ABSTRACT

REID, AUSTIN RANDOLPH. Helium-3 Polarization and Origami Folded Cryogenic Actuators and Metamaterials. (Under the direction of Paul Huffman.)

We want to know how the stuff everywhere we can see came to be. At the beginning of time, there was a lot of light, and later the things that would be the world and stars and sky turned from light into stuff. That isn’t a full picture, because light becomes stuff and not-stuff which turn back into light when they touch! When we look anywhere, we don’t see not-stuff. We want to know where it all went. Stuff and not-stuff are different, and it is easier for light to become stuff.

The language of stuff, not-stuff, and light is called the “Standard Model,” and it doesn’t let enough light become stuff, so idea people have added new parts to make more stuff. New parts act on everything, so we can tell if the idea people are wrong by looking at other things. That’s why we’re looking for an “electric dipole moment,” which is one way that a “neutron” can talk to light. The SM says it is tiny, but most new parts make “nEDM” much larger.

The world has two kinds of funny voice air: the heavy kind and the light kind. We use the light kind, because it can find little problems in our looking boxes. When you put lots of the light kind in a box, their hearts point in all directions. Funny voice air is clear, but we want it to talk to light. We play a radio and make some of the funny voice air very excited and some of them become stuck, or “metastable.” These excited ones can see the light then change clouds with ground state ones. With light and a radio, you can make the hearts in a cloud of funny voice air point the same way. This is called “MEOP.” The light funny voice air will go back to normal when you stop pushing them. By timing how long it takes the hearts to go from pointing the same direction back to normal, you can see how the cloud touches itself and its bottle.

Bending a piece of paper changes it. With many bends, you can make it into something very different. Most people use this bent paper art (“origami”) to make a land or sky animal or a pretty colored growing thing that a girl might wear in her hair. I like pretty things, but I want to make the world better. People have already used paper art ideas in space cars. We can use use paper art to build big houses that are strong and light, or we can use it to make better things that fix peoples insides.

If you roll paper into a long round thing, it can still make some shapes with flat faces, but it can't make as many kinds of things as it could before you rolled it. We make special shapes on these long round things to change how they can move. Certain shapes can pop between two states, while others can be made so that they do not pop. Some are not straight on the side. These things with not straight bends are special and very strong. When you press the ends together, all of it moves slowly, and will not break even when cold. I will tell you how to make one of these, and how we tried to break them. By looking at how we could and couldn’t break them, you might find new ways to use them.
Helium-3 Polarization and Origami Folded Cryogenic Actuators and Metamaterials

by
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DEDICATION

To Emily.
BIOGRAPHY

The author was born in a small town that happens to be full of nuclear physicists. After graduating from Oak Ridge High School, he attended Davidson College, where he majored in Physics with a minor in mathematics, while singing on the side.

After completing his BS, he fulfilled a childhood dream by going racing: A few years playing NASCAR was enough, and he happily returned to the nurturing womb of academia to pursue a physics PhD.
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Without the talents of craftsmen, none of this could have been built. Thanks to Richard Nappi at the Duke Physics Instrument Shop for teaching me how to work in a machine shop—this would have taken at least a year longer had I not been able to make my own prototypes. Thanks to Frank Milkowski, Chris Hewett, and Terry Briggs at the NCSU College of Sciences Instrument shop—your ability to turn our most half-baked prototypes and drawings into usable devices made all of my bellows work possible. Thanks to Mike Gnann and Merge Scientific—my rudimentary glassblowing is enough to make a prototype, but you’re the one we trust to run a torch next to the DR.

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<td>Automated Current Control</td>
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<td>BAU</td>
<td><strong>Baryon Asymmetry Of The Universe</strong>. A measure of how much more prevalent baryonic matter was relative to baryonic antimatter in the early universe. Written as $\eta \equiv \frac{B - \bar{B}}{B + \bar{B}} \approx \frac{n_B}{n_{\gamma}}$.</td>
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<td><strong>Electric Dipole Moment</strong></td>
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<td><strong>Function Generator</strong>. NI VirtualBench function generation subsystem.</td>
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<td>GePD</td>
<td><strong>Germanium Photodiode</strong>. Photodiode sensitive to 1083 nm pumping light. Used to observe light scattered from metastables, which is used to calibrate laser.</td>
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<td>ILL</td>
<td><strong>Institut Laue-Langevin</strong>. Neutron science center in Grenoble, France. Home of Steyr turbine, most recent laboratory to successfully measure a null EDM.</td>
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NMR  Nuclear Magnetic Resonance .................................................. 11, 129
OFHC  Oxygen Free High Thermal Conductivity Copper ................... 73, 75
PDG  Particle Data Group. The Particle Data Group (PDG) is an international collaboration that provides a comprehensive summary of Particle Physics and related areas of Cosmology: the Review of Particle Physics [Pat16]. ........................................... 1
PEEK  Polyether Ether Ketone. Thermoplastic superpolymer that can be machined or molded to high degree of precision, with a CTE that is similar to PMMA. viii, x, xi, 73, 75, 76, 108–110
PMMA  Poly(methyl Methacrylate). Commonly known as acrylic, PMMA is a brittle but extremely clear plastic. UV-transparent acrylic sheets are used to form $^3\text{He}$-nEDM sample cells. .............................................................. xii
PWS  Power Supply. NI VirtualBench subsystem with three discrete DC power supply channels, capable of delivering $-25\text{ V}$, $6\text{ V}$, and $25\text{ V}$. ......................................................... 181
QWP  Quarter Wave Plate .......................................................... 27
SEOP  Spin Exchange Optical Pumping ........................................... 16, 17, 30, 53
SiPD  Silicon Photodiode. Photodiode used in optical polarimeter. This particular Thor-Labs model has a high fixed gain, a low noise level, and a relatively slow response. 27, 65, 183, 186
SOS  Systematic And Operational Studies At PULSTAR. Next generation assembly commissioned as part of the UCN-$^3\text{He}$ nEDM collaboration to study systematic effects ix, 11–14, 17, 63, 68, 73, 76, 98, 104, 109, 110, 114, 116, 117, 181
SQUID  Superconducting Quantum Interference Device ..................... 11, 68
TPB  Tetraphenyl Butadiene. Electroluminescent dye, used in the full-scale apparatus to shift UV photons from $^4\text{He}$ scintillation to 430 nm blue light. ....................... x, 11, 73
TUNL  Triangle Universities Nuclear Laboratory. Originally Duke's Nuclear Structures Laboratory, now a facility collaboratively staffed by graduate students and faculty from NC State, Duke, UNC-Chapel Hill, and NC Central .................... 11, 116
UCN  Ultra Cold Neutron. A free neutron with $T < 360\text{ neV} (v < 8\text{ m}^{-1})$. Can be stored in a material, magnetic, or gravitational bottle of appropriate design. ...... xii, 8–12, 128
In the beginning the Universe was created. This has made a lot of people very angry and been widely regarded as a bad move. — Douglas Adams, *The Restaurant at the End of the Universe*

This work is motivated primarily by an interest in deepening our knowledge of how the universe came to be. More accurately, this work intends to provide some clarity to the question of where the universe’s antimatter went. A brief aside will help clarify the interest in so esoteric a topic.

### 1.1 Dinner Table Cosmology

In order to talk about antimatter, it’s easiest to start by discussing matter, as it lacks the “anti” prefix that is so troubling. We call our stuff “matter,” but what makes an “electron” different from a “proton” or a “neutron?” We can tell that those different particles are different because of their force interactions. Leptons (including electrons and neutrinos) have a somewhat small mass and appear to interact via the electromagnetic, weak, and gravitational forces. Baryons (such as protons and neutrons) have far more mass than the leptons, and interact by the same forces, as well as the strong nuclear force.

We measure intrinsic properties using those aforementioned force interactions. With a clever experiment, it’s possible to probe mass, electric charge, spin, or any other number of useful quantities that are recorded in Particle Data Group (PDG) books [Pat16] so future generations don’t have to repeat those clever experiments.

But even that just starts us in the right direction—we need to know how these particles interact with one another. For example, combining an electron with a proton yields a single hydrogen atom. Combining (carefully, of course) a proton, a neutron, and an electron gets you deuterium. An isolated neutron, left to its own devices, will decay into an electron, a proton, and an electron anti-neutrino. There are several contributing processes that have to be summed to calculate decay rate, but here we can just picture the process illustrated in fig. 1.1.
Imagine, for a moment, a particle that is almost identical to one of our ordinary pieces of “matter.” It has the same mass, same spin magnitude, but an opposite charge. Combining this particle with its counterpart yields pure light.

This sort of particle is what we call “antimatter.” It’s almost exactly the same as “matter,” but it, when combined with the right “matter” particle, can turn into pure energy. By convention, our stuff is called “matter” because we knew about it first. Our universe started with a surplus of energy that condensed into matter and antimatter. Then, that matter and antimatter recombined and mostly annihilated, somehow leaving behind the matter that we observe and a background glow of photons. Because this process ended with more baryons than it started with, and with baryons instead of anti-baryons, this asymmetry is called the Baryon Asymmetry of the Universe (BAU), and is indicated by \( \eta \):

\[
\eta = \frac{B - \bar{B}}{B + \bar{B}} \approx \frac{n_B}{n_\gamma}. \tag{1.1}
\]

If the laws of the universe had perfect matter-antimatter symmetry, \( B = \bar{B} \), and \( \eta = 0 \). Instead, \( \eta = 6.08 \times 10^{-10} \) [Dub11]. Although there is some matter-antimatter asymmetry included in the Standard Model of physics, it evidently does not contain enough, because \( \eta_{SM} \approx 10^{-18} \).

By now the word symmetry has come up enough to need definition or clarification. A system with a particular symmetry is the same after that symmetry is acted on. Geometric examples are easiest to visualize: a snowflake, for example, looks the same under rotations of 60 degrees, under
reflection, or with any combination of those operations.

Figure 1.3 An ideal snowflake has several different symmetries. It has a rotationally periodic symmetry, as well as several axes of discrete reflectional symmetry.

A brief side note on the synthesis of physical theories and their subsequent experimental vetting is due, as the process by which symmetries have been explored can otherwise appear ad-hoc.

It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience. [Ein33]

Or, in other words, “make things as simple as possible, but no simpler.” Including symmetries in the theory makes them dramatically simpler, and the ultimate triumph of the Standard Model is attributable to the discovery of powerful and deep symmetries. Before those came to be, a handful of simple symmetries were individually explored, and ultimately individually discarded, although they have found redemption in their combined effect.

Many features of the world that we take for granted can be traced to continuous symmetries (rotation: conservation of angular momentum and ice skaters, time: conservation of energy) [Noe18], constructing a fundamental theory of physics requires consideration of three discrete symmetries. These symmetries are called discrete because they cannot be generated smoothly. Just like how you can't be a little bit pregnant, you can't be a little bit reflected.

First, consider Charge conjugation, which takes a particle to its antiparticle.

\[
\mathcal{C} \cdot \psi(\vec{r}, t) \rightarrow \bar{\psi}(\vec{r}, t).
\]  

Protons become antiprotons, electrons become positrons, photons remain photons, etc. Interestingly, many individual reactions are very nearly the same after a charge swap. Consider hydrogen: a positively charged proton attracts a negatively charged electron. Under \( \mathcal{C} \), hydrogen becomes anti-hydrogen: a negatively charged anti-proton attracting a positively charged positron. The energy levels are very nearly the same, recently measured to be within 5 kHz of \( 2.5 \times 10^{15} \) Hz by Ahmadi et al. in [Ahm18]. The remarkable advancement in the field of antimatter synthesis and study is best
summarized by Kleppner’s summary of a 1992 antihydrogen workshop: “In the past six years the creation of antihydrogen has advanced from the totally visionary to the merely very difficult. One could hardly ask for more.” [Kle93]

Next we look at Parity, or mirror reflections. In an even number of spacetime dimensions, $\mathcal{P}$ reflects the system’s spatial coordinates through its origin. In 1 or 3D, $x \to -x$, or more generally:

$$\mathcal{P} \cdot \psi(\mathbf{r}, t) \to \psi(-\mathbf{r}, t). \quad (1.3)$$

It is tricky to represent the action of $\mathcal{P}$ in print, because in 2D only one coordinate is reflected. Formally,

$$\text{det}(\mathcal{P}) = -1. \quad (1.4)$$

This symmetry operation is a remarkable one: intuitively, the moon’s orbit would be the same if it was spinning the opposite direction, and aside from the lettering on the jerseys and a remarkable abundance of southpaws, a game of baseball would look the same in a mirror. Until the 1950’s, $\mathcal{P}$-symmetry was thought to be conserved. In other words, classical systems don’t change under $\mathcal{P}$, and it was thought that quantum ones wouldn’t either. There were some puzzles hinting otherwise ($\tau$ vs $\theta$), and ultimately Wu et al. found that $\beta$-decay, a weak process, violated Parity [Wu57].

After the discovery of P-violation, it was hoped that the combined action of $\mathcal{C}$ and $\mathcal{P}$ would be a universal symmetry. This hope was dashed by the discovery of CP-violation in the decay of the $K^0$ meson by Christenson et al. [Chr64].

The last hope of a symmetric universe lies in the third discrete symmetry: Time reversal. Trying to understand whether the perception of time is due to statistical (entropic) effects or whether there is a deeper structure to the universe has kept most physicists up for a few nights [Zee01]. At a glance, there isn’t an intuitive reason for systems to be the same forwards and backwards in time, but that’s because all of our intuition is built on dissipative macroscopic systems. If a super ball was perfectly elastic, you wouldn’t be able to tell if a movie of it bouncing on the floor was being played normally or reversed. Time reversal does mostly what you’d expect:

$$\mathcal{T} \cdot \psi(\mathbf{r}, t) \to \psi(-\mathbf{r}, -t) + \text{bookkeeping}. \quad (1.5)$$

The bookkeeping is required because, among other reasons, wave packets should still disperse (and not focus) under $\mathcal{T}$.

We include $\mathcal{T}$ as well because the combined action of $\mathcal{C}$, $\mathcal{P}$, and $\mathcal{T}$ is a Lorentz-invariant. That is to say: it can’t measurably change a system if and only if Einstein’s relativity is true, a remarkably strong constraint! If we assume that the mathematics powering relativity holds, then we can discuss $\mathcal{C}\mathcal{P}$ violation and $\mathcal{T}$ violation interchangeably. There wasn’t much interest in finding T-violating processes for decades, but that changed dramatically in the 1960’s, due to the discovery of Christenson et al. and Sakharov’s publication that suggested that CP-violation was necessary to generate the observed BAU. [Sak67]

Although first proposed and studied in 1950 as a probe of $\mathcal{P}$ invariance [Pur50], Purcell & Ram-
sey failed to observe an electric dipole moment of the neutron (nEDM), which satisfied the prevailing acceptance of global P-symmetry. As a result, their analysis was not completed for publication until after Wu et al. revealed that neutron β-decay is not P symmetric, which was itself motivated by Lee & Yang’s recognition that the Parity of the weak force was still unexplored. [Lee56] This early work extended a previously small number \(3 \times 10^{-18} \text{e cm} [\text{Pur50}]\) by two full orders of magnitude, to \(5 \times 10^{-20} \text{e cm} [\text{Smi57}]\).

Smith et al. recognized that nEDM would be suppressed due to its violation of T symmetry, and again experimental searches for this elusive property were put on hold until the discovery CP-symmetry violation in the decay of the neutral kaon by Christenson et al. in [Chr64]. With the failure of CP symmetry, compelling theoretical explanations against the observation of nEDM dissipated, and searches began anew, continuing to this day. At present, the upper bound of a nEDM from [Pen15] is \(3.6 \times 10^{-26} \text{e cm}\).

The dominant source of error in Pendlebury et al. is due to a so-called false electric dipole moment (EDM) that emerges primarily from the difficulty of conducting averages over populations, which is introduced by [Pen04] and treated in detail by [Bar06].

1.2 Baryogenesis

The fundamental goal of big bang cosmology is to predict the generation of matter from a sufficiently large energy source. Our current framework is based on [Sak67], and actually makes intuitive sense. If you start with zero baryons, and you wind up with a lot of baryons, you need a reaction in the early universe that changes the total number of baryons present. Because we consist of and have only observed baryons in the universe, you need that reaction to make more matter than antimatter. And because all of that stuff is still around, you need for that reaction to not be reversible.

Formally, this is handled with the so called “Sakharov Criteria”:

1. There must exist a reaction that violates B.
2. There must be a reaction that violates C and CP symmetry.
3. These reactions must take place out of thermal equilibrium.

The last item is the least troubling. It is addressed first by inflationary cosmology, first proposed in [Gut81], and later by the electroweak phase transition, as detailed in [Dub11]. Although searches for the B-violating processes are ongoing, they are generally expected to require even larger energy scales than our TeV accelerators currently explore.

It is not yet possible to directly create B-violation, but like neutral kaons, if neutrons and antineutrons are not pure eigenstates, it may be possible to observe the spontaneous oscillation of a neutron to an antineutron. An interesting prospect for accessing higher energy scales while searching for \(n \rightarrow \bar{n}\) is the bootstrap technique proposed in [Gol89].
1.2.1 CP violation

Having explained why the first and third criteria are someone else’s problem, we’re left with the second: \(CP\) violation. When describing the dynamics of particles, it is important to distinguish between their mass eigenstate, which governs their time evolution, and their flavor eigenstate, which determines how they interact with the rest of the world. This distinction was first made to describe the \(CP\)-violating decay of neutral kaons observed in 1964 by [Chr64], with this contribution subsequently included in the Standard Model as the complex phase of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [Pat16], represented as \(\delta_{13}\) below.

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_{23} & \sin \theta_{23} \\
0 & -\sin \theta_{23} & \cos \theta_{23}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_{13} & 0 & e^{-i\delta_{13}} \sin \theta_{13} \\
0 & 0 & 0 \\
-\sin \theta_{13} & 0 & \cos \theta_{13}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_{12} & \sin \theta_{12} & 0 \\
-\sin \theta_{12} & \cos \theta_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

These have been tuned to generate the \(CP\) violation observed in neutral K and B meson systems. I should mention that there is another possible source of \(CP\) violation in the SM Lagrangian:

\[
\left(\frac{\theta}{32\pi^2}\right) e^{\mu\nu\lambda\rho} \text{tr} F_{\mu\nu} F_{\lambda\rho} .
\]

As briefly discussed on page 259 of [Zee10], this term also contributes to nEDM, leading to a surprisingly small upper bound on the \(\theta\) term: \(1 \times 10^{-8}\). That this value is so small but not zero is considered strange.

Figure 1.4 Cartoon illustration of how non-zero nEDM violates \(T\) symmetry.

One may be concerned that the existence of molecules with large electric dipoles obviates searches for tiny electric dipoles in fundamental particles, but this is not the case. As discussed in [Gol94], molecules with electric dipole moments have a degenerate ground state, admitting nuclei with \(\hat{\mu} \cdot \hat{d} = \pm 1\). On the other hand, we know that a particle like a neutron is not degenerate in its ground state due to the exclusion of identical particles via the Pauli exclusion principle. As they follow the Pauli exclusion principle, the magnetic (\(\hat{\mu}\)) and electric (\(\hat{d}\)) dipole moments must share the same quantization axis and quantum number. If this axis is written as \(\hat{J}/J\), we can write \(\hat{\mu}\) and
Rather than directly observing the arrangement of the neutron’s constituent particles, experiments measure the minute energy difference arising from a nonzero $d$ term in the Hamiltonian for a neutron held in parallel (or antiparallel) electric and magnetic fields:

$$\mathcal{H} = \mu \cdot \vec{B} + d \cdot \vec{E}. \quad (1.9)$$

This is measured by reversing the relative orientation of the electric and magnetic field$^{1}$, and observing the resulting frequency shift:

$$\delta \omega = \frac{4ednE}{\hbar}. \quad (1.10)$$

### Table 1.1 Even/Odd actions of $C$, $P$, and $T$.

<table>
<thead>
<tr>
<th></th>
<th>$C$</th>
<th>$P$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{d}$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\vec{\mu}$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\vec{E}$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\vec{B}$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\vec{J}$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

A more careful accounting of the interactions of both external and internal sources with the actions of time-reversal (table 1.1) reveals that while $\vec{\mu} \cdot \vec{B}$ is even under any combination of $C$, $P$, or $T$, $d \cdot \vec{E}$ is odd under $P$, $CP$, and $T$.

### 1.2.2 CP violation in the Standard Model and beyond

The SM’s prediction of nEDM is interesting in its own right. The SM allows CP violation, and it has been directly observed in some systems, but it’s incredibly suppressed in the neutron, by mechanisms clearly articulated in [Cza97]:

Quark EDM cannot be generated in the standard model at the one-loop level because the relevant amplitudes do not change the quark flavor and each CKM matrix element is accompanied by its complex conjugate; no $T$-violating complex phase can arise. At the two-loop level individual diagrams have complex phases and contribute to the EDM. However, the sum over all quark flavors in the intermediate states leads to the

---

$^{1}$The most recent nEDM experiment at the Institut Laue-Langevin (ILL) held their magnetic field constant and reversed the electric field for a single cell. The $^3$He-nEDM apparatus in development for Oak Ridge uses a pair of cells with opposite electric fields held in a single large (large in volume, not large in magnitude) uniform magnetic field.
vanishing of the EDM at two loops.

The fact that the quark EDM appears only at the three-loop level in the standard model greatly complicates theoretical estimates.

Because nEDM emerges from the interplay of the neutron’s constituent quarks, and the 3rd order diagrams are the first to not be identically zero or self-canceling, the standard model’s prediction for nEDM is incredibly small: on the order of $1 \times 10^{-32} \text{e} \cdot \text{cm}$ [Cza97; Dub11].

One of the leading contenders for physics beyond the standard model is the theory of supersymmetry. Supersymmetry introduces a whole flock of new particles (sneutrinos, sleptons, winos, zinos, etc.). Some of these superpartners can interact in the EDM interaction, and theorists hope they have non-canceling interactions at the one or two-loop level. Generating nEDM via lower-order interactions means that it isn’t so heavily suppressed.

Conversely, the presence (or absence) of a nEDM can be used to probe the existence of these superpartners. By making exquisitely sensitive measurements, it is often possible to infer a huge amount without needing a device so crude as a collider. In 2014, Baron et al. further refined the limit on the electron’s EDM to an upper bound of $8.7 \times 10^{-29} \text{e} \cdot \text{cm}$, which probes SUSY on the 1 TeV to 3 TeV range, comparable to what CERN’s LHC is able to explore now, several years later [Bar14].

### 1.3 Neutron Electric Dipole Searches

After more than sixty years of fruitless searching, nEDM has been constrained to a remarkably small sector. The experiment itself is deceptively simple: “you just” take a neutron, apply an electric field, and watch to see if it precesses. Of course, each of these steps is fraught with trouble. Fortunately, the technology of the first step is well established. There are several different neutron sources that could be selected. Notably, neutron beams (of decreasing temperature) were used up until Dress et al. published a systematically limited value of $3 \times 10^{-24} \text{e} \cdot \text{cm}$ [Dre77].

Beamline experiments fundamentally require a small magnetic field (17 G) and a large static electric field (100 kV cm$^{-1}$) for the length of the beam. Any deviations from the expected precession due to B can be attributed to the $\vec{d} \cdot \vec{E}$ dipole interaction. That last sentence wasn’t true. It was optimistic, but optimism is dangerous when faced with the subtleties of high precision searches for fundamental symmetry violations. It isn’t a great use of ink to recapitulate the efforts detailed in Dress et al., but suffice it to say: it’s a beautiful piece of work, with a very clever beamline apparatus. This one used a decommissioned naval gun turret to rotate the beamline by 180°, eliminating the $\vec{v} \times \vec{E}$ systematic to the leading order, and very carefully sought to reduce all other sources of systematic error. Most$^2$ nEDM searches since 1977 have utilized bottled UCN, as they have $\langle \nu \rangle = 0$.

$^2$There are ongoing beam nEDM experiments which use crystal diffraction, trading observation time (1000 s to $10^{-2}$ s) for tremendously enhanced electric fields ($5 \times 10^4 \text{Vcm}^{-1}$ to $0.7 \times 10^8 \text{Vcm}^{-1}$) [Fed09]. There are also ongoing beam experiments using a pulsed neutron source [Pie13].
1.4 Ultracold Neutrons

Consider the interactions that take place as a beam of neutrons encounter a polished nickel surface. The interaction length of a free neutron is a function of its DeBroglie wavelength \( \lambda = \frac{h}{p} \). As such, the highest energy neutrons will only "see" individual nuclei, and their behavior will be that of a particle incident on a single nucleus. As the incident neutrons slow down, though, their DeBroglie wavelength will expand until the neutrons’ interaction is effectively with an array of crystalline nuclei. Much like Bragg reflection, neutrons approaching the surface at an angle less than a critical value will be perfectly reflected. This critical value is a function of the neutron’s wavelength, and increases for slower (or colder) neutrons. Neutrons with less than a critical amount of kinetic energy will be reflected, and this critical value is known as the Fermi potential. The Fermi potential is a function of nuclear scattering length and spacing, ranging from negative values for materials that absorb even the slowest neutrons, to 300 neV for diamond [Bry06].

Table 1.2 Neutron velocity classes, collected by [Koc15].

<table>
<thead>
<tr>
<th>Class</th>
<th>Kinetic energy (eV)</th>
<th>velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast</td>
<td>&gt;500 \times 10^3</td>
<td>&gt;10^7</td>
</tr>
<tr>
<td>epithermal</td>
<td>25 \times 10^{-3} to 500 \times 10^3</td>
<td>2200 to 10^7</td>
</tr>
<tr>
<td>thermal</td>
<td>25 \times 10^{-3}</td>
<td>2200</td>
</tr>
<tr>
<td>cold</td>
<td>0.05 \times 10^{-3} to 25 \times 10^{-3}</td>
<td>100 to 2200</td>
</tr>
<tr>
<td>very cold (VCN)</td>
<td>360 \times 10^{-9} to 0.05 \times 10^{-3}</td>
<td>8 to 100</td>
</tr>
<tr>
<td>ultracold (UCN)</td>
<td>&lt;360 \times 10^{-9}</td>
<td>&lt;8</td>
</tr>
</tbody>
</table>

1.4.1 Production

Zel’dovich proposed that neutrons could be bottled in 1959 [Zel59], but getting the neutron into its bottle is not a trivial thing. It was nearly ten years before a group successfully bottled neutrons [Lus69; Ste69], and early UCN converters weren't able to generate very high densities at all. In the last several decades, though, the technology to produce UCN has become quite sophisticated. There are two different styles of UCN production, covered in detail by [Gol91]. First is the mechanical converter. There is a finely designed turbine at the ILL that converts cold neutrons into ultracold neutrons by multiple scattering [Ste86]. Modern cryogenic converters are capable of producing equivalent UCN densities and larger phase-space densities; the crystalline deuterium source at Los Alamos yields a useful density of 184(32) UCN/cc at its shield wall [Ito18].

Cryogenic converters exist in two effective types: a crystal of ortho-deuterium that downscatters incident neutrons, dissipating the energy in a complex phonon spectrum, and liquid helium-4 cooled below its \( \lambda \)-transition point. Liquid helium is a particularly attractive UCN converter, because it has a single-phonon scattering mode, which simultaneously allows production that is
effectively rate limited by the incident 8.9 Å neutron beam flux, while simultaneously suppressing upscattering of the newly-produced UCN.

![Free neutron dispersion curve](image)

**Figure 1.5** UCN with almost no kinetic energy may be produced by coherent single-phonon scattering, as indicated by the single crossing of free neutron energy and the single phonon mode of superfluid LHe, from [Lam09].

Once a free neutron is trapped in a bottle, it will remain there until it either beta decays or is lost when bouncing off a wall. The primary loss mechanisms are upscattering and absorption—both of which can be mitigated by careful material selection. For the effective lifetime of the neutron, though, you're free to study it.

### 1.4.2 Detection

In order to study the UCN that you've bottled, you need some way to detect it. Some measurements wait for the UCN to decay and observe those reaction byproducts [Pat09], while others remove the neutrons with an absorber and observe its resulting activation [Mor17]. Those studies are primarily motivated by the decay of the neutron and what that can reveal about its composition.

In order to successfully measure nEDM, you need to be able to measure a neutron's polarization. This can be done with direct or indirect means. Both Dress et al. and Pendlebury et al. used a direct method, namely Ramsey's method of separated oscillating fields, to determine the relative procession of their neutron populations [Dre77; Pen15]. The history of EDM searches is that of increasing control of magnetic and electric fields, and half a century later, UCN-based EDM searches require absurd uniformity, which must in turn be verified with precise magnetometry. Pendlebury et al. used co-habiting $^{199}$Hg, but that required a vacuum trap, and introduced a subtle source of systematic error in that the center of mass of the mercury vapor was slightly lower than that of the UCN gas, preventing the mercury from fully sampling the experimental volume.

In order to eliminate those issues, allow higher UCN density, and enable more accurate phase detection, Golub & Lamoreaux proposed an experiment conducted in pure helium-II, with $^3$He
serving as both comagnetometer and phase detector [Gol94].

1.5 $^3$He–nEDM

$^3$He($n$, $p$)$^3$H has a spin-dependent cross section, which has the cartoon explanation that the unpaired neutron in $^3$He and the free incident neutron need to combine in a spin singlet state, requiring their spins to be antiparallel. The net result of this is that if a population of UCN and a population of $^3$He are introduced at the same time in a perfect polarized state, there will not be any captures. If the population is placed in a combined magnetic and electric field, they'll process, and as their relative phase increases, the capture rate will oscillate. If the B field is well known, departures from expected phase accumulation can be attributed to nEDM. As should be expected from a broad overview, this paragraph has completely neglected systematic effects, and thus has been an exercise in optimism and some small lies.

An experiment like $^3$He-UCN lives and dies by the control of its systematics, and much of this work is dedicated to a single one of them—the false EDM that emerges from aggregate effects of bottled UCN. In an experiment with $^3$He, this false EDM winds up being dependent on density and temperature [Bar06]. Because of this dependence, a deep study of the polarization dynamics of just $^3$He needs to be conducted. Preliminary studies of the depolarization of $^3$He have been previously conducted on a small scale apparatus, but this device has limited space for modern holding fields, and is only capable of supporting small sample cells. A generational improvement has been built to overcome these difficulties and to study Systematic and Operational Studies at PULSTAR. This thesis addresses several systems necessary for the successful operation of SOS@PULSTAR.

1.6 SOS@PULSTAR

The full-scale $^3$He-UCN apparatus will contain thousands of liters of LHe, which means the full scale device will take months to cool down and warm up again. Should a fault occur, it will take nearly three months to repair and return to operation. Such a slow turn-around prevents iterative testing of all the full-scale device's subsystems and systematics. We have instead developed a small-scale test bed containing a single 3 L sample cell and no high voltage electric systems. Sample UCN densities may be lower than the full-scale apparatus, but should be sufficient to explore the dynamics of cohabiting UCN and hyperpolarized $^3$He.

The SOS apparatus is being developed with two different read-out technologies: TPB and wavelength-shifting optical fibers to observe scintillation from $^3$He UCN capture, and a superconducting quantum interference device (SQUID) package to observe $^3$He nuclear magnetic resonance (NMR). The $^3$He NMR measurements need a higher $^3$He density, and do not need UCN, allowing preliminary studies to be conducted at Triangle Universities Nuclear Laboratory (TUNL). Once the PULSTAR UCN source is online, the SOS apparatus will be moved to the PULSTAR reactor bay. These two phases are distinguished in this text as SOS vs. SOS@PULSTAR.
Biaxially oriented polypropylene 0.0013” thick has been transferred down the guides. A thin foil at the UCN vestibule will contain the He-II. A guide entrance to the vestibule will reduce radiative heat. The guide diameter is reduced in the cold section connects to the vestibule. Non-metallic material with a bend before the vestibule to reduce radiative heat. The guide from an external source will be polarized. Owing to their low kinetic energies and relatively strong magnetic interaction, extremely high UCN polarization can be achieved with a magnetic guide or by guiding the UCN through a large static field. Antiparallel (depolarized) $^3$He will readily absorb UCN, limiting the effectiveness of the SOS apparatus. $^3$He-UCN has a capture cross section of $\sigma_+ = 0$ for parallel nuclear spins or $\sigma_- = 1 \times 10^8$ barn when antiparallel. This capture rate is given by

$$\frac{1}{\tau_3(t)} = \frac{1}{\tau_3(\rho_n, \rho_3)} \left[ 1 - P_n(t)P_3(t)\cos\theta_{n,3}(t) \right],$$  \hspace{1cm} (1.11)

With $P_n(t)P_3(t)$ given by

$$P_n(t)P_3(t) = P_n(0)P_3(0)e^{-t/T_{2,\text{total}}},$$  \hspace{1cm} (1.12)

where $T_{2,\text{total}}$ combines gradient depolarization and wall depolarization rates for both UCN and $^3$He. This immediately illustrates the importance of $^3$He initial nuclear polarization—enhancements in $P_3$ enhance the effective experimental lifetime of the co-habiting UCN. A more thorough analysis is conducted by Leung et al. in an upcoming paper: [Leu18]. In short, SOS@PULSTAR needs $^3$He polarization in excess of 70%, and this sample will need to be provided multiple times per
day. A MEOP system was commissioned to deliver this hyperpolarized gas—details of its theory of operation can be found in chapter 2.

It is crucial to verify the sample's polarization. While it is possible to directly measure the sample magnetization via NMR, even highly polarized $^3$He in this system has a weak NMR signal due to its relatively low sample density (1 mbar), requiring dedicated RF shielding and carefully tuned pickup coils. Previous researchers [Lor93; Max14; Big92] have developed optical methods to measure helium nuclear polarization from plasma emissions. We were able to model and verify the calibration sensitivity of a standard optical polarimeter (section 3.4.1). Using that polarimeter, we have achieved $P_N \geq 80\%$ in a pure $^3$He sample (section 3.6).

1.6.2 Gradient-induced $^3$He relaxation apparatus

The SOS apparatus is a generational improvement of the small-scale $^3$He relaxation apparatus used by both Ye; Swank in their dissertation research: [Ye08; Swa12b]. A gas held in an imperfect magnetic field will experience a magnetic field that varies in both direction and magnitude. At high pressure, particles are effectively caged by their environment, and can't explore the varying field enough for it to flip their spin state. At very low pressure, the particles have a long mean free path, reducing the time they spend in any particular field, which also diminishes the probability of a depolarizing spin flip. In between these regimes, there is a region where the depolarization rate is maximized. McGregor gives a method for relating the autocorrelation function of a gas to its gradient-induced decay of both longitudinal polarization ($T_1$) and transverse polarization ($T_2$) [McG90]. At very low pressures (the ballistic regime), particles are extremely unlikely to interact with each other, but Swank et al. predicts an enhanced interaction probability and resulting depolarization as they reflect off the bottle's walls [Swa12a].

This apparatus injects polarized $^3$He into a helium-II bath, applies a controlled magnetic gradient, then measures the resulting $T_1$. We have spent a substantial amount of time improving the operation and reliability of this apparatus. In spite of our best efforts, we have been continually flummoxed by electronic failure and cold leaks. These setbacks prevented us from testing the low-density predictions of Swank et al. [Swa12a], but we were able to develop new applications for additive manufacturing (3D printing) as well insights on the crucial importance of the dynamics of hopper flow for cryogenic physics (section 4.1.1).
1.6.3 Origami folded cylinders

![Figure 1.7 Miura folded bellows and unit cell](image)

Origami folded cylinders (origami bellows) have found increasingly sophisticated applications in space flight and medicine. In spite of this interest, a general understanding of the mechanics of an origami folded cylinder has been elusive. With a newly developed set of geometrical tools, we have found an analytic solution for all possible cylindrical rigid-face states of both Miura-ori and triangular tessellations. Although an idealized bellows in both of these families may have two allowed rigid-face configurations over a well-defined region, the corresponding physical device, limited by nonzero material thickness and forced to balance hinge and plate-bending energy, often cannot stably maintain a stowed configuration. We have identified the parameters that control this topological bistability, and have demonstrated the ability to design and fabricate bellows with tunable deployability, as discussed in chapter 5.

Furthermore, the detailed study of these idealized bellows led to substantial improvements in the design and fabrication of the origami folded bellows developed for Systematic and Operational Studies at PULSTAR (SOS).

1.6.4 Origami-inspired design

![Figure 1.8 Large format camera with flexible bellows. Adapted from [Kal09]](image)
Because it is RF transparent and flexible in cryogenic baths, Kapton is an ideal material for building low-temperature actuators. Kapton is relatively inextensible at all temperatures, but it can easily be folded and thermoset. Needing to develop a mechanical device with a particular set of properties for the SOS sample cell, we turned to origami patterns, as the origami community has courteously contributed thousands of man-hours of creativity. From this spontaneous world-wide distributed computing project, we identified a folding pattern that converts a rigid cylinder into a springy bellows, frequently used in large-format cameras like fig. 1.8. This folding pattern is one type of several closely related patterns, none of which have any flexibility in their mathematically ideal form (chapter 5, [Rei17]). By manipulating how the material itself is creased, it is possible to recover flexibility and a surprising amount of durability in the folded bellows (chapter 6).
You don’t understand anything until you learn it more than one way
— Marvin Minsky

In a reasonable magnetic field, helium-3 does not spontaneously achieve high levels of polarization. For example, for a sample at 1 K in a 10 G field, thermal effects\(^1\) limit polarization to less than \(1 \times 10^{-6}\).

\[
P \approx \frac{\mu \cdot B}{kT} = \frac{\mu_{He} \cdot \mu_N \cdot 10 G}{k \cdot 1 K} = 0.761 \cdot \frac{\mu_p \cdot \mu_N \cdot 10 G}{k \cdot 1 K} \approx 7.78 \times 10^{-7}
\]  

(2.1)

It is often possible to conduct NMR on a Boltzmann polarized sample given a sufficiently high field (0.08 % at 1 T), but the total polarization is not high enough to be very useful. Extremely high polarizations require that the system be pushed far from thermal equilibrium. As shown by Swank in [Swa12b], relaxation of dilute \(^3\)He can be quite slow, allowing measurements to take place over hours or even days.

Magnetization (or polarization) of a gas is the accumulation of spin in one particular direction, measured as a fraction of the total population. Hyperpolarizing a gas is accomplished by flipping a huge fraction of the undesirable population’s spins. An external source of angular momentum quanta is required to flip the spins into their desired final state. These quanta come perfectly packaged as circularly polarized photons. Thanks to decades of work in lasers and optics, \(\sigma^+\) photons are readily available.

The electrons we are pumping will start in thermal equilibrium, but that population will be dramatically skewed as the pumping proceeds. The optical pumping technique used here is form of depopulation pumping, where a magnetic sublevel is effectively depopulated with a sufficiently intense optical source, first described in detail by [Kas51], and sketched in fig. 2.1.

Exciting an electron from \(1S \rightarrow 2^3S\) (helium’s lowest energy excited state) requires a 62.6 nm (19.6 eV) photon [Hod09]. Traditional optics are not transparent at this wavelength, and further-

\(^1\)In eq. (2.1) Helium’s magnetic moment is calculated in terms of the proton’s magnetic moment from [Flo93].
more, coherent photons are not available. In other words, ground state helium doesn’t have an optically accessible electronic transition, so an intermediary needs to be introduced. There are two primary methods used to generate large volumes of hyperpolarized helium-3: spin exchange optical pumping (SEOP) and metastability exchange optical pumping (MEOP).

**SEOP** methods, reviewed in detail in [Gen17], use a gaseous alkali metal as their intermediary. They function at high pressures and can generate huge amounts of polarized $^3$He, but they require a fairly high partial pressure of nitrogen, which can severely degrade a superfluid-based cryogenic system [Swa12b], due to the increased film flow along the roughened surface formed by nitrogen freezing out in the transfer line [Wil67]. This additional film conducts more heat into the cell, heating the cell in part due to its poor coupling to the refrigerator.

**MEOP** methods use helium itself as the optically active species. Once excited to the spin-triplet $^2S$ state, relaxation to the ground state is doubly suppressed. $^2S_1 \rightarrow ^2P$ is a 1083 nm transition, readily accessible via diode (100 mW) or higher power Yb-fiber (10 W) lasers. This long-lived (metastable) state can readily exchange its spin state with a ground state helium, which can then be optically pumped before transferring its electronic configuration with yet another ground state atom. This process is as fast as thermal gas collisions, and because it is a transfer of electrons from helium to helium, it is extremely efficient.

For the gradient-induced relaxation apparatus, we need highly polarized helium ($P_N = 30\%$) and we need it in some number of hours. Fortunately, we don’t need very many polarized nuclei ($N = 3 \times 10^{16}$). The requirements for Systematic and Operational Studies at PULSTAR (SOS) are substantially more stringent: $P_N = 70\%$, with multiple measurement cycles per day. This apparatus

---

**Figure 2.1** Cartoon showing 2-level 2-state system with optical depopulation. A–C show the idealized case where there is no relaxation and the $|0, -\frac{1}{2}\rangle$ sublevel can be completely depleted. D–F show a more realistic equilibrium with relaxation in the ground state and rapid spin exchange in the excited state.
will also need a relatively small \( N \) (calculated in section 3.7). All of these requirements suit MEOP perfectly.

### 2.1 Metastable helium

In its first excited state, helium’s electrons can assume one of two different spin configurations. In their parahelium configuration, their wavefunctions form a singlet state. The excited electron’s state is conventionally written as \( 2^1 S \). Orthohelium, on the other hand, is their spin-symmetric triplet state, indicated by the superscripted 3 in the excited electron’s state: \( 2^3 S \).

Orthohelium’s electrons have a symmetric spin wavefunction, and thus must have an antisymmetric space wavefunction. This antisymmetry reduces the screening provided by the 1S electron, so orthohelium’s 2S electron is more tightly bound by the nucleus, and thus resides at a slightly lower energy level than parahelium’s.

By account of its doubly suppressed transition to ground state, the \( 2^3 S_1 \) state in \(^3\)He has an extraordinarily long lifetime. In \(^4\)He, Hodgman et al. measured a lifetime of 7870 s [Hod09].

---

![Figure 2.2](https://via.placeholder.com/150)

**Figure 2.2** Named optical transitions in metastable \(^4\)He (left) and \(^3\)He (right). More precise values are listed in table 3.1. Note that the \( \Delta m_f = 2 \) transition that would be \( C_0 : 2^3 S_{1/2} \rightarrow 2^3 P_{5/2} \) is forbidden. Vertical axis is not to scale.

Transitions from \( 2^3 S \rightarrow 2^3 P \) in \(^4\)He are labeled “D,” and are indexed by their excited state \( J \) value \((0,1,2)\), which thanks to Hund’s rule means the most energetic transition is \( D_0 \). Transitions from \( 2^3 S \rightarrow 2^3 P \) in \(^3\)He are labeled “C,” and are indexed by their energy difference. Due to hyperfine splitting, there are nine C transitions are labeled \( C_1 - C_9 \), where \( C_1 \) is the least energetic transition and \( C_9 \) is the most energetic. The levels accessed by \(^4\)He’s D series and \(^3\)He’s C series are shown in fig. 2.2 The relative oscillator strengths can be estimated from the magnetic sublevels of each state. Literature values use a standard enumeration of the magnetic sublevels for helium, with the sublevels relevant to contemporary MEOP systems given in fig. 2.3.
Promotion of an electron into the $2^3S$ state is violent: a ground-state helium is bombarded with RF-accelerated plasma, exciting an electron far beyond $2^3S$, where it fluoresces as it decays, eventually becoming trapped in the $2^3S$ state. This process isotropically populates the $2^3S$ sublevels, destroying any accumulated nuclear polarization. In a system that is optimized for equilibrium polarization rather than pumping rates, plasma discharge should be tuned to its weakest sustainable level. Fortunately, metastability lasts for multiple pumping cycles because the ground state of helium is a spin singlet, relaxation from even excited states of orthohelium ($2^3P, D, \ldots$) to the ground state is suppressed by the relative improbability of an electron spin-flip.

Electrons pumped into the $2^3P$ state have a relaxation time of 5.66 ms to $1^1S$ [Hod09] versus 0.1 µs to relax to $2^3S$ [Tas04]. While pumped electrons can decay to the ground state, it’s far more likely to lose a metastable atom due to wall interactions or collision with a quenching contaminant.

Schmeltekopf & Fehsenfeld found higher interaction probabilities for parahelium with contaminants (e.g., argon, nitrogen, and oxygen), as well as an increased interaction cross section for parahelium and electrons [Sch70]. Orthohelium may be less reactive, but it is the precious pumping population, and contaminants such as argon and oxygen have large reaction probabilities and are the bane of MEOP system development.

### 2.2 Metastability Exchange Mechanism

Helium in its $2^3S$ metastable state is extremely long-lived, and it has several optically accessible transitions. That is sufficient to generate a highly polarized metastable population, but typical metastable density is on the order of $1:10^6$. The story would end there, save for a particularly useful property in helium: metastable electrons may be coherently transferred between a metastable helium (denoted He*) and a ground state helium by gas collisions. The conditions for this coherent
transfer were first detailed by Partridge & Series in [Par66], although their work focused on using coherent transfer as a probe of otherwise inaccessible atomic states.

The exchange itself is very rapid, happening on the $10^{-7}$ Hz timescale [Dup73]. This exchange is far more rapid than the hyperfine interaction ($10^{-4}$ s), so the accumulated nuclear polarization in the metastable state is not destroyed. Effectively, the metastable atom and the ground state atom swap electronic configurations. With mixed isotopic pumping, the $^4$He just returns to its ground state, and the spins (nuclear and electron) of the now metastable $^3$He will mix via their hyperfine coupling before transferring the metastable state to another atom. In both of these cases, the metastable $^3$He retains its accumulated nuclear polarization upon returning to its ground state.

The ease with which metastability is transferred from one helium to another is the key to the speed of MEOP pumping. Put succinctly, each atom in the sample is likely to be promoted to metastable multiple times per second. With metastable density $n/N = 10^{-6}$ and metastable exchange time $\tau \approx 10^{-7}$ s, the time for any given atom to exchange electronic configurations with a metastable atom is given by

$$\frac{1}{T} = \frac{n}{N} \frac{1}{\tau}, \quad (2.2)$$

$$T \approx 10^{-1} \text{s}. \quad (2.3)$$

### Table 2.1 Timescales of various processes involved in MEOP, as defined by [Dup73].

<table>
<thead>
<tr>
<th>State</th>
<th>Time Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground</td>
<td>time to exchange metastability</td>
<td>$T$ $10^{-1}$ s</td>
</tr>
<tr>
<td></td>
<td>discharge-driven relaxation time</td>
<td>$T_r$ 100 s</td>
</tr>
<tr>
<td>Metastable</td>
<td>exchange time</td>
<td>$\tau$ $10^{-7}$ s</td>
</tr>
<tr>
<td></td>
<td>relaxation time</td>
<td>$\tau_r$ $10^{-3}$ s</td>
</tr>
<tr>
<td></td>
<td>time to pump between hyperfine sub-levels</td>
<td>$\tau_p$ $10^{-4}$ s</td>
</tr>
</tbody>
</table>

### 2.3 Metastability Exchange Formalism

Sakurai briefly introduces the density operator

$$\rho \equiv \sum_i w_i \left| \alpha^{(i)} \right\rangle \left\langle \alpha^{(i)} \right|$$

and the corresponding density matrix

$$\dot{\rho} \equiv \langle b'' | \rho | b' \rangle = \sum_i w_i \langle b'' | \alpha^{(i)} \rangle \left\langle \alpha^{(i)} | b' \right\rangle$$

$$\dot{\rho} \equiv \langle b'' | \rho | b' \rangle = \sum_i w_i \langle b'' | \alpha^{(i)} \rangle \left\langle \alpha^{(i)} | b' \right\rangle$$

(2.5)
in chapter 3 of [Sak94], but much of their utility in quantum systems is left for the reader to discover. In tables 2.2 and 2.3, the initial metastable population is labeled with a subscripted $m$ or an asterisk, and the initial ground state population is undecorated or labeled with a subscripted $g$. Thus $\rho_m$ is the density operator for the incoming metastable helium $^3\text{He}^*$ and $\rho_g$ is the density operator for the incoming $^3\text{He}$.

Table 2.2 $^3\text{He}$ metastability exchange, its density operator representation, and the vector spaces described by each representation, followed by density operator representation formulae in Dirac notation. $|A_i\rangle$ are given in table 2.4. Density operators follow MEOP literature conventions, which only differ in form from the formalism given by eq. (2.4) from [Sak94].

$$\begin{align*}
^3\text{He}^* & \quad \rho_m \\ |I\rangle & \otimes |s\rangle \\ \sum_{i=1}^{6} a_i |A_i\rangle\langle A_i| \\
+ \\
^3\text{He} & \quad \rho_g \\ |I\rangle & \frac{1+M}{2} |+\rangle\langle +| + \frac{1-M}{2} |-\rangle\langle -| \\
\downarrow \\
^3\text{He} & \quad \text{Tr}_e(\rho_m) \\ |I\rangle & \sum_{i=1}^{3} \langle e_i|\rho_m|e_i\rangle \\
+ \\
^3\text{He}^* & \quad \rho_g \otimes \text{Tr}_n(\rho_m) \\ |I\rangle & \otimes |s\rangle \\ \rho_g \otimes \sum_{i=1}^{2} \langle n_i|\rho_m|n_i\rangle \\
\end{align*}$$

Table 2.3 $^4\text{He}$ metastability exchange with $^3\text{He}$, its density operator representation, the vector spaces described by each representation, followed by density operator representation formulae in Dirac notation.

$$\begin{align*}
^4\text{He}^* & \quad \rho_m \\ |s\rangle & \sum_{i=1}^{3} y_i |Y_i\rangle\langle Y_i| \\
+ \\
^3\text{He} & \quad \rho_g \\ |I\rangle & \frac{1+M}{2} |+\rangle\langle +| + \frac{1-M}{2} |-\rangle\langle -| \\
\downarrow \\
^4\text{He} & \quad \mathbb{1} \\
+ \\
^3\text{He}^* & \quad \rho_g \otimes \rho_m \\ |I\rangle & \otimes |s\rangle \\
\rho_g \otimes \rho_m \\
\end{align*}$$

The collision between a metastable helium and a ground state helium has a finite probability to exchange the metastable state. When the metastability is lost from the incoming metastable $\rho_m$, it
Table 2.4 Low-field $^3\text{He}^2S_1$ levels $|A_i\rangle$ in $|m_j, m_I\rangle$ basis, used in eq. (2.6a)

\[
\begin{align*}
F = \frac{3}{2} & \quad m_f = \frac{3}{2} \quad |A_1\rangle = |1, +\rangle \quad (2.8a) \\
m_f = \frac{1}{2} & \quad |A_2\rangle = \sqrt{\frac{1}{3}} |1, -\rangle + \sqrt{\frac{2}{3}} |0, +\rangle \quad (2.8b) \\
m_f = -\frac{1}{2} & \quad |A_3\rangle = \sqrt{\frac{2}{3}} |0, -\rangle + \sqrt{\frac{1}{3}} |1, +\rangle \quad (2.8c) \\
m_f = -\frac{3}{2} & \quad |A_4\rangle = |1, -\rangle \quad (2.8d) \\
F = \frac{1}{2} & \quad m_f = \frac{1}{2} \quad |A_5\rangle = \sqrt{\frac{2}{3}} |1, -\rangle - \sqrt{\frac{1}{3}} |0, +\rangle \quad (2.8e) \\
m_f = -\frac{1}{2} & \quad |A_6\rangle = \sqrt{\frac{1}{3}} |0, -\rangle - \sqrt{\frac{2}{3}} |1, +\rangle \quad (2.8f)
\end{align*}
\]

ceases to have an electronic spin state that contributes to its total spin quantum number. Its density operator needs to be reduced from the $|I\rangle \otimes |s\rangle$ space to just the space spanned by nuclear spin $|I\rangle$. Formally this is accomplished by $\text{Tr}_e(\rho_m)$, read as “tracing out” or “tracing over” the electronic sublevels, leaving only the nuclear sublevels. This operation is trivial if the incoming metastable is $^4\text{He}$, per eq. (2.7c).

When $^3\text{He}$ receives the metastability, it is promoted to a compound system with both nuclear and electronic spins. The transfer does not affect nuclear spins, so only the electronic configuration is transferred. The electronic configuration of the incoming metastable is given by $\rho_m$ in the case of $^4\text{He}^*$ and by tracing out (or tracing over) the nuclear states instead, written as $\text{Tr}_n(\rho_m)$ before combining the two spin systems’ density operators with a tensor product.

It is reasonable to be concerned that the efficiency of this exchange is diminished by identical particle effects when pumping with pure $^3\text{He}$. Pinard & Laloë have handled this in detail, and at room temperature where $\text{MEOP}$ is typically conducted, indistinguishable particle corrections do not yield significant insight to pumping efficiencies [Pin80].

### 2.4 Polarization and Spin Temperature

Instead of accounting for the populations present in six different metastable sublevels, it can be far more elegant to relate a cell’s polarization to its spin temperature. In an unperturbed (unpumped) $\text{MEOP}$ system, the metastable spin populations ($a_i$) follow a Boltzmann distribution [Nac85]. The spin temperature of the ensemble is dominated by the nuclear polarization—there are $1 \times 10^6$ more atoms in the ground state than in the metastable state, and those ground state atoms’ spin is purely nuclear. Batz et al.; Courtade et al. introduce $T = \frac{1}{\beta}$ as the spin temperature [Bat11; Cou02], and use it to express the population of each sublevel as a function of overall sample polarization.

\[
e^{\beta} \equiv \frac{1 + M}{1 - M} \quad (2.9)
\]
Population sublevels are partitioned as one would expect for $^3\text{He}$ ($a_i$) and $^4\text{He}$ ($y_i$):

$$a_i = \frac{e^{\beta m_i}}{e^{-3\beta_2} + 2e^{-\beta_2} + e^{2\beta_2} + e^{3\beta_2}}$$  \hspace{0.5cm} (2.10)

$$y_i = \frac{e^{\beta m_2}}{e^{-\beta} + 1 + e^{\beta}}$$  \hspace{0.5cm} (2.11)

These sublevel expressions may be simplified to yield populations as a function of sample magnetization. We can use these expressions to visualize various sublevel populations and the equivalent aggregate magnetization, drawn in fig. 2.4. Spin temperature populations yield a ready insight to the relative effectiveness of pumping $^3\text{He}$ via C$_9$ vs C$_8$.

Table 2.5 Formulae for spin temperature populations as a function of $M$, applicable in low field (less than 1619 G) MEOP [Bat11].

<table>
<thead>
<tr>
<th>Formulae</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1 = \frac{(1 - M)^3}{2M^2 + 6}$</td>
<td>(2.12a)</td>
</tr>
<tr>
<td>$a_5 = a_2 = \frac{(1 + M)(1 - M)^2}{2M^2 + 6}$</td>
<td>(2.12b)</td>
</tr>
<tr>
<td>$a_6 = a_3 = \frac{(1 - M)(1 + M)^2}{2M^2 + 6}$</td>
<td>(2.12c)</td>
</tr>
<tr>
<td>$a_4 = \frac{(1 + M)^3}{2M^2 + 6}$</td>
<td>(2.12d)</td>
</tr>
</tbody>
</table>

Figure 2.4 Spin temperature-based population distribution in $^3\text{He}$ $^3S$ metastable state. Population bars are logarithmic: At $M = 0$ each sublevel contains 16.7 % of the metastable population. For each $M$, $\sum_{i=1}^{6} a_i = 1$. 

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2.5 $^3$He pumping transitions

There are nine possible transitions from $^2S_1 \rightarrow ^3P$ in $^3$He. Of those, only the most energetic (named $C_8$ and $C_9$) yield the highest degree of polarization. In a 300 K sample, $^3$He transitions are Doppler broadened to nearly 2 GHz. $C_1$–$C_7$ are poor choices of pumping transitions, because they are either too weakly coupled to pump effectively or because they overlap with another transition. $C_1$ is an extraordinarily weak transition, and $C_2$–$C_5$ and $C_6$–$C_7$ are separated by less than 2 GHz (as seen in table 3.1), and are thus overlapping lines at room temperature. Pumping on any of these lines with a spectrally broadened laser splits the laser’s power among multiple transitions, effectively incompletely pumping different velocity classes in those transitions. Nacher & Leduc show that pumping on a single transition is more efficient than divvying up laser power between multiple transitions [Nac85], leaving our best hope in $C_8$ and $C_9$.

Table 2.6 Selected $^3$He transition matrix elements from [Nac85]. Naming convention follows that of fig. 2.3, where $A_i \rightarrow B_j$.

<table>
<thead>
<tr>
<th></th>
<th>$C_9$ Transitions</th>
<th>$C_8$ Transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$i$</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>18</td>
<td>0.0934 $\sigma^-$</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>17</td>
<td>0.2801 $\sigma^-$</td>
</tr>
<tr>
<td>$m_f$</td>
<td>$\frac{3}{2}$</td>
<td>$\gamma_2$</td>
</tr>
</tbody>
</table>

Naively, one might expect $C_9$ to yield higher total polarization, as it simultaneously depletes both $A_1$ and $A_2$. Note, however, that the strongest coupling when pumping $C_9$ is from $A_1$, which is the least populated sublevel once the sample has accumulated any degree of polarization, as illustrated in fig. 2.4. $C_8$, on the other hand, interacts exclusively with $A_5$, a sublevel that remains populated at much higher polarizations.

Metastable atoms pumped to a $^2P$ ($B_i$) sublevel also experience enhanced collisional mixing and thermalization. Because their lifetime is relatively short, however, they are unable to fully equilibrate in this excited state. They mix more readily than the $^2S$ $A_i$ levels. This is an additional factor that benefits low polarization $C_9$ pumping: $B_{18}$ is mixed into $B_{17}$, corresponding to $\Delta m_f = 2$ when the laser only delivered $\Delta m_f = 1$. Because the system’s angular momentum increased by more than what was delivered by the laser, photon efficiency $\eta_9 = \langle \frac{\Delta m_f}{\text{photon}} \rangle > 1$ when $P < 0.5$ [Bat11].

Several of the magnetic sublevels that interact with $C_8$ or $C_9$ photons are mixed states, but the
net result of depopulating the states that are more spin-down \((A_6)\) in favor of states that are more spin-up \((A_5)\) is that the total metastable population accumulates polarization, which is projected into pure nuclear states, first by hyperfine coupling and then by metastability exchange.

2.6 Cohabiting \(^4\)He: \(D_0\) pumping

An isotopic mixture of \(^3\)He and \(^4\)He can be pumped to high degrees of polarization (80\%) with a rather weak (50 mW) laser [Sto96]. By pumping the \(\lambda_{vac} = 1083.206\) nm \([Cou02; Can12]\) \(D_0\) transition with circularly polarized light, an entire magnetic sublevel (e.g. \(A_9\) in fig. 2.3) may effectively be depopulated. As was discussed in section 2.3, electron clouds are exchanged during metastable exchanges. By exchanging polarized electrons with a ground-state \(^3\)He, the now metastable \(^3\)He is polarized by hyperfine coupling to the polarized electron.

Only \(D_0\) is used for MEOP. The two lower energy levels are Doppler broadened into one another, and overlap with \(C_9\) as well. If \(^4\)He contamination is a concern, or if pumping is being conducted on \(C_9\), it is best practice to probe the \(D_0\) transition with the laser first to ensure a clean sample.

2.7 Overview of Polarization via Metastability Exchange

\[ |A_5\rangle = \sqrt{\frac{2}{3}} |1, -\rangle - \sqrt{\frac{1}{3}} |0, +\rangle \]  

(2.13)

Absorbing this photon promotes the \(A_5\) metastable to \(B_{18}\), a \(J = 0\) P-orbital sublevel. While this level could decay all the way back to the \(1S_0\) ground state, it is far more likely to decay back to the \(2^3 S_1\) state it started in. This decay is isotropic, but the individual branching ratios of \(B_{18} \rightarrow \)
[A_2, \ldots, A_6] \text{ vary due to angular momentum selection rules}^2. \text{ Should the } B_{18} \text{ state decay back to } A_2 \text{ or } A_5, \text{ the system's angular momentum remains unchanged. If it decays to } [A_3, A_4, A_6], \text{ then the system has accumulated polarization. Using a sufficiently powerful laser to deplete } A_5 \text{ forces the population into a polarized state.}

\text{When a metastable } ^3\text{He encounters a ground state } ^3\text{He, they have a high probability of coherently exchanging their electronic states, converting the polarized metastable atom to a nuclear polarized ground state, and promoting an unpolarized ground state atom to a polarizable metastable atom. This process happens at the rate of gas collisions, which in a 1 mbar sample at 300 K is very rapid.}

2.8 MEOP Polarizer Basics

In order to generate polarized $^3$He nuclei using the metastability exchange process illustrated in this chapter, one needs to generate and sustain a relatively small population of metastable helium atoms and illuminate them with circularly polarized light in a reasonably uniform magnetic field. Exciting nuclei to the metastable state destroys the initial nuclear polarization of the atom, so the metastable rate of production (i.e. plasma discharge intensity) should be as low as possible. A long metastable lifetime is only possible in an environment that lacks relaxing third species like oxygen, nitrogen, or argon.

One can optimize a MEOP polarizer to maximize polarization rate or equilibrium polarization. At low polarization values and in less pure systems, C_9 yields the fastest polarization rate, due to its enhanced photon efficiency when $P_N < 0.5$. In clean systems, C_8 yields the highest total polarizations, primarily because the sublevel it interacts with maintains a larger population even as the sample polarization becomes quite large.

The next chapter details the systems necessary to generate metastable helium and to optically pump it. It also introduces and characterizes a polarimeter that may be used to observe the sample's accumulated nuclear polarization.

\textsuperscript{2} B_{18} \rightarrow A_1 \text{ has } \Delta m_f = 2
Real difficulties can be overcome; it is only the imaginary ones that are unconquerable.

— Theodore Vail

Given the relatively simple mechanics of MEOP polarization (discussed in chapter 2), building a functioning polarizer should be relatively straightforward. Straightforward is not quite the same thing as easy, but at worst one just has to repair all its subsystems, which is simply a matter of diligence.

In order to generate a hyperpolarized sample, a glass sample cell (section 3.1) is filled with high-purity $^3$He (section 3.5). A plasma discharge is initiated in the cell (section 3.2), and reduced in intensity as much as possible, while sustaining that discharge. Circularly polarized photons ($\sigma^+ / \sigma^-$) provided by a fiber laser pump the metastable elements of the sample. Exchange of polarized metastable states leads to polarized nuclear states. To measure the polarization of the sample, a plasma transition sensitive to nuclear polarization is monitored, and the ratio of left-circular to right-circular photons emitted by the sample can be mapped (as a function of sample pressure) to the total sample nuclear polarization.

These two sketches exist to illuminate and guide the reader through this chapter. A diagram containing all of the control systems organized by control groups may be found at the end of this chapter as section 3.7.2. A detailed listing of the elements used to construct this MEOP system is provided in table 3.3.

Nearly every element introduced so far has needed design, modification, or repair. This chapter details the repair of these subsystems and the pumping performance of the resulting system.
Figure 3.1 Picture of benchtop MEOP apparatus looking down polarimeter axis. A few things to note: cell valve (G3) is open while cell is cleaning, left circular polarizer (LCP) filter is attached to polarimeter, and one of the support posts next to the quarter wave plate (QWP) has been detached to allow access to the QWP lock screw.

Figure 3.2 Schematic of optical elements that comprise our MEOP polarizer. Laser frequency shifts are monitored by a 10 GHz Fabry-Perot interferometer. Pumped laser light is linearly polarized and transmitted along a PANDA fiber which is terminated by a FC/APC connector, which is coupled to a collimator. This collimated light is passed through a linearly polarizing beam cube, and the resulting horizontally polarized light is rotated to a circular mode by a quarter wave plate before passing through a beam expander and illuminating the sample cell. Light that interacts with the sample is scattered isotropically and can be observed with a Germanium Photodiode (GePD).
Figure 3.3 Schematic of optical polarimeter placed with respect to polarization cell. Light emitted by discharge in cell enters stack of optical elements, with left circular polarizer (LCP) only present during calibration of liquid crystal retarder (LCR). LCR converts incident circularly polarized photons to linear polarization, which are either transmitted or blocked by linear polarizer (LP). Of these remaining photons, a stack of filters strongly selects for fluorescence photons over reflected laser photons. The photons that survive are collected and amplified by a Silicon Photodiode (SiPD).
3.1 Polarization Cells

Being the result of multiple simultaneous optimizations, MEOP cell sizes are extremely consistent. Cell walls rapidly relax $^3$He*, so larger cells allow longer metastable lifetimes. Cells that are too large are susceptible to radiation trapping [Gen93], and require increasingly expensive optical elements to illuminate optimally. Glass composition itself can also affect the ultimate polarization. Alumino-silicate glass has a less permeable surface, and so is typically used in SEOP cells that take much longer to accumulate polarization [Gen17]. MEOP generates polarization much faster, allowing its cells to be fabricated out of readily available and easily worked borosilicate glass.

A product of generations of development, our sample cell is cylindrically shaped, 5 cm long and 5 cm in diameter, with a volume of approximately 100 mL, capped by optically flat windows for undistorted laser entry and egress. The plasma used in MEOP cells has extremely low optical density, so an admixture of the depolarizing circularity of light will affect the full sample being pumped, severely limiting the maximum polarization attainable in a given sample. The polarizing system is tuned to generate right-handed circular polarized light, which will be labeled $\sigma^+$, the desired optical polarization. $\sigma^-$ will destroy precious nuclear polarization.

Although the cell windows are nominally flat, they are capable of converting a fraction of $\sigma^+$ to $\sigma^-$, due to either varying thickness or resulting from birefringence in imperfectly worked glass. If an exhaustive study of the optical elements has been conducted, it may be possible to eke slightly more polarization out of a pumping system by testing a fleet of polarization cells.

3.2 Metastable generation

Optical methods don’t exist yet for directly promoting electrons from $^1S$ to $^3S$. While it may be possible to drive electrons into the metastable state with a tuned thermionic emitter, cleaning the electrodes poses a substantial challenge. Instead, standard practice is to drive a high voltage RF signal, typically 1 MHz to 10 MHz, although some work uses frequencies in the kHz range [Gen93], across a capacitively coupled set of ring electrodes outside the cell. The plasma environment is tremendously complicated, and is generally not treated with any degree of rigor in MEOP literature. That tradition is continued here.

It requires a substantially more powerful RF excitation to initiate a discharge than to sustain that same discharge. When starting a discharge, the helium in the cell is resting in its ground state. Once a discharge has started, in addition to ground state heliums, there are heliums in various ionic configurations (He+, He$_2^+$, etc.), as well as liberated electrons. These charged species are readily accelerated by the applied RF excitation, colliding with other cell residents and exciting their electrons.

The RF system developed here uses an antenna tuner to improve the RF coupling between the RF amplifier and the cell. Because this tuner must be manually adjusted, this work exclusively used a 10 MHz excitation frequency.
The cells, geometries, and pressures used by this work lend themselves to one of two different ignition methods. It is simplest to ramp the RF excitation power until a discharge spontaneously starts ($\sim 5$ W). The excitation can then be ramped down to generate a reasonable pumping fraction of metastables. This procedure can reliably ignite plasma discharges over the full operating range of this system (0.4 mbar to 2.1 mbar).

Generating plasma is inherently depolarizing; it isotropically populates the magnetic sublevels of the metastable atom. The highest degrees of polarization are found with weak discharges ($T_1 \geq 100$ s), and driving a high-power discharge effectively destroys a sample's remaining nuclear polarization. This is particularly problematic if one is attempting to study the less depolarizing but still significant contributions of the sample's cell walls and the system's holding field. These should be measured in a dark cell, one without a plasma discharge. A weak plasma discharge has $T_1 \approx 100$ s to 200 s, and a typical gradient relaxation $T_1$ in a cylindrical MEOP cell is typically at least 1000 s. Plasma relaxation typically dominates all other relaxation processes in a cell eq. (3.1).

$$\frac{1}{T_1} \approx \frac{1}{T_{1\text{plasma}}} + \frac{1}{T_{1\text{gradient}}} + \frac{1}{T_{1\text{walls}}} \tag{3.1}$$

It is useful to measure the $T_1$ without a plasma contribution, for which a more reliable and less depolarizing plasma ignition device proves useful. A kHz excitation can often initiate a discharge more easily than a 10 MHz one, and a reasonably high-quality kHz source is the flyback transformer from a $10$ plasma ball desk toy. By removing the circuit from the plasma ball (fig. 3.4) and coupling the excitation wire directly to a cell, a plasma discharge may be initiated at the flick of a switch. By wiring the flyback transformer's power supply to a programmable power supply such as the one found on the NI VirtualBench, that switch can be in software.

### Figure 3.4
The flyback transformer from a plasma ball desk toy serves as an excellent plasma ignition device. The white wire emerging from behind the black potted transformer connects to the glass sample cell between its two RF loops. The circuit is energized from the VB addressable power supply, as drawn in section 3.7.2.

At pressures above 0.9 mbar, the plasma ball igniter reliably starts a discharge in less than 0.5 s.
It is then possible to measure the $T_1$ of a system without plasma: polarize a cell, stop the plasma discharge, disable the laser, and wait some amount of time, and relight the discharge with the same discharge intensity used for polarization\(^1\), then calculate the remaining polarization from an exponential fit to its plasma-driven decay. Varying the “dark” wait time generates a set of points to fit to find the depolarizing contributions of the walls and magnetic field.

While developing this system, we attempted to streamline the RF coil design by attaching BNC ports to the cells themselves, allowing a single BNC to connect the antenna tuner to the cell itself, pictured in fig. 3.5. This was a terrible idea, because BNC connectors have an extremely high magnetic susceptibility.

![Figure 3.5](image)

**Figure 3.5** Left: cell with BNC port attached directly. Don’t do this. Right: Nonmagnetic coaxial cable wired to copper tape loops on cell.

Next, a coaxial cable was wired directly to the sample cell. This was also a terrible idea. It took substantially longer to discover, but we found that some BNC cables use nickel and iron wires for their core and/or shielding.

Both of these highly susceptible components were identified by their contribution to plasma $T_1$. A weak plasma discharge depolarizes a cell over hundreds of seconds, independent of the applied magnetic field ($<50$ G). This cell’s plasma lifetime was quadratic with respect to the applied magnetic field $(B_z, T_1) = (20$ G, $9$ s), $(25$ G, $13$ s), $(30$ G, $21$ s), which from McGregor eq. (3.24), manipulated to eq. (3.2), indicates that a constant and large perpendicular source is depolarizing the sample instead of a gradient emerging from a field defect.

$$\frac{B_\perp}{B_z} = \sqrt{ \frac{R^2}{T_1D} }$$  \hspace{1cm} (3.2)

This source was the BNC bayonet and cable. Neither component has a large amount of material, so both saturated well before the field reached 20 G. Once this wire and bayonet were removed

\(^1\)Data acquisition with the Mixed Signal Oscilloscope (MSO) has to stop to change the device sensitivity, so it is easiest to keep the optical signal at the same level to avoid losing data due to clipping from the oscilloscope.
and replaced with a copper-core (RG-58) cable, cell polarization improved dramatically, as did polarization lifetime, measured in a sealed cell in fig. 3.6. The test bed was built in a pair of large (26") Helmholtz coils that yielded the 10 G holding field used for system development with 1 A applied current.

![Graph showing electron polarization and relaxation time constant](image)

*Figure 3.6* Dark $T_1$ data from sealed cell. Note that because pressure of cell is unknown, only electron polarization is reported here. For measuring the relaxation time constant, $eP$ is sufficient.

### 3.3 Pumping Elements and Optics

Extensive interest in MEOP has led to the development of tunable Yb-doped fiber lasers with behavior optimized for MEOP [Tas04]. In order to efficiently deliver the laser’s output in a useful and safe manner, Keopsys coupled the laser to a 3 m polarization-maintaining optical fiber. The laser naturally generates linearly polarized light, and PANDA fibers are similarly designed to transmit linearly polarized light [Nod86]. This light is then passed through a quarter wave plate to rotate it to right-hand-circular polarized mode (RCP or $\sigma^+$ photons). This beam is technically capable of polarizing $^3$He, but it illuminates a tiny volume fraction of the pumping cell. To utilize most of those expensive photons, expand the beam so it uniformly illuminates the optical windows of the cell.

If laser power is limited, or you have extremely high pumping requirements, you can reflect the laser back through the cell with a dielectric mirror, gaining additional pumping from the now $\sigma^-$ photons as they pass back through the cell. This system functions well with a single pass, but the polarizing beamcube would allow a dielectric mirror to be added in the future without worrying about damaging the laser.
3.3.1 Beam Expander

We use a Galilean beam expander to illuminate the full sample cell. The beam emerging from the FC/APC collimator is 4 mm in diameter, and the sample cell is 5 cm in diameter. We want to expand the beam to illuminate as much of the cell as possible without illuminating the cylindrical glass side walls. With a $f_1 = 100$ mm converging lens and a $f_2 = -8.9$ mm diverging lens, we can build a beam expander with magnification of

$$\frac{f_1}{f_2} = 11.24 \times ,$$

with a corresponding beam size of

$$11.24 \cdot 4\,\text{mm} = 45\,\text{mm}$$

From a system length of

$$f_1 + f_2 = 91.1\,\text{mm}.$$ (3.5)

3.3.2 Laser tuning

This laser has both coarse and fine wavelength controls. The fine control is the manual dial on its face panel, and the coarse control is a BNC port on its back plate, with a strict 0 V to 4 V input range. Exceeding 4 V will damage the laser’s circulator, and it will need to be returned to France for repair. I control the coarse voltage with the variable 5 V power supply on a VirtualBench, and use LabVIEW’s In Range and Coerce function to ensure that voltages delivered to the laser are always $\leq 4$ V.

The simplest way to tune the laser’s output is to sweep the output wavelength and look for interactions with the metastable population. Metastables will scatter resonant light, but we were initially unable to observe any of the expected transitions.

Unable to observe any of the expected transitions in a $^3$He sample and suspicious that our laser tuning was ineffective, we secured a 10 GHz Fabry-Perot interferometer and coupled it to the laser’s seed laser tap to directly observe the frequency shift in the laser’s output. This second fiber conducts an unpolarized beam from the laser to the interferometer. The output is not collimated, but there are no elements and there is not much open air between the fiber and the interferometer’s input. When the laser is switched to pump, it substantially increases the seed laser power, requiring the apertures on the Fabry-Perot to be closed to avoid saturating its photodiode. By marking the times corresponding to one free spectral range with an oscilloscope, we determined that the laser was sweeping through 16.8 free spectral ranges as the laser’s coarse tuning voltage was incremented from 0 V to 4 V. This corresponds to a tuning range of 168 GHz, with a linear frequency response to applied voltage.

Manufacturer tests certified the laser’s tunable range to be 1082.680 nm to 1083.337 nm at maximum power. Most works measure the $C_8$ and $C_9$ transitions in terms of their frequency shift from $C_1$ at zero field [Bat11], but Cancio Pastor et al. measured the $^23\,P - ^23\,S$ transitions in absolute terms [Can12].

The laser is only certified to reach $C_8$ at maximum output power. At lower output power levels,
Table 3.1 Frequency and vacuum wavelengths of $^3$He transitions from $^2S_1$, from [Can12] Table 1.

<table>
<thead>
<tr>
<th>Line</th>
<th>$\nu$ (kHz)</th>
<th>$\lambda_{\text{vac}}$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C$_3$</td>
<td>276698164610.4</td>
<td>1083.4638</td>
</tr>
<tr>
<td>C$_4$</td>
<td>276698611209.1</td>
<td>1083.4621</td>
</tr>
<tr>
<td>C$_5$</td>
<td>276698832617.9</td>
<td>1083.4612</td>
</tr>
<tr>
<td>C$_6$</td>
<td>276700392099.8</td>
<td>1083.4551</td>
</tr>
<tr>
<td>C$_7$</td>
<td>276704904311.7</td>
<td>1083.4375</td>
</tr>
<tr>
<td>C$_8$</td>
<td>276726257468.9</td>
<td>1083.3539</td>
</tr>
<tr>
<td>C$_9$</td>
<td>276732997170.4</td>
<td>1083.3275</td>
</tr>
</tbody>
</table>

Figure 3.7 Relative strengths and energy separation of $^3$He and $^4$He metastable transitions from [Cou02].

the seed laser's power is also reduced, increasing the required coarse tuning voltage level to achieve the same emission wavelength. Operating at less than maximum power, the laser is incapable of reaching C$_8$. The laser's useful range, fortunately, is slightly larger. The certified range was tested with the front panel fine-adjustment potentiometer set to 50. The 0 V to 4 V coarse input range is independent of this control. Frequency increases with this dial, and setting it to 100 brings both C$_8$ and C$_9$ well within the tuning range of the laser for all power levels.

These are weaker transitions than the combined C$_3$-C$_7$, so they may be somewhat more difficult to locate, especially in a sample cell with low metastable density. With a clean sample cell, these transitions are readily apparent, centered around 3.485 V and 3.639 V for C$_9$ and C$_8$, respectively. The laser is temperature compensated, but these maxima may shift by $\pm$3 mV. When in doubt, just sweep the coarse tuning voltage, and pick out the peaks again.

If another laser needs to be purchased, be sure to specify the frequency you need. If you would rather specify wavelength, use both the vacuum and air figures.

### 3.3.3 Optical Fiber Structure and Repair

Once the laser has generated photons of the desired energy spectrum, they need to be delivered to the device. The laser generates linearly polarized light, which is passed into an optical fiber. In
Figure 3.8 $C_8$ and $C_9$ transitions, captured by light scattered to Ge Photodiode. The relative height of the $C_8$ and $C_9$ peaks is determined by the metastable density, which is in turn affected by sample contaminants. With a high-power laser and a very clean sample, background is around 20% of the $C_9$ peak.

an isotropic fiber, linear polarization modes are degenerate. Small perturbations from bending the fiber, for example, would lift this degeneracy, allowing photons to rotate out of their purely linear mode. These rotated photons would be wasted—discarded by the polarizing beamcube.

Figure 3.9 Polarizing beamsplitter splits arbitrarily polarized light into vertically polarized and horizontally polarized beams with a proprietary dielectric coating. The dielectric layer reflects vertically polarized photons and transmits horizontally polarized photons. In this system, the vertically polarized photons are directed into a beam dump.

Instead, the laser is injected into an anisotropic PANDA fiber, sketched in fig. 3.10. Aligned with the fast axis, the light's propagation mode is relatively immune to perturbation. The subtleties of single-mode polarization-maintaining fiber are beyond the scope of this work, but see [Nod86] for a comprehensive review of the technology.

As shown in fig. 3.2, the beam exits the fiber via an attached FC/APC connector and is collected by a collimator, which directs the beam into the polarization optics. Particular care should be taken when mounting the fiber in the collimator. If the fiber is not fully seated, the beam can reflect off
Figure 3.10 PANDA Polarization maintaining fiber. The core (black circle) is surrounded by cladding (gray) with two embedded off-centered cylinders of a higher-n material.

the collimator's metallic housing back onto the emission surface of the fiber. This is a dangerous situation! Reflecting light back into the laser can damage it, and directing this much additional optical power on the fiber tip can burn it, as seen in fig. 3.11. This damage was found after the laser output dimmed noticeably and the collimating assembly became warm to the touch.

Figure 3.11 Burnt and pitted PANDA fiber. Pictures are taken with 200× magnifying fiber inspection scope.

It is possible to repair the fiber tip on site, which is documented in fig. 3.13. Due to the exacting angular tolerances, coupling a new optical fiber is not possible by an end-user. The fiber repair roughly followed the procedure for constructing a new fiber. The ferrule was mounted in an aligning fixture, and was sanded with a coarse grit until the damaged areas had been removed (fig. 3.13 A). Finer grits (fig. 3.13 B,C) were then used, taking special care to clean the ferrule and the polishing mat (rubber on top of a glass plate) when changing grits. Like any polishing procedure, the larger grit should be removed as completely as possible before proceeding. Finally, the fiber was polished with a 0.02 μm diamond lapping sheet (fig. 3.13 D). The expanded beam was visibly brighter after polishing, and has not been disturbed since.

3.3.4 Circularity and Jones Calculus

Optical pumping relies on a circular polarization mode to transfer angular momentum into the target. This system uses a combination polarizing beamcube and indexable quarter wave plate
(ThorLabs VBC05-1064) to filter and rotate these photons. Although this device was factory calibrated, the quarter wave plate had been set to 0 deg, and testing revealed that the produced light was elliptical. It was not clear if the polarizer had been damaged or if the calibration value had been lost, so the polarization optics were moved to illuminate a large linear polarizer and the Germanium photodiode, sketched in fig. 3.14.

It is straightforward to predict the transmission of this system via Jones transfer matrices [Hec98], which represent the action of linear optical elements with $2 \times 2$ matrices that act on a column vector of the incident electric fields $\langle E_x; E_y \rangle$ perpendicular to beam's propagation axis. Rotation of the element is accomplished with a standard rotation matrix transformation. The effect of a stack of optical elements is calculated by multiplying the incident polarization by their respective transfer matrices.
Figure 3.13 Repair of damaged fiber tip. Pictures are taken with 200× magnifying fiber inspection scope. In order, lapping grit sizes were 6 µm, 3 µm, 1 µm, and 0.02 µm.
Transformation to and from local orientations is accomplished with rotation matrix

\[ R_\theta = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \] (3.6)

A rotated polarizer’s action is given by

\[ P_\theta = \begin{pmatrix} \cos^2 \theta & \cos \theta \cdot \sin \theta \\ \cos \theta \cdot \sin \theta & \sin^2 \theta \end{pmatrix}. \] (3.7)

A quarter waveplate shifts the relative phase of \( E_x \) and \( E_y \), and is given by

\[ Q = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \] (3.8)

This system starts with purely linearly polarized light, written as

\[ E_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \] (3.9)

Allowing the cumulative transmission intensity of the elements in the test bed to be written as the following product:

\[ T = |P_\theta \cdot R(\phi - \alpha)QR(\phi - \alpha) \cdot E_0|^2. \] (3.10)

Figure 3.15 Transmission data on left, model predictions on right. Model data has been fit to data with one free parameter.
Equation (3.10) calculates the intensity of light that is initially linearly polarized, passed through a quarter wave plate that has been rotated by $\phi - \alpha$ degrees, then passed through a linear polarizer that has been rotated $\theta$ degrees. The mean square deviation of its predicted transmission from measured transmission is minimized at $\alpha = 17.5^\circ$, which corresponds to 62.5° on the indexing ring for the quarter wave plate, using the aligning tick on the designed “bottom” of the VBC05. At this fitted minimum, the model’s predicted transmission is within 10% of the data for all measured configurations, shown in fig. 3.16. This indicates that the polarizer is functioning normally and should be set to $62.5^\circ \pm n \cdot 90^\circ$ to generate circularly polarized light. Although powerful, this technique is not capable of determining whether the generated light is right or left-handed. With a polarization signal from a functioning polarimeter (section 3.4), it is possible to determine the polarization of the pump beam.

**Figure 3.16** Fractional deviation of Jones calculus model from data.

### 3.3.5 Circularity Measurement

With a 1/2" quarter wave plate, the pumping light could be easily characterized. Viewed from the cube face opposite the polarization etchings, light polarized by the beam cube and expanding array is > 99.9% circular polarized with correct QWP alignment: $\sigma^- : 62.5^\circ$, $\sigma^+: 332.5^\circ$. Curiously, the light quality appears to be slightly better for $\sigma^+$ production, but this small effect may be due to variations in the testing laser’s output power.
3.4 Fluorescence Polarimetry

Because we are operating in a low field, the Zeeman shift is minuscule and we can directly measure
the electron polarization from fluorescence in the plasma. Helium’s red line at 668 nm is a bright
line, produced by electron fluorescence as they transition from $3^1D_2 \rightarrow 2^1P_1$. Exciting electrons to
the $3^1D_2$ state doesn’t affect the nuclear spin, but the electron is able to sample the nuclear spin via
hyperfine coupling during its 15 ns lifetime. Some of this information is lost during gas collisions,
so the signal weakens as sample pressure increases.

$$P_N = f(P) \cdot P_e$$ \hspace{1cm} (3.11)

The notation confusingly uses $P$ for both polarization and pressure, and reads as the nuclear polar-
ization $P_N$ is related to the measured electron polarization $P_e$ by a pressure-dependent weighting
factor $f(P)$. The particular weighting factor was measured by Lorenzon et al. in [Lor93], and also in
[Big92]. This work uses the fitted values from Lorenzon et al. The coupling is pressure dependent,
so absolute polarization measurements depend on an accurate pressure measurement. While its
functional form is complicated, the resulting behavior is not. It isn’t easy to key in all the relevant
fitting parameters from Lorenzon et al., so the relevant code (for pressure in torr!) is calculated as
a LabVIEW code block. The code itself is given in E.5.2 on page 217, but a simple plot of $f$ vs $P$ is
included as fig. 3.17.

![Figure 3.17](image)

Figure 3.17 Pressure-dependent $P_N/P_e$ correction factor from [Lor93].

Note that this technique has an unavoidable error of around 2 %. The polarimeter optical de-
sign is from [Max14], but the electronics differ somewhat from that prescription. The optical ele-
ments are rendered in fig. 3.19, separated by function. Photons are read by a silicon photodiode
coupled to a fixed high-gain amplifier. The optical signal is modulated by the polarimeter’s control
software at 2 Hz, and the sample’s polarization evolves over seconds, so this photodiode’s relatively
low 20 Hz bandwidth is in no way limiting.
The fluorescence line is relatively dim, especially compared to the watts of power flowing into the system from the laser. It is necessary to remove as much background light as possible, so a series of three optical filters are located in front of the photodiode. As seen in fig. 3.18, the filters are rated to transmit 40% of incident 667 nm light while suppressing 1083 nm photons by a factor of $1 \times 10^{16}$. Maxwell et al. observed laser photons leaking even with this filter stack, and we do too. The background term in eq. (3.12) varies with laser power and, in laboratories with windows, with time of day.

Additionally, the bandpass coatings used on the 650 nm and 670 nm filters is slowly degraded by atmospheric moisture. Thorlabs predicts two years at full performance before the filters start to degrade. As they degrade, they transmit less light, which has the effect of reducing the signal seen with a particular discharge intensity. The filters in the polarimeter were purchased on July 1, 2015.

![Figure 3.18](image)

**Figure 3.18** Top: Manufacturer supplied transmission curves of each filter used in polarimeter. Bottom: cumulative transmission of stacked filters. Note that this collection of filters transmits roughly half of the 667 nm fluorescence light while suppressing the 1080 nm laser pumping light by more than 16 orders of magnitude.

Instead of using two different photodiodes and working to compensate their differing response characteristics [Big92] or assembling a mechanically rotated waveplate read with a lock-in amplifier [Lor93], this polarimeter has a fixed linear polarizer and a LCR with its variable optical axis mounted 45 deg offset from it. The LCR is modulated from $\lambda/4$ to $3\lambda/4$. In the $\lambda/4$ mode, the LCR rotates LCP photons to a linearly polarized mode parallel to the polarizer's optical axis, while rotating
RCP photons to an orthogonal mode. $3\lambda/4$ mode is the converse, yielding transmitted RCP photons and absorbed LCP photons. Thus, by modulating the LCR tuning voltage and the resulting retardance from $\lambda/4$ to $3\lambda/4$, the relative population of right circular polarized photons to left circular polarized photons may be directly measured.

\[
P_N = \frac{f_P}{\cos \theta} \frac{I_{1/4} - I_{3/4}}{I_{1/4} + I_{3/4} - 2I_B}
\]  

where $f_P$ is the Lorenzon et al. pressure factor, $\theta$ is the azimuthal angle between the polarimeter's axis and the system's quantization axis as illustrated in fig. 3.3, and $I_{1/4}$, $I_{3/4}$, $I_B$ are the signal intensities for $\Delta_{LCR} = \lambda/4$, $\Delta_{LCR} = 3\lambda/4$, and background.

Standard error for eq. (3.12) is given by
\[
\sigma_p = \frac{2 f_p}{\cos \theta_m (I_{3/4} + I_{1/4})} \sqrt{I_{3/4}^2 \sigma_{3/4}^2 + I_{1/4}^2 \sigma_{1/4}^2},
\]

where errors \(\sigma_{1/4}\) and \(\sigma_{3/4}\) are extracted from their respective channels while the polarization system is held in equilibrium.

Equation (3.13) clearly illustrates the benefit of increasing total signal intensity. Equation (3.12), on the other hand, shows how stray photons reduce the sensitivity of this style of polarimeter. Figure 3.20 demonstrates the mixture of absorbing surfaces installed behind the cell to reduce this background signal. A large sheet of black safety foamcore absorbs in the visible spectrum. The patch illuminated with the laser is supplemented with a corrugated piece of infrared-absorbing aluminum foil.

![Absorbing foam-core board reduces polarimeter’s background by absorbing visible photons. A small patch of coated IR-absorbing aluminum foil absorbs laser.](image)

The LCR’s retardance is susceptible to drift over time, primarily driven by changes in the ambient temperature. The curve shown in fig. 3.21 is thus only a rough guide for where to tune the LCR controller. In order to tune the LCR, the polarimeter is supplied with pure left-handed circular polarized light by attaching a LHC polarizing filter. The tuning voltage to the LCR is swept over the values around 1/4 (3/4) wave retardance, and the voltages corresponding to the maximal (minimal) optical signals are recorded.

While it was simplest to calibrate the quarter wave plate in section 3.3.4 with Jones matrices, modeling the behavior of this optical polarimeter lends itself instead to Mueller matrices. Using Mueller calculus, a sample calibration signal may be approximated:

\[
\vec{S} = P \cdot R_{\pi/4} \cdot \Delta V \cdot \vec{R} \cdot \left(1, 0, 0, -1\right)^T
\]

where \(\Delta V\) is the voltage difference between the maximal and minimal signals.

\[\text{Equation (3.14)}\]

\[\text{\textsuperscript{2}}\text{Jones matrices act on the complex-valued } E_{x,y} \text{ vectors, but Mueller matrices operate on the Stokes vector, which can easily handle incoherent sources as well as mixtures of polarized light.}\]
Figure 3.21 Extrapolated LCR retardance in waves as a function of supplied voltage from Thorlabs listed retardance. $\frac{1}{4}$ and $\frac{3}{4}$ are found around 1 V and 2 V.

where $\Delta V$ is the Mueller matrix for an LCR, as given in eq. (3.18), $P$ is the Mueller matrix for a linear polarizer, and $R$ is a rotation matrix given by [Azz99]. Note that the action of the calibrating filter has been taken into account in the incident Stokes vector.

$$R_\phi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ \hspace{1cm} (3.15)

The Mueller matrix representation of an imperfect linear polarizer is given by

$$P(s, p) = A \left( \begin{pmatrix} s & 0 \\ 0 & p \end{pmatrix} \Phi \begin{pmatrix} s & 0 \\ 0 & p \end{pmatrix} \right) A^{-1}$$ \hspace{1cm} (3.16)

With $A$ defined in eq. (3.20) below, and $s$ ($p$) corresponding to the % transmission values for $s$ ($p$) polarized light.
3.4.1 Calibration Sensitivity Analysis

The polarimeter dynamically samples the emitted $\sigma^+$ and $\sigma^-$ populations, and calculates the plasma’s electron polarization ($P_e$) from that ratio.

It is possible for an uncorrected background source to artificially inflate or deflate the calculated polarization. It is relatively easy to check for background light: simply turn off the discharge and observe the dark signal. Polarization is calculated as a ratio of circularly polarized modes, but it is possible for ellipticity in the fluorescence signal to skew the measured polarization. The LCR is mounted in a rotating mount to expedite this test: the observed polarization ratio should not vary as the polarimeter is rotated. A more concerning source is thermal drift of the LCR. The optical depth of the nematic crystal is a function of temperature, which may drift over time. As the optical depth drifts, the LCR will admit increasingly large fractions of the wrongly-polarized photons.

We can predict the LCR’s response with Mueller matrices, tuned to parameters from ThorLabs. It is often easier to construct Jones matrices and transform them into Mueller matrices with eqs. (3.19) and (3.20) [Azz99]. Because Mueller matrices don't preserve phase information, we are able to omit global phase factors in favor of readability. Thus, the Jones matrix for an arbitrary phase retarder $\Delta J$ is

$$\Delta J = \begin{pmatrix} 1 & 0 \\ 0 & e^{-2i\pi\delta} \end{pmatrix}$$

(3.17)
which is transformed to its Mueller representation by
\[
\Delta_M = A (\Delta \otimes \Delta^*) A^{-1}.
\] (3.18)

More generally,
\[
\Gamma_M = A (J \otimes J^* ) A^{-1}
\] (3.19)

where \(J\) is the Jones matrix to be transformed and

\[
A = \begin{pmatrix}
1 & 0 & 0 & 1 \\
1 & 0 & 0 & -1 \\
0 & 1 & 1 & 0 \\
0 & -i & i & 0
\end{pmatrix}.
\] (3.20)

Phase retardance factors for this particular LCR shown as are available from ThorLabs [Tho16], and transmission data for the linear polarizer is similarly available [Tho18].

The behavior of the resulting stack of optical elements can be simulated (as shown in fig. 3.24), and reveals that it is reasonably insensitive to poor calibration. Furthermore, the measured asymmetry achieves a local maximum at the correct calibration values for the LCR. In other words, because any reasonable deviation in calibration reduces the measured \(\sigma^+ / \sigma^-\) asymmetry, the true nuclear polarization is not worse than your measured value.

Figure 3.23 LabVIEW shuffled 2D meshgrid design pattern.

It is possible to verify this simulation: a LabVIEW program generated a 2d mesh of LCR tuning voltages containing \(x = V_1 \equiv 3 \lambda / 4\) and \(y = V_2 \equiv 3 \lambda / 4\). These values were then shuffled (fig. 3.23) to guard against drifts in background illumination, laser output power, or plasma quality. The plasma discharge and laser were tuned using the VirtualBench control panel, allowing the LCR meshgrid control panel to capture the sample's polarization. The least squares paraboloid fit (fig. 3.25) is
shown below. Notably, the fitted parameters match the values determined using a more typical calibration routine.

### 3.4.2 LCR Timing Control

A DC offset would damage the nematic crystal panel in the LCR. A Thorlabs liquid crystal controller (LCC25) supplies the tuning voltage to the LCR, and can be operated in three different modes: Steady, square wave, and externally triggered.

Steady output does exactly what it describes: it drives the LCR at the requested voltage. Square wave output modulates between the two requested voltages at a user-requested frequency, typically 2 Hz. A USB interface with the LCR allows VIs to dynamically adjust the tuning voltage and the operating mode. There’s an unpredictable network lag when adjusting the output voltage by USB, making it not practical to programatically switch between $V_{3\lambda/4}$ and $V_{\lambda/4}$. The LCR itself introduces a bit of noise into the transmitted signal, and it is extremely difficult to extract this square wave modulated signal from a sample with relatively low polarization.

In order to switch the LCR at a precisely known time relative to the oscilloscope, the external triggering mode is used. In this mode, the LCR resides at $V_1$, the first specified voltage, switching to $V_2$ as long as the “EXT” signal supplied to the LCC25 is high (5 V). The VirtualBench oscilloscope offers a trigger signal, but this pulse is around 10 ns long—insufficient to integrate a $\lambda/4$ signal from the polarimeter.

This pulse is extended with a 74123 monostable vibrator pictured in fig. 3.28, tuned to deliver a 155 ms pulse when triggered. The pre-trigger acquisition of the oscilloscope yields the $3\lambda/4$ measurement, and after discarding the rise-time signal ($\approx$ 50 ms due to the LCR and the photodiode’s response rate [Tho16]), the $\lambda/4$ signal may be captured. The oscilloscope is configured to capture 100 ms pre-trigger and 150 ms once triggered. Functionally, a 250 ms signal is split into 5 parts, and the middle is discarded. This is illustrated in fig. 3.29, which was generated with a longer acquisition time to illustrate the slow decay from $\lambda/4$ to $3\lambda/4$. The LCR acquisition loop executes at 2 Hz, allowing the LCR and photodiode to equilibrate to their $3\lambda/4$ level between acquisitions.
Figure 3.24 Simulated nuclear polarization measurement from a 10% Optical polarization signal with a pressure factor of 8, as a function of calibration accuracy.

Figure 3.25 Best fit paraboloid to $\sigma^+ / \sigma^-$ asymmetry data from 2D parameter sweep of LCR polarimeter tuning voltages. $V_0, \lambda_{1/4}$ and $V_0, \lambda_{3/4}$ values repeated values measured with LCP calibration filter.
Figure 3.26 Electron polarization data from LCR parameter tuning sweep. Figure 3.25 is a paraboloid fit to this data.

\[ \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \propto P_e \]

Figure 3.27 74123 monostable vibrator pinout and logic table. The chip is run in the boxed configuration to generate a top hat shaped output.
Figure 3.28 74123 monostable vibrator pulse extender. The trigger pulse is received by the left-hand BNC port where it switches the 74123 chip to its high output state. The duration of the “high” part is established by the coupled capacitor and resistor. The chip’s Q output pin is connected to the LCC25.

Figure 3.29 Oscilloscope trace of LCR driven by triggered monostable vibrator. Here you can also clearly see the LCR’s rapid switch to higher modulating voltages and slower decay to lower modulating voltages.
3.5 MEOP Sample Preparation

Refillable SEOP cells are generally sealed with chem-thread valves using either viton or ethylene-propylene o-rings. Similarly, the cells themselves are connected to their filling manifold with o-ring quick-connect. Due to their high internal pressure and alkali content, SEOP cells are much less sensitive to contamination, allowing the resulting cells to be filled at one site, polarized at another, and deployed (emptied) at yet another [Swa12b].

As discussed earlier, MEOP relies on long-lived metastable states, which are rapidly relaxed by atmospheric gases such as oxygen, nitrogen, and argon [Sch70]. A substantial amount of water is typically adsorbed by glass and metal surfaces, so a high-purity vacuum system will need to be baked prior to use.

When possible, it is far easier to use MEOP to polarize pure $^3$He in a sealed cell, a (typically) cylindrical polarization cell that has been baked under vacuum to 300 C to 400 C, which effectively desorbs all of the internal glass surfaces. The cell is then filled with $^3$He before its filling stem is melted shut with a glassblowing torch. If the sample pressure needs to be known to an extremely high degree of precision, care should be taken at this phase: the sample pressure is likely disturbed somewhat by the extremely high heat of the molten glass stem [Lor93].

Initially, we attempted to adapt a SEOP fill station to fill our MEOP cells. Between the small diffusivity of the o-ring in the quick-connect joint and the low pumping rate at the cell joint’s manifold on the SEOP fill station, filled MEOP cells were too dirty to pump to extremely high degrees of polarization, achieving nuclear polarizations of around 40 % while pumping on C9, and 20 % with C8. C8 pumping is capable of producing higher total polarization, but it is more susceptible to contamination. Because the SEOP fill system did not have accurate pressure readings for the cell, preliminary pressure values were determined by the shape of the plasma discharge:

![Figure 3.30](image)

Figure 3.30 At higher pressures, charge mobility is reduced, and plasma discharge is concentrated around electrodes. This annular discharge distribution indicates pressure of at least 10 mbar. The cell in the rightmost image is also badly contaminated.

The true pressure of a filled cell could be determined by attaching it to a manifold with the 10 mbar baratron, and pumping out the cell in stages, and measuring the resulting pressure with the baratron.

The polarization cell needs to be filled in situ to avoid breaking and re-connecting ultra-clean
Table 3.2 Pinout for type 626 Baratron and Granville-Phillips type 316 controller.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Color</th>
<th>Baratron pin</th>
<th>GP 316 controller pin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>White</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>Power Com.</td>
<td>Black</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>−15 V</td>
<td>Green</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>15 V</td>
<td>Red</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>Signal Com.</td>
<td>Brown</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Chassis</td>
<td></td>
<td>15</td>
<td>9</td>
</tr>
</tbody>
</table>

vacuum joints. Once any polarization was achieved in a refillable cell, we focused on completing the gas handling system. The system is designed with the following goals, in order: supply clean $^3$He at 1 mbar to the cell, supply clean $^4$He for flushing/cleaning the system, and to be able to regenerate the getter and cold trap without having to vent the entire low-pressure manifold. The cell itself is mounted in a nylon and aluminum bracket, illuminated by the clean discharge in fig. 3.36 and also visible in fig. 3.20.

![Diagram of gas handling system](image_url)

Figure 3.31 Benchtop MEOP gas handling system. Pressure indicator to the left of the regulator is solely to prevent inadvertent overpressure of regulator input line. Getter is marked “G”, and blanked ports are indicated as well. Thick lines between valves are metal, thin connecting line are glass. Magnetic contamination from gas bottles is reduced with indicated 0.5 µm filters. G1 and G2 are Chem-thread valves with Viton o-rings. G3 is a greased ground glass stopcock.
3.5.1 Glass joints

It is convenient to be able to detach and reattach glass components at will, but the o-ring in a quick-connect joint isn't impermeable enough to keep a MEOP system really clean. Ground glass joints, greased with ultra low vapor pressure grease like Krytox LVP, can be leak-tight on any measurable scale. Ball and socket joints allow two degrees of freedom in a glass joint, which is desirable, but this same freedom is limiting: if they shift too far out of axial alignment, they inevitably leak. Some systems have effectively employed glass KF-style joints, but we have avoided them, attempting to reduce surfaces that are permeable by atmospheric contaminants. Torch-worked, or blown connections are the preferred sealing technology for glass MEOP gas handling components. After fighting persistent leaks due to incompletely cleaned o-ring joints and imperfectly aligned ball and socket joints, we committed to all-blown connections. This freezes the gas handling system in place, but this minor inconvenience is a fair trade. Once I torch-worked all the previously demountable joints, the gas handling system could be baked (keeping Viton o-ring seals below 130 °C), flushed with research grade $^4$He, and pumped clean.

3.5.2 Cleaning samples

Helium's $^3S$ state is only long-lived when it is deprived of third bodies to relax its excited electron. Bottled $^3$He is only isotopically pure, and may have small amounts of nitrogen in its mix. Also, regardless of how carefully a vacuum system has been constructed, it may have a small leak from atmosphere. This nitrogen and oxygen need to be removed from the sample before effective polarization can take place.

This system uses a SAES type St 707 chemical getter, which is fully activated by heating to 850° F for 10 min. Once activated, the alloy getters at 20 °C [SAE18]. It captures oxygen, hydrogen, nitrogen, H$_2$O and CO, but inert gases are not sorbed. This is a non-evaporative getter, so captured oxygen, carbon, and nitrogen are locked in the alloy matrix even at “activating” temperatures. Heating the getter doesn't release sorbed gases so much as it expedites their diffusion into the bulk of the getter's metallic alloy.
Because the getter is used at room temperature, its effectiveness diminishes as it accumulates a passivated layer of gettered atoms. Reactivating the getter simply moves these atoms into the bulk, exposing a fresh chemically active surface. This can be continued until the getter saturates, at which point it will need to be replaced.

This getter does not remove noble gases, so if a leak to atmosphere is present, argon will accumulate in the gas handling system. A liquid nitrogen cold trap will condense argon, allowing a refillable cell to operate for even longer without accumulating a metastable-toxic concentration of argon.

On its bench top testbed, the MEOP system is clean enough to achieve high polarization without a cold trap, so the nitrogen cold trap's port is sealed with a blank.

Once a sample is loaded, its cleanliness can be determined by its plasma discharge spectrum. A clean helium sample is typically light pink to almost white, and nitrogen contamination makes the plasma more blue. Instead of simply relying on the human eye, we can measure the discharge spectrum using an optical to near-infrared spectrometer (Ocean Optics USB2000+VIS-NIR).

Our bottle of $^3\text{He}$ is quite clean, but it still has small amounts of contaminants. These contaminants broaden helium's plasma lines and introduce peaks in the 400 nm to 600 nm range. After several hours, the getter effectively removes a huge fraction of this "dirt." The vast majority of this cleaning takes place in the first two hours, with minor changes for the next several days.

The 667 nm red fluorescence line used for polarimetry is clearly seen in this sample, and its brightness rapidly changes as the sample cleans. Because it isn't stable over seconds, much less minutes, it is not possible to calibrate the LCR with a brand-new fill. After about 30 minutes, the

---

**Figure 3.33** Optical spectra of a freshly filled cell (top, blue) and a gettered sample (bottom, red).
Figure 3.34 Temporally log-distributed spectra showing gettering. Note that the majority of absorption has occurred prior to 2 h mark.

sample is stable enough to run a reasonable calibration routine.

At this point, polarization of around 70 % is possible, with rapid improvements as the gas cleans. After two hours, 80 % polarization is achievable.

Figure 3.35 Pumping effectiveness rapidly improves as sample is cleaned.

Further cleaning beyond this two hour mark does not hurt polarization, but it doesn't appear to substantially improve pumping performance either.
3.6 Pumping Performance

While collecting the data reported in sections 3.6.2 and 3.6.3, the laser’s power control module faulted, rendering it unable to operate in its preferred automated power control (APC) mode. Instead, these runs were conducted in automated current control (ACC) mode, recording the power reported by the laser’s front panel. When received, the laser had a maximum power output of 11 W, but in its diminished capacity, we have only had 7.2 W of useful optical power. Control and analysis code is included in appendix E. Detailed polarization procedures and LabVIEW documentation may be found at appendix E.2. Sample Python analysis code (including a converted Jupyter notebook [Klu16]) is reproduced in appendix E.4.

3.6.1 Pumping Rates

Hyperpolarized $^3$He is often used as a beamline target or as a neutron spin filter. In these cases, the rate at which polarization can be produced is the limiting rate in the allowable flux through the target. This apparatus requires a relatively small $N_3 \approx 1 \times 10^{16}$, but ideal operating conditions for the cell will produce $1 \times 10^{18}$ polarized spins (calculated in section 3.7.1). The initial pumping rate of a MEOP system operating near 1 mbar with 1.5 W laser power is around $0.5 \times 10^{18}$ s$^{-1}$ [Gen03]. This is clearly not sustained for the duration of the pumping, but it gives us a reasonable timescale to consider when discussing pumping rates.

While per-photon efficiency for $C_8$ is independent of total polarization [Bat11], the overall pumping efficiency changes as a function of polarization, because the pumpable population is depleted as polarization is accumulated, as shown in fig. 3.37. As a result of this, it will take substantially longer than 1 s to reach equilibrium polarization.

When the system is pumping, $^3$He$^*$ is pumped from $A_6$ to $B_{17}$, where it decays isotropically. This “scattered” light is seen by the GePD, which yields a reasonable estimate of the time required for the electron polarization to reach equilibrium. Figure 3.38 illustrates this calculation and the fluorescence polarimeter’s simultaneously observed polarization. The “equilibrium GePD” value...
Figure 3.37 Relative shift in photon absorption for C\textsubscript{8} pumping as polarization is accumulated, calculated with spin temperature distributions given by table 2.5.

is determined by averaging the GePD signal over the last 100 s of acquisition, at which time the sample has been in equilibrium for at least 50 s. The time between when pumping starts and the GePD signal crosses this value is designated the pumping time. Figure 3.39 illustrates the variation in pumping time for different laser powers. This time to reach equilibrium is less than 100 s for all laser powers.

3.6.2 Equilibrium polarization

As a MEOP sample pressure decreases, \(^3\text{He}^*\) mean free path and the corresponding probability of a wall interaction increases. However, at lower pressures, there is less collisional mixing of \(^2^3\text{P}^\text{excited}\) states, improving photon efficiencies for C\textsubscript{8} pumping. As a result of these competing sources of depolarization, low pressure MEOP systems are best behaved around 0.4 mbar to 1.4 mbar, with the highest polarizations obtained in sealed cells filled to the low end of that range. The ratio of \(^3\text{He}\) partial pressure to the plasma-driven decay rate is given as the parameter \(P_3/T_1\) and, as a rough proxy for the number of metastable atoms in the sample, allows samples of different pressure to be plotted simultaneously. Lower values of this parameter tend to yield higher polarizations.

At lower pressures, it can be more difficult to initiate a discharge, and the system is more sensitive to the presence of contaminants. If the pumping beam has a measurable admixture of \(\sigma^-\) photons, the equilibrium polarization will reach a global maximum before falling off as more laser power is delivered to the system [Lar91]. \(^3\text{He}\) MEOP is conducted at 1083 nm, but the closest commercially available elements are tuned for 1064 nm Nd:YAG optics. This \(\approx 2\%\) deviation combined with photon depolarization in the beam optics and cell walls prevent MEOP systems from achieving \(\eta_\sigma \equiv P_{\sigma^-}/P_{\sigma^+} = 0\), but the quarter wave plate characterization studies of section 3.3.4 suggest that
Figure 3.38 Left column shows highest observed polarization, its pumping time, and its instantaneous $P_N$. Right column overlays that data for seven datasets. Each sample was pumped and observed for 250 s, but tailing 150 s of steady-state data is not shown.

Figure 3.39 Pumping time doesn't change dramatically as laser power increases or as plasma discharge increases in power. An astute reader will notice that the $>80\%$ polarization values reported in fig. 3.40 are not reproduced in this figure. The gas sample was a different age, and the GePD was moved between these runs, making it unsuitable to compare their scattered optical signal.
we won’t be too badly impacted, and the polarizations achieved with 7 W pumping power (figs. 3.40 and 3.41) bear that out.

Equilibrium Polarization, $P = 0.995 \text{ mbar}$

![Graph showing equilibrium polarization achieved in 0.995 mbar $^3$He sample with 7 W laser power. $\sigma^-$ photons were generated by adjusting QWP on polarizing beamcube.]

**Figure 3.40** Equilibrium polarization achieved in 0.995 mbar $^3$He sample with 7 W laser power. $\sigma^-$ photons were generated by adjusting QWP on polarizing beamcube.

### 3.6.3 Isotopic Mixtures (D$_0$ pumping)

Stoltz et al. demonstrated extremely high polarizations (>80%) in 3:1 isotopic mixtures ($P_3 \approx P_{\text{tot}}/4$) with approximately 100 mW pumping power [Sto96]. Courtade et al. derive the expected behavior of MEOP in isotopic mixtures [Cou02]. Measuring this polarization is less straightforward in an isotopic mixture because both $^3$He and $^4$He fluoresce at 667 nm. The pressure factor derives from collisional mixing of $^3$He in the 3D level, which isn’t strongly affected by the isotope of helium that contributes those gas collisions. Stoltz et al. found that a simple scaling of the pressure factor by the isotopic concentration $X = P_3/P_{\text{tot}}$, as in eq. (3.21) with nuclear polarization $M$ and observed electron polarization $\mathcal{P}$, was accurate to within 5% to 10%.

$$[M/\mathcal{P}]_{\text{mix}}(P_{\text{tot}}, X) = \frac{1}{X} [M/\mathcal{P}]_{\text{pure}}(P_{\text{tot}}) \quad (3.21)$$

Researchers typically source high purity isotopic mixtures when studying D$_0$ pumping. Because this system was envisioned to use polarize pure $^3$He, such a mixture was not available. As a proof of principle, however, it is possible to use the gas handling system illustrated in fig. 3.31 to prepare an isotopic mixture. Per the work of Stoltz et al. [Sto96], ideal isotopic pumping is achieved with
Figure 3.41 Ultimate polarization achieved as a function of laser power, sample pressure, and discharge intensity. $P_N$ is calculated with Lorenzon’s $f_P$, so all points should be further decorated with a ±2% error bar.
With a sample mixed with research grade $^4$He and much cleaner $^3$He, the equilibrium polarizations seen in fig. 3.42 are far less repeatable than what was observed in fig. 3.40. An attempt to replicate the low-power pumping of [Sto96] failed when $D_0$ could not be found with an un-pumped beam, and the pumped beam destroyed a neutral-density filter that was intended to lessen the laser’s power.

![Figure 3.42](image)

**Figure 3.42** Ultimate polarization achieved in 2.1 mbar, 24.8 % $^3$He sample with 7 W laser power.

### 3.7 Cryogenic Sample Preparation

This polarization system is destined for the SOS apparatus, which has requirements for $^3$He independent of what can be generated with MEOP. It is not feasible to change the amount of $^3$He that is polarized with each batch. Instead, it is best to adjust the number of $^3$He nuclei ($N$) after polarization is complete. This low pressure $^3$He sample then needs to be supplemented with $^4$He in order to overcome the vapor pressure of the liquid bath it is destined for.

#### 3.7.1 What $N$ achieves $X$?

The SOS@PULSTAR sample cell and associated hardware (i.e. vestibule, bellows, etc) will hold approximately 51 of helium-II. Below the lambda transition point, $\rho = 0.145 \text{g/cm}^3$, or approximately $2.2 \times 10^{22}$ atoms/mL, yielding $N_4 \approx 1.1 \times 10^{26}$. With a desired isotopic $^3$He/$^4$He fraction $X$ of $3 \times 10^{-10}$, we need $N_3 \approx 3 \times 10^{10}$.

With a 100 mL cell, 0.4 mbar to 1.3 mbar of purified $^3$He is $9.66 \times 10^{17}$ to $3.14 \times 10^{18}$ atoms. This
is several orders more than is needed in SOS, so only a small fraction of this sample should actually be injected. The easiest method of reducing N is with volumetric dilution, but care must be taken because T<sub>1</sub> relaxation rates are a function of field uniformity, cell size, and pressure.

Samples at high pressure and low pressure have longer T<sub>1</sub> lifetime than samples at intermediate pressures. These pressure regions are defined by Cates et al., calculable with the ratio reproduced here as eq. (3.23) [Cat88]. In a high pressure system (κ ≫ 1), spins are not free to sample the field’s defects, so gradients have a vanishing effect on T<sub>1</sub> relaxation. As pressure decreases, the spins are better able to explore varying fields. At low enough pressures (κ ≪ 1), spins have long free paths and have correspondingly long polarization lifetime. The region where τ<sub>d</sub> ≈ τ<sub>I</sub> is the maximally depolarizing one with the most rapid T<sub>1</sub> as you traverse this pressure range. Diffusion coefficient D is a function of pressure:

\[ D(P) = D_0 P_0 / P, \text{ with } D_0 = \frac{1370 \text{ cm}^2/\text{s}}{} \text{ and } P_0 = 1 \text{ torr} \]  

(3.22)

\[ \kappa = \frac{\Omega_0 R^2}{D} = \frac{p}{p^*} = \frac{\tau_d}{\tau_I} \]  

(3.23)

For a cell that is 5 cm long and 5 cm in diameter, filled to 1 mbar in a 5 G field, this ratio is 55, placing our sample initially in the “high pressure” regime. Diluting this sample reduces this ratio proportionally, so a naively conducted 100-fold pressure dilution could enter the maximally depolarizing region where τ<sub>d</sub> ≈ τ<sub>I</sub>.

### 3.7.2 Dilution Induced Depolarization

A more detailed calculation of the expected T<sub>1</sub> follows, using equations 14 and 16 from McGregor.

\[ \frac{1}{T_1} = \frac{\gamma^2}{2} (|\nabla H_r|^2) S_x(\omega_0) \]  

(3.24)

\[ S_x(\omega_0) = \frac{2\langle u_x^2 \rangle \tau_c}{\omega_0^2(1 + \omega_0^2 \tau_c^2)} \]  

(3.25)

\[ \omega_0 = \gamma \cdot H_0 \quad \langle u_x^2 \rangle = \frac{kT}{2M} \quad \tau_c = \frac{DM}{kT} \]  

(3.26)

In order to calculate our predicted T<sub>1</sub>, this can be simplified somewhat. The term given by \[ |\nabla H_r|/H_0 \] is the fractional gradient, and is in units of %/cm.

\[ \frac{1}{T_1} = \left| \frac{\nabla H_r}{H_0} \right|^2 \frac{D}{\left[ 1 + \gamma^2 H_0^2 \frac{D^2 M^2}{k^2 T^2} \right]} \]  

(3.27)
This is truly a worst case scenario—the sample will reside in the region with smallest $T_1$ after dilution. Fortunately, this trough may be avoided by pressurizing the sample with $^4$He prior to dilution. This will have the effect of decreasing $\tau_c$ in $^3$He’s autocorrelation function eq. (3.25), preventing the behavior of fig. 3.43. After the dilution, more $^4$He will need to be added to complete the injection procedure.

It is not unreasonable to be concerned about gradient induced depolarization while the sample is being injected. We expect to have fairly large gradients present, and by the intermediate value theorem, the sample pressure must pass through the maximally depolarizing value as it transitions from several mbar to a few $\mu$mbar. Fortunately, this calculation is not applicable there, because the autocorrelation function given in eq. (3.25) is for a bottled gas at a reasonably high pressure.
Figure 3.44 MEOP electronics and control systems. Computer controlled devices are marked with USB symbol. Virtual Bench subsystems are independently accessed in LabVIEW, so they have been separated spatially for clarity.
### Table 3.3 MEOP system parts list

**Gas Handling**
- 10 mbar Baratron: MKS, Type 626
- Chemical getter: SAES, S5H0380 MAP/7/20 mini CFF
- Heater & Thermocouple: SAES, 5H0590
- Temperature Control: Athena, XT16JFB000
- 0.5µm filter: Swagelok, SS-4F-T7-05
- Bellows valve: Swagelok, SS-4H
- Vacuum regulator: Matheson, SEQ3494
- All-metal angle valve: VAT (Lesker), 54124-GE02
- Chem-vac valve: Chemglass, CG-925-01
- Glass stopcock: Chemglass, CG-473-02
- Vacuum grease: Chemours, Krytox LVP
- Spectrometer: Ocean Optics, USB2000+VIS-NIR

**Pump**
- Collimator: Thor, F280APC-1064
- Polarizer: Thor, VBC05-1064
- Diverging lens: Thor, LC2067-C
- Focusing lens: Thor, LA1050-C
- Beam dump: Thor, LB2
- Laser: Keopsys, CYFL-GIGA-10-LP-1083-WT1-FM1-ST1-OM1-B301-FA

**Control/RF**
- VirtualBench: NI, VB-8012
- Amplifier: Minicircuits, LZY-22+
- Antenna Tuner: MFJ, MFJ-945E
- Non-magnetic cables: RG-58

**ν Monitor**
- Fabry-Perot Interferometer: Thor, SA210-8B (10 GHz FSR)
- F-P Controller/Amplifier: Thor, SA201
- Germanium Photodiode: Thor, PDA50B

**Polarimeter**
- Silicon Photodiode: Thor, PDF10A
- 650 nm Bandpass filter: Thor, FB650-40
- 670 nm Bandpass filter: Thor, FB670-10
- 800 nm Shortpass filter: Thor, FES0800
- Linear polarizer: Thor, LPNIRED100-B
- Liquid Crystal Retarder: Thor, LCC1112-A
- LCR Controller: Thor, LCC25
- Left Circular Polarizer: Edmund Optics, 88-096
- Timing Chip: 74123
CHAPTER 4

DEVELOPMENT OF APPARATUS TO STUDY GRADIENT INDUCED $^3$HE RELAXATION

Always assume that the thermal connections will be at least a factor two worse than you calculate. Cooling powers will be less than calculated, while heat leaks will be higher. Assume that every factor that can discomfit you will be several times worse than what you calculate. Cryogenic physicists are naturally pessimists.

— David Haase

The work detailed in this chapter was done in collaboration with Adam Dipert. We have built substantial changes and improvements on the $T_1$ gradient relaxation apparatus used by Swank in [Swa12b]. The apparatus may be decommissioned without a successful data collection run, but the techniques developed here should prove helpful to future generations of experiments and experimentalists.

Swank successfully measured samples with $X$ ranging from $1.04 \times 10^{-6}$ to $3.12 \times 10^{-3}$, which is sufficient to observe the divergence of relaxation rates predicted by diffusion theory and high frequency theory, but is just on the cusp of the divergence predicted by Swank et al. in [Swa12a] as the sample transitions to the ballistic regime, which occurs at and below $X = 1 \times 10^{-6}$. A successful measurement of this behavior requires a sample that is more than 10 times as dilute, which yields a 10x reduction in NMR signal as well.

These sensitivities are rapidly approaching what can be readily observed with conventional pickup coils, and the planned concentrations for Systematic and Operational Studies at PULSTAR (SOS) are well beyond their feasible sensitivities. This is within the expected sensitivity for a superconducting quantum interference device (SQUID), which offers substantially improved pickup sensitivity at the cost of substantially increased shielding and cooling needs. Adam has pioneered SQUID capabilities for this device and for SOS.

Readers interested in SQUID and pickup coil design should see Adam Dipert’s upcoming dissertation (Arizona State University est. Fall 2018) which covers this apparatus in far more detail. The
system improvements detailed in this chapter are cryogenic system design, cell fabrication, and cell cooling.

4.1 Dilution Refrigerator

The $^3$He-phonon dynamics used in the full-scale $^3$He-electric dipole moment of the neutron (nEDM) require a sample of pure helium-II at 400 mK to 600 mK. Continuous cooling at these temperatures is provided by a dilution refrigerator. A well-established technology, dilution refrigerators are constructed with several stages arranged along a thermally isolated column [Lou79]. Most of these stages operate with recirculated $^3$He and $^4$He, but the first stage, a $^4$He evaporator called a 1K Pot, is supplied with liquid helium from the main cooling bath. The 1K pot is supplied with long capillaries from the main bath, and is connected to a dedicated vacuum pump. The pressure in the pot, and its corresponding temperature, is a function of the pumping speed, the liquid flow rate into the pot, and the heat load supplied by the rest of the dilution refrigerator.

4.1.1 1K Pot Clogging

Over the course of several years, we have regularly experienced total clogging of the 1K pot in a Leiden Cryogenics dilution refrigerator, preventing cooling below 4 K. Blockages in the 1K pot’s capillaries have two measurable effects: the pressure drops from its typical operating value of 20 mbar, and the temperature climbs from 1.8 K to 4 K. These clogs could generally be cleared by warming the DR to 77 K, wasting days of work and several hundred liters of liquid helium in the process. By carefully ruling out other sources of ice and establishing that the clog could be temporarily cleared by changing the applied pressure, we have determined that these clogs are formed by crystalline neon, and can be eliminated by filtering crystals large enough to form jams out of the helium while transferring it to the cryostat’s main bath.

4.1.2 Remediation

We started with the simplest remedies, and tuned our efforts to protect from the usual suspects. In order to keep contaminants (e.g. air, hydrogen, and water) from freezing in the 1K pot as the helium bath is filled, we simultaneously pressurized the pot with elementally pure helium (supplied by LHe boil-off) to 1250 mbar until the liquid level was several cm above the 1K pot’s intake ports. Although this is a good practice, it did not prevent clogging. Our dilution refrigerator (Leiden Cryogenics Minikelvin 126-TOF) has two capillaries that refill its 1K pot with helium from the surrounding liquid bath. The first capillary has a 0.3 mm inner diameter, and provides a fixed flow impedance. The second feeds a needle valve, and has a 0.5 mm ID. Replacing both capillaries with new clean tubing failed to prevent clogging. Unfortunately, there were not enough discrete clogging events recorded to calculate the clogging events’ characteristic timescales\(^1\) before and after this change.

\(^1\)This may not even be a well-defined parameter.
Hydrogen can outgas from aged pump oil before diffusing up the vacuum line and freezing in the 1K pot's capillaries. As it had been several years since the pump's last service, we flushed and replaced its oil. Wary of contaminants that are often introduced along with liquid helium, we used a coarse (100 µm) filter on our LHe transfer bayonet (fig. 4.2). Clogging events persisted in spite of these changes.

Although we typically stopped pumping and began warming our cryostat after the first clogging event was observed (as in fig. 4.1 A), we recorded two additional events where we were able to restore liquid flow into the 1K pot by different means. By opening the needle valve, we were (unsurprisingly) able to temporarily restore liquid flow into the 1K pot. A representative run is shown in fig. 4.1 B. After opening the needle valve, liquid flow resumed and the 1K pot operated normally for several hours. The needle valve's larger capillary also clogged after several hours of flow.

We found that we could restore liquid flow by back-pressurizing the 1K pot with pure helium gas. This operation, called “burping” in [Mal87], is able to temporarily restore fluid flow, as seen in fig. 4.1 C. Our burping procedure was as follows: the 1K pot (typically at 20 mbar during normal operation) was charged with 1311 mbar of pure helium. The pumping line was sealed under pressure, allowing the excess helium to flow through the capillaries into the LHe bath. As the 1K pot’s pressure equilibrated with the bath (at approximately 1000 mbar), the pot was pressurized again. In total, the pot was charged with pressurized helium 5 times. Partial cooling was observed within ten hours.

Figure 4.1 1K Pot Thermistor data from three different cool-downs. Higher values indicate lower temperatures, and (blue) shaded region indicates typical operating temperatures (1.8 K to 2 K). All time units are in hours; the boxed units indicate hours of pumping prior to blockage event. Flow was restored in B and C by opening the needle valve and “burping”, respectively.
minutes, and full cooling (and normal operation) was observed within twenty minutes of the burping procedure, indicating the resumption of liquid flow into the pot. This did not prevent future clogging events, and subsequent clogs occurred faster. Pressurizing the 1K pot with elementally pure helium during the initial fill ensured that the blockage was almost certainly not any form of frozen air, and changing the vacuum pump's oil eliminated the only source of hydrogen that could be frozen into a solid plug. The blockage's sensitivity to pressure fluctuations suggests that it is not a solid plug at all. In fact, this clogging behavior is remarkably similar to the clogging detailed in [Mal87], and is characteristic of a static granular heap.

Our cryostat has a full compliment of RuOx thermistors to monitor the DR's operation, and we also have a pair of diode thermometers (LakeShore DT-670C-CU) to monitor the liquid level in our main helium bath. We are unable to measure the temperature of the 1K pot's capillaries directly, but their temperature must be between that of the 1K pot and the main bath. During one run where the 1K pot clogged, we continued pumping as we warmed up the cryostat. We observed a sudden increase in 1K pot pressure (indicating that the blockage had cleared) when the capillaries were around 30 K.

This clogging behavior cannot be explained by hydrogen ice plugs — these would not be cleared by burping the pumping line, furthermore, they should melt well before this temperature is attained. The jam clears itself when the 1K pot's fill lines are in the range of 30 K to 40 K. This temperature range suggests we are dealing with neon, which matches what Mallory et al. observed with their RGA [Mal87].

4.1.3 Hopper Flow

In order to treat and ultimately prevent this clog, we must first understand its general properties. It is clear that the clog is not a solid ice plug of any kind. Instead, there is a flow impedance caused by an agglomeration of grains of some kind. While it is possible for these grains to merely impede flow in a dynamic fashion, that is not what we are experiencing: our grains are dilute in their suspension (as evidenced by the density of the liquid), and should thus be subject to substantial density fluctuations. Once a “clog” has been established, however, it never spontaneously clears itself. These clogs also persist after the liquid level has dropped below the 1K pot's fill line.

This sort of clogging behavior mirrors that of hopper jamming and static granular heap formation. Unlike most of the literature that deals with glass beads, grain in silos, and mixtures of pharmaceuticals, we appear to be dealing with small neon crystals. We cannot easily change the geometry of our 1K pot or its capillaries, so we instead control the size of the grains we allow into our system. We will select a grain size that is too small to form stable heaps, and only allow those right ones in.

The probability that a heap of grains will clog is a function of their shape, their radius, and the radius of the orifice through which they are passing [Zur05]. It is most convenient to consider a unit-free ratio of radii \( R \equiv r_{\text{orifice}} / r_{\text{grain}} \). Unsurprisingly, as \( R \) increases, the clogging event’s timescale increases. This increase is not linear, and appears to have asymptotic behavior [To05].
One can thus define a “critical radius”: the smallest value of $R$ for which clogging does not occur, i.e. for which the jamming time is infinite. For our purposes, this establishes a minimum acceptable dimension for grain size, beyond which clogging can not occur. This value depends on the geometry of the orifice and of the grains, but it appears to be around 6.5 in two-dimensional systems [To05] and around 5 in three-dimensional silos [Zur05]. These values assume smooth grains, and this system is affected by coarse crystals. Increasing particle roughness increases their dilatancy [Hsi17]. To be safe, we require $R \geq 7$. Our smallest capillary is 300µm in diameter, so our contaminant particles must be less than 43µm. Our initial sintered filter was coarser than this, and proved insufficient.

![Diagram](image)

**Figure 4.2** Transfer line filter mount schematic. Spacer is sized to ensure tight seal between filter and weld bead on threaded tip. The filter’s added flow impedance necessitates pressurizing the LHe dewar to 8 psi to achieve reasonable transfer times.

To prevent the accumulation of particles capable of clogging our system. We have replaced this filter (as indicated in fig. 4.2) with a 5µm sintered stainless steel filter ($R = 60$), which is pressed between the bayonet tip and the vacuum jacketed transfer line’s terminal weld bead seen in fig. 4.3, preventing particles from flowing around the filter.

![Images](image)

**Figure 4.3** Left: the welded tip of the vacuum-jacketed LHe transfer bayonet. A fine filter (right) is pressed against this weld bead by the threaded bayonet tip extension.
Since the introduction of this filter, we have run the 1K pot for hundreds of hours without incident. Due to the exponential distribution of clogging events, one cannot say with certainty that a clogging event can not happen, but it appears to be vanishingly unlikely.

4.2 Cell Designs

While various systems were being repaired, we were able to substantially revamp the sample cell for this measurement. During his research, Swank found that a heat leak limited how cold he could run his cell, thus limiting the phonon density range he could explore. The cell is subjected to radiant heating from the LHe bath, but an even larger heat load is from the glass inlet connection. In order to remove this heat, the sample cell must be coupled to the dilution refrigerator's mixing chamber. Previous generations of this experiment used a bundle of oxygen free high thermal conductivity copper (OFHC) wires mated to grooves machined outside the sample cell, described in [Swa12b]. Because the thermal conductivity through PMMA is extremely poor, this cell cooling design struggled to remove heat introduced by superfluid film in the glass fill line, limiting the temperature that the cell could achieve.

4.2.1 Cell Construction

Surface interactions can be a substantial source of \(^3\)He-relaxing interactions [Yod10], so it is crucial that the surface of these sample cells be as consistent as possible. Although this experiment does not use cohabiting neutrons or any form of optical readout, it does still have the same wall coating as the cells for SOS@PULSTAR and the full-scale \(^3\)He-nEDM experiment: deuterated tetraphenyl butadiene (TPB), embedded in a polystyrene (PS) matrix.

While other coating techniques such as spray application and CVD have been explored, but we have found that so-called “swing coating” yields the highest quality surface finish, with minimal crystallization of TPB.

![Figure 4.4](image-url) A: Plate is lowered in shallow tray containing TPB and PS. B: Plate is swung out of liquid bath. Meniscus sweeps along face, leaving a uniformly thin coating behind.
**Figure 4.5** Assembly of sample cell with diffusion-limiting capillary. Thermally conductive Kapton is glued to the open face of the PMMA cell plate, and a needle is inserted through the Kapton to prevent Stycast from forming a solid plug.

**Figure 4.6** Two assembled 1” sample cells. Cell on left is glued to polyether ether ketone (PEEK) column, as discussed in section 4.2.3.
4.2.2 Sample Cooling

Rather than OFHC and acrylic, a new cell was designed using a direct helium-II column connection between the buffer volume and the sample cell. It is necessary to hold the $^3$He in the sample cell long enough to actually conduct a measurement, so a sheet of high-conductivity Kapton was added as a barrier between the cell and the cooling column. This helium-II will also supply the interior of the cell, so the Kapton film has a pinhole in it. To further impede diffusion of $^3$He out of the cell, a long 2 mm tube was glued to the Kapton.

Due to damage incurred in shipping from Berlin to Durham, the mixing chamber of our dilution refrigerator is not perpendicular to the column of the rest of the DR. When the sample cell was cooled by a cluster of OFHC wires, this was not an issue. The glass fill inlet could be bent as it was blown in place, and the wires themselves could bend to support the cell in its location near the center of the coil package. With a superfluid-containing tube holding the cell, initial alignment is far more critical. With a careful measurement of the buffer cell alignment, we built a CAD model of the mixing chamber, buffer cell, and sample cell all in their desired final locations. It was then possible to model a PEEK tube to connect the buffer cell to the sample cell.

Figure 4.7 Section view of glued sample cell and cooling assembly. Note that Kapton capillary extends several inches into PEEK tube.

4.2.3 Assembly/Alignment

Starting with the CAD model used in section 4.2.2, a frame was added as a new solid. This frame holds the cell tightly and bolted on to the buffer volume for alignment, but as it was going to be
epoxied together, the PEEK tube fit the buffer and sample cells very loosely. The frame was isolated and exported for 3D printing and printed on NCSU’s Stratasys uPrint SE Plus 3D printer. This particular printer was the most precise model available, with a 0.010 in layer height and soluble support structure. Figure 4.8 shows the assembly and gluing process for the cell in its frame. All components were glued with Stycast 1266, mixed with a ratio by weight of 100:26 (A:B).

Figure 4.8 Assembled PMMA sample cell, PEEK cooling/alignment column, and buffer volume. Parts are held in place with 3D printed frame prior to gluing with Stycast 1266.

4.3 Future Work

With a cold leak somewhere in the vicinity of the sample cell and ongoing difficulties with a SQUID driven pickup coil, this cell is not likely to be used in its current cryostat. With judicious epoxy application, it may be possible to seal the cell and run a simpler measurement using the new SOS cryostat.
5.1 Folding; the problem

The content of this chapter is a minor extension of published work [Rei17].

Origami typically calls to mind images of flowers, leaves, birds and ever more sophisticated and beautiful sculptures. Reluctant to cut, glue, or stretch their medium, artists have developed a stunning family of fold patterns and techniques to reshape flat sheets into imagined forms. Literally “folding paper”, the art of origami is fundamentally the study of the generation of dramatic changes of a material’s appearance and bulk mechanical properties via the application of a sequence of highly localized deformations.

Origami lattices are a prototypical metamaterial, readily converting an unwieldy film into a robust device capable of reliable and simple actuation [Leb15]. A remarkably small fraction of a sheet is deformed when it is creased, but the mere existence of a crease dramatically changes its deformation modes. Several creases acting in concert can govern a device’s kinematics—the final device’s degrees of freedom depend more on its geometry than on its local structural properties [Sch13a]. Like other metamaterials, origami lattices have several exotic mechanical properties that can be tuned with small variations in their design [Sil14]. Of particular interest, the Miura-ori chevron tessellation expands in all directions when pulled apart, exhibiting a negative Poisson’s ratio [Wei13]. Furthermore, several classes of flat-foldable origami are known to exhibit bistable behavior [Sil15; Bru16]. Origami lattices are amenable to rigorous mathematical handling [Dem01], which generates a direction for designers exploring the space of accessible patterns. Subsequent work in that quantitative endeavor has led to a set of powerful theorems and tools, in particular to deal with the
non self-intersection constraint [Bel02]. Classical “rigid face” origami tessellations are constructed with planar faces linked by flexible but well-defined hinge-like creases. These origami metamaterials have raised interest in disparate fields [Dur12], where further theoretical development is driven by a multitude of creative applications.

It is important to distinguish between origami geometry and origami mechanics. Origami geometry concerns itself with mathematically ideal objects. These bodies are inextensible, uniformly flat, and generally defined with a degree of internal symmetry. With surfaces and linkages so constrained, the base configuration of the device may be perfectly specified. Considering each crease as a hinge mechanism, it is possible to count up the degrees of freedom, which is sufficient to determine if the device is rigid or if it can be smoothly actuated. For example, the crystalline silicon cells found in deployable solar panels are effectively inelastic, and the mechanism can be analyzed much like its idealized mathematical analog. These techniques are useful for generating starting geometries and framing problems of interest, but they are insufficient to yield mechanical insight. They are forced, by their base assumptions, to ignore the differing energetic costs of various fold configurations, and they are thereby unable to predict or explain the complete mechanical response of an origami folded device. In particular, they fail for structures folded out of materials with nonzero elasticity, as both the faces and the creases have innate elastic energy. Indeed, creases have a preferred angle of repose [Lec14], which can either stabilize a particular configuration or drive it far from its rigid-face equilibrium. Moreover, creases and faces tend to bend on different energy scales, and the competition of these effects leads to dramatically different behavior than what geometric models might predict. For example, Ref. [Sil15] demonstrates how face bending can generate a pathway for an origami mechanism to follow while transitioning through a geometrically forbidden configuration to a lower energy state, while in Ref. [Lec14] a straightforward technique to measure the competition between crease and plate bending is demonstrated.

Much of the previous theoretical work was focused on planar lattices [Wei13], although cylindrical configurations are of considerable technical interest. Origami folded cylinders have found applications in space technology for deployable sails and booms [Sch14; Gue94b], medical devices such as stents [Kur06], and even nuclear physics chapter 6. Although these fields are very sensitive to reliability and cost concerns, pattern development has been conducted in a largely ad-hoc manner [Sch13b] due to the absence of a general predictive framework for their performance. Recent work with such configurations has led to several remarkable theoretical developments. In Ref. [Tac12], a family of rigid foldable cylindrical bellows is identified and a mechanism whereby the mechanics of such bellows could be tuned is demonstrated [Yas15]. It was later proved that a cylinder constructed out of radially arranged Miura corrugations is incapable of rigid foldability [Bös15]. Recent developments have explored the bistability of bellows patterned with Miura-ori folds [Cai15] as well as a Kresling pattern [Jia15]. These works utilize an elastic rod framework to explore the dynamic response of a folded bellows. This suffices to illustrate the existence of geometrically allowed bistable configurations, but it fails to capture the behavior of a folded device as it actuates.
In the present work, we use properties established by a general solution for allowable configurations to predict and explain the responses of real bellows. To this purpose, we will first define and solve the geometry at hand, detailing each constraint and assumption. We will then explore the mechanical response of their physical manifestations, necessitating the construction of a series of tunable cylindrically symmetrical bellows before subjecting them to controlled actuation and collapse. We will conclude finally with a discussion of the behavior observed during actuation, with a special focus on its possible applications.

5.2 Origami bellows Geometry

Given an ideal origami pattern represented as a system of linked rigid polyhedra, there exist multiple families of frameworks that may be used to analyze its allowable configurations. If an origami system is represented as a mechanism of rigid linkages [Gue94b], it can be subjected to classical constraint-counting techniques. Some of these frameworks invoke quaternionic algebra to generate the relative rotations of their linkages [Gue94b] or faces [Wu10]. Other treatments of Miura-ori sheets and cylinders use the angle between faces as their control parameters [Wei13; Wu10]. In the following, we instead parametrize origami tessellations using the set of vectors associated with the crease network [Bru16]. By solving the fully-constrained behavior of a periodic fundamental origami cell, we have found an analytic solution for all possible rigid-face states accessible from both cylindrical Miura-ori and Kresling patterns.

To illustrate the benefits of this method, consider the folding pattern depicted in fig. 5.1. As shown in fig. 5.2, the tessellations of several types of rotationally symmetric bellows can be derived from variations on this unit cell. Planar tilings of these unit cells have been studied extensively, but relatively few works have studied cylindrical configurations [Jia15; Bös15]. Some developable patterns on the cylinder lack rotational symmetry [Yas15], but here we will focus on regular cylindrical tilings of the above-mentioned cells, whose regularity will be formally introduced as uniform rotational symmetry.
The unit cells illustrated in fig. 5.2 generate bellows like those shown in fig. 5.3. They have been organized by class and type leading to four major domains. Type A and type B differ in the value of the continuously variable angle $\phi_1$, where type A patterns correspond to $\phi_1 < \pi/2$ and type B patterns are characterized by $\phi_1 > \pi/2$. Class 1 and 2 patterns differ in the existence of the crease vector $\vec{w}_0$, which leads to a different set of fold parametrizations and constraint counts, but will not dramatically alter the solutions for rigid-face states. When $0 < \phi_2 < \phi_1 < \pi/2$, the generated pattern is the familiar Miura corrugation for class 1 and the Kresling pattern for class 2. If $\pi/2 < \phi_1 < \pi$ and
$0 < \phi_2 < \frac{\pi}{2}$, a hexagonal grid is generated instead for class 1, and a Yoshimura pattern is generated for class 2. These angular ranges are nearly sufficient to define the class of figures, but one additionally needs to maintain positive edge lengths. As far as class 2 fold patterns are concerned, thin-walled cylinders under axial compression tend to develop a rigid triangular tessellation known as a Yoshimura pattern, first described in [Yos55]. This tessellation is described by this framework as class 2–type B. Under torsion, thin-walled cylinders of certain lengths buckle into a twisted triangular configuration, known as a Kresling pattern [Kre02], described here as class 2–type A.

Figure 5.3 First row: Miura-Ori (left) and Kresling (right) bellows models annotated with example unit cell vectors. Second row: annotated vector illustration of unit cell in XY plane. Photographs are at approximately the same scale and these structures correspond to $n = 5$ unit cells.

Figure 5.1 shows that a class 1 unit cell is described by a total of five fold vectors $\vec{\omega}_i$, with $|\vec{\omega}_i| = \ell_i$. By setting $\ell_0 + \ell_1 + \ell_4 = 1$, a global length scale is defined allowing the parametrization of all edge lengths as:

$|\vec{\omega}_0| = \ell_0$, 

$|\vec{\omega}_1| = |\vec{\omega}_4| = \ell_1 = \frac{1}{2} (1 - \ell_0)$ ,

$|\vec{\omega}_2| = \ell_2 = \frac{h}{2} \csc \phi_1$ ,

$|\vec{\omega}_3| = \ell_3 = \frac{h}{2} \csc \phi_2$ .

(5.1)

(5.2)

The free length parameters $\ell_0$ and the lateral size of a single band $h$ are geometrically restricted to
Inequality (5.3) ensures that $\ell_1$ remains positive and that the edge defined by $\omega_3$ does not cross $\omega_2$ in an adjacent unit cell. Note that class 2 fold patterns are described by this same framework but with $\ell_0 = 0$ and $h = 1/(\cot \phi_2 - \cot \phi_1)$.

When the imprinted network of creases is rigidly folded, two degrees of freedom can be fixed because of global rotational symmetry. A first degree of freedom can be eliminated by taking $\omega_0 = \ell_0 \hat{i}$, where $\hat{i}$ is a unit vector parallel to the X-axis. Second, the vectors $\omega_1$ and $\omega_4$ also lie in the XY plane, locking the final degree of freedom and expediting the generation of the remaining vectors. Here, we utilize $SO(3)$ rotation matrices in lieu of Wu & You’s quaternionic approach [Wu10].

\begin{align*}
\omega_1 &= \ell_1 R_z(-\theta_1) \hat{i}, \\
\omega_2 &= \ell_2 R_x(\psi) R_y(-\phi_1) \hat{i}, \\
\omega_3 &= \ell_3 R_x(\psi) R_y(-\phi_2) \hat{i}, \\
\omega_4 &= \ell_4 R_z(\theta_2) \hat{i},
\end{align*}

Figure 5.4 Relationship of $\omega_0$, $\omega_2$, and $\psi$

Where $\theta_1$ describes the angular deflection of $\omega_1$ with respect to $\omega_0$, and $\theta_2$ parameterizes the opening angle between $\omega_0$ and $\omega_4$, as seen in fig. 5.3. To handle the opening angle of the pattern about $\omega_0$, the angular deflection $\psi$ from vertical (XZ-plane) is a measure of the panel’s angular deviation, also illustrated in fig. 5.4. By this definition, $\pi - 2\psi$ is the opening angle between adjacent bands of folds. To ensure a symmetric cylindrical configuration of class 1 constructions, we introduce an additional constraint on $\theta_1$ and $\theta_2$.

\[ \theta_2 = \pi - 2\frac{\pi}{n} - \theta_1. \] (5.8)

Here $n$ is the number of unit cells. Condition (5.8) is necessary to generate well-behaved closed
tubes for all \( n \geq 3 \). It is easily shown that the parametrization described by eqs. (5.4) to (5.7) reduces to a system of two equations, with the as-yet unspecified parameters \( \theta_1 \) and \( \psi \):

\[
\mathbf{w}_1 \cdot \mathbf{w}_3 = \ell_1 \ell_3 \cos \theta_1 \cos \phi_2 + \sin \theta_1 \sin \phi_2 \sin \psi = \ell_1 \ell_3 \cos \phi_2, \quad (5.9)
\]

\[
\mathbf{w}_2 \cdot \mathbf{w}_4 = \ell_2 \ell_4 \left[ \cos \theta_1 \cos \phi_1 - \sin \theta_1 \sin \phi_1 \sin \psi \right] = -\ell_2 \ell_4 \cos \phi_1. \quad (5.10)
\]

Using eq. (5.8), one can show (as seen in appendix B.1) that this system of equations is satisfied by

\[
\tan \frac{\theta_1}{2} = \frac{1}{2 \tan \frac{\pi}{n}} \left[ 1 - \frac{\tan \phi_2}{\tan \phi_1} \pm \sqrt{\left( \frac{\tan \phi_2}{\tan \phi_1} - 1 \right)^2 - 4 \frac{\tan \phi_2}{\tan \phi_1} \tan^2 \frac{\pi}{n}} \right], \quad (5.11)
\]

\[
\sin \psi = \frac{\tan \frac{\theta_1}{2}}{\tan \phi_2}. \quad (5.12)
\]

Equations (5.11) and (5.12) determine \( \theta_1 \) and \( \psi \) as functions of \( \phi_1 \) and \( \phi_2 \), the only control parameters of bellows geometry. One can show that the solution of eq. (5.11) satisfies \( \theta_1 \leq \pi - \frac{2 \pi}{n} \), and thus self-intersection of the faces is implicitly avoided. However for eq. (5.11) and eq. (5.12) to yield physically meaningful solutions, one must still satisfy that \( \theta_1 \) is real and \( |\sin \psi| \leq 1 \). Using these conditions one can show that there are no more than two geometrically allowed rigid face configurations for a closed band constructed of at least 3 unit cells. The structure of the phase diagram is illustrated in fig. 5.5. The regions of the \( \phi_1, \phi_2 \) parameter space where physical solutions exist are bounded by several functions:

\[
f(\phi_1, n) = \arctan \left( \tan(\phi_1) \cdot \frac{1 - \sin \frac{\pi}{n}}{1 + \sin \frac{\pi}{n}} \right), \quad (5.13)
\]

\[
g_1(\phi_1, n) = \phi_1 - \frac{\pi}{n}, \quad (5.14)
\]

\[
g_2(\phi_1, n) = \phi_1 - \pi + \frac{\pi}{n}. \quad (5.15)
\]

\( f(\phi_1, n) \) is found by limiting the solutions of eq. (5.11) to be real, and \( g_1 \) (resp. \( g_2 \)) corresponds to the case \( \sin \psi = 1 \) (resp. \( \sin \psi = -1 \)).

Although the number of faces is identical, class 1 and class 2 tessellations differ in their total number of folds requiring careful verification of allowable rigid-face configurations for class 2 folded patterns. Using the same approach one can show that the corresponding solution space is indeed also bounded by eqs. (5.13) to (5.15), generating identical diagrams as the solutions detailed in fig. 5.5, indicating that the bistability of these bellows is topological in nature. These results dovetail with Connelly’s Bellows Theorem [Con97], which states that continuous deformations of a closed triangulated surface cannot change its volume, and that the number of enclosed volumes attainable by reconfiguring a closed triangulated surface is finite. By assuming rotational symmetry, this solution does not explore the ability for a single band of folds to continuously deform. These
Figure 5.5 Parameter space for class 1 and class 2 family bellows for a number $n \geq 3$ of unit cells.

Figure 5.6 For $\phi_1, \phi_2$ selected from the red “bistable” region of fig. 5.5, a class 1 (Miura Ori) bellows has a stowed and a deployed configuration that can be achieved without deforming the faces in any way.
folding modes require $\omega_0$, $\omega_1$, and $\omega_4$ to not be coplanar, a folding mode which is blocked by the presence of the cells corresponding to $\omega_2$ and $\omega_3$’s mirrored edges.

The classification by type in fig. 5.2 is chosen to mirror the structure of the parameter space in fig. 5.5. As there are zero free parameters in the system of equations given by eqs. (5.11) and (5.12), we will only find a finite number of allowable solutions at any point in parameter space. The figures constructed with $\phi_1 > \frac{\pi}{2}$ are at best monostable. The boundary defined by $g_2(\phi_1, n)$ generates a flat-folded monostable figure.

The type A bellows is the one generated in the left half of fig. 5.5, with a bistable region neighboring a larger monostable region. The bistable region is bounded on the left by $f(\phi_1, n)$ and below by $g_1(\phi_1, n)$. Along the curve $g_1$ one of the configurations is flat-folded. At $(\phi_1, \phi_2) = (\frac{\pi}{2}, \frac{\pi}{2} - \frac{\pi}{n})$ the deployed state is completely extended, with perfectly flat walls along the bellows’ axial direction. Moving to smaller values of $\phi_1$ along $g_1$, the deployed configuration’s hinge angle decreases until $\phi_1 = \frac{\pi}{4} + \frac{\pi}{2n}$, at which point the bistable states are degenerate. The deployable bellows explored in [Sch13b] all lie along $g_1$, with increasing deployability as $\phi_1$ increases from $\frac{\pi}{4} + \frac{\pi}{2n}$ to $\frac{\pi}{2}$.

Figure 5.7 Detailed figure of the (red) bistable region of fig. 5.5. Greyscale shading indicates the angle difference $\Delta \psi$ (in radians) between collapsed and deployed solutions, with colored call outs for the contours indicated on the figure’s right. The purple curve represents zero separation between the solutions, indicating the solution’s degeneracy along the curve $f(\phi_1, n)$.

In the region of existence of two states, the difference between more collapsed and more extended configurations for an origami cylinder may be colloquially referred to as its “deployability”. The magnitude of extension between the two states is easiest to measure from the angle difference $\Delta \psi$ between the two allowed rigid face configurations by eq. (5.11) and eq. (5.12). Figure 5.7 illustrates the parameter space governing these tubes. Along the line $\phi_2 = \phi_1 - \frac{\pi}{n}$, one of the solutions is flat foldable and therefore lies entirely within the XY-plane, and along the line $\phi_1 = \frac{\pi}{2}$, one of the solutions is maximally extended, perpendicular to the XY-plane. The pair of states found at $(\phi_1, \phi_2) = (\frac{\pi}{2}, \frac{\pi}{2} - \frac{\pi}{n})$ therefore correspond to flat foldable and maximally extended states. This point
is the maximally deployable configuration, with $\Delta \psi = \frac{\pi}{2}$. The deployability decreases monotonically as one moves away from this maximal configuration. This overall behavior is independent of the number of facets in the cylinder’s folding pattern. Figure 5.7 also hints that small variations in $\phi_1$ or $\phi_2$ can have dramatic changes in the mechanical stability of these objects.

Individual bands of a class 2 tessellation do not affect the configuration of their neighbors because the angle parameter for the polygonal footprint, $\theta = \pi - \frac{2\pi}{n}$, does not vary during the pattern’s collapse (see fig. 5.3). This behavior is different from that of a class 1 folded cylinder: $\theta_1$ and $\theta_2$ change from one stable configuration to the other, so each band of the bellows is forced to move in lockstep with its neighbors. While both of the individual bands have the same allowable configurations, the deployment behavior of an aggregate bellows should not be the same. Each stage of a Kresling bellows is able to snap between its bistable states independently of its neighbors, but a Miura-ori cylinder’s only stable rigid face configurations are completely collapsed or completely deployed. This lack of intermediate stable configurations contributes to the appearance of smooth deployability. To demonstrate this most clearly, a physical bellows should be constructed.

5.3 Mechanics

5.3.1 Device fabrication

In order to generate regularly folded bellows, a laser cutter was used to etch flat panels of a substrate, which could then be assembled into a uniform cylinder. Cylinders were initially constructed by cutting a complete fold pattern from a single sheet that included a row of tabs with which the sheet was glued into a cylinder. Once glued, the panels’ hinges could be folded so that the entire device collapsed into its preferred configuration. While useful for exploring various fold patterns, this assembly technique is not acceptable for device fabrication. The height of the energy barrier between bistable configurations is a function of the face bending energy [Sil15], and this technique, although rapid, generates a single column of particularly stiffer walls.

Instead, our bellows are constructed much like a glued paper lantern. Individual strips of acetate sheets with thickness 0.1 mm (transparency slides) are cut out. Instead of scoring creases, the sheets are perforated: leaving a constant length fraction attached for each crease ensures a hinge energy that scales appropriately with the length of the crease and allows for a small torsional stiffness of the crease compared with the bending stiffness of the faces. This energy scale separation has been checked by observing that faces of a single actuated crease do not bend. Notice that such an estimate would only provide a lower bound since in a bellows configuration the kinematics rigidify the faces. Each strip has a set of tabs that connect it to its neighbor with double-sided tape (fig. 5.8A). Panels are aligned where laser-cut lines converge, and global alignment is double-checked with the flatness of the assembled sheet of strips before they are connected into a tube. This generates a device with an isotropic cross-section, as seen in fig. 5.8B. Once shaped into a tube, the top and bottom regions are reinforced with a layer of scotch tape. Without this tape, the tubes tend to pull apart from the ends under the stress of the initial folding. Edges are gently pre-creased,
working along the entire tube (fig. 5.8C). As the tube approaches the desired shape, more force is used until the panels buckle flat (fig. 5.8D). With this design, one edge of each face is stiffer than the remainder of the bellows. Fortunately, the stiffened region is very close to the crease, where mechanics are dominated by the hinge energy. The bellows is collapsed during assembly, giving a preliminary indication of how it will respond to forced cycling.

![Figure 5.8 Isotropic bellows construction. Individual strips are patterned with the laser cutter before being attached by the small tabs seen at right-most edge of (A). Strips are then rolled into a tube (B) which is then collapsed one cell at a time (C) to form the final bellows (D)](image)

The naming convention used in fig. 5.2 may be adapted for device fabrication by considering a single band of circumferential rigid panels (see fig. 5.1). A class 2 (triangular tessellation) bellows can be constructed with any number of these bands. An odd number of bands must twist during compression, but mirrored pairs of bands will have no net twist. A class 1 (Miura-ori) bellows should be designed with an even number of bands, as each one needs to be paired with its mirror image. To average out small defects from construction while keeping the bellows as large as possible, each bellows is constructed with 6 total bands per strip. In addition to the bands themselves, a bit of extra material is required to seal the end of the bellows. Furthermore, a transition pattern is required for a Miura-Ori folded bellows to connect smoothly to these convex and rigid end caps. Finally, five-panel \((n = 5)\) bellows were selected in the following study.

Although Miura-ori and triangular tessellated unit cells are described by identical phase diagrams, the aggregate behavior of a collection of unit cells may be different during extension and collapse. Exploring this divergent behavior requires the careful selection of a control parameter to hold constant among multiple test geometries. Because the total number of unit cells will vary during testing, the total fold length and the folds length relative to the overall base size were conserved.

### 5.3.2 Experimental Observations

Because the bellows are fabricated with nonzero hinge energy and panels of finite thickness, a mathematically bistable configuration may not be mechanically bistable, i.e. able to remain in one metastable state without external forcing. As a matter of fact, fig. 5.5 shows that bellows constructed along the curve \(f(\phi_1, n)\) are degenerate and thus monostable, and bellows constructed
along the segment \((\phi_1 = \pi/2, \frac{\pi}{2} - \frac{\pi}{n} \leq \phi_2 \leq \frac{\pi}{2})\) have a barrier to their collapse, and they are thus mechanically bistable. Therefore, one expects that a transition between these regimes should exist for a critical design angle \(\phi_1 = \phi_c\). When \(\phi_1 < \phi_c\), the bellows is unable to remain in its collapsed state and is expected to deploy itself to very near its rigid-face extended state after its collapse. This property provides with a large design space for actuation, as will be discussed below.

Inspired by [Sch13b; Kan00], initial values of \(\phi_1 = \frac{2\pi}{5}\) and \(\phi_2 = \phi_1 - \frac{\pi}{n}\) were selected. This generates a flat-foldable bellows that is very close to the minimal distortion geometry described in [Kan00]. We found that such a “flat foldable” bellows is unable to maintain a stowed configuration, instead demonstrating a self-deployability. Although it would be possible to move into the non-flat stowed region \((\phi_2 > \phi_1 - \frac{\pi}{n})\), it is more interesting to explore various values of \(\phi_1\) by keeping \(\phi_2 = \phi_1 - \frac{\pi}{n}\). The exact value of the critical \(\phi_1\) for this transition is dependent on assembly, but it appears that \(\phi_c \gtrsim \frac{2\pi}{5} = 72°\). Class 1 and 2 bellows were designed on either side of this transition \((\phi_1 = 68°\) and \(76°)\) in order to demonstrate its existence.

The mechanical response of the bellows was performed using an Instron® test frame and its corresponding deformation was captured with a Nikon D800 both as high resolution still images and as movie shots at 720p, 60 fps. Bellows were designed for their endcaps to have zero net rotation as they actuate, and they were mounted to the Instron® on irrotational compression plates. Movies (available as supplemental material) were synchronized to the Instron® test frame’s data after their capture via motion tracking. On all the following force versus displacement curves, the zero displacement point is chosen at the extended position of the bellows where the applied force is nearest to zero after attaching the bellows. The zero load reference point is established by the dynamic force value at the moment the test frame reverses direction from extension to compression.

As seen in fig. 5.9, after the initial compression’s dramatic collapse behavior, subsequent cycles follow each other closely, with only minor aging of the device as it cycles, and the mechanical response of the bellows converges towards a limiting cycle. This typical behavior is not reported in figs. 5.10 to 5.13 which only illustrate a cycle consisting of a single extension followed by compression.

The class 1–type A bellows with \(\phi_1 = 68° < \phi_c\) in fig. 5.10 is the softest of the bellows examined here. As seen in its force-displacement curve as well as Movie-S1 in [Rei17], it can be fully collapsed with only 5 N applied force. Unfortunately, we are unable to resolve face-bending behavior during the collapse of this pattern. Its class 2–type A equivalent (fig. 5.11 and Movie-S2 in [Rei17]) does show small steps as the paired cells collapse, but the smooth extension stroke (the lower half of the force-displacement plot) indicates that this pattern does not lock in its collapsed state, instead deploying itself after collapse.

The class 1–type A bellows with \(\phi_1 = 76° > \phi_c\) in fig. 5.12 exhibits minor locking (see also Movie-S3 in [Rei17]), but this is an artifact of the transition end pieces, which convert the variable concave cross-section to a uniform convex hexagon that can be mounted rigidly to the Instron®.

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1See Supplemental Material at http://link.aps.org/supplemental/ for movies of the mechanical testings corresponding to fig. 5.10 (Movie-S1), fig. 5.11 (Movie-S2), fig. 5.12 (Movie-S3), fig. 5.13 (Movie-S4), fig. 5.14 (Movie-S5) and fig. 5.15 (Movie-S6).
**Figure 5.9** Typical mechanical fatigue curve as a bellows (here a Class 2–Type A) is repeatedly actuated. The snap-through transition rapidly fatigues the thin polymer bellows, leading to softening of the device on subsequent (darker) cycles without modifying qualitatively the mechanical response of the device. Top (red) curves are from multiple compressions, and the bottom (blue) curves are data from the corresponding extensions.

**Figure 5.10** Class 1–Type A (Miura-ori) bellows with $\phi_1 = 68^\circ$. This device collapses smoothly, with only a small step as its transition panels collapse. Force measurement corresponding to image on left is indicated as a solid point. The colored gradient trailing the ball indicates previous force/position measurements, and the shadowed line indicates future measurements. See also Movie-S1 in [Rei17].
class 2–Type A bellows in fig. 5.13 with $\phi_1 = 76^\circ > \phi_c$ demonstrates the most interesting behavior of this set: clear face bending, crease bending, and self-locking behavior. An overview of its cycling behavior is displayed in Movie-S4 [Rei17]. The sharp steps on the extension stroke indicate locking of individual cells and the failure of those cells as they extend to their other rigid-faced state.

Looking closely at the ends of the bellows in the last frame of fig. 5.13 reveals that the outer edges parallel to $\omega_2$ (see fig. 5.2) are bent. The bellows has been driven out of its rigid face equilibrium, and the only way to access the stable collapsed state is by bending the faces. Class 2 bellows, however, are triangular and can only bend their faces by also bending a crease. The regularity of this crease bending is evidence of the suitability of the construction techniques detailed earlier. Crease bending also is far more energetically expensive than plate bending, which largely accounts for why the class 2 bellows are so much more rigid than their class 1 counterparts. After the crease bend emerges, it does not remain in the same location. By traversing the face, the vertex of the crease bend introduces a mobile crease to the fold pattern, thereby bypassing the preconditions required for the bellows theorem to apply [Gue94a].

In all cases, it is easier to extend a collapsed bellows than it is to collapse a rigidly extended bellows. This is readily explained by the interplay between the hinge bending and plate bending: When extended, the hinge is held relatively near its preferred angle of repose. The plate’s dramatically higher folding energy then dominates the dynamics of the device, leading it to equilibrate near its rigid face state. When collapsed, the hinges are compressed far from their angle of repose, leading to dramatic deformations of the plates, as seen in fig. 5.13. Bent, the plates are driven away from their rigid-face equilibrium towards the potential barrier between the two states. Because the collapsed state is driven far from its equilibrium state by hinge energy, it is easier to overcome the
Figure 5.12 Class 1–Type A (Miura-ori) bellows with $\phi_1 = 76^\circ$. This device collapses with a single large step at its transition panels, and deploys in a fairly smooth manner. See also Movie-S3 in [Rei17].
Figure 5.13 Class 2–Type A (Kresling) bellows with $\phi_1 = 76^\circ$ shows the most dramatic behavior of the lot. Contrary to class 1–type A shown in fig. 5.12, this bellows must be pulled apart at each step, and snaps shut as it collapses with increasing rapidity. See also Movie-S4 in [Rei17].
remainder of the barrier.

Special attention should be paid to the mechanical response shown in fig. 5.13. In particular, note that there are three collapse events, with each one corresponding to a pair of panels collapsing. If the unit cells were identically manufactured and unable to communicate with one another, these progressive collapsing events would occur at roughly the same force levels. That they do not is potentially evidence that some cells are less rigid than others, due to minor variations in their manufacture. Moreover, careful inspection of video data Movie-S4 in [Rei17] shows that the cells do not always collapse in the same order, suggesting that while some cells may be temporarily softer than others, it is not entirely explainable by their permanent connections to one another. Instead, we suspect that the remaining deployed cells are able to flex and deform slightly, easing the transition for the cells that will collapse. Because the number of available face configurations falls off as cells collapse, we see that each subsequent event requires more and more force. Notice that the force-displacement curve of fig. 5.13 has a dramatically negative slope at each band’s collapse, which originates in the device’s multistability: once the system passes through the barrier to the transition, the bellows is pushing on the clamp.

That each unit cell must twist in order to access its collapsed state suggests analyzing the torsion from one state to another using geometric considerations [Din16]. However, our bellows are designed to have zero net helicity and were mounted to plates that could not rotate. Therefore there is a coupling between the geometric torque and boundary conditions which in some cases inhibits allowed geometrical transitions. Therefore, a general statement using only geometric theory can not be achieved for the present study. Finally, bellows without a geometrically allowed collapsed state for both classes behave like typical thin-walled cylinders. As seen in figs. 5.14 and 5.15, once the panels’ capacity to deform is exceeded, the rotational symmetry breaks and one side of the bellows buckles and crumples in on itself. Returning the bellows to its original position does not restore the symmetry present initially (see also Movie-S5 and Movie-S6 in [Rei17]).

Within these devices, the underlying mathematical deployability not only allows physical actuation, but is also largely enriched by the mechanics which in turn provides extended functional capabilities. Deployable designs with $\phi_1 < \phi_c$ can be smoothly actuated with hardly any snap-through effects and with small forces: they can be held collapsed and they would self-deploy as the confining force is released. On the other hand, designs with $\phi_1 > \phi_c$ will remain in a given metastable state until actuated to change configuration, and the available configurations can be made to be either fully collapsed versus fully deployed, or they can alternatively exhibit several intermediate states with well defined energy barriers between states. Interestingly, the nature of the energy barrier between metastable states depends on the geometry of the faces: quadrangular faces decouple the bending of the face and that of the crease, leading to soft actuation, whereas triangular face bending requires the adjacent creases to deform, yielding an overall much stiffer actuation.
Figure 5.14 Class 1–Type B bellows with $\phi_1 = 105^\circ$ and $\phi_2 = 69^\circ$. This hexagonal bellows is expected to have a single accessible rigid-face configuration. Without a geometrically allowed collapsed state, this bellows buckles under load, breaking its rotational symmetry. See also Movie-S5 in [Rei17].
Figure 5.15 Class 2-Type B bellows with $\phi_1 = 105^\circ$ and $\phi_2 = 69^\circ$. Much like the hexagonal bellows of fig. 5.14, without a geometrically allowed collapsed state this bellows crumples, breaking its rotational symmetry. See also Movie-S6 in [Rei17].
5.4 Discussion

We have presented a unified geometrical description of several previously disconnected classes of origami bellows, and we uncovered their allowed states, painting a phase diagram with a peculiar island of bistability. We then proceeded to craft the corresponding physical models using an original technique, which allowed us to investigate the effect of these kinematics on the mechanical response of the models. We have shown that it is possible to rapidly generate precisely folded origami bellows using a laser cutter, plastic film, and double-sided tape. This technique can be easily used to make arbitrarily complicated bellows with finely tuned fold parameters. We found that the "deployability" of a given pattern, a characterization of the distance between two geometrically allowed states, yields quantitative insight into the behavior of the physical bellows, though the mechanics of face and crease bending and self-exclusion govern the detailed characteristics of their deployment.

More importantly, we have demonstrated the existence of a critical design angle that controls the bistability of a mechanical origami bellows. Furthermore, nonzero hinge energy drives these bistable devices away from the flat-folded configuration predicted by geometry, clearly illustrating the necessity of mechanics in a design phase. Class 1 bellows tend to be much more flexible than their Class 2 counterparts. As would be expected, the figures with larger $\Delta \psi$ have more stable collapsed configurations, and some are capable of holding themselves in a collapsed state.

Bellows with specific properties can be designed with knowledge of the geometric limitations as well as the mechanical properties of their substrate. While the fabrication technique described earlier is optimal for exploring bellows configurations, it necessarily generates perforated bellows, which are unable to displace fluids or be actuated by internal pressure. Construction of a sealed bellows is more complicated, requiring machined intermediate forms and molds to shape the cylindrical substrate as needed. It is thus a more efficient use of time to only generate these construction tools once a design has been adequately prototyped by using our paper lantern construction technique.

Given the importance of plate-bending in the collapse of an origami-folded bellows and the difficulty faced by finite-element computations of thin shells undergoing bending [Cha98], physical measurements of origami patterned bellows as they collapse or deploy are crucial for the verification and development of simulation tools. As faces and hinges bend, accurate measurements of surface curvature would allow a more complete accounting of the effective mechanical response of the bellows.

Among promising fields of application for next generation bellows design are architecture, mechanical design, and cryogenic devices. In these arenas, reliability and weight are primary design goals in the development of deployable origami bellows. Materials age and fail dramatically faster when subjected to crease bending, so patterns that minimize this damage are more reliable. With a full understanding of the importance of face bending, more robust origami patterned actuators may be designed. Bellows designs to date have rightfully started with purely geometric consider-
ations, followed by iterative physical prototyping. Unfortunately, as the practical foldability of a bellows depends on the interplay between the established creases and the device's ability to plate-bend its faces, this process is dependent on prototyping and intuition to generate future designs with predetermined functionalities.

5.5 Acknowledgments

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In chapter 5, we developed a theory of mechanical stability in origami folded bellows. In this chapter, we will apply that theory to a cryogenic device destined for use in Systematic and Operational Studies at PULSTAR (SOS).

We have developed a method of folding a seamless Kapton tube into a flexible bellows. This bellows is capable of superfluid leak-tight cryogenic actuation. A sample bellows has withstood 500 cycles of compression by 5mm (14% \( \Delta \ell / \ell \)) in liquid nitrogen. While it is not as resilient as a conventional beryllium copper (BeCu) device, this folded bellows is both RF transparent and nonmagnetic. It is being developed for use with the hyperpolarized \(^3\)He generated with the metastability exchange optical pumping (MEOP) system detailed in chapters 2 and 3 in a low-magnetic cryostat \(^1\).

6.1 Introduction

Cryogenic devices often need a degree of mechanical flexibility, either to ease alignment of an overconstrained mechanism or to allow motion. This flexibility is often introduced via a commercially hydroformed bellows. Typically stainless steel, bellows can be made of BeCu when magnetic susceptibility is a concern.

Next generation searches for symmetry violating effects require additional material constraints. For example, the SNS-nEDM collaboration’s search for a permanent neutron Electric Dipole Moment monitors the phase of UCN in a helium-II bath with \(^3\)He that also serves as as a comagnetometer [Gol94]. NMR techniques allow fine control of both spin species [Chu15] and preclude the use of conductors near the experimental cell.

\(^1\)i.e. a cryostat constructed as nearly as possible out of nonmagnetic materials.
In order to conduct Systematic and Operational Studies at PULSTAR, we have commissioned a small-scale apparatus to deploy on the UCN source at our PULSTAR reactor [Kor14]. This device requires a helium leak-tight vacuum feedthrough actuator, sketched in fig. 6.1. The conventional solution of a BeCu bellows is unacceptable due to the field distortions and parasitic heating a conductor would introduce to the fiducial volume. The bellows testing procedures were designed to meet the needs of this apparatus, which is introduced in section 6.4 and explained in detail in an upcoming paper by Leung on behalf of the SOS@PULSTAR collaboration.

Needing a flexible nonmetallic device in a superfluid helium bath, we have developed a new class of cryogenic actuator using an origami folded metamaterial. Because these terms are not commonly used by cryogenicists, some clarification is warranted: In its original sense, “origami” means “folded paper”, but in contemporary materials research, the meaning is expanded to admit any substrate that can be folded, while still banning cutting or gluing. While mechanical properties are traditionally determined by the substrate material, mechanical metamaterials are objects with mechanical properties that have been tuned by the application of a regular deformation. Although the term originated in the design of electromagnetic devices [Smi00], its use inevitably expanded to mechanical structures [Kad12], ultimately proving to be a useful concept for origami folded structures as well.

Kapton is DuPont’s trade name for a thin-film polyimide preparation. Measured perpendicularly to the film, Kapton has a measurable (read: finite) elastic modulus even at 4.2 K [Dav92]. The film’s flexibility at such low temperatures is what makes this bellows’ mechanism of action possible. Ter Haar et al. generated a type of Kapton bellows for use as a heat exchanger by gluing dozens of sheets together [Ter95], but a bellows formed in this manner will be extremely delicate. Repeated actuation will degrade the glue joints over time. A seamless Kapton tube will be leak-tight but relatively inextensible, especially at cryogenic temperatures. Crisply folded bellows are commercially available, but these are folded from a flat sheet and bonded along one seam. Such a joint tends to fail and leak during cryogenic actuation. Furthermore, a hexagonally folded bellows is incapable of smooth flat-faced actuation [Con97; Rei17], leading to large stress concentrations in the sharp creases, which in turn causes rapid deterioration and mechanical failure of such a device.
We have developed a method to fold seamless Kapton tubes, produced by Concentric Microtubing, into a flexible leak-tight bellows. This chapter will present the design, manufacture, and testing of these bellows.

6.2 Design

Early development of the bellows used 0.5 in (1.27 cm) diameter Kapton tubing. Attempts to stretch the tubing with something similar to a hydroforming mandrel failed due to Kapton's relative inextensibility even at elevated temperatures.

Gaussian curvature \( K \) is an intrinsic property of a surface\(^2\), given by the product of the principal curvatures at a point.

\[
K = \kappa_1 \kappa_2
\]  

(6.1)

Surfaces with zero Gaussian curvature everywhere are called “developable surfaces,” and include flat sheets, cylinders (as shown in fig. 6.3), and cones. It is possible to impart a small degree of Gaussian curvature to a Kapton sheet, as demonstrated by the target described in [Ken16], but the curvature is extremely small, and the corresponding functionalized structure would be impossibly large. Kapton can be thermoset, however. In other words, while you can't really stretch Kapton, you can fold it, and the desired resulting geometry should be developable.

As a result of Connelly’s bellows theorem [Con97], most bellows formed with regular fixed patterns on a cylinder cannot be collapsed smoothly [Rei17]. Fortunately, fixed crease positions are a precondition of Connelly’s theorem, and allowing the creases on a bellows to move introduces an unconstrained degree of freedom and the resulting mechanism can be smoothly collapsed. It is critical for the continued operation of a folded bellows that the folds are not sharply creased, lest they be unable to translate and cause the device to be rigid.

Extensive cryogenic testing was conducted with small-diameter bellows. While it proved a robust and reliable device, its small internal clearance proved unworkable. A thin rod connects a

\[^2\]K is preserved by cutting, which explains why it is impossible to generate an undistorted projection of a world map: the globe has \( K > 0 \), and a flat sheet of paper has \( K = 0 \).
Figure 6.3 Gaussian curvature of a sphere, cylinder, and hyperboloid

Figure 6.4 Detail of crease pattern of 0.7” bellows.
stopper between the vestibule and sample cell, as illustrated in figs. 6.1 and 6.9. When a thin rod is held in compression, buckling is its primary failure mode. The maximum safe load is primarily determined by the rod’s geometry, and is nearly material independent.

The largest rod that the small OD bellows could accommodate was still too weak to reliably seal the sample cell. Rather than redesign the vestibule and sample cell isolation mechanism, we explored scaling the bellows.

Initially, we tried simply making everything about the bellows 40% larger. This generates a sufficiently large internal cross-section for the necessary sealing force and safety factor, and was easily accomplished by directly scaling the engineering model for the mandrel. Scaling the mandrel in all dimensions made the folds both larger and deeper, and the resulting inner creases were far too sharp, as shown in fig. 6.5.

![Figure 6.5 Poorly designed 0.7” bellows which proved too brittle. Note that the inner and outer radii of curvature are dramatically different.](image)

The irregular folds of fig. 6.5 clarified what parameters were critical to effectively scale the size of one of these folded bellows. The method of folding these bellows relies on the substrate's tendency to bend rather than crease. Thus, the local folding/bending interaction should be conserved for a particular wall thickness. The 0.7” bellows has the same wall thickness as the 0.5” bellows, so it is successfully folded with the same convolution depth and pitch.

Thinner films are far more resilient to folding [DuP11], but they are also more permeable to helium at room temperature, complicating leak testing. This bellows’ geometry was empirically determined for film thickness of around 4 mil (0.1 mm). Kapton tubing may be produced in a wide variety of thicknesses, allowing devices with dramatically different wall thickness to be formed with this technique. However, changing the material thickness changes its characteristic bending length [Lec14], so the convolution depth and spacing will need to be adjusted accordingly.
6.3 Bellows Folding Procedure

![Image A](image1.png)
![Image B](image2.png)
![Image C](image3.png)
![Image D](image4.png)

Figure 6.6 Components used to assemble 0.5” bellows. We start with a 5 cm long mandrel that sits inside the tube to guide folds (A). The Kapton tube is then clamped onto mandrel and guided into the correct shape (B). The clamped bellows is suspended inside an oven with an internal thermocouple (C). After reaching 550 °F, the oven is allowed to cool, and the thermoset bellows can be removed from its mandrel (D).

We have developed a set of tools—some are shown in fig. 6.6, with all technical drawings included in appendix D.2—to consistently fold our bellows into the desired hexagonal fold pattern, shown in detail in fig. 6.4. Starting with a machined aluminum mandrel inside the Kapton tube, apply small clamps so that the tube is pressed into the first groove in the mandrel. For each clamp after the first, while the tube is still able to slide past the mandrel, press extra material into the crease. This extra material helps smooth out the creases, preventing them from becoming kinked or sharp. Those fold defects generate localized stress concentrations where the film splits when actuated at low temperatures. Gradually tighten the clamps in turn until all are uniformly snug. Suspend the clamp and tube assembly by thin wires in a cool oven, then gradually heat it until the mandrel reads 550 °F (288 °C). Upon reaching the target temperature, remove the assembly from heat and allow it to cool, completing the thermosetting of the Kapton. First remove the clamps, then, having lubricated the mandrel by pouring a bit of ethanol inside the tube, pull the tube from one end³ to unfold and remove it from the mandrel. Press the mandrel out of the tube with a smooth rod (we use a chopstick), and press the folds in the tube back into place. This process does not introduce leaks into the bellows. Before installing the device, carefully inspect all of the folds. Slight variations

³The tube will crinkle like a bendy straw here. That is normal.
in fold depth are acceptable, but creases or kinks like that shown in fig. 6.7 in the folds themselves are not.

**Figure 6.7** Bellows that failed to fold correctly. This device will fail immediately if actuated cryogenically.

**Figure 6.8** Various views of one long 0.7” bellows. Note the bellows has a square internal cross-section, slightly more than 0.3” to a side.

### 6.4 Mechanical and Cryogenic Properties of Bellows

*SOS@PULSTAR* is built around the experimental volume of a rectangular PMMA cell filled with 3 L of LHe, operating from 300 mK to 500 mK. This cell is separated from a staging vestibule by a mechanically actuated plug, shown in figs. 6.1 and 6.9. This stopper is actuated by a rod that passes from the surrounding inner vacuum into the sample bath. The bellows will serve as a flexible feedthrough from vacuum to helium-II, as shown in figs. 6.1 and 6.9. Under normal operation, the bellows will be surrounded by vacuum, and will be internally pressurized by around 10 mbar due to the hydrostatic pressure of the helium-II it is containing. Due to the design of the cryostat,
failure modes where excess pressure to the bellows is applied externally are not considered. During the warm-up phase of a typical experimental cycle, the bellows needs to be able to withstand elevated internal pressure as the liquid expands and boils. The vacuum system is designed to regularly operate with 1 bar of differential pressure, so the bellows needs to withstand at least 2 bar without rupturing. Although the bellows needs to actuate while subjected to a small pressure differential, applying a larger differential yields both a more taxing test on the material as well as an experimental signal that is a function of gas pressure rather than material stiffness.

Testing of the device proceeded in stages. First, the pressure resilience and durability of the bellows was tested at room temperature. Having passed those tests, the bellows was mounted into cryogenic fixtures that allowed actuation in a liquid nitrogen bath, and was suitable for superfluid liquid helium leak-checking. The room temperature tests as well as the liquid nitrogen actuation tests were conducted with an Instron 5944, a commercially available system combining a precise linear actuator, an digital force gauge, and control/logging software.

### 6.4.1 Room Temperature Pressure Resilience

In order to determine the amount of pressure a folded bellows can withstand without failure, a short 0.5” bellows was mounted on the piston-sealed endcaps drawn in fig. 6.10 with the test frame locked in place. Because the bellows is sealed on both ends, the internal pressure may be extracted from the load cell attached to one end. By gradually increasing the internal pressure, we are able to observe the primary failure modes of the bellows. Without bracing, a new bellows will resist 125 kPa internal pressure before squirming. Squirm events rapidly fatigue the polymer bellows, and it is far less resistant to repeated tests: failure was observed at 127 kPa, 97 kPa, 75 kPa, and 81 kPa. With an internal support rod, the squirm mode is suppressed, and as illustrated in fig. 6.11, the bellows accommodates the internal gas pressure by incrementally unfolding from the ends, pressing the
Figure 6.10 Room temperature test setup. Bellows is sealed with double piston seals. The top fixture which attaches to the Instron’s cross head is blank, while the lower fixture which attaches to the Instron's base is connected to a pressure regulator.

remaining folds towards the center. The tubing withstood pressures of 305 kPa without rupturing, but should those pressures be encountered while operating, the bellows will need to be replaced.

Figure 6.11 Progressive unfolding of bellows due to internal pressure. A: 11 kPa, B: 258 kPa, C: 305 kPa

6.4.2 Room Temperature Durability

A longer bellows can compress the same amount with less localized strain, but is harder to manufacture, as the failure of any one fold dooms the device. During normal operation, the bellows will be actuated in a cryogenic bath less than 100 times. In order to assess the durability of the bellows, the bellows was mounted on the same Instron test frame and piston-sealed fixtures as before. The Instron was then used to subject the bellows to increasingly strenuous deformations. Thinner Kapton films are dramatically more resilient to fatigue and failure from repeated folding [DuP11]. Our test bellows is constructed of 4 mil (0.1 mm) Kapton tubing. From the durability of 3 mil and 5 mil films, it is reasonable to expect over 1000 compression cycles before failure.
6.4.2.1 0.5” Diameter Bellows

A 0.5” diameter bellows was subjected to the room temperature test schedule detailed in table 6.1.

<table>
<thead>
<tr>
<th>∆ℓ (mm)</th>
<th>∆ℓ/ℓ (%)</th>
<th>Cycles</th>
</tr>
</thead>
<tbody>
<tr>
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<td>8.6</td>
<td>278</td>
</tr>
<tr>
<td>6</td>
<td>17.1</td>
<td>300</td>
</tr>
<tr>
<td>9</td>
<td>25.7</td>
<td>300</td>
</tr>
<tr>
<td>12</td>
<td>34.2</td>
<td>1000–2000</td>
</tr>
<tr>
<td>15</td>
<td>43.0</td>
<td>260</td>
</tr>
</tbody>
</table>

At least 1000 cycles of the 12 mm test were completed, but that data was lost due to computer failure. While testing at 15 mm compression (43% ∆ℓ/ℓ), failure was observed after 260 cycles. This failure first manifested in an uncompressed bellows, as compressing the device presses the cracks closed. Subsequent compressions caused these cracks to grow, further decreasing the pressure observed inside the bellows, as shown in fig. 6.12.

Figure 6.12 Failure of bellows initiated after 260 cycles.
6.4.2.2 0.7” Diameter Bellows

Similar tests were conducted on a first generation 0.7” bellows similar to that shown in fig. 6.5. Several bellows split when actuated under pressure, and these failures were attributable to both overbaking of the Kapton and less resilience due to an improperly scaled folding pattern. The undamaged bellows could be tested by altering the testing procedure somewhat: instead of actuating the bellows under pressure and waiting for a pressure drop to indicate failure (a la fig. 6.12), the bellows was actuated without a pressure differential, then pressurized with 5 psi of LN boil-off to check for major leaks.

Table 6.2 Room temperature fatigue tests for short and long 0.7 in diameter bellows. Bellows integrity was tested after every 100 test cycles. The same long bellows was subjected to all the tests reported here.

<table>
<thead>
<tr>
<th>ℓ (mm)</th>
<th>Δℓ (mm)</th>
<th>ℓ (mm s⁻¹)</th>
<th>Δℓ/ℓ (%)</th>
<th>Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.1</td>
<td>5</td>
<td>0.5</td>
<td>13.1</td>
<td>1700</td>
</tr>
<tr>
<td>76.2</td>
<td>5</td>
<td>0.5</td>
<td>6.6</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.5</td>
<td>13.1</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>10.0</td>
<td>13.1</td>
<td>500</td>
</tr>
</tbody>
</table>

Both bellows were tested for Δℓ/ℓ ≈ 13%. The “short” (11 total folds) 0.7” bellows survived 500 cycles of 5 mm compression at 0.5 mm s⁻¹, while the “long” (22 total folds) bellows survived substantially more: 1000 cycles of 0.5 cm compression followed by 1000 cycles of 1 cm compression. Both devices are fairly linear in their spring response, exhibiting a slight softening nonlinearity. The long bellows is twice as long as the short, so it is not surprising that it has half the spring constant.

Figure 6.13 Force-displacement plot of short and long 0.7” bellows. Note that long bellows has twice as many convolutions, is twice as long, and has half the spring constant as the short bellows.
### 6.4.3 Durability in liquid nitrogen

Having seen promising room-temperature results, a newly folded bellows was mounted in a small vacuum can as shown in figs. 6.16 and 6.17. From the 4 K to 300 K thermal expansion coefficients given in table 6.3, Kapton’s coefficient of thermal expansion (CTE) is lower than unfilled polyether ether ketone (PEEK) and Stycast 1266. An external PEEK retaining ring is necessary to prevent delamination as the assembly cools from room temperature to 4 K. All gluing surfaces (PEEK and Kapton alike) are scuffed tangentially around their openings with Scotch-brite before being rinsed, first with alcohol, and then with DI water. The bellows and endcaps are assembled, and Stycast is applied to the seam between the two.

<table>
<thead>
<tr>
<th>Substrate</th>
<th>(\Delta \ell/\ell) (%)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kapton</td>
<td>0.81</td>
<td>[Sch88]</td>
</tr>
<tr>
<td>PEEK</td>
<td>0.98</td>
<td>[Sch88]</td>
</tr>
<tr>
<td>Stycast 1266</td>
<td>1.15</td>
<td>[Swi79]</td>
</tr>
</tbody>
</table>

**Figure 6.14** Short 0.7” bellows being glued into PEEK fixtures. 1/8” rod is too small to prevent bellows from squirming, so a square teflon support rod is also present. Outer rings are moved away from ends when epoxy is first applied to ensure even wetting of the PEEK end pieces. Nota bene: scale is in inches.

After the epoxy cures, the vacuum can is assembled with indium gaskets, and is mounted in a 5 l bucket dewar in turn. This geometry allows LN to reach and cool the interior of the bellows, and loads the bellows with a full atmosphere of internal pressure, which corresponds to a worst-case pressure differential for normal operation of SOS@PULSTAR.

We have not seen small failures of the bellows during cryogenic testing—stress concentrations generate small cracks which generate further stress concentration which rapidly leads to large...
Tests were conducted separately for the 0.5” and the 0.7” bellows. The 0.5” device survived a more rigorous testing regimen:

A pressure gauge indicates the pressure of the vacuum surrounding the bellows. Should the bellows break, the pressure would spike as cryogen vents into the vacuum. The Instron machine was again used to actuate the bellows, this time via a rod extending through the inside of the bellows, seen in figs. 6.16 and 6.18. Additionally, the force data from the Instron indicates a 1 bar pressure differential. In this configuration, the can was evacuated to a rough vacuum, and the bellows was cycled 100 times at a rate of 0.5 mm/s and a total displacement of 5 mm (14% $\Delta \ell / \ell$). At the end of this cycling, the can assembly was moved to a liquid helium dewar for subsequent testing, as detailed in section 6.4.4.

After determining the bellows was superfluid tight, the assembly was returned to the Instron for more cold cycling. The previous test compressions and rates were used again for 410 cycles without measurable failure of the bellows.

Following the pressure-induced failure of the early 0.7” bellows designs, the testing regimen was modified to more accurately reflect the actual use case of the bellows in the SOS cryostat. Namely, the device needs to withstand static pressurization of 1 bar for normal operation and 2 bar for safety, but it does not need to support a pressure differential while actuating. This complicates the test setup somewhat: the test can is evacuated to a rough vacuum while the system is cooled by LN, but the can is pressurized with pure nitrogen to prevent oxygen or water from condensing in the can or on the bellows. A more substantial test frame was also designed, allowing substantially more force to be exerted on the bellows without worry of damaging the test can’s vacuum line. Assembly of this test frame is shown in fig. 6.20.

Testing was then conducted in 100 cycle increments: a bellows was compressed by 13 % (5 mm) at 10 mm s$^{-1}$, held compressed for 2 s, returned to $\ell_0$ at $-10$ mm s$^{-1}$ and held for 2 s to relax. After 100 cycles of this, the vacuum can was evacuated again, checking for major leaks in the bellows. The can was then refilled with 1 bar of nitrogen, and the process was repeated 5 times. A leak from the liquid bath into the vacuum can was observed at this point (implying a 400-500 cycle count...
**Figure 6.16** Left, schematic of Instron LN test assembly. Right, cross-section of bellows vacuum test can engineering model. Bellows is filled with liquid and surrounded by vacuum. Actuating rod extends through bellows, and pulls on the far end of the bellows. This ensures axial compression while suppressing the squirm failure mode.

**Figure 6.17** 0.5" bellows epoxied into machined PEEK endcaps for cryogenic testing. Note that this shows an older endcap design that proved extremely difficult to assemble. The endcap drawings given in appendix D.2 are the improved design.
Figure 6.18 Instron with liquid nitrogen bucket dewar assembly used to test 0.5” bellows. A more substantial frame is used for 0.7” bellows testing.

Figure 6.19 Single cycle of 5 mm compression at 0.5 mm s$^{-1}$. The 10 N jump near 0.5 mm occurs when the pull rod engages with the bellows: that loading corresponds to atmospheric pressure distributed across a 0.5” circle. Once loaded, the bellows has a stiffness of 1.5 N mm$^{-1}$.
lifetime), so testing was terminated.

6.4.4 LHe Leak Tests

At room temperature, a 0.5” diameter bellows has a leak rate of $10^{-5}$ mbar L s$^{-1}$. With a surface area of roughly 30 cm$^2$, room temperature helium readily permeates the 4 mil thick Kapton used for the bellows, preventing accurate leak detection. Fortunately, this permeation is driven by thermal motion of the polymer chains, which is exponentially suppressed in liquid nitrogen.

In order to measure the superfluid leak rate of the bellows, the vacuum can test assembly seen in fig. 6.16 was mounted in a small helium dewar. The volume surrounding the bellows is connected to the leak detector, while the inside of the bellows is connected to the main liquid bath. During a helium fill, the bath is vented to atmosphere. As such, the inside of the bellows is exposed to LHe with 1 bar of differential pressure. To ensure superfluid-tightness, the bath is pumped below the lambda transition point, and the leak rate is observed to be on the order of $10^{-9}$ mbar L s$^{-1}$ throughout.
6.5 Conclusion

We have developed a simple method for producing a remarkably robust RF transparent and entirely nonmagnetic polymer bellows that is superfluid leak-tight through repeated actuation in liquid nitrogen and liquid helium. Folding the device requires a small amount of fixtures that can be produced in any machine shop, and the folding process itself requires only patience and care. Crucially, the bellows is folded into smooth convolutions, which allows the bellows to compress continuously. The bellows described herein is nonmetallic, flexible at cryogenic temperatures, leak-tight at 1.8 K, and inexpensive to boot.

6.6 Discussion

The measured durability for the 0.7” bellows and the 0.5” bellows are substantially different. The 0.5” bellows withstood nearly 1000 slow compression cycles under vacuum, but the 0.7” bellows only withstood 400 fast compressions without a pressure differential. It is not clear whether the 0.7” bellows is globally less resilient than the 0.5” design, or if rapid actuation (1 cm s\(^{-1}\) vs 1 mm s\(^{-1}\)) is more fatiguing, but it is most likely a combination of the two effects. These tests were conducted with compression and extension rates of 1 cm s\(^{-1}\), with only two seconds dwell time between the actuations. These are far more challenging conditions than the proposed operating cycles of SOS@PULSTAR, which only benefits from a rapid compression. Subsequent tests should be conducted with longer pauses and a slower relaxation, which should substantially improve the lifespan of the bellows.

If actuated with helium-3, A pair of bellows can function as a zero-displacement actuator capable of delivering several Newtons of force in a superfluid bath. The maximum force that the system can deliver is a function of the vapor pressure of \(^{3}\)He, as illustrated in fig. 6.22. Because adding convolutions reduces the stiffness of the bellows, a longer bellows could generate a larger displacement at any given pressure level. Additionally, this forming technique may be scaled to generate larger diameter bellows, which would deliver even more force at any given pressurization. This system is not suitable to SOS because it is incapable of generating the necessary force to seal the vestibule's

Figure 6.21 Schematic of helium-II test bed.
The hexagonally folded bellows discussed here deliberately has zero helicity. Twisting the tube instead of collapsing the tube directly onto the mandrel should generate a triangular tessellated bellows, known as a Kresling pattern [Kre02]. Kresling patterns are incapable of continuous actuation [Con97], but the severity of their bistable behavior is geometrically driven and is geometrically tunable [Rei17]. With design parameters chosen so the bellows lies in the Kresling bellows’ geometrically excluded region, a “triangular twist” bellows could generate rotary motion instead of the linear motion seen here.

![Figure 6.22](image)

**Figure 6.22** Maximum force achievable with 0.5” bellows as a function of bath temperature, assuming $^3$He working gas. Pressure calculated with eq. 6 of [Hua06]. Assumes pre-compressed pair of bellows operating as a zero displacement pair.
CHAPTER 7

CONCLUSION

The world is full of magical things patiently waiting for our wits to grow sharper.

— Bertrand Russell

Over the course of this work, we have addressed topics which are crucial to the successful operation of the Systematic and Operational Studies at PULSTAR (SOS) cryostat both at Triangle Universities Nuclear Laboratory (TUNL) at at NCSU’s PULSTAR reactor. With an established procedure for generating highly polarized $^3$He with metastability exchange optical pumping (MEOP), measurements depend on downstream contributions.

Contemporary MEOP research primarily explores metastable pumping in high field/high pressure systems (1 T/1 bar). Low field MEOP systems are a well-established technology, and with a developed and debugged gas supply and electronic control system, our MEOP setup consistently generates high degrees of nuclear polarization (>80 %) with a hobbled laser. We expect service to increase the laser’s output power by almost 50 % (from 7 W to 11 W), which may lead to slightly higher $P_N$.

Successful operation of this cryogenic measurement depends on continued operation of a dilution refrigerator. After being hampered by intermittent clogging for years, we have identified and eliminated the source of those clogs: crystalline neon contamination in our purchased liquid helium. These crystals appear to be capable of forming structures that can clog a much larger aperture than may be expected from their size. Particles are only able to spontaneously form bridges across a gap that is $5 \times$ their diameter, so this jamming can be entirely eliminated by installing a fine sintered filter that only allows particles that are $1/7$ the aperture diameter into the system.

By reducing several different types of origami bellows to an essentially identical unit cell, and then expressing that unit cell’s constraints as a set of geometric relationships, we were able to describe the complete configuration space for several different types of folded origami cylinders. This configuration space perfectly describes a mathematically ideal bellows, which is a device with perfectly flat faces, zero wall thickness, and perfectly sharp creases. Deviating from those idealizations
leads to a bellows with mechanics only roughly described by the rigid-face configuration. The rigidity of these bellows is explainable by Connelly's Bellows Theorem, which states that the volume contained by a triangulated surface is conserved under continuous deformation.

A necessary precondition of Connelly's theorem is that the edges of the triangulation be fixed. It is possible to bypass this requirement, and with care, transform a kapton cylinder into an extraordinarily flexible bellows via folding. We have developed a set of tools and techniques that allow the fabrication of bellows of different diameters. These bellows have been tested extensively at room temperature and in liquid nitrogen—they have even proven leak-tight in a helium-II (superfluid) bath.

7.1 Outlook

Over the upcoming months to years, all the technology developed here should see use in next-generation measurements. We expect to use the MEOP system developed here to generate a relatively large amount of hyperpolarized $^3$He for SOS, and the cryogenic bellows developed here will likewise be deployed for SOS.

Origami folded bellows are integral to the vestibule design for SOS@PULSTAR, and even larger diameter devices may find use in the full-scale $^3$He-electric dipole moment of the neutron (nEDM) apparatus at Oak Ridge. Per the calculations of chapter 6, these bellows are flexible enough to be actuated entirely by gas pressure well below 4 K. The proven folding pattern could be used as gas-powered linear actuators that are entirely nonmagnetic and RF-transparent, and adapting the folding technique to generate Kresling (twisted) bellows may lead to rotary motion actuators. Actuators of this style may prove a potential boon for future generations of high-precision experiments, including searches for beyond Standard Model physics.

The bellows explored in chapter 5 should guide future development of origami actuators for robotics and spacecraft. Further developments in the world of cryogenic origami would be a surprise, but a welcome one to be sure.
BIBLIOGRAPHY


In order to function as the neutron polarimeter needed for the nEDM search described in section 1.4.2, it is critical that the polarization of the helium itself be well understood. Although the formalism used to describe the time evolution of a bottled gas has existed for decades, new results continue to be discovered. A particularly important one, a false electric dipole moment (EDM) arising from the interplay of magnetic gradients and $\vec{v} \times \vec{E}$, which was discovered while analyzing data of a bottled ultra cold neutron (UCN) [Gol91]-based search for nEDM at the Institut Laue-Langevin (ILL) [Pen15].

The text below details the development of an unsuccessful extension to a lovely computational tool introduced in [Cal97] and refined by [Suk02]. These matrix-based simulation tools discretize time and simulate time-steps via sequential matrix multiplication. Computationally, the heavy lifting is conducted only once–generating the discrete time propagator. Once generated, one can study the effects of various field configurations and spin distributions. Inverse problems are even possible. But first, some theory.

### A.1 Diffusion Theory

Although individual atoms are jostled and slosh, we assume that they are not able to leave their container. In other words,

$$\frac{d}{dt} \iiint dV \rho(\vec{x}, t) = 0$$  \hspace{1cm} (A.1)

Not only are atoms globally conserved, they are locally conserved too! The rate at which atoms pass any given point is given by the “current” $J$ eq. (A.3). The current is given by the gradient of the
density, scaled by a factor $D$, the diffusion constant.

$$ \partial_t \rho = -\nabla J$$  \hspace{1cm} (A.2)

$$ J \equiv -D \nabla \rho$$  \hspace{1cm} (A.3)

$$ \partial_t \rho = D \nabla^2 \rho + \nabla D \cdot \nabla \rho$$  \hspace{1cm} (A.4)

So if $D$ is spatially constant

$$ \partial_t \rho = D \nabla^2 \rho$$  \hspace{1cm} (A.5)

Equation (A.3) is known as Fick’s First Law of diffusion, and eq. (A.5) is Fick’s Second Law [Fic95]. Fick’s Second Law is also simply known as the diffusion equation, and is typically handled with standard techniques. First, we assume $\rho$ is separable into a spatial part and a time-dependent part:

$$ \rho = U(\vec{x}) T(t)$$  \hspace{1cm} (A.6)

$$ \frac{U}{D} \frac{dT}{dt} = DT \nabla^2 U$$  \hspace{1cm} (A.7)

$$ \frac{1}{D} \frac{1}{T} \frac{dT}{dt} = \frac{1}{U} \nabla^2 U$$  \hspace{1cm} (A.8)

$$ \frac{1}{D} \frac{1}{T} \frac{dT}{dt} = -k^2$$  \hspace{1cm} (A.9)

$$ \frac{1}{U} \nabla^2 U = -k^2$$  \hspace{1cm} (A.10)

We can directly integrate the time-dependent eq. (A.9) to yield eq. (A.11).

$$ T = e^{-k^2D t}$$  \hspace{1cm} (A.11)

Equation (A.10) becomes the Laplace equation eq. (A.12).

$$ \nabla^2 U = -k^2 U$$  \hspace{1cm} (A.12)

## A.2 NMR and Diffusion

There is a large body of research into finite propagator methods to simulate nuclear magnetic resonance (NMR) signals [Cal97; Suk02; Gre07]. These works use an elegant formalism to generate high fidelity simulations without a tremendous computational overhead. Unfortunately, they are all formulated with magnetic gradients that exist solely in the $z$-direction. Hoping to improve the physical grounding of this technique, we attempted to expand from a parallel gradient to a fully realized zero divergence magnetic field.

Because this was an attempted extension of [Suk02], the derivation and notation are as identical
as possible. If a spin is tipped by an RF signal, it will process about the sample’s holding field \( B_0 \). The holding field inevitably will not be perfect, and furthermore the particles magnetic moments themselves will tend to depolarize each other. The signal we expect is not just a sinusoid, but rather

\[
S(t) = S_0(t) s(t), \quad s(t) = \langle e^{i \phi(t)} \rangle_x. \tag{A.13}
\]

Where \( S_0(t) \) and \( s(t) \) are the parts of the signal independent of depolarizing gradients and resulting from those gradients, respectively. Both of these components describe the longitudinal relaxation of the processing signal, where \( s \) is far more complicated, as it depends on the interplay between the diffusivity of the sample and its field homogeneity.

The phase accumulated by a single spin element results from the spin’s interaction with its inhomogeneous holding field \( H = H(\vec{r}, t) \), with the Larmour frequency \( \omega = \gamma H \).

\[
\phi(t) = \int_{t_0}^{t_f} dt \omega(\vec{r}, t) \tag{A.14}
\]

\[
H_G = G \cdot r, \quad G = \text{const} \tag{A.15}
\]

The gradient under consideration is parallel to the holding field \( H_0 \), so moving to the rotating frame used here is trivial;

\[
[H_0, H_G] = ab[\sigma_z, \sigma_z] = 0 \tag{A.16}
\]

The rate \( \omega \) at which particle accumulates phase \( \phi \) is a continuous function of time and the particle’s spatial coordinate \( \vec{r}(t) \). This continuous accumulation can be discretized with the trapezoidal approximation:

\[
\phi(t) = \int_0^t dt' \omega(\vec{r}(t'), t') \tag{A.17}
\]

\[
\approx \Delta t \cdot \left[ \frac{\omega(\vec{r}_0, 0) + \omega(\vec{r}_N, N)}{2} + \sum_{n=1}^{N-1} \omega(\vec{r}_n, n) \right] \tag{A.18}
\]

\[
\approx \gamma G \Delta t \left( \frac{r_0 + r_N}{2} + \sum_{n=1}^{N-1} r_n \right). \tag{A.19}
\]

The probability that a particle is at any particular point in the sample at any given time is given by

\[
\rho(r_0) \cdot P(r_1, r_0, \Delta t) \cdot P(r_2, r_1, \Delta t) \cdot \ldots \cdot P(r_N, r_{N-1}, \Delta t) \tag{A.20}
\]

Which just gives the initial density modified by the conditional probability that a particle at \( r_n \) reaches \( r_{n+1} \) in \( \Delta t \) seconds. All that is needed at this point is to average out the contribution of each time step.
The gradient strength determines the rate of evolution of the system, so it is convenient to combine the time step and magnetic gradient contributions for calculations:

\[ Q = \gamma G \cdot \Delta t \]  

(A.21)

\[ s = \frac{1}{V} \int \int \int \ldots \int \exp \left[ iQ \left( \frac{r_0 + r_n}{2} + \sum_{n=1}^{N-1} r_n \right) \right] \times (A.22) \]

By using an eigenfunction expansion of the propagator:

\[ P(r, r', t) = \frac{1}{V} \sum_{k=0}^{\infty} u_k(r) u_k^*(r') \exp(-\lambda_k t) \]  

(A.23)

eq (A.22)

eq. (A.22) can be broken into a series of independent integrals:

\[ s(t) = \sum_{k_1,k_2,\ldots,k_N} \frac{1}{V} \int dr u_{k_1}(r) e^{iQr} e^{-\lambda_{k_1} \Delta t} \times \]  

(A.24)

For a linear gradient, the time evolution is as follows.

\[ s(t) = \sum_{k_1,k_2,\ldots,k_N} F_{k_1} \left( \frac{Q}{2} \right) \Lambda_{k_1,k_2} U_{k_1,k_2} \Lambda_{k_2,k_3} U_{k_2,k_3} \ldots \]  

(A.25)

\[ U_{k_{N-1},k_N}(Q) \Lambda_{k_N,k_N} F_{k_N} \left( -\frac{Q}{2} \right) \]

where

\[ F_{k}(Q) = \frac{1}{V} \int_V dr u_k(r) \exp(iQr) \]  

(A.26)

\[ \Lambda_{kk} = \exp(-\lambda_k \Delta t) \]  

(A.27)

\[ U_{kk'} = \frac{1}{V} \int_V dr u_k^*(r) u_{k'}(r) \exp(iQr) \]  

(A.28)

Although Sukstanskii & Yablonskiy use a linear gradient as their example, the numeric integra-
tions of eqs. (A.26) and (A.28) can have any functional form:

\[
F_k(Q) = \frac{1}{V} \int_V dr \ u_k(r) \exp(i \gamma G(r) \Delta t)
\]

(A.29)

\[
\Lambda_{kk} = \exp(-\lambda_k \Delta t)
\]

(A.30)

\[
U_{kk'} = \frac{1}{V} \int_V dr \ u_k^*(r) u_{k'}(r) \exp(i \gamma G(r) \Delta t)
\]

(A.31)

This is obviously an approximation in their 3D examples: presumably the biological systems this technique is designed to simulate are relatively small, especially compared to the strength of the applied gradient. The fundamental reduction is this: an SU(2) system that is restricted to the \(z\) axis may instead be written as a U(1) rotation about that axis. That is to say, when your gradient is in more than just the \(\hat{k}\) direction, you need to use Pauli spin matrices instead of just complex phases.

### A.3 SU(2) operators

While it isn’t explicitly stated, the derivation of eq. (A.24) relies on the fact that generators commute with themselves. Consider the expression

\[
e^{-i \sigma_z \int dt \omega(t)} \approx e^{-i \sigma_z \Delta t\left(\frac{\omega_0 + \omega_N}{2} + \sum_{n=1}^{N} \omega_n\right)}
\]

(A.32)

\[
\approx e^{-i \sigma_z \Delta t \omega_0} e^{-i \sigma_z \Delta t \frac{\omega_N}{2}} \prod_{n=1}^{N} e^{-i \sigma_z \Delta t \omega_n}
\]

(A.33)

Which is a manifestation of the group multiplication law\(^1\) given in [Geo99]. Given a one parameter family of group elements given by

\[
U(\lambda) = \exp(i \lambda a_a X_a),
\]

(A.34)

group multiplication is straightforward:

\[
U(\lambda_1) U(\lambda_2) = U(\lambda_1 + \lambda_2).
\]

(A.35)

If a mixture of generators is present, it isn’t so straightforward

\[
\exp(i \alpha_a X_a) \exp(i \beta_b X_b) \neq \exp(i (\alpha_a + \beta_a) X_a)
\]

(A.36)

\(^1\) equations
Instead,
\[
\exp(i \alpha_a X_a) \exp(i \beta_b X_b) = \exp(i \delta_a X_a) \tag{A.37}
\]
\[
= \exp \left( i \alpha_a X_a + i \beta_b X_b - \frac{1}{2} [\alpha_a X_a, \beta_b X_b] + \mathcal{O}(3) \right) \tag{A.38}
\]

This technique fundamentally needs to convert \( \exp(i \sum_n \phi_n) \rightarrow \prod_n e^{i \phi_n} \). If the generators don't commute, this can't be done exactly in a finite product. This is an approximation technique already, so we don't need to look beyond an approximation that is accurate to 2nd order in \( x \). In [Suz91], Suzuki gives the second-order approximant of \( \exp(i x (A + B)) \), listed here as eq. (A.39). Applying this identity repeatedly on a sum will yield the full approximant for a series of time-ordered operations, e.g. \( \omega_0 - \omega_N \).

\[
e^{i x (A+B)} \approx e^{i \frac{x}{2} A} e^{i x B} e^{i \frac{x}{2} A} \tag{A.39}
\]
\[
e^{i x (A+B+C)} \approx e^{i \frac{x}{2} A} e^{i x (B+C)} e^{i \frac{x}{2} A} \tag{A.40}
\]
\[
e^{i x (A+B+C)} \approx e^{i \frac{x}{2} A} e^{i \frac{x}{2} B} e^{i x C} e^{i \frac{x}{2} B} e^{i \frac{x}{2} A} \tag{A.41}
\]
\[
\vdots \tag{A.42}
\]
\[
e^{i \Delta t (\omega_0 + \omega_1 + \ldots + \omega_n)} \approx e^{i \frac{\Delta t}{2} \omega_0} e^{i \frac{\Delta t}{2} \omega_1} \ldots e^{i \Delta t \omega_n} \ldots e^{i \frac{\Delta t}{2} \omega_1} e^{i \frac{\Delta t}{2} \omega_0} \tag{A.43}
\]

This presents a problem for any extension of eqs. (A.22) to (A.24) to a fully parameterized magnetic field: a full representation of the field and its action involves \( \sigma_x, \sigma_y, \sigma_z \) which obviously don't commute. The second order approximation has a structure that ultimately makes this approach not feasible, because the individual integrals \( \int dr_n \) are not separable.
In chapter 5, a solution to a system of equations is given. The trigonometric manipulations required to generate that solution are non-obvious, and are presented in full below.

**B.1 Class 1 (Miura and Hexagonal) solution**

Beginning with eqs. (5.9) and (5.10):

\[
\hat{\omega}_1 \cdot \hat{\omega}_3 = \cos \theta_1 \cos \phi_2 + \sin \theta_1 \sin \phi_2 \sin \psi = \cos \phi_2 \tag{B.1}
\]

\[
\hat{\omega}_2 \cdot \hat{\omega}_4 = \cos \theta_2 \cos \phi_1 - \sin \theta_2 \sin \phi_1 \sin \psi = -\cos \phi_1 \tag{B.2}
\]

\[
(\cos \theta_1 - 1) \cos \phi_2 + \sin \theta_1 \sin \phi_2 \sin \psi = 0 \tag{B.3}
\]

\[
(\cos \theta_2 + 1) \cos \phi_1 - \sin \theta_2 \sin \phi_1 \sin \psi = 0 \tag{B.4}
\]

\[
-2 \sin^2 \frac{\theta_1}{2} \cos \phi_2 + 2 \sin \frac{\theta_1}{2} \cos \frac{\theta_1}{2} \sin \phi_2 \sin \psi = 0 \tag{B.5}
\]

\[
2 \cos^2 \frac{\theta_2}{2} \cos \phi_1 - 2 \sin \frac{\theta_1}{2} \cos \frac{\theta_1}{2} \sin \phi_1 \sin \psi = 0 \tag{B.6}
\]

\[
-\tan \frac{\theta_1}{2} \cos \phi_2 + \sin \phi_2 \sin \psi = 0 \tag{B.7}
\]

\[
\cot \frac{\theta_2}{2} \cos \phi_1 - \sin \phi_1 \sin \psi = 0 \tag{B.8}
\]
\[
\tan \frac{\theta_1}{2} = \sin \psi \tan \phi_2 \quad \text{(B.9)}
\]

\[
\cot \frac{\theta_2}{2} = \sin \psi \tan \phi_1 \quad \text{(B.10)}
\]

Let \(z = \frac{\tan \phi_2}{\tan \phi_1}\)

\[
\tan \frac{\theta_1}{2} = \frac{\tan \phi_2}{\tan \phi_1} \quad \text{(B.11)}
\]

\[
\tan \frac{\theta_1}{2} \cot \frac{\theta_2}{2} = \tan \phi_2 \tan \phi_1 \quad \text{(B.12)}
\]

\[
\tan \theta_1 \left(1 - \tan \frac{x}{n} \tan \frac{\theta_1}{2}\right) = z \left(\tan \frac{\pi}{n} + \tan \frac{\theta_1}{2}\right) \quad \text{(B.13)}
\]

Let \(t = \tan \frac{\theta_1}{2}\)

\[
t(1 - t \tan \frac{\pi}{n}) = z(\tan \frac{\pi}{n} + t) \quad \text{(B.14)}
\]

\[
t - t^2 \tan \frac{\pi}{n} - z \tan \frac{\pi}{n} - z t = 0 \quad \text{(B.15)}
\]

\[
-t^2 \tan \frac{\pi}{n} + (1 - z) t - z \tan \frac{\pi}{n} = 0 \quad \text{(B.16)}
\]

\[
t^2 \tan \frac{\pi}{n} + (z - 1) t + z \tan \frac{\pi}{n} = 0 \quad \text{(B.17)}
\]

\[
t = \tan \frac{\theta_1}{2} = \frac{1}{2 \tan \frac{\pi}{n}} \left[1 - z \pm \sqrt{(z - 1)^2 - 4z \tan^2 \frac{\pi}{n}}\right] \quad \text{(B.18)}
\]

\[
\tan \frac{\theta_1}{2} \text{ (B.23) needs to be real, which is satisfied by equations (B.27) or (B.26)}
\]
\[(z-1)^2 - 4z \tan^2 \frac{\pi}{n} \geq 0\]  
(B.24)

\[z^2 - z \left(2 + 4 \tan^2 \frac{\pi}{n} \right) + 1 \geq 0\]  
(B.25)

\[1 + 2 \tan^2 \frac{\pi}{n} - \sqrt{\left(1 + 2 \tan^2 \frac{\pi}{n} \right)^2 - 1} \geq z\]  
(B.26)

\[1 + 2 \tan^2 \frac{\pi}{n} + \sqrt{\left(1 + 2 \tan^2 \frac{\pi}{n} \right)^2 - 1} \leq z\]  
(B.27)

Additionally, \(|\sin \psi| < 1\) and \(\text{Im}(\sin \psi) = 0\), or by using eq. (5.12): \[\left| \frac{\tan \frac{\theta_1}{\tan \phi_2}}{2 \tan \frac{\pi}{n} \tan \phi_2} \left[1 - z \pm \sqrt{(z-1)^2 - 4z \tan^2 \frac{\pi}{n}} \right] \right| \leq 1\]  
(B.28)

### B.2 Class 2 (Yoshimura and Kresling) Solution

The text of chapter 5 only introduces the constraints and generation of Class 1 unit cells, simply stating that the resulting configuration spaces for Class 1 and Class 2 are identical. Here there is plenty of space to spell out the exact solutions for

The class 2 unit cells defined in fig. 5.2 have 8 degrees of freedom. Specified values for \(\phi_1\) and \(\phi_2\) provide two constraints, and four more may be defined with the scalar products relating the unit vectors in eqs. (B.29) to (B.32). The two global rotational symmetry terms are locked out by the same declarations as before: \(\hat{\omega}_1 = \hat{i}\), \(\hat{\omega}_1 \times \hat{\omega}_4 = \hat{k}\)

\[
\hat{\omega}_1 \cdot \hat{\omega}_4 = \cos \theta \]
(B.29)

\[
\hat{\omega}_1 \cdot \hat{\omega}_3 = \cos \phi_2
\]
(B.30)

\[
\hat{\omega}_2 \cdot \hat{\omega}_4 = -\cos \phi_1
\]
(B.31)

\[
\hat{\omega}_2 \cdot \hat{\omega}_3 = \cos (\phi_1 - \phi_2)
\]
(B.32)

\[
\hat{\omega}_1 = \hat{i}
\]
(B.33)

\[
\hat{\omega}_3 = R_x(-\psi) R_y(-\phi_2) \hat{\omega}_1
\]
(B.34)

\[
\hat{\omega}_4 = R_z(\theta) \hat{\omega}_0
\]
(B.35)

\[
\hat{\omega}_{4,1} = R_z \left( \theta - \frac{\pi}{2} \right)
\]
(B.36)

\[
\hat{\omega}_2 = R_{04}(\psi) R_{04,1} (\pi - \phi_1) R_z(\theta) \hat{\omega}_0
\]
(B.37)
\[ \hat{\omega}_2 \cdot \hat{\omega}_3 = \cos(\phi_1 - \phi_2) \]  
(B.38)

\[
\begin{align*}
\sin \frac{2\pi}{n} \sin \psi \sin(\phi_1 - \phi_2) & + \\
\cos \frac{2\pi}{n} \left( \sin^2 \psi \sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2 \right) &= \cos(\phi_1 - \phi_2) \\
\quad &+ (1 - \sin^2 \psi) \sin \phi_1 \sin \phi_2 \\
\end{align*}
\]  
(B.39)

\[
\sin \psi = 
\begin{aligned}
&\sin^2 \frac{2\pi}{n} \sin^2(\phi_1 - \phi_2) - \\
&\cos \frac{2\pi}{n} \cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2 - \cos(\phi_1 - \phi_2) \times \\
&B(\sin \phi_1 \sin \phi_2 - \sin \phi_1 \sin \phi_2) \\
&\cos \frac{2\pi}{n} \sin \phi_1 \sin \phi_2 - \sin \phi_1 \sin \phi_2 \\
&\cos \frac{2\pi}{n} \sin \phi_1 \sin \phi_2 - \sin \phi_1 \sin \phi_2 \\
&2 \left( \cos \frac{2\pi}{n} \sin \phi_1 \sin \phi_2 - \sin \phi_1 \sin \phi_2 \right) \\
\end{aligned}
\]  
(B.40)

\[ A = -2 \sin \frac{n}{\pi} \cos \frac{n}{\pi} \left( \sin \phi_1 \cos \phi_2 - \sin \phi_2 \cos \phi_1 \right) \]  
(B.41)

\[ B = 4 \sin^2 \frac{n}{\pi} \cos^2 \frac{n}{\pi} \left( \sin^2 \phi_1 \cos^2 \phi_2 + \sin^2 \phi_2 \cos^2 \phi_1 - 2 \sin \phi_1 \sin \phi_2 \cos \phi_1 \cos \phi_2 \right) \]  
(B.42)

\[ C = -2 \sin^2 \frac{n}{\pi} \cos \phi_1 \cos \phi_2 \]  
(B.43)

\[ D = -2 \sin^2 \frac{n}{\pi} \sin \phi_1 \sin \phi_2 \]  
(B.44)

\[ E = -4 \sin^2 \frac{n}{\pi} \sin \phi_1 \sin \phi_2 \]  
(B.45)

\[ \sin \psi = \frac{1}{2} \cot \frac{n}{\pi} \left( \cot \phi_2 - \cot \phi_1 \right) \pm \]  
(B.46)

\[ \sqrt{\frac{1}{4} \cot^2 \frac{n}{\pi} \left( \cot^2 \phi_2 + \cot^2 \phi_1 - 2 \cos \phi_1 \cos \phi_2 \right) - 4 \cos \phi_1 \cos \phi_2} \]

\[ \sin \psi = \frac{1}{2 \tan \frac{n}{\pi} \tan \phi_2} \left[ 1 - \frac{\tan \phi_2}{\tan \phi_1} \pm \sqrt{\left( \frac{\tan \phi_2}{\tan \phi_1} - 1 \right)^2 - 4 \frac{\tan \phi_2}{\tan \phi_1} \tan^2 \frac{n}{\pi}} \right] \]  
(B.47)
Again setting $z = \frac{\tan \phi_2}{\tan \phi_1}$

$$\sin \psi = \frac{1}{2 \tan \frac{\pi}{n} \tan \phi_2} \left[ 1 - z \pm \sqrt{(z - 1)^2 - 4 z \tan^2 \frac{\pi}{n}} \right]$$

(B.48)

Which yields the same parameter space as eq. (B.23).
APPENDIX C

GEOMETRY AND DESIGN OF ORIGAMI BELLOWS WITH TUNABLE RESPONSE (AS PUBLISHED IN PHYSICAL REVIEW E)

The following paper may be found at [Rei17].
Geometry and design of origami bellows with tunable response

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Origami folded cylinders (origami bellows) have found increasingly sophisticated applications in space flight and medicine. In spite of this interest, a general understanding of the mechanics of an origami folded cylinder has been elusive. With a newly developed set of geometrical tools, we have found an analytic solution for all possible cylindrical rigid-face states of both Miura-ori and triangular tessellations. Although an idealized bellows in both of these families may have two allowed rigid-face configurations over a well-defined region, the corresponding physical device, limited by nonzero material thickness and forced to balance hinge and plate-bending energy, often cannot stably maintain a stowed configuration. We have identified the parameters that control this emergent bistability, and we have demonstrated the ability to design and fabricate bellows with tunable deployability.

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I. INTRODUCTION

Origami typically calls to mind images of flowers, leaves, birds, and ever more sophisticated and beautiful sculptures. Reluctant to cut, glue, or stretch their medium, artists have developed a stunning family of fold patterns and techniques to reshape flat sheets into imagined forms. Literally “folding paper,” the art of origami is fundamentally the study of the generation of dramatic changes of a material’s appearance and bulk mechanical properties via the application of a sequence of highly localized deformations. Origami lattices are a prototypical metamaterial, readily converting an unwieldy film into a robust device capable of reliable and simple actuation [1]. A remarkably small fraction of a sheet is deformed when it is creased, but the mere existence of a crease dramatically changes its deformation modes. Several creases acting in concert can govern a device’s kinematics—the final device’s degrees of freedom depend more on its geometry than on its local structural properties [2]. Like other metamaterials, origami lattices have several exotic mechanical properties that can be tuned with small variations in their design [3]. Of particular interest, the Miura-ori chevron tessellation expands in all directions when pulled apart, exhibiting a negative Poisson’s ratio [4]. Furthermore, several classes of flat-foldable origami are known to exhibit bistable behavior [5,6]. Origami lattices are amenable to rigorous mathematical handling [7], which generates a direction for designers exploring the space of accessible patterns. Subsequent work in that quantitative endeavor has led to a set of powerful theorems and tools, in particular to deal with the non-self-intersection constraint [8]. Classical “rigid face” origami tessellations are constructed with planar faces linked by flexible but well-defined hingelike creases. These origami metamaterials have raised interest in disparate fields [9], where further theoretical development is driven by a multitude of creative applications.

It is important to distinguish between origami geometry and origami mechanics. Origami geometry concerns itself with mathematically ideal objects. These bodies are inextensible, uniformly flat, and generally defined with a degree of internal symmetry. With surfaces and linkages so constrained, the base configuration of the device may be perfectly specified. Considering each crease as a hinge mechanism, it is possible to count up the degrees of freedom, which is sufficient to determine if the device is rigid or if it can be smoothly actuated. For example, the crystalline silicon cells found in deployable solar panels are effectively inelastic, and the mechanism can be analyzed much like its idealized mathematical analog. These techniques are useful for generating starting geometries and framing problems of interest, but they are insufficient to yield mechanical insight. They are forced, by their base assumptions, to ignore the differing energetic costs of various fold configurations, and they are thereby unable to predict or explain the complete mechanical response of an origami folded device. In particular, they fail for structures folded out of materials with nonzero elasticity, as both the faces and the creases have innate elastic energy. Indeed, creases have a preferred angle of repose [10], which can either stabilize a particular configuration or drive it far from its rigid-face equilibrium. Moreover, creases and faces tend to
bend on different energy scales, and the competition of these effects leads to dramatically different behavior than what geometric models might predict. For example, Ref. [5] demonstrates how face bending can generate a pathway for an origami mechanism to follow while transitioning through a geometrically forbidden configuration to a lower energy state, while in Ref. [10] a straightforward technique to measure the competition between crease and plate bending is demonstrated.

Much of the previous theoretical work was focused on planar lattices [4], although cylindrical configurations are of considerable technical interest. Origami folded cylinders have found applications in space technology for deployable sails and booms [11,12], medical devices such as stents [13], and even nuclear physics [14]. Although these fields are very sensitive to reliability and cost concerns, pattern development has been conducted in a largely ad hoc manner [15] due to the absence of a general predictive framework for their performance. Recent work with such configurations has led to several remarkable theoretical developments. In Ref. [16], a family of rigid foldable cylindrical bellows is identified and a mechanism whereby the mechanics of such bellows could be tuned is demonstrated [17]. It was later proved that a cylinder constructed out of radially arranged Miura corrugations is incapable of rigid foldability [18]. Recent developments have explored the bistability of bellows patterned with Miura-ori folds [19] as well as a Kresling pattern [20]. These works utilize an elastic rod framework to explore the dynamic response of a folded bellows. This suffices to illustrate the existence of geometrically allowed bistable configurations, but it fails to capture the behavior of a folded device as it actuates.

In the present work, we use properties established by a general solution for allowable configurations to predict and explain the responses of real bellows. Toward that end, we will first define and solve the geometry at hand, detailing each constraint and assumption. We will then explore the mechanical response of their physical manifestations, necessitating the construction of a series of tunable cylindrically symmetrical bellows before subjecting them to controlled actuation and collapse. We will conclude finally with a discussion of the behavior observed during actuation, with a special focus on its possible applications.

II. ORIGAMI BELLOWS GEOMETRY

Given an ideal origami pattern represented as a system of linked rigid polyhedra, there exist multiple families of frameworks that may be used to analyze its allowable configurations. If an origami system is represented as a mechanism of rigid linkages [12], it can be subjected to classical constraint-counting techniques. Some of these frameworks invoke quaternionic algebra to generate the relative rotations of their linkages [12] or faces [21]. Other treatments of Miura-ori sheets and cylinders use the angle between faces as their control parameters [4, 21]. In the following, we instead parametrize origami tessellations using the set of vectors associated with the crease network [6]. By solving the fully constrained behavior of a periodic fundamental origami cell, we have found an analytic solution for all possible rigid-face states accessible from both cylindrical Miura-ori and Kresling patterns.

To illustrate the benefits of this method, consider the folding pattern depicted in Fig. 1. As shown in Fig. 2, the tessellations of several types of rotationally symmetric bellows can be derived from variations on this unit cell. Planar tilings of these unit cells have been studied extensively, but relatively few works have studied cylindrical configurations [18, 20]. Some developable patterns on the cylinder lack rotational symmetry [17], but here we will focus on regular cylindrical tilings of the above-mentioned cells, whose regularity will be formally introduced as uniform rotational symmetry.

The unit cells illustrated in Fig. 2 generate bellows like those shown in Fig. 3. They have been organized by class and type leading to four major domains. Type A and type B differ in the value of the continuously variable angle $\phi_0$, where type A patterns correspond to $\phi_0 < \frac{\pi}{2}$ and type B patterns are characterized by $\phi_0 > \frac{\pi}{2}$. Class 1 and 2 patterns differ in the existence of the crease vector $\mathbf{e}_0$, which leads to a different set of fold parametrizations and constraint counts, but will not dramatically alter the solutions for rigid-face states. When $0 < \phi_2 < \phi_1 < \frac{\pi}{2}$, the generated pattern is the familiar Miura corrugation for class 1 and the Kresling pattern for class 2. If

![Fig. 1. Cylindrical Miura-ori corrugation: thick left and right edges connect to one another. The framework used in this paper does not specify whether edges are valley or mountain folded, a property that instead emerges from the solution of its geometry.](image1)

![Fig. 2. Various unit cells (delimited by red dashed rectangles) derived from the Miura-ori folding patterns. A global scaling factor determines the length of the unit cell. At least three circumferential cells are required to make a bellows.](image2)
Inequality (3) ensures that \( \ell_1 \) remains positive and that the edge defined by \( \omega_5 \) does not cross \( \omega_2 \) in an adjacent unit cell. Note that class 2 fold patterns are described by this same framework but with \( \ell_3 = 0 \) and \( h = 1/(\cot \phi_2 - \cot \phi_1) \).

When the imprinted network of creases is rigidly folded, two degrees of freedom can be fixed because of global rotational symmetry. A first degree of freedom can be eliminated by taking \( \omega_0 = t \hat{t} \), where \( \hat{t} \) is a unit vector parallel to the \( X \) axis. Second, the vectors \( \omega_1 \) and \( \omega_4 \) also lie in the \( XY \) plane, locking the final degree of freedom and expediting the generation of the remaining vectors. Here, we utilize SO(3) rotation matrices in lieu of Wu and You’s quaternionic approach [21].

\[
\begin{align*}
\omega_1 &= \ell_1 R_z(-\theta_1) \hat{t}, \\
\omega_2 &= \ell_3 R_x(\psi) R_z(-\theta_2) \hat{t}.
\end{align*}
\]

Here \( n \) is the number of unit cells. Condition (8) is necessary to generate well-behaved closed tubes for all \( n \geq 3 \).

It is easily shown that the parametrization described by Eqs. (4)–(7) reduces to a system of two equations, with the as-yet unspecified parameters \( \theta_1 \) and \( \psi \):

\[
\begin{align*}
\omega_1 \cdot \omega_2 &= \ell_1 \ell_3 [\cos \theta_1 \cos \phi_2 + \sin \theta_1 \sin \phi_2 \sin \psi], \\
\omega_2 \cdot \omega_3 &= \ell_2 \ell_4 [\cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2 \sin \psi],
\end{align*}
\]

Using Eq. (8), one can show that this system of equations is satisfied by

\[
\tan \frac{\theta_1}{2} = \frac{1}{2 \tan \frac{\phi_1}{2}} \left[ 1 - \frac{\tan \phi_2}{\tan \phi_1} \right] \pm \sqrt{\left( \frac{\tan \phi_2}{\tan \phi_1} - 1 \right)^2 - \frac{4 \tan \phi_2 \tan \phi_1 \tan^2 \frac{\pi}{n}}{n}},
\]

\[
\sin \psi = \tan \frac{\pi}{n} \frac{\tan \phi_2}{\tan \phi_1},
\]

Equations (11) and (12) determine \( \theta_1 \) and \( \psi \) as functions of \( \phi_1 \) and \( \phi_2 \), the only control parameters of bellows geometry. One can show that the solution of Eq. (11) satisfies \( \theta_1 \leq \pi - \frac{2\pi}{n} \), and thus self-intersection of the faces is implicitly avoided.
However, for Eqs. (11) and (12) to yield physically meaningful solutions, one must still satisfy that $\theta_1$ is real and $|\sin \psi| \leq 1$. Using these conditions, one can show that there are no more than two geometrically allowed rigid face configurations for a closed band constructed of at least three unit cells. The structure of the phase diagram is illustrated in Fig. 4. The regions of the $\phi_1, \phi_2$ parameter space where physical solutions exist are bounded by several functions:

$$f(\phi_1, n) = \arctan \left( \frac{1 - \sin \phi_1}{1 + \sin \phi_1} \right),$$

$$g_1(\phi_1, n) = \phi_1 - \frac{\pi}{n},$$

$$g_2(\phi_1, n) = \phi_1 - \frac{\pi}{n} + \frac{\pi}{n}.$$

$f(\phi_1, n)$ is found by limiting the solutions of Eq. (11) to be real, and $g_1$ ($g_2$) corresponds to the case $\sin \psi = 1$ ($\sin \psi = -1$).

Although the number of faces is identical, class 1 and class 2 tessellations differ in their total number of folds requiring careful verification of allowable rigid-face configurations for class 2 folded patterns. Using the same approach, one can show that the corresponding solution space is indeed also bounded by Eqs. (13)–(15), generating identical diagrams to the solutions detailed in Fig. 4, indicating that the bistability of these bellows is topological in nature. These results dovetail with Connelly’s bellows theorem [24], which states that continuous deformations of a closed triangulated surface cannot change its volume, and that the number of enclosed volumes attainable deformations of a closed triangulated surface cannot change its volume.

These folding modes require $\omega_0, \omega_1$, and $\alpha_0$ to not be coplanar, a folding mode that is blocked by the presence of the cells corresponding to the mirrored edges of $\omega_2$ and $\alpha_1$.

The classification by type in Fig. 2 is chosen to mirror the structure of the parameter space in Fig. 4. As there are zero free parameters in the system of equations given by Eqs. (11) and (12), we will only find a finite number of allowable solutions at any point in parameter space. The figures constructed with $\phi_1 > \frac{\pi}{2}$ are at best monostable. The boundary defined by $g_1(\phi_1, n)$ generates a flat-folded monostable figure.

The type A bellows is the one generated in the left half of Fig. 4, with a bistable region neighboring a larger monostable region. The bistable region is bounded on the left by $f(\phi_1, n)$ and below by $g_1(\phi_1, n)$. Along the curve $g_1$ one of the configurations is flat-folded. At $(\phi_1, \phi_2) = (\frac{\pi}{2}, \frac{\pi}{2} - \frac{\pi}{2})$, the deployed state is completely extended, with perfectly flat walls along the bellows’ axial direction. Moving to smaller values of $\phi_1$ along $g_1$, the deployed configuration’s hinge angle decreases until $\phi_1 = \frac{\pi}{2} + \frac{\pi}{2}$, at which point the bistable states are degenerate. The deployable bellows explored in [15] all lie along $g_1$, with increasing deployability as $\phi_1$ increases from $\frac{\pi}{2} + \frac{\pi}{2}$ to $\frac{\pi}{2}$.

In the region of existence of two states, the difference between more collapsed and more extended configurations for an origami cylinder may be colloquially referred to as its “deployability.” The magnitude of extension between the two states is easiest to measure from the angle difference $\Delta \psi$ between collapsed and deployed solutions, with colored call outs for the contours indicated on the figure’s right. The purple curve represents zero separation between the solutions, indicating the solution’s degeneracy along the curve $f(\phi_1, n)$. Grayscale shading indicates the angle difference $\Delta \psi$ (in radians) between collapsed and deployed solutions, with colored call outs for the contours indicated on the figure’s right.
from that of a class 1 folded cylinder: \( \beta_1 \) and \( \beta_2 \) change from one stable configuration to the other, so each band of the bellows is forced to move in lockstep with its neighbors. While both of the individual bands have the same allowable configurations, the deployment behavior of an aggregate bellows should not be the same. Each stage of a Kresling bellows is able to snap between its bistable states independently of its neighbors, but a Miura-ori cylinder’s only stable rigid face configurations are completely collapsed or completely deployed. This lack of intermediate stable configurations contributes to the appearance of smooth deployability. To demonstrate this most clearly, a physical bellows should be constructed.

III. MECHANICS

A. Device fabrication

To generate regularly folded bellows, a laser cutter was used to cut flat panels of a substrate, which could then be assembled into a uniform cylinder. Cylinders were initially constructed by cutting a complete fold pattern from a single sheet that included a row of tabs with which the sheet was glued into a cylinder. Once glued, the panels’ hinges could be folded so that the entire device collapsed into its preferred configuration. While useful for exploring various fold patterns, this assembly technique is not acceptable for device fabrication. The height of the energy barrier between bistable configurations is a function of the face bending energy [5], and this technique, although rapid, generates a single column of particularly stiffer walls.

Instead, our bellows are constructed much like a glued paper lantern. Individual strips of acetate sheets with thickness 0.1 mm (transparency slides) are cut out. Instead of scoring creases, the sheets are perforated: leaving a constant length fraction attached for each crease ensures a hinge energy that scales appropriately with the length of the crease and allows for a small torsional stiffness of the crease compared with the bending stiffness of the faces. This energy scale separation has been checked by observing that faces of a single actuated crease do not bend. Notice that such an estimate would only provide a lower bound since in a bellows configuration the kinematics rigidify the faces. Each strip has a set of tabs that connect it to its neighbor with double-sided tape [Fig. 6(a)]. Panels are aligned where laser-cut lines converge, and global alignment is double-checked with the flatness of the assembled sheet of strips before they are connected into a tube. This generates a device with an isotropic cross section, as seen in Fig. 6(b). Once shaped into a tube, the top and bottom regions are reinforced with a layer of Scotch tape. Without this tape, the tubes tend to pull apart from the ends under the stress of the initial folding. Edges are gently precreased, working along the entire tube [Fig. 6(c)].

As the tube approaches the desired shape, more force is used until the panels buckle flat [Fig. 6(d)]. With this design, one edge of each face is stiffer than the remainder of the bellows. Fortunately, the stiffened region is very close to the crease, where mechanics are dominated by the hinge energy. The bellows is collapsed during assembly, giving a preliminary indication of how it will respond to forced cycling.

The naming convention used in Fig. 2 may be adapted for device fabrication by considering a single band of circumferential rigid panels (see Fig. 1). A class 2 (triangular tessellation) bellows can be constructed with any number of these bands. An odd number of bands must twist during compression, but mirrored pairs of bands will have no net twist. A class 1 (Miura-ori) bellows should be designed with an even number of bands, as each one needs to be paired with its mirror image. To average out small defects from construction while keeping the bellows as large as possible, each bellows is constructed with six total bands per strip. In addition to the bands themselves, a bit of extra material is required to seal the end of the bellows. Furthermore, a transition pattern is required for a Miura-ori folded bellows to connect smoothly to these convex and rigid end caps. Finally, five-panel (\( n = 5 \)) bellows were selected in the following study.

Although Miura-ori and triangular tessellated unit cells are described by identical phase diagrams, the aggregate behavior of a collection of unit cells may be different during extension and collapse. Exploring this divergent behavior requires the careful selection of a control parameter to hold constant among multiple test geometries. Because the total number of unit cells will vary during testing, the total fold length and the folds length relative to the overall base size were conserved.

B. Experimental observations

Because the bellows are fabricated with nonzero hinge energy and panels of finite thickness, a mathematically bistable configuration may not be mechanically bistable, i.e., able to remain in one metastable state without external forcing. As a matter of fact, Fig. 4 shows that bellows constructed along the curve \( f(\phi, n) \) are degenerate and thus monostable, and bellows constructed along the segment \( \phi_1 = \pi/2, \pi < \phi_2 < \frac{\pi}{2} \) have a barrier to their collapse, and they are thus mechanically bistable. Therefore, one expects that a transition between these regimes should exist for a critical design angle \( \phi_1 = \phi_2 \). When \( \phi_1 < \phi_2 \), the bellows is unable to remain in its collapsed state and is expected to deploy itself to very near its rigid-face extended state after its collapse. This property provides a large design space for actuation, as will be discussed below.

Inspired by [15,25], initial values of \( \phi_1 = \frac{3\pi}{4} \) and \( \phi_2 = \frac{\pi}{2} \) were selected. This generates a flat-foldable bellows that is very close to the minimal distortion geometry described in [25]. We found that such a “flat foldable” bellows is unable to maintain a stowed configuration, instead demonstrating a self-deployability. Although it would be possible to move into
the nonflat stowed region ($\phi_2 > \phi_1 - \pi/n$), it is more interesting to explore various values of $\phi_1$ by keeping $\phi_2 = \phi_1 - \pi/n$. The exact value of the critical $\phi_1$ for this transition is dependent on assembly, but it appears that $\phi_2 \gtrsim \frac{2\pi}{72} = 72^\circ$. Class 1 and 2 bellows were designed on either side of this transition ($\phi_1 = 68^\circ$ and $76^\circ$) in order to demonstrate its existence.

The mechanical response of the bellows was performed using an Instron® test frame, and its corresponding deformation was captured with a Nikon D800 both as high-resolution still images and as movie shots at 720p, 60 fps. Bellows were designed for their end caps to have zero net rotation as they actuate, and they were mounted to the Instron® on irrotational compression plates. Movies (available as Supplemental Material [26]) were synchronized to the Instron® test frame’s data after their capture via motion tracking. On all the following force versus displacement curves, the zero displacement point is chosen at the extended position of the bellows where the applied force is nearest to zero after attaching the bellows. The zero load reference point is established by the dynamic force value at the moment the test frame reverses direction from extension to compression.

As seen in Fig. 7, after the initial compression’s dramatic collapse behavior, subsequent cycles follow each other closely, with only minor aging of the device as it cycles, and the mechanical response of the bellows converges toward a limiting cycle. This typical behavior is not reported in Figs. 8–11, which only illustrate a cycle consisting of a single extension followed by compression.

The class 1–type A bellows with $\phi_1 = 68^\circ < \phi_c$ in Fig. 8 is the softest of the bellows examined here. As seen in its force-displacement curve as well as Movie-S1 in [26], it can be fully collapsed with only 5 N applied force. Unfortunately, we are unable to resolve face-bending behavior during the collapse of this pattern. Its class 2–type A equivalent (Fig. 9 and Movie-S2 in [26]) does show small steps as the paired cells collapse, but the smooth extension stroke (the lower half of the force-displacement plot) indicates that this pattern does not lock in its collapsed state, instead deploying itself after collapse.

The class 1–type A bellows with $\phi_1 = 76^\circ > \phi_c$ in Fig. 10 exhibits minor locking (see also Movie-S3 in [26]), but this is an artifact of the transition end pieces, which convert the variable concave cross section to a uniform convex hexagon that can be mounted rigidly to the Instron®. The class 2–Type A bellows in Fig. 11 with $\phi_1 = 76^\circ > \phi_c$ demonstrates the most interesting behavior of this set: clear face bending, crease

![Image](image-url)
bending, and self-locking behavior. An overview of its cycling behavior is displayed in Movie-S4 [26]. The sharp steps on the extension stroke indicate locking of individual cells and the failure of those cells as they extend to their other rigid-faced state.

Looking closely at the ends of the bellows in the last frame of Fig. 11 reveals that the outer edges parallel to \( \vec{\omega}_2 \) (see Fig. 2) are bent. The bellows has been driven out of its rigid face equilibrium, and the only way to access the stable collapsed state is by bending the faces. Class 2 bellows, however, are triangular and can only bend their faces by also bending a crease. The regularity of this crease bending is evidence of the suitability of the construction techniques detailed earlier. Crease bending also is far more energetically expensive than plate bending: When extended, the hinge is held relatively near its preferred angle of repose. The plate’s dramatically higher folding energy then dominates the dynamics of the device, leading it to equilibrate near its rigid face state. When collapsed, the hinges are compressed far from their angle of repose, leading to dramatic deformations of the plates, as seen in Fig. 11. Bent, the plates are driven away from their rigid-face equilibrium toward the potential barrier between the two states. Because the collapsed state is driven far from its equilibrium state by hinge energy, it is easier to overcome the remainder of the barrier.

Special attention should be paid to the mechanical response shown in Fig. 11. In particular, note that there are three collapse events, with each one corresponding to a pair of panels collapsing. If the unit cells were identically manufactured and unable to communicate with one another, these progressive collapsing events would occur at roughly the same force levels. That they do not is potentially evidence that some cells are less rigid than others, due to minor variations in their manufacture. Moreover, careful inspection of video data Movie-S4 in [26] shows that the cells do not always collapse in the same order, suggesting that while some cells may be temporarily softer than others, it is not entirely explainable by their permanent connections to one another. Instead, we suspect that the remaining deployed cells are able to flex and deform slightly, easing the transition for the
FIG. 12. Class 1–Type B bellows with $\phi_1 = 105^\circ$ and $\phi_2 = 69^\circ$. This hexagonal bellows is expected to have a single accessible rigid-face configuration. Without a geometrically allowed collapsed state, this bellows buckles under load, breaking its rotational symmetry. See also Movie-S5 in [26].

cells that will collapse. Because the number of available face configurations falls off as cells collapse, we see that each subsequent event requires more and more force.

Notice that the force-displacement curve of Fig. 11 has a dramatically negative slope at each band’s collapse, which originates in the device’s multistability: once the system passes through the barrier to the transition, the bellows is pushing on the clamp.

That each unit cell must twist in order to access its collapsed state suggests analyzing the torsion from one state to another using geometric considerations [28]. However, our bellows are designed to have zero net helicity and were mounted to plates that could not rotate. Therefore, there is a coupling between the geometric torque and boundary conditions, which in some cases inhibits allowed geometrical transitions. Therefore, a general statement using only geometric theory cannot be achieved for the present study. Finally, bellows without a geometrically allowed collapsed state for both classes behave like typical thin-walled cylinders. As seen in Figs. 12 and 13, once the panels’ capacity to deform is exceeded, the rotational symmetry breaks and one side of the bellows buckles and crumples in on itself. Returning the bellows to its original position does not restore the symmetry present initially (see also Movie-S5 and Movie-S6 in [26]).

FIG. 13. Class 2–Type B bellows with $\phi_1 = 105^\circ$ and $\phi_2 = 69^\circ$. Much like the hexagonal bellows of Fig. 12, without a geometrically allowed collapsed state this bellows crumples, breaking its rotational symmetry. See also Movie-S6 in [26].

Within these devices, the underlying mathematical deployability not only allows physical actuation, but is also largely enriched by the mechanics, which in turn provides extended functional capabilities. Deployable designs with $\phi_1 < \phi_c$ can be smoothly actuated with hardly any snap-through effects and with small forces: they can be held collapsed and they would self-deploy as the confining force is released. On the other hand, designs with $\phi_1 > \phi_c$ will remain in a given metastable state until actuated to change configuration, and the available configurations can be made to be either fully collapsed versus fully deployed, or they can alternatively exhibit several intermediate states with well-defined energy barriers between states. Interestingly, the nature of the energy barrier between metastable states depends on the geometry of the faces: quadrangular faces decouple the bending of the face and that of the crease, leading to soft actuation, whereas triangular face bending requires the adjacent creases to deform, yielding an overall much stiffer actuation.

IV. DISCUSSION

We have presented a unified geometrical description of several previously disconnected classes of origami bellows, and we uncovered their allowed states, painting a phase diagram with a peculiar island of bistability. We then proceeded to craft the corresponding physical models using an original technique, which allowed us to investigate the effect of these kinematics on the mechanical response of the models. We have shown that it is possible to rapidly generate precisely folded origami bellows using a laser cutter, plastic film, and double-sided tape. This technique can be easily used...
to make arbitrarily complicated bellows with finely tuned fold parameters. We found that the “deployability” of a given pattern, a characterization of the distance between two geometrically allowed states, yields quantitative insight into the behavior of the physical bellows, though the mechanics of face and crease bending and self-exclusion govern the detailed characteristics of their deployment.

More importantly, we have demonstrated the existence of a critical design angle that controls the bistability of a mechanical origami bellows. Furthermore, nonzero hinge energy drives these bistable devices away from the flat-folded configuration predicted by geometry, clearly illustrating the necessity of mechanics in a design phase. Class 1 bellows tend to be much more flexible than their Class 2 counterparts. As would be expected, the figures with larger \( \Delta \psi \) have more stable collapsed configurations, and some are capable of holding themselves in a collapsed state.

Bellows with specific properties can be designed with knowledge of the geometric limitations as well as the mechanical properties of their substrate. While the fabrication technique described earlier is optimal for exploring bellows configurations, it necessarily generates perforated bellows, which are unable to displace fluids or be actuated by internal pressure. Construction of a sealed bellows is more complicated, requiring machined intermediate forms and molds to shape the cylindrical substrate as needed. It is thus a more efficient use of time to only generate these construction tools once a design has been adequately prototyped by using our paper lantern construction technique.

Given the importance of plate-bending in the collapse of an origami-folded bellows and the difficulty faced by finite-element computations of thin shells undergoing bending [29], physical measurements of origami patterned bellows as they collapse or deploy are crucial for the verification and development of simulation tools. As faces and hinges bend, accurate measurements of surface curvature would allow a more complete accounting of the effective mechanical response of the bellows.

Among promising fields of application for next generation bellows design are architecture, mechanical design, and cryogenic devices. In these arenas, reliability and weight are primary design goals in the development of deployable origami bellows. Materials age and fail dramatically faster when subjected to crease bending, so patterns that minimize this damage are more reliable. With a full understanding of the importance of face bending, more robust origami patterned actuators may be designed. Bellows designs to date have rightfully started with purely geometric considerations, followed by iterative physical prototyping. Unfortunately, as the practical foldability of a bellows depends on the interplay between the established creases and the device’s ability to plate-bend its faces, this process is dependent on prototyping and intuition to generate future designs with predetermined functionalities.

ACKNOWLEDGMENTS

This material is based upon research supported by the Chateaubriand Fellowship of the Office for Science & Technology of the Embassy of France in the United States (A.R.), French National Research Agency Grant No. ANR-14-CE07-0031 METAMAT (F.L.) and FONDECYT (Chile) Grant No. 1130709 (S.R.).


[26] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.95.013002 for movies of the mechanical testings corresponding to Fig. 8 (Movie-S1), Fig. 9 (Movie-S2), Fig. 10 (Movie-S3), Fig. 11 (Movie-S4), Fig. 12 (Movie-S5), and Fig. 13 (Movie-S6).


These files were generally sized for 8.5x11” paper. Their decorations often protrude into the 1” margin specified by NCSU’s Graduate School, so the drawings have been reduced by 75% to fit.

D.1 MEOP

151–155: Nylon Cell Holder

D.2 Cryogenic Bellows

156: Origami Mandrel

157: Origami Bellows Clamps

158–167: Instron Cryogenic Test Frame (n.b. Drawing on page 160 is reduced by 50 %. Frame crossbars are too narrow to accommodate larger bellows test can. Verify this clearance when designing future components.)

168–174: Bellows Cryogenic Vacuum Can and PEEK fixtures

180: Alternate design for PEEK fixture with larger bore

175–176: Piston Seal Bellows End-pieces for Instron tests

177: Square Teflon support (prevents bellows squirm)

178: Loose support to keep bellows base concentric with test can

179: Compression sleeve to keep Kapton from delaminating during cool-downs.
<table>
<thead>
<tr>
<th>SIZE</th>
<th>COMPONENT NAME</th>
<th>QUANTITY</th>
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<tbody>
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<td>Threaded</td>
<td>2</td>
</tr>
<tr>
<td>Nylon</td>
<td></td>
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**Austin**  
**1/17/2017**

**DIMENSIONS ARE IN INCHES**
**TOLERANCES UNLESS SPECIFIED:**

- $(0.X)$: ±0.05
- $(0.0X)$: ±0.01
- $(0.00X)$: ±0.003

**TITLE**
- (from CellHolder)

**SCALE**  
1:1  
**SHEET**  
5 OF 5
DIMENSIONS ARE IN INCHES
TOLERANCES UNLESS SPECIFIED

(0.0X): ±0.05
(0.00X): ±0.003
DIMENSIONS ARE IN INCHES
TOLERANCES UNLESS SPECIFIED:
(0.X): ±0.05
(0.0X): ±0.01
(0.00X): ±0.003

Austin 6/2/2017
(from Instron-CryoFrame)

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(0.0X): ±0.01
(0.00X): ±0.003

Title: Instron-CryoFrame

Austin 6/2/2017

1/8 scale Outer Frame Assembly
COMPONENT NAME
MATERIAL
SIZE
TITLE
SS
Outer Top Plate
:

:

:
6/2/2017
... (from Instron-CryoFrame)
1:1
SCALE SHEET 4 OF 10

1/4-20 UNC - 2B

4.413
4.41

.375
.38

.75

.38

DIMENSIONS ARE IN INCHES
TOLERANCES UNLESS SPECIFIED:
(0.X): ±0.05
(0.0X): ±0.01
(0.00X): ±0.003

A Outer Top Plate

SS

1:1

Austin 6/2/2017

TITLE: (from Instron-CryoFrame)

QUANTITY

2

MATERIAL

SHEET 4 OF 10

161
Holes correspond to threaded holes in Sheet 3 (Outer U-bracket)
Holes correspond to plate dimensioned on SHEET 4

<table>
<thead>
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<th>Component Name</th>
<th>Material</th>
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<th>Quantity</th>
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Austin

6/2/2017

DIMENSIONS ARE IN INCHES
TOLERANCES UNLESS SPECIFIED
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<td>8</td>
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(0.X): ±0.05
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<th>TITLE</th>
<th>QUANTITY</th>
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</thead>
<tbody>
<tr>
<td>Vertical Brackets</td>
<td>(from Instron-CryoFrame)</td>
<td>4</td>
</tr>
</tbody>
</table>

MATERIAL: 1/2" Stainless L

SCALE: 1:4

SHEET 9 OF 10
L-shaped grooves correspond to L-brackets dimensioned in SHEET 9.
SECTION A-A
SCALE 1 / 2

<table>
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<tbody>
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<td>Al6061 &amp; PEEK</td>
<td>A</td>
<td>1</td>
</tr>
</tbody>
</table>
Turn from single piece of aluminum.

Please protect sealing surfaces with masking tape after machining.

SECTION B-B
SCALE 1 / 2
Please protect sealing surfaces with masking tape after machining.
Please protect sealing surfaces with masking tape after machining.
Please protect sealing surfaces with masking tape after machining.
Please protect sealing surfaces with masking tape after machining.
R.07

R.12

Make paired set:
6-32 tapped & 6-32 clearance (Ø0.144)
0.10" groove for -015 o-ring (2x)

0.578
0.690
0.236 thru

Ø0.472 (12mm)

.05 X 45° Chamfer

.15 .25
.45

1.25
1.76
2.0

For pickup: Ekaterina Korobkina 919-515-3302

Drawing/design clarification: Austin Reid 865-680-2627

0.7" Kapton Bellows Fixture

Instron Air part

Size A

Qty: 1

DWG NO BigD-Bellows-airpart

Scale 1:1

Aluminum

Sheet 2 of 2
Austin 4/9/2018

DIMENSIONS ARE IN INCHES
TOLERANCES UNLESS SPECIFIED

(0.X): ±0.05
(0.0X): ±0.01
(0.00X): ±0.003

TITLE
(from Sleeve)

A Glue Sleeve

QUANTITY 8

MATERIAL PEEK

SCALE 1:1 SHEET 1 OF 1
Chapter 2 addressed the theory, and chapter 3 addressed the design and performance of the MEOP system developed for SOS. The finer details regarding the operation of the device are unique to it, and have been collected in this appendix.

The information presented here is split into several sections. First, a line-by-line procedure for operating the system is presented appendix E.2. Next you’ll find the crude procedure to generate a $^{3}\text{He}/^{4}\text{He}$ isotopic mixture in appendix E.3. Python scripts are used to analyze the data after each successful run. This code is given in appendix E.4. This miniature “library” is used in a Jupyter notebook to generate numeric results and their corresponding figures. A sample notebook is given in appendix E.4.2. Finally, LabVIEW’s autogenerated documentation for all the programs used in the polarization operation is attached, starting with appendix E.5.

### E.1 Instrumentation notes

The National Instruments VirtualBench is an all-in-one device that contains a Mixed Signal Oscilloscope (MSO), a Function Generator (FGEN), and a Power Supply (PWS). Each of these systems is controlled from either a NI-supplied interface or from a calling LabView VI. When a VI requests one of these subsystems, LabView blocks other VIs or the NI-supplied interface from changing that subsystem's settings. The subsystem may be monitored from NI's interface until the calling VI relinquishes control of the resource. The exclusivity of VirtualBench subsystems prevents multiple control VIs from running simultaneously—they can’t share the resource.

### E.2 Polarization Procedure

#### E.2.1 Regenerate Getter

1. Close valve between getter and low-pressure $^{3}\text{He}$. 
Figure E.1 Benchtop MEOP gas handling system. Pressure indicator to the left of the regulator is solely to prevent inadvertent overpressure of regulator input line. Getter is marked “G”, and blanked ports are indicated as well. Thick lines between valves are metal, thin connecting line are glass. Magnetic contamination from gas bottles is reduced with indicated 0.5 \( \mu m \) filters. Volume between V2 and V1 is roughly 5 mL. G1 and G2 are Chemthread valves with Viton o-rings. G3 is a greased ground glass stopcock.

1. Close (V6), (G2).
2. Open getter to pump station valve.
   open (V7).
3. Power on Athena thermal controller, set to 800 F for 10 minutes.
4. Wait for getter to cool.
5. Close getter to pump station valve.
   close (V7).
6. Open getter to low pressure \(^3\)He valve.
   open (V6).
7. Fill system to desired pressure, adding \(^3\)He until system has reached desired pressure as read on MKS Baratron (typically 1 mbar to 1.2 mbar).
   (a) open (V3)
   (b) close (V3)
   (c) open (V2)
   (d) close (V2)
(e) open (V1)
(f) close (V1)
(g) repeat c–f until target pressure is achieved.¹

8. Wait for system to clean (chemical getter needs 1 hr to 2 hr)

9. Check sample cleanliness with Ocean Optics spectrometer (Accessed via SpectraSuite), as detailed in section 3.5.2 on page 55.

10. Calculate pressure-calibration factor with Pfactor.vi (E.5.2), which uses Lorenzon et al.’s pressure calibration values, with an option to scale by³He concentration for isotopic mixes, as suggested by Stoltz et al. [Lor93; Sto96]

**E.2.2 Power on peripherals**

Peripherals are listed in a recommended order. Devices where order matters are enumerated.

- Fabry-Perot Amplifier
- Rigol Oscilloscope (to read Fabry-Perot Amplifier)
  1. RF Amplifier
  2. VirtualBench *(Amplifier must be powered on before Function Generator specifically)*
    1. Ge Photodiode power supply
    2. Si Photodiode power supply
    3. Ge Photodiode
    4. Si Photodiode
- LCC25
- Laser (following Keopsys procedure)

You are now ready to calibrate the polarimeter and tune the laser. We will start the plasma discharge and set its intensity to a uniform level for both of these procedures.

**E.2.3 Start plasma discharge**

This can be done manually, but it is most convenient to use StartCalibrationDischarge.vi, documented as E.5.1 on pg. 216.

In general, plasma discharge is initiated by first turning on the VB’s function generator (10 MHz, 0.2 V Amplitude), then energizing the plasma ball igniter circuit (−25 V channel set to deliver −5 V)

¹You may need to adjust the regulator if V1 and V2 are not delivering enough gas.
for 500 ms. The igniter is then grounded. This same procedure is used in MonitorPolarization.vi (221).

Control of the MSO is reserved, and the liquid crystal retarder (LCR) photodiode signal (channel 1) is set to −0.1 V to 0.9 V, with an threshold maximum of 95 %. The Thorlabs liquid crystal controller (LCC25) is set to 2.005 V, which is reasonably close to the \( \frac{\lambda}{4} \) maximum transmission tuning voltage. The discharge is then reduced in intensity until its observed signal in the Silicon Photodiode (SiPD) is less than the threshold maximum.

**E.2.4 Calibrate LCR**

Make sure the left-circular calibration filter is attached to the polarimeter, then run FastCalibrate.vi, documented as E.5.3 on pg. 219.

Contaminants in the sample will generally introduce additional relaxation pathways, reducing metastable lifetime and the brightness of the helium lines. While the sample is cleaning, the red line used for polarimetry will dramatically change in brightness. This VI expects the discharge intensity to be relatively constant in time, and so can only be conducted with a sample that is not actively cleaning. The VI sweeps both \( V_{3/4} \) and \( V_{1/4} \) simultaneously. The values of \( V_{3/4} \) are specified by the user in start, stop, increment. \( V_{1/4} \) start and stop are set by the user, and increment is calculated by the VI so that both arrays have the same number of elements. The LCC is then set with the zeroth elements of both, and the corresponding transmission values are measured, then the first element of both, etc etc. After all 10-15 pairs have been measured, the VI tries to find the local minimum (\( V_{3/4} \)) or local maximum (\( V_{1/4} \)) with a parabolic fitting routine. These are the calibration values used in MonitorPolarization.vi, documented as E.5.5 on pg. 221.

Remove the left-circular polarizing filter from the polarimeter.

**E.2.5 Calibrate Laser**

You should be wearing safety goggles, or the laser should be fully enclosed. Turn on the laser, switch on the seed laser, and start pumping the powered beam. Each subsystem needs a bit of time to switch on. It takes the laser 20 minutes to 30 minutes to warm up and stabilize. After this time, you can check its calibration with Fancy-LaserSweep.vi, documented as E.5.4 on pg. 220.

This VI tries to read Fabry-Perot data from the Rigol oscilloscope, and displays its peaks (inverted) in what is effectively real-time. The user has the option to scan over various different tuning ranges, which correspond to \( D_0 \), \( C_8 \), \( C_9 \) in the laser’s current state. After service, these ranges should be verified.

The VI sets the laser’s coarse tuning voltage to its waiting voltage of 3 V, and waits while the user selects what range they want the sweep to progress over. The VI assumes the laser is in equilibrium when the sweep is started, so the user should not start the sweep until the Fabry-Perot signal looks relatively stable.
The VI sets the laser to the starting voltage $V_0$ and waits\(^2\) 12 s $\cdot |\Delta V|$, where $\Delta V = V_0 - V_{\text{wait}}$. The laser is incremented from starting voltage to the ending voltage with a $\text{d}V$ of 7 mV with 8 s between each step. The Germanium Photodiode (GePD) signal is recorded at the end of those 8 seconds.

After the sweep, the VI scales the signal by the largest value, and attempts to find any peaks present. If more than one peak is located, the VI tunes the laser to the higher voltage one, which as seen in fig. 3.8 is typically $C_8$.

A Fabry-Perot interferometer is not capable of generating absolute frequency measurements, but it excels at measuring relative frequency shifts. The process of measuring $C_8$ and $C_9$ yields useful reference values from which a frequency shift could be calculated. A rudimentary version of the necessary frequency shift calculation code is present in Fancy-LaserSweep.vi, where it tracks the locations of the Fabry-Perot peaks (which corresponds to seconds on the oscilloscope, which can in turn be converted to GHz after scaling by the 10 GHz separation of peaks), but this functionality is disabled.

E.2.6 Turn on magnetic field

E.2.7 Start pumping

All pumping control and logging runs through MonitorPolarization.vi, documented as E.5.5 on pg. 221.

- Set the “Initial laser tuning V” to the tuning value measured in appendix E.2.5.
- Set the Vmin and Vmax to the values measured in appendix E.2.4.
- Set Pressure Factor to calculated value.
- Start VI

The “raw responses” tab of this VI displays the voltages corresponding to $\lambda_{1/4}$ and $\lambda_{3/4}$ without any additional filtering. Without a discharge, these two values are only showing unpolarized background light, so their average value should be about the same. If they are very different, either the calibration filter is still attached, or the LCR calibration itself is very bad.

“Light ‘em up” starts a discharge using the plasma ball igniter. You can monitor the accumulated polarization with the “Nuclear Polarization” tab, which generates a histogram of the previous 100 polarization samples (50 s). The polarization reaches its equilibrium value very quickly, but it takes time to accumulate enough data to have reasonable statistics on the measured $P_N$. This is convenient, because it ensures that the $P_N$ and $P_e$ are in equilibrium.

Once you have your desired $P_N$, click “stop discharge” to shut off discharge while keeping the other systems logging. If you’re done with this pass, you can click “stop everything”, which will log all data (including timestamped user interaction events) to a JSON file.

---

\(^2\)It takes the laser time to equilibrate to a thermal tuning voltage, so the VI sets the laser to an initial value $V_{\text{wait}}$ that is close to, but not necessarily the same, as the first value used in the scan $V_0$. 

185
This VI is capable of automated test execution: you can use this to measure the dark $T_1$ of the cell, or to measure the background and polarization performance (including plasma $T_1$) without needing user interaction.

E.3 Preparation of Isotopic Mixture

During development of the fill system, higher pressure $^3$He was filled upstream of the regulator (between V3 and V5), then all valves to the $^3$He bottle were closed. This small charge of gas was supplied through the regulator then V2 & V1 to fill the system to known pressures. Filling both $^3$He and $^4$He to known pressures requires flushing out the gas between V3 and V1, so the isotopic mixture runs were conducted when the supply upstream of the regulator was nearly exhausted. A useful storage volume exists in the cell (controlled by G3) and around the getter (V6, V7). Shooting for $P_3 = 0.525$ mbar, the low pressure system was charged with the residual $^3$He, typ. 2 mbar. G3, V6, and V7 were closed, storing this $^3$He in the cell and between the metal valves used to isolate the getter. The rest of the gas handling system was then pumped out: G1, G2, V1, V2, V5 were opened and the remaining low pressure $^3$He was discarded. After the system reached $10 \times 10^{-5}$ bar, V5, V2, V1, and G2 were closed. Opening V6 and G3 re-introduced the stored $^3$He into the low-pressure system. If the pressure was higher than 0.525 mbar, it could be reduced in small increments by pumping the small volume between G1, G2, and G3. $P_3$ was recorded, and high pressure $^4$He was filled via the regulator and V1/V2. A substantial pressure gradient, a small volume between V1 and V2, and brief opening of those same valves minimized diffusion of the $^3$He sample while the system’s total pressure was raised to the target 2.1 mbar. The $^4$He supply is quite clean research grade gas, but this is substantially dirtier than MEOP needs, so gettering is essential.

The $D_0$ line is well within the tuning range of our laser, located around 2.6 V to 2.8 V.

E.4 Python Analysis Code

Data is captured in LabView, but I found it much more straightforward to run my analysis in Python after the run completed. If you just need the polarization and its error, the LabView code will handle that. This Python code was written to execute more sophisticated analyses, such as comparisons of multiple pumping runs and to calculate the $T_1$ decay due to plasma discharge or magnetic field inhomogeneities.

E.4.1 MEOP_utilities.py

The block of python code included below is a collection of functions designed to expedite analysis of MEOP data, and used in generating the figures seen in chapter 3.

MonitorPolarization.vi as shown on page 221 is fastidious but not very smart. Whenever the user changes one of the “control” parameters (laser tuning V, coil power supply I, discharge level),
the VI logs the new value and a ms timestamp. Simultaneously, a list of all the GePD and SiPD values are logged, with the same ms clock as the control parameters. These logged values are loaded into two pandas dataframes: “control” and “photoData”.

This ms clock is an arbitrary timer—it is easy to call, accurate enough for our purposes, and is a reasonably short representation. Unfortunately, the zeroth entry is probably not at \( t = 0 \). Also, because this is an unsigned 32-bit integer, it may wrap around mid-way through the measurement. An unsigned 32-bit millisecond timer has a maximum value of 4294967295 ms, or about 49.7 days. It isn't likely to wrap around while you're measuring, but because it's better to be safe than sorry, the time value is loaded as a uint, and then the first entry is subtracted from all of the entries, making the time array start at 0 ms, and making all the entries sequential.

A MEOP system can be in one of three modes: pumping, decaying (plasma), or coasting. If the coil is off, the system won't polarize anything, so that is tracked as well. This code looks at all the control parameter change events, and determine which mode is active with the new control parameters. These parameters are queried via “activetime”, which returns the initial and final times for any particular mode. These can be addressed in order. For example, this returns the \((t_0 = \text{start}) \) and \((t_f = \text{end})\) for the first “coasting” event in the control dataframe:

\[
\text{start}, \text{end} = \text{activetime(control,'coasting')[0]}
\]

The first few functions defined in the file are just standard forms of exponential decay and some polarization calculators. Timestamps are often converted to seconds in these functions, just to make later reporting simpler.

```python
from scipy.optimize import curve_fit
from scipy.stats import norm
from scipy.interpolate import interp1d
from json import load as jload
import pandas as pd
import numpy as np

'''
Family of functions that will be used to calculate polarization
and fit pump and decay behavior.
'''

def decayfunction(x,A,T1):
    return A * np.exp(-0.001 * x / T1)

def pumpfunction(x,A,tau):
    return A*(1-np.exp(-0.001 * x / tau))

def electronPcalc(I1,I3,Ib=0,theta=12):
    return (I1 - I3) / (I1 + I3 - 2 * Ib) / np.cos(np.deg2rad(theta))

def Pcalc (I1,I3,Ib=0,theta=12,Pfactor=8):
```

```
return Pfactor*electronPcalc(I1,I3,Ib,theta)

def decayfit(time, polarization):
    plottime = time - time[0]
    popt, pcov = curve_fit(decayfunction, plottime, polarization, bounds = ([-1,1], [1, np.inf]))
    return popt, pcov

def pumpfit(time, polarization):
    plottime = time - time[0]
    popt, pcov = curve_fit(pumpfunction, plottime, polarization, bounds = ([-1,1], [1, np.inf]))
    return popt, pcov

def load_then_split_datafile(logfile, C8thresh = 3.69, I_thresh = 0.01):
    with open(logfile) as f:
        pumpdata = jload(f)

    lasttime = pumpdata['I3data']['time'][-1]
    Poltheta = pumpdata['Polarimeter']['angle']
    PressureFactor = pumpdata['Polarimeter']['PressureFactor']
    Polbackground = pumpdata['Polarimeter']['background']

    config_params = {
        "poltheta": Poltheta,
        "PressureFactor": PressureFactor
    }

    control = pd.DataFrame({
        't':np.array([lasttime],dtype=np.uint32)
    })

    control = pd.merge_ordered(
        control, pd.DataFrame(
            {
                't':np.array(pumpdata['Helmholtz']['time'], dtype=np.uint32),
                'I':np.array(pumpdata['Helmholtz']['current'])
            }
        ), on='t', fill_method='ffill')

    control = pd.merge_ordered(
        control, pd.DataFrame(

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{'t': np.array(pumpdata['LaserTuning']['time'], dtype=np.uint32),
 'LaserV': np.array(pumpdata['LaserTuning']['laserV'])}

control = pd.merge_ordered(
    control, pd.DataFrame(
        {'t': np.array(pumpdata['DischargeStatus']['time'],
            dtype=np.uint32),
        'plasma': np.array(pumpdata['DischargeStatus']['discharge'],
            dtype=np.bool_)}
    ), on='t', fill_method='ffill')

control = pd.merge_ordered(
    control, pd.DataFrame(
        {'t': np.array(pumpdata['plasma']['time'],
            dtype=np.uint32),
        'PlasmaV': np.array(pumpdata['plasma']['discharge'])}
    ), on='t', fill_method='ffill')

GeLCRoffset = (len(np.array(pumpdata['I3data']['I3']))-
    len(np.array(pumpdata['GeData']['Ge'])))

# Sometimes LCR data collection stops before Ge data collection.
# Just discard the leftover stuff--nothing valuable is happening there.

elements = np.min((len(np.array(pumpdata['I3data']['I3'])),
    len(np.array(pumpdata['GeData']['Ge']))))

photoData = pd.DataFrame(
    {'t': np.array(pumpdata['I3data']['time'],
        dtype=np.uint32)[:, elements],
     'I1V': np.array(pumpdata['I3data']['I1'])[:, elements],
     'I3V': np.array(pumpdata['I3data']['I3'])[:, elements],
     'GeV': np.array(pumpdata['GeData']['Ge'])[:, elements]}
)

pumpseconds = 60e3
pumpstable = 30e3

first_time = np.min([control['t'][0], photoData['t'][0]])
photoData['t'] -= first_time
control['t'] -= first_time

# print("Data collection started at "+pumpdata['Timestamp']["Time"]+
# " on "+pumpdata['Timestamp']["Date"])

# control = control.assign(
#     C9 =lambda control: np.abs(control['LaserV']-C9)<0.03)

control = control.assign(
    C8 =lambda control: control['LaserV']<C8thresh)

```
We care about particular cases: "pumping", "coasting", "decaying", and "off"

- [2] decaying: [coil, plasma] are True, [laser] is False
- [1] coasting: [coil] is True, [plasma] is False (don't care about laser)
- [0] off: [coil] is False, nothing else matters.

We will generate logical arrays for each of these states, indicating when
they become true (active) and false (inactive).
```

control = control.assign(
    pumping = lambda control: np.all([control['I']]>I_thresh,
                                      control['plasma'],
                                      control['C8'],
                                      axis=0))

control = control.assign(
    decaying = lambda control: np.all([control['I']]>I_thresh,
                                       control['plasma'],
                                       ~control['C8'],
                                       axis=0))

control = control.assign(
    coasting = lambda control: np.all([control['I']]>I_thresh,
                                       ~control['plasma'],
                                       axis=0))

control = control.assign(
    off = lambda control: control['I']<=I_thresh)
activestate = np.array(['_']*(control.last_valid_index()+1))

for key in ['coasting', 'decaying', 'pumping']:
    activestate[np.asarray(control[key])] = key
# assert ~np.any('_'==activestate), 'Indeterminate state!

simpleActiveState = np.array([activestate[0]])
for mode in activestate[1:]:
    if simpleActiveState[-1] != mode:
        simpleActiveState = np.append(simpleActiveState,mode)

photoData = photoData.assign(eP = electronPcalc(photoData['I1V'],
                                              photoData['I3V'],
                                              Ib=Polbackground,
                                              theta=Poltheta))

photoData = photoData.assign(P = Pcalc(photoData['I1V'],
                                        photoData['I3V'],
                                        Ib=Polbackground,
                                        theta=Poltheta,
                                        Pfactor = PressureFactor))

return photoData,control,simpleActiveState,config_params

# generate paired lists with start/stop times for major events

def nextValidChange(dataframe, key, index, value = True):
    if dataframe.last_valid_index() == index:
        # Current value is last element.
        return dataframe['t'][index]+1
    else:
        j = index
        while dataframe[key][j] == value:
            j += 1
            if dataframe.last_valid_index() == j:
                if dataframe[key][j]:  # Last value is True!
                    return dataframe['t'][j]+1
                else:
                    return dataframe['t'][j]
        return dataframe['t'][j]

def activetime(dataframe, key):
    starts = np.asarray(dataframe.query(key+' == True').index.tolist())
mask = np.ones(len(starts), dtype=bool)
doubles = np.where(1 == np.diff(starts)) [0] + 1
mask[doubles] = False

i = starts[mask][0]
ti = dataframe['t'][i]
tf = nextValidChange(dataframe, key, i)
window = np.array([[ti, tf]])

for i in starts[mask][1:]:
    ti = dataframe['t'][i]
    tf = nextValidChange(dataframe, key, i)
    window = np.append(window, [[ti, tf]], axis=0)

return window

E.4.2 Sample Analysis Jupyter Notebook

The next several pages have a converted notebook used to analyze the JSON files generated by the LabVIEW and python tools discussed previously. Once the “start” and “stop” times for the data are known, plots and analysis can be easily generated with standard Pandas commands.
MEOP parameter scan

Austin Reid 2018-03-13

Several substantial improvements since my last report:

- Fixed several related bugs/errors in laser calibration routine
- Adjusted timing for LCR polarimeter, improving data quality
- Started using the true statistical error measure

Laser Calibration

Laser wavelength is tuned by an externally applied voltage. This heats the diode and shifts its wavelength. After changing the it takes some time for the laser to stabilize again. This stabilization time changes as a function of laser output power: higher o powers require slightly more time.

LCR Polarimeter

The previous measurements sampled \( \lambda_{1/4} \) and \( \lambda_{3/4} \) from only 20% of sampled points. Adjusting the timing hardware increase 40%. Data is discarded due to LCR switching and photodiode charge accumulation.

Statistical error measure

From Maxwell's LCR paper:

\[
\sigma_p = \frac{2f_p}{\cos \theta_m (I_{3/4} + I_{1/4})^2} \sqrt{I_{3/4}^2 \sigma_{3/4}^2 + I_{1/4}^2 \sigma_{1/4}^2}
\]

This value is calculated with 150 s of steady-state data, taken from the tail end of the pumping run.

Load Libraries, set flags, paths, etc...
In [26]:
%pylab inline
import itertools
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import ...
`%matplotlib` prevents importing * from pylab and numpy

```
```

```
```

I
P

Sample Analysis

Load sample:

In [2]:
sample = sorted(glob('data/*.JSON'))[0]
photoData,control,simpleActiveState,config_params = load_then_split_datafile(sample)
photoData['t_s'] = photoData['t']/1e3
f_p = config_params['PressureFactor']
theta = config_params['poltheta']*np.pi/180

stringy = sample.split('-')
waveplate = float(stringy[-1].replace('..JSON',('').replace('W','')))-17
if waveplate>90:
    waveplate=90

sample_laser_power = float(stringy[-3].replace(',','').replace('W',''))
sample_laser_current = float(stringy[-4].replace(',','').replace('A',''))
sample_pressure = float(stringy[-5].replace(',','').replace('mbar',''))

Photodiode data, rough calculated P
In [3]:
ax = photoData.plot.scatter(x='t_s', y='I1V', s=5)
photoData.plot.scatter(x='t_s', y='I3V', ax=ax, color='r', s=5)
# plt.fig()
photoData.query('abs(P)<1').plot.scatter(x='t_s', y='P', s=5)
Out[3]:
<matplotlib.axes._subplots.AxesSubplot at 0x10b9c5278>

Background measurement

![Graph showing background measurement](image-url)
In [4]:
start, end = activetime(control, 'coasting')[0]
flatslice = photoData.query('@start<t<@end')
I1vec = flatslice['I1V']
I3vec = flatslice['I3V']
I1bg, I1bgsigma = norm.fit(I1vec[I1vec<0])
I3bg, I3bgsigma = norm.fit(I3vec[I3vec<0])

x = np.linspace(-1, 0, 300)
I1_PDF = norm.pdf(x, loc=I1bg, scale=I1bgsigma)
I3_PDF = norm.pdf(x, loc=I3bg, scale=I3bgsigma)

# latex = latexplots(width=15, height=10, fontsize=12)
# plt.axvline(0, color='k')

binwidth=.001
plt.plot(x, I1_PDF, '-', color=brand['ReynoldsRed'])
plt.hist(I1vec, bins=np.arange(min(I1vec), max(I1vec) + binwidth, binwidth),
        density=True, color=brand['InnovationBlue'], label='1/4 background')

plt.plot(x, I3_PDF, '-', color=brand['ReynoldsRed'])
plt.hist(I3vec, bins=np.arange(min(I3vec), max(I3vec) + binwidth, binwidth),
        density=True, color=brand['GenomicGreen'], label='3/4 background')

ax1 = plt.gca()
# if np.abs(Pmean)<.3:
#    ax1.set_xlim([-0.06, -0.03])
# else:
#    ax1.set_xlim([0, np.sign(Pmean)*1])
# ax1.set_xlim([.55,.75])
ax1.set_xlabel(r'$I_{1,3}$ background')
_= ax1.get_yaxis().set_ticks([])
_= plt.legend()
# ylims = ax1.get_ylim()
In [5]:
start, end = activetime(control, 'pumping')[-1]
pumpslice = photoData.query('@start<t<@end')
germaniumsteady = pumpslice['GeV'].tail(150).mean()
crossing = (pumpslice.query('@GeV<@germaniumsteady')['t'].values[0] - start) / 1e3
stoptime = start + crossing * 1e3
# popt, pcov = pumpfit(np.array(pumpslice['t'])-offsetms, np.array(pumpslice['P']))
# estpol, pumprate[plotroot] = popt
pumping = photoData.query('@start<t<@stoptime')
ax = pumpslice.plot.scatter(x='t_s', y='GeV', color='g')
pumpslice.tail(150).plot.scatter(x='t_s', y='GeV', color='r', ax=ax)
pumping.plot.scatter(x='t_s', y='GeV', color='k', ax=ax)

Out[5]:
<matplotlib.axes._subplots.AxesSubplot at 0x10bc33fd0>

Steady State Polarization measurement
In [6]:
steady_start = end - 100e3
flatslice = photoData.query('@steady_start<t<@end')
I1vec = flatslice['I1V']-I1bg
I3vec = flatslice['I3V']-I3bg

I1mean, I1sigma = norm.fit(I1vec)
I3mean, I3sigma = norm.fit(I3vec)
I1error = I1sigma/np.sqrt(I1vec.shape[0])
I3error = I3sigma/np.sqrt(I3vec.shape[0])

x = np.linspace(.5,.7,200)
I1_PDF = norm.pdf(x,loc=I1mean,scale=I1sigma)
I3_PDF = norm.pdf(x,loc=I3mean,scale=I3sigma)

# latex = latexplots(width=15,height=10,fontsize=12)
# plt.axvline(0, color='k')

binwidth=.005
plt.plot(x,I1_PDF,'-',color=brand['ReynoldsRed'])
plt.hist(I1vec,bins=np.arange(min(I1vec), max(I1vec) + binwidth, binwidth),
        density=True,color=brand['InnovationBlue'])
plt.plot(x,I3_PDF,'-',color=brand['ReynoldsRed'])
plt.hist(I3vec,bins=np.arange(min(I3vec), max(I3vec) + binwidth, binwidth),
        density=True,color=brand['GenomicGreen'])

ax1 = plt.gca()
# if np.abs(Pmean)<.3:
#    ax1.set_xlim([-0.5,0.5])
# else:
#    ax1.set_xlim([0,np.sign(Pmean)*1])
ax1.set_xlim([.5,.7])
ax1.set_xlabel(r'$I_{1,3}$')
ax1.get_yaxis().set_ticks([])
ylims = ax1.get_ylim()

\[
\sigma_P = \frac{2f_P}{\cos \theta_m (I_{3/4} + I_{1/4})^2} \sqrt{I_{3/4}^2 \sigma_{3/4}^2 + I_{1/4}^2 \sigma_{1/4}^2}
\]

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\[
P = \frac{f_p}{n \cos(\theta)} \left( \frac{I_{1\text{mean}} - I_{3\text{mean}}}{I_{1\text{mean}} + I_{3\text{mean}}} \right) \times 100
\]
\[
\sigma_p = \left( \frac{2 \times f_p}{n \cos(\theta) \times (I_{3\text{mean}} + I_{1\text{mean}})^2} \right) \times n \sqrt{(I_{1\text{mean}} \times I_{1\text{error}})^2 + (I_{3\text{mean}} \times I_{3\text{error}})^2} \times 100
\]

**Plasma T1 Decay Calculation**

```
In [7]:
P = f_p / np.cos(theta) * (I1mean - I3mean) / (I1mean + I3mean) * 100
sigma_p = (2 * f_p / (np.cos(theta) * (I3mean + I1mean)**2) * 
           np.sqrt((I1mean * I1error)**2 + (I3mean * I3error)**2)) * 100

In [8]:
start, end = activetime(control, 'decaying')[1]
decayslice = photoData.query('@start<t<@end')
popt, pcov = decayfit(decayslice['t'].values, decayslice['P'].values)
exx = (np.linspace(start, end, 200) - start) / 1e3
ax = decayslice.plot.scatter(x='t_s', y='P')
ax.plot(exx + start / 1e3, decayfunction(exx * 1e3, *popt), color='r')
print(popt)

[-0.54321445 168.00021874]
```

Load log files into dataframe

![Graph showing plasma T1 decay calculation](image-url)
In [9]:
qwp = [] ## laser_power = [] ## laser_current = [] ## pressure = [] ## pressure_factor = [] ## polarization = []## polarization_error = []##
pump_time = [] ##
total_pump_time = [] ##
plasma_T1 = [] ##
background1 = [] ##
background3 = [] ##
Ge_steady = [] ##
logfile = [] ##

datafiles = sorted(glob('data/*.JSON'))

    for i, test in enumerate(datafiles):
        logfile.append(test)
        photoData, control, simpleActiveState, config_params = load_then_split_datafile(test)
f_p = config_params['PressureFactor']
theta = config_params['poltheta'] * np.pi / 180
pressure_factor.append(f_p)

stringy = test.split('-')
waveplate = float(stringy[-1].replace('.JSON', '')) - 17
if waveplate > 90:
    waveplate = 360
qwp.append(waveplate)
laser_power.append(float(stringy[-3].replace(',','').replace('W','')))
laser_current.append(float(stringy[-4].replace(',','').replace('A','')))
pressure.append(float(stringy[-5].replace(',','').replace('mbar','')))

###############
## Background
start, end = activetime(control,'coasting')[0]
flatslice = photoData.query('@start<t<@end')
I1vec = flatslice['I1V']
I3vec = flatslice['I3V']
I1bg, _ = norm.fit(I1vec[I1vec<0])
I3bg, _ = norm.fit(I3vec[I3vec<0])

background1.append(I1bg)
background3.append(I3bg)

###############
## Pump Rate
start, end = activetime(control,'pumping')[-1]
total_pump_time.append((end-start)/1e3)
pumpslice = photoData.query('@start<t<@end')
 germaniumsteady = pumpslice['GeV'].tail(150).mean()
 Ge_steady.append(germaniumsteady)
crossing = (pumpslice.query('@GeV<@germaniumsteady')['t'].values[0]-start)/1e3
pump_time.append(crossing)
# popt, pcov = poptfit(np.array(pumpslice['t'])-offsetms,np.array(pumpslice['P']))
# estpol, pumprate[plotroot] = popt

###############
## Steady state
steady_start = end - 80e3
flatslice = photoData.query('@steady_start<t<@end')
I1vec = flatslice['I1V']-I1bg
I3vec = flatslice['I3V']-I3bg
I1mean, I1sigma = norm.fit(I1vec)
I3mean, I3sigma = norm.fit(I3vec)
I1error = I1sigma/np.sqrt(I1vec.shape[0])
I3error = I3sigma/np.sqrt(I3vec.shape[0])

P = f_p/np.cos(theta)**(I1mean-I3mean)/(I1mean+I3mean)*100
sigma_p = (2*f_p/np.cos(theta)**(I3mean+I1mean)**2)*
            np.sqrt((I1mean*I1error)**2+(I3mean*I3error)**2))*100
polarization.append(P)
polarization_error.append(sigma_p)

###############
## Plasma T1 Decay
```python
start, end = activetime(control, 'decaying')[-1]
decayslice = photoData.query('@start<t<@end')
popt, pcov = decayfit(decayslice['t'].values, decayslice['P'].values)
plasma_T1.append(popt[1])

print("Parsed and processed {} samples".format(i))

Parsed and processed 87 samples

In [10]:
meop = pd.DataFrame(
    {'waveplate': qwp,
     'Laser_W': laser_power,
     'Laser_I': laser_current,
     'pressure': pressure,
     'f_p': pressure_factor,
     'P_N': polarization,
     'sigma_P': polarization_error,
     't_pump': pump_time,
     'pumpingtime': total_pump_time,
     'plasma_T1': plasma_T1,
     'Background 1/4': background1,
     'Background 3/4': background3,
     'Ge_steady': Ge_steady,
     'logfile': logfile}
)
meop['P3T1'] = meop.eval('pressure/plasma_T1')
print('Sample entry: ')
meop.tail(1)

Sample entry:

Out[10]:

<table>
<thead>
<tr>
<th>Background 1/4</th>
<th>Background 3/4</th>
<th>Ge_steady</th>
<th>Laser_I</th>
<th>Laser_W</th>
<th>P_N</th>
<th>f_p</th>
<th>logfile</th>
<th>plasma_T1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.032107</td>
<td>-0.033302</td>
<td>0.006342</td>
<td>1.0</td>
<td>2.4</td>
<td>76.901305</td>
<td>7.05694</td>
<td>data/20180312T023420_-FPcontrol-0,530mbar-1A-2...</td>
<td>95.363767</td>
</tr>
</tbody>
</table>
```

Polarization vs circularity
In [11]:
Pfig,ax = plt.subplots(figsize=(10,8))
marker = itertools.cycle(('v', '^', '<', '>', '*'))

for current,mark in zip(sorted(meop['Laser_I'].unique()), marker):
    lowpow = meop.query('Laser_I==@current')['Laser_W'].values.min()
    laserlevel = str(current) + ' A'
    exx = meop.query('Laser_I==@current')['waveplate']
    eye = meop.query('Laser_I==@current')['P_N']
    plot(exx, eye, mark, label=laserlevel)
    # plot(exx[eye<0], eye[eye<0], mark, label=r'$\sigma^-$' + laserlevel)
    # legend()
xlabel(r'Quarter waveplate (deg)')
ylabel(r'\text{P}_N ($)')
title("Ultimate Polarization")
# plt.legend()
plt.legend(loc='upper left', prop={'size':12}, bbox_to_anchor=(1,1))
plt.tight_layout(pad=0)

Power, T1, Pressure vs maximum polarization
In [12]:

    from matplotlib.legend_handler import (HandlerLineCollection, HandlerTuple)
    # plt.figure()
    latex = latexplots(width=7, height=6, fontsize=12)
Pfig, ax = plt.subplots() marker = itertools.cycle((v', '"', '<', '>', '*'))
pressmark = ('o','D','s') cuts = ['pressure == 0.53', '.9<pressure<1', 'pressure == 1.39'] pressures = [0.53, 1.39]

for cut, pressure, mark in zip(cuts, pressures, pressmark):
exx = meop.query(cut+'& (waveplate==45 | waveplate==45)')['P3T1']
eye = meop.query(cut+'& (waveplate==45 | waveplate==45)')['P_N']
plot(exx[eye>0], eye[eye>0], mark, label=r'$\sigma^+$ '+laserlevel)
plot(exx[eye<0], -eye[eye<0], mark, label=r'$\sigma^-$ '+laserlevel)

# for current, mark in zip(sorted(meop['Laser_I'].unique()), marker):
#  lowpow = meop.query('Laser_I==@current')['Laser_W'].values.min()
#  laserlevel = str(lowpow)+ 'W'
#  exx = meop.query('Laser_I==@current & (waveplate==45 | waveplate==45)')['P3T1']
#  eye = meop.query('Laser_I==@current & (waveplate==45 | waveplate==45)')['P_N']
#  plot(exx[eye>0], eye[eye>0], mark, label=r'$\sigma^+$ '+laserlevel)
#  plot(exx[eye<0], -eye[eye<0], mark, label=r'$\sigma^-$ '+laserlevel)

xlabel(r'P3/T1 $\text{mbar s}^{-1}$')
ylabel(r'\text{P}_N$ (%)')
title("Ultimate Polarization")

printhandle = []
printlabel = []
for i in range(len(handles)-1):
  if i<3:
    printhandle.append(handles[i])
    printhandle.append(labels[i])
  elif i==2:
    printhandle.append((handles[i], handles[i+1]))
    printhandle.append((labels[i], labels[i+1]))
    printhandle.append((labels[i].replace('+', '{+/-}'), labels[i+1].replace('-', '{+/-}'))

print(printlabel)

# print(printlabel)
plt.legend(printhandle, printhandle, loc='upper left', prop={'size':11},
  bbox_to_anchor=(1,1), scatterpoints=1, numpoints=1,
  handler_map={tuple: HandlerTuple(ndivide=None)})
plt.tight_layout(pad=0)
if latex:
  plt.savefig('pictures/UltimateP.pdf',bbox_inches='tight',pad_inches=0.05)

205
In [13]:
meop['waveplate'].unique()

Out[13]:

In [14]:
meop.nlargest(2, 'P3T1')

Out[14]:

<table>
<thead>
<tr>
<th>Background 1/4</th>
<th>Background 3/4</th>
<th>Ge_steady</th>
<th>Laser_J</th>
<th>Laser_W</th>
<th>P_N</th>
<th>f_p</th>
<th>logfile</th>
<th>plasma_T1</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 -0.036854</td>
<td>-0.039054</td>
<td>0.095028</td>
<td>4.7</td>
<td>7.20</td>
<td>-0.633949</td>
<td>8.49280</td>
<td>data/20180310T211225_-FPcontrol-1.39mbar=4.7A=-...</td>
<td>1.000000</td>
</tr>
<tr>
<td>58 -0.035530</td>
<td>-0.036849</td>
<td>0.038779</td>
<td>1.0</td>
<td>2.39</td>
<td>-0.731873</td>
<td>7.82899</td>
<td>data/20180311T212442_-FPcontrol-0.946mbar=1A-2...</td>
<td>2.115349</td>
</tr>
</tbody>
</table>
In [53]:
latex = latexplots(width=3.5)
fig,ax = plt.subplots()
scheme = meop.query('45-abs(waveplate)<3 & .8<pressure')
colorcode = plt.get_cmap('viridis')
colorvec = scheme.Laser_W.values
scat = plt.scatter(scheme.t_pump,np.abs(scheme.P_N), c=colorvec, cmap = colorcode, s=16)
# title("Pumping time vs Laser power")
plt.xlabel('Pumping time (s)')
plt.ylabel('Equilibrium P$_\mathrm{N}$ %')
# cbar = plt.colorbar([0,2,4,6,8],scat)
# cbar.ax.set_yticklabels(['< -1', '0', '> 1'])

from mpl_toolkits.axes_grid1 import make_axes_locatable
divider = make_axes_locatable(ax)
cax = divider.append_axes('right', size='5%', pad=0.05)

# im = ax.imshow(data, cmap='bone')
cbar = fig.colorbar(scat, cax=cax, orientation='vertical')
cbar.set_label('Laser Power (W)')
# plt.show()

if latex:
    filename = 'pumping_time_scatter'
    for suff in extensions:
        if suff == '.png':
            dpi = 200
        else:
            dpi = 100
    pl.savefig(impath+'/'+filename+suff,
               bbox_inches='tight',
               pad_inches=0.05,
               dpi=dpi)

fig size:[3.5, 2.163118960624632]

Largest observed nuclear polarizations:
In [22]:
meop.nlargest(4, 'P_N')[[P_N, 'pressure', 'Laser_W', 'plasma_T1', 't_pump']]

Out[22]:

<table>
<thead>
<tr>
<th>P_N</th>
<th>pressure</th>
<th>Laser_W</th>
<th>plasma_T1</th>
<th>t_pump</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>82.912476</td>
<td>0.944</td>
<td>3.95</td>
<td>201.393157</td>
</tr>
<tr>
<td>79</td>
<td>81.053965</td>
<td>0.530</td>
<td>7.19</td>
<td>109.671668</td>
</tr>
<tr>
<td>52</td>
<td>80.944673</td>
<td>0.946</td>
<td>2.39</td>
<td>181.802179</td>
</tr>
<tr>
<td>45</td>
<td>80.426232</td>
<td>0.944</td>
<td>3.95</td>
<td>156.821821</td>
</tr>
</tbody>
</table>

Worst observed value:

In [59]:
worstfile = meop.query('waveplate==45').nlargest(1, 'P_N')['logfile'].values[0]
otherworst = meop.query('waveplate==45').nsmallest(1, 'P_N')['logfile'].values[0]

In [61]:
meop.query('(waveplate==45 | waveplate==-45) & abs(P_N)<70')

Out[61]:

<table>
<thead>
<tr>
<th>Background</th>
<th>Background</th>
<th>Ge_steady</th>
<th>Laser_I</th>
<th>Laser_W</th>
<th>P_N</th>
<th>f_p</th>
<th>logfile</th>
<th>plasma_T1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.042881</td>
<td>-0.041916</td>
<td>0.007137</td>
<td>0.0</td>
<td>1.57</td>
<td>-65.329203</td>
<td>data/20180310T162149_-FFcontrol-1,39mbar-0A-1,...</td>
<td>176.84481</td>
</tr>
<tr>
<td>6</td>
<td>-0.042964</td>
<td>-0.042470</td>
<td>0.006719</td>
<td>0.0</td>
<td>1.56</td>
<td>68.066423</td>
<td>data/20180310T170808_-FFcontrol-1,39mbar-0A-1,...</td>
<td>168.56206</td>
</tr>
<tr>
<td>27</td>
<td>-0.012006</td>
<td>-0.010839</td>
<td>0.022153</td>
<td>4.7</td>
<td>7.20</td>
<td>-69.038235</td>
<td>data/20180310T221931_-recal-FFcontrol-1,39mbar...</td>
<td>177.14009</td>
</tr>
</tbody>
</table>

In [44]:
worstfile

Out[44]:
'data/20180310T162149_-FPcontrol-1,39mbar-0A-1,57W-QWP-062.JSON'
In [55]:

photoData, control, simpleActiveState, config_params = load_then_split_datafile(worstfile)
photoData['t_s'] = photoData['t']/1e3

f_p = config_params['PressureFactor']
theta = config_params['poltheta']*np.pi/180

stringy = worstfile.split('-
waveplate = float(stringy[-1].replace('.JSON',''))-17
if waveplate>90:
    waveplate=360

sample_laser_power = float(stringy[-3].replace(',','').replace('W',''))
sample_laser_current = float(stringy[-4].replace(',','').replace('A',''))
sample_pressure = float(stringy[-5].replace(',','').replace('mbar',''))

**Photodiode data, rough calculated P**
In [56]:

ax = photoData.plot.scatter(x='t_s', y='I1V', s=5)
photoData.plot.scatter(x='t_s', y='I3V', ax=ax, color='r', s=5)
# plt.fig()
photoData.query('abs(P)<1').plot.scatter(x='t_s', y='P', s=5)
photoData.plot.scatter(x='t_s', y='GeV', s=5)
weak_exx = photoData['t_s'].values
weak_eye = photoData['GeV'].values
In [62]:

bestweak = meop.query('Laser_I==0').nlargest(1,'P_N')['logfile'].values[0]
photoData,control,simpleActiveState,config_params = load_then_split_datafile(bestweak)
photoData['t_s'] = photoData['t']/1e3

# f_p = config_params['PressureFactor']
# theta = config_params['poltheta']*np.pi/180

# stringy = bestweak.split('-')
# waveplate = float(stringy[-1].replace('.JSON',''))-17
# if waveplate>90:
#     waveplate-=360

# sample_laser_power = float(stringy[-3].replace(',','.').replace('W',''))
# sample_laser_current = float(stringy[-4].replace(',','.').replace('A',''))
# sample_pressure = float(stringy[-5].replace(',','.').replace('mbar',''))

# ax = photoData.plot.scatter(x='t_s',y='I1V',s=5)
# photoData.plot.scatter(x='t_s',y='I3V',ax=ax,color='r', s=5)
# # plt.fig()
# photoData.query('abs(P)<1').plot.scatter(x='t_s',y='P',s=5)
fig,ax = plt.subplots()
ax.plot(weak_exx,weak_eye,'.r',ms=4)
photoData.plot.scatter(x='t_s',y='GeV',s=5,ax=ax)

Out[62]:
<matplotlib.axes._subplots.AxesSubplot at 0x1a1d881b70>

Out[63]:
0.007086044581549501

Background measurement
In [28]:

start, end = activetime(control, 'coasting')[0]
flatslice = photoData.query('@start<t<@end')
I1vec = flatslice['I1V']
I3vec = flatslice['I3V']
I1bg, I1bgsigma = norm.fit(I1vec[I1vec<0])
I3bg, I3bgsigma = norm.fit(I3vec[I3vec<0])

x = np.linspace(-1, 0, 300)
I1_PDF = norm.pdf(x, loc=I1bg, scale=I1bgsigma)
I3_PDF = norm.pdf(x, loc=I3bg, scale=I3bgsigma)

# latex = latexplots(width=15, height=10, fontsize=12)
# plt.axvline(0, color='k')

binwidth = .001
plt.plot(x, I1_PDF, '-', color=brand['ReynoldsRed'])
plt.hist(I1vec, bins=np.arange(min(I1vec), max(I1vec) + binwidth, binwidth),
        density=True, color=brand['InnovationBlue'], label='1/4 background')

plt.plot(x, I3_PDF, '-', color=brand['ReynoldsRed'])
plt.hist(I3vec, bins=np.arange(min(I3vec), max(I3vec) + binwidth, binwidth),
        density=True, color=brand['GenomicGreen'], label='3/4 background')

ax1 = plt.gca()
# if np.abs(Pmean)<.3:
#     ax1.set_xlim([-0.06, -0.03])
# else:
#     ax1.set_xlim([0, np.sign(Pmean)*1])
ax1.set_xlabel(r'$I_{1,3}$ background')
_ = ax1.get_yaxis().set_ticks([])
_ = plt.legend()
# ylims = ax1.get_ylim()
Pump Rate Calculation
In [29]:
start,end = activetime(control,'pumping')[-1]
pumpslice = photoData.query('@start<t<@end')
bergainsteady = pumpslice["GeV"].tail(150).mean()
Ge_steady = pumpslice[\'GeV\'\].tail(150).mean()


crossing = (pumpslice.query('GeV<@bergainsteady')[\'t\'].values[0]-start)/1e3
stopftime = start+crossing*1e3
pump_time.append(crossing)

# popt, pcov = pumpfit(np.array(pumpslice[\'t\'])-offsetms,np.array(pumpslice[\'P\']))
# estpol, pumprate[plotroot] = popt

pumping = photoData.query('@start<t<@stopftime')
ax = pumpslice.plot.scatter(x='t_s',y='GeV',color='g')
pumpslice.tail(150).plot.scatter(x='t_s',y='GeV',color='r',ax=ax)
pumping.plot.scatter(x='t_s',y='GeV',color='k',ax=ax)

Out[29]:
<matplotlib.axes._subplots.AxesSubplot at 0x1a1c565748>

Steady State Polarization measurement
In [ ]:

```
steady_start = end - 100e3
flatslice = photoData.query('@steady_start<t<@end')
I1vec = flatslice['I1V']-I1bg
I3vec = flatslice['I3V']-I3bg

I1mean, I1sigma = norm.fit(I1vec)
I3mean, I3sigma = norm.fit(I3vec)
I1error = I1sigma/np.sqrt(I1vec.shape[0])
I3error = I3sigma/np.sqrt(I3vec.shape[0])

x = np.linspace(.5, 7, 200)
I1_PDF = norm.pdf(x, loc=I1mean, scale=I1sigma)
I3_PDF = norm.pdf(x, loc=I3mean, scale=I3sigma)

# latex = latexplots(width=15, height=10, fontsize=12)
# plt.axvline(0, color='k')

binwidth=.005
plt.plot(x, I1_PDF, '-', color=brand['ReynoldsRed'])
plt.hist(I1vec, bins=np.arange(min(I1vec), max(I1vec) + binwidth, binwidth),
        density=True, color=brand['InnovationBlue'])
plt.plot(x, I3_PDF, '-', color=brand['ReynoldsRed'])
plt.hist(I3vec, bins=np.arange(min(I3vec), max(I3vec) + binwidth, binwidth),
        density=True, color=brand['GenomicGreen'])

ax1 = plt.gca()
# if np.abs(Pmean)<.3:
#     ax1.set_xlim([-0.5,.5])
# else:
    # ax1.set_xlim([0,np.sign(Pmean)*1])
ax1.set_xlim([.5, 7])
ax1.set_xlabel(r'$I_{1,3}$')
ax1.get_yaxis().set_ticks([])
ylims = ax1.get_ylim()

\[
\sigma_p = \frac{2f_p}{\cos \theta_m (I_{3/4}+I_{1/4})^2} \sqrt{I_{3/4}^2 \sigma_{3/4}^2 + I_{1/4}^2 \sigma_{1/4}^2}
\]

In [ ]:

```
P = f_p/np.cos(theta)*(I1mean-I3mean)/(I1mean+I3mean)*100
sigma_p = (2*f_p/(np.cos(theta)*(I3mean+I1mean)**2)*
    np.sqrt((I1mean*I1error)**2+(I3mean*I3error)**2))*100
polarization.append(P)
polarization_error.append(sigma_p)

```
Plasma T1 Decay Calculation

```
In [ ]:

```python
start, end = activetime(control, 'decaying')[-1]
decayslice = photoData.query('@start<t<@end')
popt, pcov = decayfit(decayslice['t'].values, decayslice['P'].values)
plasma_T1.append(popt[1])
exx = (np.linspace(start, end, 200) - start) / 1e3
ax = decayslice.plot.scatter(x='t_s', y='P')
ax.plot(exx + start / 1e3, decayfunction(exx * 1e3, *popt), color='r')
print(popt)
```
E.5 LabVIEW Documentation

E.5.1 Calibration Discharge LabVIEW front panel

![Diagram of StartCalibrationDischarge.vi](image)

**Figure E.2** Front panel of StartCalibrationDischarge.vi, located at C:\Users\Bob\Documents\LabView\AustinCode.
P (torr) 0

Pressure Factor 0
X Pressure Factor 1
Total Pressure Factor 0

P

P unit torr

Error In
source

status code

0

Error Out
source

status code

0

float Z,n,v,s1,s2,s0,a,G,Gp,g1,g2,num,denom,P;

Z = 0.748;
n = p*3.22e22*1e-6;
v = 2.03e3*1e2;
s1 = 148e-16;
s2 = 198e-16;
s0 = 97e-16;
a = (2/5)*8.74e8;
G = 6.58e7;
Gp = G+n*s0*v;
g1 = n*s1*v;
g2 = n*s2*v;

num = Z*3*a**2*(Gp+g2)/2;
denom = (Gp+g2)*(Gp+g1)**2+6*a**2*(Gp+g1)*(Gp+2*g2/3)/Gp+a**2*(Gp-g1+2*g2)/4;
P = denom/num;

P = 1.333223684211

P (torr) 1.333223684211 "mbar"
Supplied System Pressure out of range

float Z,n,v,s1,s2,s0,a,G,Gp,g1,g2,num,denom,P;
Z = 0.748;
n = p*3.22e22*1e-6;
v = 2.03e3*1e2;
s1 = 148e-16;
s2 = 198e-16;
s0 = 97e-16;
a = (2/5)*8.74e8;
G = 6.58e7;
Gp = G+n*s0*v;
g1 = n*s1*v;
g2 = n*s2*v;
um = Z*3*a**2*(Gp+g2)/2;
denom =(Gp+g2)*(Gp+g1)**2+6*a**2*(Gp+g1)*(Gp+2*g2/3)/Gp+a**2*(Gp-g1+2*g2)/4;
P = denom/num;
P
E.5.3 Fast LCR Calibration LabVIEW front panel

Figure E.3 Front panel of Fast-Calibrate.vi, located at C:\Users\Bob\Documents\LabView\AustinCode. Note that neither $V_{\text{min}}$ nor $V_{\text{max}}$ is located at the edge of their range.
E.5.5 MonitorPolarization LabVIEW front panel

E.5.5.1 Raw Response tab

Figure E.4 Front panel of MonitorPolarization.vi showing the data directly from the LCR polarimeter. VI is located at C:\Users\Bob\Documents\LabView\Homebrew. Initial discharge was too intense, so Discharge Amplitude was reduced until both channels were well within range (before 34623...). Magnetic field was switched on shortly before 34624...
Figure E.5 Front panel of MonitorPolarization.vi showing the calculated electron polarization. VI is located at C:\Users\Bob\Documents\LabView\Homebrew. Pumping occurs between 525 s to 625 s.
E.5.5.3 Nuclear Polarization tab

Figure E.6 Front panel of MonitorPolarization.vi showing the calculated nuclear polarization. VI is located at C:\Users\Bob\Documents\LabView\Homebrew. Initially, this panel only bins the trailing 100 data points (50 s), but once the system has equilibrated, the user may select “accumulate” to start binning all new data, improving the quality of the measured $P_N$. 

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