Abstract

EROGLU, YUSUF SAID. Multi-Element Visible Light Communications. (Under the direction of Dr. Ismail Guvenc).

Visible light communications (VLC) is a communication medium using unlicensed vast visible light spectrum promising high data rates, and therefore can be a solution to the spectrum crunch problem in radio frequency networks. It has been studied thoroughly in recent years as an alternative or complementary technology to radio frequency communications. In this dissertation, we study some main challenges about VLC, which are light emitting diode (LED) assignment to users, analysis of random receiver orientation, beam steering, and mobile user localization and tracking.

First, we study the problem of assigning multiple LEDs to each user in an environment where the number of LEDs is much larger than the number of users. Future LED illumination infrastructure is envisioned to have a large number of low power LEDs. The presence of many LEDs can be exploited for indoor VLC networks, to serve each user by multiple LEDs for improving link quality and throughput. In this study, we group LEDs and assign to the users based on the received signal strength from each LED, for which we propose different solutions to achieve maximum throughput, proportional fairness, and quality of service. Additionally, we investigate the power optimization of LEDs for a given assignment and derive Jacobian and Hessian matrices of the corresponding optimization problem. Lastly, an efficient calculation of channel response is presented to simulate the multipath VLC channel with low computational complexity.

Second, we study the effects of random receiver orientation and mobility on the link quality of VLC. The reliability of VLC channels highly depends on the availability and alignment of the line of sight links. In this work, we study the effect of random receiver orientation for mobile users over VLC downlink channels, which affects the existence of the line of sight links and the receiver field of view. Based on the statistics of vertical receiver orientation and user mobility, we develop a unified analytical framework to characterize the statistical distribution of VLC downlink channels, which is then utilized to obtain the outage
probability and the bit error rate. Our analysis is generalized for arbitrary distributions of receiver orientation/location for a single transmitter and extended to multiple transmitter case for certain scenarios.

Third, we study VLC beam steering considering a scenario where VLC beam directions are assumed to be fixed during a transmission frame. We find the steering angles that simultaneously serve multiple users within the frame duration and maximize the data rates. Subsequently, we consider multiple steerable beams with a larger number of users in the network and propose an algorithm to cluster users and serve each cluster with a separate beam. We optimize the transmit power of each beam to maximize the data rates. Finally, we propose a non-orthogonal multiple access (NOMA) scheme for the beam steering and user clustering scenario to further increase the user data rates.

Last, we study VLC user localization and tracking with adaptive Kalman filter. A VLC network can be used to localize mobile users in indoor environments, where the global positioning system (GPS) signals cannot penetrate. In this dissertation, we study tracking a VLC user when the availability of VLC access point (AP) link changes over the user’s route. We propose a localization method for a single available AP and use known estimation methods when a larger number of APs are available. The generic Kalman filter does not consider instant changes in the measurement method. In order to include this information in the position estimation, we implement an adaptive Kalman filter by modifying the filter parameters based on the available AP configurations.
Biography

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Chapter 1

Introduction

1.1 Background

Light emitting diodes (LEDs) have become increasingly popular within the last decade as light sources due to their decreasing cost, low energy use and compatibility with different features such as dimming. One of the important applications enabled by the proliferation of LEDs is visible light communications (VLC) which transmit data in the visible light spectrum with the wavelength interval of 380 - 780 nm. The light intensity of LEDs can be modulated with high-frequency, which allows data transmission that is not perceivable to human eye [1]. VLC takes advantage of light used for illumination as a communication channel and does not require additional signals for data transmission, which provides an opportunity for energy efficiency.

The proliferation of smart phones and other mobile devices increased the need for high-speed wireless data transfer, which pushes radio frequency (RF) based wireless technologies to their limits. To increase wireless data capacity, different solutions are proposed including heterogeneous networks (HetNet) with small cells to utilize frequencies more efficiently, or
millimeter waves (mmWave) to increase available spectrum. The RF spectrum crunch along with the old capacity gap between the RF wireless last mile and the optical fiber Internet backbone speeds have motivated the research community to look for alternative solutions and spectrum resources. VLC has received attention as an alternative or complementary technology to RF communications primarily for indoor environments. Most wireless data usage occurs indoors, where LEDs are commonly used for illumination.

Wireless communication through visible light goes back to 1880 when Alexander Graham Bell invented the photophone, which transmitted speech on modulated sunlight over several hundred meters [2]. This pre-dates the transmission of speech over RF. However, research on wireless optical communication did not receive much attention for the next century. This has changed with the development of LEDs in the last few decades. Today VLC can use off-the-shelf white LEDs to modulate visible light and provide data rates as high as multiple Gigabits per second [3, 4]. Some of the advantages of VLC over RF can be summarized as follows [5]:

- Visible light has unregulated bandwidth over more than 350 THz for the wavelengths of range 380-780 nm.

- Visible light cannot penetrate walls and other opaque objects, which provides inherent security by preventing interception and eavesdropping [6]. It also allows for frequency reuse in different rooms of a building.

- VLC signals are harder to jam. It is safe against jamming attacks from outside of the environment.

- The VLC provides a much higher signal-to-noise ratio (SNR) compared to RF communication systems thanks to the high brightness of white LEDs and illumination requirements of indoor environments. The short distance between the transmitter and receiver is also a factor for high SNR.
• Wide utilization and low cost of LEDs and photo-detectors (PDs) are an economical solution to high density indoor networks.

• VLC signals do not cause electromagnetic interference, therefore can be installed in RF sensitive environments such as hospitals and airplanes.

• VLC signals do not cause health hazards such as heating of biological tissues.

• VLC channel is not affected from multipath fading as much as RF channel since the direct line-of-sight (LOS) link is much stronger compared to multipath components.

While VLC has many advantages, there are also several challenges that need to be addressed for successful deployment. First, application of VLC requires appropriate modulation schemes that do not cause flicker and color variation and provide the highest possible rates within the transmission capability of LEDs. Second, VLC is highly dependent on the LOS link since the multipath channel components are weak compared to the LOS component, and LOS link can easily be blocked by objects. Having a dominant LOS link also decreases the efficiency of multi-input-multi-output (MIMO) techniques since the subchannels between different PD and LED couples are strongly correlated. Third, it is difficult to provide uplink data transmission using VLC, because transmit antenna should face the receiver which cannot be guaranteed in case of a mobile device.

In this dissertation, we study some of these challenges. In the first part, we address the challenges of multi-element VLC, that is VLC with multiple LEDs and multiple receiver PDs. In the second part, we address the challenges of the mobility and random orientation of receiver devices.
1.2 Multi-Element VLC

In the first chapter, we address the challenges of multi-element VLC. As mentioned before, the LOS channel component is dominant in VLC, which causes MIMO subchannels to be highly correlated. Therefore, MIMO techniques are not as efficient as RF communication technologies. In this study, we propose making use of the high number of transmitter LEDs in an alternative way by assigning multiple LEDs to each user and increasing the signal-to-interference-plus-noise-ratio (SINR). We investigate the optimal assignment of LEDs to users and power optimization of the LEDs. We also study diversity reception in case of multiple LED assignment to a single user.

1.2.1 Literature Review

In VLC networks, in order to provide ubiquitous illumination/wireless coverage, improve link quality, and provide higher throughput, a large number of LEDs with directional propagation characteristics can be deployed in indoor environments. Therefore, there might be a significantly larger number of LEDs than the number of users in the network, and it is possible to serve each user with multiple directional LEDs over the same bandwidth [7, 8]. In RF communications, on the other hand, a single transmitter may serve many users at the same time simultaneously, and users may connect to the transmitter with the highest received power [9]. Coordinated multipoint transmission (CoMP) technique serves a user with more than one base station [10], which is used in LTE-Advanced for interference management. However, typically, the number of users connected to a base station is significantly higher than the number of antennas deployed at the radio. The assignment techniques developed for RF communications are hence not directly applicable to VLC networks, and developing new assignment techniques is necessary. The problem of assigning LEDs to users has similarities with the subchannel allocation in multiuser OFDM systems [11, 12]. However, while
assigning extra subchannel to a user affects its bandwidth, assigning an extra LED to a VLC user affects its SINR, and hence the impact on the system performance will be different.

In [13, 14], the joint transmission of information from multiple VLC nodes is studied, in order to mitigate cell-edge interference without investigating which VLC nodes will serve which users. An LED allocation scheme is presented in [15], which proposes an LED array as VLC transmitter, assigning LEDs with respect to the number and location of the users. However, it only provides a few fixed assignment options and cannot be generalized to transmitter deployments with different geometries. In [16], a spatial division multiple access (SDMA) scheme is described to serve multiple users simultaneously by allocating them different LEDs, where the LED allocation is decided based on the location of the user. However, as mentioned in the introduction, location information itself might not always be sufficient to assign LEDs to users, since the RSS can dramatically decrease in case of an obstruction or a change in the receiver orientation. A more accurate and robust method for LED assignment is using RSS information at the receivers, and developing such techniques and evaluating their performance is one of the main contributions of this study. The studies on resource allocation for VLC networks [17, 18], reported in the literature focus on power allocation of LEDs and do not address how to assign multiple LEDs to a single user.

In addition to the use of multi-element LED assignments to users, using multiple PDs at the receiver side is shown to improve the SINR performance [19]. The studies on imaging angle diversity receiver for infrared communications have been a baseline for VLC related extensions in this area [20, 21]. In [21], the performance of angular diversity receivers with multibeam infrared transmitters is investigated and it is concluded that diversity reduces the ambient noise dramatically when used with maximum ratio combining (MRC). The optimal combining (OC) is another technique [22] that considers the correlation between interference signals received from different PDs, and it suppresses the interference by using optimum combining weights. In [19], it is shown that the OC provides the highest SINR for multi-
PD VLC and it is followed by the MRC, which weighs the PDs proportionally with their individual SINR measurements. In [23], it is shown that the OC provides even higher gain in comparison to MRC when multi-LED transmitters, as also considered in this study, are used. However, these studies do not consider the grouping of LEDs to serve users. When LEDs are grouped, interfering LEDs will also be grouped to serve other users, which will further increase the correlation between interference signals. In this case, interference correlation matrix modeling of OC also needs to change too, which is also studied in this dissertation.

1.2.2 Contributions

The contributions of this study can be summarized as follows:

i. We study the assignment of multiple LEDs to users in a multi-LED transmitter VLC network. The number of LEDs is assumed to be much larger than the number of users, and multiple highly directional LEDs are assigned to each user. Using this scheme, simultaneous transmission to multiple users and hence higher SINR are aimed for. A possible method is assigning LEDs to users based on their locations. However, in VLC networks, location information itself might not be sufficient. VLC connectivity is highly dependent on direct LOS signals and the received signal strength (RSS) may decrease dramatically in case of an obstacle between the LED and the user. Also, the angle of arrival of a signal and a receiver’s orientation can significantly affect the signal strength. Therefore, we directly use the RSS information at the receiver to assign LEDs to users to provide a robust assignment scheme against obstructions. The LED assignment is studied for two different scenarios: pre-allocated QoS rates and opportunistic sum rate maximization. We examine the tractability of finding an optimal assignment algorithm in both cases and propose heuristic algorithms that find suboptimal solutions with low computational complexity.
ii. We optimize the transmit powers of the LEDs to improve the sum rate and fairness for a given LED to user assignment. We formulate the corresponding optimization problem, present Lagrangian dual function, and derive Jacobian and Hessian matrices.

iii. We utilize multi-element receiver diversity using OC and propose a novel correlation matrix calculation method to capture the correlation between interference signals more accurately. The OC proposed in [22] is shown to provide higher SINR than its counterparts. It uses a correlation matrix of interferences received from different PDs to suppress interference. Assigning many LEDs having different locations and directions to the same user as in our proposed scheme creates a more diverse channel. If the user knows which LEDs are assigned together, this information can help to suppress interference more successfully. Simulation results show that the proposed OC calculation method improves the SINR by 2 dB to 5 dB over the OC proposed in [22] at no additional cost or computational complexity.

1.3 Random Receiver Orientation

In the second chapter of the dissertation, we investigate the effects of random receiver orientation and user mobility on the VLC channel quality.

1.3.1 Literature Review

The propagation through VLC channels can be highly directional [24] and communication mainly relies on the availability of LOS links. In practice, however, the field-of-view (FOV) of VLC receiver is usually limited, which in turn appears as a barrier in providing seamless network connectivity. The hybrid RF/VLC networks [25, 26] and relay-assisted cooperative VLC systems [27, 28] are two main research directions to circumvent FOV constraints
and extend the network coverage as desired. Furthermore, as the density and mobility of VLC receivers increase along with the use of wearable sensors and Internet-of-Things (IoT) devices [29], sophisticated dynamics are emerging with FOV constraints and LOS reliability.

The receiver orientation and mobility are two major obstacles affecting the availability of LOS links in VLC networks. Their direct influence on the existence of LOS links and signal quality becomes even more significant especially when both these features are varying randomly. It is therefore vital to investigate the effect of the receiver orientation and mobility over VLC networks with practical FOV constraints. In [30, 31], a mobile VLC channel is considered with the goal of characterizing the channel impulse response (CIR) through ray-tracing simulations and laboratory measurements. The user mobility is handled by considering probabilistic noisy and outdated channel state information (CSI) models in [32]. The ergodic capacity of a mobile VLC scenario is evaluated in [33] for randomly distributed user locations. Although these recent studies consider the mobility over VLC networks, they all assume fixed and vertically upward receiver orientation without any variation.

The impact of receiver orientation on VLC networks has received very limited attention in the literature. In [34], a cellular light-fidelity (Li-Fi) network is considered for access point (AP) selection, where the receiver orientation appears to have a significant effect on the user quality of service (QoS) and overall load balancing. The handover mechanism is investigated in [31] for mobile Li-Fi networks, and the effect of receiver orientation is evaluated through a geometric approach involving rotation matrix computations. None of these works consider the effect of random receiver orientation on VLC channel statistics. The outage performance of an indoor VLC system with random receiver orientation is studied in [35] with experimental evaluations. However, this study does not analytically evaluate the effect of random receiver orientation on outage performance.
1.3.2 Contributions

In this work, we investigate the effect of the receiver orientation and mobility on the statistics of VLC downlink channels in single and multiple LED scenarios, which has not been studied in the literature before within a broad scope. The contributions of this study can be summarized as follows:

i. We develop a unified analytical framework which derives the statistical distribution of VLC downlink channels explicitly in the presence of random receiver orientation and mobility. The statistical distribution includes the cumulative distribution function (cdf) and the probability density function (pdf) of the channel gain, which enables obtaining the outage probability and the bit error rate (BER), respectively. The channel distribution is characterized in a general form so that any random statistics of the orientation and mobility can be employed directly. The analytical findings are verified through extensive simulation data matching in all cases of interest.

ii. The nonlinear effect of the receiver FOV is integrated into the analytical framework parametrically, which enables the analysis of channel statistics and error performance for specific FOV chosen from a broad range of values.

iii. The proposed framework rigorously handles the single LED and two LEDs cases. In addition, an extension of the statistical findings to multiple LED settings is also investigated. The results verify the immediate intuitions that wider FOV and multiple LED deployment can be viable solutions in coping with the adverse effects of random receiver orientation and mobility.
1.4 Slow Beam Steering and NOMA

In the third chapter, we study VLC beam steering in a scenario where the beam steering cannot be fast enough to efficiently steer multiple users for each TDMA slot. We, therefore, propose a scheme where the LEDs are steered once so that the signal strength of all users maximizes the sum rate without requiring further steering. We also study a non-orthogonal multiple access (NOMA) scheme to further increase the data rates of users in the given setting.

1.4.1 Literature review

VLC networks can provide highly accurate localization information [7, 36], and this location information can be used to steer the light beam towards user location by manipulating the orientation of the light source to further enhance the performance of the communication. It has been shown in the literature that using a steerable directional beam maximizes both the overall signal strength and the coverage area [37]. In [23], tracking users by steering LEDs is shown to provide a much higher SINR in the VLC cell borders, which provides smoother handovers between adjacent VLC APs. A beam steering scheme is studied with angle diversity receivers in [38], where the beam can be steered in some certain orientations which are predetermined depending on the user location distribution. The study is extended for imaging receivers in [39, 40]. However, these studies assume that each user is tracked with a dedicated LED or multiple LEDs. When the number of users is lower than or equal to the number of steerable beams the steering is relatively simple because each user can be assigned a single beam that tracks the user. However, in some cases, the number of users can be higher than the number of steerable beams. In such cases, how to steer the LEDs and distribute time allocation to users is an open problem which has not been addressed in the literature.
In recent years, NOMA schemes have received significant attention for cellular networks [41, 42]. The primary reason for adopting NOMA is its ability to serve multiple users using the same time and frequency resources. NOMA achieves this by assigning different power levels to users that have distinctive channel gains. In [43], the use of NOMA is investigated for VLC, and it was found that NOMA can serve multiple users to provide higher data rates compared to orthogonal multiple access (OMA) such as time or frequency division. In [44], VLC NOMA is studied for two users case, and it is shown that the gain of NOMA over OMA further increases when users with more distinctive channel gains are paired. In [45], NOMA user selection and power allocation are studied, and the power coefficients are derived considering fairness among users in [46]. However, use of NOMA has not been addressed in the VLC literature for a beam steering scenario, and it has not been studied considering the inter-beam interference caused from other steerable beams.

1.4.2 Contributions

In this study, we investigate the optimal beam steering parameters for proportionally fair rate allocation, especially for the case where the number of users is higher than the number of steerable beams. The contributions of this study can be summarized as follows:

i. We define the steering problem for a single beam and multiple users. The optimization parameters are the steering angles, the directivity index of the beam, and the time allocation of each user. We propose a solution for the non-convex problem using a grid search based optimization and majorization-minimization (MM) procedure. Our results show that the proposed beam steering improves the data rates significantly by increasing the users’ signal strength. While the data rate gain can be more than four times with a single user, a higher number of users can also be served by a single beam with a lower data rate gains.
ii. We propose a method for decreasing the search space to reduce the computation time for the mentioned problem.

iii. We evaluate the case where there are multiple steerable beams. As a solution for steering and multiple access in this scenario, we propose a user grouping algorithm which is an extension of the $k$-means clustering algorithm. In particular, we cluster the users and assign a single beam to each cluster. With this method, the time allocation of each user is increased by exploiting the spatial diversity of the users. The simulation results show that ten users can be best served with three independent beams, and the data rate gain due to steering is four times for this case.

iv. We find the optimum transmit power of each beam with respect to a total power constraint. We do it by solving a maximization problem that finds the transmit powers that maximize the sum rate or provide proportionally fair rates. The power optimization provides an additional sum rate gain between 30 - 70 Mbps, where the total gain over no steering scheme can be up to 10 times.

v. Finally, we propose a NOMA scheme by coupling users in the same cluster to further improve the data rates. We find the optimum NOMA power coefficients for a user pair again utilizing the MM procedure. The MM procedure has not been utilized to achieve the VLC NOMA coefficients in the literature. With the coefficients found by our method, the user pair has a 10 Mbps sum rate gain in a proportionally fair allocation, where the weaker user gets a larger portion of the gain.

1.5 Adaptive Kalman Tracking

In the fourth chapter, we study tracking VLC users with adaptive Kalman filter.
1.5.1 Literature Review

There is an increasing demand for indoor positioning and navigation systems due to the expansion of location-based services. The global positioning system (GPS) does not work well in indoor environments because of the strong signal attenuation. Dedicated indoor localization systems can help to provide accurate location information needed for these services at the expense of deployment and operation costs. A number of such indoor positioning systems have been proposed using different wireless technologies including radio frequency identification (RFID), infrared (IR), and wireless local area network (WLAN).

Another alternative method for indoor localization is visible light positioning (VLP), which has lately received attention [47, 48, 49, 7]. The energy-efficient LEDs have been increasingly replacing older light sources such as incandescent or fluorescent bulbs. Besides energy efficiency, LEDs can conveniently offer many new applications such as VLC and positioning. VLC provides an additional wireless communication spectrum which is wide and unregulated, and can complement RF on providing wireless connectivity. The additional advantages of VLC are, the energy efficiency due to using the same signal for illumination and communication, the absence of radio interference, and high spectral reuse. As VLC starts to serve users for communication purposes, an inherent application would be to use it for indoor positioning. As shown in [7], VLP can provide very accurate localization with root-mean-square-error (RMSE) under 5 cm when sufficient number of LEDs are available to the user.

VLP accuracy can suffer when there are not a sufficient number of available visible light APs or LEDs. In order to further enhance the accuracy, tracking the mobile user with Kalman filter (KF) has been studied in the literature [36, 50, 51, 52]. The KF predicts the user location based on previous locations and combines the prediction information with the incoming measurement to cancel out the negative effects of instantaneous bad measurements. It can track the user efficiently in case the localization method does not change over the path. However, in many applications, the estimation methods and/or parameters change
frequently. In order to incorporate such information into the KF, adaptive estimation can be used [53]. The main advantage of the adaptive technique is its weaker reliance on the a priori statistical information. An adaptive filter formulation tackles the problem of imperfect a priori information and provides a significant improvement in performance over the fixed filter through the filter learning process [54]. It has been used for improving the accuracy of GPS [55], and for tracking mobile Wi-Fi users with intermittent link availability [56].

The VLC signal highly depends on LOS link, and in case the LOS links are blocked or misaligned [57], some of the APs may not be available for some time. The varying number of available APs may cause the device to use different localization methods with different expected accuracies over the route. Therefore, the implementation of adaptive KF is crucial for efficient tracking of the VLC users, which have not been addressed in the literature.

1.5.2 Contributions

We have implemented an adaptive filter for VLC user tracking while the number of accessible APs vary. We propose a localization method for the case with a single available AP and adapt existing techniques with adaptive KF framework for the cases with a larger number of available APs. We assume each AP can be blocked with a random probability, and decide certain estimation models based on the available AP configuration. We propose a heuristic method to weight each model based on their expected estimation accuracy and use these for adaptive KF based user tracking. We track the user over a random waypoint (RWP) route with conventional and adaptive Kalman filters. The results show that the adaptive filter provides a significant increase in the accuracy of localization compared to the conventional filter.
1.6 Publications

This work has been published in two peer-reviewed journals [24, 57] and five conference proceedings [23, 58, 59, 60, 36]. Two more publications are under review [61, 62]. The author has also co-authored three additional papers [63, 7, 64] with his peers during this work, and patented an innovation [65]. The studies related to the Chapter 3 can be found in [24, 23, 59, 64], the Chapter 4 can be found in [61, 58], the Chapter 5 is in [61, 60], and the Chapter 6 is in [62, 36, 7, 63].

1.7 Outline and Notations

The remaining chapters of this dissertation are organized as follows. We introduce the Lambertian channel model in Chapter 2, and we study multi-element VLC in Chapter 3. We investigate the effects of random receiver orientation to channel quality in Chapter 4, and we present the beam steering and NOMA work in Chapter 5. We study VLC user tracking in Chapter 6, and finally, we conclude the dissertation with a summary of contributions in Chapter 7.

Notations: $\mathcal{N}(\mu, \sigma^2)$ denotes the real valued Gaussian distribution with the mean $\mu$ and the variance $\sigma^2$, $\mathcal{U}[a, b]$ denotes the continuous uniform distribution over the interval $[a, b]$, and $\mathcal{R}(\sigma)$ denotes the Rayleigh distribution with the scale parameter $\sigma$. The trigonometric functions $\cos^{-1}(\cdot)$ and $\tan^{-1}(\cdot)$ represent the inverse $\cos(\cdot)$ and $\tan(\cdot)$, respectively. $\delta(a, b)$ is the Kronecker delta function taking 1 if $a = b$, and 0 otherwise. Similarly, $\delta(a)$ is the Dirac delta function taking 1 if $a = 0$, and 0 otherwise.
Chapter 2

Wireless Optical Channel Model

In this chapter, we introduce the well-known Lambertian model for VLC channel propagation [66]. We consider a multipath propagation environment based on the well-established model in the literature for LOS and non-line-of-sight (NLOS) scenarios. In order to characterize locations, orientations, and directionality of the LEDs, without loss of generality, the $n$th LED $S_n$ can be defined with three parameters as $S_n = \{r_{tx}^{(n)}, q_{tx}^{(n)}, \gamma\}$, where $r_{tx}^{(n)} \in \mathbb{R}^{3\times1}$ is the location of the $n$th LED, $q_{tx}^{(n)} \in \mathbb{R}^{3\times1}$ is the orientation of $n$th LED, and $\gamma$ is the parameter that specifies the directionality of the light source based on Lambertian pattern. Higher $\gamma$ means more directional LED, as illustrated in Fig. 2.1. Similarly, the $k$th receiver is modeled as $R_k = \{r_{rx}^{(k)}, q_{rx}^{(k)}, A_R, \Theta\}$, where $r_{rx}^{(k)} \in \mathbb{R}^{3\times1}$ is the location of the PD, $q_{rx}^{(k)} \in \mathbb{R}^{3\times1}$ is the orientation of the PD, $A_R$ is the area of PD in m$^2$, and $\Theta$ is the FOV of PD. All orientation vectors are denoted with the letter $q$ and are unit vectors.
2.1 LOS Impulse Response

The LOS component of the channel impulse response between the source $S_n$ and the receiver $R_k$ is modeled by [66]

$$h^{(0)}(t; S_n, R_k) = \frac{\gamma + 1}{2\pi} \cos^\gamma(\phi_{k,n}) \cos(\theta_{k,n}) \frac{A_R}{d_{k,n}^2} \Pi \left( \frac{\theta_{k,n}}{\Theta} \right) \Pi \left( \frac{\phi_{k,n}}{\pi/2} \right) \delta(t - \tau), \quad (2.1)$$

where $\phi_{k,n}$ is the angle between the source orientation vector $q_{tx}^{(n)}$ and the incidence vector, $\theta_{k,n}$ is the angle between the receiver orientation vector $q_{rx}^{(k)}$ and the incidence vector, $d_{k,n}$ is the distance between the source and the receiver, $\tau = d_{k,n}/c$ is the propagation delay, $c$ is
the speed of light, \( \delta(\cdot) \) is the Dirac function, and \( \Pi(\cdot) \) is the rectangle function defined as

\[
\Pi(x) \triangleq \begin{cases} 
1 & \text{for } |x| \leq 1 \\
0 & \text{for } |x| > 1
\end{cases}
\] (2.2)

While \( \Pi(\theta_{k,n}/\Theta) \) in (2.1) implies that the receiver can detect the light only when \( \theta_{k,n} \) is less than \( \Theta \), \( \Pi(\phi_{k,n}/(\pi/2)) \) ensures that the location of the receiver is in the FOV of the source. The distance \( d_{k,n} \) is the length of the incidence vector, which is given as \( (r_{tx}^{(n)} - r_{tx}^{(k)}) \). The terms in (2.1) can be obtained as \( \cos(\phi_{k,n}) = q_{tx}^{(n)}(r_{rx}^{(k)} - r_{tx}^{(n)})/d_{k,n} \), and \( \cos(\theta_{k,n}) = -d_{tx}^{(k)}(r_{rx}^{(k)} - r_{tx}^{(n)})/d_{k,n} \). Some of the parameters are illustrated in Fig. 2.2.

### 2.2 NLOS Impulse Response

The NLOS components of the channel between a LED and a PD is obtained based on *multiple-bounce impulse response* model described in [66]. In this model, light from a source \( S_n \) can reach a receiver \( R_k \) after infinite number of diffuse reflections and the channel impulse response is expressed as

\[
h(t; S_n, R_k) = \sum_{d=0}^{\infty} h^{(d)}(t; S_n, R_k),
\] (2.3)

where \( t \) is the time index. Theoretically, \( h^{(d)}(t; S_n, R_k) \) can be expressed as a recursive function given by

\[
h^{(d)}(t; S_n, R_k) = \int_{S} \rho_{\text{ref}} h^{(0)}(t; S_n, \{r_{\text{ref}}, q_{\text{ref}}, dA, \pi/2\}) \ast h^{(d-1)}(t; \{r_{\text{ref}}, q_{\text{ref}}, 1\}, R_k),
\] (2.4)

where \( \ast \) denotes the convolution operation. In (2.4), the vector \( r_{\text{ref}} \in \mathbb{R}^{3\times1} \) and the vector \( q_{\text{ref}} \in \mathbb{R}^{3\times1} \) correspond to the location and the orientation of the reflector, respectively, \( dA \) is
the infinitesimal area of the reflector, and $\rho_{\text{ref}} \in [0, 1)$ is the reflection coefficient. In addition, the mode and the FOV of the reflector are set to 1 and $\pi/2$, respectively. The real-valued DC channel gain [19, 16] between $k$th user and $n$th LED is then given by

$$h_{kn} = \int_0^\infty h(t; S_n, R_k)dt.$$  \hspace{1cm} (2.5)
Chapter 3

Multi-Element VLC Networks: LED Assignment, Power Control, and Optimum Combining

In this chapter, we present the multi-element VLC network study. We study the problem of assigning multiple LEDs to each user in an environment where the number of LEDs is much larger than the number of users. We group LEDs and assign to the users based on the received signal strength from each LED, for which we propose different solutions to achieve maximum throughput, proportional fairness, and quality of service. Additionally, we investigate the power optimization of LEDs for a given assignment and derive Jacobian and Hessian matrices of the corresponding optimization problem. Moreover, we propose an improved optimal combining method for multi-element receivers with LED grouping at the transmitter.

The chapter is organized as follows. In Section 3.1 we introduce the network and SINR model, in Section 3.2 we study LED assignment to the users and power optimization of the LEDs, in Section 3.3 we study the diversity combining at the receiver side, and in Section 3.4
we present the simulation results.

3.1 Network and SINR Model

We consider a VLC network as shown in Fig. 3.1 where $K$ users are served by $N$ LEDs such that $N \gg K$. When more than a single LED is assigned to a user for a given case, the SINR of the $k$th user can be improved, and expressed as:

$$\text{SINR}(k) = \frac{\left( \sum_{n=1}^{N} r_{kn} h_{kn} p_n \right)^2}{N_0 B + \sum_{j=1}^{K} \left( \sum_{n=1}^{N} r_{jn} h_{kn} p_n \right)^2}, \quad (3.1)$$

where $r$ is the responsivity of the PD, $p_n$ is the standard deviation of the transmitted signal, $N_0$ is the spectral density of the additive white Gaussian noise (AWGN), and $B$ is the communication bandwidth. The connectivity variable is denoted by $\alpha_{kn}$, which is equal to 1 when $n$th LED is assigned to $k$th user, and equal to 0 otherwise:

$$\alpha_{kn} \triangleq \begin{cases} 
1 & \text{if } n\text{th LED serves } k\text{th user} \\
0 & \text{if } n\text{th LED does not serve to } k\text{th user}
\end{cases}. \quad (3.2)$$

To clarify our assumptions for the SINR in (3.1) and provide further insights, two remarks are in order.

Remark 1. Assumptions are made for the SINR in (3.1) to hold. First, it is assumed that the transmission times of all LEDs are synchronized. Moreover, we assume that energy from the signals arriving to user-$k$ from the LEDs serving to that user can be coherently aggregated as in the numerator of (3.1). This may for example be possible considering a guard period among consecutive symbols [67] where the delays from different LEDs can be
absorbed, and inter-symbol-interference (ISI) effects [8, 68] are neglected. For modulation formats such as orthogonal frequency division multiplexing (OFDM) based VLC, delays of the signals arriving from different LEDs to a user can introduce phase changes at different subcarriers [69], which is not explicitly considered here, and their impact on the SINR is left as a future work. Finally, we also assume that the interference signals coming from a group of LEDs serving to the $j$th user are assumed to add up linearly at the desired user (for example, again considering a guard interval to absorb the energy).

Remark 2. As shown in the conceptual illustration of Fig. 3.2, we consider that the communication signal $s_n(t)$ for the $n$th LED is carried on the background DC light intensity (with power $P_{ave}$) that is normally used for illumination. Zero mean communication signal $s_n(t)$ has a power $P_n = \text{var}(s_n(t))$, and therefore a standard deviation of $p_n = \sqrt{P_n} = \text{std}(s_n(t))$. 

Figure 3.1: An example for LED assignment with $K$ users and $N$ LEDs clustered at two VLC access points. All red dashed lines are assigned to user 1, all blue dotted lines are assigned to user $k$, while all solid green lines are assigned to user $K$.

Figure 3.2: Conceptual illustration for the background DC level and communication signal on $n$th LED for a transmitted VLC signal.
The $p_n$ will be referred in the rest of the chapter as the power coefficient for $n$th LED, and \( \mathbf{p} = [p_1, ..., p_N] \) will be referred as the power coefficient vector. We assume that all LEDs provide the same background light intensity ($\sqrt{P_{\text{ave}}}$), which can be adjusted as desired, i.e., dimmed. On the other hand, we consider that $p_n$’s for different LEDs can be adjusted individually, considering $0 \leq p_n \leq p_{\text{max}}$ where $p_{\text{max}}$ is the maximum power attainable based on the LED saturation output and $P_{\text{ave}}$.

The numerator term of (3.1) characterizes the received total signal power from multiple LEDs serving user $k$. These LEDs transmit the same signal, and we sum all components to find the aggregate signal strength. The second term of the denominator represents the interference from LEDs serving simultaneously to $K - 1$ other users. Likewise, the LEDs serving user $j$ ($j \neq k$) transmit the same signal, therefore their sum is considered as one interference component. Finally, using $\alpha_{kn}$’s, we can construct a connectivity matrix $\mathbf{A}$ as follows:

$$
\mathbf{A} = \begin{bmatrix}
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1N} \\
\alpha_{21} & \alpha_{22} & \cdots & \alpha_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{K1} & \alpha_{K2} & \cdots & \alpha_{KN}
\end{bmatrix}, \quad (3.3)
$$

which can also capture the assignment of multiple LEDs to each user. Note that only a single element in a column of $\mathbf{A}$ can be one, and all other elements are zeros, since an LED is assumed to serve at most one user at a time. While techniques such as non-orthogonal multiple access (NOMA) [70] and multi-user MIMO [69] are available where one LED may serve to more than one user, they require higher complexity, and our main motivation in this study is to take advantage of large number of directional LEDs for a simple yet efficient design. On the other hand, sum of each row in (3.3) is an integer greater or equal to one, since multiple LEDs can serve a single user.
One of our goals in this study is to find the matrix $A$ under different optimization criteria to maximize the capacity of users considering different constraints, where the capacity of the $k$th user is given by

$$R_k = B \log_2 \left( 1 + SINR(k) \right).$$

(3.4)

We only study the downlink VLC capacity in this study, and assume that users may have uplink connectivity through an RF technology such as Wi-Fi. Moreover, we also assume that all LEDs can transmit data in synchronization, which can be accomplished by using a central processor that controls all LEDs as well uplink RF reception from users. LEDs can be driven via Power over Ethernet (PoE) directly by the central processor [71].

### 3.2 LED Assignment to Users

In this section, we consider the problem of LED assignment to users for a network as in Fig. 3.1 considering two different scenarios. In the first scenario studied in Section 3.2.1, there are no QoS guarantees and the network provides the highest sum rate or the highest proportionally fair sum rate. LED power control for this first scenario is also investigated in Section 3.2.2. In the second scenario in Section 3.2.3, there are QoS guarantees and all users get data rates proportional to their QoS ratios.
3.2.1 LED Assignment without QoS Guarantees

When there are no QoS guarantees, the problem of maximizing throughput can be expressed as

\[
[A', p'] = \underset{A, p}{\arg \max} \sum_{k=1}^{K} R_k,
\]

subject to
\[
0 \leq p_n \leq p_{\text{max}} \quad \forall n,
\]
\[
\alpha_{kn} \in \{0, 1\},
\]
\[
\sum_{k=1}^{K} \alpha_{kn} \leq 1 \quad \forall n,
\]

where \(A'\) and \(p'\) are the connectivity matrix and the power coefficient vector which maximize the sum capacity over all possible values of \(A\) and \(p\), respectively.

While the solution of (3.5) maximizes the total throughput, it does not consider the fairness among the users, and it will assign most of the LEDs to the users with good SINRs. It has been shown that maximizing the total logarithmic throughput achieves proportional fairness [72]. If we aim at providing proportional fairness among users, the problem can be modified as follows:

\[
[A', p'] = \underset{A, p}{\arg \max} \sum_{k=1}^{K} \log(R_k),
\]

subject to the same constraints as (3.5). Solution of this problem maximizes the throughput while distributing LEDs in a proportionally fair manner with respect to users’ channel conditions. This solution is also a special case of QoS-oriented LED assignment to be studied in Section 3.2.3.

The optimization problems in (3.5) and (3.6) are generally hard to solve since they include both discrete and continuous variables. To simplify the problem, we follow a suboptimal
approach and solve the problem in two steps. We first assume identical power level to all LEDs, which is the maximum power coefficient $p_{\text{max}}$, and solve the LED assignment problem alone. In the second step, we optimize the powers coefficients of LEDs, which is addressed in the next section.

In [9], the problem of assigning users to base stations for 3G networks considering proportional fairness is shown to be an NP-hard problem, which means there is no algorithm that can find the optimum solution in polynomial time [73]. The problem in [9] is not the same as (3.6); however if we switch the role of users and LEDs, we can establish a connection between the two problems. In 3G networks, there are large number of users and fewer number of base stations, and one user is connected to one base station at a given time. In our model, we assume that there are larger number of LEDs than users, and an LED will be assigned to a single user at a time. It can be shown that the problem in [9] can be reduced to (3.6), hence, assigning LEDs to users is also an NP-hard problem.

NP-hard problems can be solved via exhaustive search; however, when the number of elements (LEDs, users) increase, running time for exhaustive search becomes very large. In Section 3.2.4, we present computational complexity of the exhaustive search along with the proposed assignment algorithms, and show that the exhaustive search is computationally infeasible. Even though one may find more efficient solutions than the exhaustive search, it is not possible to find a solution with a running time proportional to a polynomial function of the problem size. Thus, we focus on developing two different heuristic techniques that find close-to-optimal solutions at low computational complexity. We also compare the performance of these heuristics with exhaustive search for small number of users/LEDs in Section VII.
Highest RSS based assignment (HRS)

In this algorithm, we assign an LED to the user that receives the highest RSS from that LED. Considering that all LEDs provide the same transmit power and all users have the same PD responsivity, the RSS is proportional to the channel gain. Therefore, the user who will be served by the \( n \)th LED is given by

\[
\hat{k} = \arg \max_k h_{kn},
\]  
(3.7)

and hence, we have \( \alpha_{\hat{k}n} = 1 \) for the particular LED. For all the other \( k \neq \hat{k} \), we have \( \alpha_{kn} = 0 \). All LEDs are assigned in the same way without explicitly considering the fairness among users. This is the simplest algorithm and gives high sum throughput, and we will refer to this algorithm as the HRS algorithm.

Weighted signal strength based assignment (WSS)

In this algorithm, we scale RSS with the inverse of the total received signal power by that user. In particular, the weighted RSS for \( n \)th LED by \( k \)th user is given by

\[
\Psi_{kn} = \frac{r_p n h_{kn}}{\sum_{m=1}^{N} (r_p n h_{km})^2} \propto \frac{h_{kn}}{\sum_{m=1}^{N} h_{km}^2}.
\]  
(3.8)

The proportionality in (3.8) holds because power coefficients of the LEDs are assumed to be identical at this stage of the problem. Afterwards, each LED is assigned to the user with highest weighted signal strength. In other words, for the \( n \)th LED, the user who will be served by that LED is given by

\[
\hat{k} = \arg \max_k \Psi_{kn},
\]  
(3.9)
and hence, we have $\alpha_{k_\text{e}} = 1$. For all the other $k \neq \hat{k}$ and LED $n$, we have $\alpha_{kn} = 0$. This method is expected to provide a fairer allocation than just assigning each LED to the user which receives highest RSS from that LED. That is because it considers the aggregate signal power a user receives and gives priority to users with low overall RSS.

### 3.2.2 LED Power Control

In this subsection, assuming that LEDs are assigned using an approach as discussed earlier, we consider the problem of power control over the assigned LEDs to all users. In order to solve the optimization problem in either (3.5) or (3.6) to find the optimal power coefficients $p_n$, we formulate the respective Lagrange dual function as follows

$$
\mathcal{L}(p, \lambda) = \sum_{k=1}^{K} \tilde{R}_k + \sum_{n=1}^{N} \lambda_n (p_n - p_{\text{max}}) - \sum_{n=1}^{N} \lambda_{n+N} p_n ,
$$

(3.10)

where $\tilde{R}_k$ is the generic rate function given as

$$
\tilde{R}_k = \begin{cases} 
R_k , & \text{for the optimization in (3.5)} \\
\log(R_k) , & \text{for the optimization in (3.6)}
\end{cases}
$$

(3.11)

and all the Lagrange multipliers ($\lambda_n$’s) are stacked in the vector $\lambda$. The optimal power coefficient can be solved either by i) finding roots of the derivative of the Lagrange function in (3.10) with respect to unknowns $p$ and $\lambda$ via Newton based methods, or ii) directly minimizing the optimization problem in (3.5) or (3.6) by interior-point or trust-region methods [74].

For either approach, we need to derive the Jacobian and the Hessian matrices\(^1\) which will be provided in Corollary 1. Before that, we first give the first and second-order derivatives of the rate functions in Theorem 3.1, which will be necessary for the derivation in Corollary 1.

**Theorem 3.1.** Defining $f(\cdot)$ to be the mapping function for the LED assignment scheme

\(^1\)Jacobian and the Hessian matrices have been achieved in cooperation with Dr. Yavuz Yapici.
such that \( j = f(m) \) is the index for the user served by the \( m \)th LED, the first-order derivative of the rate \( R_k \) with respect to the power coefficient is given as
\[
\frac{\partial R_k}{\partial p_m} = \frac{B}{\ln 2} \frac{2 S_j h_{km}}{T_k} C^m_k, \tag{3.12}
\]
where \( S_{jk} = \sum_{n=1}^{N} \alpha_{jn} h_{kn} p_n \), \( T_k = N_0 B/r + \sum_{j=1}^{K} S^2_{jk} \), and \( C^m_k = \delta(k, l) - \text{SINR}(k)(1 - \delta(k, l)) \) with \( \delta(\cdot, \cdot) \) being the Kronecker delta function. Similarly, the second-order derivative of the rate is given as
\[
\frac{\partial^2 R_k}{\partial p_m \partial p_n} = \frac{B}{\ln 2} \frac{4 S_{jk} h_{km} S_{j'k} h_{kn}}{T_k^2} E_{k}^{m,n}, \tag{3.13}
\]
where \( j' = f(n) \), and,
\[
E_{k}^{m,n} = \begin{cases} 
-1 + \frac{T_k}{2 S^2_{jk}} \delta(j, j'), & k = j \\
\text{SINR}(k) \left( 2 + \text{SINR}(k) - \frac{T_k}{2 S^2_{jk}} \delta(j, j') \right), & k \neq j
\end{cases}
\]

While the derivatives of the generic rate function \( \tilde{R}_k \) in (3.11) is given directly by (3.12) and (3.13) for the optimization problem in (3.5), respective derivatives for the optimization problem in (3.6) are given as
\[
\frac{\partial \tilde{R}_k}{\partial p_m} = \frac{1}{R_k} \frac{\partial R_k}{\partial p_m}, \quad \frac{\partial^2 \tilde{R}_k}{\partial p_m \partial p_n} = \frac{1}{R_k^2} \frac{\partial R_k}{\partial p_m} \frac{\partial R_k}{\partial p_n} + \frac{1}{R_k} \frac{\partial^2 R_k}{\partial p_m \partial p_n}
\]

Proof. See Appendix A. \(\square\)

**Corollary 1.** The Jacobian of the Lagrange dual function in (3.10) is given as
\[
J_m = \frac{\partial \mathcal{L}(p, \lambda)}{\partial p_m} = \sum_{k=1}^{K} \frac{\partial \tilde{R}_k}{\partial p_m} + \lambda_m - \lambda_{m+N},
\]

29
for $1 \leq m \leq N$, and,

$$J_m = \frac{\partial \mathcal{L}(p, \lambda)}{\partial \lambda_{m-N}} = \begin{cases} p_{m-N} - p_{\text{max}}, & N+1 \leq m \leq 2N \\ -p_{m-2N}, & 2N+1 \leq m \leq 3N \end{cases}.$$  

Similarly, the Hessian of (3.10) is given as

$$G_{m,n} = \frac{\partial J_m}{\partial p_n} = \begin{cases} \sum_{k=1}^{K} \frac{\partial^2 \tilde{R}_k}{\partial p_m \partial p_n}, & 1 \leq m \leq N \\ \delta(m-N,n), & N+1 \leq m \leq 2N \\ -\delta(m-2N,n), & 2N+1 \leq m \leq 3N \end{cases},$$

for $1 \leq n \leq N$, and,

$$G_{m,n} = \frac{\partial J_m}{\partial \lambda_{n-N}} = \delta(m,n-N)-\delta(m+N,n-N),$$

for $1 \leq m \leq N, N+1 \leq n \leq 3N$, and 0 otherwise.

**Proof.** The Jacobian and the Hessian are the first and the second-order derivatives of (3.10), which can be readily computed using the derivatives in Theorem 3.1.

### 3.2.3 LED Assignment with QoS Guarantees

To make sure all users are allocated sufficient resources to satisfy their QoS, in this section we also study the LED assignment problem with predetermined QoS ratios among the users. Due to the complexity of the problem and space limitations, we leave the power control for the LED assignment with QoS constraints as a future study. QoS guarantees enable users with higher priorities to receive higher data rates by proportionally allocating the resources to the users based on their QoS requirements. When QoS ratios are provided, the problem
in (3.6) can be modified as follows

\[
A' = \arg \max_{A} \sum_{k=1}^{K} R_k, \quad \text{(3.14)}
\]

subject to \( \alpha_{kn} \in \{0, 1\} \),

\[
\sum_{k=1}^{K} \alpha_{kn} \leq 1 \quad \forall n, \quad \text{(3.15)}
\]

\[
\frac{R_1}{\nu_1} = \frac{R_2}{\nu_2} = \ldots = \frac{R_K}{\nu_K}, \quad \text{(3.17)}
\]

where \( \nu_k \) is the QoS ratio for user \( k \). Assigning all users the same QoS ratios means maximizing the sum rate while making all \( R_k \)'s equal and it corresponds to max-min problem, which is maximizing the rate of the minimum rate user [12]. Therefore, max-min problem is a special case of the problem in (3.17). The problem has similarities with the subchannel block allocation of multiuser OFDM systems [11], assuming frequency selective quasistatic channels where channels do not vary within a block of transmission. However, while assigning different number of LEDs to the users alters the SINR of the users, assigning different number of subchannels to the users changes the assigned bandwidth to the users.

As a solution to problem (3.17), we present a new LED assignment heuristic which we refer as Proportional Rate Algorithm (PRA) as summarized in Algorithm 1. We define \( \delta_{k,n} \triangleq (p_n h_{kn})^2/N_0 \) as the signal-to-noise ratio (SNR) for user \( k \) and LED \( n \), \( T_k \) as the set of LEDs assigned to user \( k \), and \( \mathcal{U} \) as the set of unassigned LEDs. The algorithm initially assigns one LED to each user, which is chosen based on the highest SNR to that user. Then, iteratively, the algorithm lets the user with the least proportional capacity to pick up an LED. The user picks up the LED that provides highest SNR from the available LEDs (\( \mathcal{U} \)), and the algorithm iterates until all LEDs are assigned.

**Remark 3.** In order to do the LED assignment, the central controller needs to learn the
**Algorithm 1** PRA implementation

1. Initialize, $R_k = 0$, $T_k = 0$ for $k = 1, 2, ..., K$ and $\mathcal{U} = \{1, 2, ..., N\}$
2. For $k = 1$ to $K$
   a) find $n$ providing $\delta_{k,n} \geq \delta_{k,j}$ for all $j \in \mathcal{U}$
   b) set $T_k = T_k \cup \{n\}$, $\mathcal{U} = \mathcal{U} - \{n\}$, update $R_k$ using (3.4)
3. While $\mathcal{U} \neq \emptyset$
   a) find $k$ providing $\frac{R_k}{\nu_k} \leq \frac{R_i}{\nu_i}$ for all $i$, $1 \leq i \leq K$
   b) for the found $k$, find $n$ providing $\delta_{k,n} \geq \delta_{k,j}$ for all $j \in \mathcal{U}$
   c) for the found $k$ and $n$, set $T_k = T_k \cup \{n\}$, $\mathcal{U} = \mathcal{U} - \{n\}$ and update $R_k$ using (3.4)

---

Figure 3.3: Comparison of time complexities of different algorithms for $N = 20$ on logarithmic scale.

RSS at each user observed from each LED. This information can be measured at each user and reported to the central controller using the uplink RF channel. For example, similar to LTE, periodically transmitted downlink synchronization/discovery sequences can uniquely characterize the LED identity [75, Ch. 7]. For each identified LED, users can measure the RSS from that LED over some orthogonal pilot symbols, and such measurements can be reported by the user to the central controller processor if they satisfy a triggering condition [75, Ch. 3]. Alternatively, measurements may also be reported periodically, which are then used for updating the LED assignments considering also the measurements from other users. For LED assignment with QoS guarantees, such measurement reports may also include $\nu_k$ for user $k$. Addressing unique challenges for implementing such a protocol for the multi-LED VLC framework specifically considered in this chapter is left as a future study.
3.2.4 Computational Complexity for LED Assignment

In this section, we provide some remarks on the computational complexities for LED assignment techniques with and without QoS guarantees. First, the running time to find overall optimal LED assignment by exhaustive search is $O(K^{N+2} \times N)$. The reason for that is, there are $K^N$ different possibilities to assign LEDs to users, which requires $K^N$ iterations. For each iteration, the data rate needs to be calculated for $K$ users, and each rate calculation requires SINR computation as given in (3.1). The most complex operation in (3.1) is the calculation of interference that takes $N \times K$ iterations, and the total running time of the algorithm becomes proportional to $K^{N+2} \times N$. The running time of different algorithms are compared in Fig. 3.3, which confirms that the exhaustive search is computationally infeasible even for few number of users.

Second, the running time of the HRS algorithm is $O(N \times K)$, because the time to find the maximum RSS value for an LED observed at $K$ different users is proportional to $K$, which is repeated $N$ times by the number of LEDs. Running time for the WSS algorithm is $O(N^2 \times K)$. The required time for the calculation of weighted signal strength as in (3.8) is proportional to the number of LEDs, i.e., $N$ (assuming the squares and the division are constant operations), and it will be performed for $N$ LEDs and $K$ users, which takes time proportional to $N^2 \times K$. Then, choosing the maximum weighted strength from $K$ users and repeating it for $N$ LEDs also takes time proportional to $N \times K$, which may be considered a smaller order function and insignificant.

Finally, the running time of the LED assignment with QoS guarantees in Section 3.2.3 is $O(N^2 \times K)$. There is a while loop and a for loop in the algorithm, and total number of iterations in both loops is $N$. In both loops, the most complicated operation is updating the rate of a user, which requires SINR calculation that takes time proportional to $N \times K$. It is executed once in any of $N$ iterations, so the total running time is proportional to $N^2 \times K$ (See Fig. 3.3).
3.3 Diversity Combining

In Section 3.2 we discussed the LED assignment problem, presented our solutions, demonstrated their time complexities, and provided a power control approach for improved performance. In this section, we propose an advanced receiver combining method that further improves the SINR, by taking advantage of the LED assignment information to the users. When multiple PDs are used at a receiver, the SINR of $k$th user after combining over multiple PDs can be calculated as:

$$SINR(k) = \frac{\left( \sum_{n=1}^{N} \sum_{m=1}^{M} r\alpha_{kn} p_n w_m h_{k(m)n} \right)^2}{\sum_{m=1}^{M} w_m^2 N_0 B + \sum_{j=1, j\neq k}^{K} \left( \sum_{n=1}^{N} \sum_{m=1}^{M} r\alpha_{jn} p_n w_m h_{k(m)n} \right)^2},$$  \hspace{1cm} (3.18)

where $w_m$ is the weight for $m$th PD, $M$ is the number of PDs on a receiver, $h_{k(m)n}$ is the channel attenuation between $n$th LED and $m$th PD of $k$th user, and the second term in the denominator represents the sum of all interference signal powers at all PDs from all LEDs excluding the LED group which serves the $k$th user. Choosing of $w_m$ for combining signals at the receiver can be achieved using the MRC or the OC approaches, as will be discussed next.
3.3.1 Maximum Ratio Combining

The MRC uses the signals received from different PDs with a proportional weight to the SINR observed at each PD. Weight of \( m \)th PD is calculated as

\[
w_m = \frac{\left( \sum_{n=1}^{N} r\alpha_{kn}p_n h_{k(m)n} \right)^2}{N_0B + \sum_{j=1 \atop j \neq k}^{K} \left( \sum_{n=1}^{N} r\alpha_{jn}p_n h_{k(m)n} \right)^2}.
\]

The numerator of (3.19) is for the received signal power at \( m \)th PD, and the denominator is for the sum of noise and interference. The MRC is a heuristic to use all data with a proportional ratio to maximize the combined SINR, however, it assumes that the signals received at different PDs are uncorrelated. While this approach is successful at suppressing the white noise, it yields suboptimal performance for correlated noise or interference.

3.3.2 OC with Unknown Grouping Information

The OC provides higher SINR performance than the MRC by suppressing correlated interference. In order to calculate the weights of the \( k \)th user, denote

\[
H_{k,(m)} = r \sum_{n=1}^{N} \alpha_{kn}p_n h_{k(m)n},
\]

(3.20)

to be the sum of the received desired signals at \( k \)th user’s \( m \)th PD. We can build a vector

\[
v_{kk} = [H_{k,(1)}, H_{k,(2)}, \ldots, H_{k,(M)}]^T
\]

(3.21)
which includes the received desired signals through different PDs. The weighting vector for
OC can then be calculated as
\[ w = R^{-1} v_{kk}, \]  \hspace{1cm} (3.22)
where \( R \) is the interference-plus-noise correlation matrix of the received signal, explicitly
given by
\[ R = N_0 B I + \sum_{n=1}^{N} (1 - \alpha_{kn}) E[h_{kn} h_{kn}^T], \]  \hspace{1cm} (3.23)
where \( h_{kn} = r p_n [h_{k(1)n}, h_{k(2)n}, ..., h_{k(M)n}]^T \), which is the signal vector that the \( k \)th user
captures from the \( n \)th LED through different PDs. In (3.23), while the first term of the
summation is the noise correlation matrix, the second term is the interference correlation
matrix. The expression \((1 - \alpha_{kn})\) ensures that only interference signals will be added, because
\( \alpha_{kn} \) is the assignment flag and if \( n \)th LED is assigned to \( k \)th user, \((1 - \alpha_{kn})\) is equal to zero.

### 3.3.3 OC with Known Grouping Information

If we calculate the interference without considering assignment information, we will ignore
the correlation between LEDs that are transmitting to the same interfering user. When the
grouping information between interfering LEDs is known, rather than using (3.23), \( R \) for
the \( k \)th user can be calculated as
\[ R = N_0 B I + \sum_{j=1}^{K} \sum_{j \neq k} E[v_{jk} v_{jk}^T], \]  \hspace{1cm} (3.24)
where \( I \) is the identity matrix,
\[ v_{jk} = [H_{j,k(1)}, H_{j,k(2)}, ..., H_{j,k(M)}] \]  \hspace{1cm} (3.25)
is the received interference signal from the LEDs which are transmitting to \( j \)th user, and

\[
H_{j,k(m)} = r \sum_{n=1}^{N} \alpha_{jn} p_n h_{k(m)n},
\]

(3.26)

is the received interference from \( j \)th user at \( k \)th user’s \( m \)th PD. We will refer this method as grouping based OC (GB-OC).

Fig. 3.4 helps to further explain the differences between the calculation of the OC and the GB-OC. Assuming LEDs are numbered with respect to assignment to the users, green LEDs are showing the LEDs assigned to \( k \)th user, and white LEDs shows the LEDs assigned to other users which are interference sources for \( k \)th user. The vector \( \mathbf{h}_{kn} \) includes RSS values transmitted from the \( n \)th LED and received at different PDs of the \( k \)th user. As shown at the bottom of Fig. 3.4, \( \mathbf{v}_{jk} \) is the sum of \( \mathbf{h}_{kn}s \) transmitted from LEDs that are assigned to \( j \)th user. While \( \mathbf{v}_{kk} \) includes the desired signals received at user \( k \), \( \mathbf{v}_{jk}s \) for \( j \neq k \) includes interference. To calculate interference-plus-noise correlation matrix \( \mathbf{R} \), classical OC sums \( \mathbf{h}_{ks}\mathbf{h}_{ks}^T \)s for interference LEDs, while GB-OC sums \( \mathbf{v}_{jk}\mathbf{v}_{jk}^T \)s for \( j \neq k \). Classical OC correlation matrix values are lower because cross elements from multiplication of the sum are missing. Therefore, classical OC weight calculation does not include the interference correlation caused by the simultaneous transmission from multiple sources.
Remark 4. In order for a receiver to implement GB-OC, the LED assignments to all individual users need to be known by each user, which is characterized by the sparse matrix $A$ in (3.3). Since all users need the same information, LED assignment information can be simultaneously broadcast from all the LEDs. The broadcast should be done right after a new LED assignment is computed, and before users start being served with the new assignment. In the simplest approach, the assignment matrix $A$ can be broadcast from the LEDs to users, which corresponds to an overhead of $N \times K$ bits. Due to the sparse nature of the matrix $A$ (each LED serving only one user), each column of $A$ can be replaced by a single bit vector of size $\lceil \log_2(K + 1) \rceil$ representing the identity of the user that is served by a particular LED, and a bit sequence of all zeros if the LED does not serve any user. This second approach therefore requires an overhead on the order of $N \times \log_2 K$. LED assignment overhead can be further reduced by grouping LEDs and assigning them to users in groups, which is reminiscent to group-based resource block assignment in LTE networks [75, Table 9.4].

Figure 3.5: The 12m by 12m room and the transmitter locations for simulation evaluations.
Table 3.1: Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LED directivity index, $\gamma$</td>
<td>7.0459</td>
</tr>
<tr>
<td>Maximum power coefficient of an LED, $p_{\text{max}}$</td>
<td>1 W</td>
</tr>
<tr>
<td>Responsivity, $r$</td>
<td>0.5 A/W</td>
</tr>
<tr>
<td>Modulation bandwidth, $B$</td>
<td>20 MHz</td>
</tr>
<tr>
<td>AWGN spectral density, $N_0$</td>
<td>$2.5 \times 10^{-20}$ A$^2$/Hz</td>
</tr>
<tr>
<td>Effective surface area (single PD Rx), $A_R$</td>
<td>40 mm$^2$</td>
</tr>
<tr>
<td>Effective surface area (7 PD Rx), $A_R$</td>
<td>10 mm$^2$</td>
</tr>
<tr>
<td>Reflection coefficient (walls)</td>
<td>0.8</td>
</tr>
<tr>
<td>Reflection coefficient (floor, ceiling)</td>
<td>0.3</td>
</tr>
</tbody>
</table>

3.4 Simulation Results

For simulations, a square room with dimensions 12 m × 12 m × 4 m is considered as in Fig. 3.5. Four multi-element transmitters are considered, each having seven LEDs. In a multi-element transmitter, while one LED is directed downwards, there is a second layer of LEDs having a 45° divergence angle with the center LED. The transmitters are located at the ceiling facing downwards; the receivers are assumed to be at 0.85 m height and facing upwards. Other simulation parameters are provided in Table 3.1. Up to four multipath reflection orders, i.e., $d = 4$, are considered which are generated using the method discussed in Section II.C. Users are placed at random locations in the room, and the sum rates or fairness indices are calculated over 1000 realizations. For fairness criteria, we present Jain’s fairness index (JFI) which takes values between $1/K$ and 1 for $K$ users and the index is given by

$$JFI = \frac{\left( \sum_{k=1}^{K} R_k \right)^2}{K \sum_{k=1}^{K} R_k^2}, \quad (3.27)$$

where a larger index means a fairer distribution.
3.4.1 LED Assignment without QoS Constraints (Small Room)

In Fig. 3.6, the sum rate and fairness performance of the proposed assignment schemes and the optimal assignment schemes, which are found by exhaustive search, are shown. For the simulations in Fig. 3.6, a room with half the size of the room in Fig. 3.5 is considered with dimensions $12 \times 6$ m. Two multi-element transmitters are used instead of four, and up to four users are simulated. The reason for that is exhaustive search takes exponentially longer time for additional number of LEDs and users, which makes it hard to simulate the scenarios with large number of elements.

In Fig. 3.6(a), sum rates are given for the maximum sum rate assignment that solves (3.5) by exhaustive search, the maximum log rate that solves (3.6) by exhaustive search, the proposed HRS and WSS assignments, and time-division-multiple-access (TDMA). In Fig. 3.6(b), the sum of log of the rate of the users for the same assignment schemes are given. In Fig. 3.6(c) the fairness indices are shown. In TDMA case, all LEDs send the same signal and serve one user at a time. All users are served by time division among users with equal length of time slots.

As expected, maximum sum rate gives the highest rate in Fig. 3.6(a) and maximum log rate gives highest logarithmic rate in Fig. 3.6(b). In general, WSS is comparable to maximum logarithmic throughput assignment since they both prioritize proportional fairness criteria, and HRS is comparable to maximum sum rate assignment since they both prioritize maximization of sum rate. HRS and WSS algorithms provide lower sum rate in comparison to the corresponding optimal assignment schemes; however, they both outperform TDMA in terms of data rate. Since the proposed assignment scheme makes use of space diversity, in a larger room or with higher number of users the rate gain over TDMA is expected to be higher (see Fig. 3.7). In terms of logarithmic rate, which is a parameter for both higher rate and fair distribution, WSS gives close results to the maximum log rate and TDMA, and it is followed by HRS with a larger margin. Maximum sum rate assignment fails the logarithmic
rate criteria and provides the lowest results. In terms of the JFI criteria, WSS provides close results to the max log rate and HRS provides close results to the max sum rate.

3.4.2 LED Assignment without QoS Constraints (Large Room)

In Fig. 3.7, the room setup in Fig. 3.5 is used, where exhaustive simulation results are excluded due to extensively long (on the order of months) simulation duration with today’s
Figure 3.7: Sum rate, sum of logarithmic rate, and JFI for different assignment schemes and different number of users (12 m × 12 m room size with four multi-element transmitters as in Fig 3.5).

high-performance computers, even with as low as four users. In Fig. 3.7(a), the sum rate for HRS and WSS based algorithms are shown. The results with TDMA are also shown for reference. While HRS shows the highest sum rate performance, WSS has slightly lower rate. As HRS assigns each LED to the user with the highest received signal, and does not consider any fairness criterion, it is expected to yield the highest sum throughput. Sum rates with optimized power coefficients are also shown with dotted lines. All power optimization
simulations in this section maximize the sum of logarithmic throughput as in (3.6), using interior-point method. The results show that optimization provides significant gain in the sum rate of both algorithms. Both algorithms constantly improve the sum rate when the number of users increase, by making use of spatial diversity.

In Fig. 3.7(b), the sum of logarithmic rate performances are shown. WSS shows a similar performance as TDMA by means of logarithmic sum, while HRS yields lower results, especially for higher number of users. Power optimization slightly increases logarithmic sum rate of both algorithms. In Fig. 3.7(c), the fairness index for the same algorithms is given. The WSS provides significantly higher fairness index in comparison to the HRS, since it considers the whole received signal power by a user and provides a fairer LED assignment. Power optimization increases the fairness index of the HRS significantly, since it maximizes the logarithmic sum rate. Contribution of power optimization to the fairness index of WSS is not that significant. It even causes fairness index to decrease for low number of users. The reason is that, WSS already provides a high fairness index before optimization, and improving logarithmic sum slightly may not improve the fairness index of WSS in all cases. The advantage of power optimization on WSS is mostly visible on the sum rate.

### 3.4.3 Power Control and Illumination

Fig. 3.8(a) shows the histogram of the power coefficients of the simulations in Fig. 3.7 for WSS (with Power Optimization) case. The histograms show that the optimum power coefficient tends to be either 1 or 0, depending on if the LED provides more signal power or more interference (to other users), respectively. A similar power control problem is studied in [76] under multiple interfering RF links, where they identify scenarios which binary power control is optimal. It might as well be the case in VLC scenarios with high number of LEDs, which we leave as a future study to investigate. In Fig. 3.8(a), when the number of users increases, lower number of LEDs takes 0 power, which is probably due to decreasing LEDs
per user. Since the optimization maximizes sum of logarithmic throughput, it makes sure every user is served by LEDs that are assigned a power coefficient larger than zero.

(a) Histogram of power coefficients after optimization. The x-axes show the power values, and y-axes show total number of users taking those values ($p_{\text{max}} = 1$ W).

(b) Power coefficients of LEDs for different $p_{\text{max}}$ values in a single optimization realization. Different users are shown with different markers.

Figure 3.8: Power coefficients after optimization.

In case of dimming, $P_{\text{ave}}$ needs to be decreased and it may also limit $p_{\text{max}}$. To evaluate effects of dimming on power optimization, Fig. 3.8(b) shows the power levels of all LEDs for a single optimization realization, for different $p_{\text{max}}$ values, that are 1 W, 0.6 W, and 0.2 W. The WSS algorithm is used to assign LEDs to six users, and different users are shown with different markers. While some users are assigned a single LED, some others are assigned as much as 10 LEDs, depending on the distribution of users in the room. For $p_{\text{max}} = 1$ W, many LEDs are assigned a power coefficient of zero, which can also be seen from the histograms in Fig. 3.8(a). When $p_{\text{max}}$ is decreased to 0.2 W, no LED is assigned zero power, and most LEDs are assigned the maximum power value of $p_{\text{max}}$. This shows that for lower values of $p_{\text{max}}$, noise becomes the dominant factor rather than interference, which makes $p_{\text{max}}$ optimal for most LEDs.
3.4.4 LED Assignment with QoS Constraints

In Fig. 3.9, the sum rate with the PRA algorithm for two different QoS ratios (defined in (3.17)) are compared with TDMA. For the results with square marker, half of the users’ QoS ratios are 5, and the remaining ones have a ratio of 1. With TDMA, users can be provided proportional rates by assigning them proportionate time. However, after averaging over large number of realizations, the sum rate is the same for any ratio allocation among users, since users are randomly located at each realization. The PRA provides higher sum rate with respect to TDMA for any number of users and any QoS ratio distribution. As the number of users increases, the capacity gain also increases with respect to TDMA, which is due to increased spatial diversity between LEDs and users.

With PRA algorithm, QoS ratio difference between users affects sum throughput negatively when the number of users is less than four. In this case, the distribution with equal ratios gives higher sum rate. However, with higher number of users, different QoS ratios do not affect the sum rate negatively. It even provides a slight gain over equal ratio distribution among users. One of the reasons for this behavior is that, since the SINR is proportional
to the square of the received signal strength at the receiver, assigning more LEDs on some of the users may increase the rate of those users more than the amount of decrease at the other users.

![CDF of SINR for four user case.](image1)

![50% and 10% SINRs for varying number of users.](image2)

Figure 3.10: SINR for different combining techniques.

### 3.4.5 Diversity Combining

In Fig. 3.10(a), the CDF of SINR for different diversity combining schemes are shown for the deployment scenario in Fig. 3.5. Four receivers with each having 7 PDs are randomly placed in the room and WSS is used to assign LEDs. CDF data is obtained by averaging over large number of realizations. As expected, OC outperforms MRC: While the gain is around 2 dB for low SINR region, it is more than 10 dB for some high SINR realizations. The OC provides higher gain over MRC for higher SINR regions which causes a stepwise CDF. When the assignment information of LEDs is known, SINR can be further improved by including this information in the calculation of OC weights. The GB-OC, which is OC with known assignment information, outperforms classical version, and a 2 dB to 3 dB gain is observed by using the assignment information.
In order to give further insights, we plot the average SINR of MRC by location in Fig. 3.11(a), the SINR difference between OC and MRC at different room locations in Fig. 3.11(b), and the SINR difference between GB-OC and OC in Fig. 3.11(c). To obtain SINR values, similar method as in Fig. 3.10(a) is used. Four users are placed randomly and LEDs are assigned by WSS. The average of 40,000 iterations is considered to decide SINR by location. The total received signal from all LEDs by the receiver is also shown in Fig. 3.11(d). Although the received signal is relatively uniform in the room, the high SINR region of MRC is concen-
trated under the transmitters. The reason for that is since the user beneath a transmitter is close to many LEDs, it has higher chance to be assigned more LEDs which provides higher signal strength.

Fig. 3.11(b) shows that OC provides gain over MRC especially in two different areas. The first one is beneath the transmitters, where users already have high SINR. This behavior results in stepwise CDF by providing additional gain to the high SINR users. In these locations, the user is probably assigned all the LEDs of the transmitter above it, and receiving interference signals only from other transmitters, which are at far distance. In this case, since the direction of desired signal and interference is separated, interference correlation is higher which causes OC to perform better. The second area that OC provide higher gain is the location near the walls. This is due to the interference signals caused by wall reflections. The OC suppresses correlated interference reflected from walls and provides higher gain. The gain is doubled at the corners where the reflection signals increase. Fig. 3.11(c) shows that while GB-OC provides a relatively uniform gain over OC, the gain follows a similar pattern. That is, the gain is higher beneath the LEDs and near the walls.

Fig. 3.11(d) shows the total RSS by location, assuming $p_{\text{max}}$ power coefficient to all LEDs. This is equivalent to the RSS in TDMA case which all LEDs serve a single user at a time. The total RSS values are also proportional to the illumination level in the room which is perceived by the receiver. Note that the receiver has a FOV constraint and can only receive the light within its FOV. Both the illumination and the RSS have a close to a uniform distribution within the room for the given simulation setup.

Fig. 3.10(b) shows the 50% and 10% of SINR CDF for different number of users, again while utilizing the WSS-based LED assignment. 50% stands for median SINR and 10% is for the lowest 10% of all SINR values. At least a few dB gain is provided by GB-OC for any number of users in both cases. The gain decreases with increasing number of users, especially for 10% SINR. The reason for that is when there are more users in the room, LED groups are
smaller. Therefore, coordination caused by grouping of LEDs decreases. We can observe that when the number of users becomes closer to the number of LEDs, GB-OC converges to classical OC.
Chapter 4

Effects of Random Receiver Orientation on VLC Channel

In this chapter, we present the random receiver orientation study. We study the effects of random receiver orientation and mobility on the channel quality of VLC. Based on the statistics of vertical receiver orientation and user mobility, we develop a unified analytical framework to characterize the statistical distribution of VLC downlink channels, which is then utilized to obtain the outage probability and the BER. The analysis is generalized for arbitrary distributions of receiver orientation/location for a single transmitter and extended to multiple transmitter case for certain scenarios.

The chapter is organized as follows. We derive the on-off-keying (OOK) BER of the VLC in Section 4.1, we analyze the channel distribution for single LED in Section 4.2, we present the two LED case in Section 4.3, we expand the results to more than two LEDs case in Section 4.4, and we provide the numerical results in Section 4.5.
4.1 BER of VLC

The observation model for the point-to-point transmission scenario at \( \kappa \)th discrete time instant is given as

\[
y_\kappa = h a_\kappa + v_\kappa,
\]

where \( y_\kappa \) is the received signal, \( a_\kappa \) is the transmitted symbol chosen from a modulation alphabet \( \mathcal{A} \) with the average transmit energy \( E_s \), and \( v_\kappa \) is the white Gaussian noise with zero mean and variance \( N_0/2 \). Without loss of generality, assuming the binary OOK modulation, the average probability of bit error is given as

\[
P_e = \int_{0}^{\infty} Q \left( \sqrt{\frac{E_s}{N_0}} \varphi \right) f_{h^2}(\varphi) \, d\varphi,
\]

where \( f_{h^2}(\varphi) \) is the pdf of the square-channel expression denoted as \( h^2 \), and \( Q(\cdot) \) is the Q-function [77]. In order to calculate the BER in (4.2), the pdf of \( h^2 \) needs to be known.

In the sequel, we will characterize the point-to-point VLC channel in (2.1) when the user orientation fluctuates randomly around the vertical axis, which is represented by the random incidence angle \( \theta \). Assuming a stochastic distribution for this fluctuation in the vertical direction, we will derive the distribution of the square-channel \( h^2 \), and evaluate the impact of this random behavior on the BER statistics via (4.2). To this end, we will consider single and multiple LED scenarios with the deterministic and random user deployment cases, separately, in the subsequent sections.
4.2 Square-Channel Distribution for Single LED

In this section, we derive the channel statistics for a single LED scenario. We consider only LOS component of the channel since other components are insignificant. Then, we can rearrange (2.1) as follows:

$$h = \frac{(\gamma + 1) A_R \ell^m}{2\pi} (\ell^2 + d^2)^{-\frac{\gamma + 2}{2}} \cos(\theta) \Pi \left( \frac{\theta}{\Theta} \right),$$

(4.3)

where $\ell$ and $d$ are the vertical and horizontal distances between the LED and the user, respectively, which are also shown in Fig. 4.1. We removed the user and LED indexes from parameters since we are dealing with a single LED and a single user. We also removed the components $\Pi \left( \frac{\phi}{\pi/2} \right)$ and $\delta(t - \tau)$ since we assume the $\phi$ is always smaller than $\pi/2$, which is the case for an LED installed on ceiling, and we are not interested in the time delay in this chapter. In (4.3) we employ the geometrical relations $r = \sqrt{d^2 + \ell^2}$ and $\cos(\phi) = \ell / \sqrt{d^2 + \ell^2}$, which can also be seen on Fig. 4.1.

We assume two different scenarios regarding the FOV effect of the receiver while analyzing the statistical behavior of the square-channel under receiver orientation fluctuations. In the first scenario, we assume that the FOV of the receiver is wide enough (characterized by a large $\Theta$), therefore the LED is always within the FOV. This is a simplistic scenario that
enables the derivation of square-channel statistics without any nonlinear effects arising from FOV restrictions [78], and referred to as “wide FOV” scenario in this chapter. The second scenario assumes a more general setting by assuming “narrow FOV”, where the LED might be either inside or outside the FOV depending on the vertical user orientation and specifics of the geometry in Fig. 4.1. This second scenario considers all possible geometrical interactions between the LED and user, but complicates the derivation of the desired channel statistics.

4.2.1 Deterministic User Location and Wide FOV

We first assume wide FOV where the incidence angle $\theta$ in (4.3) is always smaller than $\Theta$, which implies $\Pi(\theta/\Theta) = 1$. In addition, the user location is assumed to be chosen in a deterministic fashion such that the horizontal distance $d$ is a nonrandom variable. Then, the random part of the channel gain in (4.3) is $h_\theta = \cos(\theta)$. The distribution of the square-channel $h^2$ can be derived by considering the cdf of $h^2 = \cos^2(\theta)$ given as

$$F_{h^2}(x) = \Pr \{ \cos^2(\theta) < x \} = \Pr \left\{ \theta > \frac{1}{2} \cos^{-1}(2x - 1) \right\}.$$  (4.4)

Note that, the probability in (4.4) is always 1 for $x \geq 1$, and 0 for $x < 0$, and we therefore limit $x$ to the interval $[0, 1]$ while analyzing (4.4). Defining $F_\theta(.)$ to be the cdf of the random incidence angle $\theta$, the cdf of the $h^2_\theta$ is obtained by rearranging (4.4) as follows

$$F_{h^2_\theta}(x) = 1 - F_\theta \left( \frac{1}{2} \cos^{-1}(2x - 1) \right).$$  (4.5)

The corresponding pdf can be computed by taking derivative of (4.5) with respect to $x$, and is given as

$$f_{h^2_\theta}(x) = \frac{c_\theta}{\sqrt{4x(1-x)}} f_\theta \left( \frac{1}{2} \cos^{-1}(2x - 1) \right).$$  (4.6)
for $0 \leq x \leq 1$, and 0 otherwise. In (4.6), $c_\theta$ is the normalization constant, and $f_\theta(.)$ is the pdf of the random angle $\theta$. Denoting the deterministic part of (4.3) as $h_c$ such that $h = h_c h_\theta$, the cdf and pdf of the square-channel is readily given as

$$F_{h^2}(x) = F_{h^2_c} \left( \frac{x}{h^2_c} \right), \quad f_{h^2}(x) = \frac{1}{h^2_c} f_{h^2_c} \left( \frac{x}{h^2_c} \right),$$

(4.7)

which are defined in the most general form such that any distribution for the random angle $\theta$ can be used directly via (4.5)-(4.6).

### 4.2.2 Deterministic User Location with Narrow FOV

When we assume a narrow FOV, the point-to-point LOS link in Fig. 4.1 can be outside the receiver FOV because of the random orientation of the user around the vertical axis. In this case the square-channel $h^2$ can be derived by considering the associated random part $h^2_\theta = \cos^2(\theta) \Pi(\theta/\Theta)$, with the cdf given as

$$F_{h^2_\theta}(x) = \Pr \{ \cos^2(\theta) \Pi(\theta/\Theta) < x \}$$

$$= \Pr \{ \cos^2(\theta) < x, 0 \leq \theta \leq \Theta \} + \Pr \{ x > 0, \Theta < \theta \},$$

(4.8)

where the first and second probabilities in (4.8) represent the cases where the LOS link is within the FOV and outside the FOV, respectively.

For ease of representation, we define the function $\triangle_\theta(a, b)$, which represents the probability of the random variable $\theta$ being in an interval $(a, b]$ with arbitrary real-valued variables $a, b \in \mathbb{R}$, and is given as follows

$$\triangle_\theta(a, b) = \Pr \{ a < \theta \leq b \} = \begin{cases} F_\theta(b) - F_\theta(a) & \text{for } a \leq b \\ 0 & \text{for } a > b \end{cases}.$$
Then, following the strategy in obtaining (4.4), the cdf in (4.8) becomes

$$F_{h_2^2}(x) = \Pr \left\{ \frac{1}{2} \cos^{-1}(2x-1) < \theta \leq \Theta \right\} + \Pr \{ \theta > \Theta \} \quad (4.10)$$

$$= \Delta_\theta \left( \frac{1}{2} \cos^{-1}(2x-1), \Theta \right) + 1 - F_\theta(\Theta), \quad (4.11)$$

for $0 \leq x \leq 1$, equal to 0 for $x < 0$, and 1 for $x > 1$. Note that, when the FOV takes a large value, the random angle $\theta$ is always less than $\Theta$ implying $\Pr \{ \theta > \Theta \}$ = 0 and $F_\theta(\Theta)$ = 1, and (4.10) readily yields the cdf expression in (4.5) of the wide FOV scenario, as expected.

We observe that the first term involving the function $\Delta_\theta(\cdot)$ is zero over the interval $0 \leq x < \cos^2(\Theta)$ by the definition in (4.9), and the cdf in (4.11) becomes equal to $1 - F_\theta(\Theta)$ over this interval. The cdf is 0 for $x < 0$, which means the function in (4.11) is discontinuous at $x = 0$. With this observation, we can give the corresponding pdf as follows

$$f_{h_2^2}(x) = c_\theta \frac{\partial}{\partial x} \Delta_\theta \left( \frac{1}{2} \cos^{-1}(2x-1), \Theta \right) + (1 - F_\theta(\Theta)) \delta(x), \quad (4.12)$$

for $0 \leq x \leq 1$, and 0 otherwise. In (4.12), $c_\theta$ is the normalization constant\(^1\), and the Dirac delta function $\delta(x)$ appears as a result of the discontinuity of the cdf in (4.11) at $x = 0$. The partial derivative in (4.12) is given as

$$\frac{\partial}{\partial x} \Delta_\theta \left( \frac{1}{2} \cos^{-1}(2x-1), \Theta \right) = \frac{1}{\sqrt{4x(1-x)}} f_\theta \left( \frac{1}{2} \cos^{-1}(2x-1) \right), \quad (4.13)$$

for $\cos^2(\Theta) \leq x < 1$, and 0 otherwise. Finally, the desired cdf and pdf of the square-channel $h^2$ can be obtained readily by (4.7).

Note that, the impact of the narrow FOV on the square-channel pdf can be interpreted by comparing (4.6) and (4.12) with the help of (4.13). We observe that limiting the receiver FOV with a narrow angle $\Theta$ introduces a Dirac delta weighted by $1 - F_\theta(\Theta)$ and discards

\(^1\)Since the pdf has a Dirac delta function term of size $1 - F_\theta(\Theta)$, the normalization constant normalizes the integral sum of the other term to $F_\theta(\Theta)$ so that overall integral sum is equal to one.
the pdf portion within the interval of \(0 \leq x < \cos^2(\Theta)\), which in turn causes the magnitude of the pdf to be weighted over the interval \(\cos^2(\Theta) \leq x < 1\). To illustrate this issue, we have provided square-channel pdfs for different receiver orientation distributions in Fig. 4.2 for two different FOV angles. In the top figures, pdfs for \(\Theta = 60^\circ\) are given. The delta function does not occur for uniform distribution, and it is arbitrarily small for normal distribution. It is because the FOV is large, and probability of leaving LED out of FOV (\(\Pr\{\theta > \Theta\}\)) is zero or arbitrarily small. In the bottom figures, pdfs for \(\Theta = 35^\circ\) are given. We observe that pdf shapes are clipped from left side (from \(x = \cos^2(\Theta)\)), and the area under the clipped shape is accumulated at \(x = 0\) which appears as delta function. The smaller values of \(x\) corresponds to the larger values of \(\theta\), and when \(\theta\) is larger than \(\Theta\) the channel gain is equal to zero.

### 4.2.3 Random User Location with Wide and Narrow FOVs

In this section, we consider a mobile user over the \(xy\)-plane as in Fig. 4.1, and the associated mobility is captured by choosing the horizontal distance \(d\) at random, which corresponds
to the random user deployment strategy. This random effect in channel due to the user mobility is represented by \( h_d = (\ell^2 + d^2)^{-(\gamma+2)/2} \), and the channel in (4.3) accordingly becomes \( h = h_c h_d h_\theta \) where \( h_\theta = \cos(\theta) \Pi (\theta/\Theta) \) represents the random part due to the vertical orientation including the effect of the narrow FOV, and \( h_c \) is the remaining deterministic part, as before.

The distribution of the square-channel can now be computed by exploiting the independence of \( h_\theta \) and \( h_d \), and by employing the property on the distribution of the product of independent random variables as follows [79]

\[
F_{h^2}(x) = 1 - F_\theta(\Theta) + \frac{1}{h_c^2} \int_{\mathcal{R}_y} \frac{1}{y} f_{h_d^2}(y) F_{h_\theta^2} \left( \frac{x}{h_c^2 y} \right) \, dy, \tag{4.14}
\]

\[
f_{h^2}(x) = \begin{cases} \frac{c_h}{h_c^2} \int_{\mathcal{R}_y} \frac{1}{y} f_{h_d^2}(y) f_{h_\theta^2} \left( \frac{x}{h_c^2 y} \right) \, dy & \text{for } 0 < x \leq 1, \\ 1 - F_\theta(\Theta) & \text{for } x = 0 \end{cases}, \tag{4.15}
\]

where \( \mathcal{R}_y \) is the set of \( y \) values for which the function being integrated takes nonzero value, and \( c_h \) is the normalization constant. Defining \( F_d(.) \) to be the cdf of the random distance \( d \), the desired pdf of \( h_d^2 \) can be found by considering the cdf given as follows

\[
F_{h_d^2}(y) = \Pr \left\{ (d^2 + \ell^2)^{-(\gamma+2)} < y \right\} = \Pr \left\{ d^2 > y^{-\frac{1}{\gamma+2}} - \ell^2 \right\} = 1 - F_d \left( \left[ y^{-\frac{1}{\gamma+2}} - \ell^2 \right]^{1/2} \right), \tag{4.16}
\]

for \( 0 \leq y \leq \ell^{-2(\gamma+2)} \), and 1 for \( y > \ell^{-2(\gamma+2)} \). Taking the derivative of (4.16), the desired pdf of \( h_d^2 \) is then found to be

\[
f_{h_d^2}(y) = c_d y^{-\frac{\gamma+3}{\gamma+2}} \left[ y^{-\frac{1}{\gamma+2}} - \ell^2 \right]^{-\frac{1}{2}} f_d \left( \left[ y^{-\frac{1}{\gamma+2}} - \ell^2 \right]^{1/2} \right), \tag{4.17}
\]
for $0 \leq y \leq \ell - 2(\gamma + 2)$, and 0 otherwise. In (4.17), $c_d$ is the normalization constant, and $f_d(\cdot)$ is the pdf of the random distance $d$. Incorporating (4.11) and (4.17) into (4.14), we can obtain the cdf of the square-channel $h^2$, which can be applied to both the wide and the narrow FOV settings. Likewise, incorporating (4.12) and (4.17) into (4.15), we can obtain the pdf of the square-channel $h^2$. As before, the distribution in (4.15) is defined in general form such that any distribution for the random incidence angle $\theta$ and the horizontal distance $d$ can be incorporated directly.

4.3 Square-Channel Distribution for Two LEDs

In this section, we discuss the distribution of the square-channel for a specific two LED transmitters scenario, where the user location varies between two points which are beneath the LEDs, and user orientation is random around the vertical axis. While this scenario does not cover all possible two LED geometries, it might be useful for evaluating the channel performance when a person holding a VLC receiver device is walking in a corridor equipped with VLC transmitters.
4.3.1 Two LEDs Scenario

In this scenario two LEDs are available to serve a single user as in Fig. 4.3, and the user is assigned to the LED with the strongest signal. The resulting instantaneous effective channel is given as

\[ h_{\text{eff}} = \max \{ h_1^2, h_2^2 \}, \]  

(4.18)

where \( h_1 \) and \( h_2 \) are the point-to-point channel gains from the first and the second LEDs, respectively, to the user. The channel gains are jointly represented as \( h_i = h_c h_{d_i} h_{\theta_i} \) for \( i = 1, 2 \), where \( h_c = (\gamma + 1) A_R \ell_m / 2\pi \) is the constant multiplier not depending on either the location or the orientation of the receiver. Other multipliers are

\[ h_{d_i} = (\ell^2 + d_i^2)^{-\frac{\gamma+2}{2}}, \quad h_{\theta_i} = \cos(\theta_i) \Pi \left( \frac{\theta_i}{\Theta} \right), \]  

(4.19)

where \( \theta_i \) and \( d_i \) are the incidence angle and the horizontal distance of the user with respect to the \( i \)th LED, respectively, as shown in Fig. 4.3. Without loss of generality, we define the random angles \( \theta_1 \) and \( \theta_2 \) in the clockwise and counter-clockwise directions, respectively.

The actual degrees of freedom for both \( \{\theta_1, \theta_2\} \) and \( \{d_1, d_2\} \) are 1, so that we have only one independent random variable from each set. We therefore choose \( d_1 = d \) and \( \theta_1 = \theta \) as the independent random variables of interest, and \( d_2 = D - d \) and \( \theta_2 = \Phi - \theta \) accordingly become the dependent random variables, where \( D \) is the distance between the LEDs, and the non-negative angle \( \Phi \) is defined geometrically by Fig. 4.3 as follows

\[ \Phi = \phi_1 + \phi_2 = \tan^{-1}(d/\ell) + \tan^{-1}((D-d)/\ell). \]  

(4.20)

Note that \( \theta_1 \) and \( \theta_2 \) can take negative values, but both be negative at the same time, and their sum is equal to \( \Phi \) in any case.
4.3.2 Two LEDs with Fixed User Location

When the user location is not considered to be random, the horizontal distance $d$ turns out to be a deterministic variable. In this case, the cdf of the square-effective channel gain is then given as

$$ F_{h_{2}^{2}_{\text{eff}}}(x) = \Pr \left\{ \max \left\{ h_{1}^{2}, h_{2}^{2} \right\} < x \right\} = \Pr \left\{ h_{1}^{2} < x, h_{2}^{2} < x \right\}, \quad (4.21) $$

where (4.21) directly follows from [80]. As the horizontal distance $d$ is not random, we define a new variable $c_{i} = h_{i}^{2} h_{i}^{2}$ for $i = 1, 2$, which captures the deterministic feature of the square-effective channel, and is always non-negative by definition. Then, the cdf in (4.21) becomes

$$ F_{h_{2}^{2}_{\text{eff}}}(x) = \Pr \left\{ h_{\theta_{1}}^{2} < \frac{x}{c_{1}}, h_{\theta_{2}}^{2} < \frac{x}{c_{2}} \right\}, \quad (4.22) $$

and employing (4.19) yields

$$ F_{h_{2}^{2}_{\text{eff}}}(x) = \begin{cases} 
\Pr \{ E_{1}, E_{2} \}, & \text{for } |\theta_{1}| \leq \Theta, |\theta_{2}| \leq \Theta \\
\Pr \{ E_{1} \}, & \text{for } |\theta_{1}| \leq \Theta, |\theta_{2}| > \Theta \\
\Pr \{ E_{2} \}, & \text{for } |\theta_{1}| > \Theta, |\theta_{2}| \leq \Theta \\
1, & \text{for } |\theta_{1}| > \Theta, |\theta_{2}| > \Theta 
\end{cases}, \quad (4.23) $$

where the event $E_{i}$ is defined for $i = 1, 2$ as follows

$$ E_{i} : \left\{ \theta_{i} \in \Omega_{\theta_{i}} \left| \cos^{2} \theta_{i} < \frac{x}{c_{i}} \right. \right\}, \quad (4.24) $$

with $\Omega_{\theta_{i}}$ being the sample space of $\theta_{i}$. Note that $E_{i}$ happens with probability 1 whenever $x \geq c_{i}$, and probability 0 whenever $x < 0$ (does not happen at all). We therefore safely assume
\[ x = c_i \text{ for } x \geq c_i \text{ and } x = 0 \text{ for } x < 0, \text{ to keep the argument of the inverse cosine function within the definition interval.} \]

Before further elaborating the cdf in (4.23), we define a new function in the following Lemma, which is actually an extension of (4.9).

**Lemma 1.** Let \( \nabla_\theta(a, b, c, d) \) be a function defined as

\[ \nabla_\theta(a, b, c, d) = \Pr\{a < \theta \leq b, c < \theta \leq d\}, \quad (4.25) \]

which represents the probability of the random variable \( \theta \) being in the intervals \((a, b]\) and \((c, d]\) jointly, with arbitrary real-valued variables \(a, b, c, d \in \mathbb{R}\). Then, (4.25) can be computed in terms of the cdf of the random variable \( \theta \) as follows

\[ \nabla_\theta(a, b, c, d) = \begin{cases} 
F_\theta(b) - F_\theta(a) & \text{for } c \leq a, \ d > b \\
F_\theta(d) - F_\theta(c) & \text{for } c > a, \ d \leq b \\
F_\theta(d) - F_\theta(a) & \text{for } c \leq a, \ d \leq b \\
F_\theta(b) - F_\theta(c) & \text{for } c > a, \ d > b \\
0 & \text{otherwise}
\end{cases} \quad (4.26) \]

**Proof.** See Appendix B. \( \square \)

Defining \( z_i(x) = \frac{1}{2} \cos^{-1}(2 \frac{x}{c_i} - 1) \) for \( i = 1, 2 \), the desired cdf and pdf expressions are given in the next Theorem.

**Theorem 4.1.** The cdf of the square of the effective channel given in (4.26) can be expressed as follows

\[ F_{h_{\text{eff}}^2}(x) = P_1(x) + P_2(x) + P_3(x) + P_4, \quad (4.27) \]
where

\[
P_1(x) = \nabla_\theta(-\Theta, -z_1(x), \Phi - \Theta, \Phi - z_2(x)) + \nabla_\theta(z_1(x), \Theta, \max(0, \Phi - \Theta), \Phi - z_2(x)) \\
+ \nabla_\theta(z_1(x), \Theta, \Phi + z_2(x), \Phi + \Theta),
\]

(4.28)

\[
P_2(x) = \nabla_\theta(-\Theta, \min(-z_1(x), \Phi - \Theta)) + \nabla_\theta(z_1(x), \min(\Theta, \Phi - \Theta)),
\]

(4.29)

\[
P_3(x) = \nabla_\theta(\Phi - \Theta, \Phi - z_2(x), \Theta, \Phi) + \nabla_\theta(\max(\Phi + z_2(x), \Theta), \Phi + \Theta),
\]

(4.30)

\[
P_4 = \nabla_\theta(\Theta, \Phi - \Theta) + F_\theta(\Phi + \Theta) + 1 - F_\theta(\Phi + \Theta).
\]

(4.31)

Furthermore, taking derivative of (4.27) yields the desired pdf as follows

\[
f_{h_{\text{eff}}}^2(x) = c_\theta \sum_{j=1}^3 \frac{\partial P_j(x)}{\partial \theta} + P_4 \delta(x),
\]

(4.32)

The individual derivatives \( \frac{\partial P_j(x)}{\partial \theta} \)'s in (4.32) can be computed using the following derivative expression

\[
\frac{\partial}{\partial x} F_\theta(u + vz_i(x)) = \frac{-v}{\sqrt{4x(c_i - x)}} f_\theta(u + vz_i(x)),
\]

(4.33)

where the argument \( u + vz_i(x) \) of the cdf function \( F_\theta(\cdot) \) is the linear transformation of \( z_i(x) \) with the arbitrary transformation variables \( u \) and \( v \).

Proof. See Appendix C.

Note that the cdf expression in (4.27) handles all possible combinations of the incidence angles \( \theta_i \)'s and the receiver FOV \( \Theta \). The \( P_j(x) \) functions given in (4.28)-(4.30) correspond to the first three cases of (4.23), in the same order. The \( P_4 \) expression given in (4.31) corresponds to fourth case of (4.23), which suggests both LEDs are out of sight. In this case channel gain is zero, which shows as a Dirac delta function at \( x = 0 \) in pdf.

It is possible to simplify (4.27) further if some particular scenarios of interest are assumed.
These scenarios involving wide receiver FOV and non-negative incidence angles $\theta$'s are discussed with the simplified cdf expressions in the following remarks.

**Remark 5.** When the receiver is pointing relatively upward and does not change this orientation dramatically, the incidence angles $\theta_1$ and $\theta_2$ take non-negative values only, which implies $\theta > 0$ and $\theta < \Phi$. In this particular case, the cdf in (4.27) can be expressed by using the following simplified probabilities

\[
P_1(x) = \nabla_\theta(z_1(x), \Theta, \max(0, \Phi - \Theta), \Phi - z_2(x)),
\]

\[
P_2(x) = \Delta_\theta(z_1(x), \min(\Theta, \Phi - \Theta)),
\]

\[
P_3(x) = \nabla_\theta(\Phi - \Theta, \Phi - z_2(x), \Theta, \Phi),
\]

\[
P_4 = \Delta_\theta(\Theta, \Phi - \Theta),
\]

which can be obtained directly from (4.28)-(4.31) by assuming the condition $0 < \Theta < \Phi$ and following the respective derivation steps in Appendix C. The pdf of this case can be obtained using (4.32) and (4.33).

**Remark 6.** When the receiver FOV is sufficiently large, the LEDs are always in the FOV, and the cdf in (4.27) now becomes

\[
F_{h_{\text{crit}}}^2(x) = \Pr\{z_1(x) < |\theta_1|, z_2(x) < |\theta_2|\}
\]

\[
= \Delta_\theta(z_1(x), \Phi - z_2(x)) + F_\theta(\min(-z_1(x), \Phi - z_2(x)))
\]

\[
+ 1 - F_\theta(\max(z_1(x), \Phi + z_2(x))),
\]

which is equivalent to (4.28) with a large $\Theta$.

**Remark 7.** When the incidence angles $\theta_1$ and $\theta_2$ take non-negative values and the receiver FOV is sufficiently wide, we have the following simplified cdf expression

\[
F_{h_{\text{crit}}}^2(x) = \Pr\{z_1(x) < \theta_1, z_2(x) < \theta_2\}
\]
\[ \Pr\{z_1(x) < \theta, z_2(x) < \Phi - \theta\} = \Delta_\theta(z_1(x), \Phi - z_2(x)), \quad (4.34) \]

which is equivalent to (4.28) with $0 < \theta < \Phi$ and large $\Theta$.

The desired pdf expressions for the simplified cdf’s in Remark 6-7 can be computed using (4.33) considering the detailed explanations in Appendix C. Note that for these two remarks, wide FOV assumption is made. Therefore their cdfs are expected to be continuous, and their pdfs does not have a delta function.

### 4.3.3 Two LEDs with Random User Location

In the two LEDs case, the parts of the channel $h_{di}$ and $h_{\theta i}$ in (4.19) are now correlated through the geometric relation (4.20). Therefore, when the horizontal distance $d$ is assumed to be random, it is not possible to derive the statistics using the identity employed in (4.14) and (4.15) for the product of the independent random variables. We therefore resort to a more conventional way of taking average over the distribution of the random distance $d$. As a result, once we obtain the distribution of the square-effective channel gain parametrically for a given $d$, the desired cdf and pdf can be calculated as

\[ F_{h^2_{\text{eff}}}(x) = P_4 + \int_{\mathcal{R}_y} F_{h^2_{\theta}}(x|y) f_d(y) dy, \quad (4.35) \]

\[ f_{h^2_{\text{eff}}}(x) = \begin{cases} c_h \int_{\mathcal{R}_y} f_{h^2_{\theta}}(x|y) f_d(y) dy & \text{for } x > 0 \\ P_4 \delta(x) & \text{for } x = 0 \end{cases}, \quad (4.36) \]

respectively, where $f_d(\cdot)$ is the pdf of the random location $d$, and $\mathcal{R}_y$ is the support set.
4.4 Channel Statistics for Multiple LEDs

We have considered the single LED and two LEDs scenarios so far, and derived the square-channel distribution when the receiver orientation is random in the vertical direction. In this section, we consider a multiple LED scenario with more than two LEDs, and discuss the extension of the findings for the two LEDs case presented in Section 4.3 to a multiple LED scenario. Our purpose here is not to provide a complete derivation for the channel statistics, but to consider two representative scenarios to gain some insight.

![Diagram](image)

Figure 4.4: A representative VLC downlink with 4 LEDs deployed along a line with an equal spacing. The vertical orientation is characterized by the random angle $\varphi$.

![Diagram](image)

Figure 4.5: Channel gains for 4 LEDs of the configuration in Fig. 4.4 with $D = 3$ m, $\ell = 3$ m, and $\Theta = \{60^\circ, 90^\circ\}$.

We consider a representative multiple LED scenario with 4 LEDs each of which are deployed
along a line with equal spacing $D$, as shown in Fig. 4.4. The receiver is assumed to be located between two inner LEDs, which are labeled as LED-2 and LED-3, and is away from LED-2 by a distance $d$. Without any loss of generality, we assume that the receiver is facing upward with a deviation from the vertical axis by a random angle $\varphi$, and that $D = 3$ m, and $\ell = 3$ m. As before, we assume that the user is assigned to the LED with the strongest signal, which depends on the relative distances between the user and the LEDs, as well as the incidence angles. Note that, the effect of the incidence angle on the signal strength does not appear only due to the Lambertian pattern of the LED, but also because of the FOV evaluation, and both will be examined in the sequel.

We first assume that the user in Fig. 4.4 is located in the middle of LED-2 and LED-3 with $d = 1.5$ m, and that $\Phi_{1/2} = 60^\circ$ and $A_R = 1$ cm$^2$. The associated signal qualities of the four LEDs at the receiver are depicted in Fig. 4.5(a) with varying vertical deviation $\varphi$, and for two different FOV choices of $\Theta = \{60^\circ, 90^\circ\}$. We partitioned Fig. 4.5(a) into the regions $\mathcal{D}_i$, $i = 1, \ldots, 6$, with respect to the LEDs having the first two strongest signal level among the others, and tabulate them in Table 4.1 for each region.

<table>
<thead>
<tr>
<th>Region</th>
<th>Wide FOV</th>
<th>Narrow FOV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strongest</td>
<td>2nd-Strongest</td>
</tr>
<tr>
<td>$\mathcal{D}_1$</td>
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<td>LED-1</td>
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<tr>
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<td>LED-2</td>
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</tr>
<tr>
<td>$\mathcal{D}_6$</td>
<td>LED-3</td>
<td>LED-4</td>
</tr>
</tbody>
</table>

We observe from Table 4.1 that comparing the signal strength of the same two LEDs (LED-2 and LED-3) is sufficient to find the strongest of the 4 LEDs in either $\mathcal{D}_2-\mathcal{D}_5$ for the wide FOV, or $\mathcal{D}_3-\mathcal{D}_4$ for the narrow FOV. This result implies that when the vertical deviation $\varphi$ falls into one of these group of regions, the findings of the two LEDs scenario in Section 4.3...
can be directly used to derive the channel statistics for this multiple LED case. Otherwise, the findings for the two LEDs scenario may need to be extended to the joint statistics of three or four LEDs. For example, \( \varphi \) range corresponding to \( \mathcal{D}_2-\mathcal{D}_5 \) under the narrow FOV requires the joint cdf statistics of all 4 LEDs to find the strongest signal. In general, as the receiver FOV is relatively wide or the receiver orientation does not vary much, the two LEDs results apply for the multiple LED cases like the one in Fig. 4.4.

As a marginal example, we consider a second scenario where the user in Fig. 4.4 is located directly under LED-2 with \( d = 0 \). Based on the respective signal strength results depicted in Fig. 4.5(b), the results of the two LEDs case directly apply if \( \varphi \) takes either negative (\( \varphi \in \mathcal{D}_1 \)) or positive values (\( \varphi \in \mathcal{D}_2 \)), and otherwise requires an extension to the joint statistics of three LEDs.

## 4.5 Numerical Results

In this section, we present the numerical results for the distribution of the square-channel with the single and two LEDs setting under various statistics for the random vertical orientation. In both LED settings, we provide numerical results for only the random choice of the horizontal distance \( d \), and consider the relatively simple deterministic \( d \) case as a subset. We assume that \( D = 4 \) m, \( \ell = 3 \) m, \( \Phi_{1/2} = 60^\circ \), \( g = 1 \), and \( A_R = 1 \text{ cm}^2 \), without any loss of generality. Like [32], we consider \( E_s/N_0 \) to characterize the transmit signal-to-noise ratio (SNR), and we study the impact of random receiver orientation and location on the BER and outage probability performance.
(a) $\theta$ is uniformly distributed with $\mathcal{U}[20^\circ, 40^\circ]$, and $d$ is uniformly distributed with $\mathcal{U}[0, 5]$ m. The magnitude of the Dirac delta is $c_\delta = \{0.25, 0\}$ for $\Theta = \{35^\circ, 60^\circ\}$, respectively.

(b) $\theta$ is Gaussian distributed with $\mathcal{N}(30^\circ, 20^\circ)$, and $d$ is Rayleigh distributed with $\mathcal{R}(1)$ m. The magnitude of the Dirac delta is $c_\delta = \{0.1318, 9.85 \times 10^{-12}\}$ for $\Theta = \{35^\circ, 60^\circ\}$, respectively.

Figure 4.6: The analytical and simulation data for the pdf of the square-channel in the single LED setting, where both the vertical orientation $\theta$ and the horizontal distance $d$ are random.

### 4.5.1 Single LED Case

In Fig. 4.6, the pdf of the square-channel $h^2$ is depicted for the single LED case, where both the incidence angle $\theta$ representing the vertical orientation of the user and the horizontal distance $d$ are random variables with various distributions. We assume $\theta$ follows uniform distribution with $\mathcal{U}[20^\circ, 40^\circ]$ or Gaussian with $\mathcal{N}(30^\circ, 20^\circ)$, both of which have the same mean values, whereas $d$ follows $\mathcal{U}[0, 5]$ m or $\mathcal{R}(2)$ m, both of which have similar mean values.

We observe a perfect match between the analytical and simulation data of Fig. 4.6 for all distributions of $\theta$ and $d$, which verifies the analytical derivation of the square-channel distribution. Note that, the FOV value of $\Theta = 60^\circ$ covers the incidence angle $\theta$ range completely for $\mathcal{U}[20^\circ, 40^\circ]$, and with a very high probability for $\mathcal{N}(30^\circ, 20^\circ)$. On the other hand, the relatively narrow FOV value of $\Theta = 35^\circ$ does not cover the angle span of $\theta$ completely in either case. As a result, the Dirac delta $\delta(x)$, which arises from the FOV values smaller than the angle span of $\theta$, does not appear at all or appears with a very small magnitude $c_\delta$ for $\Theta = 60^\circ$, whereas we have $c_\delta = \{0.25, 0.1318\}$ for $\mathcal{U}[20^\circ, 40^\circ]$ and $\mathcal{N}(30^\circ, 20^\circ)$, respectively,
Figure 4.7: The analytical and simulation data for the cdf of the square-channel in the single LED setting, where $\theta$ and $d$ follow the distributions in Fig. 4.6.

for $\Theta = 35^\circ$. These results exemplify the mechanism how the narrowing FOV introduces a nonzero probability of the LOS communication link being out of the FOV, which in turn causes a Dirac delta at the pdf of the square-channel, as explained in Section 4.2. We also observe that the pdf statistics of different FOV configurations are almost the same after relatively large input values along $x$-axis. This is because the high channel gains can be observed when the receiver orientation is well aligned ($\theta$ is small), and in this case LED being out of FOV is a small probability for all FOV configurations. The respective cdf statistics for Fig. 4.6 are provided in Fig. 4.7, which also correspond to outage probability for a given channel gain requirement. Outage probability difference can be most explicitly observed for lower channel gain requirement values, where we can observe up to ten times higher outage probability with low FOV receivers.

Before providing the BERs for given channel pdfs, we provide the BERs of some non-mobile and fixed user orientation scenarios in Fig. 4.8 to give intuition about the effect of horizontal distance and orientation on the BER and provide benchmarks for later results. The FOV is assumed to be larger than $\theta$ in all scenarios. In order to achieve lower BERs, more than
120 dB $\frac{E_s}{N_0}$ is required for all scenarios\(^2\). The scenario with $\theta = 0^\circ$, and $d = 0$ m is the case when the receiver is located immediately underneath and facing the LED. This scenario provides the highest possible channel gain, thus lowest possible BER. The orientation change of $30^\circ$ increases BER slightly, while a change of $60^\circ$ causes a much higher BER. On the other hand, horizontal distance of 2.5 m causes a significant BER increase too.

While Fig. 4.8 illustrates BERs for fixed location and orientation scenarios, Fig. 4.9 illustrates the BERs for random location and orientation scenarios with the channel pdfs shown in Fig. 4.6. A representative scenario with non-mobile and fixed user orientation is also provided, where $\theta = 30^\circ$ and $d = 2.5$ m. Note that all scenarios in Fig. 4.9 has the same mean $\theta$ and approximately same mean horizontal distance. The random fluctuations increase the BER significantly compared to the fixed setting with the same mean $\theta$ and $d$. Although having much higher BER than the fixed setting, the curve of the relatively wide FOV value of $\Theta = 60^\circ$ show monotonically decaying characteristics. On the other hand, the narrow FOV of $\Theta = 35^\circ$ cause the BER curves to saturate, where the saturation value is related to the

\(^2\)For a representative study on typical transmit SNR range for indoor deployment of multi-LED VLC systems, the reader is referred to [81].
probability of the LOS link being out of the FOV, which is equal to $c_\delta$ shown in Fig. 4.6. Since the error probability of OOK is 0.5 for zero channel gain, the BERs saturate to $c_\delta/2$ for each case. This result shows that a reliable communication is not possible when there is a high possibility of LED being out of FOV. This problem can be solved by increasing the FOV of the receiver, or deploying more LEDs to make sure receiver is in contact to at least one LED in FOV.

### 4.5.2 Two LEDs Case

The pdf of the square-effective channel gain for the two LEDs setting is presented in Fig. 4.10 under various distributions for $\theta$ and $d$. In particular, we assume $\mathcal{U}[20^\circ, 40^\circ]$ and $\mathcal{N}(30^\circ, 20^\circ)$ for $\theta$, as before, and $\mathcal{U}[0, D]$ m for $d$. We observe that the analytical results follow the simulation data successfully. In this two LEDs setting, since the LEDs are never out of the receiver FOV simultaneously for the given FOV values of $\Theta = \{35^\circ, 60^\circ\}$, there is no possibility in which the effective channel gain becomes zero, and we therefore do not have
Figure 4.10: The analytical and simulation data for the pdf of the square-channel in the two LEDs setting, where both the vertical orientation $\theta$ and the horizontal distance $d$ are random.

(a) $\theta$ is uniformly distributed with $U[20^\circ, 40^\circ]$, and $d$ is uniformly distributed with $U[0, D]$ m.

(b) $\theta$ is Gaussian distributed with $N(30^\circ, 20^\circ)$, and $d$ is uniformly distributed with $U[0, D]$ m.

any Dirac delta appearing in Fig. 4.10. We observe that as the FOV becomes smaller, the pdf curve broadens towards the origin and covers much smaller $x$ values. Smaller receiver FOV angles increase the possibility of the LOS links being out of the FOV. When stronger link is blocked due to FOV limit, the receiver connects to the LED with weaker link, causing the effective channel gain taking much smaller values. Note that even more smaller FOV angles will eventually cause the broadened pdf to include the origin, which will accordingly appear as the Dirac delta at the origin, as in the single LED scenario. The respective cdf statistics (outage probabilities) are also provided in Fig. 4.11.

We finally depict the BER results of the two LEDs setting in Fig. 4.12. Since the user does not lose contact with both the LEDs simultaneously, the effective channel gain is always nonzero, and the BER statistics do not saturate at all. Although the narrower FOV of $\Theta = 35^\circ$ deteriorate the BER performance, it is better than that for the single LED case depicted in Fig. 4.9 with the same FOV value. This result highlights the multiple LED deployment [24] as an effective direction to cope with adverse effects of the random orientation.
Figure 4.11: The analytical and simulation data for the cdf of the square-channel in the two LEDs setting, where $\theta$ and $d$ follow the distributions in Fig. 4.10.

Figure 4.12: BER results for the two LEDs setting, where $\theta$ follows the distributions in Fig. 4.10, and $d \sim \mathcal{U}[0,D]$ m.
Chapter 5

Slow Beam Steering and NOMA for VLC

In this chapter, we present the VLC beam steering and NOMA study. We find the steering angles that simultaneously serve multiple users within the frame duration and maximize the data rates. Subsequently, we consider multiple steerable beams with a larger number of users in the network and propose an algorithm to cluster users and serve each cluster with a separate beam. We optimize the transmit power of each beam to maximize the data rates. Finally, we propose a NOMA scheme for the beam steering and user clustering scenario, to further increase the data rates of the users.

The chapter is organized as follows. The Section 5.1 introduces the beam steering model and the assumptions, Section 5.2 presents the beam steering problem for a single steerable beam, Section 5.3 proposes a solution to the problem, Section 5.2(b) investigates beam steering and power allocation for multiple steerable beams case, Section 5.5 studies VLC NOMA in the beam steering context, and Section 5.6 presents the simulation results.
Piezoelectric beam steering is proposed in [23] in order to track the user, improve the signal strength, and provide smoother handover between different VLC APs. Piezo actuators convert electrical signal into precisely controlled physical displacement. This property of piezo actuators is used to finely adjust machining tools, camera lenses, mirrors, or other equipment [82]. Piezoelectric actuators can also be used to tilt LEDs or lenses, to steer the beam directed towards user location. In Fig. 5.1, two different beam steering schemes using piezo actuators are illustrated. In Fig. 5.1(a), whole LED is tilted using a set of piezo actuators, while in Fig. 5.1(b), only the lens is steered. The setup in Fig. 5.1(b) makes it possible to change the directivity of the light beam by shifting the lens forward or backward. In order to tilt an LED to any angle, two sets of piezo actuators can be used: while one provides steering on one direction, the other provides steering in a perpendicular direction.

Another method to steer LED light is to use micro-electro-mechanical system (MEMS) based mirrors [37, 83, 84], where the direction of the beam is controlled by changing the orientation of micromirrors. In [84], a setup with LEDs and MEMS mirrors is presented with steering angles of ±40° with a settling time under 5 ms, additionally featuring adaptable beam directivity. As a similar method to MEMS mirrors steering, in [85], optical gratings are used to change the beam direction. MEMS mirrors are also studied in the context of steering laser beams for indoor free space optical (FSO) communications [86, 87, 88]. The phased
In this study, without explicitly assuming any of the aforementioned beam steering methods, we consider a VLC AP with a limited number of steerable beams that can be steered within a given range. Additionally, we consider two scenarios where: 1) the beam directivity is fixed, or 2) the beam directivity can be changed within a given range. Note that steering the beams in VLC systems may cause uneven light distribution in indoor environments, and joint design of communications and illumination has been studied extensively in the literature [91, 40, 37]. The uneven light distribution can be overcome by installing other lighting equipment that does not steer beams, or that steers beams towards darker areas [37]. In this study, we focus on maximizing the achievable data rates with VLC beam steering without explicitly taking into account the illumination constraints, and assume that illumination requirements can be satisfied by the mentioned approaches.

5.2 Slow Beam Steering for Multiple Access VLC

Initially, we consider an AP with a single steerable light beam and $K$ users, and Fig. 5.2(a) shows an example scenario for $K = 3$. The AP serves all users with time division multiple access (TDMA), and $k$-th user is served with time ratio $\tau_k$, where $0 \leq \tau_k \leq 1$. We aim at finding the steering angles and LED directivity index which maximizes logarithmic sum
rate of all users. In 3D model, we need two angles to specify the orientation of the beam, which are the elevation and the azimuth angles, denoted by \( \alpha \) and \( \beta \), respectively, as shown in Fig. 5.4. We can convert these angles to an orientation vector given as:

\[
\mathbf{q}_{tx} = [q_{x(tx)}, q_{y(tx)}, q_{z(tx)}]^T = [\cos(\beta) \cos(\alpha), \sin(\beta) \cos(\alpha), \sin(\alpha)]^T.
\] (5.1)

The location of the AP is \( \mathbf{r}_{tx} = [x_{tx}, y_{tx}, z_{tx}]^T \). Likewise, the location and the orientation of the \( k \)-th user are \( \mathbf{r}_k = [x_k, y_k, z_k]^T \), and \( \mathbf{n}_k = [n_{x(k)}, n_{y(k)}, n_{z(k)}]^T \), respectively. Then, the vector from the AP to \( k \)-th user is \( \mathbf{v}_k = \mathbf{r}_k - \mathbf{r}_{tx} = [v_{x(k)}, v_{y(k)}, v_{z(k)}]^T \). The distance between the LED and the \( k \)-th user is \( r_k = ||\mathbf{v}_k||_2 \), while the angle between the LED orientation and \( \mathbf{v}_k \) is denoted as \( \phi_k \), and we can write:

\[
\cos(\phi_k) = \frac{\mathbf{n}_{tx}^T (\mathbf{r}_k - \mathbf{r}_{tx})}{d_k} = \frac{\mathbf{v}_k^T \mathbf{n}_k}{||\mathbf{v}_k||_2}.
\] (5.2)

Similarly, the angle between the receiver orientation and \( \mathbf{v}_k \) is \( \theta_k \), and we can write:

\[
\cos(\theta_k) = \frac{\mathbf{n}_k^T (\mathbf{r}_{tx} - \mathbf{r}_k)}{d_k} = -\frac{\mathbf{v}_k^T \mathbf{n}_k}{||\mathbf{v}_k||_2}.
\] (5.3)

Then, assuming the receiver has a wide field of view (FOV), we can remove the FOV constraint, and the LOS channel gain of the \( k \)-th user given in (2.1) can be simplified as follows:

\[
h_k = \gamma + \frac{1}{2\pi} A_r r \cos(\phi_k)\cos(\theta_k) \frac{1}{r_k^2} = -\gamma + \frac{1}{2\pi} A_r r \frac{\mathbf{v}_k^T \mathbf{n}_k}{||\mathbf{v}_k||_2} \cos(\phi_k)\cos(\theta_k)\frac{1}{r_k^2}
\]

\[
= -\gamma + \frac{1}{2\pi} A_r r \left( \frac{v_{x(k)}n_{x(k)} + v_{y(k)}n_{y(k)} + v_{z(k)}n_{z(k)}}{v_{x(k)}^2 + v_{y(k)}^2 + v_{z(k)}^2} \right) \frac{1}{r_k^2}
\]

\[
\times \left( v_{x(k)} \cos(\beta) \cos(\alpha) + v_{y(k)} \sin(\beta) \cos(\alpha) + v_{z(k)} \sin(\alpha) \right)^\gamma,
\] (5.4)

where \( A_r \) is the detection area of the PD, and \( r \) is the responsivity coefficient. Using (5.4),
the rate of the $k$-th user is given as \[92\]

$$R_k = B \log \left( 1 + \frac{(ph_k)^2}{N_0 B} \right), \quad (5.5)$$

where $N_0$ is the spectral density of additive white Gaussian noise, and $B$ is the communication bandwidth, and $p$ is the the standard deviation of the transmitted signal. As conceptually illustrated in Fig. 5.3, we consider a transmitted optical signal $s(t)$, where the mean of the signal $E[s(t)]$ corresponds to the average emitted light from the source, and the variation of the signal is used for transmitting data. We will refer $p = \text{std}(s(t))$ as the transmit power of the beam for the rest of the chapter. Note that when the signal is scaled with a coefficient, the mean and the standard deviation is also scaled with the same coefficient. In case the dimming is required, this method can be used to decrease the average emitted light, which would decrease the transmit power in the same ratio.
with TDMA without changing the beam orientation towards each user. There are two reasons not to consider changing beam orientation each time slot for each user. The first one is, there will be time loss between each time slot for orientation change. The shortest reported settling time for LED beam steering is 5 ms [84], which is close to the whole TDMA frame length used for Wi-Fi systems. The second reason is that it is not possible to do such a switching without a flickering effect. The human eye can capture changes up to 200 Hz [1], which means the the periodic changes to the signal should settle under 5 ms. Considering that just one steering takes around 5 ms, it is not possible to quickly switch the beam between users without flickering.

In this study, we propose a solution where the beam is steered once for a given set of user locations, and no more steering is needed unless the location and orientation of the users change significantly. If any user movement occurs, new steering parameters are computed and the slow beam steering is carried out. Accordingly, the steering parameters can be found by solving the following constrained optimization problem:

$$\tilde{\tau}, \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma} = \arg \max_{\tau, \alpha, \beta, \gamma} \sum_{k=1}^{K} \log(\tau_k R_k),$$

s.t. \( c_1 : \alpha_{\min} \leq \alpha \leq \alpha_{\max}, \)

\( c_2 : 0 \leq \beta \leq 360^\circ, \)

\( c_3 : \gamma_{\min} \leq \gamma \leq \gamma_{\max}, \)

\( c_4 : \sum_{k=1}^{K} \tau_k = 1, \)

where \( \alpha, \beta \) and \( \gamma \) are beam steering and directivity parameters as captured in (5.1), (5.4) and illustrated in Fig. 5.4. The constraint \( c_1 \) limits the elevation angle within the steering capacity of the beam. The constraint \( c_2 \) is the azimuth limit, which shows that the beam can be steered towards any direction as long as the elevation angle allows. The constraint \( c_3 \) is for the limits of beam directivity index, which is decided by the device capabilities.
The $\tau = [\tau_1, ..., \tau_K]$ is the time division coefficient vector whose elements add up to 1. To make sure all users are served and the resources are distributed fairly, the objective function is the sum of logarithmic rate instead of sum rate [72]. If the logarithm is removed from objective function, a single user with the largest channel gain gets all time allocation and the beam is steered towards that user, leaving other users unserved. The solution of (5.6) will be discussed in Section 5.3.

### 5.2.1 NOMA Signal Model

The TDMA serves each user on different time slots. Alternatively, NOMA serves all users simultaneously by exploiting the channel gain differences of users. Let there be $K$ NOMA users served by the same transmitter LED. The users are ordered based on the magnitude of their channel gains so that $h_1 < h_2 < ... < h_K$. The transmitter sends the signal to all users simultaneously by superposing the symbols in the power domain and adding a DC bias. The signal to be transmitted by the LED is

$$x = p \sum_{k=1}^{K} \rho_k s_k + I_{DC}$$

(5.7)

where $p$ is the transmit power of the LED, $I_{DC}$ is the DC bias added to the signal to ensure positive intensity, $s_k$ is the modulated message symbol for the $k$-th user, and $\rho_k$ is the NOMA power allocation coefficient for the $k$-th user. The message symbol signals are assumed to have zero mean and unit variance. In NOMA, users with poor channel conditions are allocated higher power. Therefore,

$$\rho_1 \geq \rho_2 \geq ... \geq \rho_K$$

(5.8)

to make interference cancellation possible, and $\sum_{k=1}^{K} \rho_k^2 = 1$ to satisfy total electricity power constraint[44]. Removing the DC bias at the receiver, the remaining received signal at $\ell$-th
user is given by
\[ y_\ell = p h_\ell \sum_{k=1}^{K} \rho_k s_k + z_\ell, \]  
(5.9)
where \( z_\ell \) is the real-valued Gaussian noise with zero mean and variance \( \sigma_\ell^2 \). A constant noise power spectral density \( N_0 \) is assumed so that \( \sigma_\ell^2 = N_0 B \). Successive interference cancellation (SIC) is carried out to remove the signals of users with weaker channel gains. This is possible because the NOMA power coefficients of these signals are higher, therefore the symbol can be detected and removed from the received signal, as will be discussed in Section 5.5. On the other hand, the signals of stronger users are not canceled and treated as noise.

### 5.3 Single Steerable Beam

In this section, we solve the optimization problem in (5.6) for a single steerable beam and multiple users and introduce a method for decreasing the complexity of the solution. Subsequently, Section 5.4 will study the multiple steerable beam scenario.

#### 5.3.1 Solution to the Optimization Problem using MM

We can divide the problem in (5.6) into two independent maximization problems by rewriting the objective function as follows:
\[ \sum_{k=1}^{K} \log(\tau_k R_k) = \sum_{k=1}^{K} \log(\tau_k) + \sum_{k=1}^{K} \log(R_k). \]  
(5.10)

Then, using the first summation in (5.10), the first problem in (5.6) becomes:
\[ \mathbf{\bar{\tau}} = \arg \max_{\tau} \log \left( \prod_{k=1}^{K} \tau_k \right), \]  
(5.11)
subject to only $c_4$ in (5.6). The answer to this trivial problem is $\tilde{\tau}_k = 1/K, \forall k$. The second problem based on (5.10) is given by

$$\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma} = \arg \max_{\alpha, \beta, \gamma} \sum_{k=1}^{K} \log(R_k),$$

subject to $c_1, c_2,$ and $c_3$ in (5.6). The problem in (5.12) is non-convex, hence gradient based optimization methods get stuck in a local optima. This can be seen in the channel gain in (5.4), which has sine, cosine, and exponential functions of optimization parameters.

In order to not use (5.4) in the objective function, we follow a grid search based method and calculate the channel gain for discrete values of $\alpha, \beta,$ and $\gamma$. To give an example, we separate all available range for $\alpha$ to discrete values with a small sampling interval $\delta$, hence we have $\alpha = [\alpha_{\min}, \alpha_{\min} + \delta, \ldots, \alpha_{\max}]$. A similar sampling is also used for $\beta$ and $\gamma$, and the sizes of $\alpha, \beta$ and $\gamma$ are $s_\alpha, s_\beta,$ and $s_\gamma,$ respectively. We calculate the channel gain for all possible $\alpha, \beta,$ and $\gamma$ combinations and form a column vector $h^{(k)}_{\alpha,\beta,\gamma}$, whose length is $s_\alpha \times s_\beta \times s_\gamma$, and its indices can be mapped back to $\alpha, \beta,$ and $\gamma$.

As a result, we can restructure the optimization problem in (5.12) as follows:

$$\tilde{d} = \arg \max_d \sum_{k=1}^{K} \log \left( B \log \left( 1 + \frac{(p d^T h^{(k)}_{\alpha,\beta,\gamma})^2}{N_0 B} \right) \right),$$

subject to

$$c_1 : \sum_{i=1}^{s_\alpha s_\beta s_\gamma} d_i = 1,$$

$$c_2 : \{d_i = \{0, 1\} \ \forall \ i,$$

where $d$ is a vector same size as $h^{(k)}_{\alpha,\beta,\gamma}$. The constraints $c_1$ and $c_2$ in (5.13) enforce that only one element of $d$ is equal to one, and the others are all equal to zero. Therefore, the vector multiplication results in choosing an element of $h^{(k)}_{\alpha,\beta,\gamma}$. The problem with (5.13) is the combinatorial nature of the problem due to the binary constraint $d_i$'s.
We can solve (5.13) by calculating the data rates for all entries of \( d \) and picking the one maximizing the objective function. This calculation has the complexity of \( s_\alpha \times s_\beta \times s_\gamma \), which mostly depends on the sampling interval for each parameter. The sampling interval can be decided by the physical device steering resolution or the computation capacity. In order to decrease the computation complexity, we propose a method to decrease the search space in the next subsection.

5.3.2 Decreasing the Steering Angle Search Space

The optimization in (5.13) operates over the whole search space in \( h^{(k)}_{\alpha,\beta,\gamma} \) to find the optimal steering angles and LED directivity index. However, searching all possible angles is unnecessary in many cases. For example, if all users are at one side of the room, we can narrow down the search space to that side of the room only, and decrease the computing complexity of the problem. In order to propose a method for narrowing down the search space, we provide the following propositions. In most use case scenarios, the transmitter LED is at a higher location than all of the users, and users are at a similar height. For these propositions, we assume users’ heights are the same and therefore they all lie on the same plane.

**Proposition 1.** When there are two users in the system, optimal steering angle points to a location on the line segment between the location of the two users.

*Proof.* See Appendix D.1.

**Proposition 2.** When there are more than two users that are on the same plane, optimal steering angle points to a location in the convex hull of the user locations, which is the smallest convex polygon that includes all locations.

*Proof.* See Appendix D.2.
Algorithm 2 The Graham Scan algorithm[93].

1: Find the point with lowest $y$ value. If there are two points with the same $y$ value, then choose the point with smaller $x$ coordinate value. Make a list of points, and make this point the first one ($P[0]$).
2: Sort the remaining $k-1$ points by the polar angle in counterclockwise order around $P[0]$, and add them to the list. If polar angles to two points are the same, then delete the nearest point.
3: For each point, if going to that point from the previous one takes a left turn keep that point in place. If it takes a right turn, remove previous points from the list until going to that point becomes a left turn.
4: In the end, remaining points define the convex hull.

According to the Proposition 1 and Proposition 2, the optimal steering angle always points to some location within the convex hull or line segment that includes all user locations. To decrease the search space, we propose the following solution. We calculate the convex hull of user locations using Graham scan [93], which finds the convex hull of a finite set of points on the same plane. Let us assume we have $k$ points, where $k \geq 3$, and their Cartesian coordinates are $(x_k,y_k)$. The implementation of Graham scan for these points is explained in Algorithm 2. After finding the convex hull, we search within $\alpha$ and $\beta$ angles that point to the hull. If there are more than three users that are on the same line segment, then the Graham scan returns two points, which results in a line segment instead of a hull. In this case, or in case there are only two users, the search space should be the $\alpha$ and $\beta$ angles that point to the line segment in between.

In Fig. 5.5, the ratio of the decreased search space (DSS) compared to the whole search space is shown. The simulation is done for an LED installed in the center of a room, at 4 m height, looking downwards. Users are located at $(x,y,0.85)$ m, where $x$ and $y$ are uniformly distributed in the room. Three room sizes are considered: $5 \times 5$ m, $10 \times 10$ m, and $20 \times 20$ m. The elevation angle limits are $\alpha_{\text{min}} = 200$ and $\alpha_{\text{min}} = 340$. In Fig. 5.5, the dotted line with circle marker shows the ratio of the whole search space to itself, which is equal to one. The black dotted line with square markers shows the search space for the whole $20 \times 20$ m room, and the black solid line with square markers shows the DSS for the same room. For a
low number of users, the search space is decreased significantly compared to the whole room, which means the algorithm provides a large gain in the computing time. When the number of users increases, they spread to a larger area and required search space increases too. Even when there are 10 users in the room, the algorithm reduces the search space about 10%. The result is similar for smaller rooms, but they require a smaller search space compared to 20 × 20 m room. Overall, the proposed solution reduces the search space to 90% - 1% of the whole room depending on the number of users or the room size.

5.4 Multiple Steerable Beams

In this section, we extend the solution in the previous section to multiple independently steerable beams case. Multiple steerable beams create a new problem, which is assigning users to the beams. We first propose a solution to this problem, then optimize the transmit power of each beam.
5.4.1 Steering Parameters for Multiple Beams

In this subsection, we consider a transmitter that can steer multiple beams independently and therefore can track multiple users. When the number of users is equal or lower than the number of steerable beams, each user can be allocated a separate beam\(^1\). In case number of users are larger than the number of beams, users can be separated to clusters, and each cluster can be served with a separate beam as illustrated in Fig. 5.2(b). In order to cluster users, we introduce the VLC user clustering (VUC) algorithm, which is a modified \(k\)-means clustering technique. Each cluster corresponds to a separate beam. The VUC algorithm assigns users to the clusters based on the signal strength received from each beam, and finds the steering parameters for each beam, as described next in more detail.

VLC User Clustering Algorithm

Let there be \(N\) steerable beams, and the steering angles and the directivity index of the \(n\)-th beam are \(\alpha^{(n)}, \beta^{(n)}, \) and \(\gamma^{(n)}, \) respectively. To initiate the algorithm, we randomly assign a single user to each cluster (i.e., assign first \(N\) users to one cluster each). Initially, there are some unassigned users, but all users will be assigned to a cluster after the algorithm is completed. We have a total of \(N\) clusters, and we repeat the following steps iteratively to find conclusive clusters and cluster centers. In the first step, we calculate the optimal steering parameters for the \(n\)-th beam, which are \(\alpha^{(n)}, \beta^{(n)}, \) and \(\gamma^{(n)}, \) solving the optimization problem in (5.6) as described in Section 5.3.A, for the users in the \(n\)-th cluster. We repeat this and find the steering parameters for each beam. While solving (5.6), the search space should be decreased as described in Section 5.3.B to reduce the computation time.

In the second step, we assign each user to the cluster whose beam provides the maximum signal strength to the user. We repeat these two steps until the steering parameters stay

\(^1\)In this study, we do not address the problem of multiple beams serving to the same user.
Algorithm 3  The proposed VUC algorithm.

1: Initialize: Assign user $n \rightarrow J(n)$ for $n = 1, \ldots, N$
2: repeat
3:   for $n = 1$ to $N$ do
4:      Solve (5.6) for the $n$-th beam and users in $J(n)$ to find the steering parameters of $n$-th beam $(\alpha^{(n)}, \beta^{(n)}, \gamma^{(n)})$.
5:   end for
6:   for $k = 1$ to $K$ do
7:      Find $n$ maximizing $h_{k,n}$, then assign user $k \rightarrow J(n)$.
8:   end for
9: until Steering parameters stay the same for two consecutive iterations.

the same for two consecutive iterations. The proposed VUC algorithm is summarized in Algorithm 3, where $J(n)$ represent the set of users assigned to the $n$-th cluster, and $h_{k,n}$ denotes the channel gain between the $n$-th beam and the $k$-th user.

### 5.4.2 Power Optimization of Beams

In this subsection, we discuss the optimal power allocation to different beams in order to maximize the sum rate of all users. The VUC algorithm works for a given transmit power of each beam and does not optimize the transmit powers. It is because the VUC algorithm aims at efficiently clustering users, and finding optimal steering parameters for each cluster. Considering that each cluster can have a different number of users, or users can have different received signal strength, we can improve the overall rate capacity by assigning different transmit powers to each beam. In a scenario with multiple beams serving different user clusters, each beam causes interference to users in other clusters. In this case, the SINR of the $k$-th user in the $n$-th cluster is given as

$$\xi_{k,n} = \frac{(p_n h_{k,n})^2}{N_0 B + \sum_{m=1, m \neq n}^{N} (p_m h_{k,m})^2}, \quad (5.14)$$
where \( p_n \) is the transmit power allocated to the \( n \)-th beam. The rate capacity of this user is

\[
R_{k,n} = B \log(1 + \xi_{k,n}).
\]  

(5.15)

Then, the power optimization problem can be formulated as

\[
\tilde{p} = \arg \max_p \sum_{k=1}^K \log(\tau_k R_{k,n}),
\]

s.t. \( c_1 : 0 \leq p_n \ \forall n \),

\( c_2 : \sum_{n=1}^N p_n \leq p_{\text{max}} \),

(5.16)

where \( p \) is the power allocation vector including power allocation of all beams. The constraint \( c_1 \) makes sure all power coefficients are positive, and the constraint \( c_2 \) makes sure their sum does not exceed the limit \( p_{\text{max}} \). The transmit power limit \( p_{\text{max}} \) can be dictated by the illumination constraints such as dimming or the maximum amplitude capacity of the source.

Considering the conceptual transmitted signal in Fig. 5.3, both constraints can be expressed as a sum transmit power limit. The \( \tau_k \) is the time allocation of the \( k \)-th user as addressed in (5.11), and it is equal to \( 1/K_n \) where \( K_n \) is the number of users served by the \( n \)-th beam.

Note that using log at objective function is optional in this case. Even when we do not use it, more than one LED can be allocated some power level to maximize the overall sum rate. However, using logarithm can be preferred for fairness.

The problem in (5.16) is non-convex because of the objective function. There are optimization parameters both on the numerator and the denominator of (5.14), and linearizing the sum of logarithm of non-convex functions is not possible. To avoid this structure, we introduce auxiliary variables \( \zeta_{k,n} \) and \( \eta_{k,n} \) such that [94]

\[
B \log(1 + \zeta_{k,n}) \geq \eta_{k,n} \ \forall k, \quad \text{and} \quad \xi_{k,n} \geq \zeta_{k,n} \ \forall k,
\]

(5.17)
where \( \eta_{k,n} \) is a lower bound for the rate of \( k \)-th user in the \( n \)-th cluster, and \( \zeta_{k,n} \) is a lower bound for the SINR of that user.

Using (5.17), the problem in (5.16) becomes

\[
\hat{p} = \arg \max_p \sum_{k=1}^{K} \log(\tau_k \eta_{k,n}),
\]

\[
\text{s.t. } c_1 : 0 \leq p_n \quad \forall n,
\]

\[
c_2 : \sum_{n=1}^{N} p_n \leq p_{\text{max}},
\]

\[
c_3 : B \log(1 + \zeta_{k,n}) \geq \eta_{k,n} \quad \forall k,
\]

\[
c_4 : \xi_{k,n} \geq \zeta_{k,n} \quad \forall k,
\]

where \( c_3 \) and \( c_4 \) are added to satisfy (5.17). While the objective function in (5.18) is now convex, the constraint \( c_4 \) is still non-convex, and the SINR expression is still there. To address this, we introduce another auxiliary variable set \( \kappa_{k,n} \) which is an upper bound for the denominator of the \( \xi_{k,n} \) given in (5.14). Now we can replace \( c_4 \) with \( c_5 \) and \( c_6 \) which are given as:

\[
c_5 : \frac{(h_{k,n}p_n)^2}{\kappa_{k,n}} \geq \zeta_{k,n} \quad \forall k,
\]

\[
c_6 : N_0 B + \sum_{m=1}^{N} (p_m h_{k,m})^2 \leq \kappa_{k,n} \quad \forall k.
\]

Finally, the problem becomes

\[
\hat{p} = \arg \max_p \sum_{k=1}^{K} \log(\tau_k \eta_{k,n}),
\]

\[
\text{s.t. } c_1, c_2, c_3, c_5, \text{ and } c_6.
\]

The constraint \( c_5 \) in (5.20) is still non-convex because of the expression \( \frac{p_n^2}{\kappa_{k,n}} \), but it is in a
simpler form and hence we can use MM procedure on this constraint. In order to use MM, we approximate the expression \( \frac{p_n^2}{\kappa_{k,n}} \) for \( k \)-th user with multivariate first order Taylor series. This is a function of variables \( p_n \) and \( \kappa_{k,n} \), therefore we can express it as

\[
\frac{p_n^2}{\kappa_{k,n}} = f(p_n, \kappa_{k,n}) \quad (5.21)
\]

The first order Taylor approximation for this function at point \( p_n = a_n \) and \( \kappa_{k,n} = b_{k,n} \) is given by:

\[
f(p_n, \kappa_{k,n}) \approx f(a_n, b_{k,n}) + \frac{\partial f}{\partial p_n}(a_n, b_{k,n})(p_n - a_n) + \frac{\partial f}{\partial \kappa_{k,n}}(a_n, b_{k,n})(\kappa_{k,n} - b_{k,n})
\]

\[
= \frac{a_n^2}{b_{k,n}} + \frac{2a_n}{b_{k,n}}(p_n - a_n) - \frac{a_n^2}{b_{k,n}^2}(\kappa_{k,n} - b_{k,n}) = 2 \frac{a_n}{b_{k,n}} p_n - \left( \frac{a_n}{b_{k,n}} \right)^2 \kappa_{k,n} \quad (5.22)
\]

The function in (5.21) can be approximated by (5.23) which is a convex expression. Inserting this expression into \( c_5 \), the constraint becomes

\[
c_5 : \quad (h_{k,n})^2 \left( 2 \frac{a_n}{b_{k,n}} p_n - \left( \frac{a_n}{b_{k,n}} \right)^2 \kappa_{k,n} \right) \geq \zeta_{k,n} \quad \forall \, k .
\]

The MM procedure on (5.20) operates iteratively. We first solve the problem for some initial values of \( a_n \) and \( b_{k,n} \). We do not need to carry out the iterations in two dimensions, because the constraint is only dependent on the division of \( a_n \) and \( b_{k,n} \). Therefore we update the value of \( \frac{a_n}{b_{k,n}} \) at each iteration until it stays the same for two consecutive iterations, or the change between two consecutive iterations is not appreciable.
5.5 VLC NOMA for Beam Steering and User Clustering

In the previous section, we assumed that all users in a cluster are served by TDMA. An additional approach to improve user rates is to implement NOMA for some of the users. Since the LEDs are directional, not all users in a cluster receive similar signal strength. Even though the optimization problem in (5.6) considers the fairness among users, due to the Lambertian pattern of the signal, we expect some users to receive much higher signal strength compared to others. In this case, an opportunistic approach would be to employ NOMA to exploit this uneven signal strength distribution and improve the overall data rate of users.

In this section, we consider NOMA application for two users that are served by \( n \)-th LED, by coupling users whose channel gains are distinctively different. We will denote these two users as user-1 and user-2, where user-1 has the weaker channel gain \( (h_{1,n} \ll h_{2,n}) \). The achievable data rate for these two users are given as follows:

\[
R_{i,n} = \log_2(1 + \xi_i) \tag{5.24}
\]

where the SINRs for user-1 and user-2 are given as

\[
\xi_1 = \frac{(h_{1,n}p_1p_n)^2}{N_0B + \sum_{m=1}^{n} (h_{1,m}p_m)^2 + (h_{1,n}p_2p_n)^2}, \quad \xi_2 = \frac{(h_{2,n}p_2p_n)^2}{N_0B + \sum_{m=1}^{n} (h_{2,m}p_m)^2}. \tag{5.25}
\]

The first interference term of both SINR expressions come from the interference caused by other beams. The user-1 has another interference component, which is the signal message of user-2. The user-2, on the other hand, does not have this interference since it detects and cancels the message of user-1. This rate is conditioned on the event that user-2 successfully
detects and cancels the signal of user-1. Let $\xi_{2\to1}$ denote the SINR for user-2 to detect the message for user-1, and $\xi^*_1$ as the targeted SINR for successful message detection at user-1. Then the condition can be expressed as $\xi_{2\to1} \geq \xi^*_1$, or explicitly

$$\frac{(h_{2,n}\rho_1p_n)^2}{N_0B + \sum_{m=1}^{N} (h_{2,m}\rho_m)^2 + (h_{2,n}\rho_2p_n)^2} \geq \xi^*_1.$$  \hspace{1cm} (5.26)

### 5.5.1 NOMA Parameter Optimization Problem

In order to maximize the user rate, the parameters to be optimized include the power allocation of each beam and NOMA power coefficients of NOMA users. However, such an optimization problem would be too complex to solve. Firstly, in such a problem, deciding which user pair will utilize NOMA is difficult because the power allocation of each beam is unknown. An iterative solution can be proposed where the solution updates the power allocation, NOMA user pairs, and NOMA power coefficients of these users at each iteration. However, this solution would show erratic behavior since the selected NOMA pairs would introduce a non-continuous objective function because user pair selection is a binary optimization problem. Due to the complexity of this problem, we do not propose a single step solution.

Instead of solving the problem in a single step, we can use the power optimization that is proposed in Section 5.4 as the first step of the solution, choose NOMA user pairs, and optimize the NOMA coefficients of these users as the second step of the problem. This solution is guaranteed to improve the overall sum rate as long as the sum rate of NOMA users is improved because the NOMA coefficients of a user pair do not affect the interference received by other users. The same statement is also valid for logarithmic sum rate. The sum of the logarithm of the user rates is proportional to the multiplication of the user rates. If the multiplication of the rates of two users increases, overall multiplication of the user rates
increases too.

In order to implement the proposed solution, we need to decide the NOMA user pairs. It is well known that NOMA is more efficient when the channel gains are more distinct [95]. The simplest solution is to pair the users with the highest and lowest channel gains in each beam [95, 44] if they meet the SINR threshold criteria. After that, remaining users with the most distinctive channel gains can be paired if they meet the same criteria. For any user pair, the optimal NOMA coefficients can be found by solving the following problem:

\[
\hat{\rho} = \arg \max_{\rho} \sum_{i=1}^{2} \log(R_{i,n}),
\]

\text{s.t.} \quad c_1 : \quad \rho_1^2 + \rho_2^2 = 1,

\quad c_2 : \quad \xi_{2\rightarrow1} \geq \xi_1^*,

(5.27)

where the constraint \( c_1 \) is for the preservation of energy, and the constraint \( c_2 \) is to ensure successful interference cancellation at user-2. The logarithm at the objective function is optional again.

### 5.5.2 Proposed Solution for NOMA Parameter Optimization

In (5.27), the objective function and both constraints are non-convex. In the objective function, the only non-convex expression is the \( \xi_1 \), because there are optimization parameters both in the nominator and the denominator[94]. In order to handle this expression, as we did in the solution of (5.16), we introduce an auxiliary parameter \( \zeta \) as a lower bound of SINR of user-1:

\[
\xi_1 \geq \zeta.
\]

(5.28)
Now the problem in (5.27) becomes

\[
\tilde{\rho} = \arg \max_{\rho} \log(\log(1 + \zeta)) + \log(\log(1 + \xi_2)),
\]

s.t. \( c_1 : \rho_1^2 + \rho_2^2 = 1, \)  

\( c_2 : \xi_{2 \rightarrow 1} \geq \xi_1^*, \)  

\( c_3 : \xi_1 \geq \zeta, \)  

(5.29)

where \( c_3 \) is added to satisfy (5.28), and the \( \xi_1 \) is replaced with \( \zeta \) in the objective function. In order to handle the constraint \( c_1 \), we introduce the another auxiliary variable \( \eta \) such that

\[
\eta = \rho_1^2,
\]

(5.30)

and we replace the all \( \rho_1^2 \)s with \( \eta \), all \( \rho_2^2 \)s with \( 1 - \eta \) in (5.25) and (5.26). We also replace the constraint \( c_1 \) as \( c_1 : 0 \leq \eta \leq 1 \), to make sure coefficients stay within the limit \([0, 1]\).

In (5.29), the constraints \( c_2 \) and \( c_3 \) are non-convex. For these two expressions, we introduce two more auxiliary variables \( \kappa_1 \) and \( \kappa_2 \), and replace the constraint \( c_2 \) with the following expressions:

\[
c_{2.1} : \frac{(h_{2,n}p_n)^2\eta}{\kappa_1} \geq \xi_1^*,
\]

(5.31)

\[
c_{2.2} : \kappa_1 \geq N_0 B + \sum_{m=1 \atop m \neq n}^N (h_{2,m}p_m)^2 + (h_{2,n}p_n)^2 (1 - \eta),
\]

while the constraint \( c_3 \) is replaced with:

\[
c_{3.1} : \frac{(h_{1,n}p_n)^2\eta}{\kappa_2} \geq \zeta,
\]

(5.32)

\[
c_{3.2} : \kappa_2 \geq N_0 B + \sum_{m=1 \atop m \neq n}^N (h_{1,m}p_m)^2 + (h_{1,n}p_n)^2 (1 - \eta).
\]
The constraints $c_{2.2}$ and $c_{3.2}$ are convex because they are in the form of comparison of two optimization variables with some constant multipliers. The $\kappa_1$, $\kappa_2$, and $\eta$ are the optimization parameters in these constraints, while all other parameters are constants. The constraint $c_{2.1}$ can be expressed in the form of comparison of two optimization parameters by sending the $\kappa_1$ to the other side of the equation. On the other hand, the constraint $c_{3.1}$ is non-convex, because it includes the division of two optimization parameters, and another optimization parameter $\zeta$ on the other side of the inequality. In order to deal with that, we replace the expression $\frac{\eta}{\kappa_2}$ with its multivariate first order Taylor series expansion as we did with (5.21). The Taylor series expansion of the expression $\frac{\eta}{\kappa_2} = f(\eta, \kappa_2)$, when evaluated at point $\eta = a$ and $\kappa_2 = b$ is given as:

$$f(\eta, \kappa_2) \approx f(a, b) + \frac{\partial f}{\partial \eta}(a, b)(\eta - a) + \frac{\partial f}{\partial \kappa_2}(a, b)(\kappa_2 - b) = \frac{a}{b} + \frac{1}{b}(\eta - a) - \frac{a}{b^2}(\kappa_2 - b) = \frac{\eta}{b} - \frac{a}{b^2}\kappa_2 - \frac{a}{b}.$$  

(5.33)

Now we can insert the Taylor series expansions to constraint $c_{3.1}$, and the optimization problem is finally convex.

In order to solve the problem, we need to implement MM procedure over parameters $a$ and $b$. We start with some initial values of these parameters, solve the convex optimization problem, update the MM parameters for the found values of $\eta$ and $\kappa_2$, and repeat until the parameters stay unchanged for two consecutive iterations. With the suggested changes, the final optimization problem becomes the following:

$$\bar{\eta}, \bar{\zeta}, \bar{\kappa}_1, \bar{\kappa}_2 = \arg\max_{\eta, \zeta, \kappa_1, \kappa_2} \log(1 + \zeta) + \log(1 + \xi_2),$$  

(5.34)

s.t. $c_1 : 0 \leq \eta \leq 1$,  

$$c_{2.1} : (h_{2,n}p_n)^2 \eta \geq \xi_1^* \kappa_1,$$
Table 5.1: Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The transmit power, $p$</td>
<td>1 W</td>
</tr>
<tr>
<td>The receiver responsivity, $r$</td>
<td>1 A/W</td>
</tr>
<tr>
<td>Effective receiver surface area, $A_r$</td>
<td>1 cm²</td>
</tr>
<tr>
<td>The modulation bandwidth $B$</td>
<td>20 MHz</td>
</tr>
<tr>
<td>The AWGN spectral density, $N_0$</td>
<td>$2.5 \times 10^{-20}$ A²/Hz</td>
</tr>
<tr>
<td>LED directivity limits $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$</td>
<td>1 and 15</td>
</tr>
<tr>
<td>Default LED directivity $\gamma_{\text{def}}$</td>
<td>5</td>
</tr>
<tr>
<td>Elevation angle limits $\alpha_{\text{min}}$ and $\alpha_{\text{max}}$</td>
<td>200° and 340°</td>
</tr>
</tbody>
</table>

\[
c_{2.2} : \quad \kappa_1 \geq N_0 B + \sum_{\substack{m=1 \atop m \neq n}}^{N} (h_{2,m}p_m)^2 + (h_{2,n}p_n)^2(1 - \eta),
\]
\[
c_{3.1} : \quad (h_{1,n}p_n)^2 \left( \eta \frac{a}{b^2 \kappa_2} - \frac{a}{b} \right) \geq \zeta,
\]
\[
c_{3.2} : \quad \kappa_2 \geq N_0 B + \sum_{\substack{m=1 \atop m \neq n}}^{N} (h_{1,m}p_m)^2 + (h_{1,n}p_n)^2(1 - \eta),
\]

where $\eta$ and $\kappa_2$ are evaluated at $a$ and $b$. This problem can be directly solved with convex optimization tools such as CVX [96].

5.6 Simulation Results

We conduct computer simulations using MATLAB, where we consider a square room of dimensions 8 m × 8 m × 4 m. The LED transmitter is located in the center at ceiling level, and receivers are distributed at uniformly random locations at 0.85 m height, facing upwards. A wide FOV angle is considered so that the LED is always within LOS of the receivers. The remaining simulation parameters are given in Table 5.1.
5.6.1 Single LED Beam Steering

In Fig. 5.6(a), the sum rate of users are shown when the transmitter has a single steerable beam. We simulate three different scenarios. The first one is labeled as “No Steering”, where the LED beam is not steered and faced downwards with the default directivity index $\gamma_{\text{def}}$. The second one is labeled as slow beam steering (SBS), where the beam is steered as described in Section 5.2. In this scheme, we assume the directivity index cannot be changed, and equal to $\gamma_{\text{def}}$. The third scenario is labeled as slow beam steering and focus (SBSF), where both beam orientation and directivity index are optimized. For comparison, we also consider a genie-aided fast beam steering (GA-FBS) approach as an upper bound on the sum rate. In particular, while settling time for steering may be on the order of 5 ms in practice [84], we assume that we can instantaneously steer beams to each scheduled user so that the beam is completely steered towards the user that is being served at each time slot.

Results in Fig. 5.6(a) show that when there is a single user, a significant gain on the sum rate can be achieved with steering and focusing. In this case, optimal steering angles point to the direction of the user, and the optimal directivity index is high since the user is on the
exact direction of the beam. When the number of users increases, the total rate achievable with steering decreases. The optimization maximizes the sum of the logarithm of rates to serve all users simultaneously; therefore the beam orientation does not point to a single user, and the optimal directivity index gets lower. Since users are not in the exact direction of the beam, the sum rate decreases as the number of users increases. The sum rate for GA-FBS schemes does not decrease, because the beam is steered towards the receiving user at each time interval, and we consider the average rate over a large number of user locations.

5.6.2 Multiple LEDs and User Clustering

In Fig. 5.6(b), the sum rate of users are shown when the AP has three independently steerable beams. The transmit power of these beams are $p/3$ (versus $p$ that was used in Fig. 5.6(a)) for a fair comparison. For this simulation, we consider two different multiple access schemes. The first one is labeled as “single stream” and shown with dashed blue lines, where all beams transmit the same signal to avoid any interference. In this scheme, the signal strength is higher, and the interference is zero. However, all the users are served with time division of a single stream, therefore they are allocated a lower amount of TDMA time resources. In the multi-stream scheme shown with solid red lines, all beams transmit a different stream to the users assigned to them. Since Fig. 5.6(b) shows results for an AP with three independently steerable beams, the multi-stream scheme has three different streams. If multiple users are assigned to the same beam, they share the channel with TDMA. Due to the use of spatial diversity and higher time allocation to the users, this scheme may offer higher rates than the single stream scheme.

As seen in Fig. 5.6(b), the SBSF multi-stream provides the highest sum rates. The SBS multi-stream does not relatively perform well, especially with the lower number of users. In this scheme, some users suffer heavy interference because the directivity of the beams cannot be adjusted as needed. With the SBS single stream scheme, the sum rate decreases
and approaches to no steering scheme with the increasing number of users. Since the ratio of users to the number of beams increases significantly, steering becomes less effective. Note that in Fig. 5.6(b) the sum rates do not decrease rapidly as in Fig. 5.6(a), especially sum rates of multi-stream schemes. This is due to VUC algorithm clustering users together that can receive high signal strength through a single beam. In Fig. 5.7(a), the cumulative distribution function (CDF) of user rates are shown for six users and three steerable beams, as in the case of Fig. 5.6(b). The steering provides more uniform distribution of user rates in comparison to no steering scheme since the optimization problem maximizes the sum of the logarithm of rates and provides a fairer resource allocation.

In Fig. 5.7(b), the sum rates are shown for 10 users with a varying number of independently steerable beams. The transmit power of each beam is $p/N$, where $N$ is the number of beams. SBSF with multi-stream provides the highest sum rate, which is maximized at three beams per 10 users where the sum rate exceeds the four times of no steering scheme. The higher number of beams means better steering accuracy and higher received signal strength, however, it also causes higher interference in the multi-stream scheme and a lower transmit
Figure 5.8: The sum rate of users with power optimization where the beams transmit different streams (multi-stream).

power per beam. The ideal user count per beam ratio may change based on the size of the room or the total number of users in the room. The SBS multi-stream scheme provides a lower data rate than no steering scheme if the number of steerable beams is high. This is because the signal strength of each user is low, and the interference from other beams is high. In no steering scheme and single stream schemes, there is no interference since all users are served in turn with TDMA. The SBS single stream scheme falls below no steering for 3 or 4 beams, which is possible because the optimization maximizes the sum of logarithmic rates instead of the sum rate.

5.6.3 Beam Power Optimization

In Fig. 5.8(a), the sum rates of users are shown for an AP with 3 steerable multi-stream beams when the beam power optimization as in (5.18) is used. The results for single-stream beams are not included due to poor performance and the requirement of a separate optimization solution. The results of power optimization for maximizing the sum rates
are labeled as "Max. Sum Rate", and shown in dotted lines. The power optimization significantly increases the sum rate of the users, where the rate gain is between 30 - 70 Mbps for both SBS and SBSF. The rate gain is provided by assigning more power to the LEDs that have stronger LOS connection with users or serving more users overall. The sum rates of power optimization for maximizing the sum of the logarithm of the rates are labeled as "Max. Log Rate", and shown in dashed lines. In this case, there is a sum rate gain compared to "No Power Opt.", but the gain is not as high as the maximization of the sum rate.

In Fig. 5.8(b), the sum rates are shown for an AP with varying number of steerable beams. When there is no power optimization, the sum rate decreases for a high number of steerable beams. However, with the power optimization, the sum rate increases consistently. This is thanks to the interference adjustment feature of the power optimization solution. Since the power allocation is done considering the interference to other users, the higher number of beams can be utilized more efficiently. With 10 beams, the sum rate of the maximum sum rate case reaches to 10 times of the sum rate of no steering scheme.

In Fig. 5.9 the CDF of individual user rates are shown for six user and three steerable beams case. The figure includes the data rates after power optimization of the beams for maximizing the sum rate and the logarithmic sum rate. Power optimization for maximum sum rate leaves some users without service but provides some other users much higher data rates. On the other hand, optimization for maximizing the logarithmic sum rate serves all users and increases the data rates overall. In this case, the low-rate users have more gain compared to high-rate users. It is also seen that users with the highest data rates actually lose some data rate after this optimization.
5.6.4 NOMA

In this subsection, simulations are conducted for three steerable beams setting and 10 users, with other parameters being the same as previous simulations. Users in the same cluster are paired to be served by NOMA. The users with most distinctive channel gains are paired if they meet the SINR threshold $\xi^*_1 = 3$ as described in (5.26). Then the remaining users are paired if they meet the same threshold. The users that are not paired are served by TDMA and get half the time allocation of NOMA user pairs.

In Fig. 5.10(a), sum and individual data rates of two users using NOMA are shown for different small power coefficient ($\rho_2$) values. For comparison, the TDMA rates are also shown for the same users if they were not served by NOMA. The reason TDMA rates slightly increases with $\rho_2$ is that as $\rho_2$ increases fewer user pairs can achieve the threshold for NOMA, and their corresponding TDMA rates are slightly higher. The NOMA provides a gain in the sum rate compared to TDMA for all cases, except the case where $\rho_2$ is nearly equal to zero. When $\rho_2$ is between 0.1 and 0.12, both users have data rate gain compared to
TDMA. As $\rho_2$ increases the sum NOMA rate slightly increases, however it causes an unfair allocation since the weak user’s data rate decreases even further. The power coefficients should be selected by considering the trade-off between fairness and higher sum rate.

Another parameter for designing NOMA is the SINR threshold that needs to be achieved to implement NOMA. The SINR for the second user to decode the signal of the first user is given in (5.26). A threshold $\xi_{2\rightarrow 1} \geq \xi_1^*$ should be chosen as a design parameter for NOMA preference over TDMA. Fig. 5.10(b) shows the ratio of NOMA users that achieves the threshold for different $\rho_2$ and $\xi_1^*$ levels. For small values of $\rho_2$, the SINR threshold is easily achieved even for a high threshold. Small $\rho_2$ causes the $\xi_{2\rightarrow 1}$ to be larger, which provides less error probability for successive interference cancellation. For larger values of $\rho_2$, the SINR threshold should be decreased to allow NOMA. This also increases the risk of erroneous interference cancellation. Fig. 5.10(a) suggests that the sum rate can be increased by increasing $\rho_2$. However, Fig. 5.10(b) also shows that high $\rho_2$ may cause most users to not to use NOMA, which may diminish the sum rate gain. Overall, making $\rho_2$ smaller, or making the power coefficient of the users as distinct as possible, provides fairer rate increase.
of users and decreases the probability of erroneous interference cancellation.

In Fig. 5.11(a), we present the CDF of NOMA user rates with optimized coefficients as in (5.34). The black line with circle markers shows the data rates of user pairs in case these users are served with TDMA. The solid line with lower data rate is for the weak user, and the dashed line with higher data rate is for the strong user. On the x-axis, between $10^7$ and $10^8$, each vertical line means a $10^7$ bps data rate. There is about a 10 Mbps data rate difference between weak and strong TDMA users. The red line with triangle markers shows the data rates for the same users when they are served by NOMA, and the NOMA coefficients are calculated to maximize the sum rate using (5.34). The strong user rate that is shown with dashed lines has a significant rate gain compared to TDMA in the high data rate region, which is the upper parts of the line. The weak user does not have better data rates than TDMA in the high rate region, but it is better at low rate region. Overall, the NOMA provides gain for some users, but it decreases the data rates for some other users, which might be both weak or strong user in the pair.

In Fig. 5.11(a), the blue line with diamond markers show the data rates for the same users...
when they are served by NOMA, and the NOMA coefficients are calculated to maximize the sum of the logarithm of the rates. In this case, the weak user has a significant data rate gain compared to TDMA in all regions. The strong user has a small data rate gain compared to TDMA in most regions. Only the bottom 18% of the strong users has a rate loss compared to TDMA, but they still do better than weak users. Overall, this NOMA scheme provides a significant data rate gain for all weak users, and a slight data rate gain for most strong users, with a small data rate loss for some strong users.

In Fig. 5.11(b), the CDF of the sum rates are shown for the same users in Fig. 5.11(a). Both NOMA schemes provide a significant sum rate gain over TDMA. The sum rate difference between TDMA and NOMA is about 10 Mbps for most users. The NOMA coefficients that maximize sum rate provides a slight sum rate gain over the coefficients that maximize the sum of the logarithm of the rates.
Chapter 6

Adaptive Kalman Tracking for Indoor Visible Light Positioning

In this chapter, we study localization and tracking of mobile VLC users. We assume the APs are not always available, and propose localization methods for any number of available APs. We track the user with adaptive Kalman Filter (KF). The conventional KF does not consider the accuracy of the estimation at each time instance. However, the accuracy might dramatically change if some of the APs are blocked and unavailable. In order to incorporate this information in the tracking, we implement adaptive KF and decide the adaptive coefficients based on AP availability.

This chapter is organized as follows. We present the AP configurations and localization models considered in the study in Section 6.1, while we introduce the considered localization methods Section 6.2. We discuss the KF procedures and implementation of adaptive filter in Section 6.3, and we present the simulation results in Section 6.4.
6.1 System Model

In this section we introduce the APs and localization models that is used in the simulations of this chapter.

Figure 6.1: Three example multi-LED AP architectures that are considered for the simulations. All transmitters are mounted at the corners of the room at the ceiling height, facing the center of the room with a 45\degree separation from all three of the walls. Fig. 1(a) shows the side view of 3-element transmitter, while Fig. 1(b) and Fig. 1(c) are the top views of 7-element and 19-element transmitters, respectively.

6.1.1 Access Points

This subsection explains the AP configurations we consider for the simulations in this manuscript. We consider APs with multiple co-located LEDs that have different orientations as examined in [97, 59, 24, 49]. Some of the example AP configurations are illustrated in Fig. 6.1. The Fig. 6.1(a) shows the configuration with 3 LEDs. The LEDs are vertically tilted with $\alpha$ degree from the center orientation of the AP. For APs with 4 to 7 LEDs, we use the configuration in Fig. 6.1(b), where one LED is in the middle, and other LEDs surround it with again $\alpha$ degree vertical tilt, and they are separated from each other with equal degrees on the horizontal plane. For APs with more than seven LEDs, we use the configuration in Fig. 6.1(c), where there is a single LED in the middle, 6 LEDs surround it in the first layer with $\alpha$ degree vertical tilt, and the rest of the LEDs surround them in the second layer with an additional $\beta$ degree vertical tilt. Again, the LEDs on any layer are separated from each other with equal degrees, with the exception of the 8 LED configuration that has a single LED on the second layer.
Figure 6.2: Different localization models based on the availability of the APs. Blue corner means the AP is available to the user, empty corner means the AP is blocked or not in the field of view of the receiver.

### 6.1.2 Multiple Localization Models

We use the channel model introduced in Chapter 2 for localizing the VLC users. Without loss of generality, we consider a square room with four available APs located at the corners of the room at the ceiling height. The APs are oriented so that they face the center of the room, with 45° separation from all three walls. Each AP has multiple co-located LEDs with different orientations as shown in Fig. 6.1. As the user moves around the room, some of the APs might be blocked due to intervening objects, changing orientation of the receiver [57], or the limited FOV of the PD. The receiver then localizes itself using the signal that it receives from all the available APs.

As some of the APs are blocked, or not in the FOV, we can consider six different models for the accessible APs as shown in Fig. 6.2. We refer to the case with no available APs as Model-0. In this case, there cannot be any location measurements, and localization has to rely solely on the prediction from the previous location estimates. Model-1 refers to the one available AP case, in which the localization will be done as we will describe in Section 6.2.2. For the rest of the models, the hybrid localization in [7] will be used as we will describe in Section 6.2.1. Model-2 is the case where there are two available APs on adjacent corners.
Model-3 is the case where two available APs are on the corners that are across each other. Model-4 is the case with 3 available APs, and Model-5 is the case where all four APs are available.

6.2 Localization Methods and AP Availability

In this section, we explain the localization method when there are at least two available APs, and when there is a single available AP, respectively.

6.2.1 Two or More APs

There are many available localization methods in the literature for the case with multiple APs [7, 36, 49]. In this study, we use the hybrid localization method proposed in [7]. This method uses received signal strength information from all LEDs which is modeled as in (2.1), and utilizes a maximum likelihood (ML) estimation. The iterative solution in this method requires an initial point. As the initial point, the angle of arrival (AOA) based localization result in [36] is used, hence the name hybrid localization. The hybrid localization provides high accuracy results especially when there are a large number of APs and LEDs available.

6.2.2 Single AP

When there is only a single AP available at a given time, neither ML estimation nor AOA based localization works. In this case, we use the AOA information from the single AP to estimate the location of the receiver. In most indoor applications, the mobile receiver is either on a hand-held device or placed on a desk, and the AP is located higher than the receiver. By assuming a constant height for the receiver, and using the AOA information
from the available AP, we can estimate the user location by finding the intersection of a vector and a plane. Assume that the available AP is the $k$th AP, and the estimated AOA information from the $k$th AP is $\mathbf{p}_k = [p_k^{(x)}, p_k^{(y)}, p_k^{(z)}]$, which is a unit vector that points to the direction of the receiver from the AP. The height of the receiver is denoted by $\nu$. Then, the estimated location of the user is the intersection of $\mathbf{p}_k$ and the horizontal plane defined by $\nu$, which is given as

$$r'_R = r_k - \mathbf{p}_k (\nu - r_k^{(z)}) / p_k^{(z)},$$

where $r'_R$ is the estimated location of the receiver, $r_k$ is the location of available AP, and $r_k^{(z)}$ is the height of it. The AOA information from the AP can be estimated by a weighted average of the orientation vectors of the LEDs on the AP, where the weight is the signal strength that the user receives from that LED [7]. The normalized AOA estimation vector can therefore be written as

$$\mathbf{p}_k = \frac{\bar{\mathbf{p}}_k}{\|\bar{\mathbf{p}}_k\|}, \text{ where } \bar{\mathbf{p}}_k = \sum_{n=1}^{N} h_{kn} q_{kn},$$

with $N$ being the number of LEDs on the $k$th AP.

### 6.3 Tracking the Mobile VLC User

In this section, we present our conventional and adaptive KF models and techniques for tracking a mobile VLC user.

#### 6.3.1 Conventional Kalman Filter

The KF is a Bayesian filter that can track the VLC user by exploiting the prior location information. Let $(x_t, y_t, z_t)$ denote the location estimate of the user at time step $t$, and
\((V_x^t, V_y^t, V_z^t)\) denote the velocity of the user in \(x\), \(y\) and \(z\) coordinates during time step \(t\). Then, the prediction stage for the KF estimate for user location and velocity at time instant \(t\) can be written as

\[
\begin{bmatrix}
    x_t \\
    y_t \\
    z_t \\
    V_x^t \\
    V_y^t \\
    V_z^t
\end{bmatrix}
= \begin{bmatrix}
    1 & 0 & 0 & 1 & 0 & 0 \\
    0 & 1 & 0 & 0 & 1 & 0 \\
    0 & 0 & 1 & 0 & 0 & 1 \\
    1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    x_{t-1} \\
    y_{t-1} \\
    z_{t-1} \\
    V_x^{t-1} \\
    V_y^{t-1} \\
    V_z^{t-1}
\end{bmatrix}
+ \begin{bmatrix}
    e_x \\
    e_y \\
    e_z \\
    e_{V_x} \\
    e_{V_y} \\
    e_{V_z}
\end{bmatrix},
\]

where the \(\hat{x}_t\) is the prediction of state at time \(t\), \(B\) is the state transition matrix, \(x_{t-1}\) is the estimation from previous state, and \(e_x \sim \mathcal{N}(0, E_x)\) is the estimation error vector. Moreover, let \((z_x^t, z_y^t, z_z^t)\) denote the observation of the user at time step \(t\) before applying the KF. Then, the observation equations can be written as

\[
\begin{bmatrix}
    z_x^t \\
    z_y^t \\
    z_z^t
\end{bmatrix}
= \begin{bmatrix}
    1 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    x_t \\
    y_t \\
    z_t \\
    V_x^t \\
    V_y^t \\
    V_z^t
\end{bmatrix}
+ \begin{bmatrix}
    v_x \\
    v_y \\
    v_z
\end{bmatrix},
\]

where the \(v \sim \mathcal{N}(0, E_v)\) is the observation error. The \(z_{t*}^{\text{est}}\) is only a temporal estimate. At time \(t\), we also get measurement values for \(x_t\), \(y_t\), and \(z_t\), and they form our measurement vector \(z_t^{\text{meas}}\). Then we calculate our final estimate using

\[
x_t = \hat{x}_t + K_t(z_t^{\text{meas}} - z_t^{\text{est}}),
\]

(6.3)
where $K_t$ is the Kalman gain. The Kalman gain is a matrix that provides balance between estimate from previous steps, and measurement from the localization at current step. It can be iteratively calculated at each step by

$$P_t = BP_t^T + E_x, \quad (6.4)$$

$$K_t = P_tC^T (CP_tC^T + E_v)^{-1}, \quad (6.5)$$

$$P_{t+1}' = (I - K_tC)P_t, \quad (6.6)$$

where $E_x$ is the constant covariance matrix of $e_x$ and can be modeled as $\text{diag}(\sigma_x^2)$ assuming the errors of the parameters are independent and of similar amplitude. The $E_v$ is the covariance matrix of $v$ and can be modeled as $\text{diag}(\sigma_v^2)$, and $P$ is an error covariance matrix which is updated twice in an iteration.

In some prediction models, the transition between the previous state to the next one is not linear. For these nonlinear systems, extended Kalman filter (EKF) can be used which has a differential function for transition instead of a matrix. An improvement over EKF is the unscented Kalman filter (UKF), which does a better approximation of nonlinear transition. However, in our model, the transition is linear for the considered settings and can be represented with matrices $B$ and $C$. In this case, EKF and UKF are identical to the conventional KF and do not promise any performance improvement over it. In the next subsection, we discuss the adaptive KF.

### 6.3.2 Adaptive Filtering

The conventional KF assumes that we do not have information about the accuracy of the measurement at each state, and it keeps track of the estimated error with covariance matrix $P$. However, the localization method or available number of APs might change at any time, and we can use such information to improve the tracking performance with adaptive
filtering. Since we have classified all possible estimation configurations under 6 different models as shown in Fig. 6.2, we can represent the adaptive filter as follows. The KF is implemented for each model as explained in Section 6.3.1, with a difference where we replace the $E_v$ in (6.5) with model order dependent $E^j_v$. The $3 \times 3$ covariance matrix $E^j_v$ for the adaptive filter can be expressed as follows

$$E^j_v = \text{diag}(\sigma_v^2/\eta(j)),$$

(6.7)

where $\sigma_v^2$ represents the variance of the error in the location observation and $\eta(j)$ is the adjustment parameter for each localization model. For models with high accuracy, we expect $\eta(j)$ to be high, which causes the diagonals of $E^j_v$ to be low. This will result a $K_t$ with higher diagonals in (6.6), implying that a low measurement error is expected. For example, we should use a high $\eta(5)$ due to the high accuracy of Model-5, and low $\eta(1)$ due to the expected low accuracy of Model-1. The $\eta(0)$ can be set to a very small positive number that is close to zero, implying that the measurement error is very high and should not be weighted in at all. Note that by using a diagonal $E^j_v$ we assume the estimation error in different directions is uncorrelated as considered in the conventional KF model.

In the next step of the adaptive KF, a weighted combination of updated state estimates is obtained, yielding a final estimate for the state $x_t$ as in (6.3), and a covariance state $P'_{t+1}$ as in (6.6). Based on different covariance matrices $E^j_v$ in (6.7), the corresponding $x_t$ and $P'_{t+1}$ can be written as follows:

$$x_t = \tilde{x}^{(j)}_t \quad \text{and} \quad P'_{t+1} = \tilde{P}'^{(j)}_{t+1},$$

(6.8)

where $\tilde{x}^{(j)}_t$ is the state estimate and $\tilde{P}'^{(j)}_{t+1}$ is the state covariance matrix for Model-$j$ used in state $t$. Using the covariance matrix as in (6.8) for different models provides a weighted balance between prediction and measurements. In particular, the prediction is weighted
more when the model provides a low accuracy, and the measurement is weighted more when it promises a high accuracy.

6.3.3 Coefficients of Adaptive Filter

Calculation of $E^j_v$ for adaptive KF is studied in [55, 98]. However, these studies do not assume certain estimation models as we do in this study. Instead, they calculate the optimal $E^j_v$ or $E^j_x$ matrices at each filter state with up to date parameters. Moreover, the maximum likelihood (ML) estimation used in these studies has high computation complexity, and it is not possible to use them in real time without high computing power. Our problem differentiates from these studies by simplifying the adapting conditions to six distinct models unique to indoor VLC scenarios. We also simplify the problem of finding $E^j_v$ to finding coefficients $\eta(j)$ in (6.7), by assuming the estimation error in different directions being uncorrelated. Therefore we follow a different approach to find these coefficients.

While calculating the coefficients $\eta(j)$, we need to give higher weight to the models with lower localization errors. To examine this, we have simulated the average localization error for each localization model, which are shown in Fig. 6.3. The mean errors are calculated by changing the location of the receiver and calculating the localization error, and averaging it all over the room for different localization models and different number of LEDs on each AP. The simulation parameters are as shown in Table 6.1. Model-0 is not included as this model does not provide any location estimation and relies only on prediction. As the figure shows, the models with a higher number of APs provide a lower error. The only exception is that Model-1 performs better than Model-2 for 3 LEDs case. This is because the ML estimation in [7] needs a large number of LEDs and APs to perform properly. While comparing the models with two available APs, we see that Model-3 performs better than Model-2. The reason for that is having both APs side by side may cause a bias in the estimation, i.e., the user can be estimated to be closer to the APs than the actual location. All models perform
better when the number of LEDs on APs are higher, with a few exception data points.

A similar problem with distinct estimation models is studied in [56], where the authors heuristically chose the coefficients so that each model has half the coefficient of the one with one higher accuracy. Relying on a similar approach, we choose the coefficients of our model as follows:

$$\eta(0) = \frac{1}{32}, \quad \eta(1) = \frac{1}{16}, \quad \eta(2) = \frac{1}{8}, \quad \eta(3) = \frac{1}{4}, \quad \eta(4) = \frac{1}{2}, \quad \eta(5) = 1. \quad (6.9)$$

The coefficient of the Model-5 is chosen to be equal to 1, because this model has all APs available, and setting $\eta(5)$ equal to 1 provides a fair comparison with conventional KF. Model-4 has half the coefficient, and it is halved for each model with one lower accuracy. Since the coefficients in (6.9) do not depend on the exact expected error of each model, we will call them fixed coefficients for the rest of the section.

The fixed coefficients in (6.9) do not use all the information about how accurate each model is. Since we know expected errors for each model, we can use this information to calculate better $\eta(j)$ coefficients. With this purpose, we propose a heuristic method, where the coefficient of each model is inversely proportional to the expected error of the model. Denote the $\omega(j,N)$ as the mean error for model $j$ and for $N$ LEDs per AP as shown in Fig. 6.3. Denote the $\tilde{\eta}(j,N)$ as the $\eta(j)$ for $N$ LEDs case. Then, we have

$$\tilde{\eta}(j,N) = \frac{\omega(5,N)}{\omega(j,N)}.$$

(6.10)

Note that the $\eta(5)$ is always equal to 1. For other models, the $\eta(j)$ is inversely proportional to the average error of Model-$j$.

In Fig. 6.3, the expected errors saturate as the number of LEDs increase. We can consider these saturation errors as the lower bound of each model. As a third option to decide the
adaptive coefficient, we can evaluate each model at the best performance, where

$$\eta_\infty(j) = \tilde{\eta}(j, \infty).$$  \hspace{1cm} (6.11)

For the rest of the simulations, we will use the coefficients in (6.9), (6.10), (6.11), and compare their performances to study the sensitivity of adaptive KF to different coefficients. The coefficients in (6.10) and (6.11) are calculated using the data given in Fig. 6.3.

6.4 Numerical Results

For simulations, we assume that a user walks in a random waypoint path. The user starts walking from one side of the room and visits waypoints that are randomly chosen. The height of the waypoints is also random with $\mathcal{U}(0.7, 1.1)$ m. The waypoints are chosen so that the turn angle cannot exceed 90°. If this condition is not satisfied, the waypoint is randomly selected again until it is satisfied. The path ends at the maximum length of 30 m, or when new waypoint selection is not possible. The user speed is 0.1 m per time step, and
therefore the VLC user’s location is estimated at each 0.1 m. Other simulation parameters are provided in Table 6.1. The simulations are repeated for 2000 routes and average results are provided.

The APs are assumed to be blocked with a random probability at each time instant. This is a scenario where the available APs change frequently (e.g., due to random receiver orientation [57]) and is chosen to show at what extent the adaptive filter can improve the accuracy. Fig. 6.4(a) shows the RMSE for localization over the path for increasing probability of each AP being blocked. The number of LEDs on each AP is 7, with the configuration shown in Fig. 6.1(b). The solid line shows the RMSE for unfiltered localization, the dashed line shows the RMSE after KF, and the dotted lines show the RMSE after adaptive KF.

The localization mode in Fig. 6.4(a) is decided based on available AP configuration at each time step as shown in Fig. 6.2. When the blocking probability is equal to zero, the estimation model is always Model-5. In this case, the conventional and adaptive filters are identical, and they do not provide a visible improvement on the accuracy over the unfiltered localization. The reason for that is, the error is already low, and it is not necessarily Gaussian distributed, so the KF does not necessarily provide improvement. When the blocking probabilities increases, the RMSE increases too. In this case, the KF improves accuracy significantly. Moreover, the adaptive filter decreases RMSE even more by utilizing the estimation model information. Using the heuristic in (6.10) or (6.11) provides additional gain.

### Table 6.1: Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LED directivity index, $\gamma$</td>
<td>10</td>
</tr>
<tr>
<td>The vertical tilt angle of LEDs, $\alpha$</td>
<td>$25^\circ$</td>
</tr>
<tr>
<td>Additional vertical tilt angle of LEDs, $\beta$</td>
<td>10°</td>
</tr>
<tr>
<td>Receiver FOV, $\Theta$</td>
<td>$25^\circ$</td>
</tr>
<tr>
<td>Effective surface area of a PD, $A$</td>
<td>1 cm$^2$</td>
</tr>
<tr>
<td>Room size</td>
<td>6×6×3 m</td>
</tr>
<tr>
<td>Default device height, $\nu$</td>
<td>0.9 m</td>
</tr>
<tr>
<td>The standard deviation of estimation error, $\sigma_x$</td>
<td>0.005</td>
</tr>
<tr>
<td>The standard deviation of measurement error, $\sigma_y$</td>
<td>0.05</td>
</tr>
</tbody>
</table>
(a) For varying probability of blockage for APs, while each AP has 7 LEDs as shown in Fig. 6.1(b).

(b) For varying number of LEDs per AP. The blocking probability of each AP is set to 0.25.

Figure 6.4: The RMSE for varying parameters.

compared to using fixed coefficients. The coefficients in (6.10) are $\tilde{\eta}(j, 7)$, which considers the mean error for 7 LED case. This coefficients yield lower error than $\eta_\infty(j)$, which shows that the proposed heuristic utilizes the expected error information accurately.

Fig. 6.4(b) shows the RMSE for varying number of LEDs per AP. Similar to Fig. 6.4(a), the receiver moves over the random waypoint path, and the blocking probability of any AP is 0.25. When the number of LEDs increases, the RMSE steadily decreases thanks to larger number of measurements that can be used for hybrid localization. The only exception is the 8 LED case, where there is a single LED at the outermost layer in Fig. 6.1(c), which might be causing bias in the estimation. This is also visible in Fig. 6.3 with Model-2 to Model-4, where the 8 LED case performs worse than 7 LED case. Overall, on Fig. 6.4(b) we see that the adaptive KF provides a significant improvement over the conventional KF for any LED configuration. The RMSE of adaptive KF is about 25% to 40% of unfiltered localization RMSE for any number of LEDs. Using the coefficients $\tilde{\eta}(j, N)$ provides the least RMSE in general. As $N$ goes to 20, $\tilde{\eta}(j, N)$ and $\eta_\infty(j)$ becomes identical, and they yield the same RMSE.
Chapter 7

Conclusion

In this dissertation, we investigate different challenges of multi-element VLC systems. First, we investigate LED-user assignment problem in a downlink VLC scenario where multiple LEDs serve multiple users in Chapter 3. We study suboptimal but computationally efficient LED assignment algorithms and power optimization techniques while taking proportional fairness and QoS requirements into account. Our simulation results show that the investigated LED assignment algorithms with equal power distribution among LEDs perform almost as well as the optimal resource allocation schemes. In addition, power control techniques are shown to provide substantial gains in sum rate and fairness, especially for a larger number of users. We introduce an improved method for optimal diversity combining at a receiver, that considers the LED grouping information at the transmitter. With the new approach for calculating the combining weights, SINR gains between 2 dB to 5 dB are obtained in all scenarios.

Second, we investigate the statistics of a VLC downlink channel when the orientation of the user is varying randomly around the vertical axis in Chapter 4. Mobility is also considered through the random deployment of the user which results in a random distance to the source LED. The random fluctuations in the vertical user orientation can adversely affect the achiev-
able user data rate. We propose an analytical framework which successfully characterizes the channel statistics when both the vertical orientation and the user location are randomly varying. This analytical framework serves as a practical basis to develop strategies to deal with the destructive effects of random vertical orientation with a random user location. Our results show that for a receiver horizontally located 2.5 meters away from the transmitter with an orientation angle of 30°, random deviations in the receiver orientation/location results in more than 7 dB of SNR loss at a BER of $10^{-3}$ dB for a wide FOV. For a narrow FOV, the effects are even more catastrophic, where the BER quickly converges to an error floor as the transmit power increases.

Third, we study the optimal beam steering parameters for VLC when there are more users than the steerable components in Chapter 5. We find the optimal steering angles and LED directivity for a single LED and multiple users. The results show that steering VLC beams and changing the directivity can improve the user rates significantly. Although serving a single user maximizes the user rates, multiple users can also be served using a single steerable beam with a significant sum rate gain over no steering scheme. We also propose a method for decreasing the search space, thus the computation time of the optimization solution. This method decreases the search space to 1%-90% depending on user distribution. In case of a multiple steerable beam setting, we cluster users and serve each cluster with a separate beam. This setting allows higher data rates by clustering close users together and providing more accurate steering. Additionally, we optimize the transmit power of each beam to increase the sum rate or proportionally fair sum rate. With the clustering and power optimization, the sum rate can reach ten times the sum rate of no steering scheme. Finally, we propose a user clustering and NOMA scheme to utilize the space diversity of the users and further increase the data rates. NOMA can provide an additional 10 Mbps rate gain for two users that are paired together.

Last, we propose the use of adaptive KF for multi-element VLP systems with intermittent
AP availability in Chapter 6. The VLP heavily depends on LOS links, in whose absence, an AP could be completely inaccessible. Due to frequent change in the available APs to the user, localization accuracy may also change frequently. The adaptive KF takes this into account and makes a final estimation considering the different localization methods and expected accuracies. To evaluate the adaptive KF, we consider a scenario where APs can be randomly blocked, and choose our localization models based on available AP configuration. We use simulations to decide the localization accuracy in each configuration, and as a heuristic, we choose the adaptive coefficient of each model inversely related to their expected error. The simulation results show that the adaptive implementation improves the performance of the KF significantly and yields much lower RMSE in all considered scenarios.
Bibliography


Appendix A

Derivatives of Rate Function

In this appendix, we derive the first and the second-order derivatives of the rate $R_k$. We begin with expressing the SINR expression as $SINR(k) = S_{kk}^2/(T_k - S_{kk}^2)$, where the first-order derivatives of $S_{tk}$ and $T_k$ with respect to the power coefficients are $\partial s_{tk}/\partial p_m = h_{km} \delta(\ell, f(m))$ and $\partial T_k/\partial p_m = 2s_{tk}h_{km} \delta(\ell, f(m))$. Taking derivative of $R_k$ in (3.4) is then given as

$$\frac{\partial R_k}{\partial p_m} = \frac{B/\ln 2}{1 + SINR(k)} \frac{\partial SINR(k)}{\partial p_m}, \quad (A.1)$$

where

$$\frac{\partial SINR(k)}{\partial p_m} = \begin{cases} \frac{2S_{kk}h_{km}}{T_k - S_{kk}^2}, & k = \ell \\ \frac{-2S_{tk}h_{km}}{T_k - S_{kk}^2} SINR(k), & k \neq \ell \end{cases} = \frac{2S_{tk}h_{km}}{T_k - S_{kk}^2} C_{km}. \quad (A.2)$$

Realizing $[1+SINR(k)]^{-1} = (T_k - S_{kk}^2)/T_k$ and employing (A.2) in (A.1) obtain the first-order derivative of $R_k$ in (3.12). 

\[ \blacksquare \]
The second-order derivative of $R_k$, which is given as
\[
\frac{\partial^2 R_k}{\partial p_m \partial p_n} = \frac{B}{\ln 2} 2h_{km} \frac{\partial}{\partial p_n} \left( \frac{S_{tk}}{T_k} C_{mk} \right),
\]
(A.3)
can be evaluated by examining the following two conditions where we employ the derivative of the SINR given in (A.2) when necessary.

**Case 1.** Assuming $\ell = \ell'$, where $\ell = f(m)$ and $\ell' = f(n)$, we have
\[
\frac{\partial}{\partial p_n} \left( \frac{S_{tk}}{T_k} C_{mk} \right) = \begin{cases} 
  h_{kn} \frac{T_k - 2S_{tk}^2}{T_k^2}, & k = \ell \\
  h_{kn} \frac{\text{SINR}(k)}{T_k} \left( \frac{2S_{tk}^2}{T_k - S_{kk}^2} - \frac{T_k - 2S_{tk}^2}{T_k} \right), & k \neq \ell
\end{cases}
\]
(A.4)

**Case 2.** When $\ell \neq \ell'$, the desired derivative becomes
\[
\frac{\partial}{\partial p_n} \left( \frac{S_{tk}}{T_k} C_{mk} \right) = S_{tk} \frac{\partial}{\partial p_n} \left( \frac{C_{mk}}{T_k} \right) = \begin{cases} 
  -\frac{2S_{tk}S_{\ell k} h_{kn}}{T_k^2}, & k = \ell \text{ or } k = \ell' \\
  2S_{tk}S_{\ell k} h_{kn} \text{SINR}(k) \left(2 + \text{SINR}(k)\right), & k \neq \ell, k \neq \ell'
\end{cases}
\]
(A.5)

Combining (A.4) and (A.5) in (A.3) obtains the second-order derivative of $R_k$ given in (3.13). ■
Appendix B

Proof of Lemma 1

In (4.25), there are two support intervals of $\theta$, which are $S_1 = (a, b]$ and $S_2 = (c, d]$, and the function $\nabla_\theta (a, b, c, d)$ is equivalent to $\Pr \{ \theta \in S_1 \cap S_2 \}$. In order to have a nonzero value for this joint probability, the conditions $a \leq b$ and $c \leq d$ have to be satisfied simultaneously, which guarantees that the support intervals are non-empty sets. In such a case, there are six different possibilities for the intersection of $S_1$ and $S_2$, as shown in the illustration of Fig. B.1.

Realizing that for the first two cases in Fig. B.1, represented with the labels (1) and (2), the intersection of the support sets is empty, i.e., $S_1 \cap S_2 = \emptyset$. In order to circumvent these two cases, we introduce two additional conditions $d > a$ and $c < b$ to be satisfied. As a result, all the conditions for non-empty intersection can be written jointly as

$$\{a, c \mid a < \min(b, d), c < \min(b, d)\}.$$  \hfill (B.1)
For the cases (3)-(6) in Fig. B.1, the non-empty intersections are given as follows

\[
S_1 \cap S_2 = \begin{cases} 
(a, b) & \text{for } c \leq a, d > b \quad \text{(Case 3)} \\
(c, d) & \text{for } c > a, d \leq b \quad \text{(Case 4)} \\
(a, d) & \text{for } c \leq a, d \leq b \quad \text{(Case 5)} \\
(c, b) & \text{for } c > a, d > b \quad \text{(Case 6)}
\end{cases} 
\]

which directly comes from the geometrical comparison of the support sets \( S_1 = (a, b) \) and \( S_2 = (c, d) \). Because \( \Pr\{\theta \in (x, y]\} = F_\theta(y) - F_\theta(x) \), we readily obtain (4.26) using (B.2) and the cdf function \( F_\theta(\cdot) \). Note that, the cases in (B.2) satisfy the non-empty intersection condition in (B.1).
Appendix C

Proof of Theorem 4.1

In order to prove the cdf expression in Theorem 4.1, we first represent the events defined in (4.24) by using the transformation $z_i(x) = \frac{1}{2} \cos^{-1}(2 \frac{x}{c_i} - 1)$ for $i = 1, 2$ as follows

$$E_{z_i} : \{ z_i(x) \in \Omega_{z_i} | z_i(x) < |\theta_i| \}, \quad (C.1)$$

where $\Omega_{z_i}$ is the sample space of $z_i(x)$, and we employ the fact that $\cos(\cdot)$ is an even function.

In the following, we separately derive the expressions for the probabilities corresponding to each of the four cases in (4.23). Note that, because all these four cases represent disjoint events, the desired probability (4.27) is readily obtained by summing up their respective probability expressions via the law of total probability [80].

As a remark, since all of the four cases in (4.23) have inequalities of the form $|\theta_i| \leq \Theta$, which have the roots $\theta = \{0, \Theta\}$, we analyze each probability of interest by considering the three
disjoint support regions for the random angle $\theta$ given as

$$S^i_\theta = \begin{cases} 
(-\infty, 0] & \text{for } i = 1 \\
(0, \Theta] & \text{for } i = 2 \\
(\Theta, \infty) & \text{for } i = 3 
\end{cases} \quad (C.2)$$

**Case 1.** Consider $P_1(x) = \Pr \{E_{z_1}, E_{z_2}, |\theta_1| \leq \Theta, |\theta_1| \leq \Theta \}$. 

$$P_1(x) = \Pr \{z_1(x) < |\theta| < \Theta, z_2(x) < |\Phi - \theta| < \Theta \} = \sum_{i=1}^{3} P_1(x, \theta \in S^i_\theta), \quad (C.3)$$

where the individual probabilities $P_1(x, \theta \in S^i_\theta)$'s are computed with the help of the function $\nabla_\theta(\cdot)$ in (4.26) as follows

$$P_1(x, \theta \in S^1_\theta) = \Pr \{-\Theta \leq \theta < -z_1(x), \Phi - \Theta < \theta \leq \Phi - z_2(x), \theta < 0 \}$$

$$= \Pr \{-\Theta \leq \theta < -z_1(x), \Phi - \Theta < \theta \leq \Phi - z_2(x) \}$$

$$= \nabla_\theta(-\Theta, -z_1(x), \Phi - \Theta, \Phi - z_2(x)). \quad (C.4)$$

$$P_1(x, \theta \in S^2_\theta) = \Pr \{z_1(x) < \theta \leq \Theta, \Phi - \Theta < \theta \leq \Phi - z_2(x), 0 < \theta < \Phi \}$$

$$= \Pr \{z_1(x) < \theta \leq \Theta, \max(0, \Phi - \Theta) < \theta \leq \Phi - z_2(x) \}$$

$$= \nabla_\theta(z_1(x), \Theta, \max(0, \Phi - \Theta), \Phi - z_2(x)). \quad (C.5)$$

$$P_1(x, \theta \in S^3_\theta) = \Pr \{z_1(x) < \theta \leq \Theta, \Phi + z_2(x) < \theta \leq \Phi + \Theta, \theta > \Phi \}$$

$$= \Pr \{z_1(x) < \theta \leq \Theta, \Phi + z_2(x) < \theta \leq \Phi + \Theta \}$$

$$= \nabla_\theta(z_1(x), \Theta, \Phi + z_2(x), \Phi + \Theta). \quad (C.6)$$

As a result, (C.3) with (C.4)-(C.6) readily yields (4.28). \[\blacksquare\]
Case 2. Consider \( P_2(x) = \Pr\{E_{z_1}, |\theta_1| \leq \Theta, |\theta_2| > \Theta\} \).

\[
P_2(x) = \Pr\{z_1(x) < |\theta| \leq \Theta, |\Phi - \theta| > \Theta\} = \sum_{i=1}^{3} P_2(x, \theta \in S^i_\theta), \tag{C.7}
\]

where \( P_2(x, \theta \in S^i_\theta) \)'s are computed as follows

\[
P_2(x, \theta \in S^1_\theta) = \Pr\{-\Theta < \theta < -z_1(x), \theta < \min(0, \Phi - \Theta)\}
= \Pr\{-\Theta < \theta \leq \min(-z_1(x), \Phi - \Theta)\} = \Delta_\theta(-\Theta, \min(-z_1(x), \Phi - \Theta)). \tag{C.8}
\]

\[
P_2(x, \theta \in S^2_\theta) = \Pr\{z_1(x) < \theta < \Theta, 0 < \theta \leq \Phi - \Theta\}
= \Pr\{z_1(x) < \theta < \min(\Theta, \Phi - \Theta)\} = \Delta_\theta(z_1(x), \min(\Theta, \Phi - \Theta)). \tag{C.9}
\]

\[
P_2(x, \theta \in S^3_\theta) = \Pr\{z_1(x) < \theta < \Theta, \theta > \Phi + \Theta\} = 0. \tag{C.10}
\]

Similarly, \( (C.7) \) with \( (C.8)-(C.10) \) yields \( (4.29) \). \qed

Case 3. Consider \( P_3(x) = \Pr\{E_{z_2}, |\theta_1| > \Theta, |\theta_2| \leq \Theta\} \).

\[
P_3(x) = \Pr\{z_2(x) < |\Phi - \theta| \leq \Theta, |\theta| > \Theta\} = \sum_{i=1}^{3} P_3(x, \theta \in S^i_\theta), \tag{C.11}
\]

where \( P_3(x, \theta \in S^i_\theta) \)'s are given as follows

\[
P_3(x, \theta \in S^1_\theta) = \Pr\{\Phi - \Theta < \theta < \Phi - z_2(x), \theta < -\Theta\} = 0. \tag{C.12}
\]

\[
P_3(x, \theta \in S^2_\theta) = \Pr\{\Phi - \Theta < \theta < \Phi - z_2(x), \Theta < \theta < \Phi\}
= \nabla_\theta(\Phi - \Theta, \Phi - z_2(x), \Theta, \Phi). \tag{C.13}
\]
\[ P_3 (x, \theta \in S_\theta^3) = \Pr\{ \Phi + z_2(x) < \theta < \Phi + \Theta, \theta > \Theta, \theta > \Phi \} \]
\[ = \Pr\{ \max(\Phi + z_2(x), \Theta) < \theta \leq \Phi + \Theta \} \]
\[ = \triangle_\theta(\max(\Phi + z_2(x), \Theta), \Phi + \Theta). \quad \text{(C.14)} \]

As before, (C.11) with (C.12)-(C.14) yields (4.30).

**Case 4.** \( P_4 = \Pr \{ |\theta_1| > \Theta, |\theta_2| > \Theta \} \).

\[ P_4 = \Pr\{ \theta < -\Theta, \Phi - \theta > \Theta, \theta < 0 \} + \Pr\{ \theta > \Theta, \Phi - \theta > \Theta, 0 < \theta < \Phi \} \quad \text{(C.15)} \]
\[ + \Pr\{ \theta > \Theta, \Phi - \theta < -\Theta, \theta > \Phi \} \]
\[ = \Pr\{ \theta < -\Theta \} + \Pr\{ \Theta < \theta < \Phi - \Theta \} + \Pr\{ \theta > \Phi \} \]
\[ = \triangle_\theta(\Theta, \Phi - \Theta) + F_\theta(-\Theta) + 1 - F_\theta(\Phi + \Theta), \quad \text{(C.16)} \]

which verifies (4.31), and completes the proof of the cdf in (4.27).

In order to derive the desired pdf, we first observe that the cdf in (4.27) is composed of the functions \( \triangle_\theta(a,b) \) and \( \nabla_\theta(a,b,c,d) \), given in (4.9) and (4.26), respectively, for which the possible nonzero output terms involve the cdf of the random angle \( \theta \) appearing as \(-F_\theta(a), F_\theta(b), -F_\theta(c), \) and \( F_\theta(d) \). Because the partial derivative of \( \triangle_\theta(a,b) \) and \( \nabla_\theta(a,b,c,d) \) can be expressed as

\[ \frac{\partial}{\partial x} \triangle_\theta (a,b) = \begin{cases} \frac{\partial F_\theta(b)}{\partial x} - \frac{\partial F_\theta(a)}{\partial x} & \text{for } a \leq b, \\
0 & \text{otherwise} \end{cases} \quad \text{(C.17)} \]
and

\[
\frac{\partial}{\partial x} \nabla_\theta (a, b, c, d) = \begin{cases} 
\frac{\partial F_\theta(b)}{\partial x} - \frac{\partial F_\theta(a)}{\partial x} & \text{for } c \leq a, d > b \\
\frac{\partial F_\theta(d)}{\partial x} - \frac{\partial F_\theta(c)}{\partial x} & \text{for } c > a, d \leq b \\
\frac{\partial F_\theta(b)}{\partial x} - \frac{\partial F_\theta(c)}{\partial x} & \text{for } c \leq a, d \leq b \\
0 & \text{otherwise}
\end{cases}
\]

(C.18)

any individual derivative in (4.32) can be computed using (4.33) with proper choice of the transformation variables \( u \) and \( v \). Note that for constant entries which are function of only \( \Theta \) and \( \Phi \), we have \( v = 0 \) by definition, and the partial derivative in (4.33) yields 0.

As an example, the derivative of (C.4), which is actually the first term of the desired derivative \( \frac{\partial P_1(x)}{\partial \theta} \) to be involved in the pdf expression in (4.32), can be given by (C.18) where

\[
a = -\Theta \rightarrow (u, v) = (-\Theta, 0) \text{ and } \frac{\partial F_\theta(a)}{\partial x} = 0,
\]

\[
b = -z_1(x) \rightarrow (u, v) = (0, -1) \text{ and } \frac{\partial F_\theta(b)}{\partial x} = [4x (c_1 - x)]^{-\frac{1}{2}} f_\theta (-z_1(x)),
\]

\[
c = \Phi - \Theta \rightarrow (u, v) = (\Phi - \Theta, 0) \text{ and } \frac{\partial F_\theta(c)}{\partial x} = 0,
\]

\[
d = \Phi - z_2(x) \rightarrow (u, v) = (\Phi, -1) \text{ and } \frac{\partial F_\theta(d)}{\partial x} = [4x (c_2 - x)]^{-\frac{1}{2}} f_\theta (\Phi - z_2(x)).
\]

All the other required derivatives for (4.32) can be computed similarly. For the situations where the argument of the cdf \( F_\theta(\cdot) \) involve \( \min(\cdot) \) and \( \max(\cdot) \) functions, the above strategy should be applied to the argument qualifying to be the minimum or the maximum, respectively.

As a final remark, the effective channel can only be zero when both LEDs are outside the receiver FOV, where this situation is represented by \( P_4 \) in the cdf expression (4.27). Because this intuition implies that \( \Pr\{h_{\text{eff}}^2 = 0\} = P_4 \), and since \( \Pr\{h_{\text{eff}}^2 < 0\} = 0 \), the cdf function has
a discontinuity at $x = 0$, which appears as a Kronecker delta $\delta(x)$ in the pdf expression (4.32) with the magnitude $P_4$. ■
Appendix D

Proof of Propositions

D.1 Proof of Proposition 1

Consider the LOS channel gain between the transmitter LED and the $k$-th receiver in (5.4). The only parameter in (5.4) affected by the orientation is $\phi_k$, which is the angle between the LED orientation and the vector $v_k$ from LED to the $k$-th receiver. When the LED points to the receiver, $\phi_k = 0$, and the channel gain is maximized. When $\phi_k$ increases, the channel gain decreases.

Now consider that we have two users in the system as shown in Fig. D.1(a), and denote them user-1 and user-2. Let the line segment between the locations of two users is denoted by $\mathcal{K}$. Let $\phi_{1-2}$ be the angle between $v_1$ and $v_2$, the vectors towards user-1 and user-2 from the LED, respectively. Note that, the $\phi_{1-2}$ is independent of the orientation of the beam. First, assume that the LED is either pointing to user-1 or user-2, or somewhere on $\mathcal{K}$. In this case $\phi_{1-2} = \phi_1 + \phi_2$. When the LED is steered towards user-1, the $\phi_1 = 0$, and $\phi_2 = \phi_{1-2}$.

Now assume that the LED is steered to a point not on the line segment $\mathcal{K}$. This steering is not Pareto efficient because $\phi_1 + \phi_2 > \phi_{1-2}$. We can find a steering orientation with smaller
Figure D.1: Steering the LED towards users in two or three user scenarios. In both scenarios, steering the LED towards point B instead of point A provides higher LOS channel gain to all users in the system.

\( \phi_1 \) and \( \phi_2 \) if the LED is steered towards \( K \). In Fig. D.1(a) we illustrate a scenario where the LED is steered towards point A, which is not on \( K \). If the LED is steered towards point B (the projection of point A onto \( K \)) instead of point A, both angles \( \phi_1 \) and \( \phi_2 \) will decrease, which means (based on (5.4)) that both users will have higher channel gains. Therefore steering the LED towards point A cannot be optimal. The optimal steering angles that maximize (5.6) point towards either one of the two users, or somewhere on \( K \).

D.2 Proof of Proposition 2

In case there are three users that are not on the same line, the optimal steering orientation of the LED that is described in (5.6) has to point somewhere within the triangle defined by the user locations. Let us denote the triangle \( L \). For any orientation of the LED that does not point to \( L \), we can find some orientation that points to it and has smaller \( \phi_1 \), \( \phi_2 \), and \( \phi_3 \). To prove that, assume the LED is steered towards point A, which is not on \( L \) but on the plane that includes \( L \). Now change the intersection point to the closest point to A within \( L \), and steer the LED towards that point. All three angles \( \phi_1 \), \( \phi_2 \), and \( \phi_3 \) will decrease, which
means the previous steering angle was not Pareto efficient. It can be seen in Fig. D.1(b) that, steering the LED towards point B provides all three angles to be smaller compared to point A. If the LED orientation does not intersect the plane at all, the steering is not good and should be changed towards the user locations.

Similarly, if there are more than 3 users that are on the same plane, the optimal steering angle points to somewhere within the convex hull of location points. It can be shown using the same method applied to two user and three user cases.