ABSTRACT

PRABHA RAMAKRISHNAN, APARNA. Harmonic Performance of Re-configurable N-path Mixer-first Receiver Architecture with Circulant-Symmetric Baseband Feedback. (Under the direction of Dr. Brian A. Floyd).

Software defined radios (SDRs) are among the dominant technology in radio communications and can be defined as the software implementation of traditional hardware system. It can be used to overcome the issue of limited spectrum assumption in several radio architectures. It is flexible enough to account for wideband techniques that allow multiple transmitters to transmit in the same place on the same frequency with very little interference. It uses error detection and correction techniques to fix all the errors caused by signal interference and provides programmable interference cancellation. It can also be used to realize bandwidths ranging from few megahertz to few gigahertz.

An SDR with programmable baseband RF bandpass filter can be implemented using re-configurable N-path passive mixer-first receiver architecture, which has opened up a wider possibility in terms of impedance matching and achieving wider bandwidths for frequency tuning. The transparency of passive mixers between RF and baseband ports can be thus exploited to translate the functions of RF matching network, LNA and RF filter to the baseband. In this thesis, the performance of an N-path passive mixer-first receiver in terms of harmonic-suppression, bandwidth-tuning etc., are explored by using complex RC baseband feedback networks by utilizing this idea of frequency translation from baseband to RF.

Various baseband feedback paths have been explored using different RC combinations. The thesis also validates impedance translation of baseband with higher order RC network using circulant-symmetric baseband feedback theory. The idea of circulant symmetric baseband feedback which utilizes the DFT relation of the baseband termination admittance can provide multiple degrees of freedom in terms of impedance matching, harmonic tuning, controlling the overall RF 3dB bandwidth of the receiver, improving the baseband filter order and so on. The conversion gain transfer
functions have been derived to explain the theoretical work involving more complex structures and pole-zero analysis has been provided to explain the work better.

Harmonic blocker tolerance of the receiver can be improved using different RC combinations as circulants thus avoiding an extra harmonic blocker in the receiver. Here, inter-phase cross feedback has been explored, which along with the resultant phase shift in each of the N-paths leads to complex baseband impedance by using only resistors and by appropriately choosing the feedback paths. By adequate designs, it is possible to obtain a flat conversion gain response, giving rise to higher bandwidths, spanning over the entire fundamental harmonic range. An analytical versus simulation result comparison has also been included for a clock frequency of 1GHz, thus validating impedance translation of baseband with higher order RC network using circulant-symmetric baseband feedback theory.
Harmonic Performance of Re-configurable N-path Mixer-first Receiver Architecture with Circulant-Symmetric Baseband Feedback

by
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DEDICATION

To my parents, grandparents and Aishu.
BIOGRAPHY

The author was born in a small town in Kerala, India in 1995. She received her Bachelor of Technology degree (First Class with Distinction) in Electrical and Electronics Engineering from the National Institute of Technology, Calicut, India in 2017. In the same year, she joined North Carolina State University, Raleigh to pursue M.S. in Electrical Engineering, and joined Dr. Brian A. Floyd’s group, iNCS2 in 2018 as an MS Thesis student. She is specializing in Analog and RF domain, and her research interests include ultra lower power circuits and millimeter-wave radio designs.
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CHAPTER 1

INTRODUCTION

1.1 Motivation

With the advancements in the telecommunication fields, there is a demand for high data rates leading to growing interest in millimeter wave systems. Now with 5G coming into picture, there are a variety of spectrum bands coming into picture and can be divided into three main categories: sub-1GHz, sub-6GHz and above 6GHz [GSM18]. To accelerate this transition to 5G, the existing 4G-LTE infrastructure is being made to support 5G bearers [Eve18]. 5G mobile broadband services will be available to users in a primarily 4G network via mobile terminals that support Dual Connectivity (DC) to 4G LTE and 5G New-Radio (NR) base stations at the same time [Eve18],[Wor18].

The attractions of this 5G NR DC (5G En-DC technology) comes with one major concern: higher-order harmonic interference. It can be interference between multiple users based on the carrier dependent frequency range where the carrier of one user can fall in the third or higher-order harmonic frequency of another user. Another major concern is the self-interference in a transceiver,
where the harmonics and sub-harmonics of transmitter can affect the receiver or vice-versa. This necessitates the need for harmonic filters/ harmonic blockers in addition to baseband amplifier stage. This increases the overall area of the transceiver chip along with an increase in power consumption and total noise. This can be avoided if the harmonic suppression can be implemented along with the low-noise baseband amplifier. In this thesis, such a baseband feedback has been explored. Design strategies for implementing such a harmonic suppression baseband amplifier has also been presented along with simulation results verifying the theory.

Multi-band 5G communication explores the millimeter wave bands at 24, 28, 32, 37, 39, 42, 60, … GHz [5G 19]. Conventional GSM and LTE network uses sub-4 GHz frequency range. Due to the overcrowded spectrum in those frequencies, there is a major bandwidth limitation. To support enhanced mobile broadband experience with significant data rates, wider bandwidths are required. At higher frequency range such as in millimeter wave bands, the spectrum is largely unused. Efficient utilization of this available spectrum facilitates in providing a higher bandwidth [Raj18].

In this thesis, a baseband feedback topology to get very wide bandwidths spanning the spectrum has explored for VHF, UHF, L and S Radar-frequency bands according to IEEE standard.

1.2 Overview

This thesis is organized as follows:

- **Chapter 2 - Literature Review**: This chapter reviews the idea of passive mixers and mixer-first receivers. It also reviews the IQ cross feedback used to provide complex poles; thus referred here as complex feedback. It looks into the application of this cross feedback for frequency tuning in mixer-first receivers. It also reviews the idea of circulant-symmetric baseband feedback across multiple phases of a mixer. It also reviews the harmonic termination admittance for R and RC circulants.
• **Chapter 3 - R-RC circulants:** This chapter introduces the R-RC circulants and explains the different design constraints with respect to harmonic tuning, bandwidth tuning and higher-order filter response. It presents a pole-zero analysis for the conversion gain and also compares the analysis and simulation results. It also talks about the constant phase shift introduced at the output by the LO clock waveforms based on clock definition. Finally, a comparison between the different circulants considered is presented.

• **Chapter 4 - Conclusion:** In this chapter, the advantages and limitations of circulant symmetric baseband feedback with R-RC circulant has been discussed along with the limitations of circulant-symmetric baseband feedback in general. It also mentions the scope for future work.
LITERATURE REVIEW

2.1 N-phase Passive Mixer

A Mixer is one of the main components of a receiver which performs frequency translation by multiplying two waveforms and in many cases, also their associated harmonics [Raz12]. It mainly consists of three ports. Along the receiver path, for a down-conversion mixer, radio frequency (RF) input is sensed through the RF port, local oscillator (LO) input through the LO port and based on the type of receiver, the output is called intermediate frequency (IF) port in case of a heterodyne receiver or a baseband port in case of a down-conversion receiver (Fig. 2.1). The mixing operation can be visualized as multiplication of the RF input ($V_{RF}$) with a square wave ($V_{LO}$) that varies between 0 and 1.

An N-phase passive mixer consists of N-CMOS transistor switches. Compared to active mixers (Gilbert cells), ideally, passive mixers have no DC bias current in them and also fewer number of transistors [Floria],[Raz12].
\[ V_{RF}(t) = A_{dc} + A_{RF}\cos(\omega_{LO}t + \Phi) \]

\[ V_{LO}(t) = B_{dc} + B_{LO}\cos(\omega_{LO}t) \]

\[ V_{IF}(t) = V_{RF}(t)V_{LO}(t) \]

**Figure 2.1** Mixer

**Figure 2.2** A Four-phase Mixer; \( V_{in} \) is the RF input; \( vi^+ \) & \( vi^- \) are the in-phase outputs and \( vq^+ \) & \( vq^- \) are the quadrature phase outputs

Consider the case for a four-phase, single balanced mixer, as seen in Fig. 2.2, it is clear that four switches are being used. The LO signal given to switch 1 and switch 3 should be 180° out of phase so as to get the in-phase differential signals \( VI^+ \) and \( VI^- \) and to prevent RF feedthrough. The quadrature phase is orthogonal to in-phase, thus the LO signals given to switch 2 should be 90° out of phase from LO signal at switch 1 and the LO signal at switch 4 should be 180° out of phase from the same
at switch 2 giving the differential signals, VQ+ and VQ-. It can also be noticed that the LO signals given to switches 3 and 4 have a 90° phase shift from each other.

![Non-overlapping LO signals](image)

**Figure 2.3** Non-overlapping LO signals.

![Overlapping LO signals](image)

**Figure 2.4** Overlapping LO signals.

From Fig. 2.3, it can be observed that the LO signals are non-overlapping, each having 25% duty ratio with a 90° phase shift from each other [Floria],[Mir10]. As a result, only one mixer switch is turned on at a time. If there is an overlap between any two LO signals (Fig. 2.4), the RF input gets shared amongst those stages. This is not desired as it will lead to a direct path between those two basebands. Hence, I and Q will not be orthogonal to each other. This results in problems such as IQ crosstalk and can lead to signal attenuation and loss.

Limitation of a four-phase mixer are its low harmonic re-radiation impedance [Floria],[AM10a],[AM10b],[MA12],[Mai18] and [OS17] talks about five-seven phase mixers, the advantages being higher harmonic re-radiation impedance and at the same time, lesser constraints on the clock. The advantages of an eight-phase mixer including improved harmonic rejection, lower noise are
highlighted in [AM10a]. These N-phase mixers give N baseband outputs Fig. 2.5. These N-output baseband signals can be combined to form differential I and Q baseband outputs. [Floria], [AM10a], [Wel01a], [Wel01b] and [PR14] discusses ways in which the N-baseband output signals can be combined such that the goal is a circuit which combines the fundamental downconversion signal but cancels its harmonics. [Shr06] presents a harmonic cancellation/suppression method for a 3-phase mixer by providing adequate phase shift.

One of the main features of passive mixers is negligible flicker noise due to the absence of DC bias current. Detailed analysis in mixer flicker noise has been presented in [Bag06],[Che04],[RL01].

2.2 Mixer-first Receiver Architecture

With advancement in wireless technology, the available spectrum for communication is getting crowded. As a result, there is higher interference for receivers making high-linearity a very critical requirement. Fig. ?? shows a traditional receiver which employed a low-noise amplifier as its first stage. However, its poor linearity restricts the receiver’s performance. Innovations in both circuit architectures and process technologies have enabled greater bandwidth tuning [Kit09], oscillation
frequency [Soe09],[van08b],[Bag06] and gain. However, tuning the antenna interface of receivers, including the RF LNA, matching network and RF band-pass filter remains very hard [van08a],[GV06]. Mismatch of the antenna impedance results in reduced maximum field strength and deteriorates modulation quality, receiver sensitivity and power amplifier efficiency.

Mismatch of the antenna impedance results in reduced maximum field strength and deteriorates modulation quality, receiver sensitivity and power amplifier efficiency.

![Diagram](image)

**Figure 2.6** Receiver architectures.

Early receivers included only a mixer and a local oscillator to function [Tuc54], i.e, in principle, a homodyne (direct conversion) receiver does not require any RF amplifier shown in Fig. ?? This approach is becoming popular as connecting the antenna directly to a CMOS passive mixer has many benefits. [Klu17] talks about the evolution of mixer-first architecture. This architecture provides tunable input impedance without resonators [AM10c], [And13], [Yao17]. Also, connecting the antenna directly to the mixer helps in achieving a wider bandwidth for tuning [Kit09],[AM10c] apart from improving the linearity. A direct interface with the antenna provides highly flexible baseband controlled impedance [AM10b] and aids in matching without affecting the performance much. By reducing the value of the switch resistance of the mixer transistors, the noise figure can also be improved. Also, at the cost of conversion loss, passive mixers offer high linearity [Kit09]. Utilizing the transparency between RF and baseband ports [Kim09],[Mir10],[Mir09] of the passive mixers, the functions of RF matching network, LNA and RF filter can be translated to the baseband [AM10a].
2.2.1 Input Impedance

To ensure minimum reflection at the center frequency, the source impedance (here antenna impedance) should be matched to $Z_{in}$ as can be seen from Fig. 2.7. From [MA12], [AM10a] and [Floria], the input impedance $Z_{in}$, of the mixer-first receiver can be defined as:

$$Z_{in}(\omega) = R_{sw} + \sum_n \gamma Z_{BB}(\omega - n\omega_o)||Z_{sh}$$  \hspace{1cm} (2.1)

where,

- $R_{sw}$ is the switch resistance
- $\gamma = N C_{n,N}^2 = \frac{1}{N} \sin c^2(\frac{\pi}{N})$, where $C_{n,N}$ is the mixing coefficient
- $N$ refers to the mixer phase and $n$ refers to the $n^{th}$ harmonic.
- $\omega_o$ is the center frequency
- $Z_{BB}(\omega - n\omega_o)$ is the upconverted baseband termination impedance. With a baseband amplifier gain of $A_{BB}$ and a feedback impedance $Z_f$, $Z_{BB} = Z_f/(1 + A_{BB})$
- $Z_{sh} = (Z_A + R_{sw})\frac{N^2}{N}$ is the harmonic re-radiation impedance

2.2.2 Conversion Gain

The gain of a current-mode passive mixer with TIA baseband is defined as $CG_n = \frac{v_{out,k}}{v_{RF}}$ [Floria]. If the output is taken differentially, it is given by, $CG_n = \frac{v_{out,k} - v_{out,k+1}}{v_{RF}}$. Here $k = 0, 1, \ldots N-1$ and $n$ refers to the $n^{th}$ harmonic.

From Fig. 2.8 and [Floria], the fundamental conversion gain can be defined as:

$$CG_1 = \frac{v_{out,0}}{v_{RF}} = \frac{i_{in}}{v_{RF}} \cdot \frac{i_B}{i_{in}} \cdot \frac{v_{out}}{i_B}$$  \hspace{1cm} (2.2)

Substituting the expressions for each case,
\[ CG_1 = \frac{2}{Z_{in} + Z_A} \cdot C_{1,N} \cdot \frac{Z_f}{1 + \frac{1}{Z_{in} + Z_A} \cdot C_{1,N} \cdot Z_{BB} \cdot \frac{A_{BB}}{\omega_p + 1}} \] (2.3)

where,

- Baseband impedance \( Z_{BB} = \frac{Z_f}{1 + \frac{A_{BB}}{\omega_p + 1}} \)

- \( \omega_p \) is the 3 dB bandwidth of the baseband amplifier

- \( C_{1,N} = \frac{1}{N} \text{sinc}(\frac{\pi}{N}) \)

- Input impedance, \( Z_{in} = R_{sw} + \gamma Z_{BB} || Z_{sh} \) as given in section 2.2.1 and \( Z_A \) is the antenna

Figure 2.7 N-phase passive mixer-first receiver
In case of a fully differential TIA, the conversion gain gets modified as:

\[ CG_1 = 4C_{1,M} \cdot \frac{Z_{BB}}{Z_{in} + Z_A} \cdot \frac{A_{BB}}{s + \frac{1}{\omega_p}} \]  
(2.4)

### 2.3 Complex Feedback

The cross-coupled IQ feedback is being referred to as complex feedback as it helps to introduce a complex pole. The idea of complex feedback from I to Q can be traced back to [CS98]. Here, the complex signal technique was used to analyze the analog zero-IF topology and a complex pole...
was realized using active-RC filter where the in-phase output was cross-coupled with quadrature phase input and vice-versa. A similar feedback technique has been used in [Erd05] for implementing complex pole stages for filtering the IF signal and in [Che10] to modify the phase of a filter. These additional feedback paths present a $90^\circ$ phase shifted and scaled version of the original signals back to the amplifier inputs.

### 2.3.1 Complex Feedback for matching to a Complex Antenna Impedance

The input impedance of a mixer-first receiver as a function of IF frequency, shows that the imaginary component of $Z_{in}$ looks negative for positive IF and positive for negative IF as in Fig 2.9. On the upper sideband of the LO, the antenna port sees the impedance presented by the baseband port as a function of the IF; but the lower sideband sees the complex conjugate of this impedance [AM10b], [Kha09]. This implies that the required complex conjugate match for a complex antenna impedance can only exist at a single IF frequency.

In principle, the imaginary component of this match is tunable since we have control over the value of the capacitor. However, using the sampling capacitor to provide a complex impedance match has the disadvantage that it will limit the bandwidth of a good match. Worse, it can only be used to match one polarity of imaginary antenna impedance, or in other words it can only match that impedance on one of the side bands of the LO.

In order to solve the problem of matching to complex antenna impedances, [AM10a], modifies the original feedback amplifiers (Fig. 2.11) to provide complex feedback. The feedback resistors from the output of the I-channel of the amplifier to the input of the Q-channel, and vice versa. These additional feedback paths present a $90^\circ$ phase shifted and scaled versions (scaled by resistance in the complex feedback path) of the original signals back to the amplifier inputs. This phase translates to a complex impedance presented to the antenna port through the passive mixer. As in [AM10a], [AM10b] and [MA12] this complex feedback can be used for fine tuning the $S_{11}$ notch such that
**Figure 2.9** Simulation result of the imaginary component of the impedance presented at the RF port

**Figure 2.10** Simulation result of the resistive component of the impedance presented at the RF port

It is centered at the required LO frequency. The $S_{11}$ notch can be shifted to the left or the right by employing positive and negative imaginary complex feedback. Implementing the complex feedback
resistance \((R_{FI})\) in the same way as the negative feedback resistance \((R_F)\), and allowing for its polarity to be switched (as in Fig. 2.11) provides a programmable complex impedance match.

From [AM10b], the effective complex baseband impedance, \(Z_{BB}\) can be expressed as,
\[ Z_B = \left[ \left( 1 + \frac{A}{R_F} + \frac{1}{R_{FI}} \right) \pm j \frac{A}{R_{FI}} \right]^{-1} \]  

(2.5)

where, the sign of the imaginary term depends on whether the feedback is connected in positive or negative complex feedback as in Fig. 2.11.

### 2.4 Circulant Symmetric Baseband Feedback [WF19]

The real part of the baseband impedance \( Z_B \) can be controlled using the negative feedback resistor \( R_F \) across the baseband amplifier. The complex part can be controlled by an additional resistor \( R_{FI} \) between the BBA output and its quadrature input [AM10a],[MA12]. In case of a four-phase mixer, all the available feedback paths are thus utilized. But as the number of phases increase, such a feedback network does not utilize the additional degrees of freedom offered by the remaining N-2 paths [WF19].

In [WF19], the authors present a way in which these additional degrees of freedom can be incorporated in the selection of baseband polyphase feedback by the Discrete Fourier Transform (DFT) relationship which exists between the baseband feedback admittance vector, or circulant, and the equivalent baseband termination admittance seen by the receiver at harmonics of the LO frequency. The baseband termination admittance can be represented as \( Y_B(k\omega_o) \), where \( k \) represents the harmonic number and \( \omega_o \) represents the fundamental LO angular frequency.

Using the added degrees of freedom of N-path architecture, for \( N > 5 \) as in [WF19], it is possible to improve blocker tolerance of the receiver and control bandwidth its bandwidth. This is further discussed in the upcoming chapter. Complex feedback can be implemented using a resistive network or RC combinations. R-RC and R-RC-C complex feedback gives an added control over the bandwidth and stability of the system over R feedback.
By giving the same loading and feedback conditions for each baseband phase, receiver symmetry can be ensured resulting in a network with circulant symmetry. As this circulant symmetry is associated with the baseband network, it can be referred to as circulant-symmetric baseband (CSB).

![Figure 2.12](image)

**Figure 2.12** Equivalent circuit of a baseband amplifier with tunable baseband coupling feedback network using all phases; N - mixer phase, $y_n$ corresponds to the admittance coupled between phases [WF19].

Fig. 2.12 shows the N available feedback paths where N corresponds to the number of mixer phases. As it can be observed, this feedback connects baseband output phase i to baseband input phase j, for any i and j between 0 and N1 i.e, from each baseband output to all the baseband inputs for all phases giving an $N \times N$ network. This feedback is provided through an array of tunable resistors and capacitors to form an admittance, beginning with $y_0$ to $y_{N-1}$. The subscript/index of the admittance corresponds to the phase difference between the coupled baseband phases and is given by the output phase minus input phase, i.e, $i - j$. This index is referred to as the baseband
coupling index. The phase difference between two nodes at fundamental harmonic is nothing but \( \frac{2\pi(i-j)}{N} \). Fig. 2.13 illustrates the 8 available feedback paths from a single phase in an eight-phase mixer-first receiver. The dependence of this angle and LO harmonics is discussed in Subsection 2.4.1. The baseband amplifier can be single ended or fully-differential.

The overall admittance matrix \( Y \) for an \( N \)-phase system can be represented as:
\[
Y = \frac{I_{fb}}{V_{out} - V_{in}} = \begin{bmatrix}
    y_0 & y_1 & y_2 & y_3 & \cdots & \cdots & y_{N-1} \\
    y_{N-1} & y_0 & y_1 & y_2 & \cdots & \cdots & y_{N-2} \\
    y_{N-2} & y_{N-1} & y_0 & y_1 & \cdots & \cdots & y_{N-3} \\
    \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\
    y_1 & y_2 & y_3 & \cdots & \cdots & y_{N-1} & y_0
\end{bmatrix}
\] (2.6)

As discussed in [WF19], the property of circulant symmetry can be observed from equation 2.6. Each row in the admittance matrix represents the feedback from the output of a single mixer phase and each column is composed of the admittance connecting to the input of a single phase.

The matrix is composed of a single circulant,

\[
Y = \begin{bmatrix}
    y_0 & y_1 & y_2 & y_3 & \cdots & \cdots & y_{N-1}
\end{bmatrix}
\] (2.7)

and its circular shifted variations.

### 2.4.1 Harmonic Termination Admittance for Circulant-Symmetric Baseband

This section directly cites the work given in [WF19]. The phase relationship between the baseband signals at fundamental harmonic of LO is shown in Fig. 2.14(a). This phase relation depends on the harmonic of the LO signal used to downconvert the input RF signal to baseband. Fig. 2.14(b) gives the phase relation between baseband signals at the third harmonic of the LO. Evaluating the phase relationship versus harmonics between the non-overlapping clock signals results in a general form for the phase difference with an arbitrary value for N. This baseband harmonic phase parameter can be derived using Fourier series and can be defined as:
\[ \phi_{n,k} = \frac{2\pi n k}{N} \] (2.8)

where \( n \) refers to the baseband coupling index (mentioned previously as \( i-j \)) and \( k \) refers to the harmonic index.

The equivalent baseband termination admittance, \( Y_{B,k} \), can be defined as the closed-loop admittance looking into the input node of the baseband which is presented to the receiver at harmonics of the LO. Using equations 2.7 and 2.8,

\[ Y_{B,k} = \sum_{n=0}^{N-1} y_n + A_{BB,k} \sum_{n=0}^{N-1} y_n e^{-j\phi_{n,k}} = y_{sum} + A_{BB,k} Y_k \] (2.9)

The DFT relates the baseband coupling admittance \( y_n \), as a function baseband coupling index, \( n \), to the harmonic baseband termination admittance, \( Y_k \), as a function of the harmonic index, \( k \). They are a Fourier transform pair, and relationship allows synthesis of harmonic termination admittance through selection of appropriate baseband coupling admittance.
The baseband coupling admittance is given by

\[ y_n = g_n + jb_n \] (2.10)

It can be purely real as in a resistive only coupling where, \( y_n = g_n \). Capacitive feedback can be introduced to provide complex admittance which changes as a function of harmonic offset frequency, \( \omega_{IF,k} \), which is defined as the difference between RF input frequency \( \omega_{in} \) and the respective LO harmonic \( k\omega_{LO} \). Thus, equation 2.10 can be modified as:

\[ y_n = g_n + j\omega_{IF}c_n \] (2.11)

Alternative topologies for polyphase coupling include R in parallel with a series RC, R in parallel with C which is in parallel with a series RC and so on. These are discussed in detail in Chapter 3.

### 2.4.2 Harmonic Tuning using Circulant-Symmetric Baseband Feedback

As presented in Section 2.4.1, the baseband termination admittance depends on the LO harmonic. This idea can be exploited further to suppress higher order harmonics without introducing extra harmonic termination blocks [WF19]. By ensuring a perfect impedance match, S11 can be reduced to < 10 dB, thus reducing the amount of power reflected back to the RF-port/antenna[Florib]. This way the power lost can be reduced and maximum power is transferred to the output. According to maximum power transfer theorem, maximum power is transmitted to the output only when the load and source impedances are the complex conjugates of each other.

Keeping maximum power transfer in mind, if there is no impedance matching, there is a power loss. So theoretically, if there is no impedance matching at higher-order harmonics, the effect of these higher-order harmonics can be suppressed without an additional harmonic blocker. From equation 2.9, the baseband termination admittance at fundamental frequency for an 8-phase system
can be given by:

\[ Y_{B,1} = \sum_{n=0}^{7} y_n + A_{BB,1} \left( y_0 + \left( \frac{1-j}{\sqrt{2}} \right) y_1 - j y_2 + \left( \frac{-1-j}{\sqrt{2}} \right) y_3 - y_4 + \left( \frac{-1+j}{\sqrt{2}} \right) y_5 + j y_6 + \left( \frac{1+j}{\sqrt{2}} \right) y_7 \right) \]  

(2.12)

Here, from equation 2.8, \( \phi_{n,1} = \frac{2\pi n \cdot 1}{8} \).

Similarly, the third harmonic baseband termination admittance can be given by:

\[ Y_{B,3} = \sum_{n=0}^{7} y_n + A_{BB,3} \left( y_0 + \left( \frac{-1-j}{\sqrt{2}} \right) y_1 + j y_2 + \left( \frac{1-j}{\sqrt{2}} \right) y_3 - y_4 + \left( \frac{-1-j}{\sqrt{2}} \right) y_5 - j y_6 + \left( \frac{-1-j}{\sqrt{2}} \right) y_7 \right) \]  

(2.13)

Here, \( \phi_{n,3} = \frac{2\pi n \cdot 3}{8} \).

Due to the DFT relation of the baseband termination admittance, considering only the conductance, it can be observed that \( y_1 \) and \( y_7 \) adds to the conductance at fundamental harmonic whereas it reduces the total conductance at the third harmonic. An opposite effect can be observed in the case of \( y_3 \) and \( y_5 \). This effect can be visualized using Fig. 2.15(a) and 2.15(b).

Thus, the circulants can be designed such that there is impedance matching only at the fundamental harmonic. For this purpose, a harmonic ratio can be defined, \( H_k \), which is nothing but the ratio of conductance at \( k^{th} \) harmonic to that at fundamental.

\[ \text{i.e. } H_k = \frac{Re(Y_{B,k})}{Re(Y_{B,1})} \]  

(2.14)

By deciding the value of \( |H_k| \), the higher-order harmonics can be suppressed or enhanced.
2.5 R circulants

Fig. 2.16 shows the case of CSBF with resistive only coupling. Here, the harmonic termination admittance given in equation 2.9 can be modified as:
\begin{equation}
Y_{B,k} = \sum_{n=0}^{N-1} g_n + A_{BB,k} \sum_{n=0}^{N-1} g_n e^{-j\phi_{n,k}} = g_{sum} + A_{BB,k} G_{B,k}
\end{equation}

where \( G_{B,k} \) is a complex-valued DFT of the R-circulant given by,

\begin{equation}
G_{B,1} = R_0^{-1}\left(1 + b_1 m + 2 b_1 + 2 b_3 + A_{BB}(1 + \sqrt{2}(b_1 - b_3))\right) + j A_{BB}(g_6 - g_2)
\end{equation}

Here, \( b_n \) is the normalized value of conductance with respect to \( R_0^{-1} \).

Compared to complex matching discussed in Section 2.3.1, the additional feedback paths used here not only provides complex matching, but also aids in suppressing higher order harmonics.

As given in [WF19], the effective negative feedback gets modified as:

\begin{equation}
R_0 = \frac{R_b\left(1 + A_{BB}\right)(1 + x^2)}{H_3 + 1 - \frac{H_3 - 1}{2} \frac{x}{\sqrt{2}A_{BB}} - \frac{1}{A_{BB}}}
\end{equation}

Its derivation as in [WF19] is given in Appendix B.

### 2.5.1 Conversion Gain Analysis

As given in Section 2.2.2, the conversion gain in the case of R-circulant was modelled in MATLAB. From equation 2.1, the baseband termination impedance \( Z_{BB} \) becomes \( Z_{BB} = 1/Y_{B,1} \) given from equation 2.15.

Fig. 2.17 shows the analytical vs simulation results for conversion gain at an LO and input frequency of 1 GHz, for a baseband amplifier gain of 25dB. It results in a first-order baseband response. Here only the circulants \( b_3 \) and \( b_5 \) are considered. For a baseband gain of 25 dB and a harmonic ratio, \( H_3 = 5 \), \( b_3 = b_5 = 0.52 \).

The MATLAB code used for calculating the conversion gain is given in Appendix A.
Figure 2.17 Fundamental conversion gain: Analysis vs Simulation

### 2.5.2 Conclusion

By deciding the gain \(A_{BB}\) and the harmonic ratio \(H_3\), the other parameters can be calculated. The value of \(x\) can be chosen so as to fine tune the \(S_{11}\) notch to center at the LO frequency. Based on the value of \(x\), \(b_6\) or \(b_2\) can be calculated so as to achieve positive or negative reactive matching respectively. The procedure is applicable for any mixer with phases \(N > 4\). An advantage of R only circulant is that the system remains stable irrespective of the values of \(b_n\).

### 2.6 RC circulants

Capacitive coupling introduces a complex admittance which changes as a function of harmonic offset frequency, \(\omega_{IF,k}\), which is defined as the difference between RF input frequency \(\omega_{in}\) and the corresponding LO harmonic \(k\omega_{LO}\).

\[
\text{Effective impedance for RC circulant branch as in Fig. 2.18 is, } z = R_\parallel \frac{1}{j\omega C}. \text{ So, from equation}
\]
2.11, the admittance is obtained to be $y_n = g_n + j\omega I_{F,k} c_n$, where $g$ and $c$ are defined as $g = R_0^{-1} \cdot [1, b_1, b_2, b_3, b_4, b_5, b_6]$ and $c = [0, C_1, 0, C_3, 0, C_3, 0, C_1]$. Here, $C_1$ and $C_3$ are the capacitance values. The fundamental harmonic baseband termination admittance obtained using Equation 2.9 will be:

$$Y_{B,1} = G_{B,1} + j\omega I_{F,1} C_{Bp,1}$$  \hspace{1cm} (2.18)

where,

$$C_{Bp,1} = (2C_1 + 2C_3 + A_{BB}(1 + \sqrt{2}(C_1 - C_3)))$$ \hspace{1cm} (2.19)

Here, $G_{B,1}$ is given by equation B.7. $C_2$ and $C_6$ if included will contribute in balancing the real part of the complex admittance. Moreover, its effect will be dependent on the frequency. To simplify the analysis and to maintain the real part of the baseband termination admittance as a constant over frequency, $C_2$ and $C_6$ are not taken into account here.

By including the frequency-shifted baseband capacitor (sampling capacitor) which is added in
parallel with the termination admittance, the total frequency-dependent termination impedance will become,

\[ Y'_{B,1} = Y_{B,1} + j \omega_{IF,1} C_B \]  

(2.20)

i.e, \[ Y'_{B,1} = R_0^{-1} \left( 1 + b_{im} + 2b_1 + 2b_3 + A_{BB}(1 + \sqrt{2}(b_1 - b_3)) \right) + jA_{BB}(g_6 - g_2) \]

\[ + j\omega_{IF,1} \left( C_B + 2C_1 + 2C_3 + A_{BB}(1 + \sqrt{2}(C_1 - C_3)) \right) \]

(2.21)

Similarly, at third harmonic termination admittance will become,

i.e, \[ Y'_{B,1} = R_0^{-1} \left( 1 + b_{im} + 2b_1 + 2b_3 + A_{BB}(1 + \sqrt{2}(b_3 - b_1)) \right) + jA_{BB}(g_2 - g_6) \]

\[ + j\omega_{IF,1} \left( C_B + 2C_1 + 2C_3 + A_{BB}(1 + \sqrt{2}(C_3 - C_1)) \right) \]

(2.22)

So, \( C_3 \) provides negative capacitance at fundamental and positive capacitance at third harmonic. Likewise, \( C_1 \) provides positive capacitance at fundamental and negative at third harmonic.

### 2.6.1 Conversion Gain Analysis

The conversion gain in the case of RC-circulant was modelled in MATLAB similar to the case with R-circulant following the same procedure as in Section 2.2.2. \( Z_{BB} = 1/Y_{B,1} \) given from equation 2.21. Here, \( b_{im} = 0. \)

For a constant value of \( b_3 = b_5 = 0.52, \) the capacitance \( C_3 \) has been varied for the circuit in Fig. 2.18. As expected, it can be observed that the fundamental conversion gain bandwidth increased with increase in \( C_3. \) However, the system becomes unstable for higher values of \( C_3. \)
The upper bound of $C_3$ is given by,

$$C_{3\text{max}} < \frac{C_B}{A_{BB} \sqrt{2} - 2}$$  \hspace{1cm} (2.23)

Thus it can be ensured that the system always remains stable. This value of $C_3$ corresponds to the maximum obtainable bandwidth around fundamental LO frequency. Higher values of $C_3$ result in right-hand side pole which is not desired.

From Fig. 2.19 it can be observed that the bandwidth increases with increase in $C_3$ and for values closer to 1 pF, conversion gain starts to have double peaks as it starts becoming unstable.

The increase in bandwidth becomes more clear with the semilog plot shown in Fig. 2.20
2.6.2 Conclusion

Introducing a circulant with a capacitor in parallel to the resistor provides us the added freedom in bandwidth tuning. By appropriately choosing the value of the capacitance for $C_3$ with $C_1 = 0$, the fundamental bandwidth can be widened narrowing third harmonic bandwidth for an increase in $C_3$ and vice versa for a decrease in $C_3$. A behaviour opposite to that seen in the case of $C_3$ can be observed by changing $C_1$ keeping $C_3 = 0$. Introducing $C_2$ and $C_6$ results in a frequency dependent conductance.
2.7 Conclusion

Purely resistive matching to the antenna impedance results in an offset by several megahertz from the desired center frequency in the $S_{11}$ notch. Using complex feedback, a $90^\circ$ phase shift can be introduced between the output signal to the input by using just resistors which results in a complex pole. This idea was further explored in [AM10a], [MA12], [AM10b] to fine tune the $S_{11}$ notch to the LO frequency by better impedance matching.

Extending the idea of complex feedback for $N$-phase mixers with this $N > 4$, more feedback paths are available. Utilizing these additional feedback paths keeping in mind the phase difference contributed by each mixer phase, results in the harmonic input admittance of the mixer-first receiver to be related to the Discrete Fourier Transform of the baseband admittance. This Fourier relationship can be used to suppress higher order harmonics which eliminate the need for a harmonic blocker. Realizing this baseband feedback admittance with a symmetric network gives rise to the term circulant-symmetric baseband feedback. It also helps in providing an additional control in tuning the bandwidth of the receiver when RC circulant is used. However, the order of the baseband amplifier filter response remains unchanged.
CHAPTER 3

R-RC CIRCULANT

Compared to the baseband amplifier having just a negative feedback resistor, by employing I-Q cross feedback as proposed in [AM10b], [AM10a] and [MA12], a better impedance matching can be achieved which further helped in fine tuning the $S_{11}$ notch to the required center frequency.

The idea of circulant-symmetric baseband feedback (CSBF) introduced in [WF19] presents a feedback topology from each output to every other input phase. Owing to the transparency property of passive mixers and the phase shift at each mixer output introduced by the non-overlapping LO signals help in achieving a better impedance match by just using resistors for baseband feedback. Further work from the paper is cited in Appendix B and C. There, the effects of R and RC circulants in CSBF topology that can be employed in the baseband amplifier of an N-path mixer-first receiver are analyzed. Here, by adequately choosing the feedback network, along with fine tuning the $S_{11}$ notch, it is also possible to suppress higher order harmonics. Adding a capacitor in parallel to the resistor forming the RC circulant added an additional degree of freedom in terms of bandwidth tuning.
Here, in this chapter, R-RC circulant topology is introduced and various cases of its performance have been analyzed. The added degrees of freedom by using different RC combinations in improving the harmonic blocker tolerance of the receiver, the overall bandwidth of the system, baseband filter order etc. are explored in this chapter. Theoretical explanations are provided along with the simulations results to verify these results further. Finally, the performance of all these different circulants are compared.

3.1 Baseband Termination Admittance

Here a series resistor capacitor (RC) branch is added in parallel to the R circulant as shown in figure 3.1. The capacitor will block low frequencies resulting in the RC series branch to act as an open circuit. Thus effective impedance will be just $R || \infty$, which is nothing but $R$ for low frequencies. However, at very high frequencies, the capacitor acts as a short circuit resulting in an overall impedance of $R || R_s$. One of the aims in this chapter is to see if this R-RC circulant can result higher order filter response with wider bandwidth.

![Figure 3.1 R parallel R_s C_s: R-RC circulant](image-url)
The overall impedance, 

\[ z = R \left| \left( R_s + \frac{1}{j \omega C_s} \right) \right| = R \left| \frac{1 + j \omega C_s R_s}{j \omega C_s} \right|. \]

Thus, the admittance is given by \( Y_B = G_B + \frac{sG_C s}{G + sC_s} \). Here \( G = \frac{1}{R} \) and \( G_B = \frac{1}{R} \)

Therefore, the total admittance including all the baseband phases as given in equation 2.11 gets modified to,

\[ y_n = g + \frac{j \omega g_s c_s}{g_s + j \omega c_s} \] (3.1)

This can be further modified and written as,

\[ y_n = g + \frac{g_s^{-1} \omega^2 c^2_s}{1 + \frac{\omega^2 c^2_s}{g^2_s}} \] (3.2)

where \( g = R_o^{-1}\{1, b_1, b_2, b_3, 0, b_3, b_1\} \) as before. \( g_s = R_o^{-1}\{b_{s0}, b_{s1}, b_{s2}, b_{s3}, 0, b_{s3}, b_{s6}, b_{s1}\} \) and \( c_s = \{C_{s0}, C_{s1}, C_{s2}, C_{s3}, 0, C_{s3}, C_{s6}, C_{s1}\} \) are the circulants corresponding to the series RC branch \( R_s C_s \) given in Fig. 3.1. Equation 3.2 can be directly compared to the admittance seen in the case of RC circulant. Here an additional frequency dependent conductance term is present along with a frequency dependent susceptance term. Thus for a given resistance and capacitance, the total admittance varies with frequency. However, this effect is negligible for very small values capacitance.

The fundamental harmonic baseband termination admittance obtained using Equation 2.9 will be:

\[ Y_{B,1} = R_o^{-1}(1 + 2b'_1 + b'_2 + 2b'_3 + b'_6 + A_{BB}(1 + \sqrt{2}(b'_3 - b'_1))) + \omega A_{BB,1}(C'_0 - C'_2) + jA_{BB,1}R_o^{-1}(b'_2 - b'_6) + j\omega(C'_0 + 2C'_1 + C'_2 + 2C'_3 + C'_6 + A_{BB}(C'_0 + \sqrt{2}(C'_3 - C'_1))) \] (3.3)

where \( b'_n = b_n + \frac{R_o^2 \omega^2 c^2_{s n}}{1 + \omega^2 c^2_{s n}} \) and \( c'_n = \frac{c_{s n}}{1 + \omega^2 c^2_{s n}} \)
This can be simplified and written as:

\[ Y_{B,1} = G_{B,1} + \frac{j \omega G_1 C_1}{G_1 + j \omega C_1} \]  
(3.4)

Here,

\[ G_{B,1} = R_0^{-1}(1 + 2b_1 + b_2 + 2b_3 + b_6 + A(1 + \sqrt{2}(b_3 - b_1) + j(b_2 - b_6))) \]  
(3.5)

\[ G_1 = R_0^{-1}(b_{s0} + 2b_{s1} + b_{s2} + 2b_{s3} + b_{s6} + A(b_{s0} + \sqrt{2}(b_{s1} - b_{s3}) + j(b_{s2} - b_{s6})) \]  
(3.6)

\[ C_1 = (C_{s0} + 2C_{s1} + C_{s2} + 2C_{s3} + C_{s6} + A(C_{s0} + \sqrt{2}(C_{s1} - C_{s3}) + j(C_{s2} - C_{s6})) \]  
(3.7)

where, \( A = \frac{A_{BB}}{s} \omega_p + 1 \)

\( A_{BB} \) is the DC gain of the baseband amplifier and \( \omega_p \) is its 3dB frequency.

Considering the circulants, \( g = R_0^{-1}\{1, b_1, 0, b_3, 0, b_3, b_1\} \), \( g_s = R_0^{-1}\{0, b_{s1}, 0, b_{s3}, 0, b_{s3}, 0, b_{s1}\} \) and \( c_s = \{0, C_{s1}, 0, C_{s3}, 0, C_{s3}, 0, C_{s1}\} \) as in Fig. 3.1, equation 3.3 gets modified to:

\[ Y_{B,1}(\omega) = R_0^{-1}(1 + 2b_1 + 2b_3 + A_{BB}(1 + \sqrt{2}(b_3 - b_1))) + \]

\[ + \omega^2 \left( 2 - \frac{R_o C_{s1}^2}{b_{s1}} \right) + 2 - \frac{R_o C_{s3}^2}{b_{s3}} + A_{BB,1} \sqrt{2} \left( \frac{R_o C_{s1}^2}{b_{s1}} - \frac{R_o C_{s3}^2}{b_{s3}} \right) \]

\[ + j \omega \left( 2 - \frac{C_{s1}}{1 + \omega^2 C_{s1}^2(b_{s1})^2} + 2 - \frac{C_{s3}}{1 + \omega^2 C_{s3}^2(b_{s3})^2} + A_{BB,1} \sqrt{2} \left( \frac{C_{s1}}{1 + \omega^2 C_{s1}^2(b_{s1})^2} - \frac{C_{s3}}{1 + \omega^2 C_{s3}^2(b_{s3})^2} \right) \right) \]

(3.8)

This baseband termination admittance, \( Y_{B,1}(\omega) \), when upconverted to RF gives a bandpass
response proportional to \( Y_{B,1}(\omega - n\omega_o) \) centered at the LO frequency and its harmonics. Considering the fundamental harmonic, for \( \omega = \omega_o \), the admittance can be simplified to the R-circulant case. Unlike R and RC case, here the additional resistor in series with the capacitor results in a zero:

\[
\begin{align*}
    z_1 &= -\frac{G}{C} = -\frac{R_o^{-1}(2b_{s1} + 2b_{s3} + A_{BB}(\sqrt{2}(b_{s1} - b_{s3}))}{(2C_{s1} + 2C_{s3} + A_{BB}(\sqrt{2}(C_{s1} - C_{s3}))}
    
\end{align*}
\]

(3.9)

### 3.2 Harmonic Tuning Using R-RC Circulants

It is very similar to the harmonic tuning using R circulant. Proceeding the same way as in Section B.1, now the harmonic admittance ratio becomes,

\[
H_3 = \frac{1 + 2b'_3 + 2b'_3 + A_{BB}(1 + \sqrt{2}(b'_3 - b'_3))}{1 + 2b'_3 + 2b'_3 + A_{BB}(1 + \sqrt{2}(b'_3 - b'_3))}
\]

(3.10)

For \( H_3 > 1, b'_3 = 0 \), we get,

\[
b'_3 = b_3 + b'_3 = b_3 + \frac{R_o^2 C_{s3}^2}{1 + \omega^2 C_{s3}^2(\frac{R_o}{b_{s3}})^2} = \frac{1 + b_{i3} + A_{BB}}{H_3 + \frac{1}{H_3 - 1} \sqrt{2}A_{BB} - 2}
\]

(3.11)

In terms of impedance matching, from the above expression, \( b_3 \) is still the dominant term. For small capacitance values, the second term can be neglected. Here, for comparing the performance of each of these circulants, \( b_3 \) is taken as 0.52 as simulated for R and RC cases.

The values of \( b_3, b_{s3} \) and \( C_{s3} \) can be chosen as per the requirement. These become more clear from Section 3.2.1.

#### 3.2.1 Conversion Gain

Considering the \( b_3 \) and \( b_5 \) coupling along with the parallel RC branch, \( Z_{BB} = 1/Y_{B,1} \) where \( Y_{B,1} \) is given by equation 3.8.
Baseband amplifier gain, \( A = \frac{A_{BB}}{\omega + 1} \) where \( \omega_p \) is its 3dB bandwidth.

Substituting for \( Y_{B,1} \) we get total conversion:

\[
CG_1 = 4C_{1,M} \cdot \frac{Z_{BB}}{Z_{in} + Z_A} \cdot A
\]

\[
= \frac{4C_{1,M}A(\gamma + Z_{sh}Y_{B,1})}{([Z_A + R_{sw})(\gamma + Z_{sh}Y_{B,1}) + \gamma Z_{sh})Y_{B,1}}
\]

\[
= \frac{4C_{1,M}A\left(\gamma + Z_{sh}\left(G_{B,1} + sC_B + \frac{sG_1C_1}{G_1 + sC_1}\right)\right)}{(Z_A + R_{sw})(\gamma + Z_{sh}\left(G_{B,1} + sC_B + \frac{sG_1C_1}{G_1 + sC_1}\right)) + \gamma Z_{sh})\left(G_{B,1} + sC_B + \frac{sG_1C_1}{G_1 + sC_1}\right)}
\]

In the R and RC case, which are first order circulants with a single pole whose magnitude is determined mainly by the sampling capacitance \( C_B \). Here it is controlled by tuning \( c_s, g \) and \( g_s \). It is also dependent on the baseband gain and number of mixer phases.

### 3.2.2 Pole-Zero Analysis

From the conversion gain expression, it can be inferred that there are three zeros and five poles. To understand the trend with respect to bandwidth tuning and baseband filter order, the dominant poles and zeros are considered.

The dominant zero is given by:

\[
z_1 = -\frac{G_1}{C_1} = -\frac{R_{o}^{-1}(b_{s3}(2 - A_{BB}\sqrt{2}))}{(C_{s3}(2 - A_{BB}\sqrt{2}))} = -\frac{1}{R_{s3}C_{s3}} \tag{3.12}
\]

There are three dominant poles:

\[
p_1 = \frac{-(G_{B,1}C_1 + G_1(C_B + C_1)) + \sqrt{(G_{B,1}C_1 + G_1(C_B + C_1))^2 - 4C_BC_1G_{B,1}G_1}}{2C_BC_1} \tag{3.13}
\]
\[ p_2 = \frac{-(G_{B,1}C_1 + G_1(C_B + C_1)) - \sqrt{(G_{B,1}C_1 + G_1(C_B + C_1))^2 - 4C_BC_1G_{B,1}G_1}}{2C_BC_1} \]  \tag{3.14}

\[ p_3 = -\omega_p \]  \tag{3.15}

\( p_3 \) is nothing but the baseband pole.

From equation 3.12, it can be observed that the zero lies in the left-half plane irrespective of the values of the resistor and capacitor. Based on the values of \( G_{B,1}, G_1 \) and \( C_1 \) for a given sampling capacitor \( C_B \), the poles \( p_1 \) and \( p_2 \) can be either in the left or right half plane.

Following figures represent the variation in poles and zeros with change in the series resistor and capacitor values in RRC circulants.

For larger values of capacitance, the poles \( p_1 \) and \( p_2 \) are complex conjugates of each other. For even larger values, they become right-half plane poles which is not desirable as it would lead to peaking of the conversion gain response.
**Figure 3.2** Poles $p_1$ and $p_2$: Variation with $R_{s3}$ for a constant $C_{s3} = 800 \, f \, F$

**Figure 3.3** Poles $p_1$ and $p_2$: Variation with $R_{s3}$ for a constant $C_{s3} = 800 \, f \, F$ for larger resistor values
Figure 3.4 Poles $p_1$ and $p_2$: Variation with $C_{s3}$ for a constant $R_{s3} = 500\Omega$

Figure 3.5 Poles $p_1$, $p_2$ and $p_3$ and Zero $z_1$: Variation with $C_{s3}$ for a constant $R_{s3} = 1.1\, k\Omega$
3.3 Analytical Results

The conversion gain has been calculated and plotted using MATLAB for various cases of R-RC circulants. The baseband bandwidth trends and order can be observed.

![Conversion gain and phase plot for $R_{s3} = 5.5 \, k\Omega$ for variable values of $C_{s3}$](image)

**Figure 3.6** Conversion gain and phase plot for $R_{s3} = 5.5 k\Omega$ for variable values of $C_{s3}$

Here also, the coupling with respect to circulants $b_3$ and $b_5$ along with it’s parallel RC branch has
been considered.

### 3.3.1 Constant value of resistor $R_{s3}$ for varying values of capacitor $C_{s3}$

Figs. 3.6, 3.7 and 3.8 are the analytical results obtained from MATLAB. It can be observed that peaking of the conversion gain is lower for larger values of $R_{s3}$ for the same capacitance values. This can further be understood by seeing the trend in the poles and zeroes in Fig. 3.5.
Also, an increase in bandwidth with increase in the value of capacitor $C_{s3}$ can also be observed. This bandwidth increase is greater for smaller series resistance value. It can thus be inferred that the maximum possible bandwidth can be achieved in case of RC circuits for it is equivalent to the $R_{s3} = 0\Omega$, which is the smallest value possible.
3.3.2 Constant value of capacitor $C_{s3}$ for varying values of resistor $R_{s3}$

![Conversion Gain and Phase Plot](image)

**Figure 3.9** Conversion gain and phase plot for $C_{s3} = 500fF$ for variable values of $R_{s3}$

Conversion gain trend for constant values of capacitance with varying series $R_{s3}$ can be observed in Figs. 3.9, 3.10 and 3.11.

As expected there is no peaking in conversion gain for smaller values of capacitance, which is
From Figs. 3.10 and 3.11, it can be observed that with decrease in the value of $R_{s3}$, there is a considerable increase in the bandwidth. Also, for lower values of $R_{s3}$, a second order baseband response can be observed. However, from Fig. 3.9, it can be inferred that there is no major change in bandwidth with varying resistance.
3.4 Bandwidth Tuning

In the case of passive mixer-first receiver with just negative feedback for the baseband amplifier and for R-circulants, the baseband bandwidth is primarily dependent on the value of the sampling capacitor $C_B$. In case of RC circulants, based on the feedback path, by increasing the value of the additional parallel capacitance, the bandwidth can increase or decrease. Increasing $C_{s3}$ increases the bandwidth, whereas increasing $C_{s1}$ decreases the fundamental bandwidth and having opposite
effect in terms of third harmonic bandwidth.

As given in Section C.2, the bandwidth of the receiver can be controlled by controlling the location of poles and zeros. The baseband harmonic tuning can be used to introduce harmonic-dependent positive or negative capacitance to adjust bandwidth.

Here, for R-RC circulants, the total closed loop baseband bandwidth depends on $G_B$, $C$ and $G$, given from equations 3.5, 3.6 and 3.7 respectively, which in turn depends on the values of $R_n$, $R_{sn}$ and $C_{sn}$.

The values of $C_{s1}$ and $C_{s3}$ are chosen so as to achieve the desired bandwidth. Similar to what was observed for the RC case, here, taking $C_{s1} = 0$ aids in reducing the third harmonic bandwidth. Similarly, $C_{s3} = 0$ results in greater third harmonic bandwidth than the fundamental bandwidth. Thus it can be observed that increasing $C_{s3}$ increases the fundamental bandwidth and reduces the third harmonic bandwidth and opposite is the case with $C_{s1}$.

The maximum achievable RF bandwidth further depends on the values of the 3dB bandwidth of the baseband amplifier. Gain-Bandwidth product being a constant, one general way to improve the 3dB bandwidth of the baseband amplifier will be to lower the gain, thus increasing the overall 3dB bandwidth.

Overall bandwidth can be increased by reducing the sampling capacitance. But this results in a wider third harmonic bandwidth as well. However, $C_B$ along with a higher $C_{s3}$ can help widen only the fundamental bandwidth and reduce the third harmonic bandwidth at the same time. Also, by adjusting the location of the zero which is introduced due to the presence of the additional series resistance of the capacitor, it is possible to achieve a wider and a flatter conversion gain as shown in Fig. 3.12.
3.4.1 Second-order bandpass response

From Fig. 3.13 it can be observed that for a constant value of series resistance, $R_{s3}$, with increase in the value of capacitance, $C_{s3}$, the bandpass order also increases giving a second order bandpass response for larger capacitance values.

From Fig. 3.14, for a constant value of series capacitance $C_{s3}$, lowering the value of $R_{s3}$ results in second order baseband response. Here, as $R_{s3}$ lowers, from Fig. 3.2 and 3.5 in Section 3.2.2, it is clear that the three poles get closer to each other and with the presence of zero, $z_1$ given by equation B.5, the resultant is a second order baseband response.
Figure 3.13 Analysis plot: Conversion gain and phase plot for $C_s = 1 \mu F$ for variable values of $R_{s3}$
Figure 3.14 Analysis plot: Conversion gain and phase plot for $R_{s3} = 220\Omega$ for variable values of $C_{s3}$
3.5 Simulation Results

Here the effect of RRC circulants has simulated for an eight-phase mixer with the circulants given by: \( g = 2.2k^{-1} \cdot [1, 0, 0, 0.52, 0, 0.52, 0, 0] \), \( g_s = 2.2k^{-1}[0, 0, 0, b_{s3}, 0, b_{s3}, 0, 0] \) and \( c_s = [0, 0, 0, c_{s3}, 0, c_{s3}, 0, 0] \).

Simulation parameters:

- Baseband amplifier gain, \( A_{BB} = 25 \text{dB} \)
- Baseband amplifier 3dB bandwidth = 0.5GHz
- Negative feedback resistor, \( R_o = 2.2k\Omega \)
- Sampling capacitor, \( C_B = 30pF \)
- LO frequency = 1GHz
- Switch Resistance, \( R_{sw} = 8.4\Omega \)
- MOSFET width, \( W = 56\mu\text{m} \)
- MOSFET length, \( L = 40\text{nm} \)

Figs. 3.15 and 3.16 shows the simulated conversion gain for a constant value of \( R_{s3} \) with variable \( C_{s3} \) for a baseband amplifier 3dB bandwidth of 1GHz. Here, \( R_{s3} < R_o \). As expected, it can be observed that with the increase in the value of capacitance of \( C_{s3} \), the fundamental harmonic bandwidth increases whereas third harmonic bandwidth reduces.

It can also be observed that the third harmonic is suppressed by 7.5dB when compared to fundamental.
Figure 3.15 Conversion gain at fundamental frequency (1 GHz): Constant value of $R_{s3}$ with variable $C_{s3}$

Figure 3.16 Conversion gain at third harmonic frequency (3 GHz): Constant value of $R_{s3}$ with variable $C_{s3}$
Taking into account the different scenarios possible, variation in conversion gain has been plotted for two different cases:

1. Constant values of resistor $R_{s3}$ with variable capacitor $C_{s3}$.

2. Constant values of capacitor $C_{s3}$ with variable resistor $R_{s3}$.

### 3.5.1 Constant values of resistor $R_{s3}$ with variable capacitor $C_{s3}$

Figs. 3.18 to 3.21 are the simulation results of the first case where for a constant value of $R_{s3}$, capacitor $C_{s3}$ is varied. For both values of resistance, 500Ω and 2.2$kΩ$, with increase in $C_{s3}$ fundamental
Figure 3.18 Conversion gain at fundamental frequency (1 GHz): Constant value of $R_{s3} = 500\Omega$ with variable $C_{s3}$.

Figure 3.19 Conversion gain at third harmonic frequency (3 GHz): Constant value of $R_{s3} = 500\Omega$ with variable $C_{s3}$. 
Figure 3.20 Conversion gain at fundamental frequency (1 GHz): Constant value of $R_{s3} = 500 \Omega$ with variable $C_{s3}$.

Figure 3.21 Conversion gain at third harmonic frequency (3 GHz): Constant value of $R_{s3} = 500 \Omega$ with variable $C_{s3}$. 

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Figure 3.22 Conversion gain at fundamental frequency (1 GHz): Constant value of $R_{s3}$ with variable $C_{s3}$ with $C_B = 5\text{pF}$

Figure 3.23 Conversion gain at third harmonic frequency (3 GHz): Constant value of $R_{s3}$ with variable $C_{s3}$ with $C_B = 5\text{pF}$
bandwidth increases and third harmonic bandwidth reduces. It can also be observed that as mentioned in Section 3.3, the RF bandwidth is greater for lower values of $R_{s3}$.

From Figs. 3.22 and 3.23, it can be observed that it is possible to obtain wider bandwidths by tuning the sampling capacitance along with the capacitor in the RC series branch. It can also be observed that by adequate design, it is possible to achieve bandwidths that span the total spectrum. Maximum achievable bandwidth is set by the baseband amplifier bandwidth.

3.5.2 Constant values of resistor $C_{s3}$ with variable capacitor $R_{s3}$

![Figure 3.24 R-RC circulant: $S_{11}$ Smith chart representation for variable $R_{s3}$ with $C_{s3} = 1.15\, pF$](image-url)
**Figure 3.25** Conversion gain at fundamental frequency (1 GHz): Constant value of $C_{33} = 1.15 \mu F$ with variable $R_{33}$

**Figure 3.26** Conversion gain at third harmonic frequency (3 GHz): Constant value of $C_{33} = 1.15 \mu F$ with variable $R_{33}$
As predicted, ringing is high for larger values of series capacitance $C_{s3}$.

From Fig. 3.25, it can be observed that the undesirable peaking of conversion gain increases with decrease in the value of the series resistor $R_{s3}$. But, it can be observed that with increase in the value of $R_{s3}$, the 3dB bandwidth also reduces.
Figure 3.28 Conversion gain and phase: Baseband response at $C_{s3} = 500 f F$ and $R_{s3} = 5.5 k\Omega$

### 3.6 Simulation vs. Analytical result

In this section, the simulation results obtained from Cadence are verified by comparing it with the analysis results obtained from MATLAB.
It can be observed from Fig. 3.31, the simulated phase is leading by $22.5^\circ$ throughout frequency. This phase shifted is caused by the LO clock waveform. For an eight-phase mixer, each LO waveform corresponds to a phase shift of $360^\circ / 8 = 45^\circ$. In simulation, the LO waveform has been defined as in Fig. 3.29.

Figure 3.29 LO waveform for an eight-phase mixer
Mixing between RF input signal and LO clock waveform results in an average value of the input signal voltage at the mixer output and the corresponding charge stored across the sampling capacitor. According to 3.29, sampling happens around time $T/16$, which in turn corresponds to a phase shift of $360°/16 = 22.5°$.

This phase shift was observed because LO waveform has been defined in [Floria], [AM10c] as
given in Fig. 3.30. As a result, this additional phase shift will be present irrespective of the clock frequency.

**Figure 3.31** Conversion gain and phase: Baseband response at $C_{i3} = 500fF$ and $R_{i3} = 5.5\,k\Omega$ for phase shifted LO waveform corresponding to Fig. 3.30
By simulating again, this time with the LO waveforms defined as given in Fig. 3.30 with a phase lead of 22.5°, it can be inferred that the simulated phase aligns with the analytical phase response.

**Figure 3.32** Conversion gain and phase: Baseband response at $C_{s3} = 500f F$ with variable values for $R_{s3}$
Figs. 3.32 to 3.34 further compares the analytical results obtained with the simulated results. It can be observed that the analytical model can predict the behavior of the simulation.

Comparing Fig. 3.32 and 3.33, it can be observed that with an increase in the value of $C_{s3}$, the bandwidth increases, but at the same time, there is an undesirable overshoot in the conversion gain.
This peaking is clearer from Fig. 3.34.

3.7 Comparison

In this section, the RC and R-RC circulants are compared. Conversion gain at fundamental and third harmonic frequencies are plotted in Figs. 3.35 and 3.36 for the same value of $R_3 = 4.2\, k\Omega$ corresponding to $b_3 = 0.52$ with $C_3 = 825 f F$ in case of RC circulant and $C_{s3} = 825 f F$ in case of R-RC circulant.

It can be observed from Fig. 3.35 that with increase in the value of series resistor, $R_{s3}$, the order of the R-RC baseband response becomes a second order response, thus improving the filter order.
Figure 3.35 Comparison: Conversion gain at fundamental harmonic frequency (1 GHz)

Figure 3.36 Comparison: Conversion gain at third harmonic frequency (3 GHz)
Table 3.1 Comparison between different feedback types

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<th>Feedback Type</th>
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<th>Bandwidth Tuning</th>
<th>Complex Pole</th>
<th>Bandpass Filter Order</th>
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</table>

3.8 Conclusion

Introducing a circulant with a capacitor in parallel to the resistor provided the added freedom of bandwidth tuning. By adding a resistor in series to that capacitor, resulting in R-RC circulants further helped improve the baseband filter order giving a second order filtering. Thus it can be concluded that more complex network can translate to RF providing added degrees of freedom. Also, the addition of the series resistor translated to a higher-order roll-off in the conversion gain, thus negating the limitation of simple N-path circuit.

Also, R-RC circulants retains the benefits offered by R and RC circulants in suppressing the higher order harmonic thus removing the need to add an additional harmonic blocker stage in the receiver. Also, like RC circulant, R-RC provides flexibility in tuning the bandwidth by changing either the series capacitor $C_s$ or the series resistor $R_s$.

R-RC results in additional poles and zeros when compared to R and RC circulants. By choosing
appropriate values for the resistors and capacitors such that the extra poles and zeroes present lie closer to each other helps in achieving a flatter conversion gain with a wider bandwidth.

The LO clock wave-forms can result in a constant phase shift across all frequencies depending on how it is defined. This phase shift is dependent only on the number of phases of the mixer and is independent of the input RF frequency and the LO clock frequency.
CHAPTER 4

CONCLUSION

In this chapter, the advantages and limitations of circulant symmetric baseband feedback with R-RC circulant have been discussed along with the limitations of circulant-symmetric baseband feedback in general. It also mentions the scope for future work.

4.1 Advantages of CSBF with R-RC circulants

The advantages of R-RC circulants include:

- R-RC circulant retains the benefits offered by RC circulant. Also, the complexity introduced by the additional series resistor can translate to RF providing added degrees of freedom.

- Higher order harmonic suppression tuning similar to R and RC circulants.

- Better control over 3dB bandwidth without changing the value of the sampling capacitance. For R-RC circulant, bandwidth can be tuned by varying the values of either the series resistor $R_s$ or the series capacitor $C_s$. It is more flexible due to the presence of the additional poles and zeroes introduced by the R-RC circulant.
• Better impedance matching. The presence of the series resistor \( R_s \) results in a frequency dependent conductance which becomes more prominent at higher frequencies for smaller capacitances.

• Addition of the series resistor translated to higher-order roll-off in the conversion gain, thus negating the limitation of simple N-path circuit.

• Fine tuning of \( S_{11} \) notch to center frequency similar to R, RC and complex feedback case.

4.2 Limitations of CSBF with R-RC circulants

The main limitation of R-RC circulant includes:

• Higher values of the series capacitance results in complex conjugate poles which results in overshoot in the conversion gain which is undesirable.

• Higher capacitance values can also result in right-half plane poles.

• Similar to RC circulant, it is possible to tune the RF bandwidth. For a constant value of capacitance, lower value of the series resistance results in wider bandwidths. The lowest value possible is \( R_s = 0 \Omega \), which is nothing but the RC case. Hence, R-RC always has lower bandwidth when compared to RC circulant.

4.3 Limitation of Circulant-Symmetric Baseband Feedback

A limitation of this feedback technique is that with increase in the number of mixer phases, the overall complexity increases with an increase in the total available variable for harmonic and bandwidth tuning. This however is the case with R and RC circulants as well. Unlike R and RC case, since the poles and zeros for R-RC circulant are dependent on the additional resistance, care should be taken in designing such that the value of series resistor is not very large. This can result in right-hand poles and zeros which are not desired.
A major limitation for circulant-symmetric baseband feedback in general is that it cannot be employed for a four-phase system. In this case, it just results in frequency shifting. No harmonic suppression is observed. This becomes more clear from Figs. 4.1(a) and 4.1(b).

In general, four-phase mixing has limitations when compared to eight-phase mixing in terms of lower $Z_{sh}$ as mentioned in $[\text{AM10b}]$.

This can be further clarified using equation B.3. For an R-circulant case,

$$H_3 = \frac{Re(Y_{B,3})}{Re(Y_{B,1})} = \frac{1 + b_1 + b_2 + b_3 + A_{B,R}(1 - b_2)}{1 + b_1 + b_2 + b_3 + A_{B,R}(1 - b_2)} = 1 \quad (4.1)$$

It can be seen that $H_3 = 1$ always, hence harmonic suppression cannot be achieved using CSBF for a four-phase system.

### 4.4 Future Work

The idea of CSBF can be expanded to systems that require harmonic suppression and bandwidth tuning. One such application is in MIMO systems where a similar approach can help in decoupling
the interference between multiple antennas.

More complex network can translate to RF providing added degrees of freedom. So far R, RC and R-RC circulants have been discussed all of which results in first order systems. Using other R and C variations, higher order systems can be designed to give more sharper filtering. The extra degrees of freedom resulting from these circulants can be studied to see it’s impact in harmonic and bandwidth tuning of an N-phase system.
BIBLIOGRAPHY


[Eve18] EventHelix. 5G non-standalone access: Signaling flow for 5G access via LTE-5G NR dual connectivity (EN-DC). 2018. URL: https://medium.com/5g-nr/5g-non-standalone-access-4d48cea5db5f (visited on 09/16/2018).


APPENDICES
function [CG] = Conversion_gain(b3, Cs3, bs3, ABB, loop)

% This function is used to plot the conversion gain for R, RC and R–RC
% circulants. Feedback paths corresponding to y3 and y5 are considered.
%  b3 is the normalized resistance of R3 with respect to Ro
%  Cs3 is the parallel capacitor → RC circulant
%  Cs3 is the series capacitance → R–RC circulant
%  bs3 is the normalized value of the series resistance with
%  respect to Ro→ R–RC circulant
%  ABB is the DC gain of the baseband amplifier
%  loop corresponds to the set of simulation data to be accessed
Loading Simulation Data

CG_Gain = csvread("./CG1_c3_1150f_b3_var.csv",2,0);
CG_Phase = csvread("./CG1ph_c3_1150f_b3_var.csv",2,0);
CG_Gain = csvread("./cs3_500f_bs3_var/CG1.csv",2,0);
CG_Phase = csvread("./cs3_500f_bs3_var/CG1ph.csv",2,0);

Conversion Gain Model Parameters

H3 = 5; % Harmonic Ratio
Rf = 6.425e3; % Negative feedback resistor (w/o circulants case)
Ro = 2200; % Negative feedback resistor (with circulants)
CB = 30e-12; % Sampling capacitor
N=8; % Mixer phase
Za = 50; % Port impedance
Rsw = 8.4; % Switch resistance

Baseband Amplifier

wp = 0.5e9 2 pi; % 3dB freq of baseband amplifier
A = tf(ABB, [1/wp 1]); % Baseband amplifier transfer function
s = tf('s');

Circulants

GB = (1+2(b3)+A(1-sqrt(2)(b3)))/Ro; % R-only case
\[ G = \frac{(2(\text{bs3})-A \sqrt{2(\text{bs3})})}{\text{Ro}}; \quad \% \text{RRC case (corresponding to the series resistor, Rs)} \]

\[ C = (2(\text{Cs3})+A(-\sqrt{2(\text{Cs3})})); \quad \% \text{RC and RRC case} \]

\[ n = 1; \quad \% \text{Harmonic number} \]
\[ k = \text{sinc}(n \pi/N); \]
\[ C_{nN} = k/N; \]
\[ \gamma_{\text{nN}} = \frac{k^2}{N}; \]
\[ Z_{\text{sh}} = \frac{(Z_a+R_{\text{sw}})N}{(1-k^2)}; \quad \% \text{Shunt impedance} \]

%%% Harmonic Termination Admittance

%%% Harmonic Termination Admittance

\[ Y_{BB} = 1/R_f + CB s; \quad \% \text{Rf case} \]
\[ Y_{BB} = GB + CB s; \quad \% \text{R circulant} \]
\[ Y_{BB} = GB + (CB + C) s; \quad \% \text{RC circulant} \]
\[ Y_{BB} = GB + s CB + \frac{((C G s)/(G+s C))}{(G+s C)}; \quad \% \text{RRC circulant} \]
\[ Z_{BB} = 1/Y_{BB}; \quad \% \text{Total baseband impedance} \]

% Total input impedance looking into the mixer
\[ Z_{\text{in}} = R_{\text{sw}} + \frac{1}{((1/(\gamma_{\text{nN}} Z_{BB}))+1/Z_{\text{sh}})}; \]

%%% Conversion Gain

\[ C_{G} = 4C_{nN} \left( \frac{Z_{BB}}{(Z_{\text{in}}+Z_a)} \right) A; \]
[gain, phase, freq] = bode(CG);

for i = 1:length(freq)
    GCG(i) = 20 * log10(gain(:, :, i));
    PCG(i) = phase(:, :, i);
end

%% Conversion Gain Plots: Analysis vs Simulation

figure(1)

subplot(2, 1, 1)

s1 = size(CG_Gain);
x = CG_Gain(1:s1(1));
x = (x - 1.0e9) * 2 * pi;  % Shifting the bandpass response to DC
y = CG_Gain(loop s1(1)+1:(loop+1) s1(1));
l = (s1(1)+1)/2;

semilogx(freq, GCG', 'Linewidth', 2)  % Analysis
hold on
semilogx(x(1:l), y(1:l), '—k', 'Linewidth', 2)  % Simulation data
grid on

%ylim([-5 35])
xlim([5 1e7 1e9])

subplot(2, 1, 2)

s1 = size(CG_Phase);
x = CG_Phase(1:s1(1));
x = (x−1.0e9) . 2 . pi; % Shifting the bandpass response to DC
y = CG_Phase(loop s1(1)+1:(loop+1) s1(1));
l = (s1(1)+1)/2;

%%%% Phase plot %%%%%
semilogx(freq,PCG,'Linewidth',2) % Analysis
hold on
ph = 22.5; % Corresponds to the error factor, the additional
           % phase shift due to LO waveforms
semilogx(x(1:l),y(1:l)−ph,'−−k','Linewidth',2) % Simulation Data
xlim([5e7 1e9])
grid on

%%%% Transfer function to matrix
[n,d] = tfdata(CG);
um = cell2mat(n);
den = cell2mat(d);

z = roots(num); % Zeros of the conversion gain
p = roots(den); % Poles of the conversion gain

%%%% Pole−zero plot
figure()
hold on
zplane(z,p,'g')
end
APPENDIX B

R-CIRCULANT TUNING

B.1 Harmonic Tuning using Real Circulants [WF19]

As presented by the authors in [WF19], CSB feedback can be used for harmonic tuning. The feedback network can be implemented so as to obtain the desired fundamental admittance response and get either a reduced or increased admittance at higher harmonics. The role of the circulants in affecting the third harmonic admittance is discussed. From equation 2.14, the third harmonic to fundamental harmonic admittance ratio is defined as $H_3$,

\[
H_3 = \frac{Re(Y_{B,3})}{Re(Y_{B,1})} \tag{B.1}
\]

where $Y_{B,1}$ is the baseband termination admittance at fundamental frequency and $Y_{B,3}$ is the baseband termination admittance at third harmonic frequency.
Considering an 8-phase mixer, the baseband circulant is defined as

\[ g = R_0^{-1} \cdot [1, b_1, b_2, b_3, 0, b_3, b_6, b_1] \] (B.2)

From Equations 2.9 and B.2,

\[ H_3 = \frac{1 + b_{im} + 2b_1 + 2b_3 + A_{BB}(1 + \sqrt{2}(b_3 - b_1))}{1 + b_{im} + 2b_1 + 2b_3 + A_{BB}(1 + \sqrt{2}(b_1 - b_3))} \] (B.3)

where \( b_{im} = b_2 - b_6 \). If \( b_2 > 0, b_6 = 0 \) and vice versa.

The value of \( H_3 \) decides whether the third harmonic admittance should be suppressed or increased. As observed from Equation B.3, setting \( b_1 \) to zero suppresses the third harmonic admittance resulting in \( H_3 > 1 \) whereas setting \( b_3 \) to zero increases the third harmonic admittance giving \( H_3 < 1 \).

From the matching condition given in Equation F.1, redefining harmonic dependent impedance, we get

\[ Z_{in}(\omega) = R_{sw} + \sum_{k=1}^{\infty} \gamma_{N,k}[Z'_{B,k}(\omega_{IF,k})||Z_{rad,k}] \] (B.4)

where \( \gamma_{N,k} = \frac{1}{N} \sin c\left( \frac{N k}{N} \right) \) is the harmonic scaling factor and \( Z_{rad,k} = \frac{1}{Y_{rad,k}} \approx (Z_{a,k} + R_{sw}) \frac{N}{1 - N \gamma_{N,k}} \) is the harmonic dependent re-radiation impedance.

Using the above equations, the baseband termination impedance can be defined as:

\[ Z_1 = \left( \frac{\gamma}{Z_{in}(\omega_o) - R_{sw}} - Y_{rad,1} \right)^{-1} = R_B(1 + j \chi) \] (B.5)
Solving for $b_{im}$

The fundamental harmonic baseband termination admittance is given by:

$$Y_{B,k} = R_0^{-1}(1 + b_{im} + 2b_1 + 2b_3 + A_{BB}(1 + \sqrt{2}(b_3 - b_1)) - j b_2 + j b_6) \quad (B.6)$$

and

$$\text{Re}(Y_{B,k}) = G_{B,k} = R_0^{-1}(1 + b_{im} + 2b_1 + 2b_3 + A_{BB}(1 + \sqrt{2}(b_3 - b_1))) \quad (B.7)$$

By equating the imaginary parts of $Y_{B,1}$ and $Z_{1}^{-1}$, i.e, $Im(Y_{B,1}) = Im(Z_{1}^{-1})$ we get $b_{im}$.

Case I  $b_6 = 0$

$$\Rightarrow -j R_0^{-1} A_{BB} b_2 = \frac{-j x}{R_B(1 + x^2)}$$

i.e,

$$b_2 = \frac{x}{1 + x^2} \frac{R_0}{A_{BB} R_B} = b_{im} \quad (B.8)$$

Case II  $b_2 = 0$

$$\Rightarrow -j R_0^{-1} A_{BB} b_2 = \frac{-j x}{R_B(1 + x^2)}$$

i.e,

$$b_6 = \frac{-x}{1 + x^2} \frac{R_0}{A_{BB} R_B} = -b_{im} \quad (B.9)$$

Thus we get,

$$b_{im} = \frac{|x|}{1 + x^2} \frac{R_0}{A_{BB} R_B} \quad (B.10)$$

Solving for $b_3$

For the condition $H_3$ greater than or equal to 1, $b_1 = 0$.

$$H_3 = \frac{1 + b_2 + 2b_3 + A_{BB}(1 + \sqrt{2}(b_3))}{1 + b_2 + 2b_3 + A_{BB}(1 + \sqrt{2}(-b_3))}$$
\[
(H_3 - 1)(1 + b_2 + A_{BB}) + b_3 \left[ 2(H_3 - 1) - (H_3 + 1)\sqrt{2}A_{BB} \right] = 0
\]

\[
1 + b_2 + A_{BB} = \frac{H_3 + 1}{H_3 - 1} \sqrt{2}A_{BB} b_3 - 2b_3
\]
i.e.
\[
b_3 = \frac{1 + b_2 + A_{BB}}{\frac{H_3 + 1}{H_3 - 1} \sqrt{2}A_{BB} - 2}
\]

**Solving for \( b_1 \)**

For the condition \( H_3 < 1 \) and \( b_3 = 0 \),

\[
H_3 = \frac{1 + b_2 + 2b_1 + A_{BB}(1 + \sqrt{2}(-b_1))}{1 + b_2 + 2b_1 + A_{BB}(1 + \sqrt{2}(b_1))}
\]

\[
(H_3 - 1)(1 + b_2 + A_{BB}) + b_1 \left[ 2(H_3 - 1) + (H_3 + 1)\sqrt{2}A_{BB} \right] = 0
\]

\[
1 + b_2 + A_{BB} = \frac{H_3 + 1}{H_3 - 1} \sqrt{2}A_{BB} b_1 - 2b_1
\]
i.e.
\[
b_1 = \frac{1 + b_2 + A_{BB}}{\frac{H_3 + 1}{H_3 - 1} \sqrt{2}A_{BB} - 2}
\]

**Solving for \( R_0 \)**

**Case 1 \( H_3 > 1 \)**

Here \( b_1 = 0 \). Equating the real parts of \( Y_{B,1} \) and \( Z_1^{-1} \), i.e. \( Re(Z_1^{-1}) = Re(Y_{B,1}) \)

\[
\frac{1}{R_B(1 + x^2)} = R_0^{-1} \left[ 1 + b_2 + 2b_3 + A_{BB}(1 + \sqrt{2}(-b_3)) \right]
\]

\[
\frac{R_0}{R_B(1 + x^2)} = 1 + b_2 + A_{BB} + (2 + \sqrt{2}A_{BB})(b_3)
\]
\[= 1 + b_2 + A_{BB} + (2 - \sqrt{2}A_{BB}) \left( \frac{1 + b_2 + A_{BB}}{H_3 + 1} \right) \frac{1}{\sqrt{2}A_{BB} - 2} \]  

[from eq B.11]

\[= 1 + A_{BB} + \left( \frac{2 - \sqrt{2}A_{BB}}{H_3 + 1} \right) \frac{1}{\sqrt{2}A_{BB} - 2} + \left( \frac{2 - \sqrt{2}A_{BB}}{H_3 + 1} \right) b_2 \]

Substituting for \( b_2 \) from eq B.9

\[R_0 = \frac{R_B}{1 - x^2} \left( 1 + A_{BB} \right) \left( \frac{2 - \sqrt{2}A_{BB}}{H_3 + 1} \right) \frac{1}{\sqrt{2}A_{BB} - 2} A_{BB} + \frac{H_3 + 1}{H_3 - 1} \]  

\[\Rightarrow R_0 = \frac{R_B}{1 - x^2} \left( 1 + A_{BB} \right) \left( \frac{2 - \sqrt{2}A_{BB}}{H_3 + 1} \right) \frac{1}{\sqrt{2}A_{BB} - 2} A_{BB} + \frac{H_3 + 1}{H_3 - 1} \]

\[\Rightarrow R_0 = \frac{R_B(1 + x^2)(1 + A_{BB})}{ \left( \frac{2 + \sqrt{2}A_{BB}}{H_3 + 1} \right) \frac{1}{\sqrt{2}A_{BB} - 2} A_{BB} + \frac{H_3 + 1}{H_3 - 1} } \]

\[= \frac{R_B(1 + A_{BB})(1 + x^2)}{ \frac{H_3 + 1}{H_3 - 1} \frac{x}{\sqrt{2}A_{BB}} - \frac{A_{BB}}{2} - \frac{x}{\sqrt{2}A_{BB}} } \]

\[\Rightarrow R_0 = \frac{R_B(1 + x^2)(1 + A_{BB})}{ \frac{H_3 + 1}{H_3 - 1} \frac{x}{\sqrt{2}A_{BB}} - \frac{A_{BB}}{2} - \frac{x}{\sqrt{2}A_{BB}} } \quad \text{(B.13)} \]

Case 2 \( H_3 < 1 \)

Here \( b_3 = 0 \). Equating the real parts of \( Y_{B,1} \) and \( Z_{1^{-1}} \), i.e. \( Re(Z_{1^{-1}}) = Re(Y_{B,1}) \)
\[
\frac{1}{R_B(1 + x^2)} = R_0^{-1} \left[ 1 + b_2 + 2b_1 + A_{BB}(1 + \sqrt{2}b_1) \right]
\]

\[
\Rightarrow \quad \frac{R_0}{R_B(1 + x^2)} = 1 + b_2 + A_{BB} + (2 + \sqrt{2}A_{BB})b_1
\]

\[
= 1 + b_2 + A_{BB} + (2 + \sqrt{2}A_{BB}) \left( \frac{1 + b_2 + A_{BB}}{H_3 + 1 - \sqrt{2}A_{BB} - 2} \right)
\]

\[
\text{from eq B.12}
\]

\[
= 1 + A_{BB} - \frac{(2 + \sqrt{2}A_{BB})(1 + A_{BB})}{H_3 + 1 - \sqrt{2}A_{BB} + 2} + \left( 1 - \frac{2 + \sqrt{2}A_{BB}}{H_3 + 1 - \sqrt{2}A_{BB} + 2} \right) b_2
\]

Substituting for \( b_2 \) from eq B.9

\[
\Rightarrow \quad \frac{R_0}{R_B(1 + x^2)} =
\]

\[
(1 + A_{BB}) \left( 1 - \frac{2 - \sqrt{2}A_{BB}}{H_3 + 1 - \sqrt{2}A_{BB} + 2} \right) + \left( \frac{H_3 + 1}{H_3 - 1} \sqrt{2}A_{BB} - \sqrt{2}A_{BB} \right) x \frac{R_0}{1 + x^2 A_{BB} R_B}
\]

\[
\Rightarrow \quad \frac{R_0}{R_B(1 + x^2)} \left( 1 - \frac{2\sqrt{2}A_{BB}}{H_3 - 1 \sqrt{2}A_{BB} + 2} \right) = (1 + A_{BB}) \left( \frac{2\sqrt{2}A_{BB}}{H_3 - 1 \sqrt{2}A_{BB} + 2} \right)
\]

\[
\Rightarrow \quad \frac{R_0}{R_B(1 + x^2)(1 + A_{BB})} = \frac{2\sqrt{2}A_{BB}}{H_3 - 1}
\]

\[
= \frac{H_3 + 1}{H_3 - 1} \sqrt{2}A_{BB} + 2 - \frac{x2\sqrt{2}}{H_3 - 1} 2\sqrt{2}A_{BB}
\]

\[
= \frac{(H_3 + 1)\sqrt{2}A_{BB} + (H_3 - 1)2 - 2\sqrt{2}x}{(H_3 - 1)\sqrt{2}A_{BB} + (H_3 - 1)2 - 2\sqrt{2}x}
\]

\[
\Rightarrow R_0 = \frac{R_B(1 + A_{BB})(1 + x^2)}{H_3 + 1} \frac{H_3 - 1}{x} \frac{x}{\sqrt{2}A_{BB} - A_{BB}}
\]

\[\text{(B.14)}\]
From equations B.13 and B.14,

\[ R_0 = \frac{R_B (1 + A_{BB})(1 + x^2)}{H_3 + 1} \frac{|H_3 - 1|}{2} \frac{x}{\sqrt{2} A_{BB}} \text{ for all values of } H_3 \]  

(B.15)

## B.2 Simulation Results

For a fundamental frequency of 1 GHz, using an 8-phase mixer with \( R_s \approx 8.4 \Omega \), the circulant values were calculated for \( H_3 = 5 \). Since \( H_3 > 1 \), \( b_1 = 0 \).

**Figure B.1** Schematic of the CSBF with \( g = 2.2k^{-1} \cdot [1, 0, 0, 0.52, 0, 0.52, 0, 0] \)

**Case I**  
Assuming the imaginary part of the baseband termination impedance given by Equation B.5 to be 0, i.e., assuming it to be purely real. Solving for the circulants, the values were obtained as \( b_{im} = 0, b_3 = 0.52 \) and \( R_0 = 2.2k \Omega \). Thus \( g = 3.9k^{-1} \cdot [1, 0, 0, 0.52, 0, 0.52, 0, 0] \)

Figure B.1 shows the baseband amplifier with the given circulant feedback. Since the baseband termination impedance was assumed to be purely real, the center frequency of \( S_{11} \) graph is not at the required 1 GHz, but is off by a few MHz as can be seen from Figure B.3. Compared to the conversion gain obtained in the previous cases without the circulant, it can be observed from
figure B.4 that the third harmonic gain is suppressed by 11.5 dB. Thus, the need for external harmonic blockers can be reduced.

**Figure B.2** $S_{11}$ Smith chart representation

**Figure B.3** $S_{11}$ in dB vs frequency
**Figure B.4** Conversion gain at fundamental frequency (1 GHz)

**Figure B.5** Conversion gain at third harmonic frequency (3 GHz)
Case II Here, the value of $x$ in Equation B.5 is taken to be 0.25. The circulant was obtained as:

$$g = 2.2 k^{-1} \cdot [1, 0, 0.08, 0.5, 0, 0.5, 0, 0]$$

By tuning the value of $x$, $S_{11}$ center frequency was shifted to obtain the required 1 GHz as seen in Figure B.7. The introduction of $b_{1m}$ did not affect the conversion gain and bandwidth and it was found to be the same as in case I.
Figure B.7 $S_{11}$ in dB vs frequency

Figure B.8 Smith chart of $S_{11}$
APPENDIX C

RC CIRCULANT TUNING

This is the work presented in [WF19]

C.1 Harmonic Tuning Using RC Circulants

An advantage of using RC circulant over R-only circulant is that, here both the harmonic termination admittance as well as the bandwidth of the receiver can be controlled by using suitable values of resistors and capacitors.

Harmonic termination admittance can be controlled just like the R-only case. The expressions for $b_1$ and $b_3$ remains unchanged as shown in Section B.1. However, the $b_{im}$ term can be modified as:

$$b_{im} = \frac{|x|}{1+x^2} \frac{R_0}{A_{BB}R_B} - \frac{\omega C_3 R_0}{A_{BB}} (2 - A_{BB} \sqrt{2})$$  \hspace{1cm} (C.1)

Here, $C_1 = 0$. By incorporating the effect of the parallel capacitor, $b_{im}$ becomes dependent of
frequency.

Deriving the expression for $R_0$ by equating the real part of $Y_{B,1}$ and $Z_{1}^{-1}$, we get

$$R_0 = R_B (1 + A_{BB}) \frac{(1 + x^2)}{K_3 - \frac{x}{A_{BB}} - \frac{\omega C_3 (2 - A_{BB} \sqrt{2}) R_B (1 + x^2)}{A_{BB}}}$$  \hspace{0.5cm} (C.2)

where

$$K_3 = 0.5 \left(1 + H_3 - \frac{|H_3 - 1|}{A_{BB}} \sqrt{2}\right); \quad \text{for all values of } H_3 \hspace{0.5cm} (C.3)$$

For very small values of capacitance, its effect in $R_0$ can be neglected.

### C.2 Bandwidth Tuning Using RC Circulants

The bandwidth of the receiver can be defined by the baseband time constant, $\tau_B = R_{eq} C_{eq}$. This time constant can be dominated by $C_B$ and hence baseband harmonic tuning can be used to introduce harmonic-dependent positive or negative capacitance to adjust bandwidth.

The time constant at fundamental frequency is,

$$\tau_{B,1} = \frac{C_B + 2C_1 + 2C_3 + A_{BB} \sqrt{2} (C_1 - C_3)}{R_0^{-1} (1 + b_2 + 2b_1 + 2b_3 + A_{BB} (1 + \sqrt{2} (b_1 - b_3)))}$$  \hspace{0.5cm} (C.4)

The time constant at third harmonic frequency is,

$$\tau_{B,3} = \frac{C_B + 2C_1 + 2C_3 + A_{BB} \sqrt{2} (C_3 - C_1)}{R_0^{-1} (1 + b_2 + 2b_1 + 2b_3 + A_{BB} (1 + \sqrt{2} (b_3 - b_1)))}$$  \hspace{0.5cm} (C.5)

The values of $C_1$ and $C_3$ are chosen so as to achieve the desired $\tau$. Taking $C_1 = 0$ aids in reducing the third harmonic bandwidth as bandwidth is inversely proportional to $\tau$. Similarly, $C_3 = 0$ results
in greater third harmonic bandwidth than the fundamental bandwidth. Thus it can be observed that increasing \( C_3 \) increases the fundamental bandwidth and reduces the third harmonic bandwidth and opposite is the case with \( C_1 \).

### C.3 Limitations in bandwidth tuning

As mentioned previously, \( C_1 = 0 \) gives the maximum possible bandwidth for a given value of \( C_3 \).

\[
C_1 = 0, \quad \Rightarrow \quad \tau_{B,1} = \frac{C_B + C_3(2 - A_{BB}\sqrt{2})}{G_{B,1}}
\]

where, \( G_{B,1} = R_0^{-1}(1 + b_2 + 2b_1 + 2b_3 + A_{BB}(1 + \sqrt{2}(b_3 - b_1))) \) given in equation B.7.

To get a considerable conversion gain of around 25dB, the baseband gain should be greater than 20dB. Therefore, \( 2 - A_{BB}\sqrt{2} \) will always be negative. To maintain a positive value of capacitance \( C_3 \),

\[
\frac{C_B - G_{B,1}\tau_{B,1}}{A_{BB}\sqrt{2} - 2} > 0 \quad (C.6)
\]

\[
i.e, \quad \tau_{B,1} < \frac{C_B}{G_{B,1}} \quad (C.7)
\]

For getting higher values of \( \tau_{B,1} \), (lower fundamental bandwidth), \( C_3 \) should be replaced by \( C_1 \) circulant.

The upper bound of \( C_3 \) is given by,

\[
C_{3,\text{max}} < \frac{C_B}{A_{BB}\sqrt{2} - 2} \quad (C.8)
\]

This is to ensure that the system always remains stable and \( \tau_{B,1} > 0 \). This value of \( C_3 \) corresponds to the maximum obtainable bandwidth around fundamental LO frequency.
This value of $C_{3_{\text{max}}}$ can be either increased by introducing negative feedback capacitance $C_0$ or decreased by positive feedback using $C_4$, resulting in a modified circulant $c = [C_0, C_1, 0, C_3, C_4, C_3, 0, C_1]$. This however does not aid in increasing the bandwidth to a value greater than the one obtained with just $C_3$.

### C.4 Simulation Results

Fundamental frequency is chosen to be 1 GHz, $\omega_{\text{in}} = \omega_{LO} = 1$ GHz. Mixer switch resistance, $R_{sw} = 8.4\Omega$, $R_{BB} = 355\Omega$, $C_B = 30\, pF$ and baseband gain is taken as 25dB.

The circulant values were evaluated to be:

$$g = 2.2k^{-1} \cdot [1, 0, 0, 0.52, 0, 0.52, 0, 0] \text{ and } c = [0, C_1, 0, C_3, 0, C_3, 0, C_1]F.$$

Here $C_1 = 0$ and the value of $C_3$ was varied from 500 fF to 1pF. Based on equation C.7, $\tau_{B,1}$ varies from approximately 6.4ns to 2.3ns.

For simplicity, $b_{lm}$ was taken to be 0, thus keeping it a constant across frequency.

From Figures C.5 and C.6, it can be observed that there is an approx 12dB reduction in the third harmonic gain similar to the case with R circulants. Thus the aim of baseband harmonic tuning was achieved.

Also, from Figures C.5 and C.6, it can be observed that with increase in $C_3$, the fundamental bandwidth increases and the third harmonic bandwidth decreases as expected. Here the value of $C_3$ was varied from 500fF to 1pF. The desired bandwidth can be obtained by design the RC circulants accordingly as per the requirement. Using larger capacitor values for $C_3$ account towards increasing the fundamental bandwidth, unlike what is expected as that corresponding $\phi_{n,k}$ from equation 2.8 translates $C_3$ to an equivalent negative capacitance, thus reducing the capacitive effect of the
Figure C.1 Schematic of the CSB with $g = 2.2k^{-1} \cdot [1, 0, 0, 0.52, 0, 0.52, 0, 0]$ and $c = [0, 0, 0, C_3, 0, C_3, 0, 0]F$

From the conditions given in (C.7) and (C.8), for a baseband gain of 25dB and sampling capacitor, $C_B = 30\text{pF}$, $\tau_{B,1_{max}} \approx 8\text{ns}$ and $C_{3_{max}} \approx 1.1\text{pF}$. 
Figure C.2 Variation in $S_{11}$ dB vs frequency with RC circulants for variable $C_3$

Figure C.3 RC circulants: Variation in input-port resistance (Ω) vs frequency for variable $C_3$
Figure C.4 RC circulant: $S_{11}$ Smith chart representation for variable $C_3$
Figure C.5 RC circulant: Conversion gain at fundamental frequency (1 GHz) for variable $C_3$

Figure C.6 RC circulant: Conversion gain at third harmonic frequency (3 GHz) for variable $C_3$
A behavioral TIA has been used whose gain $A_{BB} = 17.78 \text{ V/V} = 25\text{dB}$, with a 3dB bandwidth = 0.5GHz.
The magnitude and phase response of the TIA is given in the figure below.

**Figure D.1** Simulated gain magnitude and phase plots of the baseband amplifier used.
Assuming an input-source resistance of 50Ω, the required baseband resistance ($R_{BB}$) for impedance match was calculated to be 355Ω for a frequency of 1 GHz using,

$$R_{in} = R_{sw} + \gamma R_{BB}||R_{sh}$$ \hspace{1cm} (E.1)

For a baseband gain ($A_{BB}$) of 25dB, the feedback resistance $R_F$ was found to be 6.7$k\Omega$, where

$$R_F = R_{BB}(1 + A_{BB})$$ \hspace{1cm} (E.2)

### E.1 Simulation Results

An 8-phase mixer was simulated at 1 GHz with only a negative feedback resistance as shown in Fig. E.1.
Figures E.2-E.5 shows the $S_{11}$ and conversion gain plots for a mixer with $\omega_{RF}=1$ GHz and $\omega_{LO}=3$ GHz for the circuit shown in figure E.1. It can be observed that the fundamental and third harmonic conversion gain peaks vary only by a small difference of 2dB. Also, the $S_{11}$ peaks at 999MHz instead of the required 1 GHz. This is because the antenna impedance was assumed to be purely real and the matching was done only for the real part of the impedance.
Figure E.3 Smith chart of $S_{11}$ without CSBF

Figure E.4 Conversion gain at fundamental frequency (1 GHz) without CSBF
Figure E.5 Conversion gain at third harmonic frequency (3 GHz) without CSBF

Figure E.6 Real/resistive part of the input port impedance ($Re(Z_m)$) without CSBF
Figure E.7 Reactance part of the input port impedance ($Re(Z_m)$) without CSBF
APPENDIX F

BASEBAND AMPLIFIER WITH COMPLEX FEEDBACK

F.1 Simulation Results

Figure F.1 Schematic of an 8-phase mixer with IQ cross-feedback (complex matching).
Using the antenna across wide frequency ranges, parasitic capacitance associated with the switches and the presence of sampling capacitor \( (C_B) \) leads to a complex antenna impedance [AM10b] and by incorporating it for impedance matching helps in fine tuning the \( S_{11} \) peak, equation E.1 can be modified to:

\[
Z_{in} = R_{sw} + \gamma Z_B \| Z_{sh}
\]  

(F.1)

![Figure E.2](image.png) \( S_{11} \) in dB vs frequency with IQ cross-feedback (complex matching).

In this formulation, the antenna impedance and shunt impedance can both modelled to take on complex values, and a complex conjugate match will only be achieved if the complex component of \( Z_{BB} \) is tunable. This can be achieved by tuning the value of the sampling capacitor. But it only acts a negative reactance and also affects the bandwidth. Considering the case of a 4-phase mixer, a complex term can be introduced by providing additional feedback resistors \( (R_{FI}) \) from I to Q channel and vice versa as shown in Figure 2.11. This type of complex feedback can be used to change the center frequency to either side of the LO by varying the value of \( R_{FI} \) and by changing its polarity.
**Figure F.3** Smith chart of $S_{11}$ with IQ cross-feedback (complex matching).

**Figure F.4** Conversion gain at fundamental frequency (1 GHz) with IQ cross-feedback (complex matching).

[AM10b], [AM10a], [MA12].
Figure E.5 Conversion gain at third harmonic frequency (3 GHz) with IQ cross-feedback.

Figure E.6 Real/resistive part of the input port impedance ($Re(Z_m)$) with IQ cross-feedback.

The above idea can be extended for an 8-phase mixer as shown in Figure E.1.
Figure F.7 Reactance part of the input port impedance ($Re(Z_m)$) with IQ cross-feedback.

From figure F.2, it can be observed that by introducing the complex matching resistor feedback, the center frequency can be shifted to the required 200MHz. There is no major change in fundamental and third harmonic conversion gains, also the bandwidth also remains unchanged.