*Transactions*, SMiRT-24
BEXCO, Busan, Korea - August 20-25, 2017
Division II

# DEVELOPMENT AND VALIDATION OF AN ANALYTICAL SCHEME FOR THE FRACTURE MECHANICS ASSESSMENT OF PIPING SYSTEMS SUBMITTED TO IMPOSED DISPLACEMENTS

**Stéphane Chapuliot[1], Edith Marquès Vieira[1], Philippe Gilles[2]**

[1] Research & Development Engineer, Mechanics & Material, AREVA-NP, France
[2] Consultant, GEP-INT(SA), France

## ABSTRACT

This paper deals with the developments performed by AREVA for the definition of an analytical methodology for the evaluation of secondary bending moment relaxation in piping systems.
After a first presentation of the problematic and the background of the methodology, the paper presents the already existing approach codified in the A16 appendix of the RCC-MRx and the specific improvements performed for the consideration of load history effects.
The pertinence of the approach is finally shown through the comparison of analytical applications to reference Finite Element Modelling solutions specifically developed for validation purpose.

## 1 INTRODUCTION

Piping systems are submitted to a complex set of primary and secondary loadings which have to be combined together within Fracture Mechanics Assessments (FMA) methodologies in order to justify, at design level, the circumferential welds of piping systems. This set of complex loadings, which are due to seismic loading, thermal expansion or stratification, pipe break… are generally determined independently through elastic modelling of the loop, and then cumulated together through accumulation rules which insure the overall conservatism of the demonstration.

Based on those accumulation rules, the conventional practice for FMA is to directly impose those loadings to the cracked section: all loading are then considered as primary loadings. However, for situations where the secondary part of the loading is dominant, this practice could be over-conservative since the elastically determined secondary loading could drastically over-estimate the real loading imposed to the piping system. In such situation, it is important to take into account the relaxation of the secondary loadings due to plasticity in the piping system.

The purpose of this paper is to present the numerical developments and methodology improvements performed by AREVA-NP in order to define an improved analytical scheme for the assessment of secondary loadings. This work focuses on simple piping systems where no elastic follow up is expected. This development relies on:

- The Finite Element Modelling (FEM), with or without cracks, developed in order to provide reference solutions for the analytical scheme development and validation;
- The proposition of the analytical scheme, specifically defined here for small surface cracks in simples piping systems;
- The validation of this proposition through a comparison to FEM reference solutions.

A detailed description of the problematic is firstly given in the paper. Then, the three main steps of the work are described. Finally, the befit of this approach is illustrated through the example of a pipe submitted to different situations of loading.

## 2 GENERAL DESCRIPTION OF THE PROBLEMATIC

### *2.1 Illustration of the problematic*

The FMA performed at design level relies on postulated defects and the assessment of their stability for every possible loading situation encountered during the complete life of the components. Those defects are postulated in locations where the manufacturing process could create defects (e.g. in welds and in casts components) and the assessment is performed comparing the critical defect size (in terms of fracture) to the performance of Non-Destructive Examination (NDE).
For piping systems, the definition of loading situations is complex since it is a mix of normal loading (e.g. pressure, thermal expansion) and abnormal/accidental situations (e.g. seismic load, thermal stratification, pipe break), some of them being statics, others being dynamics, some of them corresponding to imposed load, others to imposed displacements.
For simplification purpose, the stresses and strains associated to all those loading are determined independently through an elastic modelling. An accumulation rule is then necessary in order to combined all those loading components.

Following this loading accumulation, the FMA is generally performed through simplified analytical schemes such as the one provided by the R6 rule [1] or the appendix 5.4 of the RSE-M [2]. In a first simple approach, the load set provided by the accumulation rule is directly imposed to the cracked section, that is to say the complete loading set is considered as a primary loading. But in practice, such approach is over conservative since the contribution in J of secondary loading may be drastically over-estimated. In that case, the consequence is that FMA provides very small critical defect sizes (not in adequacy with expected NDE performance).

The fact that the evaluation of secondary loading through conventional analytical schemes is over-conservative is well known for a long time and described in [3]. For a piping system, a simple way to illustrate the problematic is to consider the case of a pipe submitted to a combined Pressure and bending Moment loading. Internal pressure is a primary loading by nature, but the bending moment can be either an imposed load or an imposed rotation (e.g. thermal expansion). Assuming an elastic-perfectly plastic behaviour, the limit load curve of the pipe can be defined by the plastic potential $\Phi$ in which (see fig. 1):

- If the bending moment is an imposed load, the plastic collapse of the pipe is reached when the elastically determined loading path crosses the limit load curve.
- If the bending moment is an imposed rotation, the elastic-plastic loading path follows the elastic one up to the limit curve, then follow this limit curve while both pressure and imposed rotations continue to increase.
  A relaxation of the bending moment (ratio between the elastically determined bending moment and the elastic-plastic one) is then observed.

$$\Phi(P,M) = \left(\frac{P}{P_0}\right)^2 + \left(\frac{M}{M_0}\right)^2 - 1 = 0$$

On fig. 1, one can see that the reduction of the secondary bending moment can be consequent. As a consequence, it appears evident that an elastic-plastic J determined from the elastic load path is drastically higher than the one determined from the elastic-plastic one.

So finally there are strong potential benefits in evaluating the relaxation of secondary bending moments. But in practice, the problem is not as simple as the one presented here since the encountered material are hardening ones (in particular the stainless steel pipes) and in general cases, a spring effect may appear between the most loaded sections and the elastic parts of the piping systems. In the present paper we are focussing on the problematic of hardening materials: <u>the spring effect is not investigated. This means that,</u>

the methodology described here applies only to simple piping systems in which no spring effect is expected.

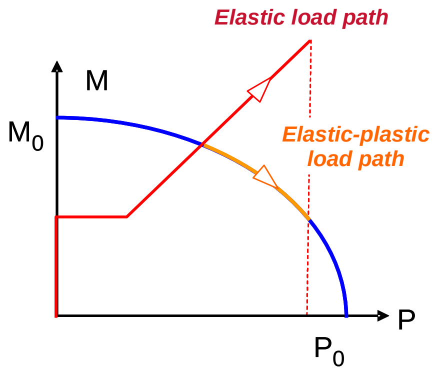



***Fig. 1:*** *Illustration of the secondary bending moment relaxation*

### *2.2 The theoretical background*

The theoretical background of the approach described here relies on limit load analysis, and more generally the Hencky-Mises theory [4] completed by the following Ilyushin theorem [5]:
For structures made from a material following an exponential hardening law $\varepsilon_{eq} \propto \sigma_{eq}^{\ n}$ and submitted to a radial loading path (which can be described by a unique parameter $\Lambda$ increasing monotonically):

- Stress, strain and displacement fields are increasing monotonically following an exponential relation, e.g. for a stress proportional to $\Lambda$ we have:

$$\sigma_{ij} \propto \Lambda \ \rightarrow \varepsilon_{ij} \propto \Lambda^n \ \ \textit{and} \ \ u_i \propto \Lambda^n$$

- Since each point of the structure is following this relation, the fully plastic solution of the nonlinear elastic theory is the exact solution of the problem in the plastic flaw theory.

Following this theorem and assuming a partition between elastic and plastic displacements,

$$q^{tot} = q^e + q^p$$

and for a material behaviour described by the following exponential law,

$$\frac{\varepsilon^p}{\varepsilon_0} = \alpha \cdot \left(\frac{\sigma}{\sigma_0}\right)^n$$

elastic and plastic displacements become:

$$q^e = \varepsilon_0 \cdot C. \frac{Q}{Q_0} \ \ \textit{and} \ \ q^p = \alpha \cdot \varepsilon_0 \cdot C \left(\frac{Q}{Q_0}\right)^n$$

where $Q_0$ is the limit load of the structure and C the elastic compliance. From those relations, the ratio between elastic and plastic displacements becomes:

$$\frac{q^p}{q^e} = \alpha \cdot \left(\frac{Q}{Q_0}\right)^{n-1}, \textit{or} \ \frac{q^p}{q^e} = \frac{\varepsilon^p(\sigma_R)}{\varepsilon^e(\sigma_R)} \ \textit{with} \ \sigma_R = \sigma_0 . \frac{Q}{Q_0}$$

$\sigma_R$ represents here a stress representative of the overall behaviour of the structure (commonly called the reference stress). From that definition, it finally appears that this ratio can be expressed as a ratio between an elastic-plastic strain and an elastic one. This relation can be generalized to every hardening law with the relation:

$$q^{tot} = q^p + q^e = q^e . \frac{\varepsilon^{tot}(\sigma_R)}{\varepsilon^e(\sigma_R)} \textit{ or } \varepsilon^{tot}(\sigma_R) = \frac{\sigma_R}{E} + g(\sigma_R),$$

where $g(\sigma_R)$ represents the stress-plastic strain law of the material.

### *2.3 The A16 appendix of RCC-MRx formulation*

The formulation provided in A16 appendix of the RCC-MRx [6] is directly following this analytical scheme, but adapted for the consideration of large through-wall defects in pipes (the A16 appendix is dedicated to FMA and LBB). For that purpose, the previous relation becomes:

$$q^{tot} = \left[ C_{no} . \frac{\varepsilon^{tot}(\sigma_{no\text{-}R})}{\varepsilon^e(\sigma_{no\text{-}R})} + C_{def} . \frac{\varepsilon^{tot}(\sigma_{def\text{-}R})}{\varepsilon^e(\sigma_{def\text{-}R})} \right] . Q ,$$

where *no* and *def* are respectively describing the nominal part of the pipe (part of the pipe without crack) and the cracked section. For a small surface defect, we simply have:

$$C_{def} << C_{no} \textit{ and } \sigma_{def-R} \approx \sigma_{no-R} \textit{ then: } q^{tot} = C_{no} . \frac{\varepsilon^{tot}(\sigma_{no\text{-}R})}{\varepsilon^e(\sigma_{no\text{-}R})} . Q = q^e . \frac{\varepsilon^{tot}(\sigma_{no\text{-}R})}{\varepsilon^e(\sigma_{no\text{-}R})}$$

which exactly corresponds to the previous formulation.

In pipe sections, the global behaviour is commonly described through a quadratic function $L_R$ describing the global plasticity of the section and expressed in terms of global loads. The reference stress is then:

$$\sigma_R = L_R . \sigma_Y \textit{ and } L_R = \frac{\sigma_{eq}}{\sigma_y} = \Phi ,$$

Following the RSE-M/5.4 appendix [2] notations using non-dimensional loadings:

$$p = \frac{\sqrt{3}}{2} \cdot \frac{P \cdot r_m}{t \cdot \sigma_y} , \ m_1 = \frac{\sqrt{3}}{2} \cdot \frac{M_1}{\pi \cdot r_m^2 \cdot t \cdot \sigma_y} , \ m_2 = \frac{M_2}{4 \cdot r_m^2 \cdot t \cdot \sigma_y} \textit{ and } m_3 = \frac{M_3}{4 \cdot r_m^2 \cdot t \cdot \sigma_y} ,$$

it becomes, for a pipe without crack, the following $L_R$ function:

$$L_R = \sqrt{p^2 + {m_1}^2 + {m_2}^2 + {m_3}^2} = \sqrt{p^2 + {m_{eq}}^2}$$

### *2.4 Application for the determination of the relaxation ratio*

For a fully secondary loading, the displacement field $q^{tot}$ is imposed. The associated elastic and elastic-plastic loadings are thus defined by:

$$q^{tot} = C . Q^{el} \textit{ and } q^{tot} = C . \frac{\varepsilon^{tot}(\sigma_R)}{\varepsilon^e(\sigma_R)} . Q$$

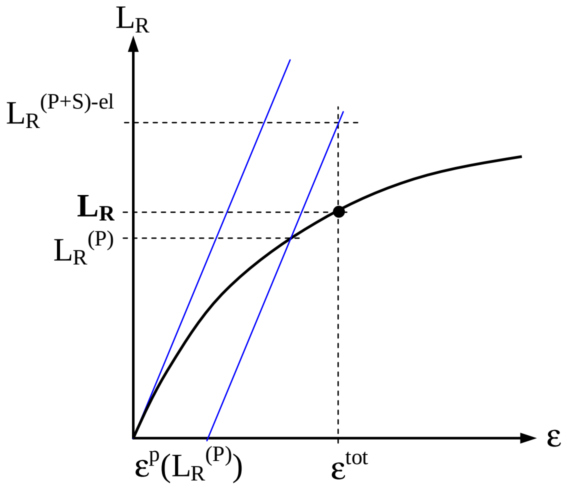
I we define the relaxation ratio $\lambda$ as the ratio between elastic-plastic and elastic loading, since the loading Q is proportional to the $L_R$ function:

$$\lambda = \frac{Q}{Q_e} = \frac{L_R}{L_R^{(S)-el}}$$

Thus, λ simply becomes the solution of the following equation:

$$\frac{L_R}{L_R^{(S)-el}} = \frac{\varepsilon^e(L_R)}{\varepsilon^{tot}(L_R)}$$

where $L_R$ is here a function of λ.



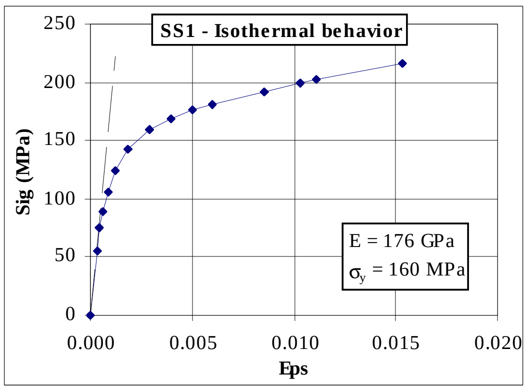




***Fig 2:*** *On left: Representation of the scheme in a stress-total strain diagram*
*On right: Stainless-steel material behaviour at 300°C*

For a more complex situation including primary and secondary loadings, a strong assumption has to be made on the relaxation ratio which has to be considered as the same for all the secondary loading components. For a pipe formulation, the decomposition of the primary and secondary bending and torsional bending moments becomes:

$$m_i^{(P+S)} = m_i^{(P)} + \lambda . m_{i-el}^{(S)} \quad \textit{(i=1, 2 ou 3)}$$

where *el* defined the elastically determined loading, *(P)* its primary part, *(S)* the secondary one and *(P+S)* the combination of both. The $L_R$ function becomes:

$$L_R = \sqrt{p^2 + \sum \left(m_i^{(P)} + \lambda . m_{i\text{-}el}^{(S)}\right)^2}$$

and the relaxation ratio becomes the solution of the equation:

$$\frac{L_R}{L_R^{(P+S)-el}} = \frac{\varepsilon^e(L_R)}{\varepsilon^{tot}(L_R) - \varepsilon^p\left(L_R^{(P)}\right)}$$

where $L_R^{(P)}$ corresponds to the contribution of primary loadings (λ = 0 in the $L_R$ definition), $L_R^{(P+S)-el}$ corresponds to the complete load without any relaxation (λ = 1 in the $L_R$ definition) and $L_R$ corresponds to the parameter taking into account the relaxation. Represented in a *stress-total strain* graph (fig. 2), one should note that this equation is equivalent to the following relation:

$$\varepsilon^{tot}\left(L_R\right)=\varepsilon^{e}\left(L_R^{(P+S)-el}\right)+\varepsilon^{p}\left(L_R^{(P)}\right)$$

In a general case (for realistic materials), the constitutive law of the material is not simple and thus a numerical scheme is needed to solve this equation.
As a complement, this graph illustrates that in such approach the secondary loading is systematically added to the primary loading, which constitutes a source of conservatism since the secondary loading may be discharged totally or partially if the both loadings are acting along the same axis.

## 3 MODELS FOR THE DEVELOPMENT OF REFERENCE SOLUTIONS

### *3.1 Materials under consideration*

Two different material behaviours are used for the analytical developments and FEM presented in this report. Those are described by a Ramberg-Osgood (RO) formulation (plate test-case) or given point by point (see fig. 2). Both of those two types of formulation is corresponding a stainless steel material at warm temperature. The RO material is defined by:

$$\frac{\varepsilon}{\varepsilon_y}=\frac{\sigma}{\sigma_y}+\left(\frac{\sigma}{\sigma_y}\right)^n \text{, with: } \varepsilon_y=\frac{\sigma_y}{E}\text{, } \sigma_y = 100 \text{ MPa, } E = 176.5 \text{ GPa, } n = 6$$

### *3.2 Plate submitted to combined tension and bending*

A simple plate model was firstly used with the objective to validate the protocol for loading application in the following modelling. A 2D plane stress assumption adopted here.
In this configuration, one side of the plate is embedded whether the other side is submitted to a membrane stress (noted $\sigma_m$) combined with an imposed rotation. This rotation is imposed considering that the straight loaded section remains straight during the loading phase. This rotation is quantified by the associated elastic stress (noted $\sigma_{b-el}$ and corresponding to the bending stress associated to the imposed rotation for a linear elastic behaviour).
The main advantage of this problem is that a quasi-analytical solution can be provided. This allows validating the FEM protocol used further for more complex cases.

### *3.3 Straight pipe submitted to combined pressure and bending moment*

The straight pipe model is defined by its internal radius $r_i$ = 368 mm and its thickness t = 70 mm. The pipe length represented by the model is L = 3364 mm.
In the FEM, this pipe is represented by 20 nodes brick element, in 3D, with 4 elements through the thickness. Only a ¼ of the structure is represented thanks to the symmetrical loading configurations.
The straight pipe boundary conditions are similar to those imposed to the plate: one end is embedded in axial and circumferential directions (radial displacements are free) whether the other is submitted to the imposed rotation (bending rotation). Again, that rotation is imposed assuming that the plane section submitted to the loading remains plane. This rotation is quantified by its associated elastic bending $M_{el}$ corresponding to an elastic behaviour.
Additionally to that imposed rotation, the pipe is submitted to an internal pressure and the associated end-effect. And in some loading cases, the bending moment is discomposed in a primary part and a secondary part. In that case, the primary part is quantified by its ratio with the total elastic bending moment.

### *3.4 Loading sequence*

For combined loading configurations (i.e. primary + secondary loadings), two different loading sequences are investigated (see fig. 3)

- Proportional loading where both the primary and the secondary loadings remain proportional;

- Non proportional loading sequence where the primary loading is imposed in a first phase then maintained constant while the secondary loading is imposed.
- When the moment is discomposed in two parts (primary + secondary contributions), only non-radial configurations are considered. In that case, the primary moment is imposed at the same time than the internal pressure (see fig. 3) then the expected rotation is added to the rotation resultant from the imposed primary moment.

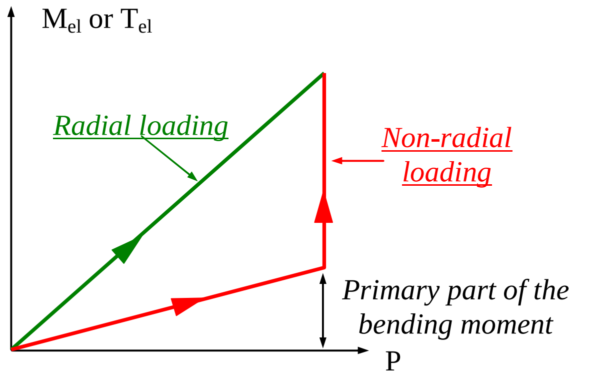



***Fig. 3:*** *Proportional and non-proportional bending moment*

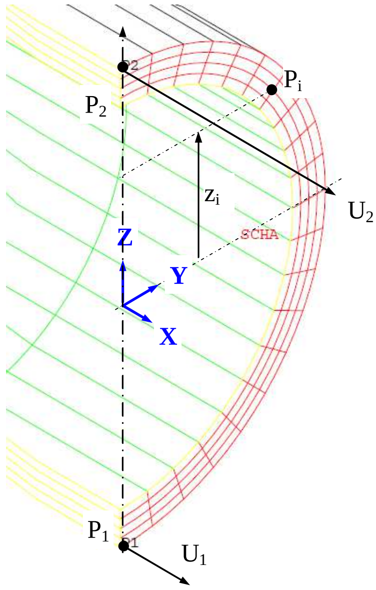



***Fig. 4:*** *Kinematic relation in the loaded section*

Next table 1 gives the details of loading conditions adopted for the FEM results provided in this paper. In this table, the loading is normalized following the RSE-M notations [2] (see §2.3).

| # | $m_{el}$ | p | Radial (Y/N) | %$M_P$ |
|---|---|---|---|---|
| **P1** | 1.37 | 0 | --- | 0 |
| **P2** | 1.37 | 0.75 | N | 0 |
| **P3** | 1.37 | 0.75 | Y | 0 |
| **P5** | 1.37 | 0.75 | N | 50 |

### *3.5 Loading protocol*

The difficulty of the FEM presented here is due to the fact that a load (e.g. end-effects) and a rotation are to be imposed on the same section. Additionally, a kinematic relation (the plane section remains plane) has to be imposed. For that purpose the following procedure is applied:

- The load is imposed through a pressure on the section (red section on fig. 4). For an effort equal to N, this pressure is simply:

$$\sigma_m = \frac{N}{\pi.\left(r_e^{\ 2} - r_i^{\ 2}\right)}$$

- The kinematic relation is imposed through a geometric relation between each node of the section and the two extreme nodes (P1 and P2 in fig. 4). For a given bending load and for each point $P_i$ of the section, the axial displacement is then defined by:

$$U(P_i) = \frac{r_e - z_i}{2.r_e}.U_1 + \frac{r_e + z_i}{2.r_e}.U_2$$

- The rotation is then imposed through the difference between the displacements of the two extreme points:

$$R = \frac{U_2 - U_1}{2.r_e}$$

This procedure allows the pipe to extend with the imposed end effect and with a superposed rotation. When the pipe is submitted to a primary bending moment, this loading is applied through a variable pressure on the loaded section. In that frame, for each point $P_i$ of the section, the imposed stress is defined by:

$$\sigma(P_i) = \frac{z_i.\sigma_{gb}}{r_e}, \textit{ with: } \sigma_{gb} = \frac{\pi.r_e.M}{4.(r_e^2 - r_i^2)}$$

This variable pressure is added to the end-effect. The resultant rotation is determined through the calculation during the first loading phase when only primary loading is applied. The rotation corresponding to the secondary loading is then simply added in the second phase.
Next fig. 5 validates this numerical protocol by comparing the quasi-analytical solution of the plate to the same calculation obtained by FEM. As it can be shown in this figure, the accordance between the F.E. modeling and the analytical solution is good. The slight difference between the two solutions for the case '$\sigma_m$ = 75' is due to the discretization of the section which is different for the two approaches.

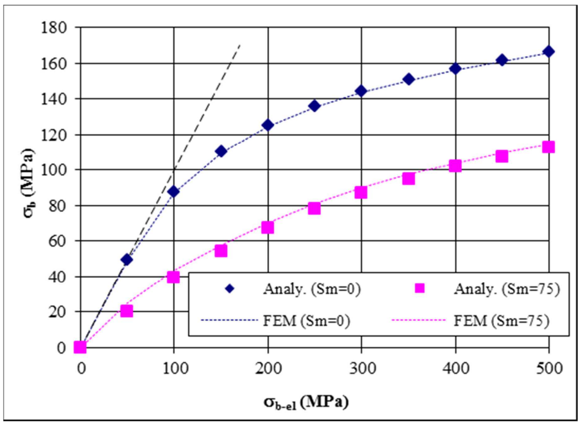



***Fig. 5:*** *Validation of the FEM protocol through comparison to an analytical solution*

### *3.6 Model for J calculation*

For the straight pipe configuration, a cracked model was developed in order to validate the complete G calculation through the analytical scheme. The investigated configurations are different combination of internal pressure and bending. Next fig. 6 gives a representation of the mesh (pipe configuration with an internal circumferential semi-elliptical surface crack – TUB-CDSI in RSE-M notation [2]). The objective of this FEM is to validate that the secondary loading determined through the relaxation modules remains pertinent for the J calculation (in particular in non-proportional configurations).

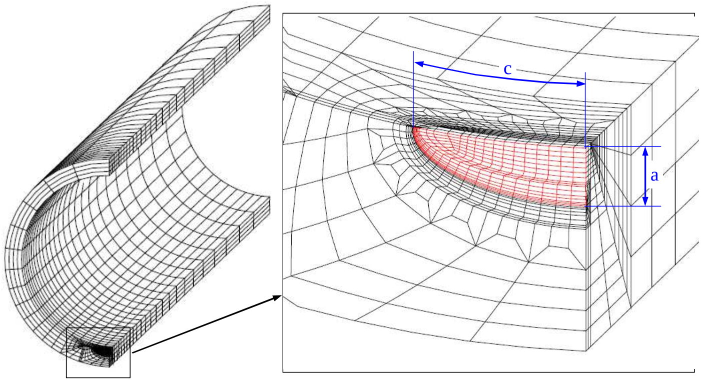



***Fig. 6:*** *Mesh of the cracked pipe with an internal semi-elliptical surface defect (CDSI)*

# 4 VALIDATION OF THE ANALYTICAL SCHEMES

## *4.1 Confrontation of the RCC-MRx formulation to reference solutions*

The fig. 7 compares an application of the RCC-MRx/A16 [6] solution (as described in §2.4) to reference FEM solutions obtained on pipes. In this graph, two configurations are compared:

- #P1 configuration corresponding to a secondary bending moment without pressure;
- #P2 configuration corresponding to a non-proportional combination of internal pressure and secondary bending moment.

In #P1 configuration, the analytical model gives a very good evaluation of the elastic-plastic bending moment. This configuration is exactly within the assumptions of the analytical model and thus an excellent agreement is obtained.
In #P2 configuration, the RCC-MRx/A16 application under-estimates the FEM solution. In that case, the proportionality assumption is not respected, explaining this difference. In a general case, the secondary loading corresponding to thermal expansion is imposed at the same time than internal pressure. This thus does not constitute a difficulty. It is more problematic when additional loading are superposed in a second step (e.g. secondary part of the seismic loading).

## *4.2 Introduction of an history effect within the analytical scheme*

The RCC-MRx/A16 formulation relies explicitly the $L_R$ function to the imposed loading. By the way, it could not take into account any load history effect. The only way to represent such history effect is to define a plastic potential $\Phi$ relying on imposed loading and associated dual variables. For a pipe section, if the model is limited to the internal pressure and the equivalent bending moment ($m_{eq}$ as defined in §2.3), the loading vector becomes:

$$Q = \begin{Bmatrix} p \\ m_{eq} \end{Bmatrix}$$

Dual variables can be defined through $\varepsilon_m$ and $\varepsilon_b$ strains as follows:

$$\varepsilon_m = \frac{P.r_m}{2.E.h} = \frac{\sqrt{3}}{3}\frac{\sigma_y}{E}.p\,,\quad \varepsilon_b = \frac{\sigma_y}{E}.m_{eq}\ \textit{and:}\ \begin{Bmatrix} p \\ m_{eq} \end{Bmatrix} = [D].\begin{Bmatrix} \varepsilon_m \\ \varepsilon_b \end{Bmatrix} = \frac{E}{\sigma_y}.\begin{bmatrix} \sqrt{3} & 0 \\ 0 & 1 \end{bmatrix}.\begin{Bmatrix} \varepsilon_m \\ \varepsilon_b \end{Bmatrix}$$

From that definition, the plastic potential can then be defined as follows:

$$\Phi = p^2 + m_{eq}{}^2 - L_R{}^2(\varepsilon_{pl}) = 0, \ \textit{or} \ \Phi = \begin{Bmatrix} p \\ m_{eq} \end{Bmatrix}^T [I].\begin{Bmatrix} p \\ m_{eq} \end{Bmatrix} - L_R{}^2,$$

where $L_R$ is no more an explicit function of the loading but a function of the cumulated plastic strain $\varepsilon_{pl}$. When plastic flaw appears, it has to follow the normality law. The plastic flaw is thus normal to $\Phi$ and we have:

$$\begin{Bmatrix} d\varepsilon_m \\ d\varepsilon_b \end{Bmatrix}_{pl} = d\zeta.\frac{\partial F}{\partial \begin{Bmatrix} p \\ m_{eq} \end{Bmatrix}} = 2.d\zeta.\begin{Bmatrix} p \\ m_{eq} \end{Bmatrix},$$

where $d\zeta$ is a scalar. In addition, the plastic work is relied to the cumulated plastic strain $\varepsilon_{pl}$ and we have:

$$\begin{Bmatrix} p \\ m_{eq} \end{Bmatrix}^T . \begin{Bmatrix} d\varepsilon_m \\ d\varepsilon_b \end{Bmatrix}_{pl} = L_R.d\varepsilon_{pl} \rightarrow 2.d\zeta.\begin{Bmatrix} p \\ m_{eq} \end{Bmatrix}^T . \begin{Bmatrix} p \\ m_{eq} \end{Bmatrix} = 2.d\zeta.L_R{}^2 = L_R.d\varepsilon_{pl} \rightarrow d\varepsilon_{pl} = 2.d\zeta.L_R$$

Finally, after the increment the potential must be respected and thus:

$$\Phi = (p + dp)^2 + (m_{eq} + dm_{eq})^2 - L_R{}^2(\varepsilon_{pl} + d\varepsilon_{pl}) = 0 \ \textit{with :} \ \begin{Bmatrix} dp \\ dm_{eq} \end{Bmatrix} = [D].\left( \begin{Bmatrix} d\varepsilon_m \\ d\varepsilon_b \end{Bmatrix} - \begin{Bmatrix} d\varepsilon_m \\ d\varepsilon_b \end{Bmatrix}_{pl} \right)$$

With such formulation, the resolution becomes incremental which means that the loadin has to be discretized in a sufficient number of steps. The application of this model is represented on fig. 7 (red points denoted *non-prop*). As it is shown this simple modification allows determining an accurate value of the bending moment, demonstrating again the pertinence of the proposed model relaying on global loadings.

### *4.3 Validations of the J calculation*

Next fig. 8 gives the evolution of the J parameter for a secondary bending moment without pressure. The first result illustrated by this figure is the large difference in terms of J between a primary and a secondary loading, one inducing a sharp amplification of the elastic J when approaching to the limit load, the second a slight reduction: at the end of the secondary loading, for $m_{el}$ larger than 1, the elastic plastic J is about 15 kJ/m², showing the strong interest to account for relaxation of secondary bending moment.
In this fig. 8 and the following figures, the Js solution corresponds to the RSE-M [2] analytical scheme (CLC option) applied with the relaxed bending moment (determined by FEM without crack): the objective is here to show the pertinence of this bending moment.

The same comparison is provided on fig. 9 for a secondary bending moment combined with internal pressure (with a max pressure of 24 MPa). On that figure, red curves are corresponding to the non-radial configuration (pressure is imposed at first then the rotation) whether the blue curve corresponds to the radial configuration (imposed pressure and rotation increase simultaneously).
For the proportional configuration (#P3), the RSE-M analytical scheme [2] provides again a very good evaluation of the J parameter. For that configuration, it can also be noticed that for the final loading, the J value is around 25 kJ/m² when the bending moment is considered as secondary loading (#P3) whether it is larger than 1200 kJ/m² when the bending moment is considered as primary loading! This again illustrates the strong benefit obtained when considering secondary loading.

For the non-radial configuration, this figure illustrates that the RSE-M/5.4 analytical scheme [2] over-estimates the J parameter determined through the FEM. This can be relied to the fact that, for a complex load history and for an elastic-plastic behaviour, the link between the local stresses imposed to the defect and the global loading is lost due to plastic flow in the structure.

When comparing the radial and non-radial configurations, this figure illustrates that the load history has also a significant effect on the J parameter (in coherence with the effect observed on the loading).

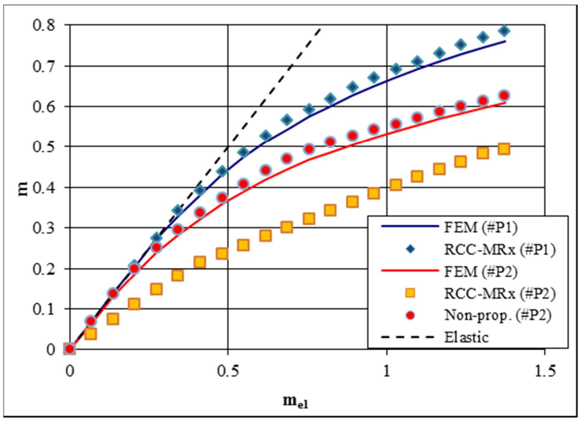



***Fig. 7:*** *Elastic-plastic bending moment calculation*

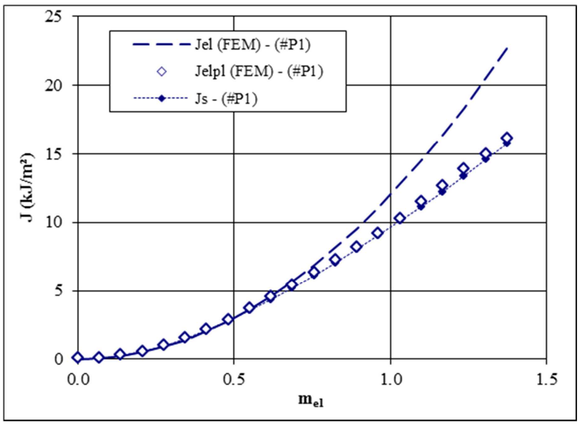



***Fig. 8:*** *J calculation for the secondary bending moment without pressure*

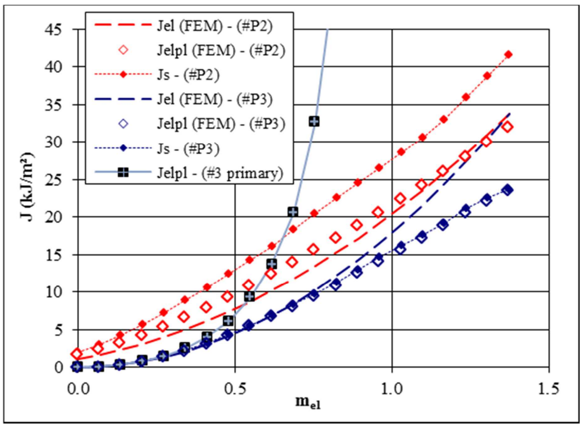



***Fig. 9:*** *J calculation for the secondary bending moment cumulated with pressure (radial and non-radial configurations)*

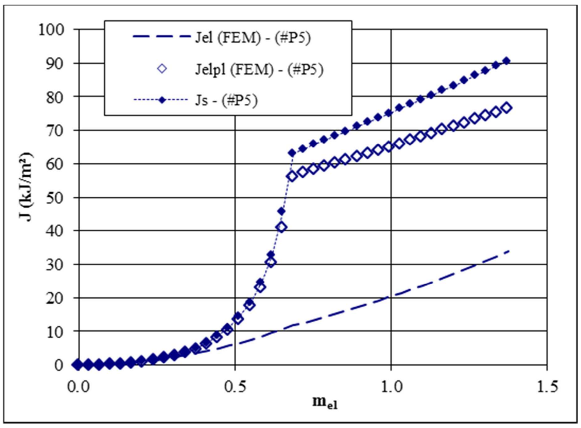



***Fig. 10:*** *J calculation for 50% primary bending moment cumulated with pressure (non-radial configurations)*

## 5 SYNTHESIS AND CONCLUSIONS

This paper presents the developments performed with AREVA for the development and the validation of an analytical scheme for the evaluation of the relaxation of secondary bending moments due to plasticity.

After a presentation of the problematic and the background of the approach, the paper shows that:

- This kind of approach is able to provide an accurate solution for simple lines where no spring effect is expected. This means that its application has to be limited to simple piping systems;
- For non-proportional loading, an improvement to the RCC-MRx/A16 methodology [6] is proposed in order to take into account the load-history effect. This improvement provides very good results in terms of relaxation;

- An illustration of the pertinence of this relaxed bending moment is provided through a comparison of the J parameter determined directly from FEM and the one determined from the RSE-M analytical scheme.
  In addition to the pertinence, this last comparison show the large benefit on the J parameter when considering the relaxation in comparison to a fully primary classification.

## 6 REFERENCES

[1] R6 – 'Assessment of the integrity of structures containing defects', Revision 4 April 2001. British Energy.
[2] 5.4 appendix of the RSE-M, 2010 edition
[3] W.S. Blackburn, Assessment of cracks under secondary and combined loading, *Int. J. Pres. Ves. & Piping 26, 213-260 (1986)*
[4] H. Hencky, Zur Theorie plästicher Deformationen und der Hierdurch im Material hervorgerufenen Nachspannungen. Z. Angnew, *Math. Mech. 4, 323-334 (1924)*
[5] A A. Illyushin, The theory of small elastic-plastic deformations, *Prikadnaja Matematika I Mekhanika, PMM Vol. 10, p. 347 (1946)*
[6] A16 appendix of the RCC-MR, 2010 edition