ABSTRACT

WANG, SHUYANG. Essays on the Principal-Agent Problem in Agricultural Contracts. (Under the direction of Tomislav Vukina and Zheng Li).

The three essays in this dissertation consider the principal-agent problem in agricultural production contracts.

In Chapter 2, we set up a principal-agent model for the integrator and grower relation in the hog production industry. We assume that the grower is risk-neutral but is subject to the limited liability constraint. We derive the optimal effort level, the incentive effects of the contract parameters and optimal contract parameters. The main goal is to analyze why the principal adjusted the contract parameters during the period covered by the hog production contract data. We first test whether there has been technological change represented by the parameter in our model, then evaluate how the principal should respond. The result shows that the principal did not adjust both contract parameters in the optimal direction.

In Chapter 3, we propose a nonparametric approach to identify the unobserved heterogeneity in cardinal tournament by exploiting the recent advancement in measurement error models. We show that the unobserved heterogeneity in growers’ abilities is nonparametrically identified, and so is the output distributions conditional on growers’ abilities. We apply the proposed method to broiler production tournament data and investigate whether integrators discriminate among growers by allocating variable quality inputs based on their abilities. The empirical results indicate that there exists (1) heterogeneity in growers’ abilities, and (2) input discrimination.

In Chapter 4, we compare the profitability of a two-part piece-rate cardinal tournament and an alternative cardinal tournament where bonuses and penalties exactly cancel out. This paper is motivated by the speculation that more able growers will excel in both the settlement cost margin and the weight gain margin, leading to bonuses outweigh penalties. We analyze whether the principal is losing money with the existing contract compared to an alternative contract. Using broiler production contract data, we estimate the structural model with heterogeneous growers. We use the estimates to conduct counterfactual analysis. The result shows that the principal’s profit is higher under the alternative contract mainly driven by the lower grower payments.
Essays on the Principal-Agent Problem in Agricultural Contracts

by
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A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Economics

Raleigh, North Carolina
2019

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Co-chair of Advisory Committee Co-chair of Advisory Committee
DEDICATION

To my parents, family and friends.
BIOGRAPHY

Shuyang Wang was born and raised in Henan, China. In 2014, she received her bachelor's degree in Economics from Shanghai University of Finance and Economics. In fall 2014, she started pursuing a PhD in Economics at North Carolina State University.
ACKNOWLEDGEMENTS

I would like to thank my co-advisors Dr. Tomislav Vukina and Dr. Zheng Li for their insightful guidance, diligent advising and kind support. They are always supportive, responsive and willing to help whenever I come across difficulties in my research. Without their help, I would not have been able to make it.

I would like to thank my committee members Dr. Xiaoyong Zheng, Dr. Umut Dur for their time and valuable comments. I would also like to thank all the faculty and staff members in Department of Economics and Department of Agricultural and Resource Economics at NC State University.

Last but not least, I would like to thank my family and friends for their unconditional support and love. Thank you, Wenhao, for making my PhD life a better one.
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It has become common practice to use contracts to vertically coordinate the production and marketing of agricultural commodities in agricultural sectors. The moral hazard problem is prevalent in agricultural contracts. The integrator company contracts with growers to grow animals; growers choose their effort level, which affects the performance such as the weight gain, feed conversion and survivability. Effort is costly to growers and unobservable. The integrator relate compensation to performance. However, the compensation scheme still entails a loss since performance is a noisy signal of effort. The three essays in this dissertation analyze the principal-agent problem with heterogeneous agents in production tournaments.

In Chapter 2, the main goal is to analyze why the principal adjusted the contract parameters during the period covered by the hog production contract data. In the theoretical model, we set up a principal-agent problem for the integrator and grower relation in the hog production industry. We assume that the grower is risk-neutral but is subject to the limited liability constraint. We derive the optimal effort level, the incentive effects of the contract parameters and optimal contract parameters. Over the period of the dataset, there has been
a change in contract parameters. Since contract parameters provide incentives for growers to exert effort, we analyze the incentive effects of the parameter change and explain what might have caused the observed contract parameter change. With the parameter estimates, we first test whether there has been technological change represented by the parameter in our model, then evaluate how the principal should respond. The result shows that the principal did not adjust both contract parameters in the optimal direction.

Chapter 2 is structured as follows. In Section 2.2, we provide stylized facts of the hog industry and describe the dataset. In Section 2.3, we present several canonical models of moral hazard as a background. In Section 2.4, we present the theoretical model. Section 2.5 is the empirical part of the paper. Section 2.6 is the conclusion.

In Chapter 3, we switch to broiler production contracts and detect the unobserved heterogeneity from the data. When analyzing players' decision problems, researchers often cannot observe some of the inputs that play a role in their decision-making. Those inputs are referred to as unobserved heterogeneity. Contract is an economic context in which identifying unobserved heterogeneity is an important issue. We propose a nonparametric approach to identify the unobserved heterogeneity in cardinal tournament by exploiting the recent advancement in measurement error models. We show that the unobserved heterogeneity in growers' abilities is nonparametrically identified, and so is the output distributions conditional on growers' abilities. We apply the proposed method to broiler production tournament data and investigate whether integrators discriminate among growers by allocating variable quality inputs based on their abilities. The empirical results indicate that there exists (1) heterogeneity in growers' abilities, and (2) input discrimination. Chapter 3 is organized as follows. Section 3.2 presents the theoretical model of the principal-agent problem in the broiler production tournaments. Section 3.3 discusses the nonparametric identification and estimation of the output distributions conditional on type. Section 3.4 conducts the empirical analysis where we detect the heterogeneity from the data, determine the number of types, recover the conditional output distributions, estimate the theoretical model and test the existence of agent discrimination. Section 3.5 presents the counterfactual analysis of principal's input allocation strategies. Section 3.6 concludes.

In Chapter 4, we compare the profitability of a two-part piece-rate cardinal tournament and an alternative cardinal tournament where bonuses and penalties exactly cancel out. In two-part piece-rate tournament, the bonus payment is calculated as the difference
between group average settlement cost and grower’s individual settlement cost multiplied by the weight gain. Thus, it could be the case that the winners of a tournament have greater differences between group average settlement cost and grower individual settlement cost or greater weight gain or both than those who happen to be on the losing end of the tournament, resulting in the overall bonuses being greater than the overall penalties. This chapter is motivated by the speculation that more able growers will excel in both the settlement cost margin and the weight gain margin, leading to bonuses outweigh penalties. We analyze whether the principal is losing money with the existing contract compared to an alternative contract. Using broiler production contract data, we estimate the structural model with heterogeneous growers. We use the estimates to conduct counterfactual analysis. The result shows that the principal’s profit is higher under the alternative contract mainly driven by the lower grower payments.

The structure of Chapter 4 is as follows. Section 4.2 presents the agent’s problem in the original contract. Section 4.3 solves for the optimal effort and contract parameters in the alternative contract. Section 4.4 conducts the empirical analysis. Section 4.5 is the counterfactual study where we compare the profitability of the existing contract with the alternative contract. Section 4.6 concludes.
In this paper, we set up a principal-agent model for the integrator and grower relation in the hog production industry. We assume that the grower is risk-neutral but is subject to the limited liability constraint. We derive the optimal effort level, the incentive effects of the contract parameters and optimal contract parameters. The main goal is to analyze why the principal adjusted the contract parameters during the period covered by the hog production contract data. We first test whether there has been technological change represented by the parameter in our model, then evaluate how the principal should respond. The result shows that the principal did not adjust both contract parameters in the optimal direction.
2.1 Introduction

It has become common practice to use contracts to vertically coordinate the production and marketing of agricultural commodities in agricultural sectors. In this paper, we model agricultural production contract in the principal-agent moral hazard problem framework with heterogeneous agents. Specifically, the dataset that we use is from hog production contract. The hog production industry is vertically integrated. Integrator companies contract with growers to grow hogs to desired market weight. The contract specifies the compensation scheme and the division of responsibilities for providing inputs.

The moral hazard problem is prevalent in the agricultural production contracts. The integrator company contracts with growers to grow animals; growers choose their effort intensity, which affects the performance such as the weight gain, feed conversion and survivability. The integrator is only interested in the performance. Effort is costly to growers and the integrator compensates growers for incurring effort-related costs. Since effort is unobservable, the best the integrator can do is to relate compensation to performance. This compensation scheme still entails a loss since performance is only a noisy signal of effort.

There are three margins of performance in the hog production contract: the weight gain margin, the feed conversion margin and the survivability margin. All three margins enter the compensation scheme. In our model, we assume agents to be heterogeneous in the sense that the disutility of effort varies from agent to agent. Based on the compensation scheme of the hog production contract, we derive the optimal effort level and optimal contract parameters. Then we estimate the effort that growers exert by nonlinear least squares. Over the period of the dataset, there has been a change in contract parameters. Since contract parameters provide incentives for growers to exert effort, we analyze the incentive effects of the parameter change and explain what might have caused the observed contract parameter change.

This paper is mainly related to the literature on moral hazard and incentive effects. [28] compares contracting with independent market production and shows that growers who enter into production contracts reduce risks associated with variable income. [8] develop an analytical framework for the estimation of parameters of a structural model of an incentive contract under moral hazard with heterogeneous agents. They find that contracting growers are heterogeneous and the heterogeneity affects the principal’s allocation of production inputs across growers.
In the incentive effects literature, based on data from Safelite Glass Corporation [21] shows that when workers faced a new compensation scheme, they responded by altering effort, turnover, and labor-supply behavior in the way predicted by the theory. [33] measure the elasticity of worker effort with respect to changes in the piece rate using payroll records of a British Columbia tree-planting firm. The elasticity is estimated to be approximately 2.14. [32] develop an agency model of worker behavior under piece rates and fixed wages. They estimate an upper and lower bound to the incentive effect of paying workers piece rate rather than fixed wages to be in the 8.8 and 60.4 percent range of increase in productivity. Structural estimation suggests that incentives caused a 22.6 percent increase in productivity. However, they also find that only part of the productivity increase represents valuable output because workers respond to incentives, in part, by reducing quality.

This paper is also related to the literature that studies the effect of technological advances on contract. [16] examines how on-board computer use has affected capacity utilization in the trucking industry. [4] investigates how contractibility and truck ownership has changed with the diffusion of on-board computers in the trucking industry.

In our paper, we assume that the agent is risk-neutral but subject to a limited liability constraint. The limited liability constraint introduces ex post limitations on the minimum payment the agent can accept or the maximum penalty that can be imposed on him ([17]).

This paper proceeds as follows. In Section 2.2, we provide stylized facts of the hog industry and describe the dataset. In Section 2.3, we present several canonical models of moral hazard as a background. In Section 2.4, we present the theoretical model. Section 2.5 is the empirical part of the paper. Section 2.6 concludes.

### 2.2 Stylized facts and data

Traditionally, hogs were raised in farrow-to-finish operations in small diversified farms where hogs provided price risk protection for grain production ([37]). Hogs are now commonly produced by specialized operations that separate production facilities for each phase of production. Hog production is categorized into 3 phases: farrow-to-wean, wean-to-feeder and feeder-to-finish.

Hog production in the United States has historically been concentrated in the Corn Belt States. In 1990, Iowa, Illinois, Minnesota, Indiana and Nebraska had the largest hog
inventories in the country (USDA/NASS, 1994). However, by 1994, North Carolina had the second largest hog inventory in the country (USDA/NASS, 1998), thus indicating a shift in production locations. As of December 1, 2014, Iowa continued to be the largest pork producing state and accounted for 31.4% of the United States hog and pig inventory. North Carolina (13.0%), Minnesota (12.0%), Illinois (6.9%), and Indiana (5.5%) round out the top five pork producing states (USDA/NASS, 2015). There are three organizational structures for swine production in the United States: a) independent producers, b) vertically integrated firms with company owned production facilities, and c) vertically integrated firms with production contracts with independent farmers. ([18])

A production contract is an agreement between an integrator company and a farmer (grower) that binds the farmer to specific production practices. Different stages of the production of animals are typically covered by different contracts and farmers generally specialize in the production of animals under one contract\(^1\).

All production contracts have two main components: one is the division of responsibility for providing inputs, and the other is the compensation scheme.\(^{[18]}\) Growers provide land, housing facilities, utilities (electricity and water) and labor. The integrator provides animals, feed, medications. Typically, companies also own and operate feed mills and processing plants and provide transportation of feed and live animals. The decision of the volume of production both in terms of the rotations of batches on a given farm and the density of animals inside the house are made by the integrator company.\(^{[18]}\)

For the empirical analysis, we use the unbalanced panel dataset from [28]. It is a sample of contract settlement data for individual growers who contracted the finishing stage of hog production with an integrator in North Carolina. There are a total of 802 observations, with each observation representing one contract realization, i.e., the payment received and the grower performance associated with one batch of animals delivered to the integrator's processing plant. The dataset spans from December 1985 to April 1993. There are 122 growers in the data set and the number of observations per grower ranges from 2 to 37.

The number of houses varies across growers between one and five houses. All houses under contract have approximately the same capacity. The average number of animals per house is 1,234. The average length of the production cycle is approximately 19 weeks. Counting one additional week for necessary cleanup gives a maximum of 2.6 batches of finished hogs per house per year.\(^{[18]}\)

\(^1\)For details, see ([18])
The finishing contract that generated the data requires that growers furnish fully equipped housing facilities and that they follow the management and husbandry practices specified by the integrator. The integrator provides the grower with feeder pigs, feed, medication, veterinary services and services of the field personnel. The quality of all inputs as well as the time of placement of feeder pigs and shipment of grown animals are exclusively under control of the integrator.[18]

Compensation to grower $i$ for his husbandry and housing facilities rental is paid on a per pound of gain basis with bonuses earned on a per head basis. The bonus is based on the difference between the individual’s feed conversion, expressed as pounds of feed divided by pounds of gain $F_i/q_i$, and a standard feed conversion ratio $\phi$. If the grower’s ratio is above the standard, he receives no bonus and simply earns the base piece rate $\alpha$ multiplied by the total pounds gained $q_i$. If the grower’s ratio is below the standard ratio, the difference is multiplied by a constant $\beta$ to determine the per head bonus rate. The total bonus payment is then determined by multiplying the bonus rate by the number of pigs marketed, where the marketed pigs $(1 - m_i)P_i$ are those feeder pigs that survived the fattening process. $m_i$ denotes the mortality rate. $P_i$ denotes the number of feeder pigs placed. Algebraically, the exact formula for the total compensation is:

$$ R_i = \alpha q_i + \max[0, \beta(\phi - \frac{F_i}{q_i})(1 - m_i)P_i]. \tag{2.1} $$

Over the period of the dataset, some parameters of the payment mechanism varied. First, there is a variation in the base piece rate having to do with the type of feeder pigs placed for grow-out. In the event that commingled feeder pigs were placed with a grower $\alpha = 0.0315$, whereas in the event that integrator own nursery feeder pigs were placed with a grower $\alpha = 0.0275$. Second, as a result of technological progress in nutrition and genetics, over the period of the dataset the feed conversion standard was lowered from $\phi = 3.50$ to $\phi = 3.35$. However, after the lower feed conversion standard was introduced, the higher standard of 3.50 remained in effect for commingled pigs. As a result, three essentially different contracts were in effect during that time period: Type I ($\alpha = 0.0315, \phi = 3.50$), Type II ($\alpha = 0.0275, \phi = 3.50$) and Type III ($\alpha = 0.0275, \phi = 3.35$).

In Table 1, we compare the contract parameters of the different types and the effort the principal aims to induce. Comparing Type I and Type II, we can see that $\alpha$ in higher in Type I while $\phi$ is the same. Under Type I contract, the feeder pigs placed are commingled...
Table 2.1 Contract type comparison

<table>
<thead>
<tr>
<th>Types</th>
<th>Contract parameters</th>
<th>Effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I vs. II</td>
<td>$\alpha_1 &gt; \alpha_2, \phi_1 = \phi_2$</td>
<td>$\text{effort}_1 &gt; \text{effort}_2$</td>
</tr>
<tr>
<td>Type II vs. III</td>
<td>$\alpha_2 = \alpha_3, \phi_2 &gt; \phi_3$</td>
<td>$\text{effort}_2 &gt; \text{effort}_3$</td>
</tr>
<tr>
<td>Type I vs. III</td>
<td>$\alpha_1 &gt; \alpha_3, \phi_1 &gt; \phi_3$</td>
<td>$\text{effort}_1 &gt; \text{effort}_3$</td>
</tr>
</tbody>
</table>

pigs. The principal raises $\alpha$ to induce higher effort. Comparing Type II and Type III, $\phi$ in Type II is higher while $\alpha$ is the same for both types. The effort level should be higher under Type II. Between Type I and Type III, both $\alpha$ and $\phi$ are higher in Type I. $\alpha$ is higher in Type I because the feeder pigs placed with growers under Type I are commingled pigs. $\phi$ is kept at the higher standard for commingled pigs. So the effort level should be higher under Type I.

Summary statistics of the data are reported in Table 2. The mean, standard deviation, maximum and minimum are calculated for per house data. The mean weight gain is 223,147 pounds per house. The mean number of animals shipped is 1,185 per house. The mean feed consumption is 614,007 per house. The mean feed conversion is 2.77. The mean grower payment per house is 11,168 USD.

2.3 Moral hazard problem: canonical models

Before we dive into the more complicated compensation scheme as used by the contract that generated the data, we present several canonical models of moral hazard. We first describe the first-best case, where effort is observable. Then, we discuss the case where effort is not observable but the grower and the principal are both risk-neutral. In this case, the grower can be made the residual claimant and the first-best effort level can be achieved. The third case is one where the grower is risk-averse, and the optimal effort level is lower than the first-best effort level. In the last case, we assume that the grower is risk-neutral but is subject to the limited liability constraint. In this case, the optimal effort level is also lower than the first-best effort level, which means the moral hazard problem exists.
Table 2.2 Data summary

<table>
<thead>
<tr>
<th>Contract type</th>
<th>Feeder pigs placed $P$ (head/house)</th>
<th>Weight gain $q$ (lbs/house)</th>
<th>Feeder pigs shipped $H$ (head/house)</th>
<th>Feed consumption $F$ (lbs/house)</th>
<th>Feed conversion $f$ (ratio)</th>
<th>Grower payment $R$ (USD/house)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I Mean</td>
<td>1,227</td>
<td>199,570</td>
<td>1,146</td>
<td>604,798</td>
<td>3.04</td>
<td>9,756</td>
</tr>
<tr>
<td>St. dev.</td>
<td>22.36</td>
<td>11,664</td>
<td>38.00</td>
<td>37,600</td>
<td>0.15</td>
<td>1,363</td>
</tr>
<tr>
<td>Max</td>
<td>1,348</td>
<td>230,923</td>
<td>1,199</td>
<td>717,300</td>
<td>3.49</td>
<td>13,121</td>
</tr>
<tr>
<td>Min</td>
<td>1,118</td>
<td>176,786</td>
<td>986</td>
<td>523,100</td>
<td>0.15</td>
<td>5,642</td>
</tr>
<tr>
<td>II Mean</td>
<td>1,235</td>
<td>225,043</td>
<td>1,193</td>
<td>615,902</td>
<td>2.75</td>
<td>12,018</td>
</tr>
<tr>
<td>St. dev.</td>
<td>22.05</td>
<td>11,207</td>
<td>28.60</td>
<td>34,543</td>
<td>0.12</td>
<td>1,120</td>
</tr>
<tr>
<td>Max</td>
<td>1,341</td>
<td>264,122</td>
<td>1,288</td>
<td>770,610</td>
<td>3.12</td>
<td>14,735</td>
</tr>
<tr>
<td>Min</td>
<td>1,204</td>
<td>199,554</td>
<td>1,122</td>
<td>513,360</td>
<td>2.43</td>
<td>8,574</td>
</tr>
<tr>
<td>III Mean</td>
<td>1,235</td>
<td>226,254</td>
<td>1,187</td>
<td>614,424</td>
<td>2.73</td>
<td>11,033</td>
</tr>
<tr>
<td>St. dev.</td>
<td>32.31</td>
<td>12,892</td>
<td>36.55</td>
<td>37,788</td>
<td>0.12</td>
<td>1,107</td>
</tr>
<tr>
<td>Max</td>
<td>1,530</td>
<td>272,049</td>
<td>1,454</td>
<td>776,000</td>
<td>3.22</td>
<td>14,540</td>
</tr>
<tr>
<td>Min</td>
<td>1,163</td>
<td>185,289</td>
<td>1,033</td>
<td>519,170</td>
<td>2.39</td>
<td>6,183</td>
</tr>
<tr>
<td>Total Mean</td>
<td>1,234</td>
<td>223,147</td>
<td>1,185</td>
<td>614,007</td>
<td>2.77</td>
<td>11,268</td>
</tr>
<tr>
<td>St. dev.</td>
<td>28.07</td>
<td>14,488</td>
<td>36.52</td>
<td>36,697</td>
<td>0.15</td>
<td>1,327</td>
</tr>
<tr>
<td>Max</td>
<td>1,530</td>
<td>272,049</td>
<td>1,454</td>
<td>776,000</td>
<td>3.49</td>
<td>14,735</td>
</tr>
<tr>
<td>Min</td>
<td>1,118</td>
<td>176,786</td>
<td>986</td>
<td>513,360</td>
<td>2.39</td>
<td>5,642</td>
</tr>
</tbody>
</table>
2.3.1 Observable effort

First, we assume that effort is observable. $e$ denotes effort. $q$ denotes the output that the agent produces. A contract here specifies the agent’s effort $e$ and the payment as a function of the observed output $w(q)$. $f(q|e)$ is the conditional distribution function of $q$. $u(\cdot)$ is the agent’s utility function, where $u'() > 0$, $u''() \leq 0$. $c(e)$ is the cost of effort, where $c'(\cdot) > 0$, $c''(\cdot) \geq 0$. We assume the principal to be risk-neutral. The principal maximizes his expected payoff subject to the agent’s individual rationality constraint. The optimal contract for the principal solves the following problem,

$$\max_{e, w(q)} \int (q - w(q)f(q|e))dq $$  \hspace{1cm} (2.2)

subject to

$$\int u(w(q))f(q|e)dq - c(e) \geq \bar{u}. $$

Given that this contract specifies effort level $e$, choosing $w(q)$ to maximize $\int (q - w(q))f(q|e)dq$ is equivalent to minimizing the expected value of the principal’s compensation costs $\int w(q)f(q|e)dq$, so the optimal compensation scheme solves

$$\min_{w(q)} \int w(q)f(q|e)dq $$  \hspace{1cm} (2.3)

subject to

$$\int u(w(q))f(q|e)d\pi - c(e) \geq \bar{u}. $$

Setting up the Lagrangian associated with this problem,

$$\mathcal{L} = -\int w(q)f(q|e)dq + \lambda(\int u(w(q))f(q|e)dq - c(e) - \bar{u}),$$

and taking the first-order condition with respect to $w(q)$,

$$\frac{\partial \mathcal{L}}{\partial w(q)} = \int (-f(q|e) + \lambda u'(w(q))f(q|e))dq = 0.$$
According to the first-order condition, the agent’s wage $w(q)$ must satisfy

$$\frac{1}{u'(w(q))} = \lambda.$$  

The implication is that the optimal compensation scheme $w(q)$ is a constant. Denote the optimal constant wage as $w^*_e$.

From the individual rationality constraint, we can get that

$$w^*_e = u^{-1}(c(e) + \bar{u}). \tag{2.4}$$

Principal specifies effort $e$ that maximizes

$$\int qf(q|e)dq - u^{-1}(c(e) + \bar{u}). \tag{2.5}$$

If we assume that output is equal to effort plus noise, so that $q = e + \epsilon$, the agent is risk-neutral, so that $u(x) = x$, the cost of exerting effort is quadratic in effort, so that $c(e) = ce^2$, the agent’s reservation utility is zero, so that $\bar{u} = 0$, then

$$e \in \text{argmax} e - \frac{ce^2}{2}.$$  

The first-best effort level in this case $e^* = \frac{1}{c}$, indicating that the optimal effort equals the reciprocal of the disutility of effort.

### 2.3.2 Risk-neutral principal and agent when effort is unobservable

In this case, we will find that even with unobservable effort, the first-best effort level can still be reached when the risk-neutral principal “sells the store” to the risk-neutral agent.

Suppose that the principal offers a compensation schedule of the form $w(q) = q - \alpha$. The agent’s problem is as follows.

$$\max_e \int w(q)f(q|e)dq - c(e) \tag{2.6}$$

$$= \int qf(q|e)dq - \alpha - c(e) \tag{2.7}$$
Comparing (2.7) with (2.5), we see that $e^*$ maximizes (2.7). Thus, this contract induces the first-best (full observability) effort level $e^*$. The agent is willing to accept this contract as long as it gives him an expected utility of at least $\bar{u}$, that is, as long as $\int q f(q|e^*)dq - \alpha - c(e^*) \geq \bar{u}$. Let $\alpha^*$ be the level of $\alpha$ at which the individual rationality constraint is binding. The principal’s payoff is then $\alpha^*$, where $\alpha^* = \int q f(q|e^*)dq - c(e^*) - \bar{u}$. Hence with compensation scheme $w(q) = q - \alpha^*$, both the principal and the agent get the exact same payoff as when effort is observable.

### 2.3.3 Risk-averse agent, risk-neutral principal

Now we assume that the agent is risk-averse with CARA utility function, $u(R, e) = -e^{-\eta[R-c(e)]]}$. $R$ denotes the payment, which is assumed to be of the simple linear form $R = \alpha + \beta q$, where $\alpha$ is the fixed compensation level and $\beta$ is the variable, performance-related component of the compensation. $\eta$ is the coefficient of absolute risk aversion, $\eta = -\frac{u''}{u'}$. The disutility of effort is assumed to be quadratic in $e$, $c(e) = \frac{ce^2}{2}$, $c'(e) = ce > 0$, $c''(e) = c > 0$. The output is assumed to be equal to effort plus noise, $q = e + \epsilon$, where $\epsilon$ is the normally-distributed shock $\epsilon \sim N(0, \sigma^2)$. The principal’s problem is to maximize expected output minus the payment to the agent subject to the incentive compatibility constraint and individual rationality constraint.

$$\max_{e, \alpha, \beta} \mathbb{E}(q - R)$$

s.t. $\mathbb{E}(-e^{-\eta[R-c(e)]}) \geq u(\bar{w})$ (IR)

$e \in \text{argmax} \mathbb{E}(-e^{-\eta[R-c(e)]})$ (IC)

Substituting in the expression for the payment $R$ and disutility of effort $c(e)$, the IC constraint becomes $e \in \text{argmax} \mathbb{E}(-e^{-\eta(\alpha + \beta(e + \epsilon) - \frac{ce^2}{2})})$. Using the mean-variance approach\(^2\), it is equivalent to $e \in \text{argmax} \mathbb{E}(-e^{-\eta(\alpha + \beta\epsilon) - \frac{ce^2}{2} - \frac{\eta\beta^2\epsilon^2}{2}})$. Taking the first-order condition with respect to $e$, we can get $e^* = \frac{\beta}{c}$.

---

\(^2\)When an uncertain income aspect is normally distributed and the von Neumann-Morgenstern utility function takes the constant absolute risk aversion (CARA) form, the preferences can be represented by a function involving the mean and variance of the normal income distribution only. See p.406-408 in [36] 2000.
Knowing that \( e^* = \frac{\beta}{c} \), the principal solves the following problem.

\[
\max_{\alpha, \beta} \frac{\beta}{c} - (\alpha + \frac{\beta^2}{c}) \tag{2.9}
\]

s.t. \( \alpha + \beta e - \frac{c e^2}{2} - \frac{\eta \beta^2 \sigma^2}{2} \geq \bar{w} \)

Note that here we replace the (IC) constraint with the first-order condition of the agent’s problem. In general one cannot substitute the (IC) constraint with the agent’s first-order condition. [30] illustrates that by replacing the agent’s (IC) constraint by only the first-order conditions of the agent’s problem, we are in fact relaxing some constraints in the principal’s optimization problem. As a result, we may identify results that are actually not attainable by the principal. If the solution to the agent’s first-order condition is unique and the agent’s optimization problem is concave, then it is legitimate to substitute the agent’s first-order condition for the agent’s (IC) constraint ([5]). The case we have here satisfies the conditions for using the first-order approach.

Set up the Lagrangian associated with this problem and take first-order conditions.

\[
\mathcal{L} = \frac{\beta}{c} - (\alpha + \frac{\beta^2}{c}) + \lambda (\alpha + \frac{\beta^2}{c} - \frac{\beta^2}{2c} - \frac{\eta \beta^2 \sigma^2}{2} - \bar{w}) \tag{2.10}
\]

\[
\frac{\partial \mathcal{L}}{\partial \alpha} = -1 + \lambda = 0 \tag{2.11}
\]

\[
\frac{\partial \mathcal{L}}{\partial \beta} = \frac{1}{c} - \frac{2\beta}{c} + \lambda (\frac{\beta}{c} - \eta \beta \sigma^2) = 0 \tag{2.12}
\]

\[
\beta^* = \frac{1}{1 + \eta c \sigma^2} < 1 \tag{2.13}
\]

As a result, \( e^* = \frac{\beta^*}{c} < \frac{1}{c} \), the optimal effort level in this case is lower than the first-best effort level. The difference between this level of effort and the first-best effort level represents the cost of moral hazard.

### 2.3.4 Risk-neutral agent with limited liability constraint

Now we assume that the agent is risk-neutral but subject to the limited liability constraint. The output is equal to effort plus noise, \( q = \mu + e + \epsilon \). We add a positive intercept \( \mu \) to the output \( q \) to make sure that \( q \) is positive. \( \epsilon \) follows a uniform distribution with mean 0, \( \epsilon \sim U[-u, u] \). When effort equals 0 and \( \epsilon \) is at its worst realization \(-u\), \( q = \mu - u \) and
we denote \((\mu - u)\) as \(l\) which indicates lower bound. The payment scheme has a fixed component and a variable component, \(R = \alpha + \beta q\). The agent's problem is to maximize his expected payoff.

\[
\max_{e} \mathbb{E}(R - c(e))
\]

\[
= \max_{e} \mathbb{E}(\alpha + \beta (\mu + e + \epsilon) - c(e))
\]

\[
= \max_{e} \alpha + \beta \mu + \beta e - \frac{c e^2}{2}
\]

The first-order-condition with respect to \(e\) gives \(e^* = \frac{\beta}{c}\).

The principal maximizes his expected payoff subject to the individual rationality constraint, the limited liability constraint and the incentive compatibility constraint. If the payment satisfies the limited liability constraint in the lowest possible state, then it satisfies the constraint in all states because the payment scheme is increasing in the state. Therefore, if the constraint is binding in the lowest state, then it is non-binding in all other states. From the agent's perspective, given that the principal controls incentives through the payment scheme, the worst state is the one in which the principal provides him no incentives to perform and the production state turns out to be the worst ([27]), that is, \(e = 0\) and \(q = l\), where \(l\) is the lower bound of \(q\).

\[
\max_{\alpha, \beta} \mathbb{E}(q - R)
\]

\[
= \max_{\alpha, \beta} \mathbb{E}(\mu + e + \epsilon - \alpha - \beta (\mu + e + \epsilon))
\]

\[
s.t. \alpha + \beta e - \frac{c e^2}{2} + \beta \mu \geq \bar{w} \quad (IR)
\]

\[
\alpha + \beta l = \bar{w} \quad (LL)
\]

\[
e \in \arg\max \mathbb{E}(R - c(e)) \quad (IC)
\]

Limited liability constraint is binding in the worst case, where the agent exerts no effort and the shock is at its worst realizations, so output \(q\) is at its lower bound denoted by \(l\). Next we show that (IR) is not binding by comparing the left-hand sides of (IR) and (LL). The
difference between the left-hand side of (IR) and the left-hand side of (LL) is as follows.

\[
(\alpha + \beta e - \frac{c e^2}{2} + \beta \mu) - (\alpha + \beta l) = \beta e - \frac{c e^2}{2} + \beta(\mu - l) \tag{2.19}
\]

\[
= \beta e - \frac{c e^2}{2} + \beta(\mu - l) \tag{2.20}
\]

\[
= \frac{\beta^2}{c} - \frac{\beta^2}{2c} + \beta(\mu - l) \tag{2.21}
\]

\[
= \frac{\beta^2}{2c} + \beta(\mu - l) > 0 \tag{2.22}
\]

After some simplifications, we can see that the difference reduces to the sum of \(\frac{\beta^2}{2c}\), which is positive, and \(\beta(\mu - l)\), which is also positive since \((\mu - l) = \mu - (\mu - u) = u > 0\). As a result, the difference between the left-hand sides of (IR) and (LL) is positive.

\[
\alpha + \beta e - \frac{c e^2}{2} + \beta \mu > \alpha + \beta l = \tilde{w} \tag{2.23}
\]

So \(\alpha + \beta e - \frac{c e^2}{2} + \beta \mu > \tilde{w}\), the individual rationality constraint is not binding.

Now we substitute in the first-order condition of (IC), set up the Lagrangian and derive the Kuhn-Tucker conditions.

\[
\mathcal{L} = \frac{\beta}{c} - \alpha - \frac{\beta^2}{c} + (1 - \beta)\mu + \lambda_1(\alpha + \frac{\beta^2}{c} - \frac{\beta^2}{2c} + \beta \mu - \tilde{w}) + \lambda_2(\alpha + \beta l - \tilde{w}) \tag{2.24}
\]

\[
\lambda_1 \geq 0 \tag{2.25}
\]

\[
\alpha + \beta e - \frac{c e^2}{2} + \beta \mu \geq \tilde{w} \tag{2.26}
\]

\[
\lambda_1(\alpha + \beta e - \frac{c e^2}{2} + \beta \mu - \tilde{w}) = 0 \tag{2.27}
\]
Since we have shown that (IR) is not binding, we can get $\lambda_1 = 0$.

\[
\frac{\partial L}{\partial \alpha} = -1 + \lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = 1
\]

\[
\frac{\partial L}{\partial \beta} = \frac{1}{c} - \frac{2\beta}{c} - \mu + \lambda_1 \left( \frac{\beta}{c} + \mu \right) + \lambda_2 l = \frac{1}{c} - \frac{2\beta}{c} - \mu + l = 0
\]

$\Rightarrow \beta^* = \frac{1 + (l - \mu)c}{2}$

$\Rightarrow e^* = \frac{\beta^*}{c} = \frac{1 + (l - \mu)c}{2c} = \frac{1}{2c} \left( \frac{l - \mu}{2} < \frac{1}{2c} \right)$

As we can see from the result, the optimal effort level in this case is also lower than the first-best effort level. So the moral hazard problem exists when the agent is risk-neutral but subject to the limited liability constraint.

### 2.4 The model of production contract

In this section, we develop the model of the hog production contract that generated the data. First, we solve the agent's problem. Once we get the agent's optimal effort, we can derive the incentive effects of the contract parameters. Then we solve the principal's problem and derive the optimal contract parameters. Once we have the formula for optimal contract parameters, we can evaluate how the optimal contract parameter respond to changes in its shifters.

We model the integrator-grower relationship in a principal-agent framework. The timing of the contracting is as follows. The integrator proposes a contract that specifies the division of responsibilities for production inputs and the compensation scheme to the grower. The integrator is required to provide feeder pigs and feed and the grower is required to provide housing facilities and labor. After the grower observes the contract terms, the number and the weight of feeder pigs, he decides whether or not he will accept the contract. If he does, feeder pigs and feed will be delivered to his farm and the production (grow-out) process begins.

We assume both sides to the contract as risk-neutral, but the agent is subject to the limited liability constraint. Limited liability is a reasonable assumption because growers are
Figure 2.1 Feed conversion and weight gain scatter plot

Figure 2.2 Number of survived animals and weight gain scatter plot
small farmers and the limited liability constraint ensures that payment to them cannot fall short of a certain level ex post. As shown in the previous section, moral hazard problem still exists when we have risk-neutral agent with limited liability constraint. \(q\) denotes weight gain. \(H\) denotes number of survived animals. \(f\) denotes feed conversion, which is feed consumption divided by the weight gain. \(q\), \(f\), and \(H\) are three important measures of a grower’s performance. These three performance measures are specified as linear in effort. The specification of weight gain \(q\) as linear in effort is commonly used in principal-agent models.\(^3\) As for the modeling of feed conversion \(f\) and number of survived animals \(H\), Figure 2.1 plots feed conversion \(f\) on the vertical axis with weight gain \(q\) on the horizontal axis and Figure 2.2 plots number of survived animals on the vertical axis with weight gain on the horizontal axis, where weight is expressed in pounds. \(f\) has no units since it is the ratio of feed and weight gain. The units for \(q\) are pounds per house. The units for \(H\) are heads per house. As can be seen from the graphs, the observations are clustered but the trend can be considered linear. So we specify feed conversion and number of survived animals as linear in effort. Moreover, the relationship between feed conversion and weight gain is clearly negative. Feed conversion decreases as weight gain increases. On the other hand, the relationship between the number of survived animals and weight gain is positive. More survived animals will, other things being equal, generate more live weight.

By exerting effort, agents can control the mean of their performance. However, the performance measure realization also depends on some shocks that are beyond agent’s control. The shocks are assumed to be uniformly distributed. We assume the bounded support for the shocks so that in the worst case the limited liability constraint will be satisfied.\(^4\) The model specification is as follows:

\[
q = \mu_1 + k_1 e + \epsilon_1, \quad \text{E}(\epsilon_1) = 0, \quad \epsilon_1 \sim \text{U}[-u_1, u_1];
\]
\[
f = \mu_2 + k_2 e + \epsilon_2, \quad \text{E}(\epsilon_2) = 0, \quad \epsilon_2 \sim \text{U}[-u_2, u_2];
\]
\[
H = \mu_3 + k_3 e + \epsilon_3, \quad \text{E}(\epsilon_3) = 0, \quad \epsilon_3 \sim \text{U}[-u_3, u_3].
\]

\(^3\)See [22], [13] for example.

\(^4\)Note that even though [5] point out that with bounded support, the first best can always be achieved because the principal can punish the agent very severely for performance outcomes outside the support, the agency problem still exists in our model because the limited liability constraint of the agents prevents severe punishment of them. This is also discussed in [27].
Weight gain $q$ is increasing in effort, so $\frac{\partial q}{\partial e} = k_1 > 0$. As effort increases, feed conversion should decrease, so $\frac{\partial f}{\partial e} = k_2 < 0$. Number of survived animals is increasing in effort, so $\frac{\partial H}{\partial e} = k_3 > 0$. We include an intercept for the specification of feed conversion $f$ to ensure that $f$ is positive since $k_2 e$ is negative. We have intercept terms for weight gain $q$ and number of survived animals $H$ as well to capture the scales of different performance margins. The disutility of effort for agent $i$ is $C_i(e) = \frac{c_i e^2}{2}$, $C_i'(e) = c_i e > 0$, $C_i''(e) = c_i > 0$.

In this paper, we analyze two types of technological change that resulted in the adjustment of contract parameters. The technological change over time leads to the change in the contract parameter $\phi$. Since technological change over time mainly affects the feed conversion margin, we represent this change by the intercept term of feed conversion. Another one is the difference between commingled and nursery pigs, which leads to the change in the contract parameter $\alpha$. We represent this difference by the intercept term of number of survived animals.

The compensation scheme we use is slightly different from that in (1). Here we denote number of survived animals by $H_i$ instead of $(1 - m_i)P_i$. We also lose the maximum operator in (1) because the feed conversion performances in the data are all below the fixed standard, in other words, $(\phi - f)$ are all positive. Hence:

$$R_i = \alpha q_i + \beta (\phi - f_i) H_i. \quad (2.28)$$

Substituting the expressions of the three performance measures in the compensation scheme, the agent’s payoff becomes

$$R = \alpha q + \beta (\phi - f) H$$

$$= \alpha (\mu_1 + k_1 e + \epsilon_1) + \beta (\phi - (\mu_2 + k_2 e + \epsilon_2))(\mu_3 + k_3 e + \epsilon_3).$$

Next, we can solve the agent’s problem.
2.4.1 Agent’s problem

The agent’s problem is to choose effort $e$ to maximize expected payoff given contract parameters. Agent $i$’s problem is as follows:

$$\max_{e_i} E(R_i - C_i(e_i))$$

$$= \max_{e_i} \alpha \mu_1 + \alpha k_1 e_i + \beta \phi k_3 e_i + \beta \phi \mu_3 - \beta(k_2 k_3 e_i^2 + k_2 e_i \mu_3 + k_3 e_i \mu_2 + \mu_2 \mu_3) - \frac{c_i e_i^2}{2}.$$  \hfill (2.29)

We take the F.O.C. w.r.t. $e_i$,

$$\alpha k_1 + \beta k_3 \phi - \beta(2k_2 k_3 e_i + k_2 \mu_3 + k_3 \mu_2) - c_i e_i = 0.$$  \hfill (2.30)

The S.O.C. w.r.t. $e_i$ equals $-(2\beta k_2 k_3 + c_i)$. In order to have a maximum instead of minimum, $-(2\beta k_2 k_3 + c_i)$ should be negative, otherwise the agent’s problem is not well defined.

Rearranging the F.O.C., we can get the optimal effort level for agent $i$ in the following form,

$$e_i^* = \frac{\alpha k_1 + \beta k_3 \phi - \beta(k_2 k_3 e_i + k_2 \mu_3 + k_3 \mu_2) - c_i e_i}{2 \beta k_2 k_3 + c_i}. \hfill (2.32)$$

The optimal effort that an agent exerts depends on $c_i$, contract parameters $\alpha$, $\beta$ and $\phi$, and intercepts and slopes of production functions $k_1$, $k_2$, $k_3$, $\mu_2$ and $\mu_3$. With different $c_i$’s for different agents, the optimal effort level varies from agent to agent.

Now that we have derived the optimal effort level, we examine how the optimal effort level changes in response to contract parameters. Since contract parameters $\alpha$ and $\phi$ are the two that have changed over the period of the dataset, we focus on the effects of $\alpha$ and $\phi$ on effort $e$. We look at the derivatives of effort $e$ with respect to $\alpha$ and $\phi$ to see if we can analytically evaluate their signs. We expect the effect of $\alpha$ on $e$ to be positive since $\alpha$ is the piece-rate for $q$. If $\alpha$ increases, $e$ should increase. We also expect the effect of $\phi$ on $e$ to be positive since in the compensation scheme $\phi$ multiplies $H$ so if $\phi$ increases, $e$ should increase.

For contract parameter $\alpha$,

$$\frac{\partial e_i}{\partial \alpha} = \frac{k_1}{2 \beta k_2 k_3 + c_i}. \hfill (2.33)$$

The numerator $k_1$ is positive. In order for the sign of $\frac{\partial e_i}{\partial \alpha}$ to be in line with our expectation, we need $2\beta k_2 k_3 + c_i$ to be positive.
For contract parameter $\phi$,
\[
\frac{\partial e_i}{\partial \phi} = \frac{\beta k_3}{2\beta k_2 k_3 + c_i}. \tag{2.34}
\]

The numerator $\beta k_3$ is positive. In order for the sign of $\frac{\partial e_i}{\partial \phi}$ to be in line with our expectation, we need $2\beta k_2 k_3 + c_i$ to be positive. Note that the second-order condition is $-(2\beta k_2 k_3 + c_i)$, which is negative for the maximization problem, otherwise our problem is not well defined. We will verify the signs of the derivatives (2.33) and (2.34) after the parameters of the model have been empirically estimated.

### 2.4.2 Principal’s problem

Assuming perfectly competitive hog market, the integrator should earn zero profit. Principal’s profit $\Pi$ is equal to revenue $V q$ minus the payment to agent $R$ minus the cost of feed and feeder pigs since the integrator provides the feed and feeder pigs to growers, $\Pi = V q - w_H P - w_F F - R$. $V$ denotes the price of the output, $F$ denotes feed, $P$ denotes the number of feeder pigs that are placed at the beginning of a contract. The zero-profit constraint is as follows.

\[
E(\Pi) = E(V q - w_H P - w_F F - R) = 0. \tag{2.35}
\]

Rearranging (2.35), we can get that expected payment to agent is equal to expected revenue minus cost of feeder pigs and feed.

\[
E(R) = E(V q - w_H P - w_F F). \tag{2.36}
\]

Now we can define principal’s problem. The problem for the principal is to pick contract parameters that maximizes agents’ expected utility subject to the incentive compatibility constraint and the limited liability constraint.

\[
\max_{\alpha, \phi} E(R_i - C(e_i)) \tag{2.37}
\]
Substituting the right-hand side of (2.36) for $E(R_i)$,

$$\begin{align*}
E(R_i - C_i(e_i)) &= E(V q - w_H P - w_F F - C(e_i)) \\
&= E(V q - w_H P - w_F (c q - C(e_i))) \\
&= E(V(\mu_1 + k_1 e_i + \epsilon_1) - w_H P - w_F (\mu_2 + k_2 e_i + \epsilon_2)(k_1 e_i + \epsilon_1) - \frac{c e_i^2}{2}) \\
&= V \mu_1 + V k_1 e_i - w_H P - w_F (k_1 k_2 e_i^2 + k_1 \mu_2 e_i) - \frac{c e_i^2}{2}
\end{align*}$$

s.t.

$$e_i \in \arg\max E(R_i - C_i(e_i)) \quad (IC) \quad (2.38)$$

$$a l_1 + \beta (\phi - u_2) l_3 = \bar{u} \quad (LL) \quad (2.39)$$

where $\bar{u}$ is the agent’s reservation utility.

Here the maximization problem is subject to the binding limited liability constraint at the worst case. The worst case occurs when $q$ reaches its lower bound $l_1$, $f$ reaches its upper bound $u_2$ and $H$ reaches its lower bound $l_3$. In this case, the payment $a l_1 + \beta (\phi - u_2) l_3$ equals the reservation utility $\bar{u}$.

Using the first-order approach, we substitute in the first-order condition of the incentive compatibility constraint and set up the Lagrangian associated with this problem,

$$\mathcal{L} = V \mu_1 + V k_1 e_i - w_H P - w_F (k_1 k_2 e_i^2 + k_1 \mu_2 e_i) - \frac{c e_i^2}{2} + \lambda (a l_1 + \beta (\phi - u_2) l_3 - \bar{u})$$

where

$$e_i = \frac{a k_1 + \beta k_3 \phi - \beta (k_2 \mu_3 + k_3 \mu_2)}{2 \beta k_2 k_3 + c_i}.$$
The first-order conditions are

\[
\frac{\partial L}{\partial \alpha} = [V k_1 - w_f (2 k_1 k_2 \alpha k_1 + \beta k_3 \phi - \beta (k_2 \mu_3 + k_3 \mu_2) + k_1 \mu_2)] - c_i \frac{\alpha k_1 + \beta k_3 \phi - \beta (k_2 \mu_3 + k_3 \mu_2)}{2 \beta k_2 k_3 + c_i} \left( \frac{k_1}{2 \beta k_2 k_3 + c_i} \right) + \lambda l_1 = 0,
\]

\[
\frac{\partial L}{\partial \phi} = [V k_1 - w_f (2 k_1 k_2 \alpha k_1 + \beta k_3 \phi - \beta (k_2 \mu_3 + k_3 \mu_2) + k_1 \mu_2)] - c_i \frac{\alpha k_1 + \beta k_3 \phi - \beta (k_2 \mu_3 + k_3 \mu_2)}{2 \beta k_2 k_3 + c_i} \left( \frac{\beta k_3}{2 \beta k_2 k_3 + c_i} \right) + \lambda \beta l_3 = 0,
\]

\[
\frac{\partial L}{\partial \lambda} = \alpha l_1 + \beta (\phi - u_2) l_3 - \bar{u} = 0.
\]

Solving for the contract parameters gives

\[
\phi^* = \frac{(2 \beta k_2 k_3 + c_i) (V k_1 - w_f k_1 \mu_2)}{2 w_f k_1 k_2 + c_i} + \beta (k_2 \mu_3 + k_3 \mu_2) - \frac{k_1}{l_1} (u + \beta l_3 u_2) \beta (k_3 - \frac{k_1 l_3}{l_1})
\]

(2.40)

\[
\alpha^* = \frac{u + \beta l_3 u_2}{l_1} - \frac{l_3}{l_1} \frac{(2 \beta k_2 k_3 + c_i) (V k_1 - w_f k_1 \mu_2)}{2 w_f k_1 k_2 + c_i} + \beta (k_2 \mu_3 + k_3 \mu_2) - \frac{k_1}{l_1} (u + \beta l_3 u_2)
\]

(2.41)

From the optimal contract parameter formulas, we can see that \( \alpha^* \) and \( \phi^* \) depend on \( c_i \) and thus vary from agent to agent. The optimal contract parameters should be grower specific. \( \alpha_i^* \) and \( \phi_i^* \) depend on \( c_i \), production function parameters \( \mu_2, \mu_3, k_1, k_2, k_3 \), minimum permissible payment \( \bar{u} \), lower bound of \( q \), \( l_1 \), lower bound of \( H \), \( l_3 \), and upper bound of \( f \), \( u_2 \).

Now that we have derived the optimal contract parameters \( \alpha \) and \( \phi \), we can investigate the parameter change of the production contract that generated our data. As discussed in Section II, the integrator lowered the feed conversion standard \( \phi \) from 3.50 to 3.35 as a result of technological progress in nutrition and genetics. Feed conversion is an important measure of the production efficiency, which is also why bonus payment is based on the feed conversion. In our model, the technological progress is represented by the decrease in \( \mu_2 \). With \( \mu_2 \) representing technological changes, the derivatives of contract parameters \( \alpha \) and \( \phi \) with respect to \( \mu_2 \) represent how contract parameters should respond to technological
changes.

First, we look at the effect of \( \mu^2 \) on \( \phi \):

\[
\frac{\partial \phi}{\partial \mu^2} = \frac{\beta k_3 - \frac{u_F k_1 (2\beta k_2 k_3 + c_j)}{2u_F k_1 k_2 + c_i}}{\beta (k_3 - \frac{k_1}{l_1})} = \frac{\beta k'_3 - \frac{u_F (2\beta k'_2 k'_3 + c'_j)}{2u_F k'_1 k'_2 + c'_i}}{\beta (k'_3 - \frac{l_3}{l_1})}
\]

(2.42)

where \( k'_3 = \frac{k_3}{k'_1}, k'_2 = \frac{k_2}{k'_1} \) and \( c'_j = \frac{c_j}{k'_1} \).

From the binding limited liability constraint, we know that \( \alpha = \frac{u + \beta l_3 u_2}{l_1} - \frac{\beta l_1}{l_1} \phi \). So \( \frac{\partial \alpha}{\partial \mu^2} = -\frac{\beta l_1}{l_1} \frac{\partial \phi}{\partial \mu^2}, \frac{\partial \alpha}{\partial \mu^3} = -\frac{\beta l_3}{l_1} \frac{\partial \phi}{\partial \mu_3} \). Recall that \( l_3 \) denotes the lower bound of number of survived animals per house, \( l_1 \) denotes the lower bound of weight gain per house and they are both positive. As a result, \( -\frac{\beta l_1}{l_1} \) is negative, \( \frac{\partial \alpha}{\partial \mu^2} \) has the opposite sign of \( \frac{\partial \phi}{\partial \mu^2} \) and \( \frac{\partial \alpha}{\partial \mu^3} \) has the opposite sign of \( \frac{\partial \phi}{\partial \mu_3} \).

Next, we look at the effect of \( \mu_3 \) on \( \alpha \):

\[
\frac{\partial \alpha}{\partial \mu_3} = \frac{\beta l_3}{l_1} \frac{\partial \phi}{\partial \mu^2} = \frac{\beta l_3}{l_1} \left( \frac{l_3 (\beta k'_3 - \frac{u_F (2\beta k'_2 k'_3 + c'_j)}{2u_F k'_1 k'_2 + c'_i})}{k'_3 - \frac{l_3}{l_1}} \right)
\]

(2.43)

From the derivatives, we can see that the response of contract parameters \( \alpha \) and \( \phi \) to changes in \( \mu_2 \) is individual grower specific since the heterogeneous disutility of effort parameter is in the derivatives. We will evaluate the signs of the derivatives in the empirical section.

There are also two values for contract parameter \( \alpha \). For feeder pigs that are of the commingled type, \( \alpha \) is set at 0.0315. For nursery feeder pigs, \( \alpha \) is set at 0.0275. The difference of the type of feeder pigs can be represented by the change in \( \mu_3 \), the intercept term of \( H \).

Now we take the derivatives of \( \phi \) and \( \alpha \) to see how the optimal contract parameters should respond to the change in the type of feeder pigs.

\[
\frac{\partial \phi}{\partial \mu_3} = \frac{k_2}{k_3 - \frac{k_1 l_3}{l_1}}
\]

(2.44)
\[
\frac{\partial a}{\partial \mu_3} = -\frac{l_3}{l_1} \frac{\beta k_2}{k_3 - \frac{k_1}{k_1}} \tag{2.45}
\]

Again, we will evaluate the signs of the optimal response in the empirical section.

### 2.5 Estimation

In this section, we estimate the model and evaluate the incentive effects of the contract parameters.

#### 2.5.1 Parameter estimates

In the model, the agents are heterogeneous with respect to their disutilities of effort. In the estimation, we specify agent \(i\)'s \(c_i\) as \(c_i = c + b_1 N_i\), where \(c\) is common to every agent and \(N_i\) denotes agent \(i\)'s number of houses. Growers with more houses grow more broilers each time, thus we use number of houses to represent grower's experience in growing broilers. \(c, b_1\) are parameters that needs to be estimated. We expect \(b_1\) to be negative in sign since more experienced growers will have smaller disutilities of effort.

Before estimating the model, we need to figure out whether all the unknown parameters are identified. By further examining (2.32), the optimal effort formula that we derived in the previous section can be written in the following way:

\[
e_i^* = \frac{\alpha + \beta \frac{k_2}{k_1} \phi - \beta \left( \frac{k_2}{k_1} \mu_3 + \frac{k_3}{k_1} \mu_2 \right)}{k_1 \left( 2\beta \frac{k_3}{k_1} + \frac{c_i}{k_1} \right)}. \tag{2.46}
\]

Substituting (2.46) in \(k_1 e, k_2 e, k_3 e\) that appear in production functions, we can get

\[
k_1 e_i^* = \frac{\alpha + \beta \frac{k_2}{k_1} \phi - \beta \left( \frac{k_2}{k_1} \mu_3 + \frac{k_3}{k_1} \mu_2 \right)}{2\beta \frac{k_3}{k_1} + \frac{c_i}{k_1}}; \tag{2.47}
\]

\[
k_2 e_i^* = \frac{k_2}{k_1} \frac{\alpha + \beta \frac{k_2}{k_1} \phi - \beta \left( \frac{k_2}{k_1} \mu_3 + \frac{k_3}{k_1} \mu_2 \right)}{2\beta \frac{k_3}{k_1} + \frac{c_i}{k_1}}; \tag{2.48}
\]

\[
k_3 e_i^* = \frac{k_3}{k_1} \frac{\alpha + \beta \frac{k_2}{k_1} \phi - \beta \left( \frac{k_2}{k_1} \mu_3 + \frac{k_3}{k_1} \mu_2 \right)}{2\beta \frac{k_3}{k_1} + \frac{c_i}{k_1}}. \tag{2.49}
\]

As can be seen from above, \(k_1 e, k_2 e,\) and \(k_3 e\) are in fact functions of \(\frac{k_2}{k_1}, \frac{k_3}{k_1},\) and \(\frac{c_i}{k_1}\). As a
Table 2.3 Parameter estimates

<table>
<thead>
<tr>
<th></th>
<th>$c'$</th>
<th>$b'_1$</th>
<th>$k'_2$</th>
<th>$k'_3$</th>
<th>$\mu_2$</th>
<th>$\mu_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.5807***</td>
<td>-1.5865***</td>
<td>-0.5371***</td>
<td>0.1607***</td>
<td>3.9648***</td>
<td>0.8267***</td>
</tr>
<tr>
<td></td>
<td>(0.7459)</td>
<td>(0.2711)</td>
<td>(0.0398)</td>
<td>(0.0113)</td>
<td>(0.0899)</td>
<td>(0.0250)</td>
</tr>
</tbody>
</table>

Note: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

result, we will only be able to identify $k_2$, $k_3$, and $c_i$. Denote $k'_2 = \frac{k_2}{k_1}$, $k'_3 = \frac{k_3}{k_1}$, $c' = \frac{c}{k_1}$, $b'_1 = \frac{b_1}{k'_1}$, $b'_2 = \frac{b_2}{k'_1}$. $k'_2$, $k'_3$, $c'$, $b'_1$ and $b'_2$ can be identified. $\mu_2$ and $\mu_3$ are the intercepts of $f$ and $H$ respectively and we expect them to be positive. $k_2$ is equal to $\frac{\partial f}{\partial e}$. We expect $k_2$ to be negative since $f$ is decreasing in $e$. $k_3$ is equal to $\frac{\partial H}{\partial e}$ and we expect it to be positive since $H$ is increasing in $e$. $c$ is the cost parameter that is common to each agent and we expect it to be positive.

$\mu_1$ and $k_1 e$ appear together as a sum, and thus they can not be identified separately. Identifying these parameters requires a normalization assumption. We normalize $\mu_1$ to be 0 such that only $c'$, $b'_1$, $k'_2$, $k'_3$, $\mu_2$, $\mu_3$ remain to be estimated. Let $\theta$ be the unknown parameters. $\theta = \{c', b'_1, k'_2, k'_3, \mu_2, \mu_3\}$.

We estimate the unknown parameters with GMM by matching the three population moments implied by the model with the sample moments of our data. The standard deviations are acquired by drawing samples from the dataset, estimating the model 1000 times and then calculating the standard deviations of the estimated parameters. The estimation results are reported in Table 3. The standard deviations are listed in brackets below the estimates. $k'_2$ is negative since as effort increases, feed conversion will decrease. $k'_3$ is estimated to be 0.1607. The intercept of feed conversion, $\mu_2$, is estimated to be 3.9648. The intercept of number of survived animals, $\mu_3$, is estimated to be 0.8267. $c'$ is estimated to be 6.5807. $b'_1$ is estimated to be -1.5865, which is as expected. Since as number of houses increase, the grower can be considered more experienced in growing feeder pigs thus having a lower $c_i$.

Now we separate the data into three groups by contract types and estimate $\theta$ for each group. Recall that the three different contract types are: Type I ($\alpha = 0.0315, \phi = 3.50$), Type II ($\alpha = 0.0275, \phi = 3.50$) and Type III ($\alpha = 0.0275, \phi = 3.35$). The estimation results are reported in the table below.

From Table 4, we can see that $b'_1$’s are all estimated to be negative.
Table 2.4 Parameter estimates

<table>
<thead>
<tr>
<th></th>
<th>$c'$</th>
<th>$b'_1$</th>
<th>$k'_2$</th>
<th>$k'_3$</th>
<th>$\mu_2$</th>
<th>$\mu_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>6.1304***</td>
<td>-1.4154*</td>
<td>-0.4505***</td>
<td>0.2152***</td>
<td>3.9357***</td>
<td>0.7164***</td>
</tr>
<tr>
<td></td>
<td>(1.9591)</td>
<td>(0.7591)</td>
<td>(0.1388)</td>
<td>(0.0351)</td>
<td>(0.2780)</td>
<td>(0.0720)</td>
</tr>
<tr>
<td>II</td>
<td>6.4251***</td>
<td>-1.7901**</td>
<td>-0.2894***</td>
<td>0.1382***</td>
<td>3.4007***</td>
<td>0.8821***</td>
</tr>
<tr>
<td></td>
<td>(1.7970)</td>
<td>(0.8409)</td>
<td>(0.0630)</td>
<td>(0.0155)</td>
<td>(0.1420)</td>
<td>(0.0343)</td>
</tr>
<tr>
<td>III</td>
<td>2.5947***</td>
<td>-0.5607**</td>
<td>-0.2473***</td>
<td>0.1621***</td>
<td>3.2869***</td>
<td>0.8207***</td>
</tr>
<tr>
<td></td>
<td>(0.6223)</td>
<td>(0.2487)</td>
<td>(0.0490)</td>
<td>(0.0214)</td>
<td>(0.1108)</td>
<td>(0.0478)</td>
</tr>
</tbody>
</table>

Note: *p < 0.1; **p < 0.05; ***p < 0.01

Table 2.5 Hypothesis testing for the incentive effects

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>p-value</th>
<th>Test result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: 2\beta k_2 k_3 + c_i &lt; 0$</td>
<td>0.00</td>
<td>Reject the null</td>
</tr>
</tbody>
</table>

2.5.2 Incentive effects

Incentive effects refer to the effects that compensation schemes have on worker productivity. During the period of the dataset, contract parameters $\alpha$ and $\phi$ have both changed. We have derived the derivatives of $e_i$ with respect to $\alpha$ and $\phi$ in the previous section, as in (2.33) and (2.34).

From the theoretical model, we know that the numerators of the derivatives are positive. The reason we cannot sign from the theoretical model alone is that $2\beta k_2 k_3$ and $c_i$, which are in the denominator, have opposite signs and we don't know the relative magnitude of these two. Now that we have done the estimation and bootstrapping, we can test whether or not $2\beta k_2 k_3 + c_i$ is positive as reported in Table 2.5. With the bootstrapped estimates, the p-value of $2\beta k_2 k_3 + c_i$ being negative is 0.00 for all agents. So we can reject $H_0: 2\beta k_2 k_3 + c_i < 0$. As a result, increasing $\alpha$ and $\phi$ has positive effects on effort. This result is in line with our comparison of the effort level the principal aims to induce with different contract parameters presented in Table 1. The S.O.C. of the agent's maximization problem is also satisfied since the S.O.C. is $-(2\beta k_2 k_3 + c_i)$.

2.5.3 Contract parameter change

To examine why the principal changed contract parameters, we check whether $\mu_2$ in one type of contract is smaller than the other, determine the optimal response of $\alpha$ and $\phi$ to
Table 2.6 Hypothesis testing for $\mu_2$ among three contracts

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>p-value</th>
<th>Test result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: \hat{\mu}_I^I &gt; \hat{\mu}_I^I$</td>
<td>0.046</td>
<td>Reject the null</td>
</tr>
<tr>
<td>$H_0: \hat{\mu}_I^I &gt; \hat{\mu}_I^I$</td>
<td>0.022</td>
<td>Reject the null</td>
</tr>
<tr>
<td>$H_0: \hat{\mu}_I^I &gt; \hat{\mu}_I^I$</td>
<td>0.268</td>
<td>Can not reject the null</td>
</tr>
</tbody>
</table>

changes in $\mu_2$, and check whether the principal made the correct adjustment of contract parameters or not.

We first test the difference in $\mu_2$ among the three types of contract as reported in Table 2.6. Between Type I and Type II, by bootstrap, the p-value of $\hat{\mu}_2$ in Type II being greater than $\hat{\mu}_2$ in Type I is 0.046. Between Type I and Type III, the p-value of $\hat{\mu}_2$ in Type III being greater than $\hat{\mu}_2$ in Type I is 0.022. Between Type II and Type III, the p-value of $\hat{\mu}_2$ in Type III being greater than $\hat{\mu}_2$ in Type II is 0.268. According to the bootstrap result, the hog production technology is more efficient with Type II and Type III contract than with Type I.

Then we determine how $\alpha$ and $\phi$ should respond to changes in $\mu_2$. We have derived $\frac{\partial \phi}{\partial \mu_2}$ and $\frac{\partial \alpha}{\partial \mu_2}$ as in (2.42) and (2.43). With $c_i$ in the derivatives, the response of $\phi$ and $\alpha$ to changes in $\mu_2$ should be grower specific. According to (2.42), we construct $\frac{\partial \phi}{\partial \mu_2}$ using the estimates that we got by bootstrap and the result shows that $\frac{\partial \phi}{\partial \mu_2}$ is significantly negative for each grower (the p-value of it being positive is smaller than 0.01). According to (2.43) and as discussed in the previous section, $\frac{\partial \alpha}{\partial \mu_2}$ and $\frac{\partial \phi}{\partial \mu_2}$ are opposite in signs. So $\frac{\partial \alpha}{\partial \mu_2}$ is significantly positive for each grower.

Combining the changes in $\mu_2$ and the optimal contract parameter response, we can check whether the principal made the right decisions when they changed the contract parameters. For Type I contract and Type II contract, $\mu_2$ is smaller in Type II. Since $\frac{\partial \phi}{\partial \mu_2}$ is negative, the optimal response to a decrease in $\mu_2$ should be to increase $\phi$. Since $\frac{\partial \alpha}{\partial \mu_2}$ is positive, the optimal response to a decrease in $\mu_2$ should be to lower $\alpha$. The principal made the right choice of having a smaller $\alpha$ for Type II but did not adjust $\phi$ as $\phi$ equals 3.5 for both Type I and Type II contracts. For Type I and Type III contracts, $\mu_2$ is smaller in Type III. So the optimal response would be to lower $\alpha$ and increase $\phi$ for Type III while the principal lowered both $\alpha$ and $\phi$. As a result, for Type II and Type III, the principal changed $\alpha$ in the correct direction but failed to do so for $\phi$.

We now test the difference in $\mu_3$ among the three contracts as reported in Table 2.7. Between Type I and Type II, by bootstrap, the p-value of $\hat{\mu}_3$ in Type II being smaller than $\hat{\mu}_3$
Table 2.7 Hypothesis testing for \( \mu_3 \) among three contracts

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>p-value</th>
<th>Test result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0: \hat{\mu}_3^{II} &lt; \hat{\mu}_3^I )</td>
<td>0.022</td>
<td>Reject the null</td>
</tr>
<tr>
<td>( H_0: \hat{\mu}_3^{III} &lt; \hat{\mu}_3^I )</td>
<td>0.119</td>
<td>Can not reject the null</td>
</tr>
<tr>
<td>( H_0: \hat{\mu}_3^{II} &lt; \hat{\mu}_3^{III} )</td>
<td>0.123</td>
<td>Can not reject the null</td>
</tr>
</tbody>
</table>

In Type I is 0.022. Between Type I and Type III, the p-value of \( \hat{\mu}_3 \) in Type III being smaller than \( \hat{\mu}_3 \) in Type I is 0.119. Between Type II and Type III, the p-value of \( \hat{\mu}_3 \) in Type II being smaller than \( \hat{\mu}_3 \) in Type III is 0.123. According to the bootstrap result, the intercept term of the number of survived animals is significantly higher under Type II contract than that under Type I contract.

Then we determine how \( \alpha \) and \( \phi \) should respond to changes in \( \mu_3 \). We have derived \( \frac{\partial \phi}{\partial \mu_3} \) and \( \frac{\partial \alpha}{\partial \mu_3} \) as in Eq. (2.44) and Eq. (2.45). We bootstrap and estimate the model 1000 times and construct \( \frac{\partial \phi}{\partial \mu_3} \) using the parameter estimates based on Eq. (2.44). The result is that \( \frac{\partial \phi}{\partial \mu_3} \) is significantly positive (the p-value of it being negative is smaller than 0.01). Based on Eq. (2.44) and Eq. (2.45), \( \frac{\partial \alpha}{\partial \mu_3} \) has the opposite sign to that of \( \frac{\partial \phi}{\partial \mu_3} \). As a result, \( \frac{\partial \alpha}{\partial \mu_3} \) is significantly negative.

Combining the difference in \( \mu_3 \) between Type I and Type II contracts and the optimal parameter change, the principal should reduce \( \alpha \) and increase \( \phi \) for Type II contract. The optimal parameter change due to the difference in \( \mu_3 \) between Type I and Type II contracts is in the same direction as the change due to the difference in \( \mu_2 \). By comparing the observed contract parameters, it turns out that the principal changed \( \alpha \) in the correct direction but failed to increase \( \phi \) for Type II contract.

2.6 Conclusions

In this paper, we set up a principal-agent model for the integrator and grower relation in the hog production industry. We assume that the grower is risk-neutral but is subject to the limited liability constraint. The agency problem still exists when we have risk-neutrality and limited liability constraint. The limited liability constraint guarantees the minimum payment the agent can accept ex post. We derive the optimal effort by solving the agent’s problem, estimate the optimal effort level and derive the incentive effects of the contract parameters. The optimal contract parameters are also derived by solving the principal’s
problem subject to the binding limited liability constraint at the worst case.

During the period covered by the data, there has been contract parameter changes. It is technological advances and different type of feeder pigs that led to the changes in contract parameters.

Feed conversion is an important efficiency measure of the hog production technology and we focus on the change in $\mu_2$. We first check whether $\mu_2$ is different between contract types and find that $\mu_2$ in Type II and Type III is smaller than that in Type I. Then we sign the response of optimal contract parameters to changes in $\mu_2$ by bootstrap. After we determine the signs, we can predict how the principal should adjust contract parameters to decrease in $\mu_2$ and check whether the changes the principal did make to the contract parameters are in the right directions or not. The result shows that the direction the principal changed $\alpha$ is correct but it is not for $\phi$.

Similarly, we use $\mu_3$, the intercept term of number of survived animals $H$, to represent different types of feeder pigs. The result shows that $\mu_3$ is significantly higher under Type II contract than that under Type I contract and the principal should reduce $\alpha$ and increase $\phi$ for Type II contract. The principal changed $\alpha$ in the correct direction but failed to increase $\phi$ for Type II contract.
In this paper, we propose a nonparametric approach to identify the unobserved heterogeneity in cardinal tournament by exploiting the recent advancement in measurement error models. We show that the unobserved heterogeneity in growers’ abilities is nonparametrically identified, and so is the output distributions conditional on growers’ abilities. We apply the proposed method to broiler production tournament data and investigate whether integrators discriminate among growers by allocating variable quality inputs based on their abilities. The empirical results indicate that there exists (1) heterogeneity in growers’
abilities, and (2) input discrimination. The integrator’s strategy stimulates career concern type of incentives, which is optimal based on the results of our counterfactual analysis.

3.1 Introduction

When analyzing players’ decision problems, researchers often cannot observe some of the inputs that play a role in their decision-making. Those inputs are referred to as unobserved heterogeneity. Games with heterogeneous players have caught researchers’ attention in recent years. Auction and contract are two economic contexts in which identifying unobserved heterogeneity is an important issue. In the auction literature, [11], [3], [6], [20], [15], [1], and [26] study bidder asymmetry. Fewer studies have been conducted in a contract framework. [23] investigate the dynamic incentives in agricultural contracts with heterogeneous agents. This papers adds to the literature by nonparametrically detecting the unobserved heterogeneity in a principal-agent context.

Cardinal tournaments, also referred to as relative performance compensation schemes ([31]) or yardstick competition ([35]), are compensation schemes in which an individual player’s reward is a continuous function (typically linear) of the difference between the individual player’s performance and the group average performance ([39]). Cardinal tournaments are widely adopted in various business environments, such as broiler production contracts. The broiler industry is entirely vertically integrated and often considered a role model for the industrialization of agriculture ([23]). The finishing stage of the production process is organized almost entirely through contracts between integrators and independent growers. Broiler production contracts are agreements between an integrator and growers that bind growers to tend for the company-owned chickens until they reach market weight by strictly following specific production practices in exchange for monetary compensation ([23]). A production contract stipulates the division of responsibility and the payment scheme. Growers are responsible for constructing and equipping chicken houses and supplying labor and management. The integrator’s responsibility is to provide baby chicks, feed, and medication. The quality of the integrator-supplied inputs are not stipulated in the contract. Growers usually observe only the quantity of inputs they receive at the time of delivery, but not necessarily the quality of inputs. The quality will be fully

---

1The finishing stage is the final stage of the production process where 1-day old chicks are brought to the farm and grown to market weight.
revealed to them during the production process. Most of the modern broiler contracts are settled using a two-part piece-rate cardinal tournament consisting of a base payment per pound of live meat produced and a bonus payment based on the grower’s relative performance ([23]).

The main motivation for this research comes from the documented evidence regarding broiler contract growers complaining about the unfair distribution of variable quality inputs (feed and chicks) they receive from their principals ([23]). A preliminary inspection of the broiler contract settlement data revealed an interesting fact that a substantial number of tournaments is systematically won by the same growers. This leads to a natural question of whether the same growers won a disproportionate number of tournaments because they were truly better than others, or because they were (for whatever reason) given superior quality inputs.

In principal-agent framework, agents often have private information about their preferences or intrinsic abilities. In many business environments, including agriculture, principal and agents often interact through a series of short-term contracts. The principal can learn about agents’ abilities over time if they contract repeatedly. If there is possibility for the principal to treat agents differently, the principal as a profit maximizer will take advantage of the opportunity to increase his profits. In general, implicit incentives arise when there is some possibility for the principal to respond to agents’ performances ex post ([29]).

An important feature of broiler production contracts is that agents contracting with the same principal will operate under formally identical contract provisions, and contracts cover one flock or one batch of animals at a time ([24]). When the principal and agents contract repeatedly, an explicitly uniform but incomplete contract leaves the possibility for the principal to treat agents differently after developing a fairly precise knowledge of their abilities over time.

In this paper, we propose a theoretical model and solve the agent’s problem in the most general case where there are \( K \) types of growers and each grower maximizes his payoff, which is the payment he can receive when the contract settles based on the tournament payment scheme. The principal makes the input allocation decision among different types of growers with the aim of maximizing profit.

Based on the theoretical work, we propose a nonparametric approach to identify and estimate the cardinal tournament used in broiler production. In the rest of the paper, we

\[2\] For more details regarding broiler production contracts, see [23], or [25].
refer to the unobserved and discrete heterogeneity of growers as types. The distributions of output conditional on the type are nonparametrically identified and estimable from the observed distribution using the recently developed methodology in measurement error literature by [14]. This methodology has been applied to the auction literature (e.g., [2], [20], [15], [1], [26]). Identity of growers and three output levels from each grower are required for the identification. These enable us to recover the joint output distribution. It can be shown that the number of types is equal to the rank of a matrix constructed from the observed output levels. We pin down the rank of the matrix using the procedure proposed by [34], which is a sequential rank test. The test result gives the number of types.

For the empirical analysis, we use broiler production contract data from a large broiler company in the United States. The dataset includes contract settlement information from July 1995 to July 1997. With the nonparametric identification and estimation, we are able to categorize the growers as different types. We can then examine whether the principal, assuming that they can learn about growers’ abilities in repeated contracts, systematically discriminate among growers by strategically distributing production inputs of varying quality. The inputs can be sorted according to the quality and partitioned into different categories depending on the number of types of growers. The discrimination may take one of the following two forms: (1) the quality of inputs allocated to different types of agents is increasing in agent’s type, or (2) the quality of inputs allocated to different types of agents is decreasing in agent’s type. Our empirical analysis shows that there are two types of agents in the broiler tournaments that generated our data. The low type probability is 0.5442 and the high type probability is 0.4558. We also test the principal’s input allocation strategy and found that input discrimination exists and the quality of the inputs the principal allocates to different types of agents is increasing in agent’s type. The strategy is confirmed by the counterfactual analysis to be more profitable compared to the alternative strategy that allocates lower quality inputs to higher type agents.

This paper is closely related to [23] who investigate the post-contractual opportunism on the part of the principal that can take the form of strategic allocation of variable quality inputs among growers of different types by modeling two types of agents in a two-period contract. This paper contributes to the research question by employing the newly developed method in the measurement error models (e.g., [14]). To the best of our knowledge, this paper is the first to detect the existence of heterogeneity in agricultural contracts and

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3The same dataset has been used in [23].
recover the number of types using a fully nonparametric approach.

Another contribution of this paper is to test the strategy that the principal uses in input allocation. We use first-week mortality rates of the broilers as the indication of input quality\(^4\). With the *ex post* type probabilities of growers that we obtained from the nonparametric estimation, we categorize the growers as different types. We conduct bootstrap and obtain the result that the quality of the inputs the principal assigns to different types of agents is increasing in agent's type.

The remainder of the paper proceeds as follows. Section 3.2 presents the theoretical model of the principal-agent problem in the broiler production tournaments. Section 3.3 discusses the nonparametric identification and estimation of the output distributions conditional on type. Section 3.4 conducts the empirical analysis where we detect the heterogeneity from the data, determine the number of types, recover the conditional output distributions, estimate the theoretical model and test the existence of agent discrimination. Section 3.5 presents the counterfactual analysis of principal's input allocation strategies. Section 3.6 concludes.

### 3.2 The Model

In this section, we first describe the cardinal tournament that is used in the broiler production contracts. Based on the cardinal tournament, we solve the agent's problem of maximizing payoff and the principal's problem of maximizing profit by choosing how to allocate varying quality inputs to different types of agents.

#### 3.2.1 The Tournament

The cardinal tournament is a two-part piece-rate payment scheme. The piece rate consists of a base payment per pound of weight produced and a bonus payment based on the grower's relative performance. The bonus payment is calculated based on the difference between group average settlement cost and grower's individual settlement cost. Each grower's settlement cost is the sum of the costs of integrator supplied inputs (chicks, feed, medication, etc.) divided by the total pounds of broilers produced. The calculation of the group average settlement cost includes growers whose flocks were harvested within the

\(^4\)The same proxy for input quality has been adopted in [41] and [23]
same week. For the below-average settlement cost, the grower receives a bonus; for the above-average settlement cost, the grower receives a penalty. The payment for grower $i$ in one tournament is calculated as

$$ R_i = [I + r(\frac{1}{N} \sum_{i}^{N} c_i - c_i)]q_i $$

(3.1)

where $I$ and $r$ are the two contract parameters with $I$ being the fixed part of the piece rate and $r$ being the slope of the variable part of the piece rate. $N$ denotes the number of growers in the tournament. $c_i$ denotes grower $i$’s settlement cost. $\frac{1}{N} \sum_{i=1}^{N} c_i$ gives the average settlement cost of growers that are in the same tournament group. The difference between group average settlement cost $\frac{1}{N} \sum_{i=1}^{N} c_i$ and grower $i$’s settlement cost $c_i$ multiplied by the contract parameter $r$ gives the variable part of the piece rate.

### 3.2.2 Agent’s Problem

Suppose there are $K$ types of growers. Grower $i$ of type $k$ maximizes his expected payoff. The grower's payoff is determined by the cardinal tournament given in Eq. 3.1.

$$ \max_{q_{ki}} E \{[I + r(\frac{1}{N} \sum_{l=1}^{K} \sum_{j=1}^{N_l} c_{lj} - c_{ki})]q_{ki}] \} $$

(3.2)

where $N_l$ denotes the number growers in type $l$, $l=1, 2, ..., K$ and $N = \sum_{l=1}^{K} N_l$.

We specify grower settlement cost $c$ as $c = q \frac{s}{x}$, where $q$ denotes the output (weight produced), $s$ denotes the type-specific parameter and $x$ denotes the input quality the grower gets. With this specification, total production cost denoted by $C$ is quadratic in $q$, $C = cq = q \frac{q}{sx}$. Grower settlement cost $c$ is increasing in weight gain $q$ and decreasing in grower’s type $s$ and input quality $x$. Note that smaller $c$ indicates higher cost efficiency.

The output realization of growers is subject to an i.i.d. shock, denoted by $\epsilon$, which follows $N(0, \sigma^2)$. $\epsilon$ summarizes the discrepancy between the expected output and the realized output as generated by the mortality rate margin.

---

5The target weight and head started for a flock are both exogenous. The output realization is affected by the mortality rate.
Substituting in $c = \frac{q_s}{s_k x_k}$, Eq. 3.2 is equivalent to

$$\max_{q_{ki}} [I + r(\frac{1}{N} \sum_{l=1}^{K} \sum_{j=1}^{N_l} q_{l,j} + \epsilon_{l,j} - q_{ki} + \epsilon_{ki})][q_{ki} + \epsilon_{ki}]].$$

(3.3)

Taking the first-order condition with respect to $q_{ki}$, we obtain

$$E\{I + r(\frac{1}{N} \sum_{l=1}^{K} \sum_{j=1}^{N_l} q_{l,j} + \epsilon_{l,j} - q_{ki} + \epsilon_{ki}) + (q_{ki} + \epsilon_{ki}) r(\frac{1 - N}{N} \frac{1}{s_k x_k})\} = 0.$$  

(3.4)

Similarly, for grower $i'$ of type $k$, we obtain

$$E\{I + r(\frac{1}{N} \sum_{l=1}^{K} \sum_{j=1}^{N_l} q_{l,j} + \epsilon_{l,j} - q_{ki'} + \epsilon_{ki'}) + (q_{ki'} + \epsilon_{ki'}) r(\frac{1 - N}{N} \frac{1}{s_k x_k})\} = 0.$$  

(3.5)

Combining Eq. (3.4) and Eq. (3.5), the following holds.

$$E(\frac{q_{ki} + \epsilon_{ki}}{s_k x_k}) = E(\frac{q_{ki'} + \epsilon_{ki'}}{s_k x_k}).$$

(3.6)

Eq. (2.6) reduces to

$$q_{ki} = q_{ki'}.$$  

(3.7)

Same type growers will make the same output choice. Substituting Eq. (3.6) into Eq. (3.4) or Eq. (3.5), the solution to grower $i$ of type $k$’s problem can be obtained as follows.

$$q_{ki} = \frac{I}{r(1 - \frac{1}{N})} s_k x_k$$

(3.8)

The choice of $q_{ki}$ is a function of contract parameters $I$ and $r$, type-specific parameter $s_k$, input quality $x_k$, and number of growers $N$.

### 3.2.3 Principal's problem

The principal’s problem is to allocate a given set of inputs to different type growers with the goal of maximizing profit. The inputs can be partitioned according to the number of grower types and the number of growers for each type. Let $x_k$ denote the input quality type-$k$ growers get, $k = 1, 2, ..., K$. Let $x$ denote the average input quality. The following
relationship holds.
\[ \sum_{k=1}^{K} N_k x_k = N x \]  \hspace{1cm} (3.9)
where \( N_k \) denotes the number of type \( k \) growers and \( N \) denotes the total number of growers.

The principal’s profit \( \Pi \) is equal to the revenue, price per pound \( P \) times weight produced \( q \) subtracted by the production cost \( C \) and the payment to growers for producing the broilers \( R \).
\[ \Pi = P q - C - R \]  \hspace{1cm} (3.10)

Total weight produced, production cost and payments are made up of those of the growers of different types. Substituting in the expressions for \( q \), \( C \) and \( R \), the principal’s problem is as follows.
\[
\max_{x_1, x_2, \ldots, x_K} \ E \sum_{k=1}^{K} \sum_{i=1}^{N_k} \left\{ P(q_{ki} + \epsilon_{ki}) - \frac{(q_{ki} + \epsilon_{ki})^2}{s_k x_k} \right\} \\
- E \sum_{k=1}^{K} \sum_{i=1}^{N_k} \left\{ (I + r\left(\frac{1}{N} \sum_{l=1}^{K} \sum_{j=1}^{N_l} q_{lj} + \epsilon_{lj} \right) - \frac{q_{ki} + \epsilon_{ki}}{s_k x_k} ) (q_{ki} + \epsilon_{ki}) \right\}
\]
\[ s.t. \sum_{k=1}^{K} N_k x_k = N x \]  \hspace{1cm} (3.11)

where \( q_{ki} = \frac{I}{r(1 - \frac{1}{N})} s_k x_k \).

The objective function can be simplified as
\[ E(\Pi) = (P - I - \frac{I}{r(1 - \frac{1}{N})} ) \left( \frac{I}{r(1 - \frac{1}{N})} \right) \sum_{k=1}^{K} N_k s_k x_k + (r(1 - \frac{1}{N}) - 1) \sigma^2 \sum_{j=1}^{K} \frac{N_k}{s_k x_k} \]  \hspace{1cm} (3.12)

The Lagrangian associated with this maximization problem is as follows.
\[ L = (P - I - \frac{I}{r(1 - \frac{1}{N})} ) \left( \frac{I}{r(1 - \frac{1}{N})} \right) \sum_{k=1}^{K} N_k s_k x_k + (r(1 - \frac{1}{N}) - 1) \sigma^2 \sum_{k=1}^{K} \frac{N_k}{s_k x_k} \]
\[ + \lambda \left( \sum_{k=1}^{K} N_k x_k - N x \right) \]  \hspace{1cm} (3.13)

where \( \lambda \) is the Lagrangian multiplier.
The first-order condition with respect to $x_k$ is as follows.

$$\left(P - I - \frac{I}{r(1 - \frac{1}{N})} N_k s_k - (r(1 - \frac{1}{N}) - 1) \frac{N_k \sigma^2}{s_k x_k^2} + \lambda N_k \right) = 0, \quad k = 1, 2, ..., K. \quad (3.14)$$

The first-order conditions with respect to $x_l$ and $x_k$ reduce to the following,

$$x_l = \left( \frac{(P - I - \frac{I}{r(1 - \frac{1}{N})} N_k s_k - (r(1 - \frac{1}{N}) - 1) \frac{N_k \sigma^2}{s_k x_k^2} + \lambda N_k)}{s_l (r(1 - \frac{1}{N}) - 1) \sigma^2 + \frac{1}{s_k x_k^2}} \right)^{-1}, \quad k \neq l. \quad (3.15)$$

Substituting Eq. (3.15) into the constraint Eq. (3.9), we obtain

$$N_k x_k + \sum_{l \neq k} N_l \left( \frac{(P - I - \frac{I}{r(1 - \frac{1}{N})} N_k s_k - (r(1 - \frac{1}{N}) - 1) \frac{N_k \sigma^2}{s_k x_k^2} + \lambda N_k)}{s_l (r(1 - \frac{1}{N}) - 1) \sigma^2 + \frac{1}{s_k x_k^2}} \right)^{-1} = N x. \quad (3.16)$$

The input quality allocated to type-$k$ growers $x_k$ is the solution to Eq. (3.16).

### 3.3 Identification

In this section, we present the identification and estimation of the unobserved heterogeneity among growers and the output distribution conditional on grower’s type.

#### 3.3.1 Number of Types

In the previous section, we assumed that there are $K$ types of agents. The existence of heterogeneity is equivalent to the number of types being greater than one. Therefore, testing the existence of heterogeneity is equivalent to the test of $K = 1$ against $K > 1$. We follow the method proposed in [14]. This method requires three different measures of output $q$ to identify agent's types and output distributions. As shown in [1], one of the observation acts as an instrumental variable for the agent's type. Therefore, two observations for each agent are sufficient for identification if there is suitable instrumental variable for agents’ types available. We need the following restrictions on the data structure to employ the method.
Assumption 1. (1) Each grower participates in at least three homogeneous tournaments and his outputs in the three tournaments are independent. (2) The type is invariant across tournaments.

Since the integrator and growers interact through a series of short-term repeated contracts, more often than not a grower has participated in more than three contracts. We assume the conditional independence of outputs on the type. We may allow the growers’ outputs in different tournaments to be correlated through some observed heterogeneity where we can exploit the procedure in [12] to control the heterogeneity such that the "residual values" are independent. The type refers to grower’s intrinsic ability and it is not likely to change over the course of three tournaments.

By Assumption 1, we obtain three outputs for each grower, denoted by an output vector \((q_1, q_2, q_3)\). To identify the discrete types, we need to first discretize two outputs. Without loss of generality, we discretize \(q_1\) and \(q_2\) into \(d_1\) and \(d_2\). For the discretization, we divide the support of \(q_i\) into \(M\) bins, \(i=1,2\). Label the bins from smallest to largest as \(bin_1, bin_2, ..., bin_M\).

\[
d_i = \begin{cases} 
1, & q_i \in bin_1 \\
2, & q_i \in bin_2 \\
... & \\
M, & q_i \in bin_M 
\end{cases}
\]

\(d_i\) ranges from 1 to \(M\) depending on which bin \(q_i\) is in. Denote the joint probability mass function of \(d_1\) and \(d_2\) as \(g(d_1, d_2)\). A matrix form of the joint probability mass function is expressed as

\[
B_{d_1,d_2} \equiv [g(d_1 = i, d_2 = j)]_{i,j}
\]

where \(B_{d_1,d_2}\) is an \(M \times M\) matrix. This matrix is the key to the identification of the number of types.

Proposition 1. \(\text{Rank}(B_{d_1,d_2}) = K\) if \(M \geq K\).

Proof. See proof of Lemma 2 in [1].

According to Proposition 1, as long as the dimension of \(B_{d_1,d_2}\) is greater than \(K\), its rank
equals $K$. If the dimension of $B_{d_1,d_2}$ is less than $K$, it has full rank\(^6\). The procedures for determining $K$ is as follows.

**Step 1.** Start from $M = 2$ and test if $B_{d_1,d_2}$ has full rank. If $B_{d_1,d_2}$ has full rank, it implies that $M \leq K$.

**Step 2.** Increase $M$ by 1 and test if $B_{d_1,d_2}$ has full rank.

**Step 3.** Repeat Step 2 until $B_{d_1,d_2}$ has not full rank.

The final value of $M$ equals $K + 1$. To test the rank of $B_{d_1,d_2}$, we follow the method proposed by [34]. The main idea is to test the number of eigenvalues that are significantly different from zero. For the $M \times M$ matrix $B_{d_1,d_2}$, we sequentially test the null hypothesis $H_r : r k(B_{d_1,d_2}) = r$ against the alternative hypothesis $H'_r : r k(B_{d_1,d_2}) > r$, $r = 0, 1, ..., M - 1$, to reveal the true rank $r^*$ of the matrix $B_{d_1,d_2}$. More details of the test could be found in Appendix A.

### 3.3.2 Type-specific Output Distribution

After we determine the number of types, we can move on to the identification of the output distributions conditional on the type and the corresponding type probabilities. Let $g(q_1, q_2, q_3)$ denote the joint density of the three outputs. By the law of total probability, we have

$$g(q_1, q_2, q_3) = \sum_{k \in \mathcal{X}} g(q_1, q_2, q_3, k) = \sum_{k \in \mathcal{X}} g(q_1 | q_2, q_3, k) g(q_3 | q_2, k) g(k, q_2). \quad (3.17)$$

By Assumption 1, outputs are independent across tournaments, indicating that $g(q_1 | q_2, q_3, k) = g(q_1 | k)$ and $g(q_3 | q_2, k) = g(q_3 | k)$. Substituting these into (3.17), we have

$$g(q_1, q_2, q_3) = \sum_{k \in \mathcal{X}} g(q_1 | k) g(k, q_2) g(k, q_3). \quad (3.18)$$

Replacing $q_1$ and $q_2$ with $d_1$ and $d_2$ respectively in the above equation gives

$$g(d_1, d_2, q_3) = \sum_{k \in \mathcal{X}} g(d_1 | k) g(k, q_3) g(k, d_2). \quad (3.19)$$

---

\(^6\)The proof is in Appendix B
Since \( d_1, d_2 \) and type \( k \) are discrete, it is convenient to express (3.19) in matrix form

\[
B_{q_3, d_1, d_2} = B_{d_1 | k} D_{q_3 | k} B_{k, d_2}, \tag{3.20}
\]

where the matrices are defined as follows

\[
B_{q_3, d_1, d_2} \equiv [g(q_3, d_1 = i, d_2 = j)]_{i,j}
\]

\[
B_{d_1 | k} \equiv [g(d_1 = i | k = l)]_{i,l}
\]

\[
D_{q_3 | k} \equiv \text{diag}[g(q_3 | k = 1), g(q_3 | k = 2), ..., g(q_3 | k = K)]
\]

\[
B_{k, d_2} \equiv [g(k = l, d_2 = j)]_{l,j}.
\]

By Proposition 1, we can choose \( M = K \) such that the matrices \( B_{d_1, d_2}, B_{d_1 | k} \) and \( B_{k, d_2} \) are all full rank and invertible. Note that we have \( B^{-1}_{d_1, d_2} = B^{-1}_{k, d_2} B^{-1}_{d_1 | k} \). Post-multiplying \( B^{-1}_{d_1, d_2} \) to both sides of (3.20), we obtain

\[
B_{q_3, d_1, d_2} B^{-1}_{d_1, d_2} = B_{d_1 | k} D_{q_3 | k} B^{-1}_{d_1 | k}. \tag{3.21}
\]

Note that the left-hand side consists of the probability density of \( q_3 \) and the probability mass of \( d_1 \) and \( d_2 \), which are observable. The right-hand side is a form of eigenvalue-eigenvector decomposition of the left-hand side. As a result, we can identify \( B_{d_1 | k} \) from the eigenvalue-eigenvector decomposition. The identification equation for the type probabilities is given by

\[
p(d_1) = B_{d_1 | k} p_k, \tag{3.22}
\]

where \( p(d_1) = (p(d_1 = 1), p(d_1 = 2), ..., p(d_1 = M))' \) and \( p_k \) denotes the type probability for type \( k \). With \( B_{d_1 | k} \) being invertible, we can rewrite (3.22) as

\[
p_k = B^{-1}_{d_1 | k} p(d_1). \tag{3.23}
\]

The probability \( p(d_1) \) can be estimated by the sample analog. The equation for the identification of the type-specific output distributions is given by

\[
\begin{cases}
G(q_3 | k = l) = G(q_3, k = l) / p(k = l) \\
g(q_3 | k = l) = g(q_3, k = l) / p(k = l) l = 1, 2, ..., M
\end{cases} \tag{3.24}
\]
where \( G(q_3, k = l) \) can be estimated by empirical CDF estimator and \( g(q_3, k = l) \) can be estimated by kernel methods.\(^7\)

### 3.4 Empirical Analysis

In this section, we first conduct the rank test to detect the existence of the heterogeneity and determine the number of types. Subsequently, we recover the output distributions conditional on the type and check the type separation. With the conditional densities obtained from kernel methods, we estimate the grower’s \( \text{ex post} \) type probabilities. We also estimate the unknown parameters in the agent’s problem. Finally, we test the principal’s input allocation strategy and whether there exists input discrimination.

#### 3.4.1 Data

We use broiler contract settlement data of a large broiler company in the United States. The dataset contains contract settlement information of broilers from July 1995 to July 1997. There are in total 3,194 observations and each observation contains contract settlement information for one flock of birds including date settled, head started, head sold, weight sold, first-week mortality rate, feed conversion and so on. Tournaments are formed based on the settlement date. Growers whose flocks are settled on the same day form a tournament. During the period of the data, there are in total 104 tournaments. The number of growers in each tournament ranges from 18 to 43, with a mean of 30.6 and a standard deviation of 5.2. Growers’ performances in the production of broilers are summarized in Table 3.1. The average weight per broiler is 4.81 pounds. The feed conversion averages around 2, indicating that 2 pounds of feed need to be consumed in order for a bird to grow one pound of weight. The mean of the mortality rate is 2.87%. Head started denotes the number of baby chicks delivered to growers for them to produce. The mean of head started on each grower’s farm is 51,424 and the standard deviation is 27,604. Due to the variation of head started, we divide total weight harvested of the flock by head started to obtain a normalized weight measure for the empirical analysis.\(^8\) The number of birds delivered to growers varies

\(^7\)For more estimation details, see [2].
\(^8\)Note that head started and target weight are not at the grower’s discretion. Growers find out head started upon delivery of baby chicks. Broilers are shipped to processing units when the principal decides that the broilers have reached the target weight on average. Hence weight harvested essentially captures the mortality margin in the sense that the only way for a grower to increase weight harvested is to make sure that mortality
Table 3.1 Production performance summary statistics

<table>
<thead>
<tr>
<th>Weight (lbs)</th>
<th>Feed conversion</th>
<th>Mortality rate (%)</th>
<th>Head started</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4.81</td>
<td>2.03</td>
<td>2.87</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.29</td>
<td>0.08</td>
<td>3.25</td>
</tr>
</tbody>
</table>

Figure 3.1 Distribution of Tournament Winners

from 10,500 to 163,200 depending on the farm size.

The observation that a fairly large number of tournaments are won by the same growers is manifested in Figure 3.1. We plot the distribution of tournament winners for growers that have participated in at least three tournaments. On the horizontal axis are the ratios of tournaments won to tournaments played and on the vertical axis are the frequencies. By law of large numbers, if the outcomes of tournaments are completely random, the distribution of winners should be centered around 0.5 while in the figure the distribution of winners has fat tails indicating some yet undetected regularities in tournament outcomes.

The quality of input supplied by the principal can vary significantly across growers and tournaments (see [23]). The 1-day old chicks and feed are obtained from different rate of the flock is low. The normalized weight measure can perfectly reflect the mortality margin.
Table 3.2 Rejection rates of rank test for M=3

<table>
<thead>
<tr>
<th>n</th>
<th>r=1</th>
<th>r=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>342</td>
<td>0.999</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 3.3 Rejection rates of rank test for M=4

<table>
<thead>
<tr>
<th>n</th>
<th>r=1</th>
<th>r=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>342</td>
<td>1.000</td>
<td>0.096</td>
</tr>
</tbody>
</table>

hatcheries and feed mills at different points in time. Therefore, there is reason to believe that the quality of input is not homogeneous. We use the first-week mortality rate that is observed in the data to indicate the varying input quality, with higher first-week mortality rate indicating lower input quality and lower first-week mortality rate indicating higher input quality.

3.4.2 Rank Test Results

In this section, we sequentially test the rank of $B_{d_1,d_2}$ in order to determine the number of types. We use the normalized output measure divided by input quality $x$ to filter out the impact of input quality on grower’s output. The input quality $x$ is constructed as one minus first-week mortality rate. There are 342 growers that have at least 3 observations in our data. Recall that the null hypothesis is $H_r : r k(B_{d_1,d_2}) = r$ and the alternative hypothesis is $H'_r : r k(B_{d_1,d_2}) > r$, $r = 0, 1, ..., M - 1$.

For $M=2$, $B_{d_1,d_2}$ is a 2×2 matrix. The rejection rate for $r = 1$ is 0.994. As a result, we can reject $H_r : r k(B_{d_1,d_2}) = 1$ for $M=2$. Then we continue to perform the rank test for $M=3$.

The result of the rank test for $M=3$ is presented in Table 3.2. In this case, the dimension of $B_{d_1,d_2}$ is 3×3. For $r = 1$, the rejection rate is equal to 0.999. For $r=2$, the rejection rate is 0.002. From the test results, we cannot reject the null hypothesis $H_r : r a n k(B_{d_1,d_2}) = 2$ for $M=3$. We continue to conduct the test for $M=4$ and 5. The results are presented in Table 3.3 and Table ???. In both cases, we cannot reject the null hypothesis $H_r : r a n k(B_{d_1,d_2}) = 2$. As a result, we can determine that the number of types $K$ is 2. We have two types of growers that contracted with the integrator.

---

9From Eq. (3.8), $\frac{q_k}{s_k} = \frac{1}{r(1-\alpha)} s_k$. 

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Table 3.4 Rejection rates of rank test for M=5

<table>
<thead>
<tr>
<th></th>
<th>r=1</th>
<th>r=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=342</td>
<td>1.000</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Table 3.5 Type probabilities

<table>
<thead>
<tr>
<th></th>
<th>Type 1 (Low)</th>
<th>Type 2 (High)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_k$</td>
<td>0.5442</td>
<td>0.4558</td>
</tr>
</tbody>
</table>

### 3.4.3 Type-Specific Output Distributions

Following Eq. (3.8), we can estimate the type probabilities from the data. The result is presented in Table 3.5. The probability that a grower is low-type is 0.5442 and the probability that a grower is high-type is 0.4558. Multiplying by the total number of growers, we obtain the number of growers of each type. The number of low-type growers equals 186 and the number of high-type growers equals 156.

Following Section 3.3.2, we can obtain the estimates of type-specific output distributions. Figure 3.2 presents the estimated type-specific output distributions. The four curves to the right are those of high-type growers, labeled as Type 2. The four curves to the left are those of low-type growers, labeled as Type 1. The means and the confidence intervals are generated by bootstrap. At each output $q$, the probability of having a performance of at least $q$ for high-type growers is always higher than that of low-type growers, indicating that the output distribution for the high type first-order stochastically dominates that of the low type. From the confidence intervals, we can observe that the confidence intervals of the two types have no overlap most of the time, which indicates the robustness of the type separation. Figure 3.3 depicts the type-specific output densities, where the curve to the right represents the densities of the high type and the curve to the left represents those of the low type. Compared to the low type, the distribution of the high type is left-skewed, which is also a sign of the type separation.

We conduct the K-S test to check whether the performances of two types are from the same distribution. The result we obtained is that we can reject the null, which also supports the type separation.
Figure 3.2 Type-Specific Output Distributions

Figure 3.3 Type-Specific Output Distributions
3.4.4 \textit{Ex Post} Type Probabilities

By Bayes’ rule, we can obtain the type probability of growers after observing their performances, i.e. the \textit{ex post} type probabilities.

\begin{equation}
Pr(k|q_3) = \frac{g(q_3|k)p_k}{g(q_3)}
\end{equation}

\(g(q_3|k)\) and \(g(q_3)\) are estimated with kernel methods. Growers’ \textit{ex post} low-type probabilities range from 0.3051 to 1. The estimated \textit{ex post} low-type probabilities are illustrated in Figure 3.4. We plot the \textit{ex post} low-type probabilities of each grower from smallest to largest. The horizontal axis shows the number of growers with a smaller \textit{ex post} low-type probability.

3.4.5 Model Estimation

In this section, we estimate the parameters in the theoretical model. Conditional on the grower’s type, \(q_i\) follows a normal distribution with mean \(\frac{L}{r(1-\eta)}s_kx_k\) and standard deviation
Table 3.6 MLE Result

<table>
<thead>
<tr>
<th></th>
<th>$\hat{s}_H$</th>
<th>$\hat{s}_L$</th>
<th>$\hat{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates</td>
<td>132.312</td>
<td>120.810</td>
<td>0.227</td>
</tr>
<tr>
<td>Std. Err.</td>
<td>(0.688)</td>
<td>(0.648)</td>
<td>(0.017)</td>
</tr>
</tbody>
</table>

$\sigma$ based on the agent’s problem.

$$q_i|k \sim N\left(\frac{I}{r(1-\pi)} s_k x_k, \sigma^2\right)$$  \hspace{1cm} (3.26)

The likelihood for $q_i$ is the sum of type probability weighted normal density functions,

$$f(q_i; \theta) = \sum_k p(q_i|k)p(k),$$  \hspace{1cm} (3.27)

where $p(q_i|k)$ denotes the normal PDF, $p(k)$ denotes the type probability as obtained in Section ?? and $\theta$ denotes the parameter in the model, $\theta = (s_H, s_L, \sigma)$.

The likelihood function is formulated as,

$$L(\theta|q) = \prod_{i=1}^{N} f(q_i; \theta).$$  \hspace{1cm} (3.28)

We obtain the estimates for $\theta$ using maximum likelihood estimation.

$$\hat{\theta} \in \{\arg \max_{\theta} L(\theta|q)\}$$  \hspace{1cm} (3.29)

The estimation result is presented in Table 3.6. $\hat{s}_H$ and $\hat{s}_L$ are 132.312 and 120.810 respectively. $\hat{\sigma}$ is 0.227. $\hat{s}_H$ is larger than $\hat{s}_L$, which is consistent with our model. The type-specific parameter for high-type growers is greater than that of low-type growers since higher $s$ leads to lower settlement cost $c$ holding everything else constant, which indicates high type in the model specification.
3.4.6 Testing the Principal's Input Allocation Strategy

With two types of growers, total weight produced, production costs and payments are made up of those of the low-type growers and the high-type growers. Thus we have the following

\[ q = q_L + q_H = \sum_{i=1}^{N_L} q_{Li} + \sum_{i=1}^{N_H} q_{Hi} \]  
(3.30)

\[ C = C_L + C_H = \sum_{i=1}^{N_L} C_{Li} + \sum_{i=1}^{N_H} C_{Hi} \]  
(3.31)

\[ R = R_L + R_H = \sum_{i=1}^{N_L} R_{Li} + \sum_{i=1}^{N_H} R_{Hi} \]  
(3.32)

where \( N_L \) and \( N_H \) denote the number of low-type and high-type growers respectively. Substituting in the expressions for \( q \), \( C \) and \( R \), the principal's profit \( \Pi \) becomes

\[
\Pi = P(\sum_{i=1}^{N_L} q_{Li} + \sum_{i=1}^{N_H} q_{Hi}) - (\sum_{i=1}^{N_L} C_{Li} + \sum_{i=1}^{N_H} C_{Hi}) - (\sum_{i=1}^{N_L} R_{Li} + \sum_{i=1}^{N_H} R_{Hi}).
\]  
(3.33)

The principal's maximization problem is as follows.

\[
\max E \sum_{k=L,H} \sum_{i=1}^{N_k} \{ P(q_{ki} + \epsilon_{ki}) - (q_{ki} + \epsilon_{ki})^2 \} - E \sum_{k=L,H} \sum_{i=1}^{N_k} \{ (I + r(\frac{1}{N} \sum_{i=L,H} \sum_{j=1}^{N_j} q_{ij} + \epsilon_{ij} - \frac{q_{ki} + \epsilon_{ki}}{s_k x_k})(q_{ki} + \epsilon_{ki})) \} \]
\[
\text{s.t. } N_L x_L + N_H x_H = N x \]  
(3.34)

where \( q_{ki} = \frac{I}{r(1-\frac{1}{N})} s_k x_k \).

Similarly, we obtain

\[
N_L x_L + N_H \left( \frac{(P - I - \frac{i}{r(1-\frac{1}{N})})}{r(1-\frac{1}{N}) - \frac{1}{s_L x_L^2}} + \frac{1}{s_L x_L^2} \right) = N x. \]  
(3.35)

The solution to Eq. (3.35) gives \( x_L^* \).
Table 3.7 Summary Statistics of Mortality Rates

<table>
<thead>
<tr>
<th></th>
<th>$m_H$ (ratio)</th>
<th>$m_L$ (ratio)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0127</td>
<td>0.0141</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.0074</td>
<td>0.0085</td>
</tr>
<tr>
<td>Min</td>
<td>0.0034</td>
<td>0.0024</td>
</tr>
<tr>
<td>Max</td>
<td>0.0550</td>
<td>0.0689</td>
</tr>
</tbody>
</table>

With $x^*_L$, $x^*_H$ can be obtained based on the first-order conditions as follows.

$$x^*_H = \left( \sqrt{s_H \left( \frac{(P - I - \frac{1}{r(1 - \frac{1}{N})})}{r(1 - \frac{1}{N}) - 1}\sigma^2 + \frac{1}{s_L x^*_L^2} \right)} \right)^{-1}.$$  

(3.36)

In the data, we have observations of the first-week mortality rates of the broilers allocated by the principal to each grower. The first-week mortality rate is an important indicator of the input quality. Based on the ex post low-type probabilities calculated in Section 3.4, we can sort the 342 growers with the ex post low-type probabilities from low to high. The first 156 growers with the smaller 156 ex post low-type probabilities are categorized as high-type and the rest are low-type. The summary statistics of the first-week mortality rates of the broilers allocated to each type are presented in Table 3.7, where $m_H$ and $m_L$ denote the first-week mortality rates of the broilers allocated to high-type growers and low-type growers respectively. The mean of $m_H$ is 0.0127, which is smaller than 0.0141, the mean of $m_L$. Note that a smaller first-week mortality rate indicates better input quality. The standard deviation of $m_H$ is 0.0074. The standard deviation of $m_L$ is 0.0085. The smallest first-week mortality rate for high-type growers is 0.0034. The largest is 0.0550. The smallest first-week mortality rate for low-type growers is 0.0024 while the largest is 0.0689. Table 3.8 gives an idea of the magnitude of the effect input allocation has on output. The differences between the outputs from best-quality input and worst-quality input for high-type growers and low-type growers are 0.2645 and 0.3080 respectively.

We bootstrap the first-week mortality rates for both types from the mortality rates associated with the respective type 1000 times and compare the means of the mortality rates of the two types each time. The null hypothesis is that $H_0$: mean($m_H$) - mean($m_L$) > 0. The p-value we obtained is 0.044. As a result, we can reject the null. We can conclude that
Table 3.8 The Effect of Input Quality on Output

<table>
<thead>
<tr>
<th></th>
<th>$q_H$ (lbs per unit)</th>
<th>$q_L$ (lbs per unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min($m_i$)</td>
<td>5.1087</td>
<td>4.6197</td>
</tr>
<tr>
<td>Max($m_i$)</td>
<td>4.8442</td>
<td>4.3117</td>
</tr>
<tr>
<td>Difference</td>
<td>0.2645</td>
<td>0.3080</td>
</tr>
</tbody>
</table>

the first-week mortality rates of the broilers allocated to high-type growers are smaller than the first-week mortality rates of the broilers allocated to low-type growers, which shows that the principal allocates higher quality inputs to high type growers, lower quality inputs to low type growers, not the other way around.

3.5 Counterfactual

We define two input allocating strategies as follows and we analyze how different strategies affect principal's profit.

**Strategy 1.** Allocate high-quality inputs to high-type growers and low-quality inputs to low-type growers.

**Strategy 2.** Allocate low-quality inputs to high-type growers and high-quality inputs to low-type growers.

In order to calculate principal's profit, we use broiler meat prices for the period covered by the data. The prices are weekly composite 12 cities market average prices. During the period between July 1995 and July 1997, broiler prices were reasonably stable with an average of 61 cents per pound. These prices are based on dressed (processed) weight and we convert them into live weights using industry average processing yields.

In both strategies, we stick to the set of inputs in the data but allocate them differently. To implement Strategy 1, we first sort the first-week mortality rates observed in the data and assign the broiler chicks with the smallest 156 first-week mortality rates to the 156 high-type growers and the 186 low-type growers end up with the broiler chicks with the larger mortality rates. By doing so, high-type growers get the high-quality inputs and vice versa. With the estimated parameters and the assigned input qualities, we can simulate

---

10 For details regarding the price data, see [40].

11 Processing yields are positively related to the size of the birds. We adopted the processing yield 71% for 5-pound birds.
Table 3.9 95% Confidence Intervals of the principal’s profits

<table>
<thead>
<tr>
<th></th>
<th>Strategy 1</th>
<th>Strategy 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>$(1.4078 \times 10^{7}, 1.4578 \times 10^{7})$</td>
<td>$(1.4093 \times 10^{7}, 1.4592 \times 10^{7})$</td>
</tr>
</tbody>
</table>

the profit 1,000 times according to Eq. (??) and construct the 95% confidence intervals as reported in Table 3.9. The 95% confidence interval of the principal’s profit under Strategy 1 is $(1.4078 \times 10^{7}, 1.4578 \times 10^{7})$.

To evaluate the effect of Strategy 2 on principal’s profit, we assign the broiler chicks with the smallest 186 first-week mortality rates to the 186 low-type growers and the broiler chicks with the 156 larger mortality rates to the 156 high-type growers. With the estimated parameters and the assigned input qualities, we simulate the principal’s profit 1,000 times and report the 95% confidence interval in Table 3.9. The 95% confidence interval of the principal’s profit under Strategy 2 is $(1.4093 \times 10^{7}, 1.4592 \times 10^{7})$.

3.6 Conclusion

In this paper, we propose a nonparametric approach to identify the unobserved heterogeneity in broiler production tournaments by exploiting the recent advancement in the measurement error models. We nonparametrically test the existence of the grower heterogeneity in agricultural contracts and recover the output distributions conditional on growers’ unobserved heterogeneity. The result we obtained is that there exists heterogeneity in agent’s abilities and there are two types of growers in the broiler production contract that generated our data. With the ex post type probabilities, we categorize growers into different types and test the strategy the principal uses in input allocation. Our result shows that the input quality allocated to growers of different types is increasing in grower’s type. With the estimated type-specific parameters as defined in the theoretical model, we conduct counterfactual analysis to compare the profit under different input allocation strategies. Confidence intervals of the profits are constructed.
CHAPTER 4

PROFITABILITY OF CARDINAL PIECE RATE VS CARDINAL TOURNAMENT WITH HETEROGENEOUS GROWERS

In this paper, we compare the profitability of a two-part piece-rate cardinal tournament and an alternative cardinal tournament where bonuses and penalties exactly cancel out. This paper is motivated by the speculation that more able growers will excel in both the settlement cost margin and the weight gain margin, leading to bonuses outweigh penalties. We analyze whether the principal is losing money with the existing contract compared to an alternative contract. Using broiler production contract data, we estimate the structural model with heterogeneous growers. We use the estimates to conduct counterfactual analysis. The result shows that the principal’s profit is higher under the alternative contract mainly driven by the lower grower payments.
4.1 Introduction

Tournaments are labor contracts in which an individual’s payoff depends on the performance relative to others ([39]). There are rank-order tournaments and cardinal tournaments. Rank-order tournaments are compensation schemes that pay according to an individual’s ordinal rank in an organization rather than the individual performance, as those considered in [22]. As in some sporting events such as soccer or tennis tournaments, what matters in the allocation of prizes is only a player’s rank. Cardinal tournaments, also referred to as relative performance compensation schemes ([31]) or yardstick competition ([35]), are compensation schemes in which an individual player’s reward is a continuous function (typically linear) of the difference between the individual player’s performance and the group average performance ([39]).

The broiler industry is entirely vertically integrated and often considered a role model for the industrialization of agriculture ([23]). Most of the modern broiler contracts are settled using a two-part piece-rate cardinal tournament consisting of a base payment per pound of live meat produced and a bonus payment based on the grower’s relative performance ([23]). The bonus payment is calculated as the difference between group average settlement cost and grower’s individual settlement cost multiplied by the weight gain. Settlement cost for each grower is the sum of the costs of integrator-supplied inputs (chicks, feed, medication, etc.) divided by the total pounds of live broilers produced. Group average settlement cost is the average settlement cost of growers whose flocks were harvested within the same week. For the below-average settlement cost, the grower receives a bonus, while for the above-average settlement cost, the grower receives a penalty.

Since bonuses and penalties depend on (1) weight gain, and (2) the difference between group average settlement cost and grower’s individual settlement cost, it could be the case that the winners of a tournament have greater differences between group average settlement cost and grower individual settlement cost or greater weight gain or both than those who happen to be on the losing end of the tournament, resulting in the overall bonuses being greater than the overall penalties. Tournaments are competitions between players of heterogeneous abilities. In broiler production tournaments, it has been shown that growers are heterogeneous in [19] and [25]. As a result, the above scenario is very likely to happen since winners are supposedly more experienced in growing out broilers.
or of higher ability and they will excel in both the settlement cost margin and the weight gain margin. Based on this speculation, the goal of this paper is to examine whether the integrator is losing money under the two-part piece-rate tournament payment scheme with heterogeneous growers than an alternative payment scheme where bonuses and penalties cancel out precisely by conducting counterfactual analysis.

The profitability of a payment scheme depends on two factors: (1) the weight gain that can be attained under the contract, and (2) the payment to growers. In the counterfactual analysis, we will compare the payment to growers as well as the incentive effect, demonstrated in the weight gain between the alternative contract and the original contract.

This paper is closely related to the literature that examines the welfare effects of tournament payment schemes. [25] compares the welfare effects of tournaments and piece rates in contracts with heterogeneous growers. [39] studies the welfare effects of mixing players of different abilities in piece rate tournaments. They show that the principal wins by mixing contestants of varying abilities rather than sorting them into more homogeneous groups. [40] analyzes the principal’s optimal response to the regulation proposal of truncating growers’ bonus payments in a piece rate tournament at zero. The result is that the potential welfare losses due to the tournament truncation could be mitigated by adjusting the contract parameters. [41] analyzes the efficiency gains associated with switching from a rank-order tournament to a cardinal tournament. However, no work regarding the non-zero sum of bonuses and penalties under the two-part piece-rate tournament payment scheme has been reported. This paper adds to the literature by focusing on the non-zero sum property of bonuses and penalties in the two-part piece-rate tournament with heterogeneous growers.

This paper also adds to the growing literature on the structural estimation of tournament models. [10] estimates the parameters of a sequential tournament game of championship series using sports data. [9] estimates an elimination tournament model of promotions in partnerships. [7] estimates a tournament model with data on employment levels and wages. [38] uses the estimates obtained from a structural model of a rank-order tournament to simulate how changes in the tournament characteristics impact welfare.

In the theoretical model, we solve for the optimal effort level in the existing contract in two cases. In the first case, we assume there is no heterogeneity among growers and show that bonuses and penalties cancel out each other precisely. In the second case, we assume that growers are heterogeneous in ability and disutility of effort. Then we propose
an alternative tournament payment scheme which leads to a zero sum of bonuses and penalties. We solve the agent's problem to obtain the optimal effort level in this contract. We also solve the principal's problem subject to the individual rationality constraint and the incentive compatibility constraint to obtain the optimal contract parameters.

In the empirical section, we estimate the model by maximum likelihood estimation. With the estimated parameters, we are able to construct the counterfactual scenario. We first compute the optimal contract parameters of the alternative tournament payment scheme. With the optimal contract parameters, we can obtain the optimal effort level and the weight gain. The payment to growers, principal's revenue and principal's profit under the alternative contract are constructed for the same number of observations as in the data. The result shows that the payment to growers under the alternative contract is smaller than the payment under the existing contract. The principal’s revenue under the alternative contract is slightly higher than that under the existing contract. The principal’s profit under the alternative contract is larger than the existing contract, which is mainly driven by the difference in payments to growers.

The rest of the paper is structured as follows. Section 4.2 presents the agent's problem in the original contract. Section 4.3 solves for the optimal effort and contract parameters in the alternative contract. Section 4.4 conducts the empirical analysis. Section 4.5 is the counterfactual study where we compare the profitability of the existing contract with the alternative contract. Section 4.6 concludes.

### 4.2 Original contract

In this section, we solve the agent’s problem in two cases. In the first case, we assume growers are homogeneous and show that the expected bonuses exactly cancel out the expected penalties. In the second case, we assume there are heterogeneous agents and obtain the optimal effort levels for each type.

#### 4.2.1 The payment scheme

The cardinal tournament is a two-part piece-rate payment scheme. The piece rate consists of a base payment per pound of weight produced and a bonus payment based on the grower's relative performance. The bonus payment is calculated based on the difference between group average settlement cost and grower's individual settlement cost. Each
grower’s settlement cost is the sum of the costs of integrator supplied inputs (chicks, feed, medication, etc.) divided by the total pounds of broilers produced. The calculation of the group average settlement cost includes growers whose flocks were harvested within the same week. For the below-average settlement cost, the grower receives a bonus; for the above-average settlement cost, the grower receives a penalty. The payment for grower $i$ in one tournament is calculated as

$$R_i = [I + r\left(\frac{1}{N} \sum_{i=1}^{N} c_i - c_i\right)]q_i$$

(4.1)

where $I$ and $r$ are the two contract parameters with $I$ being the fixed part of the piece rate and $r$ being the slope of the variable part of the piece rate. $N$ denotes the number of growers in the tournament. $c_i$ denotes grower $i$’s settlement cost. $\frac{1}{N} \sum_{i=1}^{N} c_i$ gives the average settlement cost of growers that are in the same tournament group. The difference between group average settlement cost $\frac{1}{N} \sum_{i=1}^{N} c_i$ and grower $i$’s settlement cost $c_i$ multiplied by the contract parameter $r$ gives the variable part of the piece rate.

### 4.2.2 The case of homogeneous growers

Before we solve the agent’s problem in this case, we need the following assumptions.

**Assumption 1.** (1) Growers are risk-neutral but subject to the limited liability constraint.\(^1\) (2) Cost $C_i$ is quadratic in weight gain $q_i$ and decreasing in ability $a$, specified as $C_i = \frac{q_i^2}{a}$.

As shown in Chapter 2, the optimal effort level when agents are risk-neutral with limited liability constraint is smaller than the first-best case, thus the moral hazard problem still exists.

With cost specified as $C_i = \frac{q_i^2}{a}$, average settlement cost $c_i$ is as follows.

$$c_i = \frac{C_i}{q_i}$$

(4.2)

$$= \frac{q_i}{a}$$

(4.3)

We specify output $q$ as linearly additive in effort $e$, ability $a$, and the shock $\epsilon$ as follows.

---

\(^1\)The limited liability constraint does not affect the agent’s maximization problem and only shows up in the principal’s problem, which is not solved here.
\[ q_i = e_i + a + \epsilon_i \] (4.4)

where \( i \) denotes the grower. Since now we assume growers are homogeneous, ability \( a \) is common to all growers.

Cost of effort \( C(e_i) \) is quadratic in \( e_i \), \( C(e_i) = \frac{\theta e_i^2}{2} \), where \( \theta \) is the disutility of effort parameter. Again, \( \theta \) is not subscripted due to growers being homogeneous. A risk-neutral grower \( i \) maximizes his expected utility by choosing how much effort to exert.

\[
\max_{e_i} EU_i
\] (4.5)

where \( U_i = R_i - C(e_i) \).

\[
\max_{e_i} \{R_i - C(e_i)\}
\] (4.6)

Substituting in the contract payment scheme,

\[
\max_{e_i} \{[I + r(\frac{1}{N} \sum_{j=1}^{N} c_j - c_i)]q_i - C(e_i)\}
\] (4.7)

Substituting in the model specifications,

\[
\max_{e_i} \{[I + r(\frac{1}{N} \sum_{j=1}^{N} e_j + a + \epsilon_j) - \frac{e_i + a + \epsilon_i}{a})][e_i + a + \epsilon_i] - \frac{\theta e_i^2}{2}\}
\] (4.8)

Taking the first-order condition with respect to \( e_i \),

\[
I + r(\frac{1}{N} \sum_{j=1}^{N} e_j + a - \frac{e_i + a}{a}) + (e_i + a)r(\frac{1}{N} - 1) \frac{1}{a} - \theta e_i = 0
\] (4.9)

Similarly, for grower \( j \), the first-order condition with respect to \( e_j \) is as follows,

\[
I + r(\frac{1}{N} \sum_{i=1}^{N} e_i + a - \frac{e_j + a}{a}) + (e_j + a)r(\frac{1}{N} - 1) \frac{1}{a} - \theta e_j = 0
\] (4.10)

---

\(^2\)The disutility of effort parameter \( \theta \) takes care of the difference in units between grower payment \( R_i \) and effort \( e_i \).
Combining the first-order conditions for different growers Eq. (4.9) and Eq. (4.10) leads to

\[
\left(\frac{r}{a} - \theta\right)e_i = \left(\frac{r}{a} - \theta\right)e_j. \tag{4.11}
\]

As a result, \(e_i = e_j\), indicating that growers will exert the same level of effort.

\[
e_i = e_j = \frac{I + r(\frac{1}{N} - 1)}{\theta - \frac{r(\frac{1}{N} - 1)}{a}} \tag{4.12}
\]

Then we show that in this case, the bonuses and penalties cancel each other out precisely. Summing up the expected bonuses and penalties of all growers, we have the following.

\[
E \sum_{i=1}^{N} \left\{ r \left( \frac{1}{N} \sum_{j=1}^{N} c_j - c_i \right) q_i \right\} \tag{4.13}
\]

\[
= E \sum_{i=1}^{N} \left\{ r \left( \frac{1}{N} \sum_{j=1}^{N} \frac{e_j + a + \epsilon_j}{a} - \frac{e_i + a + \epsilon_i}{a} \right) (e_i + a + \epsilon_i) \right\} \tag{4.14}
\]

\[
= E \sum_{i=1}^{N} \left\{ r \left( \frac{1}{N} \sum_{j=1}^{N} \frac{I + r(\frac{1}{N} - 1)}{\theta - \frac{r(\frac{1}{N} - 1)}{a}} a + a + \epsilon_j - \frac{I + r(\frac{1}{N} - 1)}{\theta - \frac{r(\frac{1}{N} - 1)}{a}} a + a + \epsilon_i \right) a \right\}
\]

\[
= 0
\]

Thus when growers are homogeneous, the expected bonuses are exactly offset by the expected penalties.

4.2.3 The case of heterogeneous growers

Now we assume growers are heterogeneous. We make the following assumptions.

**Assumption 2.** (1) We consider two sources of heterogeneity: (a) growers are heterogeneous in abilities; (b) growers are heterogeneous in disutilities of effort. (2) The principal knows each grower’s type.

We assume that different type growers vary in their abilities as well as their disutilities of effort. In the rest of the paper, we refer to the unobserved and discrete heterogeneity of growers as types. Note that the number of types is smaller than the number of growers. It is reasonable to assume that among all growers, some will have the same or similar abilities and disutilities of effort.
Since broiler production contracts are short-term, repeated contracts, we assume that
the principal learn about each grower’s type through repeated contracting.
Assume that there are $K$ types of growers. The number of growers of type $k$ is denoted
by $N_k$, $k = 1, ..., K$. Denote the output of grower $i$ in type $k$ as $q_{ki}$. We specify $q_{ki}$ as linearly
additive in effort $e_{ki}$ and ability $a_k$ as follows.

$$q_{ki} = e_{ki} + a_k + \epsilon_{ki}$$  \hspace{1cm} (4.15)

where $\epsilon$ is the shock with mean 0 and standard deviation $\sigma$.

Average settlement cost $c_{ki}$ is specified as

$$c_{ki} = \frac{q_{ki}}{a_k}$$  \hspace{1cm} (4.16)

$$= \frac{e_{ki} + a_k + \epsilon_{ki}}{a_k}$$  \hspace{1cm} (4.17)

Disutility of effort $C(e_{ki})$ is quadratic in $e_{ki}$.

$$C(e_{ki}) = \frac{\theta_k e_{ki}^2}{2}.$$  

Assume that growers are risk-neutral. Grower $i$ of type $k$ solves the following problem.

$$\max_{e_{ki}} E U_{ki}$$  \hspace{1cm} (4.18)

where $U_{ki} = R_{ki} - C(e_{ki})$.

$$\max_{e_{ki}} E \{R_{ki} - C(e_{ki})\}$$  \hspace{1cm} (4.19)

Substituting in the payment scheme and model specifications,

$$\max_{e_{ki}} E \{(I + r(\frac{1}{N} \sum_{l=1}^{K} \sum_{j=1}^{N_k} \frac{e_{lj} + a_l + \epsilon_{lj}}{a_l} - \frac{e_{ki} + a_k + \epsilon_{ki}}{a_k}))(e_{ki} + a_k + \epsilon_{ki}) - \frac{\theta_k e_{ki}^2}{2}\}$$  \hspace{1cm} (4.20)

Taking the first-order condition w.r.t. $e_{ki}$,

$$I + r(\frac{1}{N} \sum_{l=1}^{K} \sum_{j=1}^{N_k} \frac{e_{lj} + a_l}{a_l} - \frac{e_{ki} + a_k}{a_k}) + (e_{ki} + a_k)(\frac{1}{N} \frac{1}{a_k} - \frac{1}{a_k}) - \theta_k e_{ki} = 0$$  \hspace{1cm} (4.21)
For grower $i'$ of type $k$, the first-order condition is as follows,

$$I + r\left(\frac{1}{N} \sum_{k=1}^{K} \sum_{j=1}^{N_k} \frac{e_{ij} + a_l}{a_k} - \frac{e_{k'i'} + a_k}{a_k}\right) + (e_{k'i'} + a_k) r\left(\frac{1}{N} \frac{1}{a_k} - \frac{1}{a_k}\right) - \theta_k e_{k'i'} = 0. \quad (4.22)$$

For grower $j$ of type $l$, the first-order condition is as follows,

$$I + r\left(\frac{1}{N} \sum_{k=1}^{K} \sum_{i=1}^{N_k} \frac{e_{ki} + a_k}{a_l} - \frac{e_{lj} + a_l}{a_l}\right) + (e_{lj} + a_l) r\left(\frac{1}{N} a_l - \frac{1}{a_l}\right) - \theta_l e_{lj} = 0 \quad (4.23)$$

Combining Eq. (4.21) and Eq. (4.22), we obtain $e_{ki} = e_{k'i}'$, indicating that growers that are of the same type will exert the same level of effort.

Combining Eq. (4.21) and Eq. (4.23), the following relationship between the effort levels holds.

$$e_{lj} = \frac{r\left(\frac{1}{N} - 2\right)}{a_l} - \theta_l e_{lj} \quad (4.24)$$

Substituting Eq. (4.24) into the first-order condition Eq. (4.21), we obtain the optimal effort level as follows,

$$e_{ki} = \frac{I + r\left(\frac{1}{N} - 1\right)}{\theta_k - \frac{r\left(\frac{1}{N} - 2\right)}{a_k} - \frac{r\left(\frac{1}{N} - 2\right) - \theta_k a_k}{a_k} \sum_{l=1}^{K} \frac{N_l}{a_l}}. \quad (4.25)$$

The optimal effort level for a grower of type $k$ depends on the contract parameters $I$ and $r$, number of growers in the tournament $N$, ability $a_k$, disutility of effort parameter $\theta_k$ and the abilities and disutility of effort parameters of other types, $a_l$’s and $\theta_l$’s, $l \neq k$.

Due to the complexity of the two-part piece-rate cardinal tournament payment scheme, we do not solve for the optimal contract parameters. Nonetheless, the optimal contract parameters will be individualized in the case of heterogeneous growers. However, individualized contracts will be hard to implement because of the high transactions costs. We here take the contract parameters observed in the data as an approximation to the optimal ones.

### 4.3 Alternative contract

In this section, we consider an alternative contract. The alternative contract is a standard cardinal tournament. Under this contract, the expected sum of bonuses and penalties is
exactly zero. Thus the possibility that the bonuses outweigh the penalties in the existing contract can be avoided. We first introduce the payment scheme. Then we solve the agent’s problem for the optimal effort level and the principal’s problem for the optimal contract parameters for this payment scheme.

### 4.3.1 The payment scheme

The payment to growers is calculated based on the following expression,

\[ R_i = \alpha + \beta (q_i - \frac{1}{N} \sum_{j=1}^{N} q_j), \]  

(4.26)

where \( \alpha \) and \( \beta \) are the contract parameters with \( \alpha \) being the salary, which is independent of the grower’s performance and \( \beta \) the piece rate. A grower’s payment depends on the difference between the output and the group average output.

The sum of bonuses and penalties of all growers in the tournament is zero because

\[
\sum_{i=1}^{N} \beta (q_i - \frac{1}{N} \sum_{j=1}^{N} q_j) \\
= \beta \sum_{i=1}^{N} (q_i - \frac{1}{N} \sum_{j=1}^{N} q_j) \\
= 0.
\]

Obviously, this scheme is favorable to the principal because the total payments to growers is perfectly predictable \textit{ex ante} and equal to the sum of salaries. The question is now the incentive effects for growers to exert effort.

### 4.3.2 Agent’s problem

Again we specify output \( q \) as linear additive in effort \( e \), ability \( a \) and shock \( \epsilon \) as follows.

\[ q_i = e_i + a_i + \epsilon_i \]  

(4.27)

where \( i \) denotes the grower.

Cost of effort \( C(e_i) \) is quadratic in \( e_i \). \( C(e_i) = \frac{\theta_i e_i^2}{2} \). A risk-neutral grower \( i \) solves the
following problem,

\[
\max_{e_i} \mathbb{E}U_i, \quad (4.28)
\]

\[
\max_{e_i} \mathbb{E}\{R_i - C(e_i)\}. \quad (4.29)
\]

\[
\max_{e_i} \mathbb{E}\{\alpha + \beta(e_i + a_i + \epsilon_i - \frac{1}{N} \sum_{j=1}^{N} (e_j + a_j + \epsilon_j)) - \frac{\theta_i e_i^2}{2}\}. \quad (4.30)
\]

Taking the first-order condition w.r.t. \(e_i\),

\[
\beta (1 - \frac{1}{N}) - \theta_i e_i = 0 \quad (4.31)
\]

The efficient effort level is as follows.

\[
e_i = \frac{\beta (1 - \frac{1}{N})}{\theta_i} \quad (4.32)
\]

The effort a grower exerts depends on the piece rate \(\beta\), number of growers in the tournament \(N\) and the disutility of effort parameter \(\theta_i\). Note that with this payment scheme, ability \(a_i\) does not enter the optimal effort while the disutility of effort parameter \(\theta_i\) does.

4.3.3 Principal's problem

The principal's problem is choosing optimal contract parameters with the goal of maximizing his profit subject to agent's individual rationality constraint and incentive compatibility constraint. Here we adopt the first-order approach.\(^3\) Denote the principal's profit by \(\Pi\), \(\Pi = pq_i - R_i\). The principal's problem is as follows.

\[
\max_{\alpha, \beta} \mathbb{E}\Pi \quad (4.33)
\]

\(^3\)Note that here we replace the (IC) constraint with the first-order condition of the agent's problem. In general one cannot substitute the (IC) constraint with the agent's first-order condition. [30] illustrates that by replacing the agent's (IC) constraint by only the first-order conditions of the agent's problem, we are in fact relaxing some constraints in the principal's optimization problem. As a result, we may identify results that are actually not attainable by the principal. If the solution to the agent's first-order condition is unique and the agent's optimization problem is concave, then it is legitimate to substitute the agent's first-order condition for the agent's (IC) constraint ([5]). The case we have here satisfies the conditions for using the first-order approach.
where $\Pi = pq_i - R_i$.

\[
\begin{align*}
\text{max}_{\alpha, \beta} & \{pq_i - R_i\} \\
\text{s.t.} & \quad e_i = \frac{\beta(1 - \frac{1}{N})}{\theta_i} \quad \text{(IC)}
\end{align*}
\]

\[
E\{\alpha + \beta(q_i - \frac{1}{N}\sum_{j=1}^{N} q_j) - \frac{\theta_i e_i^2}{2}\} \geq u \quad \text{(IR)}
\]

Let $\lambda$ denote the Lagrangian multiplier for the individual rationality constraint. We set up the Lagrangian and take the first-order conditions with respect to contract parameters $\alpha$ and $\beta$.

\[
L = p\left(\frac{\beta(1 - \frac{1}{N})}{\theta_i} + a_i\right) - (\alpha + \beta\left(\frac{1 - \frac{1}{N}}{\theta_i}\right) + a_i - \frac{1}{N}\sum_{j=1}^{N}\left(\frac{\beta(1 - \frac{1}{N})}{\theta_j} + a_j\right))
\]

\[
+ \lambda(\alpha + \beta\left(\frac{1 - \frac{1}{N}}{\theta_i}\right) + a_i - \frac{1}{N}\sum_{j=1}^{N}\left(\frac{\beta(1 - \frac{1}{N})}{\theta_j} + a_j\right)) - \frac{\beta^2(1 - \frac{1}{N})^2}{2\theta_i} - u_i)
\]

The optimal contract parameters $\alpha^*$ and $\beta^*$ are as follows.

\[
\alpha^*_i = u_i + \frac{p^2}{2\theta_i} - \frac{p}{1 - \frac{1}{N}}\left(\frac{p}{\theta_i} + a_i - \frac{1}{N}\sum_{j=1}^{N}\left(\frac{p}{\theta_j} + a_j\right)\right)
\]

\[
\beta^* = \frac{p}{1 - \frac{1}{N}}
\]

The optimal piece rate $\beta^*$ is a function of broiler price $p$ and number of growers in a tournament $N$. The optimal base payment $\alpha^*$ is a function of grower’s reservation utility $u_i$, ability $a_i$, disutility of effort parameter $\theta_i$, broiler price $p$ and number of growers $N$. Note that $\beta^*$ is common to all growers while $\alpha^*$ is subscripted by $i$, indicating that the principal should offer individualized base payment to each grower.

\[\text{Details are in Appendix A.}\]
4.4 Empirical analysis

In this section, we first introduce the data and then estimate the model using the structural approach. Since we are interested in estimating the unobserved abilities and disutility of effort parameters and then conducting counterfactual analysis, the structural estimation enables us to do that. A caveat of the structural estimation is that the results depend on the model specifications. However, our specifications of the model are fairly standard in the literature. As a result, we will conduct structural estimation.

4.4.1 The data

We use broiler contract settlement data of a large broiler company in the United States. The dataset contains tournament settlement information of broilers from July 1995 to July 1997. In the tournaments, growers compete against each other along two margins, the weight gain margin and the settlement cost margin. There are in total 3,194 observations and each observation contains contract settlement information for one flock of birds including date settled, head started, head sold, weight sold, first-week mortality rate, feed conversion and so on. Tournaments are formed based on the settling date. Growers whose birds are settled on the same day form a tournament. During the period of the data, there are in total 104 tournaments. The number of growers in each tournament ranges from 18 to 43, with a mean of 30.6 and a standard deviation of 5.2. The average weight per broiler is 4.81 pounds. The mean of head started on each grower’s farm is 51,424 and the standard deviation is 27,604. The number of birds delivered to growers varies from 10,500 to 163,200 depending on the farm size.

Summary statistics of the contract is presented in Table 4.1. For large broilers, denoted by LB, a total of 354 growers participated in the contract during the period that generated the data. The percentage of penalized growers is 49.2%. The average bonus that growers receive is $1607, while the average penalty is $1385. The base pay rate is $0.0384 per pound while the average payment per pound is $0.0394, indicating that the penalty cannot cover the bonus.
Table 4.1 Broiler production contract data summary statistics

<table>
<thead>
<tr>
<th></th>
<th>LB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of observations</td>
<td>3194</td>
</tr>
<tr>
<td>Number of growers</td>
<td>354</td>
</tr>
<tr>
<td>Number of tournaments</td>
<td>104</td>
</tr>
<tr>
<td>Penalized growers (percentage)</td>
<td>49.2</td>
</tr>
<tr>
<td>Average bonus (USD)</td>
<td>1607</td>
</tr>
<tr>
<td>Average penalty (USD)</td>
<td>-1385</td>
</tr>
<tr>
<td>Average weight per grower (lbs)</td>
<td>235490</td>
</tr>
<tr>
<td>Base pay rate (USD)</td>
<td>0.0384</td>
</tr>
<tr>
<td>Average payment per pound (USD)</td>
<td>0.0394</td>
</tr>
<tr>
<td>Base pay rate - Average payment per pound</td>
<td>-0.0010</td>
</tr>
</tbody>
</table>

4.4.2 Estimation

Based on the theoretical part of the paper, we will estimate the model using maximum likelihood estimation. We derived grower’s optimal effort level to exert based on the grower’s type, but we cannot observe the types of growers. A natural candidate for the estimation approach is maximum likelihood since we can obtain the likelihood function based the theoretical model.

Conditional on the grower’s type, \( q_i \) follows a normal distribution with mean
\[
q_i | k \sim N\left( \frac{I + r\left( \frac{1}{N} - 1 \right)}{\theta_k - r\left( \frac{1}{N} - 1 \right)a_k} + a_k, \sigma^2 \right)
\]

The likelihood for \( q_i \) is the sum of type probability weighted normal density functions,
\[
f(q_i; \Theta) = \sum_k p(q_i|k)p(k),
\]
where \( p(q_i|k) \) denotes the normal PDF, \( p(k) \) denotes the type probability and \( \Theta \) denotes the parameters in the model, \( \Theta = (a_1, ..., a_K, \theta_1, ..., \theta_K, p(1), ..., p(K - 1), \sigma) \).
The likelihood function is formulated as,

$$ L(\Theta | q) = \prod_{i=1}^{N} f(q_i; \Theta). $$ \hspace{1cm} (4.42)

We obtain the estimates for $\Theta$ using maximum likelihood estimation.

$$ \hat{\Theta} \in \{ \arg \max_{\Theta} L(\Theta | q) \} $$ \hspace{1cm} (4.43)

Based on the result in Chapter 3 that there are two types of growers for the same dataset, we estimate the model in the two type case. Due to the variation of head started as described in Section 4.1, we divide total weight harvested of the flock by head started to obtain a normalized weight measure for the empirical analysis. Note that head started and target weight are decided by the principal. Growers find out head started upon delivery of baby chicks. Broilers are shipped to processing units when the principal decides that the broilers have reached the target weight on average. Hence weight harvested essentially captures the mortality margin in the sense that the only way for a grower to increase weight harvested is to make sure that mortality rate of the flock is low. The normalized weight measure can perfectly reflect the mortality margin.

Table 4.2 presents the estimation results. Ability of the high type $\hat{a}_H$ is 5.0561, while $\hat{a}_L$ is 4.7131. The disutility of effort parameter of the high type $\hat{\theta}_H$ is 42.2672, while the disutility of effort parameter of the low type $\hat{\theta}_L$ is 166.2598. The proportion of high type $\hat{p}_H$ is 0.2788. The standard deviation of the error term is 0.2402.

### 4.5 Counterfactual

In this section, we construct the expected revenue, payment and profit under the alternative contract and compare them with those under the existing contract.

In the alternative contract, bonuses are exactly offset by penalties. As a result, the sum
of payments is only the sum of base payments. According to Eq. (4.38), the optimal base payment $\alpha$ should be individualized for each grower. Based on an important feature of broiler production contracts that contracts cover one flock or one batch of animals at a time ([24]), we assume that through repeated contracting, the principal has perfect information regarding growers’ types and offer growers the contract with the set of contract parameters corresponding to the grower’s type. Since growers’ individual rationality constraints are satisfied, they will take the contract offered to them. Thus we do not consider the case of adverse selection.

Besides the parameters we have estimated, $\alpha_i$ is also a function of grower’s reservation utility. Since growers’ reservation utilities are unobservable, a natural choice is to use the grower’s minimum utility in the data. Note that the minimum utility provides an upper bound of the reservation utility since the individual rationality constraints are satisfied in the existing contract observed in the data.

$$u_i = R_i - C(e_i)$$ (4.44)

In order to calculate principal’s profit, we use broiler meat prices for the period covered by the data. The prices are weekly composite 12 cities market average prices.\(^5\) During the period between July 1995 and July 1997, broiler prices were reasonably stable with an average of 61 cents per pound. These prices are based on dressed (processed) weight and we convert them into live weights using industry average processing yields.\(^6\)

Table 4.3 presents the comparison of the profitability between the original contract and the alternative contract when we normalize the number of broilers in each realization to 1.\(^7\) The calculation of profits is detailed in Appendix B. For the original contract, since we

---

\(^5\)For details regarding the price data, see [40].

\(^6\)Processing yields are positively related to the size of the birds. We adopted the processing yield 71% for 5-pound birds.

\(^7\)We do this to avoid the complication brought by growers of unequal abilities being placed with unequal
Table 4.4 Weight gain under the original contract vs the alternative contract

<table>
<thead>
<tr>
<th></th>
<th>Weight gain (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original contract</td>
<td>15360.14</td>
</tr>
<tr>
<td>Alternative contract</td>
<td>(15378.92, 15381.72)</td>
</tr>
<tr>
<td>Difference</td>
<td>(-21.58, -18.78)</td>
</tr>
</tbody>
</table>

normalize the number of broilers to 1, the principal’s revenue presented is the sum of the revenue per head for the 3194 observations in the data. Similarly, the grower payment is the sum of the payment per head and the principal’s profit is the sum of the profit per head. In the brackets, we report the 95% confidence intervals. Note that the payment to growers under the alternative contract is just the sum of salaries. For the counterfactual, we simulate the principal’s revenue, payment to growers and the principal’s profit 1000 times using the parameter estimates obtained from Section 4.4, the optimal effort level and contract parameters obtained from Section 4.3. The principal’s revenue under the original contract is $6652.48. The payment to growers under the original contract is $603.31, which is greater than that of the alternative contract, $450.73, by $152.58. The 95% confidence interval for the principal’s revenue under the alternative contract is ($6660.61, $6661.77). The 95% confidence interval for the principal’s profit under the alternative contract is ($6209.88, $6211.04). The 95% confidence interval for the difference between the principal’s revenues under these two contract is (-$9.34, -$8.13). The 95% confidence interval for the difference between the principal’s profits under these two contract is (-$161.92, -$160.71).

Table 4.4 presents the weight gain comparison. The 95% confidence interval for the weight gain under the alternative contract is (15378.92, 15381.72). The weight gain under the original contract is 15360.14. The weight gain under the alternative contract is slightly higher.

The greater profit under the alternative contract is mainly contributed by the smaller payment to growers. Since the reservation utilities we adopt are the upper bounds of growers’ reservation utilities, the payment is overestimated and the profit is underestimated, indicating that the actual difference in profits between the existing contract and the alternative contract is even greater.
4.6 Conclusion

In this paper, we have studied the profitability of tournament payment schemes. In the theoretical model, we set up the principal-agent problem. With the parameter estimates, we conducted counterfactual analysis comparing the profits under the existing tournament and the alternative tournament. The result shows that the profit under the alternative contract is greater than that under the existing contract, which mainly results from the difference in payments.

There are possible extensions of this analysis for future research. First, due to the complexity of the tournament payment scheme, we did not solve for the optimal contract parameters. The contract parameters the principal adopt in the existing two-part piece-rate tournament might not be optimal. The principal’s profit might have been higher with optimal contract parameters. Second, we assume that the principal is able to offer contracts to growers based on their types. While the assumption is reasonable given the fact that growers repeatedly contract with the same principal, the transaction costs associated with offering individual contracts are not taken into account in our counterfactual analysis. Or adverse selection problem could be introduced when solving for the optimal contract parameters under the alternative contract.
In Chapter 2, we analyze the principal-agent problem in hog production contract. In the theoretical model, we derive the optimal effort by solving the agent's problem, estimate the optimal effort level and derive the incentive effects of the contract parameters. The optimal contract parameters are also derived by solving the principal's problem. During the period covered by the data, two contract parameter has been adjusted. It is technological advances and different type of feeder pigs that led to the changes in contract parameters. We test whether there is technological change among contracts and evaluate how the principal should respond to the change. We find that the principal did not adjust both parameters in the optimal direction.

In Chapter 3, we propose a nonparametric approach to identify the unobserved heterogeneity in broiler production tournaments by exploiting the recent advancement in the measurement error models. We nonparametrically test the existence of the grower heterogeneity in agricultural contracts and recover the output distributions conditional on growers’ unobserved heterogeneity. The result we obtained is that there exists heterogeneity in agent's abilities and there are two types of growers in the broiler production contract that
generated our data. With the *ex post* type probabilities, we categorize growers into different types and test the strategy the principal uses in input allocation. Our result shows that the input quality allocated to growers of different types is increasing in grower’s type. With the estimated type-specific parameters as defined in the theoretical model, we are able to conduct counterfactual analysis to compare the profit under different input allocation strategies. Confidence intervals of the profits are constructed.

In Chapter 4, we compare the profitability of a two-part piece-rate cardinal tournament and an alternative cardinal tournament where bonuses and penalties exactly cancel out. In the theoretical model, we solve for the optimal effort level in the existing contract in two cases, the case of homogeneous agents and the case of heterogeneous agents. Then we propose an alternative tournament payment scheme which leads to a zero sum of bonuses and penalties. We solve the agent’s problem to obtain the optimal effort level in this contract. We also solve the principal’s problem subject to the individual rationality constraint and the incentive compatibility constraint to obtain the optimal contract parameters. With the parameter estimates, we conduct counterfactual analysis comparing the profits under the existing tournament and the alternative tournament. The result shows that the profit under the alternative contract is greater than that under the existing contract, which mainly results from the difference in payments.


Appendix A.1

We conduct the rank test in a sequential manner following the procedure proposed in [34]. In this section, we briefly introduce the test and how we apply it to the test for the rank of $B_{d_1,d_2}$.

In order to perform the rank test, we first transform $B_{d_1,d_2}$ into two quadratic forms: $B_1 = B_{d_1,d_2} B'_{d_1,d_2}$ and $B_2 = B'_{d_1,d_2} B_{d_1,d_2}$. $B_1$ and $B_2$ have identical eigenvalues and their eigenvalues are all nonnegative. $B_1$, $B_2$ and $B_{d_1,d_2}$ share the number of zero eigenvalues. Therefore, testing for the rank of $B_{d_1,d_2}$ is equivalent to testing for the rank of $B_1$.

The null hypothesis $H_r$ is that $rk(B_{d_1,d_2}) = r$. Under the null hypothesis, $B_1$ has $r$ nonzero eigenvalues and $M - r$ zero eigenvalues. Since the eigenvalues of $B_1$ are all nonnegative,
the zero eigenvalues will be the smallest. Therefore, the summation of the smallest $M - r$ eigenvalues will be equal to zero under $H_r$ and we reject $H_r$ if the summation is too large.

The test statistic is defined as $TS = n \sum_{i=1}^{M-r} e_i g_i$, where $n$ is the number of growers and $e_i g_i$’s are the smallest $M - r$ estimated eigenvalues of $B_1$.

Denote the eigenvectors that correspond to the smallest $M - r$ estimated eigenvalues of $B_1$ as $C_1, C_2, ..., C_{M-r}$ and put them in $C=(C_1 \ C_2 \ ... \ C_{M-r})$, which has a dimension of $M \times (M-r)$. Similarly for $B_2$, we denote the eigenvectors that correspond to the smallest $M - r$ estimated eigenvalues as $D_1, D_2, ..., D_{M-r}$ and put them in $D=(D_1 \ D_2 \ ... \ D_{M-r})$, which has a dimension of $M \times (M-r)$.

Note that $\hat{B}_{d_1, d_2}$ is a frequency estimator of the joint probability mass function of $d_1$ and $d_2$. By Central Limit Theorem,

$$\sqrt{n}(vec(\hat{B}_{d_1, d_2})-vec(B_{d_1, d_2})) \xrightarrow{d} N_{MM}(0, \Omega) \quad (A.1)$$

where $vec(\cdot)$ is the vectorization operator and $\Omega$ is the covariance matrix. In practice, $\Omega$ can be estimated by bootstrap.

The asymptotic distribution of the test statistic is a weighted sum of chi-square distributions as follows,

$$\sum_{i=1}^{(M-r)^2} \lambda_i \chi_{1,i}^2 \quad (A.2)$$

where $\{\lambda_i\}_{i=1}^{(M-r)^2}$ are the weights and $\{\chi_{1,i}^2\}_{i=1}^{(M-r)^2}$ are i.i.d. chi-square random variables with degree of freedom one. $\{\lambda_i\}_{i=1}^{(M-r)^2}$ are the eigenvalues of $(C \otimes D)'\Omega(C \otimes D)$, which is $(M-r)^2 \times (M-r)^2$. The critical values can be generated by simulation.

**Appendix A.2**

$K$ denotes the number of types. $M$ denotes the number of bins. Given $K > M$, we prove that the rank of $B_{d_1, d_2}$ is equal to the number of bins $M$. For $k = M, ..., K$, we combine these types as $M'$. After defining the new mixture type, let $k'$ denote the type, $k' \in \{1, ..., M-1, M'\}$.

$$B_{d_1, d_2} = B_{d_1,k'} B_{k',d_2} \quad (A.3)$$
Given $K > M$, we claim that the rank of both matrices $B_{d_1|k'}$ and $B_{k',d_2}$ is equal to $M$. To prove this claim, we assume that the output distribution of any type is not a linear combination of those for other types and the output distribution of the mixture type is not a linear combination of those for other types.

The matrix $B_{d_1|k'}$ is $M \times M$ as follows.

$$B_{d_1|k'} = \begin{bmatrix}
Pr(d_1 = 1|k' = 1) & \cdots & Pr(d_1 = 1|k' = M') \\
Pr(d_1 = 2|k' = 1) & \cdots & Pr(d_1 = 2|k' = M') \\
\vdots & \ddots & \vdots \\
Pr(d_1 = M|k' = 1) & \cdots & Pr(d_1 = M|k' = M')
\end{bmatrix}$$

Because each element of the matrix $B_{d_1|k'}$ is a probability mass, each column sum is one. Consequently, the only possibility that any two columns are linearly dependent is that $Pr(d_1 = i|k' = m) = Pr(d_1 = i|k' = n)$, $i = 1, 2, \ldots, M$, $m \neq n$, which contradicts the assumption. Therefore, all the columns of $B_{d_1|k'}$ are linearly independent, and the rank is equal to $M$. Analogously, we can prove that all the rows of the matrix $B_{k',d_2}$ are linearly independent and the rank of this matrix is also equal to $M$. According to the following inequality regarding the rank of matrix $B_{d_1,d_2}$:

$$\text{rank}(B_{d_1|k'}) + \text{rank}(B_{k',d_2}) - M \leq \text{rank}(B_{d_1,d_2}) \leq \min\{\text{rank}(B_{d_1|k'}), \text{rank}(B_{k',d_2})\}, \quad (A.4)$$

we conclude that $\text{rank}(B_{d_1,d_2}) = M$ whenever $M < K$. 
Appendix B.1

The first-order condition w.r.t. $\alpha$ is as follows,

$$\frac{\partial L}{\partial \alpha} = -1 + \lambda = 0. \quad (B.1)$$
The first-order condition w.r.t. $\beta$ is as follows,
\[
\frac{\partial L}{\partial \beta} = \lambda \left( \frac{\beta(1 - \frac{1}{N})}{\theta_i} + a_i + \mu - \frac{1}{N} \sum_{j=1}^{N} \left( \frac{\beta(1 - \frac{1}{N})}{\theta_j} + a_j + \mu \right) \right) - \beta \left( \frac{1 - \frac{1}{N}}{\theta_i} - \frac{1}{N} \sum_{j=1}^{N} \frac{1 - \frac{1}{N}}{\theta_j} - \frac{\beta(1 - \frac{1}{N})^2}{\theta_i} \right) + \frac{p(1 - \frac{1}{N})}{\theta_i} \left( \frac{\beta(1 - \frac{1}{N})}{\theta_i} + a_i + \mu - \frac{1}{N} \sum_{j=1}^{N} \left( \frac{\beta(1 - \frac{1}{N})}{\theta_j} + a_j + \mu \right) \right) - \beta \left( \frac{1 - \frac{1}{N}}{\theta_i} - \frac{1}{N} \sum_{j=1}^{N} \frac{1 - \frac{1}{N}}{\theta_j} \right) = 0.
\]

From Eq. (B.1), we have $\lambda = 1$, which indicates that the individual rationality constraint binds. Substituting $\lambda = 1$ into Eq. (B.2), we can obtain the optimal contract parameter $\beta^*$ as follows.
\[
\beta^* = \frac{p}{1 - \frac{1}{N}}
\]  
(B.3)

Substituting Eq. (B.3) into the binding IR constraint, we can obtain the optimal contract parameter $\alpha^*$ as follows.
\[
\alpha^*_i = u_i + \frac{p^2}{2\theta_i} - \frac{p}{1 - \frac{1}{N}} \left( \frac{p}{\theta_i} + a_i - \frac{1}{N} \sum_{j=1}^{N} \left( \frac{p}{\theta_j} + a_j \right) \right)
\]  
(B.4)

**Appendix B.2**

The principal’s profit, denoted by $\Pi$ is obtained by subtracting the payments to growers from the revenue attained by sales of broilers at the prevailing market price.
\[ E \Pi = E \sum_{i=1}^{N} (pq_i - R_i) \]
\[ = E \sum_{i=1}^{N} pq_i - E \sum_{i=1}^{N} R_i \]
\[ = E \sum_{i=1}^{N} pq_i - E \sum_{i=1}^{N} \alpha_i \]
\[ = E \sum_{i=1}^{N} pq_i - E \sum_{i=1}^{N} \{ u_i + \frac{p^2}{2\theta_i} - \frac{p}{1-\frac{1}{N}} (\frac{p}{\theta_i} + a_i - \frac{1}{N} \sum_{j=1}^{N} (\frac{p}{\theta_j} + a_j)) \} \]
\[ = N(p_H pq_H + p_L pq_L) - \left( \sum_{i=1}^{N} R_i - N_H \frac{\theta_H e_H^2}{2} - N_L \frac{\theta_L e_L^2}{2} \right) \]
\[ - N p_H \left( \frac{p^2}{2\theta_H} - \frac{p}{1-\frac{1}{N}} p_H \left( \frac{p}{\theta_H} + a_H - \frac{p}{\theta_l} - a_L \right) \right) - N p_L \left( \frac{p^2}{2\theta_L} - \frac{p}{1-\frac{1}{N}} p_L \left( \frac{p}{\theta_L} + a_L - \frac{p}{\theta_H} - a_H \right) \right) \]