ABSTRACT

BANERJEE, ABHISEK. High Temperature Heat Extraction from Heat Recirculating Porous Burners (Under the direction of Dr. Alexei Saveliev).

Decreasing fossil fuel reserves and increasing demand for energy has driven researchers to explore alternative pathways for energy generation. This has led to the development of various portable energy generating systems based on Stirling engines and thermoelectric generators. Advantages like instant recharge, no moving parts and high efficiency of power generation have led thermoelectric modules to be used in space applications and remote off-grid power generators. Thermoelectric devices require high-temperature heat to operate efficiently. In the last few decades, researchers have been working to combine various types of combustion devices with thermoelectric modules. Considering the advantages of porous medium combustion, integrating these power generators with porous medium burners would lead to efficient energy generating systems. Even though people have reported studies towards heat extraction from porous medium combustors, they are mostly limited to low temperature heat extraction. This research investigates high-temperature heat extraction from porous medium burners. This would lead to integration of porous medium combustors with thermoelectric modules.

This research work investigates excess enthalpy burners for optimum position of heat exchangers to extract energy at high temperature efficiently. 2-D numerical studies are conducted on counterflow burner, reciprocal flow burner and a combination of counterflow and reciprocal flow burner named as reciprocal counterflow burner. Steady-state governing equations are solved to predict the thermal properties of the systems during the oxidation of reactants. The burners are operated with lean mixtures of methane and air in laminar flow regimes. Flame stabilization is studied inside the burner for the cases with and without high-temperature heat extraction. The heat extraction temperatures in the range from 300 K to 1300 K are studied. The impact of heat
recirculation inside the burner is investigated on the high-temperature heat extraction. It has been found that higher heat extraction temperatures diminish the maximum flame temperatures, and reduce the heat extraction efficiency. The effect of flame location on the heat extraction efficiency is discussed in this thesis. The numerical simulation predicted greater high-temperature heat extraction efficiencies for reciprocal counterflow porous burner than counterflow burner by a factor of nearly 15%. Furthermore, this thesis presents NO\textsubscript{x} emission characteristics of counterflow burner and reciprocal counterflow burner. The numerical simulation indicates a decrease in the NO\textsubscript{x} generation with an increase in heat extraction temperature.
High Temperature Heat Extraction from Heat Recirculating Porous Burner

by
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DEDICATION

This dissertation is dedicated to my parents and grandparents for their care and support.
BIOGRAPHY

The author was born in West Bengal, India, a state known for its food and festivities. He went to Kendriya Vidyalaya in many cities across the country. He earned his bachelor in mechanical engineering from Kalyani Government Engineering College in West Bengal. After that, he pursued his masters in the same field from Indian Institute of Technology (I.I.T.) Guwahati. Finally, he joined North Carolina State University to continue his doctoral research in the field of mechanical engineering. He worked under the guidance of Dr. Alexei Saveliev in the domain of combustion.

Outside academics, Abhisek enjoys trying various cuisines, playing and travelling.
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CHAPTER 1

Introduction

Present century is witnessing increasing demand for energy for applications in industries, transportation and to improve quality of life. With limited global reserves for fossil fuels, researchers are working to develop alternative means to generate electricity. This perspective explains the recent developments for portable energy generating systems. Portable energy devices based on thermoelectric generators and Sterling engines rely on high-temperature energy sources working at higher efficiencies. Thermoelectric generators comprise of a set of thermoelectric modules inserted between two heat exchangers. Each thermoelectric module is made of several pairs of thermoelectric couples connected together electrically in series and thermally in parallel. This arrangement converts a part of the thermal energy that passes through them into electricity [1].

1.1. Portable Energy Generation and Porous Medium Combustion

Hydrocarbon fuels have high energy density and are preferred for high-temperature operation. However, the low efficiency of high-temperature systems based on conventional burners poses a great challenge. Additionally, the conventional burners are known to release considerable pollutants as their exhaust. Based on these operational issues, researchers have developed porous medium combustion systems to challenge the threats posed by the conventional burners. Porous medium combustion (PMC) has emerged as a feasible option to deal with the aforementioned problems. PMC technology has certain advantages over homogenous combustion as mentioned below:
(i) The highly developed inner surface of the porous medium results in efficient heat transfer between the fluid and the solid phase.

(ii) Higher heat transport properties of the solid matrix result in higher flame velocities.

(iii) Dispersion of the reactants flowing through porous matrix results in better mixing, enhances effective diffusion and heat transfer between two phases.

(iv) Enlargement of the reaction zone increases or decreases the maximum flame temperature based on the fuel equivalence ratio.

(v) The high heat capacity of the porous medium results in stable combustion at changing thermal loads and equivalence ratios.

(vi) Pollutant emissions are reduced.

Filtration combustion is known to extend the flammability limits of the fuel-air mixture [2-4]. The heat recuperation from the flame zone to the reactants through porous matrix sets apart filtration combustion from the homogenous flames [5,6]. Inside the porous burner, heat is transferred to the solid phase from the gas via convection [7]. A part of the heat received by the solid phase is conducted to the reactants through the solid matrix [8]. The hot solid phase also radiates heat within the combustor [8,9]. Due to higher thermal conductivity and emissivity of the solid matrix, conduction and radiation become a significant mode of heat transfer in PMC. Additionally, the large surface of the solid matrix and high heat transfer coefficient results in better convective heat transfer for PMC when compared to the same in free flame combustion.

Porous medium combustion is majorly categorized into two subdivisions namely stationary and transient combustion. Stationary combustion [10] is used for surface-heaters and radiant burners where the combustion flame is stabilized into a finite region in the porous matrix based on the boundary conditions. In the second approach, also known as a transient approach [11-13], the
reaction zone possesses the liberty to displace through the porous medium in upstream and downstream directions. Positive enthalpy flux results in downstream displacement of the flame zone whereas the negative flux results in upstream displacement. This type of PMC is also known as filtration combustion. The movement of an unsteady reaction zone often results in “excess enthalpy” flames.

1.2. Excess Enthalpy Flames

In 1971, Weinberg [14,15] came with the idea of excess enthalpy flames. He proposed that the maximum flame temperature of a system can reach beyond the theoretical adiabatic flame temperature if the heat from hot products is circulated to the incoming reactants. In recent times, “superadiabatic combustion” has been used synonymously with “excess enthalpy flames”.

Later on, Weinberg and co-workers [2,16,17] used this concept to extend the flammability range of the fuels. Figure 1.1 shows the effect of heat recirculation on the thermal structure of the flame. They achieved excess enthalpy flames by heat recuperation from the hot exhaust to the incoming reactants. Takeno and Sato [18] proposed generation of excess-enthalpy flames by inserting high-conductivity, inert porous solid into the reaction zone, and thus, regenerated the heat inside the combustor. Takeno et al. [9] studied internal heat recirculation and found the existence of a critical mass flow rate for sustained combustion.
In recent times, researchers have developed various types of excess enthalpy burners. Out of many designs prescribed, two types namely counterflow burner (CFB) [19,20] and reciprocal flow burner (RFB) [21,22] have gained popularity amongst scientists and engineers for various types of applications. Figure 1.2 shows a counterflow reactor used for the combustion of lean heptane and propane [19]. A CFB is an excess enthalpy burner where heat recirculation between firing and exhaust channel occurs through fluid flow in opposite directions. However, in RFB periodic switching of the flow direction does heat recirculation (Fig. 1.3).
1.3. Applications of PMC

PMC finds its way in a wide range of applications. These days PMC is used in micro turbines [23], IC engines [24], hydrogen and syngas generation [25], fuel cell [26] thermoelectric generators [27], household systems [28] etc. One of the most promising and commercial uses of PMC is to function as a power source for thermoelectric modules. A thermoelectric device works on the basis of the Seebeck effect. The principle states that when two or more electrodes are subjected to two separate temperatures, a potential is developed and correspondingly, current flows. Porous medium burners are used to maintain the hotter end of the thermoelectric module to an elevated temperature. Porous burners working in the superadiabatic regime produces temperatures larger than the adiabatic flame temperatures. This helps the thermoelectric material achieve a higher temperature. Although thermoelectric power generation is limited by lower efficiencies, the energy recirculation in PMC together with its ability to burn ultralean fuels makes the system a potential energy generating product. Since the energy density of hydrocarbon fuels is
more than that of transitional batteries, a thermoelectric power generation system can produce more electricity than its counterpart in transitional batteries [29] (Fig. 1.4). Additionally, instant recharge and low maintenance make a PMC powered thermoelectric module better choice compared to the battery powered module.

![Figure 1.4 Idealized diagram of a thermoelectric device [29].](image)

One of the earliest ideas of integrating combustion chambers with thermoelectric devices was given by Weinberg et al. [30] (Fig. 1.5). That device had an efficiency of 19.7 % by burning ultralean fuel mixture. The burner was operating at adiabatic conditions. Later on, he modified the design to achieve 21.83 % efficiency [31].
One of the initial ideas to create a thermoelectric module using a porous medium was introduced by Tada et al. [32]. They performed 1-D numerical analysis of a novel thermoelectric device operating with a reciprocal flow system. In the early 2000s, Hanamura et al. [27,33] generated electrical power using superadiabatic combustion of methane-air mixture. Jiang et al. [34] worked with a lean fuel-air equivalence ratio in a centimeter long thermoelectric power generation module and reported a maximum chemical to electrical conversion efficiency of 1.25 %. Many researchers studied the effect of fuel-air equivalence ratios on the maximum combustion temperature and corresponding electricity generation efficiency [29, 35-38]. Figure 1.6 shows the schematic representation of a power generating system based on porous burner.
Bubnovich et al. [40] studied a cylindrical porous burner operating with a lean propane-air mixture. They reported a conversion efficiency of 2.93 % with a voltage of 5.9 V leading to 1.59 A current. Henriquez-Vargas et al. [41] studied a 2-D numerical model of reciprocal flow porous media burner integrated with a thermoelectric module. They reported flammability limits for stable operation of the device. Recently, Aravind et al. [42] developed a novel thermoelectric power generator working with lean liquefied petroleum gas (LPG) and reported a conversion efficiency of 4.66 %. They worked with a dual micro-combustor system and reported its thermal characteristics and flame stability limits. Figure 1.7 shows the porous medium burner with thermoelectric cells designed by Ismail et al. [43].
1.4. Heat Extraction from PMC

Many modern-day applications of PMC require heat extraction from the burners filled with porous medium. As mentioned above, one of the potential applications being portable energy generators. PMC can be used to maintain the hotter end of the power generating module at sufficiently higher temperatures. In the present century, researchers have worked with various designs of superadiabatic burners for extracting the energy released in combustion. Contarin et al. [7,23] performed a numerical and experimental investigation on a reciprocal flow porous combustor with heat exchangers. They studied the effect of extraction efficiencies of the burner system as a function of heat exchanger position, filtration velocity of fuel and equivalence ratios. They developed a high efficiency and low emission system. Similar findings have been reported by Delalic et al. [27] and Farzaneh et al. [28] using a unidirectional flow porous burner. Malico and Pereira [29, 30] performed 2-D numerical simulation of a porous burner integrated with the heat exchanger. They investigated the effects of excess air ratio, the conductivity of the solid
matrix, convective heat transfer coefficient and radiative properties on the heat extraction efficiency.

As mentioned above, though many researchers have worked with heat extraction from superadiabatic burners, there is a need to explore high-temperature heat extraction.

1.5. Thesis Objectives

In this thesis, three configurations of excess enthalpy burners namely, counterflow burner or CFB, reciprocal flow burner or RFB and reciprocal counterflow burner or RCFB have been studied to extract energy at high temperatures. This research is conducted to address the following objectives:

- Study the flame stabilization and heat transfer in heat recirculating porous burner.
- Study the heat extraction efficiency of the CFB and RCFB at high temperatures.
- Study the heat extraction efficiency of CFB and RCFB at various equivalence ratios.
- Investigate the effect of the separation wall on high-temperature heat extraction efficiency of CFB and RCFB.
- Identify the most efficient location of heat extraction at high temperatures for CFB and RCFB.
- Study the variation of heat extraction efficiency with changing superficial velocity for CFB and RCFB.
- Investigate the emission characteristics of CFB and RCFB during high temperature heat extraction.
- Investigate the staged combustion to reduce NO\textsubscript{x} emission from the combustion of rich fuel-air mixtures.
Chapter 2 presents the numerical model developed to simulate the combustion in excess enthalpy burners filled with porous medium. It describes the boundary conditions, the chemical mechanisms used and the solution procedure implemented for the numerical simulations.

Chapter 3 describes the flame stabilization in a counterflow burner (CFB) filled with a porous medium. Flame stabilization is also studied for the case when heat exchanger is operated for high-temperature heat extraction. The heat extraction efficiency is analyzed for six different heat exchanger positions inside the burner and heat extraction temperatures from 300 K to 1300 K. Based on the heat extraction efficiencies; optimum position for high-temperature heat extraction is identified. The heat recirculation in the burner is studied for two materials of the burner wall. The influence of superficial velocity of the fuel-air stream on the heat extraction efficiency is evaluated.

In Chapter 4, a novel excess enthalpy porous burner is developed by combining RFB and CFB. This new type of burner known as reciprocal counterflow burner (RCFB) works on the principle of periodic flow reversals in CFB. Time averaged numerical simulations are conducted to study combustion and heat extraction from RCFB. 2-D numerical simulations have been conducted to identify the most efficient energy extraction position in the RCFB configuration. The effect of heat recirculation and superficial velocity of reactants on the extraction efficiency is also studied.

Chapter 5 describes the NO\textsubscript{x} emission characteristics of CFB and RCFB. NO\textsubscript{x} profiles for CFB and RCFB are studied and compared with each other. The effect of heat extraction on NO\textsubscript{x} generation is also investigated for both the types of excess enthalpy burners.

Chapter 6 presents a method for minimizing NO\textsubscript{x} generation from rich premixed flames using staged combustion. The description for experimental set-up and numerical model for staged
combustion is discussed. It studies staged combustion of methane in a rich premixed porous flame followed by a non-premixed homogeneous flame. The experimental studies are performed using the porous flame with heat extraction controlled by convective losses to cooling water. The numerical study considers the coupled models of premixed porous flame and non-premixed homogeneous flame. The studied combustion process is very critical for the reduction of emissions in a variety of gas-fired equipment widely used across the industrial and commercial applications such as process heating, direct-fired baking ovens, material thermoforming, surface treatment, lamination and others.
CHAPTER 2

Numerical Model

The numerical simulations are conducted by solving steady state governing equations. Laguerre et al. [44] used two types of numerical simulations for simulating heat transfer in porous medium and compared the results with experiments. The two types of modelling approaches are pore level approach and volume average approach [45]. This numerical study is conducted using the volume average approach. For numerical simulations, alumina spheres of 0.3 cm diameter are used to for a packed bed with 40 % porosity. The numerical simulations conducted for this thesis are based on the following assumptions:

(i) Thermodynamic properties of the fuel-air mixture and combustion products are functions of temperature and concentration.
(ii) Thermal equilibrium exists between the gas and the solid phase.
(iii) The solid spheres comprising the porous medium are homogeneously distributed.
(iv) The gaseous radiation is neglected compared to the solid radiation.
(v) The solid spheres forming the porous medium are non-catalytic.

2.1. Governing Equations

The continuity equation:

\[ \varepsilon \frac{\partial \rho_g}{\partial t} + \nabla \cdot (\rho_g \vec{v}_m) = 0 \]  

(2.1)

The momentum equation:
\[
\varepsilon \frac{\partial \left( \rho_g \vec{v}_{in} \right)}{\partial t} + \varepsilon \left( \rho_g \vec{v}_{in} \cdot \nabla \vec{v}_{in} \right) = -\varepsilon \nabla p + \varepsilon \mu \nabla^2 \vec{v}_{in} - \left( \frac{\mu}{\alpha} + \frac{C \rho_g \vec{v}_{in}}{2} \right) \vec{v}_{in}
\]  
\hspace{1cm} (2.2)

The term \( -\left( \frac{\mu}{\alpha} + \frac{C \rho_g \vec{v}_{in}}{2} \right) \vec{v}_{in} \) in Eq. (2.2) represents the pressure drop due to porous medium as given by [46]. The coefficients \( \alpha \) (coefficient for viscous resistance) and \( C \) (coefficient for inertial resistance) are given by

\[
\alpha = \frac{D_p^2 \varepsilon^2}{150(1-\varepsilon)^2} \quad \text{and} \quad C = \frac{3.5(1-\varepsilon)}{D_p \varepsilon}
\]

The energy equations for gas and solid phases:

\[
\varepsilon \rho_g c_g \frac{\partial T_g}{\partial t} + \left( 1 - \varepsilon \right) \rho_s c_s \frac{\partial T_s}{\partial t} + \varepsilon \rho_g c_g \vec{v}_{in} \nabla T_g + \varepsilon \sum_i \rho_g Y_i c_{g,i} \nabla T_g = \varepsilon \nabla \cdot \left( k_{g,eff} \nabla T_g \right) + \varepsilon \nabla \cdot \left( \rho_g c_g D_{g,eff} \nabla T_g \right) + \varepsilon \sum_i \hat{\omega}_i h_i W_i
\]  
\hspace{1cm} (2.3)

The radiation heat transfer of the porous medium is modeled with radiant conductivity \( k_{rad} \) [47] defined as \( k_{rad} = 4F \sigma D_p T^3 \), where \( F \) is the radiation exchange factor and is taken as 0.4 [48].

The effective thermal conductivity of the system is taken as \( k_{eff} = 0.005 k_s + k_{rad} + k_f \).

The species transport equation:

\[
\varepsilon \rho_g \frac{\partial Y_i}{\partial t} + \varepsilon \rho_g \vec{v}_{in} \nabla \cdot Y_i = -\nabla \cdot j_i + \varepsilon \hat{\omega}_i W_i
\]  
\hspace{1cm} (2.4)

\hspace{1cm} 2.2. Boundary Conditions

A set of boundary conditions are used to simulate the computational model in Fluent 14.5. The inlet to the combustor is defined as the velocity inlet whereas the outlet is considered as the pressure outlet. Superficial velocity of the methane-air mixture is set as a constant. The outer wall of the combustor is treated as an adiabatic wall to prevent heat loss to the surroundings. The heat
exchangers are modeled as walls maintained at a fixed temperature. *Fluent 14.5* calculates the heat transfer based on the temperature difference between adjacent cells. The value thus predicted, is integrated over the complete heat exchanger domain. Mathematically the boundary conditions can be rewritten as:

At the inlet

\[ T = T_o = 300K, Y_{CH_4} = Y_{CH_4}, Y_{O_2} = Y_{O_2}, v_{in,y} = 0, v_{in,x} = \frac{v_{in}}{\epsilon} \]

At the outlet

\[ v_{in,y} = 0, p = p_o \]

### 2.3. Chemical Mechanism

Premixed combustion always proceeds through many intermediate chemical reactions and chemical species. Many of these reactions are endothermic while many are exothermic reactions. Thus, numerical simulation of combustion phenomenon is sensitive to the choice of chemical mechanisms. Chemical mechanisms are known to influence the flame temperature as well as the flame velocity for filtration combustion. Detailed chemical mechanisms are known to predict porous flame temperatures accurately, but they often come along with huge computational times to predict the solution. Chemical mechanisms with moderate number of chemical species simulate filtration combustion in a shorter time, with good precision. More often, the tradeoff for computational time and accuracy of the results, computational time is chosen to be the governing factor for the choice of chemical mechanism. However, it is important to validate the findings with previously published results. For the present analysis, 30 species chemical mechanism [49,50] is chosen to simulate the filtration combustion. Various user-defined functions (to accommodate
radiation and dispersion in porous medium) are coded and integrated with Fluent 14.5, to achieve numerical results within a shorter time without compromising the accuracy.

2.4. Solution procedure

The numerical simulation of the RCFB is conducted using Fluent 14.5. Steady state methodology is used with the pressure-based solver and absolute velocity formulation. The pressure-based solver uses an algorithm belonging to the projection method. The governing equations are non-linear and coupled with each other. Pressure based solver uses the iterative approach to obtain convergence while continuing the solution loop.

The solution strategy is set up using the energy model with viscous laminar flow. Solution approach is followed to simulate the combustion process. The pressure velocity coupling of the model is done using SIMPLE scheme. The spatial discretization for the species is performed using second order upwind scheme. Momentum and energy equations are solved using second order upwind scheme as well. The steps for the SIMPLE algorithm are mentioned below:

(i) Initialize the boundary condition.
(ii) Calculate velocity and pressure gradients.
(iii) Discretization of momentum equation for calculation of flow velocity.
(iv) Solving the pressure correction equation for computing pressure correction terms.
(v) Update the pressure field: $p^{k+1} = p^k + \gamma p'$ ($\gamma$ being the under-relaxation factor for pressure)
(vi) Update pressure values at boundaries.
(vii) Correction of mass flux at every cell of the mesh.
(viii) Correction of velocity at every cell of the mesh.
2.5. Model Validation

The modeling approach discussed above is used to simulate the combustion of the methane-air mixture using the experimental results of [48]. For superficial velocity of 0.36 m/s, the numerical predictions of the maximum flame temperature remain within the range of 1250 K to 1480 K for $0.33 < \phi < 1.0$. The experiments [48] report the maximum flame temperatures to 1360 K to 1470 K for the same range of equivalence ratio.

**Figure 2.1** Variation of maximum flame temperature with equivalence ratio. The superficial velocity of fuel-air mixture is kept as 0.36 m/s.
CHAPTER 3
Heat Extraction Using Counterflow Porous Burner

Energy extraction from an adiabatic regenerative porous burner is studied numerically. Steady state governing equations are solved to predict the fluid and thermal properties of the system. The temperature of heat extraction is varied from 300 K to 1300 K. The numerical simulation predicts the effect of efficiency of energy extraction on the location and extraction temperature of heat exchangers. Higher extraction temperatures tend to decrease the extracted energy and consequently raise the exhaust temperature of the burner. Two burner configurations are studied comparatively by changing the properties of the wall separating the incoming reactants from the exhaust gases. Out of the two materials used to study the effect of the separation wall on energy extraction, the study predicts higher gains for alumina as compared to silicon carbide. The maximum heat extraction efficiency of 35% is reported for extraction at 1300 K when silicon carbide separation wall is used in the burner. Whereas for a porous burner with alumina separation wall, 60% of the heat can be extracted at 1300 K.

3.1. Computational Model and Burner Geometry

The schematic diagram of the computational model is shown in Fig. 3.1. A premixed methane-air mixture enters the burner of dimension 20 x 2.5 cm. The inlet and outlet of the porous burner is separated by a ceramic wall whose thickness is varied from 0.1 cm to 0.5 cm. The separation wall is assumed to be made of silicon carbide and alumina in two separate designs. This separation wall prevents the intermixing of incoming reactants with the combustion products. It also plays a significant role in transferring heat from the flame zone to reactant zone and from hot
products to reactants. Ceramic alumina spheres with a diameter of 0.3 cm are assumed to form the porous matrix inside the burner resulting in homogeneous porosity of 40 % throughout the burner. The heat exchangers are modelled as 2-D cylinders with a diameter of 0.6 cm. Two heat exchangers (P_i and P_o) are located at 1 cm from the inlet and outlet to stabilize the flame inside the combustor. Cylindrical heat exchangers are located at 5 cm from each other, along the length of the combustor. A lean premixed mixture of methane-air is fed to the combustor. The porous combustor operates in the laminar regime.

Figure 3.1 Schematic of the porous burner showing the positions of the heat exchangers. The dimensions are expressed in cm.
3.2. Temperature Distribution

The effect of alumina and silicon carbide separation wall is studied on the maximum flame temperature achieved during the combustion inside the porous burner, as shown in Fig. 3.2. For numerical simulations maximum flame temperatures of premixed porous flames depend mainly on the superficial velocity of fuel-air mixture and the chemical mechanism used to simulate the phenomenon. In case of the burner with alumina separation wall, the numerical simulation predicts the peak temperature of the flame wave between 1280 K and 1500 K, for the equivalence ratio ranging between 0.4 and 1.0. However, for the burner configuration consisting of silicon carbide separation wall, the maximum flame temperature ranges from 1250 K to 1420 K, for the given range of equivalence ratios. Higher conductivity of the silicon carbide helps to recuperate the heat generated in the flame zone, thus, reducing the peak temperature of the flame.

![Figure 3.2 Maximum flame temperature as a function of equivalence ratio. The superficial velocity is maintained at 0.36 m/s.](image)
The porous burner is simulated for heat extraction by oxidizing methane-air mixture in a range of equivalence ratios from 0.4 to 1.0 (Fig. 3.3). The heat exchanger is placed at P-2 and is operated at 900 K. The superficial velocity of the methane–air mixture is maintained at 0.36 m/s. The heat extraction efficiency of the burner is defined as the ratio of heat extracted from the burner to the total energy released by the fuel:

\[
\eta = \frac{q_e}{mY_{CH_4}LHV}
\]

(3.1)

**Figure 3.3** Extraction efficiency of porous burner with varying equivalence ratio of the premixed methane-air mixture. The heat exchanger is placed at P-2 and maintained at 900 K.
Fluent’s algorithm is coupled with user defined functions to account for heat transfer through conduction, convection and radiation. The heat transferred between two cells are calculated based on their average temperatures. This value is then integrated over the surface of the heat exchanger, and $q_e$ is obtained as an output of Fluent 14.5. The numerical simulation predicts no significant variation of heat extraction efficiency at this range of equivalence ratios. The trend remains similar for separation walls made of silicon carbide and alumina. Further studies on the burner are conducted at $\phi = 0.7$.

3.3. **Flame Stabilization**

The flame location inside the burner is studied (Fig. 3.4). The separation wall thickness is considered as 0.5 cm and the material used is assumed as alumina and silicon carbide in two separate cases. Initially, the heat exchangers placed near inlet and outlet ($P_i$ and $P_o$) are operated at 300 K to restrict the flame inside the burner. The investigation is performed between $0.4 < \phi < 1.0$. For $0.4 < \phi < 1.0$, the numerical predictions show an upstream displacement of the flame front with an equivalence ratio increase for both the designs. For the design with silicon carbide separation wall the flame front for the fuel mixture of $\phi = 0.4$ is stabilized 11 cm away from the burner inlet whereas, for the burner with the alumina separation wall, flame positions itself at 9.8 cm from the inlet. Upon increasing the equivalence ratio, the energy content of the flame increases and the flame tends to move upstream. For $0.5 < \phi < 1.0$, the location of flame moves from 5 cm to 1.8 cm away from the inlet. A similar trend is observed for the design with alumina separation wall. Methane-air mixture with stoichiometric composition burns with a flame releasing higher energy. So, it tends to displace towards the inlet to lose heat to the surroundings. However, for the lean mixtures, the flame releases less energy. This energy is consumed to heat
the porous matrix in the firing and exhaust channels of the burner. Hence, the flame tends to displace deeper in the firing channel.

**Figure 3.4** Flame displacement within the porous burner with a change in equivalence ratio. The heat exchangers (P<sub>i</sub> and P<sub>o</sub>) are maintained at 300 K. High-temperature heat exchangers are not activated. The superficial velocity is 0.36 m/s.

Figure 3.5 shows the flame stabilization using methane and carbon dioxide profiles for the burner design with the alumina separation wall. It is observed that the location of methane consumption and carbon dioxide generation is the same, thus, it is inferred as the flame location. Numerical simulation predicts additional carbon dioxide generation downstream the flame. This is mainly due to oxidation of carbon monoxide to carbon dioxide. The species contours provide evidence of no secondary flame inside the burner. The flame stabilization profiles are similar for silicon carbide separation wall.
Figure 3.5 Species profiles inside the burner with alumina separation wall at various equivalence ratios. The fuel (methane) is consumed in the same region where products (carbon dioxide) are generated. The predictions remain similar for the range of equivalence ratios studied.

The species contours inside the combustor are shown in Fig. 3.6. The species profiles exhibit similar behavior inside the porous combustor for both the designs studied. For methane contour, the porous burner design with silicon carbide shows a curved profile at the separation wall interface unlike the design with alumina separation wall. This behavior is governed by the thermal conductivity of the separation wall. With high thermal conductivity, the burner with silicon carbide separation wall \( (k_{sc} = 90 \text{ W/mK}) \) has a tendency to spread the flame zone resulting in a thicker flame compared to the design with alumina separation wall. Lean premixed combustion in porous media is generally characterized by high combustion efficiency, i.e. complete combustion of the supplied gaseous fuel. In porous combustion, the extended combustion zones and active dispersion of the reactant and combustion products augment the complete oxidation of the fuel. The porous medium itself effectively works as a distributor enhancing heat and mass transfer across the combustion front. Numerical simulation predicts complete combustion of the fuel for all the cases studied with the present designs of porous burner.
Figure 3.6 Species distribution inside the porous combustor: (a) silicon carbide separation wall, (b) alumina separation wall. Methane-air mixture at $\phi = 0.7$, is fed at a superficial velocity of 0.36 m/s. Heat exchanger at P-1 is operated at 1100 K.

3.4. Heat Extraction

Figure 3.7 shows the numerical prediction of the temperature distribution inside the porous burner. Flame localization inside both the configuration of porous burners are in accordance with the predictions of [51]. Temperature distribution inside the burner is assumed to be invariable in z-axis. With alumina separation wall, thermal distribution inside the porous burner shows higher flame temperature and greater area comprising of high-temperature. Whereas the burner with silicon carbide separation wall has lower flame temperature. This is attributed to the higher thermal conductivity of the silicon carbide ($k_{sc} = 90$ W/mK) as compared to ($k_a = 15$ W/mK). Due to the high thermal conductivity of silicon carbide the heat from the flame gets regenerated over a larger domain. As a result, the exhaust gas temperature increases. However, the exhaust gas temperature for burner with alumina separation wall remains lower than that with the silicon carbide separation wall. The maximum flame temperature and the exhaust gas temperature influence the efficiency of the burner.
Figure 3.7 Temperature contours of the porous burner (a) burner with silicon carbide separation wall; (b) burner with alumina separation wall. The numerical simulations are performed at an equivalence ratio of 0.7 with superficial velocity at 0.36 m/s.

The efficiency of heat extraction is studied for different locations of heat extraction. The temperature of heat extraction varies in the range from 300 K to 1300 K, with a step of 200 K. The choice for upper limit of temperature of heat extraction is subjected to the maximum temperature of the flame, achieved during the combustion at the given mass flow rate. Figure 3.8 (a) shows the heat extraction plots for different positions of the heat exchanger for the porous burner with silicon carbide separation wall. For silicon carbide wall, the heat exchangers placed at location P-6 show the maximum efficiency of 60% for extraction at 300 K. With increase of extraction temperature, the efficiency drops non-linearly to 38% at 900 K. For extraction temperatures more than 900 K, the efficiency of extraction drops sharply and reaches to a point of no gain at 1300 K. The difference in the efficiency is due to the amount of heat recirculated. Silicon carbide wall conducts heat from the flame zone to the exhaust gases, leading to increase in the exhaust gas temperature. High exhaust temperatures for silicon carbide separation wall tends to increase the losses from the burner.
At position P-5, the efficiency of extraction drops from 63% at 300 K to no extraction at 1300 K. For position P-4, the numerical simulation predicts the drop in extraction efficiency from 25% at 500 K to 3% at 1300 K. The numerical simulations indicate that the flame cease to exist if the heat exchanger is positioned at P-4, extracting heat at 300 K. However, for extraction temperatures higher than 500 K the flame stabilizes far away from the heat exchanger making energy extraction difficult. The energy extraction trend for heat exchanger position at P-3 shows a behavior similar to P-4 except for extraction at 300 K and 500 K. For heat exchanger at 300 K and 500 K, the flame stabilizes in front of the heat exchanger. This results in higher extraction efficiencies, but as the extraction temperature increases, the flame moves upstream, towards the incoming reactants, and stabilizes farther from the extraction location. This decreases the extraction efficiency.
Figure 3.8 Efficiency of heat extraction as a function of heat extraction temperature for the porous burner with (a) silicon carbide separation wall; (b) alumina separation wall. The superficial velocity is maintained at 0.36 m/s.
Similarly, for extraction at P-2, the flame stabilizes in front of the heat exchanger for extraction at 300 K to 1100 K. Nevertheless, it displaces upstream for extraction at 1300 K. This leads to a maximum extraction efficiency of 68 % at 300 K and remains approximately independent of the extraction temperature, until 900 K. Increasing the extraction temperature further, leads to a drop in the efficiency value till it reaches 3 % for extraction at 1300 K. Heat extraction at position P-1 cause the flame to stabilize in front of the heat exchanger for the entire temperature range investigated. This decreases the efficiency variation as the extraction temperature is increased. The maximum extraction efficiency values to 55 % at 300 K and decreases until 35 % at 1300 K. The numerical simulations predicts an increase in exhaust temperature as the extraction temperature is increased. This adds to loss of energy from the burner.

Figure 3.8(b) shows the efficiency of energy extraction when the heat exchangers are placed at six different locations in the same porous burner but with alumina separation wall. For heat exchanger located at P-6, the numerical simulation predicts 78 % of energy extraction, for heat exchangers at 300 K. The efficiency for this position drops to 7 % at 1300 K. Heat exchangers placement at P-5 results in 35% energy efficiency at 300 K and 5 % at 1300 K. Lower thermal conductivity of alumina restricts heat regeneration between the inlet channel of the burner and the exhaust channel. In addition, the flame location becomes distant from the heat exchanger position. For heat exchangers located at P-4, the efficiency of extraction varies between 18 % and 1 % when the temperature is increased from 300 K to 1300 K. The flame stabilizes at a distance from the heat exchanger position making the extraction process inefficient. A significant amount of heat is transferred from the flame to the exhaust through the separation wall. This increases the exhaust gas temperature and, thus, the losses.
The extraction efficiency improves greatly when the heat exchanger is placed at P-3. For this position, the flame stabilizes ahead of the heat exchanger, for temperature range of 300 K to 1100 K. This leads to a high extraction efficiency of 80% to 65% for this temperature range. However, when the extraction temperature is increased to 1300 K the flame moves upstream and stabilizes at a distant position from the heat exchanger. This leads to a drop in extraction efficiency to ~20%. Due to the localization of the stabilized flame zone near the inlet of the burner, a significant fraction of the total heat generated gets transferred to the exhaust. The exhaust temperature increases significantly. On placing the heat exchanger at P-2, the extraction efficiency trend remains the same as for position P-3. A very efficient extraction process can be achieved with heat exchanger positioned at P-1. The flame positions itself in front of the heat exchanger for the entire range of temperatures studied. Due to close proximity of the flame zone and the heat exchanger, a significant fraction of heat is extracted by the heat exchanger. This reduces the amount of heat transferred from the flame to the exhaust gases through the alumina separation wall. This helps in reducing the exhaust gas temperature.

The heat extraction efficiency of the CFB design with alumina separation wall is studied for reactant at $\phi = 0.4$ (Fig. 3.9a) and compared with that when $\phi = 0.7$ (Fig. 3.9b). For $\phi = 0.4$, the reactant has lower maximum flame temperature than that for $\phi = 0.7$. This leads to reduction in heat extraction efficiency from the CFB operating at $\phi = 0.4$. Similar to the case with $\phi = 0.7$, heat exchanger position P-1 remains the most efficient most position of heat extraction for $\phi = 0.4$ as well. Numerical simulation predicts maximum heat extraction temperature for $\phi = 0.4$ to 1200 K. This is limited by the maximum flame temperature.
Figure 3.9 A comparison for the efficiency of high-temperature heat extraction between (a) $\phi = 0.4$ and (b) $\phi = 0.7$ is presented for CFB design with alumina separation wall. Superficial velocity of reactants is 0.36 m/s.
Figure 3.10 shows the variation of heat extraction efficiency for CFB design with alumina separation wall. For extraction at lower temperatures, the efficiency increases with decrease in the equivalence ratio. The maximum flame temperature for $0.3 < \phi < 0.4$ is approximately 1280 K. This temperature is suitable for good amount of energy extraction, when heat exchanger is maintained at 300 K. However, with decreasing equivalence ratio, the energy content of the fuel mixture decreases. So, the efficiency of heat extraction is observed to increase with the decrease in the equivalence ratio of the reactant mixture. Numerical simulation predicts that 76% energy extraction for the heat exchanger position P-1 when CFB is operated at $\phi = 0.3$. This remains similar for $\phi = 0.4$ and 0.5. Nevertheless, for $\phi = 0.7$ the heat content (8.52 kW) in the fuel is increased so, the extraction efficiency drops to 70%. However, for heat extraction at 1300 K, the maximum flame temperature for the reactant mixture at $\phi = 0.3$ and 0.4, is not sufficient for heat extraction. The numerical prediction indicates, negative efficiency, or in other words, the heat exchanger pumps in heat into the CFB design. As the equivalence ratio is increased to $0.5 < \phi < 0.7$, heat extraction efficiency increases to a maximum value of 68%. 
Figure 3.10 Variation of heat extraction efficiency with heat extraction temperature for different equivalence ratios. Heat exchanger at P-1 is operated. CFB design with alumina separation wall is used with superficial velocity of reactants equal to 0.36 m/s.
3.5. Heat Flux Analysis

Analysis of heat extraction efficiency requires a deep understanding of internal heat transfer in the CFB. To analyze the heat transfer inside CFB heat flux studies are conducted as shown in Fig. 3.11. Heat flux inside the CFB is studied for four different cases, namely:

Case 1: CFB with alumina separation wall operating at $\phi = 0.3$,

Case 2: CFB with alumina separation wall operating at $\phi = 0.7$,

Case 3: CFB with alumina separation wall operating at $\phi = 0.7$ when heat is extracted using P-1 at 1300 K and

Case 4: CFB with silicon carbide separation wall operating at $\phi = 0.7$ when heat is extracted using P-1 at 1300 K.

In all of these cases, the heat exchangers $P_i$ and $P_o$ are operated at 300 K to restrict the flame inside the CFB. The energy extracted by these heat exchangers are treated as losses.
Case 1: CFB with alumina separation wall operating at $\phi =0.3$

Heat transfer analysis of CFB with alumina separation wall operating at $\phi = 0.3$ is shown in Fig. 3.11. At $\phi = 0.3$, the flame is stabilized at approximately 9 cm from the inlet of the burner and releases 3.5 kW of heat energy. Approximately 86% of this heat energy is extracted by the heat exchanger $P_i$ which operates at 300 K to stabilize the flame inside the burner. The alumina separating wall regenerates 32% heat from the hot exhaust back to the firing channel. This heat is used to preheat the incoming reactants. The heat exchanger placed near outlet ($P_o$) extracts 13% heat from the hot products. Remaining 1% of the heat is transferred to the surroundings through the products. The details of the heat transfer across various components of the burner is shown in Fig. 3.11.

![Figure 3.11 Heat flux across various components of CFB design with alumina separation wall when it is operated with fuel-air mixture at $\phi = 0.3$.]
Case 2: CFB with alumina separation wall operating at $\phi = 0.7$

Figure 3.12 shows the heat transfer across various components inside the CFB for $\phi = 0.7$. Numerical simulation predicts flame stabilization at approximately 3 cm from the inlet. The combustion of the reactant mixture releases 8.52 kW out of which 5.16 kW (63 %) is extracted by the heat exchanger $P_i$ and the rest goes into the exhaust channel. About 11 % of energy (0.94 kW), gets transferred back to the firing channel from the exhaust channel and 2.94 kW (33 %) is extracted by $P_o$. Rest 0.34 kW (4 %) goes out of the burner with the exhaust. The details are shown in Fig. 3.12.

Figure 3.12 Heat flux across various components of CFB design with alumina separation wall when it is operated with fuel-air mixture at $\phi = 0.7$. 
Case 3: CFB with alumina separation wall operating at $\phi = 0.7$ when heat is extracted using P-1 at 1300 K

In this case CFB having alumina separation wall is simulated with P-1 operating at 1300 K, as shown in Fig 3.13. The reactants are fed in the burner at $\phi = 0.7$. Numerical simulation predicts flame stabilization at approximately 6.5 cm from the burner inlet. The flame releases 8.53 kW inside the burner, out of which, 5.27 kW (62 %) is absorbed by the heat exchanger P-1. Heat exchanger placed at position Pi extracts 0.51 kW (6 %). Rest of the energy goes to the exhaust channel. The exhaust channel then transfers 1.19 kW (14 %) back to the firing channel and 2.13 kW (25 %) is extracted by the heat exchanger P_o. About 7 % energy (0.6 kW), is lost to the environment as the exhaust gases leave the burner.

**Figure 3.13** Heat flux across various components of CFB design with alumina separation wall when it is operated with fuel-air mixture at $\phi = 0.7$. Heat exchanger at position P-1 is operated at 1300 K.
Case 4: CFB with silicon carbide separation wall operating at $\phi = 0.7$ when heat is extracted using P-1 at 1300 K

Heat transfer in the CFB design with silicon carbide separation wall is studied when heat exchanger placed at position P-1 is operated at 1300 K. Fuel mixture at $\phi = 0.7$ is fed into the burner. Similar to case 3, the flame stabilizes at approximately 6 cm from the inlet downstream the heat exchanger. The heat exchanger at P-1 extracts 3.23 kW (38 %) heat whereas the heat exchanger P_i harness 1.61 kW (19 %). The rest of the energy goes to the exhaust channel. Out of the total energy received by the exhaust channel, 2.04 kW (24 %) is transferred from the exhaust channel back to the firing channel and 2.3 kW is absorbed by the heat exchanger P_o. Remaining 1.36 kW (16 %) of heat leaves the CFB system with the products.

![Figure 3.14](image)

**Figure 3.14** Heat flux across various components of CFB design with silicon carbide separation wall when it is operated with fuel-air mixture at $\phi = 0.7$. Heat exchanger at position P-1 is operated at 1300 K.
The above analysis indicates that as the fuel equivalence ratio decreases the amount of internal heat recirculation is increased.

The wall separating the incoming reactants and outgoing exhaust gases affects the flame temperature, exhaust gas temperature and the steady state location of the flame. Figure 3.15(a) shows the transverse temperature profiles inside the burner with silicon carbide separation wall. The transverse temperature profile is plotted for various along axial locations along the length of the burner. The figure shows the flame location to be at 7 cm from the inlet with heat exchanger is operating at 900 K at position P-1.

The maximum flame temperature is close to 1360 K. The conductivity of the silicon carbide helps to recirculate the heat inside the combustor, thus reducing the flame temperature and increasing the exhaust gas temperature. Figure 3.15(b) shows the transverse thermal profile for the burner with alumina separation wall, under the same condition. The steady state flame location is predicted at 7 cm along the axis of the burner. Nevertheless, the low thermal conductivity of alumina, reduces the heat recirculation inside the burner, thus, leading to higher flame temperature after the heat is extracted at 900 K. The comparison of Figs. 3.15 (a) and (b) shows that, the exhaust gas temperature (at x = 1 cm and x = 3 cm) of the burner for the two configuration, are higher for the burner with silicon carbide separation wall compared to the one with alumina separation wall.
Figure 3.15 Transverse temperature profile for the burner with (a) silicon carbide separation wall; (b) alumina separation wall. The heat exchanger is located at P-1 and maintained at 900 K. The superficial velocity of the methane-air mixture is 0.36 m/s.
3.6. Effect of Superficial Velocity

The variation of extraction efficiency with velocity is studied in the same system for two different separation wall materials. Firstly, the superficial velocity is varied in the combustor with silicon carbide separation plate and, then, the study is extended to the system with alumina separation wall. The difference in the thermal properties of the two materials is affecting the heat transfer inside the porous burner and influence the position of the flame zone. The extraction efficiency at different velocities is plotted against extraction temperature in Fig. 3.16 for both configurations. For the burner with silicon carbide separation wall, the increase in superficial velocity increases the extraction efficiency at high-temperature. With velocity increase, mass flow rate increases and thus, the flame temperature rises. This enhances the extraction efficiency. On the other hand, the rise of the flame temperature increases the amount of heat transferred to the exhaust gases through the silicon carbide separation wall. Thus, the exhaust gas temperature rises and so does the losses.

For alumina separation wall, the numerical data exhibits a mixed trend when the extraction efficiency at different velocities is plotted against varying extraction temperature. At lower extraction temperature, reducing the velocity increases the extraction efficiency. However, for extraction temperatures exceeding 900 K, the efficiency plot for methane-air velocity, $v = 0.16$ m/s shows a decrease from 65 % to 43 % at 1300 K. For $v = 0.36$ m/s the model predicts the efficiency drop from 68 % to 58 % within the same temperature range. This increase in efficiency can be attributed to the increase in the combustion temperature, which in turn increases the heat extraction efficiency. The numerical simulation for the model predicts higher losses when the velocity is increased further to 0.56 m/s and 0.76 m/s.
Figure 3.16 Variation of heat extraction efficiency with the extraction temperature at different velocities for the porous burner (a) with silicon carbide separation wall; (b) with alumina separation wall.
Figure 3.17 presents the axial thermal distribution inside the burner when the heat exchanger is placed at position P-1 and maintained at 900 K. For both configurations, the flame front stabilizes ahead of the heat exchanger. Figure 3.17 (a) shows the thermal distribution of the burner with silicon carbide separation wall. The exhaust temperature of the combustion products is 570 K. Whereas, for burner with alumina separation wall the combustion products exit the burner at 465 K, as shown in Fig. 3.17 (b). Figures 3.17 (a) and (b) show lower maximum flame temperature for the burner with silicon carbide wall in comparison with the burner with alumina separation wall. The burner with alumina wall shows higher temperature difference between the inlet and exhaust channel. This suggests higher heat regeneration in the system compared to the burner with silicon carbide wall.
Figure 3.17 Axial temperature distribution inside the burner with (a) silicon carbide separation wall and (b) alumina separation wall. The heat exchanger is placed at position P-1 and maintained at 900 K. The superficial velocity of the fuel-air mixture is 0.36 m/s.
Temperature distributions inside the burner are shown in Fig. 3.18. With heat exchanger P-1 working at 1000 K, the flame tends to lose most of the energy to the heat exchanger and stabilizes close to the heat exchanger for both the designs of the burner. With a 0.5 cm thick separation wall, the exhaust gas temperature for the burner with silicon carbide separation wall is higher than that of the alumina separation wall. In addition, the average temperature inside the burner is lower in silicon carbide separation wall than compared to the design with the alumina separation wall. Owing to higher thermal conductivity ($k_{sc} = 90$ W/mK) 0.5 cm thick silicon carbide separation wall tends to recirculate the heat from high-temperature region to relatively colder regions. Whereas due to lower thermal conductivity ($k_a = 15$ W/mK) this phenomenon is not so significant for alumina separation wall. Upon decreasing the separation wall thickness to 0.1 cm, the effect of thermal conductivity of the material becomes negligible. Thus, with decreasing thickness, the difference in thermal profile vanishes for both the designs.

**Figure 3.18** Thermal distribution inside the burner for varying thickness of the separation wall. The heat exchanger at position P-1 is activated to extract heat at 1000 K and heat exchangers at $P_i$ and $P_o$ are operated at 300 K. Superficial velocity of fuel-air mixture is maintained at 0.36 m/s with $\phi = 0.7$. 
Figure 3.19 shows the extraction efficiency of the burner with alumina and silicon carbide separation wall. The filtration velocity is maintained at 0.36 m/s and the extraction is performed at 1000 K. For 0.5 cm of separation wall thickness, maximum efficiency of energy extraction is predicted as 43% for silicon carbide and 68% for alumina. The highly conductive silicon carbide separation wall conducts the heat from the flame zone to the relatively colder zones. In doing so, it diminishes the maximum flame temperature and increases the exhaust gas temperature. Higher exhaust gas temperature causes higher energy loss to the surroundings. With lower thermal conductivity, alumina separation wall restricts the conduction of heat from high-temperature flame zone to the relatively colder zones. This results in a larger high-temperature region in the burner with the alumina separation wall. Upon decreasing the separation wall thickness, the efficiency of high-temperature energy harvesting shows minor improvements for the design with the alumina separation wall. Whereas for silicon carbide separation wall, the decrease in the wall thickness increases the maximum efficiency of energy extraction from 43% to 65%. This is because with decreasing separation wall thickness, the effect of thermal conductivity decreases. Hence, separation walls made of both the materials show similar behavior. It leads to heat transfer enhancement between the firing channel and exhaust channel of the burner. On a contrary, an increase of thermal resistance along the axial direction (due to the decrease in the cross-sectional area of the separation wall) restricts the axial heat conduction from the flame zone to the outlet of the burner. Thus, it restricts the heat inside the burner available for extraction by the heat exchangers.
Figure 3.19 Variation of heat extraction efficiency with the separation wall thickness for (a) silicon carbide separation wall; (b) alumina separation wall. The extraction is performed at 1000 K at position P-1. Numerical simulations are performed at the superficial flow velocity of 0.36 m/s and $\phi = 0.7$. 
CHAPTER 4

High Temperature Heat Extraction from Reciprocal Counterflow Porous Burner

This chapter reports numerical findings on combustion and heat extraction in a reciprocal counterflow porous burner. Lean methane-air mixture enters the porous burner in laminar flow regime and undergoes complete combustion. Thermal energy released during the combustion is extracted using the heat exchangers operating at extraction temperatures ranging from 300 K to 1300 K. Steady state governing equations are solved to generate the thermal map of the burner. Numerical simulation is used to predict the energy extraction efficiencies of the heat exchangers placed at various positions along the burner. This research identifies the optimum location of heat exchanger for high temperature energy extraction. Numerical results indicate a strong interdependence of the heat extraction efficiency and the stabilized flame positions. It is observed that for efficient extraction at high temperature, flames stabilizes downstream the heat exchanger maintaining proximity with it. The effect of heat recirculation, heat extraction temperature, and superficial velocity of the incoming reactants on the efficiency of extraction for the burner is studied. Effect of heat recirculation is analyzed by changing the material of the wall separating the incoming reactants from the combustion products. This chapter presents a comparison of the efficiencies of high temperature energy extraction using counterflow porous burner, reciprocal flow burner and reciprocal counterflow porous burner. The reciprocal counterflow porous burner yields the maximum efficiency (15 % more than counterflow porous burner) for extraction at high temperature.
4.1. Computational Model and Burner Geometry

Figure 4.1 shows the schematic representations of a reciprocal counterflow burner combining CFB and RFB concepts. The major principle of operation is periodic flow reversal applied to a CFB. In this investigation, an RCFB system filled with porous medium is studied numerically. Numerical simulations of quasi-steady periodic reciprocal flow porous burners are computationally expensive. Hence, to emulate the phenomenon of periodic flow reversals, a time averaged approach is adopted for this study [52]. For time averaged simulation of combustion in an RCFB, the computational model considers two counterflow porous burners arranged in a stack. One of the two burners is simulated with reactant flow in counter-clockwise direction whereas the other with clockwise flow, as shown in Fig.4.2. The model also assumes a wall placed between these two counterflow burners with high thermal conductivity in z-direction \((k_z = 10^4 \text{ W/Mk})\). High thermal conductivity of the wall in z-direction leads to thermal coupling of the two counterflow burners. This system of two counterflow burners together with thermally coupled wall, results in a time averaged reciprocal flow system [52].

![Figure 4.1](image)

**Figure 4.1** Schematic representation of a reciprocal counterflow burner filled with porous medium.
The schematic diagram of the computational model of the burner is shown in Fig. 4.2. The porous burner has dimensions 20 x 2.5 cm. The incoming lean mixture of methane-air entering the burner is separated from the combustion products using a ceramic wall of 0.5 cm thickness. This ceramic wall also helps in heat transfer from the firing channel of the combustor to the exhaust channel and vice-versa. There are six heat exchangers placed inside the burner 5.0 cm apart from each other. The combustor on top has the fuel flow in clockwise (CW) direction while the bottom one has it in counterclockwise (CCW) direction. Heat transfer between the two burners with opposite flow direction is achieved by modelling a wall between them. The wall is modelled to have high thermal conductivity along z-axis ($k_z = 10^4$ W/mK) and zero in x and y-axes. In both the combustors, the heat exchangers are switched on in symmetrical pairs (A-A’, B-B’ etc.) to achieve faster flame stabilization. A pair of heat exchanger is placed near the inlet and outlet of the burner at a distance of 1 cm. These heat exchangers are placed inside the combustor to restrict the flame movement out of the combustion chamber. Alumina spheres of 0.3 cm diameter are used to form the porous matrix inside the combustor with 40 % porosity. The temperature of heat extraction is varied in heat exchangers placed inside within a range of 300 K to 1300 K.

Figure 4.2 Computational model of the reciprocal burner system filled with porous medium. The diagram shows the position of heat exchangers. All dimensions are in cm. The coordinate axes shown in the diagram describe the arrangement of counterflow burner in the stack.
The heat extraction inside the RCFB is analyzed for different locations of the heat exchangers. The operational temperature of the heat exchanger is varied from 300 K to 1300 K, in steps of 200 K. The fluid flow direction on the upper burner is directed in CW direction whereas the bottom one is set up with CCW fuel flow (Fig. 4.3). A heat conducting wall is made between the two burners to prevent intermixing of the reactant and the product of each burner with another one. This setup together produces the effect of a reciprocal counterflow porous burner, where the inlet and outlet of the burner is switched in short time intervals (in the order of 100 s). This modelling approach results in flame stabilization inside the porous burner in symmetrical positions, as shown in Fig. 4.3.

![Burner with clockwise flow](image1)

**Burner with clockwise flow**

![Burner with counter-clockwise flow](image2)

**Burner with counter-clockwise flow**

*Figure 4.3* Thermal profile of the porous burners with CW and CCW flow. Heat exchangers B-B’ are operating at 900 K. Temperature contours for the CW and CCW burners represent similar thermal profiles resembling the steady state operation of a reciprocal flow porous burner with a short switching cycle. The superficial velocity is 0.36 m/s. Equivalence ratio is 0.7.
4.2. Temperature Distribution and Energy Extraction Efficiency

The RCFB design is studied for the maximum flame temperatures with silicon carbide and alumina separation wall. Type of fuel and the superficial velocity of the fuel-air mixture mainly govern the flame temperature in filtration combustion. However, for numerical simulation the choice of chemical mechanism used to simulate the oxidation plays an important role in deciding the maximum flame temperature. Figure 4.4 shows the plot of maximum temperature of the porous flame as a function of equivalence ratio. The nature of variation of maximum flame temperature with equivalence ratio remains similar to the results published for CFB filled with porous medium [50].

![Maximum flame temperature with variation of equivalence ratio](image)

**Figure 4.4** Maximum flame temperature with variation of equivalence ratio. Superficial velocity is maintained at 0.36 m/s.
For temperature analysis, the RCFB burner is simulated for the equivalence ratio ranging from 0.4 to 1.0. RCFB design with silicon carbide separation wall shows the maximum flame temperature ranging from 1380 K at $\phi = 0.4$ to 1560 K at $\phi = 1.0$. However, for burner configuration with alumina separation wall, the maxima in the porous flame temperature is higher than that compared to the one with silicon carbide separation wall. Using alumina, the peak flame temperature predicted by the numerical simulation ranges from 1440 K to 1595 K for the same range of equivalence ratio. This rise in the peak temperature of the filtration flame for the design using alumina when compared to the configuration with silicon carbide separation wall is mainly due to higher thermal conductivity of the later. With a higher thermal conductivity, silicon carbide ($k_{sc} = 90$ W/mK, $k_a = 15$ W/mK), conducts more heat from the flame zone to the other parts of the combustor, thus, distributes the heat generated from the hotter flame to the colder regions of the combustor.

The heat exchanger A-A’ is used for heat extraction at 900 K. The results (Fig. 4.5) shows monotonic increase in efficiency as the equivalence ratio is increased. This is mainly governed by the rise in the maximum flame temperature with increase in equivalence ratio of the reactant mixture. Similar behavior is obtained for the burner under both the configurations. The results indicate lower extraction efficiency for the burner with silicon carbide separation wall compared to the design using alumina. This is due to the lower maximum flame temperature of the burner with silicon carbide separation wall as compared to the burner with alumina wall. The entire numerical analysis is conducted with superficial velocity of fuel-air mixture at 0.36 m/s.
Figure 4.5 Variation of extraction efficiency as a function of equivalence ratio. Energy is extracted with heat exchanger A-A’ operating at 900 K.
Thermal profile inside the porous burner is shown in Fig. 4.6. The temperature distribution inside the RCFB with silicon carbide separation wall shows the flame region as the hot spot. The temperature distribution in the post flame region is uniform with average temperature significantly lower than the flame. While in the design with alumina separation wall, thermal distribution inside the burner reflects a high-temperature region in the post-flame zone. In addition, the flame temperature is like that of the post flame region. This is mainly due to the difference in thermal conductivity of the two ceramics. For silicon carbide higher thermal conductivity \((k_{sc} = 90 \text{ W/mK})\) distributes the heat from the flame to the pre-flame region as well as to the post-flame region. With lower thermal conductivity, alumina separation wall \((k_a = 15 \text{ W/mK})\), restricts heat conduction inside the burner, resulting in a greater patch of high-temperature zone.

![Thermal burner maps for various heat exchanger positions (CCW burner is shown). Superficial velocity - 0.36 m/s, equivalence ratio is maintained at 0.7](image)

**Figure 4.6** Thermal burner maps for various heat exchanger positions (CCW burner is shown). Superficial velocity - 0.36 m/s, equivalence ratio is maintained at 0.7
The heat extraction is analyzed for three different heat exchanger positions for the burner with silicon carbide separation wall. Out of all six positions of heat exchangers, three positions of heat exchangers are studied based on the efficiency trends reported previously [50]. For the heat exchanger at A-A’, the burner and heat exchanger assembly show heat extraction efficiency of 69 % when the energy is extracted at 300 K (Fig. 4.7a). However, the efficiency drops to 55 % when the extraction temperature is set at 1300 K. For heat exchanger at position B-B’, the extraction efficiency curve shows a larger dip than the previous one. At this position, the energy extraction efficiency has its maximum value of 67 % at 300 K and decreases monotonically to 39 % at 1300 K. However, the heat extraction efficiency curve for the heat exchanger positioned at C-C’, proves it to be the least efficient position. At this position, numerical simulation predicts flame blow-off for heat extraction from the RCFB below 700 K. For heat extraction at position C-C’, the extraction efficiency drops from 30 % at 700 K to merely 4.2 % at 1300 K.

Figure 4.7 (b) shows the extraction efficiency when alumina separation plate is used in the burner system. For the burner system with alumina separation wall, extraction efficiency reaches 86 % when energy is harvested using the heat exchanger at position A-A’ maintained at 300 K. Nevertheless, thereafter the value decreases to 69 % monotonically when the extraction temperature is increased to 1300 K. The burner design shows similar trend when the heat exchanger is placed at B-B’. At this position numerical prediction for heat extraction starts at 78 % when harvested at 300 K. Following the similar trend, the extraction curve dips to 59 % when the heat exchangers are kept at 1300 K. For the extraction from position C-C’, numerical simulation predicts the position to be least effective in comparison to those already analyzed in the article. At this position, the heat extraction is 24 % efficient when the heat exchanger is set a300
K. The increase in heat extraction temperature further reduce the efficiency of the system reducing it to 11.4% at 1300 K.

RCFB design with silicon carbide separation extracts heat at an efficiency slightly lower than that with the alumina separation wall under similar conditions. This is governed by the maximum flame temperature achieved in the burner design. Since, the maximum temperature of the flame in porous medium for silicon carbide separation wall is lower than that of the alumina separation plate, therefore, the extraction efficiency is reduced. For silicon carbide separation wall, numerical data indicates better performance of heat exchanger at A-A’ for the extraction temperatures from 300 K to 1300 K. The trend remains similar for the design with alumina separation wall. Thermal conductivity of the separation wall plays a significant role in deciding the maximum flame temperature together with the length of high-temperature region. With higher thermal conductivity, silicon carbide tends to transfer heat from the flame zone to the relatively colder zones in the burner, thus, reducing the maximum flame temperature. This dissipation of heat also reduces the length and average temperature of high-temperature zone in the RCFB. It can be seen that, for high extraction efficiency, the heat exchanger should be located in upstream direction to porous flame, and in close vicinity to the flame.
Figure 4.7 Heat extraction efficiency of the burner with (a) silicon carbide and (b) alumina separation wall. Superficial velocity is equal to 0.36 m/s. Equivalence ratio is 0.7.
4.3. Effect of Superficial Velocity

Superficial velocity is known to be one of the key parameters for filtration combustion. Not only it affects the flame temperature but also the exhaust gas temperature and the flame location. A high superficial velocity is known to push the flame towards the exhaust while simultaneously increasing the flame temperature. Hence, it becomes significant to study the burner configuration for the energy extraction efficiency with varying superficial velocity. Figure 4.8 shows the variation of heat extraction efficiency as a function of extraction temperature when the superficial velocity is varied in a range of 0.16 m/s to 0.76 m/s. The increase in velocity increases the mass flow rate of the fuel-air mixture, resulting in rise in maximum flame temperature. This increases the extraction efficiency. However, increasing the mass flow rate enhances the average temperature of the exhaust gases at the outlet. This raises the losses in the burner. For silicon carbide separation wall, increasing the superficial velocity from 0.16 m/s to 0.36 m/s increases the heat extraction efficiency at extraction temperatures ranging from 300 K to 1300 K. However, increasing the superficial velocity beyond 0.36 m/s shows a reverse trend. Therefore, for silicon carbide separation wall, the RCFB gives optimum performance at superficial velocity of 0.36 m/s.

For the burner design with alumina separation wall, RCFB shows a mixed trend for heat extraction efficiency. For lower extraction temperatures i.e. from 300 K to 900 K, increase in the superficial velocity of the methane-air mixture results in decrease in heat extraction efficiency. This is mainly due to a larger fraction of the energy lost to the surroundings through hot exhaust gases. Lower thermal conductivity of alumina and shorter residence time of the hot products reduces the heat recuperation between the exhausts and the reactants. The phenomenon becomes significant when the mass flow rate of the fuel mixture entering the burner increases. However, at higher extraction temperatures (1100 K-1300 K), a greater mass flow rate comes with the
advantage of increased maximum flame temperature. This helps in harvesting more energy from the flame, consequently the efficiency of extraction increases. For alumina separation wall, numerical simulation predicts heat gain at 1300 K to be 68 % efficient, when the superficial velocity is maintained at 0.36 m/s. At this velocity, system performs most efficiently for high-temperature heat extraction. For superficial velocity of 0.16 m/s, numerical simulation predicts that the RCFB harvests energy at 42 % when heat exchangers operate at 1300 K. Whereas, this efficiency value is approximately 62 % for the case with superficial velocity of 0.56 m/s. On increasing the velocity further to 0.76 m/s extraction efficiency value drops further to 39 % making it to be the least performing choice.
Figure 4.8 Heat extraction efficiency as a function of superficial velocity for (a) silicon carbide and (b) alumina separation wall. Heat exchangers A-A’ are operating at 900 K. Equivalence ratio is 0.7.
4.4. Comparative Analysis of CFB and RCFB

The heat extraction efficiency of the RCFB is compared with that of RFB and CFB [50]. For numerical simulations, RCFB is modified to RFB by changing the thermal conductivity of the separation wall from a conducting material to an insulator. The results show that for all the three types of excess enthalpy burners studied here (CFB, RFB and RCFB), the heat extraction efficiency drops with an increase in heat extraction temperature. The numerical study predicts an efficiency of 69 % for CFB when heat extraction is performed at 300 K. However, upon increasing the extraction temperature to 1300 K, the efficiency drops to 62 %. For RFB this decrease is in the range of 89 % to 70 % for the studied range of extraction temperature. The extraction efficiency results for RFB obtained using the present numerical model is compared with that published by Yao and Saveliev [52]. The comparison between the two results for RFB shows a similar trend i.e. decrease in the extraction efficiency with increase in energy extraction temperature. The comparison between the RFB result and that of [52] also shows good agreement for extraction at high temperatures. However, for low extraction temperatures, i.e., below 800 K, the data shows some deviation. This is mainly due to the fact that the RFB results published in [52] have been simulated using a 1-D model however, the present investigation is conducted using a 2-D model. This trend of decreasing extraction efficiency remains unchanged for the newly developed RCFB as well. The results indicate that the energy extraction efficiency decrease from 86 % to 76 % when the extraction temperature is increased from 300 K to 1300 K. Numerical simulation predicts that the maximum efficiency of energy extraction for RCFB is higher than that of RFB and CFB within the studied range of heat extraction temperatures. One of the primary reason for higher energy extraction efficiency for RCFB than that of CFB is higher maximum flame temperature.
Figure 4.9 shows the energy extraction efficiency of RCFB and CFB for a superficial velocity of 0.36 m/s when operated at $\phi = 0.7$.

**Figure 4.9** Comparison of extraction efficiency for RCFB, RFB and CFB [50] for alumina separation wall and optimal heat exchanger position. Superficial velocity of fuel-air mixture is maintained at 0.36 m/s. Equivalence ratio is 0.7.
In the last few decades, many researchers [48,53,54] have investigated NO formation in porous medium combustion. NO formation in porous medium combustion is categorized into three major mechanisms namely thermal NO mechanism (Zeldovich mechanism), prompt NO mechanism (Fenimore mechanism) and N$_2$O intermediate mechanism. The following section describes these three mechanisms in detail.

### 5.1. Thermal NO

The thermal NO mechanism also known as Zeldovich mechanism consists of chemical reactions mentioned below:

\[
\text{N}_2 + O \rightleftharpoons NO + N \quad (5.1)
\]

\[
N + O_2 \rightleftharpoons NO + O \quad (5.2)
\]

\[
N + OH \rightleftharpoons NO + H \quad (5.3)
\]

When reaction temperature rise to 1700 K or more, these reaction gain dominance over other pathways for NO formation. Thermal NO is generated inside the burner wherever there is a local hot spot inside the burner. In PMC, the thermal interaction between the solid matrix and the fluid helps to dissipate the heat energy, thus, reduces the thermal NO formation. Generally, for PMC this mechanism is active in the flame region.
5.2. Prompt NO

Fenimore [55] was the first researcher to explore prompt NO generation, hence this mechanism of NO formation is also known as Fenimore mechanism. Prompt NO formation is activated at combustion temperatures lower than that required by thermal mechanism. This mechanism is sensitive to the hydrocarbon concentration in the reactant mixture. Prompt NO generation is active near the flame region inside a porous medium burner. This mechanism is an important pathway for NO formation during low temperature combustion. The reactions leading to NO generation through prompt mechanism are shown below:

\[
\begin{align*}
CH + N_2 &\rightleftharpoons HCN + N \quad (5.4) \\
C + N_2 &\rightleftharpoons CN + N \quad (5.5) \\
HCN + O &\rightleftharpoons NCO + H \quad (5.6) \\
NCO + H &\rightleftharpoons NH + CO \quad (5.7) \\
NH + H &\rightleftharpoons N + H_2 \quad (5.8) \\
N + OH &\rightleftharpoons NO + H \quad (5.9)
\end{align*}
\]

5.3. N\textsubscript{2}O-intermediate Pathway

\(\text{N}_2\text{O}\) is generally used as a combustion agent in military and racing car for a sudden increase in the speed. \(\text{N}_2\text{O}\) often known as laughing gas is a major green house gas. Major concentration of \(\text{N}_2\text{O}\) is generated in the flame zone and are converted to NO. The set of reactions for this mechanism are:

\[
\begin{align*}
\text{N}_2\text{O} + M &\rightleftharpoons \text{N}_2 + O + M \quad (5.13) \\
NH + NO &\rightleftharpoons \text{N}_2\text{O} + H \quad (5.14) \\
NCO + NO &\rightleftharpoons \text{N}_2\text{O} + CO \quad (5.15)
\end{align*}
\]
5.4. Numerical Model with Detailed Chemistry

The 30 species mechanism adopted in the previous chapters results in fast computations for thermal energy transport in CFB and RCFB. However, it is not adequate to analyze the NO\textsubscript{x} formation in the flame. PMC leads to NO generation, which is considered as an environmental pollutant. Hence, it is important to understand the NO generation pathways quantitatively. A detailed chemical mechanism needs to be used to quantify the NO formation during combustion in porous medium burners. Therefore, GRI 3.0 mechanism is used for the simulation. This mechanism includes 53 species and 325 elementary reactions. The comparison of temperature profiles inside the burner for 30-species mechanism and GRI 3.0 mechanism is shown in Fig 5.1.

Figure 5.1 Comparison of temperature distribution of CFB filled with porous medium using 30-species chemical mechanism and GRI 3.0. Superficial velocity of fuel-air mixture is 0.36 m/s and equivalence ratio is 0.7.
5.5. Validation of NO$_x$ Generation Model

The quantity of NO$_x$ generation predicted by the numerical simulation needs to be compared with previously known results. For this purpose, the porous medium burner used by Kennedy et al. [48] is simulated using GRI 3.0. The NO$_x$ generated from the combustion is compared with the results published in [48] as shown in Fig. 5.2. The formulation of NO$_x$ emission index (EI) is done in accordance with Kennedy et al. [48]. Figure 5.2 shows that for ultra-lean regime NO$_x$ EI is insensitive to the change in equivalence ratio. However, for $\phi$ > 0.4 NO$_x$ EI increases rapidly. Since the flame temperature is below 1700 K, the major production of NO$_x$ happens through the prompt mechanism.

![Figure 5.2](image)

**Figure 5.2** Comparison of NO$_x$ generation with change in equivalence ratio predicted by the model and experiments reported by Kennedy et al. [48]. The numerical simulation is conducted on the burner design studied in [48].
5.6. **Routes of NO Formation**

The contribution of various NO generation pathways are studied and compared against the NO concentration predicted by GRI 3.0 mechanism. This investigation is performed using CFB in two cases: (i) CFB operated at $\phi = 0.4$ and (ii) CFB operated at $\phi = 0.7$.

For CFB operated at $\phi = 0.4$, the NO concentration predicted by the numerical model is little less than 1.0 ppm (0.75 ppm). Of all the reaction pathways studied, NO formation by $N_2O$ intermediate mechanism serves as the most dominating pathway. The summation of all the NO generation pathway leads up to the NO concentration of 1.1 ppm. However, simulating the case with GRI 3.0 mechanism shows a decrease in the concentration of NO. This is mainly because part of the NO formed inside the burner transforms to other chemical species before getting out of the burner. This is shown in Fig. 5.3 (a).

For CFB operated at $\phi = 0.7$, the concentration of NO formed inside the burner increases to 5.5 ppm. Similar to the previous case, $N_2O$ intermediate reaction pathway remains the major mechanism for NO generation. Numerical simulation predicted an increase in the concentration of NO generated from prompt mechanism. This is mainly due to the increase in the hydrocarbon concentration in the reactant mixture. The net summation of the NO concentration formed by all the three pathways overshoots the value predicted by GRI 3.0 mechanism as shown in Fig. 5.3 (b). This is attributed to the fact that direct summation of the NO concentrations formed using three pathways doesn’t take into account the conversion of NO to other chemical species before exiting the burner. This conversion is taken into consideration in GRI 3.0 mechanism.
Figure 5.3 Contribution of NO generation mechanisms active in the CFB design with alumina separation wall operating with (a) $\phi = 0.4$ and (b) $\phi = 0.7$. The superficial velocity of reactants is 0.36 m/s.
5.7. Variation of NO\textsubscript{x} Generation with Equivalence Ratio for CFB and RCFB

The amount of NO\textsubscript{x} generated during the combustion of methane-air mixture is studied for a CFB and RCFB. Figure 5.4 shows the variation of NO\textsubscript{x} concentration as a function of equivalence ratio is investigated separately for CFB and RCFB when heat energy is extracted from the burners. This variation is investigated for two temperatures of heat extraction namely 500 K and 1100 K. Figure 5.4 (a) shows the variation of NO\textsubscript{x} concentration for CFB and RCFB when heat is extracted from the burner. The NO\textsubscript{x} concentration values for the CFB and RCFB are nearly similar. The maximum flame temperature of RCFB is little more than that of CFB. This small increase in the maximum flame temperature is insufficient to cause significant increase in the NO\textsubscript{x} generation through thermal NO mechanism. Furthermore, same reactant composition leads to same amount of NO\textsubscript{x} generated through prompt mechanism in both the burners. This trend remains unchanged when the heat extraction is performed at 1100 K in both CFB and RCFB as shown in Fig 5.4 (b). However, comparison of Figs. 5.4 (a) and (b) shows that NO\textsubscript{x} formed in the burners for the case when heat is extracted at 1100 K is lesser than that when heat is extracted from the burner at 500 K. This trend remains same for CFB and RCFB. This is attributed to the decrease in the maximum flame temperature which leads to decrease the NO generated through the thermal pathway. This can also be seen in Fig. 5.5(a), where NO\textsubscript{x} generation is studied as a function of heat extraction temperature in CFB and RCFB. In Fig. 5.5(b), it is evident that the increase in the heat extraction temperature decreases the maximum flame temperature for both CFB and RCFB. Furthermore, Fig. 5.5 shows that the trend for decrease in NO\textsubscript{x} concentration as a function of heat extraction temperature varies the same way as the maximum flame temperature. This is applicable to both CFB and RCFB.
Figure 5.4 Variation of NO$_x$ generation with equivalence ratio for CFB and RCFB when heat extraction is performed at (a) 500 K and (b) 1100 K using heat exchanger at position P-1.
Figure 5.5 (a) NO\textsubscript{x} generation is studied as a function of heat extraction temperature for CFB and RCFB. (b) Maximum flame temperature is studied for CFB and RCFB designs at various heat extraction temperatures. Heat exchanger placed at position P-1 is operated. Reactants are fed at 0.36 m/s maintaining $\phi = 0.7$
5.8. CO Emission from CFB and RCFB

CO emission from CFB and RCFB is investigated for CFB and RCFB. Similar to NO\textsubscript{x} generation, CO emission also decreases as the heat extraction temperature is increased, as shown in Fig. 5.6. The numerical simulation predicts that CO concentration decreases from 16 ppm to 3.8 ppm for CFB with alumina separation wall, when the heat extraction is performed between 300 K and 1300 K. The trend of decreasing CO concentration, with increase of extraction temperature remains unchanged for RCFB. CO generation is also studied as a function of equivalence ratio when the heat extraction is performed at 500 K and 1100 K for CFB and RCFB (Fig. 5.7). Numerical simulation shows that for both CFB and RCFB the concentration of CO increases as a function of equivalence ratio. For the case with CFB when heat is extracted at 500 K, numerical simulation predicts that CO increases from 0.6 ppm to 38 ppm when the equivalence ratio is increased from 0.3 to 1.0. However, for energy extraction at 1100 K, the variation remains between 0.6 ppm to 7 ppm. The increase in CO with equivalence ratio is mainly due to increase in the methane concentration in the fuel. Also, increasing the equivalence ratio leads to decrease in the oxygen concentration. This prohibits oxidation of CO to CO\textsubscript{2} leading to an increase in the concentration of CO. The variation of CO concentration for RCFB remains in the similar range as that of CFB. This is due to similar operating condition for the burners.
Figure 5.6 Variation of CO concentration as a function of energy extraction temperature for CFB and RCFB. Both the burners are simulated for the design with alumina separation wall. The heat exchanger at location P-1 is operated. Reactants ($\phi = 0.7$) are fed in the burner at 0.36 m/s superficial velocity.
Figure 5.7 Variation of CO concentration for CFB and RCFB with equivalence ratio. The computational model is simulated for the case when energy extraction is performed at 500 K and 1100 K for the two burners. Superficial velocity of fuel-air mixture is kept at 0.36 m/s.
CHAPTER 6

NO\textsubscript{x} Minimization in Staged Combustion Using Rich Premixed Flame in Porous Medium

This chapter presents research regarding NO\textsubscript{x} emissions from the oxidation of rich methane-air mixtures in staged combustion when the exhaust from the primary flames have been cooled down to 1000K. The primary flame is a rich premixed flame in the porous medium. The flame is stabilized in the range of equivalence ratios from 1.1 to 1.7 using upstream heat extraction. The products of the primary flame are rich in the partial oxidation and reforming products such as CO, H\textsubscript{2}, and CH\textsubscript{4}. Due to the self-regulated heat losses from the flame the maximum flame temperature of the primary flame remains close to 1700K. The rich flame environment and low flame temperatures limit NO\textsubscript{x} formation in the primary flame. The NO\textsubscript{x} emission index of the primary flame shows a maximum at the equivalence ratio of 1.1 and reduces for richer mixtures. The products of the primary flame are burned in the secondary non-premixed flame. The products could be intercooled to reduce temperature and minimize NO\textsubscript{x} formation in the secondary non-premixed flame. The emission index of the secondary flame increases with the equivalence ratio. Variation of combined emission index shows a minimum value at the equivalence ratio of 1.2. The trend remains consistent with the intercooling of the primary flame products.

6.1. Background

Recently, increasingly stringent air emission regulations were introduced that target pollution emissions from combustion systems. Significant reductions in atmospheric discharge of nitrous oxides are expected to be implemented in a near future. With a substantial amount of work
to analyze the concentration and pathway for generation of NO\textsubscript{x} in premixed, non-premixed and partially premixed flames, there is a great interest in developing new concepts of combustion such as mild combustion [56,57], catalytic combustion [58,59] and staged combustion [60,61].

Staged combustion naturally occurs in partially premixed flames created by adding air in less than stoichiometric proportions to the fuel stream prior to the fuel injection into an oxidizer stream. The partial premixing takes place either due to deliberate addition of air to fuel line for enhancing stability of flame or due to flame liftoff from the fuel injector. In turbulent regimes, partial premixing often occurs because of local extinction and reignition events. Several studies [62-69] analyzed the behavior of partially premixed flames. Libby and Economos [63] predicted a finitely thick reaction zone for partially premixed flames in contrast to a thin reaction sheet. Yamaoka and Tsuji [69] performed experimental investigations into the structure of partially premixed flames revealing the presence of two separate reaction zones depending on the stoichiometry of the premixed stream. Several experimental and analytical investigations [62,65,66] have been reported to understand the dynamics and extinction limits of the partially premixed flames. Based on the fuel types, NO\textsubscript{x} emissions have been reported to increase with partial premixing caused by reduced radiative heat loss [70] or due to availability of oxygen and radicals responsible for prompt NO\textsubscript{x} production over a larger region [71]. With increasing partial premixing NO\textsubscript{x} emission can be decreased by reducing the residence time of the fuel at the flame exit [70,72]. Several researchers have been working to report and analyze the NO\textsubscript{x} generation [73,74]. Gore [75] reported minimum NO\textsubscript{x} emissions for laminar flames of methane-air at an optimal level of partial premixing. Gore and Zhan [76] found minimum NO\textsubscript{x} emission index (EI NO\textsubscript{ox}) at equivalence ratio of 2.0. Kim et al. [77] reported minimum EI NO\textsubscript{x} at equivalence ratio of 2.2 for ethane-air flames. Some of the recent developments in the field of partial premixed
flames reported the effects of partial premixing on NO\textsubscript{x} emissions from laminar and turbulent flames [70,72]. Driscoll et al. [72] found that NO\textsubscript{x} emission properties of turbulent partially premixed methane-air flames are complicated and do not follow a definite pattern. Williams and co-workers [78] reported a non-monotonic behavior for EI NO\textsubscript{x} as a function of fuel stream equivalence ratio, qualitatively in agreement with earlier experimental observations. The structure of a partially premixed flame is controlled by the fuel stream equivalence ratio \( \phi \). A single flame front forms at the limits of \( \phi \rightarrow 1 \) and \( \phi \rightarrow \infty \). At the intermediate range of the equivalence ratios the primary premixed flame and the secondary non-premixed flame are established in the proximity and interact through thermal and chemical mechanisms. The close interaction of flame zones limits the individual control of the flame zones. At the same time, a separation of the primary and secondary flames is difficult to achieve for free homogeneous flames. An alternative possible concept involves using heterogeneous porous combustion to stabilize the primary premixed flame. Rich premixed flames in porous medium are known to have ultralow NO\textsubscript{x} emissions. In addition, porous flames can be easily controlled by a heat extraction through radiation and convective cooling. Thus, a combination of the primary premixed porous flame and the secondary non-premixed heterogeneous flame can be a useful way of minimizing NO\textsubscript{x}.

The present work, studies staged combustion of methane in a rich premixed porous flame followed by a non-premixed homogeneous flame. The experimental studies are performed using the porous flame with heat extraction controlled by convective loses to cooling water. The numerical study considers the coupled models of premixed porous flame and non-premixed homogeneous flame. The studied combustion process is very critical for reduction of emissions in variety of gas-fired equipment widely used across the industrial and commercial applications.
such as process heating, direct-fired baking ovens, material thermoforming, surface treatment, lamination and others.

6.2. Experimental Setup

The schematic of the experimental setup is shown in Fig. 6.1. This experimental setup was created by researchers from Gas Technology Institute, Des Plaines, IL, USA at their facility. The major components of the setup are two-stage burner, flow control system, water cooled flame jacket, monitoring and control systems, and Horiba PG-250 gas analyzer.

The body of the burner is made from 316 stainless steel. Its inner walls are covered with the high-temperature insulation (Kaowool 3000). The central combustion cavity is filled with 5-mm-diameter aluminum oxide pellets. The pellet bed rests on the water cooled flow distribution plate. Heat transfer to the plate positioned upstream from the flame zone stabilizes the premixed combustion in porous media and prevents flash back. Rich methane-air mixtures are supplied using the flow control system. The system is designed to produce a wide range of gaseous mixtures by continuously blending metered mass flow rates (using Brooks mass flow controllers) of individual components such as natural gas and air.

The flue gas coming out of the porous medium is rich in the unburned fuel and is made to go through the water cooled jacket for cooling it down. The water cooled jacket is fabricated from the 6.35 mm outer diameter copper tubing, rolled in the shape of a right cylinder with the conical head. Water flow to the jacket is connected countercurrent to the flue gas stream. The height of the water jacket is 15 cm. An R type thermocouple (273 K – 1723 K) is used at a height of 10 mm above the porous burner. Inlet and exhaust temperatures for the water jacket are recorded using T type thermocouples (73 K – 673 K). The experiments with cooled and uncooled flue streams were
conducted. The flue gas coming out of the jacket is reignited to form secondary non-premixed flame. The conical quartz chimney is applied to prevent non-premixed flame disturbances by the outside air currents.

**Figure 6.1** Experimental set-up (all dimensions are expressed in millimeters).
6.3. Numerical Model

Half section of the two-stage flame is modeled to study the behavior of the primary premixed porous flame and the secondary homogeneous non-premixed flame. The computational domains are shown in Fig. 6.2. The exhaust of the porous flame is fed to the secondary non-premixed flame through a nozzle keeping the mass flow rate constant. The combustion of premixed methane-air mixture has been studied in a domain filled with porous medium and fitted with a tubular heat exchanger (Fig. 6.2a). The passage of free stream of air to the non-premixed flame is modeled using co-flow region at the bottom of the domain (Fig. 6.2b). The boundary conditions imposed at the inlets are constant velocity-inlet. The co-flow region is given pressure-inlet boundary condition to help the diffusion of free stream of air to sustain the flame. The outlet of the domain is modeled with pressure-outlet boundary condition. The vertical boundaries on the left are described by the symmetry boundary conditions. Pressure-inlet boundary condition is set on the right vertical side of the computational domain for the non-premixed flame to allow free air entrainment.
The secondary non-premixed flame is intended to burn the products of the rich porous flame. The products of the porous flame are fed as a fuel to the secondary non-premixed flame to react with oxygen from the stream of air supplied through the bottom of the flame. The 2-D steady state CFD analysis is performed using computational domain shown in Fig. 6.2 (b). The product temperature is varied to account for intercooling of the product stream in the area covered by the water jacket. The maximum temperature corresponds to the uncooled exhaust of the primary premixed flame. The minimum temperature of 300 K is set by the temperature of the cooling water. GRI 3.0 mechanism is used for the present analysis [79]. It consists of 53 species and 375 reactions. This mechanism also incorporates NO\textsubscript{x} chemistry.
6.4. Results and Discussion

Computational models are studied to analyze the oxidation of ultrarich premixed mixture of methane air in a range of equivalence ratios $1.1 < \phi < 1.7$. The products of the porous flame could not be burned in the secondary flame for $\phi < 1.1$. Upon cooling down the porous flame exhaust to 1000 K before burning again, the lower flammability limit decreases to $\phi = 1.4$.

Figure 6.3 gives an idea about the temperature distribution inside the porous flame and the secondary flame. For the porous flame, as the premixed fuel mixture get richer, there is a downstream displacement of the flame. The increase in the concentration of the H$_2$, CO and unburnt CH$_4$ causes the maximum flame temperature to be higher in the secondary flame than in the porous flame. In addition, cooling down the exhaust of the porous flame to 1000 K, reduces the maximum flame temperature approximately by 400 K. The height of the secondary flame increases, when the fuel equivalence ratio is increased.
Figure 6.3 Temperature distribution inside (a) secondary flame; (b) porous flame.
Figure 6.4 shows the concentration of the major species at the exit of the porous flame for the range of equivalence ratios from 1.1 to 1.7. The concentration of CO increases with equivalence ratio but that of CO$_2$ decreases. Partial oxidation of rich methane-air mixture proceeds with scarcity of O$_2$. As a result, only a limited amount of CO is oxidized to CO$_2$. Significant amount of water is formed, during initial stages of methane oxidation, but as the unburned methane concentration increases, water reacts with it, forming more CO and H$_2$. The concentration of CH$_4$ is seen to increase with increase in equivalence ratio, indicating an increase in concentration of the unreacted fuel. Concentration of the major species recorded at the outlet of the porous burner are in agreement with the experimental results of Kennedy et al. [80]. These chemical species together with N$_2$ form about 99% of the exhaust of the porous flame.

![Figure 6.4 Concentration of the major chemical species in the exhaust stream for the rich premixed porous flame in a range of equivalence ratios from 1.1 to 1.7.](image)
Figure 6.5 (a) shows the evolution of NO profile along the axis of the second stage combustor. Methane diluted with nitrogen (CH₄/N₂=51.5:48.5 by mass) is burnt in the diffusion flame. The fuel flow rate is constant at 0.264 SLM. GRI 3.0 mechanism overpredicts the NO formation when compared with results of Smooke et al. [81]. The experimental results show that NO formation starts 10 mm from the fuel nozzle and increases up to 30 mm before a gradual decrease thereafter, until 60 mm. The simulation results show a steep increase in the NO concentration near 30 mm axial distance, which signifies the flame front. Numerical data predicts no generation of NO in the post-flame region. The decrease in NO concentration after 40 mm is mainly due to the dilution of NO generated in the flame with other combustion products. Figure 5 (a) shows that the NO generation trend predicted by the model traces patterns similar to that reported by Smooke et al. (1996). Numerical model predicts NO formation near the flame region with no generation in post flame region. This is in accordance with the findings of Smooke et al. [81].

Figure 6.5 (b) gives the EI NOₓ of the second stage burner, when it is used to burn rich premixed methane-air mixture. The co-flow combustion system studied for the present analysis is capable to oxidize the entire fuel mixture to CO₂. EI formulation is done in accordance with the methods used by Kennedy et al. [48]. The fuel flow rate is maintained at 0.30 SLM with 8 mm nozzle diameter. The results are compared with the experimental investigation of Lee et al. [82]. The experimental readings show an increase in EI NOₓ from 0.87 g/kg at φ = 1.0 to 1.3 g/kg at φ = 1.5. Numerical simulation captures the similar trend. The simulation results show EI NOₓ increasing from 1.12 g/kg at φ = 1.0 to 1.8 g/kg at φ = 1.5. However, for richer mixtures, i.e. φ > 2.0, numerical model shows a greater deviation from experimental values. The experimental data indicates 1.6 g/kg of EI NOₓ at φ = 2.0 whereas, GRI 3.0 predicts it as 2.1 g/kg.
Figure 6.5 (a) NO profile evolution along the axis of a diffusion flame; (b) EI NOx generated by burning rich premixed methane-air mixture as a function of equivalence ratio. The data is compared with experimental results.
Burning a rich premixed fuel mixture in two stage combustors results in three types of efficiencies. The first stage efficiency \( \eta_1 \) is defined as the ratio of the heat extracted from the flame by the upstream heat exchanger and water cooled jacket to the energy supplied to its inlet at steady state. Similarly, the secondary efficiency \( \eta_2 \) is the ratio of heat released during the combustion in second reactor to the energy supplied at its inlet. The efficiency of reforming \( \eta_R \) is the ratio of the fuel energy available to the secondary flame in comparison to the first one. Mathematically the efficiencies are defined as:

\[
\eta_1 = \frac{Q_1}{\dot{m} \cdot LHV_1} \quad (6.1)
\]

\[
\eta_2 = \frac{Q_2}{\dot{m} \cdot LHV_1} \quad (6.2)
\]

\[
\eta_R = \frac{LHV_2}{LHV_1} \quad (6.3)
\]

where \( Q_1 \) denotes the power extracted from the primary premixed porous flame and \( Q_2 \) represents the power extracted from the secondary non-premixed flame. Equations (6.1,6.2) result in \( \eta_1 \eta_2 = 1 \).

Figure 6.6 represents the comparison of \( \eta_1, \eta_2 \) and \( \eta_R \) for the exhaust at two different conditions: the uncooled exhaust of the primary porous flame and the exhaust cooled to 1000 K. With the increase of equivalence ratio, the flame shows a downstream displacement in the porous combustor, thus, increasing its distance from the heat exchanger. Consequently, the heat extraction by the heat exchanger decreases. Furthermore, the absence of water jacket for uncooled exhaust causes its first stage efficiency value to be lesser compared to that for cooled exhaust, for the same equivalence ratio. For the cooled exhaust case, the first stage efficiency spans in the range of 40 % at \( \phi = 1.1 \) to 26 % at \( \phi = 1.7 \). However, the absence of cooling water jacket for uncooled
exhaust causes the efficiency value to drop for the uncooled exhaust. The downstream flame displacement causes a further reduction when the equivalence ratio increases from 1.4 to 1.7. At \( \phi = 1.7 \) the first stage efficiency value is found to be close to zero.

Burning rich premixed methane-air mixture in a porous flame produces significant concentration of unburnt CH\(_4\) with considerable concentrations of H\(_2\) and CO as combustion products. The concentration of these species collected at the outlet of the porous combustor increases as the equivalence ratio is increased. Significant concentration of H\(_2\) and CO together with unburnt CH\(_4\) increase the calorific value of the porous flame exhaust. This trend is evident from the variation of \( \eta_R \) plots as a function of equivalence ratio as shown in Fig. 6.6. The variation of \( \eta_R \) shows a steep increase (21 % to 41 %) until \( \phi = 1.4 \) followed by a gradual increase (41 % to 56 %) till \( \phi = 1.7 \) is reached. This variation is analogous to the evolution of CO as a function of equivalence ratio. Since the heat extraction using water jacket do not affect the calorific value of the fuel, so the variation of \( \eta_R \) remains unaltered in both the studied cases. With increase in equivalence ratio, the concentrations of H\(_2\), CO and unburnt CH\(_4\), increases in the exhaust of the porous flame. Since the energy content of the exhaust mixture is increased, burning it in the secondary flame liberates more energy. Burning uncooled porous exhaust result in secondary efficiency value ranging from 80 % to 96 % for 1.1 < \( \phi < 1.7 \). But by burning the cooled exhaust the secondary efficiency value gets reduced and the variation occurs in a range of 60 % to 78 %. For both cases, the secondary efficiency trace peaks up as the equivalence ratio is increased (Fig. 6.6).
Figure 6.6 Variation of energy efficiencies with equivalence ratio: (a) the exhaust of the primary flame is cooled to 1000 K; (b) the exhaust of the primary flame is uncooled
Figure 6.7 shows the variation in the EI NO, for 1.1 < φ < 1.7. The data obtained for the primary flame has been compared with Kennedy et al. [45]. For the premixed porous flame, a decrease in the NOx concentration is observed for high equivalence ratios. EI NOx decreases from 0.5 g/kg at φ = 1.1 to 0.092 g/kg at φ = 1.7. This decrease is due to the burning of methane in a rich environment i.e. in scarcity of oxygen. For 1.1 < φ < 1.7 the premixed mixture burns with temperature approximately 1700 K, making thermal NO generation mechanism, insignificant for the system. Kennedy et al. [45] concluded that the ultrarich combustion of methane-air mixture is dominated by C2 intermediate species as compared with C1 species. This suppression creates a shortage of CH radicals, which suppress the NO formation via prompt mechanism. Bachmaier et al. [83] observed similar behavior in his work.

For the secondary flame NOx generation is governed mainly by Zeldovich mechanism, which is very sensitive to temperature. With increase in the equivalence ratio for the porous flame, unburnt CH4 concentration increases at the flame outlet, together with H2 and CO. This result in higher maximum flame temperature for the secondary flame compared to the porous flame. Maximum flame temperature recorded is in the range of 2300 K. As a result, the NOx generation in the secondary flame starts becoming a stronger function of thermal NO generation mechanism compared to prompt mechanism. With increase in equivalence ratio of the premixed mixture at porous flame inlet, the maximum flame temperature increase in the secondary flame, leading to an increasing contribution of thermal mechanism of NO generation compared to the prompt mechanism. At φ = 1.2, the increasing dominance of the thermal mechanism becomes significant to the prompt generation pathway. However, for 1.2 < φ < 1.7, thermal mechanism becomes decisive for NOx generation. This leads to a minimum in the EI NOx data when plotted as a function of equivalence ratio. This minimum value corresponds to 0.71 g/kg at φ = 1.2. The EI NOx values
decrease from 0.77 g/kg to 0.71 g/kg when $\phi$ is increased from 1.1 to 1.2. Beyond $\phi = 1.2$, the emission index data shows a continuous increase to 1.18 g/kg for $1.2 < \phi < 1.7$. Gore [84] reported similar trend for the EI NO$_x$. The combination of porous flame and the secondary flame reduces the EI NO$_x$ approximately by 38 %, when compared with that for a co-flow flame. The behavior of EI NO$_x$ as a function of equivalence ratio, remains unchanged when the exhaust of the porous flame is cooled down before reburning in the secondary flame. However, due to decrease in maximum flame temperature during reburning the cooled exhaust of porous flame, thermal NO mechanism produces a lesser concentration of NO$_x$ at the combustor exit, when compared to the same formed during reburning of uncooled exhaust. However due to cooling, the exhaust of the porous burner could not be burned below $\phi = 1.4$. Overall, the EI NO$_x$ gets reduced approximately by 54 % by cooling the porous flame exhaust.
Figure 6.7 EI NO\textsubscript{x} generated by burning cooled and uncooled fuel mixtures at various equivalence ratios. The EI NO\textsubscript{x} data for the staged combustion is compared with that for homogeneous partially premixed flame under similar conditions. 38 % drop in EI NO\textsubscript{x} is achieved by using staged combustion. Cooling down the porous flame exhaust to 1000 K further reduces the EI NO\textsubscript{x} by 54 % when compared with the data for uncooled porous exhaust.
CHAPTER 7
Conclusions and Future Work

7.1. Conclusions

This thesis presents research related to high temperature heat extraction from excess enthalpy burners. A numerical investigation is conducted on three different types of excess enthalpy porous burners using Fluent 14.5. The three variants of excess enthalpy porous burners studied here are counterflow burner (CFB), reciprocal flow burner (RFB) and reciprocal counterflow burner (RCFB). The reciprocal counterflow burner is a novel type excess enthalpy burner designed by combining RFB and CFB. It works on the principle of periodic flow reversal in a CFB. The heat extraction efficiency is studied for extraction temperatures ranging from 300 K - 1300 K. The effect of heat recirculation is studied by changing the material of the wall separating the inlet and exhaust channel.

Numerical simulations are performed on the CFB for the equivalence ratio ranging from 0.4 to 1.0. The numerical simulation performed on CFB predicts an upstream flame displacement when the fuel equivalence ratio is increased from 0.4 to 1.0. The energy content of the flame and the internal heat regeneration is found to define the flame location. Flames with high energy content move closer to the burner inlet and lose its energy to the environment. However, those with lower energy content propagate deeper inside the burner and regenerate the heat internally through the solid porous matrix. It is found that the addition of a heat exchanger operating at high-temperature affects the flame stabilization. Placing a heat exchanger extracting energy at high temperature at positions P-1 and P-2 restricts the flame downstream the heat exchangers. Whereas for other heat exchanger positions the flame shifts downstream or upstream based on the
temperature of energy extraction. This behavior remains unchanged for the CFB design studied with alumina and silicon carbide separation wall.

The effect of heat recirculation is analyzed on the excess enthalpy burners, by changing the material of the wall separating the inlet and exhaust channel of the burner. Two types of material are chosen namely alumina \((k_a = 15 \text{ W/mK})\) and silicon carbide \((k_{sc} = 90 \text{ W/mK})\). The material with lower thermal conductivity recirculates lesser heat from the flame zone to the other parts of the burner. This results in a flame temperature of 1470 K in CFB design with alumina separation wall when the superficial velocity of the fuel-air stream is maintained at 0.36 m/s. Higher flame temperatures result in better extraction efficiencies when energy is extracted in the range of 900 K to 1300 K. A moderately higher thermal conductivity of silicon carbide reduces maximum flame temperature to approximately 1360 K. This decreases the efficiency of the energy extraction from the burner. Numerical simulation predicts position P-1 as the location for most efficient high-temperature heat extraction. This remains unchanged when the amount of heat recirculation inside the burner is varied. The numerical simulations predict 62 % energy extraction with alumina separation wall for heat exchanger at P-1 harnessing energy at 1300 K. Whereas with silicon carbide separation wall the CFB extracts 35 % heat under similar operating conditions. Similar investigation is performed on RCFB. For RCFB, the location of most efficient heat extraction remains position A-A’ which is analogous to position P-1 in CFB. Numerical simulations predict maximum heat extraction efficiency for RCFB design with alumina to be 78 % when the superficial velocity for fuel is maintained at 0.36 m/s. A comparison of high-temperature heat extraction using the most efficient heat exchanger position shows that RCFB operates with 15 % more efficiency than CFB. The primary reason is attributed to the greater maximum flame temperature in RCFB than in CFB under similar conditions.
The effect of the superficial velocity of the fuel-air mixture on the heat extraction efficiency is also studied. The increase in superficial velocity results in higher flame temperatures. However, increasing the reactants velocity also results in rise in the temperature of products leaving the burner. The results suggest 0.36 m/s as the optimum superficial velocity for both CFB and RCFB designs with alumina separation wall when these are used for high temperature heat extraction.

The results for NO\textsubscript{x} emission from the excess enthalpy burners studied here shows that the NO\textsubscript{x} estimation performed by post-processing the data overshoots that predicted by detailed chemistry mechanisms. This is because post-processing does not include the transformation of NO to other chemical species. Analysis of numerical results shows that for the combustion of ultralean fuel-air mixtures N\textsubscript{2}O-intermediate pathway is the major NO contributor. Whereas for lean fuel mixtures prompt mechanism is responsible for major NO formation. The results indicate that the NO\textsubscript{x} emission from CFB and RCFB decreases with increase in the heat extraction temperatures. This is mainly due to a decrease in the maximum flame temperature. The decrease in maximum flame temperature restricts NO generation through thermal NO generation mechanism.

NO\textsubscript{x} emission is also studied for the combustion of rich fuel mixtures using numerical and experimental approach. A novel approach for reduction of NO\textsubscript{x} emission from rich flame is considered. The main idea of this approach is to extract heat from the porous burner used to oxidize the rich methane-air mixture. Subsequently the exhaust from the porous burner is burned again in a diffusion flame after cooling down to 1000 K. The numerical simulations have been conducted to study the NO\textsubscript{x} emission from rich fuel mixtures (1.1 < \phi < 1.7) in staged combustion for two cases namely (a) burning the exhaust from primary flame in second stage and (b) burning the exhaust from the primary flame after cooling it down to 1000 K.
Initially the fuel mixture is burned in a porous burner having a heat exchanger operating at 300 K. This heat exchanger was placed 1 cm away from the inlet. Presence of heat exchanger helps to stabilize the flame in the porous reactor. The NO$_x$ concentration recorded at the exit of the porous burner shows a decrease as the equivalence ratio of the mixture is increased. This decrease is due to reduced maximum flame temperature and scarcity of oxygen in a rich burning environment. Upon reburning the exhaust of the porous burner in a secondary non-premixed flame, NO$_x$ concentration increases with increase in equivalence ratio. At the porous flame exhaust, concentration of H$_2$, CO and unburnt CH$_4$ increase with increasing level of premixing in the fuel mixture. This leads to high flame temperatures for the non-premixed flame. Elevated flame temperature in the secondary flame triggers the thermal NO generation mechanism together with the previously active prompt mechanism. At $\phi = 1.2$, the thermal NO generation mechanism becomes significant compared to the prompt mechanism. For $1.2 < \phi < 1.7$ the thermal NO generation mechanism gains dominance over the prompt generation pathway. Thus, a minimum is observed for the EI NO$_x$ when plotted as a function of equivalence ratio.

Cooling down the exhaust of the porous burner to 1000 K before reburning in the secondary flame, leads to a reduction in maximum flame temperature approximately by 400 K. This decrease hinders in thermal NO generation for the non-premixed flame, causing EI NO$_x$ to reduce from a value ranging between 0.8 g/kg - 1.18 g/kg to 0.40 g/kg – 0.55 g/kg when compared to the same equivalence ratio. However, cooling the exhaust of the porous burner decreases the lower flammability limit to $\phi = 1.4$. Thus, cooling down the porous burner exhaust results in approximately 54 % drop in the EI NO$_x$. The numerical results obtained in the research shows good agreement with the experimental data.'
7.2. Future Work

This thesis presents studies related to high temperature heat extraction from heat regenerating porous burners. It would be interesting to study these burners for fuels having low calorific values (e.g. gasifier gas, landfill gas, etc.) and investigate the high temperature heat extraction. This numerical study has been conducted using the solid-fluid thermal equilibrium. It would be exciting to study these burners using two-temperature model or solid-liquid non-equilibrium. These porous burners could also be studied in the high velocity regimes. It would be exciting to simulate these burners together with power generating modules to study the efficiency of the entire system. Insights into the CO and NO\textsubscript{x} emissions from these burners under low calorific value fuels would be of great interest. Another motivating research would be to understand the limit of operation for heat exchangers integrated with these burners when rich fuel mixtures are burned.
REFERENCES


NOMENCLATURE

\( C \) specific heat at constant pressure \( \varepsilon \) porosity
\( C \) coefficient of inertial resistance \( \eta \) efficiency
\( D_{ax} \) thermal dispersion in porous media \( \phi \) equivalence ratio
\( D_m \) diffusion coefficient of species \( \Phi \) viscous dissipation
\( D_p \) pellet diameter \( \rho \) density
\( E \) specific energy \( \sigma \) Stephan-Boltzmann constant
\( F \) radiative heat exchange factor \( \tau \) stress tensor
\( h_i \) molar enthalpy of species \( \mu \) mass averaged viscosity
\( h_v \) volumetric convective heat transfer coefficient \( \omega \) molar rate of production of species
\( J \) diffusion flux
\( k \) thermal conductivity
\( \dot{m} \) mass flow rate
\( S \) source term
\( T \) temperature
\( q_e \) rate of heat extraction by heat exchanger
\( v \) velocity
\( \nu \) diffusion velocity
\( p \) pressure
\( W \) molar mass
\( x \) distance along the burner axis
\( X \) mole fraction
\( Y \) mass fraction

Subscripts
\( LHV \) lower heating value
\( m \) mass flow rate
\( 1 \) Primary flame
\( 2 \) Secondary flame
\( \text{eff} \) effective
\( \text{f} \) fluid
\( \text{g} \) gas
\( \text{i} \) species
\( \text{in} \) interstitial
\( \text{o} \) ambient
\( \text{R} \) reformer
\( \text{rad} \) radiation
\( \text{s} \) solid
\( \text{su} \) superficial

Greek Symbols
\( \alpha \) coefficient for viscous resistance
Appendix A

Temperature form of energy equation

The energy equation

\[
\frac{\partial (\rho E)}{\partial t} + \nabla \cdot (v(\rho E + p)) = \nabla \cdot (k_{\text{eff}} \nabla T_g) - \sum_i h_{i,j} + \tau \cdot v
\]  

(A1.1)

From left hand side

\[
\frac{\partial (\rho E)}{\partial t} + \nabla \cdot (v(\rho E + p)) = E \frac{\partial \rho}{\partial t} + \rho \frac{\partial E}{\partial t} + E \nabla \cdot (vp) + p \nabla \cdot (vE) + \nabla \cdot (vp)
\]

\[
= E \frac{\partial \rho}{\partial t} + \rho \frac{\partial E}{\partial t} + E \nabla \cdot (vp) + p \nabla \cdot (vE) + \nabla \cdot (vp) = \rho \frac{DE}{Dt} + \nabla \cdot (vp)
\]  

(A1.2)

Using \( E = h - \frac{p}{\rho} + \frac{v^2}{2} \) in Eq. (A1.2) left hand side becomes:

\[
\rho \frac{DE}{Dt} + \nabla \cdot (vp) = \rho \frac{Dh}{Dt} - \rho \frac{Dp}{Dt} \left( \frac{p}{\rho} \right) + \rho \frac{Dv}{Dt} \left( \frac{v^2}{2} \right) + p \nabla v + v \nabla p
\]

\[
\rho \frac{DE}{Dt} + \nabla \cdot (vp) = \rho \frac{Dh}{Dt} - \rho \frac{Dp}{Dt} \left( \frac{p}{\rho} \right) - \rho \left( \frac{Dp}{Dt} \right) + \rho \left( \frac{Dv}{Dt} \right) + v \nabla p
\]

\[
\rho \frac{DE}{Dt} + \nabla \cdot (vp) = \rho \frac{Dh}{Dt} - \rho \frac{Dp}{Dt} + v \cdot \nabla \tau
\]  

(A1.3)

Using Eq. (A1.1) and Eq. (A1.4)

\[
\rho \frac{Dh}{Dt} - \frac{Dp}{Dt} + v \cdot \nabla \tau = \nabla \cdot (k_{\text{eff}} \nabla T_g) - \sum_i h_{i,j} + \tau \cdot v
\]

\[
\rho \frac{Dh}{Dt} - \frac{Dp}{Dt} + v \cdot \nabla \tau = \nabla \cdot (k_{\text{eff}} \nabla T_g) - \sum_i \nabla \cdot (h_{i,j}) + \nabla \cdot (\tau v)
\]
\[
\rho \frac{Dh}{Dt} = \nabla \cdot (k_{\text{eff}} \nabla T) - \sum_i \nabla \cdot (h_i j_i) + \nabla \cdot (\tau v) + \frac{Dp}{Dt} - \nabla \cdot \tau
\]

\[
\rho \frac{Dh}{Dt} = \nabla \cdot (k_{\text{eff}} \nabla T) - \sum_i \nabla \cdot (h_i j_i) + \Phi + \frac{Dp}{Dt}
\]  

(A1.4)

For low speed flows, the mechanical compression \((Dp/Dt)\) and the viscous dissipation \((\Phi)\) are very low hence, can be neglected.

So, Eq. (A1.4) can be rewritten as

\[
\rho \frac{Dh}{Dt} = \nabla \cdot (k_{\text{eff}} \nabla T) - \sum_i \nabla \cdot (h_i j_i)
\]  

(A1.5)

Also, 

\[
h = \sum_i h_i Y_i
\]

\[
\frac{Dh}{Dt} = \sum_i \left( Y_i \frac{Dh_i}{Dt} + h_i \frac{DY_i}{Dt} \right)
\]  

(A1.6)

Since,

\[
\frac{Dh_i}{Dt} = c_{\text{m}} \frac{DT}{Dt}
\]

So,

\[
\frac{Dh_i}{Dt} = \sum_i \left( Y_i c_{\text{m}} \frac{DT}{Dt} + h_i \frac{DY_i}{Dt} \right)
\]  

(A1.7)

From species continuity equation

\[
\rho \frac{DY_i}{Dt} = -\nabla \cdot j_i + \dot{\omega}_i W_i
\]  

(A1.8)

Using Eqs. (A1.5-A1.8)

\[
\rho c_{\text{p}} \frac{DT}{Dt} + \sum_i h_i (-\nabla \cdot j_i + \dot{\omega}_i W_i) = \nabla \cdot (k_{\text{eff}} \nabla T) - \sum_i \nabla \cdot (h_i j_i)
\]  

(A1.9)
\[
\sum_i \nabla \cdot (h_i j_i) = \sum_i (h_i \nabla j_i + j_i \nabla h_i)
\]

\[
\rho c_p \frac{DT}{Dt} + \sum_i h_i (-\nabla \cdot j_i - \dot{\omega}_i W_i) = \nabla \cdot \left( k_{\text{eff}} \nabla T \right) - \sum_i (h_i \nabla j_i + j_i \nabla h_i) 
\]  \hspace{2cm} (A1.10)

\[
\rho c_p \frac{DT}{Dt} + \sum_i j_i \nabla h_i = \nabla \cdot \left( k_{\text{eff}} \nabla T \right) + \sum_i h_i \dot{\omega}_i W_i
\]  \hspace{2cm} (A1.11)

Using \[ j_i = \rho \dot{V}_i \]

\[
\rho c_p \frac{DT}{Dt} + \sum_i \rho \dot{V}_i c_p \nabla T = \nabla \cdot \left( k_{\text{eff}} \nabla T \right) + \sum_i h_i \dot{\omega}_i W_i
\]  \hspace{2cm} (A1.12)

Thus, the energy equation is transformed to the temperature form, as shown in Eq. A1.12.