ABSTRACT

SWANSE, KOLE. Feeder Cattle Price Dynamics: A Study of Price Relationships in the Beef Cattle Industry. (Under the direction of Dr. Walter Thurman).

This study examines price relationships in interrelated markets that support beef cattle production and considers a variety of factors that contribute to differences in relative prices and values for groups of cattle. The market dynamics that underlie cattle markets are both complex and interesting, providing vast opportunity for economic inquiry.

An overview of beef cattle production included in the first chapter identifies some important industry and biological features that shape relationships between markets for cattle at various stages of production and for related inputs (i.e., feed). Analysis presented in subsequent chapters is based on a derived demand framework and breakeven, zero-profit equilibrium model of feeder cattle prices.

The relationship whereby per pound prices for lighter cattle tend to be higher than for heavier cattle is commonly known as the feeder cattle “price slide.” Chapter 2 focuses on price-weight relationships and the complex interactions between feed input markets, fed cattle output markets, and the price of feeder cattle inputs. I use a hedonic regression model of feeder cattle basis with a large dataset of feeder cattle transactions to explore price-slide dynamics related to weight. The empirical work incorporates futures price information for related markets (e.g., futures contract prices for feeder cattle, live cattle, and corn) that would be available to buyers at time of sale.

Chapter 3 develops a “bio-economic” model that incorporates known biological growth relationships. It extends the two-input breakeven feeding model to accommodate nonlinear growth rates for feeder cattle. Biological growth parameters are embedded in an expression for the remaining feed input, $\bar{F}$. This parameterized model provides useful insights but is too rigid
for empirical implementation. A more flexible functional form designed for this purpose is the subject of chapter 4.

The combination of futures price information and feeder cattle transaction data facilitates computation of an implied breakeven feed requirement (IBEFR) variable that is the empirical counterpart of $\bar{F}$. A model based on IBEFR, developed in chapter 4, is used to explore factors that influence feeder cattle price relationships. An important innovation of this model is to separate effects into two components: those that are related to feed costs and those that are related to non-feed factors. The IBEFR model differentiates simultaneous feed and non-feed effects of several important cattle characteristics.

Research presented this study contributes new ideas and approaches to the study of cattle price relationships and contributes to the body of literature that seeks to understand the dynamics of beef cattle markets.
Feeder Cattle Price Dynamics: A Study of Price Relationships in the Beef Cattle Industry

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For my dad, Ward Swanser.
BIOGRAPHY

Kole Swanser was born and raised in Billings, Montana. He received his undergraduate education at Carleton College in Northfield, Minnesota where he attained a Bachelor of Arts degree in Economics. He later returned to Montana, where he earned a Master of Science in Applied Economics at Montana State University in Bozeman, MT.
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CHAPTER 1

The Beef Cattle Industry, Markets, and Price Differentials

An Oklahoma cattle auction in October is a busy place to even a casual observer, but a market expert will recognize that much more is going on than meets the untrained eye. Calves that were born in the spring and raised on grass during the summer are typically weaned and sold in the fall, creating a seasonal increase in auction volume. Farmers and ranchers from large and small cow-calf operations sell groups of cattle (frequently called “lots” or “pens”1) numbering from three to more than one hundred steers, heifers, or bulls, weighing from the low-300 lb. to mid-800 lb. range, and varying in breed, color, quality, and uniformity of the group. Sellers bring cattle from near and far and their travel and timing depends on rain and weather conditions. Many different types of buyers are on hand to take the other side of these market transactions. These include small and large feedlot operations who are looking for feeder steers over 650 lbs. that can be finished and ready to slaughter by April and farmers who run stocker operations seeking light calves to purchase and feed on wheat pasture over the winter. The 2018 calf crop is large, but fall 2018 winter wheat pasture conditions are good and rainy weather has reduced the number of sellers so prices are relatively high – 450-500 lb. steers average $183 per hundredweight (cwt). Meanwhile, prices for 700-750 lb. steers are also considered to be strong at $163/cwt, but the market for 550-650 lb. steers is considered weak at the same $163/cwt price.

The casual observer might wonder why the smallest cattle described above would sell for $20/cwt more than the largest and why cattle in the middle of the weight range are considered to be facing a weak market. The market dynamics underlying these price differentials are complex

1 Throughout this volume, the terms “lot” and “pen” will be used in reference to groups of cattle that are sold together in one transaction. In forming such groups, cattle are typically sorted according to sex and weight.
and interesting. They are influenced, among other factors, by markets for cattle feed inputs, seasonal beef supply and demand conditions, and the biology of cattle growth and beef production.

This dissertation is an investigation of cattle price differentials and of price dynamics in the beef supply chain. The study of price relationships in cattle markets is not novel. However, the complex nature of the interrelated markets that support beef cattle production provide vast opportunity for exploration. I begin in this chapter with a discussion and general overview of cattle markets and some distinct features that make them particularly interesting. In chapter 2 I develop a derived demand framework that is used to analyze and empirically model relative feeder cattle prices. In particular, this section explains the “price slide,” a key market feature, and its relationship to input price dynamics. A more complex model of feeder cattle price relationships that incorporates biological growth relationships is developed in chapter 3. An alternative empirical strategy for estimating the cattle price relationship is explored in chapter 4.

The prices of cattle in the beef supply chain are almost always quoted on a per pound (or per hundredweight), rather than per animal (per head) basis. This provides an important signal about valuation and reflects the focus of the market in an industry in which the valued output is quantity of beef, rather than numbers of cattle. Per pound prices facilitate useful price comparisons across various stages of production. This study will consider the many factors that determine per pound price differentials for cattle based on type, stage of production, location, and point in time.

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2 Per pound pricing is logical within the context of cattle growth as a process of beef production. Steers and heifers intended for eventual slaughter are nearly always described by their current weight and almost never by current age. In contrast, breeding stock are valued as biological units and described according to age rather than weight. For example, bulls are typically sold individually at one or two years of age on a per head basis. Their prices reflect lifetime breeding potential and range from $800 to more than $100,000 (in February 2019, a single seedstock bull sold for over $1.5 million). Breeding replacement heifers are females that have not yet had a calf, and the age of a bred cow would typically be noted at time of sale.
The industry standard of per pound pricing naturally reflects the importance of weight as the most important trait differentiating cattle throughout the beef production process. Weight, rather than age, marks an animal’s stage of development and value to a potential buyer. In this sense the animal is a beef production factory for which weight indicates the stages of progression through the supply chain from birth to slaughter. Cattle growth during the beef production process is measured in pounds of gain. A common measure of production efficiency in the industry is the rate at which the primary input to the beef production factory, feed, is converted into gain. As such, per pound pricing facilitates important production cost calculations as the animals progress through stages of the production chain. Important industry benchmarks include the cost of gain (COG), value of gain (VOG), and average daily gain (ADG).

The derived demand framework that will be used to analyze cattle prices models the combination of cattle and feed inputs that produce a fed or slaughter cattle output.

Overview of Beef Cattle Production

This subsection describes the decentralized process by which calves born on diverse farm and ranch operations throughout the U.S. are transformed into beef. The beef cattle industry is comprised of many different segments as described below.

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3 There exists a rich literature on the relationship between the manner in which goods are priced and the attributes or characteristics that are most valued by buyers. Goods tend to be transacted in ways that minimize the cost of measuring valued attributes (Barzel 1982, Leffler 1982). As is the case for many goods, feeder and stocker cattle represent a bundle of priced and unpriced characteristics. In particular, a stocker calf or feeder steer is valued not only for its weight in pounds of beef if slaughtered today, but also for its frame and potential to gain additional pounds of beef in the future. Per pound pricing of beef cattle is an example of market organization that reduces overall costs of measurement.

4 Cost of gain (COG) is calculated as the cost (especially the feed component) required to add weight to a group of cattle. Value of gain (VOG) is calculated by subtracting the total current per head value of cattle (current per pound price x current weight) from the expected future per head value of the same cattle (future per pound price x future weight). Total COG and VOG are usually divided by the total amount of weight gain and expressed on a per pound basis for comparison with one another. Average daily gain (ADG) simply refers to the average amount of weight (in pounds) an animal or group of animals gain per day, especially while they are in a feeding program. These benchmarks are used to guide feedlot purchasing and placement decisions. For illustrative applications of these concepts, see <https://www.agmanager.info/sites/default/files/Value-of-Gain_FactSheet_AM-GTT_2011_0.pdf>.
**Seedstock**

The seedstock industry produces the genetics used for beef cattle production. It stands apart from the rest of the industry, supplying the raw materials used to produce beef. The main output from this segment of the industry, genetics, is sold in the form of breeding bulls, breeding females, and bull semen for artificial insemination (A.I.).

**Cow-calf**

The cow-calf sector produces calves that enter the beef production supply chain. These operations maintain cow herds; each cow is bred (by bulls that are either owned or rented) to produce one calf per year (typically) for an average productive life of about seven years. The calves are typically raised alongside their mothers on milk and then some type of grass pasture forage until their digestive systems can process whole feeds and they are able to be weaned; this typically occurs around 7-8 months of age at a weight of between 500 and 600 lbs.\(^5\) In various regions, cow-calf operations might breed cows to calve during the spring, winter, or even fall months. Decisions on when to calve are generally based on climate conditions and market timing. Most northern calves are born in the first quarter of the calendar year, allowing operations to take advantage of available grazing on pasture during the spring and summer months.\(^6\) Fall-born calves are more common in the southeastern U.S. where warmer climates allow winter grazing.

Many operations retain a portion of their female calves (heifers) for herd replacement. The decision to retain heifers or raise and sell them as beef cattle is driven by long-term considerations regarding desired herd size and capacity and is therefore influenced by

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\(^6\) According to USDA-NASS 2016, 72 percent of calves were born during the first six months of the year in 2015.
expectations about future cattle and feed (or forage) prices. Male calves (bulls) are frequently castrated (at which point they become steers) some time prior to weaning.

Cow-calf operations vary in size from large ranches with thousands of cows to small operations having only a few head. Management and methods of production also differ greatly. While some cow-calf operations emphasize genetic quality and standardization, other cattle operations (especially small-scale operations) include a diverse mix of breeds, sizes, and genetic characteristics.

The market for higher quality cuts of beef (e.g., steak) is primarily supplied by beef calves raised specifically for that purpose. However, other types of cattle are also slaughtered for beef production. For example, cull animals from the cow-calf sector are sold to packing plants. These include aging breeding stock (bulls and cows) and open cows (those not bred during the annual cycle) that are replaced by younger breeding animals as part of normal herd maintenance. Cull animals are slaughtered to produce lower quality beef products such as ground beef. Cull animals and calves from dairy operations make up 10-15% of U.S. beef production.7 These calves, most commonly Holstein steers, tend to follow a different production path from cattle bred for beef.

Stockers and backgrounding

After they have been weaned, calves intended for beef production (instead of breeding) are considered stocker cattle. Stockers typically weigh in the range of 350-600 lbs. and may be purchased and placed into a wide variety of feeding arrangements. This next phase of the production process is sometimes called “backgrounding,” and generally includes a diet high in forage (e.g., pasture grazing, hay, etc). During the initial backgrounding phase, young cattle

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7 See Boetel (2017).
become accustomed to being separated from their mothers and develop immunity to diseases; they often undergo “preconditioning” programs that involve a set of post-weaning vaccinations.

The length of time and corresponding weight gain for cattle during the backgrounding phase can vary substantially. The amount of weight that cattle gain and time that they spend consuming a diet that is more forage-based varies with feeding programs which depend, in large part, on relative prices of corn-based feed inputs and availability of pasture-based forage.

Most commonly, steers and heifers intended for beef production spend between five and seven months on stocker or backgrounder farms and ranches. However, beef cattle follow different paths through this intermediate production stage between cow-calf and feedlot. The following are four common examples for calves that are born in the spring and weaned in October at a weight of about 550 lbs.8

- Pre-conditioning direct to feedlot: Calves are placed in a short pre-conditioning program intended to prepare them for a feedlot environment. These calves can be ready for November feedlot placement at around 600 lbs. (approximately nine months of age).
- Background yard: Calves are placed into a background yard, consisting of pens or lots where they are fed a backgrounding ration that includes more grain and sileage for three to four months. This program serves as a warm-up for a feedlot program. Depending on the type of ration and rate of gain produced, feedlot placement might occur around January.
- Wheat pasture: Calves are grazed through the winter on farm pasture that has been planted to wheat. This practice is common in the southern plains (including Kansas,

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8 Examples follow USDA-NASS 2016.
Oklahoma, and Texas). These cattle come off wheat pasture between February and April for placement in feedlots at around 850 lbs. and just over one year of age.

- **Grass cattle:** Following weaning, calves might spend the winter months consuming a forage-based ration in an enclosed area (this is sometimes referred to as “dry lot wintering”). The following spring these animals can be placed as “yearlings” at around 700-800 lbs. and run on grass pasture through the summer months. These cattle can then be placed in a feedlot in the fall at around 1,000 lbs. where they are fed grain for finishing.

- **Grass finished:** A very small percentage of U.S. beef cattle (less than 3%) remain on pasture until they reach finished weight and are ready for slaughter. This process is known as “grass-finishing.”

The cow-calf operator may retain ownership of their cattle during some portion (or all) of the stocker/backgrounding stage. However, it is common for ownership to change several times during this phase of the production process as cattle are purchased and later resold by various types of interim cattle feeding operations.

**Feedlot or finishing phase - feeder cattle**

Most beef cattle are eventually placed into a feedlot for the finishing phase of the beef production process. Feedlot operations range in size from small farmer/feeders with limited capacity to large commercial feedlots that can feed thousands of cattle at a time.

The time cattle will spend in a feedlot to be finished depends on the type of feeding and finishing program and their placement weight. Cattle are finished when they meet slaughter

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9 Cheung, McMahon et al. (2017) estimate the percent of U.S. slaughter cattle that were finished on grass, including cull cows and bulls, in 2015 to be 3%. In contrast, this number is much higher in countries such as Australia with access to large amounts of less expensive pasture.
marketing specifications and are ready to be sold to a meat packing facility.\textsuperscript{10} The weight at which cattle are finished depends on several factors including the size or frame of the animal, desired carcass characteristics, and the cost of feed inputs. Average beef cattle finishing weights have been increasing for the past several years; in 2017 average live weight for commercial slaughter cattle was 1,349 lbs.\textsuperscript{11}

Cattle in feedlots are fed a ration of grain, sileage, hay, and/or protein – the ration is comprised of 70-90\% grain and protein concentrates.\textsuperscript{12} Higher grain content translates into more energy in feedlot rations, which accelerates the rate of weight gain. Whereas forage diets during earlier phases of production produce 1.5-2.0 lbs. per day of average daily gain (ADG), cattle on feedlot rations achieve much more rapid gain rates of 2.5-4.0 lbs. ADG. Feedlot rations can be adjusted to produce higher or lower rates of gain depending on the relative grain, protein, and forage content.

The time when cattle will reach finished slaughter weight depends on the production path. Because the production process spans several months and many different paths are available, the beef industry is able to respond to market signals that encourage seasonal distribution of the final output. The majority of calves are born during the first three months of each year, but demand for beef is spread throughout the year. Through various processes that vary the pace of growth toward finishing, lumps in the supply chain can be smoothed to manage seasonality of supply. Price signals are transmitted up the production chain and the intermediate

\textsuperscript{10} The terminology for finished cattle can be confusing. Other terms frequently used to refer to finished cattle include fed cattle, slaughter cattle, and live cattle (as in the live cattle futures contract).


cattle producers (e.g., feedlots) can respond by either accelerating growth or “holding calves back” to change marketing windows for their finished cattle. In recent years, cattle growth implants have become common as another important tool aimed at increasing rates of growth and feed conversion for cattle in feedlots.\textsuperscript{13}

Cattle are typically placed into feedlots when they weigh between 600 and 850 lbs. Cattle placed at lighter weights and given grain rations to produce more rapid growth early in development will reach finishing at a younger age than cattle that remain on pasture and forage-based diets longer. As a result, the age at which beef cattle reach finished weight and are slaughtered varies from around twelve months to just under two years of age for grain-fed cattle. Grass-finished animals are typically slaughtered after two years of age.

\textit{Fed cattle / finished cattle / slaughter cattle}

Once cattle have reached the desired finishing weight at around 1,350 lbs., they are sold to beef processing facilities, which are generally owned by large meat packing corporations. The meat packing industry is extremely concentrated. Approximately 80\% of beef cattle are processed at facilities owned by four large meat packing corporations.\textsuperscript{14} More vertical coordination occurs at this stage of the beef production process. In some cases meat packing firms have ownership stake in feedlot operations. In other cases, packers enter into marketing agreements with feeders to set prices for finished cattle in advance of delivery. The increase in prevalence of these arrangements over time has given rise to concerns within the beef industry about “captive supplies” and the possible adverse effects on cattle market price information.

\textsuperscript{13} See Schroeder and Tonsor (2011) for a study of the impact of implants on cattle feeding. The use of implants is not without tradeoffs; studies suggest that they can result in depression of quality grades for finished cattle. Sexten, WLJ (2018).

\textsuperscript{14} The “big four” meat processing companies are Tyson Foods, Cargill, JBS USA, and National Beef.
Because relatively few finished cattle are sold in open auction market transactions, the sample size of publicly available price data from such sales tends to be quite small.\textsuperscript{15}

After slaughter and processing, beef carcasses are evaluated according to official USDA grading standards for yield and quality. Yield Grade indicates the number of high-value cuts of beef yielded by the beef carcass, with 1 being the greatest and 5 being the least. Grades for beef carcass quality are designed to measure the palatability of the end beef product and are based on marbling and maturity. In descending order, beef quality grades are USDA Prime, Choice, Select, and Standard.

An alternative method of pricing fed cattle sold to beef processors involves constructing a table of all the animals sold according to yield and quality grade. The parties agree on the base price for the grid (for example at Choice and Yield Grade 3) and a schedule of premiums and discounts for animals that fall in other grading combinations.

\textit{Ownership from birth to slaughter}

The beef cattle industry is comprised of many different types of buyers and sellers who may engage at various points during the production process by buying (and later selling) cattle, and ownership of a steer may change several times before slaughter. A common example would be for the animal to be purchased from the cow-calf operator by a stocker operation and subsequently sold to a feedlot that eventually markets to a meat packing facility. As a result, a steer could easily have four or more different owners within two years from birth to slaughter.

\textsuperscript{15} In response to concerns about price transparency, Congress passed the Livestock Mandatory Reporting Act of 1999 requiring large packers to report sales transaction details to the USDA. In 2003, Congress also directed the USDA's Grain Inspection, Packers and Stockyards Administration (GIPSA) to study the effects of alternative marketing arrangements (AMAs) on livestock markets. The “Livestock and Meat Marketing Study” conducted by RTI and released in 2007 found that such arrangements in fact yield a net benefit to livestock producers. (www.gipsa.usda.gov). Nonetheless, questions about competition in beef packing and their market effects remain <http://www.choicesmagazine.org/magazine/article.php?article=121>. 
Alternatively, a cow-calf producer might choose to retain ownership of the animal throughout the entire process, paying for custom feeding and eventually marketing their fed cattle to the packer.

**Biology of Cattle Development and Growth**

Two aspects related to the biology of beef cattle production merit particular attention in an economic analysis of cattle markets and cattle prices. The first of these is the importance of biological lags that constrain the rate at which the cattle industry and its participants are able to respond to market forces and price signals. Although cows may be bought and sold by different cow-calf operators, the total size of the U.S. beef cow herd can only expand as quickly as retained heifer calves can reach maturity and produce their first calves at around 24 months of age. If standing heifers intended for beef production are repurposed as breeding stock, they will require a year to breed and produce new calves and at least another year before those calves can be finished for beef production. Further, any expansion of U.S. beef production capacity requires that heifer calves be retained for breeding, causing a near-term reduction in beef supply. This biological constraint gives rise to something commonly known as the cattle cycle: periodic herd expansion and contraction in lagged response to current prices causing beef prices to rise and fall over time (Rucker et al. (1984), Rosen (1987), and Rosen et al. (1994)). Cow-calf producers also respond to changes in regional and individual production costs. For example, drought conditions that reduce availability of grazing pasture across cattle-producing areas of the country can cause herd liquidation as herd cows are culled and heifer calves are raised for slaughter and not retained.

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16 Heifers become ready for first breeding at around one year of age. The bovine gestation period is approximately 280 days, and cows generally produce one calf per year.
Because of the importance of biological lags, any such decisions have long-term effects on the industry. At any given time and current U.S. herd capacity, cow-calf producers will make stocking decisions for their individual operations that determine how the herd will be geographically distributed. Cows bred in May or June will produce calves the following spring. Then, depending on the chosen production path, these calves can be grown to finishing and slaughtered for beef production between two and three years after breeding.

The biology of how cattle transform feed inputs into growth is critical to understanding beef production and, as such, has an important impact on cattle price differentials. As beef cattle grow, they transform feed inputs into pounds of beef. The rate and efficiency of this process is important because it determines both the length of time and quantity of feed consumed before the animal can be finished and sold for slaughter. As a result, it determines future market timing and the total feed input cost of production. Cattle price differentials reflect the interaction of feed conversion biology with current and future feed input and fed cattle output prices. These interactions and relationships will be exploited in chapter 2 and extended in chapters 3 and 4.

**Feeder Cattle Price Determinants: A review of the literature**

Prices and price dynamics across commodities have been of great interest to economists. Per pound prices facilitate comparisons across the beef cattle production landscape for animals (more commonly groups of animals) that differ in many important aspects. What is reflected in the price of one pound of “cattle?” This dissertation will explore that question in order to shed light on some of the complex and nuanced answers, and it begins with a discussion of several important previous contributions from the extant literature.

In his seminal work on feeder cattle price differentials, Buccola (1980) noted that a wide variety of characteristics are used to differentiate prices for cattle of different types. Certain
factors, such as location, time of sale, and quality characteristics, are important for most commodities. In addition to these, the sex and weight of cattle are particularly important. Buccola’s work emphasized the important relationship between cattle price and weight. In normal market conditions, lightweight cattle sell at a per pound price premium to heavier cattle. In the cattle industry, this relationship is commonly called the price slide. Factors affecting the shape of the price slide will be a focus of analysis presented in chapters 2 through 4.

Buccola constructed breakeven profit models for both sides of the feeder cattle market – buyers and sellers – and simulated the influence of several factors on the relative price of different weights of feeder cattle. This conceptual model included feed input costs, fed cattle prices, feed conversion and gain rates, and other cattle characteristics and demonstrated the dynamic and interdependent influence these factors exert on breakeven prices. The accompanying empirical analysis found that live cattle prices (for fed cattle), corn prices, current pasture conditions and cattle inventories, and the sex of the animals sold were statistically significant determinants of feeder cattle prices and the shape of the price-weight relationship.

The work of Buccola influenced subsequent analysis of feeder cattle price differentials. Empirical work by Marsh (1985) focused on differences in prices for steer calves and yearlings and confirmed the importance of fed cattle prices and cost of gain in determining the premiums paid for lighter weight feeder cattle.17

Langemeier, Schroeder, and Mintert (1992) studied the impact of various factors on cattle feeding profitability. Using a breakeven analysis and linear regression, they considered the impact of input and output prices for cattle, feed conversion, and average daily gain and determined that these factors explained 86 percent of the difference in profits between steers and

17 See Tonsor and Mollolan (2017) for more recent work on this topic.
heifers. Overall, they found that most of the variability in profits for cattle feeders were explained by movements in fed cattle output prices (50 percent) and feeder cattle input prices (25 percent), while changes in corn prices accounted for 22 percent of feeding profit variability. Unsurprisingly, the weight at which cattle were placed on feed exerted a primary influence on input costs. Albright, Schroeder, and Langemeier (1994) analyzed several factors that influenced the variability in the cost of gain for Kansas cattle feeding operations using feedlot data. They attributed the greatest influence to corn prices (60 percent), but also included feed conversion (26-29 percent) and average daily gain (1.9-2.5 percent).

Anderson and Trapp (2000) extended analysis of corn price effects on feeder cattle prices beyond breakeven analysis by demonstrating that slaughter and placement weights as well as feed conversion rates also change in response to changes in the price of corn. They constructed a recursive model to accommodate more complex relationships and feedback effects between the production and price variables. From this model, they dynamically simulated outcomes to show that changes in corn prices (as the primary feed input) would lead to changes in feeding programs. Specifically, they showed that when corn prices rise, cattle placement weights increase as they gain more weight on forage; feedlots also change ration compositions in response. Slaughter weights increase because a certain amount of gain must be produced from grains to meet finishing grades (marbling). However, the overall amount of gain on feed is lower and feed conversion rates are lower. The overall impact of corn price changes on the price of feeder cattle prices, which they refer to as the “corn multiplier,” is, to some extent, attenuated by changes in the cattle feeding equilibrium.

Some studies have developed dynamic equilibrium supply response models of feeder cattle markets (Shonkwiler and Hinckley (1985) and Marsh (2001)). This work provides further
evidence regarding feeder cattle price responses to changes in fed cattle prices and feed costs. It also identifies additional factors that influence feeder cattle prices, including finance costs, profit risk, technologies used by cow-calf producer and feedlots, and feeder cattle imports.

Other research has focused on empirical analyses of static short-run price differentials using feeder cattle sales data gathered from various regions (Bailey and Peterson (1991), Dhuyvetter and Schroeder (2000), Dhuyvetter et al. (2008), Faminow and Gum (1986), Sartwelle et al. (1996), Schroeder et al. (1988), Schultz et al. (2010), Turner and McKissick (1991)). These studies have identified a number of physical and market characteristics that impact feeder cattle prices. In addition to sex and weight of cattle sold, these physical factors include lot size, health, preconditioning status, breed and genetics, frame size, muscling, fill, condition, presence of horns, location, and time of sale. Several of these studies have used linear regression with quadratic terms to model nonlinear relationships for the price-weight slope and other variables (e.g., lot size). Further, many factors interact with one another and with the price-weight relationship. Schroeder et al. (2010) find that the impacts of many individual factors and the shape of the price-weight slope vary seasonally. Empirical results from Dhuyvetter and Schroeder (2000) and others also find that futures prices for live cattle, feeder cattle, and corn have significant impacts on cash prices for feeder cattle.

The literature discussed to this point has, for the most part, attempted to explain cash price differences observed for groups of cattle being sold at a given point in time. Another area of research has focused on the relationship between cash prices for feeder cattle (sold at various weights) and cattle futures prices.

For many commodities, the factors that explain the difference between cash and futures prices (or basis, defined as cash minus futures) are quite distinct from those factors that explain
cash price differentials. Cash corn prices, for example, differ according to location and quality or variety. Meanwhile, the additional differences that arise between cash spot prices and forward (futures) prices for corn (the spot-forward spread) are primarily attributed to the cost of storage and carrying charges (see Working (1949)). In contrast, cattle (live beef) cannot be stored but will change form over time. Paul and Wesson (1967) proposed that the spot-forward price spread should be analyzed as “the market price of converting one form into the other.” For cattle, a nonstorable commodity, they hypothesized that the future price of fed cattle was explained by the price of feeder cattle (as inputs) and the cost of feeding those cattle (feed and feedlot services).

Note that the factors that explain intertemporal price differences are the same factors that explain the price-weight relationship for cash cattle. Similarly, consider that two relevant dimensions exist for comparing differences in cattle weights. First, the weight of different groups of steers sold in the feeder cattle market can be compared at a point in time. As described previously, these differences in weight class determine available finishing options for a group of cattle and, thus, the type of buyer. Alternatively, the weight of cattle in a particular group is not static - they will gain weight in some type of feeder program and progress through the beef production process until their future value is determined by the market price for fed slaughter cattle that meet the description of the live cattle futures contract. Described in this way, an important component of feeder cattle basis is conceptually related to the price-weight relationship. Ehrich (1969) presented a theory of cash-futures price relationships for beef cattle that connected the prices of fed cattle, feeder cattle, and feeding costs.

The first futures contract for live cattle (fed slaughter cattle) began trading on the Chicago Mercantile Exchange in 1964. A feeder cattle futures contract was added to the
exchange in 1971. Participants in various stages of the production process can therefore hedge risk using any combination of these two products (including options). Some feedlot owners employ a hedging strategy known as the “cattle crush,” incorporating corn futures in a hedging portfolio to simulate the ownership risks of cattle feeding activities. Work by Belasco (2008) quantified the risk involved in producing fed cattle and demonstrated that price risk management could substantially reduce overall risk exposure.\(^\text{18}\) However, the ability of cattle feeders to use futures contracts for risk management is limited by their ability to understand and minimize basis risk. This, in turn, requires an understanding of the price differentials that link cattle of different types, locations, and points in the beef production process. Cattle basis is one of many related and interconnected types of cattle price differential.

\(^{18}\) See also Belasco, Schroeder and Goodwin (2009) and Belasco, Ghosh, and Goodwin (2009).
CHAPTER 2

A Derived Demand Model of Feeder Cattle Basis and Price Slides

Changing cattle prices contribute to fluctuations in the value of beef cattle and represent an important source of risk for all involved in the cattle industry. Futures markets can be useful for managing this risk only to the extent that cattle basis – defined as the difference between the per pound price for a standardized feeder cattle futures contract and the cash price for any specific group (lot) of cattle – can be predicted. Two of the most important factors that influence feeder cattle prices and values are the price of finished cattle and the cost of feed inputs. The former represents output price – it measures the value of fed cattle that have reached a finished slaughter weight at the end of their production cycle. The latter represents the most substantial variable cost component of producing finished cattle from lighter feeder cattle.

Prices of standardized futures contracts for fed slaughter cattle and corn (a primary cattle feeding input) are determined almost continuously on the Chicago Mercantile Exchange (CME). In the same way that the value of a farmer’s standing crop depends on current futures price for the commodity that can only be delivered after harvest, the value of a group of cattle today depends on fed cattle prices in the future, when those animals are ready for slaughter. After adjusting for time to slaughter, changes in fed cattle prices indicate changes in the expected ending value of lighter weight cattle. The impact of corn prices on the value of unfinished cattle is more complex. Changes in the cost of corn (a proxy for feed input cost) will have a disparate effect on cattle according to weight. Because lighter animals require more feed to reach finishing weight, the value of these animals is more sensitive to future feed costs. This has direct implications for the risk associated with owning cattle. Specifically, lightweight cattle are more exposed to value fluctuation resulting from corn price variability.
The fact that per pound prices for feeder cattle tend to decrease as sale weight increases is well known to market observers and has been studied and documented by agricultural economists (Ehrich (1969), Buccola (1980), Marsh (1985), Dhuyvetter and Schroeder (2000)). Within the cattle industry, this price-weight relationship is commonly called the “price slide.” The slope of the price slide is heavily influenced by the price of the feed inputs that will be used to add weight to cattle. Changes in the shape of the cattle price slide reflect complex interactions between dynamic markets for feed inputs and fed cattle and other feeder cattle price determinants. Previous researchers have even documented extreme market conditions when per pound prices for lightweight cattle have been discounted relative to heavier animals. These unusual periods when the market exhibits an “inverted price slide” correspond to very high corn prices relative to live cattle prices. Recent years have produced record high cattle and corn prices and significant price fluctuations. Ethanol became an important competing use for corn. These developments suggest the need for updated analysis.

My analysis of cattle and corn price dynamics begins by first developing predictions about the relationship between expectations of future prices and fed cattle price using a two-input slaughter cattle production model within a simple derived demand framework. I then empirically test the predictions of the model using a subset of data from a very large database of transaction-level feeder cattle sales. The analysis in this chapter concludes with a discussion of fruitful areas of future research.
A Two-Input Derived Demand Model of Cattle Feeding

I generate predictions for the effects of fed cattle and corn prices on the slope of the feeder cattle price slide using a straightforward two-input derived demand model. In this model, one finished steer for slaughter is produced by combining one unfinished (feeder) steer with the requisite amount of a corn feed input. I make the simplifying assumption that cattle and corn are used in fixed proportions, and that the quantity of corn input required can be calculated directly based on the amount of weight gain required for the input steer to reach slaughter weight (assumed here to be exactly 1,250 lbs.).

The value of the output good, one finished steer, is determined by the market price of live cattle. Market prices for live cattle are determined by supply and demand conditions for finished slaughter cattle. The beef production process is characterized by long biological lags. This means that the current supply of live cattle for slaughter is comprised of cattle that were born over 18 months earlier. Consequently, the future supply of live cattle can be predicted long in advance of delivery based on current cattle inventories. In this model I assume that cattle gain weight and progress through the beef production and feeding process at a known constant rate. Thus, the supply of cattle in any given weight group today will be equal to the supply of live cattle on the known future date when they will all reach 1,250 lbs. This implies that current cash cattle prices

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19 Two futures contracts for beef cattle are traded on the CME (Chicago Mercantile Exchange). The feeder cattle contract (FC) included 650-849 lb. steers until November 2016 and 700-899 lb. steers for subsequent contracts. The live cattle futures contract (LC) is for live steers that are “finished” (fed to the appropriate weight) and ready to be slaughtered. The live cattle contract does not specify a weight range.

20 A note on terminology is appropriate here. Curiously, there is no singular form of the word “cattle” in the English language that does not refer to a specific sex. Thus, I have chosen to use one feeder steer as the subject of my model illustration. Cattle that have not yet reached slaughter weight are typically called either feeder cattle (if 650 lbs. or heavier) or stocker cattle (if less than 650 lbs.). Feeder and stocker cattle are typically sold as either steers (castrated males) or heifers (females that have not yet given birth to a calf), although bulls (males that have not been castrated) are commonly sold in some regions. The model presented here applies to heifers and bulls as well as steers.
for lighter weight animals are linked to market conditions that are reflected in prices for heavier animals to be delivered in the future.

Derived demand for the steer input in the two-input model can be obtained by subtracting the cost of the corn feed input from the value of the finished steer output (the derived demand model presented here follows Friedman (1962), chapter 7). The difference between the total value of the finished steer and the total value of the input steer will be, in equilibrium, equal to the corn feed input cost. This difference is commonly known as the gross feeding margin (GFM). This application of the derived demand model for cattle will be illuminated using specific example calculations and graphical analysis.

*Example feeder cattle model calculations*

An example using specific values for production coefficients, prices, and input variables helps to illustrate the price slide relationship implied by the two-input derived demand model. Table 1 displays feed requirement and cost calculations for two different weights of input steer: a 750 lb. “feeder” and a 550 lb. “stocker.” The calculations use a standard feed conversion factor – seven pounds of corn grain are required to produce one pound of steer weight gain. Gray-shaded values for input and output weights and corn prices (set initially at $7.50/bushel) are considered to be exogenous. Because feed costs are linear with respect to weight, gross feeding margin (GFM) is proportional to the weight of the input steer and per pound cost of gain (COG) is equal for both steer types.

Derived values for the two weights of input steer are displayed in table 2. These are calculated by subtracting the expected gross feeding margin calculated in table 1 from the

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21 This average conversion factor is common in the industry literature. See, for example, Anderson and Trapp (2000). I will consider a model in which this strict assumption of a fixed feed conversion rate is relaxed in chapter 3. Also note that one bushel of corn weighs 56 lbs.
expected value of a finished 1,250 lb. steer. A live cattle futures price \((LC1250)\) of $1.24 per pound is used to calculate slaughter value for both weights of cattle in this example, implying a static price of the final output.

Note that additional pounds of weight gain will always translate into greater value for heavier steers in the two-input derived demand model as long as cost of gain is non-zero. At the same time, it is apparent that the per pound price of a steer declines as sale weight increases. The empirical analysis focuses on these per pound price-weight relationships (the cattle price slide).

Figure 2.1 plots calculated values from the two-input derived demand model over a spectrum of input steer weights ranging from 200 to 1,250 lbs. using example price and production input values. The required gross feeding margin declines with input weight starting from approximately $1,000 GFM for a 200 lb. steer input. Steer input value increases linearly at the same rate until steer input weight equals slaughter weight, GFM is zero, and the steer input value equals slaughter value \((Value_{1250})\). The implied per pound price for each weight of steer input is also plotted on the right vertical axis. Note that the model parameters used in this example produce a downward sloping price slide relationship with a convex curvature.

This numerical example is also useful for illustrating the effect of corn input price on both steer input value and the price slide. Figure 2.2 plots new model outputs calculated using identical values for all parameters except the corn price, which has been reduced from $7.50 to $3.50 per bushel. Holding all else equal, including the price of live cattle, a sharp reduction in the price of corn lowers the GFM and raises the value of the input steer at all weights. In addition, the slope of the price slide becomes steeper (more negative) over the entire range of input steer weights.
Figure 2.3 depicts a familiar representation of the derived demand model using supply and demand graphs. Here, $V_{1250}$ represents the final output “price” (value per 1,250 lb. steer) that is determined at the intersection of supply and final output demand. Because the quantity of steers available for slaughter today were born months earlier and few alternative uses exist, the short-run supply is assumed to be fixed (a vertical supply curve). The gross feeding margin can also be interpreted as the marginal cost or supply curve for the feed input, which I assume does not depend on the quantity of steers (a horizontal marginal cost curve). The derived demand for steers as inputs can be obtained as the difference between the final output demand curve for 1,250 lb. steers and the supply curve for feed inputs.

Graphing derived demand relationships normally requires that the input and output goods be converted into equivalent units. No conversion is required in this case because one unit of the steer input is necessarily equal to one unit of the slaughter steer output. Further, if figure 2.3 is considered as a representation of derived demand for the one cohort of steers over different stages of weight throughout the beef production cycle, then the supply of steers (the number of steers in the cohort) will be the same at any given input weight and will equal the final output supply of slaughter steers. Thus, the value (price per steer) of a steer input is determined at the intersection of a single vertical supply curve and a derived demand curve that corresponds to the input weight. Consistent with the previous example calculations, value per steer increases and per pound price decreases with steer input weight. The group of 550 lb. steers (valued at $893.75/steer and $1.63/lb.) in figure 2.3 will become the 750 lb. feeder steers (valued at $1,081.25/steer and $1.44/lb.) approximately three months later and become 1,250 lb. slaughter steers (valued at $1,550/steer and $1.24/lb.) in roughly one year. Analysis of cattle prices at
different weights must therefore incorporate the future price expectations relevant to a particular weight cohort of cattle.

Predictions about the shape of the price slide that have been derived from this two-input derived demand model cannot be applied to a cross-section of prices for cattle of different weights at a given point in time. This is because supply and demand conditions are likely to vary for different cattle weight groups due to factors such as seasonality and cattle production trends. Differences in inventory for different steer weights will affect supply in the final slaughter steer output market and thus impact the derived demand price of steer inputs.

Figure 2.4 illustrates this point using separate derived demand markets for the 550 lb. stockers and 750 lb. feeder steer inputs. The market for 550 lb. steers shown on the left is identical to figure 2.3. However, the market shown in the panel on the right shows a larger supply of 750 lb. feeder steers. Both input and output prices for the 750 lb. steer are lower relative to the market conditions shown in figure 2.3 when a smaller quantity of steers was supplied. (For reference, the supply curve and prices from figure 2.3 are shown by dashed lines in figure 2.4.) Notice that the value of a 750 lb. feeder steer input in figure 2.4 is less than the value of the 550 lb. stocker steer. This results directly from the lower expected live cattle price for the weight cohort due to greater supply expectations.

This example clearly illustrates why a cross-section of prices for cattle of different sale weights at a given point in time need not necessarily exhibit a downward-sloping price slide relationship. Similar outcomes can be shown to result from differences in future demand for slaughter steers. Because live cattle futures prices depend on expected supply and demand conditions for a particular weight cohort of cattle, it will be important to link current cash transactions to appropriate future price expectations in any empirical analysis.
Empirical Predictions

I use the two-input derived demand model to generate empirical predictions related to the shape and dynamics of the price slide relationship. A mathematical representation of the model facilitates a derivation of comparative static relationships. Recalling that, in equilibrium, the value of the output reflects the combined value of both inputs, I begin with the following expression:

\[
\begin{align*}
Value_{1250} &= Value_{InputWt} + GFM_{InputWt} \\
1250 \cdot P_{InputWt} &= P_{1250} \cdot InputWt + COG(1250 - InputWt)
\end{align*}
\]  

(2.1)

I have assumed that the weight of the slaughter steer is fixed at exactly 1,250 lbs. as described previously. \(InputWt\) is the weight of the steer at the time it is sold as a feeder cattle input, and \(P\) always refers to a per pound price that is denoted by a subscript to identify the weight of cattle for which the price pertains. \(COG\) is per pound cost of gain, which I assume to be constant over all weight ranges and to depend only on the cost of the feed corn input, and \(GFM\) refers to gross feeding margin based on steer input weight. Rearranging this relationship gives us the price of the steer input in terms of known weights and exogenously determined output and corn prices.

\[
P_{InputWt} = P_{1250} \left( \frac{1250}{InputWt} \right) - COG \left[ \left( \frac{1250}{InputWt} \right) - 1 \right]
\]

(2.2)

---

22 The mathematical model used here to develop empirical predictions is similar to those used by Buccola (1980) and Ehrich (1969).
The per pound price spread at different sale weights is further isolated by expressing this relationship in terms of the difference between input and output prices:

\[
(P_{\text{InputWt}} - P_{1250}) = (P_{1250} - \text{COG}) \left[ \frac{1250}{\text{InputWt}} - 1 \right]
\]  

(2.3)

Given that input steers must gain weight before slaughter and InputWt < 1250, the term in brackets clearly must be positive. Therefore, the model predicts that price of the lighter input steer will always be greater than the live cattle price \( P_{\text{InputWt}} > P_{1250} \) when per pound price of the output steer exceeds the marginal per pound cost of gain \( P_{1250} > \text{COG} \). This implies that the price slide will be negatively sloped (as expected) except in cases where feed costs become exceptionally high relative to live cattle prices.

For the empirical analysis I normalize the dependent variable to remove variation in overall price levels and focus more directly on differentials between prices for different weights of cattle. This normalization is accomplished by replacing the cash price of the input steer, \( P_{\text{InputWt}} \), with basis to the feeder cattle contract, \( \text{Basis}_{\text{InputWt}}^{750} = P_{\text{InputWt}} - P_{750} \). By substituting for the price of a 750 lb. steer and again rearranging the expression, I obtain the following equation for feeder cattle basis:

\[
\text{Basis}_{\text{InputWt}}^{750} = (P_{\text{InputWt}} - P_{750}) = \left( P_{1250} - \text{COG} \right) \left[ \frac{750}{\text{InputWt}} - 1 \right] \left( \frac{1250}{750} \right)
\]

(2.4)

This basis equation relates directly to an empirical strategy that predicts basis as the dependent variable using sale weights, live cattle prices, and corn prices as explanatory variables. Empirical predictions can now be generated by deriving comparative statics from feeder cattle basis relationship. First derivatives with respect to the three explanatory variables are shown below.
\[ \frac{\partial \text{Basis}_{\text{InputWt}}^{750}}{\partial \text{InputWt}} = -(P_{1250} - \text{COG}) \left( \frac{750}{\text{InputWt}^2} \right) \left( \frac{1250}{750} \right) \quad \text{(negative*)} \]  

(2.5.a)

\[ \frac{\partial \text{Basis}_{\text{InputWt}}^{750}}{\partial \text{COG}} = -\left[ \left( \frac{750}{\text{InputWt}} \right) - 1 \right] \left( \frac{1250}{750} \right) \quad \text{(negative)} \]  

(2.5.b)

\[ \frac{\partial \text{Basis}_{\text{InputWt}}^{750}}{\partial P_{1250}} = \left[ \left( \frac{750}{\text{InputWt}} \right) - 1 \right] \left( \frac{1250}{750} \right) \quad \text{(positive)} \]  

(2.5.c)

The expected sign of each derivative is given in parentheses. Note that model predictions for the impact of both cost of gain (negative) and live cattle price (positive) on the magnitude of basis are unambiguous. However, the asterisk (*) next to the predicted sign of the first partial with respect to weight indicates that the slope of the price slide (expressed here in terms of basis) will be negative only when \( P_{1250} > \text{COG} \).

As shown previously, our model implies a price slide relationship that is nonlinear. Taking the second derivative with respect to weight allows us to sign the change in slope over weight (which will be represented in the empirical model by a quadratic term).

\[ \frac{\partial^2 \text{Basis}_{\text{InputWt}}^{750}}{\partial \text{InputWt}^2} = (P_{1250} - \text{COG}) \left( \frac{750}{\text{InputWt}^3} \right) \left( \frac{1250}{750} \right) \quad \text{(positive*)} \]  

(2.6.a)

The positive expected sign of the expression in equation (2.6.a) suggests that the price slide will have a negative slope which becomes less negative (flattens) as weight of the input steer increases. Cross-partial derivatives with respect to weight and prices yield two additional predictions about dynamics of the price slide relationship that will be of particular interest for the empirical analysis.

\[ \frac{\partial^2 \text{Basis}_{\text{InputWt}}^{750}}{\partial \text{InputWt} \cdot \partial \text{COG}} = \text{COG} \left( \frac{750}{\text{InputWt}^2} \right) \left( \frac{1250}{750} \right) \quad \text{(positive)} \]  

(2.6.b)
Two unambiguous model predictions are evident based on the sign of these expressions.

First, an increase in \( COG \) will cause the slope of the price slide to flatten (become less negative). This implies that prices (and feeder cattle basis) for lighter weights of cattle will decline disproportionately relative to heavier cattle. Second, an increase in the expected live cattle output price will have the opposite effect: the slope of the price slide will increase as the prices of lighter cattle increase relative to prices of heavier cattle.

**Data**

This analysis makes use of an extensive database of cash cattle transactions obtained from the USDA’s Agricultural Marketing Service (AMS). The AMS feeder cattle data include information about all individual lots (sale groups) of cattle sold at hundreds of public auction locations in more than 20 states across the continental U.S. In its entirety, the database contains over 13 million sales transactions for cattle weighing between 300 and 900 pounds from 1996 to 2013. The cattle sold represent a variety of stages in the beef production process, from light stockers to heavy feeders, although buyer information and intended use are not available.

A subset extracted from the larger AMS dataset has been used to conduct the empirical analysis described in this chapter. The sample data contain 282,152 transactions from seven public auction locations in Kansas over a 10-year period. Each sales transaction record in the dataset contains the date of sale, cash price, and several individual lot characteristics. Lot characteristics about the group of cattle sold in each transaction include the sales location, number of cattle in the group (lot size), average weight, and sex of the cattle (steers, heifers, or bulls).
The AMS cash feeder cattle market data have been combined with CME futures price data. Relevant cattle futures contracts were identified for each group of cattle sold, and prices were linked to each transaction in the database. Sale date and average lot weight were used to establish the future dates on which cattle in each sale lot would be predicted to attain weights of 750 and 1,250 pounds assuming a constant rate of weight gain. These weights were chosen because they generally correspond to the weight specification of the feeder cattle futures contract and the average slaughter weight of finished cattle (as described by the live cattle futures contract) during the sample period. The future date at which a group of cattle will reach each weight benchmark was projected by assuming graduated rates of average daily gain (ADG) at different stages of growth.23

As a specific example, consider a lot of cattle sold on 10/1/2012 with an average weight of 550 lbs. Assuming average gain of 2 lbs. per day, these cattle would be expected to weigh 750 lbs. after 100 days on January 9, 2013. As of that date, the JAN2013 feeder cattle contract would be the next to expire (nearby contract) on 1/31/2013. The same group of cattle would be expected to weigh 1,250 pounds after another 250 days \( ((1,250 \text{ lbs.} - 750 \text{ lbs.}) / 2 \text{ lbs./day}) \) on 9/16/2013 when the OCT2013 live cattle contract would be the next to expire on 10/31/2013.

The observed daily settlement prices for these contracts on 10/1/2013 determine the values of feeder cattle and live cattle price variables for the cattle:

\[
\begin{align*}
FC_{750} &= \text{Price of JAN2013 feeder cattle contract on 10/1/2012} = $148.125 / \text{cwt} \\
LC_{1250} &= \text{Price of OCT2013 live cattle contract on 10/1/2012} = $133.75 / \text{cwt}
\end{align*}
\]

23 The following graduated ADG rates were used for future weight/date projections: 1.8 lbs./day for cattle weighing less than 500 lbs., 2.0 lbs./day for cattle between 500 and 800 lbs., and 3.14 lbs./day for cattle weighing more than 800 lbs. This relatively simple and straightforward method for calculating future weights to project relevant futures contracts was provided by cattle industry expert Brett Crosby, who considers these to be the industry standard gain rates based on 30+ years of personal observation and market experience as a buyer, seller, and hedger of feeder cattle.
These prices are subscripted for sale date $t$ and individual lot $i$. They reflect the expected future supply and demand conditions for a specific group of cattle described in the sales transaction. In a broader sense, these are specific market price expectations for a single weight cohort of cattle that are assumed to reach slaughter weight at the same time. For empirical estimation, $FC750_t$ is used as the current estimate of $P_{750}$ (the expected future price of the group of cattle at 750 lbs.) and $LC1250_t$ is used as the current estimate of $P_{1250}$ (the expected future price of the group of cattle at 1,250 lbs.). Current basis is calculated using the feeder cattle contract reference price of $FC750_t$ for each cattle sales transaction as follows:

$$Basis_{750}^{t} = Cash_{t} - FC750_{t}$$ (2.7)

The current price of the nearby corn contract, $CN0_t$, proxies for feed input costs in the empirical model. For lighter weight animals, prices of deferred corn futures contracts (contracts expiring after the nearby) might also be considered. However, the nearby contract price was chosen as the best corn price expectation at all sale weights for two reasons: 1) remaining feed input requirements are easily forecast in advance based on the average weight of the group of cattle purchased, and 2) corn for grain is a storable commodity. Together, these imply that a cattle buyer will always have the option of purchasing the expected feed corn requirement at the same time as the cattle.

Table 4 displays summary statistics for the sample Kansas dataset that was used in the empirical analysis. These data span a period from January 6, 1999 to March 26, 2009 and include data for sale lots of feeder and stocker cattle weighing 300 to 900 pounds. The time period also included substantial variation in the prices of feeder cattle ($69.97 to $119.57 per cwt), live cattle ($60.67 to $117.70 per cwt), and corn ($1.75 to $7.61 per bushel). Feeder cattle basis in the
sample data ranged from a low of -$63.62 (when cash price was lower than FC750) to a high of $87.33.

**Estimation and Results**

To test predictions developed from the two-input derived demand model, I estimate a hedonic regression of feeder cattle basis. My analysis focuses primarily on the price relationship between cattle basis and weight, as described previously. The basic regression specification in Model 1 below describes this relationship and accommodates the expected convex shape of the price slide.

**Model 1:** \( Basis_{it}^{750} = a + b \cdot Weight_{it} + c \cdot Weight_{it}^2 + \varepsilon_{it} \)

The expected shape of the price slide relationship from figures 2.1 and 2.2 suggest that \( a \) and \( c \) will be positive, while \( b \) should be negative. Our model also generated predictions about the relationship between basis and expected prices for the live cattle output and feed corn input. Model 2 gives another simple linear specification that includes these variables of interest:

**Model 2:** \( Basis_{it}^{750} = a + b \cdot Weight_{it} + c \cdot Weight_{it}^2 + d \cdot CN0_i + e \cdot LC1250_{it} + \varepsilon_{it} \)

Comparative statics derived previously predicted that the sign on coefficient \( d \) will be negative and coefficient \( e \) will be positive. Parameter estimates for models 1 and 2 are shown in table 5. All four coefficients exhibit the predicted signs in both models and are highly significant (p-values < 0.0001).

Models 1 and 2 generate parameter estimates that can be easily interpreted and directly compared to model predictions. However, these simple specifications have some obvious limitations. In particular, they do not facilitate testing of any dynamic interaction between market prices and weight. To explore these relationships requires the introduction of additional
interaction terms between weight and price. Model 3 allows the slope of the price slide to depend on expected corn and live cattle prices.

Model 3:

\[
\text{Basis}_{it}^{750} = a + d \cdot CN0_t + e \cdot LC1250_{it} \\
+ (b + f \cdot CN0_t + g \cdot LC1250_{it}) \cdot Weight_{it} \\
+ (c + h \cdot CN0_t + l \cdot LC1250_{it}) \cdot Weight_{it}^2 + \varepsilon_{it}
\]

Interpreting the meaning of coefficients estimated from Model 3 requires first taking derivatives of the regression equation. First, second, and cross-partial derivatives of basis with respect to the independent variables of Model 3 are shown below, followed by predicted signs.

First derivatives:

\[
\frac{\partial \text{Basis}_{\text{InputWt}}^{750}}{\partial Weight} = (b + f \cdot CN0_t + g \cdot LC1250_{it}) \\
+ 2 \cdot (c + h \cdot CN0_t + l \cdot LC1250_{it}) \cdot Weight_{it} \\
\]

\[
\frac{\partial \text{Basis}_{\text{InputWt}}^{750}}{\partial CN0_t} = d + f \cdot Weight_{it} + h \cdot Weight_{it}^2 \\
\]

\[
\frac{\partial \text{Basis}_{\text{InputWt}}^{750}}{\partial LC1250_{it}} = e + g \cdot Weight_{it} + l \cdot Weight_{it}^2 \\
\]

Second and cross partial derivatives:

\[
\frac{\partial^2 \text{Basis}_{\text{InputWt}}^{750}}{\partial Weight^2} = 2 \cdot (c + h \cdot CN0_t + l \cdot LC1250_{it}) \\
\]

\[
\frac{\partial^2 \text{Basis}_{\text{InputWt}}^{750}}{\partial Weight \cdot \partial CN0_t} = f + 2 \cdot h \cdot Weight_{it} \\
\]

\[
\frac{\partial^2 \text{Basis}_{\text{InputWt}}^{750}}{\partial Weight \cdot \partial LC1250_{it}} = g + 2 \cdot l \cdot Weight_{it} \\
\]
Equations (2.8.a) – (2.8.c) and (2.9.a) – (2.9.c) are empirical counterparts to (2.5.a) – (2.5.c) and (2.6.a) – (2.6.c), respectively. These derivatives must be evaluated at specific values of the explanatory variables to determine their sign. The relationships are consistent with the comparative static results. First and second partials of basis with respect to weight depend on exogenous corn and live cattle prices and sale weight. The predicted sign of these derivatives also depends on the relative per pound price of gain and slaughter cattle. First partials and cross-partial with respect to both prices also exhibit consistency with the comparative static results in the sense that they depend only on weight. These maintain their predicted sign when sale weight is at or below the feeder cattle weight of 750 lbs.

The plots in figure 2.5 show predicted values of the dependent variable, $Basis_{InputWt}^{750}$, and the partial derivative relationships from Model 3. Predicted values are calculated at mean values of the live cattle and corn price variables and plotted against weight. These graphs make it visually apparent that the slope of the price slide (2.8.a) becomes more steep (negatively sloped) at lighter sale weights. An increase in the corn price produces a decrease in basis at all weights (2.8.b) and “flattens” the negative slope of the price slide to a greater degree at lighter weights (2.9.b). Similarly, live cattle price increases increase basis at all weights (2.8.c) and increase the “steepness” of the price slide to a greater degree for lighter cattle.

In order to more easily interpret and visualize the price slide dynamics, figures 2.6 and 2.7 plot predicted basis from Model 3 at different values of the corn and live cattle price. The price slide shown in the middle of both figures plots basis predictions at the sample mean values of both price variables (equivalent to figure 2.5). In figure 2.6, the corn price is held constant and two additional predictions are plotted for the live cattle price at one standard deviation above and below the sample mean of $78.33/cwt. Figure 2.7 holds live cattle price constant and plots
predictions for corn prices one standard deviation above and below the mean value of $2.61/bushel. These figures clearly show the manner in which changes in live cattle and corn prices make the price slide steeper or flatter and have larger impacts on the price slide at lighter cattle weights.

Lot characteristics

Several previous studies have demonstrated the importance of other lot characteristics in determining feeder cattle prices and basis. I control for and test the significance of several lot characteristics by adding them as additional explanatory variables in models 2 and 3. Table 5 displays estimates for the price and weight variables when other lot characteristics are included as Model 2(o) and Model 3(o). Coefficients for individual lot characteristics are suppressed. A comparison of estimated parameters from models with and without lot characteristics is informative. Most notably, the inclusion of lot characteristics produces a considerable improvement in the adjusted R-square for both models, suggesting that these factors are indeed important basis determinants. Further, the inclusion of lot characteristics does not cause appreciable change in the values of any of the price-weight coefficients.

Coefficients for the individual lot characteristic variables from the Model 3(o) specification are displayed separately in table 6. All of these estimates exhibit strong statistical significance, which is not surprising in light of the large number of observations. The sign of the coefficient on lot size is consistent with many previous studies (for example Dhuyvetter and Schroeder, 2000), which have shown that larger groups of cattle sell at a premium compared to smaller lots. Dummy variables were included for four additional characteristics: sex, class, month, and location. The estimates suggest that buyers pay a premium for steers relative to heifers or bulls and for cattle grading at class 1. Monthly dummy variables capture seasonal price
differences and show that basis is lowest for feeder cattle that are sold in the fall months. Figure 2.8 displays coefficients for these monthly coefficients. The results are consistent with previous research and also consistent with the notion that a large supply of spring calves results in a large supply of fall stocker cattle. Location dummies reveal that basis also differs based on where the cattle are sold. This might result from geographic price differences as well as from differences in the type of cattle that are sold at different auction locations.

Conclusions and Future Research

The research presented in this chapter has demonstrated that empirical relationships between price and weight are consistent with predictions derived from a simple two-input derived demand model. Dynamics of the cattle price slide are clearly linked to changing conditions in the markets for corn feed inputs and live cattle outputs. Further, these relationships depend on future expectations of prices for cattle at different sale weights. Information about expected future market conditions for specific groups of cattle was incorporated by identifying appropriate futures contract prices.

The empirical analysis presented here suggests many possible areas for future exploration. For example, several explanatory variables could potentially be added to the model. These might include proxies for pasture feed inputs (ex., Palmer drought index), cattle inventory numbers, fuel costs, and recent feedlot profit margins.

Most obviously, the analysis could be expanded to include a vast AMS database of feeder cattle transactions. This introduces regional variation in price slide dynamics that might stem from proximity to corn production and feedlot facilities or from regional differences in production practices and types of animals sold. Expanding the dataset will also introduce additional temporal variation by the inclusion of recent years with high and variable prices.
Table 2.1. Example feed cost calculations for two weights of the steer input

<table>
<thead>
<tr>
<th></th>
<th>Feeder</th>
<th>Stocker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output (slaughter) weight</td>
<td>1250</td>
<td>1250 lbs</td>
</tr>
<tr>
<td>- Input Weight</td>
<td>750</td>
<td>550 lbs</td>
</tr>
<tr>
<td>Gain Required</td>
<td>500</td>
<td>700 lbs</td>
</tr>
<tr>
<td>( \times ) Corn Conversion</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Corn Input (lbs)</td>
<td>3,500</td>
<td>4,900 lbs</td>
</tr>
<tr>
<td>( \div ) corn bushels / lb</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td>Corn Input (bushels)</td>
<td>62.5</td>
<td>87.5 bushels</td>
</tr>
<tr>
<td>( \times ) Corn Price ($/bushel)</td>
<td>$7.50</td>
<td>$7.50 per bushel</td>
</tr>
<tr>
<td>Gross Feeding Margin (GFM)</td>
<td>$468.75</td>
<td>$656.25 per head</td>
</tr>
<tr>
<td>Cost of Gain (COG)</td>
<td>$0.94</td>
<td>$0.94 per lb</td>
</tr>
</tbody>
</table>

Table 2.2. Example derived steer input values and prices

<table>
<thead>
<tr>
<th></th>
<th>Price LC1250</th>
<th>x Weight 1250</th>
<th>1.24</th>
<th>1.24 per lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slaughter value</td>
<td>$1,550.00</td>
<td>1250</td>
<td>$1,550.00</td>
<td>$/hd</td>
</tr>
<tr>
<td>- GFM</td>
<td>$468.75</td>
<td>1250</td>
<td>$656.25</td>
<td>$/hd</td>
</tr>
<tr>
<td>Steer Input Value (derived)</td>
<td>$1,081.25</td>
<td>750</td>
<td>550 lbs</td>
<td>$893.75 $/hd</td>
</tr>
<tr>
<td>Steer Input Price (derived)</td>
<td>$1.44</td>
<td>1.63</td>
<td>per lb</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3. Summary of example steer prices by weight

<table>
<thead>
<tr>
<th>Steer type</th>
<th>Weight</th>
<th>Price</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slaughter</td>
<td>1250</td>
<td>$1.24</td>
<td>$1,550.00</td>
</tr>
<tr>
<td>Feeder</td>
<td>750</td>
<td>$1.44</td>
<td>$1,081.25</td>
</tr>
<tr>
<td>Stocker</td>
<td>550</td>
<td>$1.63</td>
<td>$893.75</td>
</tr>
</tbody>
</table>
Table 2.4. Summary statistics for Kansas sample data 1999-2009

Number of observations = 282,152

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>SaleDate</td>
<td>YYYYMMDD</td>
<td></td>
<td>19990106</td>
<td>20090326</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>Sale weight</td>
<td>pounds</td>
<td>648</td>
<td>141</td>
<td>300</td>
<td>900</td>
</tr>
<tr>
<td>Price Variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basis750</td>
<td>[Cash price] - [FC750]</td>
<td>$/cwt</td>
<td>2.48</td>
<td>10.42</td>
<td>-63.62</td>
<td>87.33</td>
</tr>
<tr>
<td>Cash Price</td>
<td>Cash sale price</td>
<td>$/cwt</td>
<td>95.17</td>
<td>16.30</td>
<td>34.75</td>
<td>199.00</td>
</tr>
<tr>
<td>FC750</td>
<td>Price on sale date for feeder cattle futures contract that will be nearby when cattle weigh 750 lbs</td>
<td>$/cwt</td>
<td>92.32</td>
<td>12.52</td>
<td>69.97</td>
<td>119.57</td>
</tr>
<tr>
<td>CN0</td>
<td>Price on sale date for nearby corn futures contract price</td>
<td>$/bushel</td>
<td>2.61</td>
<td>0.91</td>
<td>1.75</td>
<td>7.61</td>
</tr>
<tr>
<td>LC1250</td>
<td>Price on sale date for live cattle contract that will be nearby when cattle weigh 1250 lbs</td>
<td>$/cwt</td>
<td>78.33</td>
<td>10.37</td>
<td>60.67</td>
<td>117.70</td>
</tr>
<tr>
<td>Interaction/Quadratic Terms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Wt2</td>
<td></td>
<td></td>
<td>439982</td>
<td>178863</td>
<td>90000</td>
<td>810000</td>
</tr>
<tr>
<td>CN0*Wt</td>
<td></td>
<td></td>
<td>1697</td>
<td>733</td>
<td>526</td>
<td>6811</td>
</tr>
<tr>
<td>LC1250Wt</td>
<td></td>
<td></td>
<td>50843</td>
<td>13373</td>
<td>19740</td>
<td>100598</td>
</tr>
<tr>
<td>CN0*Wt2</td>
<td></td>
<td></td>
<td>1156605</td>
<td>664828</td>
<td>157725</td>
<td>6095800</td>
</tr>
<tr>
<td>LC1250Wt2</td>
<td></td>
<td></td>
<td>3456105</td>
<td>15142460</td>
<td>5922000</td>
<td>9033667</td>
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<tr>
<td>Lot Characteristics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lot Size</td>
<td># head sold</td>
<td></td>
<td>25</td>
<td>1</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>Str</td>
<td>Steers</td>
<td></td>
<td>0.5335</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Hfr</td>
<td>Heifers</td>
<td></td>
<td>0.4636</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Bul</td>
<td>Bulls</td>
<td></td>
<td>0.0029</td>
<td>0</td>
<td>1</td>
<td></td>
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<td>Class1</td>
<td>Class 1</td>
<td></td>
<td>0.6654</td>
<td>0</td>
<td>1</td>
<td></td>
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<tr>
<td>Class12</td>
<td>Class 1-2</td>
<td></td>
<td>0.1944</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Class2</td>
<td>Class 2</td>
<td></td>
<td>0.1357</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo1</td>
<td>January</td>
<td></td>
<td>0.1169</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo2</td>
<td>February</td>
<td></td>
<td>0.0930</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo3</td>
<td>March</td>
<td></td>
<td>0.1228</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo4</td>
<td>April</td>
<td></td>
<td>0.1015</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo5</td>
<td>May</td>
<td></td>
<td>0.0651</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo6</td>
<td>June</td>
<td></td>
<td>0.0254</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo7</td>
<td>July</td>
<td></td>
<td>0.0493</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo8</td>
<td>August</td>
<td></td>
<td>0.0757</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo9</td>
<td>September</td>
<td></td>
<td>0.0704</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo10</td>
<td>October</td>
<td></td>
<td>0.1130</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo11</td>
<td>November</td>
<td></td>
<td>0.1056</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo12</td>
<td>December</td>
<td></td>
<td>0.0612</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Loc01</td>
<td>Syracuse, KS</td>
<td></td>
<td>0.0378</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Loc02</td>
<td>Junction City, KS</td>
<td></td>
<td>0.0777</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Loc03</td>
<td>Winter Livestock Video</td>
<td></td>
<td>0.0018</td>
<td>0</td>
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<tr>
<td>Loc04</td>
<td>Winter Livestock Auction</td>
<td></td>
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<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Loc05</td>
<td>Pratt Livestock Auction</td>
<td></td>
<td>0.2659</td>
<td>0</td>
<td>1</td>
<td></td>
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<tr>
<td>Loc06</td>
<td>Kansas Direct Feeder Cattle</td>
<td></td>
<td>0.0554</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Loc07</td>
<td>Farmers &amp; Ranchers Livstk Co</td>
<td></td>
<td>0.3686</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.5. Model parameter estimates for price-weight variables

Dependent Variable = Basis\textsuperscript{750}_it

*Number of observations* = 282,152

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 2(o)</th>
<th>Model 3</th>
<th>Model 3(o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adj. R-square</td>
<td>0.4834</td>
<td>0.4887</td>
<td>0.7438</td>
<td>0.5096</td>
<td>0.7581</td>
</tr>
<tr>
<td>a Intercept</td>
<td>66.8342(0.25949)</td>
<td>64.01814(0.28440)</td>
<td>54.13294(0.21004)</td>
<td>-94.74554(2.32976)</td>
<td>-82.56878(1.64035)</td>
</tr>
<tr>
<td>b Weight</td>
<td>-0.15744(0.00084697)</td>
<td>-0.15813(0.00084271)</td>
<td>-0.12886(0.00060393)</td>
<td>-0.2677(0.00747)</td>
<td>0.24727(0.00526)</td>
</tr>
<tr>
<td>c Wt2</td>
<td>0.000085666(6.686718E-07)</td>
<td>0.00008634(6.653638E-07)</td>
<td>0.00005586(4.789582E-07)</td>
<td>-0.00018488(0.0000580)</td>
<td>-0.0001905(0.00000408)</td>
</tr>
<tr>
<td>d CN0</td>
<td>-1.27259(0.2355)</td>
<td>-2.23911(0.01743)</td>
<td>-27.73627(0.45203)</td>
<td>-25.64551(0.31815)</td>
<td></td>
</tr>
<tr>
<td>e LC1250</td>
<td>0.08021(0.00206)</td>
<td>0.19665(0.00158)</td>
<td>2.99828(0.04014)</td>
<td>2.73032(0.02827)</td>
<td></td>
</tr>
<tr>
<td>f CN*Wt</td>
<td>0.07012(0.00144)</td>
<td>0.06225(0.00101)</td>
<td>48.69</td>
<td>61.63</td>
<td></td>
</tr>
<tr>
<td>g LC1250*Wt</td>
<td>-0.0078(0.00012775)</td>
<td>-0.0069(0.00008997)</td>
<td>-61.06</td>
<td>-76.69</td>
<td></td>
</tr>
<tr>
<td>h CNWt2</td>
<td>-0.00004417(0.00000111)</td>
<td>-0.00003922(7.813848E-07)</td>
<td>-39.79</td>
<td>-50.19</td>
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</tr>
<tr>
<td>i LC1250Wt2</td>
<td>0.00000495(9.863877E-08)</td>
<td>0.00000447(6.943237E-08)</td>
<td>50.18</td>
<td>64.38</td>
<td></td>
</tr>
</tbody>
</table>

Lot Characteristics | Included | Included

Notes: Estimated coefficients (standard errors) and t-statistics are shown. All estimates are statistically significant with P-values <0.0001. Individual parameter estimates for lot characteristics included in models 2(o) and 3(o) have been suppressed.
Table 2.6. Individual parameter estimates for lot characteristics - Model 3(o)

Dependent Variable = Basis_{t}^{750}

*Number of observations* = 282,152

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>StdErr</th>
<th>T-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td># Head Sold (Lot Size)</td>
<td>0.00169</td>
<td>(0.00017732)</td>
<td>9.53</td>
</tr>
<tr>
<td><strong>Sex Dummies</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steer</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heifer</td>
<td>-8.14089</td>
<td>(0.01982)</td>
<td>-410.74</td>
</tr>
<tr>
<td>Bull</td>
<td>-6.88621</td>
<td>(0.17916)</td>
<td>-38.44</td>
</tr>
<tr>
<td><strong>Class Dummies</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class1</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class12</td>
<td>-4.18968</td>
<td>(0.02570)</td>
<td>-163.02</td>
</tr>
<tr>
<td>Class2</td>
<td>-7.0359</td>
<td>(0.02988)</td>
<td>-235.47</td>
</tr>
<tr>
<td><strong>Month Dummies</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>January</td>
<td>4.00651</td>
<td>(0.04096)</td>
<td>97.82</td>
</tr>
<tr>
<td>February</td>
<td>5.37661</td>
<td>(0.04361)</td>
<td>123.29</td>
</tr>
<tr>
<td>March</td>
<td>5.7799</td>
<td>(0.04084)</td>
<td>141.53</td>
</tr>
<tr>
<td>April</td>
<td>6.73215</td>
<td>(0.04242)</td>
<td>158.70</td>
</tr>
<tr>
<td>May</td>
<td>5.35235</td>
<td>(0.04830)</td>
<td>110.81</td>
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<td>June</td>
<td>4.13316</td>
<td>(0.06803)</td>
<td>60.75</td>
</tr>
<tr>
<td>July</td>
<td>4.48984</td>
<td>(0.05262)</td>
<td>85.33</td>
</tr>
<tr>
<td>August</td>
<td>3.76185</td>
<td>(0.04575)</td>
<td>82.23</td>
</tr>
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<td>September</td>
<td>2.22642</td>
<td>(0.04654)</td>
<td>47.84</td>
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Notes: Estimated *coefficients* (standard errors) and *t-statistics* are shown. All estimates are statistically significant with P-values <0.0001. Price and weight coefficients shown on table 5.
Figure 2.1. Example value, price, and feed cost calculations by steer input weight
Figure 2.2. Example calculations with reduced corn price
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CHAPTER 3

A Nonlinear Bio-economic Price Slide Model

The two-input derived demand model of cattle feeding used to analyze price slides in chapter 2 relies on several simplifying assumptions. This includes a strict assumption about cattle growth such that cattle and feed inputs are combined in fixed proportions to produce constant growth throughout the life of the animal. A numerical example in chapter 2 uses a constant ratio of seven pounds of feed (corn) per pound of weight gain regardless of the steer’s current weight at time of sale.

The biology of feeder cattle growth suggests that rates of gain and feed consumption are likely to change over the life of a feeder animal. In this chapter I extend the two-input derived demand model of feeder cattle prices to accommodate nonlinear relationships between feed inputs and growth. A “bio-economic” model of cattle feeding incorporates biological growth parameters to facilitate exploration of the effect that biological growth factors might exert on the price slide relationship.

Biological Growth Model Assumptions Reconsidered

The science of beef cattle nutrition and feeding programs is complex. Modern cattle feeding operations incorporate a variety of feeding regimens, possible ration combinations, and animal growth response functions to optimize profitability. The theoretical model developed here will abstract from many of these complexities while still accommodating some of the most important aspects of cattle growth.

Consider the growth curve of a steer as a simple plot of weight versus age (time) as depicted in figure 3.1. This growth curve will typically have a sigmoidal shape as the steer...
exhibits slow growth immediately after birth, followed by a rapid acceleration and eventual tapering off as the animal approaches maturity (Berg and Butterfield (1976) and Boggs et al. (1998)). The period of life during weaning and feeding, during which cattle are bought and sold, occurs during the period of acceleration and tapering (when the steer weighs more than 250 lbs). During this period, cattle continue to grow (rate of gain is always positive), but they do so at a constantly decreasing rate. This implies that the first derivative of an observed feeder cattle growth curve is always positive and the second derivative always negative.

In practice, feedlots use well-known performance indicators to measure cattle feeding performance against industry benchmarks. One such measure tracks the rate at which cattle progress along the growth curve depicted in figure 3.1. Feedlots calculate the average daily gain
(ADG) for a pen of cattle\textsuperscript{24} as the difference between beginning and ending weight divided by the total number of days the cattle are on feed. The number of days fed, pounds gained, and beginning and ending weights for which ADG is measured will vary by type of feeding operation. Some sophisticated feeding operations collect substantial amounts of data to measure performance on a per head per day basis, while others simply consider the average for a pen of animals over the entire feeding period.

In addition to rate of gain over time, cattle feeders also consider the relationship between feed intake and weight gain when assessing cost and performance. Two related measures of this relationship are commonly used in the industry. Feed conversion efficiency (FCE) is defined as the total pounds of cattle growth per pound of feed. Cattle biology (and simple logic) dictates that FCE must be less than one (i.e., one pound of feed can produce no more than a pound of gain). Another commonly used indicator, feed conversion ratio (FCR) is simply the mathematical inverse of FCE. FCR is defined to be the pounds of feed required to produce a pound of gain.\textsuperscript{25}

A report published by the National Research Council since 1944 entitled, “Nutrient Requirements of Beef Cattle,” is considered by many to be the industry standard reference guide on cattle growth and nutrition (National Research Council 2000). Beef nutrition experts and others use this report, with associated software, tables, and feed equations, to develop and compare feeding strategies for cattle of different types and weights. A fundamental relationship found in the NRC report is that beef cattle require more energy for body maintenance as their

\textsuperscript{24} A “pen” refers to a group of cattle that are grouped together and fed in the same location. To optimize feedlot performance, feeders tend to value consistency among cattle in a pen. Thus, cattle that are generally grouped together in pens that are similar in sex, type (frame and class), and weight. Further, this pen of similar cattle typically enter and exit the lot together; new cattle are usually not added to existing pens. In many cases, a group of cattle that are purchased together (as a “lot”) are placed together in a pen, fed together, and sold for slaughter at the same time.

\textsuperscript{25} As an example, the linear growth model assumption of seven pounds of feed (corn) per pound of weight gain from chapter 2 equates to $FCE = 0.1429$ and $FCR = 7.0$. 

50
body weight increases. Correspondingly, cattle must consume more feed to maintain their essential physiological functions as they grow heavier. As a direct result of this biological fact, FCE is expected to decrease with weight (and, conversely, FCR is expected to increase with weight) because growing steers will require increasingly more pounds of feed to achieve a pound of gain.

It follows that a steer’s rate of daily gain would decline over weight and over time if its daily ration remained constant. However, the effective amount of the feed ration a steer consumes under ad libitum feed conditions\(^\text{26}\) will increase as it grows. In practice, this may not equate to greater quantities of the same ration. Energy content, which is needed to produce gain, depends on the composition of the feed ration (e.g., energy, protein, and other nutrients), which may contain several different feedstuffs (such as corn, soybean meal, sileage, hay, etc.). Further, the quantity of ration is ultimately bounded by the amount of feed the animal will consume (dry matter intake, DMI), which varies based on a wide variety of biological factors that are beyond the scope of this study. Nonetheless, increased daily feed consumption will, to some extent, offset declines in the rate of daily gain as FCR (pounds of feed used to produce a pound of gain) increases or, equivalently, as FCE (pounds of gain produced by a pound of feed) declines.

**Conceptual Framework for a Nonlinear Bio-economic Model**

The derived demand model of feeder cattle price is based on value of the output (live cattle) minus value of the feed input. Prices for the output (live cattle futures) and the inputs (cash feeder cattle and corn futures) can be obtained from market data. Here, I relax the assumption that marginal productivity of the feed input (in terms of weight gain) remains

\(^{26}\) Cattle are normally fed under *ad libitum* feeding conditions, which means that they are allowed to eat according to their appetite.
constant over the life and growth of the animal. In doing this, I allow for a more realistic scenario in which the amount of feed input required to produce additional weight gain increases as the animal grows from calf to feeder to finished.

The model can be visualized as a two-input transformation whereby a steer is a simple factory producing added pounds of beef through consumption of the feed input (corn). Instead of fixed proportions, the production function allows for diminishing returns to the feed input. The “steer factory” is harvested in its entirety once total production reaches a desired finishing weight of approximately 1,350 pounds. For a given steer at time $t$, the total amount of remaining feed input, $F_t$, required to produce the finished live cattle output is a function of the steer’s current weight, $w$, finishing weight, $\bar{w}=1,350$, and parameters that determine the animal’s rate of gain over time. Those factors affecting the shape of $F_t$ for a given current and finished weight are biological growth parameters.

The instantaneous rate of gain per unit time that is produced by the fixed ration depends on the steer weight and is assumed to decrease according to the following linear relationship:

$$\dot{w} = \frac{dw}{dt} = b - aw_t,$$  \hspace{1cm} (3.1)

Note that if $a = 0$, equation 3.1 is equivalent to the linear growth model where rate of gain is constant over weight. However, positive values of $a$ allow for a negatively sloped marginal gain function in which the rate of gain decreases as the animal grows. This expression for the marginal rate of change in weight per day, $\dot{w}$, can be solved as a differential equation to yield the following expression for time remaining until the steer reaches finished weight,

$$t^*_{w_t}(\bar{w}, w, b, a) = \frac{1}{a} \cdot \ln \left( \frac{b - w_t a}{b - \bar{w} a} \right).$$  \hspace{1cm} (3.2)
Equation (3.2) represents the time required for the steer to grow from some initial weight, \( w_i \), to the greater finished weight of \( \bar{w} \) given growth parameters \( a \) and \( b \) from (3.1).\(^{27}\) If the rate of consumption is assumed to be fixed at constant rate \( f \), the remaining feed input required to grow the animal from current to finished weight can be written as,

\[
\tilde{F}_t = \tilde{F}_{\bar{w}}(\bar{w}, w_i, a, b, f) = f \times t^{\bar{w}}_{w_i}(\bar{w}, w_i, a, b) = f \times \left[ \frac{1}{a} \cdot \ln \left( \frac{b - w_i a}{b - \bar{w} a} \right) \right]
\] (3.3)

Equation (3.3) expresses the total amount of feed input that must be combined with the feeder cattle input of weight \( w_i \) to produce the live cattle output of a target weight \( \bar{w} \) in terms of the fixed ration \( f \) and biological growth parameters. Model parameters \( a \) and \( b \) allow for nonlinear growth reflected in a changing rate of gain over weight.

A fixed ration assumption is convenient but, as noted in the previous discussion of cattle growth literature, not realistic. In fact, cattle will consume more feed as they grow larger (i.e., daily feed or dry matter intake, DMI, increases). To accommodate a rate of consumption that increases with weight, the instantaneous rate of feed consumption can be expressed as follows:

\[
f(w_i) = \mu + \delta w_i
\] (3.4)

The more general expression given by equation (3.4) allows \( f(w_i) \) to increase with steer weight when parameter \( \delta > 0 \). These two additional biological growth parameters are bounded in a realistic model for two reasons: 1) because it is impossible for an animal to consume a negative ration, implying \( f \geq 0 \ \forall \ w_i \) and \( \mu \geq 0 \), and 2) because, due to biological maintenance requirements, larger animals will consume as much or more feed than smaller animals, implying that \( \delta \geq 0 \).

\(^{27}\) Appendix A contains a more detailed derivation of this expression.
Figure 3.2 gives a graphical representation of the relationships between rate of gain (from equation (3.1)) and rate of feed consumption (from equation (3.4)) as they depend on current weight, \( w_t \), for acceptable (positive) values of the four parameters.\(^{28}\)

![Graphical representation of the model](image)

**Figure 3.2.** Rate of gain and feed consumption by weight

A more general expression for the total remaining feed input can be derived by incorporating the equations for both rate of gain and rate of feed consumption, as given in equation (3.4), into (3.3).

\[
\bar{F}_t = \bar{F}_{w_t}(\bar{w}, w_t, a, b, f(w_t)) = f_{w_t}(w_t, \mu, \delta) \ast f_{w_t}(\bar{w}, w_t, a, b) \quad (3.5)
\]

Given rates of gain and feed consumption that vary over the life of the animal, the expression for remaining feed input in the more general model incorporates all four biological growth parameters as follows\(^{29}\),

---

\(^{28}\) Graphical representations of the model displayed in figures 3.2 – 3.4 were generated using the following parameter values: \( a=0.004, b=10, \mu=5, \delta=0.033 \).

\(^{29}\) A derivation of the more generalized model is provided in chapter appendix B.
Notice that the more general equation (3.6) is similar to the expression given previously in (3.3). When the ration is fixed, such that $\delta = 0$ and $f(w) = \mu$, the two expressions are equivalent.

Terms from equation (3.6) can be interpreted to better understand this expression. The first term in parentheses, $\mu + \delta \left( \frac{b}{a} \right)$, represents the rate of feed consumption from equation (3.4) evaluated at $w = b/a$. From (3.1), $b/a$ represents the model-implied maximum limit of weight (denoted $\omega_{\text{lim}}$) for the animal when rate of gain decreases to zero: $\dot{w} = 0$ at $w = \frac{b}{a} = \omega_{\text{lim}}$. The bracketed term in equation (3.6) can be recognized as $t_{w}^{\omega} (\bar{w}, w, b, a)$ (from equation (3.2)).

Note that the model can be consolidated by scaling to fewer parameters. With slight rearrangement of the terms in (3.6), the expression contains only biological growth parameters $b$, $\mu$, and $\delta$ scaled by parameter $a$:

$$F_t(\bar{w}, w, a, b, \mu, \delta) = \left( \frac{\mu}{a} + \frac{\delta}{a} \left( \frac{b}{a} \right) \right) \left[ \frac{1}{a} \ln \left( \frac{b - w_i}{b - \bar{w}} \right) \right] + \frac{\delta}{a} \left( \bar{w} - w_i \right)$$

Thus, the new model can be rewritten using only the scaled parameters,

$$F_t(\bar{w}, w, \omega_{\text{lim}}, u, d) = (u + d \omega_{\text{lim}}) \left[ \ln \left( \frac{\omega_{\text{lim}} - w_i}{\omega_{\text{lim}} - \bar{w}} \right) \right] - d \left( \bar{w} - w_i \right)$$

where $u = \frac{\mu}{a}$, $d = \frac{\delta}{a}$, and $\omega_{\text{lim}} = \frac{b}{a}$. 0.
Two cattle feeding performance measures mentioned previously, feed conversion efficiency (FCE) and feed conversion ratio (FCR), can also be expressed in terms of model parameters:\(^{30}\)

\[
FCE = \frac{dw}{dF} = \frac{\Delta \text{Lbs of Steer Weight}}{\Delta \text{Lbs of Feed Consumption}} ; \quad FCR = \frac{dF}{dw} = \frac{\Delta \text{Lbs of Feed Consumption}}{\Delta \text{Lbs of Steer Weight}},
\]

where \(FCR = FCE^{-1}\).

In contrast to average daily gain, FCE and FCR are measured relative to the animal’s weight rather than age. The future date at which a feeder steer reaches finished weight depends on the feeding program to be chosen by the future owner (buyer), and this information is unknown to the market observer at time of sale and price discovery. Feeder cattle are priced according to their current weight (not their age) and the expected per pound cost of feed. Feed conversion measures therefore include the two pieces of information that are most relevant to the model and obviate the need for unnecessary timing assumptions.

Substituting previously discussed expressions for the rates of weight gain and feed consumption over time, we obtain an expression for “instantaneous FCE,” which is the linear relationship between weight and marginal gain per increment of feed:

\[
FCE = \frac{dw}{dF} = \frac{\frac{d}{dt}w(w_i)}{\frac{d}{dt}f(w_i)} = \frac{\frac{\delta_i w_i}{\mu + \delta_i w_i}}{\frac{\mu}{\mu + \delta_i w_i}} = \frac{b - aw_i}{\mu + \delta i w_i}.
\]  \tag{3.8}

Equation (3.8) makes clear that the marginal change in weight per unit of feed consumption is equivalent to the ratio of the marginal rates of change in weight and feed consumption per unit time (from equations (3.1) and (3.4)). FCE varies with steer weight

\(^{30}\) In practice, FCE and FCR may refer to pounds of gain relative to feed consumption over a variety of time periods (perhaps most commonly, as the average conversion over the entire feeding period). Here, however, FCE is defined as an instantaneous rate of change.
according to a nonlinear relationship defined in terms of parameters from a model based on these two relationships.

**Bio-economic Model Dynamics**

Several features of the structure imposed on cattle growth by the model above bear further examination. Importantly, the efficiency with which a steer converts feed into weight gain (FCE) is allowed to decrease as the animal approaches finished weight. The model accommodates a feed regime in which lighter calves convert corn into weight gain more rapidly than larger feeder cattle.

![Figure 3.3: FCE and FCR by weight](image)

**Figure 3.3:** FCE and FCR by weight

Figure 3.3 displays both instantaneous and average rates of FCE and FCR by weight. The instantaneous change in weight per unit time, \( FCE = \frac{dw}{df} = \frac{\dot{w}}{f} \), is given by the ratio of slopes of \( w(t) \) (the cattle growth curve) and \( F(t) \) (total feed remaining) at time \( t \) when the steer weighs \( w_t \).
As discussed previously (and shown in figure 3.2 above) rate of gain, \( \dot{w} \), is decreasing in weight and rate of feed consumption, \( f \), is increasing in weight. Therefore, it follows from equation (3.8) that FCE will be decreasing in weight and its inverse, FCR, will be increasing in weight as shown in figure 3.3. Average feed conversion efficiency (\( \overline{FCE} \)) and average feed conversion ration (\( \overline{FCR} \)) for the remaining feeding period from \( w_i \) to \( \bar{w} \) are as follows:

\[
\overline{FCE} = \frac{\int_{w_i}^{\bar{w}} \dot{w}(w)dw}{\int_{w_i}^{\bar{w}} f(w)dw} = \frac{\bar{w} - w_i}{\overline{F_t}} \quad \text{and} \quad \overline{FCR} = \frac{\int_{w_i}^{\bar{w}} f(w_i)dw}{\int_{w_i}^{\bar{w}} \dot{w}(w)dw} = \frac{\overline{F_t}}{\bar{w} - w_i}.
\]

These represent the overall feedlot performance that would be expected for a pen of animals placed at \( w_i \) and fed to finishing.

From the current time, \( t_0 \), when the steer weighs \( w_i \), until it reaches slaughter weight \( \bar{w} \) at time \( T \), the steer will consume total remaining feed in the amount of \( \overline{F_t} \). As shown in figure 3.4, \( \overline{F_t} \) declines to zero as the animal approaches finished weight \( \bar{w} \). The slope of \( \overline{F_t} \) is the reduction in remaining feed per pound of gain, which is also \( -FCR = -\frac{dF}{dw} \), which is shown decreasing at an increasing rate in the nonlinear bio-economic model. In contrast to the linear model with a fixed rate of gain, \( \overline{F_t} \) is concave to the origin as shown in figure 3.4 because greater amounts of feed are required to produce marginal pounds of gain as the steer approaches slaughter weight.
The Nonlinear Bio-economic Price Slide

The biological growth model yields an expression for total amount of feed remaining to be consumed by a steer of known current and ending weight and three unknown biological growth parameters (from equation (3.7)): \( \bar{F}_t = F_{w_t}^{\pi}(\bar{w}, w_t, \omega_{\text{lim}}, u, d) \)

An equation for empirical estimation of the unknown biological growth parameters can be obtained by substituting the feed cost expression into the two-input derived demand model.

\[
\text{[Current Value]} = \text{[Final Value]} - \text{[Remaining Feed Cost]}
\]

or

\[
w_i \cdot P_i = \bar{w} \cdot P_\pi - P_c \cdot F_{\bar{w}}(\bar{w}, w_t, \omega_{\text{lim}}, u, d)
\]

Equation (3.9) represents the general derived demand relationship: a steer’s current value (on a per head basis) is equal to its expected final sale value less the remaining cost of feeding it to
finished weight. If all profitable investment opportunities in feeder cattle are exploited, equation (3.9) reflects an equilibrium, no profit condition. By rearranging equation (3.9) we obtain an expression for the cash price,

\[ P_i = \frac{\bar{w}}{w_i} * P_\pi - \frac{P}{w_i} * F_w(\bar{w}, w_i, \omega_{lim}, u, d), \]  

(3.10)

For a given group (lot) of steers, i, equation (3.10) contains several variables that are known from data, including current weight (\( w_i \)), current price (\( P_i \)), the output price for live cattle (\( P_\pi \)), and the corn price (\( P_c \)) as a proxy for feed cost. The desired finishing weight, \( \bar{w} \), can be treated as a known constant of 1,350 pounds.

**Predicted Price**

Cash price equation (3.10) incorporates biological growth parameters but does not accommodate any differences in basis between an individual lot of cattle, \( i \), and the CME contract value for the output price. Several factors have been shown to influence feeder cattle basis, as discussed in chapter 2. Therefore, the nonlinear “bio-economic” model must include some means of accounting for these effects.

A straightforward method of incorporating basis into the model from equation (3.10) is to simply include an additive parameter that can reflect basis relative to the output price:

\[ P_i = \frac{\bar{w}}{w_i} * (P_\pi + Basis_i) - \frac{P}{w_i} * F_w(\bar{w}, w_i, \omega_{lim}, u, d) \]

where \( Basis_i \) corresponds to a specific lot of cattle.

**Summary**

The bioeconomic price slide model yield valuable insights into the nature of cattle asset price determination. It is too restrictive for empirical implementation, however, as it imposes a
rigid functional form throughout the life of the fed animal and up to $\omega_{\text{lim}}$, its maximum possible weight, which in general will be well beyond the economically optimal slaughter weight. In applying the model to the data observed in the 300 to 900-pound weight range, I take an empirically flexible approach that incorporates the time variation in growth and feeding rates illustrated by the theoretical model, but does so in a less restrictive way. This flexible parametric approach is the subject of the following chapter.
CHAPTER 4

Empirical Analysis of Nonlinear Feeder Cattle Growth

The value of a feeder steer in the derived demand model is calculated starting from expected fed (output) value of the steer when it will be slaughtered for beef production and subtracting the total expected cost of feeding that steer from feeder to fed slaughter weight. From the theoretical model developed in chapter 3, $F_i$ represents the total amount of feed required to produce a finished live cattle output of weight $\bar{w}$ from a feeder steer input having current weight $w_i$. Under reasonable assumptions of feed conversion and average daily gain, $F_i$ was shown to be a nonlinear function of the current and finished feeder cattle weights, $w_i$ and $\bar{w}$, and biological growth parameters ($\theta$). The empirical model developed in this chapter will focus on observable factors that influence $F_i$ using an estimation equation based on the bio-economic derived demand model.

Development of the Empirical Model

A slight rearrangement of the equilibrium no-profit condition from the derived demand model yields the following expression for total remaining feed cost,

$$[\text{Total Remaining Feed Cost}] = [\text{Final Value}] - [\text{Current Value}]$$

$$P_{Feed} \ast F_i = (P_i \ast \bar{w}) - (P_w \ast w_i) \quad (4.1)$$

At a given point in time, the amount of total remaining feed cost implied by the two-input model is the difference between current (input) and expected future (output) values of the feeder animal. The future value of fed steers can be approximated using current per pound prices of

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31 The difference between value of the purchased feeder cattle input and the value of the finished (fed) cattle sold is known to hedgers using the “cattle crush,” feeders, and others in the cattle industry as Gross Feeding Margin (GFM). <https://www.cmegroup.com/trading/agricultural/files/AC-378_CattleFeedingWhitePaper_r2.pdf>
deferred live cattle futures contracts ($P_w$) for steers with an assumed average finishing weight of 1,350 pounds ($\overline{w}$). Per pound cash price ($P_w$) paid for a group of steers along with reported average weight ($w_i$) can be used to calculate the current value (per head) of a given feeder animal. Total remaining feed cost is calculated as total remaining feed quantity $\overline{F}_i$ multiplied by a unit price. Relying as before on the substantial importance of corn prices in the overall cost of feeding cattle, the current per pound corn price, $P_{Corn}$, is used as the unit price of feed. Dividing both sides of (4.1) by the cost of feed results in an expression for $\overline{F}_i$ directly,

$$\overline{F}_i(w_i, \overline{w}, \theta) = \frac{(P_w * \overline{w}) - (P_w * w_i)}{P_{Corn}}.$$ (4.2)

Equation (4.2) suggests an empirical model for analyzing biological growth based on the derived demand model discussed in chapter 3. All terms on the right-hand side of (4.2) are either assumed constants ($\overline{w}$=1,350) or variables that can be observed in data. Because the feed price, $P_{Corn}$, is expressed in dollars per pound of corn, $\overline{F}_i$ represents the total remaining pounds of feed (corn) input required by feeder steer (or group/lot/pen of feeder cattle), $i$. This empirical expression for $\overline{F}_i$, derived directly from the zero-profit condition of the derived demand model, is calculated for each individual lot sale transaction in the AMS cash feeder cattle dataset. The resulting variable is defined here as the implied breakeven feed requirement (IBEFR) for lot $i$:

$$IBEFR_i = \frac{(P_w * \overline{w}) - (P_w * w_i)}{P_{Corn}}$$ (4.3)

---

32 Corn prices for CME futures contracts are quoted in dollars or cents per bushel (e.g., $4.00/bu or 400 cents/bu). These prices are converted to a per pound basis using a standard conversion rate of 56 pounds per bushel of corn, and $P_{corn}$ is expressed in dollars per pound. For example, at $4.00/bu corn price, equals $P_{corn} = 0.0714/lb ($4.00 / 56 lb/bu).
IBEFR will serve as the dependent variable in a regression model that will be used to explore biological growth factors that influence $\bar{F}$. One empirically noteworthy feature of IBEFR as a model variable is that prices (in $/lb.) cancel when factored out of the numerator and denominator. As a result, IBEFR is expressed in total pounds (of corn) and is unaffected by inflationary price changes. Thus, there is no need to index input and output prices for inflation when using prices from different time periods in empirical estimation.

**Incorporating non-feed factors**

The two-input derived demand model is a powerful tool for analyzing feeder cattle price dynamics. However, it also imposes a somewhat unreasonable limitation of the model as constructed here. Specifically, equation (4.3) implies that all variation in IBEFR must be explained by factors that relate directly to remaining feed consumption and exert influence through $\bar{F}$. Feed cost factors are primary, but not exclusive, determinants of relative feeder cattle prices. Other factors have previously been found to influence feeder cattle basis. As an example, transportation cost differentials at different locations are a standard component of basis for most commodities. To the extent that transportation costs affect relative cash prices but not the total amount of remaining feed cattle are expected to consume, the empirical model must accommodate these adjustments in some manner. Otherwise, they will be relegated to the error term.

Returning to the zero-profit derived demand equation, the current value of a feeder steer is defined as the residual value once remaining feed input costs have been deducted from the expected final (future) value of the slaughter steer output:

$$[\text{Current Value}] = [\text{Final Value}] - [\text{Feed Cost}] .$$
A separate value adjustment that includes factors not related to feed (non-feed) can be incorporated into the model directly as an additional component of the difference between input and output values:

\[
[\text{Current Value}] = [\text{Final Value}] - \{[\text{Feed Costs}] + [\text{Non-feed Value Adjustment (NfVA)}]\}.
\]

Incorporating this change gives the following,

\[
P_{w_i} * W_i = P_{w} * \overline{W} - \left\{ \left( P_{\text{corn}} * \overline{F} \right) + NfVA_i \right\}
\]

(4.4)

The term \{in braces\}, which is deducted from expected final (output) value to determine current value, now includes two components: feed and non-feed. The feed component represents the cost of feed inputs; the non-feed adjustment encompasses all other factors. Non-feed factors may include, for example, returns to feedlot operations, premiums or discounts for expected quality differences, lot characteristics (e.g., uniformity), and all other components of output value that do not depend on the price of feed inputs. Acting through the model in the same manner as higher feed costs, positive non-feed adjustments have the effect of reducing current feeder cattle value. By contrast, negative values for NfVA reduce the total amount that is deducted from expected output value and therefore lead to an increase in feeder cattle input value. Simply put, the non-feed value adjustment has a one-to-one negative relationship with current value of the feeder cattle input,

\[
\frac{\partial[\text{Current Value}]_i}{\partial NfVA_i} = -1.
\]

Each individual observation in the AMS data contains the cash price paid for a lot (pen) of feeder cattle with a given average weight and other observed lot characteristics on a specific date. Market participants are assumed to observe all these factors in addition to current (as of sale date) market expectations based on relevant futures prices for corn inputs (\(P_{\text{corn}}\)) and
slaughter cattle outputs \((P_i)\). Cash transactions between buyers and sellers responding to observed market factors and lot characteristics reveal prices for individual groups of feeder cattle; this in turn drives variation of the dependent variable in the model, \(IBEFR_i\).

For convenience when interpreting empirical results, non-feed value adjustment is converted from an amount per animal to an amount per pound of the animal’s input weight \(($/w_i)\). Define per pound Non-feed Price Adjustment as \(NfPA_i = \frac{NfVA_i}{w_i}\), which can be interpreted as the non-feed component of the feeder cattle price. Substituting for the value adjustment \((NfVA_i = NfPA_i \cdot w_i)\) and rearranging as before results in the following expression for \(\bar{F}_i\):

\[
\bar{F}_i(w_i, \bar{w}, \theta) = \frac{(P_i \cdot \bar{w}) - (P_{w_i} \cdot w_i)}{P_{Corn}} - \frac{w_i}{P_{Corn}} \cdot NfPA_i.
\] (4.5)

Substituting for \(IBEFR_i\) and rearranging (4.5) gives a new expression for the implied breakeven feed requirement that includes the non-feed component:

\[
IBEFR_i = \frac{(P_i \cdot \bar{w}) - (P_{w_i} \cdot w_i)}{P_{Corn}} = \bar{F}_i(w_i, \bar{w}, \theta) + \frac{w_i}{P_{Corn}} \cdot NfPA_i
\] (4.6)

Equation (4.6) will form the basis for empirical estimation in which \(IBEFR_i\) is the dependent variable. The equation suggests classifying explanatory variables for the model into two groups corresponding to separate components of the implied breakeven feed variable:

\(IBEFR_i = IBEFR_i^{Fd} + IBEFR_i^{Nf}\). The first group includes variables \((Fd)\) that measure differences in \(IBEFR_i\) that are caused by changes in remaining feed consumption, \(\bar{F}_i\), and are related to biological growth factors. The second group includes variables \((Nf)\) that measure differences in \(IBEFR_i\) caused by non-feed factors. Some explanatory variables in the model will measure a
characteristic or factor that may be expected to exert both feed and non-feed marginal effects on \( IBEFR_i \). These variables will included in both groups, \( Fd \) and \( Nf \), and the empirical model will be used to differentiate between feed and non-feed impacts.

**Estimation equation**

The general structure of an ordinary least squares estimation equation to be used for empirical analysis is as follows,

\[
IBEFR_i = \left[ \alpha + \left( \beta^p \cdot w_i^p \right) + \phi_f^p \left( w_i^p \cdot Fd_{i,f} \right) \right] + \left[ B^* \frac{w_i}{P_{corn}} \right] + N_s^* \left( \frac{w_i}{P_{corn}} \cdot Nf_{i,s} \right) + \varepsilon_i,
\]

(4.7)

In addition to the error term, \( \varepsilon_i \), the OLS equation shown in (4.7) consists of two bracketed terms that correspond to the feed and non-feed component terms in (4.6).\(^{33}\)

**Feed cost component**

The first term in equation (4.7) represents the feed cost component of \( IBEFR_i \), which is explained by biological factors that influence remaining feed:

Feed Cost Component = \( IBEFR_{i,Fd} = F_i(w_i, \bar{w}, \theta) = \alpha + \left( \beta^p \cdot w_i^p \right) + \phi_f^p \left( Fd_{i,f} \cdot w_i^p \right) \).

Following from the discussion in chapter 3, the feed cost component of the empirical model, representing \( F_i(w_i, \bar{w}, \theta) \), is expected to be a nonlinear function of current weight, \( w_i \). This empirical relationship is modeled using an ordinary least squares regression equation with

---

\(^{33}\) Equation (4.7) uses compact notation to represent groups of variables that are will be included in the model. These include \( p \) polynomial forms of the current sale weight variable, \( f \) separate variables in the \( Fd \) group (all of which will be interacted with each of the \( p \) weight variables), and \( s \) different variables in the \( Nf \) group (each of which is interacted with the ratio of current weight \( w_i \) to corn price, \( P_{corn} \). For convenience, the ratio will be denoted in subsequent text and tables as \( Wi_{-P_{corn}} \) (where \( Wi_{-P_{corn}} = \frac{w_i}{P_{corn}} \)). Linear parameters corresponding to variables \( P_{corn} \) within each group are indexed accordingly: \( \beta^p, \phi_f^p \), and \( N_s^p \). Parameters for the model intercept \( \alpha \) and coefficient \( B \) on the ratio \( Wi_{-P_{corn}} \) (with no interaction) do not require indexing. An expanded form of the regression equation showing all variables included in the model is included as appendix 4.A.
intercept $\alpha$, weight variables, and interaction terms between the weight variables and other variables in the $Fd$ group. Polynomial forms of the current weight variable ($w_i, w_i^2, ..., w_i^p$) are used to give a linear-in-parameters approximation of the expected nonlinearity in $w_i$. The included $Fd$ variables measure attributes or factors that are expected to reflect marginal differences between biological growth functions of animals in separate sale lots of the AMS data. Variables in the $Fd$ group include sex, muscle grade (class), location, and month of sale.

Sex and muscle thickness grade (class) both indicate biological characteristics that may be expected to influence feedlot performance. Heifers, for example, are expected to gain weight more slowly (lower ADG) and less efficiently (lower FCE or higher FCR) than steers. Feeder cattle that are graded class 1 muscle thickness exhibit superior muscle and breed composition and might also be expected to perform more efficiently in the feedlot than cattle given lower muscle thickness grades (class 2).

Sale locations often differ from one another in the type and quality of cattle sold. For example, some locations attract higher quality “production sales” and some others mainly provide an outlet for odd lots of lower quality cattle. To the extent that systematic differences in the quality of cattle of various sale weights at different locations are associated with differences in future feedlot performance, dummy variables for each individual sale location may help explain the feed cost component of $IBEFR_i$.

The month of the year in which cattle of various weights are sold can reflect biological differences. For example, cattle weighing between 400 and 600 pounds in October are likely to

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34 Muscle thickness grades of thrifty cattle are measured by designation as class 1, 2, or 1-2 (where 1-2 indicates a mix of class 1 and 2 animals). Feeder cattle graded class 1 exhibit a higher proportion of beef breeding and a greater degree of muscle thickness relative to class 2. A complete definition of standards for each grade is published by AMS <https://www.ams.usda.gov/grades-standards/feeder-cattle-grades-and-standards> last accessed 3/11/2019
be spring-born calves, whereas cattle sold in the same weight range during March most likely are not. As discussed in chapter 1, feeder cattle follow a variety of paths from birth to slaughter. The interaction term between month of sale and weight contains information about the type and background of cattle in the sale lot. In addition, seasonal differences in feeding conditions are known to have important effects on feedlot performance of cattle. For example, cattle gain weight more slowly and less efficiently during cold winter months. As a result, the amount of feed remaining for a particular lot of feeder cattle will depend on which months they are expected to spend gaining weight in the feedlot and when they will reach finishing weight.

Overall, the \( Fd \) variable characteristics and factors that indicate less efficient expected feedlot performance for a particular lot of feeder cattle will exhibit positive coefficients in the empirical model, indicating higher amounts of remaining feed consumption (for a given weight) and larger \( IBEFR^{Fd}_i \). In contrast, characteristics and factors associated with superior feedlot performance lead to lower expected amounts of remaining feed consumption (for a given weight) and reduced \( IBEFR^{Fd}_i \). From chapter 3, remaining feed \( F_i(w_i, \bar{w}, \theta) \) is expected to be monotonically decreasing in the weight of the feeder cattle. As a result, the net sign of the coefficients on all weight variable polynomial terms, \( \beta^p \cdot w_i^p \), must produce a function that is everywhere negatively sloped in \( w_i \).

**Non-feed adjustment component**

The second bracketed term in (4.7) measures the non-feed component of \( IBEFR \),

\[
\text{Non-feed Adjustment Component} = IBEFR^{\text{NF}}_i = \frac{w_i}{P_{\text{corn}}} \cdot NFP_A_i = \frac{w_i}{P_{\text{corn}}} \left( B + N_s \cdot Nf_{i,s} \right). 
\]

A positive value for this term indicates that non-feed price adjustment factors account for some portion of feeder cattle prices. All \( Nf \) variables in regression equation (4.7) are multiplied by
(interacted with) the term \( Wi_{-}Pcorn \) \((w_i/P_{corn})\) and have coefficients \( N_s \). In addition, \( Wi_{-}Pcorn \) is included directly as a model variable without an interaction variable (with coefficient \( B \)).

When interpreting results from the empirical model it will be important to understand the units in which the non-feed price adjustment coefficients, \( B \) and \( N_s \), are expressed. Isolating these coefficients and recalling that \( IBEFR \) is expressed in pounds of corn makes clear that the coefficients will be expressed as dollars per pound of current feeder cattle weight.

\[
\left( B + N_s \times Nf_{i,s} \right) = \frac{1}{w_i} \times P_{corn} \times IBEFR_{i}^{Ny}.
\]

Substituting units \( \Rightarrow \left( \frac{1}{w_i} \right) \times \left( \frac{\$}{LbsCorn} \right) \times (LbsCorn) = \frac{\$}{w_i} \)

This result leads us to a very straightforward and important interpretation for the non-feed coefficients in the empirical model. The coefficient on \( Wi_{-}Pcorn \), \( B \), can be interpreted as the non-feed price adjustment (\( NfPA \)) component of feeder cattle prices for the base animal (absent any marginal effects measured by other \( Nf \) variables). In addition, each coefficient \( N_s \) can be interpreted directly as a marginal feeder cattle price discount (if \( N_s \) is positive) or premium (if \( N_s \) is negative) that is attributed to a unit change in the corresponding \( Nf \) variable.\(^{35}\)

\(^{35}\) As an example, consider the effect on \( IBEFR_{i}^{Ny} \) of a lot characteristic or trait that increases the expected value of a particular group of feeder cattle to a prospective buyer: cattle that are graded higher quality will command a price premium at slaughter. An expected future quality premium will manifest in the empirical model through an increase in the current cash feeder cattle price \( (P_{w}) \). Recall that the combination of expected output price \( (P_{wbar}) \) and weight \( (W) \) used in calculating \( IBEFR \), represent the expected output value for any group of cattle sold at a particular date and current weight. This output value does not reflect any individual future premiums or discounts that might be expected for specific cattle in lot \( i \) when they are sold as finished cattle. Therefore, an expected cattle quality premium will result in a lower calculated value for \( IBEFR \), because the difference between input and output value will be reduced. Lot premiums such as this will, in effect, squeeze the implied margin represented in the numerator of \( IBEFR \), because the current value increases in relation to the fixed expected output value. This does not, however, suggest that buyers anticipate realizing lower real margins by paying premiums for feeder cattle. These buyers expect to receive a commensurate price premium for those finished animals (higher realized \( P_{wbar} \)) when they are sold for slaughter at a future date. Conversely, cattle that buyers expect to receive price discounts as a result of low quality grades when they are eventually finished and sold as slaughter cattle should also bring a discounted price at the time they are sold as feeder cattle. The larger implied margins for these cattle will result in larger values of \( IBEFR \) in the model but do not imply that higher actual margins will be realized for feeding lower quality feeder cattle.
The $Nf$ group includes variables that have also been included in the $Fd$ variable group: sex, class, location, and month. The coefficients $N_s$ on $Nf$ variables measure separate non-feed effects exerted by these same factors as described below.

Heifers raised for beef production not only exhibit less efficient feedlot performance when compared to steers, they also tend to grade less favorably as slaughter animals, resulting in price discounts. Muscle thickness grades indicate groups of feeder cattle having higher ($Class1$) or lower ($Class2$) expected carcass quality. The influence of these distinguishing quality characteristics on $IBEFR_i$ will be measured by values of the non-feed price adjustment coefficients, $B$ and $N^s$. Positive coefficients indicate price discounts; negative coefficients indicate price premiums.

Differences in transportation cost between sales locations (locational basis) will be measured as non-feed effects in the current model. If a location requires greater transportation cost than the base location, the corresponding $N^s$ coefficient will be positive. Coefficients on monthly dummy variables will reflect any price discounts (if positive) or premiums (if negative) paid for cattle based on any seasonal factors that are not related to the remaining cost of feeding the animals to finishing. For example, if future death loss were expected to be more common for cattle sold in certain months, coefficients on those monthly dummy variables should be positive.

One additional variable included in the $Nf$ variables, total number of head sold in the lot ($Head$), is not included in the $Fd$ variables. Previous research has shown that cattle buyers pay price premiums for larger groups of cattle for at least two reasons. First, cattle grouped together tend to be more uniform in breed, quality, performance, etc., which facilitates better management in the feedlot. Second, the unit cost of transporting cattle is reduced to the extent that trucks are
filled to capacity. Lot sizes that are sufficient to fill truckloads therefore command a price premium. The coefficient on Head in the IBEFR model is therefore expected to be negative.

Data

Empirical analysis of the IBEFR model was conducted using individual feeder cattle lot sales transaction data from USDA-AMS. A data subset extracted for estimation of the IBEFR model includes 179,867 observations from three Kansas public auctions over a ten-year period. Each observation contains information from the sale of a specific group (lot) of feeder cattle, including date, location, total number of head sold in the lot, average weight per head, sales price, sex, and muscle grade (class).

The cash sales transaction data were combined with CME futures price data to incorporate market information that would have been available to buyers on the date of sale. The method used to identify relevant live cattle futures contracts based on current weight and projected gain rate of cattle in the lot has been described in chapter 2. 36 The method has been slightly modified for the current sample to accommodate a trend of increasing slaughter weights for finished cattle over time. From the beginning of the data sample period used for chapter 2 (January 1999) to the end of the current period (November 2017), the national average monthly slaughter weight increased from 1,224 lbs. to 1,376 lbs. To account for this shift, the expected fed cattle price (\(P_{wbar}\)) for this data sample has incorporated the price from the live cattle contract that would be nearby when cattle are projected to reach 1,350 lbs. (i.e., LC1350). When

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36 Recall from the discussion of the data in chapter 2 that future live cattle output price (\(P_{wbar}\)) for a group of cattle is obtained as follows. A future date when cattle having weight \(w_i\) on the sale date will reach expected finished weight is projected using assumed rates of average daily gain (ADG) over specified weight ranges. For these calculations, cattle are assumed to gain 1.8 lbs./day until reaching 500 lbs., 2.0 lbs./day while weighing between 500 and 800 lbs., and 3.14 lbs./day while weighing more than 800 lbs. This future date is used to identify the live cattle futures contract that will be next to expire (nearby) when cattle are finished and sold for slaughter. The closing price as of the date at which the cattle are sold (as feeder cattle) for this (deferred) live cattle contract is used as \(P_{wbar}\).
prices were available on the sale date for both LC1350 and LC1250 deferred contracts, $P_{wbar}$, was calculated as a simple average of the two prices.37 38

Table 4.1 displays summary statistics for the sample data used in the IBEFR empirical analysis. The sample includes nearly 180,000 sales transactions from three Kansas public cattle auction locations from December 2007 through November 2017. To maintain consistency and integrity of the data, the sample was restricted to steers and heifers graded class 1,1-2, or 2, weighing between 300 and 900 lbs., with lot size less than 200.39 The sample period includes substantial variation in corn prices ($3.02 to $8.30 per bushel), fed (live) cattle feeder cattle prices as measured by $P_{wbar}$ ($80 to $170 per cwt), and cash prices ($56 to $400 per cwt).

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37 LC1350 prices were not available for a small percentage of observations (<0.15%) having very light lot weights because the more deferred contract had not yet begun trading. In such cases, only the LC1250 price was used for $P_{wbar}$.

38 Calculating $P_{wbar}$ as the average of two different deferred contract prices has the beneficial effect of smoothing the resulting price series. Predicted future finishing dates and associated nearby live cattle contracts (LC1250 or LC1350) may differ for cattle exhibiting only small differences in weight or sale date. Incorporating information from two adjacent contracts helps to minimize these discrete changes in $P_{wbar}$ for similar animals.

39 Observations that were removed because they contained bulls, dairy, other muscle grades, or lot sizes >200 head totaled less than 3% of the sample.
Table 4.1 Summary statistics for Kansas sample data used for IBEFR model estimation

Number of observations = 179,867

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>SaleDate</td>
<td>YYYYMMDD</td>
<td>20071205</td>
<td>20171130</td>
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<td></td>
</tr>
<tr>
<td>Weight</td>
<td>Sale weight</td>
<td>pounds</td>
<td>657</td>
<td>142</td>
<td>300</td>
</tr>
<tr>
<td>Price Variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pwi (Cash Price)</td>
<td>Cash sale price</td>
<td>$/cwt</td>
<td>144.03</td>
<td>43.55</td>
<td>56</td>
</tr>
<tr>
<td>PwBar (LC1350)</td>
<td>Price on sale date for live cattle contract that</td>
<td>$/cwt</td>
<td>116.41</td>
<td>19.64</td>
<td>80.075</td>
</tr>
<tr>
<td></td>
<td>will be nearby when cattle weigh 1350 lbs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CME Corn Price</td>
<td>Price on sale date for nearby CME corn futures</td>
<td>cents / bushel</td>
<td>481.40</td>
<td>140.16</td>
<td>301.5</td>
</tr>
<tr>
<td></td>
<td>contract in cents per bushel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pcorn</td>
<td>Price on sale date for nearby CME corn futures</td>
<td>$ / lb</td>
<td>0.0860</td>
<td>0.0250</td>
<td>0.0538</td>
</tr>
<tr>
<td></td>
<td>contract in dollars per lb. (56#/bu)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Dependent Variable (Calculated)</td>
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<td></td>
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<tr>
<td>IBEFR1350</td>
<td>Implied breakeven feed requirement for cattle to</td>
<td>pounds of corn</td>
<td>7679</td>
<td>2116</td>
<td>441</td>
</tr>
<tr>
<td></td>
<td>reach 1,350 lb. slaughter weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lot Characteristics</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Head</td>
<td>Lot size (# head sold in lot)</td>
<td># head</td>
<td>19</td>
<td>21</td>
<td>1</td>
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<tr>
<td>Str</td>
<td>Steers</td>
<td>0.5417</td>
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<td>1</td>
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<td>Hfr</td>
<td>Heifers</td>
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<td>Class1</td>
<td>Class 1</td>
<td>0.5726</td>
<td>0</td>
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<td></td>
</tr>
<tr>
<td>Class12</td>
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<td>0</td>
<td>1</td>
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</tr>
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<td>Winter Livestock Auction</td>
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<td>Pratt Livestock Auction</td>
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</tr>
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<td>Loc3</td>
<td>Farmers and Ranchers (F&amp;R) Livestock Auction</td>
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<td>1</td>
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<td>Mo1</td>
<td>January</td>
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<td>0</td>
<td>1</td>
<td></td>
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<td>Mo2</td>
<td>February</td>
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<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo3</td>
<td>March</td>
<td>0.1150</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo4</td>
<td>April</td>
<td>0.0957</td>
<td>0</td>
<td>1</td>
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</tr>
<tr>
<td>Mo5</td>
<td>May</td>
<td>0.0630</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo6</td>
<td>June</td>
<td>0.0204</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo7</td>
<td>July</td>
<td>0.0486</td>
<td>0</td>
<td>1</td>
<td></td>
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<tr>
<td>Mo8</td>
<td>August</td>
<td>0.0705</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Mo9</td>
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<tr>
<td>Mo10</td>
<td>October</td>
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<td>0</td>
<td>1</td>
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<tr>
<td>Mo11</td>
<td>November</td>
<td>0.1152</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mo12</td>
<td>December</td>
<td>0.0741</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Implied breakeven feed requirement (IBEFR)

$\text{IBEFR}_i$ has been calculated as shown in equation (4.3) using the sale weight $w_i$, cash price ($P_{wi}$), and CME market price information ($P_{wbar}$ and $P_{corn}$) for each feeder cattle sales transaction in the Kansas data sample. Figure 4.1 displays a scatter plot of these individual observations by sale weight over the sample range of 300 to 900 lbs. and color coded by year (2007-2017).

**Figure 4.1:** Plot of IBEFR by weight for the Kansas data sample

$\text{IBEFR}_i$ is the empirical analog of $\bar{F}_i$. Under reasonable assumptions of the nonlinear bio-economic model described in chapter 3 (and as displayed in figure 3.4), $\bar{F}_i$ is expected to exhibit concavity to the origin and a downward slope (reaching zero at $\bar{w}=1,350$ lbs.) when plotted over current (input) weight $w_i$. Visually, the $\text{IBEFR}_i$ data in figure 4.1 displays a pattern that is
consistent with the predicted shape of $F_i \cdot IBEFR_i$ appears to be decreasing in weight, and the relationship appears to be nonlinear over the sample range.

**IBEFR Model Estimation and Results**

Parameter estimates and standard errors for the full IBEFR regression model outlined in appendix C are displayed in table 4.2. The large number of $Fd$ and $Nf$ model variables and corresponding interaction terms complicate interpretation of these results. To assist in this process, the parameter estimates have been organized according to $Fd$ and $Nf$ variable groups. Estimated coefficients for each of the $Fd$ variables and corresponding weight polynomial interactions are presented in rows, (with each row corresponding to a line in the equation representing $IBEFR_{Fd}$ in appendix C). The model includes dummy variables for sex, class, location, and month, requiring that one from each group be omitted to avoid perfect collinearity. The values for the omitted dummy variables describe the “base lot” of cattle in the model, which contains steers ($Str$) with mixed muscle grade 1s and 2s ($Class12$) sold at Winter Livestock Auction ($Loc1$) during the month of October ($Mo10$).
Table 4.2. IBEFR Model Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Intercept</th>
<th>Weight</th>
<th>Wt2</th>
<th>Wt3</th>
<th>NON-FEED (Nf)</th>
<th>Nf*wiPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>20882.38</td>
<td>-4.73</td>
<td>0.04368</td>
<td>-2.11944E-05</td>
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<tr>
<td></td>
<td>499.31</td>
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<td>FEED (Fd)</td>
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<tr>
<td>Hfr_Fd</td>
<td>-3446.20</td>
<td>21.7485</td>
<td>-0.0391</td>
<td>1.993E-05</td>
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</tr>
<tr>
<td></td>
<td>266.04</td>
<td>1.3564</td>
<td>0.0022</td>
<td>1.193E-06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class1_Fd</td>
<td>-310.37</td>
<td>-1.2823</td>
<td>0.0050</td>
<td>-2.857E-06</td>
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<td></td>
<td>289.27</td>
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| Notes: Estimated coefficients are shown with standard errors below. Statistical significance of the individual coefficients is indicated as follows: bold (p-value < 0.0001), bold italic (0.0001 < p-value < 0.01), italic (0.01 < p-value < 0.05), plain text (p-value > 0.05).

The model fits the data very well overall, explaining a large amount of variation as measured by an adjusted R-square of 0.8574. The sample size is extremely large, and most of the individual parameter estimates are highly significant (70 of 93 model coefficients are significant at p-values <0.01; 62 of these are significant at p-values < 0.0001). More generally, each of the
model variable groups (i.e., sex, class, location, month, head, weight) proved to be highly significant when subjected to tests for joint significance.\textsuperscript{40}

Predicted IBEFR for the feed variable groups can be calculated by taking the product of the vector of coefficients in a single row of table 4.2 and the weight polynomials \((1 + Weight + Wt2 + Wt3)\).\textsuperscript{41} The result is a series of predicted values for the feed component of IBEFR \((IBEFR_{Fd})\) at a given weight. Including only the first row of \(Weight\) coefficients gives \(IBEFR_{Fd}\) for the base lot. Marginal effects for each of the \(Fd\) variable groups can be calculated by repeating these vector calculations in each row of table 4.2 to produce a series that is the difference in \(IBEFR_{Fd}\) attributed to the lot characteristic measured by that binary variable (ex., \(Hfr_{dFd}\)). A more accessible graphical interpretation of model results for the feed variables is provided later in this chapter.

Interpretation of marginal effects for the non-feed variables is more straightforward because the coefficients are expressed in dollars per pound of the feeder animal. For example, the value of the coefficient on \(Wt\_Pcorn\ (B)\) in table 4.2 is 0.40, which indicates that the model attributes 40 cents of the per pound price paid for cattle in the base lot to the non-feed adjustment component. Non-feed marginal effects of lot characteristics are similarly transparent. For example, the estimated coefficient on \(Hfr\_wiPC\ (\delta_{h}^{0} = 0.1772)\) suggests that the non-feed component discount (relative to steers) for feeder heifers is nearly 18 cents per pound. A

\textsuperscript{40} F-tests for joint significance were conducted for several combinations of the model variables. These tests supported the notion that, although a few individual model coefficients were not statistically significant, coefficients for each of the variable groups were highly significant as a group. For example, although not every monthly dummy variable coefficient is statistically significant, the joint significance of the monthly variables is high.

\textsuperscript{41} Appendix C details these model calculations in equation form. Rows in results table 4.2 correspond to rows in the equation presented for \(IBEFR_{Fd}^{i}\)
graphical analysis of individual non-feed \((Nf)\) model results will also be included in the subsequent section.

**Graphical analysis of model results**

As a result of the large number of independent variables and complex weight variable interactions, the model results are more easily understood and interpreted by graphing predicted IBEFR as a function of the lot weight for cattle lots with different characteristics. Predicted values for \(IBEFR_i\) corresponding to the base lot of feeder cattle are displayed in figure 4.2 for a range of lot weights from 300 to 1,350 lbs. IBEFR predictions for the base lot are calculated using base values for the model dummy variables and sample mean values for lot size \((Head=19)\) and corn price \((Pcorn=$0.086/lb.).^42

**Figure 4.2.** Model predicted IBEFR for base lot of feeder cattle (0 to 1,350 lbs.)

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42 Unless otherwise noted, references and graphs of the “base lot” will always refer to predictions calculated at the base values of the dummy variables and sample means for any other variable. As such, the base lot consists of 19 steers with class 1-2 muscle grade sold at Winter Livestock Auction with a nearby CME corn price of $4.81/bu. (equivalent to \(Pcorn = $0.086/lb)\) and deferred live cattle futures of $116/cwt \((Pwbar).\)
The model predicted values of IBEFR shown in figure 4.2 have been extended to weights well beyond the range of the sample data (the sample range is indicated by a blue box overlay). This highlights an empirical challenge of the bioeconomic price slide modeling that was discussed at the end of the previous chapter. Once cattle have been placed in a feedlot to be put on a finishing ration, the animals are rarely sold again until they are ultimately marketed as finished slaughter cattle. Feeder cattle data therefore do not contain reliable observations of price and weight combinations for animals weighing more than 900 lbs. Thus, the shape of $F_i$ over this range is not observed. The more rigid structure of the bioeconomic price slide parametric model imposed the restriction that remaining feed $F_i$ must reach zero at the intended slaughter weight (1,350 lbs.). However, the linear OLS model with polynomial weight interaction terms presented in this chapter is more empirically flexible. Although these model predictions should not be extrapolated to heavy feeder cattle, the model appears well suited to explaining variation within the sample range.

**Visualizing IBEFR and related price adjustments**

A series of four graphs are presented in figures 4.3 and 4.4 to illustrate the relationship between predictions for implied breakeven feed requirement (IBEFR) and feeder cattle prices. All four graphs represent model results for the base lot in a sequence starting from predicted IBEFR and leading to other relevant feeder cattle value and price counterparts.
Figure 4.3. IBEFR and total value components and feeder value by weight (base lot)

The two separate components of IBEFR are displayed in figure 4.3.a. as IBEFR_Fd and IBEFR_Nf, which constitute IBEFR_TOTAL when added together. IBEFR_Fd is everywhere downward sloping, which is consistent with a constant decline in total remaining feed consumption as animals approach finishing weight. IBEFR_Nf exhibits a positive constant (linear) slope. For the base lot,

$$IBEFR_\text{Nf} = B^0 \cdot \left( \frac{W_i}{P_{\text{Corn}}} \right) + h_i^0 \cdot \left( \text{Head}_i \cdot \frac{W_i}{P_{\text{Corn}}} \right) + h_i^2 \cdot \left( \text{Head}_i^2 \cdot \frac{W_i}{P_{\text{Corn}}} \right)$$

$$= \left( B^0 + h_i^0 \cdot \text{Head}_i + h_i^2 \cdot \text{Head}_i^2 \right) \cdot \frac{W_i}{P_{\text{Corn}}}$$

where the slope term preceding $w_i$ is a constant calculated from data and estimated coefficients. Given the estimated coefficients ($h_i^0 = -0.0015$ and $h_i^2 = 9.58E-06$), the effect of sample average lot size on IBEFR_Nf is negative.\(^{43}\) Non-feed coefficients are estimated in the model as fixed feeder cattle price adjustments regardless of weight.\(^{44}\) Consequently, the total additional corn

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\(^{43}\) The total value of the combined $N_f$ coefficient for lot size at the sample mean is \((-0.0015*19)+(9.58E-06*19^2)\)\(^2\) = -0.03, or roughly 3 cents per pound. For these coefficients, the maximum $N_f$ premium for lot size is 0.0612 at 80 head.

\(^{44}\) This modeling assumption facilitates clear interpretation of $N_f$ coefficients as price discounts/premiums. However, it is also restrictive, as it precludes the possibility that non-feed price discounts may also vary according to weight. I return to this possibility briefly in concluding remarks to this chapter.
requirement \((IBEFR_{Nf})\) that is implied by this adjustment becomes larger (or smaller, if the coefficient is negative) for heavier weights \((w_i)\).

Multiplying IBEFR, expressed in pounds of corn per head, by the price of corn gives an expression for total value of IBEFR and its \(Fd\) and \(Nf\) components in dollars per head. The solid lines displayed in figure 4.3.b display the analogous per head values for \(IBEFR_{TOTAL}\) (where \(IBEFR_{TOTAL}_{shd} = IBEFR_{TOTAL}*P_{corn}\)) the and components \(IBEFR_{Fd}\) and \(IBEFR_{Nf}\) from 4.3.a. From the preceding discussion, we know that \(IBEFR_{TOTAL}_{shd}\) is equivalent to the difference between final output value and feeder cattle value, or GFM. To illustrate this relationship, figure 4.3.b also plots (using dashed lines) the total output value, \(FedValue_{shd}\), and the value of feeder cattle, \(FeederValue_{shd}\), over the weight range:

\[
FeederValue_{shd} = FedValue_{shd} - IBEFR_{TOTAL}_{shd}.
\]

The graphs in figure 4.3 show how an increase in the weight of the feeder animal corresponds to an increase in total (per head) value of the animal and a decrease in IBEFR. If \(IBEFR_{Nf}\) has a steeper slope (as a result of some larger positive \(Nf\) coefficient), \(IBEFR_{TOTAL}\) will decline more slowly and the value of the feeder animal will be lower at any given weight.

Plots showing total value are easily transformed into plots of feeder cattle price simply by dividing weight into value. The graphs included in figure 4.4 are calculated from the same model results for the base lot used in figure 4.3; however, they express IBEFR components as familiar prices in dollars per pound of feeder cattle rather than pounds of corn (IBEFR) or total value per head.
Figure 4.4. IBEFR price adjustment and price slide components

Figure 4.4.a displays only the IBEFR price adjustment components:

\[ IBEFR_{Fd}_{Swi} + IBEFR_{Nf}_{Swi} = IBEFR_{TOTAL}_{Swi}. \]

These same components are included again in figure 4.4.b, which adds two additional reference prices as dashed lines. *FedOutputPrice*_{Swi} (in gray) is the total expected future value of the fed cattle output expressed in dollars per pound of the current feeder animal:

\[ FedOutputPrice_{Swi} = \frac{FedValue_{Shd}}{Weight} \]

The green dashed line in figure 4.4.b is the feeder cattle price slide. It represents the predicted feeder cattle price for the base lot over the input weight range.

\[ FeederPrice_{Swi} = FedOutputPrice_{Swi} - IBEFR_{TOTAL}_{Swi} \]

*FeederPrice*_{Swi} exhibits the characteristic shape of the feeder cattle price slide and is decreasing in weight. Figure 4.4.b also illustrates the manner in which changes in IBEFR translate to changes in feeder cattle price. It is apparent from figure 4.4 (and the equation above) that any upward shift in *IBEFR_{TOTAL}_{Swi}* (as a result of any factor that might increase *Fd* or *Nf* components of IBEFR) would correspond to a downward shift in *FeederPrice*_{Swi}.

Another feature of the model that is visually evident from the price adjustment graphs shown in figure 4.4. is that the non-feed price adjustment component *IBEFR_{Nf}_{Swi}* (indicated
by a solid blue line) exhibits a constant value for all weights, equal to

\[
(0.4021 + -0.0015 \cdot Head + 9.579 \times 10^{-6} \cdot Head^2)
\]

or $0.3764/lb.^{45}$ for the base lot. As noted previously, the model estimated price adjustments do not vary with weight.

**Graphical Analysis of Marginal Effects**

The graphs included in figure 4.3 displayed IBEFR and total value components for the base model. In this section I use similar graphs to analyze marginal effects for variable groups included in \( Fd \) and \( Nf \) components of the model. Figure 4.5 below includes the first of these four panel graph combinations and displays results for the sex variables (heifers and steers).

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**Figure 4.5** Model-predicted IBEFR, value, and price comparison and differences by SEX

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45 Calculated from model coefficients, \( B_0=0.4021, h_1=-0.0015, h_2=9.579E-06 \), and sample average \( Head=19 \).
**Marginal effects of sex variables**

Panel a. of figure 4.5 plots total predicted IBEFR for the base lot (steers – in blue) and heifers, showing that predicted IBEFR is higher for heifers at all weights. Panel b. displays IBEFR\_TOTAL\_Shd for steers and heifers in addition to FedValue\_$/hd and FeederValue\_$/hd (as described for figure 4.3). Relative values of FeederValue\_$/hd shown in panel b. show that higher values of IBEFR translate into lower per head values (discounts) for feeder heifers at all weights. This result is consistent with the earlier prediction that feeder cattle buyers tend to discount heifers relative to steers.

The two lower panels differentiate between feed and non-feed marginal effects for heifers relative to steers. In particular, panel c. shows the magnitude of the difference in IBEFR for heifers attributed to feed (Hfr\_dFd) and non-feed (Hfr\_dNf) effects and in total (Hfr\_dTotal). Panel d. expresses the same differences in terms of total per head value. Both panels show that Fd marginal effects indicate an expected discount for heifers (higher IBEFR\_Fd than for steers) at weights under 600 pounds. However, the model indicates a premium (lower IBEFR\_Fd) for heavier weight heifers. This unexpected result might be explained by the fact that typical slaughter weights for heifers are lower than for steers. Future feed costs may therefore become relatively less important for heifers at heavier weights because they will not require as many pounds of gain to reach finishing weight.

The marginal effect of non-feed factors is positive for heifers, as indicated by the upward slope of Hfr\_dNf and the coefficient on Hfr\_wiPC that indicates a non-feed discount of $0.1772. These non-feed discounts for heifers may include factors such as pregnancy risk during feeding and the expectation of lower grades at slaughter.
The same graphs shown in panels a. through d. from figure 4.5 above have also been generated for class, location, and month variables. Discussion of those results follows below. However, for economy of space, these graphs are included only in appendix D.

**Marginal effects of muscle grade (class) variables**

The graphs in figure D.2 illustrate results for the muscle grade variables in the model. The base lot (Class12) is plotted for comparison with higher grading Class1 and lower grading Class2 lots. Muscle thickness grades measure a quality characteristic that is primarily expected to influence carcass quality but could also have some effect on feedlot performance. Differences in expected carcass quality are expected to be observed as non-feed premiums or discounts. To the extent that buyers anticipate that these grades influence feed consumption, feed premiums or discounts should be observed.

The IBEFR graphs shown in figure D.2 a. and b. indicate that Class1 cattle exhibit lower predicted overall IBEFR and therefore receive a value and price premium relative to mixed Class12 cattle. The opposite is true for Class2 cattle. Marginal effects for Class1 cattle displayed in panels c. and d. show that Class1 feed effects as measured by $IBEFR_{dFd}$ result in reduced IBEFR for lighter weight cattle and increase to having a positive effect on IBEFR for heavier Class1 cattle. This pattern could suggest that Class1 cattle perform more efficiently over their growth cycle but are expected to finish at higher slaughter weights as a result of their muscle thickness. If this is true, feed costs would tend to have a greater influence on the price of heavier Class1 cattle (relative to lower quality Class12 or Class2) even though the ultimate final fed value of the output will be positively influence by higher quality and weight. This relationship is reversed for Class2 cattle as would be expected.
Non-feed marginal effects for class are indicated by the coefficients on $Class1_{wiPC}$ (-0.0667) and $Class2_{wiPC}$ (0.1022) and corresponding slopes of the dashed $dNf$ lines in panels c. through f. This model result suggests that $Class1$ cattle obtain a roughly 7 cent feeder cattle price premium while $Class2$ cattle receive a roughly 10 cent discount relative to the $Class12$ base. This result is also consistent with the idea that carcass quality premiums and discounts will be reflected in the non-feed coefficients.

The magnitude of total marginal effects for muscle grade are noticeably smaller than the corresponding effects for sex. This is evident from the graphs, which have been kept to the same scale to facilitate comparison across variables. Overall, the total difference in IBEFR or value between steers and heifers was roughly twice the magnitude of difference between $Class12$ and either $Class1$ or $Class2$, although the total difference between $Class1$ and $Class2$ was of similar magnitude.

**Marginal effects of location variables**

Marginal effects related to the location dummy variables are presented graphically in figure D.3. The base lot auction location is $Loc1$ (Winter), which is compared to $Loc2$ (Pratt) and $Loc3$ (F&R). Predictions for location effects were ambiguous. There are reasons to expect both feed cost and non-feed effects by location based on such factors as systematic difference in the types of lots and cattle sold at each, transportation differences, etc. However, information about what these effects may be for each location is not easily available. Instead, the data will speak for each location.

One initial feature of note for the location graphs is that the magnitude of difference in overall IBEFR between locations is not large. Panels a. and b. in figure D.3, which are scaled identically to those for sex and class, show that the overall IBEFR and value are very similar for
all three locations. Panels c. through d. show the same relationship for total IBEFR but indicate slightly more difference for the feed cost and non-feed component difference. Relative to Loc1, results for both Loc2 and Loc3 suggest feed cost discounts ($dFd$ greater than Location1) and non-feed premiums (negative $Nf$ coefficients and slopes for $dNf$). Reasons why feeder cattle sold at Pratt and F&R might incur slightly higher overall future feed costs is not known but could be related to difference in the type of cattle sold. A non-feed discount of between three and six cents per pound for cattle sold at Loc1 (Winter) relative to the other two locations could be related to transportation cost, although these three locations do not appear to differ substantially in that regard. It is also possible that these discounts are the result of non-feed factors that differ by location. Overall, these $dFd$ and $dNf$ effects largely offset each other at each location. However, understanding the factors that cause the model to detect effects for the separate components may be an interesting subject for future exploration.

**Marginal effects of seasonal (month) variables**

The effects of seasonality in feeder cattle markets is measured in the IBEFR model using twelve monthly dummy variables corresponding to the month during which the lot of cattle were sold. October was chosen as the month of sale for the base lot because it is a traditional peak in the market for spring-born calves placed in feed and backgrounding programs and a natural point of transition in the yearly cattle cycle. Model results for the other eleven months relative to October are calculated using coefficients estimated for a total of 55 variable in the IBEFR model ($44 Fd$ and $11 Nf$), many of which are likely to be highly collinear. For this reason, several individual monthly variable coefficients do not show individual significance (as seen in table 4.2); taken as a group, however, the monthly dummies exhibit a high degree of joint significance.
A graphical treatment of the individual monthly dummy variables is presented in figure D.4. Displaying all eleven series on a single graph as presented in the first two panels for the other variables is not tractable for the monthly variables. Instead, figure D.4 contains four panels, corresponding to quarters of the year, with each panel displaying three months in comparison to the base month of October (except Q4, which contains only two other months).

Charts of the month variables support the notion of seasonality in feeder cattle markets. A review of the four quarterly panels reveals patterns in the shape of the IBEFR and value series that seem to persist from month to month. For example, a pattern of lower relative IBEFR for lighter cattle (indicating a value premium) begins to emerge in January and becomes accentuated moving into the late spring and especially the summer months. This trend is most pronounced from May through July and tapers off beginning in August and continuing through December. This pattern seems consistent with an abundance of spring-born calves weighing 300 to 500 lbs. in October relative to the spring and summer months when calves in this weight range are increasingly less common.

Another pattern that stands out from the graphs of monthly variables is that the model estimates larger non-feed premiums in the summer months of June through August (as indicated by negative $N_f$ coefficients corresponding to more negatively sloped $dN_f$ component series). At the same time, feed effects become more pronounced with larger feed cost premiums for light cattle and discounts for heavier cattle. One possible explanation may be that there is relatively greater demand at this time for lighter cattle that can take advantage of summer grazing as compared to the fall when cattle are coming off pasture and moving to other feed environments.

These graphs suggest interesting questions for future research.
Further Research

The IBEFR model presented in this chapter yields several interesting results that merit further investigation. In particular, the functional form used to estimate non-feed parameters could be explored further. The model assumption of fixed non-feed price discounts that apply to all weights of feeder cattle facilitates convenient interpretation of model coefficients directly as price adjustments. However, this model structure imposes limits on the non-feed parameters that need to be considered and possibly refined.

An alternate form of the model was estimated in which $N_f$ variables were interacted with weight polynomial terms to allow for a nonlinear relationship between the $N_f$ price discounts and weight. An expanded form of the IBEFR model (referenced as IBEFR-NfWt) that includes more flexible non-feed parameters is included in appendix 4.C along with preliminary model output and graphs. A complete discussion and interpretation of these alternate results and how they compare to the original IBEFR model results described here will be left to future research.

Concluding Remarks

The analysis presented in this study is based on a derived demand framework and breakeven, zero-profit equilibrium model of feeder cattle prices. The model presented in chapter 2 focuses on price-weight relationships, including the complex interactions between feed input markets, fed cattle output markets, and the price of feeder cattle inputs at various stages of the beef production process (i.e., different weights). The model demonstrates factors that give rise to the commonly observed feeder cattle “price slide,” whereby per pound cattle prices tend to decrease as the weight of cattle increases. The analysis in chapter 2 uses deferred futures contract prices for corn, live cattle, and feeder cattle to incorporate available information about future market price expectations at time of sale. The hedonic regression on basis calculated using a
time-relevant feeder cattle contract is used to analyze price slide dynamics that relate to weight cohorts rather than a simple current price-weight cross-section.

Chapter 3 extends the inquiry to consider the question of biological growth relationships contained in feeder cattle price data. Strict assumptions of linear growth from chapter 2 give way to a more flexible model that incorporates variable rates of gain and feed consumption over time. The nonlinear model includes biological growth parameters and is constructed using basic assumptions about cattle feedlot performance that are known from animal science and nutrition literature. A bio-economic price slide model developed in chapter 3 includes a term for total remaining feed consumption, \( F_i (\bar{w}, w_i, \theta) \), that depends on biological growth factors. Reasonable assumptions for the growth parameters define the expected shape of \( F_i (\bar{w}, w_i, \theta) \), which is shown to be a nonlinear function of current weight and convex to the origin. Because the structure of the parameterized model in chapter 3 is too rigid for empirical implementation, a more flexible form is employed for this purpose in chapter 4.

Empirical analysis presented in chapter 4 builds on the bio-economic model and develops an estimation strategy based on calculating the implied breakeven feed requirement (IBEFR) as an empirical counterpart to \( F_i (\bar{w}, w_i, \theta) \). An important innovation of this model is to separate model effects into two components: those that are related to feed costs and those that are related to non-feed factors. The importance of corn prices and feed costs to the formulation of feeder cattle prices has been much researched. Previous studies have also found that other non-feed factors, including differences in quality, transportation costs, lot sizes, etc. have an influence on prices paid for feeder cattle. However, the nature of separate simultaneous effects on cattle price relationships that are caused by each of these components is less well understood. The research presented in chapter 4 takes a step forward by analyzing the importance of lot characteristics.
such as sex, class, location, and month in terms of both feed cost and non-feed components of value over the feeder cattle growth cycle. The empirical method used to identify and separate these value components from price relationships in feeder cattle data differs from previous studies.

The beef cattle “complex,” which is comprised of several interrelated input and output markets, remains a rich and fertile field for economic inquiry. Research presented in this study contributes new ideas and approaches to the study of these price relationships and seeks to advance our understanding of the interesting dynamics underlying beef cattle markets.
REFERENCES


U.S. Beef Production Practices (PowerPoint)


Appendix A

Chapter 3 Supplement:

Derivation of time left on feed with constant feeding rate
Appendix A. Derivation of time left on feed with constant feeding rate

The equation for instantaneous rate of change in weight over time is a first-order differential equation:

\[ \dot{w} = \frac{dw}{dt} = b - aw \]

A general solution to the non-homogenous differential equation is given as:

\[ w(t) = Ae^{-at} + \frac{b}{a} \]

We establish an initial condition for the expression at \( w_0 \), where \( t_0 = 0 \),

\[ w_0 = A + \frac{b}{a}, \text{ or} \]

\[ A = w_0 - \frac{b}{a}, \]

where \( w_0 \) is defined as the beginning weight of the animal.

Evaluating the expression at arbitrary \( w_1 \) (where \( t_1 > t_0 \)), we obtain the following:

\[ w_1 - \frac{b}{a} = Ae^{-at_1} \]

Solving for \( t \) yields,

\[ t_1 = \frac{\ln \left( \frac{w_1 - \frac{b}{a}}{A} \right)}{-a} \]

Now we can substitute the initial condition into the equation for \( A \) to obtain an expression for the total time it takes the animal to grow from \( w_0 \) to \( w_1 \):
\[
\ln \left( \frac{w_i - \frac{b}{a}}{w_0 - \frac{b}{a}} \right) = \frac{\ln \left( \frac{b - w_0 a}{b - w_i a} \right)}{-a} = \frac{\ln \left( \frac{b - w_j a}{b - w_i a} \right)}{a} = t_i
\]

For ease of notation, \( w(t) = w \) refers to the weight of an individual animal at any particular time \( t \), and \( w(\bar{t}) = \bar{w} \) refers to the terminal weight for slaughter at time \( \bar{t} \).

The expression for total feeding time can be generalized as the time it takes the animal to grow from any starting weight, \( w_i \) at time \( t_i \) to any heavier weight \( w_j \) at time \( t_j \) (where \( t_j > t_i \)):

\[
t_{w_i}^{w_j}(w_i, w_j, b, a) = \frac{\ln \left( \frac{b - w_i a}{b - w_j a} \right)}{a}.
\]

More commonly, we are interested in the time it takes for a steer to grow from a current weight of \( w_t \) at time \( t \) until a known slaughter (terminal) weight of \( \bar{w} \) as given below:

\[
t_{w_t}^{\bar{w}}(w_t, \bar{w}, b, a) = \frac{1}{a} \ln \left( \frac{b - w_t a}{b - \bar{w} a} \right)
\]
Appendix B

Chapter 3 Supplement:

*Derivation of feed remaining with variable rates of gain and feed*
Appendix B. Derivation of feed remaining with variable rates of gain and feed

In the more general model, the rates of both feed consumption and weight gain are assumed to vary with steer weight as follows:

\[ \dot{w} = \frac{dw}{dt} = b - aw_i, \text{ and} \]

\[ f(w(t)) = f(t) = \mu + \delta w_i, \]

We can once again utilize the general solution to a non-homogenous differential equation,

\[ w_i = w(t) = Ae^{-at} + \frac{b}{a}, \text{ and obtain} \]

\[ f(t) = \mu + \delta \left( Ae^{-at} + \frac{b}{a} \right) \]

\[ = \left( \mu + \frac{\delta b}{a} \right) + \delta Ae^{-at} \]

At time \( t \), when the steer weighs \( w_i \), the total amount of feed required to produce the live cattle output (i.e., a steer of weight \( \bar{w} \) at time \( \bar{T} \)) can be calculated by integrating marginal feed consumption over the time period from \( t \) to \( \bar{T} \) (as shown in figure 3.B.1):

\[ F_{w_i} = \int_t^{\bar{T}} f\ (t)dt \]

\[ = \int_t^{\bar{T}} \left[ \left( \mu + \frac{\delta b}{a} \right) + \delta Ae^{-at} \right] dt \]

\[ = \left[ \left( \mu + \frac{\delta b}{a} \right) (\bar{T} - t) \right] + \left[ -\frac{\delta}{a} Ae^{-at} + C \right]_{t}^{\bar{T}} \]
As seen above, the integration divides into two parts that will be considered separately. For the first part, recall the expression derived in Appendix 3.A giving total feeding time required for an animal to grow from \( w_i \) to \( \bar{w} \):

\[
t_w^\sigma (\bar{w}, w_i, b, a) = (\bar{t} - t) = \frac{1}{a} \ln \left( \frac{b - w_i a}{b - \bar{w} a} \right).
\]

Substituting this expression into the first integral yields,

\[
\left( \mu + \frac{\delta b}{a} \right) (\bar{t} - t) = \left( \mu + \frac{\delta b}{a} \right) \times \left( \frac{1}{a} \ln \left( \frac{b - w_i a}{b - \bar{w} a} \right) \right).
\]

The second integral depends on \( t \) and can be evaluated at \( t \) and \( \bar{t} \) and simplified as follows,

\[
\left[ \frac{\delta}{-a} A e^{-\sigma t} + C \right] - \left[ \frac{\delta}{-a} A e^{-a t} + C \right] = \frac{\delta A}{-a} (e^{-\sigma t} - e^{-a t})
\]

Note from Appendix 3.A that,

\[
\ln \left( \frac{w_i - b}{a} \right) \quad \text{and} \quad \ln \left( \frac{\bar{w} - b}{a} \right),
\]

which can be used to simplify,

\[
\frac{\delta A}{-a} \left[ \frac{\bar{w} - b}{a} - \left( \frac{w_i - b}{a} \right) \right] = \frac{\delta}{-a} \left( \frac{\bar{w} - w_i}{a} \right).
\]

When both components of the integral are combined, we obtain the following expression for total remaining feed at \( t \),

\[
\bar{F}_t = \bar{F}_t(\bar{w}, w_i, a, b, \mu, \delta) = \left( \mu + \delta \left( \frac{b}{a} \right) \right) \times \left[ \frac{1}{a} \ln \left( \frac{b - w_i a}{b - \bar{w} a} \right) \right] + \left( \frac{\delta}{-a} \right) \times (\bar{w} - w_i).
\]
Figure B.1: Total feed ($F$) as the integral of marginal feed ($f$) over time
Appendix C

Chapter 4 Supplement:

*IBEFR model detail*
Appendix C. IBEFR Model Detail
Expanded IBEFR Modeling Equation, Notation, and Variable Names

\[ IBEFR_i = \alpha + \left( \beta^1 \cdot w_i + \beta^2 \cdot w_i^2 + \beta^3 \cdot w_i^3 \right) + \]

\[
\begin{aligned}
\delta^0_i \cdot Hfr_i + \delta^1_i \cdot \left( Hfr_i \cdot w_i \right) + \delta^2_i \cdot \left( Hfr_i \cdot w_i^2 \right) + \delta^3_i \cdot \left( Hfr_i \cdot w_i^3 \right) + \\
\gamma^0_i \cdot Class1_i + \gamma^1_i \cdot \left( Class1_i \cdot w_i \right) + \gamma^2_i \cdot \left( Class1_i \cdot w_i^2 \right) + \gamma^3_i \cdot \left( Class1_i \cdot w_i^3 \right) + \\
\gamma^4_i \cdot Class2_i + \gamma^5_i \cdot \left( Class2_i \cdot w_i \right) + \gamma^6_i \cdot \left( Class2_i \cdot w_i^2 \right) + \gamma^7_i \cdot \left( Class2_i \cdot w_i^3 \right) + \\
\lambda^0_i \cdot Loc2_i + \lambda^1_i \cdot \left( Loc2_i \cdot w_i \right) + \lambda^2_i \cdot \left( Loc2_i \cdot w_i^2 \right) + \lambda^3_i \cdot \left( Loc2_i \cdot w_i^3 \right) + \\
\lambda^4_i \cdot Loc3_i + \lambda^5_i \cdot \left( Loc3_i \cdot w_i \right) + \lambda^6_i \cdot \left( Loc3_i \cdot w_i^2 \right) + \lambda^7_i \cdot \left( Loc3_i \cdot w_i^3 \right) + \\
\mu^0_i \cdot Mol_i + \mu^1_i \cdot \left( Mol_i \cdot w_i \right) + \mu^2_i \cdot \left( Mol_i \cdot w_i^2 \right) + \mu^3_i \cdot \left( Mol_i \cdot w_i^3 \right) + \\
\mu^4_i \cdot Mo2_i + \mu^5_i \cdot \left( Mo2_i \cdot w_i \right) + \mu^6_i \cdot \left( Mo2_i \cdot w_i^2 \right) + \mu^7_i \cdot \left( Mo2_i \cdot w_i^3 \right) + \\
\mu^8_i \cdot Mo3_i + \mu^9_i \cdot \left( Mo3_i \cdot w_i \right) + \mu^{10}_i \cdot \left( Mo3_i \cdot w_i^2 \right) + \mu^{11}_i \cdot \left( Mo3_i \cdot w_i^3 \right) + \\
\mu^{12}_i \cdot Mo4_i + \mu^{13}_i \cdot \left( Mo4_i \cdot w_i \right) + \mu^{14}_i \cdot \left( Mo4_i \cdot w_i^2 \right) + \mu^{15}_i \cdot \left( Mo4_i \cdot w_i^3 \right) + \\
\mu^{16}_i \cdot Mo5_i + \mu^{17}_i \cdot \left( Mo5_i \cdot w_i \right) + \mu^{18}_i \cdot \left( Mo5_i \cdot w_i^2 \right) + \mu^{19}_i \cdot \left( Mo5_i \cdot w_i^3 \right) + \\
\mu^{20}_i \cdot Mo6_i + \mu^{21}_i \cdot \left( Mo6_i \cdot w_i \right) + \mu^{22}_i \cdot \left( Mo6_i \cdot w_i^2 \right) + \mu^{23}_i \cdot \left( Mo6_i \cdot w_i^3 \right) + \\
\mu^{24}_i \cdot Mo7_i + \mu^{25}_i \cdot \left( Mo7_i \cdot w_i \right) + \mu^{26}_i \cdot \left( Mo7_i \cdot w_i^2 \right) + \mu^{27}_i \cdot \left( Mo7_i \cdot w_i^3 \right) + \\
\mu^{28}_i \cdot Mo8_i + \mu^{29}_i \cdot \left( Mo8_i \cdot w_i \right) + \mu^{30}_i \cdot \left( Mo8_i \cdot w_i^2 \right) + \mu^{31}_i \cdot \left( Mo8_i \cdot w_i^3 \right) + \\
\mu^{32}_i \cdot Mo9_i + \mu^{33}_i \cdot \left( Mo9_i \cdot w_i \right) + \mu^{34}_i \cdot \left( Mo9_i \cdot w_i^2 \right) + \mu^{35}_i \cdot \left( Mo9_i \cdot w_i^3 \right) + \\
\mu^{36}_i \cdot Mo11_i + \mu^{37}_i \cdot \left( Mo11_i \cdot w_i \right) + \mu^{38}_i \cdot \left( Mo11_i \cdot w_i^2 \right) + \mu^{39}_i \cdot \left( Mo11_i \cdot w_i^3 \right) + \\
\mu^{40}_i \cdot Mo12_i + \mu^{41}_i \cdot \left( Mo12_i \cdot w_i \right) + \mu^{42}_i \cdot \left( Mo12_i \cdot w_i^2 \right) + \mu^{43}_i \cdot \left( Mo12_i \cdot w_i^3 \right)
\end{aligned}
\]

\[
\begin{aligned}
B^0 \cdot \left( \frac{w_i}{P_{Corn}} \right) + h^0_i \cdot \left( Head_i \cdot \frac{w_i}{P_{Corn}} \right) + h^2_i \cdot \left( Head_i^2 \cdot \frac{w_i}{P_{Corn}} \right) + \\
d^0_i \cdot \left( Hfr_i \cdot \frac{w_i}{P_{Corn}} \right) + g^0_i \cdot \left( Class1_i \cdot \frac{w_i}{P_{Corn}} \right) + g^2_i \cdot \left( Class2_i \cdot \frac{w_i}{P_{Corn}} \right) + \\
l^0_i \cdot \left( Loc2_i \cdot \frac{w_i}{P_{Corn}} \right) + l^2_i \cdot \left( Loc3_i \cdot \frac{w_i}{P_{Corn}} \right) + \\
m^0_i \cdot \left( Mol_i \cdot \frac{w_i}{P_{Corn}} \right) + m^2_i \cdot \left( Mol2_i \cdot \frac{w_i}{P_{Corn}} \right) + m^4_i \cdot \left( Mol3_i \cdot \frac{w_i}{P_{Corn}} \right) + \\
m^0_i \cdot \left( Mo4_i \cdot \frac{w_i}{P_{Corn}} \right) + m^2_i \cdot \left( Mo5_i \cdot \frac{w_i}{P_{Corn}} \right) + m^4_i \cdot \left( Mo6_i \cdot \frac{w_i}{P_{Corn}} \right) + \\
m^2_i \cdot \left( Mo7_i \cdot \frac{w_i}{P_{Corn}} \right) + m^4_i \cdot \left( Mo8_i \cdot \frac{w_i}{P_{Corn}} \right) + m^6_i \cdot \left( Mo9_i \cdot \frac{w_i}{P_{Corn}} \right) + \\
m^4_i \cdot \left( Mo11_i \cdot \frac{w_i}{P_{Corn}} \right) + m^6_i \cdot \left( Mo12_i \cdot \frac{w_i}{P_{Corn}} \right)
\end{aligned}
\]

\[ + \epsilon_i \]

\[ + IBEFR_{Ed} \]

\[ + IBEFR_{Nf} \]

\[ + error_i \]
**Feed Component: \( IBEFR_{Fd} \)**

<table>
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<th>Coeff</th>
<th>Variable description</th>
<th>Table name</th>
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<tr>
<td>( w(0) )</td>
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<td>Intercept</td>
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<td>( w_i )</td>
<td>( \beta^i )</td>
<td>Average weight of cattle in lot ( i ). Included quadratic and cubic terms in the equation</td>
<td>Weight Wt2 Wt3</td>
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<td>( w_i^2 = w_i \cdot w_i )</td>
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<tr>
<td>( w_i^3 = w_i \cdot w_i \cdot w_i )</td>
<td>( \beta^{i^3} )</td>
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<td>( Fd-Wt ) Interactions: ( Fd ) variables ( x ) Weight variables</td>
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<td>Class1 Class1Wt Class1Wt2 Class1Wt3 Class2 Class2Wt Class2Wt2 Class2Wt3</td>
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**Feed Component:** \( IBEFR_i^{Fd} \) (cont.)

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## Non-feed Component: $IBEFR_i^{NF}$

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<td>Non-feed price adjustment (NFPA) component of feeder cattle price for base animal (i.e., <em>Steer</em>, <em>Class12</em>, <em>Loc1</em>, <em>Mo10</em>, no lot size premium)</td>
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<td>$Nf^a$</td>
<td>$N^a$</td>
<td><strong>Non-feed variables</strong></td>
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<td>$NFPA$ for number of head of cattle sold together in the lot (Head)</td>
<td>Head wiPC</td>
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<tr>
<td>$\frac{w_i}{P_{Corn}}$</td>
<td>$H_i^a$</td>
<td>$NFPA$ for number of head of cattle sold together in the lot (Head)</td>
<td>Head2 wiPC</td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}}$</td>
<td>$H_i^c$</td>
<td>$NFPA$ dummy for number of head of cattle sold together in the lot (Head)</td>
<td>Hfr wiPC</td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}}$</td>
<td>$d_i^0$</td>
<td>$OMITTED$ base $NFPA$ sex dummy variable: <em>Steer</em></td>
<td>Str wiPC*</td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}}$</td>
<td>$d_i^a$</td>
<td>$OMITTED$ base $NFPA$ sex dummy variable: <em>Steer</em></td>
<td></td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}}$</td>
<td>$g_i^0$</td>
<td>$NFPA$ class dummy variables; $c=1$ and $c=2$ ($c=12$ omitted) Quality: $Class1&gt;Class12&gt;Class2$ positive (or negative) value of coeff indicates discount (premium) v. $c=12$</td>
<td>Class1 wiPC</td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}}$</td>
<td>$g_i^a$</td>
<td>$OMITTED$ base $NFPA$ class dummy variable: <em>Class12</em></td>
<td>Class12 wiPC*</td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}}$</td>
<td>$l_i^0$</td>
<td>$NFPA$ for location dummy variables $a=2$ and $a=3$ ($a=1$ omitted)</td>
<td>Loc2 wiPC</td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}}$</td>
<td>$l_i^a$</td>
<td>$OMITTED$ base $NFPA$ class dummy variable: <em>Class12</em></td>
<td></td>
</tr>
</tbody>
</table>

### Lot size (# head sold)

- $\frac{w_i}{P_{Corn}}$
- $H_i^0$
- $H_i^a$
- $H_i^c$
- $d_i^0$
- $d_i^a$
- $g_i^0$
- $g_i^a$
- $l_i^0$
- $l_i^a$

### Sex (steers and heifers)

- $\frac{w_i}{P_{Corn}}$
- $H_i^0$
- $H_i^a$
- $H_i^c$
- $d_i^0$
- $d_i^a$

### Muscle grade (Class)

- $\frac{w_i}{P_{Corn}}$
- $g_i^0$
- $g_i^a$

### Location

- $\frac{w_i}{P_{Corn}}$
- $l_i^0$
- $l_i^a$
- $l_i^1$
- $l_i^2$
### Non-feed Component: $IBEFR_i^{nj}$ (cont.)

<table>
<thead>
<tr>
<th>Variable/Group</th>
<th>Coeff(^\wedge)</th>
<th>Variable description</th>
<th>Table name</th>
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<tbody>
<tr>
<td>$Nf$</td>
<td>$Nf$</td>
<td>Non-feed variables (cont.)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Seasonality (Month)</td>
<td></td>
</tr>
<tr>
<td>$Mo(t)<em>i \cdot \frac{w_i}{P</em>{Corn}}$</td>
<td>$m_i^0$</td>
<td>$NFPA$ for month dummy variables $t=1$ through $t=12$ (except $t=10$, omitted).</td>
<td>Mo1_wiPC Mo2_wiPC Mo3_wiPC Mo4_wiPC Mo5_wiPC Mo6_wiPC Mo7_wiPC Mo8_wiPC Mo9_wiPC Mo10_wiPC Mo11_wiPC Mo12_wiPC</td>
</tr>
<tr>
<td>$Mo10_i(o) \cdot \frac{w_i}{P_{Corn}}$</td>
<td>$m_{10}$</td>
<td>OMITTED base $NFPA$ month dummy variable: $Mo10$</td>
<td>Mo10_wiPC*</td>
</tr>
</tbody>
</table>

* Indicates dummy variables that have been omitted from the regression model to avoid perfect collinearity and represent characteristics of the “base lot.”

\(^\wedge\) Positive (or negative) coefficients for the $Nf$ variables indicate a feeder cattle price (per lb.) discount (or premium) for the characteristic relative to the base lot.
Appendix D

Chapter 4 Supplement:

Model-predicted IBEFR, value and price comparisons and differences
Appendix D. Model-predicted IBEFR, value, and price comparisons and differences

Figure D.1. SEX: Heifers versus Steers
Figure D.2. CLASS: Class1 and Class2 versus Class12
Figure D.3. LOCATION: Location2 and Location3 versus Location1
Figure D.4.1. Q1 MONTH: Month1 – Month3 versus Month10
Figure D.4.2. Q2 MONTH: Month4 – Month6 versus Month10
Figure D.4.3. Q3 MONTH: Month7 – Month9 versus Month10
Figure D.4.4. Q4 MONTH: Month11 and Month12 versus Month10
Appendix E

Chapter 4 Supplement:

\textit{IBEFR-NfWt model}
Appendix E. IBEFR-NfWt Model
Expanded IBEFR-NfWt Modeling Equation, Notation, and Variable Names

\[
IBEFR_i = \alpha + \left( \beta^1 \cdot w_i + \beta^2 \cdot w_i^2 + \beta^3 \cdot w_i^3 \right) + \\
\left[ \delta^0 \cdot Hfr_i + \delta^1 \cdot \left( Hfr_i \cdot w_i \right) + \delta^2 \cdot \left( Hfr_i \cdot w_i^2 \right) + \delta^3 \cdot \left( Hfr_i \cdot w_i^3 \right) + \\
\gamma^0 \cdot Class_{1i} + \gamma^1 \cdot \left( Class_{1i} \cdot w_i \right) + \gamma^2 \cdot \left( Class_{1i} \cdot w_i^2 \right) + \gamma^3 \cdot \left( Class_{1i} \cdot w_i^3 \right) + \\
\gamma^0 \cdot Class_{2i} + \gamma^1 \cdot \left( Class_{2i} \cdot w_i \right) + \gamma^2 \cdot \left( Class_{2i} \cdot w_i^2 \right) + \gamma^3 \cdot \left( Class_{2i} \cdot w_i^3 \right) + \\
\lambda^0 \cdot Loc_{2i} + \lambda^1 \cdot \left( Loc_{2i} \cdot w_i \right) + \lambda^2 \cdot \left( Loc_{2i} \cdot w_i^2 \right) + \lambda^3 \cdot \left( Loc_{2i} \cdot w_i^3 \right) + \\
\lambda^0 \cdot Loc_{3i} + \lambda^1 \cdot \left( Loc_{3i} \cdot w_i \right) + \lambda^2 \cdot \left( Loc_{3i} \cdot w_i^2 \right) + \lambda^3 \cdot \left( Loc_{3i} \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo1_i + \mu^1 \cdot \left( Mo1_i \cdot w_i \right) + \mu^2 \cdot \left( Mo1_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo1_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo2_i + \mu^1 \cdot \left( Mo2_i \cdot w_i \right) + \mu^2 \cdot \left( Mo2_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo2_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo3_i + \mu^1 \cdot \left( Mo3_i \cdot w_i \right) + \mu^2 \cdot \left( Mo3_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo3_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo4_i + \mu^1 \cdot \left( Mo4_i \cdot w_i \right) + \mu^2 \cdot \left( Mo4_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo4_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo5_i + \mu^1 \cdot \left( Mo5_i \cdot w_i \right) + \mu^2 \cdot \left( Mo5_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo5_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo6_i + \mu^1 \cdot \left( Mo6_i \cdot w_i \right) + \mu^2 \cdot \left( Mo6_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo6_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo7_i + \mu^1 \cdot \left( Mo7_i \cdot w_i \right) + \mu^2 \cdot \left( Mo7_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo7_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo8_i + \mu^1 \cdot \left( Mo8_i \cdot w_i \right) + \mu^2 \cdot \left( Mo8_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo8_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo9_i + \mu^1 \cdot \left( Mo9_i \cdot w_i \right) + \mu^2 \cdot \left( Mo9_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo9_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo11_i + \mu^1 \cdot \left( Mo11_i \cdot w_i \right) + \mu^2 \cdot \left( Mo11_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo11_i \cdot w_i^3 \right) + \\
\mu^0 \cdot Mo12_i + \mu^1 \cdot \left( Mo12_i \cdot w_i \right) + \mu^2 \cdot \left( Mo12_i \cdot w_i^2 \right) + \mu^3 \cdot \left( Mo12_i \cdot w_i^3 \right) \\
\right] + 
\]
\[
\begin{align*}
B^0 \left( \frac{W_i}{P_{\text{Core}}} \right) + B' \left( \frac{W_i}{P_{\text{Core}}} \right) + B^2 \left( \frac{W_i}{P_{\text{Core}}} \right) + \\
h^0 \left( \text{Head}_1 \cdot \frac{W_i}{P_{\text{Core}}} \right) + h^1 \left( \text{Head}_1 \cdot \frac{W_i}{P_{\text{Core}}} \right) + h^2 \left( \text{Head}_1 \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
h^0 \left( \text{Head}_2 \cdot \frac{W_i}{P_{\text{Core}}} \right) + h^1 \left( \text{Head}_2 \cdot \frac{W_i}{P_{\text{Core}}} \right) + h^2 \left( \text{Head}_2 \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
d^0 \left( \text{Hfr}_1 \cdot \frac{W_i}{P_{\text{Core}}} \right) + d^1 \left( \text{Hfr}_1 \cdot \frac{W_i}{P_{\text{Core}}} \right) + d^2 \left( \text{Hfr}_1 \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
g^0 \left( \text{Class1} \cdot \frac{W_i}{P_{\text{Core}}} \right) + g^1 \left( \text{Class1} \cdot \frac{W_i}{P_{\text{Core}}} \right) + g^2 \left( \text{Class1} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
g^0 \left( \text{Class2} \cdot \frac{W_i}{P_{\text{Core}}} \right) + g^1 \left( \text{Class2} \cdot \frac{W_i}{P_{\text{Core}}} \right) + g^2 \left( \text{Class2} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
l^0 \left( \text{Loc2} \cdot \frac{W_i}{P_{\text{Core}}} \right) + l^1 \left( \text{Loc2} \cdot \frac{W_i}{P_{\text{Core}}} \right) + l^2 \left( \text{Loc2} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
l_1 \left( \text{Loc3} \cdot \frac{W_i}{P_{\text{Core}}} \right) + l_1 \left( \text{Loc3} \cdot \frac{W_i}{P_{\text{Core}}} \right) + l_1 \left( \text{Loc3} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_1 \left( \text{Mo1} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_1 \left( \text{Mo1} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_1 \left( \text{Mo1} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_2 \left( \text{Mo2} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_2 \left( \text{Mo2} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_2 \left( \text{Mo2} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_3 \left( \text{Mo3} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_3 \left( \text{Mo3} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_3 \left( \text{Mo3} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_4 \left( \text{Mo4} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_4 \left( \text{Mo4} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_4 \left( \text{Mo4} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_5 \left( \text{Mo5} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_5 \left( \text{Mo5} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_5 \left( \text{Mo5} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_6 \left( \text{Mo6} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_6 \left( \text{Mo6} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_6 \left( \text{Mo6} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_7 \left( \text{Mo7} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_7 \left( \text{Mo7} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_7 \left( \text{Mo7} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_8 \left( \text{Mo8} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_8 \left( \text{Mo8} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_8 \left( \text{Mo8} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_9 \left( \text{Mo9} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_9 \left( \text{Mo9} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_9 \left( \text{Mo9} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_{10} \left( \text{Mo11} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_{10} \left( \text{Mo11} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_{10} \left( \text{Mo11} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_{11} \left( \text{Mo12} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_{11} \left( \text{Mo12} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_{11} \left( \text{Mo12} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
m_{12} \left( \text{Mo13} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_{12} \left( \text{Mo13} \cdot \frac{W_i}{P_{\text{Core}}} \right) + m_{12} \left( \text{Mo13} \cdot \frac{W_i}{P_{\text{Core}}} \right) + \\
+ \varepsilon_i + \text{error}_i
\end{align*}
\]
## Feed Component: \textit{IBEFR}_i^{Fd}

<table>
<thead>
<tr>
<th>Variable/Group</th>
<th>Coeff</th>
<th>Variable description</th>
<th>Table name</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weight</strong></td>
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</tr>
<tr>
<td>(w(0))</td>
<td>(\alpha)</td>
<td>Model intercept</td>
<td>Intercept</td>
</tr>
<tr>
<td>(w_i)</td>
<td>(\beta_i)</td>
<td>Average weight of cattle in lot (i). Included quadratic and cubic terms in the equation</td>
<td>Weight Wt2 Wt3</td>
</tr>
<tr>
<td>(w_i^2 = w_i \cdot w_i)</td>
<td>(\beta_i^2)</td>
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<tr>
<td>(w_i^3 = w_i \cdot w_i \cdot w_i)</td>
<td>(\beta_i^3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Fd-Wt</strong></td>
<td>(\phi_i^p)</td>
<td><strong>Fd-Wt Interactions:</strong> (Fd) variables (x) (Weight) variables</td>
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<tr>
<td>(Hfr_i)</td>
<td>(\delta_s^0)</td>
<td>INCLUDED sex dummy variable; Interaction terms between weight and heifer sex dummy variable.</td>
<td>Hfr HfrWt HfrWt2 HfrWt3</td>
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<tr>
<td>(\text{Hfr}_i \cdot w_i)</td>
<td>(\delta_s^1)</td>
<td>Heifers</td>
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<td>(\text{Hfr}_i \cdot w_i^2)</td>
<td>(\delta_s^2)</td>
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<tr>
<td>(\text{Hfr}_i \cdot w_i^3)</td>
<td>(\delta_s^3)</td>
<td></td>
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</tr>
<tr>
<td>(\text{Str}_i(o))</td>
<td>(\delta_s)</td>
<td>OMITTED base model sex dummy variable: \textit{Steer}</td>
<td>Str*</td>
</tr>
<tr>
<td><strong>Muscle grade (Class)</strong></td>
<td></td>
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<tr>
<td>(\text{Class}(c)_i)</td>
<td>(\gamma_c^0)</td>
<td>INCLUDED class dummy variables; Interaction terms between weight and class dummy variables for classes (c = 1) and (c = 2) ((c = 12) omitted)</td>
<td>Class1 Class1Wt Class1Wt2 Class1Wt3</td>
</tr>
<tr>
<td>(\text{Class}(c)_i \cdot w_i)</td>
<td>(\gamma_c^1)</td>
<td>Quality: \textit{Class1} &gt; \textit{Class12} &gt; \textit{Class2}</td>
<td>Class2 Class2Wt Class2Wt2 Class2Wt3</td>
</tr>
<tr>
<td>(\text{Class}(c)_i \cdot w_i^2)</td>
<td>(\gamma_c^2)</td>
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<tr>
<td>(\text{Class}(c)_i \cdot w_i^3)</td>
<td>(\gamma_c^3)</td>
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<tr>
<td>(\text{Class12}_i(o))</td>
<td>(\gamma_{12})</td>
<td>OMITTED base model class dummy variable: \textit{Class12} (Represents a mix of Class 1s and 2s)</td>
<td>Class12*</td>
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<tr>
<td><strong>Location</strong></td>
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<tr>
<td>(\text{Loc}(a)_i)</td>
<td>(\lambda_a^0)</td>
<td>INCLUDED location dummy variables; Interaction terms between weight and location dummy variables for locations (a = 2) and (a = 3) ((a = 1) omitted).</td>
<td>Loc2 Loc2Wt Loc2Wt2 Loc2Wt3</td>
</tr>
<tr>
<td>(\text{Loc}(a)_i \cdot w_i)</td>
<td>(\lambda_a^1)</td>
<td>location dummy variables for locations (a = 2) and (a = 3) ((a = 1) omitted).</td>
<td>Loc2 Loc2Wt2 Loc2Wt3</td>
</tr>
<tr>
<td>(\text{Loc}(a)_i \cdot w_i^2)</td>
<td>(\lambda_a^2)</td>
<td></td>
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</tr>
<tr>
<td>(\text{Loc}(a)_i \cdot w_i^3)</td>
<td>(\lambda_a^3)</td>
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<tr>
<td>(\text{Loc1}_i(o))</td>
<td>(\lambda_1)</td>
<td>OMITTED base model location dummy variable: \textit{Loc1}</td>
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### Feed Component: \( IBEFR_{Fd}^{i} \) (cont.)

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<th>Variable description</th>
<th>Table name</th>
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<tbody>
<tr>
<td><strong>Fd-Wt (cont.)</strong></td>
<td>( \phi_{i}^{p} )</td>
<td><strong>Fd-Wt Interactions (cont.)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Seasonality (Month)</strong></td>
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<td></td>
</tr>
<tr>
<td>( Mo(t)_{i} )</td>
<td>( \mu_{i}^{0} )</td>
<td>INCLUDED month dummy variables; Interaction terms between weight and location dummy variables for locations ( t = 1 ) through ( t = 12 ) (except ( t = 10 ), omitted).</td>
<td>Mo1 Mo2</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i} )</td>
<td>( \mu_{i}^{1} )</td>
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<td>Mo1Wt Mo2Wt</td>
</tr>
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<td>( Mo(t)<em>{i} \cdot w</em>{i}^{2} )</td>
<td>( \mu_{i}^{2} )</td>
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<td>Mo1Wt2 Mo2Wt2</td>
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<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{3} )</td>
<td>( \mu_{i}^{3} )</td>
<td></td>
<td>Mo1Wt3 Mo2Wt3</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{4} )</td>
<td>( \mu_{i}^{4} )</td>
<td></td>
<td>Mo3 Mo4</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{5} )</td>
<td>( \mu_{i}^{5} )</td>
<td></td>
<td>Mo3Wt Mo4Wt</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{6} )</td>
<td>( \mu_{i}^{6} )</td>
<td></td>
<td>Mo3Wt2 Mo4Wt2</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{7} )</td>
<td>( \mu_{i}^{7} )</td>
<td></td>
<td>Mo3Wt3 Mo4Wt3</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{8} )</td>
<td>( \mu_{i}^{8} )</td>
<td></td>
<td>Mo5 Mo6</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{9} )</td>
<td>( \mu_{i}^{9} )</td>
<td></td>
<td>Mo5Wt Mo6Wt</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{10} )</td>
<td>( \mu_{i}^{10} )</td>
<td></td>
<td>Mo5Wt2 Mo6Wt2</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{11} )</td>
<td>( \mu_{i}^{11} )</td>
<td></td>
<td>Mo5Wt3 Mo6Wt3</td>
</tr>
<tr>
<td>( Mo(t)<em>{i} \cdot w</em>{i}^{12} )</td>
<td>( \mu_{i}^{12} )</td>
<td></td>
<td>Mo5Wt4 Mo6Wt4</td>
</tr>
</tbody>
</table>

**Mo10** (o) | \( \mu_{10} \) | OMITTED base model location dummy variable: Mo10 | **Mo10** |

---

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### Non-feed Component: $IBEFR_{i}^{Nf}$

<table>
<thead>
<tr>
<th>Variable/Group</th>
<th>Coeff</th>
<th>Variable description</th>
<th>Table name</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$Nf^0$ base</strong></td>
<td></td>
<td><strong>Base non-feed adjustment</strong></td>
<td></td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}}$</td>
<td>$B^0$</td>
<td>Non-feed price adjustment $(NFPA)$ component of feeder cattle price for base animal (i.e., Steer, Class12, Loc1, Mo10, no lot size premium)</td>
<td>Wi_Pcorn</td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}} \cdot w_i$</td>
<td>$B^1$</td>
<td></td>
<td>Wi_Pcorn_Wt</td>
</tr>
<tr>
<td>$\frac{w_i}{P_{Corn}} \cdot w_i^2$</td>
<td>$B^2$</td>
<td></td>
<td>Wi_Pcorn_Wt2</td>
</tr>
<tr>
<td><strong>$Nf^p$</strong></td>
<td><strong>$Nf_i^p$</strong></td>
<td><strong>Non-feed variables</strong></td>
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<tr>
<td><strong>Lot size (# head sold)</strong></td>
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<tr>
<td>$\frac{Head_i \cdot w_i}{P_{Corn}}$</td>
<td>$h_i^0$</td>
<td>$NFPA$ for number of head of cattle sold together in the lot (Head)</td>
<td>Head_wiPC</td>
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<td>$\frac{Head_i \cdot w_i}{P_{Corn}} \cdot w_i$</td>
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<td>Head2_wiPC</td>
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<td>$(Head_i^2 = Head_i \cdot Head_i)$</td>
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<td><strong>Sex (steers and heifers)</strong></td>
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<td>$\frac{Hfr_i \cdot w_i}{P_{Corn}}$</td>
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<td>$\frac{Str_i(o) \cdot w_i}{P_{Corn}} \cdot w_i^p$</td>
<td>$d_i$</td>
<td>OMITTED base $NFPA$ sex dummy variable: <strong>Steer</strong></td>
<td>Str_wiPC*</td>
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### Non-feed Component: $IBEFR_i^Nf$ (cont.)

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<th>Variable/Group</th>
<th>Coeff</th>
<th>Variable description</th>
<th>Table name</th>
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<tr>
<td>$Class(c)<em>i \cdot \frac{w_i}{P</em>{CornLbs}}$</td>
<td>$g_c^0$</td>
<td>$NFPA$ class dummy variables; $c = 1$ and $c = 2$ ($c = 12$ omitted)</td>
<td>Class1_wiPC, Class1_wiPC_Wt, Class1_wiPC_Wt2, Class2_wiPC, Class2_wiPC_Wt, Class2_wiPC_Wt2</td>
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<tr>
<td>$Class(c)<em>i \cdot \frac{w_i}{P</em>{Corn}}$</td>
<td>$g_c^1$</td>
<td>Quality: $Class1 &gt; Class12 &gt; Class2$ positive (or negative) value of coeff indicates discount (or premium) v $c = 12$</td>
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<td>$Class(c)<em>i \cdot \frac{w_i}{P</em>{Corn}} \cdot w_i^2$</td>
<td>$g_c^2$</td>
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<tr>
<td>$Class12_i(o) \cdot \frac{w_i}{P_{Corn}} \cdot w_i^p$</td>
<td>$g_{12}^0$</td>
<td>OMITTED base $NFPA$ class dummy variable: $Class12$</td>
<td>Class12_wiPC*</td>
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<td>$Loc(a)<em>i \cdot \frac{w_i}{P</em>{CornLbs}}$</td>
<td>$l_a^0$</td>
<td>$NFPA$ for location dummy variables $a = 2$ and $a = 3$ ($a = 1$ omitted)</td>
<td>Loc2_wiPC, Loc2_wiPC_Wt, Loc2_wiPC_Wt2, Loc3_wiPC, Loc3_wiPC_Wt, Loc3_wiPC_Wt2</td>
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<tr>
<td>$Loc(a)<em>i \cdot \frac{w_i}{P</em>{Corn}} \cdot w_i$</td>
<td>$l_a^1$</td>
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<tr>
<td>$Loc(a)<em>i \cdot \frac{w_i}{P</em>{Corn}} \cdot w_i^2$</td>
<td>$l_a^2$</td>
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<tr>
<td>$Loc1(o) \cdot \frac{w_i}{P_{Corn}}$</td>
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### Non-feed Component: $IBEFR_i^{Nf}$ (cont.)

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<td><strong>Seasonality (Month)</strong></td>
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<tr>
<td>$Mo(t) \cdot \frac{w_i}{P_{Corn}}$</td>
<td>$m_i^0$</td>
<td>NFPA for month dummy variables $t=1$ through $t=12$ (except $t=10$, omitted).</td>
<td>Mo1_wpPC Mo1_wpPC_Wt Mo1_wpPC_Wt2 Mo2_wpPC Mo2_wpPC_Wt Mo2_wpPC_Wt2 Mo3_wpPC Mo3_wpPC_Wt Mo3_wpPC_Wt2 Mo4_wpPC Mo4_wpPC_Wt Mo4_wpPC_Wt2 Mo5_wpPC Mo5_wpPC_Wt Mo5_wpPC_Wt2 Mo6_wpPC Mo6_wpPC_Wt Mo6_wpPC_Wt2 Mo7_wpPC Mo7_wpPC_Wt Mo7_wpPC_Wt2 Mo8_wpPC Mo8_wpPC_Wt Mo8_wpPC_Wt2 Mo9_wpPC Mo9_wpPC_Wt Mo9_wpPC_Wt2 Mo10_wpPC Mo10_wpPC_Wt Mo10_wpPC_Wt2 Mo11_wpPC Mo11_wpPC_Wt Mo11_wpPC_Wt2 Mo12_wpPC Mo12_wpPC_Wt Mo12_wpPC_Wt2</td>
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<tr>
<td>$Mo(t) \cdot \frac{w_i}{P_{Corn}} \cdot w_i$</td>
<td>$m_i^1$</td>
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<tr>
<td>$Mo(t) \cdot \frac{w_i}{P_{Corn}} \cdot w_i^2$</td>
<td>$m_i^2$</td>
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$Mo10_i(o) \cdot \frac{w_i}{P_{Corn}} \cdot w_i^p$ $m_i0$ OMITTED base NFPA month dummy variable: $Mo10$ Mo10_wpPC*

* Indicates dummy variables that have been omitted from the regression model to avoid perfect collinearity and represent characteristics of the “base lot.”
### IBEFR-NfWt Model Results

Dependent Variable: IBEFR<sub>i</sub>

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<th>Adj. R-Square</th>
<th>0.8995</th>
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<td>Root MSE</td>
<td>670.69</td>
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| Number of observations | 179,867 |

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<tr>
<th>Variable</th>
<th>Intercept</th>
<th>Weight</th>
<th>Wt2</th>
<th>Wt3</th>
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<td>-0.0056</td>
<td>3.163E-06</td>
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<td>Class2_Fd</td>
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<td>4.9477</td>
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<td>Loc2_Fd</td>
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<td>Loc3_Fd</td>
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<td>0.0275</td>
<td>-1.332E-05</td>
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<td>Mo1_Fd</td>
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<td>NON-FEED (Nf)</td>
<td>Nf*wiPC</td>
<td>Nf<em>wiPC</em>Wt</td>
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**Figure E.1.** IBEFR-NfWt: IBEFR and total value components and feeder value by weight (base lot)

**Figure E.2.** IBEFR-NfWt: IBEFR price adjustment and price slide components
Figure E.3. SEX: Heifers versus Steers
Figure E.4. CLASS: Class1 and Class2 versus Class12
**Figure E.5.** LOCATION: Location2 and Location3 versus Location1
Figure E.6.1. Q1 MONTH: Month1 – Month3 versus Month10
Figure E.6.2. Q2 MONTH: Month4 – Month6 versus Month10
Figure E.6.3. Q3 MONTH: Month7 – Month9 versus Month10
Figure E.6.4. Q4 MONTH: Month11 and Month12 versus Month10