ABSTRACT

YANG, RUONAN. Polarimetry Based on Tandem Organic Photovoltaics. (Under the direction of Dr. Michael Kudenov).

Polarization is one of the fundamental properties of the light field, separately defined from its frequency and intensity. Humans have long benefited from our ability to distinguish light of different frequency based on its color but our eyes are not sensitive to the polarization of light. Polarimeters are the devices capable of measuring the polarization state of light and bringing polarimetric capability to a wide range of applications. Most common polarimeter architectures, such as division of time, division of amplitude, division of aperture, division of focal plane and channeled polarimeter either suffers a loss of temporal resolution or has a greater tradeoff of spatial resolution. Alternative detector-level polarization technologies, such as the use of metasurfaces to overcome the size constraints of conventional free-space optics, still have some challenges to overcome, such as low conversion efficiency and high cost.

In this work, a full-Stokes intrinsic coincident polarimeter with high spatial and temporal resolution to measure the polarization state of light has been first designed, characterized, calibrated and experimentally verified in the laboratory. This polarimeter is based on semitransparent, strain-aligned organic photovoltaic devices, which are low-cost and easily fabricated. The description of the intrinsic coincident polarimeter, its design characterizations, system model, calibration, validation and optimization are discussed. By using Muller matrix formalism and applying appropriate calibration parameters, the intrinsic coincident polarimeter allows us to calculate the four-element Stokes vector accurately both on-axis and off-axis.
Intrinsic Coincident Polarimetry Based on Tandem Organic Photovoltaics

by
Ruonan Yang

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APPROVED BY:

Dr. Michael Kudenov
Committee Chair

Dr. Robert Kolbas

Dr. Brendan O’Connor

Dr. Michael Escuti
DEDICATION

To my parents Baoqing Yang and Yingjie Liu for their unwavering love and support.
BIOGRAPHY

Ruonan Yang was born in Baoding, Hebei, China to Baoqing Yang and Yingjie Liu. Following high school, Ruonan attended Zhejiang University in Hangzhou China for four years to earn her B.S. in optical engineering. Out of the belief that use of Optics can greatly benefit people’s life, Ruonan continued her pursuit of PhD degree at North Carolina State University under the guidance of Dr. Michael Kudenov. Upon graduation, Ruonan will dedicate herself in Optics industry to explore new applications with Optics and make Optics benefit people’s life.
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TABLE OF CONTENTS

LIST OF TABLES........................................................................................................................ vii
LIST OF FIGURES ..................................................................................................................... viii
CHAPTER 1. Introduction............................................................................................................. 1
  1.1. Overview.............................................................................................................................. 1
  1.2. Motivation............................................................................................................................ 1
  1.3. Dissertation Structure........................................................................................................... 4
CHAPTER 2. Background ............................................................................................................. 6
  2.1. Representation of Polarization............................................................................................. 6
  2.2. State of Art Polarimeters.................................................................................................... 9
    2.2.1. Division of Time Polarimeter ................................................................................. 10
    2.2.2. Division of Amplitude Polarimeter .......................................................................... 11
    2.2.3. Division of Aperture Polarimeter ........................................................................... 12
    2.2.3. Division of Focal Plane Polarimeter ........................................................................ 13
    2.2.4. Channeled Polarimeter ............................................................................................ 14
    2.2.5. Detector-level Polarimeter Technology .................................................................. 16
  2.3. Polarimeter Optimization................................................................................................... 17
    2.3.1. Optimization under Gaussian noise ........................................................................ 18
    2.3.2. Optimization under Poisson noise .......................................................................... 19
CHAPTER 3. Intrinsic Coincident Polarimeter ........................................................................... 20
  3.1. Polarization-Sensitive Organic Photovoltaics................................................................. 20
    3.1.1. Schematic Diagram ................................................................................................. 20
    3.1.2. Absorption Spectrum .............................................................................................. 21
    3.1.3. External Quantum Efficiency ................................................................................. 22
    3.1.4. Detector Linearity ................................................................................................... 23
  3.2. Intrinsic Coincident Polarimeter Design ......................................................................... 24
  3.3. Optical and Electrical Model of Organic Photovoltaic cell ............................................ 24
    3.2.1. Optical Model ......................................................................................................... 24
    3.2.2. Optical-Electrical Model......................................................................................... 26
  3.3. Intrinsic Coincident Polarimeter Model ......................................................................... 28
  3.4. System Calibration and Model Validation ....................................................................... 29
    3.4.1. OPV and Waveplates Characterization ................................................................... 29
    3.4.2. Experiment Setup.................................................................................................... 31
    3.4.2. Radiometric and Polarimetric Calibration .............................................................. 32
LIST OF TABLES

Table 1. RMS error for Stokes parameters. ................................................................. 35
Table 2. Initial and final value of the fitting variables obtained using the parametric fit.... 35
Table 3. OPV and WP parameters as determined by separate characterization experiments. 43
Table 4. Initial value and fitted value for OPVs and WPs.............................................. 44
Table 5. Initial and fitted parameter values for the OPV .............................................. 48
Table 6. Noise variance of DoT, DoAM and Ideal polarimeter .................................... 55
Table 7. Noise variance under measurement matrix $W_{\text{Case I - GN}}$ and $W_{\text{Case I - PN}}$. 58
Table 8. Noise variance under measurement matrix $W_{\text{Case II - GN}}$ and $W_{\text{Case II - PN}}$. 60
Table 9. Noise variance under measurement matrix $W_{\text{Case III - GN}}$ and $W_{\text{Case III - PN}}$. 61
Table 10. Noise variance under measurement matrix $W_{\text{Case IV - GN}}$ and $W_{\text{Case IV - PN}}$. 63
Table 11. Noise variance under measurement matrix $W_{\text{Case V - GN}}$ and $W_{\text{Case V - PN}}$. 65
Table 12. Noise variance under measurement matrix $W_{\text{Ideal}}$ and $W_{\text{Ideal - IC}}$. .......... 66
LIST OF FIGURES

Figure 1.  Polarization image enhances target detection. Vehicles under tree shadows can hardly be seen in (a) long-wave infrared image but are enhanced using (b) long-wave infrared polarization image. ................................................................. 2

Figure 2.  An elliptically polarized wave and the polarization ellipse................................. 7

Figure 3.  Schematic diagram of a DoT polarimeter.............................................................. 11

Figure 4.  Schematic diagram of DoAM polarimeter. (a) BS or prism along with Wollaston prism are used to divide the light path (b) BSs and PSAs are used to divide light path. The second detector D2 and the second PSA are out of the plane of the page. ................................................................................................................................. 12

Figure 5.  Schematic diagram of DoA polarimeter................................................................. 13

Figure 6.  Schematic diagram of DoFP polarimeter.............................................................. 14

Figure 7.  Schematic of the spectral modulated CH polarimeter............................................ 15

Figure 8.  Schematic of the spatial modulated CH polarimeter............................................. 15

Figure 9.  Detector-level polarimeter technology uses a subwavelength antenna array to generate four scattered beams light depending on the polarization states.................. 16

Figure 10. Illustration of polymer chain alignment in the film. (a) The film is isotropic and has no polarization sensitivity. (b) The polymer backbone is aligned uniaxially, resulting in anisotropic absorption................................................................. 20

Figure 11. (a) A schematic diagram of one OPV cell (b) Transition dipole along the backbone of the semiconducting polymer active layer, which leads to anisotropic absorbance of polarized light................................................................. 21

Figure 12. Absorbance spectra for strain-aligned P3HT:PCBM blend films......................... 22
Figure 13. Absorbance spectra for strain-aligned P3HT:PCBM blend films. ............................ 23

Figure 14. (a) I-V curve (b) Responsivity curve of an OPV under different incident powers.... 23

Figure 15. Schematic diagram of a full-Stokes intrinsic coincident polarimeter...................... 24

Figure 16. A schematic representation of a full-Stokes IC polarimeter................................. 29

Figure 17. (a) Transmission, reflectance and responsivity characterization experiment setup
(b) Diattenuation and reflectance characterization experiment setup (c) Waveplate retardance characterization experiment setup................................................................. 30

Figure 18. (a) Schematic of the calibration experiment setup. (b) Electrical circuit for recording electrical signal from each OPV and waveforms from oscilloscope output.......................................................... 32

Figure 19. Current outputs from 4 OPVs and the model fitting results under 3 different illumination powers. I1-I3 are the experimental data, and I1F-I3F are the model fits. ............................................................ 34

Figure 20. (a) 15708 Stokes vectors (blue data) were sampled across the Poincare sphere. (b) Current result from 4 OPVs calculated from measurement matrix A and â. To have a good visualization, only 315 of 15708 sample results were plotted. ............ 37

Figure 21. Schematic of monolithic cell with back reflections................................................. 38

Figure 22. Structure of OPV. The case where light is incident on the ITO is defined as forward incidence (FI). Conversely, the case where light is incident on the gold contact (Au) is defined as backward incidence (BI)................................................. 39

Figure 23. (a) Characterization experiment for the OPV to obtain model parameters. (b) The BI case was experimentally tested, using a mirror, to validate the model’s ability to parametrically calculate the detected current. ....................................................... 41
Figure 24. (a) Photo-generated current versus QWP angle from characterization measurement; (b) Transmitted power versus QWP angle from characterization measurement; (c) Measured and modeled output photo-generated current with a mirror behind a single OPV ................................................................. 42

Figure 25. (a) Experimental setup of monolithic polarimeter calibration in free space. (b) The electrical circuit for recording the electrical signal from each OPV ........................................ 43

Figure 26. Current outputs from 4 OPVs and the model fitting results: (a) 20 measurements under P2 (1.38mW) and P3 (1.67mW) for calibration and (b) the remaining 57 measurements under P1 (1.12 mW), P2 (1.38 mW), P3 (1.67 mW) and P4 (1.95 mW) for validation. ........................................................................................................ 44

Figure 27. 10,000 Stokes vectors (blue data) are uniformly sampled across the Poincare sphere .................................................................................................................. 46

Figure 28. (a) IC polarimeter array and (b) incidence cone ..................................................... 47

Figure 29. OPV thickness and refractive index calibration experimental setup. The angle $\mu$ can be adjusted by rotating the OPV in the x-z plane while $\nu$ is adjusted by rotating it in its local x-y plane. ......................................................................................... 48

Figure 30. Transmission and absorption experiment data and fitted value for both TM and TE polarized light when varying $\mu$ from 0˚ to 18˚ at azimuthal angle $\nu = 0$˚ ............... 49

Figure 31. The current ratio of OPV1 and OPV2 under TM and TE polarized light for zenith angle $\mu$ varying from 0˚ to 18˚ and azimuthal angle $\nu$ from 0˚ to 360˚ ...................... 50

Figure 32. Reconstruction error of the S1 component of (a) TM and (b) TE polarized light zenith angle $\mu$ varying from 0˚ to 18˚ and azimuthal angle $\nu$ from 0˚ to 360˚. ........ 51
Figure 33. 10,000 Stokes vectors (blue data) are uniformly sampled across the Poincare sphere.

Figure 34. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under PN for (a) DoT; (b) DoAM and (c) Ideal polarimeters.

Figure 35. IC polarimeter configurations: (a) Case I and Case II: Four-cell (OPV) system with two quarter-wave plates (QWPs). (b) Case III: Four-cell system with additional QWP in front. (c) Case IV: Four-cell system with retardance-relaxed waveplates. (d) Case V: Five-cell ICP polarimeter.

Figure 36. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) WCaseI-GN (b) WCaseI-PN.

Figure 37. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) WCaseII-GN (b) WCaseII-PN.

Figure 38. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) WCaseIII-GN (b) WCaseIII-PN.

Figure 39. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) WCaseIV-GN (b) WCaseIV-PN.

Figure 40. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) WCaseV-GN (b) WCaseV-PN.

Figure 41. Noise variances of the sampled Stokes vector under PN for ideal IC polarimeter.

Figure 42. Noise variance comparison of ideal, DoT, DoAM and IC polarimeters. (a) Noise variance of $S_0$, $S_1$, $S_2$, and $S_3$ under Gaussian noise. (b) Maximum noise variance of $S_0$, $S_1$, $S_2$, and $S_3$ under Poisson noise.
Figure 43. Comparison of current polarimeter architectures and proposed intrinsic coincident technique. ................................................................................................................... 70
CHAPTER 1. Introduction

1.1. Overview

This dissertation is focused on the development of a novel polarization sensing architecture based on stain-aligned semitransparent organic photovoltaic devices. Polarization as an important property of light has broad applications in the fields of remote sensing, astronomy, biomedical imaging, industrial process and computer vision. Polarization states can be measured by polarimeters. Current polarimeter architectures have a trade-off between temporal and spatial resolution, potentially losing polarization information either in time domain or in space domain. This motivates us to develop a polarimeter with high temporal and spatial resolution to resolve this issue.

1.2. Motivation

Polarized light is pervasive in our world and can provide important signatures of certain targets. We must understand it, measure it and finally be able to use it. The spatial orientation of the electric field in a Transverse Electromagnetic (TEM) wave defines the polarization angle of the propagating light wave. Like wavelength and intensity, polarization is a separate property of light but is one that we cannot see directly. When light interacts with different objects or medium, whether found in nature\textsuperscript{1–4} or manmade\textsuperscript{5–8}, the polarization state can be changed in different manners. Therefore, polarization is an important discriminator used for target detection and finds applications in various fields, such as remote sensing\textsuperscript{9,10}, astronomy\textsuperscript{11–13}, biomedical imaging\textsuperscript{14–19}, industrial process\textsuperscript{20–22} and computer vision\textsuperscript{23–25}.

In remote sensing, potential target detection, such as aircrafts or ground vehicles, is a critical step. However, background clutter makes the detection difficult when targets are embedded in a cluttered environment. The signal returned from the target does not stand out against the
background. The introduction of polarization into infrared imaging system is one of the techniques to improve the low target-to-clutter ratio since manmade objects are the sources of emitted and reflected polarized radiation while natural backgrounds are predominantly unpolarized. Figure 1 shows an example of capability of polarization to enhance the contrast when there is little contrast in intensity imagery.

![Figure 1. Polarization image enhances target detection. Vehicles under tree shadows can hardly be seen in (a) long-wave infrared image but are enhanced using (b) long-wave infrared polarization image.](image)

In astronomy, direct imaging of exoplanets is a very challenging task since the separation between a star and its companion is very small but the ratio of their intensities or the contrast is extremely high. Unpolarized light from the central star that is reflected by exoplanets becomes linearly polarized by the reflection process. Therefore polarization can be used to discriminate between the polarized light from circumstellar environments and the unpolarized light from the nearby central star.

Additionally polarization effects are extensively used in various forms of biomedical imaging. For example, dermatologists use polarization imaging to selectively concentrate on either surface irregularities or alternatively on deeper epidermal/dermal layers. Further, different biological structures affect the polarization of light differently, thus polarization can provide access to intrinsic, tissue-specific image contrast, which can be applied to various disease diagnosis, such as Alzheimer’s disease, retinal tissue disease and cancers.
In industrial process, measurement of sugar concentrations is very important to determine the sugar levels in beverages or to test the quality of agricultural crops. Sugar can rotate the plane of polarization of linearly polarized light by a certain angle that is proportional to its concentration. This behavior is known as optical rotation\textsuperscript{20}. Thus by using a polarizer pair and Malu’s law\textsuperscript{21}, sugar concentration can be derived using a polarization based non-invasive measurement\textsuperscript{22}.

The field of computer vision deals with how computers can understand digital images or videos and seeks to automate tasks that human visual system can do. One of the tasks in computer vision field is to understand the scene. Use of polarization can immensely simplify some important visual tasks, such as region and edge segmentation, material classification, which are possibly infeasible using intensity information. Wolff\textsuperscript{23} introduced a polarization-based method for discriminating between dielectric and metal surfaces. Wang et al\textsuperscript{24} used polarized camera to detect high pollutant discharge through haze. The most studied application of polarization in computer vision relates to three-dimensional shape analysis, where the polarizing effects of surface reflection are exploited to estimate geometry\textsuperscript{25}.

Given the broad applications and importance of polarization, polarimetry emerges as a field of study that quantifies the polarization state of light. An important part of polarimetry is the polarimeter which is the tool to essentially measure the polarization states of light. Current polarimeter architectures include division of time (DoT), division of amplitude (DoAM), division of aperture (DoA), division of focal plane (DoFP), channeled (CH) polarimeter and detector-level polarimeters. Division of time (DoT) polarimeters contain rotating polarization elements in front of a camera system and require at least four measurements to obtain the full Stokes vector, resulting in a loss of temporal resolution. DoAM, DoA, DoFP, and CH have advantages over DoT in simultaneous measurements. However, DoAMs have large optical systems and require accurate
image registration in post-processing. DoFP systems utilized 2 by 2 subpixels to reconstruct polarization information of one macropixel which suffer a loss of spatial resolution, while CH polarimeters offer a greater tradeoff of spatial and/or spectral resolution than comparable DoA systems. DoA can maintain both high temporal and spatial resolution, but with higher complexity. There are also alternative detector-level polarization technologies, such as the use of metasurfaces to overcome some of the size constraints of conventional free-space optics. This technology still has some challenges to overcome, such as low conversion efficiency and infeasible application in imaging system.

To resolve the issues in current polarimeter architectures, an intrinsic coincident polarimeter architecture is addressed in this dissertation. This new polarimeter architecture utilizes polarization-sensitive organic photovoltaics cells in combination with polarization elements to enable polarization information reconstruction. Its design characterizations, system model, calibration, validation and optimization are discussed in this dissertation. By using Muller matrix formalism and applying appropriate calibration parameters, this intrinsic coincident polarimeter allows us to calculate the four-element Stokes vector accurately both on-axis and off-axis.

1.3. Dissertation Structure

We approach this goal with individually focused research topics shown in following chapters.

Relevant background is given in Chapter 2 which is necessary for a better understanding of this dissertation. Mathematical representation of polarized light is introduced first to show how we can describe and quantify polarized light. Second, state of art polarimeter architectures are discussed to present to the readers the advantages and limitations of current polarimeter technology.
Finally, polarimeter optimization concept is brought up with which scientists and engineers can obtain more accurate measurements.

Chapter 3 is focused on introducing the design of the intrinsic coincident (IC) polarimeter. The most important element, the polarization-sensitive organic photovoltaics (OPVs), is first discussed in this chapter, followed by the design of IC polarimeter enabled by OPVs. Both OPV models and IC polarimeter model are derived and validated through experiments.

Chapter 4 addresses the problem of integrating the IC polarimeter where back reflection from subsequent OPVs is unavoidable through integration. Optical cross-talk model induced by back reflection is proposed and validated. For the first time, monolithic intrinsic coincident polarimeter is demonstrated with the updated model.

Chapter 5 is mainly about optimization of an IC polarimeter. Different configurations of IC polarimeters are optimized under Gaussian noise and Poisson noise. A quantitative comparison is given for existing polarimeter architectures and IC polarimeters.

Chapter 6 summarizes the conclusive results of this dissertation and suggests future research topics based on the discussions.
CHAPTER 2. Background

This chapter presents background information and an overview of the related research to the design of polarimeters. The first part provides background information on mathematical description of polarization. The second part overviews existing polarimeter architectures, including division of time, division of amplitude, division of aperture, division of focal plane, channeled polarimeter and detector-level polarimeter technology with methodology of optimizing a polarimeter given in the third part.

2.1. Representation of Polarization

The most general description of the polarization of a single light wave is in terms of an ellipse, as depicted in Figure 2. The total field is represented as the sum of two orthogonal electric fields: $E_x$ is the component in the $\hat{x}$ direction, while $E_y$ is the component in the $\hat{y}$ direction. Each of these components has a magnitude and phase. Since absolute phase of light cannot be measured readily, the phases can be reduced to a relative phase $\phi$ between the two components. The total electric field is then described as

$$E = \hat{x}E_x \cos(\omega t - kz) + \hat{y}E_y \cos(\omega t - kz + \phi)$$ (2.1)

where $\omega$ is the frequency of light, $\phi$ is the relative phase, $t$ is time, $E_x$ and $E_y$ are the maximum electric field amplitudes, and $k$ is the wavenumber. If there is zero relative phase $\phi$ between the two orthogonally oriented electric field components, the light will be linearly polarized. In this case, both components oscillate together in time. If both $E_x$ and $E_y$ are equal and have a $\pm 90^\circ$ phase shift, circular polarization will occur, causing the total electric field to rotate through space and time in a circular pattern. The general case of elliptical light can be viewed as a superposition of circularly polarized light and any linearly polarized light vector. Instead of equal $E_x$ and $E_y$...
components with exactly 90° phase shift, any combination of phase and magnitude can occur. This polarization is generally a rotating ellipse in time and space. Therefore in the general case, light is said to be elliptically polarized.

Figure 2. An elliptically polarized wave and the polarization ellipse.

The description of light in terms of the polarization ellipse is very useful because it enables us to describe various states of polarized light by means of a single equation. However, this representation is inadequate because it is only applicable to describing light that is completely polarized. It cannot be used to describe either unpolarized light or partially polarized light. Stokes parameters containing four elements are an accurate means of describing partially polarized light. Each of these elements describes a fundamentally different polarization property, as listed in Eq. (2.2).

\[
S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}
\]

(2.2)

\(S_0\) or \(I\) is the total intensity of light or more appropriately, the total irradiance of light. This is the only Stokes quantity that our eyes can see. \(S_1\) or \(Q\) is the fraction of light that is polarized more in the horizontal than in the vertical direction. \(S_1\) is negative if positive if the light is
polarized more in the horizontal than the vertical direction and $S_1$ is positive if vice versa. $S_2$ or $U$ is the fraction of light that is polarized more in the 45° direction than in −45° direction. $S_3$ or $V$ indicates the amount of circular polarization present in the measured light. Negative numbers represent right-hand circular while positive numbers represent left-hand circular polarization. Since light can be a combination of polarized states plus potentially unpolarized component, $S_1$, $S_2$, and $S_3$ are constrained by the total intensity $S_0$, following the relationship of

$$S_0^2 \geq S_1^2 + S_2^2 + S_3^2$$  \hspace{1cm} (2.3)$$

The Stokes vector is only valid for light that is incoherent. Light that is coherent could potentially interfere at the detector plane causing the Stokes representation not to be valid. This requires that the coherence time of the wavefront being measured be much shorter than the integration time of the instrument.

Stokes parameters can be applied to a transfer-function approach for optical propagation. When light travels through space, scatters through a medium, reflects off a surface, or propagates through optical components, its Stokes vector typically can change. The output Stokes vector is a function of the initial state and the interactions that occurred during propagation. The Mueller matrix is introduced as a mathematical tool to describe the effects of material reflection, propagation, and scattering on any polarized, unpolarized, or partially polarized light wave. A Mueller matrix is a 4×4 matrix that connects the original state $S_{in}$ to the output state denoted by $S_{out}$, as illustrated in Eq. (2.4).

$$S_{out} = M \cdot S_{in} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{bmatrix} \cdot S_{in}.$$  \hspace{1cm} (2.4)
Each element in Mueller matrix $M$ represents a certain intrinsic property of the medium of the system. Element $m_{00}$ scales the input intensity to the output intensity so this element can be interpreted as the simple transmittance for a transmissive system or reflectance for a reflecting system. The rest diagonal elements $m_{11}$, $m_{22}$ and $m_{33}$ describe the depolarization of the system. If the diagonal elements $m_{00}$, $m_{11}$, $m_{22}$ and $m_{33}$ are equal to 1, while the others are equal to zero, the Mueller matrix represents the free-space propagation. Element $m_{01}$ and $m_{02}$ scale the linear polarization at $0^\circ / 90^\circ$ and $45^\circ / -45^\circ$ to the output intensity respectively, which are known as linear extinction while $m_{03}$ scales the circular polarization to the output intensity which is known as circular extinction. When a sample exhibits more than one optical property, the elements of the Mueller matrix mix together and complicate the interpretation.

2.2. State of Art Polarimeters

Polarimeters use the Mueller matrix to reconstruct the Stoke vector of a system. The difficulty of polarization measurement is that the standard sensors cannot return a polarization value but return a value that corresponds to the irradiance or brightness of an object. Therefore, a novel approach must be used to determine the Stokes vector for an optical measurement. Since a Mueller matrix models an optical system, it was conceived that by measuring the irradiance of an object through different known Mueller matrices, the various pixel brightness values and the known Mueller matrix values could be used to find the polarization of the incoming light. According to Eq.(2.4), the irradiance of a pixel $S_0$ is only dependent upon the first row of the Mueller matrix. Therefore, only the first lines of each Muller matrix is useful in extracting the input polarization state. To extract a complete Stokes vector, a minimum of four independent intensity measurements with different polarization settings on the polarimeter where each setting
is represented by a unique Mueller matrix \( M \) are required. The first rows of the Mueller matrix for each of the four polarimeter states form the system matrix \(^9\), as noted by

\[
\begin{bmatrix}
M_{1}(1,1:4) \\
M_{2}(1,1:4) \\
M_{3}(1,1:4) \\
M_{4}(1,1:4)
\end{bmatrix} = \begin{bmatrix}
m_{100} & m_{101} & m_{102} & m_{103} \\
m_{200} & m_{201} & m_{202} & m_{203} \\
m_{300} & m_{301} & m_{302} & m_{303} \\
m_{400} & m_{401} & m_{402} & m_{403}
\end{bmatrix}, \tag{2.5}
\]

where \( M_i (i = 1,2,3,4) \) is the Mueller matrix for \( i \)th measurement and \( m_{i,j} (j = 1,2,3,4) \) is the elements of the first row of \( M_i \). The irradiance or intensity of a pixel obtained from four independent measurements form a signal vector \( I = [I_1, I_2, I_3, I_4]^T \) which can be calculated by

\[
I = W \cdot S, \tag{2.6}
\]

where \( S = [S_0, S_1, S_2, S_3]^T \) is the incident Stokes vector. Consequently \( S \) can be reconstructed by the inverse matrix of the measurement matrix \( W \) and the signal vector \( I \) by

\[
S = W^{-1} \cdot I, \tag{2.7}
\]

Imaging of the same scene through four different Mueller matrices is accomplished either by quickly changing polarization elements between images and assuming the scene is not changing, which is categorized as division of time (DoT) polarimeter or by using different optical path to obtain four simultaneous measurements. The latter case uses either division of amplitude (DoAM), division of aperture (DoA), division of focal plane (DoFP), channeled (CH) polarimeters and detector-level polarimeter technology.

2.2.1. Division of Time Polarimeter

Division of time (DoT) polarimeter rotates polarization elements in front of the camera system three or more times and records the pixel value sequentially, as depicted in Figure 3. The rotating elements or polarization state analyzer (PSA) in a DoT polarimeter can be a linear
polarizer to measure the linear Stokes \((S_0, S_1, S_2)^{26}\) only or a combination of one or two waveplates and a linear polarizer to obtain the full Stokes vector \((S_0, S_1, S_2, S_3)^{27,28}\). The advantage of using a DoT polarimeter to measure the input Stokes vector is its relatively straightforward system design and data reduction. However, the main issue is the systematic occurrence of artifacts as the scene is not static. In addition, artifacts could also be introduced by the beam wander induced by the rotating element. Beam wander may result from the wedge in the rotating elements or wobbling elements during data acquisition.

![Figure 3. Schematic diagram of a DoT polarimeter.](image)

Efforts have been devoted to correct the motion artifacts that are avoidable in DoT polarimeter for dynamic scene acquisition, such as applying a temporal median filter to improve the rendering of the Stokes images\(^ {29}\) or performing a motion estimation to correct the objects displacement between two polarization state pictures\(^ {30}\). However, all trials require heavy post-processing work and are not suitable for real-time observation.

2.2.2. Division of Amplitude Polarimeter

Typical division of amplitude (DoAM) polarimeters are depicted in Figure 4 (a) where it utilizes beamsplitter (BS)\(^ {31}\) or prism\(^ {32}\) and wollastron prisms and (b) where a serial of beam splitters and polarization state analyzers (PSAs) are inserted in the light path to divide the incoming light into four channels\(^ {33}\). The polarization states are measured for each light path separately.

In a DoAM polarimeter, the four detectors simultaneously capture the four images needed to compute a complete Stokes image, thus eliminating the motion artifacts inherent in DoT
polarimeters. Since the polarizing elements exist in all four measurement channels, imposing more freedoms in the system, postprocessing is required to coregister the four image. In addition, the size of the DoAM system is usually bulky and rigid mechanical mounts are required to support polarizing elements and detectors in positions.

![Figure 4. Schematic diagram of DoAM polarimeter. (a) BS or prism along with Wollaston prism are used to divide the light path (b) BSs and PSAs are used to divide light path. The second detector D2 and the second PSA are out of the plane of the page.](image-url)

2.2.3. Division of Aperture Polarimeter

The schematic diagram of a Division of aperture (DoA) polarimeter is depicted in Figure 5. A standard camera objective lens was used to form an image of the object onto the field stop. The collimation optics imaged the objective lens aperture onto the mini-lens array such that the light incident on the objective lens was evenly divided across the four mini-lens arrays which formed an image of the object onto the detector. The polarizing elements, polarizers or retarders, was
placed after each mini-lens to measure a different polarization state across the object. DoA can acquire all the polarization data simultaneously and ensure the field of view (FOV) of all the polarization channels are coboresighted\textsuperscript{34}. Compared with DoAM, the size of DoA polarimeter is much smaller and system is more stable after the components are mechanically fixed. However, similar with DoAM polarimeter, misregistration of the image and distortion can be critical issues in DoA polarimeter when the system is drifted during operation. Careful calibration is needed to match the transmission, magnification and distortion between the channels. Besides, DoA polarimeter suffers a loss of spatial resolution by dividing the aperture.

Figure 5. Schematic diagram of DoA polarimeter.

2.2.3. Division of Focal Plane Polarimeter

Division of focal plane (DoFP) polarimeter uses a polarization focal plane array (FPA) that is analogous to a Bayer color filter\textsuperscript{35} FPA in that 2 x 2 neighboring pixels are used to form one image pixel called macro pixel. Majority of DoFP polarimeters are based on wire-grid polarizer arrays which can be fabricated on a separate substrate\textsuperscript{36} or directly on the sensor\textsuperscript{37}. Most often the linear polarizers in the micropolarizer array are oriented at $0^\circ, 45^\circ, 90^\circ,$ and $135^\circ$, as depicted in Figure 6 (a). Due to the lack of retarders in the system, this kind of DoFP can only measure the linear Stokes ($S_0, S_1$ and $S_2$). Myhre et al\textsuperscript{38} demonstrated a full Stokes DoFP polarimeter using two layers of patterned liquid crystal polymer (LCP) separated by an intermediate buffer layer, as depicted in Figure 6 (b).
Figure 6. Schematic diagram of DoFP polarimeter.

Since DoFP uses 2 by 2 detector pixel to reconstruct one Stokes vector in polarization image, spatial resolution is traded off for polarization information. It also introduces the instantaneous field of view (IFOV) error induced by 1-pixel misregistration.

2.2.4. Channeled Polarimeter

Channeled (CH) polarimeters usually create wavelength-dependent or location-dependent analyzing states using optical components such as prisms, retarders, linear polarizer and polarization gratings to measure the Stokes parameters. Multiple polarization-dependent channels are formed spectrally or spatially, or a combination of both.

A spectral modulated channeled polarimeter is depicted in Figure 7. Broadband light passes successively through a pair of thick birefringent retarders, R1 and R2 and analyzer A. The fast axis of R1 and R2 intersect at an angle of 45° and the transmission axis of A is aligned with the fast axis of R1. Then the light is fed into a spectrometer followed by a photodetector D and spectrum \( P(\sigma) \) is obtained. The Fourier inversion of the channeled spectrum contains the information of one or two Stokes parameters. Through channel isolation and phase compensation, the four components in Stokes vector can be determined.
By switching the thick retarders in a spectral modulated channeled polarimeter to birefringent wedge prisms, polarization information from incident light can be modulated into spatial channels which transfers the polarization analysis from spectral domain to spatial domain, as depicted in Figure 8. PR1, PR2, PR3 and PR4 are wedge prisms with birefringence to provide spatial-dependent retardance, with fast-axis oriented at 0°, 90°, 45° and -45° respectively. LP is a linear polarizer with transmission axis parallel to the horizontal axis, which transfers the spatial-dependent retardance into spatial-dependent intensity patterns. After filtering the spatial frequency in Fourier domain and applying inverse Fourier transform, four channels containing Stokes parameters can be obtained in spatial domain.

CH polarimeter uses no mechanically movable components, which greatly reduces the system complexity. However, the need for a birefringent medium can additionally drive up the minimum cost and size of polarimeter, and suitable materials are not readily available for all wavelength ranges. Additionally spectral modulated CH polarimeter requires a full spectrum
measurements which requires a spectrometer incorporate into the system. For a spatial modulated CH polarimeter, spatial resolution is potentially lost by beam splitting.

2.2.5. Detector-level Polarimeter Technology

A detector-level polarimeter technology based on metasurface is used to monitor the polarization state of transmitted signal in telecommunications\textsuperscript{39}. It utilized two-dimensional antenna array to scatter a small part of the normally incident light into four different directions as depicted in Figure 9 (a). The scattered intensity in each channel is proportional to the strength of a different polarization component which depends on the spacing and orientation angles of the rod antennas. As shown in Figure 9 (b), two pairs of rows are superimposed at a relative angle of $45^\circ$ and in each pair the subwavelength rod antennas are orientated $\pm 45^\circ$ with respect to the row axis with a spacing of $(1+1/8)\lambda$. With this configuration, four left- and right- elliptically polarized light is directionally scattered in opposite directions. To change the polarization being scattered, simply engineering the spacing between each rows. For example, if the antenna rows spaced by $(m+1/2)(\lambda/2)$ where $m$ is an integer, circularly polarized light of different handedness will be scattered in opposite directions. If the rows are spaced by multiples of $\lambda/2$, linearly polarized light along the row axis will be scattered equally in both directions.

![Figure 9](image.png)

Figure 9. Detector-level polarimeter technology uses a subwavelength antenna array to generate four scattered beams light depending on the polarization states.
One advantage of this technique is that the polarization measurement leaves the signal mostly intact, which is especially useful in optical telecommunications. Besides, the system is compact, overcoming the size constraints of free-space polarimeters. Despite the attractiveness of the use of this detector-level polarimeter in telecommunications, it is difficult to apply the technology to imaging system since the signal in four channels are in the same plane with the antenna array and additional gratings are needed to couple the light out.

2.3. Polarimeter Optimization

Using Mueller-Stokes formalism, four different polarization modulation channels can be used to recover the full Stokes parameters. However, the Poisson and Gaussian noise in the detection system will result in inaccurate Stokes vector reconstruction and degrade the signal-to-noise (SNR) ratio on the measured Stokes parameters. Accurate reconstruction of Stokes parameters not only depends on accurate signal measurements in each polarization channel but also depends on the polarimeter system matrix.

Optimizing a polarimeter always seeks to structure the measurement matrix to minimize the noise variance on the Stokes vector, which is obtained from its covariance matrix $\mathbf{\Gamma}^s$ calculated from

$$\mathbf{\Gamma}^s = \mathbf{W}^T \mathbf{\Gamma}^t \mathbf{W}^\dagger$$

(2.8)

where $\mathbf{W}^\dagger$ is the pseudoinverse matrix of the polarimeter measurement matrix $\mathbf{W}$ and $\mathbf{\Gamma}^t$ is the covariance matrix of the measured signal vector $\mathbf{I}$\textsuperscript{62}. The measured signal vector $\mathbf{I} = [i_1, i_2, i_3, i_4]^T$ is the coupling between $\mathbf{W}$ and the incident Stokes vector $\mathbf{S} = [S_0, S_1, S_2, S_3]^T$, given by

$$\mathbf{I} = \mathbf{W} \cdot \mathbf{S}.$$ 

(2.9)

Signal-independent Gaussian noise (GN) and signal-dependent Poisson noise (PN) are usually considered in evaluating a polarimeter’s performance\textsuperscript{46,62–65}.
2.3.1. Optimization under Gaussian noise

The condition number of a matrix represents how errors propagate backwards during the matrix inversion\(^4\). As an example, if a raw image has an error of 2% and the 2-norm condition number of the Mueller matrix is 2.5, the expected typical error after matrix inversion is around 5%. SNR in reconstructed Stokes parameters can be maximized or equalized when various condition numbers of measurement matrices are minimized. The condition number of a matrix is defined in terms of the matrix norms as

\[
\kappa(A) = \|A\|_p \|A^{-1}\|_p,
\]

where \(\|A\|_p\) and \(\|A^{-1}\|_p\) represents \(p\)-norm of the matrix \(A\) and its inverse \(A^{-1}\), respectively.

Optimization of a polarimeter seeks to structure the instrument matrix based on its various condition numbers (L1\(^-\), L2\(^-\), L\(\infty\)-norm), reciprocal absolute determinant (RAD)\(^4\) and equally weighted variance (EWV)\(^4\). L1-norm represents the sum of the errors in all the Stokes parameter while L\(\infty\)-norm represents the maximum possible error across all Stokes parameters\(^4\). L2-norm equalizes the noise variance across all Stokes parameters in the presence of Gaussian noise\(^\)\(^4\). The choice of different condition numbers is left to the user based on the applications and requirements. But it should be noted that using condition number as a figure of merit cancels out the throughput of the system which is also an important factor to evaluate system performance. RAD is the multiplication of the reciprocal of all the singular values of the measurements matrix as defined by

\[
RAD = \prod_{j=1}^{R} 1/ \mu_j,
\]

for a R-channel polarimeter where \(\mu_j\) is the singular value of the measurement matrix. RAD can provide a means of visualizing the optimization process in geometric terms but it lacks a
quantitative interpretation on the basis of system performance. To overcome the issues in the above figures of merit, EWV is introduced given that the importance of the respective Stokes components to the polarimeter’s intended application are the same, which is calculated by

\[ EWV = \prod_{j=1}^{R} 1/ \mu_j^2, \]  

(2.12)

which is also the square of the Frobenius norm of measurement matrix’s inverse.

2.3.2. Optimization under Poisson noise

The above figures of merit are discussed under signal-independent noise where the noise variances on the measured Stokes parameters are insensitive to the incident polarization state. When Poisson noise which is signal-dependent is dominant in the system, the system optimized for Gaussian noise is not necessarily the optimal one. Lara and Paterson\(^{47}\) used the expectation of the Stokes vector variance as a cost function for optimizing DoAM polarimeter in the presence of combined Poisson and Gaussian noise. An existing prism-based DoAM has been optimized with this method. However, the cost function is simply related to the condition number and EWV and resulted configuration is still sensitive to Poisson noise. The main reason is that simply using this figure of merit can only minimize the Euclidean length of the rows of the synthesis matrix. Goudail\(^{48}\) optimized the EWV in the minimax sense for Poisson noise and demonstrated an ideal measurement matrix model which minimize and equalize the noise variance under both Poisson noise and Gaussian noise. However, there is no such practical configuration that forms the ideal measurement matrix. Mu et al\(^{49}\) proposed an optimization method by approaching the ideal measurements by varying the parameters such as rotation angles and retardance in DoT polarimeters and achieve the immunity to both Gaussian and Poisson noise.
CHAPTER 3. Intrinsic Coincident Polarimeter

3.1. Polarization-Sensitive Organic Photovoltaics

Organic photovoltaics (OPVs) have attracted significant research interest in its low-cost processing onto flexible substrates, tunable properties through material synthesis, and their use of earth abundant materials\(^50\). Polymer semiconductors have an optical transition dipole moment \((\pi - \pi^*)\) that is aligned along the polymer backbone. Aligning the polymer backbone uniaxially in the plane of film results in anisotropic optoelectronic properties\(^51-53\) and thus achieves polarization-sensitive detection. An illustration of the backbone alignment is shown in Figure 10, where in (a) the backbone of the polymer is oriented randomly resulting in isotropic absorption while in (b) the backbone of the polymer is preferentially aligned along \(x\)-axis, resulting in preferential absorption along \(x\)-axis than \(y\)-axis\(^54\). When the polymer backbone is aligned in plane, the devices exhibit larger optoelectronic response along the strain direction than its orthogonal direction when incident light is linearly polarized parallel to the strain direction.

![Figure 10. Illustration of polymer chain alignment in the film. (a) The film is isotropic and has no polarization sensitivity. (b) The polymer backbone is aligned uniaxially, resulting in anisotropic absorption.](image)

3.1.1. Schematic Diagram

A well-controlled level of polarization sensitivity in organic photovoltaic (OPVs) cells has been demonstrated by Awartani et al\(^55\), where poly(3 hexylthiophene):Phenyl-C61-butyric acid
methyl ester (P3HT:PCBM) blend films was used and the P3HT backbone was aligned by uniaxially straining the P3HT:PCBM film while on a polydimethylsiloxane (PDMS) elastomer substrate. A schematic diagram of a demonstrated polarization-sensitive semitransparent OPV cell is depicted in Figure 11 (a), which has the structure of transparent substrate/ transparent conduction electrodes (TCE)/ charge transport layer/ organic photoactive layer/ charge transport layer/ transparent conduction electrodes/ transparent substrate\textsuperscript{56}. The polarization sensitivity of an OPV detector results from applying uniaxial strain to the active layer, which aligns the polymer backbones in the direction of the strain, as depicted in Figure 11 (b). In addition to polarization sensitivity, the devices are also made semitransparent, which allows light detection in subsequent OPVs.

![Figure 11](image)

Figure 11. (a) A schematic diagram of one OPV cell (b) Transition dipole along the backbone of the semiconducting polymer active layer, which leads to anisotropic absorbance of polarized light.

### 3.1.2. Absorption Spectrum

The absorbance spectra of strain-aligned P3HT:PCBM measured with linearly polarized light both parallel and perpendicular to the strain direction is provided in Figure 12 for films strained from 0% to 100% in 25% increments\textsuperscript{55}. The absorption for parallel absorption increases with the percentage of the strain but plateaus after 50%. The reason is that even if the backbone of the polymer aligns more in the strain direction, resulting in more absorption for linearly polarized light in strain direction but the thickness of the film decreases with the straining process, causing
incoming photons quickly escaping from the film before being absorbed. Meanwhile, the absorbance in the perpendicular direction continuously decreases with increasing strain.

Figure 12. Absorbance spectra for strain-aligned P3HT:PCBM blend films.

3.1.3. External Quantum Efficiency

External quantum efficiency (EQE) is the ratio of the number of charge carriers collected by the detector to the number of incident photons which is important to characterize the optoelectronic properties of the OPVs. EQE of the OPV cells, measured under polarized light, is depicted in Figure 13. It is observed that even if the absolute absorption increases with strain in parallel direction and decreases in perpendicular direction, EQE is the same for both directions under 0% and 50% strain but drops under 100%. The decrease of EQE under 100% strain indicates a decrease of photon-electron conversion ability and may result from the destruction of polymer molecule with the increased strain.
3.1.4. Detector Linearity

For the photo detectors, it is desired that the photo-generated current is directly proportional the intensity of the incident light in order to obtain the light intensity directly and circumvent nontrivial post-processing. Absorption behavior of an OPV is examined under different light powers (0.25mW – 1.74mW). I-V characteristic curve and responsivity curve of an OPV are provided in Figure 14 (a) and (b) respectively. Both curves demonstrate a linear responsivity of the OPV detector.

Figure 14. (a) I-V curve (b) Responsivity curve of an OPV under different incident powers
3.2. Intrinsic Coincident Polarimeter Design

The polarization sensitivity and semi-transparent properties of an OPV enable the design of a full-Stokes intrinsic coincident (IC) polarimeter, as depicted in Figure 15. At least four OPVs and additional waveplates are cascaded along the optical axis and the current from each OPV is measured by external circuit. The function of the OPVs is to provide polarization-related signal measurement while the waveplates are used to modulate $S_3$ component to ensure the full-Stokes reconstruction. Orientation of the OPVs and selection of waveplates can provide lots of freedom to optimize an IC polarimeter.

![Figure 15. Schematic diagram of a full-Stokes intrinsic coincident polarimeter.](image)

3.3. Optical and Electrical Model of Organic Photovoltaic cell

To reconstruct the Stokes parameters from an IC polarimeter, the measurement matrix of the polarimeter should be obtained where the Mueller matrix for each polarization measurement should be known at first. In this section, the optical and electrical model of an OPV are discussed to provide the associated transmission and absorption Mueller matrix.

3.2.1. Optical Model

The Mueller matrix for each OPV should be developed to describe its transmission and absorption behavior. Polarization-sensitive OPVs have different transmission and thus absorption along strain direction (parallel direction) and its orthogonal direction (perpendicular direction). A diattenuator Mueller matrix can be used to describe the transmission and absorption difference.
between the orthogonal directions. In transmission, for an OPV with maximum transmission \( T_{\text{max}} \) or \( T_{\text{perp}} \), minimum transmission \( T_{\text{min}} \) or \( T_{\text{para}} \), rotation angle \( \theta \) with respect to \( x \)-axis (\( \theta = 0^\circ \) when perpendicular direction is along \( x \)-axis), the transmission diattenuation Mueller matrix can be described by

\[
\mathbf{M}_D(D_T, E_T, \theta) = \text{Rot}(\theta) \begin{bmatrix}
1 & D_T & 0 & 0 \\
D_T & 1 & 0 & 0 \\
0 & 0 & 2E_T & 0 \\
0 & 0 & 0 & 2E_T \\
\end{bmatrix} \text{Rot}(-\theta),
\]

(3.1)

where \( \text{Rot}(\theta) \) is the rotation matrix, defined as

\[
\text{Rot}(\theta) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(2\theta) & \sin(2\theta) & 0 \\
0 & -\sin(2\theta) & \cos(2\theta) & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}.
\]

(3.2)

The attenuation parameters associated with transmission \( D_T \) and \( E_T \) are calculated by

\[
D_T = \frac{T_{\text{perp}} - T_{\text{para}}}{T_{\text{perp}} + T_{\text{para}}},
\]

(3.3)

and

\[
E_T = \sqrt{\frac{T_{\text{perp}}T_{\text{para}}}{T_{\text{perp}} + T_{\text{para}}}},
\]

(3.4)

respectively. In addition to transmission difference, the retardance difference along perpendicular and parallel direction are also induced by the straining process. To account for the retardance difference, a general Muller matrix must be incorporated into OPV transmission model. The Mueller matrix of a general retarder with retardance \( \phi \) and rotation angle \( \theta \) is defined as
With the defined diattenuation and general retarder Mueller matrix per Eq. (3.1) and (3.5), the transmission Mueller matrix of an OPV can be characterized as a diattenuator in series with a parallel retarder, described by

\[ T_{\text{OPV}}(D_T, E_T, \phi, \theta) = M_D(D_T, E_T, \theta) \times M_R(\phi, \theta). \]

Similarly, the absorption of an OPV is determined by a diattenuator to account for the different absorption between the strain direction and its orthogonal direction, which is calculated by

\[ A_{\text{OPV}} = M_D(D_A, E_A, \theta) \]

where absorption-related diattenuation parameters \( D_A \) and \( E_A \) are calculated by

\[ D_A = \frac{(1-T_{\text{perp}} - \rho) - (1-T_{\text{para}} - \rho)}{(1-T_{\text{perp}} - \rho) + (1-T_{\text{para}} - \rho)} \]

and

\[ E_A = \sqrt{\frac{(1-T_{\text{perp}} - \rho)(1-T_{\text{para}} - \rho)}{(1-T_{\text{perp}} - \rho) + (1-T_{\text{para}} - \rho)}}. \]

where \( \rho \) is the reflectance of the OPV.

3.2.2. Optical-Electrical Model

The transmission and absorption Mueller matrix discussed so far describes OPV’s optical response when interacting with light. The optical properties \( T_{\text{perp}}, T_{\text{para}} \) and \( \rho \) associated with the transmission and absorption Mueller matrix can be obtained by simple characterization experiment. However, optical model of an OPV is insufficient to fully characterize OPV’s absorption behavior
because the signal obtained from the OPV detector is photo-generated current or electrons extracted by the external circuit instead of the photons being absorbed by the OPV. An optical-electrical model based on the optical model of an OPV is developed in this section to describe the opto-electrical response of an OPV.

Responsivity, the detector output per unit of input power in amperes, can connect the optical response and electrical response of an OPV. If the responsivity or EQE of an OPV for perpendicular direction and parallel direction are the same, a product of the responsivity $\eta$ and the absorption Mueller matrix $A_{OPV} = M_d(D_A, E_A, \theta)$ can describe the electrical current output given the input optical power. Thus optical-electrical absorption matrix $A_{OE}$ can be updated as

$$A_{OE} = \eta \cdot A_{OPV} = \eta \cdot M_d(D_A, E_A, \theta).$$  \hspace{1cm} (3.9)

For OPVs with higher strain, the responsivities or EQE will be different for orthogonal polarization\textsuperscript{55}, as depicted in Figure 13. In this case, a constant responsivity cannot be multiplied by the absorption matrix since the responsivities for strain direction and orthogonal direction are no longer the same. To model the optical-electrical response of OPVs with higher strain, the diattenuator matrix for absorption is modified as

$$A_{OE}(D_{OE}, E_{OE}, \theta) = \text{Rot}(\theta)\begin{bmatrix} 1 & D_{OE} & 0 & 0 \\ D_{OE} & 1 & 0 & 0 \\ 0 & 0 & 2E_{OE} & 0 \\ 0 & 0 & 0 & 2E_{OE} \end{bmatrix} \text{Rot}(-\theta)$$  \hspace{1cm} (3.10)

to account for the photon-electron conversion difference for orthogonal directions. The optoelectronic-related diattenuation parameters $D_{OE}$ and $E_{OE}$ are defined by

$$D_{OE} = \frac{\left[(1-T_{\text{perp}} - \rho) \times \eta_{\text{perp}}\right] - \left[(1-T_{\text{para}} - \rho) \times \eta_{\text{para}}\right]}{\left[(1-T_{\text{perp}} - \rho) \times \eta_{\text{perp}}\right] + \left[(1-T_{\text{para}} - \rho) \times \eta_{\text{para}}\right]}.$$

$$E_{OE} = \frac{\left[(1-T_{\text{perp}} - \rho) \times \eta_{\text{perp}}\right] - \left[(1-T_{\text{para}} - \rho) \times \eta_{\text{para}}\right]}{\left[(1-T_{\text{perp}} - \rho) \times \eta_{\text{perp}}\right] + \left[(1-T_{\text{para}} - \rho) \times \eta_{\text{para}}\right]}.$$  \hspace{1cm} (3.11)
and

\[ E_{OE} = \sqrt{\left[ (1 - T_{\text{perp}} - \rho) \times \eta_{\text{perp}} \right] \times \left[ (1 - T_{\text{para}} - \rho) \times \eta_{\text{para}} \right] }, \]  

(3.12)

where \( \eta_{\text{perp}} \) and \( \eta_{\text{para}} \) are the responsivities along the perpendicular and parallel direction, respectively. The diattenuator Muller matrix per Eq. (3.10) can account for the opto-electrical response difference for perpendicular and parallel directions.

### 3.3. Intrinsic Coincident Polarimeter Model

Different from other existing polarimeter techniques such as DoT, DoAM, DoA, and DoFP where each polarimeter state is independent of other states, the transmitted polarization modified by previous OPV cells and/or polarizing elements will affect the absorption in subsequent cells. Therefore the Mueller matrix for each polarimeter state in an IC polarimeter is constructed by the transmission matrices of the previous transmitting elements and the absorption matrix of the current OPV cell. For a full-Stokes IC polarimeter, as depicted in Figure 16, the Mueller matrices for OPV1-4 are described by

\[ \mathbf{M}_{\text{OPV1}} = \mathbf{A}_{\text{OPV1}} (D_{A1}, E_{A1}, \theta_1), \]  

(3.13)

\[ \mathbf{M}_{\text{OPV2}} = \mathbf{A}_{\text{OPV2}} (D_{A2}, E_{A2}, \theta_2) \times \mathbf{M}_{\text{R1}} (\phi_{R1}, \theta_{R1}) \times \mathbf{T}_{\text{OPV1}} (D_{T1}, E_{T1}, \phi_1, \theta_1), \]  

(3.14)

\[ \mathbf{M}_{\text{OPV3}} = \mathbf{A}_{\text{OPV3}} (D_{A3}, E_{A3}, \theta_3) \times \mathbf{M}_{\text{R2}} (\phi_{R2}, \theta_{R2}) \times \mathbf{T}_{\text{OPV2}} (D_{T2}, E_{T2}, \phi_2, \theta_2) \times \mathbf{M}_{\text{R1}} (\phi_{R1}, \theta_{R1}) \times \mathbf{T}_{\text{OPV1}} (D_{T1}, E_{T1}, \phi_1, \theta_1), \]  

(3.15)

\[ \mathbf{M}_{\text{OPV4}} = \mathbf{A}_{\text{OPV4}} (D_{A4}, E_{A4}, \theta_4) \times \mathbf{T}_{\text{OPV3}} (D_{T3}, E_{T3}, \phi_3, \theta_3) \times \mathbf{M}_{\text{R2}} (\phi_{R2}, \theta_{R2}) \times \mathbf{T}_{\text{OPV2}} (D_{T2}, E_{T2}, \phi_2, \theta_2) \times \mathbf{M}_{\text{R1}} (\phi_{R1}, \theta_{R1}) \times \mathbf{T}_{\text{OPV1}} (D_{T1}, E_{T1}, \phi_1, \theta_1), \]  

(3.16)

where the variables \( D_{Ai}, D_{Ti}, E_{Ai}, E_{Ti}, \phi_i, \theta_i \) refer to the corresponding parameter values for the \( i \)th OPV and \( \phi_{Ri}, \theta_{Ri} \) are the retardance and fast-axis angle of \( i \)th waveplate, respectively. The first row
from the Mueller matrix of each OPV per Eq. (3.13) – Eq. (3.16) is used to form the measurement matrix W of the IC polarimeter, as described by

\[
W_{\text{ICP}} = \begin{bmatrix}
M_{\text{OPV}_1}(1,1) & M_{\text{OPV}_1}(1,2) & M_{\text{OPV}_1}(1,3) & M_{\text{OPV}_1}(1,4) \\
M_{\text{OPV}_2}(1,1) & M_{\text{OPV}_2}(1,2) & M_{\text{OPV}_2}(1,3) & M_{\text{OPV}_2}(1,4) \\
M_{\text{OPV}_3}(1,1) & M_{\text{OPV}_3}(1,2) & M_{\text{OPV}_3}(1,3) & M_{\text{OPV}_3}(1,4) \\
M_{\text{OPV}_4}(1,1) & M_{\text{OPV}_4}(1,2) & M_{\text{OPV}_4}(1,3) & M_{\text{OPV}_4}(1,4)
\end{bmatrix}
\]  

(3.17)

By optimizing the detector and waveplate parameters, the measurement matrix \( W_{\text{ICP}} \) can be well conditioned and inverted to reconstruct the input Stokes vector.

Figure 16. A schematic representation of a full-Stokes IC polarimeter.

3.4. System Calibration and Model Validation

3.4.1. OPV and Waveplates Characterization

Before aligning OPVs and waveplates in the system, each element should be characterized first to obtain accurate model parameters, which includes maximum transmission \( T_{\text{perp}} \), reflectance \( \rho \), diattenuation \( D \), retardance \( \phi \), and responsivity \( \eta \) for each OPV and waveplate retardance in case that the waveplate retardance is not exactly 90° for used wavelength. Figure 17 (a) shows the experiment setup for obtaining the maximum transmission, reflectance and responsivity of an OPV, where the OPV is oriented at 0° and titled by a small angle. The transmission and reflectance can be easily obtained by the division of the transmitted power \( P_t \) or reflected power \( P_r \) to the incident power \( P_{\text{inc}} \). The responsivity is the division of obtained current to the incident power.
Although the diattenuation of the OPV can be simply obtained through the Eq.(3.3), the retardance of the OPV is not easy to measure. The experiment setup as depicted in Figure 17 (b) can be used to obtain the diattenuation and retardance at the same time by rotating a QWP in front of the OPV sandwiched by two crossed linear polarizers (LPs). The QWP is rotated from \(0^\circ\) to \(180^\circ\) with an increment of \(10^\circ\) and the current and transmitted power are recorded at each QWP rotation angle. This transmission Muller matrix of this experiment setup can be modeled as

\[
W_T = LP (90^\circ) \times T_{OPV} (D_T, E_T, \phi, \theta) \times M_R (90^\circ, \theta_R) \times LP (0^\circ),
\]

while the absorption Muller matrix of this system can be calculated by

\[
W_A = A_{OPV} (D_A, E_A, \theta) \times M_R (90^\circ, \theta_R) \times LP (0^\circ),
\]

where \(T_{OPV}\) and \(A_{OPV}\) are obtained by Eq. (3.6) and Eq. (3.9). The unknown parameters in this experiment system is only the minimum transmission \(T_{para}\) with which \(D_T, E_T, D_A,\) and \(E_A\) can be calculated, rotation angle \(\theta\) in case of the rotation error and the retardance \(\phi\). The unknown model parameters can be obtained by fitting calculated transmitted and reflected power from the model to the actual data.

Figure 17. (a) Transmission, reflectance and responsivity characterization experiment setup (b) Diattenuation and reflectance characterization experiment setup (c) Waveplate retardance characterization experiment setup.
The waveplates used in the system is not necessarily ideal and need to verify the retardance of the waveplates and also the fast axis angle. This can be achieved by putting waveplates between two crossed polarizers and rotating the waveplates. The waveplates is rotated from 0° to 180° by an increment of 10° and the transmitted power is recorded. Similarly with OPV retardance characterization, the waveplate retardance is obtained by fitting the power calculated by the model as per Eq. (3.20)to the actual measurement data.

$$W_T = \text{LP}(90°) \times M_{R}(\phi_R, \theta_R) \times \text{LP}(0°).$$

(3.20)

3.4.2. Experiment Setup

The demonstration of the IC polarimeter was first achieved in free space without any integration. The experiment setup for calibration and validation is illustrated in Figure 18 (a). A linearly polarized 532 nm diode laser (Thorlabs DJ532-40) was used as the source. Although this diode is temperature stabilized, a partial reflector was used as a beam-pickoff to re-direct 8% of the incident light into a radiometer to record fluctuations in its output power. These readings are used to normalize the measured currents before subsequent data processing. The chopper-oscilloscope combination is used as a detection technique to record electrical signals from each OPV cell. The chopper operates at 200 Hz to suppress flicker noise, which is equivalent to average the experiment results from multiple measurements and thus reduces the complexity of data post-processing. The external circuit for extracting current from OPVs is depicted in Figure 18(b). A reverse bias voltage of -0.5 V with a load resistance 680 Ω was used to measure the photo-generated current from each OPV. A Glan-Thompson linear polarizer (LP), oriented with its transmission axis parallel to the x-axis, is used to clean up the laser’s polarization state. Incoming light with known polarization states for calibration and validation was generated by rotating a quarter wave plate (QWP) and transmitted through OPV1, WP1, OPV2, WP2, OPV3 and OPV4.
successively. OPV cells are oriented at \( \theta_1 = 0^\circ \) (OPV1), \( \theta_2 = 36^\circ \) (OPV2), \( \theta_3 = 86^\circ \) (OPV3) and \( \theta_4 = 0^\circ \) (OPV4) and waveplates \( \theta_{R1} = 115^\circ \) (WP1), \( \theta_{R2} = 76^\circ \) (WP2).

Figure 18. (a) Schematic of the calibration experiment setup. (b) Electrical circuit for recording electrical signal from each OPV and waveforms from oscilloscope output.

### 3.4.2. Radiometric and Polarimetric Calibration

There are two ways to calibrate the ICP’s measurement matrix \( W \): (1) Characterize the parameters of each OPV and WP in the system and obtain \( W \) using the model described in previous section; or (2) Fit the coefficients of \( W \) to measured currents, generated by known polarization states, using the data reduction matrix technique\(^5^7\). The data reduction matrix technique is adopted here to take into account reflection and misalignment of the elements that could affect the accuracy of measurements. Since the OPVs used in this experiment were under 50% stain, the absorption behavior for each OPV can be modeled as per Eq. (3.9). The photo-generated current for four OPVs in the system were calculated by

\[
I = \eta \times W \times S_{in} + \beta, \tag{3.21}
\]
where $\mathbf{\beta} = [\beta_1, \beta_2, \beta_3, \beta_4]^T$, and represents the electrical offset induced by the external circuit (internal resistance of the OPVs) shown in Figure 18 (b) and $\mathbf{\eta}$ is a matrix containing the responsivities $\eta_1, \eta_2, \eta_3,$ and $\eta_4$ (amps/watts) for OPV1, OPV2, OPV3, and OPV4, given by

$$
\mathbf{\eta} = \begin{bmatrix}
\eta_1 & 0 & 0 & 0 \\
0 & \eta_2 & 0 & 0 \\
0 & 0 & \eta_3 & 0 \\
0 & 0 & 0 & \eta_4 \\
\end{bmatrix}
$$

(3.22)

The calibration process can be performed by illuminating the ICP with $Q$ known Stokes vectors and the currents from OPV1-OPV4 can be calculated as

$$
\begin{bmatrix}
\mathbf{I}_1 \\
\mathbf{I}_2 \\
\vdots \\
\mathbf{I}_Q
\end{bmatrix} = \mathbf{\eta} \times \mathbf{W} \times \begin{bmatrix}
\mathbf{S}_{\text{in}1} \\
\mathbf{S}_{\text{in}2} \\
\vdots \\
\mathbf{S}_{\text{in}Q}
\end{bmatrix},
$$

(3.23)

where $\mathbf{I}_q = [i_{1q}, i_{2q}, i_{3q}, i_{4q}]^T$ is the current output from OPV1-4 for the $q$th measurement and $\mathbf{S}_{\text{in}q} = [S_{0q}, S_{1q}, S_{2q}, S_{3q}]^T$ is the corresponding input Stokes vector. The measurement matrix $\mathbf{\eta} \times \mathbf{W}$ can be calibrated from $Q$ measurements. The unknown input Stokes vectors can be calculated from any corresponding measurement vector $\mathbf{I}$ by

$$
\mathbf{S}_{\text{in}} = [\mathbf{\eta} \times \mathbf{W}]^{-1} \times (\mathbf{I} - \mathbf{\beta}).
$$

(3.24)

Radiometric calibration is performed under three different powers: 3.85 mW (P1), 4.52 mW (P2), and 5.14 mW (P3). For each incident power, the QWP was rotated from 0° to 180° in 10° increments, thereby generating a total of $Q = 57$ unique Stokes vectors. To fit the data, a Nelder-Mead minimization function was used to determine the parameters associated with the measurement matrix $\mathbf{\eta} \times \mathbf{W}$. Figure 19 depicts the fitted results ($\mathbf{I}_1F$, $\mathbf{I}_2F$, and $\mathbf{I}_3F$) as compared to the currents obtained from the oscilloscope ($\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$).
The matrix $\eta \times W$ was determined as

$$A = \eta \times W = \begin{bmatrix}
0.0734 & -0.0334 & 0.0003 & 0.0002 \\
0.0280 & 0.0056 & -0.0046 & 0.0037 \\
0.0104 & 0.0043 & 0.0042 & 0.0008 \\
0.0046 & 0.0029 & -0.0002 & -0.0012
\end{bmatrix},$$

and the electrical offset vector $\beta = [0.0089, 0.0036, 0.0010, 0.0006]^T$ mA.

### 3.4.3. Model Validation

To validate the performance of the proposed IC polarimeter, 30 out of 57 measurements were used for calibration, while the remaining 27 measurements were used for validation. The RMS error for Stokes parameter $S_i$ between the measured and theoretical Stokes parameters is calculated as

$$RMS(S_i) = \frac{100}{\sqrt{27}} \sqrt{\sum_{j=1}^{27} \left( \frac{S_i}{S_{0j}} - \hat{S}_{ij} / \hat{S}_{0j} \right)^2},$$

(3.26)
where $S_{ij}$ and $\hat{S}_{ij}$ represent theoretical and measured Stokes parameter $S_i$ for $j^{th}$ validation measurements respectively. The RMS error for Stokes parameters $S_1$, $S_2$, and $S_3$ was tabulated in Table 1, yielding an average RMS error 0.84%.

Table 1. RMS error for Stokes parameters.

<table>
<thead>
<tr>
<th>Stokes parameter</th>
<th>$S_0$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS error</td>
<td>0</td>
<td>0.62%</td>
<td>0.49%</td>
<td>1.41%</td>
<td>0.84%</td>
</tr>
</tbody>
</table>

To validate the IC polarimeter model, the parametric model was used to fit the data directly. *A priori* parameters included $T_{\text{perp}1} = 0.52$, $T_{\text{perp}2} = 0.5$, $T_{\text{perp}3} = 0.53$, $T_{\text{perp}4} = 0.33$, $\rho_1 = 0.3$, $\rho_2 = 0.3$, $\rho_3 = 0.3$, $\rho_4 = 0.3$, $\phi_{R1} = 113.6^\circ$, $\theta_{R1} = 115^\circ$, $\phi_{R2} = 90.3^\circ$, $\theta_{R2} = 76^\circ$, which were used to constrain the fitting algorithm. These values were measured on the benchtop before assembling the IC polarimeter system. The remaining model parameters were obtained using the same fitting procedure on the remaining free parameters, the results of which are presented in Table 2.

Table 2. Initial and final value of the fitting variables obtained using the parametric fit.
It should be noted here that OPV cells were tilted to prevent reflected light affecting previous cells, which was not accounted for in the prior IC polarimeter model established in previous section. The monolithic IC polarimeter considering the reflection effect will be discussed in details in the following chapter. Remaining discrepancies between the initial and final fitted values may result from misalignment of the elements (OPVs and WPs), inaccurate retardance for WPs, and consequences from tilting the OPV cells.

The measurement matrix, obtained by the parametric fit, was calculated to be

\[
\begin{bmatrix}
0.0733 & -0.0331 & 0 & 0 \\
0.0280 & 0.0057 & -0.0046 & 0.0038 \\
0.0105 & 0.0043 & 0.0042 & 0.0008 \\
0.0045 & 0.0030 & -0.0001 & -0.0012
\end{bmatrix}
\]

One method to compare the similarity between the matrices obtained from data reduction techniques and from fitting the model parameters directly is to calculate the RMS error between the measurement matrix entries\(^56\). The method we adopted here was to compare the results obtained from the two matrices across many simulated observations. If discrepancies between the results are within an acceptable tolerance, the two matrices can be said to match each other. Thus, we compared the two measurement matrices \(A\) and \(\hat{A}\) by inputting 15708 unique Stokes vectors sampled across the Poincare sphere as illustrated in Figure 20 (a) based on MATLAB simulation. The current, calculated from each OPV, is depicted in Figure 20 (b), which demonstrates agreement between the results calculated from the two matrices. The RMS for discrepancies between reconstructed Stokes vectors were on the order of \(10^{-16}\), which is small enough to state that the measurement matrices obtained from the data reduction technique and from the model and parametric fitting are the same. This highlights that the ICP model is well defined such that
optoelectronic response for each OPV cell can be used to construct an accurate measurement matrix.

Figure 20. (a) 15708 Stokes vectors (blue data) were sampled across the Poincare sphere. (b) Current result from 4 OPVs calculated from measurement matrix $A$ and $\hat{A}$. To have a good visualization, only 315 of 15708 sample results were plotted.
CHAPTER 4. Monolithic Intrinsic Coincident Polarimeter

4.1. Monolithic IC Polarimeter Model

A full-stokes IC polarimeter was demonstrated by cascading four OPVs and two WPs first without any integration. These elements were tilted to avoid crosstalk from back-reflections. However, normal incidence is necessary for monolithic integration which means that the back reflection is unavoidable.

4.1.1. Back Reflection Induced Optical Crosstalk

Crosstalk between adjacent cells is illustrated in Figure 21. Laser light, illuminating the cells at normal incidence, can have its polarization state modulated by a linear polarizer (LP) and a quarter-wave plate (QWP). The layout of the monolithic cell is depicted to include four OPV detectors (OPV1 through OPV4) and two waveplates (WP1 and WP2). The parameters of these cells and waveplates can be adjusted to optimize the IC polarimeter’s performance\(^59\).

![Figure 21. Schematic of monolithic cell with back reflections.](image)

The absorption of one OPV, integrated into the monolithic IC polarimeter, is associated with both incoming and reflected light and therefore two incidence direction should be considered. The typical structure of a semitransparent OPV is depicted in Figure 22, which includes glass as both top and bottom substrates, gold as the anode, ITO as the cathode, P3HT:PCBM as the active layer, PEIE as an electrode work function modification layer and MoO\(_x\) as a charge transport layer\(^60\). Due to the position of the absorbing layer within the stack and the relatively high reflectivity (~30%) of the gold anode, the OPV either reflects or absorbs more light depending on whether the
light first strikes the cathode or anode. We define the case where the light is incident on the ITO as forward incidence (FI) and the case where the light is incident on the gold contact as backward incidence (BI).

Characterization of the model parameters is needed for both FI and BI to account for the back-reflection crosstalk. The \( i \)th OPV’s absorption matrix, considering the back-reflection, can be represented by

\[
A_{\text{OPVi}} \left( D_{\text{FI}}, E_{\text{FI}}, D_{\text{BI}}, E_{\text{BI}}, \theta_i, \rho_{i+1} \right) = A_{\text{FIi}} \left( D_{\text{FI}}, E_{\text{FI}}, \theta_i \right) + A_{\text{BIi}} \left( D_{\text{BI}}, E_{\text{BI}}, 180^\circ - \theta_i \right) \times R_{i+1} \times T_{\text{OPVi}} \left( D_{\text{TI}}, E_{\text{TI}}, \phi_i, \theta_i \right). \tag{4.1}
\]

where \( D_{\text{FI}}, E_{\text{FI}}, D_{\text{BI}}, E_{\text{BI}} \) are the diattenuation parameters characterized for FI and BI and \( R \) is the reflection Mueller matrix expressed by

\[
R_{i+1} = \rho_{i+1} \times \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{bmatrix}, \tag{4.2}
\]

where \( \rho_{i+1} \) is the reflection of the subsequent surface. For the BI case, the OPV’s transmission axis angle will be the complementary angle when the same OPV is modeled under the FI case because of the coordinate change after reflection. Note that only one reflection is considered in this paper since the second reflection from a previous cell is considered negligible (\( \ll 1\% \)).
The measurement matrix of each OPV is calculated by its absorption matrix concatenated by the transmission matrix of the elements preceding it \(^5\). Each OPVs’ Mueller matrix can be described by

\[
\mathbf{M}_{\text{OPV}} = \mathbf{A}_{\text{OPV}} \begin{pmatrix} D_{FI1}, E_{FI1}, D_{BI1}, E_{BI1}, \theta_1, \rho_2 \end{pmatrix},
\]

(4.3)

\[
\mathbf{M}_{\text{OPV}} = \mathbf{A}_{\text{OPV}} \begin{pmatrix} D_{FI2}, E_{FI2}, D_{BI2}, E_{BI2}, \theta_2, \rho_3 \end{pmatrix} \times \mathbf{M} \begin{pmatrix} \phi_{R1}, \theta_{R1} \end{pmatrix} \times \mathbf{T}_{\text{OPV}} \begin{pmatrix} D_{T11}, E_{T11}, \phi_1, \theta_1 \end{pmatrix},
\]

(4.4)

\[
\mathbf{M}_{\text{OPV}} = \mathbf{A}_{\text{OPV}} \begin{pmatrix} D_{FI3}, E_{FI3}, D_{BI3}, E_{BI3}, \theta_3, \rho_4 \end{pmatrix} \times \mathbf{M} \begin{pmatrix} \phi_{R2}, \theta_{R2} \end{pmatrix} \times \mathbf{T}_{\text{OPV}} \begin{pmatrix} D_{T12}, E_{T12}, \phi_2, \theta_2 \end{pmatrix}
\]

\[
\times \mathbf{M} \begin{pmatrix} \phi_{R1}, \theta_{R1} \end{pmatrix} \times \mathbf{T}_{\text{OPV}} \begin{pmatrix} D_{T11}, E_{T11}, \phi_1, \theta_1 \end{pmatrix},
\]

(4.5)

\[
\mathbf{M}_{\text{OPV}} = \mathbf{A}_{\text{OPV}} \begin{pmatrix} D_{FI4}, E_{FI4}, D_{BI4}, E_{BI4}, \theta_4, \rho_5 \end{pmatrix} \times \mathbf{T}_{\text{OPV}} \begin{pmatrix} D_{T13}, E_{T13}, \phi_3, \theta_3 \end{pmatrix} \times \mathbf{M} \begin{pmatrix} \phi_{R2}, \theta_{R2} \end{pmatrix}
\]

\[
\times \mathbf{T}_{\text{OPV}} \begin{pmatrix} D_{T22}, E_{T22}, \phi_4, \theta_4 \end{pmatrix} \times \mathbf{M} \begin{pmatrix} \phi_{R1}, \theta_{R1} \end{pmatrix} \times \mathbf{T}_{\text{OPV}} \begin{pmatrix} D_{T11}, E_{T11}, \phi_1, \theta_1 \end{pmatrix}.
\]

(4.6)

The first rows from the Mueller matrix of each OPV form the IC polarimeter’s measurement matrix, calculated as

\[
\mathbf{W} = \begin{bmatrix}
M_{\text{OPV}}(1,1) & M_{\text{OPV}}(1,2) & M_{\text{OPV}}(1,3) & M_{\text{OPV}}(1,4) \\
M_{\text{OPV}}(2,1) & M_{\text{OPV}}(2,2) & M_{\text{OPV}}(2,3) & M_{\text{OPV}}(2,4) \\
M_{\text{OPV}}(3,1) & M_{\text{OPV}}(3,2) & M_{\text{OPV}}(3,3) & M_{\text{OPV}}(3,4) \\
M_{\text{OPV}}(4,1) & M_{\text{OPV}}(4,2) & M_{\text{OPV}}(4,3) & M_{\text{OPV}}(4,4)
\end{bmatrix}.
\]

(4.7)

### 4.2. Monolithic IC Polarimeter Calibration and Validation

#### 4.2.1. Optical cross-talk model validation for one detector

To experimentally simulate the back-reflection a reflective mirror was positioned behind the OPV, as per Figure 23 (b). Characterization experiments were used to obtain the model parameters, including the cells’ transmission, reflection, and responsivities, which is illustrated in Figure 23 (a). A 532.5 nm laser was polarized by a linear polarizer (LP) with its transmission axis parallel to the x-axis. A quarter-wave plate (QWP) was rotated from 0° to 180° in 10° increments, to generate 19 known polarization states, including both linear and circular polarizations. After passing through an OPV in the FI direction, the light power was measured by a calibrated
radiometer. The absorption and transmission data from this characterization measurement are
given in Fig.24 (a) and (b), with the characterized model parameters $\theta = 86^\circ$, $T_{\text{perp}} = 0.51$, $T_{\text{para}} = 0.33$, $\rho_{FI} = 0.15$, $\rho_{BI} = 0.23$, $R_{\text{perp}} = 0.17 \text{ A/W}$ and $R_{\text{para}} = 0.28 \text{ A/W}$.

The same experiment process was used as per Figure 23 (b), except that only photo-
generated current was recorded. The experiment’s measured photo-generated current and the
current calculated from the parameterized back-reflection model are depicted in Figure 24 (c). The
measured current increased by an average of 27.9% in the back-reflection situation compared to
without back-reflection. Root mean square relative error (RMSRE) was used to evaluate the
discrepancy between the measured data and the value calculated by the model, as defined by

$$\text{RMSRE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{I_{i,c} - I_{i,m}}{I_{i,m}} \right)^2},$$

where $I_{i,c}$ and $I_{i,m}$ are the photo-generated current calculated by the model and the measured current
for ith measurement, respectively. RMSRE between the measured current and the model output is
2.17%, which is sufficient for using this model in future optimization procedures.
4.2.2. System Calibration and Model Validation

The experimental setup used for monolithic IC polarimeter calibration and validation is illustrated in Figure 25 (a). A linearly polarized 532 nm diode laser was used as the source with a partial reflector to monitor the laser light’s power fluctuations. A polarizer (LP) polarized the laser light along the x-direction. Four different incident powers, 1.12 mW (P1), 1.38 mW (P2), 1.67 mW (P3) and 1.95 mW (P4), were used for radiometric calibration and under each power a QWP was rotated from 0° to 180° in 10° increments to enable polarimetric calibration. OPV cells were oriented at \( \theta_1 = 80^\circ \) (OPV1), \( \theta_2 = 135^\circ \) (OPV2), \( \theta_3 = 180^\circ \) (OPV3) and \( \theta_4 = 0^\circ \) (OPV4) and the waveplates’ fast axes were oriented at \( \theta_{R1} = 0^\circ \) (WP1) and \( \theta_{R2} = 130^\circ \) (WP2). Encapsulated OPVs and WPs were edge-bonded with glue and refractive index matching fluid (n = 1.49) were used between the cells and waveplates. Laser light transmitted through OPV1, WP1, OPV2, WP2, OPV3 and OPV4 successively and the photo-generated current, from each OPV, was collected by the transimpedance circuit depicted in Figure 25 (b).
Rotating the QWP under four different incident powers generated 76 input Stokes vectors in total. Twenty random measurements under incident power P2 and P3 were selected for calibration and the remaining 56 measurements were used for validation. The OPVs’ absolute transmittances and reflectances and the WPs’ retardances were determined individually by separate characterization experiments. These parameters are listed in Table 3. Beyond these parameters, initial values for rotation angles, retardance, electrical offset, the responsivity of the four OPVs and WP orientation angles obtained from individual OPV characterization experiments were input into a Nelder-Mead minimization function. This function minimized the RMS error between the measured and calculated currents. The initial and fitted values for the rotation angles, retardance, electrical offset, the responsivity of the four OPVs, and the WP orientation angles are tabulated in Table 4.

Table 3. OPV and WP parameters as determined by separate characterization experiments.

<table>
<thead>
<tr>
<th>OPV</th>
<th>Transmission</th>
<th>Reflection</th>
<th>WP</th>
<th>Retardance(°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{perp}$</td>
<td>$T_{para}$</td>
<td>FI</td>
<td>BI</td>
</tr>
<tr>
<td>1</td>
<td>0.56</td>
<td>0.35</td>
<td>0.08</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>0.55</td>
<td>0.32</td>
<td>0.06</td>
<td>0.17</td>
</tr>
<tr>
<td>3</td>
<td>0.52</td>
<td>0.3</td>
<td>0.1</td>
<td>0.17</td>
</tr>
<tr>
<td>4</td>
<td>0.08</td>
<td>0.08</td>
<td>0.1</td>
<td>NA</td>
</tr>
</tbody>
</table>
Table 4. Initial value and fitted value for OPVs and WPs.

<table>
<thead>
<tr>
<th>OPV</th>
<th>Theta (°)</th>
<th>Responsivity (A/W)</th>
<th>Retardance (°)</th>
<th>Offset (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>x</td>
<td>y</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Initial Value</td>
<td>80.00</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>Fitted Value</td>
<td>86.47</td>
<td>0.14</td>
<td>0.23</td>
</tr>
<tr>
<td>2</td>
<td>Initial Value</td>
<td>135.00</td>
<td>0.14</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>Fitted Value</td>
<td>145.94</td>
<td>0.16</td>
<td>0.26</td>
</tr>
<tr>
<td>3</td>
<td>Initial Value</td>
<td>180.00</td>
<td>0.18</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>Fitted Value</td>
<td>192.10</td>
<td>0.20</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>Initial Value</td>
<td>0.00</td>
<td>0.21</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Fitted Value</td>
<td>0.00</td>
<td>0.29</td>
<td>NA</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP</th>
<th>Theta (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial Value</td>
</tr>
<tr>
<td></td>
<td>Fitted Value</td>
</tr>
<tr>
<td>2</td>
<td>Initial Value</td>
</tr>
<tr>
<td></td>
<td>Fitted Value</td>
</tr>
</tbody>
</table>

Figure 26. Current outputs from 4 OPVs and the model fitting results: (a) 20 measurements under P2 (1.38mW) and P3 (1.67mW) for calibration and (b) the remaining 57 measurements under P1 (1.12 mW), P2 (1.38 mW), P3 (1.67 mW) and P4 (1.95 mW) for validation.
Inserting the measured and fitted parameters into the model enabled the output to be calculated. Data from the model are depicted alongside the calibration data in Figure 26 (a) and alongside the validation results in Figure 26 (b). The calibrated measurement matrix was determined as

\[
W = \begin{bmatrix}
0.0973 & 0.0425 & -0.0054 & -0.0001 \\
0.0512 & -0.0242 & -0.0041 & 0.0153 \\
0.0232 & -0.0004 & 0.0099 & -0.0001 \\
0.0164 & -0.0017 & -0.0020 & -0.0054 \\
\end{bmatrix},
\] (4.9)

with the electrical offset vector \( \beta = [0.010, 0.0001, 0.0059, 0.0068]^T \) mA for OPV1-4.

The RMS error for Stokes parameters \( S_i \) \((i = 0, 1, 2, 3)\) between the measured and theoretical Stokes parameter is calculated as

\[
RMS(S_i) = \frac{100}{\sqrt{56}} \sqrt{\frac{1}{56} \sum_{j=1}^{56} (S_{ij} / S_{ij} - \hat{S}_{ij} / \hat{S}_{ij})^2}
\] (4.10)

where \( S_{ij} \) and \( \hat{S}_{ij} \) represent the theoretical and reconstructed Stokes parameter \( S_i \) for the \( j^{th} \) validation measurement, respectively. The RMS error for \( S_0, S_1, S_2, \) and \( S_3 \) are 0%, 2.80%, 3.68%, and 3.69% respectively, yielding an average RMS error 2.52%. This indicates that our parametric model is sufficiently accurate for future optimization strategies.

Calibration of the monolithic IC polarimeter can also be obtained by fitting the coefficients of \( W \) directly, as per the data reduction method 57. The calibrated measurement matrix \( \hat{W} \), calculated using the data reduction method, was

\[
\hat{W} = \begin{bmatrix}
0.0975 & 0.0402 & -0.0046 & 0 \\
0.0504 & -0.0243 & -0.0042 & 0.0154 \\
0.0238 & -0.0004 & 0.0097 & 0 \\
0.0169 & -0.0020 & -0.0017 & -0.0054 \\
\end{bmatrix},
\] (4.11)
with the electric offset vector $\beta = [0.0071, 0.0008, 0.0048, 0.0060]^T$ mA for OPV1-4. The similarity between the two matrices obtained from the IC polarimeter model and from the data reduction method can be inferred by the difference of the current obtained by the two matrices. The current was calculated across 10,000 random Stokes vectors that were uniformly distributed on the Poincare sphere, as depicted in Figure 27. The RMSRE between the current for OPV1-4, obtained by $W$ and $\tilde{W}$, are 2.48%, 0.44%, 1.72%, and 1.71%, yielding an average error of 1.59%. Thus, the parametric model and the experimentally-derived matrices are essentially equivalent.

![Figure 27. 10,000 Stokes vectors (blue data) are uniformly sampled across the Poincare sphere.](image)

### 4.3. Incidence Angle Response

In our previous modeling and validation, all experiments and theory were derived for the on-axis illumination case. However, off-axis illumination would be incident from different portions of an imaging lens’s aperture, as illustrated in Figure 28 (a). Angular definitions for this cone are depicted in Figure 28 (b), illustrating the zenith angle $\mu$ and azimuthal angle $\nu$. 
The 4×4 transfer matrix formalism can be used to simulate light propagation within an OPV stack from which transmission and absorption can be obtained across different incident angles\textsuperscript{61}.

The experimental calibration setup is depicted in Figure 29. A 520 nm multimode laser diode, with a spectral bandwidth of approximately 3 nm, was used as the light source to reduce the laser’s coherence length, which suppressed interference from the thicker glass substrates. A linear polarizer (LP) was used to polarize the incoming laser light to be either TM or TE for calibration. The laser illuminated a 1.5×1.5 mm\textsuperscript{2} aperture to limit the illuminated area of the OPV, which was mounted on a rotation stage that rotates about the \( y \)-axis. The photo-generated current was measured by the external circuit and the transmitted signal was obtained by the radiometer. Absorption and transmission data were collected under both TM and TE polarized light at \( \nu = 0^\circ \) with \( \mu \) varying from 0\(^\circ\) to 18\(^\circ\) in 2\(^\circ\) increments. The maximum rotation angle of 18\(^\circ\) was adopted to avoid beam walk off within the cell.
Figure 29. OPV thickness and refractive index calibration experimental setup. The angle $\mu$ can be adjusted by rotating the OPV in the x-z plane while $\nu$ is adjusted by rotating it in its local x-y plane.

Similar to our prior approach, a Nelder-Mead minimization function was used to determine the OPV’s parameters, such as each layers’ thickness and the active layer’s refractive index. The refractive index of other layers, including glass (1.5195), ITO (2+0.0136i), PEIE (1.56), MoOx (2.5614+0.01872i) and Au (0.608+2.12i) were fixed during the Nelder-Mead minimization fitting procedure. The experiment and fitted results for both transmission and absorption under TM and TE polarized light are plotted in Figure 30 and the fitted results are listed in Table 5. The experimental results are averaged values from five repeated measurements and the standard deviation is plotted as the error bar. Residual errors are likely caused by the remaining coherence length of the laser relative to the optical path difference of the thicker substrates, which influences the measurements most significantly at near-normal incidence.

Table 5. Initial and fitted parameter values for the OPV.

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>Thickness (nm)</th>
<th>P3HT:PCBM refractive index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top Glass</td>
<td>ITO</td>
</tr>
<tr>
<td>Initial Value</td>
<td>700000.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Fitted Value</td>
<td>700000.00</td>
<td>100.00</td>
</tr>
<tr>
<td></td>
<td>$x_n$</td>
<td>$x_h$</td>
</tr>
<tr>
<td>Initial Value</td>
<td>1.86</td>
<td>0.99</td>
</tr>
<tr>
<td>Fitted Value</td>
<td>1.77</td>
<td>0.54</td>
</tr>
</tbody>
</table>
In this section, we only discuss the effect of oblique incidence on the reconstruction of TM and TE polarized light. Two OPVs were configured as a partial polarimeter and were used to reconstruct only the $S_0$ and $S_1$ Stokes parameters for TM ($[1, 1, 0, 0]^T$) and TE ($[1, -1, 0, 0]^T$) polarization. One possible configuration of the two-cell polarimeter can be the first OPV with strain direction along the $x$-axis and the second OPV with strain direction along $y$-axis. The current from the two OPVs, at different $(\mu, \nu)$ combination where $\mu$ varies from $0^\circ$ to $18^\circ$ and $\nu$ from $0^\circ$ to $360^\circ$ under TM and TE polarized light, denoted as $i(\mu, \nu)$, can be obtained using the transfer matrix formalism. The current ratio ($CR$) of two OPVs, as measured off-axis, is normalized to the values obtained at normal incidence by

$$CR = \frac{i(\mu, \nu)}{i(\mu = 0^\circ, \nu = 0^\circ)}. \quad (4.12)$$

Results of this normalization are presented in Figure 31. Figure 31 (a) and (b) depicts $CR$ for OPV1-2 under TM polarized light and (c) and (d) under TE polarized light where the $x$- and $y$-axis represent zenith angle $\mu$ and the surface contours indicate the azimuthal angle $\nu$. Generally, TM
polarization experiences a 3.5-4% change at $\mu = 18^\circ$ while TE polarization experiences a smaller change of 1.8-2% at $\mu = 18^\circ$.

![Figure 31. The current ratio of OPV1 and OPV2 under TM and TE polarized light for zenith angle $\mu$ varying from 0° to 18° and azimuthal angle $\nu$ from 0° to 360°.](image)

To evaluate the effect of oblique incidence on the Stokes parameter reconstruction for the TM and TE polarization, the measurement matrix $W_{\text{two-cell}}$ for normal incidence was constructed using the IC polarimeter model, discussed in Section 2, using the model parameters of OPV1 and OPV2 listed in Table 3 and Table 4. The normal incidence of TM and TE currents were calculated by

$$
\begin{bmatrix}
I_{\text{TM-OPV1}} & I_{\text{TE-OPV1}} \\
I_{\text{TM-OPV2}} & I_{\text{TE-OPV2}}
\end{bmatrix} = W_{\text{two-cell}} \begin{bmatrix}
1 & 1 \\
1 & -1 \\
0 & 0 \\
0 & 0
\end{bmatrix}.
$$

(4.13)
The partial Stokes vectors $\mathbf{STM}$ and $\mathbf{STE}$, under different incident angles $(\mu, \nu)$, can be reconstructed by

$$\begin{bmatrix} S_{TM}(\mu, \nu) \\ S_{TE}(\mu, \nu) \end{bmatrix} = W_{\text{two-cell}}^{-1} \cdot \begin{bmatrix} I_{TM-OPV1} \cdot \alpha_{TM-OPV1}(\mu, \nu) \\ I_{TM-OPV2} \cdot \alpha_{TM-OPV2}(\mu, \nu) \\ I_{TE-OPV1} \cdot \alpha_{TE-OPV1}(\mu, \nu) \\ I_{TE-OPV2} \cdot \alpha_{TE-OPV2}(\mu, \nu) \end{bmatrix}, \quad (4.14)$$

where $\alpha_{TM-OPV1}(\mu, \nu)$, $\alpha_{TM-OPV2}(\mu, \nu)$, $\alpha_{TE-OPV1}(\mu, \nu)$ and $\alpha_{TE-OPV2}(\mu, \nu)$ are the current ratios of OPV1 and OPV2 under TM and TE polarized light, as per Figure 31. The absolute error of the $S_1$ component of $\mathbf{STM}$ and $\mathbf{STE}$ are plotted in Figure 32 (a) and (b), respectively after normalization to the $S_0$ component. The maximum errors for TM and TE are 1.51% and 2.45% respectively, indicating that the normal incidence model still holds well for small zenith angles $\mu$ up to 18°.

![Figure 32](image-url)
CHAPTER 5. Optimization of an Intrinsic Coincident Polarimeter

5.1. Figure of Merit

In this dissertation we used the equally weighted variance (EWV) as a metric to evaluate the performance of the system under Gaussian noise (GN) to take into account the throughput of the system. If GN imposed to each measurement channel is \(\sigma\), the total noise variance on the Stokes vector can be calculated as

\[
\text{Trace}(\Gamma^S) = \text{Trace}(\mathbf{W}^\dagger \mathbf{W}^T) \cdot \sigma^2 = \text{EWV} \cdot \sigma^2. \tag{5.1}
\]

Conversely, the noise variance of each Stokes vector, induced by signal-dependent PN, depends on the specific polarization state under test. Thus, our figure of merit under PN is the maximum variance obtained from sampled Stokes vectors. 10,000 Stokes vectors that are uniformly distributed on the Poincare sphere surface are randomly generated to evaluate noise variance under PN, as depicted in Figure 33.

![Figure 33. 10,000 Stokes vectors (blue data) are uniformly sampled across the Poincare sphere.](image)

For a polarimeter with \(N\) measurement channels, the maximum variance is defined as

\[
\max(\sigma^2_{S_j}) = \max (N \sum_{k=0}^{3} \sum_{n=1}^{N} [\mathbf{W}_{j,n}^\dagger]^2 \mathbf{W}_{n,k} S_k), \tag{5.2}
\]
where $\sigma_{Sj}^2$ is the noise variance on $S_j$ ($j = 0, 1, 2$ or $3$) induced only by PN and $N_{ph}$ is the total number of photons incident on the polarimeter.

5.2. DoT, DoFP, DoA, DoAM and an Ideal Polarimeter

Before we optimize the IC polarimeter, we first investigate the noise performance of the best DoT, DoFP, DoA, and DoAM polarimeter architectures. Since DoT, DoFP and DoA can produce the similar measurement matrices, only the DoT system was used for our comparison. Mu, et. al.\textsuperscript{70} proposed three optimal configurations for a DoT variable retarder polarimeter that had maximum immunity to both Poisson and Gaussian noise: (1) a retarder followed by a linear polarizer; (2) two retarders followed by a linear polarizer; and (3) two QWPs followed by a linear polarizer. Since these three configurations produced similar noise performance, only the first configuration (1) was considered in our analysis. This polarimeter had an optimal retardance of $(102.2^\circ, 142.1^\circ)$ and azimuthal angles $(\pm 71.9^\circ, \pm 34.95^\circ)$. The measurement matrix produced by this set of retardance and azimuthal angles is

$$W_{DoT} = \begin{bmatrix} 0.1250 & 0.0722 & -0.0722 & -0.0722 \\ 0.1250 & 0.0722 & 0.0722 & 0.0722 \\ 0.1250 & -0.0722 & 0.0722 & -0.0721 \\ 0.1250 & -0.0722 & -0.0722 & 0.0721 \end{bmatrix}, \quad (5.3)$$

where the measurement matrix is scaled by half due to the polarizer rejecting half of light.

Meanwhile, Lara et al.\textsuperscript{47} optimized Compain’s prism-based DoAM polarimeter\textsuperscript{32} by unconstraining the shape of the prism, thus minimizing the noise variance with the measurement matrix given by

$$W_{DoAm} = \begin{bmatrix} 0.2457 & -0.1324 & 0.2070 & 0 \\ 0.2457 & -0.1324 & -0.2070 & 0 \\ 0.2543 & 0.1500 & 0 & -0.2053 \\ 0.2543 & 0.1500 & 0 & 0.2053 \end{bmatrix}, \quad (5.4)$$
Assuming 100% light collection efficiency in DoAM polarimeter, we normalize the matrix to make the sum of the first column equals to 1. The EWV and noise variance of the optimal DoT and prism-based DoAM under GN and PN are listed in Table 6. For PN, only the Stokes vector with maximum noise variance is listed in the table. The noise variance is in units of $\sigma^2$ under GN and of $N_{ph}$ under PN. DoAM has less noise variance thus higher signal-to-noise ratio (SNR) than DoT due to a higher signal collection efficiency. The sampled Stokes vectors’ noise variance for DoT and DoAM under PN are plotted in Figure 34 (a) and (b) respectively, where only the envelope of the noise variance for $S_0$, $S_1$, $S_2$, and $S_3$ are shown for a better visualization. GN is independent of Stokes vector thus only noise variance under PN is plotted. Figure 34 shows that DoT has a larger noise variance than DoAM but has a better noise equalization across all the sampled Stokes vector. For DoAM measurement matrix per Eq.(5.4), the difference between maximum and minimum noise variance are $0.07N_{ph}$, $0.10N_{ph}$, $3.09N_{ph}$ and $3.56N_{ph}$ for $S_0$, $S_1$, $S_2$, and $S_3$ respectively.

Goudail\textsuperscript{62} proposed an ideal polarimeter that minimizes and equalizes the noise on all Stokes vectors, with the measurement matrix as

\[
W_{\text{ideal}} = \frac{1}{4} \begin{bmatrix}
1 & 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\
1 & -1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \\
1 & -1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \\
1 & 1/\sqrt{3} & -1/\sqrt{3} & -1/\sqrt{3}
\end{bmatrix}.
\]

We listed the noise variance of this ideal polarimeter under PN and GN in Table 6 and plotted PN noise variance in Figure 34 (c). The ideal polarimeter lists the lower bound of the noise variance under GN and PN, where the minimum noise variance for $S_0$, $S_1$, $S_2$, and $S_3$ is $4\sigma^2$, $12\sigma^2$, $12\sigma^2$ and $12\sigma^2$ under GN and $N_{ph}$, $3N_{ph}$, $3N_{ph}$ and $3N_{ph}$ under PN, respectively.
Table 6. Noise variance of DoT, DoAM and Ideal polarimeter.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>GN</th>
<th>PN</th>
<th>EWV</th>
<th>Max $\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Var($S_0$)</td>
<td>Var($S_1$)</td>
<td>Var($S_2$)</td>
<td>Var($S_3$)</td>
</tr>
<tr>
<td>DoT</td>
<td>16.00</td>
<td>47.96</td>
<td>47.96</td>
<td>48.03</td>
</tr>
<tr>
<td>DoAM</td>
<td>4.02</td>
<td>12.57</td>
<td>11.67</td>
<td>11.86</td>
</tr>
<tr>
<td>Ideal</td>
<td>4</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

*For GN, the noise variance is in the units of $\sigma^2$. For PN, the maximum noise variance of $S_0$, $S_1$, $S_2$, and $S_3$ are listed in the table, in the units of $N_{ph}$.

Figure 34. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under PN for (a) DoT; (b) DoAM and (c) Ideal polarimeters.

5.3. Optimization of an IC Polarimeter

In this section, different IC polarimeter structures are discussed and optimized. Five different optimization configurations are considered: Case I: the first three OPVs are identical and Case II: the first three OPVs have different characteristics, as shown in Figure 35 (a); Case III: an additional QWP being put in front in a four-cell system, as depicted in Figure 35 (b); Case IV: relaxing waveplates’ retarance to arbitrary values based on Case III, as illustrated in Figure 35 (c); Case V: a five-cell system shown in Figure 35 (d) and Case VI: ideal IC polarimeter without practical constraints.
Figure 35. IC polarimeter configurations: (a) Case I and Case II: Four-cell (OPV) system with two quarter-wave plates (QWPs). (b) Case III: Four-cell system with additional QWP in front. (c) Case IV: Four-cell system with retardance-relaxed waveplates. (d) Case V: Five-cell ICP polarimeter.

In the following optimization, we set the same constraints that all OPVs have (a) the same responsivity $\eta$; (b) the reflectance $\rho$ of all four OPVs is zero (the reflection of OPV can be potentially reduced with highly transparent conducting electrode such as PEDOT:PSS$^{72}$); and (c) the retardance $\phi$ of all OPVs is zero; and (d) the last OPV absorbs all the remaining light. It should be noted that the OPV’s characteristics can be modified by material selection and process parameters typical to organic electronics fabrication$^{53,73,74}$.

5.3.1. Case I: Four-Cell System with Identical OPVs

As depicted in Figure 35 (a), the optimization of a four-cell system, with the first three OPVs identical, is discussed in this section. The model’s degrees of freedom for the optimization included the $x$-eigenvector’s maximum transmission $T_x$, the diattenuation $D_T$ of the first three OPVs, and the orientation angles of the OPVs ($\theta_1, \theta_2, \theta_3$) and QWPs ($\theta_{R1}, \theta_{R2}$). The other parameters were either defined as constants ($\phi_1 = \phi_2 = \phi_3 = 0^\circ$, $\phi_{R1} = \phi_{R2} = 90^\circ$, $T_{\text{perp}4} = T_{\text{perp}4} = 0$, $D_{A4} = 0$, $E_{A4} = 0$, $\theta_4 = 0^\circ$) or calculated via a parameter sweep ($D_{A1}, D_{A2}, D_{A3}, E_{A1}, E_{A2}, E_{A3}, E_{T1}, E_{T2}, E_{T3}, T_{\text{para}1}, T_{\text{para}2}, T_{\text{para}3}$) to search for regions containing minima. The varying model parameters are listed in vector $\mathbf{C}$, expressed by
\( C = [\theta_1, \theta_2, \theta_3, \theta_{R1}, \theta_{R2}, T_{\text{perp}}, D_T]. \)  \hspace{1cm} (5.6)

\( T_x \) and \( D_T \) are constrained from 0.1 to 0.9 and the orientation angles are restricted to span 0° to 360°.

Since Gaussian and Poisson noise were considered separately, we created two designs that were optimized for each kind of noise. For additive GN with standard deviation \( \sigma \), the EWV was minimized by the following cost function

\[
C = \arg \min_c \text{EWV}(W),
\]  \hspace{1cm} (5.7)

yielded the best structure with the measurement matrix as

\[
W_{\text{Case-I-GN}} = \begin{bmatrix}
0.3601 & -0.1950 & -0.1736 & 0 \\
0.2295 & 0.0674 & 0.0617 & -0.1522 \\
0.1548 & -0.0063 & 0.1117 & 0.0651 \\
0.2547 & 0.1339 & 0.0001 & 0.0871
\end{bmatrix}.
\]  \hspace{1cm} (5.8)

The optimized model parameters were \( \theta_1 = 20.84^\circ, \theta_2 = 52.63^\circ, \theta_3 = 34.03^\circ, \theta_{R1} = 8.18^\circ, \theta_{R2} = 0.12^\circ, T_{\text{perp}} = 0.90, \) and \( D_T = 0.41 \). Optimizing under pure PN was accomplished by minimizing the maximum variance on the Stokes vector by

\[
C = \arg \min_c \text{max} (\sigma_S^2). \]  \hspace{1cm} (5.9)

The optimized measurement matrix was calculated as

\[
W_{\text{Case-I-PN}} = \begin{bmatrix}
0.3644 & -0.1729 & -0.2001 & 0 \\
0.2480 & 0.1171 & 0.0782 & -0.1438 \\
0.1425 & -0.0431 & 0.1022 & 0.0316 \\
0.2450 & 0.0990 & 0.0197 & 0.1122
\end{bmatrix}.
\]  \hspace{1cm} (5.10)

The optimized model parameters were \( \theta_1 = 24.58^\circ, \theta_2 = 56.35^\circ, \theta_3 = 32.62^\circ, \theta_{R1} = 1.44^\circ, \theta_{R2} = 10.94^\circ, T_{\text{perp}} = 0.90, \) and \( D_T = 0.42 \).
The noise variances of the Stokes vectors under GN and PN are summarized in Table 7 for the two optimized matrices $W_{\text{CaseI-GN}}$ and $W_{\text{CaseI-PN}}$ and the noise variances under PN are plotted in Figure 36 (a) and (b). The noise variance for $S_0$ follows the calculation in a conventional intensity detector: the total noise variance for $S_0$ under GN is the sum of the noise from all channels, which was calculated as $4\sigma^2$ and the noise variance under PN is $N_{ph}$, which is the number of incoming photons. EWV under GN and maximum variance under PN are comparable for the two matrices in Eq. (5.8) and (5.10). The minimum EWV under $W_{\text{CaseI-GN}}$ is $111.5\sigma^2$ while $115.8\sigma^2$ under $W_{\text{CaseI-PN}}$. A moderate improvement in PN performance is obtained from $W_{\text{CaseI-PN}}$ with a lower maximum variance of $10.66\ N_{ph}$, compared with $12.78N_{ph}$ calculated from $W_{\text{CaseI-GN}}$. The optimized results for Case I achieves better noise immunity than best DoT under GN where the EWV is $159.95\sigma^2$ under GN but the IC polarimeter still has a large noise variance under PN of best $10.66\ N_{ph}$.

Table 7. Noise variance under measurement matrix $W_{\text{CaseI-GN}}$ and $W_{\text{CaseI-PN}}$.

<table>
<thead>
<tr>
<th>Measurement Matrix</th>
<th>GN</th>
<th>PN</th>
<th>EWV</th>
<th>Max $\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{\text{CaseI-GN}}$</td>
<td>Var($S_0$) 33.05</td>
<td>Var($S_1$) 45.50</td>
<td>Var($S_2$) 28.91</td>
<td>1</td>
</tr>
<tr>
<td>$W_{\text{CaseI-PN}}$</td>
<td>4 40.29</td>
<td>41.21</td>
<td>30.13</td>
<td>1</td>
</tr>
</tbody>
</table>

*For GN, the noise variance is in the units of $\sigma^2$. For PN, the maximum noise variance of $S_0$, $S_1$, $S_2$, and $S_3$ are listed in the table, in the units of $N_{ph}$.

Figure 36. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) $W_{\text{CaseI-GN}}$ (b) $W_{\text{CaseI-PN}}$. 

58
5.3.2. Case II: Four-Cell System with Different OPVs

If we use three same OPV cells, there is less freedom for us to tune the polarimeter measurement matrix to get better noise performance. In this session, we optimize the previous configuration using different OPVs which give more freedom to improve the performance. The optimization parameters are given by

\[
\begin{bmatrix}
\theta_1, \theta_2, \theta_3, \theta_{R1}, \theta_{R2}, T_{\text{perp}1}, T_{\text{perp}2}, T_{\text{perp}3}, D_{T1}, D_{T2}, D_{T3}
\end{bmatrix}.
\]

Similarly, the optimized measurement matrix under GN, per Eq.(5.1), was calculated as

\[
W_{\text{Case II-GN}} = \begin{bmatrix}
0.3061 & -0.1677 & 0.1198 & 0 \\
0.2645 & 0.0621 & -0.0364 & -0.1882 \\
0.2237 & 0.1431 & 0.0831 & 0.1009 \\
0.2058 & -0.0375 & -0.1665 & 0.0873
\end{bmatrix},
\]

where the optimized model parameters are \(\theta_1 = 162.23^\circ, \theta_2 = 184.62^\circ, \theta_3 = 163.96^\circ, \theta_{R1} = 141.09^\circ, \theta_{R2} = 140.47^\circ, T_{\text{perp}1} = T_{\text{perp}2} = T_{\text{perp}3} = 0.90, D_{T1} = 0.30, D_{T2} = 0.46, \text{and } D_{T3} = 0.9\).

Meanwhile, the optimized measurement matrix under PN, per Eq.(5.2), was calculated as

\[
W_{\text{Case II-PN}} = \begin{bmatrix}
0.3059 & -0.1757 & 0.1045 & 0 \\
0.2671 & 0.0047 & -0.1133 & -0.1665 \\
0.2258 & 0.1683 & 0.1011 & 0.0032 \\
0.2012 & 0.0027 & -0.0923 & 0.1633
\end{bmatrix},
\]

where the optimized model parameters are \(\theta_1 = 164.63^\circ, \theta_2 = 174.45^\circ, \theta_3 = 130.51^\circ, \theta_{R1} = 114.39^\circ, \theta_{R2} = 131.31^\circ, T_{\text{perp}1} = T_{\text{perp}2} = T_{\text{perp}3} = 0.90, D_{T1} = 0.29, D_{T2} = 0.48, \text{and } D_{T3} = 0.9\).

Using different OPVs to construct the polarimeter results in a reduction of 44% in total noise variance under GN and 41% for the maximum variance under PN, compared with Case I. These results are summarized in Table 8 and plotted in Figure 37 (a) and (b). Compared with the best DoT polarimeter, this configuration of IC polarimeter has greatly improved the noise performance under GN: \(61.94\sigma^2\) in Case II while \(159.95\sigma^2\) for DoT. Additionally, the noise
variance under PN for case II and best DoT is comparable: $6.23 N_{ph}$ in Case II and $6 N_{ph}$ for best DoT. It should be noted that we examined maximum noise variance across different polarization states under PN. In best DoT, the noise variance is almost the same for sampled Stokes vectors, as illustrated in Figure 34 (a) while in case II for IC, nearly half polarization states has a less than $6 N_{ph}$ noise variance, as provided in Figure 37 (b). We can claim that IC polarimeter under case II achieves a better performance than best DoT polarimeter (or DoA, DoFP).

Table 8. Noise variance under measurement matrix $W_{\text{CaseII-GN}}$ and $W_{\text{CaseII-PN}}$.

<table>
<thead>
<tr>
<th>Measurement Matrix</th>
<th>$\text{Var}(S_0)$</th>
<th>$\text{Var}(S_1)$</th>
<th>$\text{Var}(S_2)$</th>
<th>$\text{Var}(S_3)$</th>
<th>$\text{Var}(S_0)$</th>
<th>$\text{Var}(S_1)$</th>
<th>$\text{Var}(S_2)$</th>
<th>$\text{Var}(S_3)$</th>
<th>EWV</th>
<th>Max $\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{\text{CaseII-GN}}$</td>
<td>4</td>
<td>18.82</td>
<td>20.19</td>
<td>18.96</td>
<td>1</td>
<td>6.55</td>
<td>6.74</td>
<td>6.53</td>
<td>61.94</td>
<td>6.74</td>
</tr>
<tr>
<td>$W_{\text{CaseII-PN}}$</td>
<td>4</td>
<td>17.21</td>
<td>23.97</td>
<td>18.70</td>
<td>1</td>
<td>6.23</td>
<td>6.21</td>
<td>6.23</td>
<td>63.89</td>
<td>6.23</td>
</tr>
</tbody>
</table>

*For GN, the noise variance is in the units of $\sigma^2$. For PN, the maximum noise variance of $S_0$, $S_1$, $S_2$, and $S_3$ are listed in the table, in the units of $N_{ph}$.

Figure 37. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) $W_{\text{CaseII-GN}}$ (b) $W_{\text{CaseII-PN}}$.

5.3.3. Case III: Four-Cell System with an additional QWPs

Optimization in this section focuses on adding a QWP in front of the IC polarimeter, enabling OPV1 to gain sensitivity in measuring the $S_3$ Stokes parameter. Additional model parameters for this configuration is the first QWP’s orientation angle $\theta_{RF}$. The optimization parameters are given by

$$
C = \left[ \theta_1, \theta_2, \theta_3, \theta_{RF}, \theta_{R1}, \theta_{R2}, T_{\text{perp}1}, T_{\text{perp}2}, T_{\text{perp}3}, D_{T1}, D_{T2}, D_{T3} \right].
$$

(5.14)
The optimized measurement matrix under GN was calculated as

\[
W_{\text{CaseIII-GN}} = \begin{bmatrix}
0.3061 & -0.1207 & -0.1151 & 0.1210 \\
0.2645 & -0.1102 & 0.1261 & -0.1121 \\
0.2237 & 0.1265 & 0.1011 & 0.1065 \\
0.2058 & 0.1044 & -0.1121 & -0.1153
\end{bmatrix}.
\] (5.15)

The optimized model parameters were calculated to be \(\theta_1 = 183.85^\circ\), \(\theta_2 = 195.19^\circ\), \(\theta_3 = 148.32^\circ\), \(\theta_{RF} = 21.82^\circ\), \(\theta_{R1} = 136.48^\circ\), \(\theta_{R2} = 150.78^\circ\), \(T_{\text{perp}1} = T_{\text{perp}2} = T_{\text{perp}3} = 0.90\), \(D_{T1} = 0.30\), \(D_{T2} = 0.46\), and \(D_{T3} = 0.9\). Optimization under PN produced the measurement matrix

\[
W_{\text{CaseIII-PN}} = \begin{bmatrix}
0.2980 & -0.1144 & -0.1144 & 0.1119 \\
0.2648 & -0.1164 & 0.1139 & -0.1160 \\
0.2274 & 0.1167 & 0.1094 & 0.1146 \\
0.2098 & 0.1163 & -0.1089 & -0.1105
\end{bmatrix},
\] (5.16)

with optimized model parameters of \(\theta_1 = 185.02^\circ\), \(\theta_2 = 196.97^\circ\), \(\theta_3 = 148.50^\circ\), \(\theta_{RF} = 22.22^\circ\), \(\theta_{R1} = 136.39^\circ\), \(\theta_{R2} = 152.21^\circ\), \(T_{\text{perp}1} = T_{\text{perp}2} = T_{\text{perp}3} = 0.90\), \(D_{T1} = 0.28\), \(D_{T2} = 0.46\), and \(D_{T3} = 0.89\).

These results are summarized in Table 9 and noise variances under PN are plotted in Figure 38 (a) and (b) for \(W_{\text{CaseIII-GN}}\) and \(W_{\text{CaseIII-PN}}\), respectively. Comparing with Case II, the total noise variance shows no improvement under GN, which is 61.94 \(\sigma^2\) under both cases. However, the maximum noise variance under PN was reduced from 6.23 \(N_{ph}\) to 5.08 \(N_{ph}\) by adding additional quarter waveplate in front.

<table>
<thead>
<tr>
<th>Measurement Matrix</th>
<th>GN (\text{Var}(S_0))</th>
<th>(\text{Var}(S_1))</th>
<th>(\text{Var}(S_2))</th>
<th>(\text{Var}(S_3))</th>
<th>PN (\text{Var}(S_0))</th>
<th>(\text{Var}(S_1))</th>
<th>(\text{Var}(S_2))</th>
<th>(\text{Var}(S_3))</th>
<th>EWV</th>
<th>Max (\sigma^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_{\text{CaseIII-GN}})</td>
<td>4</td>
<td>19.03</td>
<td>19.40</td>
<td>19.51</td>
<td>1</td>
<td>5.27</td>
<td>5.11</td>
<td>5.18</td>
<td>61.94</td>
<td>5.27</td>
</tr>
<tr>
<td>(W_{\text{CaseIII-PN}})</td>
<td>4</td>
<td>18.71</td>
<td>20.07</td>
<td>19.56</td>
<td>1</td>
<td>5.08</td>
<td>5.08</td>
<td>5.08</td>
<td>62.34</td>
<td>5.08</td>
</tr>
</tbody>
</table>

For GN, the noise variance is in the units of \(\sigma^2\). For PN, the maximum noise variance of \(S_0\), \(S_1\), \(S_2\), and \(S_3\) are listed in the table, in the units of \(N_{ph}\).
Figure 38. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) WCaseIII-GN (b) WCaseIII-PN.

5.3.4. Case IV: Four-Cell System with relaxed retardance waveplates

In order to use commercially available off-the-shelf components, the retarders used in the previous optimization were configured as standard QWPs. Consequently, we optimized the IC polarimeter by including the waveplates’ retardance as a degree of freedom, as illustrated previously in Figure 35 (c). The optimization parameters are given by

$$\mathbf{C} = [\theta_1, \theta_2, \theta_3, \theta_{RF}, \theta_{R1}, \theta_{R2}, T_{\text{perp}1}, T_{\text{perp}2}, T_{\text{perp}3}, D_{T1}, D_{T2}, D_{T3}, \phi_{RF}, \phi_{R1}, \phi_{R2}]$$

where $\phi_{RF}$, $\phi_{R1}$, and $\phi_{R2}$ are the retardances for WPF, WP1, and WP2, respectively. Minimizing the EWV, under GN, yielded the optimized measurement matrix

$$\mathbf{W}_{\text{CaseIV-GN}} = \begin{bmatrix} 0.3061 & -0.1215 & -0.1168 & 0.1186 \\ 0.2645 & -0.1098 & 0.1222 & -0.1167 \\ 0.2237 & 0.1208 & 0.1053 & 0.1090 \\ 0.2058 & 0.1104 & -0.1107 & -0.1110 \end{bmatrix},$$

with optimized model parameters of $\theta_1 = 184.24^\circ$, $\theta_2 = 195.71^\circ$, $\theta_3 = 147.59^\circ$, $\theta_{RF} = 21.82^\circ$, $\theta_{R1} = 135.82^\circ$, $\theta_{R2} = 151.25^\circ$, $T_{\text{perp}1} = T_{\text{perp}2} = T_{\text{perp}3} = 0.90$, $D_{T1} = 0.30$, $D_{T2} = 0.46$, $D_{T3} = 0.9$, $\phi_{RF} = 90.33^\circ$, $\phi_{R1} = 91.39^\circ$, and $\phi_{R2} = 90.86^\circ$. Optimization under PN produced the measurement matrix

62
\[
W_{\text{CaseIV-PN}} = \begin{bmatrix}
0.2966 & -0.1167 & -0.1122 & 0.1115 \\
0.2650 & -0.1163 & 0.1137 & -0.1161 \\
0.2279 & -0.1172 & 0.1094 & 0.1148 \\
0.2105 & 0.1158 & -0.1109 & -0.1101 \\
\end{bmatrix},
\]

with optimized model parameters of \(\theta_1 = 184.54^\circ\), \(\theta_2 = 196.23^\circ\), \(\theta_3 = 147.08^\circ\), \(\theta_{RF} = 21.82^\circ\), \(\theta_{R1} = 135.31^\circ\), \(\theta_{R2} = 151.44^\circ\), \(T_{\text{perp}1} = T_{\text{perp}2} = T_{\text{perp}3} = 0.90\), \(D_{T1} = 0.28\), \(D_{T2} = 0.46\), \(D_{T3} = 0.89\), \(\phi_{RF} = 90.32^\circ\), \(\phi_{R1} = 90.67^\circ\), and \(\phi_{R2} = 90.84^\circ\). The noise performance, calculated by the optimized two matrices per Eq. (5.18) and Eq. (5.19), is listed in Table 10 and the Poisson noise variances of the estimated Stokes vectors are plotted in Figure 39 (a) and (b), respectively. This shows that there is no significant improvement by relaxing the waveplates’ retardances compared with Case III with the minimum total noise variance of 61.94 \(\sigma^2\) under GN and the optimized maximum variance under PN of 5.08 \(N_{ph}\) for Case III and Case IV.

Table 10. Noise variance under measurement matrix \(W_{\text{CaseIV-GN}}\) and \(W_{\text{CaseIV-PN}}\).

<table>
<thead>
<tr>
<th>Measurement Matrix</th>
<th>GN</th>
<th></th>
<th></th>
<th></th>
<th>PN</th>
<th></th>
<th></th>
<th></th>
<th>EWV</th>
<th>Max (\sigma_S^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_{\text{CaseIV-GN}})</td>
<td>4</td>
<td>19.04</td>
<td>19.42</td>
<td>19.48</td>
<td>1</td>
<td>5.21</td>
<td>5.05</td>
<td>5.11</td>
<td>61.94</td>
<td>5.21</td>
</tr>
<tr>
<td>(W_{\text{CaseIV-PN}})</td>
<td>4</td>
<td>18.71</td>
<td>20.10</td>
<td>19.59</td>
<td>1</td>
<td>5.08</td>
<td>5.08</td>
<td>5.08</td>
<td>62.40</td>
<td>5.08</td>
</tr>
</tbody>
</table>

*For GN, the noise variance is in the units of \(\sigma^2\). For PN, the maximum noise variance of \(S_0\), \(S_1\), \(S_2\), and \(S_3\) are listed in the table, in the units of \(N_{ph}\).*

Figure 39. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) \(W_{\text{CaseIV-GN}}\) (b) \(W_{\text{CaseIV-PN}}\).
5.3.5. Case V: Five-Cell System

The final optimization case was selected by adding a OPV cell and QWP, as depicted previously in Figure 35 (d). Minimizing the EWV, under Gaussian noise, produced an optimized measurement matrix

\[
W_{\text{CaseV-GN}} = \begin{bmatrix}
0.3060 & 0.1187 & 0.1158 & 0.1221 \\
0.2644 & -0.1859 & -0.0164 & 0.0756 \\
0.2238 & -0.0086 & 0.0757 & -0.1783 \\
0.2041 & 0.0756 & -0.1748 & -0.0193 \\
0.0017 & 0.0001 & -0.0003 & -0.0001
\end{bmatrix}, \quad (5.20)
\]

with the optimized parameters \( \theta_1 = 93.97^\circ, \theta_2 = 30.95^\circ, \theta_3 = 167.76^\circ, \theta_4 = 75.26^\circ, \theta_{RF} = 112.14^\circ, \theta_{R1} = 50.37^\circ, \theta_{R2} = 76.53^\circ, \theta_{R3} = 53.98^\circ, \ T_{\text{perp1}} = T_{\text{perp2}} = T_{\text{perp3}} = 0.90, \ T_{\text{perp4}} = 0.1, \ DT_1 = 0.30, \ DT_2 = 0.46, \ DT_3 = 0.90, \) and \( DT_4 = 0.90 \). Meanwhile, the maximum variance under PN demonstrated a small improvement, where the optimized measurement matrix

\[
W_{\text{CaseV-PN}} = \begin{bmatrix}
0.3013 & 0.1134 & 0.1140 & 0.1211 \\
0.2566 & -0.1102 & -0.1084 & 0.1131 \\
0.2283 & -0.1096 & 0.1147 & -0.1164 \\
0.1096 & 0.0774 & -0.0410 & -0.0608 \\
0.1041 & 0.0290 & -0.0793 & -0.0569
\end{bmatrix}, \quad (5.21)
\]

with the optimized model parameters \( \theta_1 = 94.09^\circ, \theta_2 = 51.83^\circ, \theta_3 = 188.99^\circ, \theta_4 = 40.13^\circ, \theta_{RF} = 112.58^\circ, \theta_{R1} = 52.09^\circ, \theta_{R2} = 97.39^\circ, \theta_{R3} = 53.99^\circ, \ T_{\text{perp1}} = T_{\text{perp2}} = T_{\text{perp3}} = T_{\text{perp4}} = 0.90, \ DT_1 = 0.29, \ DT_2 = 0.44, \ DT_3 = 0.9, \) and \( DT_4 = 0.82 \). A summary of these results is presented in Table 11. While this solution minimizes the maximum variance, it introduces more noise into the system by increasing the measurement channels, yielding an increased EWV. The Poisson noise variances of the estimated Stokes vectors based on two matrices per Eq.(5.20) and Eq.(5.21), are plotted in Figure 40 (a) and (b), respectively. Using five-cell system only improves performance under PN only by 2% but amplifies the noise under GN from \( 5.21 \sigma^2 \) to \( 7.03 \sigma^2 \).
Table 11. Noise variance under measurement matrix $W_{\text{CaseV-GN}}$ and $W_{\text{CaseV-PN}}$.

<table>
<thead>
<tr>
<th>Measurement Matrix</th>
<th>GN Var($S_0$)</th>
<th>Var($S_1$)</th>
<th>Var($S_2$)</th>
<th>Var($S_3$)</th>
<th>PN Var($S_0$)</th>
<th>Var($S_1$)</th>
<th>Var($S_2$)</th>
<th>Var($S_3$)</th>
<th>EWV</th>
<th>Max $\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{\text{CaseV-GN}}$</td>
<td>4.01</td>
<td>18.42</td>
<td>20.35</td>
<td>19.24</td>
<td>1</td>
<td>6.89</td>
<td>7.03</td>
<td>7.03</td>
<td>62.03</td>
<td>7.03</td>
</tr>
<tr>
<td>$W_{\text{CaseV-PN}}$</td>
<td>5.00</td>
<td>24.90</td>
<td>24.28</td>
<td>24.06</td>
<td>1</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
<td>78.25</td>
<td>5.00</td>
</tr>
</tbody>
</table>

*For GN, the noise variance is in the units of $\sigma^2$. For PN, the maximum noise variance of $S_0$, $S_1$, $S_2$, and $S_3$ are listed in the table, in the units of $N_{\text{ph}}$.

Figure 40. 2D envelope plot of noise variances for the sampled Stokes vector across 3D Poincare sphere surface under (a) $W_{\text{CaseV-GN}}$ (b) $W_{\text{CaseV-PN}}$.

5.3.6. Case VI: Ideal IC Polarimeter

In our previous optimization, we constrain both $T_s$ and $D_T$ from 0.1 to 0.9 for optimization and achieve a minimum EWV 61.94 in Case II, III and IV and a minimum-maximum variance under PN of $5N_{\text{ph}}$ in Case V. In this section, we relaxed $T_s$ and $D_T$ to allow for the ideal situation, where these values can span 0 to 1, and perform the optimization using the Case IV architecture. The model parameters were $\theta_1 = 164.55^\circ$, $\theta_2 = 162.82^\circ$, $\theta_3 = 126.05^\circ$, $\theta_{RF} = 46.27^\circ$, $\theta_{RI} = 105.62^\circ$, $\theta_{R2} = 198.04^\circ$, $T_{\text{perp1}} = T_{\text{perp2}} = T_{\text{perp3}} = 1$, $D_{T1} = 0.33$, $D_{T2} = 0.50$, $D_{T3} = 1$, $\phi_{RF} = 136.22^\circ$, $\phi_{RI} = 103.86^\circ$, and $\phi_{R2} = 60.75^\circ$ resulted in an ideal IC polarimeter with a measurement matrix

$$W_{\text{Ideal-IC}} = \begin{bmatrix} 0.25 & 0.1441 & 0.1441 & 0.1441 \\ 0.25 & -0.1441 & -0.1441 & 0.1441 \\ 0.25 & -0.1441 & 0.1441 & -0.1441 \\ 0.25 & 0.1441 & -0.1441 & -0.1441 \end{bmatrix},$$

(5.22)
which has a similar noise performance as an ideal polarimeter per Eq.(5.5), as listed in Table 12 and plotted in Figure 41. To achieve the best polarimeter architecture, the dichroic ratio needs to be 1 ($T_{\text{perp}1} = T_{\text{perp}2} = T_{\text{perp}3} = 1$, $D_{T3} = 1$) which is currently not practical. However the analysis here provides an optimization direction of increasing the dichroic ratio to achieve less error sensitivity.

Table 12. Noise variance under measurement matrix $W_{\text{Ideal}}$ and $W_{\text{Ideal-IC}}$.

| Measurement Matrix | GN | | | | PN | | | | | | | \[ \text{Var}(S_0) \] | \[ \text{Var}(S_1) \] | \[ \text{Var}(S_2) \] | \[ \text{Var}(S_3) \] | \[ \text{Var}(S_0) \] | \[ \text{Var}(S_1) \] | \[ \text{Var}(S_2) \] | \[ \text{Var}(S_3) \] | EWV | Max $\sigma^2$ |
|--------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $W_{\text{Ideal}}$ | 4 | 12 | 12 | 12 | 1 | 3 | 3 | 3 | 40 | 3 |
| $W_{\text{Ideal-IC}}$ | 4 | 12.03 | 12.03 | 12.03 | 1 | 3.01 | 3.01 | 3.01 | 40.09 | 3.01 |

For GN, the noise variance is in the units of $\sigma^2$. For PN, the maximum noise variance of $S_0$, $S_1$, $S_2$, and $S_3$ are listed in the table, in the units of $N_{\text{ph}}$.

Figure 41. Noise variances of the sampled Stokes vector under PN for ideal IC polarimeter.

5.4. Polarimeter Architectures Comparison

Figure 42 summaries Ideal, DoT, DoAM and best IC polarimeters under PN for cases I to VI, which are denoted as ICP-I to ICP-VI, respectively. These data demonstrate that the DoAM and IC polarimeters had a smaller EWV than DoT since the signal collection ability in DoAM and IC polarimeters is nearly 100%, while the polarizer annihilated half of the light in DoT. Considering GN only, the noise variance of $S_0$, when using a four-channel IC polarimeter, is 4 times smaller than DoT, which yields a factor of 2 improvement in the SNR. Under PN, the noise variance in $S_0$ is two times smaller in the DoAM and IC polarimeters, indicating a $\sqrt{2}$ improvement in SNR in $S_0$. The ideal polarimeter gives the minimum total noise variance under
GN of $40\sigma^2$ and a minimum max noise variance under PN of $3N_{ph}$. The ideal IC polarimeter in Case-VI has the potential to achieve similar noise variance compared to the ideal polarimeter; however, the optimized parameters of an ideal IC polarimeter with maximum transmission and diattenuation of 100% are not practical using existing material systems.

These results assist in providing guidelines for future polarization-sensitive detectors to maximize the performance of IC imaging polarimetry. Most clearly, there is a need to maximize the transmittance of the detectors in the electric field direction of low absorption by the active layers ($T_x$ in our reference frame). For instance, the optimization consistently set the transmittance in the $T_x$ direction to hit the maximum bound of 0.9. This suggested that the conducting electrodes must have maximal optical transmission. Alternatively, the electrodes can be moved out of the optical path by introducing other detection elements, such as photo-transistors. An additional design requirement that becomes evident is the need to have each OPVs’ diattenuation separately optimized. This requires the optical anisotropy of the active layer to be tunable; a feature that is possible with strain aligning polymer-based detectors. In this situation (studied in case II), the diattenuation of cells 1, 2, and 3 gradually increased, spanning values of 0.3, 0.46, and 0.9, respectively. The strain alignment approach can meet the diattenuation demand of 0.45; however, achieving a diattenuation of 0.9 requires a very high degree of orientational order of the polymers. Thus, alternative alignment methods to achieve this level of order may be necessary. While achieving an ideal IC polarimeter is not possible, given a transmittance of 1 if not possible with any refractive index change of the detector element, we believe that OPV cells are well-equipped to approach this limit.
Figure 42. Noise variance comparison of ideal, DoT, DoAM and IC polarimeters. (a) Noise variance of $S_0$, $S_1$, $S_2$, and $S_3$ under Gaussian noise. (b) Maximum noise variance of $S_0$, $S_1$, $S_2$, and $S_3$ under Poisson noise.
CHAPTER 6. Discussion and Conclusions

The motivation of this work to develop a novel polarimeter architecture to achieve a high spatial and temporal resolution which the existing polarimeter architectures fail to obtain. For the first time, a full-Stokes intrinsic coincident polarimeter was developed in this dissertation. Complete models of IC polarimeter were established, including the optical model and optoelectronic model under different strain conditions, and the optical-crosstalk influenced monolithic IC polarimeter model. With the established model, a monolithic IC polarimeter was finally achieved and demonstrated through experiments. This dissertation also provided the methodology of optimizing an IC polarimeter under Gaussian and Poisson noise with which IC polarimeter was optimized for different configurations.

6.1. Discussion

In Chapter 2, we introduced the mathematical representation of polarization which can be represented either by polarization eclipse or by Stokes parameters. In polarimetry, a combination of Muller matrix and Stokes parameters is used to obtain the states of polarization of incoming light. Existing polarimeter architectures were also reviewed in Chapter 2, including division of time, division of amplitude, division of aperture, division of focal plane, channeled polarimeter and detector-level polarimeter technology. The advantages and disadvantages of current techniques and the proposed intrinsic coincident polarimeter are compared in Figure 43. The proposed intrinsic coincident configuration has the advantage of maintaining both temporal and spatial resolution at the same time as compared with existing technology. A polarimeter must/can be optimized to suppress noise propagation to the reconstructed Stokes parameters due to matrix inversion. Optimization metrics include minimizing the various condition numbers, reciprocal
absolute determinant, equally weighted variance and approaching the ideal measurement matrix under both Gaussian noise and Poisson noise.

Figure 43. Comparison of current polarimeter architectures and proposed intrinsic coincident technique.

In Chapter 3, we discussed the polarization-sensitive organic photovoltaic in terms of its structure and optoelectronic properties, such as absorption spectrum, external quantum efficiency and detector linear response. The OPVs polarization-related absorption increases with the strain percentage and finally plateaus when the thickness of the film competes with the backbone alignment induced absorption. Even if the EQE is the same for strain direction and its orthogonal direction, there is an undesired drop in EQE for films with higher strain, which should be considered in the model. Both optical and electrical model were established for one OPV cell from which the measurement matrix of IC polarimeter was derived. The concept of IC polarimeter was first verified in free space without any integration. Both radiometric and polarimetric calibration were performed and with the calibrated measurement matrix or model parameters, the IC polarimeter can measure the polarized light with a mean square error of 0.84%.

In Chapter 4, monolithic IC polarimeter’s model and validation were illustrated. Due the high reflectance of OPV’s gold layer (~20%), back-reflection from subsequent OPVs is unavoidable, resulting in additional absorption besides the one in forwarding incidence. The back-
reflection effect can be modeled by introducing a reflection Mueller matrix. It should be noted that only one time reflection from the next OPV is considered because the reflection from waveplates, OPVs after the next and current OPV itself is negligible (<1%). This optical crosstalk model was first verified by the experiment with a relative RMS error of 2.17% where the error may come from inaccurate characterized model parameters and the reflections that model does not consider. With the optical crosstalk model established, each OPV’s model in a monolithic IC polarimeter was updated and finally the polarimeter measurement matrix was derived from the absorption Mueller matrix of each OPV. Similarly with the demonstration process of unintegrated IC polarimeter, both radiometric and polarimetric calibration were performed under different light powers and different incident polarization state. With the data reduction method which is free from model parameters, the monolithic IC polarimeter was validated with an RMS error of 1.59% to measure polarization states. The IC polarimeter model was validated through fitting the results calculated from the model to the measured data by varying model parameters. The model was validated with an average RMS error of 2.52%.

Additionally, the actual length along the optical axis for measurements is elongated due to the coincidence property of IC polarimeter. Therefore incidence angle response should be considered. Light rays with large incidence angle will escape from the system before reaching the subsequent OPVs and potentially introduce errors not only in the current pixel but also in adjacent pixels in an OPV-based detector array. This dissertation provided an initial investigation of the influence of incident angle up to 18°. For a two-cell system, the maximum errors for TM and TE are 1.51% and 2.45% respectively, indicating that the normal incidence model still holds well for small zenith angles θ up to 18°. However, further analysis of the four-cell system and even an array is needed in the future.
In Chapter 5, IC polarimeter was optimized under different configurations. Since it is hard to optimize a polarimeter under both signal-independent Gaussian and signal-dependent Poisson noise, we used two figures of merit to optimize the system separately. It was demonstrated that the IC polarimeter can be easily optimized to have a comparable performance with the best DoT by using OPVs with different characteristics. For IC polarimeter itself, it can be further optimized by increasing the dichroic ratio and transmission of the OPV. We believe high transparent and high dichroic ratio of OPV can be achieved with the advance of material technology.

6.2. Future Works

6.2.1. Incidence Angle Response for an OPV Detector Array

We performed an initial incidence angle response in Chapter 4 for a two-cell system and experimentally proved that this system still had a good performance for an incidence angle of up to $18^\circ$. Further analysis of incidence angle response is needed to advance the integrated IC polarimeters. Firstly a four cell system should be studied and then further the study into the detector array. The response should be considered across different system parameters: pixel thickness, pixel width, f/# of the system and different wavelength.

6.2.2. Sensitivity Analysis

In Chapter 5, we optimized the IC polarimeters by varying the model parameters, such as transmission, diattenuation, rotation angle of OPVs and waveplates’ retardance and angle. The optimized model parameters were rounded to two decimal numbers, which may be hard to achieve in practice. Sensitivity analysis is needed to illustrate how sensitive of the system matrix is to the model parameters.
6.3. Closing

The goal of this dissertation is to develop a full-Stokes intrinsic coincident polarimeter based on polarization-sensitive semitransparent organic photovoltaics to capture polarization information with both high temporal and spatial resolution. This goal has been achieved for the first time in this dissertation through a combination of model, design, characterization, calibration, validation and optimization efforts. We believe the realized intrinsic coincident polarimeter enables more accurate polarization information acquisition by integration and extending it to an image array.
REFERENCES


