ABSTRACT

KIRSCHMEIER, BENJAMIN ANDREW. Wing-Wake Interactions in Aeroelastic Systems. (Under the direction of Dr. Matthew Bryant).

Wing-wake interactions have been a topic of considerable interest due to the complex interactions between wings and incident vorticity. Common examples of wing wake interactions occur in insect flight, such as the dragon fly, fish swimming with tail and dorsal fin interaction, and energy harvesting applications. Interactions in each of these applications are determined by the shed vorticity from an upstream oscillating wing. The transfer of vortex energy to the downstream wing dictates the aeroelastic response. The dissertation presented here discuss experimental investigations into nonlinear wing-wake aeroelastic phenomena. Specifically, this work studies the effect of vortical disturbances on an aeroelastic wing undergoing limit cycle oscillations and develops signal analysis techniques to understand those interactions. Experiments were conducted using two different upstream bodies to generate vortical disturbances on the downstream aeroelastic wing: another aeroelastic wing in a limit cycle and a fixed bluff body. In conjunction with these wake-interaction experiments, an aeroelastic inverse technique was developed to estimate the aerodynamic loads and power distribution for a wing undergoing limit cycle oscillations. The work was divided into three separate studies, as summarized below.

The first study experimentally investigated tandem aeroelastic wing-wake interactions to quantify how structural properties dictate vortex energy transfer. The pitch stiffness of the downstream wing was varied such that it was less than or equal to that of the upstream wing. It was hypothesized that pitch stiffness modulates the sensitivity of the wing to incoming vortex disturbances thus changing vortex energy transfer to the wing. The experimental results showed that the limit cycle and transient response of the downstream wing in a tandem configuration
are dependent on its pitch stiffness, while the upstream wing dictated the aeroelastic stability and flutter point of the downstream wing.

The second study focused on developing an algorithm to compute the aerodynamic forces and moments of an aeroelastic wing undergoing large amplitude heave and pitch limit cycle oscillations. The aeroelastic inverse technique is based on inverting the equations of motion to solve for the lift and moment generated on the wing. Bayesian inferencing is used to estimate the structural parameters of the system and generate credible intervals on the lift and moment calculations. The aeroelastic inverse technique is then validated against prescribed aeroelastic motion in a water tunnel. The efficacy of the inversion technique was then shown by studying the effect of mass coupling on limit cycle oscillation amplitude. By investigating the evolution of the force, power, and energy of the system, the reasons for amplitude growth with wind speed are determined.

Finally, in the third part of this work, the effect of an upstream wake generated by a rectangular cylinder bluff body on an aeroelastic wing undergoing large amplitude limit cycle oscillations was studied. Experimental results show that under certain conditions, the large amplitude limit cycle is annihilated. The conditions under which annihilation occurs are dependent on the limit cycle frequency, bluff body shedding frequency, and the magnitude of the mass coupling in the system. The kinematic and aerodynamic behavior of limit cycle annihilation was examined by applying the aeroelastic inverse technique developed in the second study, along with signal analysis methods.
Wing-Wake Interactions in Aeroelastic Systems

by

Benjamin Andrew Kirschmeier

A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the Degree of Doctor of Philosophy

Mechanical Engineering

Raleigh, North Carolina

2019

APPROVED BY:

Dr. Ashok Gopalarathnam

Dr. Andre Mazzoleni

Dr. Venkat Narayanaswamy

Dr. Matthew Bryant
Chair of Advisory Committee
DEDICATION

To my fiancée,
for showering me with endless love and support and making sure I defended before our
wedding;
To my parents,
for always encouraging me and believing in my success;
To my brother,
for always wanting the best for me;
To my friends,
whose smiles and laughter made graduate school more enjoyable
BIOGRAPHY

Ben received his BS in Mechanical Engineering with a concentration in Aerospace engineering from The George Washington University in May 2014. During his time at GWU, he gained research experience working for Dr. Adam Wickenheiser for two summers. Additionally, he started the GWU Rocket Team with his friends and they placed 3rd overall in their first ever competition. After graduation Ben went North Carolina State University to pursue a doctoral degree under the guidance of Dr. Matthew Bryant. He will complete the requirements for his PhD in Mechanical Engineering at North Carolina State University in August 2019.
ACKNOWLEDGEMENTS

First and foremost, I would like to thank my advisor, Dr. Matthew Bryant, for his support and guidance over the past five years, and for sparking my interest in aeroelasticity. I would also like to thank the members of my committee, Dr. Ashok Gopalarathnam, Dr. Andre Mazzoleni, Dr. Venkat Narayanaswamy and Dr. Ralph Smith, for their time and input into my research. To all of my labmates over the years, Marc, Punnag, Ted, Tyler, Warren, Rag, Zach, Graham, Jake, Stephan, Nick, and David, thank you for all your valuable feedback, joking around, and providing useful distractions from work.

Finally, I would like to thank all my friends and family for helping me throughout graduate school. To my parents, brother, and family, thanks for all the support, encouraging me, and making sure I was doing well. To all my friends from high school, college, and graduate school, thanks for always putting a smile on my face and putting up with my complaints about graduate school. And finally, to my wonderful fiancée, Megan, for always believing in me, for her fun-loving nature always bringing a smile to my face, and for keeping me on track to finish my graduate studies.

I gratefully support from the Air Force Office of Scientific Research under award number FA9550-17-1-0301, monitored by Dr. Gregg Abate. I gratefully acknowledge additional funding support for this research from the National Science Foundation under Award No. ECCS – 1509592 and program officer Radhakisan S. Baheti.
# TABLE OF CONTENTS

**LIST OF TABLES** ................................................................. vii

**LIST OF FIGURES** ............................................................. ix

**LIST OF SYMBOLS** ............................................................ xiii

**Chapter 1**  
**Introduction to wing wake interactions**  .............................................. 1
  1.1 Aeroelastic Behavior of a Wing or Airfoil ........................................... 1
  1.2 Research Aims .................................................................................. 3

**Chapter 2**  
**Experimental Investigation of Wake-Induced Aeroelastic Limit Cycle Oscillations in Tandem Wings** .................................................. 5
  2.1 Introduction ...................................................................................... 5
  2.2 Experimental Methods ...................................................................... 9
    2.2.1 Experimental Apparatus ................................................................. 9
    2.2.2 Data Acquisition and Reduction .................................................... 12
  2.3 Results and Discussion ..................................................................... 14
    2.3.1 Aeroelastic Response of a Single Wing .......................................... 14
    2.3.2 Tandem Wing Aeroelastic Interactions .......................................... 26
  2.4 Conclusion ....................................................................................... 42

**Chapter 3**  
**Aeroelastic Inverse: Estimation of Aerodynamic Loads During Large Amplitude Limit Cycle Oscillations** ........................................... 43
  3.1 Introduction ...................................................................................... 43
  3.2 Experimental Setup .......................................................................... 46
  3.3 Aeroelastic Inverse Method ................................................................ 48
  3.4 System Identification and Uncertainty Quantification ......................... 52
    3.4.1 Model Selection and Sensitivity Analysis ......................................... 52
    3.4.2 Uncertainty Quantification ............................................................... 56
    3.4.3 Model Calibration ........................................................................ 58
    3.4.4 Aerodynamic Force Prediction and AEI Validation ......................... 61
  3.5 Results and Discussion ..................................................................... 64
    3.5.1 Bifurcation Analysis ...................................................................... 64
    3.5.2 Limit-cycle Kinematic Analysis ...................................................... 66
    3.5.3 Limit-cycle Force Analysis ............................................................... 67
    3.5.4 Limit-Cycle Power and Energy Analysis ......................................... 71
    3.5.5 Coupling Energy Analysis ............................................................... 74
  3.6 Conclusion ....................................................................................... 77
Chapter 4  Amplitude Annihilation in Wake-Influenced Aeroelastic Limit Cycle Oscillations  78
4.1  Introduction  78
4.2  Experimental Methods  81
4.3  System Characterization and Modeling  84
  4.3.1  Parameter Estimation  84
  4.3.2  System Energies  87
4.4  Results  89
  4.4.1  Wing Limit Cycle Oscillation Behavior  89
4.5  Analysis and Discussion  95
  4.5.1  Recurrence Analysis  95
  4.5.2  Instantaneous Frequency Analysis  100
  4.5.3  Coupling and Aerodynamic Energy Analysis  104
4.6  Conclusion  107

Chapter 5  Conclusions and Future Work  108
5.1  Key Findings of Tandem Wing Aeroelastic Experiments  108
5.2  Key Findings of the Aeroelastic Inverse Method  109
5.3  Key Findings of Limit Cycle Annihilation  109
5.4  Future Work  110
5.5  Final Conclusions  112

REFERENCES  113
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table 2.1</th>
<th>Limit cycle oscillation characteristics for AW-1.</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2.2</td>
<td>Limit cycle oscillation characteristics for AW-1.</td>
<td>25</td>
</tr>
<tr>
<td>Table 2.3</td>
<td>LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 1 pitch configuration at $U_\infty = 6.21$ m/s.</td>
<td>30</td>
</tr>
<tr>
<td>Table 2.4</td>
<td>LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 1 pitch configuration at $U_\infty = 6.21$ m/s.</td>
<td>31</td>
</tr>
<tr>
<td>Table 2.5</td>
<td>LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 1 pitch configuration at $U_\infty = 6.51$ m/s.</td>
<td>31</td>
</tr>
<tr>
<td>Table 2.6</td>
<td>LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 1 pitch configuration at $U_\infty = 6.51$ m/s.</td>
<td>31</td>
</tr>
<tr>
<td>Table 2.7</td>
<td>LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 pitch configuration at $U_\infty = 6.21$ m/s.</td>
<td>33</td>
</tr>
<tr>
<td>Table 2.8</td>
<td>LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 pitch configuration at $U_\infty = 6.21$ m/s.</td>
<td>33</td>
</tr>
<tr>
<td>Table 2.9</td>
<td>LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 pitch configuration at $U_\infty = 6.51$ m/s.</td>
<td>33</td>
</tr>
<tr>
<td>Table 2.10</td>
<td>LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 pitch configuration at $U_\infty = 6.51$ m/s.</td>
<td>34</td>
</tr>
<tr>
<td>Table 2.11</td>
<td>LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.3 pitch configuration at $U_\infty = 6.21$ m/s.</td>
<td>35</td>
</tr>
<tr>
<td>Table 2.12</td>
<td>LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.3 pitch configuration at $U_\infty = 6.21$ m/s.</td>
<td>35</td>
</tr>
<tr>
<td>Table 2.13</td>
<td>LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.3 pitch configuration at $U_\infty = 6.51$ m/s.</td>
<td>36</td>
</tr>
<tr>
<td>Table 2.14</td>
<td>LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.3 pitch configuration at $U_\infty = 6.51$ m/s.</td>
<td>36</td>
</tr>
<tr>
<td>Table 3.1</td>
<td>Results from Parameter Subset Selection Algorithm with the quasi-global sensitivity matrix to determine least influential parameters of $M(\theta)$.</td>
<td>55</td>
</tr>
<tr>
<td>Table 3.2</td>
<td>Fixed structural parameter values.</td>
<td>56</td>
</tr>
<tr>
<td>Table 3.3</td>
<td>MAP estimates and credible intervals for the Config - 1 and Config - 2.</td>
<td>60</td>
</tr>
<tr>
<td>Table 4.1</td>
<td>Structural parameter values.</td>
<td>86</td>
</tr>
</tbody>
</table>
Table 4.2  MAP estimates and credible intervals for Bayesian parameter estimation of Config - 1 and Config - 2.  

87
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1</td>
<td>Typical aeroelastic section</td>
<td>2</td>
</tr>
<tr>
<td>Figure 2.1</td>
<td>Top view schematic of compound double pendulum aeroelastic device with coordinates defined. See figure 3 for an isometric view of the devices with components labeled.</td>
<td>9</td>
</tr>
<tr>
<td>Figure 2.2</td>
<td>Restoring moment in heave (a) and pitch (b) degrees of freedom, theory from Kirschmeier et al. [43].</td>
<td>11</td>
</tr>
<tr>
<td>Figure 2.3</td>
<td>(a) Schematic of tandem wing wind tunnel tests (b) Experiment in wind tunnel with hot-wire probe.</td>
<td>12</td>
</tr>
<tr>
<td>Figure 2.4</td>
<td>Total velocity signal decomposed into individual components (a) and convergence of phase-averaged velocity (b).</td>
<td>13</td>
</tr>
<tr>
<td>Figure 2.5</td>
<td>Aeroelastic modal frequency responses in freestream for (a) AW-1 and (b) AW-2 with various pitch stiffnesses.</td>
<td>15</td>
</tr>
<tr>
<td>Figure 2.6</td>
<td>Phase planes of AW-1 LCO at (a) $U_\infty = 6.21$ m/s and (b) $U_\infty = 6.51$ m/s. Shaded area represents limit cycle orbits plotted over entire testing period and black dashed line is the phase-averaged heave and pitch cycle.</td>
<td>16</td>
</tr>
<tr>
<td>Figure 2.7</td>
<td>(a) Instantaneous wing position and trace of instantaneous wing pivot location. (b) Scaled phase-averaged velocity profile 0.52c downstream of trailing edge of wing. $x$'s represent instantaneous trailing edge position while $o$'s represent instantaneous leading edge position. $U_\infty = 6.21$ m/s for both (a) and (b).</td>
<td>17</td>
</tr>
<tr>
<td>Figure 2.8</td>
<td>Spatially distributed frequency content of $&lt;U(t)&gt; / U_\infty$ at (a) $U_\infty = 6.21$ m/s and (b) $U_\infty = 6.51$ m/s. The hot-wire probe is located 0.52c downstream of trailing edge for both (a) and (b).</td>
<td>19</td>
</tr>
<tr>
<td>Figure 2.9</td>
<td>Phase-averaged velocity downstream of AW-1 with and without the pivot rod for AW-2 present.</td>
<td>20</td>
</tr>
<tr>
<td>Figure 2.10</td>
<td>(a) Time history of phase-average velocity for two streamwise locations $x/c = 0.52$ and 2.12. (b) Calculated convective speeds from both phase-averaged and turbulent velocity signals. $U_\infty = 6.21$ m/s for both (a) and (b).</td>
<td>22</td>
</tr>
<tr>
<td>Figure 2.11</td>
<td>Phase planes of AW-2 LCO at (a) $U_\infty = 7.04$ m/s and (b) $U_\infty = 7.30$ m/s. Shaded area represents limit cycle orbits plotted over the entire testing period and black dashed line is the phase-averaged heave and pitch cycle.</td>
<td>23</td>
</tr>
<tr>
<td>Figure 2.12</td>
<td>(a) Instantaneous wing position and trace of instantaneous wing pivot location. (b) Scaled phase-averaged velocity profile 0.89c downstream of trailing edge of wing. $x$’s represent instantaneous trailing edge position while $o$’s represent instantaneous leading edge position. $U_\infty = 7.04$ m/s for both (a) and (b).</td>
<td>24</td>
</tr>
</tbody>
</table>
Figure 2.13 Spatially distributed frequency content of $< U(t) > / U_{\infty}$ at (a) $U_{\infty}=7.04$ m/s and (b) $U_{\infty}=7.30$ m/s. The hot-wire probe is located 0.89c downstream of trailing edge.

Figure 2.14 Modal frequencies in tandem wing case for (a) initial deflections applied to AW-1, (b) initial deflections applied to AW-2, and (c) initial deflections applied to both devices. Pitch stiffness on AW-2 is 0.09 Nm/rad in all cases.

Figure 2.15 (a) Time traces of heave motion for initial deflection to AW-1 (top row), and initial deflection to AW-1 and AW-2 (bottom row). (b) Pitch response of AW-2 for each pitch stiffness for similar initial conditions.

Figure 2.16 (a) Phase planes of LCO for AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 1 pitch configuration. (b) Spatially distributed frequency content of $U(t)/ U_{\infty}$ 0.89c downstream of AW-2 for $k_{p_{AW-2}}/k_{p_{AW-1}}$ 1 pitch configuration. (a) and (b) at $U_{\infty}= 6.21$ m/s.

Figure 2.17 (a) Phase planes of LCO for AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 pitch configuration. (b) Spatially distributed frequency content of $U(t)/ U_{\infty}$ 0.89c downstream of AW-2 for $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 pitch configuration. (a) and (b) at $U_{\infty}= 6.21$ m/s.

Figure 2.18 (a) Phase planes of LCO for AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.3 pitch configuration. (b) Spatially distributed frequency content of $U(t)/ U_{\infty}$ 0.89c downstream of AW-2 for $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.3 pitch configuration. (a) and (b) at $U_{\infty}= 6.21$ m/s.

Figure 2.19 a) Instantaneous wing position and trace of instantaneous wing pivot location with $U_{\infty}= 6.21$ m/s for (a) AW-1, (b) AW-2 with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.1 (c) AW-2 with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 and (d) AW-2 with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.3.

Figure 3.1 Schematic of aeroelastic apparatus with relevant properties labeled and picture of aeroelastic wing in subsonic wind tunnel.

Figure 3.2 Spring torque measurements with load cell compared to Eqn. 4.3.

Figure 3.3 Joint densities and marginal posterior histograms for Config - 1.

Figure 3.4 Free decay comparison of experiment and model for (a) Config - 1 and (b) Config - 2.

Figure 3.5 Comparison of (a)$C_L$ and (b)$C_M$ from AEI method and AFRL prescribed motion measurements. Reynolds number for this case is 75,600. Note, ··· represent the 95% credible interval for each measurement.

Figure 3.6 $E(t)/E_0$ versus time for (a) Config - 1 and (b) Config - 2 for wind speeds tested up to LCO initiation. Pitch and heave response for wind speeds at and below LCO initiation for (c) Config - 1 and (d) Config - 2.
Figure 3.7  (a) Heave amplitude, (b) pitch amplitude, and (c) pitch-heave phase difference versus wind speed. Note error bars in amplitude plots represent the deviation in amplitude over the recorded time and not the measurement error. .................................................. 67

Figure 3.8  Lift coefficient and effective angle of attack versus cycle fraction for (a-c) Config - 1 at \( U_\infty = 7.2m/s, U_\infty = 10m/s \), and \( U_\infty = 12.1m/s \), respectively, and (d-f) Config - 2 at \( U_\infty = 7.2m/s, U_\infty = 10m/s \), and \( U_\infty = 12.1m/s \), respectively. \( A, A' \) represent the maximum and minimum lift while \( B, B' \) are when full lift stall occurs ([16]). Note, \( \cdots \) represent the 95% credible intervals. .................................................. 69

Figure 3.9  Moment coefficient and effective angle of attack versus cycle fraction for (a-c) Config - 1 at \( U_\infty = 7.2m/s, U_\infty = 10m/s \), and \( U_\infty = 12.1m/s \), respectively and (d-f) Config - 2 at \( U_\infty = 7.2m/s, U_\infty = 10m/s \), and \( U_\infty = 12.1m/s \), respectively. \( C, C' \) represent the maximum and minimum moment while \( D, D' \) are the corresponding moment when full lift stall occurs ([16]). Note, \( \cdots \) represent the 95% credible intervals. ............... 70

Figure 3.10  a) \( CL_{\text{max}} \) vs \( h/c \) for both mass coupling configurations. b) Dynamic \( C_L \) versus \( \theta \) compared to thin airfoil theory and static measurements performed by AFRL. c) Dynamic \( C_M \) versus \( \theta \) compared to static measurements performed by AFRL, measured at the half chord (only Config - 1 shown for clarity). Note, \( \cdots \) represent the 95% credible intervals. ............... 71

Figure 3.11  Aerodynamic energy for (a) Config - 1 and (b) Config - 2 and aerodynamic efficiency for (c) Config - 1 and (d) Config - 2 versus wind speed. Note, \( \cdots \) represent the 95% credible intervals. ............... 72

Figure 3.12  Power from aerodynamic lift and heave velocity versus cycle fraction for (a-c) Config - 1 at \( U_\infty = 7.2m/s, U_\infty = 10m/s \), and \( U_\infty = 12.1m/s \), respectively and (d-f) Config - 2 at \( U_\infty = 7.2m/s, U_\infty = 10m/s \), and \( U_\infty = 12.1m/s \), respectively. Note, \( A, A', B, B' \) are the same time stamps from Figure 3.8 and \( \cdots \) represent the 95% credible intervals. ............... 73

Figure 3.13  Power from aerodynamic moment and pitch velocity versus cycle fraction for (a-c) Config - 1 at \( U_\infty = 7.2m/s, U_\infty = 10m/s \), and \( U_\infty = 12.1m/s \), respectively and (d-f) Config - 2 at \( U_\infty = 7.2m/s, U_\infty = 10m/s \), and \( U_\infty = 12.1m/s \), respectively. Note, \( C, C', D, D' \) are the same time stamps from Figure 3.9 and \( \cdots \) represent the 95% credible intervals. .... 75

Figure 3.14  Coupling energy between the degrees of freedom for both configurations. Note, \( \cdots \) represent the 95% credible intervals. ............... 76

Figure 4.1  Schematic of aeroelastic apparatus with relevant properties labeled and picture of aeroelastic wing in subsonic wind tunnel. ............... 83

Figure 4.2  Free decay comparison of experiment with model for (a) Config - 1 and (b) Config - 2 ............... 87
Figure 4.3 (a) Pitch time histories, (b) pitch amplitude versus wind speed and (c) heave amplitude versus wind speed for wing LCO without the bluff body upstream. .......................................................... 90

Figure 4.4 a) Frequency spectrum of pitch response for five selected wind speeds without the bluff body present. b) Frequency spectrum of pitch response with the bluff body present for all wind speeds tested. $x_\theta = 0.078$ for each figure. .......................................................... 91

Figure 4.5 a) Pitch time histories, (b) pitch amplitude versus wind speed and (c) heave amplitude versus wind speed for wing LCO with the bluff body upstream. .......................................................... 93

Figure 4.6 Multiple LCO annihilations at $f_s/f_{LCO} = 2.92$ for $x_\theta = 0.078$, (a) first two seconds of motion and (b) full time history. ................................. 94

Figure 4.7 a) Kinematic recurrence of LCO annihilation event with the rest of the time signal. Regions of similarity are highlighted with blue markers. b) Kinematic and bluff body joint recurrence of LCO annihilation with the rest of the time signal. $x_\theta = 0.078$ and $f_s/f_{LCO} = 2.98$ for both figures. .............................. 98

Figure 4.8 Multi-trial cross recurrence for three different trials at $f_s/f_{LCO} = 2.98$ and $x_\theta = 0.078$. Corresponding regions of similarity between trials are highlighted with colored markers. ...................................................... 100

Figure 4.9 Continuous wavelet transforms for $x_\theta = 0.078$ at (a) $f_s/f_{LCO} = 2.72$, (b) $f_s/f_{LCO} = 2.98$, and (c) $f_s/f_{LCO} = 3.21$. .......................................................... 101

Figure 4.10 (a) Instantaneous frequency and (b) pitch heave frequency ratio versus time for $f_s/f_{LCO} = 2.98$ shown over the full time history. Instantaneous frequency and pitch-heave frequency ratio during (c) recoverable decay and (d) LCO annihilation. ................................................. 103

Figure 4.11 Coupling energy and pitch-heave difference versus time for $f_s/f_{LCO} = 2.98$ for the same trial during (a) recoverable amplitude decay and (b) LCO annihilation. Note, ••• represent the 95% credible intervals. ......... 105

Figure 4.12 Aerodynamic energy and pitch amplitude difference versus time for $f_s/f_{LCO} = 2.98$ for the same trial during amplitude (a) recoverable decay and (b) LCO annihilation. ......................................................... 106
**LIST OF SYMBOLS**

- \( \bar{U} \)  
  Mean velocity [m/s]

- \( \beta \)  
  fitting parameter for softening pitch stiffness model

- \( \langle U(t) \rangle \)  
  Phase-averaged velocity [m/s]

- \( \varepsilon \)  
  error

- \( \eta_{L,M} \)  
  aerodynamic efficiency in the heave and pitch degrees of freedom, respectively

- \( \lambda \)  
  eigenvalue

- \( \omega \)  
  Structural natural frequency [Hz]

- \( \phi_{\theta,h} \)  
  pitch-heave phase difference [°]

- \( \phi_{\theta_{1},\theta_{2}} \)  
  Pitch phase difference between AW-1 and AW-2 [°]

- \( \phi_{h,\theta} \)  
  Intra-wing heave-pitch phase difference [°]

- \( \phi_{h_{1},h_{2}} \)  
  Heave phase difference between AW-1 and AW-2 [°]

- \( \rho \)  
  freestream air density [kg/m³]

- \( \Theta \)  
  Heaviside function

- \( \theta, \theta_A \)  
  pitch displacement and pitch amplitude, respectively [°]

- \( \theta_1, \theta_2 \)  
  Heave angle displacement and wing angle relative B-axis, respectively [°]

- \( \theta_{P_1}, \theta_{P_2} \)  
  pitch transition angles [°]
\( \theta_p, \theta_{p_0} \)  pitch angle and pitch amplitude, respectively [°]

\( AR \)  wing aspect ratio

\( AW - 1, AW - 2 \)  upstream wing and downstream wing, respectively

\( b \)  airfoil semi-chord length [m]

\( c \)  airfoil chord length [m]

\( C_1, C_2, C_3 \)  fitting parameters for softening pitch stiffness model

\( c_h, c_p \)  viscous damping coefficient for heave and pitch, respectively [kg/s]

\( C_M, C_M \)  lift and pitching moment coefficients, respectively

\( D \)  bluff body characteristic length (short side) [m]

\( E_L \)  aerodynamic energy into the structure from the aerodynamic force [J]

\( E_M \)  aerodynamic energy into the structure from the aerodynamic moment [J]

\( E_{x\theta, \theta} \)  coupling energy into the heave degree of freedom [J]

\( E_{x\theta, h} \)  coupling energy into the heave degree of freedom [J]

\( f \)  frequency [Hz]

\( F_f \)  force due to kinetic friction in the heave degree of freedom (DOF) [N]

\( f_{h, LCO}, f_{\theta, LCO} \)  heave and pitch limit cycle oscillation frequency, respectively [Hz]

\( f_{LCO} \)  fundamental frequency of limit cycle oscillation [Hz]

\( f_{n, h}, f_{n, \theta} \)  wind-off heave and pitch natural frequency, respectively [Hz]
$f_{shed}$  bluff body shedding frequency [Hz]

$h$  heave displacement [m]

$h_0, h_A$  heave amplitude [m]

$I_\theta$  pitching inertia about elastic axis [kg m$^2$]

$K_h$  effective heave stiffness [N/m]

$K_{\theta_A}, K_{\theta_B}, K_{\theta_C}$  effective pitch stiffness for linear, hardening, and softening regimes, respectively [Nm/rad]

$KE$  kinetic energy [J]

$M_f$  moment due to kinetic friction in the pitch DOF [Nm]

$M_{k\theta}$  restoring moment due to structural pitch stiffness

$m_w, m_{total}$  wing mass (rotating components), and total mass (all moving components), respectively [kg]

$P$  power [W]

$P_L$  aerodynamic power into the structure from the aerodynamic force [J]

$P_M$  aerodynamic power into the structure from the aerodynamic moment [J]

$q$  parameter set

$S$  wing span [m]

$St$  Strouhal number
\( T \) oscillation period [s]

\( t \) time [s]

\( U \) potential energy [J]

\( u'(t) \) Turbulent velocity [m/s]

\( U_\infty \) freestream wind speed [m/s]

\( U_c \) Convective velocity [m/s]

\( X_b \) streamwise distance between bluff body trailing edge (TE) and wing leading edge (LE) [m]

\( x_p \) non-dimensional pitching axis location (chord length fraction from LE)

\( x_\theta \) non-dimensional distance (by semi-chord) from pitching axis to rotational center of mass [kg \cdot m^2]
Introduction to wing wake interactions

This dissertation investigates how incident vorticity affects the flow energy transfer to an aeroelastic wing with boundary layer separation. An oscillating wing or a bluff body are used to generate vorticity impinging on a downstream aeroelastic wing. A brief introduction to aeroelasticity and nonlinear aeroelastic studies will prepare the reader for the phenomena discussed in this dissertation.

1.1 Aeroelastic Behavior of a Wing or Airfoil

Aeroelastic structures are systems who response is governed by coupled structural, inertial, and fluid dynamic forces [8] and have been studied for more than 80 years. Theodorsen [99] was a pioneer of early aeroelastic research and developed some of the theoretical foundation required for aeroelastic analysis. Theodorsen’s analysis was concerned with developing stability criteria for airplanes to avoid flutter. Flutter is a dynamic instability of an elastic body, usually a wing, in a fluid stream [8]. At a certain fluid velocity, the system damping goes to zero, the system loses stability, and the structure begins to oscillate. The flutter instability and the oscillatory nature of the wing response are affected by numerous structural and aerodynamic effects. A
A canonical aeroelastic section is shown in Figure 1.1. Typical aeroelastic analysis focuses on an elastically restrained airfoil in heave ($y$-direction) and pitch ($\theta$) degrees of freedom, while the elastic axis, point $P$, is rigidly constrained in the $x$-direction. Additionally, the center of mass of the wing, point $C$, can be located at different point along the chord line from $P$. The heave motion of the wing is coupled to the pitching motion and vice versa when points $P$ and $C$ are not collocated. The placement of $P$ and $C$ relative to each other and $P$ relative to the wing aerodynamic center are important design parameters for the aeroelastician and influence the linear stability of the system. Additional parameters that affect the linear stability are the system damping, the heave and pitch natural frequencies and how close those frequencies are to each other. While aeroelastic stability is an important design parameter, modern aeroelastic research has focused on understanding the fluid structure interaction once the system loses linear stability.
The post-flutter response is concerned with studying the nonlinear limit-cycle response of the system. Limit-cycles are bounded oscillations in nonlinear systems on an isolated trajectory [96]. Depending on their amplitude and frequency, these limit cycles can range from mild sustained oscillations that can be exploited in aeroelastic energy harvesting devices [14] to dangerous oscillations that can cause structural fatigue failures in aircraft. Additionally, limit-cycles can exist above and below the flutter speed and their existence is dependent upon the nature of the bifurcation. In a subcritical hopf-bifurcation, a stable limit-cycle exists around a fixed point [96], or equilibrium. If an aeroelastic system has a subcritical hopf-bifurcation, the wing can grow to a limit-cycle if, when given a perturbation, that perturbation exceeds a certain threshold [96, 22, 92, 34, 91]. Subcritical hopf-bifurcations result in limit cycle oscillations (LCO) below the flutter speed. Additionally, a supercritical hopf-bifurcation can exist after the flutter speed. The supercritical hopf-bifurcation is characterized by the fixed point losing stability[46, 47, 55, 80]. Once in a LCO, the type of structural and aerodynamic nonlinearities present influences the amplitude, frequency, and phase kinematics.

1.2 Research Aims

Aeroelastic wing-wake interactions result in a plethora of aeroelastic responses due to the variation in freestream disturbances, nonlinear aerodynamic effects, and structural properties. How each of these characteristics influence the overall aeroelastic response requires further investigation and will be the focus of this dissertation. This dissertation is composed of experimental investigations to understand the underlying physics of aeroelastic wing-wake interactions and the development of an algorithm to compute the aerodynamic loads during wing limit cycle oscillations. The first study investigates tandem aeroelastic wing interactions, specifically focusing on how pitch stiffness on the downstream affects vortex energy transfer. The second
study develops an aeroelastic inverse technique to estimate the aerodynamic loads during limit cycle oscillations. Finally, the third study applies the aeroelastic inverse method to investigate limit cycle annihilation phenomena.
Experimental Investigation of Wake-Induced Aeroelastic Limit Cycle Oscillations in Tandem Wings

2.1 Introduction

This work is published in Journal of Fluids and Structures 2018
https://doi.org/10.1016/j.jfluidstructs.2018.04.015

Wing-wake interactions have been a topic of considerable interest recently due to the complex interactions between wings and incident vorticity. Common examples of wing wake interactions occur in insect flight, such as the dragon fly, fish swimming with tail and dorsal fin interaction, and energy harvesting applications. Interactions in each of these applications are determined by the shed vorticity from an upstream oscillating wing. The ability of the downstream wing to utilize incident vortex energy dictates the aeroelastic response. Historically,
research primarily focused on thrust producing mechanism, however the study presented here examines drag producing/energy harvesting mechanisms.

Numerous researchers investigated tandem wing-wake interactions in thrust generating systems including pure pitch oscillations[2, 9], pure plunge oscillations[28, 29, 68, 26], and combined pitch plunge motions (Akhtar et al., 2007 [12, 51, 86, 88]). For example, Gong et al.[28, 29] (2015, 2016) investigated tandem heaving wings, finding peak lift enhancement when a leading edge vortex (LEV) from the upstream wing interacts with an LEV on the downstream wing that has opposite rotation. For background information on LEVs, the reader is referred to [58, 59, 16, 25, 24, 62, 102, 17, 6, 19, 65, 66, 67, 63, 93, 49]. Broering et al. (2012) investigated combined pitching and plunging kinematics and found complex tradeoffs between LEV enhancement and destruction on total thrust and efficiency. LEV enhancement on the downstream wing resulted in a higher thrust, whereas LEV muddling resulted in higher resultant efficiencies. Their findings provided evidence that insects change their inter-wing phase difference depending on their operating flight regime. Each of the studies cited demonstrated strong dependence of the system thrust and efficiency on the wing spacing and inter-wing phase difference. This dependence occurs because the wing spacing and inter-wing phase difference dictate the timing of when shed vortex interacts with the downstream wing, thus affecting the aerodynamic state of the downstream wing.

While there have been numerous studies on tandem thrust producing oscillating wing wake interactions, few have investigated tandem wings as related to energy harvesting or aeroelastic limit cycle oscillations (LCO). Kinsey and Dumas [41] and Xu et al. [103] numerically investigated tandem oscillating wings undergoing prescribed, energy-extracting motions. Their research focused on the effects of spacing, oscillation frequency, and inter-wing phase difference on energy harvesting efficiency and aerodynamic power. Their results showed that a tandem wing system with a spacing of 5.4 chord lengths and optimized inter-wing phase difference
yielded nearly double the energy harvesting efficiency of a single wing operating in freestream. The separation distance is significantly different than traditional horizontal axis wind turbines which can require separation distances on the order of 25 rotor diameters to avoid deleterious interactions (Du Pont and Cagan [23]). Kinsey and Dumas [41] and Xu et al. [103] also found that the LEV shed by an upstream wing undergoing prescribed motion provided a vortex energy transfer mechanism, which can enhance or disrupt the LEV on a downstream wing. However, in a low aspect ratio highly tapered (backwards delta) aeroelastic wing, McCarthy et al. [57] found that a horseshoe cone vortex structure affected the downstream wing rather than a LEV. Bryant et al. [13] experimentally demonstrated that aeroelastic energy harvesters in tandem can produce more power together than if they were oscillating separately. The driving mechanism is this study was found from the flow deflection changes caused by the upstream wing and not shed LEVs. Therefore, several aerodynamic mechanisms exist that can induce a range of different aeroelastic LCOs in downstream devices.

The above literature in tandem wing aeroelastic energy harvesting systems investigated how spatial arrangement of aeroelastic devices change the wake energy transfer to a downstream wing. Additionally, prior studies have posited that for enhanced LCOs in downstream aeroelastic wings, the devices should have near identical aeroelastic behavior. However, few researchers have investigated how other structural parameters govern the aerodynamic coupling between the two devices and the wake energy transfer to the downstream wing. In this paper, we hypothesize that the pitch stiffness of the downstream wing plays an important role in the tandem wing response and can tune the energy transfer to the downstream wing. This hypothesis will be tested in wind tunnel experiments to determine how structural parameters govern the aerodynamic coupling between the shed wake and the downstream wing by varying the pitch stiffness on the downstream wing to be less than or equal to the pitch stiffness on the upstream
wing. The study is divided into three different focuses to analyze this hypothesis: aeroelastic stability, wake characterization, and LCO behavior.

Aeroelastic stability experiments are performed to characterize the pre-flutter behavior of each device as well as the relationship between pitch stiffness and flutter wind speed. Aeroelastic stability is tested for the tandem wing case as well to determine the instability mechanisms and transient response characteristic. Wake characterizations of single wing devices with similar pitch stiffness are conducted to determine the frequency content of the single wing wake as well as the type of wake structure present. The wake behind the downstream wing is also characterized in tandem configuration to determine how wake frequency content compares to single wing LCO wake. Additionally, steady state LCO behavior for both individual wing cases and tandem configurations is characterized and compared.
2.2 Experimental Methods

2.2.1 Experimental Apparatus

A pair of compound double pendulum aeroelastic apparatuses that rotate in heave ($h(t)$) and pitch ($\theta_p(t)$) degrees of freedom (Figure 1) are used to study tandem aeroelastic wing-wake interaction phenomena. A small surge velocity component is present due to the rotational heave degree of freedom kinematics considered. The two devices are referred to as aeroelastic wing one (AW-1) and aeroelastic wing two (AW-2), AW-1 is the upstream device in tandem configuration.

![Figure 2.1](image_url)  

**Figure 2.1** Top view schematic of compound double pendulum aeroelastic device with coordinates defined. See figure 3 for an isometric view of the devices with components labeled.
Both devices are constructed to have similar geometric and structural parameters including matched dimensions, center of gravity locations, wing masses, and wing pivot locations. Small differences arise from the amount of dissipative forces acting on each degree of freedom. The wing is a standard NACA 0012 airfoil profile across the entire span, $s$, with chord, $c$, equal to 0.1m, and span equal to 0.3m, giving a wing aspect ratio of 3. The wing is constructed from 3D printed plastic that has spanwise steel rods inside to move the center of gravity aft of the pivot the location. The wing pivot location is one-third chord length behind the leading edge of the airfoil, the wing mass, heave, and pitch inertia are 0.13 kg, 0.01 kgm$^2$, and 0.00013 kgm$^2$, see Kirschmeier et al. [43] for other wing details. The pitch degree of freedom is passively controlled through linear extension springs that are attached a distance 12.7mm away from the pivot axis to create a restoring moment. The heave degree of freedom also uses linear extension springs attached a distance 35mm away from the pivot location to create a heave restoring moment. The wing is mounted between two parallel aluminum arms that extend from the heave pivot rod out a distance of 200mm. The inertia of the heave degree of freedom, and consequently the natural frequency, can be modified by varying the lengths of the aluminum arms. The heave pivot rod extends the full span of the wind tunnel, with the bottom of the rod attached to the heave springs and the top portion being free to rotate but supported by ball bearings. A US Digital E6 3600 optical encoder with 0.11 degree uncertainty connected to the upper portion of heave pivot rod monitors the heave rotation. Another US Digital E6 3600 encoder is mounted beneath the lower wing assembly and records the relative angle between heave and pitch motions with 0.16 degree uncertainty.

The geometrically nonlinear stiffness characteristics of each degree of freedom in the system were measured over a range of deflections. The structural dynamics, derived in Kirschmeier et al. [43], show nonlinear coupling between the heave and pitch motions. The heave springs show a linear response for heave amplitudes up to $+ - 0.35c$ then undergo a hardening non-
linearity (Figure 2a). The pitch springs show a linear response for $+ - 35^\circ$ deflections, (Figure 2b).

**Figure 2.2** Restoring moment in heave (a) and pitch (b) degrees of freedom, theory from Kirschmeier et al. [43].

Aeroelastic experiments were performed in the NCSU closed return subsonic wind tunnel with a 0.81x1.14x1.17m$^3$ test section and maximum wind speed around 40m/s. The operating free stream velocity varied from 6m/s to 7.5m/s for LCO experiments, and 3 m/s to 10 m/s for modal frequency characterizations. The chord based Reynolds number for LCO tests was 40,000.

A Dantec miniature-wire straight CTA probe connected to a Dantec MiniCTA unit measured the velocity profile downstream of the mid-span of the oscillating wings to capture the dominant dynamics in the wake (Parker et al. [71]). For single wing tests of AW-1, the hot-wire probe was placed 0.52 and 2.12 chord lengths downstream of the trailing edge. For single wing tests of AW-2 as well as tandem tests, the probe location was 0.89 chord lengths downstream
of the trailing edge of AW-2. The probe traversed a distance of 4 chord lengths across the flow (y-direction) with a 0.16 chord step size for single wing testing, while for tandem configurations, the traversed distance was 5 chord lengths across the flow with a 0.22 chord step size. A Velmex Unislide series B6000 stepper motor with 0.00635mm resolution controlled the position of the probe. Figure 3 shows the experimental setup installed in the wind tunnel for tandem testing.

![Figure 2.3](image)

**Figure 2.3** (a) Schematic of tandem wing wind tunnel tests (b) Experiment in wind tunnel with hot-wire probe.

### 2.2.2 Data Acquisition and Reduction

Hot-wire anemometer voltage and encoder signal measurements were taken simultaneously for one minute at each y/c measurement location. A National Instruments PXIe 6363 card recorded data at a rate of 5kHz. A Keysight 335000 waveform function generator triggered the data collection. The LCO oscillation frequency for both wings was nominally 5Hz, this resulted in approximately 300 cycles for phase-averaging. A temporal resolution of $1 \times 10^{-3}$ seconds for
each cycle resulted from the 5kHz sampling rate, proving useful for phase-averaging. Cross correlation techniques calculated vortex convection speeds \((U_c)\) as well as intra pitch-heave \((\phi_{h,\theta_p})\), inter-wing heave-heave \((\phi_{h1, h2})\), and pitch-pitch phase differences \((\phi_{\theta p1, \theta p2})\). The coordinate system used in Figure 4.1 is used to define the intra and inter-wing phase differences.

The phase-averaging technique decomposed the velocity signal into three different components: mean \((\bar{U})\), phase-averaged \(<U(t)\rangle\), and turbulent velocity components \((u'(t))\) (Hussain and Reynolds [36]) (Figure 2.4(a)). The phase-averaged component required averaging the measured velocity at a given cycle fraction across all oscillation cycles. The encoder signal defined peak-to-peak oscillation cycles. The phase-averaged velocity error converged to 1e-3 for each \(y/c\) probe location in under 240 cycles, therefore the one minute recording time is sufficient to reconstruct the velocity information.

![Figure 2.4](image_url)  
**Figure 2.4** Total velocity signal decomposed into individual components (a) and convergence of phase-averaged velocity (b).
2.3 Results and Discussion

2.3.1 Aeroelastic Response of a Single Wing

2.3.1.1 Modal Frequency Behavior

Aeroelastic stability experiments determined the modal frequencies as functions of wind speed for each of the devices in freestream flow. These single-wing results highlight effects from incident wind speed and allow for isolating differences when in tandem configuration. In each test, the wing was deflected in the heave degree of freedom to $h/c = 0.2$ and consequently in the pitch degree of freedom to $\theta_p = 6$ deg, and then released. The frequencies of the resulting oscillations were recorded and segregated into heave and pitch degrees of freedom. From these experiments, both AW-1 and AW-2 were found to exhibit a modal convergence type response with increasing wind speed (Figure 2.5(a,b)). It should be noted that no self-excited oscillations were observed in this velocity range, rather the initial perturbations described above were required to initiate LCO at the point of modal convergence. Therefore, the wing response to these perturbations is characteristic of a sub-critical Hopf bifurcation. We also note that even for the case of equal pitch stiffness, AW-1 and AW-2 had slightly different flutter wind speeds due to differences in damping and wind-off heave natural frequency. Other pitch stiffnesses tested on the downstream wing were $K_p = 0.046$ Nm/rad and 0.03 Nm/rad. Figure 2.5(b) shows that, as expected, the flutter speed increased with reducing pitch stiffness because the zero-wind speed modal frequencies are spaced further apart. The error bars in the plot represent the modal frequency resolution. The resolution improves with flow speed because as wind speed increases the modal damping decreases, and therefore the number of oscillation cycles and consequently sample time increases as well.
2.3.1.2 AW-1 Limit Cycle behavior for $K_p = 0.09 Nm/rad$

The limit cycle behavior of the individual wings was studied to explore how LCO characteristics evolved with incident wind speed. The resulting limit cycle of AW-1 is consistent during the entire test, as shown in Figure 2.6. The resultant LCO produced large displacements well above static stall angles (Table 2.1) thus providing initial indications that nonlinear aerodynamic phenomena are present. $h_0$ and $\theta_{p_0}$ refer to the heave and pitch oscillation amplitude where $f$ is the oscillation frequency. Heave displacements do not extend in the nonlinear restoring force range (Figure 2.2) therefore nonlinear pitching moment and aerodynamic phenomena must be affecting the wing for limit cycles to be present. The limit cycle shows small increases in heave amplitude while the pitch amplitude and intra-wing pitch heave phase difference remain constant within the wind speed range tested. These indicate that the shape of the limit cycle is relatively insensitive over the range of incident flow speeds tested.
Table 2.1 Limit cycle oscillation characteristics for AW-1.

<table>
<thead>
<tr>
<th>$U_\infty$ [m/s]</th>
<th>$h_0/c$</th>
<th>$\theta_{p0}$</th>
<th>$\phi_{h,\theta_p}$</th>
<th>$f_c/U_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.21</td>
<td>0.26 ± 0.01</td>
<td>42.67 ± 2.59</td>
<td>-165.40 ± 0.78</td>
<td>0.08</td>
</tr>
<tr>
<td>6.51</td>
<td>0.29 ± 0.02</td>
<td>42.58 ± 1.59</td>
<td>-165.42 ± 0.27</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Figure 2.6 Phase planes of AW-1 LCO at (a) $U_\infty = 6.21$ m/s and (b) $U_\infty = 6.51$ m/s. Shaded area represents limit cycle orbits plotted over entire testing period and black dashed line is the phase-averaged heave and pitch cycle.

The hot-wire anemometer was traversed $0.52c$ downstream of the oscillating wing allowing for insight into the type of wake structure shed from the wing, as well as the flow over the wing. The phase-averaged velocity profiles, Figure 2.7, depict two characteristic velocity deficit regions occurring around $t/T = 0.2$ and 0.7. These regions are indicative of a vortex structure being convected past the anemometer probe. Vortex rotation direction can be inferred from the phased average velocity plots by examining $dU/dy$ to see the direction of the vortex (Sullivan...
and Pollard [98]). Since only a single direction velocity probe is used; true calculations of vorticity are not possible, and only directional information can be determined. In Figure 2.7(b) a vortex with sign \( -k \), as defined by Figure 4.1(a), is seen near \( t/T = 0.2 \), whereas as vortex with sign \( +k \) is seen near \( t/T = 0.7 \). The wake pattern corresponds to a drag-producing wake, as expected for a passively controlled oscillating wing in free stream flow. The vortex structures at \( t/T = 0.2 \) and \( 0.7 \) are derived from large leading edge vorticity, which can be determined by examining the instantaneous wing position (Figure 2.7a) accounting for convective speed. Therefore, LEV’s provide nonlinear aerodynamic phenomena to help facilitate the limit cycle oscillation.

![Figure 2.7](image)

**Figure 2.7** (a) Instantaneous wing position and trace of instantaneous wing pivot location. (b) Scaled phase-averaged velocity profile 0.52c downstream of trailing edge of wing. x’s represent instantaneous trailing edge position while o’s represent instantaneous leading edge position. \( U_\infty =6.21 \text{ m/s} \) for both (a) and (b).
Analysis of the spatially distributed frequency content of the phase-averaged velocity reveals two characteristic regions, an inner wake region under the influence of the shed vortices, and an outer wake region under the effect of either a single vortex structure or air deflected by the wing. The inner wake dominant frequency is a harmonic of the oscillation frequency and can clearly be seen by the spike at $2f_{LCO}$. As noted in Jung and Park ([37]) the x-component of the hot-wire velocity produces a frequency doubling effect for wake vortices. Therefore, the wake vortex forcing frequency is equivalent to $1f_{LCO}$. Wake frequency content is also distributed to higher harmonics with significant frequency content decaying by $10f_{LCO}$. Young and Lai [105, 104] demonstrated that as oscillation amplitude grew larger the frequency content of the wake dropped to match the oscillation frequency of the wing. This is because significant separation occurred over the wing and leading edge vortices were produced. Therefore, the wake frequency content predicts vortex structure growth and separation over the wing.

The inner and outer wake regions change width with increasing wind speed respectively due to the larger swept area of the wing oscillation at higher wind speed. The wake width estimated by the transition from $2f_{LCO}$ to $1f_{LCO}$ dominant frequency is $+ - 1.12c$, while wake width estimated using the half width method based on the mean velocity profile is $+ - 1.09c$ for $U_\infty = 6.21m/s$. The wake width estimates for both are similar in magnitude and will be used to discuss tandem wing behavior.
Figure 2.8 Spatially distributed frequency content of $< U(t) > / U_\infty$ at (a) $U_\infty=6.21 \text{ m/s}$ and (b) $U_\infty=6.51 \text{ m/s}$. The hot-wire probe is located 0.52$c$ downstream of trailing edge for both (a) and (b).

Periodic vortex shedding from the heave pivot rod exists for all wind speeds tested. Measurements of the pivot rod wake were conducted with the hot-wire anemometer. Measurements were taken at $x/d = 22$, which would correspond to the 10mm behind the wing leading edge if the wing was present. The measured shedding frequency was $35f_{LCO}$ with a wake width of $+ - 0.5c$. The measured centerline velocity is $0.8U_\infty$. This indicates that the wing oscillates through a shear flow during LCO. However, during wing LCO, spectral analysis of the hot-wire anemometer, heave deflection, and pitch deflection signals show no peak at $35f_{LCO}$. Therefore, the vortex shedding frequency from the pivot rod has negligible effects on the wing.
LCO, such that the wing either destroys or significantly disperses the vortices such that the overall wake flow is not appreciably affected. Additionally, measurements were taken of the single wing wake with the downstream rod present to determine what effect, if any, there was on the measured wake response due to the downstream rod. As shown in Figure 2.9, the phase averaged velocity profiles $x/c = 1.9$ downstream do not appreciably change with the presence of the rod downstream nor do the measured wake statistics such as wake width and vortex size. Thus, vortices forming over the wing dominate the response.

**Figure 2.9** Phase-averaged velocity downstream of AW-1 with and without the pivot rod for AW-2 present.

### 2.3.1.3 Convective Time Scales and Vortex Size

Wake vortex convection speed fundamentally influences the wing-wake interaction and the response of a downstream wing. Vortex structure convective speed is calculated by cross correlating the velocity signal measured at two streamwise probe positions. Convective speeds were determined with both the phase-averaged velocity and the turbulent velocity components mea-
sured 0.52c and 2.12c downstream of the oscillating AW-1. Representative phased-average velocity signals for $y/c = 0.48$ are plotted in Figure 2.10(a), and demonstrate a time shift between signals of 0.05s. Convective velocity was calculated using this time shift and the distance between the two sample locations (Figure 2.10b) which is equivalent to $0.5U_\infty$ for $y/c = + - 0.5c$. Carr et al. [16] found the vortex convection speed in two-dimensional pitching wings significantly differs from free stream velocity, with estimated convection speeds around $0.4U_\infty$. Park et al. [70] investigated vortex convection speeds for a lightly stalled two-dimensional oscillating wing, discovering vortex wake convection speed was a constant $0.6U_\infty$. Conversely, Rival et al. [87] showed that LEV convection speed asymptotically approached free stream velocity as the vortex convected downstream. From Rival et al.’s [87] experiments, the vortex convection speed for similar streamwise locations is around $0.5U_\infty$, therefore measurements are comparable with their previous two-dimensional experimental counterparts. Vortex convection speeds calculated from the phase-averaged and turbulent velocity signals begin to differ as the measurement locations move further away from $y/c = 0$. Differences occur because the phase-averaged velocity is sensitive to the transition between inner and outer wake regions causing inaccurate measurements of convective speeds. Convective speeds will be used to analyze vortex interactions in tandem wing experiments.
Figure 2.10 (a) Time history of phase-average velocity for two streamwise locations $x/c = 0.52$ and 2.12. (b) Calculated convective speeds from both phased-averaged and turbulent velocity signals. $U_\infty = 6.21 \text{ m/s}$ for both (a) and (b).

Estimates of mid-plane vortex size are also possible from the turbulent hot-wire signal. Autocorrelation performed on the turbulent velocity signal finds the typical time required for the energy containing eddies (Bradshaw [11]) to traverse past the hot wire probe at a single streamwise location. A time scale is calculated for each oscillation cycle, then averaged over the entire measurement time for an estimate of the mid-plane vortex size time scale at each $y/c$ location. Measurements from the inner region of the wake are averaged together to find an estimated time scale of the vortex. Mid-plane vortex structure size is estimated as the product of the vortex size time scale and convective speed, which results in a length scale of $0.9c$.

2.3.1.4 AW-2 LCO characteristics for $K_p = 0.09Nm/rad$

AW-2 with $K_p = 0.09Nm/rad$ was tested in similar fashion to AW-1 to quantify its freestream LCO behavior and wake characteristics. The LCO results provide a baseline for comparing the response of AW-2 in freestream to the tandem case. LCO behavior for AW-2 is similar to that...
of AW-1 but with slightly higher oscillation amplitudes, due to the higher operating flow speed range. Non-dimensional frequencies and intra-wing phase differences are similar between AW-2 and AW-1; therefore, similar vortex formation can be expected over the wing. In this case, a combination of nonlinear restoring forces and nonlinear aerodynamics dictate the response.

![Figure 2.11](image-url) Phase planes of AW-2 LCO at (a) $U_\infty = 7.04$ m/s and (b) $U_\infty = 7.30$ m/s. Shaded area represents limit cycle orbits plotted over the entire testing period and black dashed line is the phase-averaged heave and pitch cycle.

Similar to AW-1, the passing of two large-scale vortex wakes affect the phase-averaged velocity profile (Figure 2.12). These vortices form from the leading edge of the wing and convect downstream past the anemometer probe, with vortex events occurring around $t/T = 0.2$ and 0.7. Despite AW-2 having a higher flutter wind speed and kinematic displacement, the overall shape and structure of the wake is similar to that of AW-1, thus they are influenced by similar aerodynamic nonlinearities.
Figure 2.12 (a) Instantaneous wing position and trace of instantaneous wing pivot location. (b) Scaled phase-averaged velocity profile 0.89c downstream of trailing edge of wing. x’s represent instantaneous trailing edge position while o’s represent instantaneous leading edge position. $U_\infty=7.04$ m/s for both (a) and (b).

The frequency content of the wake shows a similar trend to that of AW-1 with an inner wake region dominated by the $2f_{LCO}$ wake vortices and an outer wake region at $1f_{LCO}$. The inner wake region width is greater for AW-2 due to the larger swept area of the wing, and owing to the measurement location further downstream of the wing. This wake region is similar to that of AW-1 indicating similar aerodynamic effects for both wings in both cases.
Figure 2.13 Spatially distributed frequency content of $< U(t) > / U_\infty$ at (a) $U_\infty = 7.04$ m/s and (b) $U_\infty = 7.30$ m/s. The hot-wire probe is located $0.89c$ downstream of trailing edge.

Table 2.2 Limit cycle oscillation characteristics for AW-1.

<table>
<thead>
<tr>
<th>$U_\infty$ [m/s]</th>
<th>$h_0/c$</th>
<th>$\theta_{p0}$</th>
<th>$\phi_{h,\theta}$</th>
<th>$f c / U_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.21</td>
<td>0.26 ± 0.01</td>
<td>42.67 ± 2.59</td>
<td>-165.40 ± 0.78</td>
<td>0.08</td>
</tr>
<tr>
<td>6.51</td>
<td>0.29 ± 0.02</td>
<td>42.58 ± 1.59</td>
<td>-165.42 ± 0.27</td>
<td>0.08</td>
</tr>
</tbody>
</table>
2.3.2 Tandem Wing Aeroelastic Interactions

To investigate the global effect of tandem wing-wake vortex energy transfer, the problem is broken into two different regimes, flutter characterizations and limit cycle behavior. Experimental flutter characterization and transient response experiments are useful to determine the forcing mechanism on the downstream wing and how the aeroelastic stability of the downstream wing is affected by the wake of the upstream wing. Additionally, limit cycle behavior defines how pitch stiffness affects the energy transfer from the wake to the downstream wing after steady oscillation is established. In tandem configuration, AW-2’s leading edge is located 179mm downstream of AW-1’s trailing edge.

2.3.2.1 Modal Frequency Behavior and Transient Response

Three different initial conditions were tested to investigate the wake vortices effect on the stability and response of the downstream wing. The different conditions were: 1) initial heave deflection applied to the upstream device, 2) initial heave deflection applied to the downstream device, and 3) initial heave deflection applied to both devices. Figure 2.14 demonstrates that the choice of initial conditions influences the frequency behavior of the system. In the first set of initial conditions, when the upstream device only is deflected, the downstream wing exhibits oscillations in both degrees of freedom near the heave natural frequency of the upstream device for all nonzero wind speeds. The converged modal frequencies in the downstream wing demonstrate that the heave motion of the upstream device dictates the motion of AW-2 via its oscillating wake. In the second set of initial conditions, when the downstream wing only is initially deflected, the downstream wing modal frequencies are similar to those observed in single wing experiments, thus negligible upstream aerodynamic coupling exists because no oscillations are induced in the upstream wing. Furthermore, this initial condition case shows that the
vortex shedding from the static wing upstream does not induce oscillations in the downstream wing. The third set of initial conditions shows distinct modes in the downstream wing for the first nonzero wind speed tested, but then converged modes for all other wind speeds tested, as the wake from the upstream device dominates the response of AW-2.

**Figure 2.14** Modal frequencies in tandem wing case for (a) initial deflections applied to AW-1, (b) initial deflections applied to AW-2, and (c) initial deflections applied to both devices. Pitch stiffness on AW-2 is 0.09 Nm/rad in all cases.
These results demonstrate that the downstream wing is forced to move due to the unsteady wake of the oscillating upstream device. Therefore, the stability of the downstream wing is dependent on the stability of the upstream device, aerodynamically coupling the two devices in one direction. In the first and third initial condition cases tested, the wake of the upstream device effectively acts as an external forcing function applied to the downstream wing. Thus, there exists an inherent frequency-lock between the devices. Notably, although the single wing tests revealed that the pitch stiffness strongly affected the modal convergence behavior of AW-2 when operating in freestream, the same trends in Figure 2.14 were observed for all pitch stiffnesses tested.

While the modal convergence plots are useful in determining the frequency content of the structural motion, the transient response provides insight into the uniqueness of LCOs. The downstream wing LCO response showed independence to the initial condition, and settled into the same inter-wing heave-heave phase difference relative to the upstream wing for both case 1 and 3 initial conditions (Figure 2.15(a)). Independence to initial condition indicates that there exists a unique LCO for the downstream wing when in the unsteady wake of the upstream wing. The existence of only one limit cycle is shown in Figure 2.15(a) where two distinct inter wing phase differences are quickly changed within 8 flapping cycles to a phase difference much closer to the final LCO inter wing phase difference. Additionally, the response time of the downstream wing changes with pitch stiffness for cases 1 and 3 (Figure 2.15(b)). Lower pitch stiffness values respond quicker to incident vorticity, while higher pitch stiffnesses respond more slowly. Under attached flow theory, this has a compounding effect due to the location of the pivot axis, whereby increases in effective angle of attack increase lift, which tends to rotate the wing to a larger angle of attack. With higher pitch stiffness, a larger flow angle is required to deflect the wing, and therefore, in the transient response, a stiffer downstream wing responds more slowly to the growing oscillations and wake strength of the upstream wing. Therefore,
wing pitch stiffness introduces an additional lag in response to incoming vortical wakes and defines wing susceptibility to vortical disturbances.

Figure 2.15 (a) Time traces of heave motion for initial deflection to AW-1 (top row), and initial deflection to AW-1 and AW-2 (bottom row). (b) Pitch response of AW-2 for each pitch stiffness for similar initial conditions.

2.3.2.2 Tandem LCO Behavior

Steady LCO behavior of the tandem aeroelastic wings was tested for each of the three downstream wing pitch stiffnesses to examine pitch stiffness effects on vortex energy transfer. The wake of the oscillating upstream wing destabilizes the downstream wing in every case tested, exciting sustained oscillations in the downstream wing even though the wind speed is below the freestream flutter speed of the downstream wing. However, due to differences in pitch stiffness, the system exhibits either a high vortex energy transfer (HVET) regime resulting in a large LCO and a high maximum kinetic energy or a low vortex energy transfer (LVET) regime resulting in significantly smaller LCOs and lower maximum kinetic energy in the structure. Experiments when both wings have similar pitch natural frequencies produce a LVET interaction.
in the downstream wing (Figure 2.16(a)). The resultant heave amplitude is 80% less than the heave amplitude of the upstream wing for \( U_\infty = 6.21 \text{ m/s} \) and 64% less for \( U_\infty = 6.51 \text{ m/s} \) (Table 2.3-2.6)) demonstrating weak vortex energy transfer. From the previous wake-width estimates, the upstream wing shed vortex wake fully envelopes the downstream wing oscillation cycle. The previous vortex size estimates also indicate that during these interactions, the downstream wing is under the influence of one vortex structure at a given time. In these wind speed ranges, the intra-wing pitch-heave phase difference in AW-2 shows a change of 12% between single and tandem wing tests as a result of the incident wake. Inter-wing heave-heave phase difference shows that the stable LCO for the downstream wing trends towards out of phase motion with increasing wind speed. During these LVET interactions, there exists little backward aerodynamic coupling from the downstream device on the upstream device even in LCO conditions.

This is exhibited by the similar LCOs achieved in AW-1 during tandem as compared to single wing testing. Ristroph et al. [85] postulated that when two flapping flags interact in a destructive mode, the tandem wake pattern is less structured as compared to the single wing wake. This can be seen in the spatially distributed frequency plot of the phase-averaged velocity at \( U_\infty = 6.21 \text{ m/s} \), which now has dominant frequency components in the \( 1f_{LCO} \) range compared to the single wing frequency plot. The dominant frequency at \( 1f_{LCO} \) characterizes that the flow downstream of the wing is dominated by the deflection of both of the wings and muddles inner and outer wake regions defined by the spatially distributed frequency.

<table>
<thead>
<tr>
<th>( U_\infty ) [m/s]</th>
<th>( h_0/c )</th>
<th>( \theta_{p0} )</th>
<th>( fc/U_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1 6.21</td>
<td>0.25 ±0.02</td>
<td>41.46±2.81</td>
<td>0.08</td>
</tr>
<tr>
<td>AW-2 6.21</td>
<td>0.05 ±0.01</td>
<td>24.37±1.67</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3 LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with \( k_{p_{w-2}}/k_{p_{w-1}} \) 1 pitch configuration at \( U_\infty = 6.21 \text{ m/s} \).
Table 2.4 LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{aw-2}}/k_{p_{aw-1}}$ pitch configuration at $U_\infty = 6.21$ m/s.

<table>
<thead>
<tr>
<th></th>
<th>$U_\infty$ [m/s]</th>
<th>$\phi_{h, \theta_p}$</th>
<th>$\phi_{h_1, h_2}$ [°]</th>
<th>$\phi_{\theta_p, 1, \theta_p, 2}$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.21</td>
<td>-163.29±0.58</td>
<td>106.11</td>
<td>129.31</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>-140.12±5.67</td>
<td>± 4.46</td>
<td>± 1.37</td>
</tr>
</tbody>
</table>

Table 2.5 LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{aw-2}}/k_{p_{aw-1}}$ pitch configuration at $U_\infty = 6.51$ m/s.

<table>
<thead>
<tr>
<th></th>
<th>$U_\infty$ [m/s]</th>
<th>$h_0/c$</th>
<th>$\theta_p$</th>
<th>$f c/ U_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.51</td>
<td>0.25 ±0.02</td>
<td>41.22 ±2.10</td>
<td>0.08</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>0.09 ±0.01</td>
<td>30.13 ±1.76</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.6 LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{aw-2}}/k_{p_{aw-1}}$ pitch configuration at $U_\infty = 6.51$ m/s.

<table>
<thead>
<tr>
<th></th>
<th>$U_\infty$ [m/s]</th>
<th>$\phi_{h, \theta_p}$</th>
<th>$\phi_{h_1, h_2}$ [°]</th>
<th>$\phi_{\theta_p, 1, \theta_p, 2}$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.51</td>
<td>-165.01±0.58</td>
<td>119.46</td>
<td>128.21</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>-154.42±1.52</td>
<td>± 3.10</td>
<td>± 0.84</td>
</tr>
</tbody>
</table>
When the downstream wing pitch stiffness is approximately half that of the upstream wing, a HVET interaction exists for AW-2 and large limit cycle oscillations exist in the downstream wing for the range of wind speeds tested (Table 2.7-2.10). The resulting AW-2 oscillations are characterized by having similar heave amplitudes to that of AW-1 (Figure 2.17(a)). Furthermore, intra-wing pitch-heave phase difference in the downstream wing is almost identical to the intra-wing phase difference on the upstream wing. Pitch amplitude on the downstream wing is 25% less than that of the upstream wing, indicating that pitch amplitude is not a sufficient proxy for aerodynamic force; heave structural properties of AW-1 and AW-2 are similar, thus, similar aerodynamic force magnitudes must exist on both wings to produce the same heave

Figure 2.16 (a) Phase planes of LCO for AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 1 pitch configuration. (b) Spatially distributed frequency content of $U(t)/U_\infty$ 0.89c downstream of AW-2 for $k_{p_{AW-2}}/k_{p_{AW-1}}$ 1 pitch configuration. (a) and (b) at $U_\infty = 6.21$ m/s.
amplitude. Varying pitch amplitudes introduces ambiguity into aerodynamic forces because it presents a departure from classical single wing experimental results of Carr et al. [16].

Table 2.7 LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with \( k_{p_{aw-2}}/k_{p_{aw-1}} \) 0.5 pitch configuration at \( U_\infty = 6.21 \) m/s.

<table>
<thead>
<tr>
<th></th>
<th>( U_\infty ) [m/s]</th>
<th>( h_0/c )</th>
<th>( \theta_{p_0} )</th>
<th>( f_c/U_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.21</td>
<td>0.26 ±0.01</td>
<td>45.34±2.10</td>
<td>0.08</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>0.28 ±0.02</td>
<td>33.40 ±1.61</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.8 LCO phase characteristics of AW-1 and AW-2 in tandem configuration with \( k_{p_{aw-2}}/k_{p_{aw-1}} \) 0.5 pitch configuration at \( U_\infty = 6.21 \) m/s.

<table>
<thead>
<tr>
<th></th>
<th>( U_\infty ) [m/s]</th>
<th>( \phi_{h,\theta_p} )</th>
<th>( \phi_{h_1,h_2} ) [°]</th>
<th>( \phi_{\theta_{p,1},\theta_{p,2}} ) [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.21</td>
<td>-167±0.52</td>
<td>45.48</td>
<td>46.85</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>-166.24±0.77</td>
<td>± 4.19</td>
<td>± 4.86</td>
</tr>
</tbody>
</table>

Table 2.9 LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with \( k_{p_{aw-2}}/k_{p_{aw-1}} \) 0.5 pitch configuration at \( U_\infty = 6.51 \) m/s.

<table>
<thead>
<tr>
<th></th>
<th>( U_\infty ) [m/s]</th>
<th>( h_0/c )</th>
<th>( \theta_{p_0} )</th>
<th>( f_c/U_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.51</td>
<td>0.26 ±0.02</td>
<td>41.22±2.10</td>
<td>0.08</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>0.28 ±0.01</td>
<td>34.50 ±1.48</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.10 LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 pitch configuration at $U_\infty = 6.51$ m/s.

<table>
<thead>
<tr>
<th></th>
<th>$U_\infty$ [m/s]</th>
<th>$\phi_{\theta_1, \theta_p}$</th>
<th>$\phi_{\theta_1, \theta_2}$ [°]</th>
<th>$\phi_{\theta_1, \theta_{p_1, \theta_2}}$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.51</td>
<td>-164.74±0.42</td>
<td>85.96±</td>
<td>82.99</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>-167.64±0.71</td>
<td>1.07</td>
<td>± 0.59</td>
</tr>
</tbody>
</table>

Figure 2.17 (a) Phase planes of LCO for AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 pitch configuration. (b) Spatially distributed frequency content of $U(t)/U_\infty$ 0.89c downstream of AW-2 for $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.5 pitch configuration. (a) and (b) at $U_\infty = 6.21$ m/s.

In the case where downstream wing pitch stiffness is roughly one third that of the upstream wing pitch stiffness, heave amplitudes greater than or equal to the upstream wing heave am-
plitude are achieved over the range of wind speeds test as seen in Figure 2.18(a) and Table 2.11-2.14. Intra-wing phase difference of AW-2 is slightly higher than AW-1, and is closer to in phase pitch-heave oscillations. The pitch amplitude in the downstream wing is 50% less than the upstream wing for $U_\infty = 6.21\,\text{m/s}$, a significant departure from the previous HVET oscillation case. Again, the incident vortices must be affecting the flow over the airfoil to generate the required aerodynamic forces to oscillate the wing with a large heave amplitude.

**Table 2.11** LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.3 pitch configuration at $U_\infty = 6.21\,\text{m/s}$.

<table>
<thead>
<tr>
<th></th>
<th>$U_\infty$ [m/s]</th>
<th>$h_0/c$</th>
<th>$\theta_{p0}$</th>
<th>$f_c/U_{\infty}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.21</td>
<td>0.27 ± 0.02</td>
<td>45.74 ± 2.06</td>
<td>0.08</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>0.27 ± 0.02</td>
<td>23.87 ± 1.83</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2.12** LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p_{AW-2}}/k_{p_{AW-1}}$ 0.3 pitch configuration at $U_\infty = 6.21\,\text{m/s}$.

<table>
<thead>
<tr>
<th></th>
<th>$U_\infty$ [m/s]</th>
<th>$\phi_{h,\theta_{p}}$</th>
<th>$\phi_{h1,h2}$ [°]</th>
<th>$\phi_{\theta_{p1,\theta_{p1,2}}}$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.21</td>
<td>-166 ± 0.53</td>
<td>2.91 ± 0</td>
<td>-0.13</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>-169.47 ± 1.08</td>
<td>2.29 ± 2.62</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.13 LCO amplitude and frequency characteristics of AW-1 and AW-2 in tandem configuration with $k_{p,AW-2}/k_{p,AW-1}$, 0.3 pitch configuration at $U_\infty = 6.51$ m/s.

<table>
<thead>
<tr>
<th></th>
<th>$U_\infty$ [m/s]</th>
<th>$h_0/c$</th>
<th>$\theta_{p0}$</th>
<th>$f c/U_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.51</td>
<td>0.27 ± 0.01</td>
<td>41.57 ± 1.88</td>
<td>0.08</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>0.31 ± 0.02</td>
<td>26.11 ± 1.73</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.14 LCO phase characteristics of AW-1 and AW-2 in tandem configuration with $k_{p,AW-2}/k_{p,AW-1}$, 0.3 pitch configuration at $U_\infty = 6.51$ m/s.

<table>
<thead>
<tr>
<th></th>
<th>$U_\infty$ [m/s]</th>
<th>$\phi_{h,\theta_p}$</th>
<th>$\phi_{h_1,h_2}$ [$^\circ$]</th>
<th>$\phi_{\theta_{p1},\theta_{p2}}$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW-1</td>
<td>6.51</td>
<td>-164.97± 0.54</td>
<td>61.01±</td>
<td>52.08</td>
</tr>
<tr>
<td>AW-2</td>
<td></td>
<td>-173.93± 0.83</td>
<td>10.19</td>
<td>± 9.75</td>
</tr>
</tbody>
</table>
Distinct inter-wing phase differences exist for each pitch stiffness combination. It is expected that a LVET interaction and a HEVT interaction would have disparate inter-wing phase differences. However, both HVET interaction cases have distinct inter-wing phase difference. This suggests that inter-wing phase differences are not the only variable in determining the type of interaction and that structural parameters strongly influence the resultant inter-wing phase difference. Counterintuitively, while reducing the pitch stiffness of AW-2 makes the device less susceptible to flutter in freestream as seen in Figure 2.5(b), it makes the device more susceptible to HVET wake-induced oscillations in tandem configuration. This result occurs because the freestream aeroelastic stability is determined by the relative spacing of the heave and pitch
modal frequencies; however, in tandem configuration, the pitch stiffness alone dominates the response.

In wake induced vibration studies of tandem oscillating cylinders, high and low vortex energy transfer have been reported in cylinders with identical structural properties (Ma et al.[54]; Prasanth and Mittal [79]). Ma et al. [54] demonstrated that tandem cylinder vortex energy transfer is dependent on incident wind speed, differing from our experimental results, which demonstrate vortex energy transfer dependence on pitch stiffness. Prasanth and Mittal [79] also observed that the vibration amplitude of the downstream cylinder depends on incident wind speed; however, they also note distinct phase difference trends between high and low vortex energy transfer regimes. They report low vortex energy transfer regime phase differences are between 100 to 210 degrees, whereas high vortex energy transfer phase differences range from -30 to 90-degrees. These phase difference trends are similar to the experimental results presented here and demonstrate that a range of phase difference exists for high vortex energy transfer.

Furthermore, the timing of the wing motion relative to the vortex passage affects the vortex energy transfer to the downstream device. Based on the calculated vortex convective velocity, the distance between the tandem wings, and assuming that the LEV sheds from AW-1 at pitch reversal, the phase of the vortex structure passage and the heave velocity can be estimated. In the case of equal pitch stiffnesses, this analysis shows that the LEV structure will interact with the surface of AW-2 corresponding to its heave velocity direction (Figure 2.19(b)). During this interaction, the velocity induced by the vortex decreases the effective angle of attack, thus reducing the lift generated in the heave direction. Conversely, in the cases of lower pitch stiffness in the downstream wing, the vortex passes on the opposite side of AW-2, and the induced velocities act to enhance the effective angle of attack. The enhanced effective angle of attack generates the required flow conditions over the wing to generate the larger heave deflec-
tions. These interactions are similar with the two-dimensional results of Lua et al. [52, 53] who showed that destructive or constructive interactions can occur if the effective angle of attack is decreased or increased, respectively, by the vortex-induced velocity.

Wake frequency content downstream of AW-2 also exhibits dependence on the pitch stiffness of AW-2. In the two lower pitch stiffness experiments, which also correspond to higher vortex energy capture, more energy is distributed to the $2f_{LCO}$ frequency when compared to the highest pitch stiffness case. This shift represents a change in the wake structure downstream of AW-2 as a result of the changing aerodynamic interaction and corresponding LCO motion.
Figure 2.19 a) Instantaneous wing position and trace of instantaneous wing pivot location with \( U_\infty = 6.21 \) m/s for (a) AW-1, (b) AW-2 with \( k_{p_{aw-2}}/k_{p_{aw-1}} = 0.1 \) (c) AW-2 with \( k_{p_{aw-2}}/k_{p_{aw-1}} = 0.5 \) and (d) AW-2 with \( k_{p_{aw-2}}/k_{p_{aw-1}} = 0.3 \).

The three pitch stiffness cases presented above suggest that decreasing pitch stiffness tends to increase vortex energy transfer and therefore heave amplitude in the downstream wing. Exploring the limits of this trend, a zero-pitch stiffness case and a pitch stiffness 1.25 times greater than the upstream wing were also tested. The zero-pitch stiffness case found inconsistent oscillation amplitudes and stall-flutter type phenomena in which the pitch angle oscillated about
a non-zero mean. The higher pitch stiffness case resulted in a low amplitude LCO similar to the equal pitch stiffness case. The results from these tests suggest that a non-zero optimal pitch stiffness exists for maximum vortex energy transfer to downstream wings.
2.4 Conclusion

Wing-wake interactions remain a fundamental and inherently complex subject with applications ranging from energy harvesting, to formation flight, to bioinspired locomotion. While previous researchers have studied how incident wakes affect thrust or energy harvesting performance in prescribed motion systems, we have investigated aeroelastic limit cycle oscillations in two aerodynamically interacting, fully passive systems. This study has shown that wing-wake aeroelastic phenomena presents unique events due to the wake induced limit cycle oscillations. Single-wing wake analysis was performed to detail the type of wake structure being shed from the wings. The tandem-wing experimental results show the complexity in the wing wake interactions and the sensitivity of those interactions to downstream wing pitch stiffness. The resultant limit cycle in the downstream wing is stable and was found to be the only attractor state at that wind speed. The results found that the interaction mode is independent of the initial conditions and is dependent on the pitch stiffness. The results also found that aeroelastic stability in freestream does not intuit an understanding of the resultant wing wake LCO, demonstrating that higher flutter speed configurations in single wing tests yielded high vortex-energy transfer interactions in the downstream wing. Finally, the results demonstrate that resultant inter-wing phase differences are sensitive to the pitch stiffness, and high vortex-energy transfer interactions can be produced over a range of inter-wing phase differences.

In practicality, the findings demonstrate a way to tune vortex flow interactions to either produce higher or lower energy transfer to a downstream aeroelastic structure. The enhanced energy transfer would prove fruitful for aeroelastic energy harvesting systems, while energy transfer reduction would prove useful in formation flight aerodynamic interactions.
Chapter 3

Aeroelastic Inverse: Estimation of Aerodynamic Loads During Large Amplitude Limit Cycle Oscillations

3.1 Introduction

Nonlinear aeroelastic flutter phenomena remain a topic of considerable interest due to their complex interactions between dynamic systems, structural mechanics, and aerodynamics. Nonlinear interactions can manifest in limit cycle oscillations (LCO) such that the aeroelastic structure oscillates at a bounded amplitude. Significant research efforts have investigated how structural and aerodynamic nonlinearities affect the aeroelastic system response in LCO. [21], [84], [73], [72], [74], [10], [94], and [1], and [75] experimentally investigated stall-influenced LCOs. These experiments demonstrated significant interplay between structural properties and stall phenomena. While the aforementioned studies have measured the effects of structural properties and aerodynamic nonlinearities on LCO kinematics, experimental data on the aerodynamic
forces histories during LCO remains largely unavailable. This limitation arises because direct measurement of aerodynamic forces and moments in aeroelastic LCOs is difficult, even in controlled wind tunnel experiments, because body inertia terms can be significant relative to the airloads, leading to poor measurement resolution.

While researchers such as [20] measured forces using piezoelectric force balances and [21] measured forces using piezoresistive pressure transducers, both technologies implementation in aeroelastic experiments is not wide spread. Limited use of either technology is due to challenges developing models of the inertial and stiffness contributions in the piezoelectric force balances, or potentially prohibitive wing geometry in the case of the pressure transducers. Alternatively, a few researchers have explored inverting the dynamic equations to solve for the lift and moment based on kinematic measurements, called the aeroelastic inverse (AEI) method. [77], [76], [10], and [33] have each applied AEI methods to their systems. As an example of the efficacy of the AEI method, [77] demonstrated that the nonlinear moment associated with transitional Reynolds number aeroelastic LCOs is caused by laminar separation bubbles. We expand off AEI principles from previous researchers by combining uncertainty quantification techniques into the AEI method to build credible intervals on the aerodynamic force and moment of a two degree-of-freedom wing undergoing large amplitude LCOs influenced by stall phenomena.

The AEI method presented here incorporates statistical uncertainties from position, velocity, acceleration, and system parameter estimates directly into the inverse calculations for aerodynamic lift and moment during large amplitude heave and pitch motions. The stiffness, damping, friction, and mass coupling parameters in the aeroelastic system are estimated using the Delayed Rejection Adaptive Metropolis (DRAM) algorithm [31], as part of a Markov Chain Monte Carlo (MCMC) simulation. The parameter estimates from the DRAM algorithm represent a distribution of acceptable parameter values. The AEI method is validated against
measured lift and moment from aeroelastic motion profiles prescribed in the Air Force Research Lab (AFRL) water tunnel at Wright-Patterson Air Force Base. The utility of the AEI method is demonstrated by investigating the affects of pitch-heave mass coupling on stall, power flow, and the motion kinematics in large amplitude pitch-heave LCOs. The power and force analysis demonstrate that the correlation between heave and pitch amplitude and pitch-heave phase difference is not causal. Additionally, the results demonstrate how pitch-heave phase difference not only controls the aerodynamic energy transfer but also the distribution of aerodynamic energy between the heave and pitch degrees of freedom. It is found that the distribution of aerodynamic energy is crucial in understanding how the LCO amplitude varies with wind speed. Furthermore, it is found that increased mass coupling does not equate to more energy transfer between the degrees of freedom.
3.2 Experimental Setup

The aeroelastic apparatus, Figure 4.1, consists of a wing section constrained to have an elastic translation degree of freedom (plunge/heave) and an elastic rotation degree of freedom (pitch). The apparatus is mounted to an external frame that surrounds the wind tunnel test section but does not contact the wind tunnel to eliminate the possibility of fan vibrations being transferred to the apparatus. The translation degree of freedom is provided by linear rails and ball-bearing carriages mounted above and below the test section. The carriages serve as housing for the heave encoders, pitch encoders, pitch restoring moment apparatus, and mounting connections for heave springs. Heave stiffness is governed by four extension springs while the pitch stiffness is governed by a set of parallel extension springs attached to the wing pivot rod via a constant radius pulley. Heave and pitch encoders are mounted to both the upper and lower carriages to serve as redundant measurements to ensure that no significant wing flexing occurs. Two Renishaw LM10 magnetic linear encoders with 0.03 mm resolution measure the heave displacement while two U.S. Digital E6-10000 optical encoders with 0.07 ° uncertainty record the pitching motion of the wing. The wing is a 3D printed symmetric SD 7003 with chord length, \( c \), equal to 150 mm and span, \( S \), equal to 600 mm. Endplates are attached to the wing tips to produce two-dimensional flow over the wing ([?]). The wing section is designed to accommodate several pitch axis locations, but for this study the pitch axis location is fixed at the half chord. The mass coupling, and consequently the wing inertia, is varied by moving the ballasts relative to the pitch axis. The two mass coupling configurations tested in the wind tunnel are referred to Config - 1 and Config - 2, where Config - 1 has lower mass coupling than Config - 2. Specific system parameter values are described in Section 3.4.

Aeroelastic experiments were conducted in the North Carolina State University Subsonic wind tunnel which has a \( 0.81 \times 1.14 \times 1.17 \) m\(^3\) test section and maximum wind speed of 40
m/s. The aeroelastic apparatus was tested at wind speeds ranging from 4 – 12.1 m/s. The chord-based Reynolds number for these experiments ranged from 70,000 – 120,000.

Figure 3.1 Schematic of aeroelastic apparatus with relevant properties labeled and picture of aeroelastic wing in subsonic wind tunnel.
3.3 Aeroelastic Inverse Method

The aeroelastic apparatus, Figure 4.1, is modeled as a coupled two degree-of-freedom mass-spring-damper system. The aeroelastic equations of motion derived previously in [42] are reproduced in Eqns. (4.1) and (4.2),

\[ m_{total} \ddot{h} + m_w b x_{\theta} \dot{\theta}^2 \sin(\theta) - m_w b x_{\theta} \dot{\theta} \cos(\theta) + k_h \dot{h} + c_h \dot{h} + F_f \text{sign}(\dot{h}) = C_L \frac{1}{2} \rho U_\infty^2 c S \]  

(3.1)

\[ I_\theta \dot{\theta} - m_w b x_{\theta} \cos(\theta) \ddot{h} + k_\theta (\theta) \dot{\theta} + c_\theta \dot{\theta} + M_f \text{sign}(\dot{\theta}) = C_M \frac{1}{2} \rho U_\infty^2 c^2 S \]  

(3.2)

Where the \( (\dot{\cdot}) \) notation is used for time derivatives. \( m_{total} \) is the sum of \( m_h \) and \( m_w \). \( m_h \) is the mass of all translating but non-rotating components (e.g. the carriages), \( m_w \) is the mass of all rotating parts. \( I_\theta \) is the moment of inertia about the elastic axis. \( x_\theta \) is the distance between the elastic axis and center of mass of rotating parts, nondimensionalized by half chord, \( b \), and with positive defined towards trailing edge. \( k_h \) and \( k_\theta \) are the effective stiffnesses in the heave and pitch DOF, respectively. \( c_h \) and \( c_\theta \) are the viscous damping coefficients for the heave and pitch DOF, respectively. \( F_f \) and \( M_f \) are the force and moment due to kinetic friction for the pitch and heave DOF, respectively. Finally, \( C_L \) is the aerodynamic force coefficient in the heave direction, \( C_M \) is the aerodynamic moment coefficient, \( \rho \) is the air density, and \( U_\infty \) is the freestream wind speed. The AEI method is applied by solving Eqn. 4.1 and 4.2 for \( C_L \) and \( C_M \), where all the state variables \( \vec{X} = \{ h, \dot{h}, \ddot{h}, \theta, \dot{\theta}, \ddot{\theta} \} \), are experimental measurements. This results in

\[ C_L = 2 \rho U_\infty^2 c S m_{total} \ddot{h} + m_w b x_{\theta} \dot{\theta}^2 \sin(\theta) - m_w b x_{\theta} \dot{\theta} \cos(\theta) + k_h \dot{h} + c_h \dot{h} + F_f \text{sign}(\dot{h}) \]  

(3.3)


\[ C_M = 2 \rho U_\infty^2 c^2 S \left[ I_\theta \ddot{\theta} - m_w b x_\theta \cos(\theta) \ddot{h} + k_\theta(\theta) \theta + c_\theta \dot{\theta} + M_f \text{sign}(\theta) \right] \]  

(3.4)

Parameter estimation is done using data from free decay experiments. Experiments with and without the wing attached were performed to determine if there was significant aerodynamic interaction during the free decays. The results found negligible aerodynamic interaction, thus, for free decay, (4.1) and (4.2) reduce to

\[ m_{\text{total}} \dddot{h} + m_w b x_\theta \dot{\theta}^2 \sin(\theta) - m_w b x_\theta \ddot{\theta} \cos(\theta) + k_h h + c_h \dot{h} + F_f \text{sign}(\dot{h}) = 0 \]  

(3.5)

\[ I_\theta \dddot{\theta} - m_w b x_\theta \cos(\theta) \dddot{h} + k_\theta(\theta) \theta + c_\theta \dot{\theta} + M_f \text{sign}(\dot{\theta}) = 0 \]  

(3.6)

and can be used to determine the unknown parameters in the system. We next develop expressions for the system energy transfer from the flow into the structure. We formulate the system energies following ([4]) to examine how aerodynamic energy, power, and energy transfer between the two degrees of freedom affect the LCO amplitude. The kinetic energy of the system is given by

\[ KE = \frac{1}{2} m_h (\ddot{\bar{o}} v_{cm/o} \cdot \ddot{\bar{o}} v_{cm/o}) + \frac{1}{2} m_w (\ddot{\bar{o}} v_{cm/o} \cdot \ddot{\bar{o}} v_{cm/o}) + \frac{1}{2} I_{cm} \dot{\theta}^2 \]  

(3.7)

where \( \ddot{\bar{o}} v_{cm/o} \) is the velocity of the center of mass relative to the inertial frame, and \( I_{cm} \) is the inertia about the center of mass of the wing. The velocity of the wing center of mass, \( \ddot{\bar{o}} v_{cm/o} \), can be found by differentiating the position of the center of mass, \( \ddot{\bar{o}} x \),

\[ \ddot{\bar{o}} x = -b x_\theta \cos \theta \hat{i} + h - b x_\theta \sin \theta \hat{j}, \quad \ddot{\bar{o}} v_{cm/o} = \dot{\theta} b x_\theta \sin \theta \hat{i} + \ddot{h} - \dot{\theta} b x_\theta \cos \theta \hat{j} \]  

(3.8)
Combining Eqns. 3.7-3.8 the total kinetic energy is

\[ KE = \frac{1}{2} m_{total} \dot{h}^2 - m_{wing} b \dot{\theta} \cos(\theta) \dot{h} + \frac{1}{2} m_{total} I_{\theta} \dot{\theta}^2 \]  

(3.9)

The first and last terms represent the kinetic energy in the heave and pitch degrees of freedom, respectively. The middle term is the kinetic energy due to the coupling between the two degrees of freedom. The potential energy is found by integrating the restoring forces in the heave and pitch degrees of freedom,

\[ U = \int_0^h k_i h dh + \int_0^\theta M k_\theta (\theta) d\theta \]  

(3.10)

Evaluating the flow of energy into the aeroelastic structure from the aerodynamic forces, the aerodynamic power is given by

\[ P_L = C_L \frac{1}{2} \rho U^2 c S \dot{h}, \quad P_M = C_M \frac{1}{2} \rho U^2 c^2 S \dot{\theta} \]  

(3.11)

where \( P_L, P_M \) are positive when energy is being added to the structure, and negative when the aerodynamic forces are dissipating energy from the structure. Integrating the power flow over a cycle gives the aerodynamic energy input or dissipated per cycle.

\[ E_L = \int_0^T P_L dt, \quad E_M = \int_0^T P_M dt \]  

(3.12)

Where \( T \) is the oscillation period. Since wind speed is an independent variable in the experiment used to control the kinetic energy of the flow, \( E_L \) and \( E_M \) are translated to a cycle-average aerodynamic efficiency, following a similar definition from [40]. The aerodynamic efficiency
is,

\[ \eta_{LM} = \frac{E_{LM}}{(1/2)T \rho U_\infty^3 dS} \]  \hspace{1cm} (3.13)

where \( d \) is the largest peak to peak excursion of any point on the wing. Additionally, the coupling energy transfer is found by multiplying the mass coupling terms in Eqn. 4.1 and Eqn. 4.2 by \( \dot{h} \) and \( \dot{\theta} \), respectively. The coupling energy provides insights into the distribution of aerodynamic energy throughout the structure. The coupling energy per cycle for each degree of freedom is given by:

\[
E_{x_\theta, h} = \int_0^T (m_w b x_\theta \dot{\theta} \cos \theta - m_w b x_\theta \dot{\theta}^2 \sin(\theta)) \dot{h} dt, \quad E_{x_\theta, \theta} = \int_0^T m_w b x_\theta \dot{h} \cos(\theta) \dot{\theta} dt \]  \hspace{1cm} (3.14)
3.4 System Identification and Uncertainty Quantification

There are numerous techniques used to estimate the structural parameters in a typical aeroelastic system. [75], and [10] estimated damping parameters using log decrement methods. [10] modified existing log decrement methods to account for both friction and viscous damping. [73] implemented a modified unifying least squares method ([3]) to estimate viscous damping coefficients. [39] used MCMC Bayesian inferencing to estimate the stiffness and damping parameters of their system. MCMC methods construct Markov chains whose stationary distribution is the posterior distribution. Evaluating realizations of the converged Markov chains will sample the posterior parameter distributions conditioned on observed measurements ([95, 30]). [39]’s approach is well suited to our coupled free decay model because it easily incorporates the system nonlinearities, gives credible intervals on the estimated parameters, and allows us to propagate the uncertainties through to the model response. Our approach couples MCMC Bayesian inverse methods with load cell measurements to determine the form and values of the structural parameters. Those values are then implemented to estimate the aerodynamic force and moment coefficients during limit cycle oscillations.

3.4.1 Model Selection and Sensitivity Analysis

A model of the structural stiffness and damping is required before MCMC parameter estimation can occur. A series of load cell and free decay measurements were used to develop a model of the system. The heave stiffness was measured via tensile testing of the heave springs using an Instron 4400R machine, while an ATI Gamma six axis load cell was used to measure the moment generated by the pitch spring and pulley arrangement. As demonstrated in Figure 3.2, the restoring moment measurements capture a piece-wise nonlinearity when the pitch angle is
high enough such that the springs on one side of the pulley lose tension. Additionally, during pitch-only and coupled free decay tests where the initial deflection angle did not go into the piece-wise stiffness regime, a shift in the decay frequency was observed. A softening model was incorporated into the piecewise stiffness model to capture this frequency shift. The softening portion of the pitch stiffness model influences only a small pitch angle range (< 6 deg).

The pitch spring moment equation is

\[
M_{k_0}(\theta) = \begin{cases} 
K_{\theta_L}(\theta)\theta & \text{if } |\theta| < \theta_{P_1} \\
K_{\theta_H}(\theta_{T_1} + (\theta - \theta_{P_1})) & \text{if } \theta_{P_1} \leq |\theta| < \theta_{P_2} \\
K_{\theta_S}(\theta_{T_2} + (\theta - \theta_{P_2})) & \text{if } \theta_{P_2} \leq |\theta| \\
\text{where } K_{\theta_L} = K_{\theta_L}(\theta_{P_1})/2 & 
\end{cases}
\]

(3.15)

where \(K_{\theta_L}, K_{\theta_H}, \text{and } K_{\theta_S}\) are the spring constants for the three different regions and \(\theta_{P_1,2}, \theta_{T_1,2}\) are the geometric transition angles and modified transition angles to ensure a continuous moment when the stiffness changes. The modified transition angles are defined as

\[
\theta_{T_1} = \frac{K_{\theta_L}(\theta_{P_1})\theta_{P_1}}{K_{\theta_H}}, \quad \theta_{T_2} = \frac{K_{\theta_H}(\theta_{T_1} + \theta_{P_2} - \theta_{P_1})}{K_{\theta_S}};
\]

(3.16)
Based on the envelope of the free decay response, friction along with viscous damping dissipate energy from the system. Therefore, the form of the stiffness and dissipation elements have been determined and are incorporated into Eqn 4.1-3.6. The unknown structural parameters in our model are the set \( q = \{ k_h, C_1, C_2, C_3, \beta, c_h, c_p, F_f, M_f, x_\theta \} \). However, from Eqn. (4.3), the contributions from \( \{ C_1, C_2, \beta \} \) cannot be individually distinguished through a Bayesian framework. As a result, the parameter subset selection (PSS) algorithm described in ([81, 50]) is used to evaluate the relative importance of the parameters \( \{ C_1, C_2, \beta \} \) and determine which parameter to retain. The PSS method is based on a quasi-global gradient analysis evaluating the effect of small perturbations in parameters, \( q \), on the overall model response. A scaled gradient method is employed to account for the differences that may arise in parameter scales. The PSS method builds a sensitivity matrix of the form

\[
R = \begin{bmatrix}
\frac{\partial Q_1}{\partial q_1}(t_1; q^*) & \cdots & \frac{\partial Q_1}{\partial q_p}(t_1; q^*) \\
\vdots & \ddots & \vdots \\
\frac{\partial Q_l}{\partial q_1}(t_N; q^*) & \cdots & \frac{\partial Q_l}{\partial q_p}(t_N; q^*)
\end{bmatrix}, \tag{3.17}
\]
Where $Q_I$ is a quantity of interest, specifically $h(t), \theta(t)$; $q^*$ represents a nominal set of parameter values. The PSS algorithm compares the eigenvalues of the Fisher Information matrix ($R^T R$) with respect to a specific threshold to determine parameter significance. The results from the PSS algorithm are shown in Table 3.1. The algorithm is a sifting process that removes the parameter most aligned with the least informative eigenvector. After a parameter is removed, the eigenvalues and eigenvectors are recomputed and the sifting process repeated until no eigenvalue is below the threshold. The parameter $\beta$ should be retained for Bayesian inferencing based on the results of the PSS algorithm, so our reduced parameter set is

$$q = \{k_h, C_3, \beta, c_h, c_p, F_f, M_f, x_0\}$$

(3.18)

<table>
<thead>
<tr>
<th>Table 3.1 Results from Parameter Subset Selection Algorithm with the quasi-global sensitivity matrix to determine least influential parameters of $M(\theta)$.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Eigenvector $\Delta\theta_1$ with corresponding parameters</strong></td>
</tr>
<tr>
<td><strong>Iteration</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Result: $C_1, C_2$ are less influential than $\beta$.

The values of the structural parameters $m_{total}, m_w, I_\theta, \theta_p, \theta_T$ are fixed for Bayesian inferencing, with values provided in Table 3.2.
Table 3.2 Fixed structural parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Config - 1</th>
<th>Config - 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{tot}$</td>
<td>Total mass all moving parts (kg)</td>
<td>3.268</td>
<td></td>
</tr>
<tr>
<td>$m_{wing}$</td>
<td>Mass of all rotating parts (kg)</td>
<td>1.609</td>
<td></td>
</tr>
<tr>
<td>$I_{\theta}$</td>
<td>Pitching inertia about elastic axis (kg · m$^2$)</td>
<td>5.32e-03</td>
<td>6.16e-03</td>
</tr>
<tr>
<td>$c$</td>
<td>Chord length (m)</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>Span length (m)</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>$\theta_{p_1}$</td>
<td>Transition angles for $k_\theta$ (°)</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>$\theta_{p_2}$</td>
<td>Transition angles for $k_\theta$ (°)</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>$C_1$</td>
<td>Pitch stiffness coefficient (Nm/rad)</td>
<td>6.86e-04</td>
<td>7.90e-04</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Pitch stiffness coefficient (Nm/rad)</td>
<td>9.74e-02</td>
<td>1.13e-01</td>
</tr>
</tbody>
</table>

### 3.4.2 Uncertainty Quantification

A Bayesian inference framework is implemented to estimate the parameters and quantify their uncertainty for both mass coupling configurations. Utilizing a Bayesian framework serves two purposes: (1) to construct marginal densities for the parameter set $q$ in Eqn. 3.18 and (2) to assess parameter identifiability and correlation. In Bayesian inverse problems, parameters are considered to be random variables whose densities incorporate information obtained through acquired measurements ([38]). The solution to this inverse problem is to find the posterior density $P(q|D(t))$ whose marginal densities of $q$ reflect the distribution of parameters based on the measured observations. From Bayes’ relation we observe

$$P(q|D(t)) = \frac{P(D(t)|q)P_0(q)}{P(D(t))} = \frac{P(D(t)|q)P_0(q)}{\int_{R^p} P(D(t)|q)P_0(q)dq} \quad (3.19)$$
where \( \mathcal{P}(D(t)|q) \) is the likelihood function. The prior function \( \mathcal{P}_0(q) \) encodes information known \textit{a priori} about the parameters \( q \). The denominator is a normalization term that ensures the probability distribution integrates to unity. The statistical model employed is assumed to have identically and independently distributed (i.i.d.) errors, \( \varepsilon_i \)

\[
D_i(t) = f_i(q) + \varepsilon_i, \quad i = 1, \ldots, n
\]  
(3.20)

This is an additive noise model, where the data observed, \( D_i = \{h(t), \theta(t)\} \), is assumed to be generated from the parameter-dependent model, \( f_i(q) \), (Eqns. 3.5, 3.6). The likelihood function utilized is the sum-of-squares likelihood, which has the form.

\[
\mathcal{P}(D(t)|q) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{N}{2\sigma^2} \sum_{i=1}^{N} [p_i(t) - f(q)]^2}
\]  
(3.21)

where \( \sigma^2 \) is the variance estimate and \( N \) is the number of data points. Structural parameter estimates are determined in MATLAB using the Delayed Rejection Adaptive Metropolis (DRAM) MCMC algorithm ([31, 32]). DRAM extends the traditional Metropolis-Hastings algorithm by combining two ideas, delayed rejection and an adaptive covariance. The delayed rejection step considers second or higher order steps after rejected candidates allowing for local adaptation in the Markov chain and increased candidate acceptance requiring fewer model evaluations to converge. The Adaptive Metropolis updates the proposed distribution by updating the covariance matrix using the past chain, which helps the chain to mix more quickly.
3.4.3 Model Calibration

Initial estimates of $k_h$ are found from tensile test measurements, while the other initial parameter estimates are found using MATLAB’s `fmincon` routine. Initial estimates are only provided to speed up the convergence of the model calibration. A non-informative uniform prior parameter distribution is employed with physical constraints, e.g. positive damping. Marginal densities are constructed with a kernel density estimate algorithm after the chains have converged to the fixed posterior distribution. Figure 3.3 presents joint densities and marginal posterior histograms. The parameters that have correlation are chosen for representation in Figure 3.3a-c, while the other two parameters are shown in Figure 3.3d. The marginal posterior densities show each of the parameters apart from the heave friction are unimodal and symmetric. The pairwise correlations demonstrate that correlation exists between damping and friction parameters, as well as the pitch spring moment parameters. In this case, Bayesian parameter inference is useful over traditional least-squares optimization schemes because correlated parameters represent local minimums. No correlations are single-valued however, supporting the conclusion that all the parameters were identifiable given the free decay data.
Figure 3.3 Joint densities and marginal posterior histograms for Config - 1

Figure 4.2 shows the predicted system responses using the maximum a posteriori (MAP) parameter estimate and credible intervals, along with the experimental data to illustrate the accuracy of the resulting system dynamics. The MAP estimate is given by the parameter combination that corresponds to the maximum of the posterior distribution (maximum of main diagonal elements in Figure 3.3). This value is closely related to the maximum likelihood estimator (MLE) that maximizes the likelihood or, equivalently, minimizes the sum-of-squares error ([?]). The 95% credible intervals are constructed by sampling from the MCMC parame-
ter chains and computing the corresponding model response. The results demonstrate that the model captures the structural response and shows the estimates of the parameters are valid.

Table 3.3 MAP estimates and credible intervals for the Config - 1 and Config - 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Config - 1</th>
<th>Config - 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_h$ [Ns/m]</td>
<td>1.73e+00 [1.13e+00, 2.19e+00]</td>
<td>2.08e+00 [1.54e+00, 2.66e+00]</td>
</tr>
<tr>
<td>$c_p$ [Nm/s/rad]</td>
<td>3.36e-03 [2.75e-03, 3.93e-03]</td>
<td>3.77e-03 [3.045e-03, 4.91e-03]</td>
</tr>
<tr>
<td>$F_f$ [N]</td>
<td>1.02e+00 [9.29e-01, 1.28e+00]</td>
<td>1.77e+00 [1.56e+00, 1.90e+00]</td>
</tr>
<tr>
<td>$M_f$ [Nm]</td>
<td>8.19e-03 [6.32e-03, 1.02e-02]</td>
<td>9.56e-03 [6.43e-03, 1.16e-02]</td>
</tr>
<tr>
<td>$x_\theta$</td>
<td>6.21e-02 [6.04e-02, 6.34e-02]</td>
<td>9.44e-02 [9.09e-02, 9.84e-02]</td>
</tr>
<tr>
<td>$k_h$ [N/m]</td>
<td>2.17e+03 [2.16e+03, 2.17e+06]</td>
<td>2.18e+03 [2.17e+03, 2.19e+03]</td>
</tr>
<tr>
<td>$C_3$ [Nm/rad]</td>
<td>3.47e+00 [3.45e+00, 3.49e+00]</td>
<td>3.65e+00 [3.62e+00, 3.69e+00]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-4.39e-01 [-4.82e-01, -3.94e-01]</td>
<td>-4.11e-01 [-4.59e-01, -3.32e-01]</td>
</tr>
</tbody>
</table>

Figure 3.4 Free decay comparison of experiment and model for (a) Config - 1 and (b) Config - 2.
3.4.4 Aerodynamic Force Prediction and AEI Validation

After finding the parameter distributions, the next step in this AEI method is to compute velocity and acceleration from the heave and pitch position data. The heave and pitch position signals include measurement uncertainty, therefore, we fit a smooth curve to the signals to reduce the effects of noise while taking derivatives. Following the work of [?] and [?], a smoothing spline, using MATLAB’s spaps command, is fit to the heave and pitch signal such that the error and roughness of the spline are minimized. The formulation proposed by [?] and [?] defines the tolerance value for use in the spaps command and results in smooth velocity and acceleration derivatives. For each heave and pitch measurement point, 1000 data points are generated based on the uncertainty in the heave and pitch signals. One thousand splines are fit to the heave and pitch position by randomly sampling from the data points generated to account for error induced by curve fitting. This procedure will quantify the uncertainties in velocity and acceleration from fitting splines to the position data. Additionally, a low pass filter is applied to the heave and pitch accelerations 1 Hz below 3 times the fundamental limit cycle oscillation frequency. This low pass filter is chosen based on the comparisons done with AFRL’s measured force data (Figure 3.5). The effect of the low-pass filter is similar to the cut off frequency used in Poirel and Yuan [77] for their inversion technique.

Eqns. 3.3-3.4 are used to solve $C_L$ and $C_M$ once the position, velocity, and acceleration signals have been processed and the parameter distributions and correlations found. The lift and moment are calculated by randomly sampling the parameter distributions, $q$, for a given $\vec{X}$. This process is repeated for every realization of $\vec{X}$ to sufficiently sample the entire parameter space and build credible intervals on $C_L$ and $C_M$. The one thousand different numerical computations are then phase-averaged to get the nominal lift and moment per oscillation cycle. The phase averaging begins when the pitch angle is at a maximum and ends when the pitch angle reaches
the next maximum. The credible intervals of the phase-averaged $C_L$ and $C_M$ include the uncertainty in the measurement system, parameter distributions, and any cycle to cycle deviation.

The phase-averaging is given by

$$
C_L = \frac{1}{N} \sum_{k=1}^{N} C_{Lk}(t), \quad C_M = \frac{1}{N} \sum_{k=1}^{N} C_{Mk}(t)
$$

(3.22)

where $k$ corresponds to the $k^{th}$ oscillation cycle and $N$ is the number of oscillations over the recorded signal.

The efficacy of the AEI method was determined by comparing $C_L$ and $C_M$ estimates to direct force measurements from re-created LCO kinematics in AFRL’s free surface water tunnel. In the water tunnel experiments, LCO pitch-heave time histories are played back on the actuated pitch-heave apparatus in the water tunnel as prescribed motion trajectories. More details about AFRL’s experimental apparatus can be found in [60]. Only a small number of LCO experiments were used due to the Reynolds number and amplitude limitations of the water tunnel. Reynolds and Strouhal numbers for these LCO cases were 70,700-77,600 and 0.082-0.074 respectively.

The heave and pitch signals from an LCO were phase-averaged to generate one nominal LCO cycle for recreation in AFRL’s water tunnel. The nominal heave and pitch cycle is repeated 70 times in the water tunnel to generate sufficient $C_L$, $C_M$, and $C_D$ time histories for phase averaging ([18]). Figure 3.5 shows that good agreement in $C_L$ and $C_M$ exist between the inverse method and the prescribed motion experiments. Note that Figure 3.5 plots only the $t/T$ range when the geometric effective angle of attack ($\alpha_{eff}$), defined at the leading edge, is positive.

In particular we consider the leading edge effective angle of attack based on the work of [83], who demonstrated a criticality of leading edge velocity with leading edge vortex formation. Inconsistent matching in $C_L$ and $C_M$ exists for negative $\alpha_{eff}$ ($t/T = 0.3$ and 0.8). In this time range, flow interference due to the actuator push rods in the AFRL apparatus are known to
affect the vortex development over the wing when the effective angle of attack is negative. The force and moment matching between the two systems demonstrate that the AEI method captures the dominant flow physics of the aeroelastic LCO.

Figure 3.5 Comparison of (a)$C_L$ and (b)$C_M$ from AEI method and AFRL prescribed motion measurements. Reynolds number for this case is 75,600. Note, ··· represent the 95% credible interval for each measurement.
3.5 Results and Discussion

3.5.1 Bifurcation Analysis

A series of experiments was performed below the LCO initiation wind speed to characterize the nature of the bifurcation. In each test, the wind speed was set to a prescribed value and the wing was released from rest with a $40^\circ$ initial pitch angle. In subsequent tests, the wind speed was increased and the process repeated until a sustained limit cycle was observed. Time histories of the pitch and heave responses and energy of the system, $E(t) = U(t) + KE(t)$, over the initial potential energy input, $E_0$, are shown in Figure 3.6. The pitch and heave time histories of both mass coupling configurations before the onset of LCOs indicate that the pitch degree of freedom is lightly damped. At low wind speeds the heave degree of freedom is more heavily damped and shows little response; however, once the wind speed approaches the LCO onset, the aerodynamic coupling between pitch angle and lift force begins to increase the heave motion. Compared to Config - 1, the increased mass coupling in Config - 2 delays the wind speed at which LCOs formed to 6.9 m/s and the resultant LCO amplitude at LCO initiation decreased from $\theta_A = 50^\circ$ to $\theta_A = 46^\circ$ and $h_A/c = 0.08$ to $h_A/c = 0.05$. The time histories of the energy ratio, $E(t)/E_0$, are useful in understanding the nature of the bifurcation. Regardless of the configuration, the wing exhibits a by-pass transition([92]) without any evidence of transient growth effects ([34]) wherein $E(t)/E_0 < 1$ for all stable wind speeds. A by-pass transition means that the instability is highly dependent on the initial amplitude, and is characteristic of a sub-critical hopf-bifurcation, wherein a limit cycle exists around a stable fixed point. It is hypothesized that because the initial amplitude given is well above the static stall angle, the stall features are a necessary condition for the wing to start oscillating in this sub-critical hopf-bifurcation regime.
Figure 3.6 \( E(t)/E_0 \) versus time for (a) Config - 1 and (b) Config - 2 for wind speeds tested up to LCO initiation. Pitch and heave response for wind speeds at and below LCO initiation for (c) Config - 1 and (d) Config - 2.
3.5.2 Limit-cycle Kinematic Analysis

Beyond the bifurcation wind speed, the wing was left to oscillate for several minutes to allow the system to reach a constant amplitude LCO. The LCO motion history was then recorded for one minute at each wind speed tested. Figure 3.7 plots the mean heave amplitude, pitch amplitude, and pitch-heave phase difference as functions of free stream speed. The results show large LCO amplitudes with heave and pitch amplitudes up to $0.52c$ and $65^\circ$, respectively, in Config - 1 and $0.24c$ and $59^\circ$ in Config - 2. In Config - 1, the heave and pitch motions undergo rapid amplitude increase with incident wind speed until the piecewise nonlinearity is reached, $|\theta| > \theta_{p_1}$, depicted when $\frac{\partial h_A}{\partial U_\infty}$ and $\frac{\partial \theta_A}{\partial U_\infty}$ change abruptly at approximately $7.5m/s$ wind speed. The deviations in amplitude over the recorded time are negligible and demonstrate an unmodulated LCO. Config - 1’s LCO frequency reduces from $4.03$ Hz to $4.00$ Hz as wind speed is increased to $12.1$ m/s, while Figure 3.7c shows that the phase difference between the pitch and heave degrees of freedom, $\phi_{\theta,h}$, starts at $20^\circ$ and decreases as the wind speed increases. $\phi_{\theta,h}$ is defined positive when pitch is leading the heave degree of freedom and negative when heave is leading pitch. In Config - 2, the heave and pitch amplitudes steadily rise as the wind speed increases, however, the pitch motions did not reach the piecewise pitch stiffness transition regime, i.e. $|\theta| < \theta_{p_1}$, over the wind speed range tested. Comparing Config - 2 to Config - 1, the maximum heave amplitude is $2.3$ times less than Config - 1. Meanwhile, $\phi_{\theta,h}$ starts at $0^\circ$ and decreases to $-20^\circ$, showing a similar trend with wind speed of Config - 1. The larger mass coupling/inertia in Config - 2 results in the LCO frequency dropping to $3.89$ Hz, which further reduces to $3.80$Hz at the highest wind speed tested.

These results show that increasing the mass coupling decreases both the heave and pitch amplitudes and LCO frequency. Furthermore, the decrease in phase angle with wind speed is correlated with larger amplitude heave and pitch motions. In contrast, [74] performed a
similar parameter variation and found that increased mass coupling resulted in larger heave and pitch amplitudes, while also having a strong influence on the initial phase difference between the two degrees of freedom. The contradictory trends between mass coupling and amplitude demonstrate the difficulties in drawing causal relationships from kinematic results about the mechanisms for amplitude growth. Therefore, we propose that causality can be revealed by examining the aerodynamic forces and the energy transfer into and out of the structure.

![Figure 3.7](image)

Figure 3.7 (a) Heave amplitude, (b) pitch amplitude, and (c) pitch-heave phase difference versus wind speed. Note error bars in amplitude plots represent the deviation in amplitude over the recorded time and not the measurement error.

### 3.5.3 Limit-cycle Force Analysis

Figures 3.8-3.9 show the aerodynamic forces and moments for each configuration at 7.2, 10.0, and 12.1 m/s. The aerodynamic forces and moments reach a maximum value before the maximum heave, pitch, and geometric effective angle of attack calculated at the leading edge. After $C_L$ and $C_M$ reach a maximum, the lift and moment decrease and reach a local minimum when heave and pitch reversal occur and $\alpha_{eff}$ reaches a maximum. This loss of lift and moment is indicative of stalled conditions over the wing. The stalled regions highlighted in Figures 3.8-3.9 are based on the definitions from [16] and denote lift stall to when the boundary layer begins to
reattach. Shortly after, the flow is in an attached flow state, illustrated by the fact that $\partial C_L/\partial \theta$ and $\partial C_M/\partial \theta$ match static measurements taken by AFRL (Figure 3.10b). The entire stall and recovery process occurs between $t/T = 0.35 - 0.65$ and $t/T = 0.85 - 1.00 - 0.15$. Additionally, altering the mass coupling does not change $\partial C_L/\partial \theta$ or $\partial C_M/\partial \theta$ during the attached flow regions of the time cycle because these regions are dependent on the airfoil geometry and not the wing kinematics.

Figures 3.8-3.9 also demonstrate that $C_L$ and $C_M$ follow a similar phase-averaged profile regardless of wind speed. The cycle fraction when maximum lift occurs changes by 5% from $U_\infty = 7.2 \text{ m/s}$ to $U_\infty = 12.1 \text{ m/s}$ for Config - 1 and Config - 2. Furthermore, the cycle fraction difference between $C_L$ max, points $A,A'$, and full stall ([16]), points $B,B'$ are similar between Config - 1 and Config - 2. Additionally, the cycle fraction difference between $C_M$ max, points $C,C'$, and the corresponding moment at full stall, points $D,D'$ are also similar between Config - 1 and Config - 2. Interestingly, maximum $C_L$ and $C_M$ are inversely proportional to kinematic amplitude (Figure 3.10a), with Config - 2 having larger maximum $C_L$, and significantly lower heave amplitude. As a result, a force-only analysis does not fully illustrate the underlying aerelastic mechanism for LCO amplitude growth. Thus, a power and energy analysis is required to elucidate causal mechanisms for LCO amplitude.
Figure 3.8 Lift coefficient and effective angle of attack versus cycle fraction for (a-c) Config - 1 at $U_\infty = 7.2\, m/s$, $U_\infty = 10\, m/s$, and $U_\infty = 12.1\, m/s$, respectively, and (d-f) Config - 2 at $U_\infty = 7.2\, m/s$, $U_\infty = 10\, m/s$, and $U_\infty = 12.1\, m/s$, respectively. $A,A'$ represent the maximum and minimum lift while $B,B'$ are when full lift stall occurs ([16]). Note, ··· represent the 95% credible intervals.
Figure 3.9 Moment coefficient and effective angle of attack versus cycle fraction for (a-c) Config - 1 at $U_\infty = 7.2 \text{ m/s}$, $U_\infty = 10 \text{ m/s}$, and $U_\infty = 12.1 \text{ m/s}$, respectively and (d-f) Config - 2 at $U_\infty = 7.2 \text{ m/s}$, $U_\infty = 10 \text{ m/s}$, and $U_\infty = 12.1 \text{ m/s}$, respectively. $C, C'$ represent the maximum and minimum moment while $D, D'$ are the corresponding moment when full lift stall occurs ([16]). Note, \ldots represent the 95% credible intervals.
3.5.4 Limit-Cycle Power and Energy Analysis

The input aerodynamic energy per cycle due to lift and moment, $E_L$ and $E_M$, respectively, (Figure 3.11a-b), and the power flow into the structure from the aerodynamics, $P_L$ and $P_M$ (Figures 3.12-3.13), demonstrate how energy is distributed throughout the system. Figure 3.11a-b shows that $E_L$ and $E_M$ are positive for each wind speed tested, thus the aerodynamics are sustaining both the heave and pitch degrees of freedom. Positive energy flow into both degrees of freedom is not a necessary requirement because the mass coupling allows energy to transfer between the two degrees of freedom. The lift aerodynamic energy for Config - 1, $E_{L\,\text{Config}-1}$, is up to twice as much as the aerodynamic moment energy in Config - 1 and the aerodynamic lift and moment energy of Config - 2. However, in Config - 2, $E_{L\,\text{Config}-2}$ is less than $E_{M\,\text{Config}-2}$ for all wind speeds tested (Figure 3.11a-b). The larger $E_{L\,\text{Config}-1}$ compared to $E_{L\,\text{Config}-2}$ is associated with increased aerodynamic efficiency (Figure 3.11c-d). The aerodynamic power analysis will elucidate the causes of increased aerodynamic efficiency.

Figure 3.10 a) $C_{L_{\text{max}}}$ vs $h/c$ for both mass coupling configurations. b) Dynamic $C_L$ versus $\theta$ compared to thin airfoil theory and static measurements performed by AFRL. c) Dynamic $C_M$ versus $\theta$ compared to static measurements performed by AFRL, measured at the half chord (only Config - 1 shown for clarity). Note, $\cdots$ represent the 95% credible intervals.
Figure 3.11 Aerodynamic energy for (a) Config - 1 and (b) Config - 2 and aerodynamic efficiency for (c) Config - 1 and (d) Config - 2 versus wind speed. Note, · · · represent the 95% credible intervals.

The cycle fractions where $P_L$ and $P_M$ are positive, regardless of mass coupling, correlate to constant $\partial C_L/\partial \theta$ and $\partial C_M/\partial \theta$ and until just after maximum lift and moment (Figures 3.12-3.13). Positive $P_L$ and $P_M$ after lift and moment maximum indicate that aerodynamic energy is still added to the structure during the stall regions. Aerodynamic energy dissipation, i.e. negative $P_L$ and $P_M$, starts near the local minima of $C_L, C_m$, point B,B’,D,D’ in Figures 3.8 -3.9, and lasts until the flow reattaches over the wing. Furthermore, the time difference between peak $P_L$ and peak heave velocity is significantly greater in Config - 2 than in Config - 1 (Figure 3.12), making Config - 2 less efficient. Additionally, the difference in $P_L$ between the two configurations arises because $\phi_{\theta,h}$ modulates the timing of the lift force with the heave motion. As $\phi_{\theta,h}$ decreases, the heave position moves more in phase with maximum $C_L$, i.e. lift force is mov-
ing out of phase with heave velocity, indicating a decrease in efficient power transfer (Figure 3.8). Consequently, since the cycle fractions of maximum $C_L$ and $C_M$ are nearly constant (i.e. changing by only 5%), varying $\phi_{\theta,h}$ affects how efficiently the motion kinematics capture the kinetic energy in the flow. The efficiency-phase relationship found here has been demonstrated by previous researchers for prescribed motion ([61]), and aeroelastic motion ([10]), with pitch-heave phase differences closer to 90° being more efficient. Conversely, since $P_M$, $E_M$, and $\eta_{E_M}$ are similar for each of the mass coupling configurations, a different energy transfer mechanism causes the differences in pitch amplitude.

Figure 3.12 Power from aerodynamic lift and heave velocity versus cycle fraction for (a-c) Config - 1 at $U_\infty = 7.2\text{m/s}$, $U_\infty = 10\text{m/s}$, and $U_\infty = 12.1\text{m/s}$, respectively and (d-f) Config -2 at $U_\infty = 7.2\text{m/s}$, $U_\infty = 10\text{m/s}$, and $U_\infty = 12.1\text{m/s}$, respectively. Note, $A,A',B,B'$ are the same time stamps from Figure 3.8 and $\cdots$ represent the 95% credible intervals.
3.5.5 Coupling Energy Analysis

The coupling energy plays a significant role in determining LCO amplitude kinematics. Positive $E_{x\theta,h}$ results in energy transfer from the pitch degree of freedom to the heave degree of freedom and vice-versa for positive $E_{x\theta,h}$. Figure 4.11 shows that in Config - 1, energy passed to the structure through through the aerodynamic lift is transferred into the pitch degree of freedom for wind speeds up to $U_\infty = 11 \text{ m/s}$. On the contrary, in Config - 2, the heave degree of freedom accepts energy from the pitch degree of freedom for all wind speeds tested. At the highest wind speed tested, the heave degree of freedom receives nearly 40% of its total energy input from the coupling energy. Consequently, the differences in pitch amplitude between configurations occurs because the coupling energy in Config - 1 grows the pitch amplitude, whereas in Config - 2 it limits the pitch amplitude. Additionally, increasing the coupling between the two degrees of freedom does not equate to larger energy transfer between the two degrees freedom, as Figure 4.11 shows $E_{x\theta}$ Config - 2 is less than $E_{x\theta}$ Config - 1 for half of the wind speeds tested. Therefore, $E_{x\theta}$ is more sensitive to changes in $\phi_{\theta,h}$. The direction of the coupling flow is governed by $\phi_{\theta,h}$, therefore, modulating $\phi_{\theta,h}$ strongly effects the resultant LCO kinematics. Additionally in Config - 2, the heave amplitude saturates at high wind speeds (Figure 3.7a) because the coupling energy does not compensate for the inefficient capture of energy from the lift force. It is hypothesized that a similar amplitude saturation would occur in Config - 1 at higher wind speeds as $\phi_{\theta,h}$ continues to decrease for that configuration. Moreover, the correlational relationship between increasing amplitude and decreasing $\phi_{\theta,h}$ is not causal because $\phi_{\theta,h}$ limits the growth in the pitch degree of freedom. Furthermore, $\phi_{\theta,h}$ is crucial in influencing the LCO amplitude because it not only controls the efficiency of the aerodynamic energy capture but also how the aerodynamic energy is distributed throughout the system.
Figure 3.13 Power from aerodynamic moment and pitch velocity versus cycle fraction for (a-c) Config - 1 at $U_\infty = 7.2\text{m/s}$, $U_\infty = 10\text{m/s}$, and $U_\infty = 12.1\text{m/s}$, respectively and (d-f) Config -2 at $U_\infty = 7.2\text{m/s}$, $U_\infty = 10\text{m/s}$, and $U_\infty = 12.1\text{m/s}$, respectively. Note, $C, C', D, D'$ are the same time stamps from Figure 3.9 and $\cdots$ represent the 95% credible intervals.
Figure 3.14 Coupling energy between the degrees of freedom for both configurations. Note, the dashed lines represent the 95% credible intervals.
3.6 Conclusion

The aerodynamics of an aeroelastic wing undergoing stall-influenced limit cycle oscillations are investigated by inverting the equations of motion to solve for the aerodynamic lift and moment. The inverse method utilizes Markov Chain Monte Carlo simulations to estimate the stiffness, damping, friction, and mass coupling parameters of the system. The parameter distributions are propagated through the inverse method to generate credible intervals on the measured lift, moment, power, and energy. The aeroelastic inverse method is validated against prescribed motion experiments from the AFRL water tunnel with matched Reynolds number and Strouhal number scaled kinematics. After validation, a study of how mass coupling alters large amplitude limit cycle oscillations is conducted by examining the aerodynamic forces and energy transfer mechanisms. It is found that the mass coupling alters the phase angle between the heave and pitch degrees of freedom. This is important because the pitch-heave phase angle controls the aerodynamic efficiency. Additionally, the pitch-heave phase angle also directs how the aerodynamic energy is distributed throughout the structure by varying the influence of the coupling energy. Therefore, the pitch-heave phase difference is shown as a mechanism for controlling the amplitude growth in aeroelastic limit cycle oscillations.
Chapter 4

Amplitude Annihilation in Wake-Influenced Aeroelastic Limit Cycle Oscillations

4.1 Introduction

Since LCOs can cause structural damage and reduce aircraft performance, numerous researchers have investigated ways to mitigate LCO motion. One such way to mitigate the LCO response is through the use of nonlinear energy sinks [48, 7, 69]. To date, nonlinear sinks applied to aeroelastic systems have taken the form of nonlinear mass-spring-dampers. These devices are attached to the wing to capture and dissipate energy that would have gone to either the pitch or heave degree of freedom. Lee et al. [48] used a nonlinear energy sink that coupled to the heave degree of freedom and dissipated aerodynamic energy entering the structure. Lee et al. [48] demonstrated that for a given wind speed, as the coupling strength of the nonlinear energy sink increased, a stable fixed point emerged and the limit cycle disappeared. The results of Lee et al. [48] represent one of the first experimental applications of nonlinear amplitude death phenomena to aeroelastic LCO suppression.
Broadly, amplitude death is the emergence of a stable fixed point from a limit cycle and cessation of oscillations in systems of two or more coupled nonlinear oscillators [44, 97]. While a significant majority of amplitude death studies have focused on mathematical systems such as the Rosseler oscillator or Stuart-Landau oscillator, this phenomenon has been observed in physical systems including lasers [45], circuits [82], and clocks [5]. Research into amplitude death has investigated how different coupling mechanisms lead to the suppression of LCOs. Amplitude death mechanisms relevant to aeroelastic LCO suppression include dynamic coupling, velocity coupling, and nonlinear coupling [90]. Dynamic coupling occurs when a nonlinear oscillator is added to the system to change how the energy is exchanged between the system oscillators. The energy sink of Lee et al. [48] can be classified as an example of dynamic coupling. In this case, the system oscillators are the wing heave and pitch degrees of freedom, and the added dynamic oscillator is a passive, nonlinear, mechanical device attached to the heave degree of freedom.

The experimental results of the present study are one of the first to show LCO cessation via aerodynamic wake-structure interaction rather than the addition of a supplementary mechanical degree of freedom. Our experiments show that an aeroelastic wing undergoing large amplitude oscillations influenced by upstream vortical disturbances can exhibit spontaneous cessation of the limit cycle we call LCO annihilation. The LCO annihilation annihilation phenomenon is related to, but distinct from, classical amplitude death because it occurs by destabilizing the LCO and returning the system to a pre-existing stable equilibrium. The route to LCO annihilation happens when a vortical wake from an upstream bluff body disturbs the aeroelastic limit cycle from a heave-frequency dominated LCO to a pitch-frequency dominated LCO, which results in a large amplitude modulation and eventual cessation of the oscillations. The excitation of the pitch-frequency dominated LCO depends on the interaction of the shedding frequency and the third harmonic of the primary LCO frequency. Additionally, the results reveal that the
key differences between amplitude decay and recovery and LCO annihilation lie in reversible versus non-reversible transition to the pitch-frequency dominated LCO and show that the behavior is sensitive to the bluff body vortex wake phasing. This paper presents the time history, amplitude, and mean frequency characteristics of LCO annihilation before employing various signal analysis techniques to investigate the routes for amplitude decay, recovery and LCO annihilation, such as recurrence and instantaneous frequency analysis, and aeroelastic inverse methods to study the energy transfer behavior of the wing.
4.2 Experimental Methods

A two-degree-of-freedom pitch-heave aeroelastic apparatus is used to investigate the effects of vortical disturbances on aerelastic LCOs (Fig. 4.1). The wing is elastically constrained to permit pure heave translations and pure pitch rotations. A 3D printed symmetric SD 7003 wing with a chord length ($c$) of 150mm and aspect ratio ($AR$) of 4 is used. Endplates are affixed to the wing tips emulate 2D flow conditions. The size of the endplates are chosen based on the work of Visbal and Garman[101] and extend one chord length forward and aft of the wing section. The elastic axis for all experiments is the mid chord. An external box structure isolates the aeroelastic apparatus from the wind tunnel to prevent wind tunnel fan vibrations from interacting with the structure. PBC Linear guide rails are used to support the wing above and below the wind tunnel test section. Carriages attached to the rails provide housing for the heave and pitch encoders, and serve as mounting points for the pitch restoring moment system and the heave springs. A pair of linear extension springs provide elastic restoring forces in the heave degree of freedom, while a pair of linear extension springs attached to a cable pulley system provide a restoring moment in the pitch degree of freedom. Two U.S. Digital E6-10000 encoders with 0.07-degree resolution are used to measure the wing rotation angle, while two Renishaw LM10 magnetic linear encoders with 0.03 mm resolution measure the heave position. The distance between the elastic axis and the center of gravity of the rotating components, and thereby the pitch-heave mass coupling, is controlled through a movable ballast. Two mass coupling configurations are used to investigate how the aeroelastic response of a wing with an upstream bluff body changes with mass coupling. The low mass coupling configuration is referred to as Config - 1 while the higher mass coupling configuration is referred to as Config - 2. Aeroelastic experiments were conducted in the North Carolina State University closed-return subsonic wind tunnel with a 0.81 m x 1.14 m x 1.17 m test section. The freestream velocity for
LCO experiments ranged from 7 m/s to 12 m/s, which corresponds to an airfoil chord based Reynolds number range of $70 \times 10^3$ to $120 \times 10^3$.

A rectangular cylinder bluff body was used to create the upstream vortical disturbances interacting with the wing. A rectangular cylinder is used because the Strouhal number, $St$, is 0.085[64], which results in significantly lower shedding frequencies compared to a similarly sized circular cylinder. This allows for the bluff body vortex shedding frequency to pass through both the second and third harmonics of the wing LCO frequency. The bluff body shedding frequencies in the experiments range from $2f_{LCO}$ to $3.3f_{LCO}$ as the free stream speed is varied from 7 to 12 m/s. The bluff body dimensions are 152mm in the streamwise direction and 76mm perpendicular to the flow. The bluff body’s trailing edge is 0.49m upstream from the wing leading edge when the wing is at rest. An ATI Gamma six axis load cell mounted between the base of the rectangular cylinder and the apparatus support frame is used to measure the shedding frequency. A National Instruments NI PXIe-6363 I/O module with a sampling rate of 50 kHz is used to measure the heave and pitch kinematics as well as the force signal from the load cell.

The experimental procedure begins with determining the LCO initiation wind speed. Starting at the minimum wind tunnel speed, the wing is given an initial pitch angle displacement of $40^\circ$ and then released to determine if a limit cycle exists at that wind speed. The wind speed is increased and the process is repeated until an LCO is established. The initiation of limit cycles given a large disturbance is characteristic of a sub critical hopf-birfurcation wherein a limit cycle exists around a stabled fixed point or equilibrium. The sub-critical hopf bifurcation is characterized by requiring a minimum energy input into the aeroelastic system such that a limit cycle forms. Initial conditions below this energy threshold result in the system returning to the equilibrium position whereas initial conditions above this energy threshold result in limit cycles. Once a limit cycle is established, the motion history is recorded for one minute for cases
without the bluff body upstream and up to three minutes for cases with the bluff body present. In cases where amplitude annihilation occurred, numerous different initial angle displacements were tested to evaluate the sensitivity of the system response to the initial conditions.

Figure 4.1 Schematic of aeroelastic apparatus with relevant properties labeled and picture of aeroelastic wing in subsonic wind tunnel.
4.3 System Characterization and Modeling

4.3.1 Parameter Estimation

The system is characterized using Markov Chain Monte Carlo (MCMC) Bayesian inferencing to estimate the stiffness, damping, and coupling parameters of the aeroelastic model. This approach allows us to propagate parameter uncertainties through the model response to use in aeroelastic inverse (AEI) methods. An AEI method is used to compute the aerodynamic energy into the structure and determine how the energy is distributed throughout the structure. The inverse method has been validated previously in Kirschmeier et al. [?] in order to investigate large amplitude LCOs. The AEI method is applied by solving Eqn. 4.1 and 4.2 for $C_L$ and $C_M$, where all the state variables $\vec{X} = \{h, \dot{h}, h, \theta, \dot{\theta}\}$, are experimental measurements and all parameter values are estimated through Bayesian inference or measured. The equations of motion of the aeroelastic system are:

$$m_{\text{total}}\ddot{h} + m_w b x_0 \dot{\theta}^2 \sin(\theta) - m_w b x_0 \dot{\theta} \cos(\theta) + k_h h + c_h \dot{h} + F_f \text{sgn}(\dot{h}) = C_L \frac{1}{2} \rho U_\infty^2 c S \quad (4.1)$$

$$I_\theta \ddot{\theta} - m_w b x_\theta \cos(\theta) \ddot{h} + k_\theta(\theta) \dot{\theta} + c_\theta \dot{\theta} + M_f \text{sgn}(\dot{\theta}) = C_M \frac{1}{2} \rho U_\infty^2 c^2 S \quad (4.2)$$

Where the (\_) notation is used for time derivatives. Additionally, load cell force and torque measurements of the heave springs and pitch spring and pulley system were used to characterize the force and moment versus displacement behaviour of the elastic elements. Tensile test measurements using an Instron 4400R found the heave stiffness to remain linear over the displacements observed in LCO. Based on load cell measurements of the spring pulley system, a piecewise pitch stiffness exists at high pitch deflection angles, $\theta > 60^\circ$. Additionally, pitch-only free de-
cay experiments found a softening stiffness to occur when $\theta < 6^\circ$. The pitch spring moment model is given as

$$
M_{k\theta}(\theta) = \begin{cases} 
K_{\theta L}(\theta)\theta & \text{if } |\theta| < \theta_{P1} \\
K_{\theta H}(\theta_{T1} + (\theta - \theta_{P1})) & \text{if } \theta_{P1} \leq |\theta| < \theta_{P2} \\
K_{\theta S}(\theta_{T2} + (\theta - \theta_{P2})) & \text{if } \theta_{P2} \leq |\theta| \\
\text{where } K_{\theta L}(\theta_{P1})/2 & 
\end{cases}
$$

(4.3)

where $K_{\theta L}$, $K_{\theta H}$ and $K_{\theta S}$ are the spring constants for the three different regions and $\theta_{P1,2}$, $\theta_{T1,2}$ are the geometric transition angles and modified transition angles to ensure a continuous moment when the stiffness changes. The modified transition angles are defined as

$$
\theta_{T1} = \frac{K_{\theta L}(\theta_{P1})\theta_{P1}}{K_{\theta H}}, \quad \theta_{T2} = \frac{K_{\theta H}(\theta_{T1} + \theta_{P2} - \theta_{P1})}{K_{\theta S}};
$$

(4.4)

Structural parameter estimates for $\{k_h, x_\theta, c_h, c_\theta, \beta, C_3, F_f, M_f\}$ were determined using Markov Chain Monte Carlo simulations in MATLAB using the Delayed Rejection Adaptive Metropolis (DRAM) algorithm ([31, 32]). The DRAM algorithm estimates the distributions of parameters based on the data by using the sum of squares error between the measurement and the simulation as a likelihood. Figure 4.2 shows the maximum a posteriori (MAP) estimate along with the data and credible intervals to ensure model accuracy. The MAP estimate is given by the parameter combination that corresponds to the highest density of the parameter distributions.

Structural parameters not estimated through Bayesian inferencing include $m_{\text{total}}, m_\theta, I_\theta, C_1, C_2, \theta_{P1}$, and $\theta_{P2}$. These parameters were measured and estimated through weight scales, CAD software, free decay experiments, and torque measurements; their values are listed in Table 4.1.
This value is closely related to the maximum likelihood estimator (MLE), that minimizes the sum-of-squares error (Smith2013uncertainty). The 95% credible intervals are constructed by sampling from the parameter chains and computing the corresponding model response. The MAP estimates and 95% credible intervals for the structural parameters are listed in Table 4.2. The results demonstrate that the model captures the structural response and shows the estimates of the parameters are valid. These parameter estimates will be used in the AEI method to calculate the coupling and aerodynamic energy per cycle.

Table 4.1 Structural parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Config - 1</th>
<th>Config - 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{tot}$</td>
<td>Total mass all moving parts (kg)</td>
<td>3.268</td>
<td></td>
</tr>
<tr>
<td>$m_{wing}$</td>
<td>Mass of all rotating parts (kg)</td>
<td>1.609</td>
<td></td>
</tr>
<tr>
<td>$I_\theta$</td>
<td>Pitching inertia about elastic axis (kg · m$^2$)</td>
<td>5.32e-03</td>
<td>5.77e-03</td>
</tr>
<tr>
<td>$c$</td>
<td>Chord length (m)</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>Span length (m)</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>$C_1$</td>
<td>Pitch stiffness coefficient (kg · m$^2$)</td>
<td>6.86e-04</td>
<td>7.44e-04</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Pitch stiffness coefficient (kg · m$^2$)</td>
<td>4.87e-02</td>
<td>1.05e-01</td>
</tr>
<tr>
<td>$\theta_{P1}$</td>
<td>Transition angles for $k_\theta$ (°)</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>$\theta_{P2}$</td>
<td>Transition angles for $k_\theta$ (°)</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>$K_H$</td>
<td>Pitch stiffness at high deflection angles (Nm/rad)</td>
<td>5.92</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.2 MAP estimates and credible intervals for Bayesian parameter estimation of Config - 1 and Config - 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MAP</th>
<th>95% Credible Interval</th>
<th>MAP</th>
<th>95% Credible Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_h$ [Ns/m]</td>
<td>1.73e+00</td>
<td>[1.13e+00, 2.19e+00]</td>
<td>1.56e+00</td>
<td>[0.99e+00, 1.58e+00]</td>
</tr>
<tr>
<td>$c_p$ [Nm s/rad]</td>
<td>3.36e-03</td>
<td>[2.75e-03, 3.93e-03]</td>
<td>3.62e-03</td>
<td>[3.047e-03, 3.70e-03]</td>
</tr>
<tr>
<td>$F_f$ [N]</td>
<td>1.02e+00</td>
<td>[9.29e-01, 1.28e+00]</td>
<td>0.82e+00</td>
<td>[0.73e+00, 0.96e+00]</td>
</tr>
<tr>
<td>$M_f$ [Nm]</td>
<td>8.19e-03</td>
<td>[6.32e-03, 1.02e-02]</td>
<td>7.62e-03</td>
<td>[7.22e-03, 8.72e-02]</td>
</tr>
<tr>
<td>$x_\theta$</td>
<td>6.21e-02</td>
<td>[6.04e-02, 6.34e-02]</td>
<td>7.82e-02</td>
<td>[7.61e-02, 8.07e-02]</td>
</tr>
<tr>
<td>$K_h$ [N/m]</td>
<td>2.17e+03</td>
<td>[2.16e+03, 2.17e+06]</td>
<td>2.21e+03</td>
<td>[2.21e+03, 2.22e+03]</td>
</tr>
<tr>
<td>$c_3$ [Nm/rad]</td>
<td>3.47e+00</td>
<td>[3.45e+00, 3.49e+00]</td>
<td>3.52e+00</td>
<td>[3.52e+00, 3.53e+00]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-4.39e-01</td>
<td>[-4.82e-01, -3.94e-01]</td>
<td>-4.83e-01</td>
<td>[-4.96e-01, -4.57e-01]</td>
</tr>
</tbody>
</table>

Figure 4.2 Free decay comparison of experiment with model for (a) Config - 1 and (b) Config - 2

4.3.2 System Energies

The system energy transfer from the flow into the structure is developed following Bendisksen ([4] to examine how aerodynamic energy, and coupling energy transfer between the two degrees of freedom affect the aeroelastic system. The aerodynamic power is calculated by evalu-
ating the flow of energy into the aeroelastic structure:

\[ P_L = C_L \frac{1}{2} \rho U^2 c S \dot{h}, \quad P_M = C_M \frac{1}{2} \rho U^2 c^2 S \dot{\theta} \]

(4.5)

where \( P_L, P_M \) are positive when energy is being added to the structure, and negative when the aerodynamic forces are dissipating energy from the structure. The aerodynamic energy input or dissipated per cycle is found by integrating the power flow over a cycle.

\[ E_L = \int_0^T P_L dt, \quad E_M = \int_0^T P_M dt \]

(4.6)

Where \( T \) is the oscillation period. Additionally, the coupling energy transfer is found by multiplying the mass coupling terms in Eqn. 4.1 by \( \dot{h} \) and Eqn. 4.2 by \( \dot{\theta} \), respectively. The coupling energy provides insights into the distribution of aerodynamic energy throughout the structure. The coupling energy per cycle for each degree of freedom is given by:

\[ E_{x_\theta,h} = \int_0^T (m_w b x_\theta \dot{\theta} \cos \theta - m_w b x_\theta \dot{\theta}^2 \sin(\theta)) \dot{h} dt, \quad E_{x_\theta,\theta} = \int_0^T m_w b x_\theta \ddot{h} \cos(\theta) \dot{\theta} dt \]

(4.7)
4.4 Results

4.4.1 Wing Limit Cycle Oscillation Behavior

Figure 4.3 shows the wing pitch time history for a selected range of wind speeds and the pitch and heave amplitudes for both configurations when the bluff body is not present. The amplitude plots, Fig. 4.3b-c, are generated by plotting a semitransparent marker for the maximum of each cycle over the entire time history. Therefore, a darker marker represents a high number of LCO cycles at that amplitude while a lighter marker represents a lower number of LCO cycles with that amplitude. The time histories of the no-bluff-body LCOs for both configurations are shown in Fig. 4.3a. For Config - 1, the LCO amplitude at a given wind speed remain constant over the measured time history, however, small, transient amplitude modulations exist for Config - 2. These oscillations are represented as a larger spread of the amplitudes in Fig. 4.3(b-c) for the no-bluff-body cases. When $x_\theta = 0.062$, the no-bluff-body pitch and heave amplitudes range from $58$ to $66^\circ$ and $h/c=0.3-0.6$, respectively. When $x_\theta = 0.078$, the no-bluff-body pitch amplitude ranges from $40$ to $58^\circ$, while the heave amplitudes range from $h/c =0.1$ to $0.33$. Additionally, compared to $x_\theta = 0.062$, slightly higher wind speeds are required for LCOs to appear for $x_\theta = 0.078$. The relationship between the mass coupling and the LCO amplitude is discussed further in Kirschmeier et al. [?] and is the result of changing the pitch-heave phase difference such that the distribution of energy between the two degrees of freedom is altered and the efficiency of the aerodynamic power transfer is reduced for higher mass coupling across all wind speeds.
Figure 4.3 (a) Pitch time histories, (b) pitch amplitude versus wind speed and (c) heave amplitude versus wind speed for wing LCO without the bluff body upstream.

For each of the configurations, a fast Fourier transform of the pitch signal is used to determine the mean frequency[35] components when the bluff body is not present. Figure 4.4 shows the spectral analysis for $x_\theta = 0.078$, however both configurations show similar trends. The frequency spectrum is dominated by the fundamental LCO frequency, at 4 Hz, and lower amplitude super harmonics of the structure, with odd harmonics showing a stronger influence.
Figure 4.4 a) Frequency spectrum of pitch response for five selected wind speeds without the bluff body present. b) Frequency spectrum of pitch response with the bluff body present for all wind speeds tested. $x_\theta = 0.078$ for each figure.

The introduction of an upstream bluff body influences the aeroelastic LCO of the downstream wing. A fast Fourier transform of the pitch signal, Fig. 4.4b, shows the presence of the bluff body introduces not only frequency content at the shedding frequency, but additional side band frequencies around the LCO frequency. The difference between the side-band frequencies and the LCO frequency is determined by the difference between the shedding frequency and $3f_{LCO}$. The presence of side band frequencies in the spectral content is characteristic of amplitude modulated signals. At the lowest wind speed tested with the bluff body, the shedding frequency is below the LCO third harmonic. As the wind speed is increased, the shedding frequency passes through the third harmonic of the LCO and then moves past it. Figure 4.5a shows the amplitude modulated pitch time histories for both mass coupling configurations. The time histories for $x_\theta = 0.062$ show an amplitude modulated response for all wind speeds tested, however, the LCO is stable and does not annihilate. When $x_\theta = 0.078$, amplitude modulation exists for all wind speeds, however for a certain range of wind speeds and consequently bluff body shedding frequencies, the limit cycle annihilates. When the LCOs are annihilated, the pitch and heave amplitudes reduce to negligible deflections that are due to harmonic forcing.
from the bluff body shedding. The instances of LCO annihilation occur when the bluff body vortex shedding frequency is within $2.9 < f_s/f_{LCO} < 3.08$, and when the side band modulation frequencies are within 0.3Hz of the mean LCO frequency. Above $U_\infty = 11.2\text{m/s}$, stable limit cycles return and no LCO annihilation is found. Additionally, only one of the multiple time histories during LCO annihilation is shown in Fig. 4.5a. While $f_s/f_{LCO}$ of 2.98 and 3.02 show the longest oscillation time, other experiments performed at the same frequency ratio show much smaller time to LCO annihilation. Furthermore, during the time histories exhibiting LCO annihilation there exists a high amplitude modulation mode, where the pitch amplitude varies by up to 30° and low amplitude modulation mode where the pitch amplitude varies up to 15°. Figure 4.5(b-c) shows all the pitch and heave amplitudes over the full time history of each configuration at each wind speed tested. The results show that placing a bluff body upstream of the wing does not alter the LCO initiation speed when $x_\theta = 0.062$, however there is a 1.7 m/s increase in the LCO initiation speed when $x_\theta = 0.078$. Additionally, both configurations show significant amplitude modulation occurs once the shedding frequency is within a critical distance of the third harmonic of the LCO. Furthermore, the oscillation annihilation is not caused by singular upstream flow disturbance forcing the wing back to the stable fixed point, as can be seen by the stable LCOs at wind speeds above and below the annihilation region. Furthermore, since stable oscillations exist at wind speeds less than the LCO annihilation region, the shift in LCO initiation speed caused by the increased mass coupling does not account for LCO annihilation.
Figure 4.5 a) Pitch time histories, (b) pitch amplitude versus wind speed and (c) heave amplitude versus wind speed for wing LCO with the bluff body upstream.

At each wind speed where LCO annihilation occurred, multiple initial conditions were tested to understand the sensitivity of LCO annihilation to starting conditions. Figure 4.6 shows multiple time histories for $f_s/f_{LCO} = 2.91$. The results demonstrate that LCO annihilation does not depend on the initial conditions, with amplitude annihilation present in all trials. Therefore, the initial energy input into the structure only influences whether or not the system grows to the LCO before destabilizing and returning to the equilibrium position. Additionally, the time to LCO annihilation does not have an apparent pattern or correlation to the initial condition amplitude. The initial condition near $\theta = 20^\circ$ oscillated for 70 seconds while for the two initial
conditions near $\theta = 40^\circ$, one oscillated for approximately 10 seconds and the other 30 seconds. It is hypothesized, and discussed further in Section 4.5.1, that LCO annihilation requires a specific phase relationship between the bluff body shedding and the wing kinematics. Furthermore, the switching between the low amplitude modulation mode and the large amplitude modulation mode does not appear to have a specific time scale associated with it, suggesting a chaotic nature. Moreover, the path to LCO annihilation does not have to occur with a direct transition to the equilibrium position. Certain time histories show that LCO cessation occurs over a longer period of time, with a large amplitude drop occurring within 5 seconds followed by 5 seconds of oscillations with a pitch amplitude around $15^\circ$. The oscillations near $15^\circ$ are just below the minimum energy needed to sustain LCOs.

Figure 4.6 Multiple LCO annihilations at $f_s/f_{LCO} = 2.92$ for $x_B = 0.078$, (a) first two seconds of motion and (b) full time history.
4.5 Analysis and Discussion

The emergence of LCO annihilation in the system requires further investigation of the underlying phenomenon. It will be shown that the route to LCO cessation depends on the emergence of a pitch-frequency dominated LCO and a unique kinematic and bluff body vortex shedding phase that reduces the system energy below the threshold needed for sustained LCOs, causing annihilation.

4.5.1 Recurrence Analysis

The time histories during which LCO annihilation occurs show multiple amplitude decay and recovery intervals before LCO annihilation occurs. A recurrence analysis is used to investigate whether the LCO annihilation event is kinematically unique or if there is a unique combination of kinematic states and bluff body vortex shedding states. Recurrence analysis has been applied in recent years as a way to analyze how often a signal returns to a location within in a phase space[56]. Recurrence analysis is based on a euclidean distance formula that finds the distance between a current point in a phase space and another point in the phase space and then assigns a 1 or 0 to that combination of time instances depending on a threshold level set[56]. While there are several ways to calculate the distance threshold, we set the threshold to either be 5, 10, or 20 times the measurement error associated with the specific variable. We relax the threshold to 20 times the measurement error to ensure the comparisons are not too restrictive. The kinematic recurrence calculation is given by

\[ R_{i,j}(\epsilon) = \Theta(\epsilon^{T} - ||\bar{x}_i - \bar{x}_j||), \quad i = 1,...,N, j = 1,...,M \]  

(4.8)
where $\Theta(\cdot)$ is the Heaviside function, $||\cdot||$ is a norm, $\epsilon$ is the threshold, $i, j$ are instances in time of $\vec{x}$, where in our system $\vec{x} = \{h, \dot{h}, \theta, \dot{\theta}\}$, and $R_{i,j}$ is the recurrence matrix. The signals and errors are normalized by the maximum values of the state in the given signal. The normalization is done to account for the order of magnitude differences between the position and velocity terms as well as the difference in magnitudes between the heave and pitch states. Additionally, a kinematic and bluff body force joint recurrence is used to investigate the hypothesis that a unique combination of kinematic and bluff body shedding states results in LCO annihilation. The kinematic and bluff body force joint recurrence is defined as

$$JR^\vec{x}\vec{z}_{i,j} = \Theta(\epsilon^{\vec{x}} - ||\vec{x}_i - \vec{x}_j||)\Theta(\epsilon^{\vec{z}} - ||\vec{z}_i - \vec{z}_j||), \quad i = 1, \ldots, N, \; j = 1, \ldots, M$$  \hspace{1cm} (4.9)

Where $\vec{z}$ is the bluff body force and its derivative, as measured by the load cell. A kinematic and bluff body force joint recurrence is used instead of a traditional recurrence calculation as in Eqn. 4.8, because grouping the bluff body force signal into $\vec{x}$ results in false recurrence values due to the fact that the phase space of the force signal is at a higher frequency than the phase space of $\vec{x}$. Additionally, $\vec{z}$ is defined using the force signal and its derivative to remove false recurrences associated with sinusoidal signals. In simple sinusoidal signals, recurrence analysis will calculate that values on either side of a peak or trough are the same, however, one side is going towards the peak or trough while the other side is moving away from that peak or trough. Therefore, a rate dependence is required in the recurrence analysis to differentiate the direction of motion through the phase space. Furthermore, a multi-trial cross recurrence analysis was performed to analyze how kinematically similar the LCO annihilation events are between multiple trials at the same wind speed. The multi-trial cross recurrence is used to evaluate whether there is a kinematic trigger such that LCO annihilation occurs. The cross
recurrence is defined as

\[ CR_{i,j}^{\vec{x},\vec{y}} = \Theta(e^{\vec{x} - \|\vec{x}_i - \vec{y}_j\|}), \quad i = 1, ..., N, j = 1, ..., M \]  

(4.10)

where \(\vec{y}\) are the same states as \(\vec{x}\) but from a different trial time series. The nondimensionalization procedure is different than that used for the recurrence calculation. Since there is no guarantee that the maximum values of \(\vec{x}\) and \(\vec{y}\) are the same, the maximum values of \(\vec{x}\) are used to normalize \(\vec{x}\) and nondimensionalize \(\vec{y}\), keeping the same magnitude scale relationships between the two signals. Unfortunately, recurrence analysis is memory intensive since \(R_{i,j}\) is usually defined as an \(N \times N\) matrix where \(N\) is the number of data points in the time series. This is mitigated in two ways; by limiting the evaluation window such that each recurrence matrix only encapsulates 1.5 seconds of data, and by only evaluating the recurrence of the last 12 seconds of data with the rest of the signal. The last 12 seconds of data covers a few seconds before the annihilation starts through the entirety of LCO annihilation event. This analysis is done for both the kinematic recurrence and the kinematic and bluff body joint recurrence analysis. In the multi-trial cross recurrence analysis, only the LCO annihilation events are compared to each other and the signals are still divided into 1.5 seconds to maintain efficient computation.

Figure 4.7a shows the kinematic recurrence of one of the LCO annihilation time series. The results show that even though there are multiple instances of large amplitude modulation in the signal, only one region has noticeable kinematic similarity with the LCO annihilation event. The similarity with only a small portion of one of the handful of large amplitude recoverable decay regions demonstrates that the majority of the recoverable decay regions take on a different kinematic path compared to LCO annihilation. Additionally, since amplitude decay can take a similar path to LCO annihilation, other factors must exist such that LCO annihilation occurs. The kinematic similarity between recoverable decay and annihilation is present.
in the other trials, with few if any of the decay regions of signals showing similarity to the LCO annihilation region. The specific kinematic differences will be further discussed in Section 4.5.2 while the factors separating recoverable decay and annihilation are found in the joint recurrence.

Figure 4.7 a) Kinematic recurrence of LCO annihilation event with the rest of the time signal. Regions of similarity are highlighted with blue markers. b) Kinematic and bluff body joint recurrence of LCO annihilation with the rest of the time signal. $x_\theta = 0.078$ and $f_s/f_{LCO} = 2.98$ for both figures.

Besides kinematic factors, the influence of the bluff body on LCO annihilation is captured in the kinematic and bluff body joint recurrence analysis. The kinematic and bluff body joint recurrence analysis, Fig. 4.7b, shows that there are only 3 instances in time, for any of the tolerance values chosen, in which the bluff body force signal and the kinematics of the LCO annihilation are similar to other parts of the signal. The bluff body force signal is used as an analog for the formation and shedding of vortices from the bluff body. Therefore, the reduction in similarity of the decay region with the annihilation region aligns with the hypothesis that
LCO annihilation is caused by a unique combination of bluff body vortex and kinematic states which drives the system to the stable fixed point.

Figure 4.8 shows the multi-trial cross recurrence between three LCO annihilation regions performed over different trials at $f_s/f_{LCO} = 2.98$. As shown, there is high degree of similarity between two LCO annihilation cases, $CR^{1,2}$, where the superscripts refer to the trial number. The similarity region of $CR^{1,2}$ goes from the maximum pitch amplitude right before annihilation until the pitch amplitudes are around $10^\circ$, then the signals are dissimilar until the pitch amplitude are less than $5^\circ$. The similarity implies that both trials follow similar kinematic paths to LCO annihilation. However, there is little similarity in the LCO annihilation regions as demonstrated by $CR^{1,3}$ and $CR^{2,3}$, with the only similarity coming before the annihilation event and for oscillations near the equilibrium position. Therefore, there is not a unique kinematic threshold that results in LCO annihilation but rather a phase space of kinematic relationships that must exist for LCO annihilation. The annihilation phase space represents a region of attraction for the oscillator that will annihilate the oscillations given proper kinematic and bluff body vortex interactions. As will be shown, the annihilation phase space for kinematics is governed by the LCO frequency and pitch-heave phase difference.
4.5.2 Instantaneous Frequency Analysis

A continuous wavelet transform and a Hilbert transform are used to gather instantaneous frequency and phase information from the amplitude envelope and the LCO signal. The continuous wavelet transform of the envelope of the pitch signal is used to study the frequency interactions that govern the amplitude modulation, while the Hilbert transform of the heave and pitch time histories will be used to analyze instantaneous frequencies and phase differences. The MATLAB CWT command is used to calculate the continuous wavelet transform. Figure 4.9 shows that when $f_s/f_{LCO} < 2.9$ and $f_s/f_{LCO} > 3.1$, the amplitude modulation frequencies are dominated by the difference between the third harmonic and the bluff body vortex shedding frequency. However, when $2.9 < f_s/f_{LCO} < 3.1$, the modulation content includes durations dominated by the third harmonic and the bluff body vortex shedding frequency and durations dominated by a secondary mode of oscillation. The secondary mode of oscillation corresponds to the large amplitude modulation mentioned in Section 4.4. The modulation frequency for the secondary mode is equal to the difference between a pitch-frequency and heave-frequency.
dominated LCO, indicating a switch in the LCO frequency. The wavelet transform reveals that the secondary mode can present itself when \( f_s/f_{LCO} < 2.9 \) and \( 3.1 < f_s/f_{LCO} \), however the magnitude of such interaction is significantly lower and results only in a small amplitude change.

![Figure 4.9](image)

**Figure 4.9** Continuous wavelet transforms for \( x_\theta = 0.078 \) at (a) \( f_s/f_{LCO} = 2.72 \), (b) \( f_s/f_{LCO} = 2.98 \), and (c) \( f_s/f_{LCO} = 3.21 \).

Instantaneous frequency of each degree of freedom, \( f_{h,LCO}, f_{\theta,LCO} \), and frequency ratio, \( f_{h,LCO}/f_{\theta,LCO} \), are computed by applying the Hilbert transform to each position signal. The instantaneous frequencies are passed through a moving mean window, whose window size is 4 Hz, to understand the global frequency change as opposed to the intra-cycle frequency change. Figure 4.10a shows the LCO frequency of the each degree of freedom over the entire time history for \( f_s/f_{LCO} = 2.98 \). During each of the recoverable amplitude decay intervals and the annihilation event, both \( f_{h,LCO} \) and \( f_{\theta,LCO} \) drop from 4 Hz to 3.8 Hz. This reduction in frequency is the secondary mode of oscillation, and based on the system configuration, is a pitch-frequency dominated LCO. This is classified as the pitch-frequency dominated LCO because that is the pitch natural frequency at this wind speed. This secondary mode of oscillation can be seen as the existence of multiple LCO attractors. Poirel et al. [78] reported that for
pitch-heave natural frequency ratios near unity, amplitude modulation existed between a heave-frequency dominated LCO and a pitch-frequency dominated LCO. Poirel et al. [78] found that the heave-frequency dominated LCO was associated with larger flow energy transfer to the structure compared to the pitch-frequency dominated LCO. Examining the intervals of amplitude decay and recovery shown in Figure 4.10c, \( f_{h,LCO} \) and \( f_{\theta,LCO} \) reduce to 3.8 Hz near the amplitude trough and then recover back to the 4 Hz heave-frequency dominated LCO. During LCO annihilation, Figure 4.10d shows that \( f_{h,LCO} \) and \( f_{\theta,LCO} \) converge towards the pitch frequency dominated LCO and the frequency does not recover to the heave-frequency dominated LCO.

Figure 4.10b shows the frequency ratio of pitch to heave frequency at \( f_s/f_{LCO} = 2.98, U_{\infty} = 10.8\, m/s \). The frequency ratio when \( f_s/f_{LCO} = 3.42 \) is also plotted to show the nominal variations caused by the bluff body when annihilation does not occur. The nominal variations are used to classify the frequency spread caused by the bluff body and represent a ±1% frequency variation. Outside of this region, the pitch-heave frequency lock-on begins to break down and the heave and pitch degrees of freedom begin to move at dissimilar frequencies. Figure 4.10c shows that the amplitude decay and recovery regions incur greater than 1% mismatches in the pitch-heave frequency ratio. The noticeable mismatches in the pitch-heave frequency ratio occur just before the trough of the recoverable amplitude decay interval and during the amplitude recovery portion. Combined with the joint recurrence, the noticeable mismatches in frequency ratio before and at the amplitude trough of the recoverable amplitude decay region indicate that amplitude recovery occurs because the interactions with the bluff body wake cause the system to escape the pitch-frequency dominated LCO. By breaking the frequency lock-on to the pitch-frequency dominated LCO the system is able to escape and return to the heave-frequency dominated LCO. Conversely, Figure 4.10d shows that the pitch-heave frequencies during LCO annihilation remain locked-on. The frequency lock-on highlights why
the recurrence analysis in Figure 4.7a shows few amplitude decay regions matching with the LCO annihilation region. Therefore, LCO annihilation requires that the aeroelastic frequencies converge to a pitch-frequency dominated LCO and remain locked-on to this frequency throughout the amplitude decay.

![Graphs showing instantaneous frequency and pitch heave frequency ratio](image)

**Figure 4.10** (a) Instantaneous frequency and (b) pitch heave frequency ratio versus time for $f_s/f_{\text{LCO}} = 2.98$ shown over the full time history. Instantaneous frequency and pitch-heave frequency ratio during (c) recoverable decay and (d) LCO annihilation.
4.5.3 Coupling and Aerodynamic Energy Analysis

The aeroelastic inverse method is employed to study the evolution of the cycle-to-cycle coupling (Eqn. 4.7) and aerodynamic energy (Eqn 4.6) over the full time history. The cycle-to-cycle coupling energy is tracked over the full time history to determine how the energy is flowing between the two degrees of freedom (Fig. 4.11). Before amplitude decay occurs, $E_{\theta,h}$ is positive, indicating energy is being transferred from the pitch degree of freedom into the heave degree of freedom. During recoverable amplitude decay, $E_{\theta,h}$ remains positive until the trough of the amplitude decay interval. During amplitude recovery, $E_{\theta,h}$ is negative, therefore, energy from the heave degree of freedom is being transferred into the pitch degree of freedom. The pitch-heave phase difference switching sign, going from a heave-leading motion to a pitch-leading motion, causes the direction of the coupling energy change. Additionally, as shown in Gianikos et al. [27], $\phi_{\theta,h}$ is correlated to the power flow into the structure, with greater positive $\phi_{\theta,h}$ leading to amplitude growth and more negative $\phi_{\theta,h}$ leading to amplitude decay. Therefore, the kinematic requirements for LCO recovery are positive $\phi_{\theta,h}$ and energy flow from the heave degree of freedom to the pitch degree of freedom. At the start of the LCO annihilation event, $E_{\theta,h}$ is positive, however at a smaller energy transfer per cycle than during the recoverable decays. Additionally, $E_{\theta,h}$ becomes negative during the LCO annihilation event, however the energy transfer is not sufficient to break the frequency lock-on and grow the pitch amplitude. Furthermore, the pitch-heave phase difference is a small negative value, thus, a large negative $\phi_{\theta,h}$ is not an indicator of LCO annihilation. Therefore, additional kinematic differences that arise in amplitude annihilation as opposed to recoverable amplitude decay are due to the evolution in pitch-heave difference.
Figure 4.11 Coupling energy and pitch-heave difference versus time for $f_s/f_{LCO} = 2.98$ for the same trial during (a) recoverable amplitude decay and (b) LCO annihilation. Note, $\cdots$ represent the 95% credible intervals.

Figure 4.12 shows the aerodynamic energy into the heave and pitch degrees of freedom during a region of amplitude decay and during LCO annihilation for the same trial. The amplitude decay region is characterized by $E_L$ becoming negative, which indicates the aerodynamics are dissipating energy from the structure. However, during this time, $E_M$ remains positive. Results from other recoverable amplitude decay regions show similar effects where $E_L$ reduces to near zero or becomes negative while $E_M$ reduces to near zero but remains positive. However, the LCO annihilation event, Figure 4.12(b), is always characterized by $E_M$ becoming negative while $E_L$ is either positive or negative but near zero. Additionally, from results of multiple trials at $f_s/f_{LCO} = 2.98$, the minimum pitch amplitude needed to initiate LCOs is approximately $20^\circ$. Therefore, the time when $E_M$ becomes dissipative occurs a few cycles before $\theta_A$ drops below $20^\circ$. From the joint recurrence analysis, $E_M$ becoming negative occurs because the bluff body wake causes adverse aerodynamic interactions. Therefore, a loss of aerodynamic energy in the heave and pitch degrees of freedom combined with pitch energy transferring to the heave
degree of freedom results in a precipitous decline of system energy and eventual amplitude cessation. This LCO annihilation phenomena presented is closely related to the aforementioned amplitude death phenomena, with the exception that our system has a stable fixed point for all wind speeds tested, whereas amplitude death is the emergence of a stable fixed point.

Figure 4.12 Aerodynamic energy and pitch amplitude difference versus time for $f_s/f_{LCO} = 2.98$ for the same trial during amplitude (a) recoverable decay and (b) LCO annihilation.
4.6 Conclusion

The work presented here investigated the first aerodynamically annihilated aeroelastic limit cycle oscillation. The aerodynamic limit cycle annihilation phenomenon is caused by the presence of upstream vortical disturbances on the wing. The results show that limit-cycle annihilation depends on the magnitude of the mass coupling and the proximity of the shedding frequency to the third harmonic of the primary limit-cycle oscillation frequency. Given these conditions, the large amplitude limit cycle oscillations destabilize and the system returns to its equilibrium position. Instantaneous frequency analysis through the wavelet and Hilbert transforms shows that large amplitude modulations are caused by the excitation and lock-on of the limit cycle to a pitch-frequency dominated limit-cycle that results in large energy dissipation from the structure. Therefore, the kinematic requirements for limit-cycle annihilation are the convergence and lock-on of aeroelastic frequencies onto a pitch-frequency dominated limit cycle and a small negative pitch-heave phase difference. Combined with the results from the kinematic and bluff body joint recurrence, the results suggest that a disruption in unsteady flow features over the wing result in limit cycle annihilation. These results present unique aerodynamic interactions between vortex wakes and aeroelastic limit cycle oscillations and provide insight that can benefit engineers designing wings in vortex-dominated flows.
Conclusions and Future Work

This dissertation presented experimental investigations into different types of aeroelastic wing wake interactions and different signal analysis techniques used to understand those interactions. The findings highlight how different structural and aerodynamic mechanisms affect the fluid dynamic energy transfer in aeroelastic limit cycle oscillations.

5.1 Key Findings of Tandem Wing Aeroelastic Experiments

The first experimental campaign investigated tandem aeroelastic wings in limit cycle oscillations. This study shows that wing-wake aeroelastic phenomena present unique events due to the wake-induced limit cycle oscillations. Single-wing wake analysis using a hot-wire anemometer details the type of wake structure being shed from the wings. The tandem-wing experimental results show the complexity in wing-wake interactions and the sensitivity of those interactions to the downstream wing pitch stiffness. The results highlight that aeroelastic stability in freestream does not intuit an understanding of the resultant wing-wake limit cycle and demonstrates that higher flutter speed configurations in single wing tests yielded high vortex-energy transfer interactions in the downstream wing. Finally, the results demonstrate that resultant
inter-wing phase differences were sensitive to the pitch stiffness, and high vortex-energy transfer interactions were produced over a range of inter-wing phase differences.

5.2 Key Findings of the Aeroelastic Inverse Method

The aerodynamics of an aeroelastic wing undergoing stall-influenced limit cycle oscillations were investigated by inverting the equations of motion to solve for the aerodynamic lift and moment. The inverse method utilizes Markov Chain Monte Carlo simulations to estimate the stiffness, damping, friction, and mass coupling parameters of the system. The parameter distributions are propagated through the inverse method to generate credible intervals on the estimated lift, moment, power, and energy. It is found that the mass coupling alters the phase difference between the heave and pitch degrees of freedom. This is important because the pitch-heave phase difference controls the aerodynamic efficiency. Additionally, the pitch-heave phase difference also directed how the aerodynamic energy is distributed throughout the structure by varying the influence of the coupling energy. Therefore, the pitch-heave phase difference is shown as a mechanism for controlling the amplitude growth in aeroelastic limit cycle oscillations.

5.3 Key Findings of Limit Cycle Annihilation

The work presented in Chapter 4 investigates the first aerodynamically annihilated aeroelastic limit cycle oscillation from superharmonic frequency excitation. The presence of vortical disturbances on the wing causes the annihilation phenomenon to occur. The results show that limit-cycle annihilation depends on the magnitude of the mass coupling and the proximity of the bluff body shedding frequency to the third harmonic of the primary limit-cycle oscillation.
frequency. Given these conditions, the large amplitude limit cycle oscillations destabilize and the system returns to its equilibrium position. Instantaneous frequency analysis through the wavelet and Hilbert transforms show that the large amplitude modulations are caused by the excitation and lock-on of the limit cycle to a pitch-frequency dominated limit-cycle that results in large energy dissipation from the structure. The application of the inverse method and a kinematic and bluff body joint recurrence suggest that a disruption in unsteady flow features over the wing result in limit cycle annihilation.

5.4 Future Work

Based on the tandem wing experiments and the cited literature in Chapter 2, future research into tandem wing aeroelastic systems should investigate the dependence of the aeroelastic response on structural nonlinearities. Furthermore, additional research should address the phase angle relationship between an impinging vortex and LEV formation to fully capture the effects of vortex energy transfer. While inter-wing phase differences have been used as a proxy, fully quantifying an impinging vortex-LEV phase angle would remove uncertainties related to vortex convection velocity and be informative as to whether constructive and destructive interactions are occurring.

Future research applying the aeroelastic inverse method should investigate developing reduced order aerodynamic models to determine what forms of the aerodynamic models are necessary to capture the aeroelastic interactions observed. The plethora of unsteady aerodynamic models available in the literature each consist of fundamentally different formulations. Models such as LDVM[83], and BGG [15] incorporate switching between different aerodynamic influences dependent on the flow topology over the wing. The ONERA [100] and BGG [15] model are based on a set of second order differential equations. Models developed by Khlalil
et al. [39] and Sandhu et al. [89] focus on second order, highly nonlinear, differential equations to capture the relevant phenomena. In developing reduced order models, researchers can investigate numerous forms for the lift and moment, such as switching functions, 1st or 2nd order ordinary differential equations, partial differential equations, and nonlinear functions. A thorough investigation will determine which form of the lift and moment equations are necessary to match the physics present in the system. Additionally, the models should be compared to prescribed motion force measurement experiments performed by other researchers to determine the validity of the models to other data sets.

Future limit-cycle annihilation research should investigate a wider set of bluff body and structural parameters to determine the sensitivity of annihilation to the specific parameters already investigated. Bluff body parameters should include changing the shape of the bluff body such that the vortex shedding frequency passes through the 4th and 5th harmonic of the limit cycle oscillation frequency over the range of wind speeds tested. Additionally, constructing an oscillating bluff body can change the vortex shedding frequency at a specific wind speed as opposed to varying the wind speed to control the bluff body shedding frequency. An oscillating bluff body will allow researchers to study the effect of wake oscillation frequency separately from incident wind speed, which is currently used to change the bluff body shedding frequency. Structurally, the pitch stiffness should be varied, while keeping heave stiffness constant, to see if pitch sensitivity to incoming vortical wakes affects the annihilation response. Additionally, the heave and pitch natural frequencies should vary by the same amount to study the sensitivity of limit cycle frequency shifting on annihilation. Furthermore, the pitch-heave frequency ratio should be varied to investigate the sensitivity of annihilation to the modal convergence characteristics of the wing.
5.5 Final Conclusions

Wing-wake interactions in aeroelastic systems will continue to be investigated as scientists and engineers continue to push the operating boundaries of wings. The work presented in this dissertation expands upon previous wing-wake interaction studies and highlights the important interplay between structural and aerodynamic properties. We anticipate the continued expansion of aeroelastic wing-wake interaction research to bring many fruitful and insightful investigations.
REFERENCES


[103] Jianan Xu, Hongyu Sun, and Songlin Tan. Wake vortex interaction effects on energy 
extraction performance of tandem oscillating hydrofoils. _Journal of Mechanical Science 
and Technology_, 30(9):4227–4237, 2016.


[105] John Young and Joseph C. S. Lai. Oscillation Frequency and Amplitude Effects on the 