GHOSH, MADHUJA. Behaviorally Modeled Code-Modulation Based Embedded Test for Phased Arrays. (Under the direction of Brian Floyd.)

Emerging technologies such as fifth-generation cellular communications (5G), satellite communications, short-range communications and defense applications require high performance phased arrays. A significant portion of the production cost of these systems is contributed by the test and calibration of the phased arrays. Built-in Self-Test (BIST) provides alternative low-cost technique of performing on-site chip diagnosis of the array elements, eliminating the requirement of external vector network analyzers (VNAs) to measure gain and phase of individual array channels. In such tests, in situ measurement of each array element allows capturing of array-level degradations such as supply-voltage variations, mutual element coupling or process variations in terms of amplitude and phase response of a calibration signal. A typical approach to perform such tests is to introduce an input test signal into the RF signal path to excite an array. The response of the array is then measured with a test signal processing unit such as a power detector or a coherent quadrature down-conversion mixer.

The state-of-the-art CoMET or Code-Modulated Embedded Test, was such a BIST approach that allowed a fast and simple way to calibrate phased arrays using orthogonal codes. This technique used an on-chip embedded path to inject a millimeter-wave calibration signal into the elements through a built-in parallel distribution network. Code-modulation was employed to the test signal at the phase shifter to rotate the phase of each array element across the four Cartesian quadrants. The combined array response was then passed through a square-law device and down-converted into a baseband. The squaring operation allowed the baseband to be comprised of element-pair correlations where each correlation term was modulated by the product of two unique codes. These orthogonal code products were used to process the baseband waveform digitally from which the array-performance
parameters were extracted. The parameters extracted were each element’s output power, phase response, quadrature accuracy and relative phase offset to other elements. CoMET was implemented in 0.13\(\mu\)m SiGe BiCMOS technology for a four-element receiver array. The measurement was performed using an oscilloscope and the baseband waveform was processed in Matlab. Gain and phase of each element were extracted using CoMET, and compared with measured results using a VNA which were accurate within 1 dB and 5° respectively. To improve the extraction accuracy of CoMET, a methodical approach is necessary to establish the robustness of this testing technique. For this, it is important to account for the behavioral circuit non-idealities that affect phased array channels. Further, it needs to be investigated whether these imperfections introduce errors into the CoMET technique itself, thus degrading the accuracy of CoMET-based extraction. Finally, it needs to be explored if there exists a better approach or perhaps a modification in existing CoMET technique that can calibrate out the non-ideal response of phased array circuits.

In this work, CoMET has been extended for use in a series-feed series-combining scenario, allowing this technique to be used in a wider range of phased arrays. To achieve this, a behavioral model-based implementation of CoMET has been performed. The calibration signal is injected using an embedded ‘series-feeding’ network and extracted using a ‘series-combining’ network, employing two input and two output ports for signal injection and extraction respectively. This technique permits quadruple sets of measurement data which allow extraction of amplitude errors, phase errors and phase offsets caused due to input and output path delays in series type networks.

Inaccuracies in parameter extraction occur predominantly from imbalances in signal-encoding at the phase shifter. To analyze the accuracy of CoMET extraction, an error model comprised of the typical phase shifter non-idealities is constructed. This prototype is integrated into the CoMET-behavioral model from which non-ideal response of the array are analyzed and errors are extracted. The additional measurement capabilities provided by this method allow extraction of non-idealities in the array as well as in the CoMET
technique itself. Matlab results shows good accuracy between array channel response to the CoMET-extracted response, validating the accuracy of this approach.
Behaviorally Modeled Code-Modulation Based Embedded Test for Phased Arrays

by
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A thesis submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the Degree of Master of Science

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DEDICATION

To my parents
BIOGRAPHY

The author was born and raised in West Bengal, India. She received two undergraduate degrees, first in Physics and second in Radio Physics and Electronics in the years 2010 and 2013 respectively from the University of Calcutta, India. She was a research scientist at SAMEER Kolkata Centre, India from 2013 to 2016 where her work and interests were focused on design of millimeter-wave transceiver systems.

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At millimeter-wave frequencies, there is a vast range of unlicensed spectrum that can be used to achieve high speed wireless data communication as shown in Figure 1.1. Advancement in silicon-based semiconductor technology where maximum oscillation frequencies can go above 200 GHz have enabled design and integration of complete transceivers on-chip in the millimeter-wave regime. Such transceivers find applications in radar technology, fifth-generation new radio (5G NR) cellular communication, short-range point-to-point communication around 60 GHz, millimeter-wave imaging etc. [Kis10a].

At these high frequencies, the effective dimension of a communicating omni-directional antenna is drastically reduced which lowers the equivalent power transmitted or received by the system [Bal16]. The emergence of phased arrays or beam-formers occurred as a means to address this issue where, multiple antenna element response could be combined to form
a directive beam that could be electronically steered to a desired direction [Sko81; PZ02a; PZ02b]. Phased array systems have successfully demonstrated multi-Gbps speed with 8, 16 and 32 elements. However, as the number of elements begins to rise, a significant part of the operating cost is invested in the testing of these millimeter-wave phased arrays [Gre16]. This include the lab equipment upgrades required to perform these measurements such as vector network analyzers, signal generators, power detectors, spectrum analyzers etc. Apart from the expensive equipment, the testing time is yet another factor that primarily contributes to the cost of test [Gre17]. Since phased arrays consist of multiple elements, each element needs to be characterized individually across different phase settings, to calibrate the array as a whole. Further, a high density of circuitry on-chip impose serious challenges upon the ability to trace circuit impairments due to functional errors or ageing of the individual blocks within the transceiver [Kis10b]. To simplify testing procedures as well as perform an on-chip circuit diagnosis, a new type of test procedure called Built-In Self-Test or BIST was introduced [Akb04]. BIST enables parallel characterization of phased array elements using simple low-cost, low-frequency test equipment at the expense of additional testing circuitry area and slightly increased power consumption on-chip.

The work presented in this thesis focuses on the behavioral modeling a novel BIST technique that can be implemented using low-cost, low-frequency test equipment. The presented BIST model makes use of code-modulation based testing scheme that can be
easily implemented on-chip to simultaneously characterize multiple array elements.

1.1 Prior work on BIST techniques

Phased arrays are used to synthesize beam patterns in a desired direction. The quality of measured pattern is dependent on the accuracy of amplitude and phase response within each element of the array. Current manufacturing tests for phased arrays requires each element to be individually measured using expensive millimeter-wave test equipment. BIST techniques offer simple low-cost ways to detect errors in the phased array system that can be introduced from antenna, chip-packaging, non-linearities of amplifiers and phase shifters. Larger arrays are becoming essential in high performance communication systems. Millimeter-wave phased arrays using 16-32 elements have been demonstrated in [Val10; Coh10; Ina13; Kim12]. RF power detection in BIST techniques have been demonstrated by using power detectors in [Tak14; Sle10; Coh12] or by using I/Q downconversion mixers in [Ina12; Ina13; Kim12; Kan16; Yeh18] to generate baseband signals. Illustrated in Figure 1.2 are BIST detection approaches commonly used on phased array systems.

In [Tak14], multiple built-in blocks were used within the receiver for monitoring and measuring gain and phase response. In [Sle10] power detectors were used to calibrate a 60 GHz low noise amplifier performance by dynamic adjustment of operating points under varying conditions of PVT (Process Voltage and Temperature). In [Coh12], BIST system is embedded in the array to extract the gain, output power and phase difference of the TX and RX elements. RF signal coupled from two adjacent chains are combined and a common detector is used to sense the phase difference between them. Power detectors were also used at each element’s output to extract output power. Gain and power in both TX and RX are measured using two additional detectors and a loopback path.

The use of I/Q down-conversion receiver was presented in [Kim12]. The work demonstrated a BIST technique where, a 38-42 GHz local-oscillator (LO) input was used as a test
Figure 1.2 Prior BIST techniques for characterizing millimeter-wave phased arrays.[Gre17](a) Using power detectors shown in red and yellow to extract elemental power and relative phase information between adjacent elements. (b) A coherent IQ down-conversion receiver to extract amplitude and phase information from each array element

signal that could be distributed to a 16-element phased array receiver system as shown in Figure 1.3. The detection technique required using in-phase (I) and quadrature-phase (Q) homodyne mixers to allow measurement of phase and amplitude of individual array channels where each channel could be turned on sequentially. Drawbacks of this approach include requirement of coherent I/Q down-conversion receivers that occupy significant area on-chip. Also, the technique does not allow for parallel measurement of multiple element response.

To allow parallel characterization of array elements, orthogonal codes were used to modulate a calibration test-signal and was presented in [Sil97; Lie10; Gre18]. In [Sil97], two algorithms, UTE (Unitary Transform Encoding) and CCE (Control Circuit Encoding) were introduced for remote calibration of phased array antennas. The technique involved transmission of multiplexed orthogonally encoded signals that could be received and coherently detected, combined in vector forms, and decoded with the inverse of the encoding matrix. The UTE algorithm requiring additional encoding hardware was more suited for digital beamforming whereas, the CCE algorithm was ideally suited for analog beamform-
The CCE method encoded phased array elemental signals using Hadamard matrices [Wal23] to allow simultaneous measurement of all array elements in-situ. The CCE technique was further used to calibrate active phased array antennas in [Lie10]. This work involved free-space measurement which relied on code modulation as shown in Figure 1.4. A coherent source was applied to the input of the array, the signal was mixed with orthogonal codes. The encoded signal was radiated and received through a calibration probe and sent to a coherent receiver and processor. The merit of this approach was that each element response was obtained through correlation of the received signal with the original code. However, a key challenge for this technique was the need for a coherent receiver to perform BIST on the array. Additionally, the technique was hardware-intensive requiring a calibration probe and coherent receiver. Finally, a code-modulation based novel technique called Code-Modulated Embedded Test (CoMET) was presented in [Gre16; Gre17; Gre18]. This technique, allowed for parallel measurement of the array with very simplified signal injection and extraction capabilities. A block diagram of a receiver array
Figure 1.4 Phased-array far-field characterization using orthogonal coding technique [Lie10]

Figure 1.5 Block diagram of a receiver array using CoMET [Gre18]
using CoMET is illustrated in Figure 1.5, where corresponding waveforms are shown at key nodes. It eliminates the need for I/Q down-conversion and uses a single power detector to down-convert the combined array response into baseband. Each element is uniquely code-modulated by a sequence from the Hadamard-Walsh [Wal23; Edw75] matrix which encodes each element uniquely. A single power detector allows correlation-based extraction of performance parameters of the array. CoMET was further demonstrated for free-space characterization and calibration of an 8-element phased array transceiver packaged with antennas [Hon19]. A high-gain horn antenna in the far-field detected the code-modulated transmitted signal from the array as shown in Figure 1.6a. The signal was then amplified and sent back over free-space to the on-chip power detectors to create a cross-correlation based interference pattern from which the array performance parameters were extracted. The extracted gain and phase response in comparison to VNA measurements were accurate to within 0.4 dB and 4° respectively. Additionally, a calibration loop was used to equalize gain and phase response across the array elements which resulted in gain and phase offsets to be reduced to 1.1 dB and nearly 0° respectively. The calibrated beam pattern is shown in 1.6b.

![Figure 1.6 Characterization of 8-element TX array using over-the-air CoMET [Hon19]](image)
The benefit of CoMET is that it is a built-in approach and can be used for characterizing chip-level phased arrays. It avoids the use of I/Q down-converters and uses a single power detector for converting array response to baseband, saving significant area on-chip. The elements on-chip are active during the measurement. Thus, performance impairments due to non-ideal array operation such as mutual coupling are included in the characterization. This technique by far stands out from the other BIST approaches discussed above in terms of the simplicity of its implementation and the ease of parameter extraction. The following chapters will discuss the need to implement CoMET in a new BIST architecture. However, to better understand this testing technique, a brief review of the state-of-the-art CoMET approach is done next.

1.2 Review of CoMET state-of-the-art

CoMET was introduced as a behavioral model in [Gre16] for a four-element phased array receiver and was later designed and demonstrated on-chip in a 60 GHz receiver and transmitter as shown in Figure 1.7(a) and (b) respectively [Gre17; Gre18]. This BIST approach involved a parallel-feed network architecture embedded in a four-element receiver is shown in Figure 1.7(a). A millimeter-wave test signal (also known as calibration signal in this report), was injected into the array and equally power-divided to the front-end of each element path in the array by way of coupling the test signal into the RF input lines of each element. The signal was then amplified by the LNA of gain, $G$, and then phase modulated at the phase-shifter. At the phase-shifter the test signal was branched into in-phase (I) and quadrature-phase (Q) waves where unique Rademacher codes, $c_i$ and $c_q$ [Edw75] were applied to the signal at each element respectively as shown in Figure 1.5. The phase shifter topology used in COMET is an active vector-interpolator type [TN09]. In this topology, the codes applied to each of the I and Q channels may be phase inverted at any frequency. The code-modulated signal was then combined by a N:1 RF power combiner.
and coupled into a square-law device such as a power detector. The detector performed a squaring operation that down-converted the combined array response into a baseband signal which was comprised of the cross-correlation of two elemental responses along with their corresponding orthogonal codes mixed together. Thus each correlation term consisted of a unique orthogonal code-product (OCP). The OCPs are formed by the multiplication of two codes whose product forms a new, unique orthogonal code. One subset of the Walsh-Hadamard [Wal23] functions contain the OCP property which were used in this work. The novel concept of OCP enabled element-pair correlations to be extracted by decoding the modulated baseband signal using corresponding OCP. The orthogonal nature of the codes allowed only a single cross-correlation to be filtered out at a time. The correlations that are solutions to a system of non-linear equations were converted to digital domain and processed in Matlab [MATe ] to extract the performance parameters of the array. The non-linear equations were comprised of all possible element-pair correlations from which the relative phase difference between element paths as a result of mismatch, process variations and supply gradients across the chip could be extracted [Gre17]. A brief discussion of the encoding, decoding and equation-solving procedure used is discussed next.

### 1.2.0.1 Signal encoding and decoding

An overview of CoMET-based injection and extraction is discussed in this section which is taken from [Gre17]. The analysis although done for a receiver is equally applicable to a transmitter. The output response of the \( n^{th} \) element in Figure 1.7(a), that is phase shifted by \( \theta \) is given by,

\[
E_n(t) = [G_n A_n] \cos(\omega t - \theta_n - \phi_n) \tag{1.1}
\]

Here, \( G_n \) is the gain of the element, \( A_n \) is the amplitude of the phase shifted signal, \( \theta_n \) is the applied phase shift applied to the \( n^{th} \) element and \( \phi_n \) is the phase offset of each element. In the behavioral analysis, the calibration signal is considered to be a perfect sinusoid at the
characterization frequency. The response at the output of the \(n^{th}\) element can be written as the sum of an in-phase and a quadrature-phase signal as:

\[
E_n(t) = G_n[I_n + Q_n] \\
I_n(t) = A_{i,n}(\theta_n)\cos(\omega t - \phi_n) \\
Q_n(t) = A_{q,n}(\theta_n)\sin(\omega t - \phi_n)
\]

Where, \(A_{i,n}\theta_n = A_n\cos(\theta_n)\) and \(A_{q,n}\theta_n = A_n\sin(\theta_n)\). The I and Q signals are encoded with two different orthogonal Rademacher code sets, where details on Rademacher codes can be found in [Edw75]. Multiplication of two such unique code sets forms another unique orthogonal code product (OCP). The response becomes,

\[
E_n(t) = G_n[(c_{i,n}A_{i,n})(\theta_n)\cos(\omega t - \phi_n) + (c_{q,n}A_{q,n})(\theta_n)\sin(\omega t - \phi_n)]
\]

The sum of I and Q vectors at phase shifter output creates a resultant vector set at an angle \(\theta\) which is modulated across all the four quadrants by sign-inverting the signal by the codes as shown in Figure 1.8. This creates \(I \rightarrow \overline{I}\) and \(Q \rightarrow \overline{Q}\) components, where the primed axis
denotes a phase inverted I and Q. The response from all the parallel receiver channels are multiplexed together via an RF combiner to form an aggregate response, $E_{tot}$. The aggregate response is then passed through a power detector which performs a square-law operation, forming a resultant baseband, $E_{tot}^2$.

\[
E_{tot}^2 = [G_1^2(A_{i,1}^2 + A_{q,1}^2) + G_1^2(A_{i,1}^2 + A_{q,1}^2)] \\
+ c_{i,1}c_{q,1}[2(G_1 A_{i,1})(G_2 A_{q,1})] \cos(\omega t - \phi_{i,1}) \sin(\omega t - \phi_{q,1}) \\
+ c_{i,1}c_{i,2}[2(G_1 A_{i,1})(G_2 A_{i,2})] \cos(\omega t - \phi_{i,1}) \cos(\omega t - \phi_{i,2}) \\
+ c_{i,1}c_{q,2}[2(G_1 A_{i,1})(G_2 A_{q,2})] \cos(\omega t - \phi_{i,1}) \sin(\omega t - \phi_{q,2}) \\
+ c_{q,1}c_{i,2}[2(G_1 A_{q,1})(G_2 A_{i,2})] \sin(\omega t - \phi_{q,1}) \cos(\omega t - \phi_{i,2}) \\
+ c_{q,1}c_{q,2}[2(G_1 A_{q,1})(G_2 A_{q,2})] \sin(\omega t - \phi_{q,1}) \sin(\omega t - \phi_{q,2}) \\
+ c_{i,2}c_{q,2}[2(G_1 A_{i,2})(G_2 A_{q,2})] \cos(\omega t - \phi_{i,2}) \sin(\omega t - \phi_{q,2})
\]

The detected output is a code-modulated pattern of intermixed terms represented by equation 1.6 where each row is encoded by a unique code-product formed by the multiplication of two codes. The first line in the equation represents the codes multiplied with themselves. These terms may be called the Self-Code products that gives the total output power of the
Correlations between a single element’s I and Q components should vanish due to orthogonality of the quadrature components but quadrature accuracy is never a perfect 90 degrees. The finite value of these code-products measures the quadrature accuracy within a single element (lines 2 and 7). Correlations between two different element’s I and Q components terms have finite values because phase delay through each element is not equal. The finite value of these terms measures the phase-offset between the elements (lines 4 and 5). Correlations between two I or two Q products gives the output power and phase of each element using (lines 3 and 6).

To decode the modulated baseband signal, the OCPs are used to form pairwise elemental correlations. The composite signal is multiplied by each OCP and then integrated over the full code duration. This results in three categories of cross correlations: auto-correlations, quadrature-phase correlations and in-phase correlations. The system of equations generated using these correlations are then solved in an iterative solver in Matlab. The average I and Q vector magnitudes represented by $G_n A_{i,n}$ and $G_n A_{q,n}$ are extracted first. The in-phase terms are assumed to have little or no phase error which simplifies the extraction process incurring less than 3.5% error [Gre17]. The extracted amplitudes are then used to solve the second category of equations with the quadrature correlations from which the quadrature inaccuracies are derived. Equations containing the phase offset errors are solved to obtain inter-elemental phase offsets of the array. Finally the code-offsets are solved by exploiting the symmetry property of the OCPs. The output power, $P_{out,n}$ and phase of each element, $\theta_n$ are estimated using the following equations, the derivation details are given in [Gre17]:

\[
P_{out,n} = \sqrt{(G_n A_{i,n})^2 + (G_n A_{q,n})^2} \tag{1.7}
\]

\[
\theta_n = \arctan \left( \frac{G_n A_{q,n}}{G_n A_{i,n}} \right) \tag{1.8}
\]
1.2.1 Experimental verification

In the test setup, a millimeter-wave signal generator injected a 60 GHz single-frequency tone through the BIST path in a four-element receiver array with each element’s RF input left un-terminated. Each element path was phase-modulated by writing phase settings through an on-chip serial-programmable interface (SPI). The interference pattern obtained from the combined response of each array element was coupled to an on-chip power detector which performed the squaring function, down-converting the signal to baseband and filtering high frequency component. The power detector output was amplified by an external op-amp and captured using an oscilloscope that sampled the waveform. The modulated waveform at a particular phase setting produced by the power detector and captured by the oscilloscope is indicated in Figure 1.9a [Gre17]. Code pulses of 5 ms duration resulted in a modulated waveform of duration 1.28s. The red lines in the figure indicate the begin and end time of the phase modulation. From this collected data, the cross-correlations between I-Q, I-I and Q-Q components of the elements could be extracted using the concept of OCPs discussed previously. Shown in Figure 1.9b is an illustration of the correlation values obtained from the collected baseband waveform corresponding to the phase setting. The system of non-linear correlation equations generated in the behavioral model were used to extract amplitude error, quadrature phase error, inter-elemental phase offsets, elemental output power and elemental phase-shift.

The average gain and phase response extracted using COMET were compared to VNA measured results for different phase settings. Using swept power responses the gain and phase response of each element were compared as shown in Figure 1.10a and 1.10b respectively. The extracted average gain response of each element was within 1 dB of the measured VNA result of individual elements whereas average phase response was within 5°. Quadrature error of individual elements and phase offsets between different elements were within 2° and 3° respectively and is shown in Figure 1.10c and 1.10d respectively.
(a) Baseband waveform produced by the power detector captured by oscilloscope in COMET test setup at a certain phase setting

(b) Correlations extracted using OCPs from the modulated baseband waveform

**Figure 1.9** Illustration of baseband data processing in CoMET technique [Gre17]
Figure 1.10 Performance parameters extracted in a swept power response using CoMET compared to the measured response of individual elements using a VNA [Gre17]
1.3 Objective

The work presented in this thesis attempts to explore how CoMET-based extraction can be made more accurate than the prior art and used in a wider range of phased arrays. This is accomplished by making the CoMET behavioral model more robust by combining the attractive features of the prior approach and introducing them into a new type of BIST architecture which will be called the "series-fed" BIST architecture in this report. CoMET technique will be modified to analyse a behavioral model of the phased array system in this new BIST architecture. A new measurement scheme is introduced which increases the obtained non-linear cross-correlations to fourfold by simple manipulation of the signal injection and extraction directions. Characteristic non-linearities are introduced into the behavioral model and extracted using CoMET. The accuracy of CoMET-based extraction of an element is compared to the individual response of an array element. The cross-correlations that preserve performance parameters of the array have been used to further evaluate the circuit non-idealities.

Chapter 2 presents the limitations of CoMET in the prior art and discusses the motivation behind a new BIST architecture. Using the series-fed technique, CoMET is behaviorally embedded into a two-element phased array system and implemented using Matlab. A new measurement scheme is presented that quadruples the set of down-converted data. Mathematical analysis of the approach is described including modifications in the encoding, decoding and extraction technique.

Chapter 3 discusses the importance of phase shifter performance on the accuracy of CoMET-based extraction. It analyses typical phase shifter non-idealities that degrade the performance of an array element and introduce amplitude and phase error in the array response. The impact of each type of non-ideality on the phase shifter output is discussed and an error model of the phase shifter is thus constructed.

Chapter 4 behaviorally implements the non-ideal phase shifter model into a CoMET-
embedded phased array system with the series-fed BIST architecture and performs ex-
traction using the measurement approach introduced in Chapter 2. Finally, the extracted
performance using COMET is compared versus the response of individual elements to
quantify the accuracy to which COMET is able to characterize an array.
In Chapter 1, the CoMET technique was reviewed that uses a built-in distribution network allowing injection and extraction of calibration signals. Code-modulation applied to the elemental signals at the phase shifter uniquely encoded each array channel. The advantage was that, the combined array response was down-converted and decoded in a single measurement. The detected baseband signal was comprised of cross-correlations between all elemental signals from which amplitude and phase response of the array were extracted. However, a new methodical approach for evaluating the achievable extraction accuracy of CoMET is necessary to enhance the accuracy of this testing method.

In this Chapter, first the limitations of the CoMET technique in the prior art are dis-
cussed. This gives insight into the issues that need to be resolved to improve CoMET-based testing accuracy. Second, a new type of 'series-fed' BIST architecture motivated from [Yeh18] is behaviorally introduced into a two-element phased array system and CoMET-based analysis is performed. In the new architecture, modifications in CoMET-based measurement scheme is introduced by the use of two input BIST ports and two output baseband ports. This scheme allows routing the signal through four different BIST paths, resulting in four sets of measurement data obtained from the same array. The system uses additional CoMET-based circuitry such as pairs of RF combiner and square-law device for down-conversion of the RF signal at the two baseband ports. Each measurement data-set is comprised of cross-correlations between the I and Q components of the array element. With quadruple measurement sets, the number of correlation values increase fourfold, where each correlation equation defines an element's performance parameters. In the extraction technique, the benefit is that, each correlation term is decoded with the same-set of OCPs across all the four measurement sets, avoiding the requirement of additional OCPs. However, this comes at the expense of measurement time which is fourfold in comparison to the prior art. Third, the series topology introduces path delays in between consecutive elements that manifests as phase offsets. With four times the correlation equations, additional performance parameters can be evaluated. The mathematical analysis of the encoding, decoding and extraction procedure is discussed. CoMET-based extraction is used through a Matlab [MATe] implementation to characterize the array.

2.1 Limitations of state-of-the-art CoMET

In the prior art, CoMET was validated by comparing extracted performance parameters to VNA measurement results. While the gain of each element agreed with VNA measurements within 1 dB error, the phase response agreement occurs within 5° of error. These disagreements reveal the limitation in CoMET’s ability to accurately estimate the amplitude and
phase error to characterize the array. The following section discusses the limitations of CoMET and the implications of these limitations on the error-extraction capability using this testing technique:

1. CoMET could not capture AM-PM distortions that is correlated between the elements. The main reason for this error can be attributed to the gain dependent phase variation of the Variable Gain Amplifiers (VGAs) in the phase shifter block of each array channel. The phase of the VGAs varying as a function of its gain results in non-linear distortions in both amplitude and phase. There will be more discussions about this in Chapter 3.

2. The quadrature-phase correlations, used to determine the quadrature accuracy of individual elements and phase offsets between two different elements, were accurate to within 2° and 3° respectively. The reason can be attributed to the fact that the 90° coupler splitting the signal into inphase and quadrature-phase components can itself have imperfections in both amplitude and phase. These imbalances needs to be extracted to get an accurate estimation of the quadrature error.

3. At the phase shifter, the magnitude for the I and Q vectors are switched in sign to form \( I, \tilde{I} \) and \( Q, \tilde{Q} \) vectors and thereby obtain phase shift in all four quadrants. Amplitude offsets in I and Q signal vectors may cause defect in signal encoding manifesting as code errors.

4. The power detector in CoMET was used for the simplicity of the detection step, avoiding the need to provide coherent quadrature down-conversion mixers and local-oscillator signals. The accuracy of CoMET is dependent upon the accuracy of its detection technique that performs the squaring operation to generate the OCPs in elemental cross-correlation product terms. However, duration of CoMET measurement and its accuracy may be limited by response time of the power detector. The speed of detection has to be faster than the fastest code, otherwise the detection may be erroneous disturbing the integrity of the entire down-converted baseband pattern.
5. The passive parallel distribution network occupies area on-chip. For larger arrays, this can cause space constraint on-chip. Moreover, the $\lambda/4$ impedance matching transformers are narrow-band. So, a broadband approach is more desirable.

6. The demonstration of CoMET has been performed on 4 and 8-element arrays [Gre18], [Hon19]. The Hadamard-Walsh codes [Wal23] applied to the in-phase and quadrature-phase components for each phase-shifter result in long codes whose duration scales by a factor of $2^n$, where 'n' is the number of elements in the array. Thus, the approach is only limited to smaller arrays. As the arrays scale up with more number of elements, longer codes are required, increasing encoding and decoding time. This limitation may be overcome either by using faster on-chip code generators that reduce code duration or by using other orthogonal codes that minimize the testing time relative to the array size.

### 2.2 Motivation for a modified BIST architecture

To perform element-level as well as array-level self-test, a new type of built-in self test (BIST) architecture was introduced by [Yeh18] and is shown in Figure 2.1. The series-fed BIST network path highlighted in grey was implemented as a means to save significant area on-chip when compared to its parallel network counter-part given in [Gre17]. Moreover, adopting a series-fed network ensured a broadband matching alternative to the narrow-band limitation of the parallel network design. In the following sections CoMET has been behaviorally implemented into this series-fed BIST architecture. It will be observed that in addition to the above mentioned advantages, this BIST architecture has the capability to overcome some of the major limitations of CoMET described in Section 2.1.

The BIST architecture shown in Figure 2.1, employs an input series-feed path and an output series-distributed network common to all the array elements. Shown at the ends of the embedded path in Figure 2.1 are two BIST ports, $\textit{BIST\_RFin1}$ and $\textit{BIST\_RFin2}$. Each
BIST port has the capability to be terminated by 50 Ω. The 50 Ω resistor termination is turned ON/OFF using a controlled switching operation. The advantage is that, when one of the two BIST RF ports sends in an input calibration test signal for array characterization, the other BIST RF port can be broadband terminated. This allows the input calibration signal to be coupled to the front-end of each array element without suffering significant reflections along the length of the built-in path. To implement CoMET in this architecture, the calibration signal will be injected using the input series-fed network and extracted from the output series-distributed network. This modified technique allows injection of the test signal from either of the two BIST input ports via the built-in paths. Since the two BIST input ports are located at the opposite ends of the embedded path, the injected test signal directions are reversed. From a mathematical standpoint, we will observe that the signal accrues modified phase information due to the reversal in test signal direction. In the following section, this built-in architecture will be used to perform CoMET-based measurement behaviorally on a series-fed phased array architecture. To avoid mathematical complexity, we will be performing the analysis using a two-element phased array system.
However, this technique can be extended to larger array systems. As arrays are scaled up in size, the built-in path design simply requires the series feeding transmission line to be extended to the input signal-injection points of the array elements. We will find that the architectural advantage of sending in the calibration signal in two reverse signal path directions can be leveraged to obtain twice measured data-sets which is a benefit over the state-of-the-art technique. We will utilize this expanded data-sets to achieve better accuracy in characterizing the array elements using CoMET.

2.3 CoMET embedded series-fed phased array architecture

This section discusses the behavioral implementation and analysis of CoMET-embedded receiver architecture for a multi-element series-fed phased array receiver. First, we introduce the adaptations required in the COMET measurement technique for this type of architecture. Then, a detailed mathematical analysis of this model is performed. The analysis is completely a mathematical treatment of the system where the calibration test signals are characteristically emulated using mathematical equations.

Shown in Figure 2.2 is a behavioral block diagram of an ‘n’ element receiver array. The built-in path required to perform CoMET on this system is highlighted in green. The architecture is similar to Figure 2.1, except, here the elements are placed adjacent to each other and the input BIST path runs as one straight embedded path. First, a single-tone sine wave is injected as a calibration signal into one of the two input BIST RF ports indicated as BIST RF_{in,1} and BIST RF_{in,2}. When one BIST input port is ON, the other BIST input port is considered terminated by a matching impedance. The RF input ports given by RF_{in,1}, RF_{in,2} etc. are left open. The input calibration signal traverses through the lossy series path and is capacitively coupled into the front-end of each array element. The series feed introduces a path delay between two consecutive array elements and is denoted by a phase lag, ψ. While passing through each array channel, the signal is amplified by behavioral amplifier
blocks (LNA). The amplified signal passes through a passive 90° coupler and branches out into in-phase (I) and quadrature-phase (Q) components. Each I and Q component in the array channel is then phase-shifted by a desired angle, $\theta$. The phase shifter topology is a phase-interpolation type. It consists of variable gain amplifiers (VGAs) in both the I and Q paths. We term the two VGAs as I-VGA and Q-VGA respectively. The VGAs are used to weight the I and Q signal vectors to achieve a desired phase shift per element. Once a desired phase angle is set, the I-VGA and Q-VGA signal outputs are modulated by orthogonal codes. These are unique Rademacher codes, $(c_i(t), c_q(t))$ that are applied to each I and Q path. The codes modulate the amplitude of each I and Q path signals by a unique sequence of +1 and -1 across all gain settings of the I and Q-VGAs. The advantage of code-modulation is that each I and Q paths can be sign inverted. Thus, a summation of the weighted I and Q vectors can create phase settings across all the four Cartesian quadrants. The phase shifted signal from all the channels are collected through a shared lossy series-distributed network. The collected signal is first RF combined and then down-converted to a baseband output using

---

**Figure 2.2** Block diagram of a phased-array receiver in the series-feed series-combine BIST architecture
a square-law device. However, in this modified architecture, we introduce two different output ports at a time to capture the baseband.

Along with the two RF output ports of the array, i.e. $RF_{out,1}$ and $RF_{out,2}$, in this measurement technique we behaviorally introduce two different baseband (BB) output ports for the collection of the code-modulated baseband aggregate signal data, indicated as $BB_{out,1}$ and $BB_{out,2}$ in Figure 2.2. As the baseband signal is collected out of one of the two BB ports, the other BB port and all other RF output ports of the array are OFF or left terminated by 50 $\Omega$. This is done to ensure that the collected code-modulated signal of the array can only be obtained at one of the two BB ports at a time. Since there are two input BIST RF ports, by alternately turning ON each BIST input port, two sets of BB output data can be obtained per signal-injection operation. A pictorial representation of the calibration signal injection and extraction mechanism using the two BIST input and two BB output port is indicated in Figure 2.2. Thus, this modified approach is able to create four sets of BB measurement data. Table 2.1 indicates the port conditions for injection and extraction that are used to obtained each set of measured data. We will call them measurement sets 1, 2, 3 and 4. Figures 2.2a, 2.2b, 2.2c and 2.2d illustrate the signal routing directions for each measurement set respectively.

**Table 2.1** Port conditions for the four measurement sets

<table>
<thead>
<tr>
<th>Meas. set</th>
<th>Injection port</th>
<th>Extraction port</th>
<th>Terminated/OFF ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BIST $RF_{in,1}$</td>
<td>$BB_{out,1}$</td>
<td>BIST $RF_{in,2}$,$RF_{out,1}$,$RF_{out,2}$,$BB_{out,2}$</td>
</tr>
<tr>
<td>2</td>
<td>BIST $RF_{in,1}$</td>
<td>$BB_{out,2}$</td>
<td>BIST $RF_{in,2}$,$RF_{out,1}$,$RF_{out,2}$,$BB_{out,1}$</td>
</tr>
<tr>
<td>3</td>
<td>BIST $RF_{in,2}$</td>
<td>$BB_{out,1}$</td>
<td>BIST $RF_{in,1}$,$RF_{out,1}$,$RF_{out,2}$,$BB_{out,2}$</td>
</tr>
<tr>
<td>4</td>
<td>BIST $RF_{in,2}$</td>
<td>$BB_{out,2}$</td>
<td>BIST $RF_{in,1}$,$RF_{out,1}$,$RF_{out,2}$,$BB_{out,1}$</td>
</tr>
</tbody>
</table>

In the discussion on prior art of CoMET, it was observed that the BB data captures the array performance parameters in the cross-correlation terms. The two BB output ports in
(a) Measurement set 1

(b) Measurement set 2
Figure 2.2 Illustration of the four measurement sets. Indicated in red, are the signal injection and extraction path directions in the series-feed series-combine BIST topology.
this modified approach perform similar operations except that by doubling the BB output
data-set, the number of cross-correlation pairs are also doubled in total. The extra BB data
set are additional sets of information and will be used to evaluate the series phase offsets
introduced by the BIST input feed network and output distribution network. The following
section is a mathematical analysis of the modified CoMET approach.

2.4 Mathematical analysis of the modified approach

In this mathematical analysis we will first show how the codes are applied via the RF phase
shifters to each array element, code-modulating the I and Q paths. The modulated response
is then combined and down-converted to form a cross-correlation-mixed baseband pattern
resulting from the squaring operation. Four sets of baseband waveform is obtained from
which array performance parameters are extracted. The following analysis is restricted to a
two-element phased array, although it can be extended to ‘n’ number of array elements.

2.4.1 Encoding mechanism

Consider a single-frequency tone, \( \cos(\omega t) \) at the desired characterization frequency in-
jected into the input port, BIST RF\(_{i,n,1}\) and distributed to the two-element array. At a desired
phase setting, \( \theta_n \), the I and Q components will be amplitude weighted by the I and Q-VGAs
and may be indicated by \( A_{i,n}(\theta) \) and \( A_{q,n}(\theta) \) respectively, where ‘n’ is the index number of
the array element. The lossy series feed path embedded in the phased array architecture in-
troduces a path delay between any two array elements, we denote it by an equivalent phase
lag, \( \psi \) in the input distribution network. We consider that the input distribution signal
suffers a phase lag of \( \psi_{i,n} \) in the I-arm and \( \psi_{q,n} \) in the Q-arm of each element. Likewise, as
the signal is collected from the output series distribution network, the signal accrues path
delays between the output signal of two different elements owing to the series topology
of the network. This path delay manifests as a phase lag, \( \phi \) in the signal. Any path delay
occurring in the signal before series-combining is considered included in the phases $\psi_{i,n}$ and $\psi_{q,n}$. This assumption allows the use of a single output phase offset variable given by $\phi$ and avoids mathematical complexity.

For the following analysis, we consider a two-element phased array system. Out of the four measurement sets shown in Table 2.1, let us consider measurement set 1 where, the port $BB_{out,1}$ is ON, all other output ports remaining OFF or terminated as shown in Figure 2.3. The active input port, BIST $RF_{in,1}$ and $BB_{out,1}$ output port are indicated in red, all the other ports are inactive and have been faded out in the illustration. Ports $RF_{in,1}$ and $RF_{in,1}$ are left open, ports $RF_{out,1}$ and $RF_{out,2}$ are 50 $\Omega$ terminated, port $BB_{out,2}$ is considered OFF.

Considering two different orthogonal codes, $c_{i,n}$ and $c_{q,n}$ applied to the I and Q components

![Figure 2.3 Illustration of measurement set 1 in a two-element phased array. The active ports and path of calibration signal injection and extraction are indicated in red. The inactive ports are faded.](image)

of each element respectively, the code-modulated signal through the I and Q branches for
the elements 1 and 2 may be described by the following equations:

\[ E_{i,1}(t) = (A_{i,1} c_{i,1}) \cos(\omega t - \phi) \]  
(2.1)

\[ E_{q,1}(t) = (A_{q,1} c_{q,1}) \sin(\omega t - \phi) \]  
(2.2)

\[ E_{i,2}(t) = (A_{i,2} c_{i,2}) \cos(\omega t - \psi_{i,2}) \]  
(2.3)

\[ E_{q,2}(t) = (A_{q,2} c_{q,2}) \sin(\omega t - \psi_{q,2}) \]  
(2.4)

The above form of signal encoding corresponds to an initial phase setting, \( \theta_n \). Using code-modulation, the phase setting is cycled through all four quadrants by sign inverting \( I \) and \( Q \) components to \( \overline{I} \) and \( \overline{Q} \) depending on the sign of the code and the desired quadrant. Figure 2.4 graphically portrays this. Next, consider measurement set 2 shown in Figure 2.2b

\[ \begin{align*}
E_{i,1}(t) &= (A_{i,1} c_{i,1}) \cos(\omega t - \phi) \\
E_{q,1}(t) &= (A_{q,1} c_{q,1}) \sin(\omega t - \phi) \\
E_{i,2}(t) &= (A_{i,2} c_{i,2}) \cos(\omega t - \psi_{i,2}) \\
E_{q,2}(t) &= (A_{q,2} c_{q,2}) \sin(\omega t - \psi_{q,2})
\end{align*} \]

Figure 2.4 Polar diagram illustrating code-modulation of the in-phase and quadrature-phase components across the four Cartesian quadrants.

where, the output port BB\(_{out,2}\) is ON, all other output ports remaining OFF or terminated, the injection signal still coming in from port, BIST RF\(_{in,1}\). This will result in similar form of encoding as described by the above equations with some changes. This may be described
by the following equations:

\[ E_{i,1}(t) = (A_{i,1} c_{i,1}) \cos(\omega t) \]  
(2.5)

\[ E_{q,1}(t) = (A_{q,1} c_{q,1}) \sin(\omega t) \]  
(2.6)

\[ E_{i,2}(t) = (A_{i,2} c_{i,2}) \cos(\omega t - \psi_{i,2} - \phi) \]  
(2.7)

\[ E_{q,2}(t) = (A_{q,2} c_{q,2}) \sin(\omega t - \psi_{q,2} - \phi) \]  
(2.8)

Comparing equations 2.1-2.4 with equations 2.5-2.8, we find that the same array create modified sets of equation with difference being in the performance parameters indicated in sine and cosine arguments. This can be made useful to extract the series path delays.

Similar forms of equation can be obtained using measurement set 3 shown in Figure 2.2c, by injecting the test signal through the other input port, BIST RF\(_{in,2}\) and terminating BIST RF\(_{in,1}\). Considering the same initial phase setting, \(\theta_n\), and the output code-modulated signal collected at BB\(_{out,1}\), the signal equations change to the following:

\[ E_{i,1}(t) = (A_{i,1} c_{i,1}) \cos(\omega t - \psi_{i,1} - \phi) \]  
(2.9)

\[ E_{q,1}(t) = (A_{q,1} c_{q,1}) \sin(\omega t - \psi_{q,1} - \phi) \]  
(2.10)

\[ E_{i,2}(t) = (A_{i,2} c_{i,2}) \cos(\omega t) \]  
(2.11)

\[ E_{q,2}(t) = (A_{q,2} c_{q,2}) \sin(\omega t) \]  
(2.12)

Next, turning ON port BB\(_{out,2}\) keeping all other output ports OFF or terminated, the injection signal still coming in from port, BIST RF\(_{in,2}\) results in measurement set 4 shown in Figure 2.2d. The above equations get modified as:

\[ E_{i,1}(t) = (A_{i,1} c_{i,1}) \cos(\omega t - \psi_{i,1}) \]  
(2.13)

\[ E_{q,1}(t) = (A_{q,1} c_{q,1}) \sin(\omega t - \psi_{q,1}) \]  
(2.14)

\[ E_{i,2}(t) = (A_{i,2} c_{i,2}) \cos(\omega t - \phi) \]  
(2.15)

\[ E_{q,2}(t) = (A_{q,2} c_{q,2}) \sin(\omega t - \phi) \]  
(2.16)

Table 2.2 summarizes the modified phase arguments due to the four different measurement
sets. It can be observed from the table that the encoding procedure in the two-element array introduces 9 unknown parameters in both amplitude and phase terms. In the next section a description of the detection scheme is given that allows demodulation of the above derived equations to obtain valuable array metrics using CoMET.

Table 2.2 Summary of phase arguments and the unknown array parameters obtained from the signal equations of the four different measurement sets for a two-element phased array

<table>
<thead>
<tr>
<th>Meas. Set</th>
<th>Element</th>
<th>I/Q Path</th>
<th>Phase argument</th>
<th>Unknown parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>1</td>
<td>I</td>
<td>$\omega t - \phi$</td>
<td>$A_{i,1}, \phi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q</td>
<td>$\omega t - \phi$</td>
<td>$A_{q,1}, \phi$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>I</td>
<td>$\omega t - \psi_{i,2}$</td>
<td>$A_{i,2}, \psi_{i,2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q</td>
<td>$\omega t - \psi_{q,2}$</td>
<td>$A_{q,2}, \psi_{q,2}$</td>
</tr>
<tr>
<td>Second</td>
<td>1</td>
<td>I</td>
<td>$\omega t$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q</td>
<td>$\omega t$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>I</td>
<td>$\omega t - \psi_{i,2} - \phi$</td>
<td>$A_{i,2}, \psi_{i,2}, \phi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q</td>
<td>$\omega t - \psi_{q,2} - \phi$</td>
<td>$A_{q,2}, \psi_{q,2}, \phi$</td>
</tr>
<tr>
<td>Third</td>
<td>1</td>
<td>I</td>
<td>$\omega t - \psi_{i,1} - \phi$</td>
<td>$A_{i,1}, \psi_{i,1}, \phi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q</td>
<td>$\omega t - \psi_{q,1} - \phi$</td>
<td>$A_{q,1}, \psi_{q,1}, \phi$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>I</td>
<td>$\omega t$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q</td>
<td>$\omega t$</td>
<td>0</td>
</tr>
<tr>
<td>Fourth</td>
<td>1</td>
<td>I</td>
<td>$\omega t - \psi_{i,1}$</td>
<td>$A_{i,1}, \psi_{i,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q</td>
<td>$\omega t - \psi_{q,1}$</td>
<td>$A_{q,1}, \psi_{q,1}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>I</td>
<td>$\omega t - \phi$</td>
<td>$A_{i,2}, \phi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q</td>
<td>$\omega t - \phi$</td>
<td>$A_{q,2}, \phi$</td>
</tr>
</tbody>
</table>

2.4.2 Decoding mechanism

Each of the above set of code-modulated signals are RF combined and squared by a square-law device. Thus in this mathematical model, the $\sum I$ and $\sum Q$ signals are added from each element and squared. From the theory of CoMET we learnt that performing squaring operation on the code-modulated array response creates inter-elemental cross correlations. Following is a similar analysis on the signal equations. Using equations (2.1-2.4), the
squearing operation on the output signal, $E_{tot}(t)$ for measurement set 1 may be described by the following equations:

$$E_{tot}^2(t) = [A_{i,1}^2 + A_{q,1}^2 + A_{i,2}^2 + A_{q,2}^2]$$

$$+ 2(A_{i,1}A_{q,1})(c_{i,1}c_{q,1})\cos(\omega t - \phi)\sin(\omega t - \phi)$$

$$+ 2(A_{i,2}A_{q,2})(c_{i,2}c_{q,2})\cos(\omega t - \psi_{i,2})\sin(\omega t - \psi_{q,2})$$

$$+ 2(A_{i,1}A_{i,2})(c_{i,1}c_{i,2})\cos(\omega t - \phi)\cos(\omega t - \psi_{i,2})$$

$$+ 2(A_{q,1}A_{q,2})(c_{q,1}c_{q,2})\sin(\omega t - \phi)\sin(\omega t - \psi_{q,2})$$

$$+ 2(A_{i,1}A_{q,2})(c_{i,1}c_{q,2})\cos(\omega t - \phi)\sin(\omega t - \psi_{q,2})$$

(2.17)

Equation 2.17 shows different terms with cross-product combinations of the I and Q components of the array elements. Each term consists of an OCP as a result of the product of two codes. Each of these terms can be individually multiplied by the corresponding OCP so that the desired term is demultiplexed whereas all other terms average out to zero due to the use of orthogonal code products. Thus, the cross-correlations can be divided into three categories of correlations: a) The auto-correlations which are the result of mixing a modulated signal with itself. The auto-correlations are represented by the first line in Equation 2.17 and is proportional to the total power in the down-converted response. b) The quadrature-phase correlations formed by the cross-products of two signals in quadrature-phase to one another. These correlations are formed by the cross-product of the I and Q components of a single element or between two different elements and is represented by the second, third, sixth and seventh line in Equation 2.17. c) The in-phase correlations formed by the cross-products of two signals which are in-phase with one another. These products can take the form of two I components or two Q components shown in line four and five of Equation 2.17.
Using Matlab, the baseband composite signal is filtered, synchronously decoded by multiplying each correlation term with its corresponding OCP and integrated over a full code duration to form elemental correlations. The integration and parameter-extraction method follows the same technique as CoMET technique discussed in Chapter 1. The auto-correlation term is proportional to the total power in the baseband response, the quadrature-phase correlations capture quadrature error within the same element or phase offsets within two different elements. The in-phase correlations quantify the output power and phase of each element. The correlations that are solutions to a system of non-linear equations are processed in Matlab to evaluate the performance metrics of the array. The following is a representation of the set of cross-correlation equations obtained from Measurement set 1:

\[ R_{i,i,12} = (0.5)(A_{i,1}A_{i,2})\cos(\psi_{i,2} - \phi) \]  
\[ R_{q,q,12} = (0.5)(A_{q,1}A_{q,2})\cos(\psi_{q,2} - \phi) \]  
\[ R_{i,q,11} = (0.5)(A_{i,1}A_{q,1})\sin(\phi_{11}) \]  
\[ R_{i,q,22} = (0.5)(A_{i,2}A_{q,2})\sin(\phi_{22}) \]  
\[ R_{i,q,12} = -(0.5)(A_{i,1}A_{q,2})\sin(\psi_{q,2} - \phi) \]  
\[ R_{i,q,21} = (0.5)(A_{i,2}A_{q,1})\sin(\psi_{i,2} - \phi) \]

In the above equations, \( R_{i,i,12} \) and \( R_{q,q,12} \) represent the in-phase correlation values whereas, the remaining four correlations are the quadrature-phase correlations between same element \( (R_{i,q,11}, R_{i,q,22}) \) and two different elements \( (R_{i,q,12}, R_{i,q,21}) \). We thus have all the possible correlation equations needed to characterize the array. The correlation equations from the four different measurement scenarios modify the phase argument and are summarized in Table 2.3. Each measurement set results in six correlation equations A-F. The sign inversion in the phase arguments in sets 1,2 and 3,4 occur due to change in the signal injection direction. This property will be exploited to solve the correlation equations to obtain the performance parameters of the array.
<table>
<thead>
<tr>
<th>Meas. Set</th>
<th>Eqn.</th>
<th>Correlation type</th>
<th>Correlation value</th>
<th>Phase arguments</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>1A</td>
<td>In-phase</td>
<td>$R_{ii,12}$</td>
<td>$\psi_{i,2} \cdot \phi$</td>
</tr>
<tr>
<td></td>
<td>1B</td>
<td>In-phase</td>
<td>$R_{qq,12}$</td>
<td>$\psi_{q,2} \cdot \phi$</td>
</tr>
<tr>
<td></td>
<td>1C</td>
<td>Quadrature-phase</td>
<td>$R_{iq,11}$</td>
<td>$\phi_{11}$</td>
</tr>
<tr>
<td></td>
<td>1D</td>
<td>Quadrature-phase</td>
<td>$R_{iq,22}$</td>
<td>$\phi_{22}$</td>
</tr>
<tr>
<td></td>
<td>1E</td>
<td>Quadrature-phase</td>
<td>$R_{iq,12}$</td>
<td>$\psi_{q,2} \cdot \phi$</td>
</tr>
<tr>
<td></td>
<td>1F</td>
<td>Quadrature-phase</td>
<td>$R_{iq,21}$</td>
<td>$\psi_{i,2} \cdot \phi$</td>
</tr>
<tr>
<td>Second</td>
<td>2A</td>
<td>In-phase</td>
<td>$R_{ii,12}$</td>
<td>$\psi_{i,2} + \phi$</td>
</tr>
<tr>
<td></td>
<td>2B</td>
<td>In-phase</td>
<td>$R_{qq,12}$</td>
<td>$\psi_{q,2} + \phi$</td>
</tr>
<tr>
<td></td>
<td>2C</td>
<td>Quadrature-phase</td>
<td>$R_{iq,11}$</td>
<td>$\phi_{11}$</td>
</tr>
<tr>
<td></td>
<td>2D</td>
<td>Quadrature-phase</td>
<td>$R_{iq,22}$</td>
<td>$\phi_{22}$</td>
</tr>
<tr>
<td></td>
<td>2E</td>
<td>Quadrature-phase</td>
<td>$R_{iq,12}$</td>
<td>$\psi_{q,2} + \phi$</td>
</tr>
<tr>
<td></td>
<td>2F</td>
<td>Quadrature-phase</td>
<td>$R_{iq,21}$</td>
<td>$\psi_{i,2} + \phi$</td>
</tr>
<tr>
<td>Third</td>
<td>3A</td>
<td>In-phase</td>
<td>$R_{ii,12}$</td>
<td>$\psi_{i,1} + \phi$</td>
</tr>
<tr>
<td></td>
<td>3B</td>
<td>In-phase</td>
<td>$R_{qq,12}$</td>
<td>$\psi_{q,1} + \phi$</td>
</tr>
<tr>
<td></td>
<td>3C</td>
<td>Quadrature-phase</td>
<td>$R_{iq,11}$</td>
<td>$\phi_{11}$</td>
</tr>
<tr>
<td></td>
<td>3D</td>
<td>Quadrature-phase</td>
<td>$R_{iq,22}$</td>
<td>$\phi_{22}$</td>
</tr>
<tr>
<td></td>
<td>3E</td>
<td>Quadrature-phase</td>
<td>$R_{iq,12}$</td>
<td>$\psi_{i,1} + \phi$</td>
</tr>
<tr>
<td></td>
<td>3F</td>
<td>Quadrature-phase</td>
<td>$R_{iq,21}$</td>
<td>$\psi_{q,1} + \phi$</td>
</tr>
<tr>
<td>Fourth</td>
<td>4A</td>
<td>In-phase</td>
<td>$R_{ii,12}$</td>
<td>$\psi_{i,1} \cdot \phi$</td>
</tr>
<tr>
<td></td>
<td>4B</td>
<td>In-phase</td>
<td>$R_{qq,12}$</td>
<td>$\psi_{q,1} \cdot \phi$</td>
</tr>
<tr>
<td></td>
<td>4C</td>
<td>Quadrature-phase</td>
<td>$R_{iq,11}$</td>
<td>$\phi_{11}$</td>
</tr>
<tr>
<td></td>
<td>4D</td>
<td>Quadrature-phase</td>
<td>$R_{iq,22}$</td>
<td>$\phi_{22}$</td>
</tr>
<tr>
<td></td>
<td>4E</td>
<td>Quadrature-phase</td>
<td>$R_{iq,12}$</td>
<td>$\psi_{i,1} \cdot \phi$</td>
</tr>
<tr>
<td></td>
<td>4F</td>
<td>Quadrature-phase</td>
<td>$R_{iq,21}$</td>
<td>$\psi_{q,1} \cdot \phi$</td>
</tr>
</tbody>
</table>
2.4.3 Matlab-based extraction

The terms within the parenthesis in the correlation equations represent the parameters to be extracted. From Table 2.2, we observe that the phased array system has 9 unknown parameters. However, the four measurement scenarios result in 24 correlation equations as listed in Table 2.3. These equations are inserted into a non-linear equation solver, along with the extracted correlation values, where each element's amplitude and phase error is extracted. The solution is a step-by-step approach. First, we solve for the weighted I and Q magnitudes represented by $A_{i,n}$ and $A_{q,n}$ using Equations 2.18,2.19, 2.22 and 2.23. To do this, we first need to assign an initial value to the phases, $\psi_{i,2}$, $\psi_{q,2}$ and $\phi$. These phase values can be estimated from the series path lengths between two array elements at the input and output network in the array design. Using these, first amplitude terms given by $A_{i,1}$, $A_{q,1}$, $A_{i,2}$ and $A_{q,2}$ are solved. Once the amplitude terms are obtained, they are substituted in equations 2A and 2B to extract $\psi_{i,2}$ and $\psi_{q,2}$. Next, equation 2E extracts the output phase offset, $\phi$. Quadrature phase error of the two elements, $\phi_{11}$ and $\phi_{22}$ respectively are solved using equations 2C and 2D.

The input series phase offset value, $\psi$ is a function of the distance between two adjacent elements. To de-embed this phase offset, a test signal can be injected through one of the two input BIST RF ports and collected at the other BIST RF port. The total phase offset incurred by the test signal can be divided by the number of elements, ‘n’, to evaluate the path delay, $\psi$ (vide Figure 2.3) between two consecutive elements. However in our model, $\psi$ has been split into $\psi_{i,n}$ and $\psi_{q,n}$ to incur the elemental phase offset in each I/Q path which cannot be de-embedded easily. Shown in Figure 2.5a are the extracted input series phase offsets for both the elements in the CoMET behavioral model. The input offset, $\psi$ was varied from $-100^\circ$ to $100^\circ$ as a test value for the path delay. The benefit of CoMET is that it can extract phase offsets of each element’s I/Q path ($\psi_{i,1}$, $\psi_{q,1}$, $\psi_{i,2}$ and $\psi_{q,2}$) accurately to within $0.01^\circ$, which cannot be otherwise de-embedded. The de-embedded phase offset,
Figure 2.5 Extraction of series phase offsets in the input and output networks.
\( \psi \), can be subtracted from \( \psi_{i,n} \) and \( \psi_{q,n} \) to obtain each elemental phase offset. The output phase offset, \( \phi \) is extracted with an accuracy of within 0.01° for different values of input series offsets (i.e. \( \psi = 25^\circ, 50^\circ, 75^\circ \)) and is shown in Figure 2.5b.

Quadrature phase errors between the I and Q arms are extracted using CoMET for both the elements and is shown in Figure 2.6. The amplitude terms for I and Q path for each element are extracted from which the output power of each element are evaluated and is shown in Figure 2.7a. The behavioral gain of the LNAs in the receiver front-end are considered to be 10 dB. By varying the input power, the output power of the array could be determined by equation 1.7. The phase shift per element was determined from equation 1.8 which is the ratio of the quadrature arm amplitude to the in-phase arm amplitude term and is shown in Figure 2.7b.

![Figure 2.6 Extraction of quadrature phase errors, \( \phi_{11} \) and \( \phi_{22} \) in elements 1 and 2 respectively.](image-url)
(a) Extracted output power, $P_{out}$ versus swept input power, $P_{in}$, where the behavioral gain block (LNA) has a gain of 10 dB in channels 1 and 2.

(b) Extraction of phase response, $\theta_1$ and $\theta_2$ for elements 1 and 2 respectively.

Figure 2.7 Extraction of amplitude and phase response of elements 1 and 2.
2.4.4 Other series topology variants

CoMET in its prior art used a shunt-feed shunt-combine BIST topology for the purpose of signal injection and extraction respectively [Gre17]. The modified BIST architecture introduced in [Yeh18] and discussed above uses a series-feed series-combine topology that was chosen as a means to implement CoMET in a wider range of phased arrays. Four sets of measurements create quadruple baseband data-sets allowing extraction of a larger number of array performance parameters. Due to fourfold increase in measurement, this technique can be time consuming. This limitation can be overcome by using other topological variants of the series-type BIST architecture where, the input or output networks can also be made into a shunt topology. Shown in Figure 2.8a is a schematic for a series-feed shunt-combine BIST topology integrated with CoMET. In this topology, the output path delay, $\phi$ introduced in the signal due to series combining, is eliminated. This architecture further eliminates the use of two BB ports, reducing the number of measurement sets to only two, thus saving significant measurement time. Conversely, shown in 2.8b is the schematic of a shunt-feed series-combine BIST topology. This eliminates the need for two BIST input ports for signal injection and allows the extracted signal to be collected at the two BB ports. In this architecture, input series phase offset, $\psi$ can be eliminated and the effective I/Q path offsets can be attributed to the output path delay, $\phi$. Thus, $\phi$ can be split into $\phi_{i,n}$ and $\phi_{q,n}$ to account for the path delays in the I and Q paths respectively. A summary of the correlations and corresponding phase arguments obtained from the baseband data-sets for both the variants are listed in Table 2.4. The analysis for the two variants described is similar to the series-feed series-combine topology as discussed in this chapter. However, both the topologies allow two sets of measurement, creating twice the baseband data-sets. This can be a limitation when dealing with non-ideal phased arrays where the number of unknown parameters can be large.
Figure 2.8 CoMET employed in other series variants of BIST topology.
Table 2.4 Summary of CoMET-based measurement sets obtained from the series variants.

<table>
<thead>
<tr>
<th>Feed</th>
<th>Combine</th>
<th>Meas. Set</th>
<th>Corr. type</th>
<th>Corr. value</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series</td>
<td>Shunt</td>
<td>First</td>
<td>In-phase</td>
<td>$R_{ii,12}$</td>
<td>$\psi_{i,2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>In-phase</td>
<td>$R_{qq,12}$</td>
<td>$\phi_{22} + \psi_{q,2} \cdot \phi_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,11}$</td>
<td>$\phi_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,22}$</td>
<td>$\phi_{22} + \psi_{q,2} \cdot \psi_{i,2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,12}$</td>
<td>$\phi_{22} + \psi_{q,2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,21}$</td>
<td>$\phi_{11} - \psi_{i,2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Second</td>
<td>$R_{ii,12}$</td>
<td>$\psi_{i,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>In-phase</td>
<td>$R_{qq,12}$</td>
<td>$\phi_{22} - \psi_{q,1} \cdot \phi_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,11}$</td>
<td>$\phi_{11} + \psi_{q,1} \cdot \psi_{i,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,22}$</td>
<td>$\phi_{22}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,12}$</td>
<td>$\phi_{22} - \psi_{i,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,21}$</td>
<td>$\phi_{11} - \psi_{q,1}$</td>
</tr>
<tr>
<td>Shunt</td>
<td>Series</td>
<td>First</td>
<td>In-phase</td>
<td>$R_{ii,12}$</td>
<td>$\phi_{i,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>In-phase</td>
<td>$R_{qq,12}$</td>
<td>$\phi_{22} - \phi_{11} - \phi_{q,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,11}$</td>
<td>$\phi_{11} + \phi_{q,1} - \phi_{i,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,22}$</td>
<td>$\phi_{22}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,12}$</td>
<td>$\phi_{22} \cdot \phi_{i,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,21}$</td>
<td>$\phi_{11} + \phi_{q,1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Second</td>
<td>$R_{ii,12}$</td>
<td>$\phi_{i,2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>In-phase</td>
<td>$R_{qq,12}$</td>
<td>$\phi_{22} - \phi_{11} + \phi_{q,2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,11}$</td>
<td>$\phi_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,22}$</td>
<td>$\phi_{22} + \phi_{q,2} - \phi_{i,2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,12}$</td>
<td>$\phi_{22} + \phi_{q,2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quad-phase</td>
<td>$R_{iq,21}$</td>
<td>$\phi_{11} - \phi_{i,2}$</td>
</tr>
</tbody>
</table>
2.4.5 Accuracy of extraction

The new scheme introduced in this chapter suffers from certain limitations. First, the accuracy of CoMET is affected by its accuracy of the signal-encoding procedure which occurs at the phase shifter. In the modified extraction method we have not assumed the unbalanced nature of orthogonal codes. When we solved for I and Q vector, it was assumed that the phase shifter perfectly switches from +1 to -1, thus the amplitude weighting of the I and Q vectors are considered to be identical in magnitude in all quadrants. However, this may not be true as circuit imperfections can cause imbalances in the sign inversion. This was solved in [Gre17] exploiting the symmetrical nature of the orthogonal code sequence. Second, the quadrature errors have been assumed to be a small value. However, the source of the error and its effect on the CoMET extraction technique has not been accounted for. Thus in the presence of circuit non-idealities the above model fails to extract the unknown parameters accurately. This implies that we need a more robust model to enhance the accuracy of CoMET extraction. With respect to the above limitations, it will be established that the phase shifter performance is the key to achieving accuracy in parameter extraction.

At the phase shifter, which in CoMET is the active vector interpolator type (VI), the elements are modulated between inverted and non-inverted states to obtain phase settings across all four quadrants. However, amplitude offsets in the I and Q vectors will change the overall phase shift of a channel. Thus the non-ideal performance of the phase shifter needs to be modeled behaviorally.

2.5 Conclusion

In this chapter a new technique of CoMET-based extraction was introduced in a new BIST architecture. This modified technique uses two input BIST ports and two output BB ports for signal injection and extraction respectively. Four sets of measurements are performed which creates four different baseband data-sets. This gives four times the number of cor-
relation equations from the existing approach allowing more accurate extraction of the array parameters. However, the model fails to capture code errors and other amplitude and phase imbalances that can be attributed to the non-ideal behavior of the circuits which is a limitation of this model. In the next chapter, we will try to improve this model by introducing behavioral parameters that can imitate the non-ideal behavior of the array channel and perform a behavioral analysis of the system with CoMET.
A critical feature of CoMET affecting its accuracy is the signal-encoding procedure which occurs at the phase shifter. In the prior art i.e., [Gre18], the phase shifter design accuracy and performance was thus key to extract accurate array response from the cross-correlation equations. However, even in a perfected phase shifter design, dynamic non-idealities exist causing errors in both amplitude and phase in the CoMET-based modulated response. While phase errors manifest as quadrature errors or phase offsets, amplitude errors can lead to imbalance in the encoding procedure. The primary goal of this chapter and the following are, first, to understand if non-idealities in the phase shifter circuit introduce errors into CoMET. Second, if CoMET can be used to calibrate or correct a phase shifter when the phase shifter itself introduce errors for CoMET.

In this chapter, first the operation of a vector interpolator type phase-shifter is discussed
that allows each element to be modulated between inverted and non-inverted states to obtain phase settings across all four quadrants, essential to realize CoMET-based testing in an array. Further, the characteristic non-idealities of this type of phase shifter will be discussed in details. Each source of non-ideality is then behaviorally modeled to obtain a more realistic phase shifter response from individual channels of the array. This model also termed as ‘error model’ accounts for the typical imperfections in the phase shifter performance and gives insight into understanding the physical sources of code-imbalance in CoMET.

### 3.1 Basic theory

The basic block diagram of a vector interpolation type phase shifter is shown in Figure 3.1 [TN09]. The input signal is split into two equal amplitude in-phase (I) and quadrature-phase (Q) components through a 90° coupler. The I and Q split signals are further differentially split by the baluns. The normalized phases at the output of the first balun is 0° and 180°, whereas, the output of the second input balun is 90° and 270°. Next, the two I and Q components are independently amplified by two Variable Gain Amplifiers (VGAs). To synthesize phase in all four quadrants, the VGAs should have both positive and negative gains. This is accomplished by making the VGAs differential, so that their outputs can be flipped to cause a sign inversion of the gain [Gre17; Sar10]. The output balun converts the differential signal into single ended. By tuning the gain of the two VGAs, any phase between 0° and 360° can be synthesized. The I and Q components are weighted by the gain of the two VGAs to achieve a desired phase rotation. Graphically this is shown in Figure 3.2(a). As a result of the sign inversion and summation, vectors in all the four quadrants can be synthesized as: $\vec{I} + \vec{Q}$, $-\vec{I} + \vec{Q}$, $-\vec{I} - \vec{Q}$ and $\vec{I} - \vec{Q}$. 

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3.1.1 Mathematical analysis

In this analysis, it is assumed that the normalized gain if the VGAs range from -A to A, where, A is the maximum weight of the I and Q vectors [Sar10]. The following analysis is performed for 16 equi-distant phase states between $0^\circ$ and $360^\circ$ with a step of $22.5^\circ$. Throughout, the analysis phasor notations have been used to represent signals.

Figure 3.2 (a) Graphical representation of phase synthesis (b) Magnitude and sign of I and Q vectors across four quadrants.
The input signal to the coupler may be represented as:

\[ V_{in} = v_{in} e^{j0^\circ} \]  \hspace{1cm} (3.1)

This is split into two equal I and Q components, given by:

\[ V_I = \frac{v_{in}}{\sqrt{2}} e^{j0^\circ} \]  \hspace{1cm} (3.2)

\[ V_Q = \frac{v_{in}}{\sqrt{2}} e^{-j90^\circ} \]  \hspace{1cm} (3.3)

If the I and Q vectors are weighted by \( A_I \) and \( A_Q \) respectively, then the signal expression for the output of the phase shifter can be written as:

\[ V_{Out} = A_I V_I + A_Q V_Q \]  \hspace{1cm} (3.4)

The expression of \( A_I \) and \( A_Q \) are normalized by the maximum VGA gain, i.e. 1 or 0 dB. The following equation gives the expression for \( A_I \) and \( A_Q \):

\[ A_I = \cos \phi = \frac{1}{\sqrt{1 + \tan^2 \phi}} \]  \hspace{1cm} (3.5)

\[ A_Q = \sin \phi = \frac{\tan \phi}{\sqrt{1 + \tan^2 \phi}} \]  \hspace{1cm} (3.6)

In the above expression, \( \tan \phi = \frac{A_Q}{A_I} \). Where, \( \phi \) is a desired phase setting. Figure 3.2(a) indicates phase synthesis by weighting the I and Q vectors. The final expression at the output of the phase shifter can be written as:

\[ V_{Out} = \frac{1}{\sqrt{1 + \tan^2 \phi}} \frac{v_{in}}{\sqrt{2}} e^{j(0^\circ - k_I 180^\circ)} + \frac{\tan \phi}{\sqrt{1 + \tan^2 \phi}} \frac{v_{in}}{\sqrt{2}} e^{j(-90^\circ - k_Q 180^\circ)} \]  \hspace{1cm} (3.7)

Where, \( k_{I/Q} = 0 \) if \( A_{I/Q} \geq 0 \) and \( k_{I/Q} = 1 \) if \( A_{I/Q} < 0 \). The sign and magnitude of the \( A_I \) and \( A_Q \) vectors for different phase states are indicated in Table 3.1 and graphically represented in Figure 3.2(b).
Table 3.1 Magnitude and sign of $A_I$ and $A_Q$ vectors for different phase states across the four Cartesian quadrants

<table>
<thead>
<tr>
<th>Phase setting</th>
<th>$A_I$ (normalized)</th>
<th>$A_Q$ (normalized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.5°</td>
<td>0.9239</td>
<td>0.3827</td>
</tr>
<tr>
<td>45°</td>
<td>0.7071</td>
<td>0.7071</td>
</tr>
<tr>
<td>67.5°</td>
<td>0.3827</td>
<td>0.9239</td>
</tr>
<tr>
<td>90°</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>112.5°</td>
<td>-0.3827</td>
<td>0.9239</td>
</tr>
<tr>
<td>135°</td>
<td>-0.7071</td>
<td>0.7071</td>
</tr>
<tr>
<td>157.5°</td>
<td>-0.9239</td>
<td>0.3827</td>
</tr>
<tr>
<td>180°</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>202.5°</td>
<td>-0.9239</td>
<td>-0.3827</td>
</tr>
<tr>
<td>225°</td>
<td>-0.7071</td>
<td>-0.7071</td>
</tr>
<tr>
<td>247.5°</td>
<td>-0.3827</td>
<td>-0.9239</td>
</tr>
<tr>
<td>270°</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>292.5°</td>
<td>0.3827</td>
<td>-0.9239</td>
</tr>
<tr>
<td>315°</td>
<td>0.7071</td>
<td>-0.7071</td>
</tr>
<tr>
<td>337.5°</td>
<td>0.9239</td>
<td>-0.3827</td>
</tr>
<tr>
<td>360°</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

3.2 Non-idealities of Phase Shifter

The mathematical analysis performed above assumed ideal components in the phase shifter. In this section, non-idealities of the phase shifter are discussed and a behavioral model is thus constructed. These imperfections give rise to amplitude and phase errors in the signal at the output of the phase shifter and hence the beam-former. The sources of each characteristic error are considered separately and modeled to derive the output voltage expression of a non-ideal phase shifter, $V_{out}$. Using constellation diagrams across all the possible phase states, we will further visualize effects of the non-idealities on the phase shifter output.
3.2.1 Quadrature amplitude and phase imbalance

In a real coupler, imperfections in the model or process variations can cause unequal quadrature splitting between the two coupler outputs. Varying load conditions at the VGA output is yet another reason that can cause quadrature error [Flo18]. This imbalance results in two unequally split I and Q components which are not exactly 90° out of phase. Since the coupler is a passive model, the two quadrature components at the output of the coupler can be written as:

\[
V_{0°} = \alpha_c e^{j0°} \quad (3.8)
\]
\[
V_{90°} = \sqrt{1-\alpha_c^2} e^{j(90°+\phi_c)} \quad (3.9)
\]

Where, \( \alpha_c \) is the fraction of amplitude imbalance and \( \phi_c \) is the quadrature phase error in the coupler. Here, \( 0 \leq \alpha_c \leq 1 \) and \( -45° \leq \phi_c \leq 45° \). When there are no errors, \( \alpha_c = 1/\sqrt{2} \) and \( \phi_c = 0° \). The quadrature splitter amplitude imbalance may be defined as:

\[
A_c = 20 \log \left( \frac{\alpha_c}{\sqrt{1-\alpha_c^2}} \right) (dB) \quad (3.10)
\]

Figure 3.3 indicates the effect of non-idealities on the I-Q signal constellation points for varying VGA gain settings. Shown in Figure 3.3a and 3.3b is the effect of amplitude and phase imbalance in the quadrature arms of the 90° coupler. Quadrature amplitude imbalance of \( A_c = 1.5 \) dB cause the signal constellations to be flattened horizontally, the ideal square shape distorts to a rectangle as shown in 3.3a. To observe the effect of quadrature phase imbalance, a quadrature error of \( \phi_c = 10° \) was introduced. This error causes a shear about the vertical axis of the square and is indicated in Figure 3.3b.

3.2.2 Transformer balun amplitude and phase imbalance

Imperfections in the transformer balun may result in unequal signal splitting and combining [Sar10]. The two differential branches thus may not be not be precisely 180° out of phase. The expression of the two outputs of the transformer balun including the non-idealities
Figure 3.3 Impact of quadrature amplitude and phase imbalance on signal constellations for different I and Q VGA gain settings.

may be written as:

\[ V_{0^\circ} = \alpha_b e^{j0} \]  \hspace{1cm} (3.11)

\[ V_{180^\circ} = \sqrt{1 - \alpha_b^2} e^{j(180^\circ + \phi_b)} \]  \hspace{1cm} (3.12)

Where, \( \alpha_b \) is the fraction of amplitude imbalance and \( \phi_b \) is the phase error in the balun. Here, \( 0 \leq \alpha_b \leq 1 \) and \( -90^\circ \leq \phi_b \leq 90^\circ \). With no errors, \( \alpha_b = 1/\sqrt{2} \) and \( \phi_b = 0^\circ \). The transformer balun amplitude imbalance may be defined as:

\[ A_b = 20 \log \left( \frac{\alpha_b}{\sqrt{1 - \alpha_b^2}} \right) (dB) \]  \hspace{1cm} (3.13)

Shown in Figure 3.4a and 3.4b are the effects of amplitude and phase imbalance respectively in the transformer balun that performs differential splitting and combining of the RF signal. Amplitude imbalance of \( A_b = 1.5dB \) causes the signal constellations to be slightly spread out as shown in Figure 3.4a. With phase imbalance of \( \phi_b = 10^\circ \) in the differential arms, the entire signal constellation appears to be rotated. However, this error does not cause any significant distortion in the I and Q vectors as shown in Figure 3.4b.
3.2.3 Gain-dependent phase variation of VGAs

In the previous analysis, it was assumed that the phase shift introduced by the VGAs are constant with varying gain. However, in practical VGAs, the phase of each VGA can vary significantly with changes in amplitude setting [Flo18; TN09; Sad16]. Thus the weighting of the I and Q vectors does not follow the simple trigonometric equations as described by equations 3.5 and 3.6 above. This type of error manifests as a distortion in the original I and Q vectors. Assuming linear variation of the transmission phase of the VGA ($\phi_{VGA}$) with the gain, $\phi_{VGA}$ is given by,

$$\phi_{VGA} = \phi_{lin}(^\circ/dB)|S_{21}|(dB)$$

(3.14)

Where, $\phi_{lin}$ is the VGA phase error and $|S_{21}|$ is the variable gain of the amplifier. Figure 3.5 illustrates severe distortion in the overall shape of the constellation points when the phase error of the VGA, $\phi_{lin} = 4^\circ/dB$. The ideal rectangular shape transforms into one with ‘twisted axis’ indicating both amplitude and phase error.
3.2.4 I-Q Cross-talk

In the VGAs the output current in the I and Q paths are dotted together and connected to the load. If the output impedance of the VGAs are too low then there can be a situation when the I vector interferes and influences the state of the Q vector and vice-versa [Flo18]. Thus the impedance at the load varies as a function of the angle. The gain of the in-phase and quadrature-phase VGAs can be expressed respectively as:

\[
A_I(A_Q) = A_I + \alpha_1 A_Q
\]

\[
A_Q(A_I) = A_Q + \alpha_2 A_I
\]

Where \( \alpha_1 \) and \( \alpha_2 \) are the cross-talk coefficients in the I arm due to Q component and in the Q arm due to I component respectively. Figure 3.6 shows the effect of 10% I/Q crosstalk, i.e., (\( \alpha_1 = \alpha_2 = 0.1 \)) on the constellation points. This error distorts the rectangle along both the horizontal and vertical axis.

3.3 Impact of non-idealities on phase shifter output

In the previous section, each source of error in the phase shifter was considered separately and analysed. The expression for a real phase shifter output was thus derived. The effect
Figure 3.6 Impact of I-Q Cross-talk on phase states.

of all the possible non-idealities analysed in the above section are combined to form a behavioral expression of the phase shifter output. This behavioral error model will be further studied using constellation diagrams across all the possible phase states as shown in Table 3.1. Substituting equations 3.8-3.16 in equation 3.7 the output signal expression of the phase shifter can be obtained and is given by:

\[
V_{Out} = \alpha_c A_{I,ckt} e^{j\phi_{I,VGA}} + \sqrt{1 - \alpha_c^2} A_{Q,ckt} e^{j\phi_{Q,VGA}} e^{j(90^\circ + \phi_c)}
\]  

(3.17)

In the above equation 3.17,

\[
A_{I,ckt} = A_I + \alpha_1|A_Q|, \quad A_{Q,ckt} = A_Q + \alpha_2|A_Q|
\]  

(3.18)

\[
\phi_{I,VGA} = \phi_{lin} |A_I|, \quad \phi_{Q,VGA} = \phi_{lin} |A_Q|
\]  

(3.19)

It was observed in Figures 3.4a and 3.4b that errors in the transformer balun did not cause any significant distortion in the I and Q vectors. For this reason, in the final expression of \(V_{Out}\) given by equation 3.17, those errors are omitted to reduce mathematical complexity of dealing with a large number of errors. Shown in Figure 3.7 is a constellation diagram portraying all the signal constellations for different magnitude of I and Q VGA gain settings. In the absence of phase shifter non-idealities, the magnitude of I and Q vary linearly across all gain settings in all four quadrants and the signal constellation points form a perfect square. In the diagram the ‘real’ axis indicates the in-phase and ‘imaginary’ axis indicates
quadrature-phase component of the behavioral phase shifter signal output, $V_{\text{Out}}$.

In the two-element CoMET analysis described in Chapter 2, the code-modulated signal is rotated across 16 equi-distant phase states where consecutive states are 22.5° apart. Ideally, these phase states align along the circumference of a circle. The reason is because $\sqrt{A_I^2 + A_Q^2}$ is always 1. This comes from the following assumption for the I and Q VGAs:

$$|A_I| \propto |\cos(\theta)|$$  \hspace{1cm} (3.20)

$$|A_Q| \propto |\sin(\theta)|$$  \hspace{1cm} (3.21)

Next, polar diagrams are generated to show the degradation of the 16 phase states caused by each type of non-ideality in Figure 3.8. The constellation points represented by the solid dots represent the ideal location of the phase settings along the circumference of a circle. Non-idealities cause deviations of constellation points from these ideal points. Figures 3.8a and 3.8b show distortion in the phase states when amplitude and phase imbalance in the quadrature splitter are $A_c = 1.5$ dB and $\phi_c = 10^\circ$ respectively. In both cases, the phase states on the polar diagram are distorted and becomes elliptic from a perfect circle.

Shown in 3.8c and 3.8d are amplitude and phase imbalance in the transformer balun, which causes negligible amplitude and phase error in the phase shifter output.

Figures 3.8e shows a highly distorted polar diagram due to gain dependent transmission
(a) Effect of amplitude imbalance on phase states

(b) Effect of quadrature-phase imbalance on phase states

(c) Effect of transformer amplitude imbalance on phase states

(d) Effect of transformer phase imbalance on phase states

(e) Effect of gain dependent transmission phase of VGAs on phase states

(f) Effect of cross-talk on phase states

Figure 3.8 Polar diagrams illustrating effects of phase shifter non-idealities on the chosen 16 gain settings.
phase shift of the VGA where, $\phi_{lin} = 4^\circ$. This is the severest non-ideality causing distortions in both amplitude and phase. The circular shape appears to be a distorted rectangle.

Figure 3.8f depicts the effect of cross-talk on the phase states of the phase shifter. The cross-talk co-efficients in this case is assumed to be, $\alpha_1 = 10\%$ and $\alpha_2 = 10\%$. $\alpha_1$ being the effect of a low impedance node at quadrature output affecting the in-phase signal and $\alpha_2$ being vice-versa. The polar diagram is deformed due to cross-talk.

### 3.4 Conclusion

In this chapter, the typical non-idealities of a vector interpolator type phase shifter was discussed. The imbalances in the transformer balun was observed to be insignificant and will not be pursued further. However, other imbalances arising from the coupler, the I- and Q-VGAs and the cross-talk are the main concerns. In the next chapter, I will discuss how these non-idealities affect CoMET measurement. Further, CoMET behavioral model will be modified to include these non-ideal affects and account for them during extraction of the performance parameters of the array.
This chapter presents a Matlab-based implementation of CoMET behavioral model in a series-feed series-combine BIST architecture. The purpose is to understand if CoMET works with non-ideal phase shifters as described in Chapter-3. Further, we will explore if CoMET can be used to extract and correct for those non-idealities. The phase shifter error model derived previously, is embedded into the series-fed BIST model, considering a two-element phased array system. The non-ideal model introduced a number of amplitude and phase errors in the array, arising out of different imbalances mainly from the phase shifter. In this case, the modified CoMET technique described in Chapter-2, generating quadruple baseband data-sets, are advantageous in solving for the additional unknown parameters. To validate the method, the extracted phase response using the modified CoMET approach is compared to the response of individual elements. Finally, the accuracy to which CoMET
4.1 Non-ideal response of array elements

In the previous chapter, we studied the effects of typical non-idealities on the phase shifter output performance. Before introducing these imbalances into the behavioral system using CoMET, we will first extract the response of a single phased array channel with the non-ideal phase shifter model and estimate the error in the element response across all phase settings. From CoMET measurement [Gre18], we have an estimate of the phase error values in a real array. The non-ideal parameters are chosen such that they introduce phase errors within the response, in a typical range. Later in this Chapter CoMET will be analysed using this non-ideal model. In the following section the individual response of a channel has been studied that will serve as a platform to compare an actual element response against CoMET-based extracted response of an array element.

4.1.1 Quadrature phase error

Shown in Figure 4.1(a) and (b) are phase response of a non-ideal phased array channel in comparison to an ideal one. The figure illustrates the impact of a typical quadrature amplitude imbalance, $A_c = 1.5\, dB$ and quadrature phase error, $\phi_c = 10^\circ$ respectively on an array element output response. In Table 4.1, it is observed that for 1.5 dB amplitude imbalance, the maximum phase error is $4.92^\circ$ which occurs at the phase settings $45^\circ, 135^\circ, 225^\circ$ and $315^\circ$. For $\phi_c = 10^\circ$, the maximum phase error is $10^\circ$ occurring at the axis settings $90^\circ$ and $270^\circ$. With the chosen quadrature amplitude and phase imbalance, the mean phase error is about $2.97^\circ$ and $4.9^\circ$ respectively across all the 16 phase settings.
Figure 4.1 Comparison of ideal and non-ideal phase response of an array channel. Due to non-idealities, the ideally linear response (shown in black) get distorted (shown in red).

Table 4.1 Phase errors in element response due to quadrature amplitude and phase imbalance in coupler.

| Setting | Phase-shift ($A_c = 1.5dB$) | Error, $|\Delta|$ | Phase-shift ($\phi_c = 10^\circ$) | Error, $|\Delta|$ |
|---------|-----------------------------|------------------|---------------------------------|------------------|
| 22.5°   | 19.2°                       | 3.28°            | 23.73°                          | 1.23°            |
| 45°     | 40.07°                      | 4.92°            | 50°                             | 5°               |
| 67.5°   | 63.8°                       | 3.71°            | 76.27°                          | 8.77°            |
| 90°     | 90°                         | 0°               | 100°                            | 10°              |
| 112.5°  | 116.21°                     | 3.71°            | 120.8°                          | 8.33°            |
| 135°    | 139.9°                      | 4.92°            | 140°                            | 5°               |
| 157.5°  | 160.8°                      | 3.28°            | 159.2°                          | 1.66°            |
| 180°    | 180°                        | 0°               | 180°                            | 0°               |
| 202.5°  | 199.2°                      | 3.28°            | 203.7°                          | 1.23°            |
| 225°    | 220.07°                     | 4.92°            | 230°                            | 5°               |
| 247.5°  | 243.8°                      | 3.71°            | 256.3°                          | 8.77°            |
| 270°    | 270°                        | 0°               | 280°                            | 10°              |
| 292.5°  | 296.2°                      | 3.71°            | 300.8°                          | 8.33°            |
| 315°    | 319.9°                      | 4.92°            | 320°                            | 5°               |
| 337.5°  | 340.8°                      | 3.28°            | 339.2°                          | 1.66°            |
| 360°    | 360°                        | 0°               | 360°                            | 0°               |
4.1.2 VGA phase error

In Chapter 3, it was shown how the gain dependent transmission phase of the VGA can cause distortions in both the I and Q-VGA gain settings through constellation diagrams. Shown in Figure 4.2 is the element response due to $\phi_{lin} = 10^\circ$, where $\phi_{lin}$ is the VGA phase error described in equation 3.14. Table 4.2 indicates a comparison of the phase settings and the phase shift obtained due to this type of error. The maximum error in this case is $30.1^\circ$ with mean phase error across all settings being $15.5^\circ$. From observation, this error appears to be more severe around the lower value phase settings in each quadrant.

![Figure 4.2](image)

Figure 4.2 Comparison of ideal and non-ideal phase response of an array channel with VGA phase error, $\phi_{lin} = 10^\circ$.

4.1.3 I/Q Crosstalk error

Shown in Figure 4.3 is the effect of 25% IQ crosstalk in an element response. The linearity of the curve is observed to be distorted. The phase angles set and the phase shift obtained after introducing crosstalk between the I and Q vectors are tabulated in Table 4.3. The
Table 4.2 Phase errors in element response due to VGA phase error.

| Phase setting | Phase-shift ($\phi_{lin} = 10^\circ$) | Error, $|\Delta|$ |
|---------------|--------------------------------------|-----------------|
| 22.5°         | -2.95°                               | 25.45°          |
| 45°           | 14.89°                               | 30.1°           |
| 67.5°         | 73.96°                               | 6.46°           |
| 90°           | 90°                                  | 0°              |
| 112.5°        | 87.05°                               | 25.45°          |
| 135°          | 104.89°                              | 30.1°           |
| 157.5°        | 163.96°                              | 6.46°           |
| 180°          | 180°                                 | 0°              |
| 202.5°        | 177.05°                              | 25.45°          |
| 225°          | 194.89°                              | 30.1°           |
| 247.5°        | 253.96°                              | 6.46°           |
| 270°          | 270°                                 | 0°              |
| 292.5°        | 267.05°                              | 25.45°          |
| 315°          | 284.89°                              | 30.1°           |
| 337.5°        | 343.96°                              | 6.46°           |
| 360°          | 360°                                 | 0°              |

maximum phase error is around $14^\circ$ and occurs around the four axes settings, i.e. $90^\circ$, $180^\circ$, $270^\circ$ and $360^\circ$. The mean phase error is $8.6^\circ$ across all phase settings.

4.2 Behavioral model

In Chapter 2, we discussed how the modified ideal CoMET model implemented in Matlab suffer from strong limitations when the unbalanced nature of the orthogonal codes are considered. Thus, to build a more robust model, behavioral parameters needs to be introduced into this model. The non-ideal parameters discussed in Chapter 3 are the behavioral parameters of the phase shifter that will be next introduced into the series-fed two element array. The analysis approach get modified from Chapter 2 when including amplitude and phase non-idealities in the appropriate corners of the equations. CoMET-based extraction is then performed using the modified measurement approach involving four measurement sets as discussed in Chapter 2. The objective is to explore if CoMET can accurately extract
Figure 4.3 Comparison of ideal and non-ideal phase response of an array channel with I-Q Cross-talk co-efficients, $\alpha_1 = \alpha_2 = 25\%$.

Table 4.3 Phase errors in element response due to I/Q Crosstalk.

| Phase setting | Phase-shift ($\alpha_1 = \alpha_2 = 25\%$) | Error, $|\Delta|$ |
|---------------|------------------------------------------|-----------------|
| 22.5°         | 31.04°                                   | 8.54°           |
| 45°           | 45°                                      | 0°              |
| 67.5°         | 58.95°                                   | 8.54°           |
| 90°           | 75.96°                                   | 14.04°          |
| 112.5°        | 100.38°                                  | 12.12°          |
| 135°          | 135°                                     | 0°              |
| 157.5°        | 169.62°                                  | 12.12°          |
| 180°          | 194.04°                                  | 14.04°          |
| 202.5°        | 211.04°                                  | 8.54°           |
| 225°          | 225°                                     | 0°              |
| 247.5°        | 238.96°                                  | 8.54°           |
| 270°          | 255.96°                                  | 14.04°          |
| 292.5°        | 280.38°                                  | 12.12°          |
| 315°          | 315°                                     | 0°              |
| 337.5°        | 349.62°                                  | 12.12°          |
| 360°          | 374.04°                                  | 14.04°          |
the non-ideal response of array elements without itself being affected by the non-idealities.

### 4.2.1 Modification in encoding

Table 4.4 lists the extraction parameters to be introduced into the behavioral model. The following equations summarize the modification in the signal encoding technique of CoMET in a two-element array. For measurement set 1, equations 2.1-2.4 can be modified as:

\[
E_{i,1}(t) = (A_{i,1}^{err} c_{i,1}) \cos(\omega t - \phi_{ivga,1})
\]

\[
E_{q,1}(t) = (A_{q,1}^{err} c_{q,1}) \sin(\omega t - \phi_{c,1} - \phi - \phi_{qvga,1})
\]

\[
E_{i,2}(t) = (A_{i,2}^{err} c_{i,2}) \cos(\omega t - \psi_{i,2} - \phi_{ivga,2})
\]

\[
E_{q,2}(t) = (A_{q,2}^{err} c_{q,2}) \sin(\omega t - \phi_{c,2} - \psi_{q,2} - \phi - \phi_{qvga,2})
\]

In the above equations, \(A_{i/q,n}^{err}\) indicate the amplitude of the I and Q components in an ‘n’ element array affected by non-idealities. The expression for the amplitude terms are given by:

\[
A_{i,n}^{err} = (A_{i,n} + \alpha_1 A_{q,n})(\alpha_c)(\alpha_b)
\]

\[
A_{q,n}^{err} = (A_{q,n} + \alpha_2 A_{i,n})(\sqrt{1 - \alpha_b^2})(\alpha_b)
\]

\(A_{i/q,n}\) is the normalized VGA gain at a particular phase setting. \(\alpha_1\) and \(\alpha_2\) are the crosstalk coefficients, \(\alpha_c\) and \(\alpha_b\) are the fractional amplitude imbalances in the 90° coupler and transformer balun respectively. In equations 4.1-4.4, the term \(\phi_{ivga,n}\) and \(\phi_{qvga,n}\) indicates the phase error introduced in the signal due to phase distortions of I and Q-VGA respectively. The angle \(\phi_{c,1}\) and \(\phi_{c,2}\) are the quadrature phase errors in element 1 and 2 respectively. \(\psi_{i/q,n}\) is the series phase offset introduced due to the input distribution network and \(\phi\) is that due to the series distributed output network. To avoid adding too many similar equations in this report, the modified equations of the other three measurement sets are added in Appendix-A towards the end for interested readers.
### Table 4.4 Extraction parameters for an ‘n’ element array

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{err}^{i,n}$</td>
<td>Amplitude of I-path affected by non-idealities</td>
</tr>
<tr>
<td>$A_{err}^{q,n}$</td>
<td>Amplitude of Q-path affected by non-idealities</td>
</tr>
<tr>
<td>$\psi_{i,n}$</td>
<td>Input phase offset in the In-phase path</td>
</tr>
<tr>
<td>$\psi_{q,n}$</td>
<td>Input phase offset in the Quadrature-phase path</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Output phase offset</td>
</tr>
<tr>
<td>$\phi_{c,n}$</td>
<td>Quadrature phase error in $n^{th}$ element</td>
</tr>
<tr>
<td>$\phi_{ivga,n}$</td>
<td>Phase error at I-VGA in the $n^{th}$ element</td>
</tr>
<tr>
<td>$\phi_{qvga,n}$</td>
<td>Phase error at Q-VGA in the $n^{th}$ element</td>
</tr>
</tbody>
</table>

#### 4.2.2 Extraction of array performance metrics

Using the equations above, expressions for phase shift of elements 1 and 2 are derived. For the purpose of this report, the discussion will be limited to measurement set 1 only. The original signal phasors may be decomposed in the Cartesian representation to obtain the magnitudes of I and Q vectors. Using 2.1-2.4, the phase shift $\theta_1$ and $\theta_2$ for elements 1 and 2 respectively can be expressed as:

$$\theta_1 = \arctan \left( \frac{A_{err}^{i,1} \sin(\phi + \phi_{ivga,1}) + A_{err}^{q,1} \cos(\phi_{c,1} + \phi + \phi_{qvga,1})}{A_{err}^{i,1} \cos(\phi + \phi_{ivga,1}) - A_{err}^{q,1} \sin(\phi_{c,1} + \phi + \phi_{qvga,1})} \right)$$  \hspace{1cm} (4.7)

$$\theta_2 = \arctan \left( \frac{A_{err}^{i,2} \sin(\psi_{i,2} + \phi_{ivga,2}) + A_{err}^{q,2} \cos(\phi_{c,2} + \psi_{q,2} + \phi_{qvga,2})}{A_{err}^{i,2} \cos(\psi_{i,2} + \phi_{ivga,2}) - A_{err}^{q,2} \sin(\phi_{c,2} + \psi_{q,2} + \phi_{qvga,2})} \right)$$  \hspace{1cm} (4.8)

Equation 4.7 and 4.8 are modified expressions for elemental phase shift. It indicates that if the unknown parameters within parenthesis be extracted accurately with the modified technique, then CoMET should be able to track the non-ideal response of array elements. The output power of an element has been derived using the required modifications as follows:

$$P_{out} = \left( \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(p_1 - p_Q)} \right) a_b + \left( \sqrt{b_1^2 + b_2^2 + 2b_1b_2\cos(m_1 - m_Q)} \right) \sqrt{1 - a_b^2}$$  \hspace{1cm} (4.9)
In equation 4.9, amplitude terms $a_1$, $a_2$, $b_1$ and $b_2$ are given by:

\begin{align*}
a_1 &= \sqrt{2} A_{e_{r_{i,1}}} a_c a_b \\
a_2 &= \sqrt{2} A_{e_{q_{i,1}}} \left( \sqrt{1 - a_{c}^2} \right) a_b \\
b_1 &= \sqrt{2} A_{e_{r_{i,1}}} a_c \left( \sqrt{1 - a_{d}^2} \right) \\
b_2 &= \sqrt{2} A_{e_{q_{q,1}}} \left( \sqrt{1 - a_{d}^2} \right) \left( \sqrt{1 - a_{c}^2} \right)
\end{align*}

$p_I$, $p_Q$, $m_I$ and $m_Q$ represent the phase lag suffered by the signal when passing through I, I, Q and Q path during phase modulation.

The correlation equations of measurement set 1 modified by the errors listed in Table 4.4 and are given by:

\begin{align*}
R_{i_{i,1}i} &= (0.5) \left( A_{e_{r_{i,1}}} A_{e_{r_{i,2}}} \right) \cos \left( 90^\circ + \phi_{c_1} + \psi_{q_{a,1} - \phi_{i_{v,1}}} \right) \\
R_{i_{q,2}q} &= (0.5) \left( A_{e_{r_{q,1}}} A_{e_{r_{q,2}}} \right) \cos \left( 90^\circ + \phi_{c_2} + \psi_{q_{a,2} - \phi_{i_{v,2}}} \right) \\
R_{i_{i,2}i} &= (0.5) \left( A_{e_{r_{i,1}}} A_{e_{r_{i,2}}} \right) \cos \left( \psi_{q_{a,2} - \phi + \phi_{c_2} - \phi_{c_1} + \psi_{q_{a,2} - \phi_{i_{v,2}}} \right) \\
R_{q_{q,1}q} &= (0.5) \left( A_{e_{r_{q,1}}} A_{e_{r_{q,2}}} \right) \cos \left( \psi_{q_{a,2} - \phi + \phi_{c_2} - \phi_{c_1} + \psi_{q_{a,2} - \phi_{i_{v,2}}} \right) \\
R_{q_{q,2}i} &= (0.5) \left( A_{e_{r_{q,1}}} A_{e_{r_{q,2}}} \right) \cos \left( 90^\circ + \phi_{c_1} + \psi_{q_{a,2} - \phi_{i_{v,2}}} + \psi_{q_{a,1} - \phi_{i_{v,2}}} \right) \\
R_{q_{q,2}q} &= (0.5) \left( A_{e_{r_{q,1}}} A_{e_{r_{q,2}}} \right) \cos \left( 90^\circ + \phi_{c_1} + \psi_{q_{a,2} - \phi_{i_{v,2}}} + \psi_{q_{a,1} - \phi_{i_{v,2}}} \right)
\end{align*}

A summary of all the possible correlation equations obtained from the four baseband data-sets are listed in Table 4.5. Each measurement set forms six unique correlation equations. Thus, we have 24 correlation equations to solve with 15 unknown parameters. The non-linear system of correlation equations are solved to extract the unknown parameters following step-by-step approach. The detailed Matlab implementation is given in Appendix-B.

So far, the characteristic phase shifter non-idealities have been considered separately and analysed by introducing each non-ideal parameter one at a time into the system when other errors are absent. However, in a real phased array system, all the non-idealities co-exist simultaneously making parameter extraction from the correlation equations extremely
Table 4.5 Summary of phase arguments obtained from decoding four different measurement sets

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>First</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1A</td>
<td>2.1A</td>
<td>In-phase</td>
<td>$R_{i,i_{12}}$</td>
<td>$\psi_{i,i_{2}}+\phi_{c,1}+\phi_{i,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>1.2B</td>
<td>In-phase</td>
<td>$R_{q,q_{12}}$</td>
<td>$\psi_{q,q_{2}}+\phi_{c,2}+\phi_{c,1}+\phi_{q,vga2}+\phi_{q,vga1}$</td>
</tr>
<tr>
<td></td>
<td>1.3C</td>
<td>Quad-phase</td>
<td>$R_{i,q_{11}}$</td>
<td>$\phi_{c,1}+\phi_{q,vga1}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>1.4D</td>
<td>Quad-phase</td>
<td>$R_{i,q_{22}}$</td>
<td>$\phi_{c,2}+\psi_{q,q_{2}}+\phi_{i,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>1.5E</td>
<td>Quad-phase</td>
<td>$R_{i,q_{12}}$</td>
<td>$\psi_{q,q_{2}}+\phi_{c,2}+\phi_{q,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>1.6F</td>
<td>Quad-phase</td>
<td>$R_{i,q_{21}}$</td>
<td>$\phi_{c,1}+\phi_{i,vga1}+\phi_{i,vga2}$</td>
</tr>
<tr>
<td><strong>Second</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1A</td>
<td>2.2B</td>
<td>In-phase</td>
<td>$R_{i,i_{12}}$</td>
<td>$\psi_{i,i_{2}}+\phi_{c,1}+\phi_{i,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>2.3C</td>
<td>Quad-phase</td>
<td>$R_{i,q_{11}}$</td>
<td>$\phi_{c,1}+\phi_{q,vga1}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>2.4D</td>
<td>Quad-phase</td>
<td>$R_{i,q_{22}}$</td>
<td>$\phi_{c,2}+\psi_{q,q_{2}}+\phi_{i,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>2.5E</td>
<td>Quad-phase</td>
<td>$R_{i,q_{12}}$</td>
<td>$\psi_{i,q_{2}}+\phi_{c,2}+\phi_{q,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>2.6F</td>
<td>Quad-phase</td>
<td>$R_{i,q_{21}}$</td>
<td>$\phi_{c,1}+\phi_{i,vga1}+\phi_{i,vga2}$</td>
</tr>
<tr>
<td><strong>Third</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1A</td>
<td>3.2B</td>
<td>In-phase</td>
<td>$R_{i,i_{12}}$</td>
<td>$\psi_{i,i_{1}}+\phi_{c,1}+\phi_{i,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>3.3C</td>
<td>Quad-phase</td>
<td>$R_{i,q_{11}}$</td>
<td>$\phi_{c,1}+\psi_{q,q_{1}}+\phi_{q,vga1}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>3.4D</td>
<td>Quad-phase</td>
<td>$R_{i,q_{22}}$</td>
<td>$\phi_{c,2}+\phi_{q,vga2}+\phi_{i,vga2}$</td>
</tr>
<tr>
<td></td>
<td>3.5E</td>
<td>Quad-phase</td>
<td>$R_{i,q_{12}}$</td>
<td>$\phi_{c,2}+\psi_{q,q_{2}}+\phi_{i,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>3.6F</td>
<td>Quad-phase</td>
<td>$R_{i,q_{21}}$</td>
<td>$\phi_{c,1}+\phi_{q,q_{1}}+\phi_{q,vga1}+\phi_{i,vga2}$</td>
</tr>
<tr>
<td><strong>Fourth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1A</td>
<td>4.2B</td>
<td>In-phase</td>
<td>$R_{i,i_{12}}$</td>
<td>$\psi_{i,i_{1}}+\phi_{c,1}+\phi_{i,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>4.3C</td>
<td>Quad-phase</td>
<td>$R_{i,q_{11}}$</td>
<td>$\phi_{c,2}+\phi_{q,q_{1}}+\phi_{q,vga1}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>4.4D</td>
<td>Quad-phase</td>
<td>$R_{i,q_{22}}$</td>
<td>$\phi_{c,2}+\phi_{q,vga2}+\phi_{i,vga2}$</td>
</tr>
<tr>
<td></td>
<td>4.5E</td>
<td>Quad-phase</td>
<td>$R_{i,q_{12}}$</td>
<td>$\phi_{c,2}+\phi_{q,q_{2}}+\phi_{i,vga2}+\phi_{i,vga1}$</td>
</tr>
<tr>
<td></td>
<td>4.6F</td>
<td>Quad-phase</td>
<td>$R_{i,q_{21}}$</td>
<td>$\phi_{c,2}+\psi_{q,q_{1}}+\phi_{q,vga1}+\phi_{i,vga2}$</td>
</tr>
</tbody>
</table>
complicated. The two-step extraction method as discussed in CoMET literature [Gre17],
cannot be used here since the equations consist of multiple unknown parameters. Thus,
the solution technique needs to be modified as well. The following is a proposed approach:

Table 4.5 indicates a summary of the extracted cross-correlation values \( R_{i,q,11}, R_{i,q,22}, \)
\( R_{i,q,12}, R_{i,i,11}, R_{i,q,12} \) and \( R_{i,q,21} \) which represent the solutions to the correlation equations.
The equations themselves are composed of individual element’s performance parameters
which are to be solved. It is observed that the phase arguments consist of multiple unknown
parameters. To solve the correlation equations, certain assumptions will be made which
eliminates some of the unknown parameters, enabling equation solving. The sequence of
steps used in the modified CoMET extraction are listed in Table 4.6.

First, the amplitude of the I-vector component for both the elements \( A_{i,1}, A_{i,2} \) using
the correlation equations 1.1A and 2.1A (Table 4.5) from the inphase correlation value,
\( R_{i,i,12} \) in the baseband pattern are used. But the two correlation equations has 6 unknown
terms. Thus we make some assumptions to solve these equations: First, from the on-chip
BIST design, an estimate is made of the series path length between two consecutive array
elements at input and output from the distribution and extraction network respectively and
the equivalent phase lag is calculated \( \Delta \psi = \frac{2 \pi}{\lambda} \Delta x \), where \( \lambda \) is the operating wavelength
of the array under test. We thus have an estimate of \( \psi_{i,2} \) and \( \phi \). Second, if both the elements
are put to the same phase setting, i.e. \( \theta_1 = \theta_2 \), it can be approximated that \( \phi_{vga,1} = \phi_{vga,2} \)
since we know that:

\[
\phi_{VGA}(S_{21}) \propto |S_{21}| \quad (4.20)
\cos \theta \propto |S_{21}| \quad (4.21)
\]

In the \( R_{i,i,12} \) correlation equations, this implies that VGA phase error terms cancel out. Thus,
we are left with 2 unknown parameters, i.e. the inphase amplitude terms which are extracted
using \( R_{i,i,12} \) equations from baseband sets 1 and 2.

Next, the inphase amplitude terms are substituted in equations 3.1A and 4.1A and
similar assumptions as described above are made to extract $\psi_{i,1}$ and $\phi$ from the $R_{i,i,12}$ correlation equations from measurement set 3 and 4.

Next, quadrature correlation equations involving $R_{iq,11}$ and $R_{iq,22}$ are used to evaluate the quadrature errors $\phi_{c_1}$ and $\phi_{c_2}$ of the elements and the quadrature amplitude terms, $A_{err,1}^q$ and $A_{err,2}^q$. In this case, the phase of the two elements ($\theta_1, \theta_2$) needs to be set at 45°. This ensures that $\phi_{i,\text{vga1}}=\phi_{q,\text{vga1}}$ and $\phi_{i,\text{vga2}}=\phi_{q,\text{vga2}}$. Using equations 1.2B, 3.2B and 1.6F, we further extract $\psi_{q,1}$, $\psi_{q,2}$ and $\psi_{i,2}$. Finally, cross-correlation equations, $R_{iq,12}$ from all the four measurement sets are used to extract $\phi_{i,\text{vga1}}, \phi_{q,\text{vga1}}, \phi_{i,\text{vga2}}$ and $\phi_{q,\text{vga2}}$. The sequence of extraction is further elaborated in Table 4.6, along with the correlation equations from Table 4.5 used to extract the unknown parameters. Also, shown in the table are the assumptions made to eliminate the VGA phase errors occurring in the phase arguments.

Table 4.6 Sequence of performance parameter extraction using CoMET in a two-element phased array embedded with series-feed series-combine BIST topology

<table>
<thead>
<tr>
<th>Corr. value used</th>
<th>Eqn. pair</th>
<th>Assumption</th>
<th>Param. extracted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{i,i,12}$</td>
<td>1.1A, 2.1A</td>
<td>$[\theta_1=\theta_2] \Rightarrow [\phi_{i,\text{vga1}}=\phi_{i,\text{vga2}}]$</td>
<td>$A_{err,1,1}^i, A_{err,1,2}^i$</td>
</tr>
<tr>
<td>$R_{i,i,12}$</td>
<td>3.1A, 4.1A</td>
<td>$[\theta_1=\theta_2] \Rightarrow [\phi_{i,\text{vga1}}=\phi_{i,\text{vga2}}]$</td>
<td>$\psi_{i,1}, \phi$</td>
</tr>
<tr>
<td>$R_{iq,11}$</td>
<td>1.3C, 3.3C</td>
<td>$\theta_1=45^\circ \Rightarrow [\phi_{i,\text{vga1}}=\phi_{q,\text{vga1}}]$</td>
<td>$A_{err,1}^q, \phi_{c_1}$</td>
</tr>
<tr>
<td>$R_{iq,22}$</td>
<td>1.4D, 3.4D</td>
<td>$\theta_2=45^\circ \Rightarrow [\phi_{i,\text{vga2}}=\phi_{q,\text{vga2}}]$</td>
<td>$A_{err,2}^q, \phi_{c_2}$</td>
</tr>
<tr>
<td>$R_{q,q,12}$</td>
<td>1.2B</td>
<td>$[\theta_1=\theta_2] \Rightarrow [\phi_{q,\text{vga1}}=\phi_{q,\text{vga2}}]$</td>
<td>$\psi_{q,2}$</td>
</tr>
<tr>
<td>$R_{iq,12}$</td>
<td>3.2B</td>
<td>$[\theta_1=\theta_2] \Rightarrow [\phi_{q,\text{vga1}}=\phi_{q,\text{vga2}}]$</td>
<td>$\psi_{q,1}$</td>
</tr>
<tr>
<td>$R_{iq,21}$</td>
<td>1.6F</td>
<td>$[\theta_1=\theta_2=45^\circ] \Rightarrow [\phi_{q,\text{vga2}}=\phi_{i,\text{vga2}}]$</td>
<td>$\psi_{i,2}$</td>
</tr>
<tr>
<td>$R_{iq,12}$</td>
<td>1.5E, 2.5E</td>
<td></td>
<td>$\phi_{q,\text{vga2}}, \phi_{i,\text{vga1}}$</td>
</tr>
<tr>
<td>$R_{iq,12}$</td>
<td>3.6F, 4.6F</td>
<td></td>
<td>$\phi_{q,\text{vga1}}, \phi_{i,\text{vga2}}$</td>
</tr>
</tbody>
</table>

### 4.3 Accuracy: State-of-the-art vs. Modified approach

In this section, results of behavioral model from prior art of CoMET are compared with the modified series-fed BIST architecture introduced in this work. This section compares two
things. First, the behavioral model phase response of the parallel-fed and series-fed CoMET architecture. Second, comparison of individual array element phase response using the phase shifter error model and CoMET-based behavioral model.

Figure 4.4 plots the phase response of a phased array channel with respect to the initial phase settings. Shown is a comparison between the response of the prior art of CoMET, the series-fed CoMET and the individual element response. Shown in Figure 4.4a and 4.4b are the phase response of an array element with quadrature amplitude imbalance of 1.5 dB and quadrature phase imbalance of 10° respectively. In the plot, it is observed that the response of the prior CoMET-behavioral model depicts an ideal linear response, implying that the non-ideal response of the phase shifter could not be captured by the prior extraction technique. Comparison is made between this and the response from the series-fed CoMET model introduced in this work. The response in the latter case is non-linear with some deviations from the ideal phase setting points. The response obtained using the series-fed technique is then compared to the individual response of an element. The two responses show a very good overlap across all the 16 phase settings in the four quadrants. The absolute error between these two responses being less than 0.1° for both cases. The above comparison is indicative of two things. First, the behavioral model of the series-fed technique is capable of extracting non-ideal response of an array element using CoMET. Second, the non-ideal response thus extracted is in fact the actual response of the element, making this new testing technique more accurate and reliable. Further, it can also be concluded that even in the presence of non-idealities, this modified BIST operation does not introduce any additional errors in the CoMET technique.

The above discussion holds when the I and Q-VGAs are subjected to phase errors of 10°. Comparison is made between the response of the prior model to the response of the new model and the elemental response and is shown in 4.5a. The error in phase response between the new approach and an individual element is close to 0°. Finally, similar comparison is repeated to account for the effect of I/Q cross-talk shown in Figure 4.5b. In
Figure 4.4 Phase response of an element when it is subjected to quadrature amplitude imbalance ($A_c=1.5$dB) and phase imbalance ($\phi_c = 10^\circ$) across the 16 phase settings. Comparison is made between the phase response obtained from the extraction using CoMET prior art (shown in black squares), this work (shown in red solid line) and the individual element response (shown in yellow circles).
Figure 4.5 Comparison of phase response when array is subjected to VGA phase error and I/Q cross-talk
this comparison, it is observed that both the behavioral model response using the parallel-fed and series-fed approach are unable to detect the true response of an array element. The reason is that, in the presence of I/Q cross-talk, the I and Q vectors are not in phase-quadrature anymore according to equations 3.15 and 3.16. This violates the assumptions given by equations 3.20 and 3.21 according to which, $A_{i,n} \propto |\cos(\theta_n)|$ and $A_{q,n} \propto |\sin(\theta_n)|$, where $\theta_n$ is the phase shift of an element. The extraction of the amplitude terms using the I and Q vector are thus not able to track this defect and is a limitation of the modified approach.

In the prior art, extraction of the above non-idealities of the phase shifter was not possible due to the limitation of having only one set of down-converted baseband data-set. This drawback comes from the fact that the basic BIST architecture was of the shunt-feed shunt-combine type. Thus, only a single BIST RF port was available for signal injection and a single baseband (BB) port for signal extraction. This allowed only a single measurement to obtain a down-converted baseband data-set. For a two element array this would mean only 6 correlation equations to solve for 15 unknown parameters. However, by modifying the BIST topology to a series-feed series-combine type, the data-sets are increased fourfold. This has allowed the behavioral extraction of the non-idealities of the phase shifter.

4.4 Conclusion

Behavioral modeling of CoMET in a series-fed phase array architecture is implemented in Matlab. Modifications required in the encoding and extraction method for the new technique have been discussed. The cross-correlation equations are mixed with characteristic errors of the phase shifter which demands modifications in the extraction technique. A new approach to parameter extraction has been discussed. Behavioral tests are performed on a phased array system laden with phase shifter non-idealities. Prior CoMET approach could not account for such non-idealities resulting in phase errors between VNA and mea-
sured results. However, the new testing method introduced in this work is able to create quadruple measured data-sets than prior art, that allows fourfold increase in the correlation equations to extract performance parameters of the array. CoMET is thus able to respond to the non-ideal imbalances of the phase shifter. This has been verified by comparing the CoMET-extracted response to that of an individual array element response and a very good overlap is observed. The verification establishes that the new behavioral model is superior to the prior model in terms of accuracy in extraction of array element response, allowing a more accurate calibration of the phased array system.
The growing demand for 5G wireless communication that operates around millimeter-wave frequencies, have created the need to lower the cost of phased arrays. A significant part of this cost can be attributed to manufacturing tests for millimeter-wave phased arrays requiring each element to be characterized across multiple phase settings. The work presented in this report addresses this issue by introducing CoMET-based characterization of phased array elements in a novel series-feed series-combine BIST architecture. CoMET was first implemented in a behavioral model [Gre16] and then in a four-element phased array fabricated using the Jazz 0.13μm SBC18H3 technology [Gre18]. In this work, COMET has been demonstrated in the modified BIST architecture through a behavioral model using a two-element phased array. The theory and validation of CoMET operation in the new BIST architecture has been presented.
In prior work, CoMET was presented using a shunt-fed injection technique that allowed for multi-element characterization from a single baseband output, requiring minimal overhead on-chip (less than 2%). The technique used a novel signal encoding method where a test-signal at the characterization frequency was injected into each element and phase modulated using unique orthogonal-codes at the RF phase shifters. The modulated signals were then combined on-chip and down-converted to baseband using a single square-law power detector. The down-conversion process resulted in a baseband waveform composed of cross-correlations between each inphase and quadrature-phase components of each element where each correlation term got encoded with a unique orthogonal code product (OCP). Each correlation term was extracted using its respective OCP and then substituted as solution into system-of-equations defined by each element’s performance parameters. A step-by-step extraction was performed in which the in-phase correlations were first used to determine average vector-magnitudes. Second, the quadrature-phase correlations along with the previously determined average vector magnitudes were used to extract phase offsets between elements and quadrature accuracy of individual elements. Finally, symmetry of the original codes were used to extract code offsets.

Key limitations affecting the accuracy of this approach is that it does not account for non-idealities arising in a real circuit specifically the phase shifter, in its extraction process. These practical non-idealities can introduce coding errors which result in erroneous extraction of array performance parameters. The vector interpolator type phase shifter used in CoMET can suffer from amplitude and phase imbalances arising out of non-ideal functioning of various blocks. The important ones are quadrature amplitude and phase imbalance in the 90° coupler, gain dependent phase variation of the VGAs and I/Q Cross-talk. The non-idealities can degrade the accuracy of CoMET extraction. The primary objective of this work is to first, understand if non-idealities in the phase shifter circuit introduce errors into CoMET and second, if CoMET can be modified to calibrate out the errors in a non-ideal phase shifter which introduce errors for CoMET.
5.1 Summary of work done

CoMET has been behaviorally modeled in a new type of BIST architecture termed as "series-fed" BIST architecture in a phased array system. The idea was conceived as a means to improve the accuracy of characterizing each array element. This new BIST architecture allows for a series injection and distribution of the test signal at the characterization frequency. The series form of signal distribution accumulates path delays in the signal between two consecutive elements. The technique uses signal encoding by the phase shifter similar to the prior approach. The code-modulated signal is collected through a series distribution network at the output. The series output network accrues additional path delays in the code-modulated signal. The new technique involves use of two input BIST RF ports and two output baseband ports allowing four sets of measurement data. Thus, this technique routes the signal through four different paths. The CoMET architecture in this technique uses two RF combiners and two square-law devices to allow down-conversion of the RF signal at the two baseband ports. This modified CoMET approach results in four sets of baseband waveform each being comprised of cross-correlations between the I and Q components of the array element. The extraction of each correlation term is done with the respective OCPs. The same set of OCPs are used for all the four measurement sets because encoding procedure at each element is performed by the same orthogonal codes. This results in four different sets of correlation equations where each correlation equation define an element’s performance parameters. With four times the correlation equations additional performance parameters can be evaluated.

Prior work on CoMET, characterized each element across multiple phase settings and could extract: average output power per element, average phase shift per element, quadrature error between the I and Q components of an element, relative phase offsets between different elements and absolute I and Q vector magnitudes using unbalanced coding method. The previous CoMET behavioral model, due to limited number of solvable correlation
equations could not be used to further characterize array element for circuit impairments. This work tries to overcome the above limitations to improve the accuracy of CoMET based extraction.

The phase shifter performance is shown to be the key in achieving extraction accuracy. However, the vector modulator type phase shifter used in the CoMET technique suffers from non-ideal limitations. These non-idealities have been behaviorally modeled and implemented in Matlab to create an "error model" of the phase shifter. This error model is then integrated into the CoMET behavioral model. With the non-ideal environment characteristically embedded, the performance of the prior CoMET behavioral model and the new model are compared. The modified model is observed to predict the non-ideal performance of the array elements. To validate the integrity of the extracted response, it was compared to the response of an individual array element. Excellent overlap was observed with phase error $\leq 0.1^\circ$ across all the extractions except for the case of I/Q cross-talk which violates the quadrature assumption of the I and Q components. A new step-by-step approach has been introduced to extract the performance parameters of the array. This include the series phase offsets at the injection and extraction network accrued by the signal between two consecutive elements, quadrature phase imbalance between I and Q components of an element, VGA phase error, phase shift per element and output power of the array.

5.2 Future Work

In this work the effects of phase shifter non-idealities were modeled and extracted through CoMET behaviorally. However, other impairments may occur in the array such as, time-synchronisation mismatch in the encoding technique resulting in timing skew between the desired sampling instant of the signal by the code and the actual instant at which the code is applied, leading to erroneous extraction of correlation terms. Also, the modified
technique uses a pair of RF combiners and square-law devices to form the baseband output. These device pairs may not be identical and there may be practical mismatches which has not been accounted for in this behavioral model. The series-fed BIST architecture involves extraction of four measurement data-sets. This can significantly increase testing times. A way to overcome this limitation may be to use a single BIST input port and leave the other BIST port open-circuited. The injected test signal is initially distributed to the array elements through the BIST path. However, with the second port open-circuited, the test signal after traversing to the second port will be reflected by the open circuit end. This bounced back signal will then be re-distributed to excite the array elements and can be extracted using CoMET. This may be considered as another set of measured data thus avoiding requirement of signal injection through a second input BIST port, lowering testing time. The series-fed BIST architecture has been modeled behaviorally but it needs to be designed and practically tested on-chip. The CoMET approach has so far been tested on arrays with four and eight number of elements. Hadamard-Walsh sequences are used in CoMET to encode each element in the array such that their product formed a new unique orthogonal code product from which cross-correlations are extracted. Higher-order Hadamard-Walsh matrices greatly increase test times for arrays with a large number of elements. Expanding the viability of this approach to larger number of arrays involves exploration of unique orthogonal codes that are easily scalable with the number of elements in the array.
BIBLIOGRAPHY


Signal equations for other measurement sets are listed below: For measurement set 2, equations 2.5-2.8 can be modified as:

\[ E_{i,1}(t) = (A_{i,1}^{err} c_{i,1}) \cos(\omega t - \phi_{iva,1}) \quad (A.1) \]
\[ E_{q,1}(t) = (A_{q,1}^{err} c_{q,1}) \sin(\omega t - \phi_{c,1} - \phi_{qva,1}) \quad (A.2) \]
\[ E_{i,2}(t) = (A_{i,2}^{err} c_{i,2}) \cos(\omega t - \psi_{i,2} - \phi - \phi_{iva,2}) \quad (A.3) \]
\[ E_{q,2}(t) = (A_{q,2}^{err} c_{q,2}) \sin(\omega t - \phi_{c,1} - \phi - \phi_{qva,2}) \quad (A.4) \]
In a similar way the signal equations of set 3 given by equations 2.9-2.12 can be written as:

\[ E_{i,1}(t) = (A_{i,1}^e r r c_{i,1}) \cos(\omega t - \psi_{i,1} - \phi - \phi_{ivga,1}) \]  \hspace{1cm} (A.5)

\[ E_{q,1}(t) = (A_{q,1}^e r r c_{q,1}) \sin(\omega t - \phi_{c,1} - \psi_{q,1} - \phi - \phi_{q v g a,1}) \]  \hspace{1cm} (A.6)

\[ E_{i,2}(t) = (A_{i,2}^e r r c_{i,2}) \cos(\omega t - \phi_{ivga,2}) \]  \hspace{1cm} (A.7)

\[ E_{q,2}(t) = (A_{q,2}^e r r c_{q,2}) \sin(\omega t - \phi_{c,2} - \phi_{q v g a,2}) \]  \hspace{1cm} (A.8)

For set 4, equations 2.13-2.16 can be modified as:

\[ E_{i,1}(t) = (A_{i,1}^e r r c_{i,1}) \cos(\omega t - \psi_{i,1} - \phi_{ivga,1}) \]  \hspace{1cm} (A.9)

\[ E_{q,1}(t) = (A_{q,1}^e r r c_{q,1}) \sin(\omega t - \phi_{c,1} - \psi_{q,1} - \phi_{q v g a,1}) \]  \hspace{1cm} (A.10)

\[ E_{i,2}(t) = (A_{i,2}^e r r c_{i,2}) \cos(\omega t - \phi - \phi_{ivga,2}) \]  \hspace{1cm} (A.11)

\[ E_{q,2}(t) = (A_{q,2}^e r r c_{q,2}) \sin(\omega t - \phi_{c,2} - \phi - \phi_{q v g a,2}) \]  \hspace{1cm} (A.12)
B.1 Phase shifter error model

```matlab
arr=zeros(17,3);
phi=linspace(22.5,360,16);
R=tan(deg2rad(phi));
A_I=1./sqrt(1+R.^2);
A_Q=R./sqrt(1+R.^2);
A_I(5:12)=-A_I(5:12);
A_Q(5:12)=-A_Q(5:12);

%Non-idealities
phi_r=degtorad(0);
```
alpha_IQ = 1 / sqrt(2);  
alpha_d  = 1 / sqrt(2);  
s = circle(0, 1 / sqrt(2));  
phi_e = 0;  
phi_be = 0;  

for p = 1:1:16  
  for q = 1:1:16  
t = linspace(0, 0.01, 16);  
w = 2 * pi * 1;  
a1 = (A_I(p) - 0 * abs(A_Q(q))) * alpha_IQ * alpha_d;  
a2 = (A_Q(q) - 0 * abs(A_I(p))) * sqrt(1 - alpha_IQ^2) * alpha_d;  
p1 = deg2rad(0) - phi_r * 20 * log10(abs(A_I(p))) - deg2rad(180 - phi_be);  
p2 = deg2rad(90 - phi_e) - phi_r * 20 * log10(abs(A_Q(q))) - deg2rad(180 - phi_be);  
a = sqrt(a1.^2 + a2.^2 + 2 * a1 .* a2 .* cos(p1 - p2));  
Num1 = (a1 .* sin(p1) + a2 .* sin(p2));  
Den1 = (a1 .* cos(p1) + a2 .* cos(p2));  

if Num1 >= 0 && Den1 >= 0  
  th11 = atan(abs(Num1) ./ abs(Den1));  
elseif Num1 > 0 && Den1 < 0  
  th11 = deg2rad(180) - atan(abs(Num1) ./ abs(Den1));  
elseif Num1 < 0 && Den1 < 0  
  th11 = -deg2rad(180) + atan(abs(Num1) ./ abs(Den1));  
else  
  th11 = -atan(abs(Num1) ./ abs(Den1));
v = a \cdot \exp(l \cdot \text{th11});

\text{plot}(\text{real}(v), \text{imag}(v), 'o', 'MarkerSize', 10, 'MarkerEdgeColor', 'k', '
\text{Linewidth}', 1, 'MarkerFaceColor', [0 0 0]);

s = \text{circle}(0, 0.5);

\text{hold on};

\text{xlabel}('Real (Out)', 'FontSize', 24);

\text{ylabel}('Imaginary (Out)', 'FontSize', 24);

\text{end}

\text{end}

\text{grid on}; \text{grid minor};

\text{xlim}([-0.6 0.6]); \text{ylim}([-0.6 0.6]);

\textbf{B.2} \hspace{1em} \text{CoMET code}

\textbf{B.2.1} \hspace{1em} \text{Generating measurement set}

\textbf{function} \hspace{1em} F = \text{test_in1_test_out1_new}(ps, psi1, phi1, quad_gain_imb, 
\hspace{1em} quad_phase_er, balun_gain_imb, balun_phase_er, phase_vga)

%Initialization

n = 2; \% number of elements

dBm2mW = @(x) 10.^((x./10));

Gain_amp1_dB = 10; \% Amplifier 1 gain

Gain_amp2_dB = 10; \% Amplifier 2 gain

PS_angle = ps; \% Phase Shift in degrees for two elements

Test_in1 = 0; \% in dBm

Ain1 = \sqrt{dBm2mW(Test_in1+Gain_amp1_dB)}; \sqrt{dBm2mW(Test_in1+
\begin{verbatim}
Gain_amp2_dB) ];

psi_deg=psi1;
phi_deg=phi1;
f=1; %cycles
samples=1000; %200 n=10, 0.1
w=2 pi f;
t=linspace(0.1/200,2^(2 n),2^(2 n) f samples);

phi_r=phase_vga;
R=tan(deg2rad(PS_angle));
A_I=1./sqrt(1+R.^2);
A_Q=R./sqrt(1+R.^2);

for j=1:n
    if j==1
        off_ang=phi_deg; % el1
        phi_inp=deg2rad(phi_r) . 20 . log10(abs(A_I(j)));
        phi_qua=deg2rad(phi_r) . 20 . log10(abs(A_Q(j)));
        Ain=Ain1(j);
        quad_phase_err=quad_phase_er(j);
        balun_phase_err=balun_phase_er(j);
    elseif j==2
        off_ang=psi_deg; % el2
        phi_inp=deg2rad(phi_r) . 20 . log10(abs(A_I(j)));
        phi_qua=deg2rad(phi_r) . 20 . log10(abs(A_Q(j)));
        Ain=Ain1(j);
        quad_phase_err=quad_phase_er(j);
    end
\end{verbatim}
\[
\text{balun\_phase\_err} = \text{balun\_phase\_er}(j);
\]

end

\[
\alpha_{\text{IQ}} = \text{quad\_gain\_imb};
\]

\[
\alpha_{\text{d}} = \text{balun\_gain\_imb};
\]

\[
A_{I}(j) = A_{I}(j) - 0.25(A_{Q}(j));
\]

\[
A_{Q}(j) = A_{Q}(j) - 0.25(A_{I}(j));
\]

% El1 I+Q output from balun

\[
a_{1} = \sqrt{2} \cdot A_{I}(j) \cdot \alpha_{\text{IQ}} \cdot \alpha_{\text{d}};
\]

\[
a_{2} = \sqrt{2} \cdot A_{Q}(j) \cdot \sqrt{1 - \alpha_{\text{IQ}}^{2}} \cdot \alpha_{\text{d}};
\]

\[
b_{1} = \sqrt{2} \cdot A_{I}(j) \cdot \alpha_{\text{IQ}} \cdot \sqrt{1 - \alpha_{\text{d}}^{2}};
\]

\[
b_{2} = \sqrt{2} \cdot A_{Q}(j) \cdot \sqrt{1 - \alpha_{\text{IQ}}^{2}} \cdot \sqrt{1 - \alpha_{\text{d}}^{2}};
\]

\[
p_{I} = \text{deg2rad}(0 + \text{off\_ang}) + \phi_{\text{inp}};
\]

\[
p_{Q} = \text{deg2rad}(90 + \text{quad\_phase\_err + off\_ang}) + \phi_{\text{qua}};
\]

\[
m_{I} = \text{deg2rad}(180 + \text{balun\_phase\_err + off\_ang}) + \phi_{\text{inp}};
\]

\[
m_{Q} = \text{deg2rad}(270 + \text{quad\_phase\_err + balun\_phase\_err + off\_ang}) + \phi_{\text{qua}};
\]

\[
\text{Num1} = a_{1} \sin(p_{I}) + a_{2} \sin(p_{Q});
\]

\[
\text{Den1} = a_{1} \cos(p_{I}) + a_{2} \cos(p_{Q});
\]

\[
\text{Num2} = b_{1} \sin(m_{I}) + b_{2} \sin(m_{Q});
\]

\[
\text{Den2} = b_{1} \cos(m_{I}) + b_{2} \cos(m_{Q});
\]

if Num1\geq0 && Den1\geq0

\[
\text{th11} = \text{atan}(\text{abs}(\text{Num1}) / \text{abs}(\text{Den1}));
\]

elseif Num1\geq0 && Den1<0

\[
\text{th11} = \text{deg2rad}(180) - \text{atan}(\text{abs}(\text{Num1}) / \text{abs}(\text{Den1}));
\]
elseif Num1<0 && Den1<0
    th11 = -deg2rad(180) + atan(abs(Num1)./abs(Den1));
else
    th11 = -atan(abs(Num1)./abs(Den1));
end

if Num2>=0 && Den2>=0
    th21 = atan(abs(Num2)./abs(Den2));
elseif Num2>0 && Den2<0
    th21 = deg2rad(180) - atan(abs(Num2)./abs(Den2));
elseif Num2<0 && Den2<0
    th21 = -deg2rad(180) + atan(abs(Num2)./abs(Den2));
else
    th21 = -atan(abs(Num2)./abs(Den2));
end

vout_pIpQ(j,:) = Ain.*(sqrt(a1.^2+a2.^2+2*a1.*a2.*cos(pI-pQ))
    balun_gain_imb + ...
    sqrt(b1.^2+b2.^2+2*b1.*b2.*cos(mI-mQ))
    sqrt(1-balun_gain_imb^2)).cos(w*t-
    deg2rad(180+balun_phase_err) + 2*th11-
    th21);

% El1 –I–Q output from balun
a1 = sqrt(2).*A_I(j).*alpha_IQ.*sqrt(1-alpha_d.^2);
a2 = sqrt(2).*A_Q(j).*sqrt(1-alpha_IQ.^2).*sqrt(1-alpha_d.^2);
b1 = sqrt(2).*A_I(j).*alpha_IQ.*alpha_d;
b2 = sqrt(2) \cdot A_Q(j) \cdot sqrt(1-\alpha_{IQ}^2) \cdot \alpha_d;

Num1 = a1 \sin(mI) + a2 \sin(mQ);
Den1 = a1 \cos(mI) + a2 \cos(mQ);
Num2 = b1 \sin(pI) + b2 \sin(pQ);
Den2 = b1 \cos(pI) + b2 \cos(pQ);

if Num1 >= 0 && Den1 >= 0
  th12 = atan(abs(Num1) ./ abs(Den1));
elseif Num1 > 0 && Den1 < 0
  th12 = deg2rad(180) - atan(abs(Num1) ./ abs(Den1));
elseif Num1 < 0 && Den1 < 0
  th12 = -deg2rad(180) + atan(abs(Num1) ./ abs(Den1));
else
  th12 = -atan(abs(Num1) ./ abs(Den1));
end

if Num2 >= 0 && Den2 >= 0
  th22 = atan(abs(Num2) ./ abs(Den2));
elseif Num2 > 0 && Den2 < 0
  th22 = deg2rad(180) - atan(abs(Num2) ./ abs(Den2));
elseif Num2 < 0 && Den2 < 0
  th22 = -deg2rad(180) + atan(abs(Num2) ./ abs(Den2));
else
  th22 = -atan(abs(Num2) ./ abs(Den2));
end

vout mâŒ’mQ(j,:) = Ain \cdot (sqrt(a1^2+a2^2+2 \cdot a1 \cdot a2 \cdot \cos(mI-mQ)) )
balun_gain_imb + ... | sqrt(b1.^2+b2.^2+b1*b2.*cos(pI-pQ)) | sqrt(1-balun_gain_imb^2) * cos(w t-
deg2rad(180+balun_phase_err)+2*th12-
  th22);

% El1 I-Q output from balun
a1=sqrt(2).*A_I(j).*alpha_IQ.*alpha_d;
a2=sqrt(2).*A_Q(j).*sqrt(1-alpha_IQ.^2).*sqrt(1-alpha_d.^2);
b1=sqrt(2).*A_I(j).*alpha_IQ.*sqrt(1-alpha_d.^2);
b2=sqrt(2).*A_Q(j).*sqrt(1-alpha_IQ.^2).*alpha_d;

Num1=a1.*sin(pI)+a2.*sin(mQ);
Den1=a1.*cos(pI)+a2.*cos(mQ);
Num2=b1.*sin(mI)+b2.*sin(pQ);
Den2=b1.*cos(mI)+b2.*cos(pQ);

if Num1>=0 && Den1>=0
  th13=atan(abs(Num1)./abs(Den1));
elseif Num1>0 && Den1<0
  th13=deg2rad(180)-atan(abs(Num1)./abs(Den1));
elseif Num1<0 && Den1<0
  th13=-deg2rad(180)+atan(abs(Num1)./abs(Den1));
else
  th13=-atan(abs(Num1)./abs(Den1));
end

if Num2>=0 && Den2>=0
  th23=atan(abs(Num2)./abs(Den2));
elseif Num2>0 && Den2<0
    th23=deg2rad(180)−atan(abs(Num2)/abs(Den2));
elseif Num2<0 && Den2<0
    th23=−deg2rad(180)+atan(abs(Num2)/abs(Den2));
else
    th23=−atan(abs(Num2)/abs(Den2));
end

vout_pImQ(j,:) = Ain.* ( sqrt(a1.^2+a2.^2+2.*a1.*a2.*cos(pI−mQ)) 
    balun_gain_ imb+... 
    sqrt(b1.^2+b2.^2+2.*b1.*b2.*cos(mI−pQ)) 
    sqrt(1−balun_gain_ imb^2)) * cos(w t−
    deg2rad(180+balun_phase_err)+2*th13−
    th23);
%

% El1 −I+Q output from balun
a1=sqrt(2). A_I(j). alpha_IQ. sqrt(1−alpha_d.^2);
a2=sqrt(2). A_Q(j). sqrt(1−alpha_IQ.^2). alpha_d;
b1=sqrt(2). A_I(j). alpha_IQ. alpha_d;
b2=sqrt(2). A_Q(j). sqrt(1−alpha_IQ.^2). sqrt(1−alpha_d.^2);

Num1=a1 sin(mI)+a2 sin(pQ);
Den1=a1 cos(mI)+a2 cos(pQ);
Num2=b1 sin(pI)+b2 sin(mQ);
Den2=b1 cos(pI)+b2 cos(mQ);

if Num1>=0 && Den1>=0
    th14=atan(abs(Num1)/abs(Den1));
else if Num1>0 && Den1<0
    th14=deg2rad(180)–atan(abs(Num1)./abs(Den1));
elseif Num1<0 && Den1<0
    th14=–deg2rad(180)+atan(abs(Num1)./abs(Den1));
else
    th14=–atan(abs(Num1)./abs(Den1));
end

if Num2>0 && Den2>0
    th24=atan(abs(Num2)./abs(Den2));
elseif Num2>0 && Den2<0
    th24=deg2rad(180)–atan(abs(Num2)./abs(Den2));
elseif Num2<0 && Den2<0
    th24=–deg2rad(180)+atan(abs(Num2)./abs(Den2));
else
    th24=–atan(abs(Num2)./abs(Den2));
end

vout_mIpQ(j,:) = Ain . ( sqrt(a1.^2+a2.^2+2.*a1.*a2.*cos(ml−pQ))
    balun_gain_ imb+…
    sqrt(b1.^2+b2.^2+2.*b1.*b2.*cos(p1−mQ))
    sqrt(1−balun_gain_ imb^2)) * cos(w t−
    deg2rad(180+balun_phase_err)+2*th14−
    th24);

k1=2^(2^n) * samples;

% el1
Arr1(1,1:k1/4) = vout_pIpQ(1,1:k1/4);
Arr1(1,k1/4+1:2 k1/4) = vout_plmQ(1,k1/4+1:2 k1/4);
Arr1(1,2 k1/4+1:3 k1/4) = vout_mlpQ(1,2 k1/4+1:3 k1/4);
Arr1(1,3 k1/4+1:4 k1/4) = vout_mlMQ(1,3 k1/4+1:4 k1/4);

% el2
count=0;
c=0;
for i=1:4
    for j=0:k1/16−1
        Arr2(1,c+j+1) = vout_plpQ(2,c+j+1);
        Arr2(1,c+samples+j+1) = vout_plmQ(2,c+samples+j+1);
        Arr2(1,c+2 samples+j+1) = vout_mlpQ(2,c+2 samples+j+1);
        Arr2(1,c+3 samples+j+1) = vout_mlMQ(2,c+3 samples+j+1);
    end
    count=count+1;
c=(k1/4) count;
end
Eout=Arr1(1,:) + Arr2(1,:);
Eout_squared=Eout.^2;
for i =0:2^(2 n)−1
    Eout_squared_filter(i+1) = mean(Eout_squared((i f samples)+1:(
        i+1) f samples));
end

% Generates orthogonal codes
c_ideal=zeros([2 n 2^(2 n)]);
for i =1:(2 n)
for \( k = 0 : 2^{(2n)} - 1 \)

\[ c_{\text{ideal}}(i,k+1) = 2 \mod(k,2^i) < (2^i/2) - 1; \]

end

end

c_{\text{ideal}} = \text{flipud}(c_{\text{ideal}});

% Generates Orthogonal Code Pairs
m=1;
k=1;

for i=1:(n/2)-1
  for k=i:n/2
    if i==1
      c_{pair}(m,:) = c_{ideal}(i,:).c_{ideal}(k,:);
      m=m+1;
    elseif i~=k % not equal to
      c_{pair}(m,:) = c_{ideal}(i,:).c_{ideal}(k,:);
      m=m+1;
    else
      k=k+1;
    end
  end
end

% Decodes with Orthogonal Code Pairs
for i=1:size(c_{pair},1)
  Eout_{cp}(i) = mean(Eout_{squared_filter} . c_{pair}(i,:));
end

F=Eout_{cp};
end
B.2.2 Main solver

clear all;
clc;

% % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
% Solves Systems of equation %
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % %

options = optimoptions( 'fsolve' , 'Algorithm' , 'trust-region-reflective' , 'TolFun' , 1e-9 , 'MaxFunEvals' , 10000 , 'MaxIter' , 10000 , 'Display' , 'iter-detailed' ) ; % Option to display output

n=2;
c=1;

for y = 22.5:22.5:360
dBm2mW = @(x) 10.^((x./10));

Gain_amp1_dB = 10; % Amplifier 1 gain
Gain_amp2_dB = 10; % Amplifier 2 gain

PS_angle = [y y]; % Phase Shift in degrees for two elements

Test_in1=0; % in dBm phi, alpha_IQ, phi_e, alpha_d, phi_be, phi_r

Ain1= sqrt(dBm2mW(Test_in1+Gain_amp1_dB));
Ain2= sqrt(dBm2mW(Test_in1+Gain_amp2_dB));

quad_gain_imb= 1/sqrt(2);
balun_gain_imb= 1/sqrt(2); %
quad_phase_err=[0 0]; % in degrees
balun_phase_err=[0 0]; % in degrees
phase_vga=0; % in deg/dB

psi_deg=0; %initial values degrees
phi_deg=0;
psi=deg2rad(psi_deg);
phi=deg2rad(phi_deg);

A1=[Ain1 . cos(deg2rad(PS_angle(1))); Ain1 . sin(deg2rad(PS_angle(1)));]
    Ain2 . cos(deg2rad(PS_angle(2))); Ain2 . sin(deg2rad(PS_angle(2)))]
phi_r=phase_vga;

R=tan(deg2rad(PS_angle));
A_I=1./sqrt(1+R.^2);
A_Q=R./sqrt(1+R.^2);
ivga1=deg2rad(phi_r) . 20 . log10(abs(A_I(1)));
ivga2=deg2rad(phi_r) . 20 . log10(abs(A_I(2)));
qvga1=deg2rad(phi_r) . 20 . log10(abs(A_Q(1)));
qvga2=deg2rad(phi_r) . 20 . log10(abs(A_Q(2)));

----------------------------------------Amplitude evaluation----------------------------------------
Eout_in1_out1_cpq1=test_in1_test_out1_new(PS_angle, psi_deg, phi_deg, quad_gain_imb, quad_phase_err, balun_gain_imb, balun_phase_err, phase_vga);
Eout_in1_out2_cpq1=test_in1_test_out2_new(PS_angle, psi_deg, phi_deg, quad_gain_imb, quad_phase_err, balun_gain_imb, balun_phase_err, phase_vga);
Eout_in2_out1_cpq1=test_in2_test_out1_new(PS_angle, psi_deg, phi_deg, quad_gain_imb, quad_phase_err, balun_gain_imb, balun_phase_err, phase_vga);
Eout_in2_out2_cpq1=test_in2_test_out2_new(PS_angle, psi_deg, phi_deg, quad_gain_imb, quad_phase_err, balun_gain_imb, balun_phase_err, phase_vga);

% at 45 deg to both elements
% Ai1, Ai2
Ai=[Ain1 .* (cos(degtorad(PS_angle(1))) - 0.25 * (sin(degtorad(PS_angle(1))))) ; Ain2 .* (cos(degtorad(PS_angle(2))) - 0.25 * (sin(degtorad(PS_angle(2))))) ];
f1=@(A) inphase_amp(A, Eout_in1_out1_cpq1, Eout_in1_out2_cpq1, psi, phi, ivga2, ivga1);
[Aival, fival] = fsolve(f1, Ai, options);
% Aq1, Aq2
Aq=[Ain1 .* (sin(degtorad(PS_angle(1))) - 0.25 * (cos(degtorad(PS_angle(1))))) ; Ain2 .* (sin(degtorad(PS_angle(2))) - 0.25 * (cos(degtorad(PS_angle(2))))) ];
del=deg2rad(0);

% test in x out x
off_ang1=phi; off_ang2=psi;

Num1=Aival(1) * sin(off_ang1+ivga1)+Aqval(1) * sin(deg2rad(90+quad_phase_err(1)))+off_ang1+qvga1);
Den1=Aival(1) * cos(off_ang1+ivga1)+Aqval(1) * cos(deg2rad(90+quad_phase_err(1)))+off_ang1+qvga1);

if Num1>=0 && Den1>=0
    th11=atan(abs(Num1)./abs(Den1));
end
elseif Num1>0 && Den1<0
    th11=deg2rad(180)-atan(abs(Num1)./abs(Den1));
elseif Num1<0 && Den1<0
    th11=-deg2rad(180)+atan(abs(Num1)./abs(Den1));
else
    th11=-atan(abs(Num1)./abs(Den1));
end
ps1=rad2deg(th11);

Num2=Aival(2)*sin(off_ang2+ivga2)+Aqval(2)*sin(deg2rad(90+quad_phase_err(2))+off_ang2+qvga2);
Den2=Aival(2)*cos(off_ang2+ivga2)+Aqval(2)*cos(deg2rad(90+quad_phase_err(2))+off_ang2+qvga2);

if Num2>=0 && Den2>=0
    th21=atan(abs(Num2)./abs(Den2));
elseif Num2>0 && Den2<0
    th21=deg2rad(180)-atan(abs(Num2)./abs(Den2));
elseif Num2<0 && Den2<0
    th21=-deg2rad(180)+atan(abs(Num2)./abs(Den2));
else
    th21=-atan(abs(Num2)./abs(Den2));
end
ps2=rad2deg(th21);
Num1=Aqval(1); Den1=Aival(1);
if Num1>=0 && Den1>=0
    th11=atan(abs(Num1)./abs(Den1));
\[ \text{else if } \text{Num1} > 0 \&\& \text{Den1} < 0 \]

\[ \text{th}11 = \text{deg2rad}(180) - \text{atan}(\text{abs(Num1)}/\text{abs(Den1)}); \]

\[ \text{else if } \text{Num1} < 0 \&\& \text{Den1} < 0 \]

\[ \text{th}11 = -\text{deg2rad}(180) + \text{atan}(\text{abs(Num1)}/\text{abs(Den1)}); \]

\[ \text{else} \]

\[ \text{th}11 = -\text{atan}(\text{abs(Num1)}/\text{abs(Den1)}); \]

\[ \text{end} \]

\[ \text{ps11} = \text{rad2deg}(\text{th}11); \]

\[ \text{k1}(c) = \text{ps11}; \quad \% \text{ideal} \]

\[ \text{k2}(c) = \text{ps1}; \quad \% \text{non-ideal} \]

\[ \text{pI} = \text{deg2rad}(0 + \text{off_ang1}) + \text{ivgal}; \]

\[ \text{pQ} = \text{deg2rad}(90 + \text{quad_phase_err}(1) + \text{off_ang1}) + \text{qvga1}; \]

\[ \text{mI} = \text{deg2rad}(180 + \text{balun_phase_err}(1) + \text{off_ang1}) + \text{ivgal}; \]

\[ \text{mQ} = \text{deg2rad}(270 + \text{quad_phase_err}(1) + \text{balun_phase_err}(1) + \text{off_ang1}) + \text{qvga1}; \]

\[ \text{alpha}_{\text{IQ}} = \text{quad_gain_imb}; \]

\[ \text{alpha}_{\text{d}} = \text{balun_gain_imb}; \]

\[ a1 = \text{sqrt}(2) \times \text{Aival}(1) \times \text{alpha}_{\text{IQ}} \times \text{alpha}_{\text{d}} / (\text{Ain1}); \]

\[ a2 = \text{sqrt}(2) \times \text{Aqval}(1) \times \text{sqrt}(1 - \text{alpha}_{\text{IQ}}^2) \times \text{alpha}_{\text{d}} / (\text{Ain1}); \]

\[ b1 = \text{sqrt}(2) \times \text{Aival}(1) \times \text{alpha}_{\text{IQ}} \times \text{sqrt}(1 - \text{alpha}_{\text{d}}^2) / (\text{Ain1}); \]

\[ b2 = \text{sqrt}(2) \times \text{Aqval}(1) \times \text{sqrt}(1 - \text{alpha}_{\text{IQ}}^2) \times \text{sqrt}(1 - \text{alpha}_{\text{d}}^2) / (\text{Ain1}); \]

\[ \text{pout}(c) = (\text{Ain1} \times (\text{sqrt}(a1^2 + a2^2 + 2 \times a1 \times a2 \times \text{cos}(\text{pI} - \text{pQ})) \times \text{balun_gain_imb} + \text{sqrt}(b1^2 + b2^2 + 2 \times b1 \times b2 \times \text{cos}(\text{mI} - \text{mQ})) \times \text{sqrt}(1 - \text{balun_gain_imb}^2))); \]
pout\_dB(c) = 20 \text{log10}(pout(c));

tr(c) = y;

c = c + 1;

end

k1(9:16) = k1(9:16) + 360;

k2(9:16) = k2(9:16) + 360;

plot(tr, k2);

hold on;

xlabel('Phase setting (degrees)', 'FontSize', 24, 'FontWeight', 'bold');

ylabel('Phase shift (degrees)', 'FontSize', 24, 'FontWeight', 'bold');