …Like I said, it was a time to check in on everybody but also just see what could come up. It helped that I was behind. So, I got the benefit of everyone else’s trailblazing.

While Dr. DM’s participation was overwhelmingly active, the majority of instances in which he was coded as passive, were passive listening roles (over-hearer) rather than passive speaking roles (phatic responder). These exchanges would occur when Dr. DM did not interject himself into conversation (either by his own choice or Dr. KK was treating someone else as her conversation partner). Below is an excerpt from OFC2 where Dr. KK is having a conversation with Dr. AB. In this instance, Dr. AB was treated as Dr. KK’s conversation partner.

KK: So, AB, I was really enjoying what you had to say about, but my question that I was asking when you said you couldn’t hear me. Was how did you approach the terminology, like the \( \frac{dy}{dt} \) and \( \frac{dx}{dt} \) if somebody didn’t know what a derivative was? [Passive; Over-Hearer]

AB: So, I had a little section that said that \( \frac{dx}{dt} \) is just short for how \( x \) changes as \( t \) gets bigger. So, I gave them sort of a mantra to use. Whenever you see this symbol, say this out loud in your head.

KK: Yeah. Which should work right? And a lot of ways I wish that everybody thought that. When we were writing these materials [IODE], a lot of times, we had noticed how often he used rate of change equation. So, hopefully over the course of the semester it kind of morphs into differential equation, but at the beginning we call it rate of change equation. [Passive; Over-Hearer]

It is also important to look at what Dr. DM is speaking about when he takes on these roles. The following subsection takes a closer and deeper look at the intersection of the roles
Dr. DM took on in the OFC and what he was speaking about during those instances.

**Conversation categories.** In this section, I take results from Figure 15 and condense them to paint a picture of Dr. DM’s participation based on the conversation. Recall that the goals of the OFC were to help participants understand the IODE materials (*mathematical issues*), anticipate student thinking (*student issues*), and watch video of themselves teaching and seek feedback on their instruction (*pedagogical issues*). Thus, all conversation centered on these three issues. Thus, here I consolidate codes along these three issues. *Mathematical issues* are a combination of mathematics and materials codes. *Student issues* are a combination of all student mathematical thinking codes as well as environment codes. *Pedagogical issues* are a combination of the pedagogy code, all focal instructional component codes, and the norms code. Additionally, while not a goal of the OFC, there was also the underlying component of what it means to be in an online faculty collaboration. That is, the community needed to be a supportive environment for the participants to share and grow in their pedagogical practice and mathematical understandings. Further, there was also conversation relative to being part of a research project. Thus, this fourth conversation category is called *OFC issues*. Here *OFC issues* are a combination of OFCManage, OFCTech, OFCVideo, Orienting, Appraisal, Advise/Feedback, Community, Resources, and Update.

To focus in on these four conversation categories, Table 21 combines conversation codes according to the aforementioned scheme. Table 22 combines the roles further to be simply active and passive roles for the four conversation categories. Following these tables are excerpts that illustrate how Dr. DM’s participation varied based on the conversation category.
Table 21. Dr. DM’s conversation category frequencies by speaker and listener.

<table>
<thead>
<tr>
<th>Conversation Category</th>
<th>Speaker</th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
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<td>Author</td>
<td>Pivoter</td>
<td>Partaker</td>
<td>Phatic</td>
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<td>Mathematical Issues</td>
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<td>Student Issues</td>
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<tr>
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<table>
<thead>
<tr>
<th>Conversation Category</th>
<th>Listener</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Conversation Partner</td>
<td>Co-Hearer</td>
<td>Over-Hearer</td>
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<td>23</td>
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</tr>
<tr>
<td>OFC Issues</td>
<td>49</td>
<td>42</td>
<td>156</td>
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</table>

Table 22. Dr. DM's conversation category frequencies by active or passive roles.

<table>
<thead>
<tr>
<th>Conversation Category</th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>Passive</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>Mathematical Issues</td>
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<td>40</td>
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<tr>
<td>Student Issues</td>
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<td>2</td>
<td>20</td>
<td>23</td>
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<tr>
<td>OFC Issues</td>
<td>97</td>
<td>24</td>
<td>91</td>
<td>156</td>
</tr>
</tbody>
</table>

Here we see that in all speaking topics, Dr. DM took an active role, particularly with pedagogical, mathematical, and student issues. While the OFC issues category was active in the speaking role, his listener role for that conversation category was more passive. This is noteworthy because it shows that for OFC issues he chose to not participate in the conversation as often as he did for other conversation categories. This provides further context to Figure 17. To provide context for what conversations looked like in these four categories, I include excerpts and further details on the conversation.

**Pedagogical issues.** Dr. DM’s most frequent active speaker conversation was about pedagogical issues, and it was his second most active listener conversation. When looking at the listening roles for pedagogical issues in Table 22 we see a more active listening role as well indicating that if the conversation was about pedagogy, Dr. DM was more likely to
insert himself into the conversation than if it was about student issues or OFC issues.

Here is an excerpt from OFC5 when Dr. DM was sharing video; it begins right after all participants watched the first clip in the OFC. In this excerpt conversation codes are also listed (even more than just pedagogy as they oftentimes overlapped).

1 **KK:** So, DM, you’re question to us was did you bring up the uniqueness theorem too early? [Active; Conversation Partner; Advice/Feedback]
2 **DM:** Yeah, I never really gave them a chance to raise the issue. And it is possible that they would have gotten there but I didn’t want to spend the whole class like arguing over it. So, I kind of said, in my head, let’s just bring it up because we are going to be hitting them over the head with this over and over. So, that’s kind of my question, I jumped right back to it because I had that agenda almost. But I don’t know what you guys think about that I guess. [Active; Pivoter; Pedagogy, Advice/Feedback]
3 **KK:** So, I’m going to ask a question. It felt like if you stopped at 10:15 we didn’t actually get to the uniqueness where you put up h to the 1/3. [Active; Conversation Partner; OFCVideo, Mathematics]
4 **ST:** Yeah, we did. I saw it up to 11 minutes, I saw the \( \frac{dh}{dt} = -h^{(1/3)} \).
5 **DM:** Yeah, I just brought it up. I didn’t say, I didn’t bring up the uniqueness theorem at that point but I said here is an example of DE that we studied in the uniqueness theorem. [Active; Partaker; Pedagogy]

Part of the pedagogical issues code were all focal instructional component codes. These conversations were important to the OFC as they were the focal point of the intended direction of instruction change, that is, inquiry oriented instruction. Developing a shared understanding was Dr. DM’s most talked about focal instructional component. While not necessarily saying it by name, there were numerous instances of him talking about how he could get his students on the same page. For instance, in the following quote he talks about how his ideal instructional environment is when students can validate or agree with other students without his interjection.

1 **DM:** I was going to say I think the kind of the holy grail is to have the answer to the question what do you when the student gives the right answer, is can the rest of the class validate that answer? You don’t want to be the authority on all things that way. The real goal is to be able to turn that back on the class like PR said, ask why. You’re smiling on the inside and now you’re seeing if the
rest of the class can attack it until they realize, oh no this is going to stand.

Further, when he was discussing, in OFC2, the article they read, he described in detail how it affected his instruction and many of the focal instructional components emerged.

DM: Yeah, so, uh, one of the classroom examples for this was the second page of the first set of materials on what do the graphs of the populations look like? And, um, they used that, I had just read this [Rasmussen & Marrongelle (2006)], so that really helped, and it really helped get the conversation going in a good way because I had them draw the graphs and then as I walked around, um, there were two major themes that I saw. One was about half the groups had kind of the right graph where on the t=0 axis they had different slopes going up. But then a lot of those kind of joined together at some mysterious asymptote. And the other groups had like what I’m looking at in figure 1 where they were flat initially. And kind of rose at similar rates. And, so, I put those on the board, I said, okay I’m seeing graphs that look like this and like this and I put them on the same graph and I had a conversation about can anyone defend these sets, right? And had them go back and forth a little bit on that. And they got kind of stuck at some point because they kind of both felt, like the whole class could see, they kind of refused to agree on one. They agreed to agree on well maybe both are possible. And that got us into the conversation of what exactly do these requirements on the population, continuous reproduction, and unlimited resources, what do those really mean? And, eventually, we got to the point where I was using this idea of just writing up their thoughts and just leaving them on the board. [Active; Author; FIC_Build, FIC_Shared, FIC_Generate; Pedagogy, Resources, SMT_Describe; SMT_Evaluate]

Mathematical issues. Mathematical issues were Dr. DM’s third most participated conversation both as a speaker and listener, and in both cases, was more active than not. Mathematical issues are the combination of mathematics and materials codes. One of the components of the OFC was to ensure that the participants fully grasped the concepts of the IODE materials. As discussed in the instructional practice part of this chapter, Dr. DM has very strong mathematical content knowledge, specifically in differential equations as that is where his research lies. His participation in the OFC is reflective of that in the sense that he did not need any help on the materials or mathematics themselves, which was atypical of the
participant body. Consequently, his mathematical issues conversation was either simply talking about his understanding of mathematics or the materials (oftentimes suggestions on how to improve them). In the following excerpt, we see Dr. DM partake in a conversation about Dr. AB’s students. Note that he is not asking questions about mathematics, rather he is displaying his knowledge of the subject.

1 DM: Yeah, and one of the nice things about talking about the tangent line is that this $dF/dR$ taken as the ratio, is ambiguous. Because if they are both positive, or if they are both negative, you get the same slope and you don’t know if it is going up and right or left and down. So, usually, when I teach students about this, I tell them just to check one of them, if I am on this loop, and if I know $dR/dt$ is negative in this case, I must be going left and by virtue, I must be going up also. So, that is AB’s idea, saying, well, this is really, instead of looking at it as the ratio, think of it as going left and up. And left and down. [Active; Pivoter; Mathematics, Pedagogy, SMT_Desired]

2 KK: Oh, so that is what you are talking about the vector addition thing?

3 AB: Right.

4 KK: So, the $dR/dt$ and $dF/dt$ and then adding them up. [Active; Co-Hearer; Mathematics]

5 DM: Sort of saying, how are you go left and how far you go up. Or down. Wherever we are. [Active; Partaker; Mathematics]

Student issues. Student issues were the least participated in conversations by Dr. DM, but they were also the least talked about conversation across the entire OFC. There are two takeaways from Dr. DM here. First, unlike pedagogical issues and mathematical issues, Dr. was more often a passive listener than an active listener in the conversation revolving around students. I posit this to be the case because Dr. KK oftentimes shifted conversations to students in certain instances and in doing so it was possible that Dr. DM was an over-hearer in those instances. Second, Dr. DM would more often describe his students’ thinking.

Most of Dr. DM’s discussion of his students’ thinking came during the general share out. Whereas weeks when the OFC watched videos, conversation was more about his pedagogical issues. Below is an excerpt from an early OFC where Dr. DM describes what his
students did on a particular task.

DM: Because when tasked with the question, is this a solution to the DE? This group of students did something I hadn’t really seen before. Which is, they plugged in everywhere where \( y \) was in the equation. They plugged in the function, except for the \( dy/dt \). So, they plugged in only on the RHS. And then, integrated it, the [right hand side] RHS, and then recovered \( y = \) whatever it was, \( t + c \). And then they said oh I guess \( c \) has to be 1 for this to work.

ST: They are integrating with respect to what?

DM: With respect to \( t \). So, the equation was like \( y^2 - 1 \over t + 1 \) or something like that. And so, when you check to see if it’s solution, you plug in the RHS and you check to see if the derivative matches the simplified version. So instead they simplified the RHS so they got \( dy/dt = 1 \) and then they integrate and they get okay that’s \( t + c \), okay yes of course, this is a solution, look I can make it work.

OFC issues. Lastly, while not a goal of the OFC, the OFC issues category were inevitable conversations in an online supportive faculty collaboration revolved around reforming instruction. Recall OFC issues are a combination of OFCManage, OFCTech, OFCVideo, Orienting, Appraisal, Advise/Feedback, Community, Resources, and Update codes. An overwhelming proportion of listener codes were coded as passive here, most of them from OFCManage, OFCTech, and OFCVideo. While it was clear Dr. DM was in the OFC to enhance his pedagogical practice, he did enjoy the supportive community that the OFC provided. He concluded his exit interview reflecting on his experience in the OFC and teaching by saying:

DM: Just I really enjoyed it. Obviously, I’m excited to be working with the group over the summer and next semester again. So, out of the gate I knew that this was something great. And it was great every week, I really looked forward to the meetings. I looked forward to class every day. I was really beaten down by my schedule and the things I set out for myself but the DEs class, it energized me three days a week. I’m so tired, this is going to be a terrible class and I come out of it on Cloud 9. It was great.

Summary of Answer to Research Question 1b: Participation in OFC

How did Dr. DM’s participation unfold in the online faculty collaboration?
To summarize the answer to this research question, I situate the findings in the context of the goals of the online faculty collaboration. Recall the goal of the OFC was to support cohorts of mathematicians as they came to learn about inquiry oriented instruction and in this case also inquiry oriented differential equations and use it as the foundation of their reformed instructional practice. Specifically, the facilitator of the OFC led the participants through two different modified Japanese lesson studies (Demir et al., 2013; Lewis et al., 2009), one on IODE Unit 6 and one on IODE Unit 9. For those lesson studies participants first focused on ensuring they had a good grasp of the mathematical content itself as it may have been different from the way they have seen it before or learned it as a student. Second, the participants discussed ways to anticipate student thinking through video watching of outside instructors, reading research articles, discussing their personal experiences, or discussing inquiry oriented instruction in general. Finally, the participants focused on enhancing their instructional strategies by watching videos of themselves and their fellow colleagues actually teaching the units of focus. When sharing self-selected video clips the participants either sought feedback on what could have been improved, highlighted an exchange with student(s) that they thought went well, or focused on specific focal instructional components of inquiry oriented instruction; the choice of which rested with them. To answer this research question, I looked at how Dr. DM participated in the OFC (i.e., what roles did he take on) and then I looked at further at how his role looked in the four different themes of the OFC (pedagogical issues, mathematical issues, student issues, or OFC issues).

Rather than growth throughout the semester, Dr. DM immediately jumped into the active role in the OFC and that active role was consistent throughout the semester. Dr. DM
most often was a partaker in the conversation, but he also was an author and pivoter of
certain conversation topics as well. Similar to his classroom instruction there was not a
change but rather how his role looked depended on the content of each OFC. For example, if
the week focused on doing mathematics, he rarely authored topics because he simply was
partaking in the conversation, however, he was very active in those weeks as he has a real
passion for mathematics. Additionally, when the OFC focused on sharing of his videos, he
authored frequently those weeks and the conversation focused on pedagogy as he sought
advice on, for example, how to speed up his class because he was running out of time at the
end.

It became clear, through observations of the OFC and interview data, that Dr. DM
had strong mathematical content knowledge and did not need support for that from the OFC.
Rather, he explicitly would ask his fellow colleagues questions about pedagogical issues
related to inquiry oriented instruction as that was what he wanted to grow in: his teaching.
Even though Dr. DM was most active in pedagogical issue discussions, he was also active in
the other three conversation categories (mathematical issues, student issues, and OFC issues)
to varying degrees.

In summary, Dr. DM’s participation in the OFC was consistent, active overall across
all conversation categories, but in particular was active in pedagogical issues as that was his
focus or goal of being in the OFC. Namely, Dr. DM joined TIMES because he wanted to
learn how a differential equations classroom could be structured to center around deep
student engagement of the mathematics. During this process, he came to learn about inquiry
oriented instruction and to this day is still very passionate about IOI and has become a
facilitator of three additional OFCs.
Chapter 6: Discussion and Conclusion

The purpose of this dissertation was to explore the interplay between one mathematician’s instruction and his participation in an online faculty collaboration geared towards reforming instruction, as well as, explore both of those constructs individually. The study addressed the following research question, with sub research questions:

1. In what ways does one mathematician’s experiences in an online faculty collaboration on inquiry oriented differential equations relate to his instructional practice?
   a. How does his instructional practice unfold over his first implementation of inquiry oriented differential equations and in what ways does it align with inquiry oriented instruction?
   b. How does his participation unfold in the online faculty collaboration?

In this chapter, I discuss answers to the overall research question relating Dr. DM’s experiences in the faculty collaboration to his instructional practice, situated in relation to existing research. Furthermore, I discuss implications for the mathematics and mathematics education communities. Lastly, I conclude with numerous future research directions and an overall conclusion.

Overall Research Question: Relation between Instruction and OFC

In what ways did Dr. DM’s experiences in the online faculty collaboration on inquiry oriented differential equations relate to his instructional practice?

Very rarely was there sufficient data to say, for example, ‘in time point A in the OFC, \( x \) happened and that led to \( y \) happening in Dr. DM’s instruction’ or ‘in time point B in Dr. DM’s class \( z \) happened that he brought into the OFC at time point C.’ However, what can be considered is a holistic look at Dr. DM’s experiences as a participant in TIMES in relation to how he participated in the OFC and the form his instruction took. In this section, I discuss
five ways that Dr. DM’s instruction was related to his participation in the OFC. These relationships are situated in the context of existing research literature.

**Mathematics background.** Dr. DM’s mathematics background was a bridge between how his instruction panned out throughout the semester and how he participated in the OFC. In both cases his mathematical content knowledge (rooted in his background and research interests) was placed on top of his interest in enhancing his pedagogical practice. By that I mean, in his teaching his view of mathematics sometimes was the view of mathematics that he was guiding his students towards. Likewise, in his participation in the OFC, his mathematical understanding was one of the driving factors for his interest in enhancing his pedagogical practice. Namely, he had a deep geometric understanding of differential equations and sought support on how he can get his students to that same level of awe and understanding.

His mathematical background affected the way he used student thinking in his classroom and how he talked about student thinking in the OFC. When the mathematical content aligned with his research interests he was less likely to use his students’ thinking to advance the mathematical agenda. And similarly, when discussing his students in the OFC, oftentimes it was about how he wanted them to view mathematics, not about what they actually did in their class or their thinking about the mathematical content.

This conclusion, in particular the classroom setting conclusion, supports previous work form Speer, Wagner, and colleagues (Speer & Wagner, 2009; Wagner et al., 2007). In their work, they considered the concept of analytic scaffolding necessary for mathematicians to facilitate whole class discussions in inquiry-driven classrooms. They considered analytic scaffolding to be how one supports the mathematical goals of discussion. They remarked,
“Gage’s [their participant] analytic scaffolding … was met with only limited success, despite his strong understanding of the mathematical content, clear vision of the learning goals for the lesson, and commendable ability to elicit contributions from students” (Speer & Wagner, 2009, pp. 558–559).

In this quote, numerous parallels can be made between Gage and Dr. DM. Firstly, both had strong understanding of the mathematical content. Second, both had a clear vision of the learning goals. Third, both were very able to elicit contributions from students. Recall that Dr. DM’s most used IOI LP was LP2, eliciting student ways of reasoning and contributions. Note that at the time Speer and Wagner were conducting their analyses in 2009, the IOI framework did not exist yet. The construct of analytic scaffolding is one in which recent researchers have used to develop the inquiry oriented instruction local practices (Kuster et al., 2017). Also note that in their work their mathematician was using the same IODE curriculum as Dr. DM did in this study.

While the findings from this dissertation support findings found from Speer and Wagner’s work, there are important distinctions that sheds light on this topic and provides discussion for faculty collaborations going forward. The first difference is the amount of data in each study. Recall this study analyzed four units and approximately 11 class days of Dr. DM’s instruction. Speer and Wagner (2009) focused their choice of video data on when Gage said a discussion did not go well and in total they focused on 2 days of whole class discussion. They remarked that “these episodes are by no means representative of the full set of Gage’s practices and are not meant to be illustrative of his teaching as a whole” (Speer & Wagner, 2009, p. 539). Second, and most importantly, is the subtle notions of what a mathematician’s mathematical content knowledge is. In their work, Speer and Wagner noted
that Gage had a strong understanding of the mathematical content but that did not help in terms of his analytic scaffolding. Similarly, Dr. DM also had a strong understanding of the mathematical content across all units. However, the difference lies in the fact that in some units he was able to provide analytic scaffolding, namely, he was able to use his students’ ideas in the class (LP3: actively inquiring into student thinking, LP4: being responsive to student contributions, LP5: engaging students’ in one another’s thinking, LP6: guide the mathematical agenda). Yet, he was more likely to do that when the mathematical content was not his specific research interest. Consequently, I concur with Speer and Wagner and posit that one’s mathematics background, in the case of Gage and Dr. DM, is not sufficient to successfully use student thinking in one’s class, however, the level to which one understands that content makes a difference in their instruction.

Gage was not in a faculty collaboration for the study he participated in. Dr. DM was. Dr. DM’s participation in the OFC gave him additional supports that were necessary to be successful in implementing inquiry oriented curriculum. Where the OFC could have been improved relates to how faculty address their own understanding of the material and content itself; something the OFC did not do outside of the ‘doing mathematics’ part of the lesson studies in the OFC. The implications for this will be discussed later in this chapter.

**Tension between agenda and inquiry.** This first theme contributes to the second theme: there is a tension between what a mathematician wants to do in her/his class and inquiry oriented instruction. In the case of Dr. DM, his focus, for some of the content from the course, was to get his students to his view of the mathematics. This ultimately leads to a tension between one’s teaching agenda and inquiry. If in inquiry, student thoughts are central to the development of the mathematical agenda (Kuster et al., 2017), then imposing one’s
own view of mathematics does not align with an inquiry perspective. The reason this causes a tension is because being passionate about your research inherently is not a bad thing, nor trying to get your students to see the beauty of mathematics. However, in so doing, one privileges their understanding over that of their students. This tension is not one seen in previous literature and was not something considered by the grant designers and OFC facilitators.

**Anticipating student thinking.** A third theme that relates Dr. DM’s instruction to his participation in the OFC considers how one anticipates student thinking and how student thinking is used in instruction. We know from earlier research that mathematicians often struggle to implement novel teaching (new to them) and in particular struggle with how to respond to and deal with student contributions in a productive and successful way (Wagner et al., 2007). In Wagner and Speer’s work, the third author of the article was the instructor of an IODE class. Not only did he struggle to deal with student contributions, he was most often not able to predict what his students would do or think about (i.e., he was unable to anticipate his students’ thinking). However, this was not an issue for Dr. DM as he was in an OFC supporting his instruction. He never noted that he was unsure what his students were going to do. Yet, he seldom actively inquired into his students thinking. This indicates he either knew what his students were thinking or simply did not probe into their thinking.

We also know from extant literature on the possible successes of anticipating student thinking in professional development settings (e.g., Dyer & Sherin, 2016; Lewis, Perry, & Friedkin, 2011). In the OFC, participants spent 1-2 weeks for each lesson study doing the mathematics of the units of focus and then anticipating how their students may approach the tasks. This model was based off of the Japanese lesson study (Demir et al., 2013; Lewis et
al., 2009), which I called modified Japanese lesson study.

In this modified Japanese lesson study, IODE OFC participants were not developing materials, as they were supposed to use the IODE materials, but rather they were solving the tasks and anticipating ways students may engage with the tasks. They were told not to pretend they are students but rather solve the tasks as themselves, for the first time, so they may experience what a student feels first engaging with the task.

Many scholars who have done research with lesson studies have argued that anticipating student thinking was a critical component of their lesson study. For example, Lewis et al. (2011) argue that anticipating student thinking was important for their teachers, in particular anticipating possible various student solution strategies to given problems. Further, Dyer and Sherin (2016) argued that being responsive to student thinking makes one a sophisticated teacher. Also, in relation to instructional change, Lewis et al. (2009) found that during and following a lesson study teachers did make small changes to their instruction.

Further, Perry and Lewis (2009) found four categories of change that emerged from lesson studies with teachers. These four categories were incorporation of feedback from reflection, development of tools, focus on student thinking, and use of outside knowledge sources. Specifically, in terms of the category ‘focus on student thinking,’ they found that, at first, while watching video, teachers focused on student thinking only at a superficial level. They then argued that it was oftentimes hard for teachers to anticipate student thinking because “not all lessons include ‘thought-revealing tasks’ that enable observers to study student thinking” (Perry & Lewis, 2009, p. 377). This issue, however, is not an issue for the majority of the IODE materials as most are thought-revealing tasks.

Lastly, Ball and Cohen (1996) argue that curricula should be rooted in student
thinking. This philosophy aligns with Realistic Mathematics Education (Freudenthal, 1991) and in particular how the IODE materials were developed. Rasmussen’s early work (e.g., Rasmussen, 2001; Rasmussen & King, 2000) centered around collecting a corpus of information on how students approach differential equations content and used that information to design the IODE materials.

In most of the aforementioned work about anticipating student thinking, the curricula that was being used was standard K-12 curricula, even if being taught in inquiry-driven settings. Ultimately, I conjecture there is a tension between anticipating student thinking and inquiry oriented instruction. By that I mean that because a critical component of inquiry oriented instruction is inquiring into student thinking and engaging with unexpected contributions, if student contributions are overwhelmingly “anticipated,” mathematics faculty may struggle to engage with those unexpected contributions if they spend a large amount of time anticipating what their students will do.

In the case of Dr. DM, one possible explanation for why Dr. DM did not often actively inquire into his students’ thinking or engage with unexpected contributions was because the OFC spent a significant amount of time focusing on anticipating the student thinking that might emerge from his class. Consequently, it is possible that anticipating student thinking in the OFC took away from Dr. DM’s opportunity to actively inquire into his students’ thinking or engage with unexpected contributions. Specifically, because he spent time anticipating his students’ thinking he often did not consider the possibility of unknowable student contributions and the uniqueness of individual students’ perspectives and experiences.

In conclusion, while there is documented success of teachers anticipating student
thinking in professional development settings (e.g., Lewis et al., 2011, 2009; Perry & Lewis, 2009), certain types of inquiry instruction may be at odds with that strategy. Finally, there is also an equity argument to be had here. Which students’ ideas form the basis for the student thinking when anticipating student thinking in professional development or faculty collaboration settings? Who decides which students’ ideas are the ones to share? Inquiry environments are fraught with exciting possibilities for students to engage with mathematics, in their own way; we, as educators and researchers, cannot claim to know every way that a student may approach a certain problem. If we continue to anticipate student thinking, we must ask ourselves whose mathematics are we putting in the spotlight (Gutiérrez, 2002) and how can our inquiry be “critical inquiry” (Rasmussen, Marrongelle, Kwon, & Hodge, 2017)?

**Active participation in faculty collaborations.** The fourth theme centers on active participation in the OFC positively impacting instruction. As discussed, Dr. DM was an active participant in the OFC. Additionally, his goals were clear in that he was there to enhance his pedagogical practice. Dr. DM’s passion for differential equations and inquiry instruction bled into both his instruction and participation in the OFC.

This OFC was an example of what Borrego and Henderson (2014) defined as a faculty learning community. “STEM undergraduate instruction will be changed by groups of instructors who support and sustain each other’s interest, learning, and reflection on their teaching” (Borrego & Henderson, 2014, p. 233). Dr. DM was supported by and supported his fellow colleagues in learning about and reflecting on inquiry oriented instruction. This indicates that for successful instructional change, faculty learning communities or faculty collaborations need to be designed with ensuring active participation from all involved. Doing so would help ensure the long-term success of the faculty’s teaching endeavors.
Possible options are to play on the passions of mathematicians (or faculty in general). As discussed, Dr. DM was very passionate about IODE to the point that in some instances it affected how his instruction looked. This passion could be used in productive ways in faculty learning communities or faculty collaborations to bring various faculty together with either all the same passion or varying passions or interests.

**Reading research articles.** The final theme is less broad, yet it has large implications. In one of the OFCs, Dr. DM mentioned how reading a mathematics education research article supported his instruction that same week (see OFC2 in Chapter 4). Previous research has highlighted the importance of reading resources in professional development settings (e.g., Hayward et al., 2015; Perry & Lewis, 2009). These readings ranged from “curricular materials” (Perry & Lewis, 2009, p. 382) to the “reading and discussion of research articles” (Hayward et al., 2015, p. 62) with no further details on what those exactly were.

In the case of Dr. DM, the article he read, Rasmussen and Marrongelle (2006), was about pedagogical content tools that a teacher can use in their class (transformative records and generative alternatives). More specifically, the content used as examples of what pedagogical content tools are came from an IODE context as the first author of that article is the main author of the IODE curriculum. Even more specifically, it focused, in part, on the same unit that Dr. DM was teaching that week. Dr. DM found that particular discussion useful because it gave him insights on how he can highlight student work through the use of transformative records and generative alternatives. Interestingly, this also was Dr. DM’s highest coded frequency of using LP5, engaging students in one another’s thinking. Dr. DM specifically mentioned in the OFC how this article aided in his instruction.
Consequently, rather than a general argument for the inclusion of ‘research articles’ in professional development settings, the context of those research articles cannot be overlooked. By context I mean, where the research was situated. Namely, a differential equations OFC should focus on a mathematics education article that conducted work in an undergraduate differential equations course, not a high school calculus classroom. Here it was very important that the context of what Dr. DM, as a trained mathematician and not mathematics educator (i.e., familiar with content but not familiar with education jargon or methodology), was able to not be bogged down by extraneous detail and was able to learn about pedagogical tools that he could use in his classroom.

Further, this act of reading research articles is one aspect of scholarly teaching (Borrego & Henderson, 2014; Henderson et al., 2011). Learning about the research related to your specific instruction is what it means to be a scholarly teacher. Dr. DM was in the OFC to enhance his pedagogical practice by collaborating with colleagues. Important to him in that process was learning about the research conducted in this arena, not just hearing about personal experiences from his colleagues. The use of research based articles, aligned with the context of a professional development, supports instructor interest and the instructional change process as a whole.

**Implications**

In this section I provide implications from this work for the mathematics education research community, those developing and facilitating professional development with faculty and the community of mathematicians.

**Mathematics education research community.** In 2010, Speer and colleagues remarked on the under reporting on collegiate faculty’s instructional practice (Speer et al.,
Since then a handful of studies have conducted work on faculty’s instructional practice (e.g., E. Johnson et al., 2013; E. Johnson & Larsen, 2012). This dissertation is an example of reporting on a faculty’s instructional practice with an important distinction: how much instruction was analyzed. One implication for the mathematics education research community is that when trying to relate experiences in faculty collaboration to instruction, or just categorize instruction overall, instruction must be analyzed on a large scale (possibly even larger than that of this study) if we want to actually get an accurate representation of one’s instruction. In this study, there existed trends across the whole semester but differences among units, thus if I had only looked at one or two of the units then the picture that I would have painted of the participant’s instruction would have been woefully incomplete (cf. Speer & Wagner, 2009). Instruction is a complex act; thus, research must be done at a scale that captures that complexity.

Additionally, the mathematics education community can learn from mathematicians’ passion and how to use that passion effectively. By effectively I mean that if the mathematics education community hopes to continue instructional change research they need the support of mathematicians. Finding mathematicians who are passionate and active in the teaching part of their job can be important as they can act as change agents for reform in undergraduate mathematics education (Borrego & Henderson, 2014).

**Facilitators of faculty professional development.** This work indicates the fruitful and possible success of faculty collaborations and their ability to support instructional change. This is not to say that faculty collaborations change instruction, but rather, for faculty interested in reforming their pedagogical practice, collaborating with their colleagues about their day-to-day teaching is beneficial. Consequently, these faculty collaboration
structures should be implemented in more and different contexts (e.g., in person faculty collaborations) and with more and different mathematical contents (e.g., abstract algebra, linear algebra, calculus). We see from this work that this faculty collaboration was impactful for the participant; yet, we know ways in which they can be improved.

First, mathematical beliefs need to be addressed in the faculty collaboration. While we know from previous work that doing the mathematics in the faculty collaboration is important (Andrews-Larson et al., 2016), the faculty collaboration did not address the participant’s varying mathematical backgrounds. This may be a difficult subject to address in that mathematicians are often considered (and often do) have strong mathematical content knowledge, but learning where they are most strong in their content knowledge can shed light on where they may need the most help in being open to student thinking.

Second, there is a tension between IOI and anticipating student thinking. Facilitators of faculty professional development must work closely with the mathematics education research community and mathematicians to continuously improve how student thinking is used in faculty collaboration settings that is true to the spirit of inquiry-driven instruction.

Third, for faculty collaboration to be a successful method of supporting instructional change, the network must grow. This work highlighted how an active participant noted personal success with reforming his instruction and went on to become a future facilitator for the remaining years of the grant. While more research is needed to know if this is true in more cases, it provides facilitators of faculty collaborations with starting points of who can become involved in the change process.

Lastly, the participant in this study saw positive outcomes when reading a mathematics education research article that was from the same teaching context as his
teaching endeavors. Rather than general statements such as ‘read research articles in professional development settings,’ facilitators of faculty professional development should note the importance of those research articles being contextually synonymous with the context of the professional development.

**Mathematics community.** There are also implications for the mathematics community. First, and most notably, they should talk with their colleagues (at their own institution or elsewhere) about their instruction. This work has highlighted that collaborating on instruction is not something that normally happens in mathematics departments; this is often why participants joined TIMES. Talking with colleagues about instruction is important (in formal faculty collaborations or even informally in the office) as it can support faculty’s pedagogical practice.

Second, mathematicians must be aware of their own mathematical beliefs. If they have strong content knowledge in an area they may desire to push their understanding of said content on to their students. While this is well-intentioned, research has shown the importance of students work being the relevant mathematics (Kuster et al., 2017). Lastly, mathematicians who devote time and are active in the aforementioned endeavors will reap the benefits more than ones who are less active. Research has highlighted how mathematicians need to see instructional change efforts work to believe they will work in their classroom (Walczyk et al., 2007). Therefore, I argue that actively seeking out these resources and actively seeking to collaborate on instruction will provide the necessary ‘existence proof’ of the success of inquiry-driven instruction.

**Future Research**

There are numerous avenues for future research based on this work. First, TIMES has
additional data from the following year of Dr. DM’s instruction (not a full semester). TIMES has conducted some work on first and second implementations of IO curricula (e.g., Andrews-Larson & McCrackin, 2018); this work specifically focused on facilitation of student argumentation. A larger question can be asked about the instruction of the second implementation of IO curricula, such as, how does Dr. DM’s instruction in his second implementation compare to his first implementation of IODE? Additionally, TIMES has a large corpus of data from approximately 40 instructors from differential equations, linear algebra, and abstract algebra. The same study conducted here can be done with a different participant or multiple participants.

Within this future research direction are smaller avenues based on who the participant is that is selected. A possible direction I could take is to choose a participant who identifies as more pedagogically strong than mathematically strong (to contrast with the participant from this study), as TIMES had a few participants who were mathematics educators by trade as opposed to mathematicians by trade. Additionally, a participant could be chosen who does not feel strong in their pedagogy or mathematics. In all of these cases, what support do these participants need?

As discussed, a plethora of research indicates the importance of anticipating student thinking. It seems quite natural to prepare to teach by thinking about how one’s students would approach a task. However, is this technique truly at odds with inquiry oriented instruction? This work here indicated that there is a possible tension between these two constructs. Future research is needed on what it means to anticipate student thinking and how that truly prepares a mathematician (or faculty in general) to use that student thinking in their class. Is it possible to engage with unexpected contributions if all (or even some)
contributions have been anticipated?

Furthermore, recall the participant from this study went on to be a future facilitator for the project. TIMES had three other participants who did the exact same thing (one other in differential equations, one in linear algebra, and one in abstract algebra). I intend to look at speaking and listening roles while these faculty were participants in the OFC (not facilitators), in particular active versus passive roles and compare to these three other participants and see if there is a connection between how they all participated. If so, this can be further evidence of how to target future facilitators in a growing network of faculty who desire to collaborate on instruction.

Likewise, I intend to ask the question ‘is it better to have someone who is stronger in mathematics as a facilitator or someone who is stronger in pedagogical approaches?’ We had four mathematics education facilitators and three mathematics facilitators (with varying degrees of mathematics education experience) over the course of TIMES. Does their facilitation make a difference in how the online faculty collaboration plays out? Does their facilitation impact participants’ instruction in different ways? Do different participants benefit from facilitators with different kinds of expertise?

**Conclusion**

The purpose of this study was to explore the interplay between one mathematician’s instruction and his participation in an online faculty collaboration geared towards reforming instruction, as well as, explore both of those constructs individually. Namely, this study addressed 1) how a mathematician’s instructional practice unfolded over one academic semester while he was part of this faculty collaboration for undergraduate mathematics instructors on inquiry oriented differential equations, 2) how his participation developed in
the faculty collaboration, and 3) how his instructional practice and participation in the online faculty collaboration related.

Results indicated the participant was very capable of eliciting student ways of reasoning and contributions from his class but he less often actively inquired into their thinking, either because he had already anticipated their thinking, or was not interested in it because his mathematical opinion took precedence over that of his students. Results also indicated his participation in the online faculty collaboration was overwhelmingly active which he indicated helped him in supporting his instructional endeavors.

There also existed an interplay between his instruction and participation in the online faculty collaboration. First, both instruction and participation connected to some degree to the participant’s mathematical research and mathematics content knowledge. Second, there existed a tension between a mathematician’s agenda for his classroom and purpose in the OFC with inquiry oriented instruction as a whole. Third, anticipating student thinking in the OFC potentially took away from the unknowable student contributions and the uniqueness of individual students’ perspectives and experiences while teaching IODE. Fourth, active participation in the OFC connected to specific desires of instructional change (i.e., pedagogical support). Fifth, there was one specific instance of experience in the OFC directly correlated to supporting instruction, namely, reading a research article situated in the context of IODE.

This area of research is ripe for future research. The instruction of undergraduate mathematics courses is a hot button item in undergraduate mathematics education research today. More importantly, the research community still needs to know more about how we can support endeavors to reform instruction, how can they be scaled up, and how do we measure
success? In this qualitative instrumental case study, I conclude that the online faculty collaboration supported Dr. DM’s efforts to reform his instruction. Even though there is no evidence to claim that Dr. DM’s instruction changed throughout the study, there is evidence to frame how Dr. DM was supported by the OFC during his instructional change process. Instructional change is a large-scale endeavor. Dr. DM joined TIMES to learn about how to run a differential equations class from an inquiry oriented perspective. Collaborating with colleagues on instruction supported Dr. DM’s instructional change.

Finally, this work provides insight how OFCs can be improved moving forward and most importantly highlights that instructional change is possible if the time and effort are put into it. First, OFCs have potential to be scaled up once we know more how to successfully anticipate student thinking while being true to the spirit of inquiry oriented instruction. These OFCs can become in person faculty collaborations that departments can set up on their own to support instructional change that is very important in today’s mathematics community (MAA, 2015b). Lastly, this work lays the foundation for future work to continue to demonstrate that instructional change is possible. I posit that these ‘small’ online faculty collaborations hold immense power in the instructional change field; this power must be harnessed to its fullest potential. If time and effort are put into the instructional change process, all stakeholders, such as faculty, administrators, and most important students, will reap the benefits.
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http://doi.org/10.2307/40539355


APPENDICES
## Appendix A: Details on Chosen Video Clips from Classroom Instruction

<table>
<thead>
<tr>
<th>Unit</th>
<th>Topic</th>
<th>Dates</th>
<th>Video Clip Times</th>
<th>Total Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>Qualitative and graphical approaches &amp; Numerical Approaches</td>
<td>150914, 150918</td>
<td>150914 [0:00-51:39 end] 150918 [0:00-50:03 end]</td>
<td>87.9833 minutes</td>
</tr>
<tr>
<td>6</td>
<td>Autonomous differential equations</td>
<td>151005, 151007, 151009</td>
<td>151005 1 [17:20-26:42 end] 151005 2 [0:00-23:34 end] 151007 1 [0:00-28:54 end] 151007 2 [0:00-20:49 end] 151009 1 [0:00-21:15]</td>
<td>104 minutes</td>
</tr>
<tr>
<td>9</td>
<td>Systems of differential equations</td>
<td>151030, 151102, 151104, 151109, 151111</td>
<td>151030 1 [0:00-27:45 end] 151030 2 [0:00-19:48 end] 151102 1 [0:00-29:55 end] 151102 2 [0:00-20:40 end] 151104 1 [0:00-28:00 end] 151104 2 [0:00-20:45 end] 151109 1 [0:00-27:00 end] 151109 2 [0:00-23:55 end] 151111 1 [0:00-26:30 end] 151111 2 [0:00-20:25 end]</td>
<td>244.7167 minutes</td>
</tr>
<tr>
<td>12</td>
<td>Eigentheory applied to linear systems</td>
<td>151120, 151123, 151130</td>
<td>151120 1 [20:00-26:00 end] 151120 2 [0:00-20:30 end] 151123 1 [0:00-28:10 end] 151123 2 [0:00-24:14 end] 151130 1 [0:00-15:00 end]</td>
<td>129.7333 minutes</td>
</tr>
</tbody>
</table>

**Total** Approx. 9.44 hours
Appendix B: TIMES Entrance Interview, Differential Equations, Fall 2015

Before turning the audio recorder on:

1. Thank the participating instructor for their time.
   a. First of all, I just want to say thank you so much for agreeing to work with us on this study, and for taking the time to meet with us today. We know you’re very busy, and your insights are extremely valuable to us.

2. Explain the purpose of the interview:
   a. The goal of this study is to learn how best to support instructors of undergraduate mathematics in effectively implementing inquiry oriented instruction and design instructional and curricular support materials for these instructors. The settings in which people teach, the student populations they serve, and their views of teaching and differential equations play an important role in how individuals interpret and implement specific instructional materials. This interview aims to capture information about these important contextual features from your point of view, as a partnering instructor on our project.

Start audio recorder:
It is (date) at (time). This is (interviewer’s name) and I am interviewing ()

Differential Equations Teaching Experience & Content Expectations

I want to begin by asking you some questions about your background specific to teaching differential equations.

1. What experience do you have teaching differential equations?
   a. How many times have you taught differential equations? (If they have taught multiple different differential equations courses, probe primarily on the most introductory level course they have taught.)
   b. If they have taught multiple different differential equations courses at the same institution, ask: In your view, how do those courses fit together?

2. Can you tell us a little bit about the specific differential equations course you’ll be teaching this fall?
   a. What do you see as the role of this course in the curriculum (across their math classes) for students who take it?
   b. Tell me about the students that typically take this course. (e.g., math majors? Math ed? Engineering? Biology? Unprepared students? Etc.)
   c. Are there any specific prerequisites for this course?
   d. What course or courses does this differential equations course typically prepare your students to take next?
   e. What percent of students typically pass this class?

3. What textbook will you be using this fall?
   a. If they’ve taught before, could ask: Is this the same text you’ve used in the past? If no, probe on why they switched from one text to the other.

4. Are there any expectations from your department or client disciplines (e.g., engineering, biology) about what should be taught in the course?
5. Is your department chair supportive of efforts to try new instructional approaches? In what ways is s/he (un)supportive?

6. Are others in your department supportive of efforts to try new instructional approaches? In what ways are these colleagues supportive/unsupportive?
   a. Have you had any experiences in which your colleagues were resistant to efforts you or others have made to teach in innovative ways? If so, can you give me an example of such an experience?
   b. To what extent have students at your institution had opportunities to learn math in courses that are not lecture based?
   c. Have you experienced any student resistance to attempts you’ve made to teach in innovative ways? Can you give me an example?
   d. How important are student evaluations to how your department measures your job performance?

7. What do YOU see as the most important ideas/topics to be taught in this class?
   a. What do you anticipate will be the most challenging aspects of the course for students?
      i. Why do you think those aspects will be challenging?
   b. If instructor’s previous response focused on general issues like symbolism or struggles with proof/formality, ask: What content do you think will be most challenging for students?
      i. Why do you think that content is challenging for students?

Typical Math Instruction & Experience Using Group work
Now I’d like to ask you a few questions about your instruction.

1. When you teach, what does a typical day in your class look like? (If teacher says he/she teaches different classes differently, probe on most innovative class. If relevant, probe how this is different from when he/she teaches differential equations.)

2. How do you typically plan for class? (What is your process?)
   a. When you’re planning how to teach a new topic, what are the top things that you try to keep in mind?
   b. How does that inform your instructional decisions? (e.g. in making decisions about setting up the syllabus, teaching particular topics, or in making day-to-day decisions)

3. How often do you have students do small group work during class time?
   a. What is the point of group work? (e.g. students practice? hear different approaches before instructor formalizes? etc)
   b. What kinds of problems do you typically have students work on in small groups?
   c. What challenges did students experience?
   d. What challenges did you as instructor face in trying to teach using this format?

4. Did you typically have a whole class discussion after students do small group work?
   a. What did that discussion look/sound like?
   b. In your view, what role does that kind of discussion play in supporting students’ learning?
c. What do you do when a student says something very different from what you might expect?
d. If issues arise, ask about use of student presentations (e.g. do they use these? do they think of them as part of whole class discussion or as separate from those? who do those fit into their instruction?)

5. How do you decide what homework to assign or what you expect students to do outside of class?
   a. In your view, what role does homework play in supporting students’ learning?

Thank you for participating in this. Do you have any questions?
Appendix C: TIMES Exit Interview, Differential Equations, Fall 2015

Before turning the audio recorder on:

1. Thank the participating instructor for their time.
   a. First of all, I just want to say thank you so much for agreeing to work with us on this study, and for taking the time to meet with us today. We know you’re very busy, and your insights are extremely valuable to us.

2. Explain the purpose of the interview:
   a. The goal of this study is to learn how best to support instructors of undergraduate mathematics in effectively implementing inquiry oriented instruction and design instructional and curricular support materials for these instructors. The settings in which people teach, the student populations they serve, and their views of teaching play an important role in how individuals interpret and implement specific instructional materials. This interview aims to capture information about these important contextual features from your point of view, as a partnering instructor on our project.

Start audio recorder:
It is (date) at (time). This is (interviewer’s name) and I am interviewing ().

1. At the beginning of the semester, we asked you about your goals of participating in this project. Can you tell me a little about what you were thinking you may get out of participating in this project? To what extent do you feel like your goal(s) was met?
   a. *How did you hope to change your instruction? Why did you want to change?*

Questions about instructional supports (curricular materials, online workgroups)

2. To what extent did you use the <IODE/IOAA/IOLA> curricular materials?
   a. How effective do you think these materials were in supporting student learning?
   b. To what extent did you need to make additions or adjustments? *<follow up on the nature of additions/adjustments and reasons for them>*
   c. Do you think you’ll use these materials more or less in future semesters? Why?
      i. *If they plan to use the materials in the future: How do you think using the materials will be different for you in future semester?*
   d. What challenges did you experience in using these materials?
      i. *If not already addressed: Did you experience any challenges aligning the materials with homework, exams, and/or your course textbook?*
   e. How did you use the instructor support materials to plan for instruction?
      i. Did you read the instructor notes? Did you look at examples of student work and approaches?
      ii. Did you actually work out the tasks ahead of time, or did you tend to just look at them to get an idea? (If you worked them, did you work them using just one solving strategy, or did you work them in more than one way?)
   f. *Did the materials change the way you think about teaching this*
content? About students’ learning of the content? If so, how?

3. Let’s talk about what you spent time doing during our weekly workgroups, as well as if and how those were useful for you. Can you give me a sense of what happened in the OWG? What kinds of things did you talk about?
   a. To what extent did you talk about the math? How useful was this?
   b. To what extent did you talk about the student thinking? How useful was this?
   c. To what extent did you use video clips? How helpful were the clips?
      i. Was there anything about this use of video clips that made you uncomfortable?
      ii. What could be done to make use of clips more helpful?
   d. To what extent did you talk about the four focal instructional practices in your particular workgroup (eliciting student contributions, building on student contributions, developing a shared understanding, and formalizing)? Was this useful, and why or why not?
      i. Were there aspects of instruction that were not discussed that you think it would have been helpful to discuss?
   e. To what extent was it helpful to spend time at the beginning of each session talking about general issues?
   f. If you could change one thing about the online workgroups, what would it be?

4. What is the most useful thing you took away from participating in this OWG?
   a. Do you feel your instruction has changed as a result of OWGs? If so, how?
   b. Have your goals for instruction changed at all? If so, how?

Questions about Instructor Orientation and Goals (and social networks)

5. Can you identify any ways in which your experience teaching <DE/AA/LA> has been different from previous semesters?
   a. How have your ideas changed in terms of student thinking and how students learn?
   b. How have your views changed with regard to your role as the instructor?
      i. How did student thinking influence the instructional decisions you made this semester?
   c. What do you see as the purpose of small group work?
   d. What do you see as the purpose of whole class discussions?
      i. How did you navigate the interplay between small group and whole class?
   e. Have you made any changes to the ways in which you assess students’ understanding of course material?
   f. What do you want your “A” students to learn from your <DE/AA/LA> class? What do you want your “C” students to learn? Has this changed from the past?

6. My next question may seem a little odd, but research has shown that teachers’ professional networks have really important implications for their teaching. Because this has important implications for the future work of this project, I’d like to ask:
   a. Who do you go to for advice about math instruction? (Ask for actual names, institutions, how they know that person, why they go to them, and what kinds of things they seek advice about...)
7. When you think back about your experiences with the materials, online working groups, and summer workshops, what stands out to you?